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STABILITY OF A REINFORCED CIRCULAR CYLINDRICAL SHELL SUBJECTED TO COMPRESSION AND PURE BENDING

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1. INVESTIGATION OF THE ROLE OF SHEAR DEFORMATION IN A GENERALLY DEFORMED STATE OF A CYLINDRICAL SHELL DURING THE LOSS OF STABILITY CAUSED BY CENTRAL COMPRESSION

We shall examine the stability of a reinforced cylindrical shell with a changing circular cross-section contour under the influence of a uniformly compressing linear load P_{cir} along the generatrices, which is applied to the ends.

We will define this type of loading as central compression.

We will refer the shell to a stationary system of coordinates, with its origin at the center of the end cross-section O, the z-axis parallel to the generatrices, and φ the angle between the initial and the running radii, as shown in fig. 1. We will define the radial displacements of the points of the mean surface as

$$u(s,z) = u_1(z)u_2(s),$$

where $u_1(z)$ is the unknown function originating from the z-coordinate and $u_2(s)$ is the function of the s-coordinate, which is selected according to the type stability loss being examined.

All displacements are extremely small and are deducted from the mean cylindrical surface after applying the critical load to the shell and before the bulging effect begins.

We will concern ourselves with the role of shear deformation in the mean surface $\gamma = \frac{\partial w}{\partial s} + \frac{\partial v}{\partial z}$ and the expansion e_{ss} in various cases of bulging.

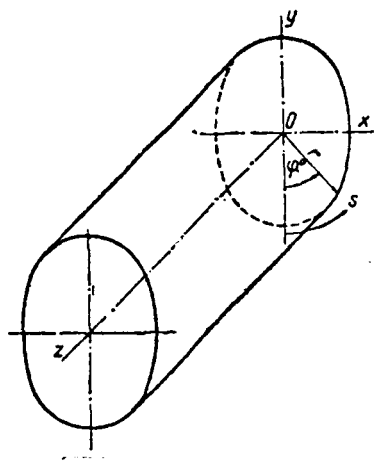

Fig. 1

We will investigate the case of asymmetrical bulging in relation to the axis of the cylinder.

We shall determine the deformed and stressed state in the /478 mean surface of the shell, following the approximate method of investigations of shear deformation described in another article [3]. Moreover, we shall preserve the same specifications.

We shall apply the unknown quantity

$$K = \frac{\frac{\partial w}{\partial s}}{\frac{\partial v}{\partial z}}$$

as a constant, and we shall show that it is possible to select it in such a way that it will not be contradictory to the following conditions of the deformed and stressed state in the mean surface of the shell:

$$1) \quad \tau = G_{\gamma}; \tag{1}$$

$$2)\quad \sigma_{zz} = Ee_{zz}; \tag{2}$$

$$3)\quad \frac{\partial\sigma_{zz}}{\partial z} + \frac{\partial\tau}{\partial s} = 0 \tag{3}$$

and so that, moreover, the critical load will be minimal.

We shall impose a less rigid condition on the shear deformation γ than in applied theories which employ $K = -1$ ($\gamma = 0$) or $K = 0 \left(\frac{\partial w}{\partial s} = 0\right)$ since we shall define the quantity K but will not employ it in that form in advance of the assigned number. As a result, we shall succeed in avoiding a discrepancy between tangent or normal stresses and the corresponding relative deformations which are observed in the above-mentioned applied theories.

We shall treat the middle line of the cross-section as nonextensible; in similar types of stability loss in thin rings, nonoblique arcs, etc., such an assumption is generally made.

On the basis of the accepted assumptions, we have

$$1)\quad \gamma = (1 + K)\frac{\partial v}{\partial z};$$

$$2)\quad e_{ss} = \frac{\partial v}{\partial s} + \frac{u}{r} = 0.$$

We shall assign the function $u_2(s)$ in the form $u_2(s) = \cos n\frac{s}{r} = \cos n\varphi$, considering that in the cross-section a number of waves can be formed ($n = 1, 2, 3, 4, \ldots$)*. We shall accept the bending rigidities of ribs and stingers as constant and shall designate them B_{rib} and B_{str}.

$EJ_{rib} = B_{rib}$ is the linear bending rigidity of ribs in relation to the attached casing;

*In the case $n = 0$, the hypothesis of nonextensibility of a middle line must be ignored.

$EJ_{str} = B_{str}$ is the linear bending rigidity of stringers in relation to the attached casing.

It is possible to imagine the shell in the form of the following model.

The casing is reinforced by centered ribs and stringers, which are located at a distance equal to unity from one another and possess bending rigidity only in the radial direction. The casing can be displaced, deformed along the generatrices and can twist about the /479 generatrices and axes parallel to the s-axis without preventing the flexing of ribs and stringers.

In this way, it is assumed that the displacement of the casing does not involve, for instance, the twisting of stringers and their flexing in the direction of the s-axis nor does it cause complementary local stretching or compression from corresponding deformations of the casing which arise in the twisting.

The selection of such a model is justified if we consider that during the twisting, rivet and other connections virtually eliminate the displacement of elements relative to one another in the direction of the mean surface; in other directions, some relative displacements are tolerated.

From the condition $e_{ss} = 0$, we shall obtain

$$v = -\int_0^s \frac{u}{r}\, r d\varphi + v_0(z)$$

From the condition of symmetry of a bulge relative to the surface, which passes over the z-axis and the origin of the s-coordinates, we have

$$v_0(z) = 0.$$

Then,

$$v = -u_1(z)\frac{1}{n}\sin n\varphi. \tag{4}$$

From the condition $\tau = G\gamma$, we shall obtain the expression of tangent stresses in the mean surface of the shell

$$\tau = G\ (1 + K)\ \frac{\partial v}{\partial z} = -\ \frac{G(1 + K)}{n}\ u_1'(z)\ \sin n\varphi.$$

From the state of equilibrium of an element ds × dz, we shall express the complementary normal stresses in the mean surface σ_{zz} through the quantities K, $u_1(z)$ and n:

$$\sigma_{zz} = -\int_0^z \frac{\partial \tau}{\partial s}\ dz + \sigma_0(s) = \frac{G(1 + K)}{r}\left[\int_0^z u_1'(z)\ dz\right]\cos n\varphi + \sigma_0(s).$$

We shall select the quantity K so that the elongation deformation e_{zz} is equal to $\frac{\partial w}{\partial z}$ to agree with the stresses σ_{zz}. For this, we shall use the relation $\sigma_{zz} = E\ \frac{\partial w}{\partial z}$. We have

$$\frac{\partial w}{\partial z} = \frac{\sigma_{zz}}{E} = \frac{G}{E}\cdot\frac{(1 + K)}{r}\ u_1(z)\ \cos n\varphi + \frac{\sigma_0(s)}{E},$$

whence

$$w = \frac{G}{Er}\left[\int_0^z u_1(z)\ dz\right]\cos n\varphi\ (1 + K) + \frac{\sigma_0(s)z}{E} + w_0(s)$$

It is known that the influence of terminal conditions in the amount of the critical load is negligible in the length of the shell provided its diameter is sufficiently great. We shall examine such a shell and will assume $\sigma_0(s) = w_0(s) = 0$, which will not influence the critical load. Then,

$$K = \frac{\frac{\partial w}{\partial s}}{\frac{\partial v}{\partial z}} = \frac{n^2 G}{r^2 E}\ (1 + K)\ \frac{\int_0^z u_1(z)\ dz}{u_1'(z)}$$ /480

As will be seen below, $u_1(z)$ can be expressed in the form

$$u_1(z) = C\ \sin\frac{m\pi}{L}\ z$$

where m is the number of halfwaves in length L and C is the greatest deviation of stringers from a right angle

Then,

$$K = - d_1 n^2 \frac{(1 + K)}{k^2}$$

where

$$d_1 = \frac{G}{Er^2}, \qquad k = \frac{m\pi}{L}.$$

Hence, it is possible to express k^2 through n and K:

$$k^2 = - d_1 n^2 (1 + K) \frac{1}{K}. \tag{5}$$

In this manner, the value of the quantity K, in which conditions (1), (2), and (3) are met simultaneously, will correspond to each value of k and n. Further, we shall select the value K and n (and consequently, also k) to be such, that the critical load P_{cir} is minimal.

We shall utilize the extreme properties of the system's potential energy for the conclusion of the differential equation.

We shall examine the system's potential energy, accumulated in the process of bulging under the effect of a load p_{cir}.

We shall compile expressions of separate potential energy (accumulated by an element of unit length and width) according to separate shapes of deformation.

The bending energy of stringers is

$$W_{0\ str} = \frac{1}{2} \cdot \frac{M_{str}^2}{B_{str}} = \frac{1}{2} B_{str} \left[u_1'' (z)\right]^2 \cos^2 n\varphi.$$

The bending energy of ribs is

$$W_{rib} = \frac{1}{2} \cdot \frac{M_{rib}^2}{B_{rib}} = \frac{1}{2} B_{rib}\, u_1^2 (z) \frac{(n^2 - 1)^2}{r^4} \cos^2 n\varphi.$$

Here, according to the theory of thin rings and curves of beams we have

$$M_{rib} = \frac{B_{rib}}{r^2}\left(\frac{\partial^2 u}{\partial \varphi^2} + u\right).$$

The energy of the shear of the casing is

$$W_{0\ shear} = \frac{1}{2}\cdot\frac{\tau^2}{G}h = \frac{1}{2}\cdot\frac{Gh}{n^2}(1+K)^2\left[u_1'(z)\right]^2\sin^2 n\varphi$$

The energy of extension — the compression of the casing is

$$W_{0\sigma} = \frac{1}{2}\cdot\frac{\sigma_{zz}^2}{E}h = \frac{1}{2}\cdot\frac{G^2h}{Er^2}(1+K)^2 u_1^2(z)\cos^2 n\varphi^*.$$

The energy of torsion of elements of the casing is /481

$$W_{0\ twist} = \frac{2}{2}\cdot\frac{M_{twist}^2}{B_{twist}} = B_{twist}\left[\frac{u_1'(z)}{r}\right]^2\frac{(n^2-1)^2}{n^2}\sin^2 n\varphi.$$

Here,

$$B_{twist} = EJ_{twist} = D(1-\mu);\quad M_{twist} = B_{twist}\,\chi_{sz},$$

where $\chi_{sz} = \frac{1}{r}\left(\frac{\partial v}{\partial z} - \frac{\partial^2 u}{\partial z\partial\varphi}\right)$ is the relative angle of torsion of element $ds \times dz$ in relation to the s- and z-axes.

Let us construct the expression of separate energy equal to the effect of stresses in the casing and the stringers, which are bulging

* If the connection between the stringers and the casing along the generatrices is sufficiently rigid (for instance, if the stringers are made as a unit with the casing) in the expressions of energy of extension (compression of the casing together with the stringers) it is necessary to assume $h_{reduced} = h + \frac{F_{str}}{b_{str}}$ instead of the thickness of the casing h.

from a critical load, on the displacements of the approach to the ends to the center line of an element one unit in length caused by its inclination due to bulging relative to the corresponding generatrix:

$$A_0 = - p_{cir} \frac{1}{2} (\omega')^2 = - \frac{1}{2} p_{cir} \left[u_1'(z)\right]^2 \left(\cos^2 n\varphi + \frac{\sin^2 n\varphi}{n^2}\right)$$

the projection on the surface of the cross-section of a complete displacement of points of a mean surface is designated here by the letter ω

$$\omega = \sqrt{u^2 + v^2} = u_1(z) \sqrt{\cos^2 n\varphi + \frac{\sin^2 n\varphi}{n^2}}$$

The complete potential energy of the system is

$$\Pi = \int_0^L dz \oint (W_o + A_0)\, ds = \int_0^L F(u_1, u_1', u_1'')dz$$

whereupon

$$F(u_1, u_1', u_1'') = \frac{1}{2}\Big\{B_{str}(u_1'')^2 + B_{rib}\, u_1^2 \frac{(n^2 - 1)^2}{r^4} +$$

$$+ \frac{Gh}{n^2}(1 + K)^2 (u_1')^2 + \frac{G^2 h}{Er^2}(1 + K)^2 u_1^2 + 2B_{twist} \frac{(n^2 - 1)^2}{n^2 r^2}(u_1')^2 -$$

$$- p_{cir}\, \eta (u_1')^2 \Big\} \pi r,$$

where

$$\eta = \frac{1}{\pi} \int_0^{2\pi} \left(\cos^2 n\varphi + \frac{\sin^2 n\varphi}{n^2}\right) d\varphi = \left(1 + \frac{1}{n^2}\right).$$

Euler's equation

$$\frac{\partial F}{\partial u_1} - \frac{d}{dz}\left(\frac{\partial F}{\partial u_1'}\right) + \frac{d^2}{dz^2}\left(\frac{\partial F}{\partial u_1''}\right) = 0$$

yields the equation of the system's equilibrium

$$B_{str}\, u_1^{V} + (p_{cir}\, \eta - B)\, u_1'' + C_1 u_1 = 0, \tag{6}$$

where the designations are introduced:

$$B = \frac{Gh}{n^2}(1 + K)^2 + 2B_{twist}\frac{(n^2 - 1)^2}{n^2 r^2};$$

$$C_1 = B_{rib}\frac{(n^2 - 1)^2}{r^4} + \frac{G^2 h}{Er^2}(1 + K)^2. \qquad /482$$

Having substituted in (6) the solution of this equation in the form $u_1(z) = C \sin kz$ and having solved the characteristic equation in relation to p_{cir}, we have

$$p_{cir} = \frac{1}{\eta}\left(B_{str}k^2 + B + \frac{C_1}{k^2}\right) \tag{7}$$

We substitute here the value k^2 from (5) and assume $E = 2G(1 + \mu)$. After an ~~uncomplicated transposition~~, we express the critical load p_{cir} by the quantities K and n:

$$p_{cir} = \frac{Gh}{\eta n^2}\left\{(1 + K) - \frac{1}{hr^2}\left[4J_{rib}(1 + \mu)^2 (n^2 - 1)^2 \frac{K}{(1 + K)} + \right.\right.$$

$$\left.\left. + J_{str}\, n^4 \frac{(1 + K)}{K} - 4J_{twist}(n^2 - 1)^2(1 + \mu)\right]\right\}. \tag{7'}$$

The minimum condition $p_{cir}\left(\frac{\partial p_{cir}}{\partial K} = 0\right)$ leads to the equation

$$f(K) = K^4 + 2K^3 + (1 + p - q)K^2 + 2pK + p = 0 \tag{8}$$

where

$$p = \frac{J_{str}}{hr^2} n^4;$$

$$q = \frac{4J_{rib}}{hr^2} (1 + \mu)^2 (n^2 - 1)^2.$$

The solution of equation (8), together with (7'), gives the dependence P_{cir} on the number n, whose quantity must be chosen from a minimum condition p_{cir}.

It is possible to search for the substantial roots of equation (8) which concern us by means of various approximation methods, numerically as well as graphically. An analytic expression of the roots of the equation of the fourth power via its coefficients also exists. But these methods cannot give conclusive results in the form of simple formulas.

We shall employ the method of approximate determination of roots indicated by A. N. Krylov [4]. This method will enable us to present the expression p_{cir} in a simple form by means of known quantities and the number of waves in the cross-section n.

We shall note first of all that if the quantity q is considered small in comparison with unity and temporarily disregard it, then equation (8) can be expressed in the following form:

$$f_1 (\bar{K}) = (\bar{K} + 1)^2 (\bar{K}^2 + p) = 0. \tag{8a}$$

Only the real roots of this equation will concern us

$$\bar{K}_{1,2} = - 1.$$

It is possible to consider this root of the second multiple as an approximate value of the roots of equation (8), for it was obtained

as a result of a small change of one of the coefficients of the equation.

We shall designate the difference between the true value of /483 the roots of equation (8) and the approximate $\overline{K}_{1,2} = -1$ via ϵ. Then, broken down into Taylor's theories, equation (8) will become

$$f(K) = f(-1+\epsilon) = f(-1) + \frac{\epsilon}{1} f'(-1) + \frac{\epsilon^2}{1.2} f''(-1) + \ldots = 0.$$

We shall define the correction ϵ from this series, while disregarding all terms which contain ϵ to the third power and higher, which gives us

$$(1 + p - q)\epsilon^2 + 2q\epsilon - q = 0.$$

A refined value of the root of equation (8) will be

$$K_{1,2} = -1 + \epsilon_{1,2} = -\left[\frac{(1+p) \mp \sqrt{q(1+p)}}{(1+p) - q}\right] \tag{9}$$

This refinement is quite practical. For the unreinforced shell, for instance, after disregarding the small terms according to comparison with $\frac{h}{r}$ and unity, we have

$$K_{1,2} = -\left[1 \mp \frac{h}{r}(1+\mu)\frac{(n^2-1)}{\sqrt{3(1-\mu^2)}}\right] \tag{9a}$$

Substituting this value in expression (7'), disregarding the small terms and retaining only the plus sign in the parentheses of (9a), and assuming that $E = 2G(1+\mu)$, we obtain

$$\sigma_{cir} = \frac{Eh}{r\sqrt{3(1-\mu^2)}} \cdot \frac{(n^2-1)}{(n^2+1)} \tag{7a}$$

or, when $n = 2$ and $\mu = 0.3$,

$$\sigma_{cir} = 0.36 \frac{E}{\left(\frac{r}{h}\right)} \tag{7b}$$

These equations were developed by S. P. Timoshenko from the general theory of shells by means of the solution of systems of three equations (in particular derivatives), whereupon the hypothesis was used as a basis for the formation of long halfwaves along the generatrices.

As a numerical example we shall estimate how much the true quantity K differs from the value of $\overline{K}_{1,2} = -1$ in the stability loss of the shell. We assume a reinforced shell with a fuselage $J_{str} \approx J_{rib} = 0.04\ cm^3$, $r = 60$ cm, $h = 0.2$ cm and $\mu = 0.3$ and obtain $K = -0.94$.

For the unreinforced shell with $\frac{h}{r} = 0.003$ and $\mu = 0.3$, $K = -0.993$.

Thus, for a broad class of shells, loss of stability is possible when the value of K is nearly -1. This implies that in bulging, shear deformation is almost absent ($\gamma \approx 0$). From an investigation conducted in the author's dissertation, it follows that equation (7a) is accurate in the values $n \leq 10$. Moreover, the shear deformation in the mean surface and the torsion of elements practically at the quantity of the critical load are not influenced and therefore can be disregarded completely; i.e., it is possible to set $\gamma = 0$ and $\chi_{zs} = 0$, implying the absence of shear deformation e_{zs} along the entire shell. In this manner, it is possible to add still another condition $e_{zs} = e_{ss} = 0$ to the basic deformation hypotheses of the general theory of shells $e_{sn} = e_{zn} = e_{nn} = 0$ in determining the critical load of compression of a cylindrical shell. /484

Such a hypothesis, while greatly simplifying the calculations, does not introduce any noticeable errors into the size of the critical load of compression up to significant numbers of n.

We shall explain the nature of the origin of the supplementary normal stresses in the casing, which plays a significant role in

determining the critical load. In stability loss, bulges are formed, the sizes and shapes of which change from one cross-section to another. The changes in cross-sections in $\gamma = 0$ involve deplanations, which cannot be accomplished freely, because they differ from section to section. As a result of the deplanation constraint, supplementary normal stresses arise along the generatrices.

We shall find the critical load, as a result of the conditions $e_{zs} = e_{ss} = 0$. In this case, the above model is retained except that now we assume that no shears occur in the casing.

On the basis of the condition $e_{zs} = 0$

$$W_{0\ \text{shear}} = W_{0\ \text{twist}} = 0.$$

We shall determine the displacements w from the condition

$$\gamma = \frac{\partial w}{\partial s} + \frac{\partial v}{\partial z} = 0.$$

Then,

$$w = -\ u_1'(z)\ \frac{r}{n^2} \cos n\varphi + w_0(z).$$

The supplementary stresses σ_{zz} will be

$$\sigma_{zz} = E\,\frac{\partial w}{\partial z} = -\ Eu_1''(z)\ \frac{r}{n^2} \cos n\varphi.$$

Here, from the condition of equilibrium σ_{zz}, we have $\dfrac{\partial w_0(z)}{\partial z} = 0$.

Then,

$$F(u_1, u_1', u_1'') = \frac{1}{2}\left[C_{str}\ (u_1'')^2 + C_{rib}\ u_1^2 + C_\sigma\ (u_1'')^2 - \right.$$

$$\left. - p_{cir}\ \eta (u_1')^2\right] \pi r,$$

where

$$C_{str} = B_{str};$$

$$C_{rib} = B_{rib} \frac{(n^2 - 1)^2}{r^4};$$

$$C_{\sigma} = Eh \frac{r^2}{n^4}.$$

Having completed the operation according to Euler's equation, we shall obtain the equilibrium equation of the system:

$$(C_{\sigma} + C_{str})\, u_1^{IV} + p_{cir}\, \eta\, u_1'' + C_{rib}\, u_1 = 0. \tag{10}$$

After substituting in it the solution $u_1 = C \sin kz$ and the /485 solution of the characteristic equation in relation to p_{cir}, we have

$$p_{cir} = \frac{1}{\eta} \left[(C_{\sigma} + C_{str})\, k^2 + C_{rib} \frac{1}{k^2} \right]. \tag{7c}$$

The minimum condition p_{cir} gives

$$k^2 = \sqrt{\frac{C_{rib}}{C_{\sigma} + C_{str}}}$$

and

$$p_{cir} = \frac{2}{\eta} \sqrt{(C_{\sigma} + C_{str}) C_{rib}}.$$

For the unreinforced shell, disregarding C_{str} in favor of C_{σ} we have

$$\sigma_{cir} = \frac{Eh}{r\sqrt{3(1 - \mu^2)}} \cdot \frac{(n^2 - 1)}{(n^2 + 1)}.$$

but

$$k^2 = \frac{h(n^2 - 1)\, n^2}{2r^3 \sqrt{3(1 - \mu^2)}}$$

We shall examine various cases of shell bulging on the basis of this solution.

1. $n = 1$ (the cross-section of the shell, which remains circular, can be displaced in the direction perpendicular to the z-axis):

$$k^2 = \left(\frac{m\pi}{L}\right)^2 = \frac{h(n^2 - 1)\, n^2}{2r^3 \sqrt{3(1 - \mu^2)}} = 0$$

Having assumed that the bulging is possible, we must consider $m \neq 0$. Then, the condition $k = \frac{m\pi}{L} = 0$ can be fulfilled only if $L = \infty$. Having substituted the value k^2 in equation (7c), we obtain $p_{cir} = 0$ when $n = 1$. In this manner, the least critical force is equal to zero in the infinite length of the halfwave.

In order to determine the critical compression load in the finite length L, it is necessary to substitute the value $k^2 = \left(\frac{\pi}{L}\right)^2$ in equation (7c) when $n = 1$. We then obtain

$$\sigma_{cir} = \frac{\pi^2 E}{\left(\frac{L}{i}\right)^2}, \tag{11}$$

where $i = \frac{r}{\sqrt{2}}$ is the radius of inertia of the cross-section of the shell.

This is Euler's equation for a rod.

In this particular case, a shall of sufficiently great length can lose stability as a flexible rod.

The result obtained is as expected, since equation (10) is a generalization of Euler's equation for a flexible rod in the case of

asymmetric bulging from axial compression of a circular cylindrical shell. In his dissertation, the author [5] demonstrates that Euler's equations for a rod supported at the ends by hinges, as well as for a rod with one end fixed and the other free, can be obtained from /486 equation (10).

2. $n = 2$ (ellipses are formed in the cross-sections). According to equation (7a), when $\mu = 0.3$, it is possible to imagine the least critical stress of compression in the form

$$\sigma_{cir} = 0.36 \frac{E}{\left(\frac{r}{h}\right)} .$$

This stress is lower than in symmetrical bulging. The length of the halfwave $\frac{L}{m}$, which corresponds to the stability loss, when $\mu = 0.3$ is

$$\frac{L}{m} \approx 1.65\ r \sqrt{\frac{r}{h}} .$$

3. With a further increase of the number of waves n in the cross-section, the coefficient $\frac{(n^2 - 1)}{(n^2 + 1)\sqrt{3(1 - \mu^2)}}$ in equation (7a), when $\mu = 0.3$, will approach value 0.6, and the length of the longitudinal half wave will decrease. Thus, for instance, when $n = 10$, we obtain

$$\sigma_{cir} = 0.6 \frac{E}{\frac{r}{h}},$$

but

$$\frac{L}{m} = 0.06\ r \sqrt{\frac{r}{h}} .$$

We shall note that Euler's solution of the problem of the stability of a flexible bar is based on the hypothesis of plane sections which corresponds to the assumption of the absence of all components of shear deformations in the rod ($e_{xz} = e_{yz} = e_{xy} = 0$).

The solutions obtained on the basis of these same conditions relative to shear deformation are a generalization of Euler's solution

in the more complicated cases of asymmetrical bulging from axial compression of the long as well as the short reinforced circular cylindrical shell.

2. LOSS OF STABILITY OF THE REINFORCED CIRCULAR CYLINDRICAL SHELL SUBJECTED TO PURE BENDING

The problem of stability of the circular cylindrical thin shell subjected to pure bending in the presence of a temporary internal pressure was examined thoroughly by Ye. D. Pletnikova in 1949. In her work [1], the possibility of bulging of shells along the long longitudinal halfwaves was indicated. However, these cases were not successfully investigated because of the practical difficulty of utilizing the determinants of a considerably higher order than eight, which would lead to still more cumbersome calculations than those made in this interesting work.

In the present investigation, we are attempting to determine the critical bending load in the bulging along the long longitudinal semi-halfwaves. The normal stress from the action of the bending moment will be distributed following the plane law according to the elementary theory of beams. The linear stresses in the cross-section will be expressed in the form /487

$$p = p_0 \cos \varphi$$

where

$$p_0 = \sigma_0 \left(h + \frac{F_{str}}{b_{str}}\right);$$

F_{str} is the area of the stringer,

b_{str} is the distance between stringers, and

$$\sigma_0 = \frac{M_{bend}\, r}{J};$$

J being the moment of inertia of the cross-section.

The mechanical picture of the origin of the complementary normal stresses in the mean surface and the functions of the ribs and stringers

of the model in the case of bulging from pure bending is similar to that of bulging along an asymmetrical shape due to central compression.

As a basis of the solution, we shall assume the deformation hypotheses $e_{zs} = 0$ and $e_{ss} = 0$, which, as has been shown above, was justified in the investigation of stability of shells subjected to central compression.

We shall employ the same method and will retain the previously calculated model. We shall accept the displacement in the form of the product of the two functions

$$u(s,z) = u_1(z)\, u_2(s).$$

The law of the change of ~~radical~~ displacements u in the cross-section is characterized by the function $u_2(s)$. We shall consider that in the cross-sections, the bulging is distributed only along the part of the shell located in the compressed zone up to a neutral line, until the stress of the shell along the generatrices against one side of the neutral line obstructs the bulging. This condition has been confirmed experimentally.

Thus, the loss of stability during bending changes the deformed and stressed states of the shell significantly only in the compressed zone. We shall search for the function $u_2(s)$ in the form of a trigonometric series

$$u_2(s) = \sum_{i=0}^{i} A_i \cos(n + i)\frac{s}{r} = \sum_{i=0}^{i} A_i \cos(n + i)\,\varphi \qquad (12)$$

We shall demand that it satisfy the following four basic conditions: the condition of nonextensibility of the mean line of the cross-section $e_{ss} = 0$, the kinematic conditions of the limits $\pm\frac{\pi}{2}$ (at the points of intersection of the neutral lines with the mean surface),

$$u_2\left(\pm\frac{\pi}{2}\right) = 0, \qquad v\left(\pm\frac{\pi}{2}\right) = 0$$

and the basic statistical condition of the disappearance of bending moments in the ribs in the same boundaries — $u_2''\left(\pm\frac{\pi}{2}\right) = 0$. It is possible to satisfy these requirements by limiting to three members /488 of the series (12) in the corresponding selection of coefficients A_i. We determine the displacement from the condition $e_{ss} = 0$ and obtain

$$v = -u_1(z)\left[\int_0^{\varphi} u_2(\varphi)\, d\varphi + v_0\right]$$

From a condition of symmetry of the bulging forms in the cross-section relative to the origin of the reading of angles φ, we have $v_0 = 0$. Then,

$$v = u_1(z)v_2(\varphi)$$

where

$$v_2(\varphi) = -\frac{A_0}{n}\sin n\varphi - \frac{A_1}{n+1}\sin(n+1)\varphi - \frac{A_2}{(n+2)}\sin(n+2)\varphi.$$

Thus, we have a uniform system of three linear equations with three unknowns: A_0, A_1, and A_2

$$u_2\left(\pm\frac{\pi}{2}\right) = A_0\cos n\frac{\pi}{2} + A_1'\cos(n+1)\frac{\pi}{2} + A_2\cos(n+2)\frac{\pi}{2} = 0;$$

$$v_2\left(\pm\frac{\pi}{2}\right) = -\frac{A_0}{n}\sin n\frac{\pi}{2} - \frac{A_1}{(n+1)}\sin(n+1)\frac{\pi}{2} -$$

$$-\frac{A_2}{(n+2)}\sin(n+2)\frac{\pi}{2} = 0;$$

$$u_2''\left(\pm\frac{\pi}{2}\right) = -A_0\ n^2 \cos n\frac{\pi}{2} - A_1\ (n+1)^2 \cos (n+1)\frac{\pi}{2} -$$

$$- A_2\ (n+2)^2 \cos (n+2)\frac{\pi}{2} = 0.$$

In order that this system have a non-zero solution for A_i, it is necessary and sufficient that its determinant be equal to zero:

$$\Delta = 0.$$

This condition is fulfilled only in odd values of $n = 1, 3, 5, \ldots$ Moreover, $A_1 = 0$. Having accepted $A_2 = 1$, which cannot influence the end result, we shall obtain

$$A_0 = \frac{n}{(n+2)},$$

consequently,

$$u_2(\varphi) = \frac{n}{n+2} \cos n\varphi + \cos (n+2)\varphi \tag{13}$$

From the condition $\gamma = 0$, we find

$$w = -u_1'(z)\left[\frac{A_0}{n} \cos n\varphi + \frac{1}{(n+2)^2} \cos (n+2)\varphi + w_0\right].$$

We obtain supplementary normal stress along the generatrices from the condition $\sigma_{zz} = E\frac{\partial w}{\partial z}$. In accordance with the hypothesis that the stability loss during bending changes the deformed and stressed states of the shell only in the compressed zone, i.e., up to the neutral line, the diagram of normal stresses should be in a state of equilibrium within the limits of the bulge. To ensure this condition, we shall add two items in the expression σ_{zz}, reduced to the moments relative to the

neutral axis and the axis perpendicular to it. These stresses will /489 be expressed in the form

$$\sigma_{zz} = -Eu_1''(z)r\left[\frac{A_0}{n^2}\cos n\varphi + \frac{1}{(n+2)^2}\cos(n+2)\varphi + w_0 + \right.$$
$$\left. + a\cos\varphi + b\sin\varphi\right]. \tag{14}$$

The integration constant w_0 and the constants a and b are defined from the equilibrium states of stresses σ_{zz}:

$$\Sigma M_y = 0; \quad \Sigma M_x = 0; \quad \Sigma Z = 0.$$

From these conditions, we obtain

$$b = 0; \qquad a = \frac{D_a}{D}; \qquad w_0 = \frac{D_{w0}}{D},$$

where

$$D = \begin{vmatrix} 1 & \frac{\pi}{2} \\ \frac{\pi}{2} & 2 \end{vmatrix}, \quad D_a = \begin{vmatrix} A & \frac{\pi}{2} \\ B & 2 \end{vmatrix}, \quad D_{w0} = \begin{vmatrix} 1 & A \\ \frac{\pi}{2} & B \end{vmatrix},$$

whereupon

$$A = -\left[\frac{A_0}{n^3}\sin n\frac{\pi}{2} + \frac{1}{(n+2)^3}\sin(n+2)\frac{\pi}{2}\right],$$

and $B = -A_0\frac{\pi}{2}$ when $n = 1$ and $B = 0$, with $n \neq 1$ – odd number.

The functional of full energy, which is accumulated by the system after the bulging, is written as

$$\Pi = \int_0^L dz \cdot r\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}(W_0 + A_0)\,d\varphi = \int_0^z F(u_1, u_1', u_1'')\,dz,$$

where

$$F\ (u_1, u_1', u_1'') = \frac{r}{2}\left[C_{str}\ (u_1'')^2 + C_{rib}\ u_1^2 + C_\sigma (u_1'')^2 - p_0 c_p (u_1')^2\right]$$

Here, the following designations are introduced:

$$C_{str} = B_{str}\ C_{str},$$

where

$$C_{str} = \left[A_0^2 + 1\right]\frac{\pi}{2}\ ;$$

$$C_{rib} = \frac{B_{rib}}{r^1}\ C_{rib},$$

where

$$C_{rib} = \{A_0^2 (n^2 - 1)^2 + [(n + 2)^2 - 1]^2\}\ \frac{\pi}{2}\ ,$$

$$C_\sigma = Er^2\ hc_\sigma,$$

when $n \neq 1$ odd number /490

$$C_\sigma = \left[\frac{A_0^2}{n^4} + \frac{1}{(n + 2)^4}\right]\frac{\pi}{2} + 4w_0\left[\frac{A_0}{n^3}\sin n\frac{\pi}{2} + \right.$$

$$\left. + \frac{1}{(n + 2)^3}\sin (n + 2)\frac{\pi}{2}\right] + \left[a^2 + 2w_0^2\right]\frac{\pi}{2} + 4\ aw_0,$$

when $n = 1$, the term $\frac{A_0 a\ 4\pi}{n^2}$ is added to the expression C_σ;

$$C_p = A_0^2 \left\{ \left[1 + \frac{1}{4n^2 - 1} \right] + \frac{1}{n^2} \left[1 - \frac{1}{4n^2 - 1} \right] \right\} + \left\{ \left[1 + \frac{1}{(2n + 3)(2n + 5)} \right] + \right.$$

$$\left. + \frac{1}{(n + 2)^2} \left[1 - \frac{1}{(2n + 3)(2n + 5)} \right] \right\} + 2A_0 \left\{ \left[\frac{1}{3} - \frac{1}{(2n + 1)(2n + 3)} \right] + \right.$$

$$\left. + \frac{1}{n(n + 2)} \left[\frac{1}{3} + \frac{1}{(2n + 1)(2n + 3)} \right] \right\}$$

From the extremum condition of the function of full energy of the bulging, we obtain the equilibrium equation

$$(C_\sigma + C_{str})\, u_1^{IV} + p_{0\ cir}\, C_p\, u_1'' + C_{rib}\, u_1 = 0. \tag{15}$$

After substituting the derivation $f = c \sin kz$ and the solution of the characteristic equation relative to $P_{0\ cir}$, we have

$$p_{0\ cir} = \frac{1}{c_p} (C_\sigma + C_{str})\, k^2 + \frac{C_{rib}}{c_p k^2} \quad . \tag{16}$$

The minimum condition of the critical load $\frac{\partial p_{cir}}{\partial (k^2)} = 0$ gives

$$k^2 = \sqrt{\frac{C_{rib}}{C_\sigma + C_{str}}} \tag{17}$$

Having substituted (17) in (16), we shall obtain a minimal value of critical load for the various numbers of n:

$$p_{0\ cir} = 2\sqrt{\frac{(C_\sigma + C_{str})\, C_{rib}}{c_p^2}} \tag{18}$$

or, after disregarding the term C_{str} in favor of C_σ, we obtain, for the unreinforced shell,

$$\sigma_{0\ cir} = \sqrt{\frac{c_{rib}\ c_\sigma}{3(1 - \mu^2)\ c_p^2}} \cdot \frac{E}{\left(\frac{r}{h}\right)} = \nu \frac{E}{\left(\frac{r}{h}\right)}, \tag{18'}$$

where

$$\nu = \sqrt{\frac{c_{rib}\ c_\sigma}{3(1 - \mu^2)\ c_p^2}}.$$

This expression of critical load disregards the minimum value when n = 1. Moreover, the bulging in the cross-section is expressed as (fig. 2)

$$u_2(\varphi) = \frac{1}{3} \cos \varphi + \cos 3\ \varphi.$$

Further, we find that /491

$$c_{rib} = 100; \qquad c_\sigma = 0.01; \qquad c_p = 1.62; \qquad \nu = 0.38$$

and

$$\sigma_{0\ cir} = 0.38 \frac{E}{\left(\frac{r}{h}\right)}\ ^*. \tag{19}$$

This value of the critical stress in the most compressed generatrix of a cylindrical shell subjected to pure bending is small compared with the values cited in the literature. The length of the longitudinal halfwave in which such a loss of stability is possible (n = 1 corresponds to the shape of the bulging, when three halfwaves are formed in the cross-section) can be determined from equation (17). It is considerably less than the length of the halfwave of a cylindrical shell with a loss of stability from central compression in a similar case (the case of formation of the longest halfwaves along the z-axis in the elliptical bulging in the cross-section).

* The calculation of the energy of shear deformation in the mean surface and of torsion yields a difference of about 1 per cent.

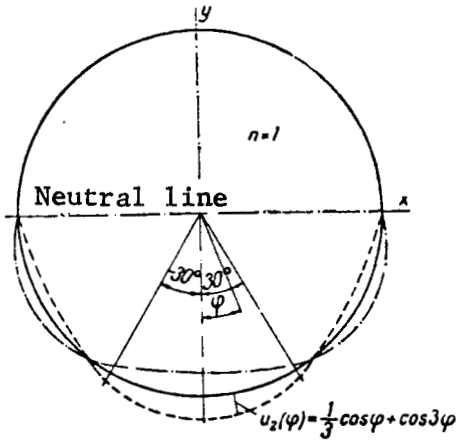

Fig. 2

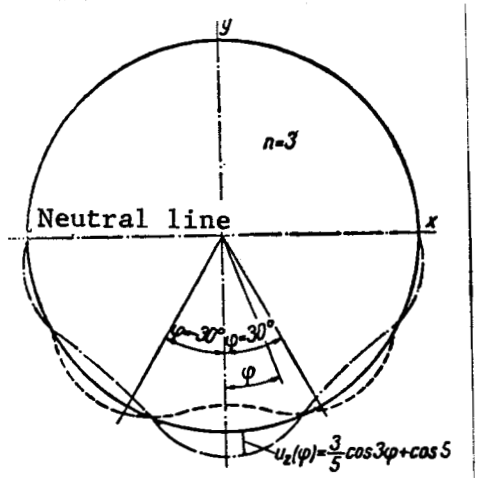

Fig. 3

For the unreinforced shell we have, from expression (17),

$$\frac{L}{m} = x\ r, \tag{17a}$$

where

$$x = \pi \sqrt{2\left(\frac{r}{h}\right)} \sqrt{\frac{3(1-\mu^2)\ c\sigma}{c_{rib}}}$$

In the shell with $\mu = 0.3$, this gives $\left(\frac{h}{r}\right) = 0.003$ and $n = 1$ the length of a halfwave $\frac{L}{m} \approx 5d$, where d is the diameter of the shell. For the reinforced shell, the length of the halfwave along the z-axis diminishes correspondingly in this type of loss of stability. For example, when $J_{rib} = J_{str} = 0.04\ cm^3$, $h = 0.2$ cm, $r = 60$ cm, and /492
$\mu = 0.3$, then $\frac{L}{m} \approx 1.8d$. Thus, a given shape of loss of stability is completely possible, for instance, for a fuselage type of shell.

When $n = 3$, the bulging in the cross-section is expressed by (fig. 3)

$$u_2(\varphi) = \frac{3}{5} \cos 3\varphi + \cos 5\varphi.$$

The critical stress will be

$$\sigma_{0\ cir} = 0.9 \frac{E}{\left(\frac{r}{h}\right)},$$

and the length of the halfwave for the shell, when $\frac{h}{r} = 0.003$, is $\frac{L}{m} \approx 3d$. In fig. 4, the magnitudes of the coefficients are indicated by the points ν [see equation (18')] in various numbers of n and corresponding dimensionless quantities $\xi = \frac{L}{mr}\sqrt{\frac{h}{r}} = x\sqrt{\frac{h}{r}}$ [see equation (17a)], in which the critical load receives minimal values. For better visibility, these points are connected by a broken line. By using this curve, it is easy to obtain the minimal critical stresses in the various numbers of n during bending, and the length of the halfwaves in which the stresses are possible for the shell with the established values h and r.

We shall follow the relation of the critical load to the length of the shell in the loss of stability in greater detail along a sufficiently long longitudinal halfwave when n = 1. Expression (16) gives value $p_{0\ cir}$ as a function of $k = \frac{m\pi}{L}$. We shall transpose this expression so that, in the assigned number n, we have the relation of the quantity $\frac{\sigma_{0\ cir}}{E}\left(\frac{r}{h}\right)$, which characterizes the critical load, to the quantity ξ, which defines the only possible length of the halfwave $\frac{L}{m}$ in the assigned values h and r.

We have

$$\frac{\sigma_{0\ cir}}{E}\left(\frac{r}{h}\right) = \frac{c_\sigma}{c_p} \cdot \frac{\pi^2}{\xi^2} + \frac{c_{rib}}{c_p 12(1 - \mu^2)\mu^2}\xi^2.$$

Where $\mu = 0.3$ and $n = 1$, we obtain

$$\frac{\sigma_{0\ cir}}{E}\left(\frac{r}{h}\right) = 6.3 \cdot 10^{-2}\ \frac{1}{\xi^2} + 5.8 \cdot 10^{-1}\ \xi^2. \qquad (20)$$

According to equation (20), the values $\frac{\sigma_{0\ cir}}{E}\left(\frac{r}{h}\right)$ were calculated as functions of ξ.

The results are shown by a continuous line in fig. 4. The graph indicates the critical load during pure bending of the shell with any parameters in the case ~~if~~ no plastic deformation occurs.

For example, if the length of the shell is such that the value ξ, when m = 1 (in the hypothesis of formation of one longitudinal half-wave) is less than 0.3, then the shell will bulge through a con- /493 siderable quantity of longitudinal and cross-sectional halfwaves (greater numbers of m and n); in actual numbers, $\xi \approx 0.03$.

Moreover, in accordance with our conclusions, the critical stress in the most compressed longitudinal filament of the shell will be close to the value $\sigma_{0\ cir} = 0.75\ \frac{Eh}{r}$.

If the length of the shell is such that when m = 1, a value of more than 0.3 is obtained, then the greatest critical stress will be in the formation of long halfwaves along the z-axis (bulging in the cross-section of three halfwaves of n = 1).

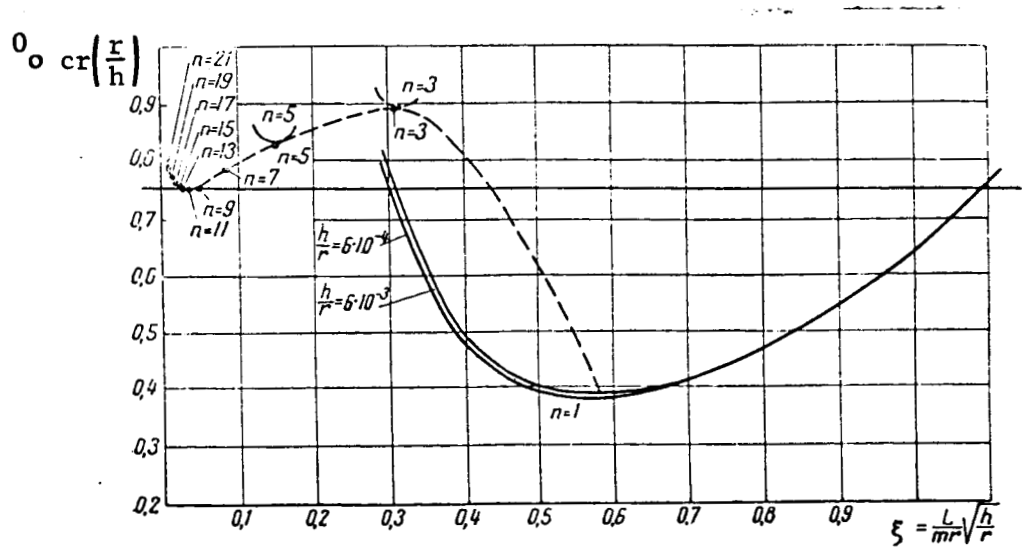

Fig. 4

According to the graph presented in fig. 4 (the continuous line), it is simple to determine the critical load for any length of shell in such a case. In a shell with $\frac{h}{r} = 3 \cdot 10^{-3}$, as we have already noted above (see page 24), the minimum $\sigma_{0\ cir} = 0.38\ \frac{Eh}{r}$ will occur when $L \approx 3d$, then, assuming that $m = 1$, $\xi = 0.3$, ~~but~~ $\sigma_{0\ cir} \approx 0.6\ \frac{E}{\frac{r}{h}}$.

Moreover, the formation of one as well as two halfwaves is possible along the z-axis, since, when $m = 1$ ($\xi = 0.8$) and $m = 2$ ($\xi = 0.4$), the minimum critical loads will have identical values. Further, if we have a shell with $L \approx 8.7d$, when $m = 1$, then $\xi \approx 1$, and when $m = 2$, then $\xi \approx 0.5$; the least critical stress $\sigma_{0\ cir} = 0.4\ \frac{E}{\left(\frac{r}{h}\right)}$ is obtained in the bulging of a shell along the z-axis. With a further increase in length, the shell will always lose stability in values of critical stresses near $\sigma_{0\ cir} = 0.4\ \frac{E}{\frac{r}{h}}$. /494

During this time the quantity of longitudinal halfwaves is equal to two or more.

Figure 4 presents cases of loss of stability of thin shells with any data provided $\xi \geq 0.3$. From the values obtained for $\sigma_{0\ cir}$, it is easy to derive the critical bending moments according to the formula

$$M_{cir} = \frac{\sigma_{0\ cir}\ J}{r},$$

where

$$J = \pi r^3 h.$$

We shall now investigate the relation of the critical load of a reinforced shell subjected to bending to its length and the loss of stability in sufficiently long longitudinal halfwaves when $n = 1$*.

* As calculations have shown, when $n = 1$, the least critical load is obtained for sufficiently long reinforced shells, as well as for unreinforced shells.

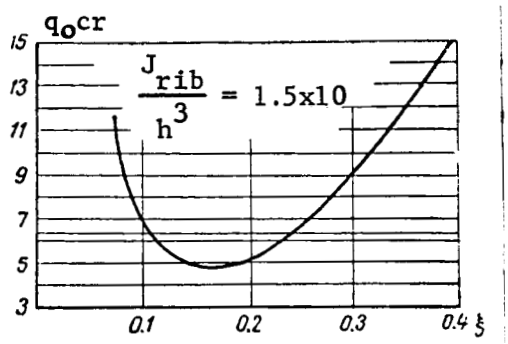

Fig. 5

We shall transpose the general expression (16), which gives the value of $p_{0\ cir} = \sigma_{0\ cir}\ h$ as a function of $k = \frac{m\pi}{L}$, in order to obtain dependence of the dimensionless quantity $q_{0\ cir} = \frac{\sigma_{0\ cir}}{E}\left(\frac{h_{reduced}}{h}\right)\left(\frac{r}{h}\right)$ (which characterizes the critical load) on another dimensionless quantity $\xi = \frac{L}{mr}\sqrt{\frac{h}{r}}$ (which characterizes in the assigned values of h and r the only possible length of a halfwave $\frac{L}{m}$) in a specific number n.

We have

$$q_{0\ cir} = \frac{c_\sigma + \frac{J_{str}}{hr^2}\, c_{str}}{c_p} \cdot \frac{\pi^2}{\xi^2} + \frac{c_{rib}}{c_p \pi^2} \cdot \frac{J_{rib}}{h^3}\, \xi^2.$$

It is possible to disregard the term $\frac{J_{str}}{hr^2}\, c_{str}$ by virtue of its low value in comparison with c_σ. Then, when $p = 1$, we have

$$q_{0\ cir} = 6.3 \cdot 10^{-2}\, \frac{1}{\xi^2} + 6.3\, \frac{J_{rib}}{h^3}\, \xi^2. \qquad (21)$$

The graph presented in fig. 5 is constructed according to equation (21) for a fuselage-type shell with the value $\frac{J_{rib}}{h^3} = 15$. This indicates

that there is an increase in the bending rigidity along the s-coordinates of more than 150 times as compared with the unreinforced shell with $E_{rib} = E_{gen}$:

$$\frac{J_{rib\ reinfor.}}{J_{rib\ unreinfor.}} = \frac{J_{rib\ reinfor.}}{h^3} 12(1 - \mu^2).$$

From this graph, it is easy to determine the critical load /495 for any reinforced shell in which the parameter $\frac{J_{rib}}{h^3} = 15$.

Similar graphs are easily constructed according to equation (21) for shells with other values of $\frac{J_{rib}}{h^3}$.

As is seen from the graph, beginning with the values $\xi = 0.115$ and above, the critical stresses will not increase the value

$$\sigma_{0\ cir} = q_{0\ cir} \left(\frac{h}{h_{reduced}}\right) \frac{E}{\left(\frac{r}{h}\right)} = 6.3 \left(\frac{h}{h_{reduced}}\right) \frac{E}{\left(\frac{r}{h}\right)}$$

Actually, if we obtain $\xi \geq 0.23$ using the hypothesis that at any length of shell one halfwave is formed (in addition, $q_{0\ cir} \geq 6.3$), then not one but two or several longitudinal halfwaves are formed, and $q_{0\ cir} \leq 6.3$ whenever $0.115 \leq \xi \leq 0.24$.

The minimum possible value for the shell in question will be $q_{0\ cir} = 4.8$ when $\xi = 0.16$.

If a shell is filled, for example, with a material with a resilience $E = 0.7 \cdot 10^6$ kg/cm^2, with $h = 0.15$ cm, $h_{reduced} = 0.3$ cm and $\frac{r}{h} = 800$, then

$$\sigma_{0\ cir} = 4.8 \cdot 0.5 \frac{0.7 \cdot 10^6}{8 \cdot 10^2} \approx 2\,100 \text{ kg/cm}^2.$$

In addition, the length of the longitudinal halfwave will be

$$\frac{L}{m} = \xi\ r \sqrt{\frac{r}{h}} = 1.6 \cdot 10^{-1} \sqrt{800}\ r = 4.5r = 2.25d.$$

The length of the half wave $\frac{L}{m} \approx 1.6d$ corresponds to a value of $\xi = 0.115$. Here, we have investigated the general stability loss of a reinforced shell during bending. The critical stresses obtained, and the smallest ones must be selected.

We note a certain analogy between the case of loss of stability of a shell during pure bending and the case of loss of stability due to central compression.

In sufficiently long shells, for example, in this as well as in another case, stability loss is possible along the long longitudinal halfwaves and a small quantity of halfwaves in the cross-section.

This stability loss also occurs in less critical stresses than in the case of short longitudinal waves (during bending, for instance, the minimum $\sigma_{0\ cir} = 0.38 \frac{E}{\frac{r}{h}}$ when $n = 1$, and in the case of central compression, $\sigma_{0\ cir} = 0.36 \frac{E}{\frac{r}{h}}$).

However, in case of bending, the minimum length of shells in which such a stability loss is considerably less than in the case of compression. Consequently, a given shape can be found considerably more often in cases of bending than in cases of central compression of shells.

3. EXPERIMENTAL RESULTS /496

Critical loads and the bulging of cylindrical shells during pure bending have been verified experimentally. For test purposes, long cylindrical shells with a diameter of 54 mm have been prepared. These cylinders are glued together with a special glue made out of steel lamina, 0.1 mm thick, with the following specifications:

$E = 2.2 \cdot 10^6$ kg/cm^2; $\sigma_{\text{т}} = 41$ kg/mm^2; $\sigma_{\text{в}} = 56$ kg/mm^2; $\mu = 0.3$.

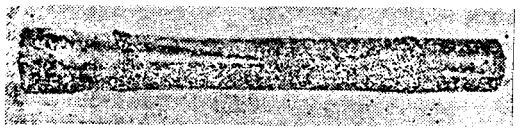

Fig. 6

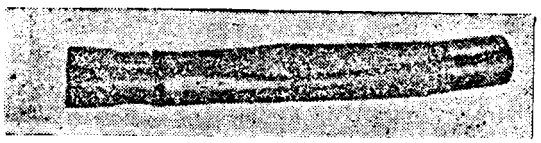

Fig. 7

Strips approximately 7 mm wide were glued together with an overlap. During the bending of the cylinder, a strip was located in the elongated zone in order to eliminate its influence of the bulging.

At both ends, continuous dural cylinders were installed, to which the shells to be tested were glued and which were strengthened by wire.

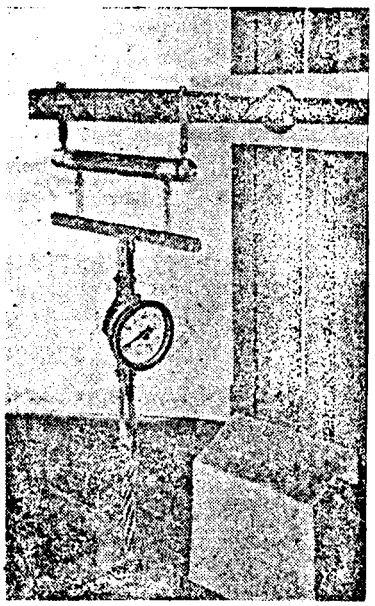

Fig. 8

Couples were attached to the ends of the cylinder in such a manner that the shell was loaded by means of pure bending. In all, 12 cylinders were tested. Their length was determined on the basis of the graph of fig. 4, which indicates that the bulging occurred along one long longitudinal halfwave in the formation of three halfwaves in a cross-section in the compressed area of the shell (n = 1).

On the basis of this theory, the expected shape of bulging was confirmed by the experiment. In fig. 6, the complete bulge of one of the images is shown, and in fig. 7, its side view.

A view of the experimental apparatus is presented in fig. 8. On the basis of the dynamometer-measured critical force p_{cir} the /497

bending moment of the shell was calculated, as was the amount of critical stress in the most compressed generatrix according to the formula

$$(\sigma_{0\ cir})_{exper} = \frac{M_{prep\ cir}\ r}{J}.$$

In fig. 9, the theoretical graph of fig. 4 is repeated, but only the curve for p = 1 is retained. The experimental critical stresses $(\sigma_{0\ cir})_{exper}$ are indicated by crosses, referred to the quantity

$\frac{Eh}{r}$. It is seen that the experimental critical stresses fluctuate within

a range of 83 to 100 per cent of the theoretical stresses.

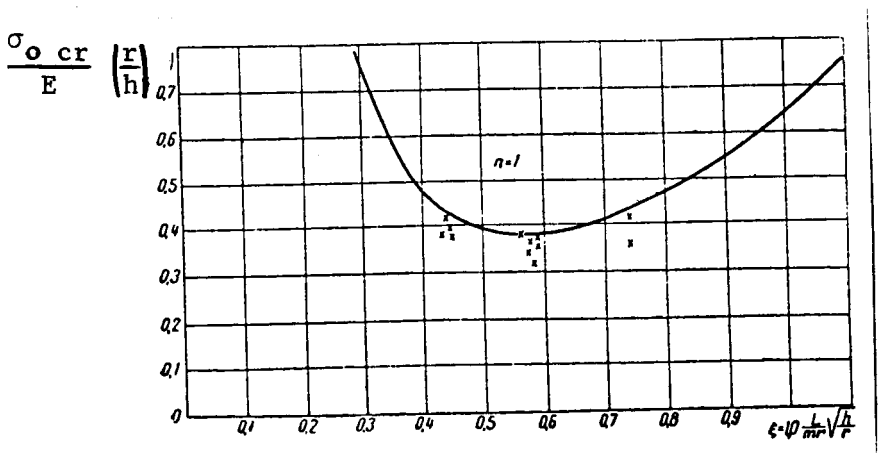

Fig. 9

Thus, the difference between the quantities of critical stresses obtained on the basis of the theory of small displacements and the

quantities of experimental critical stresses in the case of bulging along long longitudinal halfwaves during bending is considerably less than in the case of bulging along short longitudinal halfwaves due to central compression.

The last case of stability loss was investigated theoretically and experimentally in detail by A. S. Vol'mir. He showed that, in this case, the critical stresses of compression obtained by him on the basis of the theory of great displacements (very close to those obtained experimentally) comprised in all 25 per cent or more of those obtained on the basis of the theory of small displacements.

CONCLUSION

1. The introduction of function K makes possible the placement of only weak restrictions on the shift deformation and allows the role of the shift in a generally deformed state of circular cylindrical shells to be investigated during bulging due to central compression.

The opportunity of using the hypothesis of the absence of shifts $[e_{zs} = 0$ ($\gamma = 0$ and $\chi_{zs} = 0$), together with $e_{sn} = e_{sn} = 0]$ in the solution of some problems concerning the stability of shells became apparent. On the basis of this hypothesis, equation (1) was obtained, which is a generalization of Euler's equation for a flexible rod in cases of asymmetric bulging due to axial compression of long, as /498 well as short circular cylindrical shells.

2. The hypothesis allowed the problem to be resolved by determining the critical load during the pure bending of a long, circular cylindrical shell.

It has been shown that, beginning with certain lengths, the shell loses stability along long longitudinal halfwaves with the formation of three halfwaves in a cross-section in the compressed area of the shell. For a shell with $\frac{h}{r} = 0.003$ and $\mu = 0.3$, for example, this type of stability loss is possible with lengths of more than 3½ diameters; for the reinforced shell, the indicated stability loss is possible with even less length.

3. It has been shown that a thin shell, beginning with certain lengths, loses stability as a result of pure bending when

$$\sigma_{0\ cir} \leq 0.4 \frac{E}{\frac{r}{h}} .$$

The minimum possible theoretical critical stresses is $\sigma_{0\ cir} = 0.38 \frac{E}{\frac{r}{h}}$.

4. Results of the investigation were presented with reference to a similar graph for practical use (fig. 4), accurate for thin shells with any parameters. Similar graphs are easily constructed for reinforced shells using equation (21).

Experimental critical stresses fluctuate within a range of 83 to 100 per cent of the theoretical values presented by the graph.

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