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FORTRAN IV COMPUTER PROGRAM FOR THE EVALUATION OF NATURAL FREQUENCIES AND UNSTABLE VALUES OF THE THRUST FREQUENCY FOR A FREE-FREE, CIRCULAR CYLINDRICAL SHELL SUBJECTED TO A GIMBALED, PERIODICALLY-VARYING END THRUST

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1.0 INTRODUCTION

A Fortran IV computer program has been written to solve for unstable values of the thrust frequency from the thirteen stability equations derived in Section 10 of Reference 1.

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For completeness the origin and the equations themselves are presented in Sections 2.2 and 2.3. Specifically, this report presents a Fortran IV computer program that evaluates the natural frequencies of a thin cylindrical shell with free-free end conditions and then evaluates unstable values of the thrust frequency for given initial conditions. This Fortran IV computer program is operationally compatible with the Fortran Processors of the IBM 7094 and the Univac 1107 digital computers. author

Of the thirteen stability equations derived in Reference 1 and presented in Section 2.3 of this report, eleven were identical to the stability equations derived previously for the simply-supported case in Reference 2.

A program to solve for unstable values of the thrust frequency for these eleven equations was written and this program was presented in Reference 3. With some modifications this program is used to solve for the natural frequencies and unstable values of the thrust frequency of the eleven equations identical between the simply-supported case and the free-free case. The necessary modifications of the program presented in Reference 3 are given in Sections 4.1 and 4.2. Consideration of the bending action of the cylinder of Reference 1, an action not previously considered in Reference 2, has produced two additional stability equations. These two equations contain some beam parameters. A Fortran IV Computer Program presented

in Reference 4 has been modified and then used in calculating the necessary beam parameters. The modifications of this program appear in Sections 5.1 and 5.2.

All additional subroutines that did not appear in the programs presented in Reference 3 and Reference 4 are presented in Sections 6.1, 6.2, 6.3, and 6.4. The various constraints of the program are given in Sections 7.1, 7.2, and 7.3.

A complete listing of the Fortran IV Computer Program is given in Section 8.2.

2.0 STABILITY CONSIDERATIONS

2.1 Development of Equations involving only Cylindrical Parameters

This report is a companion report to the free-free analysis presented in Reference 1. All symbols unless otherwise indicated will be those used in Reference 1.

As presented in Reference 1 the Matheau Equation is

$$\frac{d^2\psi}{d\tau_1^2} + (a - 2q \cos 2\tau_1)\psi = 0 \qquad (1) \qquad \{7.1\text{-}4\}^*$$

where

$$a = \frac{8\pi rL\ T_o K}{I\Omega^2} \qquad q = \frac{\gamma\alpha}{2} \qquad (2) \qquad \{7.1\text{-}3\}$$

with ranges

$$0 < a < 1, \qquad 0 < q \leq 0.05 \qquad (3) \qquad \{7.1\text{-}5\}$$

As shown in Reference 1

$$\sin \frac{\beta\pi}{2} = \Delta(0) \sin^2 \frac{\pi a^{\frac{1}{2}}}{2} \qquad (a \neq 4r^2) \qquad (4) \qquad \{7.1\text{-}27\}$$

where for small values of q

$$\Delta(0) \simeq 1 + \frac{\pi q^2}{4a^{\frac{1}{2}}(a-1)} \cot \frac{\pi a^{\frac{1}{2}}}{2} \qquad (5) \qquad \{7.1\text{-}28\}$$

---

* If an equation appearing in this report appeared in Reference 1 also, the equation number in Reference 1 will be shown in braces.

which after substituting $q = \gamma a/2$ and simplifying becomes

$$\Delta(0) \simeq 1 + \frac{\pi a^{3/2}\ \gamma^2}{16(1-a)} \cot \frac{\pi a^{\frac{1}{2}}}{2} \tag{6}$$

The expression for β is written

$$\beta = 2/\pi\ [\sin^{-1} \{[\Delta(0)]^{\frac{1}{2}} \sin \frac{\pi a^{\frac{1}{2}}}{2}\}] \tag{7}$$

which demonstrates that β is a function of Ω by virtue of equation (6) and equation (2) in this report.

2.2 Origin of the Two Stability Equations Involving Beam Parameters

Two of the stability equations derived in Reference 1 contained beam parameters. These two stability equations are

$$\bar{\Omega} = \frac{\bar{\omega}_{0k}}{|(\alpha_r + s)|} = \frac{\Omega}{\omega_1} \tag{8}$$ {10.2-8}

$k = 1,2,\ldots,N_1$

$r = 1,2,\ldots,R$

$s = -S$ to $+S$

and

$$\bar{\Omega} = \frac{\bar{\omega}^m_{jk}}{|(\alpha_r + s)|} = \frac{\Omega}{\omega_1} \tag{9}$$ {10.2-39}

$j = 1,2,\ldots,M_1$
$k = 1,2,\ldots,N_1$
$m = 1,2,3$
$r = 1,2,\ldots,R$
$s = -S$ to $+S$

Now

$$M_1 = N_1 = 10$$

as in Reference 3.

Also

$$R = 3$$

Corresponding to

1) The first bending mode

2) The second bending mode

3) The rotational mode

and

$$S = 5$$

where S is the number of terms of a converging series that are considered. The series is

$$\{\psi_k\} = \sum_{s=-\infty}^{\infty} \{C_k\}^{(s)} e^{is\bar{\Omega}t} \qquad (10) \qquad [3.4]^*$$

This series has converged sufficiently after five terms so only

$$s = -5 \text{ to } +5$$

is considered.

Equations {10.2-8} and {10.2-39} have the two parameters α_r and s in them. These parameters were introduced into the analysis in Reference 1

* All equation numbers appearing in brackets are the equation numbers of the same equations in Reference 5.

to allow for the end displacement of the cylinder. The relative end displacement of the beam is given in terms of the parameter y in the equation

$$\psi_G = \tan^{-1} \frac{\frac{\partial y}{\partial x}}{(1 + \frac{\partial u}{\partial x})} \qquad (11) \qquad [2.2]$$

The displacement y is approximated considering the Galerkin Method by a solution of the form

$$y_N(\xi,\tau) = q_A + q_B \xi + \sum_{n=1}^{\infty} q_n(\tau)\ \phi_n(\xi) \qquad (12) \qquad \{8.2\text{-}1\} \text{ and } [2.59]$$

where

q_A is a translation term

q_B is a rotation term and

$$q_B(\tau) = u_B(\tau) = \sum_{s=-\infty}^{+\infty} C_B^{(s)}\ e^{i(\alpha_r+s)\ \bar{\Omega}\tau} \qquad (13) \qquad \{8.3\text{-}7\} \text{ and } [3.5]$$

$$r = 1,2,\ldots,R$$

$q_n(\tau)$ is a generalized coordinate associated with $\phi_n(\xi)$, the nth vibration mode shape of a free-free beam (cylinder).

$$q_n(\tau) = u_n(\tau) = \sum_{s=-\infty}^{+\infty} C_n^{(s)}\ e^{i(\alpha_r+s)\ \bar{\Omega}\tau} \qquad (14) \qquad \{8.3\text{-}9\} \text{ and } [3.5]$$

$$r = 1,2,\ldots,R$$

When the expressions {8.3-7} and {8.3-8}are substituted into the expression for u_{0k} {9.1-36} and the resulting convolution integrals are evaluated the term (α_r+s) appears. The same convolution integrals appear in the expression

for u_{jk} {9.4-6}, v_{jk} {9.4-8} and w_{jk} {9.4-10}. The two additional stability equations presented earlier in this section are

$$\bar{\Omega} = \frac{\omega_{0k}}{|\alpha_r + s|} = \frac{\Omega}{\omega_1} \qquad (8) \qquad \{10.2\text{-}8\}$$

which comes from considering u_{0k} {9.1-36} and

$$\bar{\Omega} = \frac{\omega^m_{jk}}{|\alpha_r + s|} = \frac{\Omega}{\omega_1} \qquad (9) \qquad \{10.2\text{-}39\}$$

which comes from considering u_{jk} {9.4-6}, v_{jk} {9.4-8}, and w_{jk} {9.4-10}.

2.3 Presentation of Stability Equations

2.3.1 Subroutine UNST1 Solves Equation {10.2-6}

$$\Omega_i = \overbrace{[(i-2)^2 + \varepsilon_i \beta]}^{①} \frac{\Omega}{\omega_1} = \bar{\omega}_{0k}$$

where

$$\varepsilon_i = \begin{cases} -\frac{1}{2} \text{ if } i=1 \\ \frac{1}{2} \text{ if } i=2 \\ \frac{1}{2} \text{ if } i=3 \end{cases} \qquad \begin{array}{l} k = 1,2,\ldots,N_1 \\ i = 1,2,3 \end{array}$$

i	①	k	$\bar{\omega}_{0k}$
1	$1 - \frac{\beta}{2}$	1	$\bar{\omega}_{01}$
2	$\frac{\beta}{2}$	2	$\bar{\omega}_{02}$
3	$1 + \frac{\beta}{2}$	3	$\bar{\omega}_{03}$
		⋮	⋮
		N_1	$\bar{\omega}_{0N_1}$

2.3.2 Subroutine UNST2 Solves Equation {10.2-8}

$$\bar{\Omega} = \frac{\bar{\omega}_{0k}}{|(\alpha_r + s)|} = \frac{\Omega}{\omega_1}$$

where

$k = 1,2,\ldots,N_1$

$r = 1,2,\ldots,R$

$s = -S$ to $+S$

r	s	$\bar{\omega}_{0k}$	k
1	-S	$\bar{\omega}_{0k}$	$k = 1,2,\ldots,N_1$
1	-S+1	.	.
.	.	.	.
.	.	.	.
.	.	.	.
1	-1	.	.
1	0	.	.
1	1	.	.
.	.	.	.
.	.	.	.
.	.	.	.
1	S-1	.	.
1	S	.	.
2	-S	.	.
2	-S+1	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
R	S-1	.	.
R	S	$\bar{\omega}_{0k}$	$k = 1,2,\ldots,N_1$

2.3.3 Subroutine UNST3 Solves Equation {10.2-10}

$$\bar{\Omega} = \frac{\Omega}{\omega_1} = \bar{\omega}^m_{j0}$$

where

$m = 1,2$

$j = 1,2,\ldots,M_1$

m	j	$\bar{\omega}^i_{j0}$
1	1	$\bar{\omega}^1_{10}$
2	1	$\bar{\omega}^2_{10}$
1	2	$\bar{\omega}^1_{20}$
2	2	$\bar{\omega}^2_{20}$
1	3	$\bar{\omega}^1_{30}$
2	3	$\bar{\omega}^2_{30}$
.	.	.
.	.	.
.	.	.
.	.	.
1	M_1	$\bar{\omega}^1_{M_1 0}$
2	M_1	$\bar{\omega}^2_{M_1 0}$

2.3.4 Subroutine UNST4 Solves Equation {10.2-13}

$$2\overbrace{[(i-2)^2 + \varepsilon_i \beta]}^{4} \frac{\Omega}{\omega_1} = \bar{\omega}^m_{j0}$$

where

$m = 1,2$

$j = 1,2,\ldots,M_1$

$i = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} & \text{if } i=1 \\ \frac{1}{2} & \text{if } i=2 \\ \frac{1}{2} & \text{if } i=3 \end{cases}$$

m	j	$\bar{\omega}^m_{j0}$
1	1	$\bar{\omega}^1_{10}$
2	1	$\bar{\omega}^2_{10}$
1	2	$\bar{\omega}^1_{20}$
2	2	$\bar{\omega}^2_{20}$
1	3	$\bar{\omega}^1_{30}$
2	3	$\bar{\omega}^2_{30}$
.	.	.
.	.	.
.	.	.
.	.	.
1	M_1	$\bar{\omega}_{M_1 0}$
2	M_1	$\bar{\omega}_{M_1 0}$

i	4
1	$2 - \beta$
2	β
3	$2 + \beta$

2.3.5 Subroutine UNST5 Solves Equation {10.2-16}

$$\overbrace{\left|(i-2)^2 + (p-2)^2 + (\varepsilon_i+\varepsilon_p)\beta\right|}^{5} \frac{\Omega}{\omega_1} = \bar{\omega}^m_{j0}$$

where

$m = 1,2$

$j = 1,2,\ldots,M_1$

$i = 1,2,3 \qquad i \neq p$

$p = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} \text{ if } i=1 \\ \frac{1}{2} \text{ if } i=2 \\ \frac{1}{2} \text{ if } i=3 \end{cases} \qquad \varepsilon_p = \begin{cases} -\frac{1}{2} \text{ if } p=1 \\ \frac{1}{2} \text{ if } p=2 \\ \frac{1}{2} \text{ if } p=3 \end{cases}$$

m	j	$\bar{\omega}^m_{j0}$
1	1	$\bar{\omega}^1_{10}$
2	1	$\bar{\omega}^2_{10}$
1	2	$\bar{\omega}^1_{20}$
2	2	$\bar{\omega}^2_{20}$
1	3	$\bar{\omega}^1_{30}$
2	3	$\bar{\omega}^2_{30}$
.	.	.
.	.	.
.	.	.
1	M_1	$\bar{\omega}^1_{M_1 0}$
2	M_1	$\bar{\omega}^2_{M_1 0}$

i	p	5
1	1	not admissible
2	1	1
3	1	2
1	2	1
2	2	not admissible
3	2	$1 + \beta$
1	3	2
2	3	$1 + \beta$
3	3	not admissible

2.3.6 Subroutine UNST6 Solves Equation {10.2-19}

$$\overbrace{\left|(i-2)^2 - (p-2)^2 + (\varepsilon_i - \varepsilon_p)\beta\right|}^{6} \frac{\Omega}{\omega_1} = \bar{\omega}^m_{j0}$$

where

$m = 1,2$

$j = 1,2,\ldots,M_1$

$i = 1,2,3 \qquad i \neq p$

$p = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} \text{ if } i=1 \\ \frac{1}{2} \text{ if } i=2 \\ \frac{1}{2} \text{ if } i=3 \end{cases} \qquad \varepsilon_p = \begin{cases} -\frac{1}{2} \text{ if } p=1 \\ \frac{1}{2} \text{ if } p=2 \\ \frac{1}{2} \text{ if } p=3 \end{cases}$$

m	j	$\bar{\omega}^m_{j0}$
1	1	$\bar{\omega}^1_{10}$
2	1	$\bar{\omega}^2_{10}$
1	2	$\bar{\omega}^1_{20}$
2	2	$\bar{\omega}^2_{20}$
1	3	$\bar{\omega}^1_{30}$
2	3	$\bar{\omega}^2_{30}$
.	.	.
.	.	.
.	.	.
.	.	.
1	M_1	$\bar{\omega}^1_{M_1 0}$
2	M_1	$\bar{\omega}^2_{M_1 0}$

i	p	6
1	1	not admissible
2	1	$-1 + \beta$
3	1	β
1	2	$1 - \beta$
2	2	not admissible
3	2	1
1	3	$-\beta$
2	3	-1
3	3	not admissible

2.3.7 Subroutine UNST7 Solves Equation {10.2-22}

$$\overbrace{[(i-2)^2 + \varepsilon_i \beta]}^{7} \frac{\Omega}{\omega_1} = \bar{\omega}^m_{jk}$$

where

$m = 1,2,3$

$j = 1,2,\ldots,M_1$

$k = 1,2,\ldots,N_1$

$i = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} & \text{if } i=1 \\ \frac{1}{2} & \text{if } i=2 \\ \frac{1}{2} & \text{if } i=3 \end{cases}$$

m	j	$\bar{\omega}^m_{jk}$	k	i	7
1	1	$\bar{\omega}^1_{1k}$	$k = 1,2,\ldots,N_1$	1	$1 - \frac{1}{2}\beta$
2	1	$\bar{\omega}^2_{1k}$	.	2	$\frac{1}{2}\beta$
3	1	$\bar{\omega}^3_{1k}$	.	3	$1 + \frac{1}{2}\beta$
1	2	$\bar{\omega}^1_{2k}$	.		
2	2	$\bar{\omega}^2_{2k}$	.		
3	2	$\bar{\omega}^3_{2k}$	.		
1	3	$\bar{\omega}^1_{3k}$	.		
2	3	$\bar{\omega}^2_{3k}$	.		
3	3	$\bar{\omega}^3_{3k}$	.		
.	.	.	.		
.	.	.	.		
.	.	.	.		
1	M_1	$\bar{\omega}^1_{M_1k}$	.		
2	M_1	$\bar{\omega}^2_{M_1k}$	.		
3	M_1	$\bar{\omega}^3_{M_1k}$	$k = 1,2,\ldots,N_1$		

2.3.8 Subroutine UNST8 Solves Equation {10.2-25}

$$\overbrace{\left|(i-2)^2 + 1 + \varepsilon_i \beta\right|}^{8} \frac{\Omega}{\omega_1} = \bar{\omega}^m_{jk}$$

where

$m = 1,2,3$

$j = 1,2,\ldots,M_1$

$k = 1,2,\ldots,N_1$

$i = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} & \text{if } i=1 \\ \frac{1}{2} & \text{if } i=2 \\ \frac{1}{2} & \text{if } i=3 \end{cases}$$

m	j	$\bar{\omega}^m_{jk}$	k	i	8
1	1	$\bar{\omega}^1_{1k}$	$k = 1,2,\ldots,N_1$	1	$2 - \frac{1}{2}\beta$
2	1	$\bar{\omega}^2_{1k}$	.	2	$1 + \frac{1}{2}\beta$
3	1	$\bar{\omega}^3_{1k}$	.	3	$2 + \frac{1}{2}\beta$
1	2	$\bar{\omega}^1_{2k}$	.		
2	2	$\bar{\omega}^2_{2k}$	.		
3	2	$\bar{\omega}^3_{2k}$	.		
1	3	$\bar{\omega}^1_{3k}$	.		
2	3	$\bar{\omega}^2_{3k}$	.		
3	3	$\bar{\omega}^3_{3k}$	.		
.	.	.	.		
.	.	.	.		
.	.	.	.		
1	M_1	$\bar{\omega}^1_{M_1k}$	.		
2	M_1	$\bar{\omega}^2_{M_1k}$	.		
3	M_1	$\bar{\omega}^3_{M_1k}$	$k = 1,2,\ldots,N_1$		

2.3.9 Subroutine UNST9 Solves Equation {10.2-28}

$$\overbrace{\left|(i-2)^2 - 1 + \varepsilon_i\beta\right|}^{9} \frac{\Omega}{\omega_1} = \bar{\omega}^m_{jk}$$

where

$m = 1,2,3$

$j = 1,2,\ldots,M_1$

$k = 1,2,\ldots,N_1$

$i = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} \text{ if } i=1 \\ \frac{1}{2} \text{ if } i=2 \\ \frac{1}{2} \text{ if } i=3 \end{cases}$$

m	j	$\bar{\omega}^m_{jk}$	k	i	9
1	1	$\bar{\omega}^1_{1k}$	$k = 1,2,\ldots,N_1$	1	$-\frac{1}{2}\beta$
2	1	$\bar{\omega}^2_{1k}$	.	2	$-1 + \frac{1}{2}\beta$
3	1	$\bar{\omega}^3_{1k}$	.	3	$\frac{1}{2}\beta$
1	2	$\bar{\omega}^1_{2k}$	.		
2	2	$\bar{\omega}^2_{2k}$	.		
3	2	$\bar{\omega}^3_{2k}$	.		
1	3	$\bar{\omega}^1_{3k}$	.		
2	3	$\bar{\omega}^2_{3k}$	.		
3	3	$\bar{\omega}^3_{3k}$	.		
.	.	.	.		
.	.	.	.		
.	.	.	.		
1	M_1	$\bar{\omega}^1_{M_1 0}$	.		
2	M_1	$\bar{\omega}^2_{M_1 0}$	.		
3	M_1	$\bar{\omega}^3_{M_1 0}$	$k = 1,2,\ldots,N_1$		

2.3.10 Subroutine UNST10 Solves Equation {10.2-31}

$$\overbrace{\left|(i-2)^2 + (p-2)^2 + (\varepsilon_i+\varepsilon_p)\beta\right|}^{⑩} \frac{\Omega}{\omega_1} = \bar{\omega}^m_{jk}$$

where

$m = 1,2,3$

$j = 1,2,3,\ldots,M_1$

$k = 1,2,3,\ldots,N_1$

$i = 1,2,3 \qquad i \neq p$

$p = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} \text{ if } i=1 \\ \frac{1}{2} \text{ if } i=2 \\ \frac{1}{2} \text{ if } i=3 \end{cases} \qquad \varepsilon_p = \begin{cases} -\frac{1}{2} \text{ if } p=1 \\ \frac{1}{2} \text{ if } p=2 \\ \frac{1}{2} \text{ if } p=3 \end{cases}$$

i	j	$\bar{\omega}^m_{jk}$	k	i	p	⑩
1	1	$\bar{\omega}^1_{1k}$	$k = 1,2,\ldots,N_1$	1	1	not admissible
2	1	$\bar{\omega}^2_{1k}$	.	2	1	1
3	1	$\bar{\omega}^3_{1k}$	.	3	1	2
1	2	$\bar{\omega}^1_{2k}$	.	1	2	1
2	2	$\bar{\omega}^2_{2k}$	.	2	2	not admissible
3	2	$\bar{\omega}^3_{2k}$	.	3	2	$1 + \beta$
1	3	$\bar{\omega}^1_{3k}$	.	1	3	2
2	3	$\bar{\omega}^2_{3k}$	.	2	3	$1 + \beta$
3	2	$\bar{\omega}^3_{2k}$	.	3	3	not admissible
.	.	.	.			
.	.	.	.			
.	.	.	.			
1	M_1	$\bar{\omega}^1_{M_1k}$	.			
2	M_1	$\bar{\omega}^2_{M_1k}$	.			
3	M_1	$\bar{\omega}^3_{M_1k}$	$k = 1,2,\ldots,N_1$			

2.3.11 Subroutine UNST11 Solves Equation {10.2-34}

$$\overbrace{\left|(i-2)^2-(p-2)^2+(\varepsilon_i-\varepsilon_p)\beta\right|}^{11}\frac{\Omega}{\omega_1}=\bar{\omega}^m_{jk}$$

where

$m = 1,2,3$

$j = 1,2,\ldots,M_1$

$k = 1,2,\ldots,N_1$

$i = 1,2,3 \qquad i \neq p$

$p = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} \text{ if } i=1 \\ \frac{1}{2} \text{ if } i=2 \\ \frac{1}{2} \text{ if } i=3 \end{cases} \qquad \varepsilon_p = \begin{cases} -\frac{1}{2} \text{ if } p=1 \\ \frac{1}{2} \text{ if } p=2 \\ \frac{1}{3} \text{ if } p=3 \end{cases}$$

m	j	$\bar{\omega}^m_{jk}$	k	i	p	11
1	1	$\bar{\omega}^1_{1k}$	$k = 1,2,\ldots,N_1$	1	1	not admissible
2	1	$\bar{\omega}^2_{1k}$	.	2	1	$-1+\beta$
3	1	$\bar{\omega}^3_{1k}$	.	3	1	β
1	2	$\bar{\omega}^1_{2k}$	.	1	2	$1-\beta$
2	2	$\bar{\omega}^2_{2k}$	.	2	2	not admissible
3	2	$\bar{\omega}^3_{2k}$	.	3	2	1
1	3	$\bar{\omega}^1_{3k}$	.	1	3	$-\beta$
2	3	$\bar{\omega}^2_{3k}$	.	2	3	$-\beta$
3	3	$\bar{\omega}^3_{3k}$	.	3	3	not admissible
.	.	.	.			
.	.	.	.			
.	.	.	.			
1	M_1	$\bar{\omega}^1_{M_1k}$	.			
2	M_1	$\bar{\omega}^2_{M_1k}$	.			
3	M_1	$\bar{\omega}^3_{M_1k}$	$k = 1,2,\ldots,N_1$			

2.3.12 Subroutine UNST12 Solves Equation {10.2-37}

$$\overbrace{2[(i-2)^2 + \varepsilon_i \beta]}^{12} \frac{\Omega}{\omega_1} = \bar{\omega}^m_{jk}$$

where

$m = 1,2,3$

$j = 1,2,\ldots,M_1$

$k = 1,2,\ldots,N_1$

$i = 1,2,3$

$$\varepsilon_i = \begin{cases} -\frac{1}{2} \text{ if } i=1 \\ \frac{1}{2} \text{ if } i=2 \\ \frac{1}{2} \text{ if } i=3 \end{cases}$$

m	j	$\bar{\omega}^m_{jk}$	k	i	12
1	1	$\bar{\omega}^1_{1k}$	$k = 1,2,\ldots,N_1$	1	$2 - \beta$
2	1	$\bar{\omega}^2_{1k}$	.	2	β
3	1	$\bar{\omega}^3_{1k}$	.	3	$2 + \beta$
1	2	$\bar{\omega}^1_{2k}$	.		
2	2	$\bar{\omega}^2_{2k}$	.		
3	2	$\bar{\omega}^3_{2k}$	.		
1	3	$\bar{\omega}^1_{3k}$	.		
2	3	$\bar{\omega}^2_{3k}$	.		
3	3	$\bar{\omega}^3_{3k}$	.		
.	.	.	.		
.	.	.	.		
.	.	.	.		
1	M_1	$\bar{\omega}^1_{M_1k}$	.		
2	M_1	$\bar{\omega}^2_{M_1k}$	.		
3	M_1	$\bar{\omega}^3_{M_1k}$	$k = 1,2,\ldots,N_1$		

2.3.13 Subroutine UNST13 Solves Equation {10.2-39}

$$\bar{\Omega} = \frac{\bar{\omega}^{m}_{jk}}{|(\alpha_r+s)|} = \frac{\Omega}{\omega_1}$$

where

$j = 1,2,\ldots,M_1$

$k = 1,2,\ldots,N_1$

$m = 1,2,3$

$r = 1,2,\ldots,R$

$s = -S$ to $+S$

m	j	r	s	$\bar{\omega}^{m}_{jk}$	k
1	1	1	-S	$\bar{\omega}^{1}_{1k}$	$k = 1,2,\ldots,N_1$
1	1	1	-S+1	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1	1	1	1	.	.
1	1	1	0	.	.
1	1	1	1	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1	1	1	S-1	.	.
1	1	1	S	.	.
1	1	2	-S	.	.
1	1	2	-S+1	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

m	i	r	s	$\bar{\omega}^m_{ik}$	k
1	1	R	S-1	.	.
1	1	R	S	.	.
2	1	1	-S	$\bar{\omega}^2_{1k}$	.
2	1	1	-S+1	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
2	1	R	S-1	.	.
2	1	R	S	.	.
3	1	1	-S	$\bar{\omega}^3_{1k}$	.
3	1	1	-S+1	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
3	1	R	S-1	.	.
3	1	R	S	.	.
1	2	1	-S	$\bar{\omega}^1_{2k}$	.
1	2	1	-S+1	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
3	M_1	R	S-1	$\bar{\omega}^3_{M_1k}$	.
3	M_1	R	S	$\bar{\omega}^3_{M_1k}$	$k = 1,2,\ldots,N_1$

2.4 Modifications of the Stability Equations

The stability equations as they are presented in Section 2.3 have been rendered dimensionless by the factor ω_1 (see Section 4.1). But the dimensioned unstable value Ω ($\Omega = \bar{\Omega}\omega_1$) of the thrust frequency is the desired quantity. This quantity is obtained in the FORTRAN IV computer program by using ω rather than $\bar{\omega}$ in the stability equations. This true natural frequency ω is calculated in the subroutine NAFREQ.

3.0 OUTLINE OF FORTRAN IV COMPUTER PROGRAM

3.1 General

The following FORTRAN IV Computer Program has been developed to calculate the natural frequencies and the unstable values of thrust frequency from data consisting of the parameters, H, r, t, ρ, E, ν, M_1, N_1, T_o, γ, Ω, and K. The manner in which these data words are read into the computer is presented under FORTRAN IV COMPUTER PROGRAM INPUT AND OUTPUT INFORMATION of this report.

The computer program consists of the following parts:

3.1.1 Main Program - FORTRAN IV Computer Program for the Evaluation of Natural Frequencies (OMG) and Unstable Values of the Thrust Frequency (COMG) for a Free-Free, Circular Cylindrical Shell.

3.1.2 Subroutine NAFREQ - Subroutine for the Evaluation of Natural Frequencies (OMG). Subroutine calculates 10-values of OMG(0,K,I), 20-values of OMG(J,0,I) and 300-values of OMG(J,K,I).

3.1.3 Subroutine UNST1 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-6}. Consideration of three (3) cases, involving 30 values.

3.1.4 Subroutine UNST2 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-8}. Consideration of thirty-three (33) cases, involving 363 values.

3.1.5 Subroutine UNST3 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-10}. Consideration of one (1) case, involving 20 values.

3.1.6 Subroutine UNST4 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-13}. Consideration of three (3) cases, involving 60 values.

3.1.7 Subroutine UNST5 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-16}. Consideration of nine (9) cases (3 inadmissible), involving 120 values.

3.1.8 Subroutine UNST6 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-19}. Consideration of nine (9) cases (3 inadmissible), involving 120 values.

3.1.9 Subroutine UNST7 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-22}. Consideration of three (3) cases, involving 900 values.

3.1.10 Subroutine UNST8 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-25}. Consideration of three (3) cases, involving 900 values.

3.1.11 Subroutine UNST9 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-28}. Consideration of three (3) cases, involving 900 values.

3.1.12 Subroutine UNST10 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-31}. Consideration of nine (9) cases (3 inadmissible), involving 1800 values.

3.1.13 Subroutine UNST11 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-34}. Consideration of nine (9) cases (3 inadmissible), involving 1800 values.

3.1.14 Subroutine UNST12 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-37}. Consideration of three (3) cases, involving 900 values.

3.1.15 Subroutine UNST13 - Subroutine for Unstable Values of Thrust Frequency, COMG, Equation {10.2-39}. Consideration of thirty-three (33) cases, involving 9900 values.

3.2 FORTRAN IV Legend for Input - Output Data

The FORTRAN IV designation for input data is as follows:

Input - Output Parameter	FORTRAN IV Name
H, half length	H
r, radius	R
t, thickness	T
ρ, mass density	RHO
E, modulus	E
ν, Poisson's ratio	V
ω_1, reference frequency	W
M_1, axial modal limit	M
N_1, circumferential modal limit	N
T_o, constant thrust	TO
γ, ratio, variable to constant thrust	GAM

Input - Output Parameter	FORTRAN IV Name
Ω, thrust frequency	COMG
K, control constant	CK
ω, natural frequency	OMG

Full details on the manner in which these data are read into the computer is presented under FORTRAN IV Computer Program Input and Output Information.

3.3 FORTRAN IV Computer Program Input and Output Information

3.3.1 Input

The input data which must be punched on cards are the terms H, R, T, RHO, E, V, M, N, TO, GAM, CMG, CK. These data are punched on two cards.

The first data card will contain H in spaces 1 through 10, R in spaces 11 through 20, T in spaces 21 through 30, RHO in spaces 31 through 40, E in spaces 41 through 50, and V in spaces 51 through 60.

The second data card will contain M in spaces 1 through 5, N in spaces 6 through 10, TO in spaces 11 through 20, GAM in spaces 21 through 30, COMG in spaces 31 through 40, and CK in spaces 51 through 60.

The READ STATEMENTS for the first and second data card will be as follows:

```
2     READ(5,102) H, R, T, RHO, E, V,
102   FORMAT(6F10.5)

3     READ(5,103) M, N, TO, GAM, COMG, CK
103   FORMAT (215, 4E10.5)
```

A definition of input data is given under FORTRAN IV Legend for Input - Output Data.

3.3.2 Output

The values of natural frequencies ω_{jk}^{m} [OMG(J,K,I)] are printed in floating point numbers except the values of J and K which are integer values.

The unstable values of the thrust frequency Ω_{jk}^{m} [COMG(J,K,I)] are printed in floating point numbers except the values of J and K which are integer values. A descriptive title, properly identifying the equation number and the particular case, is printed before each section of the program printout.

4.0 MODIFICATIONS OF FORTRAN IV COMPUTER PROGRAM FOR A SIMPLY-SUPPORTED CYLINDER

The program presented in Reference 3 has been modified to solve for the natural frequencies and for unstable values of the thrust frequency for the eleven stability equations that are identical between the simply-supported analysis (Ref. 2) and the free-free analysis (Ref. 1). These modifications were as follows.

4.1 Modifications of the Main Program

In the main program, subroutine NAFDEQ, the value of a parameter, with a FORTRAN IV name of W (Ref 3., Section 3.2), was read into the routine from a data card. This parameter is ω the frequency by which the stability equations are divided to obtain a dimensionless equation. For the program presented in Reference 3 an arbitrary value of 100 was selected for W. In the free-free analysis of Reference 1, ω is no longer arbitrary but now $\omega = \omega_1$ where ω_1 is the frequency of the first bending mode of a uniform beam with the same length and mass as the cylinder under consideration. A calculation of W is now performed in NAFDEQ using the relation

$$\omega_n = \left(\frac{\lambda_n}{2L}\right)\sqrt{\frac{EI}{m}} \qquad (15) \quad \{C\text{-}12\}$$

4.2 Modifications of the Subroutines

The order and names of the various subroutines that solve for unstable values of the thrust frequency were changed to conform to the order that these same eleven equations appeared in Reference 1. These changes were:

UNST1 remained UNST1

UNST2 became UNST3

UNST3 became UNST6

UNST4 became UNST5

UNST5 became UNST4

UNST6 became UNST7

UNST7 became UNST9

UNST8 remained UNST8

UNST9 became UNST11

UNST10 remained UNST10

UNST11 became UNST12

5.0 MODIFICATIONS OF A FORTRAN IV COMPUTER PROGRAM FOR A FREE-FREE BEAM

A program presented in Reference 4 has been modified to calculate the beam parameter α_r appearing in equations {10.2-8} and {10.2-39}.

5.1 Modifications of the Main Program

The name of the Main Program was changed from DRIVER to BEAM. All of the necessary data that was read into DRIVER from data cards is inserted into BEAM via defining statements. The only exception to this is the value of T_o (the FORTRAN IV name is TO), the constant thrust force. TO is calculated in BEAM and the necessary data for the calculation is inserted into BEAM via the argument of the CALL and SUBROUTINE statements of BEAM. All COMMON statements have been removed from the program presented in Reference 4, and data is now transmitted between subroutines via the arguments of the CALL and SUBROUTINE statements. All write statements appearing in DRIVER have been removed from BEAM. The natural frequencies of the beam are calculated by obtaining the eigenvalues of a matrix. A new parameter name, OMG2, is inserted into BEAM. This parameter is defined as equal to the second natural bending frequency of the beam.

5.2 Modifications of the Subroutines

The parameter α_r is related to another parameter z by the relation

$$z = \cos 2\pi\alpha \qquad (16) \qquad [3.17b]$$

The FORTRAN IV name of z is ROOTR. Three values of ROOTR, corresponding to the first and second bending modes, and the rotational mode, are calculated in the subroutine STAB using another subroutine MULLER. The program presented in Reference 4, has been modified in STAB to check for multiplicity of the three values of ROOTR derived by MULLER because the accuracy of the values from MULLER is at best only four significant digits if a multiplicity exists. If as a result of the check a multiplicity is found to exist then the values of ROOTR derived by MULLER are used as initial guesses in an additional, Newton-Raphson iteration subroutine called ROOTS that has been programmed to derive ROOTR to better than seven significant digits.

Consider the change of variables in Equation [3.17b] from

$$z = \cos 2\pi\alpha \qquad (17) \qquad [3.17b]$$

to

$$z = \cos\beta \qquad (18)$$

where now

$$\beta = 2\pi\alpha \ . \qquad (19)$$

As was established in Reference 4 the conditions of $z \leq 1$ must be satisfied or there is a beam instability. With this restriction on z, β can now be expressed as

$$\beta = \tan^{-1} \left(\frac{1-z^2}{z}\right) \qquad (20)$$

STAB has been modified to calculate three values of β, whose FORTRAN IV name is BETT, for the three values of ROOTR.

A check is performed on the three values of ROOTR to determine if one of them is negative. The variable ME2 is assigned a value of zero if the check is negative and a value of one if the check is positive.

6.0 ADDITIONAL SUBROUTINES

The two additional stability equations {10.2-8} and {10.2-3} that contain the beam parameters α_r and s, and whose origin was traced in Section 2.2, require four subroutines to be added to the program presented in Reference 3 modified as in Section 4.0 in addition to the modified beam program presented in Section 5.0. These four additional subroutines are as follows.

6.1 Subroutine UNST2

This subroutine considers thirty cases involving three hundred sixty-three values. UNST2 is designed to use when necessary the iteration technique described in Appendix A, Section II of Reference 3 to calculate for unstable values of the thrust frequency.

When $\bar{\Omega}$ is greater than 100 the value of α_r is assumed to be zero since z approaches asymptotically the value of one in the region $\Omega > 2\omega_2$ (see Section 7.1). Therefore when Ω is greater than 100 UNST2 calculates Ω using the formula

$$\Omega = \frac{\omega_{0k}}{|s|} \tag{21}$$

When Ω is less than 100 the value of α_r corresponding to a given Ω is calculated and the iteration procedure described in Reference 3, Appendix AII is used to derive a value of Ω that will satisfy {10.2-8}. By assigning a value other than zero to the parameter ME1 after the fifth iteration all subsequent values of ROOTR calculated in STAB (see Section 5.2) using MULLER will be improved upon by using ROOTS also.

The values of Ω are printed out, one at a time, in a write loop after all of the values have been calculated.

6.2 Subroutine UNST13

This subroutine considers thirty cases involving nine thousand nine hundred values. UNST13 is similar to UNST2 except that it calculates unstable value of the thrust frequency for equation {10.2-38}. The write loop of UNST13 is designed to print out all the values of $\bar{\Omega}$ three at a time. The three values being the three values of the subscripts J, K, R, and S. The subscript I is the FORTRAN IV name for the superscript m in equation

$$\bar{\Omega} = \frac{\bar{\omega}^{m}_{jk}}{|\alpha_r + s|} \qquad (9) \qquad \{10.2\text{-}39\}$$

6.3 Subroutine SORT

This subroutine is called in both UNST2 and UNST13. Its purpose is to take the values of BETT calculated in STAB (see Section 5.2) and sort these values according to relative magnitude and calculate α_r from the relation

$$\alpha_r = \beta/2\pi \qquad (22)$$

(see Section 5.2). The method of sorting depends upon the value assigned to ME2 in subroutine STAB (see Section 5.2).

6.4 Subroutine ROOTS

Increased accuracy was found to be necessary in the computing of a variable named ROOTR in the beam program (see Section 5.2). To get this desired additional accuracy the Newton-Raphson iteration subroutine ROOTS is used.

7.0 CONSTRAINTS OF THE FORTRAN IV COMPUTER PROGRAM

7.1 Origin of Limiting Value of the Thrust Frequency

In the analysis of Reference 5, and as presented in Reference 6, p220 and p228, the regions of instability occur when

$$\Omega = \frac{\omega_i \pm \omega_j}{k} \tag{23}$$

and

$$\Omega = \frac{2\omega_i}{k} \tag{24}$$

$$k = 1,2,3$$

where ω is the natural frequency of a beam and Ω is the thrust frequency. For a beam the natural frequencies are ordered

$$\omega_B < \omega_1 < \omega_2 \tag{25}$$

where

ω_B is the natural frequency of the rotational mode

ω_1 is the natural frequency of the first bending mode

ω_2 is the natural frequency of the second bending mode

Necessarily then

$$2\omega_B < 2\omega_1 < 2\omega_2 \tag{26}$$

therefore since

$$\Omega = \frac{2\omega_i}{k} \qquad \text{(Ref. 6)} \tag{24}$$

$$k = 1,2,3,\ldots$$

the value

$$\Omega = 2\omega_2$$

is the largest value of Ω for a beam instability considering the three modes ω_β, ω_1, and ω_2.

If

$$\Omega < 2\omega_2 \tag{27}$$

since by

$$\omega_B < \omega_1 < \omega_2 \tag{25}$$

$$\omega_1 < \omega_2 < 2\omega_2 \tag{28}$$

The relative order of the three values of β calculated in STAB (see Section 5.2) will remain the same.

7.2 The Constraints of Subroutines UNST2 and UNST13

The value of ω_2 is calculated in BEAM (see Section 5.1). In light of the limit imposed upon Ω in section 7.1 a test is made upon Ω to determine if

$$\Omega > 2\omega_2 \tag{27}$$

If the inequality is satisfied the iteration to determine Ω continues. If the inequality is not satisfied the iteration is terminated and Ω is assigned the value

$$\text{COMG(JKIS)} = .11111111 \tag{29}$$

and the next case is then considered. If the iteration procedure cannot satisfy the stability equation after fifteen iterations, $\bar{\Omega}$ is assigned the value

$$\text{COMG(JKIS)} = .22222222 \tag{30}$$

and the next case is then considered.

If in the iteration procedure the inequality

$$\bar{\Omega} > 1 \times 10^6$$

is satisfied, the value of $\bar{\Omega}$ that caused the inequality to be satisfied is stored in the COMG(JKIS) array and the next case is considered.

If the value of $\bar{\omega}_{0k}$ or $\bar{\omega}^m_{jk}$ is such that the formula

$$\bar{\Omega} = \frac{\bar{\omega}_{0k}}{|s|} \tag{31}$$

or

$$\bar{\Omega} = \frac{\bar{\omega}^m_{jk}}{|s|} \tag{32}$$

is being used (see Section 6.1) and if also $s = 0$. Then Ω is assigned the value

$$\text{COMG(JKIS)} = 10000000 \tag{33}$$

A test is made on each value of Ω before it is printed out. If the inequality

$$\Omega < 1 \times 10^6$$

is satisfied the value of Ω that satisfied the inequality is printed out. If the inequality is not satisfied the letters INFIN are printed out.

7.3 The Remaining Constraints

The eleven subroutines that solve the eleven stability equations that are the same in the simply-supported analysis (Ref. 2) and the free-free analysis (Ref. 1) are the same as the eleven subroutines that solved these equations in Reference 3. Only the order and names of these subroutines have been changed from the programs presented in Reference 3. These changes are outlined in Section 4.2. The constraints of these eleven subroutines are given in Reference 3, Appendix A, Section I.

8.0 FORTRAN IV SOURCE PROGRAM

8.1 General Statement

The program which defines the operations which the computer is to do and which is written by the programmer in the FORTRAN IV language is called the FORTRAN IV Source Program. With this in mind, and with the understanding of the programs presented in Reference 3 and Reference 4, the reader with the aid of the preceeding sections should have little difficulty in understanding the following Program Listing.

This program has been written to use either chain or overlay techniques. These two methods result in differences in the main subroutine NAFDEQ and in the MAP used. Duplicate listings of both NAFDEQ and MAP, one listing each for overlay and chain, are given in Section 8.2.

8.2 Program Listing

```
$I FOR NAFDEQ
      COMMON/DOG/L,S,U,V,W,PI,M,N,OMG(20,20,3),R,H,TO,CK,RI,GAM
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     FORTRAN 1V COMPUTER PROGRAM FOR THE EVALUATION OF NATURAL
C     FREQUENCIES (OMG) AND UNSTABLE VALUES OF THE THRUST FREQUENCY
C     (COMG) FOR A SIMPLY-SUPPORTED, CIRCULAR CYLINDRICAL SHELL.
      REAL L
1     WRITE (6,100)
100   FORMAT(1H1,20X,31HLEGEND OF TERMS USED IN PROGRAM,//,5X,30HH ----
     1HALF LENGTH OF CYLINDER,/,5X,25HR ---- RADIUS OF CYLINDER,/,5X,33H
     2T       THICKNESS OF CYLINDER WALL,/,5X,40HRHO -- MASS DENSITY OF C
     3YLINDER MATERIAL,/,5X,38HE ---- YOUNGS MODULUS FOR THE MATERIAL,/,
     45X,21HV ---- POISSONS RATIO,/,5X,26HW ---- REFERENCE FREQUENCY,/,5
     5X,38HM ---- NUMBER OF AXIAL VARIATION TERMS,/,5X,39HN ---- NUMBER
     60F RADIAL VARIATION TERMS,/,5X,30HJ,K -- SUBSCRIPTS OF THE ARRAY,/
     7,5X,34HOMG(J,K,1) --- NATURAL FREQUENCY 1,/,5X,34HOMG(J,K,2) --- N
     8ATURAL FREQUENCY 2,/,5X,34HOMG(J,K,3) --- NATURAL FREQUENCY 3,/,5X
     9,28HTO --- CONSTANT THRUST FORCE,/,5X,38HGAM -- VARIABLE THRUST/CO
     1NSTANT THRUST,/,5X,48HCOMG -- CAPITAL OMEGA, INSTABLE SEARCH FREQU
     2ENCY,/,5X,39HDCOMG -- DELTA CAPITAL OMEGA, INCREMENT,/,5X,20HCK--C
     3ONTROL CONSTANT)
2     READ (5,102) H,R,T,RHO,E,V
102   FORMAT (6F10.5)
3     READ (5,103) M,N,TO,GAM,COMG,DCOMG,CK
103   FORMAT (2I5,5E10.5)
      W=22.373286*SQRT(E*R**2.0/(2.0*RHO))/(4.0*H**2.0)
4     WRITE (6,104) H,R,T,RHO,W,E,V,M,N,TO,GAM,COMG,DCOMG,CK
104   FORMAT(1H1,10X,62HUNSTABLE VALUES OF THE THRUST FREQUENCY(COMG) FO
     1R THE CYLINDER,//,3X,14HHALF LENGTH = ,3PE15.5,10X,9HRADIUS = ,3PE
     215.5,10X,12HTHICKNESS = ,1PE15.5,//,3X,15HMASS DENSITY = ,E15.8,9X
     3,22HREFERENCE FREQUENCY = ,E15.5,//,3X,17HYOUNGS MODULUS = ,1PE15.
     45,7X,17HPOISSONS RATIO = ,E15.5,//,3X,34HNUMBER OF AXIAL VARIATION
     5 TERMS = ,I5,10X,35HNUMBER OF RADIAL VARIATION TERMS = ,I5,//,3X,2
     62HCONSTANT THRUST, TO = ,1PE15.5,3X,6HGAM = ,E15.5,3X,21HCAPITAL O
     7MEGA,COMG = ,E15.5,//,3X,28HDELTA CAPITAL OMEGA,DCOMG = ,E15.5,3X,
     824HCONTROL CONSTANT K,CK = ,E15.5,//)
      PI=3.141592654
      L=H/R
      S=T/H
      U=RHO*H*H*W*W/E
      CALL NAFREQ (L,S,U,V,W,PI,M,N,OMG)
5     WRITE (6,105)
105   FORMAT (20X,41HTABLE OF NATURAL FREQUENCIES, RAD PER SEC,//,4X,2HJ
     1,,4X,2HK,,8X,9HOMEGA (1),11X,9HOMEGA (2),10X,9HOMEGA (3),//)
106   FORMAT (2I5,3E20.8//)
      DO 6 J=1,M
      DO 6 K=1,N
      J1=J-1
      K1=K-1
6     WRITE (6,106) J1,K1,(OMG(J,K,I),I=1,3)
      RM=2.*H*2.*PI*R*T*RHO
      RI=RM*(.5*R*R+.333*H*H)*386.
      CALL UNST1 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST2 (PI,R,H,TO,OMG,M,N,E,T)
      CALL UNST3 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
```

```
CALL UNST4 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST5 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST6 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST7 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST8 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST9 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST10(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST11(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST12(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
CALL UNST13(PI,R,H,TO,OMG,M,N,E,T)
GO TO 2
END
```

```
$I FOR NAFDEQ
      COMMON/DOG/L,S,U,V,W,PI,M,N,OMG(20,20,3),R,H,TO,CK,RI,GAM
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     FORTRAN 1V COMPUTER PROGRAM FOR THE EVALUATION OF NATURAL
C     FREQUENCIES (OMG) AND UNSTABLE VALUES OF THE THRUST FREQUENCY
C     (COMG) FOR A SIMPLY-SUPPORTED, CIRCULAR CYLINDRICAL SHELL.
      REAL L
1     WRITE (6,100)
100   FORMAT(1H1,20X,31HLEGEND OF TERMS USED IN PROGRAM,//,5X,30HH ----
     1HALF LENGTH OF CYLINDER,/,5X,25HR ---- RADIUS OF CYLINDER,/,5X,33H
     2T       THICKNESS OF CYLINDER WALL,/,5X,40HRHO -- MASS DENSITY OF C
     3YLINDER MATERIAL,/,5X,38HE ---- YOUNGS MODULUS FOR THE MATERIAL,/,
     45X,21HV ---- POISSONS RATIO,/,5X,26HW ---- REFERENCE FREQUENCY,/,5
     5X,38HM ---- NUMBER OF AXIAL VARIATION TERMS,/,5X,39HN ---- NUMBER
     60F RADIAL VARIATION TERMS,/,5X,30HJ,K -- SUBSCRIPTS OF THE ARRAY,/
     7,5X,34HOMG(J,K,1) --- NATURAL FREQUENCY 1,/,5X,34HOMG(J,K,2) --- N
     8ATURAL FREQUENCY 2,/,5X,34HOMG(J,K,3) --- NATURAL FREQUENCY 3,/,5X
     9,28HTO --- CONSTANT THRUST FORCE,/,5X,38HGAM -- VARIABLE THRUST/CO
     1NSTANT THRUST,/,5X,48HCOMG -- CAPITAL OMEGA, INSTABLE SEARCH FREQU
     2ENCY,/,5X,39HDCOMG -- DELTA CAPITAL OMEGA, INCREMENT,/,5X,20HCK--C
     30NTROL CONSTANT)
2     READ (5,102) H,R,T,RHO,E,V
102   FORMAT (6F10.5)
3     READ (5,103) M,N,TO,GAM,COMG,DCOMG,CK
103   FORMAT (2I5,5E10.5)
      W=22.373286*SQRT(E*R**2.0/(2.0*RHO))/(4.0*H**2.0)
4     WRITE (6,104) H,R,T,RHO,W,E,V,M,N,TO,GAM,COMG,DCOMG,CK
104   FORMAT(1H1,10X,62HUNSTABLE VALUES OF THE THRUST FREQUENCY(COMG) F0
     1R THE CYLINDER,//,3X,14HHALF LENGTH = ,3PE15.5,10X,9HRADIUS = ,3PE
     215.5,10X,12HTHICKNESS = ,1PE15.5,//,3X,15HMASS DENSITY = ,E15.8,9X
     3,22HREFERENCE FREQUENCY = ,E15.5,//,3X,17HYOUNGS MODULUS = ,1PE15.
     45,7X,17HPOISSONS RATIO = ,E15.5,//,3X,34HNUMBER OF AXIAL VARIATION
     5 TERMS = ,I5,10X,35HNUMBER OF RADIAL VARIATION TERMS = ,I5,//,3X,2
     62HCONSTANT THRUST, TO = ,1PE15.5,3X,6HGAM = ,E15.5,3X,21HCAPITAL O
     7MEGA,COMG = ,E15.5,//,3X,28HDELTA CAPITAL OMEGA,DCOMG = ,E15.5,3X,
     824HCONTROL CONSTANT K,CK = ,E15.5,//)
      PI=3.141592654
      L=H/R
      S=T/H
      U=RHO*H*H*W*W/E
      CALL NAFREQ (L,S,U,V,W,PI,M,N,OMG)
5     WRITE (6,105)
105   FORMAT (20X,41HTABLE OF NATURAL FREQUENCIES, RAD PER SEC,//,4X,2HJ
     1,,4X,2HK,,8X,9HOMEGA (1),11X,9HOMEGA (2),10X,9HOMEGA (3),//)
106   FORMAT (2I5,3E20.8//)
      DO 6 J=1,M
      DO 6 K=1,N
      J1=J-1
      K1=K-1
6     WRITE (6,106) J1,K1,(OMG(J,K,I),I=1,3)
      RM=2.*H*2.*PI*R*T*RHO
      RI=RM*(.5*R*R+.333*H*H)*386.
      CALL CHAIN (2)
      END
```

```
$I   FOR NAFREQ
C      NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C      SUBROUTINE  FOR THE EVALUATION OF NATURAL FREQUENCIES(OMG).
C      SUBROUTINE CALCULATES 10 VALUES OF OMG(O,K,I), 20 VALUES OF
C      OMG(J,O,I), AND 300 VALUES OF OMG(J,K,I)
       SUBROUTINE NAFREQ (L,S,U,V,W,PI,M,N,OMG)
       DIMENSION A(3,3),ROOTR(3),ROOTI(3),OMG(20,20,3)
       REAL L
       U1=U*(1.-V*V)
       DO 1 J=1,M
       DO 1 K=1,N
       DO 1 I=1,3
 1     OMG(J,K,I)=0.0
       DO 2 J=1,M
       DO 2 K=1,N
       J1=J-1
       K1=K-1
       FJ=J1
       FK=K1
       IF(J-1)3,3,4
 3     IF(K-1)2,2,5
 5     OMG(J,K,1)=L*FK*W/SQRT(2.*U*(1.+V))
       GO TO 2
 4     IF(K-1)6,6,7
 6     A(1,1)=.25*(PI*FJ)**2/U1
       A(1,2)=.5*V*L*FJ*PI/U1
       A(2,1)=A(1,2)
       A(2,2)=(L*L+S*S*(FJ*PI)**4/192.)/U1
       CALL HESSEN(A,2)
       CALL QREIG(A,2,ROOTR,ROOTI,0)
       DO 8 I=1,2
       IF(ROOTI(I))9,10,9
 10    IF(ROOTR(I))9,11,11
 9     WRITE (6,100)J,K,I,ROOTR(I),ROOTI(I)
 100   FORMAT(23H0ROOT IS NOT ACCEPTABLE 3I5,2E15.8)
       GO TO 8
 11    OMG(J,K,I)=W*SQRT(ABS(ROOTR(I)))
 8     CONTINUE
       GO TO 2
 7     A(1,1)=.25*((PI*FJ)**2+2.*(1.-V)*(L*FK)**2)/U1
       A(1,2)=-.25*FJ*FK*PI*L*(1.+V)/U1
       A(1,3)=.5*V*L*PI*FJ/U1
       A(2,1)=A(1,2)
       A(2,2)=.125*((1.-V)*(FJ*PI)**2+8.*(L*FK)**2)/U1
       A(2,3) = -1.*L*L*FK/U1
       A(3,1)=A(1,3)
       A(3,2)=A(2,3)
       A(3,3)=(12.*L*L+S*S*((.5*FJ*PI)**2+L*L*FK*FK)**2)/(12.*U1)
       CALL HESSEN(A,3)
       CALL QREIG(A,3,ROOTR,ROOTI,0)
       DO 12 I=1,3
       IF(ROOTI(I))13,14,13
 14    IF(ROOTR(I))13,15,15
 13    WRITE (6,101)J,K,I,ROOTR(I),ROOTI(I)
 101   FORMAT(23H0ROOT IS NOT ACCEPTABLE,3I5,2E15.8)
```

```
      GO TO 12
15    OMG(J,K,I)=W*SQRT(ABS(ROOTR(I)))
12    CONTINUE
2     CONTINUE
      RETURN
      END
```

```
@I    FOR HESSEN
C      SUBROUTINE TO PUT MATRIX IN UPPER HESSENBERG FORM.
       SUBROUTINE HESSEN(A,M)
       DIMENSION A(3,3), B(2)
       DOUBLE PRECISION SUM
       IF (M - 2) 30,30,32
    32 DO 40 LC = 3,M
       N = M - LC + 3
       N1 = N - 1
       N2 = N - 2
       NI = N1
       DIV = ABS(A(N,N-1))
       DO 2 J = 1,N2
       IF(ABS(A(N,J))- DIV) 2,2,1
     1 NI = J
       DIV = ABS(A(N,J))
     2 CONTINUE
       IF(DIV) 3,40,3
     3 IF(NI - N1) 4, 7,4
     4 DO 5 J = 1,N
       DIV = A(J,NI)
       A(J,NI) = A(J,N1)
     5 A(J,N1) = DIV
       DO 6 J = 1,M
       DIV = A(NI,J)
       A(NI,J) = A(N1,J)
     6 A(N1,J) = DIV
     7 DO 26 K = 1, N1
    26 B(K) = A(N,K)/A(N,N-1)
       DO 45 J = 1,M
       SUM = 0.0
       IF (J - N1) 46,43,43
    46 IF(B(J))  41,43,41
    41 A(N,J) = 0.0
       DO 42 K = 1,N1
       A(K,J) = A(K,J) - A(K,N1)*B(J)
    42 SUM = SUM + A(K,J)*B(K)
       GO TO 45
    43 DO 44 K = 1,N1
    44 SUM = SUM + A(K,J)*B(K)
    45 A(N1,J) = SUM
    40 CONTINUE
    30 RETURN
        END
```

```
GI.   FOR QREIG
C        PROGRAM TO CALL QR TRANSFORMATION, MAXIMUM ITER IS 50.
      SUBROUTINE QREIG(A,M,ROOTR,ROOTI,IPRNT)
      DIMENSION A(3,3),ROOTR(3),ROOTI(3)
      N = M
      IF(IPRNT)  80,81,80
 80   WRITE (6,104)
 81   ZERO = 0.0
      JJ=1
 177  XNN=0.0
      XN2=0.0
      AA = 0.0
      B = 0.0
      C = 0.0
      DD = 0.0
      R=0.0
      SIG=0.0
      ITER = 0
 17   IF(N-2)     13,14,12
 13   IF(IPRNT)  82,83,82
 82   WRITE (6,105)A(1,1)
 83   ROOTR(1) = A(1,1)
      ROOTI(1) = 0.0
  1   RETURN
 14   JJ=-1
   12 X = (A(N-1,N-1) - A(N,N))**2
      S = 4.0*A(N,N-1)*A(N-1,N)
      ITER = ITER + 1
      IF(ABS(S/X).GT. 1.0E-8) GO TO 15
   16 IF(ABS(A(N-1,N-1))-ABS(A(N,N)))  32,32,31
   31 E = A(N-1,N-1)
      G = A(N,N)
      GO TO 33
   32 G = A(N-1,N-1)
      E = A(N,N)
   33 F = 0.
      H = 0.
      GO TO 24
   15 S = X + S
      X = A(N-1,N-1) + A(N,N)
      IF(S)     18,19,19
 19   SQ=SQRT(S)
      F=0.0
      H=0.0
      IF (X) 21,21,22
 21   E=(X-SQ)/2.0
      G=(X+SQ)/2.0
      GO TO 24
 22   G=(X-SQ)/2.0
      E=(X+SQ)/2.0
      GO TO 24
   18 F = SQRT(-S)/2.0
      E=X/2.0
      G=E
      H=-F
```

```
24    IF(JJ)  28,70,70
  70 D = 1.0E-10*(ABS(G) + F)
     IF(ABS(A(N-1,N-2)) .GT. D)  GO TO 26
28    IF(IPRNT)  84,85,84
84    WRITE (6,105)E,F, ITER
      WRITE (6,105)G,H
85    ROOTR(N) = E
      ROOTI(N) = F
      ROOTR(N-1) = G
      ROOTI(N-1) = H
      N=N-2
      IF(JJ)  1,177,177
  26 IF(ABS(A(N,N-1)) .GT. 1.0E-10*ABS(A(N,N)))  GO TO 50
29    IF(IPRNT)  86,87,86
86    WRITE (6,105)A(N,N), ZERO, ITER
87    ROOTR(N) = A(N,N)
      ROOTI(N) = 0.0
      N=N-1
      GO TO 177
50    IF(ABS(ABS(XNN/A(N,N-1))-1.0)-1.0E-6)  63,63,62
62    IF(ABS(ABS(XN2/A(N-1,N-2))-1.0)-1.0E-6)  63,63,700
63    VQ=ABS(A(N,N-1))-ABS(A(N-1,N-2))
64    IF(VQ)   67,67,66
66    IF(IPRNT)  88,85,88
88    WRITE (6,107)A(N-1,N-2)
      GO TO 84
67    IF(IPRNT)  89,87,89
89    WRITE (6,107)A(N,N-1)
      GO TO 86
 700 IF(ITER .GT. 50) GO TO 63
 701 Z1=          ((E-AA)**2+(F-B)**2)/(E*E+F*F)
     Z2=       ((G-C)**2+(H-DD)**2)/(G*G+H*H)
     IF(Z1-0.25)  51,51,52
51    IF(Z2-0.25)   53,53,54
53    R=E*G-F*H
      SIG=E+G
      GO TO 60
54    R=E*E
      SIG=E+E
      GO TO 60
52    IF(Z2-0.25) 55,55,601
55    R=G*G
      SIG=G+G
      GO TO 60
 601 R = 0.0
     SIG = 0.0
60    XNN=A(N,N-1)
      XN2=A(N-1,N-2)
      CALL QRT(A,N,R,SIG,D)
      AA=E
      B=F
      C=G
      DD=H
      GO TO 12
104   FORMAT(////1X, 9HREAL PART  6X  14HIMAGINARY PART, 26X
     1  13HTAKEN AS ZERO 6X 4HITER //)
```

```
105   FORMAT(1X,E15.8,3X,E15.8, 42X I3)
107   FORMAT(56X  E13.8)
       END
```

```
QI    FOR QRT
      SUBROUTINE QRT(A,N,R,SIG,D)
      DIMENSION A(3,3), PSI(3), G(3)
      N1 = N - 1
      IA = N - 2
      IP = IA
      IF(N-3) 101,10,60
 60   DO 12 J = 3,N1
      J1 = N - J
      IF(ABS(A(J1+1,J1))-D)     10,10,11
 11   IF(ABS(A(J1+1,J1)*A(J1+2,J1+1)*(ABS(A(J1+1,J1+1)+A(J1+2,J1+2)
     1-SIG)+ABS(A(J1+3,J1+2)))/(A(J1+1,J1+1)*(A(J1+1,J1+1)-SIG)+A(      J
     21+1,J1+2)*A(J1+2,J1+1)+R))-D)      10,10,12
 12   IP=J1
 10   DO  14  J=1,IP
      J1=IP-J+1
      IF(ABS(A(J1+1,J1))-D)     13,13,14
 14   IQ=J1
 13   DO  100  I=IP,N1
      IF(I-IP)    16,15,16
 15   G(1)=A(IP,IP)*(A(IP,IP)-SIG)+A(IP,IP+1)*A(IP+1,IP)+R
      G(2)=A(IP+1,IP)*(A(IP,IP)+A(IP+1,IP+1)-SIG)
      G(3)=A(IP+1,IP)*A(IP+2,IP+1)
      A(IP+2,IP)=0.0
      GO TO 19
 16   G(1)=A(I,I-1)
      G(2)=A(I+1,I-1)
      IF(I-IA)    17,17,18
 17   G(3)=A(I+2,I-1)
      GO TO 19
 18   G(3)=0.0
   19 XK = SIGN(SQRT(G(1)**2 + G(2)**2 + G(3)**2), G(1))
 22   IF(XK)    23,24,23
 23   AL=G(1)/XK+1.0
      PSI(1)=G(2)/(G(1)+XK)
      PSI(2)=G(3)/(G(1)+XK)
      GO TO 25
 24   AL=2.0
      PSI(1)=0.0
      PSI(2)=0.0
 25   IF(I-IQ)    26,27,26
 26   IF(I-IP)     29,28,29
 28   A(I,I-1)=-A(I,I-1)
      GO TO 27
 29   A(I,I-1)=-XK
 27   DO 30  J=I,N
      IF(I-IA)  31,31,32
 31   C=PSI(2)*A(I+2,J)
      GO TO 33
 32   C=0.0
 33   E=AL*(A(I,J)+PSI(1)*A(I+1,J)+C)
      A(I,J)=A(I,J)-E
      A(I+1,J)=A(I+1,J)-PSI(1)*E
      IF(I-IA)    34,34,30
 34   A(I+2,J)=A(I+2,J)-PSI(2)*E
```

```
30    CONTINUE
      IF(I-IA)    35,35,36
35    L=I+2
      GO TO  37
36    L=N
37    DO  40  J=IQ,L
      IF(I-IA)  38,38,39
38    C=PSI(2)*A(J,I+2)
      GO TO  41
39    C=0.0
41    E=AL*(A(J,I)+PSI(1)*A(J,I+1)+C)
      A(J,I)=A(J,I)-E
      A(J,I+1)=A(J,I+1)-PSI(1)*E
      IF(I-IA)  42,42,40
42    A(J,I+2)=A(J,I+2)-PSI(2)*E
40    CONTINUE
      IF(I-N+3)    43,43,100
43    E=AL*PSI(2)*A(I+3,I+2)
      A(I+3,I)=-E
      A(I+3,I+1)=-PSI(1)*E
      A(I+3,I+2)=A(I+3,I+2)-PSI(2)*E
100   CONTINUE
101   RETURN
       END
```

```
$I FOR UNST1
C     SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
     1-6)
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     CONSIDERATION OF THREE (3) CASES, INVOLVING 30 VALUES.
      SUBROUTINE UNST1(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
     1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
     2,3),F1(4),X(4)
      DO 1 J=1,M
      DO 1 K=1,N
      DO 1 I=1,3
      COMG(J,K,I)=0.0
1     CONTINUE
101   FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
     1R EQUATION (82), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
     2//)
102   FORMAT (2I5, 1E20.8//)
103   FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104   FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105   FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
      DO 2 J=1,1
      DO 2 K=1,N
      DO 2 I=1,1
      START=1.2
      START1=1.
35    X(1)=START*(OMG(J,K,I))
      DO 3 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)30,31,30
30    C2=2.**28
      IF(B(J,K,I)-C2)300,301,301
31    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 32
300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
32    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START-.1
      IF(START)325,325,35
325   C  B(J,K,I)=.33333333
      GO TO 2001
33    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(1.-BETA12(J,K,I))
3     X(L+1)=F1(L)
4     X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
```

```
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)39,39,6
39      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(3)=(OMG(J,K,I))/(1.-BETA12(J,K,I))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)390,394,394
390     IF(ABS(X(3))-100.)392,393,393
392     C1=.001
        GO TO 395
393     C1=.01
        GO TO 395
394     C1=.1
395     IF(ABS(D1)-C1)5,5,6
6       X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-1000.)4,4,201
201     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)330,501,501
330     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331     COMG(J,K,I)=.88888888
        GO TO 2001
332     COMG(J,K,I)=.11111111
        GO TO 2001
301     COMG(J,K,I)=.77777777
        GO TO 2001
401     COMG(J,K,I)=.99999999
        GO TO 2001
5       COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2001,501,501
501     COMG(J,K,I)=.66666666
2001    CONTINUE
2       CONTINUE
        WRITE(6,101)
        WRITE(6,103)
        DO 7 J=1,1
        DO 7 K=1,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,1)
7       CONTINUE
        DO 8 J=1,1
        DO 8 K=1,N
        DO 8 I=1,1
```

```
      START=1.2
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(BETA12(J,K,I))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
```

```
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE(6,101)
      WRITE(6,104)
      DO 13 J=1,1
      DO 13 K=1,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,1)
13    CONTINUE
      DO 14 J=1,1
      DO 14 K=1,N
      DO 14 I=1,1
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(1.+BETA12(J,K,I))
15    X(L+1)=F1(L)
```

```
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*T0*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(1.+BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*T0*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*T0*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 19 J=1,1
      DO 19 K=1,N
      J1=J-1
```

```
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,1)
19    CONTINUE
      RETURN
      END
```

```
ØI FOR UNST2
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
       1-8)
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       CONSIDERATION OF THREE (33) CASES, INVOLVING 363 VALUES.
        SUBROUTINE UNST2 (PI,RA,H,TO,OMG,M,N,E,TI)
        DIMENSION ALPH(4),X(4),F1(4),OMG(20,20,3),COMG(363),BETT(4)
1000    FORMAT(1X,1E20.8)
105     FORMAT(4I5,8X,A5//)
104     FORMAT (4I5,1E20.8//)
103     FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (13), RAD PER SEC.,//,4X,2HJ,,3X,2HK,,3X,2HR,,3X,2HS,,8
       1X,10HCOMEGA (1),//)
        INTEGER T,R,S,S1,S2,JKIS,Z,IJKISF,Y,XZ,P,O
        REAL MEY,MEX,MEV,MEW,MEU,MEZ
        REAL INFIN
        DATA INFIN/5HINFIN/
        DO 4 J=1,1
        DO 4 K=1,N
        DO 4 I=1,1
        DO 4 R=1,3
        T=5
        C4=1.0E+6
        S1=2*T+1
        S=1
2       IJKISF=3*S1*K+3*S1*I+S1*R+S-7*S1
        JKIS=IJKISF
        S=S-(T+1)
        IF(OMG(J,K,I)-1.0)19,20,20
20      IF(ABS(S)-4)300,301,302
302     IF(OMG(J,K,I)-520.0)17,18,18
301     IF(OMG(J,K,I)-420.0)17,18,18
300     IF(ABS(S)-2)303,304,305
305     IF(OMG(J,K,I)-320.0)17,18,18
304     IF(OMG(J,K,I)-220.0)17,18,18
303     IF(ABS(S)-1)306,307,300
307     IF(OMG(J,K,I)-120.0)17,18,18
306     IF(OMG(J,K,I)-30.0)17,201,201
17      ALPH(R)=0.07
        MEV=TO
        MEU=H
        MEW=E
        MEX=TI
        MEY=RA
        DO 8 L=1,2
        X(L)=OMG(J,K,I)/ABS(ALPH(R)+FLOAT(S))
        MEZ=X(L)
        START1=1.0
        ME1=0
        CALL BEAM  (MEV,MEU,MEW,MEX,MEY,MEZ,BETT,OMG2,ME1,ME2)
        IF(X(L)-2.0*OMG2)12,12,13
13      CALL SORT (ALPH,BETT,ME2)
        F1(L)=OMG(J,K,I)/ABS(ALPH(R)+FLOAT(S))
        IF(F1(L)-C4)500,501,501
501     COMG(JKIS)=F1(L)
```

```
      GO TO 14
500   X(L+1)=F1(L)
8     CONTINUE
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(X(1)-F1(1)-X(2)+F1(2))-C1)9,9,10
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
410   MEZ=X(3)
      CALL BEAM  (MEV,MEU,MEW,MEX,MEY,MEZ,BETT,OMG2,ME1,ME2)
      IF(X(3)-2.0*OMG2)12,12,15
15    CALL SORT (ALPH,BETT,ME2)
      F1(3)=OMG(J,K,I)/ABS(ALPH(R)+FLOAT(S))
      IF(F1(3)-C4)502,503,503
502   D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)9,9,11
11    X(1)=X(2)
      F1(1)=F1(2)
406   X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-5.0)10,10,21
21    ME1=ME1+1
401   IF(ME1-10)10,10,16
16    COMG(JKIS)=.22222222
      GO TO 14
12    COMG(JKIS)=.11111111
      GO TO 14
19    COMG(JKIS)=.00000000
      GO TO 14
503   COMG(JKIS)=F1(3)
      GO TO 14
201   COMG(JKIS)=10000000.
      GO TO 14
18    COMG(JKIS)=OMG(J,K,I)/ABS(FLOAT(S))
      GO TO 14
9     COMG(JKIS)=X(3)
14    S=S+(T+2)
      IF(S1-S)4,2,2
4     CONTINUE
      WRITE(6,103)
      S1=2*T+1
      Z=1
      DO 6 J=1,1
      DO 6 K=1,N
```

```
      DO 6 R=1,3
      DO 6 S=1,S1
      J1=J-1
      K1=K-1
      S2=S-(T+1)
      IF(COMG(Z)-C4)511,511,510
510   COMG(Z)=INFIN
      WRITE(6,105)J1,K1,R,S2,COMG(Z)
      GO TO 38
511   WRITE(6,104)J1,K1,R,S2,COMG(Z)
38    Z=Z+1
6     CONTINUE
      RETURN
      END
```

```
QI FOR UNST3
C      SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
      1-10)
C      NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C      CONSIDERATION OF ONE (1) CASE, INVOLVING 20 VALUES.
       SUBROUTINE UNST3(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
       DIMENSION OMG(20,20,3),COMG(20,20,3)
101    FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
      1R EQUATION (83), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
      210X,10HCOMEGA (2),//)
102    FORMAT (2I5, 2E20.8//)
       DO 1 J=1,M
       DO 1 K=1,N
       DO 1 I=1,3
       COMG(J,K,I)=0.0
1      CONTINUE
       DO 2 J=2,M
       DO 2 K=1,1
       DO 2 I=1,2
       COMG(J,K,I)=OMG(J,K,I)
2      CONTINUE
       WRITE (6,101)
       DO 3 J=2,M
       DO 3 K=1,1
       J1=J-1
       K1=K-1
       WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
3      CONTINUE
       RETURN
       END
```

```
@I FOR UNST4
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
      1-13)
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       CONSIDERATION OF THREE (3) CASES, INVOLVING 60 VALUES.
        SUBROUTINE UNST4(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
      1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),F1(4),X(4)
101     FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
      1R EQUATION (86), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
      210X,10HCOMEGA (2),//)
102     FORMAT (2I5, 2E20.8//)
103     FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104     FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105     FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
        DO 2 J=2,M
        DO 2 K=1,1
        DO 2 I=1,2
        START=.1
        START1=1.
35      X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
30      C2=2.**28
        IF(B(J,K,I)-C2)300,301,301
31      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
      1)))))/TAN(B(J,K,I))
32      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
34      START=START+.1
        IF(START-1.0)35,35,325
325     COMG(J,K,I)=.33333333
        GO TO 2001
33      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(2.-BETA(J,K,I))
3       X(L+1)=F1(L)
4       X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)36,37,36
36      C2=2.**28
        IF(B(J,K,I)-C2)360,401,401
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
```

```
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=ASIN(D(J,K,I))
      F1(3)=(OMG(J,K,I))/(2.-BETA(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE(6,101)
      WRITE (6,103)
      DO 7 J=2,M
      DO 7 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=1,1
      DO 8 I=1,2
      START=.6
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
```

```
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(BETA(J,K,I))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(BETA(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
```

```
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE (6,101)
      WRITE (6,104)
      DO 13 J=2,M
      DO 13 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=1,1
      DO 14 I=1,2
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(2.+BETA(J,K,I))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
```

```
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(2.+BETA(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE (6,101)
      WRITE (6,105)
      DO 19 J=2,M
      DO 19 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
19    CONTINUE
      RETURN
      END
```

```
@I FOR UNST5
C      SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
      1-16)
C      NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C      CONSIDERATION OF 9 CASES (3 INADMISSIBLE), INVOLVING 120 VALUES.
       SUBROUTINE UNST5 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
       DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
      1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
      2,3),F1(4),X(4)
101    FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THRUST FREQUENCY FO
      1R EQUATION (85), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
      210X,10HCOMEGA (2),//)
102    FORMAT (2I5, 2E20.8,//)
103    FORMAT (10X,78HFIRST CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQUA
      1L 1, (CASE NOT ADMISSIBLE).,///)
104    FORMAT (10X,56HSECOND CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
      1AL 1,//)
105    FORMAT (10X,55HTHIRD CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
      1L 1,//)
106    FORMAT (10X,56HFOURTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQU
      1AL 2,//)
107    FORMAT (10X,78HFIFTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQUA
      1L 2, (CASE NOT ADMISSIBLE).,///)
108    FORMAT (10X,55HSIXTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
      1L 2,//)
109    FORMAT (10X,57HSEVENTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQ
      1UAL 3,//)
110    FORMAT (10X,56HEIGHTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
      1AL 3,//)
111    FORMAT (10X,78HNINTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
      1L 3, (CASE NOT ADMISSIBLE).,///)
       DO 1 J=1,M
       DO 1 K=1,N
       DO 1 I=1,3
       COMG(J,K,I)=0.0
1      CONTINUE
       DO 2 J=2,M
       DO 2 K=1,1
       DO 2 I=1,2
       COMG(J,K,I)=OMG(J,K,I)
2      CONTINUE
       WRITE (6,101)
       WRITE (6,103)
       WRITE (6,101)
       WRITE (6,104)
       DO 3 J=2,M
       DO 3 K=1,1
       J1=J-1
       K1=K-1
       WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
3      CONTINUE
       DO 4 J=2,M
       DO 4 K=1,1
       DO 4 I=1,2
       COMG(J,K,I)=.5*(OMG(J,K,I))
```

```
4     CONTINUE
      WRITE (6,101)
      WRITE(6,105)
      DO 5 J=2,M
      DO 5 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
5     CONTINUE
      DO 6 J=2,M
      DO 6 K=1,1
      DO 6 I=1,2
      COMG(J,K,I)=OMG(J,K,I)
6     CONTINUE
      WRITE (6,101)
      WRITE (6,106)
      DO 7 J=2,M
      DO 7 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
7     CONTINUE
      WRITE (6,101)
      WRITE (6,107)
      DO 8 J=2,M
      DO 8 K=1,1
      DO 8 I=1,2
      START=.1
      START1=1.
35    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)30,31,30
30    C2=2.**28
      IF(B(J,K,I)-C2)300,301,301
31    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 32
300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
32    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START+.1
      IF(START-1.0)35,35,325
325   COMG(J,K,I)=.33333333
      GO TO 2001
33    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
```

```
      IF(B(J,K,I)-C2)360,401,401
37    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 38
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,12
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)10,10,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
8     CONTINUE
      WRITE(6,101)
      WRITE(6,108)
      DO 13 J=2,M
      DO 13 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=1,1
      DO 14 I=1,2
```

```
      COMG(J,K,I)=.5*(OMG(J,K,I))
14    CONTINUE
      WRITE(6,101)
      WRITE(6,109)
      DO 15 J=2,M
      DO 15 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
15    CONTINUE
      DO 16 J=2,M
      DO 16 K=1,1
      DO 16 I=1,2
      START=.1
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 17 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START+.1
      IF(START-1.0)45,45,425
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
17    X(L+1)=F1(L)
18    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,20
49    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
```

```
492.  C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)19,19,20
20    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)18,18,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
19    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
16    CONTINUE
      WRITE (6,101)
      WRITE (6,110)
      DO 21 J=2,M
      DO 21 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
21    CONTINUE
      WRITE (6,101)
      WRITE (6,111)
      RETURN
      END
```

```
$I FOR UNST6
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
     1-19)
C     CONSIDERATION OF 9 CASES (3 INADMISSIBLE), INVOLVING 120 VALUES.
      SUBROUTINE UNST6 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
     1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
     2,3),F1(4),X(4)
101   FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
     1R EQUATION (84), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
     210X,10HCOMEGA (2),//)
102   FORMAT (2I5, 2E20.8//)
103   FORMAT (10X,78HFIRST CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQUA
     1L 1, (CASE NOT ADMISSIBLE).,///)
104   FORMAT (10X,56HSECOND CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
     1AL 1,//)
105   FORMAT (10X,55HTHIRD CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 1,//)
106   FORMAT (10X,56HFOURTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQU
     1AL 2,//)
107   FORMAT (10X,78HFIFTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQUA
     1L 2, (CASE NOT ADMISSIBLE).,///)
108   FORMAT (10X,55HSIXTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 2,//)
109   FORMAT (10X,57HSEVENTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQ
     1UAL 3,//)
110   FORMAT (10X,56HEIGHTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
     1AL 3,//)
111   FORMAT (10X,78HNINTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 3, (CASE NOT ADMISSIBLE).,///)
      DO 1 J=1,M
      DO 1 K=1,N
      DO 1 I=1,3
      COMG(J,K,I)=0.0
1     CONTINUE
      WRITE (6,101)
      WRITE (6,103)
      DO 2 J=2,M
      DO 2 K=1,1
      DO 2 I=1,2
      START=.1
      START1=1.
35    X(1)=START*(OMG(J,K,I))
      DO 3 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)30,31,30
30    C2=2.**28
      IF(B(J,K,I)-C2)300,301,301
31    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 32
300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
32    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
```

```
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START+.1
      IF(START-1.0)35,35,325
325   COMG(J,K,I)=.33333333
      GO TO 2001
33    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(-1.+BETA(J,K,I)))
3     X(L+1)=F1(L)
4     X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
37    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 38
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
```

```
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE(6,101)
      WRITE(6,104)
      DO 7 J=2,M
      DO 7 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=1,1
      DO 8 I=1,2
      START=.6
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
      D1=X(3)-F1(3)
```

```
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 13 J=2,M
      DO 13 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=1,1
      DO 14 I=1,2
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
```

```
     .1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.-BETA(J,K,I)))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.-BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
```

```
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,106)
      DO 19 J=2,M
      DO 19 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
19    CONTINUE
      WRITE (6,101)
      WRITE (6,107)
      WRITE (6,101)
      WRITE (6,108)
      DO 20 J=2,M
      DO 20 K=1,1
      DO 20 I=1,2
      COMG(J,K,I)=OMG(J,K,I)
20    CONTINUE
      DO 21 J=2,M
      DO 21 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
21    CONTINUE
      DO 22 J=2,M
      DO 22 K=1,1
      DO 22 I=1,2
      START=.6
      START1=1.
65    X(1)=START*(OMG(J,K,I))
      DO 23 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)60,61,60
60    C2=2.**28
      IF(B(J,K,I)-C2)600,304,304
61    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 62
600   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
62    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)63,63,64
64    START=START-.1
      IF(START)625,625,65
625   COMG(J,K,I)=.33333333
      GO TO 2004
63    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(-1.*(BETA(J,K,I))))
23    X(L+1)=F1(L)
```

```
24    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)66,67,66
66    C2=2.**28
      IF(B(J,K,I)-C2)660,404,404
67    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 68
660   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
68    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)69,69,26
69    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.*(BETA(J,K,I))))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)690,694,694
690   IF(ABS(X(3))-100.)692,693,693
692   C1=.001
      GO TO 695
693   C1=.01
      GO TO 695
694   C1=.1
695   IF(ABS(D1)-C1)25,25,26
26    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)24,24,204
204   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)630,504,504
630   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)631,631,632
631   COMG(J,K,I)=.88888888
      GO TO 2004
632   COMG(J,K,I)=.11111111
      GO TO 2004
304   COMG(J,K,I)=.77777777
      GO TO 2004
404   COMG(J,K,I)=.99999999
      GO TO 2004
25    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2004,504,504
504   COMG(J,K,I)=.66666666
2004  CONTINUE
22    CONTINUE
      WRITE(6,101)
      WRITE(6,109)
      DO 27 J=2,M
      DO 27 K=1,1
      J1=J-1
      K1=K-1
```

```
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
27    CONTINUE
      DO 28 J=2,M
      DO 28 K=1,1
      DO 28 I=1,2
      COMG(J,K,I)=OMG(J,K,I)
28    CONTINUE
      WRITE (6,101)
      WRITE (6,110)
      DO 29 J=2,M
      DO 29 K=1,1
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,2)
29    CONTINUE
      WRITE (6,101)
      WRITE (6,111)
      RETURN
      END
```

```
$I FOR UNST7
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
       1-22)
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       CONSIDERATION OF THREE (3) CASES, INVOLVING 900 VALUES.
        SUBROUTINE UNST7 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
        DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
       1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
       2,3),F1(4),X(4)
101     FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
       1R EQUATION (87), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
       210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102     FORMAT (2I5, 3E20.8//)
103     FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104     FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105     FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
        DO 1 J=1,M
        DO 1 K=1,N
        DO 1 I=1,3
        COMG(J,K,I)=0.0
1       CONTINUE
        DO 2 J=2,M
        DO 2 K=2,N
        DO 2 I=1,3
        START=.1
        START1=1.
35      X(1)=START*(OMG(J,K,I))
        DO 3 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)30,31,30
30      C2=2.**28
        IF(B(J,K,I)-C2)300,301,301
31      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 32
300     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
32      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)33,33,34
34      START=START+.1
        IF(START-1.0)35,35,325
325     COMG(J,K,I)=.33333333
        GO TO 2001
33      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(L)=(OMG(J,K,I))/(1.-BETA12(J,K,I))
3       X(L+1)=F1(L)
4       X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)36,37,36
36      C2=2.**28
        IF(B(J,K,I)-C2)360,401,401
```

```
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)39,39,6
39      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(3)=(OMG(J,K,I))/(1.-BETA12(J,K,I))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)390,394,394
390     IF(ABS(X(3))-100.)392,393,393
392     C1=.001
        GO TO 395
393     C1=.01
        GO TO 395
394     C1=.1
395     IF(ABS(D1)-C1)5,5,6
6       X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-100.)4,4,201
201     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)330,501,501
330     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331     COMG(J,K,I)=.88888888
        GO TO 2001
332     COMG(J,K,I)=.11111111
        GO TO 2001
301     COMG(J,K,I)=.77777777
        GO TO 2001
401     COMG(J,K,I)=.99999999
        GO TO 2001
5       COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2001,501,501
501     COMG(J,K,I)=.66666666
2001    CONTINUE
2       CONTINUE
        WRITE(6,101)
        WRITE(6,103)
        DO 7 J=2,M
        DO 7 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7       CONTINUE
        DO 8 J=2,M
        DO 8 K=2,N
        DO 8 I=1,3
```

```
      START=1.
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START-.1
      IF(START)425,425,45
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(BETA12(J,K,I))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
```

```
      IF(START1-1000.)10,10,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE(6,101)
      WRITE(6,104)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(1.+BETA12(J,K,I))
15    X(L+1)=F1(L)
```

```
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(1.+BETA12(J,K,I))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 19 J=2,M
      DO 19 K=2,N
      J1=J-1
```

```
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19    CONTINUE
      RETURN
      END
```

```
$I FOR UNST8
C     SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
     1-25)
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     CONSIDERATION OF THREE (3) CASES, INVOLVING 900 VALUES.
      SUBROUTINE UNST8 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
     1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
     2,3),F1(4),X(4)
101   FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
     1R EQUATION (89), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
     210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102   FORMAT (2I5, 3E20.8//)
103   FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104   FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105   FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
      DO 1 J=1,M
      DO 1 K=1,N
      DO 1 I=1,3
      COMG(J,K,I)=0.0
1     CONTINUE
      DO 2 J=2,M
      DO 2 K=2,N
      DO 2 I=1,3
      START=.1
      START1=1.
35    X(1)=START*(OMG(J,K,I))
      DO 3 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)30,31,30
30    C2=2.**28
      IF(B(J,K,I)-C2)300,301,301
31    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 32
300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
32    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START+.1
      IF(START-1.0)35,35,325
325   COMG(J,K,I)=.33333333
      GO TO 2001
33    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(2.-BETA12(J,K,I)))
3     X(L+1)=F1(L)
4     X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
```

```
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)39,39,6
39      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(2.-BETA12(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)390,394,394
390     IF(ABS(X(3))-100.)392,393,393
392     C1=.001
        GO TO 395
393     C1=.01
        GO TO 395
394     C1=.1
395     IF(ABS(D1)-C1)5,5,6
6       X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-100.)4,4,201
201     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)330,501,501
330     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331     COMG(J,K,I)=.88888888
        GO TO 2001
332     COMG(J,K,I)=.11111111
        GO TO 2001
301     COMG(J,K,I)=.77777777
        GO TO 2001
401     COMG(J,K,I)=.99999999
        GO TO 2001
5       COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2001,501,501
501     COMG(J,K,I)=.66666666
2001    CONTINUE
2       CONTINUE
        WRITE (6,101)
        WRITE (6,103)
        DO 7 J=2,M
        DO 7 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7       CONTINUE
        DO 8 J=2,M
        DO 8 K=2,N
        DO 8 I=1,3
```

```
      START=.1
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START+.1
      IF(START-1.0)45,45,425
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA12(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(1.+BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
```

```
      IF(START1-100.)10,10,202
202.  A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE(6,101)
      WRITE(6,104)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      START=.1
      START1=1.
      START2=1.0
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(2.+BETA12(J,K,I)))
```

```
15      X(L+1)=F1(L)
16      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)56,57,56
56      C2=2.**28
        IF(B(J,K,I)-C2)560,403,403
57      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 58
560     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
58      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)59,59,18
59      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(2.+BETA12(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)590,594,594
590     IF(ABS(X(3))-100.)592,593,593
592     C1=.001
        GO TO 595
593     C1=.01
        GO TO 595
594     C1=.1
595     IF(ABS(D1)-C1)17,17,18
18      X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-100.)16,16,203
203     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)530,503,503
530     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531     COMG(J,K,I)=.88888888
        GO TO 2003
532     COMG(J,K,I)=.11111111
        GO TO 2003
303     COMG(J,K,I)=.77777777
        GO TO 2003
403     COMG(J,K,I)=.99999999
        GO TO 2003
17      COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2003,503,503
503     COMG(J,K,I)=.66666666
2003    CONTINUE
14      CONTINUE
        WRITE (6,101)
        WRITE (6,105)
        DO 19 J=2,M
        DO 19 K=2,N
```

```
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19    CONTINUE
      RETURN
      END
```

```
$I FOR UNST9
C     SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
     1-28)
C     NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C     CONSIDERATION OF THREE (3) CASES, INVOLVING 900 VALUES.
      SUBROUTINE UNST9 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
     1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),BETA12(20,20
     2,3),F1(4),X(4)
101   FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
     1R EQUATION (88), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
     210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102   FORMAT (2I5, 3E20.8//)
103   FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104   FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105   FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
      DO 1 J=1,M
      DO 1 K=1,N
      DO 1 I=1,3
      COMG(J,K,I)=0.0
1     CONTINUE
      DO 2 J=2,M
      DO 2 K=2,N
      DO 2 I=1,3
      START=1.
      START1=1.
35    X(1)=START*(OMG(J,K,I))
      DO 3 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)30,31,30
30    C2=2.**28
      IF(B(J,K,I)-C2)300,301,301
31    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 32
300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
32    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START-.1
      IF(START)325,325,35
325   COMG(J,K,I)=.33333333
      GO TO 2001
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(-1.*BETA12(J,K,I)))
3     X(L+1)=F1(L)
4     X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
37    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
```

```
      GO TO 38
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.*BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE(6,101)
      WRITE(6,103)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=.1
```

```
      START1=1.
45    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)40,41,40
40    C2=2.**28
      IF(B(J,K,I)-C2)400,302,302
41    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 42
400   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
42    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)43,43,44
44    START=START+.1
      IF(START-1.0)45,45,425
425   COMG(J,K,I)=.33333333
      GO TO 2002
43    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(-1.+BETA12(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)46,47,46
46    C2=2.**28
      IF(B(J,K,I)-C2)460,402,402
47    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 48
460   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
48    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)49,49,12
49    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.+BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)490,494,494
490   IF(ABS(X(3))-100.)492,493,493
492   C1=.001
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)11,11,12
12    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)10,10,202
```

```
202     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)430,502,502
430     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431     COMG(J,K,I)=.88888888
        GO TO 2002
432     COMG(J,K,I)=.11111111
        GO TO 2002
302     COMG(J,K,I)=.77777777
        GO TO 2002
402     COMG(J,K,I)=.99999999
        GO TO 2002
11      COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2002,502,502
502     COMG(J,K,I)=.66666666
2002    CONTINUE
8       CONTINUE
        WRITE (6,101)
        WRITE(6,104)
        DO 13 J=2,M
        DO 13 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13      CONTINUE
        DO 14 J=2,M
        DO 14 K=2,N
        DO 14 I=1,3
        START=1.
        START1=1.
55      X(1)=START*(OMG(J,K,I))
        DO 15 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)50,51,50
50      C2=2.**28
        IF(B(J,K,I)-C2)500,303,303
51      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 52
500     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
52      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)53,53,54
54      START=START-.1
        IF(START)525,525,55
525     COMG(J,K,I)=.33333333
        GO TO 2003
53      F(J,K,I)=PI-ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        BETA12(J,K,I)=.5*(BETA(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(BETA12(J,K,I)))
15      X(L+1)=F1(L)
16      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
```

```
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      BETA12(J,K,I)=.5*(BETA(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(BETA12(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE(6,101)
      WRITE(6,105)
      DO 19 J=2,M
      DO 19 K=2,N
      J1=J-1
      K1=K-1
```

```
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19    CONTINUE
      RETURN
      END
```

```
      UNST10
C      NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C      SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
     1-31)
C      CONSIDERATION OF 9 CASES (3 INADMISSIBLE), INVOLVING 1800 VALUES.
       SUBROUTINE UNST10(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
       DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
     1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),F1(4),X(4)
101    FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
     1R EQUATION (91), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
     210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102    FORMAT (2I5, 3E20.8//)
103    FORMAT (10X,78HFIRST CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQUA
     1L 1, (CASE NOT ADMISSIBLE).,///)
104    FORMAT (10X,56HSECOND CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
     1AL 1,//)
105    FORMAT (10X,55HTHIRD CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 1,//)
106    FORMAT (10X,56HFOURTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQU
     1AL 2,//)
107    FORMAT (10X,78HFIFTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQUA
     1L 2, (CASE NOT ADMISSIBLE).,///)
108    FORMAT (10X,55HSIXTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 2,//)
109    FORMAT (10X,57HSEVENTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQ
     1UAL 3,//)
110    FORMAT (10X,56HEIGHTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
     1AL 3,//)
111    FORMAT (10X,78HNINTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
     1L 3, (CASE NOT ADMISSIBLE).,///)
       DO 1 J=1,M
       DO 1 K=1,N
       DO 1 I=1,3
       COMG(J,K,I)=0.0
1      CONTINUE
       WRITE (6,101)
       WRITE (6,103)
       DO 2 J=2,M
       DO 2 K=2,N
       DO 2 I=1,3
       COMG(J,K,I)=OMG(J,K,I)
2      CONTINUE
       WRITE (6,101)
       WRITE (6,104)
       DO 3 J=2,M
       DO 3 K=2,N
       J1=J-1
       K1=K-1
       WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
3      CONTINUE
       DO 4 J=2,M
       DO 4 K=2,N
       DO 4 I=1,3
       COMG(J,K,I)=.5*(OMG(J,K,I))
4      CONTINUE
```

```
      WRITE (6,101)
      WRITE (6,105)
      DO 5 J=2,M
      DO 5 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
5     CONTINUE
      DO 6 J=2,M
      DO 6 K=2,N
      DO 6 I=1,3
      COMG(J,K,I)=OMG(J,K,I)
6     CONTINUE
      WRITE (6,101)
      WRITE (6,106)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
      WRITE (6,101)
      WRITE (6,107)
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=.1
      START1=1.
35    X(1)=START*(OMG(J,K,I))
      DO 9 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)30,31,30
30    C2=2.**28
      IF(B(J,K,I)-C2)300,301,301
31    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 32
300   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
32    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START+.1
      IF(START-1.0)35,35,325
325   COMG(J,K,I)=.33333333
      GO TO 2001
33    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
9     X(L+1)=F1(L)
10    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
```

```
37      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 38
360     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
38      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)39,39,12
39      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)390,394,394
390     IF(ABS(X(3))-100.)392,393,393
392     C1=.001
        GO TO 395
393     C1=.01
        GO TO 395
394     C1=.1
395     IF(ABS(D1)-C1)11,11,12
12      X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-100.)10,10,201
201     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)330,501,501
330     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331     COMG(J,K,I)=.88888888
        GO TO 2001
332     COMG(J,K,I)=.11111111
        GO TO 2001
301     COMG(J,K,I)=.77777777
        GO TO 2001
401     COMG(J,K,I)=.99999999
        GO TO 2001
11      COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2001,501,501
501     COMG(J,K,I)=.66666666
2001    CONTINUE
8       CONTINUE
        WRITE (6,101)
        WRITE (6,108)
        DO 13 J=2,M
        DO 13 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13      CONTINUE
        DO 14 J=2,M
        DO 14 K=2,N
        DO 14 I=1,3
        COMG(J,K,I)=.5*(OMG(J,K,I))
```

```
14      CONTINUE
        WRITE (6,101)
        WRITE (6,109)
        DO 15 J=2,M
        DO 15 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
15      CONTINUE
        DO 16 J=2,M
        DO 16 K=2,N
        DO 16 I=1,3
        START=.1
        START1=1.
45      X(1)=START*(OMG(J,K,I))
        DO 17 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)40,41,40
40      C2=2.**28
        IF(B(J,K,I)-C2)400,302,302
41      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 42
400     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
42      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)43,43,44
44      START=START+.1
        IF(START-1.0)45,45,425
425     COMG(J,K,I)=.33333333
        GO TO 2002
43      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
17      X(L+1)=F1(L)
18      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)46,47,46
46      C2=2.**28
        IF(B(J,K,I)-C2)460,402,402
47      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 48
460     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
48      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)49,49,20
49      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(1.+BETA(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)490,494,494
490     IF(ABS(X(3))-100.)492,493,493
492     C1=.001
```

```
      GO TO 495
493   C1=.01
      GO TO 495
494   C1=.1
495   IF(ABS(D1)-C1)19,19,20
20    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)18,18,202
202   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)430,502,502
430   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
19    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
16    CONTINUE
      WRITE (6,101)
      WRITE (6,110)
      DO 21 J=2,M
      DO 21 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
21    CONTINUE
      WRITE (6,101)
      WRITE (6,111)
      RETURN
      END
```

```
$I FOR UNST11
C      NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C      SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
      1-34)
C      CONSIDERATION OF 9 CASES(3 INADMISSIBLE), INVOLVING 1800 VALUES.
       SUBROUTINE UNST11(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
       DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
      1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),F1(4),X(4)
101    FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
      1R EQUATION (90), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
      210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102    FORMAT (2I5, 3E20.8//)
103    FORMAT (10X,78HFIRST CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQUA
      1L 1, (CASE NOT ADMISSIBLE).,///)
104    FORMAT (10X,56HSECOND CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
      1AL 1,//)
105    FORMAT (10X,55HTHIRD CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
      1L 1,//)
106    FORMAT (10X,56HFOURTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQU
      1AL 2,//)
107    FORMAT (10X,78HFIFTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQUA
      1L 2, (CASE NOT ADMISSIBLE).,///)
108    FORMAT (10X,55HSIXTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
      1L 2,//)
109    FORMAT (10X,57HSEVENTH CASE, LOWER CASE M EQUAL 1, LOWER CASE N EQ
      1UAL 3,//)
110    FORMAT (10X,56HEIGHTH CASE, LOWER CASE M EQUAL 2, LOWER CASE N EQU
      1AL 3,//)
111    FORMAT (10X,78HNINTH CASE, LOWER CASE M EQUAL 3, LOWER CASE N EQUA
      1L 3, (CASE NOT ADMISSIBLE).,///)
       DO 1 J=1,M
       DO 1 K=1,N
       DO 1 I=1,3
       COMG(J,K,I)=0.0
1      CONTINUE
       WRITE (6,101)
       WRITE (6,103)
       DO 2 J=2,M
       DO 2 K=2,N
       DO 2 I=1,3
       START=.1
       START1=1.
35     X(1)=START*(OMG(J,K,I))
       DO 3 L=1,2
       A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
       B(J,K,I)=.5*PI*SQRT(A(J,K,I))
       IF(A(J,K,I)-1.)30,31,30
30     C2=2.**28
       IF(B(J,K,I)-C2)300,301,301
31     DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
       GO TO 32
300    DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
      1)))))/TAN(B(J,K,I))
32     C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
       D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
```

```
      IF(ABS(D(J,K,I))-1.)33,33,34
34    START=START+.1
      IF(START-1.0)35,35,325
325   COMG(J,K,I)=.33333333
      GO TO 2001
33    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(-1.+BETA(J,K,I)))
3     X(L+1)=F1(L)
4     X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)36,37,36
36    C2=2.**28
      IF(B(J,K,I)-C2)360,401,401
37    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 38
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
```

```
501     COMG(J,K,I)=.66666666
2001    CONTINUE
2       CONTINUE
        WRITE (6,101)
        WRITE (6,104)
        DO 7 J=2,M
        DO 7 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7       CONTINUE
        DO 8 J=2,M
        DO 8 K=2,N
        DO 8 I=1,3
        START=.6
        START1=1.
45      X(1)=START*(OMG(J,K,I))
        DO 9 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)40,41,40
40      C2=2.**28
        IF(B(J,K,I)-C2)400,302,302
41      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 42
400     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
42      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)43,43,44
44      START=START-.1
        IF(START)425,425,45
425     COMG(J,K,I)=.33333333
        GO TO 2002
43      F(J,K,I)=PI-ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
9       X(L+1)=F1(L)
10      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)46,47,46
46      C2=2.**28
        IF(B(J,K,I)-C2)460,402,402
47      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 48
460     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
48      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)49,49,12
49      F(J,K,I)=PI-ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)490,494,494
```

```
490     IF(ABS(X(3))-100.)492,493,493
492     C1=.001
        GO TO 495
493     C1=.01
        GO TO 495
494     C1=.1
495     IF(ABS(D1)-C1)11,11,12
12      X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-1000.)10,10,202
202     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)430,502,502
430     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431     COMG(J,K,I)=.88888888
        GO TO 2002
432     COMG(J,K,I)=.11111111
        GO TO 2002
302     COMG(J,K,I)=.77777777
        GO TO 2002
402     COMG(J,K,I)=.99999999
        GO TO 2002
11      COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2002,502,502
502     COMG(J,K,I)=.66666666
2002    CONTINUE
8       CONTINUE
        WRITE(6,101)
        WRITE(6,105)
        DO 13 J=2,M
        DO 13 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13      CONTINUE
        DO 14 J=2,M
        DO 14 K=2,N
        DO 14 I=1,3
        START=.1
        START1=1.
55      X(1)=START*(OMG(J,K,I))
        DO 15 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)50,51,50
50      C2=2.**28
        IF(B(J,K,I)-C2)500,303,303
51      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 52
500     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
```

```
52      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)53,53,54
54      START=START+.1
        IF(START-1.0)55,55,525
525     COMG(J,K,I)=.33333333
        GO TO 2003
53      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(1.-BETA(J,K,I)))
15      X(L+1)=F1(L)
16      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)56,57,56
56      C2=2.**28
        IF(B(J,K,I)-C2)560,403,403
57      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 58
560     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
58      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)59,59,18
59      F(J,K,I)=ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(1.-BETA(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)590,594,594
590     IF(ABS(X(3))-100.)592,593,593
592     C1=.001
        GO TO 595
593     C1=.01
        GO TO 595
594     C1=.1
595     IF(ABS(D1)-C1)17,17,18
18      X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-100.)16,16,203
203     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)530,503,503
530     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
        C(J,K,I)=SQRT(B(J,K,I))
        C3=.01
        IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531     COMG(J,K,I)=.88888888
        GO TO 2003
532     COMG(J,K,I)=.11111111
        GO TO 2003
303     COMG(J,K,I)=.77777777
        GO TO 2003
403     COMG(J,K,I)=.99999999
        GO TO 2003
```

```
17      COMG(J,K,I)=ABS(X(3))
        IF(A(J,K,I)-1.)2003,503,503
503     COMG(J,K,I)=.66666666
2003    CONTINUE
14      CONTINUE
        WRITE(6,101)
        WRITE(6,106)
        DO 19 J=2,M
        DO 19 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19      CONTINUE
        WRITE (6,101)
        WRITE (6,107)
        DO 20 J=2,M
        DO 20 K=2,N
        DO 20 I=1,3
        COMG(J,K,I)=OMG(J,K,I)
20      CONTINUE
        WRITE (6,101)
        WRITE (6,108)
        DO 21 J=2,M
        DO 21 K=2,N
        J1=J-1
        K1=K-1
        WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
21      CONTINUE
        DO 22 J=2,M
        DO 22 K=2,N
        DO 22 I=1,3
        START=.6
        START1=1.
65      X(1)=START*(OMG(J,K,I))
        DO 23 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)60,61,60
60      C2=2.**28
        IF(B(J,K,I)-C2)600,304,304
61      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 62
600     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
62      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)63,63,64
64      START=START-.1
        IF(START)625,625,65
625     COMG(J,K,I)=.33333333
        GO TO 2004
63      F(J,K,I)=PI-ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(-1.*BETA(J,K,I)))
23      X(L+1)=F1(L)
24      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
```

```
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)66,67,66
66    C2=2.**28
      IF(B(J,K,I)-C2)660,404,404
67    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 68
660   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
68    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)69,69,26
69    F(J,K,I)=PI-ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(-1.*BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)690,694,694
690   IF(ABS(X(3))-100.)692,693,693
692   C1=.001
      GO TO 695
693   C1=.01
      GO TO 695
694   C1=.1
695   IF(ABS(D1)-C1)25,25,26
26    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)24,24,204
204   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)630,504,504
630   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)631,631,632
631   COMG(J,K,I)=.88888888
      GO TO 2004
632   COMG(J,K,I)=.11111111
      GO TO 2004
304   COMG(J,K,I)=.77777777
      GO TO 2004
404   COMG(J,K,I)=.99999999
      GO TO 2004
25    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2004,504,504
504   COMG(J,K,I)=.66666666
2004  CONTINUE
22    CONTINUE
      WRITE (6,101)
      WRITE(6,109)
      DO 27 J=2,M
      DO 27 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
```

```
27    CONTINUE
      DO 28 J=2,M
      DO 28 K=2,N
      DO 28 I=1,3
      COMG(J,K,I)=OMG(J,K,I)
28    CONTINUE
      WRITE (6,101)
      WRITE (6,110)
      DO 29 J=2,M
      DO 29 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
29    CONTINUE
      WRITE (6,101)
      WRITE (6,111)
      RETURN
      END
```

```
$I FOR UNST12
C      SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
      1-37)
C      NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C      SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (92).
C      CONSIDERATION OF THREE (3) CASES, INVOLVING 900 VALUES.
       SUBROUTINE UNST12(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
       DIMENSION OMG(20,20,3),COMG(20,20,3),A(20,20,3),B(20,20,3),DELO(20
      1,20,3),C(20,20,3),D(20,20,3),F(20,20,3),BETA(20,20,3),F1(4),X(4)
101    FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
      1R EQUATION (92), RAD PER SEC.,//,4X,2HJ,,4X,2HK,,8X,10HCOMEGA (1),
      210X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
102    FORMAT (2I5, 3E20.8//)
103    FORMAT (20X,33HFIRST CASE, LOWER CASE M EQUAL 1.,//)
104    FORMAT (20X,34HSECOND CASE, LOWER CASE M EQUAL 2.,//)
105    FORMAT (20X,33HTHIRD CASE, LOWER CASE M EQUAL 3.,//)
       DO 1 J=1,M
       DO 1 K=1,N
       DO 1 I=1,3
       COMG(J,K,I)=0.0
1      CONTINUE
       DO 2 J=2,M
       DO 2 K=2,N
       DO 2 I=1,3
       START=.1
       START1=1.
35     X(1)=START*(OMG(J,K,I))
       DO 3 L=1,2
       A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
       B(J,K,I)=.5*PI*SQRT(A(J,K,I))
       IF(A(J,K,I)-1.)30,31,30
30     C2=2.**28
       IF(B(J,K,I)-C2)300,301,301
31     DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
       GO TO 32
300    DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
      1)))))/TAN(B(J,K,I))
32     C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
       D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
       IF(ABS(D(J,K,I))-1.)33,33,34
34     START=START+.1
       IF(START-1.0)35,35,325
325    COMG(J,K,I)=.33333333
       GO TO 2001
33     F(J,K,I)=ASIN(D(J,K,I))
       BETA(J,K,I)=(2./PI)*(F(J,K,I))
       F1(L)=(OMG(J,K,I))/(ABS(2.-BETA(J,K,I)))
3      X(L+1)=F1(L)
4      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
       A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
       B(J,K,I)=.5*PI*SQRT(A(J,K,I))
       IF(A(J,K,I)-1.)36,37,36
36     C2=2.**28
       IF(B(J,K,I)-C2)360,401,401
37     DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
```

```
      GO TO 38
360   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
38    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)39,39,6
39    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(2.-BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)390,394,394
390   IF(ABS(X(3))-100.)392,393,393
392   C1=.001
      GO TO 395
393   C1=.01
      GO TO 395
394   C1=.1
395   IF(ABS(D1)-C1)5,5,6
6     X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-1000.)4,4,201
201   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)330,501,501
330   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)331,331,332
331   COMG(J,K,I)=.88888888
      GO TO 2001
332   COMG(J,K,I)=.11111111
      GO TO 2001
301   COMG(J,K,I)=.77777777
      GO TO 2001
401   COMG(J,K,I)=.99999999
      GO TO 2001
5     COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2001,501,501
501   COMG(J,K,I)=.66666666
2001  CONTINUE
2     CONTINUE
      WRITE (6,101)
      WRITE (6,103)
      DO 7 J=2,M
      DO 7 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
7     CONTINUE
      DO 8 J=2,M
      DO 8 K=2,N
      DO 8 I=1,3
      START=.6
      START1=1.
```

```
45      X(1)=START*(OMG(J,K,I))
        DO 9 L=1,2
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)40,41,40
40      C2=2.**28
        IF(B(J,K,I)-C2)400,302,302
41      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 42
400     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
42      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)43,43,44
44      START=START-.1
        IF(START)425,425,45
425     COMG(J,K,I)=.33333333
        GO TO 2002
43      F(J,K,I)=PI-ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(L)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
9       X(L+1)=F1(L)
10      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
        A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        B(J,K,I)=.5*PI*SQRT(A(J,K,I))
        IF(A(J,K,I)-1.)46,47,46
46      C2=2.**28
        IF(B(J,K,I)-C2)460,402,402
47      DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
        GO TO 48
460     DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
       1)))))/TAN(B(J,K,I))
48      C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
        D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
        IF(ABS(D(J,K,I))-1.)49,49,12
49      F(J,K,I)=PI-ASIN(D(J,K,I))
        BETA(J,K,I)=(2./PI)*(F(J,K,I))
        F1(3)=(OMG(J,K,I))/(ABS(BETA(J,K,I)))
        D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)490,494,494
490     IF(ABS(X(3))-100.)492,493,493
492     C1=.001
        GO TO 495
493     C1=.01
        GO TO 495
494     C1=.1
495     IF(ABS(D1)-C1)11,11,12
12      X(1)=X(2)
        F1(1)=F1(2)
        X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-1000.)10,10,202
202     A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
        IF(A(J,K,I)-1.0)430,502,502
430     B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
```

```
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)431,431,432
431   COMG(J,K,I)=.88888888
      GO TO 2002
432   COMG(J,K,I)=.11111111
      GO TO 2002
302   COMG(J,K,I)=.77777777
      GO TO 2002
402   COMG(J,K,I)=.99999999
      GO TO 2002
11    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2002,502,502
502   COMG(J,K,I)=.66666666
2002  CONTINUE
8     CONTINUE
      WRITE (6,101)
      WRITE (6,104)
      DO 13 J=2,M
      DO 13 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
13    CONTINUE
      DO 14 J=2,M
      DO 14 K=2,N
      DO 14 I=1,3
      START=.1
      START1=1.
55    X(1)=START*(OMG(J,K,I))
      DO 15 L=1,2
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(L)*X(L))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
      IF(A(J,K,I)-1.)50,51,50
50    C2=2.**28
      IF(B(J,K,I)-C2)500,303,303
51    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 52
500   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1)))))/TAN(B(J,K,I))
52    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)53,53,54
54    START=START+.1
      IF(START-1.0)55,55,525
525   COMG(J,K,I)=.33333333
      WRITE(6,4001)J,K,I,A(J,K,I),B(J,K,I),C(J,K,I),D(J,K,I),DELO(J,K,I)
4001  FORMAT(1X,3I5,5E12.8)
      GO TO 2003
53    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(L)=(OMG(J,K,I))/(ABS(2.+BETA(J,K,I)))
15    X(L+1)=F1(L)
16    X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
      A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      B(J,K,I)=.5*PI*SQRT(A(J,K,I))
```

```
      IF(A(J,K,I)-1.)56,57,56
56    C2=2.**28
      IF(B(J,K,I)-C2)560,403,403
57    DELO(J,K,I)=1.+(PI*PI*GAM*GAM)/64.
      GO TO 58
560   DELO(J,K,I)=1.+((PI*(A(J,K,I)**(3./2.))*GAM*GAM)/(16.*(1.-(A(J,K,I
     1))))))/TAN(B(J,K,I))
58    C(J,K,I)=SQRT(ABS(DELO(J,K,I)))
      D(J,K,I)=(C(J,K,I))*(SIN(B(J,K,I)))
      IF(ABS(D(J,K,I))-1.)59,59,18
59    F(J,K,I)=ASIN(D(J,K,I))
      BETA(J,K,I)=(2./PI)*(F(J,K,I))
      F1(3)=(OMG(J,K,I))/(ABS(2.+BETA(J,K,I)))
      D1=X(3)-F1(3)
      IF(ABS(X(3))-1000.)590,594,594
590   IF(ABS(X(3))-100.)592,593,593
592   C1=.001
      GO TO 595
593   C1=.01
      GO TO 595
594   C1=.1
595   IF(ABS(D1)-C1)17,17,18
18    X(1)=X(2)
      F1(1)=F1(2)
      X(2)=X(3)
      F1(2)=F1(3)
      START1=START1+1.
      IF(START1-100.)16,16,203
203   A(J,K,I)=(8.*PI*R*H*TO*CK)/(RI*X(3)*X(3))
      IF(A(J,K,I)-1.0)530,503,503
530   B(J,K,I)=(8.*PI*R*H*TO*CK)/(RI)
      C(J,K,I)=SQRT(B(J,K,I))
      C3=.01
      IF(ABS(C(J,K,I)-X(3))-C3)531,531,532
531   COMG(J,K,I)=.88888888
      GO TO 2003
532   COMG(J,K,I)=.11111111
      GO TO 2003
303   COMG(J,K,I)=.77777777
      GO TO 2003
403   COMG(J,K,I)=.99999999
      GO TO 2003
17    COMG(J,K,I)=ABS(X(3))
      IF(A(J,K,I)-1.)2003,503,503
503   COMG(J,K,I)=.66666666
2003  CONTINUE
14    CONTINUE
      WRITE (6,101)
      WRITE (6,105)
      DO 19 J=2,M
      DO 19 K=2,N
      J1=J-1
      K1=K-1
      WRITE (6,102) J1,K1,(COMG(J,K,I),I=1,3)
19    CONTINUE
      RETURN
```

END

```
$I FOR UNST13
C       SUBROUTINE FOR UNSTABLE VALUES OF THRUST FREQUENCY, EQUATION (10.2
      1-39)
C       NASA CONTRACT NAS8-11255 (LONGITUDINAL VIBRATION RESEARCH)
C       CONSIDERATION OF THREE (33) CASES, INVOLVING 9900 VALUES.
        SUBROUTINE UNST13(PI,RA,H,TO,OMG,M,N,E,TI)
        DIMENSION ALPH(4),X(4),F1(4),OMG(20,20,3),COMG(10000),BETT(4)
104     FORMAT (4I5,3E20.8//)
105     FORMAT(4I5,8X,A5,15X,A5,15X,A5//)
106     FORMAT(4I5,8X,A5,15X,A5,7X,1E20.8//)
107     FORMAT(4I5,8X,A5,7X,1E20.8,8X,A5//)
108     FORMAT(4I5,8X,A5,7X,2E20.8//)
109     FORMAT(4I5,1E20.8,8X,A5,15X,A5//)
110     FORMAT(4I5,1E20.8,8X,A5,7X,1E20.8//)
111     FORMAT(4I5,2E20.8,8X,A5//)
1000    FORMAT(1X,1E20.8)
103     FORMAT (20X,80HTABLE OF UNSTABLE VALUES OF THE THRUST FREQUENCY FO
      1R EQUATION (13), RAD PER SEC.,//,4X,2HJ,,3X,2HK,,3X,2HR,,3X,2HS,,8
      1X,10HCOMEGA (1),10X,10HCOMEGA (2),10X,10HCOMEGA (3),//)
        INTEGER T,R,S,S1,S2,JKIS,Z,IJKISF,Y,XZ,P,O
        REAL MEY,MEX,MEV,MEW,MEU,MEZ
        REAL INFIN
        DATA INFIN/5HINFIN/
        DO 4 J=2,M
        DO 4 K=2,N
        DO 4 I=1,3
        DO 4 R=1,3
        T=5
        C4=1.0E+6
        S1=2*T+1
        S=1
2       IJKISF=90*S1*(J-1)+9*S1*(K-1)+3*S1*I+S1*R+S-103*S1
        JKIS=IJKISF
        S=S-(T+1)
        IF(OMG(J,K,I)-1.0)19,20,20
20      IF(ABS(OMG(J,K,I))-14.0)17,18,18                                  4900
17      ALPH(R)=0.07                                                      1810
        MEV=TO
        MEU=H
        MEW=E
        MEX=TI
        MEY=RA
        DO 8 L=1,2
        X(L)=OMG(J,K,I)/ABS(ALPH(R)+FLOAT(S))
        MEZ=X(L)
        START1=1.0
        ME1=0
        CALL BEAM  (MEV,MEU,MEW,MEX,MEY,MEZ,BETT,OMG2,ME1,ME2)
        IF(X(L)-2.0*OMG2)12,12,13
13      CALL SORT (ALPH,BETT,ME2)
        F1(L)=OMG(J,K,I)/ABS(ALPH(R)+FLOAT(S))
        IF(F1(L)-C4)500,501,501
501     COMG(JKIS)=F1(L)
        GO TO 14
500     X(L+1)=F1(L)
```

```
      DO 6 R=1,3
      DO 6 S=1,S1
      J1=J-1
      K1=K-1
      S2=S-(T+1)
      Y=6*T+3+Z
      XZ=12*T+6+Z
      IF(COMG(Z)-C4)511,511,510
510   IF(COMG(Y)-C4)513,512,512
512   IF(COMG(XZ)-C4)515,514,514
513   IF(COMG(XZ)-C4)517,516,516
511   IF(COMG(Y)-C4)519,518,518
519   IF(COMG(XZ)-C4)521,520,520
518   IF(COMG(XZ)-C4)523,522,522
514   COMG(Z)=INFIN
      COMG(Y)=INFIN
      COMG(XZ)=INFIN
      WRITE(6,105)J1,K1,R,S2,COMG(Z),COMG(Y),COMG(XZ)
      GO TO 7
515   COMG(Z)=INFIN
      COMG(Y)=INFIN
      WRITE(6,106)J1,K1,R,S2,COMG(Z),COMG(Y),COMG(XZ)
      GO TO 7
516   COMG(Z)=INFIN
      COMG(XZ)=INFIN
      WRITE(6,107)J1,K1,R,S2,COMG(Z),COMG(Y),COMG(XZ)
      GO TO 7
517   COMG(Z)=INFIN
      WRITE(6,108)J1,K1,R,S2,COMG(Z),COMG(Y),COMG(XZ)
      GO TO 7
522   COMG(Y)=INFIN
      COMG(XZ)=INFIN
      WRITE(6,109)J1,K1,R,S2,COMG(Z),COMG(Y),COMG(XZ)
      GO TO 7
523   COMG(Y)=INFIN
      WRITE(6,110)J1,K1,R,S2,COMG(Z),COMG(Y),COMG(XZ)
      GO TO 7
520   COMG(XZ)=INFIN
      WRITE(6,111)J1,K1,R,S2,COMG(Z),COMG(Y),COMG(XZ)
      GO TO 7
521   WRITE(6,104)J1,K1,R,S2,COMG(Z),COMG(Y),COMG(XZ)
7     IF(S-S1)38,39,38
39    IF(R-3)38,40,38
40    Z=1+99*K1+990*(J1-1)
      GO TO 6
38    Z=Z+1
6     CONTINUE
      RETURN
      END
```

```
8       CONTINUE
        IF(ABS(X(3))-1000.)490,494,494
490     IF(ABS(X(3))-100.)492,493,493
492     C1=.001
        GO TO 495
493     C1=.01
        GO TO 495
494     C1=.1
495     IF(ABS(X(1)-F1(1)-X(2)+F1(2))-C1)9,9,10
10      X(3)=(X(1)*F1(2)-X(2)*F1(1))/(X(1)-F1(1)-X(2)+F1(2))
410     MEZ=X(3)
        CALL BEAM  (MEV,MEU,MEW,MEX,MEY,MEZ,BETT,OMG2,ME1,ME2)
        IF(X(3)-2.0*OMG2)12,12,15
15      CALL SORT (ALPH,BETT,ME2)
        F1(3)=OMG(J,K,I)/ABS(ALPH(R)+FLOAT(S))
        IF(F1(3)-C4)502,503,503
502     D1=X(3)-F1(3)
        IF(ABS(X(3))-1000.)390,394,394
390     IF(ABS(X(3))-100.)392,393,393
392     C1=.001
        GO TO 395
393     C1=.01
        GO TO 395
394     C1=.1
395     IF(ABS(D1)-C1)9,9,11
11      X(1)=X(2)
        F1(1)=F1(2)
406     X(2)=X(3)
        F1(2)=F1(3)
        START1=START1+1.
        IF(START1-5.0)10,10,21
21      ME1=ME1+1
401     IF(ME1-10)10,10,16
16      COMG(JKIS)=.22222222
        GO TO 14
12      COMG(JKIS)=.11111111
        WRITE(6,1000)OMG2
        GO TO 14
19      COMG(JKIS)=.00000000
        GO TO 14
503     COMG(JKIS)=F1(3)
        GO TO 14
18      IF(S)200,201,200
201     COMG(JKIS)=10000000.
        GO TO 14
200     COMG(JKIS)=OMG(J,K,I)/ABS(FLOAT(S))
        GO TO 14
9       COMG(JKIS)=X(3)
14      S=S+(T+2)
        IF(S1-S)4,2,2
4       CONTINUE
        WRITE(6,103)
        S1=2*T+1
        Z=1
        DO 6 J=2,M
        DO 6 K=2,N
```

```
$I FOR SORT
      SUBROUTINE SORT (ALPH,BETT,ME2)
      DIMENSION ALPH(4),BETT(4)
      PI=3.141592654
      IF(ME2)1,1,2
2     IF(BETT(1)-BETT(2))3,3,4
3     IF(BETT(1)-BETT(3))5,5,35
4     IF(BETT(1)-BETT(3))36,36,6
5     IF(BETT(2)-BETT(3))39,39,40
6     IF(BETT(2)-BETT(3))33,33,38
1     IF(ABS(BETT(1)-BETT(2))-.000009)31,41,41
41    IF(BETT(1)-BETT(2))30,31,32
30    IF(ABS(BETT(1)-BETT(3))-.000009)33,42,42
42    IF(BETT(1)-BETT(3))34,33,33
34    IF(ABS(BETT(2)-BETT(3))-.000009)35,43,43
43    IF(BETT(2)-BETT(3))35,35,36
31    IF(ABS(BETT(1)-BETT(3))-.000009)33,35,35
32    IF(ABS(BETT(1)-BETT(3))-.000009)38,44,44
44    IF(BETT(1)-BETT(3))38,38,37
37    IF(ABS(BETT(2)-BETT(3))-.000009)39,45,45
45    IF(BETT(2)-BETT(3))39,39,40
33    ALPH(1)=BETT(1)/(2.0*PI)
      ALPH(2)=BETT(2)/(2.0*PI)
      ALPH(3)=BETT(3)/(2.0*PI)
      GO TO 50
35    ALPH(1)=BETT(2)/(2.0*PI)
      ALPH(2)=BETT(3)/(2.0*PI)
      ALPH(3)=BETT(1)/(2.0*PI)
      GO TO 50
36    ALPH(1)=BETT(3)/(2.0*PI)
      ALPH(2)=BETT(2)/(2.0*PI)
      ALPH(3)=BETT(1)/(2.0*PI)
      GO TO 50
38    ALPH(1)=BETT(1)/(2.0*PI)
      ALPH(2)=BETT(3)/(2.0*PI)
      ALPH(3)=BETT(2)/(2.0*PI)
      GO TO 50
39    ALPH(1)=BETT(3)/(2.0*PI)
      ALPH(2)=BETT(1)/(2.0*PI)
      ALPH(3)=BETT(2)/(2.0*PI)
      GO TO 50
40    ALPH(1)=BETT(2)/(2.0*PI)
      ALPH(2)=BETT(1)/(2.0*PI)
      ALPH(3)=BETT(3)/(2.0*PI)
50    CONTINUE
      ME2=0
      RETURN
      END
```

```
$I FOR BEAM
C       SUBPROGRAM BEAM
        SUBROUTINE BEAM  (MEV,MEU,MEW,MEX,MEY,MEZ,BETT,OMG2,ME1,ME2)
        DIMENSION R(5),B(5),PH(5),DPH(5),DPHG(5)
        DIMENSION F(3,3),G(8,8),H(5,8,8),RTR(8), RTI(8)
        DIMENSION FF(3,3),FG(8,8),FH(5,8,8)
        DIMENSION BETT(4)
        INTEGER S,P,Q
        REAL MEY,MEX,MEV,MEW,MEU,MEZ,MES,MET
    5   FORMAT(I3,3E7.0,1E12.0,36X)
    6   FORMAT (4E12.0,24X)
    7   FORMAT(5E12.0,12X)
    8   FORMAT(3E12.0,36X)
        N=2
        XG=+0.5
        P1=1.0
        S1=5.0
        PI=3.141592654
        R(1)=4.7300408
        R(2)=7.8532046
        R(3)=10.9956078
        R(4)=14.1371655
        R(5)=17.2787596
        B(1)=-0.9825022
        B(2)=-1.0007773
        B(3)=-0.99996645
        B(4)=-1.00000145
        B(5)=-0.99999993
        PH(1)=2.0
        PH(2)=-2.0
        PH(3)=2.0
        PH(4)=-2.0
        PH(5)=2.0
        DPH(1)=9.29453
        DPH(2)=-15.71858
        DPH(3)=21.99045
        DPH(4)=-28.27433
        DPH(5)=34.55751
        DPHG(1)=+0.0
        DPHG(2)=+10.80066
        DPHG(3)=+0.0
        DPHG(4)=-20.017144
        DPHG(5)=+0.0
        FQ=1.0
        OMEGL=+100.0
        OMEGB=MEZ
        ETA=+0.0
        SPC=+1.0
        GAMMA=+0.05
        MET=MEV*4.0*MEU*MEU
        MES=MEW*PI*MEX*MEY*MEY*MEY
        TO=MET/MES
        LV = 0
        Q = FQ + .5
        P = P1
```

```
      S = S1 + .5
      FQ1 = FQ
      FETA = ETA
      FTO = TO
      FSPC = SPC
      FGAMMA = GAMMA
      FOMEGL = OMEGL
      N1 = N+1
      IF(SPC.EQ.0.0) N1=N
15    CALL   COEF     (R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P,Q,S,O
     1MEGL,LV,GA,F,G,H,DPHG)
      DO 10 J=1,N1
      DO 10 K=1,N1
      FF(J,K) = F(J,K)
      FG(J,K) = G(J,K)
      DO 9 I=1,5
    9 FH(I,J,K) = H(I,J,K)
   10 CONTINUE
   95 CONTINUE
      TOR4 = TO/R(1)**4
      DO 13 J=1,N1
      DO 13 K=1,N1
      F(J,K) = TOR4*FF(J,K)
      G(J,K) = TOR4*FG(J,K)
      IF(OMEGL.GT.50.0) G(J,K) = F(J,K)
      IF(J.EQ.(N+1)) GO TO 16
      IF(J.EQ.K) F(J,K) = F(J,K) + (R(J)/R(1))**4 - (ETA*ETA)/4.0
   16 CONTINUE
      DO 12 I=1,5
   12 H(I,J,K) = FH(I,J,K)*TOR4
   13 CONTINUE
      CALL HESSEN(F,N1)
      CALL QREIG(F,N1,RTR,RTI,0)
      N1M1 = N1-1
      DO 110 I=1,N1M1
      IP1 = I+1
      DO 110 J=IP1,N1
      IF(RTR(I)-RTR(J)) 110,110,120
  120 TEMP = RTR(I)
      RTR(I) = RTR(J)
      RTR(J) = TEMP
  110 CONTINUE
      DO 130 I=1,N1
  130 RTR(I) = SQRT(ABS(RTR(I)))
      OMG2=RTR(2)
      DO 14 J=1,N1
      DO 14 K=1,N1
      F(J,K) = TOR4*FF(J,K)
      IF(J.EQ.(N+1)) GO TO 14
      IF(J.EQ.K)F(J,K) = F(J,K) + (R(J)/R(1))**4 - (ETA*ETA)/4.0
   14 CONTINUE
   27 FORMAT( 8X7H(GAMMA=F4.2,            4H, N=I1,8H, OMEGL=F6.3,6H, ET
     1A=F6.3,6H, SPC=F5.2,5H, XG=F5.1,1H)/)
   90 FORMAT    (15X3HTO ,8X5HFREQ(I1,1H),8X5HFREQ(I1,1H),8X5HFREQ(I1,1H
     1)/)
  140 FORMAT(13X,1F5.1,3F15.6//)
```

```
   45 CONTINUE
  501 FORMAT (1H1)
  502 FORMAT(//22X,21HCHARACTERISTIC VALUES/22X,4H(TO=F4.1,4H LV=I1,4H G
     1A=F6.3,1H))
503   CONTINUE
504   CALL STAB (R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P,Q,S,OMEGL,L
     1V,GA,F,G,H,DPHG,ITS,STAB2,BETT,ME1,ME2)
      RETURN
      END
```

```
      GO TO 250
257   GG(1)=ROOTR(J)
      GG(2)=CKCON
      GG(3)=CKCON
272   JJ(1)=-2
      JJ(2)=200
      CC(1)=1.0E-15
      CC(2)=7.0472[illegible]9E-7
      AA(1)=A(4)
      AA(2)=A(3)
      AA(3)=A(2)
      AA(4)=A(1)
      CALL ROOTS (AA,GG,3,JJ,C,X)
      ROOTR(J)=REAL(X(1))
264   IF(ROOTR(J)-CKCON)251,252,252
251   BETT(J)=ATAN(SQRT(1.0-ROOTR(J)*ROOTR(J))/ROOTR(J))
      IF(ROOTR(J))280,250,250
280   ME2=1
1002  FORMAT(1X,1E20.8)
      GO TO 250
252   BETT(J)=ATAN(SQRT(1.0-CKCON*CKCON)/CKCON)
250   CONTINUE
      RMAX =  AMAX1(RMOD1(1),RMOD1(2),RMOD1(3),RMOD2(1),RMOD2(2),
     1RMOD2(3))
      STAB2 = ALOG(RMAX)*OMEGB/PI
      STAB1 = STAB2 - ETA
      IF(STAB1 - .1E-6) 1,1,2
    1 ITS = 1
      GO TO 3
    2 ITS = 2
   21 FORMAT(11X6HOMEGB=F6.3,5X6HSTABLE)
   22 FORMAT(11X6HOMEGB=F6.3,5X8HUNSTABLE)
3     CONTINUE
      RETURN
      END
```

```
GI FOR STAB
C       PROGRAM TO CALCULATE STABILITY CONSTANTS OF A UNIFORM BEAM SUB-
C       JECTED TO A PERIODICALLY VARYING THRUST WITH DIRECTIONAL CONTROL
        SUBROUTINE STAB (R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P,Q,S,O
       1MEGL,LV,GA,F,G,H,DPHG,ITS,STAB2,BETT,ME1,ME2)
        DIMENSION A(7), ROOTR(6), ROOTI(6)
        DIMENSION DT(6),ACHE(6),     Y(6),W(6)
        DIMENSION R(5),B(5),PH(5),DPH(5),DPHG(5),F(3,3),G(8,8),H(5,8,8)
        DIMENSION RMOD1(6),RMOD2(6)
        DIMENSION BETT(4)
        DIMENSION AA(51),GG(50),X(50),CC(2),JJ(6)
        COMPLEX AA,GG,X
        COMPLEX CZ,CARG1,CARG2,CSQRT
        INTEGER P,Q,S
        CKCON=.99999998
        N1 = N + 1
        IF(SPC.EQ.0.0)N1=N
        CALL DETER (DT,W,R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P,Q,S,O
       1MEGL,LV,GA,F,G,H,DPHG)
        DO 5  I=1,N1
        ACHE(I) = PI*SIN(2.*PI*W(I))*DT(I)/W(I)
        Y(I) = COS(2.*PI*W(I))
      5 CONTINUE
        A(1) = -1.
        A(2) = Y(1)+Y(2)+Y(3) - ACHE(1)-ACHE(2)-ACHE(3)
        A(3)=-(Y(1)*Y(2)+Y(1)*Y(3)+Y(2)*Y(3)-ACHE(1)*(Y(2)+Y(3))
       1-ACHE(2)*(Y(1)+Y(3))-ACHE(3)*(Y(1)+Y(2)))
        A(4) = Y(1)*Y(2)*Y(3) - ACHE(1)*Y(2)*Y(3) - ACHE(2)*Y(1)*Y(3) -
       1ACHE(3)*Y(1)*Y(2)
        CALL MULLER (A,ROOTR,ROOTI,3)
        DO 111 I=1,3
        IF(ABS(ROOTI(I)).LT..1E-4) ROOTI(I) = 0.0
        CZ = CMPLX(ROOTR(I),ROOTI(I))
        CARG1 = CZ     + CSQRT(CZ**2-1.)
        CARG2 = CZ -  CSQRT(CZ**2-1.)
        RMOD1(I) = CABS(CARG1)
  111   RMOD2(I) = CABS(CARG2)
        DO 250 J=1,3
 1000   FORMAT (1X,3E20.8)
        IF(ME1)270,270,271
 271    IF(ROOTR(J)-CKCON)273,252,252
 273    GG(1)=ROOTR(J)
        JJJ=J+1-(J*J-3*J+2)*3/2
        JJG=JJJ+1-(JJJ*JJJ-3*JJJ+2)*3/2
        GG(2)=ROOTR(JJJ)
        GG(3)=ROOTR(JJG)
        GO TO 272
 270    IF(ABS(ROOTR(1)-ROOTR(2))-.00001)253,254,254
 253    IF(ABS(ROOTR(1)-ROOTR(3))-.00001)255,256,256
 254    IF(ABS(ROOTR(1)-ROOTR(3))-.00001)259,260,260
 260    IF(ABS(ROOTR(2)-ROOTR(3))-.00001)263,264,264
 259    IF(J*J-4*J+3)257,255,257
 263    IF(J*J-5*J+6)257,255,257
 256    IF(J*J-3*J+2)257,255,257
 255    BETT(J)=ATAN(SQRT(1.0-CKCON*CKCON)/CKCON)
```

```
$I FOR DETER
C       SUBPROGRAM TO EVALUATE AN INFINITE DETERMINANT
        SUBROUTINE DETER (DT,W,R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P
       1,Q,S,OMEGL,LV,GA,F,G,H,DPHG)
        DIMENSION R(5),B(5),PH(5),DPH(5),DPHG(5)
        DIMENSION A(100,100),DT(6),F(3,3),G(8,8),H(5,8,8)
        DIMENSION FF(3,3),FG(8,8),FH(5,8,8)
        DIMENSION W(6)
        INTEGER P,Q,S
        SOMEGB = OMEGB**2
        N1=N+1
        IF (SPC.EQ.0.0) N1 = N
        IQ = Q
        Q = 1
        IF(LV) 11,12,11
11      CALL   COEF     (R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P,Q,S,O
       1MEGL,LV,GA,FF,FG,FH,DPHG)
        TOR4=TO/R(1)**4
        DO 13 J=1,N1
        DO 13 K=1,N1
   13   G(J,K)=FG(J,K)*TOR4
   12   CONTINUE
        DO105 I1=1,N1
        W(I1) = SQRT(ABS(   F(I1,I1)        ))/(OMEGB    )
        M1 = (SQRT(ABS(F(N,N))) + SQRT(ABS(F(I1,I1))))/OMEGB + 1.0
        MO= 2*M1+1
        L = 0
    2   L = L + 1
        DO100 M=1,MO
        DO100 J=1,N1
        DO100 K=1,N1
        IO = M1-L+1
        E = IO
        DENOM = -(W(I1) - E)**2 + (   F(J,J)        )/(SOMEGB    )
        IF(I1.NE.J) GO TO 4
        IF (IO) 4,3,4
    3   DENOM = 1.0
    4   J1= J+(L-1)*N1
        K1= K+(M-1)*N1
        A(J1,K1) = 0.0
        IF(L-M)10,5,10
    5   IF(J1-K1)55,50,55
   10   IF (IABS(L-M)-1) 20,60, 20
   20   IF(Q.EQ.1) GO TO 100
        DO 30 I=1,5
        IF(IABS(L-M).EQ.I*Q) A(J1,K1) = .5*H(I,J,K)/(DENOM*SOMEGB)
   30   CONTINUE
        GO TO 100
   50   A(J1,K1)= 1
        IF(I1.NE.J) GO TO 100
        IF (IO) 100,54,100
   54   A(J1,K1) = 0.0
        GO TO 100
   55   A(J1,K1)= F(J,K)/(DENOM*SOMEGB)
        GO TO 100
```

```
 60 A(J1,K1)= .5*GAMMA*G(J,K)/(DENOM*SOMEGB)
100 CONTINUE
    IF(L.LT.MO) GO TO 2
    MO = N1*MO
    CALL GASDET(A,MO,DET)
    DT(I1)=DET
105 CONTINUE
    Q = IQ
    RETURN
    END
```

```
GI FOR COEF
      SUBROUTINE COEF (R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P,Q,S,O
     1MEGL,LV,GA,F,G,H,DPHG)
      DIMENSION R(5),B(5),PH(5),DPH(5),DPHG(5),A(5)
      DIMENSION F(3,3),G(8,8),H(5,8,8),FJK(8,8),GJK(8,8),HIJK(5,8,8)
      INTEGER S,P,Q
      COMPLEX  CI,Z,CG,CCOS,CSIN,CSQRT
      CI = (0.0,1.0)
      N1=N+1
      SIG = OMEGB/OMEGL
      CALL    INTEG   (R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P,Q,S,O
     1MEGL,LV,GA,F,G,H,DPHG,FJK,GJK,HIJK)
      SOMEGB = OMEGB**2
      X = SIG
C     COMPUTE  A ARRAY
      DO 1  I=1,S
      SGNI = (-1.)**I
      XI= I
    1 A(I) =  2.*SGNI*(1.0/XI**2 + GAMMA/(XI**2-X**2))/PI**2
      X = PI*SIG
      DO 15 J=1,N
      DO 15 K=1,N
   10 F(J,K)=        PH(J)*DPH(K)-FJK(J,K)+SPC*PH(J)*DPHG(K)
   11 G(J,K)=        PH(J)*DPH(K)-GJK(J,K)+SPC*PH(J)*DPHG(K)
      DO 12 I=1,S
      XI= I
   12 H(I,J,K) =  A(I)*(-1.0 )*HIJK(I,J,K)
   15 CONTINUE
      DO 20 K=1,N
      F(N+1,K)= 12.*      (.5*DPH(K)-PH(K)+.5*SPC*DPHG(K))
      G(N+1,K)= 12.*      (.5*DPH(K)-PH(K)+GJK(N+1,K)+.5*SPC*DPHG(K) )
      DO 20 I=1,S
      XI= I
   20 H(I,N+1,K)=       -A(I)*12.*      HIJK(I,N+1,K)
      DO 25 J=1,N
      F(J,N+1)=       SPC*PH(J)
      G(J,N+1)=        GJK(J,N+1)+SPC*PH(J)
      DO 25 I=1,S
      XI=I
   25 H(I,J,N+1)=(-1.0 )* A(I)*HIJK(I,J,N+1)
      F(N+1,N+1)= 6.*SPC
      IF(LV)27,26,27
   26 CONTINUE
      G(N+1,N+1)= 12.*      (.5-(1.0-COS(X))/(X*SIN(X))+.5*SPC)
      GO TO 28
   27 CONTINUE
      Z = PI*CSQRT(SIG**2 - CI*GA)
      CG = (1.0 - CCOS(Z))/(Z*CSIN(Z))
      G(N+1,N+1) = 12.*(.5 - REAL(CG) + .5*SPC)
   28 CONTINUE
      DO 30 I=1,S
      SGNI = (-1.)**I
      XI= I
   30 H(I,N+1,N+1)= 12.*      (SGNI-1.)* A(I)/(PI*XI)
      RETURN
```

END

```
@I FOR GASDET
      SUBROUTINE GASDET(A,N,DET)
      DIMENSION A(100,100)
      EQUIVALENCE (D,ID),(E,IE)
      DATA IG/1207959552/
      DET=1.
      NM1=N-1
      DO 2 K=1,NM1
      TEST=ABS(A(K,K))
      KP1=K+1
      L=K
      DO 4 I=KP1,N
      IF (TEST.GE.ABS(A(I,K))) GO TO 4
      TEST=ABS(A(I,K))
      L=I
 4    CONTINUE
      IF (L.EQ.K) GO TO 3
      DO 5 J=K,N
      TEMP=A(L,J)
      A(L,J)=A(K,J)
 5    A(K,J)=TEMP
      DET = -DET
 3    DET=DET*A(K,K)
      A(K,K)=1./A(K,K)
      DO 6 J=KP1,N
      A(K,J)=A(K,J)*A(K,K)
 6    CONTINUE
      DO 7 I=KP1,N
      FAC=A(I,K)
      A(I,K)=-A(I,K)*A(K,K)
      DO 8 J=KP1,N
      E=A(I,J)
      D=E-FAC*A(K,J)
      IF (IABS(IE)-IABS(ID).GT.IG) D=0.
      A(I,J)=D
 8    CONTINUE
 7    CONTINUE
 2    CONTINUE
      DET=DET*A(N,N)
      RETURN
      END
```

```
$I FOR INTEG
C     SUBROUTINE TO EVALUATE  INTEGRALS
      SUBROUTINE INTEG(R,B,PH,DPH,N,PI,XG,SPC,TO,OMEGB,GAMMA,ETA,P,Q,S,O
     1MEGL,LV,GA,F,G,H,DPHG,FJK,GJK,HIJK)
      DIMENSION R(5),B(5),PH(5),DPH(5),DPHG(5),F(3,3),G(8,8),H(5,8,8)
      DIMENSION FJK(8,8),CGJK(8,8),GJK(8,8),CHIJK(5,8,8),HIJK(5,8,8)
      INTEGER P,Q,S
      COMPLEX CGJK,CHIJK,CI,CA,CB,CEXP
      COMPLEX CX1,CX2,CX3
      SIG = OMEGB/OMEGL
      COMPLEX     Z,CG,CCOS,CSIN,CSQRT
      X=PI*SIG
      Z = PI*CSQRT(SIG**2 - CI*GA)
      DO 31 J=1,N
      DO 31 K=1,N
      CGJK(J,K) = (0.0,0.0)
      DO 4 I = 1,5
    4 CHIJK(I,J,K) = (0.0,0.0)
      SGN= (-1.)**(J+K)
      IF(K-J)25,5,10
    5 FJK(J,K)= (B(J)*R(J)/2.0)*(B(J)*R(J)-6.0)
      GO TO 15
   10 FJK(J,K)= (4.*SGN*R(J)*R(K)/(R(K)**4-R(J)**4))*(-B(J)*R(K)**3+B(K)
     1*R(J)**3)+((16.0*R(J)**4*R(K)**4)/(R(K)**4-R(J)**4)**2)*(SGN-1.0)
   15 DO 20 L=1,4
      DO 20 M=1,4
      CI= (0.0,1.0)
      CA= R(J)*CI**(L-1) + R(K)*CI**(M-1)
      CB= (R(J)*R(K)/4.0)*(B(J) + CI**(L-1))*(B(K) + CI**(M-1))
      IF(LV)22,21,22
   21 CONTINUE
      CGJK(J,K) = CGJK(J,K) + (CB/(CA**2+X**2))*((CA*SIN(X)-X*COS(X))*
     1CEXP(CA) + X)/SIN(X)
      GO TO 23
   22 CONTINUE
      CGJK(J,K) = CGJK(J,K) + (CB/(CA**2+Z**2))*((CA*CSIN(Z)-Z*CCOS(Z))*
     1CEXP(CA) + Z)/CSIN(Z)
   23 CONTINUE
      DO 20 I=1,S
      SGNI = (-1.)**I
      Y = PI*FLOAT(I)
      CHIJK(I,J,K)=CHIJK(I,J,K)+(CB*Y/(CA**2+Y**2))*(1.0-SGNI*CEXP(CA))
   20 HIJK(I,J,K) = REAL (CHIJK(I,J,K))
      GJK(J,K) = REAL(CGJK(J,K))
      GO TO 30
   25 FJK(J,K)= FJK(K,J)
      CGJK(J,K)= CGJK(K,J)
      DO 26 I=1,S
      CHIJK(I,J,K)= CHIJK(I,K,J)
   26  HIJK(I,J,K)= REAL(CHIJK(I,J,K))
       GJK(J,K)= REAL(CGJK(J,K))
   30 IF(SPC.EQ.0.0) GJK(J,K) = FJK(J,K)
   31 CONTINUE
      DO35 K=1,N
      XK = 1.0/(R(K)**4-X**4)
```

```
      SGNK = (-1.)**K
      IF(LV)33,32,33
   32 CONTINUE
      GJK( N+1,K)= XK*((X**3)*DPH(K)*(SGNK-COS(X))-(X**4)*PH(K)*SIN(X))/
     1SIN(X)
      GO TO 24
   33 CONTINUE
      CG = 1.0/(R(K)**4 - Z**4)
      CG =CG*((Z**3)*DPH(K)*(SGNK-CCOS(Z))-(Z**4)*PH(K)*CSIN(Z))/CSIN(Z)
      GJK(N+1,K) = REAL(CG)
   24 CONTINUE
      DO34 I=1,S
      SGNI = (-1.)**I
      Y = PI*FLOAT(I)
      YK = 1.0/(R(K)**4-Y**4)
   34 HIJK(I,N+1,K) = YK*(Y**3)*DPH(K)*(SGNI-SGNK)
   35 CONTINUE
      DO 40 J=1,N
      XJ = 1.0/(R(J)**4-X**4)
      SGNJ = (-1.)**J
      IF(LV)37,36,37
   36 CONTINUE
      GJK(J,N+1)= (-XJ)*((X**2)*DPH(J)*SIN(X)+(X**3)*PH(J)*(COS(X)+
     1SGNJ))/SIN(X)
      GO TO 38
   37 CONTINUE
      CG =  1.0/(R(J)**4 - Z**4)
      CG = -CG*(Z**2*DPH(J)*CSIN(Z)+(Z**3)*PH(J)*(CCOS(Z)+SGNJ))/CSIN(Z)
      GJK(J,N+1) = REAL(CG)
   38 CONTINUE
      DO 39 I=1,S
      SGNI = (-1.)**I
      Y = PI*FLOAT(I)
      YJ = 1.0/(R(J)**4-Y**4)
   39 HIJK(I,J,N+1)=YJ*(Y**3)*DPH(J)*(SGNI-SGNJ)
   40 CONTINUE
      RETURN
      END
```

```
$I FOR MULLER
      SUBROUTINE MULLER(COE,ROOTR,ROOTI,N1)
      DIMENSION COE(7),ROOTR(6),ROOTI(6)
      N2=N1+1
      N4=0
      I=N1+1
   19 IF(COE(I))9,7,9
    7 N4=N4+1
      ROOTR(N4)=0.0
      ROOTI(N4)=0.0
      I=I-1
      IF(N4-N1)19,37,19
    9 CONTINUE
   10 AXR=0.8
      AXI=0.0
      L=1
      N3=1
      ALP1R=AXR
      ALP1I=AXI
      M=1
      GOTO99
   11 BET1R=TEMR
      BET1I=TEMI
      AXR=0.85
      ALP2R=AXR
      ALP2I=AXI
      M=2
      GOTO99
   12 BET2R=TEMR
      BET2I=TEMI
      AXR=0.9
      ALP3R=AXR
      ALP3I=AXI
      M=3
      GOTO99
   13 BET3R=TEMR
      BET3I=TEMI
   14 TE1=ALP1R-ALP3R
      TE2=ALP1I-ALP3I
      TE5=ALP3R-ALP2R
      TE6=ALP3I-ALP2I
      TEM=TE5*TE5+TE6*TE6
      TE3=(TE1*TE5+TE2*TE6)/TEM
      TE4=(TE2*TE5-TE1*TE6)/TEM
      TE7=TE3+1.0
      TE9=TE3*TE3-TE4*TE4
      TE10=2.0*TE3*TE4
      DE15=TE7*BET3R-TE4*BET3I
      DE16=TE7*BET3I+TE4*BET3R
      TE11=TE3*BET2R-TE4*BET2I+BET1R-DE15
      TE12=TE3*BET2I+TE4*BET2R+BET1I-DE16
      TE7=TE9-1.0
      TE1=TE9*BET2R-TE10*BET2I
      TE2=TE9*BET2I+TE10*BET2R
      TE13=TE1-BET1R-TE7*BET3R+TE10*BET3I
```

```
      TE14=TE2-BET1I-TE7*BET3I-TE10*BET3R
      TE15=DE15*TE3-DE16*TE4
      TE16=DE15*TE4+DE16*TE3
      TE1=TE13*TE13-TE14*TE14-4.0*(TE11*TE15-TE12*TE16)
      TE2=2.0*TE13*TE14-4.0*(TE12*TE15+TE11*TE16)
      TEM=SQRT(TE1*TE1+TE2*TE2)
      IF(TE1)113,113,112
  113 TE4=SQRT(.5*(TEM-TE1))
      TE3=.5*TE2/TE4
      GO TO 111
  112 TE3=SQRT(.5*(TEM+TE1))
      IF(TE2)110,200,200
  110 TE3=-TE3
  200 TE4=.5*TE2/TE3
  111 TE7=TE13+TE3
      TE8=TE14+TE4
      TE9=TE13-TE3
      TE10=TE14-TE4
      TE1=2.0*TE15
      TE2=2.0*TE16
      IF(TE7*TE7+TE8*TE8-TE9*TE9-TE10*TE10)204,204,205
  204 TE7=TE9
      TE8=TE10
  205 TEM=TE7*TE7+TE8*TE8
      TE3=(TE1*TE7+TE2*TE8)/TEM
      TE4=(TE2*TE7-TE1*TE8)/TEM
      AXR=ALP3R+TE3*TE5-TE4*TE6
      AXI=ALP3I+TE3*TE6+TE4*TE5
      ALP4R=AXR
      ALP4I=AXI
      M=4
      GO TO 99
   15 N6=1
   38 IF(ABS(HELL)+ABS(BELL)-1.E-20)18,18,16
   16 TE7=ABS(ALP3R-AXR)+ABS(ALP3I-AXI)
      IF(TE7/(ABS(AXR)+ABS(AXI))-1.E-7)18,18,17
   17 N3=N3+1
      ALP1R=ALP2R
      ALP1I=ALP2I
      ALP2R=ALP3R
      ALP2I=ALP3I
      ALP3R=ALP4R
      ALP3I=ALP4I
      BET1R=BET2R
      BET1I=BET2I
      BET2R=BET3R
      BET2I=BET3I
      BET3R=TEMR
      BET3I=TEMI
      IF(N3-100)14,18,18
   18 N4=N4+1
      ROOTR(N4)=ALP4R
      ROOTI(N4)=ALP4I
      N3=0
   41 IF(N4-N1)30,37,37
   37 RETURN
```

```
 30 IF(ABS(ROOTI(N4))-1.E-5)10,10,31
 31 GO TO(32,10),L
 32 AXR=ALP1R
    AXI=-ALP1I
    ALP1I=-ALP1I
    M=5
    GO TO 99
 33 BET1R=TEMR
    BET1I=TEMI
    AXR=ALP2R
    AXI=-ALP2I
    ALP2I=-ALP2I
    M=6
    GO TO 99
 34 BET2R=TEMR
    BET2I=TEMI
    AXR=ALP3R
    AXI=-ALP3I
    ALP3I=-ALP3I
    L=2
    M=3
 99 TEMR=COE(1)
    TEMI=0.0
    DO100I=1,N1
    TE1=TEMR*AXR-TEMI*AXI
    TEMI=TEMI*AXR+TEMR*AXI
100 TEMR=      TE1+COE(I+1)
    HELL=TEMR
    BELL=TEMI
 42 IF(N4)102,103,102
102 DO101I=1,N4
    TEM1=AXR-ROOTR(I)
    TEM2=AXI-ROOTI(I)
    TE1=TEM1*TEM1+TEM2*TEM2
    TE2=(TEMR*TEM1+TEMI*TEM2)/TE1
    TEMI=(TEMI*TEM1-TEMR*TEM2)/TE1
101 TEMR=TE2
103 GO TO(11,12,13,15,33,34),M
    END
```

```
$I FOR ROOTS,ROOTS
      SUBROUTINE ROOTS(A,G,N,J,C,X)
C
C     ROOTS OF A POLYNOMIAL SUBROUTINE.
C
      DIMENSION A(51), G(50), X(50), AA(51), J(6), C(2)
      COMPLEX A,G,X,AA,   X1,      FUNCT,DERIV,Q1,Q2
      LOGICAL FLOGIC,DLOGIC,ELOGIC,RLOGIC
C
C     PROGRAM INITIALIZATION
C
      IF (J(1).LT.0) GO TO 1004
      CONST1 = 1.0E-15
      CONST2 = 1.0596381E-7
      ICON = 120
      GO TO 1003
 1004 CONST1 = C(1)
      CONST2 = (C(2))**2
      ICON = J(2)
 1003 N2 = N+1
      NUM = 1
      DO 1 K = 1,N
      AA(K) = A(K)
1     X(K) = G(K)
      AA(N2) = A(N2)
      J(3) = 0
      J(4) = 0
      J(5) = 0
      J(6) = 0
      CALL OVERFL (INDCT1)
      I = 0
C
C     BEGIN COMPUTATION
C
2     I = I+1
      IF (I.EQ.N) GO TO 17
    3 N1 = I+1
      ELOGIC = .FALSE.
      RLOGIC = .FALSE.
C
C     COMPUTATION OF F(X) AND DERIVATIVE OF F(X)
C
4     DO 11 L = 1,ICON
      FUNCT = AA(N2)
      DERIV = FUNCT
      DO 5 K = N,N1,-1
      FUNCT = AA(K) + X(I) * FUNCT
5     DERIV = FUNCT + X(I) * DERIV
      FUNCT = AA(N1-1) + X(I) * FUNCT
C
C     CHECK FOR OVERFLOW IN F(X) AND DERIVATIVE OF F(X)
C
      CALL OVERFL (INDCT1)
      IF (INDCT1.EQ.1) GO TO 23
C
```

```
C     CHECK FOR F(X) = 0 AND THE DERIVATIVE OF F(X) = 0
C
      DLOGIC = .FALSE.
      FLOGIC = .FALSE.
      IF (REAL(FUNCT)) 91, ,91
      IF (AIMAG(FUNCT)) 91, ,91
      IF (NUM.EQ.2) GO TO 21
      FLOGIC = .TRUE.
91    IF (REAL(DERIV)) 92, ,92
      IF (AIMAG(DERIV)) 92, ,92
      DLOGIC = .TRUE.
92    IF (.NOT.DLOGIC.AND..NOT.FLOGIC) GO TO 93
      IF (.NOT.FLOGIC) GO TO 26
      IF (DLOGIC) GO TO 13
      GO TO 14
C
C     COMPUTATION OF A NEW ITERATE AND TEST FOR OVERFLOW
C
93    Q1 = FUNCT/DERIV
      X1 = X(I) - Q1
      CALL OVERFL (INDCT1)
      IF (INDCT1.EQ.1) GO TO 26
      IF (L.LE.2) GO TO 10
C
C     TEST FOR CONVERGENCE
C
      IF(((REAL(Q1))**2+(AIMAG(Q1))**2)-(((REAL(X1))**2+(AIMAG(X1))**2)*
     1CONST2)) , ,10
      IF (((REAL(Q1))**2+(AIMAG(Q1))**2)-((REAL(Q2))**2+(AIMAG(Q2))**2))
     110, ,
      GO TO (14,21),NUM
10    Q2 = Q1
11    X(I) = X1
C
C     ITERATION DIVERGED UNLESS NUM = 1 AND ELOGIC = .FALSE.
C
      IF (NUM.EQ.1) GO TO 29
      IF (J(5).EQ.0) J(5) = I
      GO TO 22
   29 IF (ELOGIC) J(3) = I
C
C     USE A NEW FIRST ESTIMATE X(I) IF ELOGIC = .FALSE.
C
12    IF (ELOGIC) GO TO 18
1012  ELOGIC = .TRUE.
      X(I) = (1.0,1.0)
      GO TO 4
13    RLOGIC = .TRUE.
C
C     REDUCING POLYNOMIAL BY SYNTHETIC DIVISION
C
14    DO 15 K = N,N1,-1
15    AA(K) = AA(K) + X(I) * AA(K+1)
C
C     CHECK FOR MULTIPLE ROOTS
C
```

```
      IF (.NOT.RLOGIC) GO TO 16
      X(I+1) = X(I)
      GO TO 2
C
C     COMPUTATION FOR POSSIBLE CONJUGATE ESTIMATE
C
   16 IF (IABS(J(1)).EQ.1) GO TO 2
      IF (ABS(AIMAG(X(I)))-1.0E-5) 2,2,
      IF (I.EQ.1) GO TO 98
      IF (REAL(G(I))-REAL(X(I-1))) 98, ,98
      IF (AIMAG(G(I))+AIMAG(X(I-1))) ,2,
98    X(I+1) = CONJG(X(I))
      GO TO 2
C
C     CALCULATION OF ROOT X(N)
C
17    X(N) = -AA(N)/AA(N2)
      CALL OVERFL (INDCT1)
      IF (INDCT1.EQ.1) GO TO 1017
      NORD = I
      IF (N.EQ.1) GO TO 21
      GO TO 19
 1017 J(3) = I
      J(4) = -1
      IF (N.EQ.1) RETURN
18    NORD = I-1
C
C     IMPROVING THE ROOTS BY ITERATION ON THE ORIGINAL POLYNOMIAL
C
19    NUM = 2
      DO 20 K = 1,N
20    AA(K) = A(K)
      N1 = 2
      I = 0
      GO TO 22
C
C     CHECK FOR UNDERFLOW LESS THAN CONST1 AND SET EQUAL TO ZERO
C
21    IF (CONST1) 22,22,
      IF (ABS(REAL(X(I)))-ABS(AIMAG(X(I)))) 1000,22,
      IF (ABS(AIMAG(X(I))).LT.CONST1) X(I) = CMPLX(REAL(X(I)),0.0)
      GO TO 22
1000  IF (ABS(REAL(X(I))).LT.CONST1) X(I) = CMPLX(0.0,AIMAG(X(I)))
22    IF (NORD.EQ.I) RETURN
      I = I+1
      GO TO 4
C
C     SET J(K) (K=3,4,5,6) TO SHOW OVERFLOW IN F OR ITS DERIVATIVE
C
23    IF (NUM.EQ.1) GO TO 24
      IF (J(5).NE.0) GO TO 22
      J(5) = 1
      J(6) = 1
      GO TO 22
24    IF (.NOT.ELOGIC) GO TO 1012
      J(3) = I
```

```
      J(4) = 1
      GO TO 18
26    IF (NUM.EQ.1) GO TO 27
      IF (J(5) .NE.0)GO TO 22
      J(5) = I
      J(6) = -1
      GO TO 22
27    IF (.NOT.ELOGIC) GO TO 1012
      J(3) = I
      J(4) = -1
      GO TO 18
      END
```

```
.MI MAP PRSN
        SEG NAFDEQ-*(A,B,C,D,E)-F
A       SEG UNST1-UNST2-SORT-BEAM-STAB-DETER-COEF-INTEG-GASDET-MULLER-ROO
      1TS-UNST3
B       SEG UNST4-UNST5-UNST6
C       SEG UNST7-UNST8
D       SEG UNST9-UNST10-UNST11
E       SEG UNST12-UNST13-SORT-BEAM-STAB-DETER-COEF-INTEG-GASDET-MULLER-R
      100TS
F       SEG NXPAF$-NXPAX$-EXP-ALOG-ATAN
```

```
@I MAP PRSN
      CHN      1
      SEG      NAFDEQ-NAFREQ-GREIG-GRT-HESSEN
      CHN      2
      SEG      DR1-UNST1-UNST2-SORT-BEAM-STAB-DETER-COEF-INTEG-GASDET-MUL
     1LER-ROOTS-UNST3
      CHN      3
      SEG      DR2-UNST4-UNST5-UNST6
      CHN      4
      SEG      DR3-UNST7-UNST8
      CHN      5
      SEG      DR4-UNST9-UNST10-UNST11
      CHN      6
      SEG      DR5-UNST12-UNST13-SORT-BEAM-STAB-DETER-COEF-INTEG-GASDET-M
     1ULLER-ROOTS
@I    FOR      DR1
      CALL UNST1 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST2 (PI,R,H,TO,OMG,M,N,E,T)
      CALL UNST3 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL CHAIN(3)
      END
@I    FOR      DR2
      CALL UNST4 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST5 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST6 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL CHAIN(4)
      END
@I    FOR      DR3
      CALL UNST7 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST8 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL CHAIN(5)
      END
@I    FOR      DR4
      CALL UNST9 (PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST10(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST11(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL CHAIN(6)
      END
@I    FOR      DR5
      CALL UNST12(PI,R,H,TO,CK,RI,GAM,OMG,M,N)
      CALL UNST13(PI,R,H,TO,OMG,M,N,E,T)
      CALL CHAIN(1)
      END
```

```
@ XQT PRSN
   480.        180.        1.0         2.591E-4      1.0E7      .3
   11    11      1.0E6     0.05        1.0         0.1          1.0
   480.        180.        1.0         2.591E-4      1.0E7      .333
   11    11      1.0E6     0.05        1.0         0.1          1.0
   990.        126.        .3125       2.591E-4      1.0E7      .333
   11    11      1.0E6     0.05        1.0         0.1          1.0
  2190.        198.        .5000       2.591E-4      1.0E7      .333
   11    11      1.0E6     0.05        1.0         0.1          1.0
@ FIN
@ FIN
```

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