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A REVISED SOLAR MODEL WITH A SOLAR NEUTRINO SPECTRUM

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A Revised Solar Model with a Solar Neutrino Spectrum

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ABSTRACT

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Improved formulae for the nuclear energy generation rate, together with the Los Alamos opacity code, are used to re-analyze the solar interior. The behavior of the sun in the last 4.5 billion years is displayed. The solar neutrino spectrum is obtained and the hope of detecting neutrinos from the sun is discussed.

INTRODUCTION

The analysis of stellar interiors has been going on for many years. The subject has turned out to be one of the most complicated in the field of physics. Almost every possible branch of physics appears in it, together with the intermediate domains where these branches coalesce, domains which usually happen to be the most difficult and the less extensively explored.

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At the present time, stars such as the sun (the so-called main sequence stars) are believed to be relatively well understood throughout. Detailed analyses of the solar interior are available, yielding for the physical parameters numbers which are (probably) not too far off.

It is very hard to estimate quantitatively (percentage-wise) the accuracy of these values. Although the uncertainties attached to the experimental parameters used to build up stellar models are known to be rather small (usually less than 10%), their effect on the models would be extremely difficult to evaluate. The mathematical framework is indeed far too complex.

Another possible source of error in our predicted values of the solar interiors is the (hypothetical) existence of some important physical factor which, because of ignorance, we would have completely overlooked. Although this may be a rather remote possibility, it should not be forgotten. The history of science would teach us such wisdom.

It has been pointed out several times that the detection of solar neutrino fluxes and the measurement

of their intensities would give us direct information on the conditions pertaining to the solar interior.

An experiment to that effect has been under way for some time (Davis 1959). So far, the detection power of the instrumentation has been much too low. However, a new and bigger apparatus is under construction with greatly enhanced detection power.

Our aim has been to estimate the chances of the present trial. In a model previously built by Dr. M. Schwarzschild and P. Pochoda (to be published), we have incorporated recent data of atomic and nuclear physics. The model follows the sun during the last 4.5 billion years. The results are displayed in Table I. In passing, we note that the variation in the luminosity may have had deep effects on the history of the solar system. For this reason it should be of interest to the geologist and the biologist.

From this model, we have tried to calculate the effect of the solar neutrino flux in the tank of Dr. Davis. The detection power appears to be too small by a factor of about 10. However, because of the high sensitivity of the neutrino flux to the central temperature, a rather

small increase (less than 30%) would bring the flux in the detectable range. Such a variation is still entirely possible.

Note: A similar analysis has been made by C. Marx and N. Menyhard (1960). Their values of $\bar{\sigma}$, the average cross-section, have seemingly been largely over-estimated.

Finally, J. N. Bahcall, et al. (1968), have considered the same problem. Their prediction of the fluxes differs from ours by less than 30%, which is well inside the model uncertainties.

SOLAR ENERGY SOURCES

We recognized the conversion of four hydrogen atoms into one helium atom as the basic source of energy generation in the sun. In stellar conditions, the transformation is accomplished through four main channels, the so-called PP branches. The detail of these branches is reproduced in the following paragraphs:

PP I

$$H^1 + H^1 \rightarrow D^2 + e^+ + \nu$$

$$D^2 + H^1 \rightarrow He^3 + \gamma$$

$$He^3 + He^3 \rightarrow He^4 + 2H^1 \qquad \text{or}$$

$$He^3 + He^4 \rightarrow Be^7 \quad \gamma$$

PP II

$$Be^7 + e^- \rightarrow Li^7 + \nu$$

$$Li^7 + H^1 \rightarrow Be^8 + \gamma$$

$$Be^8 \rightarrow 2He^4 \qquad \text{or}$$

PP III

$$Be^7 + H^1 \rightarrow B^8 + \gamma$$

$$B^8 \rightarrow Be^8 + e^+ + \nu$$

$$Be^8 \rightarrow 2He^4$$

PP IV

$$C^{12} + H^{1} \rightarrow N^{13} + \gamma$$

$$N^{13} \rightarrow C^{13} + e^{+} + \nu$$

$$C^{13} + H^{1} \rightarrow N^{14} + \gamma$$

$$N^{14} + H^{1} \rightarrow O^{15} + \gamma$$

$$O^{15} \rightarrow N^{15} + e^{+} + \nu$$

$$N^{15} + H^{1} \rightarrow C^{12} + He^{4}$$

The relative importance of these PPi branches is clearly a function of the temperature, the density and the chemical composition of the solar interior. As each branch contributes in its own way to the solar neutrino flux (both in number of particles and in spectral characteristics), we have followed the individual development of each branch in the sun.

In a previous work (Reeves 1963), the rate of these branches has been reconsidered in view of the recent developments in nuclear physics. The new rates have been used in the present work.

A SOLAR MODEL

The usual equations are assumed for the stellar interior (Schwarzschild, 1958). Two boundary conditions are given in the interior by expanding one of the hydrostatic equilibrium conditions and the thermal equilibrium condition in powers of "r". At the surface two more boundary conditions are derived from the adiabatic relation between pressure and temperature in a convective zone (to within a constant, K) and by eliminating the pressure and density from the hydrostatic equilibrium equation.

The basic set of four differential equations and boundary conditions is solved by using the Henyey method (Henyey, 1959). The gas equation of state is assumed throughout. The energy generation formula for hydrogen burning is given as in Reeves (1963).

The carbon-nitrogen composition was taken from a solar composition compiled by A.G.W. Cameron (private communication). Since the relative total metal concentration (Z) by weight was assumed to be 0.044, close to the value obtained by Rogerson (1961), the Cameron table, which yields a metal composition of 0.022 was doubled. The value, then, for x_{14} was 0.0091.

The opacities were derived by using the Los Alamos opacity code. With the metal content held fixed at 0.044, opacities were first computed for a wide range of pressures and temperatures at three different hydrogen compositions: $x_1 = 0.8, 0.45, 0.1$. The computer then stored these three two-dimensional tables. At a particular point in the model, the value of the opacity is determined by, first, interpolating in two of the tables for ρ and T, and then by interpolating linearly between the two results, depending on the value of x_1, at the point.

Six models were computed between the initial and present sun. The values for the initial concentration of hydrogen, and the adiabatic constant K were determined by matching the luminosity and the radius of the final model to the observed values. With an initial value of $x_1 = 0.680$ and $\log K = -2.25$, the final luminosity and radius were within 0.4% of the observed values.

Table I summarizes the history of the sun. The symbols have their usual meanings. $\epsilon_1/\epsilon_{TOT}$ is the fractional contribution of the PPI branch to the total energy rate. $\epsilon_c/\bar{\epsilon}$ is the ratio of central to mean energy

generation rate.

Fig. 1 shows the run of temperature and density of the final model (present sun), normalized to the central values, as a function of the mass fraction q. The luminosity is normalized to its surface value. Also shown is the fractional abundance of hydrogen x_1.

Fig. 2 indicates the fractional rate of energy generation contributed by each PPi branch throughout the final model. ϵ_{III} contributes very little.

NEUTRINOS FROM THE SUN

Five reactions are overwhelmingly responsible for the solar neutrino flux. (The notation ν_j identifies the mother nucleus j).

$$H^1 + H^1 \rightarrow H^2 + e^+ + \nu_H$$

$$Be^7 + e^- \rightarrow Li^7 + \nu_{Be}$$

$$B^8 \rightarrow Be^8 + e^+ + \nu_B$$

$$N^{13} \rightarrow C^{13} + e^+ + \nu_N$$

$$O^{15} \rightarrow N^{15} + e^+ + \nu_O$$

(A negligibly small amount of antineutrinos come from the reaction: $(H^3 \rightarrow He^3 + e^- + \bar{\nu})$. In the Table II the relevant energetic parameters of these reactions are given.

In a gas of free electrons (such as the solar core), the free electron capture process may occur in every one of the reactions considered, yielding a peak at an energy of 1.022 Mev above the maximum energy of the corresponding "hump" spectrum.

In the case of the sun, besides Li^7, only the peak of the ν_H spectrum $(H^1 + H^1 + e^- \rightarrow H^2 + \nu_H)$

is of any importance. The relative probability of this process over the ($H^1 + H^1 \rightarrow D + e^+ + \nu_H$) process is given by: $P_{H^1+H^1+e^-} / P_{H^1+H^1} \simeq 5 \times 10^{-5}(1+x_1)\rho T_6^{-\frac{1}{2}}$, ($T_6 = 10^6 T$ in $^\circ K$), (x_1 is the mass fraction of hydrogen). Using our model we get for this quantity an average value of about 2.6×10^{-3}.

To build up our spectrum we define a few useful quantities. Let N be the total number of reactions $4H^1 \rightarrow He^4$ (with $Q_0 = 26.730$ Mev) per gram/sec in the sun. Let a_i be the fractional number of these reactions channelled through the PPi branch, and let g_i be the effective (non-neutrinic) energy generation coming from this branch. Then clearly the average solar energy production can be written as $\bar{\varepsilon}_{TOT} = Q_0 N \Sigma a_i g_i$. The g_i are calculated from Table II and listed in Table III. For $\bar{\varepsilon}_{TOT}$ we use the experimental value 1.90 erg/gm-sec and we get : $N\Sigma a_i g_i = 4.45 \times 10^4$ $gm^{-1} sec^{-1}$. Next we write $\bar{\varepsilon}_{TOT} = \Sigma \bar{\varepsilon}_i$ and $\bar{\varepsilon}_i / \bar{\varepsilon}_{TOT} = a_i g_i / \Sigma a_i g_i$. With the help of Table I, we obtain the value of N($N = 4.6 \times 10^4$ $gm^{-1} sec^{-1}$), G($G = 0.97$; the efficiency of the solar furnace) and the a_i, listed in Table III.

Each fusion of four protons into one helium emits two neutrinos. Hence the sun emits in the average

9.2×10^4 neutrinos per gm-sec. Of this flux, a fraction $b_H = a_I + a_{II}/2 + a_{III}/2$ emits ν_H; a fraction $b_{Be} = a_{II}/2$ emits ν_{Be}; a fraction $b_B = a_{III}/2$ emits ν_B; a fraction $b_N = b_O = a_{IV}/2$ emits ν_N and ν_O. The values of b_j for our model are given in Table III. The total flux of neutrinos at the earth is

$$F = 6.5 \times 10^{10}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}.$$

For the purpose of the following discussion we write: $F = \sum F_j = \sum_j \int f_j(E)dE$, and also $f(E) = \sum_j f_j(E)$, where clearly $F_j/F = b_j$ and $f_j(E)$ is the density of neutrinos j per unit energy range. Values of F_j are given in Table III. In Fig. 3 we have drawn the neutrino energy spectrum of the sun, normalized to one particle in the entire flux.

EXPERIMENTAL DETECTION

This flux of neutrinos may induce inverse beta decays in various substances. Attempts have been made so far to detect the reaction (Davis, 1959) $\nu + Cl^{37} \rightarrow A^{37} + e^-$ (lifetime 35 days) in a tank of CCl^4 (1000 gallons).

The threshold for this reaction is $E_\nu = 814$ Kev, and the cross-section is given to about 15% accuracy (J. M. Pearson, private communication) by:

$$\sigma = 1.95 \text{ (Bahcall)} \pm 5\%$$

where W is the total energy (in units of $M_e c^2$) and p, the momentum (in unit of $M_e c$) of the outgoing electron. G is a function, tabulated for instance in (Siegbahn, 1955). To a good approximation we have: $G = 2\pi\alpha Z[1 - e^{-(2\pi\alpha ZW/p)}]^{-1}$.

In Table III we have listed $\bar{\sigma}_j$, the average cross-section for one incoming ν_j and $\int \sigma_j f_j dE = \bar{\sigma}_j F_j$ for the actual flux. The relative contributions to the total value are given in the last column.

The total value is $\Sigma \int \sigma_j f_j dE = 5.7 \times 10^{-36} \text{sec}^{-1}$.

The first experiment of Davis (with 1000 gallons of CCl^4) gave an upper limit of 5×10^{-33} for this quantity.

Davis is planning an improved experiment with 10^5 gallons of $C^2H^3Cl^4$. This should bring the lower limit

of detection to about a factor ten below our estimate.

The uncertainties in the numbers in Table III come from various sources. The energy generation rates are known to about 10%, but the (Be^7,p) (negligible for energy generation but important for neutrino fluxes) is known to only 50%.

Much bigger uncertainties could come from the model itself. It would be very difficult to make any estimate. We can however calculate that an increase of $T_c = 3 \times 10^6$ °K would bring the ν_8 flux within the reach of the proposed experiment. Such an error in our model can hardly be ruled out at the present time.

FIGURES

Figure 1: Physical parameters inside the sun. The temperature T and the density ρ are normalized to their central values T_c and ρ_c. The luminosity L is normalized to its total value $L_\odot$. Also shown is the mass fraction of hydrogen x_H. The parameter q measures the fractional mass of the sun interior to the point under consideration.

Figure 2: Fractional rate of energy generation contributed by each proton branch in the present sun.

Figure 3: Neutrino energy spectrum of the sun normalized to one particle in the entire flux. The three lines result from the capture of free electrons by Be^7 and protons. Their height is a function of the density and temperature in the solar interior.

FIGURE 1

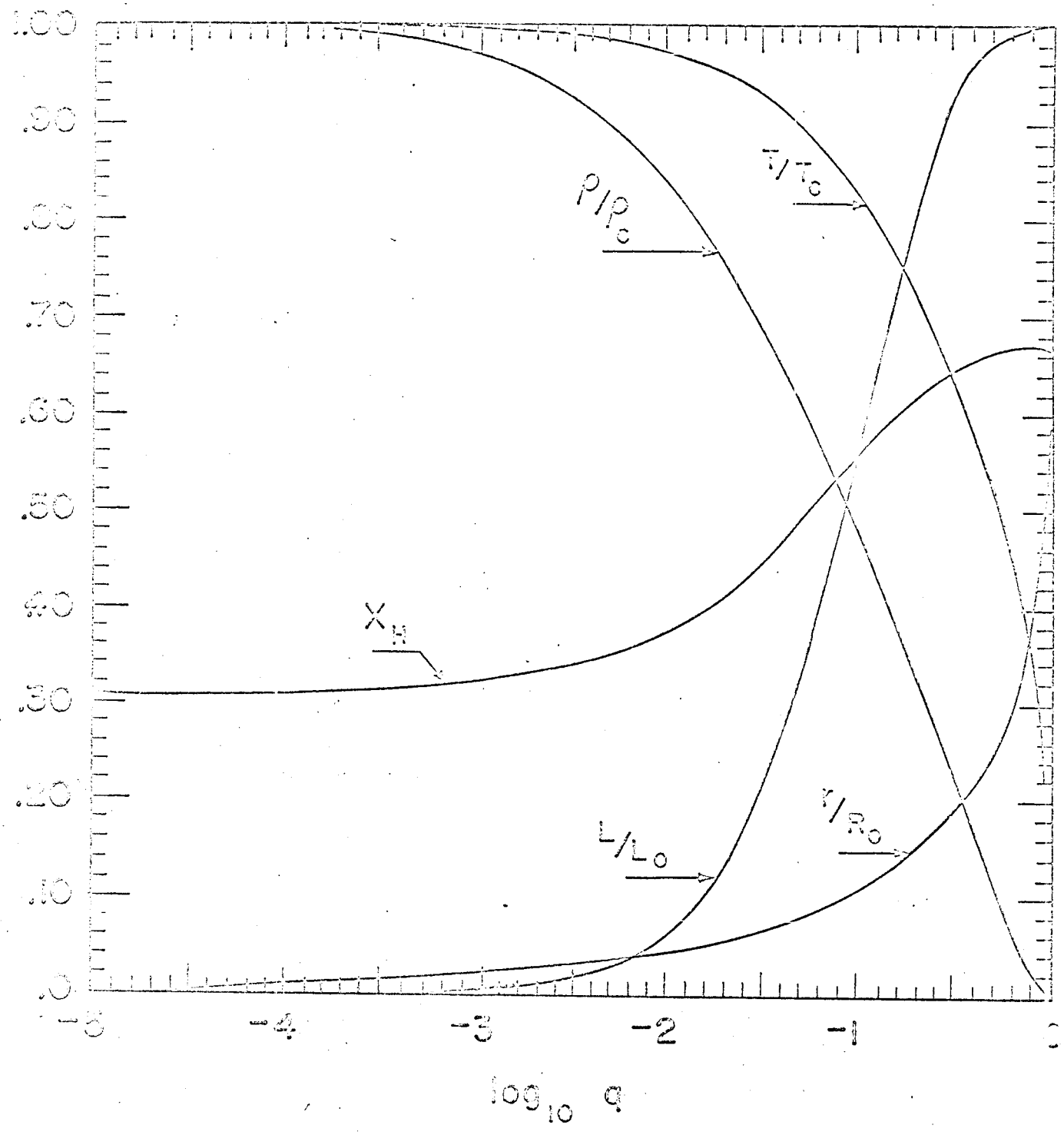
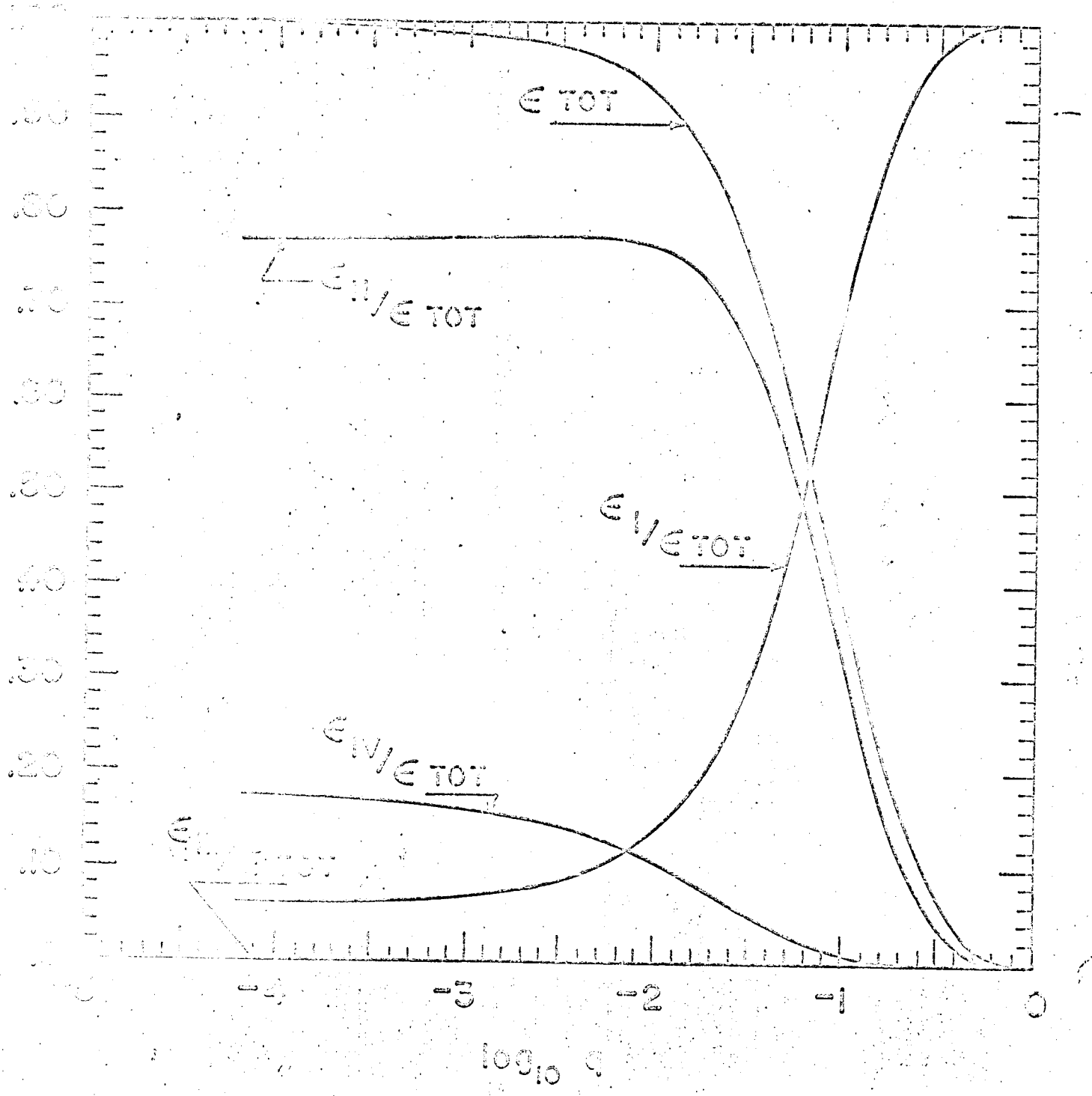

1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.0
-5
-4
-3
-2
-1
0
ρ/ρ_c
T/T_c
X_H
L/L_o
r/R_o
$\log_{10} q$

FIGURE 2


ϵ_{TOT}
$\epsilon_{II}/\epsilon_{TOT}$
$\epsilon_{I}/\epsilon_{TOT}$
$\epsilon_{IV}/\epsilon_{TOT}$
$\epsilon_{III}/\epsilon_{TOT}$
-4
-3
-2
-1
0
$\log_{10} q$

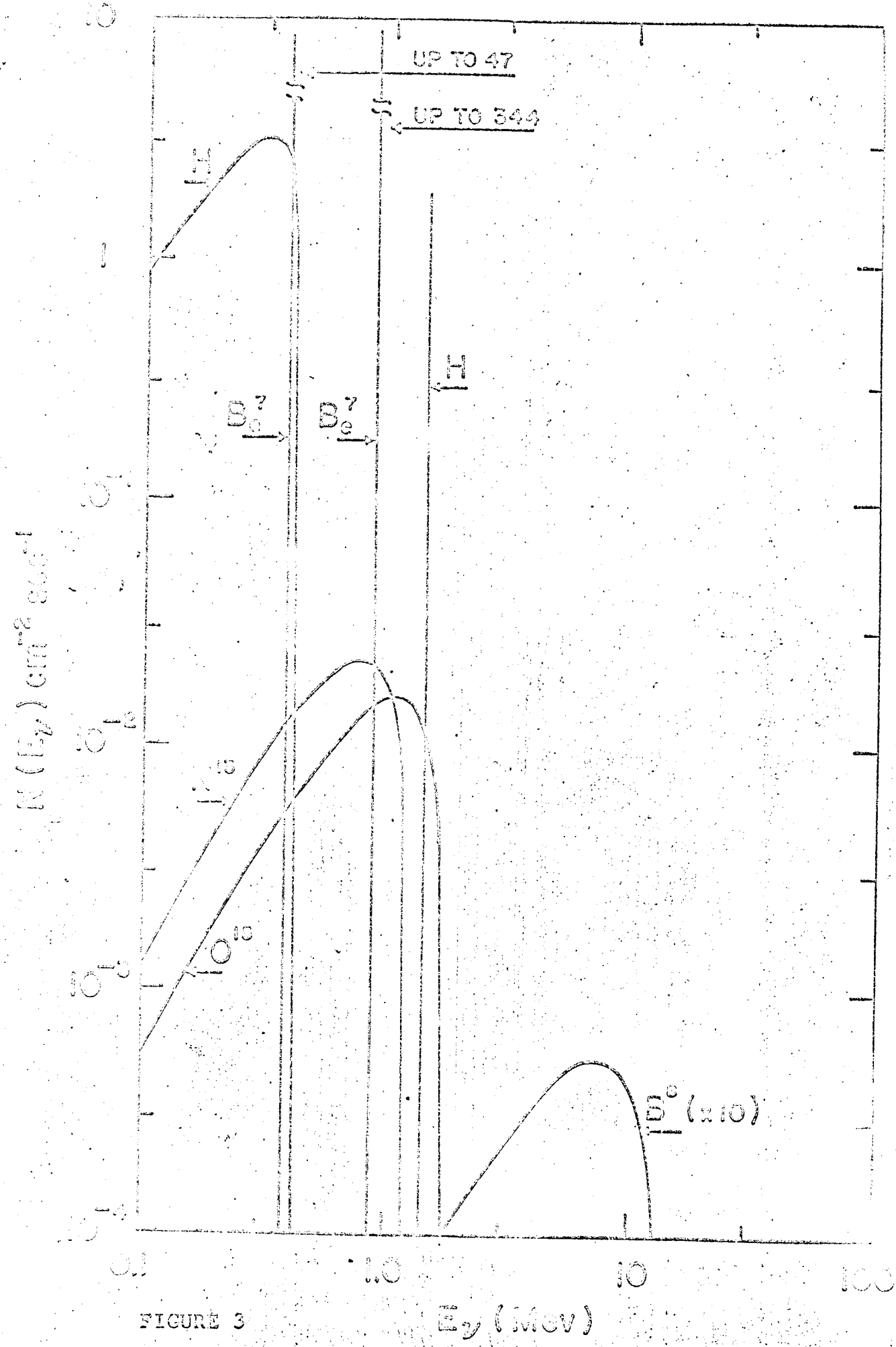

UP TO 47
UP TO 344
H
H
B_e^7
B_e^7
N^{13}
O^{15}
B^8 (x10)
$N(E_\nu)$ cm^{-2} sec^{-1}
E_ν (MeV)
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.1
1.0
10
100

FIGURE 3

TABLES

Table I: Characteristics of the sun during its evolution. The luminosity L and the radius R are normalized to their present values $L_\odot$ and $R_\odot$. T_e is the surface temperature. T_c, ρ_c and P_c are the central values of the temperature, the density and the pressure. X_1 is the fractional abundance of hydrogen (by mass) at the center. $\bar{\epsilon}_i/\bar{\epsilon}_{tot}$ is the average fractional rate of energy generation contributed by the ith branch of the proton cycle. Finally, $\epsilon_c/\bar{\epsilon}$ is the ratio of central to mean energy generation rate.

Table II: Description of various neutrino processes in the sun. The first column identifies the progenitor, e.g., ν_B describes the flux of neutrinos from the decay of B^8. The second column describes the energy extent of the continuous spectrum (hump). For B^8 the disintegration goes sometime to the excited states of the Be^8 nucleus. The third column is the position of the electron capture peak. The last column is the mean energy in the neutrino flux. Except in the case of ν_{Be}, the energy contribution of the peak is negligible in solar conditions.

Table III: Characteristics of the four proton branches in the sun. g_i is the relative efficiency of each cycle (ratio of non-neutrinic to total energy generated). The third line a_i gives the fractional number of $4H \rightarrow He^4$ conversions channeled through the ith branch.

Table IV: Experimental detection of neutrinos. b_j is the fractional amount of neutrinos from the various nuclei in the total solar flux (1.8×10^{33} neutrinos per sec). F_j is the flux at the earth (number of neutrinos per cm^2 per sec). σ_j is the average capture cross-section (cm^2) for the process $Cl^{37} + \nu \rightarrow A^{37} + e^-$. The two next lines represent the individual contributions to experimental effects in a chlorine tank.

TABLE 1

Age in 10^9 years.

	-4.5	-3.6	-2.7	-1.8	-0.9	0
$L/L_\odot$	0.73	0.77	0.83	0.87	0.92	1.00
$R/R_\odot$	0.95	0.96	0.97	0.98	0.99	1.00
$\log T_e$	3.76	3.74	3.74	3.75	3.75	3.76
$\log T_c$	7.15	7.16	7.17	7.18	7.19	7.20
$\log \rho_c$	1.95	1.99	2.03	2.09	2.15	2.23
$\log P_c$	17.21	17.24	17.26	17.30	17.33	17.40
μ_1	0.68	0.61	0.53	0.46	0.38	0.31
$\bar{\sigma}_I/\bar{\sigma}_{tot}$	0.86	0.82	0.79	0.71	0.64	0.56
$\bar{\sigma}_{II}/\bar{\sigma}_{tot}$	0.13	0.17	0.22	0.28	0.34	0.40
$\bar{\sigma}_{III}/\bar{\sigma}_{tot}$	4×10^{-5}	8×10^{-5}	10^{-4}	2×10^{-4}	3×10^{-4}	5×10^{-4}
$\bar{\sigma}_{IV}/\bar{\sigma}_{tot}$	0.004	0.006	0.009	0.013	0.020	0.032
[illegible]	11	11	11	11	10	9

TABLE II

Energies in Mev.

E_γ (Mev)	Continuous spectrum (hump)	Peak
0.303	ν_H : 0 to 0.420	1.442
0.80	ν_{Be} : None	0.861 (88%) 0.383 (12%)
7.25	ν_0 : 0 to 14.06 (– 100%)[a]	
	0 to 16.96 (< 1%)	17.98
	0 to – 5 (< 1%)	
0.710	ν_H : 0 to 1.199	2.221
1.00	ν_0 : 0 to 1.739	2.761

[a]Analysis of the alpha decay spectrum of the resultant Be^8 is in agreement with a 100% branching ratio toward the first excited level of Be^8 (class, private communication).

TABLE III

	PP I	PP II	PP III	PP IV
C_1	0.9603	0.9602	0.7191	0.9360
$\bar{\sigma}_1/\bar{\sigma}_{10}$	0.56	0.41	5×10^{-4}	0.032
ε_1	0.55	0.41	7×10^{-4}	0.033

TABLE IV

	ν_H		ν_H		ν_S	ν_H	ν_S
	hump	peak	line	line			
			0.[illegible]	0.861			
b_j	0.76	2×10^{-3}	0.02	0.18	4×10^{-4}	0.016	0.0[illegible]
F_j	5×10^{10}	1.3×10^{8}	1.3×10^{9}	1.2×10^{10}	2.6×10^{7}	10^{9}	$10^{[illegible]}$
$\bar{\sigma}_j$	0	1.5×10^{-45}	0	2.5×10^{-46}	6.6×10^{-44}	1.6×10^{-46}	6.4×10^{-46}
$\int\sigma_j f_j dE$	0	2.0×10^{-37}	0	3.0×10^{-36}	1.7×10^{-36}	1.6×10^{-37}	6.4×10^{-37}
$\frac{\int\sigma_j f_j dE}{\sum_j\int\sigma_j f_j dE}$	0	0.04		0.53	0.2[illegible]	0.03	0.11

$$\sum_j \int\sigma_j f_j dE = 5.7\times10^{-[illegible]}$$

ACKNOWLEDGMENTS

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