ON THE ATMOSPHERIC OSCILLATIONS EXCITED BY TURBULENCE

SHOJI KATO*

Institute for Space Studies

Goddard Space Flight Center, NASA

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*NAS-NASA Resident Research Associate on leave of absence from the University of Tokyo.

ABSTRACT

The atmospheric velocity field excited by turbulence is studied for frequencies near the critical frequency of an atmosphere. The atmosphere is assumed to be isothermal, convectively neutral, and unbounded. It is shown that in the solar atmosphere the power spectrum of the velocity correlation of the acoustic field can have a local peak for frequencies near the critical frequency of the atmosphere. With the help of a model of the solar convection zone, it is found that the intensity of the acoustic field near the peak agrees with that of the oscillatory motion actually. In addition, the acoustic field excited agrees qualitatively with observations in other points: the period of the oscillation decreases with increasing height; the vector velocity field consists mainly of a vertical component.

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I. INTRODUCTION

Biermann (1946, 1948) and Schwarzschild (1948) suggested that the mechanical energy supply from the sub-photospheric turbulent layer is important to an understanding of the character of the upper solar atmosphere. Thereafter, Lighthill (1952,1954) in considering the problem of aerodynamical generation of waves showed that in a homogeneous medium waves are mainly emitted by the fluctuating turbulent stress (quadrupole radiation). However, in stellar atmospheres where turbulence occurs on a scale comparable with the scale height of density, effects of inhomogeneity due to gravity become important in the problem. This problem has been studied by Moore and Spiegel (1964) for a time-harmonic point source. Estimation of source terms resulting from turbulent motions in a stratified atmosphere was made by Unno and Kato (1962, Kato 1963, Unno 1964).

An interesting feature of the problem of wave generation in stratified atmospheres is the existence of a frequency range in which waves cannot propagate (e.g. Bjerknes, Bjerknes, Salberg and Bergeron 1933). Just above the upper frequency of this range the energy flux radiated by acoustic waves is negligible because the group velocity tends to zero there.

Therefore, the bulk of the radiated acoustic energy can propagate outward only in the higher frequency range. This, however, does not mean that the kinetic energy density excited near the critical frequency is negligible. Indeed, we shall see that the non-linear sources in turbulence have a monopole component if the atmosphere is stratified (see also Unno 1964). Therefore, even at frequencies near the critical frequency the atmosphere far from the source can respond to turbulence through the monopole source. In addition, the kinetic energy density excited near the critical frequency by the monopole source can be expected to decrease with increasing frequency because the main energy in turbulent motions in the solar sub-photospheric layer is probably in frequency components somewhat below the critical frequency (e.g. Behng and Schwarzschild 1961). On the other hand, below the critical frequency, the kinetic energy density due to oscillations generated by turbulence decreases with increasing distance from the source since waves are space damped in this frequency range. Thus, if observed at some distance from the turbulent source, we can expect a peak in the kinetic energy spectrum for frequencies near the critical frequency.

The motivation of these calculations is that in the solar atmospheric velocity field, oscillatory motions with frequencies near the critical frequency of the atmosphere have been observed (Leighton, Noyes and Simon 1962; Evans and Michard 1962; Howard 1962; Evans, Michard and Servajean 1963; Jensen and Orrall 1963; Noyes and Leighton 1963). Hence, it is of interest to study quantitatively whether we can explain the observed intensity of the oscillatory motions by means of excitation due to turbulence in the solar convection zone.

Before turning to the calculations themselves, we should mention the essential difference between our approach and that of some authors (Noyes and Leighton 1963; Evans and Michard 1963) who have considered the resonance phenomenon at the critical frequency as the origin of the observed oscillatory motions. In his study of an unbounded and isothermal atmosphere, Lamb (1908) showed that for time-harmonic vertical disturbances of force over infinite horizontal planes, there is a resonance phenomenon at the critical frequency with response of infinite amplitude. In actual cases, however, the horizontal sizes of disturbances are finite, and in this case, the resonance phenomenon does not occur even if the disturbances have exactly the frequency in question, as Lamb (1908)

himself pointed out (also see Kato, 1965). Especially if the atmosphere is convectively neutral and if disturbances are point sources, there is no increased efficiency for the response of the atmosphere to those disturbances whose frequencies are equal to the critical frequency.

Furthermore, we should mention that in the actual solar atmosphere the temperature reaches a relative minimum slightly above the convection zone and increases as one goes in the photosphere or up into the chromosphere from this point. In our model, however, the temperature variation is not essential. So, in this work, we shall assume that the atmosphere is isothermal and unbounded. Moreover, for simplicity, we will assume convective neutrality. Because of the second assumption, there is no gravity wave in our model atmosphere.

II. THE SOURCE TERMS

We shall discuss the atmospheric velocity field due to turbulence using a simple model for stratified atmospheres and source terms. The atmosphere is assumed to be unbounded, isothermal, and convectively neutral. Thus, the undisturbed density is given by

$$\rho_0 = \rho_{00} \exp(-z/H) , \tag{1}$$

where the direction of the z-axis is anti-parallel to the direction of gravity, $\underset{\sim}{g}$, and where the scale height, H, is given in terms of the magnitude of gravity, g, and undisturbed temperature, T_o, as

$$H = \mathcal{R} T_0 / g = c^2 / g . \tag{2}$$

Here we have assumed that the ratio of specific heats is unity to ensure convective neutrality, and have written C for the Newtonian speed of sound.

For such an atmosphere the inhomogeneous wave equation is

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho - g \frac{\partial \rho}{\partial z} = S , \tag{3}$$

(e.g. Moore and Spiegel 1964; Unno and Kato 1962) where ρ is the density fluctuation associated with waves or oscillations excited by turbulence and S is the source term which is specified in the following paragraph.

For the case of a homogeneous medium, Lighthill (1952, 1954) studied mainly the non-linear quadrupole source terms. In stellar atmosphere, in which turbulence occurs on a scale comparable with the scale height of density, the linear source

terms in turbulence also become important for generation of acoustic waves (Unno and Kato 1962, Kato 1963). However, near the critical frequency which we have discussed above, the linear source terms are unimportant, as shown in Appendix I. Then, we can represent the source terms in equation (3) by (e.g., Lighthill 1952)

$$S = \frac{\partial^2}{\partial x_i \partial x_j}\left[(\rho_0 + \rho_t) u_i u_j \right] + \nabla^2 (p - c^2 \rho)_t \ , \tag{4}$$

where the suffix, t, denotes the fluctuation terms due to turbulence, and u_i denotes the i-component of the turbulent velocity. Following the formulation of Lighthill, we shall take the Mach number of the turbulent field to be small, so that the turbulent density fluctuation may be neglected; i.e., $|\rho_t| \ll \rho_0$. We shall also neglect the second term on the right hand side of the equation (4), which can be shown to be small since the undisturbed atmosphere is isothermal and convectively neutral. Equation (3) then becomes

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 - g \frac{\partial}{\partial z} \right) \rho = \frac{\partial^2}{\partial x_i \partial x_j} \left(\rho_0 u_i u_j \right) \ , \tag{5}$$

which we shall make the basis of our calculation.

III. THE ATMOSPHERIC VELOCITY FIELD EXCITED BY THE NON-LINEAR SOURCE TERM

To put equation (5) in a more conventional form it is convenient to introduce the variable

$$\phi = \rho / \rho_0^{1/2} \quad . \tag{6}$$

Then, equation (5) can be rewritten as

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \frac{c^2}{4H^2} \right) \phi = \rho_0^{-1/2} \frac{\partial^2}{\partial x_i \partial x_j} (\rho_0 u_i u_j) \quad . \tag{7}$$

Now the source term can be rearranged in the following way:

$$\rho_0^{-1/2} \frac{\partial^2}{\partial x_i \partial x_j} (\rho_0 u_i u_j) = \frac{\partial^2}{\partial x_i \partial x_j} (\rho_0^{1/2} u_i u_j) - \frac{\partial}{\partial x_j} \left(\frac{\rho_0^{1/2} u_z u_j}{H} \right) + \frac{1}{4H^2} (\rho_0^{1/2} u_z^2) \quad . \tag{8}$$

In this form, the various parts of the source term can be given the usual interpretation in which first, second, and the third terms on the right hand side are called quadrupole, dipole and monopole sources, respectively (cf. Unno 1964).

Let us now introduce a time-harmonic analysis, in which

$$\phi(\underset{\sim}{x}, t) = \int_{-\infty}^{\infty} \Phi(\underset{\sim}{x}, \sigma) \exp(i\sigma t) \, d\sigma , \tag{9}$$

and

$$u_i u_j (\underset{\sim}{x}, t) = \int_{-\infty}^{\infty} \sigma_{ij} (\underset{\sim}{x}, \sigma) \exp(i\sigma t) \, d\sigma , \tag{10}$$

where Φ and $\mathcal{T}_{ij}$ are the appropriate Fourier amplitudes. Then, from equation (7) we obtain

$$\left(\sigma^2+c_\sigma^2\nabla^2\right)\Phi(\underset{\sim}{X},\sigma) = -\frac{c_\sigma^2}{c^2}\rho_0^{-1/2}\frac{\partial^2}{\partial x_i\partial x_j}\left[\rho_0\mathcal{T}_{ij}(\underset{\sim}{X},\sigma)\right], \tag{11}$$

where

$$c_\sigma^2 = \frac{\sigma^2}{\sigma^2-\sigma_c^2}\,c^2 \tag{12}$$

is the square of the phase velocity, and the critical frequency of the atmosphere, σ_c, is given by

$$\sigma_c = \frac{c}{2H}. \tag{13}$$

Equation (11) can now be solved by standard techniques; if we also introduce a radiation condition, we obtain for the solution,

$$\Phi(\underset{\sim}{R},\sigma) = \frac{1}{4\pi c^2}\int_V \frac{1}{R'}\exp\left(-i\frac{\sigma}{c_\sigma}R'\right)\rho_0^{-1/2}\frac{\partial^2}{\partial x_i\partial x_j}\left[\rho_0\mathcal{T}_{ij}(\underset{\sim}{X},\sigma)\right]d\underset{\sim}{X} \tag{14}$$

for $\sigma^2 > \sigma_c^2$ and

$$\Phi(\underset{\sim}{R},\sigma) = \frac{1}{4\pi c^2}\int_V \frac{1}{R'}\exp\left(-\left|\frac{\sigma}{c_\sigma}\right|R'\right)\rho_0^{-1/2}\frac{\partial^2}{\partial x_i\partial x_j}\left[\rho_0\mathcal{T}_{ij}(\underset{\sim}{X},\sigma)\right]d\underset{\sim}{X} \tag{15}$$

for $\sigma^2 < \sigma_c^2$. Equation (14) describes the density fluctuations associated with traveling waves, and equation (15) represents those associated with oscillations excited by turbulence. The volume integral should be performed over all turbulent regions, and R' denotes the distance between the observation point, $\underset{\sim}{R}$,

and a point, $\underset{\sim}{x}$, in the turbulent region.

We note that equation (15) shows that the density fluctuation (also the kinetic energy density as shall be shown later) associated with the frequencies lower than the critical frequency, decreases with increasing distance from the source, and this spatial attenuation increases with decreasing frequency. This property agrees qualitatively with observations in the solar atmosphere where the mean period of the oscillatory motion decreases with height (e.g. Evans and Michard 1962).

Let us now concentrate our attention only upon the frequency range near the critical frequency where the phase velocity, C_σ, is infinite. In addition, let us assume that the distance between the observation point and the turbulent region is sufficiently large. Then, if partial integrations of equations (14) and (15) are performed with the help of equation (8), only the contribution of the monopole source in equation (8) remains and we have

$$\Phi(\underset{\sim}{R},\sigma) = \frac{1}{4\pi c^2}\frac{1}{4H^2}\int_V \frac{1}{R'}\exp\left(-i\frac{\sigma}{c_\sigma}R'\right)\rho_0^{1/2}\,\mathcal{V}(\underset{\sim}{x},\sigma)\,d\underset{\sim}{x} \tag{16}$$

for $\sigma^2 > \sigma_c^2$, and

$$\Phi(\underset{\sim}{R},\sigma) = \frac{1}{4\pi c^2}\frac{1}{4H^2}\int_V \frac{1}{R'}\exp\left(-\left|\frac{\sigma}{c_\sigma}\right|R'\right)\rho_0^{1/2}\,\mathcal{V}(\underset{\sim}{x},\sigma)\,d\underset{\sim}{x} \tag{17}$$

for $\sigma^2 < \sigma_c^2$, where $\mathcal{V}$ denotes $\mathcal{V}_{zz}$ and hereafter, for simplicity, this abbreviation will be used.

Now in a convectively neutral atmosphere the velocity fluctuation, $\underset{\sim}{v}$, associated with the wave mode is related to the density fluctuation, ρ, by

$$\frac{\partial \underset{\sim}{v}}{\partial t} = -\,\mathrm{grad}\left[\, c^2 (\rho/\rho_0)\right] \tag{18}$$

in linear theory (e.g. Lamb 1945). Fourier transformation of this equation with respect to t gives

$$i\sigma\, \hat{\underset{\sim}{v}}(\underset{\sim}{R},\sigma) = -c^2\left[\, \mathrm{grad}\,\Phi(\underset{\sim}{R},\sigma) + \frac{1}{2H}\, \underset{\sim}{i}_z\, \Phi(\underset{\sim}{R},\sigma)\right] , \tag{19}$$

where $\underset{\sim}{i}_z$ is the unit vector in the direction of the z-axis, and

$$\underset{\sim}{v}(\underset{\sim}{R},t) = \int_{-\infty}^{\infty} \hat{\underset{\sim}{v}}(\underset{\sim}{R},\sigma)\, \exp(i\sigma t)\, d\sigma . \tag{20}$$

Inserting equations (16) and (17) into equation (19) under the previous assumption that the distance between the observation point and the turbulent region is sufficiently large, we obtain an expression for the Fourier amplitude, $\hat{\underset{\sim}{v}}(\underset{\sim}{R},\sigma)$, of the acoustic field, $\underset{\sim}{v}(\underset{\sim}{R},t)$. The results show that the Fourier amplitude

$\hat{v}_x(\underset{\sim}{R},t)$ and $\hat{v}_y(\underset{\sim}{R},t)$, of the horizontal velocity field $v_x(\underset{\sim}{R},t)$ and $v_y(\underset{\sim}{R},t)$, are small quantities of the order of $2H\sigma/c_\sigma$ compared with that of the vertical velocity field, $v_z(\underset{\sim}{R},t)$, since the first term in the right hand side of equation (19) is smaller than the second term. Thus, we may expect that the excited oscillatory velocity field has mainly a vertical component, in agreement with the observation of the velocity field in the solar atmosphere. (e.g., Leighton, Noyes and Simon 1962).

Let us now estimate the power spectrum, $E(\underset{\sim}{R},\sigma)$, of the kinetic energy at the point, $\underset{\sim}{R}$, defined by

$$E(\underset{\sim}{R}) = \int_{-\infty}^{\infty} E(\underset{\sim}{R},\sigma)\,d\sigma = \frac{1}{2}\rho_0(\underset{\sim}{R})\langle \underset{\sim}{v}^2(\underset{\sim}{R})\rangle , \tag{21}$$

where the bracket denotes the ensemble mean. As discussed above, at frequencies near the critical frequency we have

$$\langle \underset{\sim}{v}^2(\underset{\sim}{R})\rangle \sim \langle v_z^2(\underset{\sim}{R})\rangle \quad \text{and} \quad i\sigma\hat{v}_z(\underset{\sim}{R},\sigma) \sim -c^2(1/2H)\Phi(\underset{\sim}{R},\sigma)$$

(see equation (19)). Therefore, with the use of equation (20), $E(\underset{\sim}{R},\sigma)$ is written approximately as

$$E(\underset{\sim}{R},\sigma) \cong \frac{1}{2}\frac{c^4}{\sigma^2}\left(\frac{1}{2H}\right)^2 \rho_0(\underset{\sim}{R})\langle \Phi(\underset{\sim}{R},\sigma)\Phi^*(\underset{\sim}{R},\sigma)\rangle , \tag{22}$$

where the asterisk denotes the complex conjugate. In equation (22) we also used the relation $\langle \Phi(\sigma')\Phi^*(\sigma'')\rangle = 0$ except for $|\sigma'| = |\sigma''|$, which results from the condition of statistical stationarity of turbulence $\left(\langle \mathcal{V}(\sigma')\mathcal{V}^*(\sigma'')\rangle = 0 \right.$ except for $\left. |\sigma'| = |\sigma''| \right)$, and from the relations (16) or (17) between $\Phi(\sigma)$ and $\mathcal{V}(\sigma)$.

As mentioned before, at some distance from the source region we can expect a local maximum in the power spectrum of the kinetic energy near the critical frequency, partially because of the spatial attenuation of frequencies below the critical frequency (e.g. see the term, $\exp(-|\sigma/c_\sigma| R')$, in equation (17)) and partially because $\mathcal{V}(\underset{\sim}{x},\sigma)$ in the solar convection zone is probably a decreasing function at frequencies higher than the critical frequency (e.g. Bahng and Schwarzschild, 1961). But we have no precise knowledge about $\mathcal{V}(\underset{\sim}{x},\sigma)$ in the present stage. So, if the mean width of the local maximum near the critical frequency in the power spectrum is written as $\beta\sigma_c$, then the total energy contained there is given approximately as (see Appendix II)

$$E(\underset{\sim}{R}) \sim \frac{\beta}{8\pi c}\left(\frac{1}{2H}\right)^5 D \ln \bar{R} \cdot \rho_{om} \int \langle \mathcal{V}(\underset{\sim}{x},\sigma_c)\mathcal{V}^*(\underset{\sim}{x}',\sigma_c)\rangle d\underset{\sim}{r} , \qquad (23)$$

where $\underset{\sim}{r} = \underset{\sim}{x}' - \underset{\sim}{x}$ and we have taken the source region to have a horizontal radius $\bar{R}$ and a vertical thickness D. Furthermore, in equation (23), ρ_{om} is an appropriate mean value of ρ_o in the source region. In addition, it should be mentioned that the term enclosed by the bracket in equation (23) can be written as

$$\langle \mathcal{T}(\underset{\sim}{x},\sigma)\mathcal{T}^*(\underset{\sim}{x}',\sigma)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle u_z^2(\underset{\sim}{x},t)\, u_z^2(\underset{\sim}{x}',t')\rangle \exp(-i\sigma\tau)\, d\tau \tag{24}$$

with the use of equation (10), and where $\tau = t' - t$.

IV. ESTIMATION OF INTENSITY OF THE OSCILLATORY MOTION

In estimating the value of E from equations (23) and (24) we shall assume simply that the time correlations in the turbulence has the following Gaussian form:

$$\langle u_z^2(\underset{\sim}{x},t)\, u_z^2(\underset{\sim}{x}',t')\rangle = u_o^4 \exp[-r^2/L^2]\, \exp[-(u_o/L)^2\tau^2], \tag{25}$$

where L and u_o are the characteristic length and velocity of turbulence. Then, on performing the necessary integrations with respect to $\underset{\sim}{r}$ and τ in equations (23) and (24), we obtain

$$E(\underset{\sim}{R}) \sim \frac{2}{(16)^3}\beta \ln \bar{R} \left(\frac{D}{H}\right)\left(\frac{L}{H}\right)^4\left(\frac{u_o}{c}\right)\rho_{om}\, u_o^2 \exp\left[-\frac{1}{16}\left(\frac{L}{H}\right)^2\left(\frac{u_o}{c}\right)^{-2}\right] \tag{26}$$

Under the assumption that the velocity in Vitense's model (1953) of the solar convection zone represents the characteristic velocity, u_0, we can make the numerical estimates of $E(\underset{\sim}{R})$ as given by equation (26). The values used for these estimates are summed up in Table 1. Since we cannot obtain the theoretical form of the power spectrum function accurately because of the lack of knowledge about $\mathcal{V}(\underset{\sim}{X},\sigma)$, we use $\beta = 0.4$ obtained from observations (Evans, Michard and Servajean 1963). We estimate that $\bar{R} \sim 1/10\, R_{\odot}$ and assume the effective source region to be between log P_g = 5.16 and log P_g = 5.8 (D/H = 1.5). This region corresponds to the hydrogen convection zone active in velocity. The calculated values of E are 1.35x10 erg/cm^3 and 1.63x10^2 erg/cm^3 for ℓ(=L/H)=1 and 2, respectively. At the level where the optical depth $\hat{\tau}$ is 0.1, log ρ_0 = - 7.0 (Allen 1955). Therefore, from the relation $E \sim 1/2\, \rho_0(\underset{\sim}{R}) \langle v^2(\underset{\sim}{R}) \rangle$ where v($\underset{\sim}{R}$) is the velocity of the oscillatory motion excited by turbulence, we have $\langle v^2 \rangle^{1/2} \sim$ 0.17 km/sec at $\hat{\tau}$ = 0.1 for ℓ = 1 and $\langle v^2 \rangle^{1/2} \sim$ 0.57 km/sec for ℓ = 2, respectively. On the other hand, the observed r.m.s. of velocity at $\hat{\tau}$ = 0.1 is about 0.4 km/sec (e.g. Leighton et. al. 1962).

Because of the existence of the exponential term in equation (26) the lower convection zone where the Mach number is small does not contribute to E($\underset{\sim}{R}$). This is the main reason why

we restricted the source region to the upper convection zone which is active in velocity. In addition, the above estimate is based on the approximation that the distance between the observation point and the turbulent region is sufficiently long (for example, longer than twice the scale height (see Section III)). This approximation may not be good. If terms neglected in our calculation are included, the intensity of the theoretical velocity field increases. Moreover, our calculation is based on the assumption of convective neutrality. In a convectively stable atmosphere, the degree of stability of the atmosphere has important effects upon the efficiency of the response of the atmosphere to disturbances (Kato 1965), but we suppose that the degree of stability has no important effect upon the efficiency in an unstable atmosphere. Furthermore, we should mention that our results depend strongly upon the assumption (25). If the assumption (25) is taken literally, it may be doubtful whether the acoustic flux sufficient to heat the corona can be radiated at higher frequencies because the value of $\langle \mathcal{V}(\underset{\sim}{x},\sigma)\mathcal{V}^*(\underset{\sim}{x}',\sigma)\rangle$ decreases very rapidly with increasing value of σ.

V. SOME OTHER COMPARISONS WITH OBSERVATIONS

Quantitative comparisons in other points are impossible in

the present stage because of the lack of precise knowledge about the spectrum function of the turbulence in the solar convection zone. However, our model agrees qualitatively with some characteristics of the oscillatory motions actually observed in the solar atmosphere in the following points. i) The mean period of the velocity field excited decreases with increasing height. In our model this results from the space attenuation of the frequency components below the critical frequency (see the term $\exp(-|\sigma/c_\sigma| R')$ in equation (15)). ii) The vector velocity field consists mainly of a vertical component. This is one of the characteristics of acoustic oscillations excited near the critical frequency. iii) Finally, our model also is not inconsistent with the periodic variation in the horizontal observed in the oscillatory motions. That is, in our results the components of the atmospheric velocity field whose frequencies are lower than the critical frequency oscillate statistically in phase. However, a component of the velocity field having a frequency, σ, larger than the critical frequency can propagate and its wave length is roughly $2\pi c_\sigma/\sigma$, which is equal to $4\pi H(\alpha^2-1)^{-1/2}$ for $\sigma = \alpha\sigma_c$. So, it is not difficult to obtain a few thousand kilometers as the mean half wave length, which is actually observed in the solar atmosphere.

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APPENDIX I

THE EFFECTS OF LINEAR SOURCES UPON THE VELOCITY FIELD NEAR THE CRITICAL FREQUENCY

For a stratified atmosphere, Unno and Kato (1962) showed that linear sources in turbulence can be important for the generation of acoustic waves. Near the critical frequency, however, linear sources have no important effect. We should recall that the effects of linear source terms in the generation of waves are not cancelled out even if the space distribution of these sources is homogeneous and isotropic, because the influence of different phases of decay in turbulence is superposed at the observation point due to the difference in arrival time of the effects of each turbulent element (Unno and Kato 1962). Near the critical frequency, however, the effects of time retardation is small because the phase velocity becomes infinite at the critical frequency.

More concretely speaking, in the equation evaluating the atmospheric velocity field or the energy flux radiated by turbulent

sources, we have the following term (a product of two terms similar with that in equation (18)):

$$\int \langle Q(\underset{\sim}{x},\sigma)\, Q(\underset{\sim}{x}',\sigma)\rangle \exp\left(i\frac{\sigma}{c_\sigma}\, \underset{\sim}{n}\cdot \underset{\sim}{r}\right) d\underset{\sim}{r} \tag{27}$$

for frequencies higher than the critical frequency, where the form of Q will be discussed later, and where $\underset{\sim}{n}$ is the unit vector in the direction of the observer, and $\underset{\sim}{r}$ is $\underset{\sim}{x}' - \underset{\sim}{x}$. In equation (27) the exponential term denotes an effect of the difference in arrival time of effects of each turbulent element. At the critical frequency (where $c_\sigma = \infty$), the exponential term in equation (27) becomes unity, and below the frequency the term is always unity (e.g., consider a product of two terms similar with that in equation (17)). If the source term is non-linear, Q in equation (27) is a term such as v_{ij} defined by equation (10). However, if the source term is linear, Q in equation (27) is a term such as v_i where v_i means the time Fourier component of u_i (cf. equation (10)). Here, we should mention that if v_i is spatially homogeneous in turbulence (this will be reasonable as the first approximation) the integral $\int \langle v_i(\underset{\sim}{x},\sigma)\, v_j(\underset{\sim}{x}',\sigma)\rangle d\underset{\sim}{r}$ is zero.

APPENDIX II

From equations (16) or (17), we obtain

$$\langle \Phi(\underset{\sim}{R},\sigma_c)\Phi^*(\underset{\sim}{R},\sigma_c)\rangle = \frac{1}{16\pi^2 c^4}\left(\frac{1}{2H}\right)^4 \int \frac{\rho_0(\underset{\sim}{r_0})}{|\underset{\sim}{R}-\underset{\sim}{r_0}|^2}\, d\underset{\sim}{r_0} \tag{28}$$

$$\times \int \langle V(\underset{\sim}{x},\sigma_c)V^*(\underset{\sim}{x}',\sigma_c)\rangle\, d\underset{\sim}{r} \quad ,$$

where $\underset{\sim}{r_0} = \frac{1}{2}(\underset{\sim}{x}+\underset{\sim}{x}')$ and $\underset{\sim}{r} = \underset{\sim}{x}'-\underset{\sim}{x}$. If the source region extends with a horizontal radius $\bar{R}$ and a vertical thickness D, the first integral in equation (28) can be written approximately as

$$\int \frac{\rho_0(\underset{\sim}{r_0})}{|\underset{\sim}{R_0}-\underset{\sim}{r_0}|^2}\, d\underset{\sim}{r_0} \sim 2\pi\, \rho_{0m} D \int_0^{\bar{R}} \frac{\varpi}{\varpi^2+z^2}\, d\varpi$$

$$\sim 2\pi\, \rho_{0m} D \ln \bar{R} \quad , \tag{29}$$

where ρ_{0m} is an appropriate mean value of ρ_0 in the source region. Inserting equations (28) and (29) into equation (22) and also from the relation $E(\underset{\sim}{R}) \sim 2\beta\sigma_c\, E(\underset{\sim}{R},\sigma_c)$, we obtain equation (23).

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TABLE I

Values used to estimate E, and results

Basic Quantities	β	0.4	
	$\bar{R}$	1/10 $R_\odot$	
	D/H	1.5	
	ρ_{om}	5×10^{-7} gram/cm^3	
	c	9×10^5 cm/sec	
Arbitrary Parameters	ℓ (=L/H)	1	2
	u_o	2.1×10^5 cm/sec	2.9×10^5 cm/sec
	M(= u_o/c)	0.23	0.32
Results	E	1.4×10 erg/cm^3	1.6×10^2 erg/cm^3
	$\langle v^2\rangle^{1/2}$ at $\rho_o = 10^{-7}$ gram/cm^3	0.17 km/sec	0.57 km/sec
Observations	$\langle v^2\rangle^{1/2}$ at $\rho_o = 10^{-7}$ gram/cm^3	0.40 km/sec	