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On Chandrasekhar's Limiting Mass for Rotating White Dwarf Stars*

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The theory of rotating polytropes has been considerably advanced recently by Chandrasekhar and Lebovitz,[1] Hurley and Roberts,[2,3] and James.[4] The earliest notable work in this field was by Chandrasekhar[5] in 1933. James[6] also investigated the structure of rotating white dwarfs by actual integration of the governing partial differential equation numerically. However, he did not consider the effects of rotation on Chandrasekhar's limiting mass (M). Chandrasekhar[6] proved that the maximum mass (M) of a stellar configuration which can be regarded as wholly degenerate is $M = 6.65\ \mu_e^{-2}\ M_\odot$, where μ_e is mean molecular weight and $M_\odot$ is one solar mass. In this note we discuss the effects of rotation on M using Chandrasekhar's series expansion method.[1,5] It is found that by rotations, M is increased by about 2.5% implying that the limiting mass for a rotating white dwarf is $M = 6.82\ \mu_e^{-2}\ M_\odot$.

We take the center of mass as the origin of coordinates, and denote the distance from the center of mass by r and the cosine of the colatitude by μ. We further assume that the configuration is axially symmetric about the axis of rotation. Let ω and ρ denote the angular velocity of rotation and density of the configuration. On combining Poisson's equation and the equation for

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hydrostatic equilibrium for a rotating and completely degenerate system, we get

$$\frac{1}{\eta^2}\frac{\partial}{\partial\eta}\left[\eta^2\frac{\partial\Phi}{\partial\eta}\right]+\frac{1}{\eta^2}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\Phi}{\partial\mu}\right] = -\left(\Phi^2-\frac{1}{y_o^2}\right)^{3/2}+v \qquad (1)$$

where the various quantities are defined as follows:

$$r=\alpha\xi \quad , \quad y=y_o\Phi \quad , \quad y^2=x^2+1 \quad .$$

$$\alpha=(2A/\pi G)^{\frac{1}{2}}\,(B\,y_o)^{-1} \quad , \quad y_o^2=x_o^2+1 \qquad (2)$$

$$v=\frac{\omega^2}{2\pi G\,B\,y_o^3} \quad , \quad A=0.01\times10^{22} \quad , \quad B=0.82\times10^5\,\mu_e .$$

Here x is related to the mean electron concentration by $x^3=\mu_e n_e\,H/B$ and x_o and y_o are the values at the center. A and B are the usual parameters associated with the pressure and density of complete degenerate matter.

We shall suppose that the rotation is small, so that v may be treated as a small perturbation parameter. Chandrasekhar and Lebovitz[1] obtained the solution of the polytropic problem, up to the first order in v. We have carried out this approximation to second order in v. In this case we also assume, then, for Φ, a solution of the form

$$\Phi = \varphi(\eta)+v\left[\psi_o(\eta)+\sum_{l=1}^{\infty}A_l\,\psi_l(\eta)\,P_l(\mu)\right] + v^2\left[f_o(\eta)+\sum_{l=1}^{\infty}B_l\,f_l(\eta)\,P_l(\mu)\right] \qquad (3)$$

where the A_l's and B_l's are constants which will be determined by imposing boundary conditions. Substituting this form of Φ in Eq. (1), then the radial functions φ, ψ_o, ψ_l, f_o, f_l satisfy the following differential

[redacted]

equations:

$$D_o \varphi = -(\varphi^2 - y_o^{-2})^{3/2} \tag{4}$$

$$D_o \psi_o = -3\varphi (\varphi^2 - y_o^{-2})^{\frac{1}{2}} \psi_o + 1 \quad ; \quad D_l \psi_l = -3\varphi (\varphi^2 - y_o^{-2})^{\frac{1}{2}} \psi_l \tag{5}$$

$$D_o f_o = -3\varphi (\varphi^2 - y_o^{-2})^{\frac{1}{2}} f_o \quad ; \quad D_l f_l = -3\varphi (\varphi^2 - \varphi^{-2})^{\frac{1}{2}} f_l \tag{6}$$

where

$$D_l = \frac{d^2}{d\xi^2} + \frac{2}{\xi}\frac{d}{d\xi} - \frac{l(l+1)}{\xi} . \tag{7}$$

Equation (4) is the usual structure equation for nonrotating white dwarfs.

Following Chandrasekhar and Lebovitz[1] we have obtained the solution of the problem up to second order in v for this problem. The complete mathematical details and other applications will be published elsewhere. In this case the mass relation for the rotating white dwarfs is given by

$$M_\omega = M_o (1 + \alpha_1 v + \alpha_2 v^2) \tag{8}$$

where

$$\alpha_1 = \frac{1}{3} \frac{\eta_1 - \psi_o'(\eta)}{|\varphi'(\eta_1)|} , \qquad \alpha_2 = -\frac{f_o'(\eta_1)}{|\varphi'(\eta_1)|} \tag{9}$$

$$M_o = -4\pi \left(\frac{\alpha_1}{\pi G}\right)^{3/2} \frac{1}{B^2} \eta_1^2 \varphi'(\eta_1) \tag{10}$$

where the $\eta = \eta_1$ is the boundary of the nonrotating system specified by the Eq. (4) and prime denotes differentiation with respect to η.

The effect of radiation pressure on Eq. (1) can be introduced in a similar way. For this, we have just to replace A by $\beta_e^{-1} A$ where β_e is the ratio

of gas pressure to total pressure. Now we can write Eq. (8) as[7]

$$M_\omega(\beta_e; y_o) = M(1; y_o)\, \beta_e^{-3/2} \left[1 + \alpha_1 v + \alpha_2 v^2\right] \tag{11}$$

We know that when $y_o \to \infty$, Eq. (4) reduces to the usual Lane-Emden equation of a nonrotating polytrope for $n = 3$. In this limit it follows from the definition of v, that, v becomes

$$v = \frac{\omega^2}{2\pi G \rho_o} \tag{12}$$

where ρ_o is the central density. Also Eqs. (5) and (7) reduce to the usual equation for $n = 3$ in the polytropic case. In this case Eq. (11) gives

$$M_\omega(\beta_e; \infty) = M_3\, \beta_e^{-3/2} \left[1 + (6.1038)v + (2.02003)v^2\right] \tag{13}$$

where we have substituted the values of η_1, $V_o'(\eta_1)$, $\varphi'(\eta_1)$ from reference 1. We have calculated the function $f_o'(\eta_1)$ for $n = 3$ from Eq. (6). M_3 is the limiting mass of nonrotating white dwarfs and it is given by[8]

$$M_3 = 5.75\, \mu_e^{-2}\, M_\odot \tag{14}$$

A criterion of stellar degeneracy in a white dwarf star, due to Chandrasekhar[9] is the following: If $\beta_e > \beta_\omega = 0.9078$ then for the specified β_e the electron assembly becomes degenerate for sufficiently high concentrations.

The critical values of v (say v_c) for which the rotating system has a maximum angular momentum consistent with equilibrium due to Chandrasekhar [$v_c(C)$], Hurley and Roberts [$v_c(R)$], and James [$v_c(J)$] are the following:

$$\begin{aligned} v_c(C) &= 7.0574 \times 10^{-3}, \quad & v_c(R) &= 4.3296 \times 10^{-3} \\ v_c(J) &= 8.93 \times 10^{-3}, \quad & v_c(A) &= 4.1964 \times 10^{-3} \end{aligned} \tag{15}$$

The value $v_c(A)$ has been calculated by the methods of Chandrasekhar including second order effects. These three values of v_c (excluding Chandrasekhar's original value) are different because each author's approach to the problem is different. Using these values for v_c, M_3 and β_ω, we get the following limiting masses corresponding to each v_c:

$$M(C) = 6.9339\ \mu_e^{-2}\ M_\odot \quad , \quad M(R) = 6.6229\ \mu_e^{-2}\ M_\odot$$
$$M(J) = 6.8066\ \mu_e^{-2}\ M_\odot \quad , \quad M(A) = 6.8175\ \mu_e^{-2}\ M_\odot \tag{16}$$

We suggest that the limiting mass of a hot and rotating white dwarf star is $M = 6.8175\ \mu_e^{-2}\ M_\odot$ which is 2.5% more from nonrotating mass. Also for our $v_c(A)$, Eq. (13) for $\beta_e = 1$ gives the limiting mass for a cold star as $M = 5.8975\ M_\odot$. If we take $\mu_e = 2$,[10] then the limiting mass of rotating white dwarfs is following

$$M = 1.474\ M_\odot \quad \text{(cold star)}$$
$$M = 1.704\ M_\odot \quad \text{(hot star).}$$

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