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POST-BUCKLING STRENGTH OF A PRESSURIZED CYLINDER

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CONTENTS

	Page
Summary	1
Analysis	1
Numerical Example	2
References	7

ILLUSTRATIONS

No.	Title	Page
1	Loading Condit[illegible]	4
2	Stress Dis[illegible]	4
3	Bending Strength	5
4	Moment and Tension Stress as Functions of Curvature	6

TABLES

No.	Title	Page
1	Bending Characteristics	3

POST-BUCKLING STRENGTH OF PRESSURIZED CYLINDER

D. J. Peery

Summary

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A simple analysis procedure is presented for a cylinder with axial load and bending moment. The bending stress is assumed linear in the unbuckled region and constant in the buckled region. Further analysis consists of applying the equations of statics. Flexural rigidity is obtained from conventional equations. Charts are presented to simplify numerical calculations.

Author

Analysis

The cylinder of Fig. 1 resists an axial load P_a and an external bending moment M. The assumed stress distribution and notation are shown in Fig. 2. The bending moment is resisted by the load varying from zero at angle θ to a maximum of N per unit length. The pressure, axial load, and uniform load N_c have no moment about the neutral axis. Integrating the values for the triangular distribution, the following results are obtained.

$$P_1 = 2NR\,\frac{\sin\theta - \theta\cos\theta}{1 - \cos\theta} \tag{1}$$

$$M = NR^2\,\frac{\theta - \sin\theta\cos\theta}{1 - \cos\theta} \tag{2}$$

$$\frac{M}{P_1 R} = \frac{\theta - \sin\theta\cos\theta}{2\,(\sin\theta - \theta\cos\theta)} \tag{3}$$

Values of M/P_1R and NR/P_1 are plotted in Fig. 3.

The bending deflection is obtained from the stress distribution of Fig. 2. The stress varies an amount N/t in a distance $R(1-\cos\theta)$. The cylinder bends through an angle ϕ per unit length, obtained as follows.

$$\phi = \frac{N}{EtR\,(1-\cos\theta)} \tag{4}$$

From Equations 1 and 4,

$$P_1 = 2EtR^2\phi\,(\sin\theta - \theta\cos\theta) \tag{5}$$

From Equations 2 and 4,

$$M = EtR^3\phi\,(\theta - \sin\theta\cos\theta) \tag{6}$$

It is easier to visualize the characteristics of the cylinder if M is plotted as a function of ϕ, to show a normal type of load-deflection curve. Such a curve is plotted in Fig. 4, which is similar to Fig. 4 of Reference 1. Equations 47 and 48 of Reference 1 correspond to Equations 5 and 6 above, with some change in notation. For $N_c = 0$, the preceding analysis corresponds to the membrane analysis of Reference 1.

Numerical Example

The bending characteristics will be calculated for a cylinder with $R = 60"$, $P_a = 40{,}000$ lb, $p = 16$ psi, $t = .014$ in, $\theta = 180°$, and $N_c = 30$ lb/in. The load P_1 is first calculated

$$P_1 = -P_a + \pi R^2 p + 2\pi R N_c$$

$$= -40{,}000 + 11{,}300 + 30 + 377 \times 30 = 152{,}300 \text{ lb.}$$

From Fig. 3, $M/P_1R = 0.5$, $NR/P_1 = .318$

$$M = 0.5\,P_1R = 0.5 \times 152{,}300 \times 60 = 4{,}569{,}000 \text{ in-lb.}$$

$$N = 0.318\,\frac{P_1}{R} = 0.318 \times 152{,}300/60 = 807 \text{ lb/in.}$$

$$N_t = N - N_c = 807 - 30 = 777 \text{ lb/in.}$$

$$\sigma_t = N_t/t = 777/.014 = 55{,}500 \text{ psi}$$

From Fig. 4, $M/(EI\phi) = 1$,

For $E = 33 \times 10^6$, $I = \pi R^3 t = 9.51 \times 10^3$, $EI = 314 \times 10^9$ lb-in^2

For $L = 150"$, $\phi L = ML/EI = 4{,}569{,}000 \times 150/314 \times 10^9 = .00218$ rad $= .125°$

Other values are calculated by the same procedure in Table I. The same cylinder dimensions are used, but various values of θ are considered, and also a pressure increase to 20 psi.

TABLE I

BUCKLING CHARACTERISTICS

p = 16 psi, P_1 = 152,[illegible] lb.	[illegible]	N_t	σ_t	$\phi L = 150\phi$
θ = [illegible]°	[illegible]	777	55,500	.125°
θ = [illegible]°	[illegible]	[illegible]	59,100	.141°
θ = [illegible]°	[illegible]	967	69,100	.205°
θ = [illegible]°	[illegible]	1240	88,600	.388°
p = 20 psi, P_1 [illegible]				
θ = [illegible]°	[illegible]	[illegible]	72,600	.162°
θ = [illegible]°	[illegible]	[illegible]	77,400	.185°
θ = [illegible]°	[illegible]	[illegible]	90,400	.266°
θ = [illegible]°	[illegible]	1618	115,800	.510°

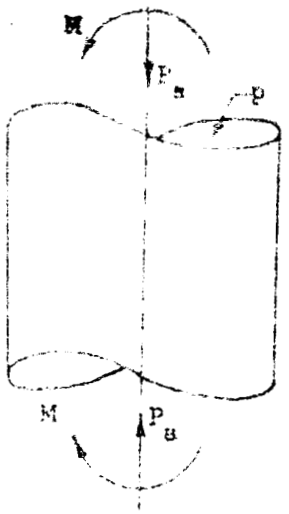

Figure 3 - Loading Conditions

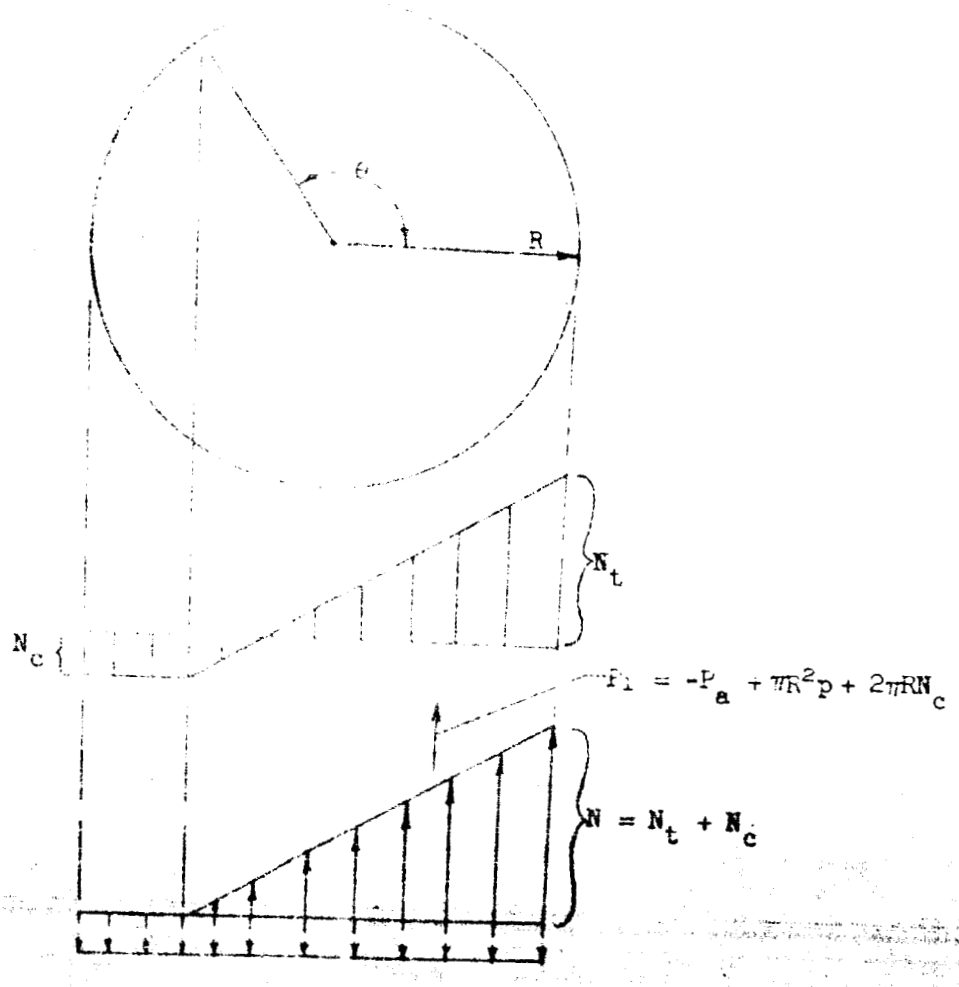

Figure 4 - Stress Distribution

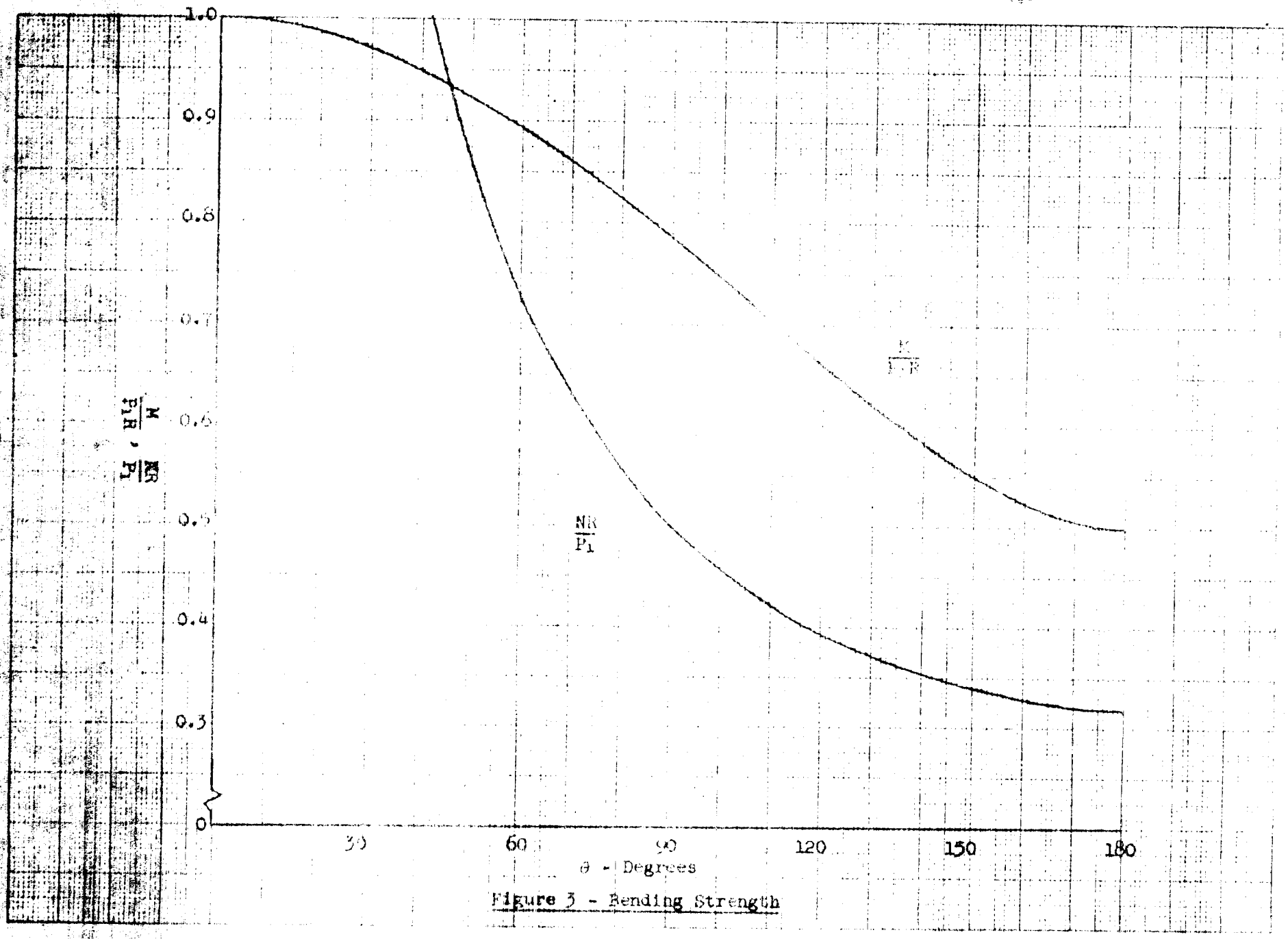

Figure 3 - Bending Strength

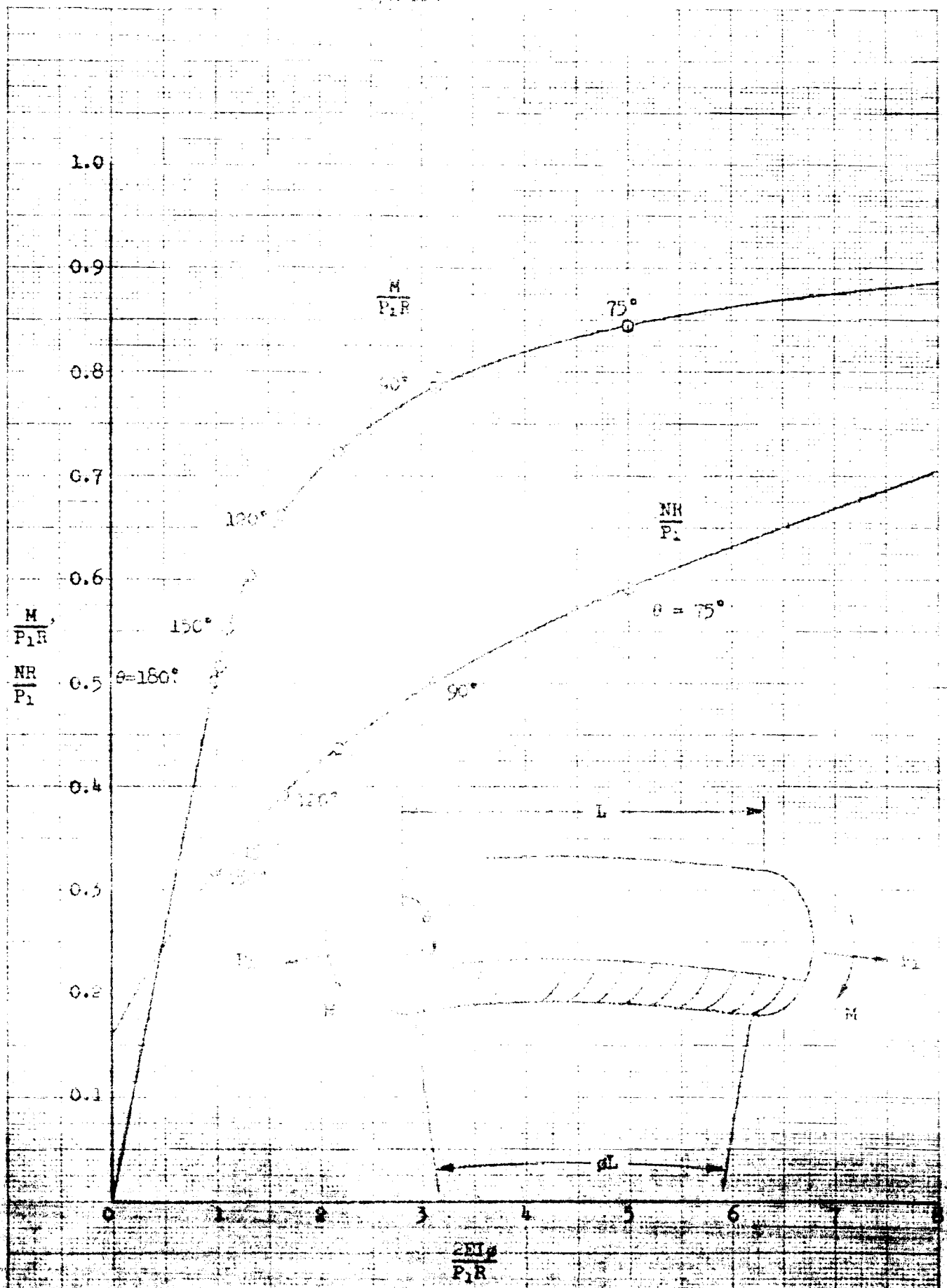

Figure 4 - Moment and Tension Stress as Functions of Curvature