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SUMMARY

A theoretical analysis is presented for the buckling and dynamic characteristics of eccentrically stiffened cylinders and flat plates. Results indicate that in the analysis of many static and dynamic plate and shell problems, the effects of any eccentricity in stiffening should be carefully evaluated. For example, for a longitudinally stiffened cylinder configuration with practical proportions the lowest natural frequency with stiffening on the outside is found to be about 35 percent higher than the corresponding frequency for the same stiffening on the inside. Results are also presented which illustrate the buckling characteristics of eccentrically stiffened cylindrical shells under a hydrostatic pressure loading, and the vibration, buckling, and flutter characteristics of stiffened flat plates.

INTRODUCTION

The understanding of the effects of stiffening on the behavior of plates and shells has grown in importance with the growing need for precision in the design of lightweight structures. The effects of stiffener eccentricities on the buckling characteristics of stiffened circular cylindrical shells are treated analytically in references 1 through 5. It has been shown experimentally (ref. 6) that for some configurations a circular cylindrical shell with stringers attached to its external surface can carry more than twice as much load in axial compression as its internally stiffened counterpart. The purpose of the present paper is to demonstrate the importance of eccentricities for a variety of plate and shell problems including buckling, vibration, and flutter.

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Nonlinear equilibrium equations and boundary conditions are derived for eccentrically stiffened cylindrical shells and flat plates. The derivation is accomplished by defining nonlinear strain-displacement relations for the plate or shell and the stiffeners, treating the stiffeners as beam elements. The stiffeners are not considered as discrete members, but their effects are averaged or "smeared out" over the plate or shell. The equilibrium equations and boundary conditions are then determined by formulating the potential energy of the system and applying the principle of minimum potential energy.

The nonlinear equations are subsequently used to derive linear equations which govern the small vibrations of a prestressed, eccentrically stiffened cylinder. A closed form solution to these equations is presented for a membrane-like prestress state, and simple support boundary conditions. When the natural frequency is taken equal to zero the linear equations and the solution apply to buckling problems.

Due to the large number of parameters involved, results of a general nature are impractical to present. Results are presented, however, for the hydrostatic pressure buckling and the free vibration of specific stiffened cylinder configurations. These results illustrate typical effects which exist in eccentrically stiffened structures. Similar results are presented for the buckling and vibration of eccentrically stiffened flat plates.

Linearized equations are also used to study the flutter characteristics of stiffened flat plates subjected to supersonic flow over one surface. Calculations are made from an approximate solution (two-mode Galerkin solution), and large eccentricity effects are again demonstrated.

SYMBOLS

A	cross-sectional area of stiffener
D	flexural stiffness of isotropic plate or isotropic cylinder wall, $\frac{Et^3}{12\left(1 - \mu^2\right)}$
D_x, D_{xy}	orthotropic plate stiffnesses

E Young's modulus

G shear modulus

I moment of inertia of stiffener about its centroid

I_o moment of inertia of stiffener about middle surface of plate or cylinder

J torsional constant for stiffener

$\bar{M}$ mass per unit area

M Mach number

M_x, M_y, M_{xy}, M_{yx} middle surface moment resultants

N_x, N_y, N_{xy} middle surface stress resultants

$\bar{N}_x$ externally applied load resultant in x-direction

$\bar{N}_y$ for plate, externally applied load resultant in y-direction; for cylinder, circumferential stress resultant due to applied pressure

R radius to middle surface of isotropic cylinder (see fig. 1)

$\bar{R}$ nondimensional parameter, $E_r A_r / Etl$

$\bar{S}$ nondimensional parameter, $E_s A_s / Etd$

Z curvature parameter, $\frac{a^2}{Rt}\left(1 - \mu^2\right)^{1/2}$

a length of cylindrical shell or plate

b width of plate

d stringer spacing (see fig. 1)

$\bar{e}$ distance from cylinder middle surface to line on which $\bar{N}_x$ acts

$\bar{e}_x, \bar{e}_y$ distance from plate middle surface to line on which $\bar{N}_x$ and $\bar{N}_y$ act, respectively

f frequency, $\omega/2\pi$

l ring spacing (see fig. 1)

m, n integers

p external pressure load

q dynamic pressure

t thickness of cylinder or plate

u, v, w shell or plate middle surface displacements in x-, y-, and z-directions, respectively

u_T, v_T displacements in x- and y-directions, respectively

x, y, z orthogonal coordinates (x and y lie in the middle surface of the cylinder or plate)

$\bar{z}$ distance from middle surface of plate or cylinder to the centroid of stiffener

α, β wavelength parameters

$\epsilon_x, \epsilon_y, \gamma_{xy}$ membrane normal strains and shearing strain

$\epsilon_{xT}, \epsilon_{yT}, \gamma_{xyT}$ normal strains and shearing strain

λ dynamic pressure parameter

μ Poisson's ratio

ν plate aspect ratio parameter, na/b

Π potential energy

ρ mass density

ω circular frequency

Subscripts:

c refers to cylinder

r refers to stiffening in y-direction

s refers to stiffening in x-direction

p refers to plate

A refers to prestress state

B refers to small changes away from prestress state

A comma indicates partial differentiation with respect to the subscript following the comma.

DERIVATION OF NONLINEAR EQUATIONS AND BOUNDARY CONDITIONS

In this section the nonlinear differential equations of equilibrium are derived for a thin-walled circular cylinder, stiffened by evenly spaced uniform rings and/or

stringers (see fig. 1). The only applied loadings considered in the analysis are an axial compressive end load and a constant external pressure load. It is assumed that the stiffener spacing is small so that the stiffener effect on the behavior of the structure may be averaged or "smeared out." In cases when stiffeners in both directions are attached to the same surface of the structure the effect of joints in the stiffener framework is ignored. The nonlinear equations of equilibrium and boundary conditions are also obtained for a stiffened flat plate with applied edge loads.

Stiffened Cylinder

The strain energy of an unstiffened, thin-walled, isotropic cylinder is

$$\Pi_c = \frac{E}{2\left(1 - \mu^2\right)} \int_{-t/2}^{t/2} \int_0^{2\pi R} \int_0^a \left(\epsilon_{xT}^2 + \epsilon_{yT}^2 + 2\mu\epsilon_{xT}\epsilon_{yT} + \frac{1 - \mu}{2} \gamma_{xyT}^2\right) dx\, dy\, dz \tag{1}$$

The Donnell type nonlinear strain-displacement relations to be used are

$$\left.\begin{aligned} \epsilon_{xT} &= u_{T,x} + \frac{1}{2} w_{,x}^2 \\ \epsilon_{yT} &= v_{T,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 \\ \gamma_{xyT} &= u_{T,y} + v_{T,x} + w_{,x}w_{,y} \end{aligned}\right\} \tag{2}$$

where

$$\left.\begin{aligned} u_T &= u - zw_{,x} \\ v_T &= v - zw_{,y} \end{aligned}\right\} \tag{3}$$

The quantities u, v, and w are the displacements of the middle surface of the cylinder wall. Thus

$$\left.\begin{aligned} \epsilon_{xT} &= u_{,x} + \frac{1}{2} w_{,x}^2 - zw_{,xx} \\ \epsilon_{yT} &= v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 - zw_{,yy} \\ \gamma_{xyT} &= u_{,y} + v_{,x} + w_{,x}w_{,y} - 2zw_{,xy} \end{aligned}\right\} \tag{4}$$

After integrating with respect to z, the cylinder strain energy becomes

$$\Pi_c = \frac{Et}{2\left(1 - \mu^2\right)} \int_0^{2\pi R} \int_0^a \left(\epsilon_x^2 + \epsilon_y^2 + 2\mu\epsilon_x\epsilon_y + \frac{1 - \mu}{2}\gamma_{xy}^2\right) dx\, dy$$

$$+ \frac{D}{2} \int_0^{2\pi R} \int_0^a \left[w_{,xx}^2 + w_{,yy}^2 + 2\mu w_{,xx}w_{,yy} + 2(1 - \mu)w_{,xy}^2\right] dx\, dy \tag{5}$$

where

$$\epsilon_x = \epsilon_{xT}\Big|_{z=0}, \qquad \epsilon_y = \epsilon_{yT}\Big|_{z=0}, \qquad \gamma_{xy} = \gamma_{xyT}\Big|_{z=0}$$

If displacements in the cylinder and stringers are continuous and the effects of the stringers are averaged over the circumference, the total strain energy for stringers of spacing d attached to the shell is assumed to be

$$\Pi_s = \frac{1}{d} \int_0^{2\pi R} \left(\frac{E_s}{2} \int_0^a \int_{A_s} \epsilon_{xT}^2\, dA_s\, dx + \frac{G_s J_s}{2} \int_0^a w_{,xy}^2\, dx\right) dy \tag{6}$$

The first term in equation (6) is the strain energy of extension and bending, while the second term is the strain energy of twisting. The quantity dA_s is an element of the cross-sectional area of the stringer and $G_s J_s$ is the twisting stiffness of the stringer section. Substitution of the first of equations (4) into equation (6) and intetrating over the area of the stiffener yields the following expression for the stringer strain energy

$$\Pi_s = \frac{1}{d} \int_0^{2\pi R} \int_0^a \left[\frac{E_s}{2}\left(A_s\epsilon_x^2 - 2\bar{z}_s A_s \epsilon_x w_{,xx} + I_{os} w_{,xx}^2\right) + \frac{G_s J_s}{2} w_{,xy}^2\right] dx\, dy \tag{7}$$

where $\bar{z}_s$ is the distance from the middle surface of the isotropic shell to the centroid of the stringer cross section, and I_{os} is the moment of inertia of the stringer with respect to an axis in the middle surface of the isotropic shell. It should be noted that $\bar{z}_s$ is positive for stringers on the outer surface of the shell and negative for stringers on the inner surface.

By a similar method of analysis it can be shown that the strain energy for rings of spacing l is

$$\Pi_r = \frac{1}{l} \int_0^{2\pi R} \int_0^a \left[\frac{E_r}{l}\left(A_r\epsilon_y^2 - 2\bar{z}_r A_r \epsilon_y w_{,yy} + I_{or} w_{,yy}^2\right) + \frac{G_r J_r}{l} w_{,xy}^2\right] dx\, dy \tag{8}$$

The potential energy of a constant external pressure p and a constant externally applied axial load resultant $\bar{N}_x$ (positive in compression) is

$$\Pi_L = p \int_0^{2\pi R} \int_0^a w\, dx\, dy + \int_0^{2\pi R} \bar{N}_x \left[u_T\Big|_{z=\bar{e}}\right]\Big|_0^a dy \tag{9}$$

The quantity $\bar{e}$ is the distance from the middle surface of the isotropic shell to the line on which the load resultant $\bar{N}_x$ acts.

If u, v, and w are amplitudes of simple harmonic motion with circular frequency ω, and if in-plane inertia is neglected, the potential energy of inertia loading at maximum deflection is

$$\Pi_\omega = -\frac{1}{2} \int_0^{2\pi R} \int_0^a \bar{M}\omega^2 w^2 dx\, dy \tag{10}$$

where $\bar{M} = \rho_c t + \rho_s \frac{A_s}{d} + \rho_r \frac{A_r}{l}$ is the averaged or "smeared out" mass per unit area of the stiffened cylinder.

By combining equations (5), (7), (8), (9), and (10), the total energy at maximum deflection of the stiffened shell can be written in terms of stress and moment resultants as

$$\Pi = \frac{1}{2} \int_0^{2\pi R} \int_0^a \left(N_x\epsilon_x + N_y\epsilon_y + N_{xy}\gamma_{xy} - M_x w_{,xx} - M_y w_{,yy} + M_{xy} w_{,xy} - M_{yx} w_{,xy}\right) dx\, dy$$

$$+ \int_0^{2\pi R} \int_0^a \left(pw - \frac{1}{2}\bar{M}\omega^2 w^2\right) dx\, dy + \int_0^{2\pi R} \bar{N}_x \left[u_T\Big|_{z=\bar{e}}\right]\Big|_0^a dy \tag{11}$$

where

$$
\left.
\begin{aligned}
N_x &= \frac{Et}{1-\mu^2}\left[u_{,x} + \frac{1}{2}w_{,x}^2 + \mu\left(v_{,y} + \frac{w}{R} + \frac{1}{2}w_{,y}^2\right)\right] + \frac{E_sA_s}{d}\left(u_{,x} + \frac{1}{2}w_{,x}^2 - \bar{z}_s w_{,xx}\right) \\
N_y &= \frac{Et}{1-\mu^2}\left[v_{,y} + \frac{w}{R} + \frac{1}{2}w_{,y}^2 + \mu\left(u_{,x} + \frac{1}{2}w_{,x}^2\right)\right] + \frac{E_rA_r}{l}\left(v_{,y} + \frac{w}{R} + \frac{1}{2}w_{,y}^2 - \bar{z}_r w_{,yy}\right) \\
N_{xy} &= Gt\left(u_{,y} + v_{,x} + w_{,x}w_{,y}\right) \\
M_x &= -\left[D\left(w_{,xx} + \mu w_{,yy}\right) + \frac{E_sI_s}{d}w_{,xx} - \bar{z}_s\frac{E_sA_s}{d}\left(u_{,x} + \frac{1}{2}w_{,x}^2 - \bar{z}_s w_{,xx}\right)\right] \\
M_y &= -\left[D\left(w_{,yy} + \mu w_{,xx}\right) + \frac{E_rI_r}{l}w_{,yy} - \bar{z}_r\frac{E_rA_r}{l}\left(v_{,y} + \frac{w}{R} + \frac{1}{2}w_{,y}^2 - \bar{z}_r w_{,yy}\right)\right] \\
M_{xy} &= \left(\frac{Gt^3}{6} + \frac{G_sJ_s}{d}\right)w_{,xy} \\
M_{yx} &= -\left(\frac{Gt^3}{6} + \frac{G_rJ_r}{l}\right)w_{,xy}
\end{aligned}
\right\} \tag{12}
$$

and $$I_s = I_{os} - \bar{z}_s^2A_s, \qquad I_r = I_{or} - \bar{z}_r^2A_r$$

The nonlinear equilibrium equations and boundary conditions are obtained from equation (11) by application of the principle of minimum potential energy ($\delta\Pi = 0$). The equations of equilibrium so obtained are:

$$
\left.
\begin{aligned}
&N_{x,x} + N_{xy,y} = 0 \\
&N_{y,y} + N_{xy,x} = 0 \\
&-M_{x,xx} + M_{xy,xy} - M_{yx,xy} - M_{y,yy} + \frac{N_y}{R} - N_xw_{,xx} - N_yw_{,yy} - 2N_{xy}w_{,xy} - \bar{M}\omega^2w + p = 0
\end{aligned}
\right\} \tag{13}
$$

A set of boundary conditions to be satisfied at each end of the cylinder are

$$M_{x,x} - \left(M_{xy,y} - M_{yx,y}\right) + N_xw_{,x} + N_{xy}w_{,y} = 0 \quad \text{or} \quad w = 0 \tag{14}$$

$$M_x + \bar{N}_x\bar{e} = 0 \quad \text{or} \quad w_{,x} = 0 \tag{15}$$

$$N_x + \bar{N}_x = 0 \quad \text{or} \quad u = 0 \tag{16}$$

$$N_{xy} = 0 \quad \text{or} \quad v = 0 \tag{17}$$

Stiffened Flat Plate

The equilibrium equations for a stiffened flat plate can be derived in a similar manner and are as follows:

$$\left.\begin{aligned} &N_{x,x} + N_{xy,y} = 0 \\ &N_{y,y} + N_{xy,x} = 0 \\ &-M_{x,xx} + M_{xy,xy} - M_{yx,xy} - M_{y,yy} - N_x w_{,xx} - N_y w_{,yy} - 2N_{xy} w_{,xy} - \bar{M}\omega^2 w + p = 0 \end{aligned}\right\} \tag{18}$$

where the moment and stress resultants are defined by equations (12) with R taken as infinite. The displacements u, v, and w are now displacements of the middle surface of the isotropic plate, and subscripts s and r refer to stiffening in the x- and y-directions, respectively. If load resultants $\bar{N}_x$ and $\bar{N}_y$ are applied to the edges of the plate along lines which are distances $\bar{e}_x$ and $\bar{e}_y$, respectively, from the plate middle surface, the boundary conditions which must be satisfied on an edge parallel to the y-axis are

$$\left.\begin{aligned} &M_{x,x} - \left(M_{xy,y} - M_{yx,y}\right) + N_x w_{,x} + N_{xy} w_{,y} = 0 \quad \text{or} \quad w = 0 \\ &M_x + \bar{N}_x \bar{e}_x = 0 \quad \text{or} \quad w_{,x} = 0 \\ &N_x + \bar{N}_x = 0 \quad \text{or} \quad u = 0 \\ &N_{xy} = 0 \quad \text{or} \quad v = 0 \end{aligned}\right\} \tag{19}$$

and for edges parallel to the x-axis

$$\left.\begin{aligned} &M_{y,y} - \left(M_{xy,x} - M_{yx,x}\right) + N_y w_{,y} + N_{xy} w_{,x} = 0 \quad \text{or} \quad w = 0 \\ &M_y + \bar{N}_y \bar{e}_y = 0 \quad \text{or} \quad w_{,y} = 0 \\ &N_y + \bar{N}_y = 0 \quad \text{or} \quad v = 0 \\ &N_{xy} = 0 \quad \text{or} \quad u = 0 \end{aligned}\right\} \tag{20}$$

In addition to the above boundary conditions $w_{,xy}$ must equal zero at a free corner of the plate.

VIBRATION AND BUCKLING EQUATIONS

In this section the nonlinear equations just derived are used to obtain linear equations which govern the small amplitude vibrations of a prestressed, eccentrically stiffened, cylindrical shell and an eccentrically stiffened flat plate. Buckling equations may be obtained from this analysis by setting the natural frequency equal to zero.

Stiffened Cylinder

The deformations associated with the vibration of a prestressed cylinder are divided into two parts as follows:

$$u = u_A + u_B, \qquad v = v_A + v_B, \qquad w = w_A + w_B \tag{21}$$

The first part, denoted by a subscript A, is an axisymmetric, static prestress deformation which occurs prior to excitation of one of the natural frequencies. The second part, denoted by a subscript B, is a small additional deformation which occurs as a result of the excitation. Since the subscript A quantities are static deformations, the term $\overline{M}\omega^2 w_A$ is zero, and the equilibrium equations which govern these deformations are found from equations (13) as

$$\left.\begin{aligned} &N_{xA,x} = 0 \\ &N_{xyA,x} = 0 \\ &-M_{xA,xx} + \frac{N_{yA}}{R} - N_{xA}w_{A,xx} + p = 0 \end{aligned}\right\} \tag{22}$$

From the second of equations (22), $N_{xyA} = 0$ if there is no applied shear.

A set of appropriate boundary conditions are found from equations (14), (15), (16), and (17) to be

$$\left.\begin{aligned} &M_{xA,x} + N_{xA}w_{A,x} = 0 \quad \text{or} \quad w_A = 0 \\ &M_{xA} + \overline{N}_x\overline{e} = 0 \quad \text{or} \quad w_{A,x} = 0 \\ &N_{xA} + \overline{N}_x = 0 \quad \text{or} \quad u_A = 0 \\ &N_{xyA} = 0 \quad \text{or} \quad v_A = 0 \end{aligned}\right\} \tag{23}$$

where in equations (22) and (23)

$$\left.\begin{aligned}
N_{xA} &= \frac{Et}{1-\mu^2}\left[u_{A,x} + \frac{1}{2}w_{A,x}^2 + \mu\left(\frac{w_A}{R}\right)\right] + \frac{E_sA_s}{d}\left(u_{A,x} + \frac{1}{2}w_{A,x}^2 - \bar{z}_s w_{A,xx}\right) \\
N_{yA} &= \frac{Et}{1-\mu^2}\left[\frac{w_A}{R} + \mu\left(u_{A,x} + \frac{1}{2}w_{A,x}^2\right)\right] + \frac{E_rA_r}{l}\left(\frac{w_A}{R}\right) \\
N_{xyA} &= Gtv_{A,x} \\
M_{xA} &= -\left[Dw_{A,xx} + \frac{E_sI_s}{d}w_{A,xx} - \bar{z}_s\frac{E_sA_s}{d}\left(u_{A,x} + \frac{1}{2}w_{A,x}^2 - \bar{z}_s w_{A,xx}\right)\right]
\end{aligned}\right\} \tag{24}$$

The equilibrium equations and boundary conditions which govern the additional deformations (subscript B) are obtained by substituting equations (21) into equations (13) through (17). If the axisymmetric prestress equations are subtracted out and if only linear terms in the additional deformations (subscript B) are retained the following equations governing the additional deformations are obtained

$$\left.\begin{aligned}
&N_{xB,x} + N_{xyB,y} = 0 \\
&N_{yB,y} + N_{xyB,x} = 0 \\
&-M_{xB,xx} + M_{xyB,xy} - M_{yxB,xy} - M_{yB,yy} + \frac{N_{yB}}{R} - N_{xA}w_{B,xx} \\
&-N_{xB}w_{A,xx} + N_{yA}w_{B,yy} - M\omega^2 w_B = 0
\end{aligned}\right\} \tag{25}$$

and the boundary conditions become

$$\left.\begin{aligned}
&M_{xB,x} - \left(M_{xyB,y} - M_{yxB,y}\right) + N_{xA}w_{B,x} + N_{xB}w_{A,x} = 0 \quad \text{or} \quad w_B = 0 \\
&M_{xB} = 0 \quad \text{or} \quad w_{B,x} = 0 \\
&N_{xB} = 0 \quad \text{or} \quad u_B = 0 \\
&N_{xyB} = 0 \quad \text{or} \quad v_B = 0
\end{aligned}\right\} \tag{26}$$

where

$$
\left.
\begin{aligned}
N_{xB} &= \frac{Et}{1 - \mu^2}\left[u_{B,x} + w_{A,x}w_{B,x} + \mu\left(v_{B,y} + \frac{w_B}{R}\right)\right] + \frac{E_sA_s}{d}\left(u_{B,x} + w_{A,x}w_{B,x} - \bar{z}_s w_{B,xx}\right) \\
N_{yB} &= \frac{Et}{1 - \mu^2}\left[v_{B,y} + \frac{w_B}{R} + \mu\left(u_{B,x} + w_{A,x}w_{B,x}\right)\right] + \frac{E_rA_r}{l}\left(v_{B,y} + \frac{w_B}{R} - \bar{z}_r w_{B,yy}\right) \\
N_{xyB} &= Gt\left(u_{B,y} + v_{B,x} + w_{A,x}w_{B,y}\right) \\
M_{xB} &= -\left[D\left(w_{B,xx} + \mu w_{B,yy}\right) + \frac{E_sI_s}{d} w_{B,xx} - \bar{z}_s \frac{E_sA_s}{d}\left(u_{B,x} + w_{A,x}w_{B,x} - \bar{z}_s w_{B,xx}\right)\right] \\
M_{yB} &= -\left[D\left(w_{B,yy} + \mu w_{B,xx}\right) + \frac{E_rI_r}{l} w_{B,yy} - \bar{z}_r \frac{E_rA_r}{l}\left(v_{B,y} + \frac{w_B}{R} - \bar{z}_r w_{B,yy}\right)\right] \\
M_{xyB} &= \left(\frac{Gt^3}{6} + \frac{G_sJ_s}{d}\right)w_{B,xy} \\
M_{yxB} &= -\left(\frac{Gt^3}{6} + \frac{G_rJ_r}{l}\right)w_{B,xy}
\end{aligned}
\right\} \qquad (27)
$$

The equilibrium equations (25) and boundary conditions (26) are the governing equations needed to solve for the natural frequencies of an eccentrically stiffened cylinder which is prestressed by the stress resultants N_{xA} and N_{yA}. The quantities having the subscript A must be determined from the solution of the prestress equations (22) and (23).

Stiffened Flat Plate

Equilibrium equations governing the additonal deformations of an eccentrically stiffened flat plate along with the definitions of the stress and moment resultants can be obtained by following a procedure similar to that already outlined for the cylinder. Note, however, that in general there are no simplifications of the equations from symmetry of the prestress state. The flat-plate equations are omitted here for brevity.

SOLUTIONS AND DISCUSSION

In this section solutions are obtained for several problems associated with stiffened cylinders and flat plates. In all cases the solutions are for simple support-type boundary conditions.

Buckling and Vibration of Stiffened Cylinder

In general, equations (25) have variable coefficients and would be quite difficult to solve for many cases. If the assumption analagous to that of classical cylinder buckling theory is made, that the prestress deformation w_A is contant prior to the excitation, the solution to equations (25) is greatly simplified. A short discussion of prestress deformations and the assumption of constant w_A is given in the appendix. With w_A constant, the solution to the prestress equations are $N_{xA} = -\bar{N}_x$ and $N_{yA} = -\bar{N}_y = -pR$. Equations (25) may now be written as

$$\left.\begin{aligned} &N_{xB,x} + N_{xyB,y} = 0 \\ &N_{yB,y} + N_{xyB,x} = 0 \\ &-M_{xB,xx} + M_{xyB,xy} - M_{yxB,xy} - M_{yB,yy} + \frac{N_{yB}}{R} + \bar{N}_x w_{B,xx} + \bar{N}_y w_{B,yy} - \bar{M}\omega^2 w = 0 \end{aligned}\right\} \tag{28}$$

where the stress and moment resultants can be found by substituting w_A = Constant into equations (27). A coordinate system is chosen having its origin at one end of the cylinder. The simple support boundary conditions to be satisfied at each end, $x = 0,a$ are:

$$w_B = M_{xB} = v_B = N_{xB} = 0 \tag{29}$$

Expressions for the displacements u_B, v_B, and w_B which satisfy these boundary conditions are given as

$$\left.\begin{aligned} u_B &= \bar{u} \cos \frac{m\pi x}{a} \cos \frac{ny}{R} \\ v_B &= \bar{v} \sin \frac{m\pi x}{a} \sin \frac{ny}{R} \\ w_B &= \bar{w} \sin \frac{m\pi x}{a} \cos \frac{ny}{R} \end{aligned}\right\} \tag{30}$$

where m is the number of axial half waves and n is the number of circumferential full waves. If equations (30) are substituted into equations (28) and the determinant of the coefficients of $\bar{u}$, $\bar{v}$, and $\bar{w}$ is set equal to zero, the following expression results

$$\frac{\bar{M}a^4\omega^2}{\pi^4 D} = m^4\left(1 + \beta^2\right)^2 + m^4\left[\frac{E_s I_s}{Dd} + \beta^2\left(\frac{G_s J_s}{Dd} + \frac{G_r J_r}{Dl}\right) + \beta^4 \frac{E_r I_r}{Dl}\right]$$

$$+ \frac{12Z^2}{\pi^4}\left(\frac{1 + \bar{S}\Lambda_s + \bar{R}\Lambda_r + \bar{R}\bar{S}\Lambda_{rs}}{\Lambda}\right) - \left(\bar{N}_x + \bar{N}_y\beta^2\right)\frac{m^2a^2}{\pi^2 D} \tag{31}$$

where

$$\left.\begin{aligned}
\Lambda_s &= 1 + 2\alpha^2\left(\frac{\bar{z}_s}{R}\right)\left(\beta^2 - \mu\right) + \alpha^4\left(\frac{\bar{z}_s}{R}\right)^2\left(1 + \beta^2\right)^2 \\
\Lambda_r &= 1 + 2n^2\left(\frac{\bar{z}_r}{R}\right)\left(1 - \beta^2\mu\right) + n^4\left(\frac{\bar{z}_r}{R}\right)^2\left(1 + \beta^2\right)^2 \\
\Lambda_{rs} &= n^2\alpha^2\left(\frac{\bar{z}_s}{R}\right)^2\left[\beta^2\left(1 - \mu^2\right) + 2(1 + \mu)\right] + n^4\left[1 - \mu^2 + 2\beta^2(1 + \mu)\right]\left(\frac{\bar{z}_r}{R}\right)^2 \\
&\quad + 2n^2\left(1 - \mu^2\right)\left(\frac{\bar{z}_s}{R}\right) + 2n^2\left(1 - \mu^2\right)\left(\frac{\bar{z}_r}{R}\right) + 2n^4(1 + \mu)^2\left(\frac{\bar{z}_r}{R}\right)\left(\frac{\bar{z}_s}{R}\right) + 1 - \mu^2 \\
\Lambda &= \left(1 + \beta^2\right)^2 + 2\beta^2(1 + \mu)(\bar{R} + \bar{S}) + \left(1 - \mu^2\right)\left[\bar{S} + \bar{R}\beta^4 + 2\beta^2\bar{R}\bar{S}(1 + \mu)\right]
\end{aligned}\right\} \tag{32}$$

$$Z^2 = \frac{a^4\left(1 - \mu^2\right)}{R^2t^2}, \qquad \beta = \frac{na}{m\pi R}, \qquad \alpha = \frac{m\pi R}{a}, \qquad \bar{S} = \frac{E_s A_s}{Etd}, \qquad \bar{R} = \frac{E_r A_r}{Etl} \tag{33}$$

Equation (31) is a closed form expression which gives the natural frequencies for a prestressed eccentrically stiffened cylinder. If the quantity ω is set equal to zero in equation (31) a stability equation which is identical to that presented in reference 5 is obtained and for the case of hydrostatic pressure, equation (31) can be shown to be identical to that presented in reference 1.

In equation (31), the effect of eccentric stiffening is reflected by the terms containing $\bar{z}_s$ and $\bar{z}_r$. The quantities $\bar{z}_s$ and $\bar{z}_r$ are positive when the stiffeners are located on the external surface of the cylinder and negative when the stiffeners are on

the inner surface so that several sign changes can occur in the equation with change of stiffener location. However, the quantity $\left(\beta^2 - \mu\right)$ in Λ_s and the quantity $\left(1 - \beta^2\mu\right)$ in Λ_r can also change sign depending upon the cylinder geometry and deflected shape. These facts suggest that caution should be exercised in drawing general conclusions as to the influence of eccentric stiffening on the behavior of stiffened cylinders.

The effects of eccentricities on the buckling of eccentrically stiffened cylinders in axial compression have been studied by many authors (for example, see refs. 1, 3, 4, 5, and 6) and it is believed that these effects are fairly well understood. In the present paper results are given on the buckling of stiffened cylinders under hydrostatic pressure to augment the results presented in reference 2. The results obtained from the present analysis for the buckling of a particular ring and stringer stiffened cylinder under a hydrostatic pressure loading are given in table I and figure 2. The cylinder and stiffener properties are the same as one of the longitudinally stiffened cylinders, investigated experimentally in reference 6; the cylinder properties are given in figure 2. From table I it can be seen that for the cylinder configuration considered, rings alone are always better on the inside. The results included in the table for external stringers are presented in figure 2, with the dashed curve giving the buckling pressures for rings attached externally and the solid curve giving the buckling pressures for rings attached internally. This figure illustrates how the effects of ring eccentricities on buckling can depend on the geometry of the shell. The curves intersect at a length-radius ratio a/R of approximately two. For $\frac{a}{R} < 2$ external rings give higher buckling pressures and for $\frac{a}{R} > 2$ internal rings give higher buckling pressures. In reference 2 only one length cylinder was considered so that the effect of length on stiffener eccentricities was not noted. The results of the present paper show that under hydrostatic pressure loadings general conclusions about the effects of stiffener eccentricities cannot be made.

The natural frequencies in the absence of prestress are plotted for a ring stiffened cylinder in figure 3. The dimensions and physical properties of the cylinder and

stiffening are given in the figure. For this case the eccentricity effects are not very large. The lowest natural frequency is 6 percent higher for rings on the inside than for the corresponding frequency for rings on the outside. Notice, however, that for $m = 2$ external rings give a higher frequency at the lower portion of the curves than internal rings. This situation can be explained by examination of the term which involves $\left(1 - \beta^2\mu\right)$ in equation (31). For a cylinder which is stiffened with only rings, this term is the only one which can change sign. For small β, the term $\left(1 - \beta^2\mu\right)$ is positive and external rings give a higher frequency than internal rings. As β increases, $\left(1 - \beta^2\mu\right)$ becomes negative at which time internal rings give a higher frequency than external rings. The crossing on the plot for either $m = 1$ or $m = 2$ can be found by setting $\left(1 - \beta^2\mu\right)$ equal to zero and solving for n.

The natural frequencies in the absence of prestress are given in figure 4 for an integrally stringer stiffened cylinder which was studied both experimentally and analytically for compressive buckling in reference 3. Dimensions of this cylinder simulate those of a projected design of a 33-foot-diameter tank for a nuclear powered upper stage of a large launch vehicle (ref. 3). The lowest natural frequency for this case was 35 percent higher for external stiffeners than for the same stiffening attached internally. The point where the curves cross is where the term $\left(\beta^2 - \mu\right)$ in equation (31) changes sign.

The situation for a cylinder which is stiffened with both rings and stringers is further complicated by the coupling terms A_{rs} in equation (31). To determine the influence of internal or external stiffening for this case, the natural frequencies must be investigated from equation (31) for the specific cylinder of interest.

These results indicate that the effects of eccentricities on the buckling and vibration behavior of a stiffened cylinder depend not only on the stiffener geometry but also on the overall shell geometry and the loading. It is difficult, therefore, to make generalizations regarding the best stiffening arrangement to be used in a particular application without making a complete parametric study to optimize the design.

Buckling and Vibration of Stiffened Flat Plate

A parallel analysis can be used to obtain the natural frequencies for a stiffened prestressed flat plate. For this case the deformation w_A is taken equal to zero. A discussion of this assumption is given in the appendix. A coordinate system is chosen having its origin at one corner of a plate of length a and width b. The simple support boundary conditions to be satisfied at $x = 0,a$ are $w_B = M_{xB} = N_{xB} = v_B = 0$ and at $y = 0,b$ are $w_B = M_{yB} = N_{yB} = u_B = 0$. Expressions for the deformations u_B, v_B, and w_B which satisfy these boundary conditions are

$$\left.\begin{aligned} u_B &= \bar{u} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\ v_B &= \bar{v} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \\ w_B &= \bar{w} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \end{aligned}\right\} \tag{34}$$

where now m and n are the number of half waves in the x- and y-directions, respectively. If equations (28) are written in terms of the deformations and R is permitted to approach infinity, substitution of equations (34) yields the following characteristic equation:

$$\begin{aligned} \frac{\bar{M}a^4\omega^2}{\pi^4 D} = {} & m^4\left(1 + \beta^2\right)^2 + m^4\left[\frac{E_s I_s}{Dd} + \beta^2\left(\frac{G_s J_s}{Dd} + \frac{G_r J_r}{Dl}\right) + \beta^4 \frac{E_r I_r}{Dl}\right] \\ & + \frac{12\left(1 - \mu^2\right)m^4\left[\bar{S}\left(1 + \beta^2\right)^2\left(\frac{\bar{z}_s}{t}\right)^2 + \bar{R}\beta^4\left(1 + \beta^2\right)^2\left(\frac{\bar{z}_r}{t}\right)^2 + \overline{RSC}\right]}{\left(1 + \beta^2\right)^2 + 2\beta^2(1 + \mu)(\bar{R} + \bar{S}) + \left(1 - \mu^2\right)\left[\bar{S} + \beta^4\bar{R} + 2\beta^2\overline{RS}(1 + \mu)\right]} \\ & - \left(\bar{N}_x + \bar{N}_y\beta^2\right)\frac{m^2a^2}{\pi^2 D} \end{aligned} \tag{35}$$

where

$$C = \beta^2\left[\beta^2\left(1 - \mu^2\right) + 2(1 + \mu)\right]\left(\frac{\bar{z}_s}{t}\right)^2 + 2\beta^4(1 + \mu)^2\left(\frac{\bar{z}_s}{t}\right)\left(\frac{\bar{z}_r}{t}\right) + \beta^4\left[1 - \mu^2 + 2\beta^2(1 + \mu)\right]\left(\frac{\bar{z}_r}{t}\right)^2 \tag{36}$$

and the following nondimensional parameters are defined:

$$\beta = \frac{na}{mb}, \qquad \bar{S} = \frac{E_s A_s}{Etd}, \qquad \bar{R} = \frac{E_r A_r}{Etl}$$

In equation (35), the terms which involve $\bar{z}_s$ and $\bar{z}_r$ are present because of eccentric stiffening. If the plate is stiffened by only transverse or only longitudinal stiffeners, all terms involving $\bar{z}_r$ or $\bar{z}_s$ are squared and the side to which the stiffening is attached is irrelevant. However, if both longitudinal and transverse stiffening are present, the term C defined by equation (36) has a term with the coefficient $\bar{z}_r\bar{z}_s$ which is negative if the transverse and longitudinal stiffeners are on opposite sides of the plate; hence, the side to which the stiffening is attached does have significance.

Results for the buckling of an eccentrically stiffened flat plate under compressive loading are presented in table II. The table shows results obtained from the present theory and results obtained from a classical orthotropic theory (see, for example, ref. 7) for various stiffening configurations. The details of the plate configuration are given in figure 5; the orthotropic constants used to obtain the results were calculated as in reference 8. The results differ for all stiffening configurations, but the largest differences occur when stiffening in both directions is considered. The classical orthotropic theory gives the same buckling load for stiffening attached to the same surface as it does for transverse stiffening on one surface and longitudinal stiffening attached to the opposite surface. The present theory, however, gives a difference in buckling load of over 30 percent. This difference results from the fact that the product $\bar{z}_r\bar{z}_s$ is negative in equation (36).

The natural frequencies in the absence of prestress are plotted in figure 5 for a flat plate which is stiffened in both directions. The plate and stiffener proportions are given in the figure. The frequencies are seen to be higher for the case when all stiffening is on the same surface of the plate. When the stiffening is on opposite surfaces, the product $\bar{z}_r\bar{z}_s$ is negative in equation (36) and the frequencies are consequently lowered. For the case where there are stiffeners in only one direction, the

surface upon which the stiffeners are attached is unimportant; however, eccentricity effects must still be considered to determine the frequencies accurately.

These results illustrate the importance of considering stiffener eccentricities even in the analysis of flat plates.

Flutter of Stiffened Flat Plate

The flutter of eccentrically stiffened flat plates is an aeroelastic stability problem which is of considerable interest to the designers of high-speed aircraft. In this paper the problem is not considered in detail but an approximate solution is presented to show the differences that can arise between the stiffening theory presented herein and an orthotropic theory which has been used in the past to analyze such problems. If the aerodynamic loading of a supersonic flow in the x-direction is assumed to be given by the simple Ackeret value $-\frac{2q}{\left(M^2 - 1\right)^{1/2}} w_{B,x}$, and no applied loads are present the equilibrium equations become

$$\left.\begin{aligned} &N_{xB,x} + N_{xyB,y} = 0 \\ &N_{yB,y} + N_{xyB,x} = 0 \\ &-M_{xB,xx} + M_{xyB,xy} - M_{yxB,xy} - M_{yB,yy} - \bar{M}\omega^2 w_B + \frac{2q}{\left(M^2 - 1\right)^{1/2}} w_{B,x} = 0 \end{aligned}\right\} \tag{37}$$

where q is the dynamic pressure and M is the Mach number. A two-term Galerkin solution is to be obtained for equations (37) for the case of eccentric stiffening which is perpendicular to the flow direction. A two-term solution which satisfies the simple support boundary conditions is given by the following:

$$\left.\begin{aligned} u_B &= \left(\bar{u}_1 \cos \frac{\pi x}{a} + \bar{u}_2 \cos \frac{2\pi x}{a}\right) \sin \frac{n\pi y}{b} \\ v_B &= \left(\bar{v}_1 \sin \frac{\pi x}{a} + \bar{v}_2 \sin \frac{2\pi x}{a}\right) \cos \frac{n\pi y}{b} \\ w_B &= \left(\bar{w}_1 \sin \frac{\pi x}{a} + \bar{w}_2 \sin \frac{2\pi x}{a}\right) \sin \frac{n\pi y}{b} \end{aligned}\right\} \tag{38}$$

Substituting equations (38) into equations (37), applying the Galerkin method, and maximizing the dynamic pressure parameter $\left(\lambda = \frac{2qa^3}{D\left(M^2 - 1\right)^{1/2}}\right)$ with respect to frequency ω, results in the following expression for the critical value of the dynamic pressure parameter:

$$\lambda_{cr} = \frac{3\pi^4}{16}\left(15 + 3\upsilon^2\left(2 + \frac{G_rJ_r}{Dl}\right) + \frac{E_rA_r\bar{z}_r^2\upsilon^6\bar{R}(1 + \mu)}{lD}\left\{\frac{2 + (1 - \mu)\upsilon^2}{\left(\upsilon^2 + 1\right)^2 + \bar{R}\upsilon^2\left[2(1 + \mu) + \upsilon^2(1 - \mu^2)\right]}\right.\right.$$

$$\left.\left. - \frac{8 + (1 - \mu)\upsilon^2}{\left(4 + \upsilon^2\right)^2 + \bar{R}\upsilon^2\left[8(1 + \mu) + \upsilon^2(1 - \mu^2)\right]}\right\}\right) \tag{39}$$

where $\upsilon = \frac{na}{b}$, $\bar{R} = \frac{E_rA_r}{Etl}$.

The corresponding equation from reference 7 (eq. (A4) in ref. 7) is the result of obtaining a two-term Galerkin solution to equilibrium equations derived from a classical orthotropic plate theory. This equation is

$$\lambda_{cr} = \frac{9\pi^4}{16}\left(5 + 2\upsilon^2\frac{D_{xy}}{D_x}\right) \tag{40}$$

where D_x is the panel stiffness in x-direction and D_{xy} is the twisting stiffness of the panel. Considering a square panel with dimensions and properties as given in figure 6, equation (39) gives a critical dynamic pressure q_{cr} equal to $\frac{472\left(M^2 - 1\right)^{1/2}}{a^3}$ while equation (40) gives a critical dynamic pressure of $\frac{247\left(M^2 - 1\right)^{1/2}}{a^3}$. The stiffening theory derived in this paper therefore gives a critical value of dynamic pressure which is almost twice the value obtained from the orthotropic theory. Although more terms must be taken in the solution to assure convergence, the difference is significant and can in part account for some of the discrepancies which exist between theory and experiment with regard to the flutter of eccentrically stiffened panels.

CONCLUDING REMARKS

Eccentric stiffening is found to have a significant effect on the buckling and vibration characteristics of eccentrically stiffened cylinders and flat plates. The expressions derived for the cylinder and plate relating natural frequency and prestress stress resultants may be used to examine a variety of prestress vibration problems and buckling problems which result from various combinations of loading. Nondimensional stiffening parameters are defined and the expressions mentioned above may be used to perform a parametric study which would optimize the stiffening configuration for a particular application.

Results obtained from the present theory are compared to results obtained from a classical orthotropic theory for the buckling and flutter of eccentrically stiffened flat plates. In both cases significant differences in results are obtained from the two theories.

It is concluded that stiffener eccentricity effects on the static and dynamic behavior of eccentrically stiffened cylinders and flat plates are important and should be considered in the design and analysis of stiffened structural components. It is rather difficult, however, to make positive statements regarding the effects that eccentricities might have in any particular application. This is particularly true for the case of stiffened cylinders. The eccentricity effects depend on the type of loading and the configuration and physical properties of both the cylinder and the stiffening. As a result, each particular configuration must be thoroughly analyzed to determine these effects. In the present study of the significance of stiffener eccentricity simple membrane prestress states are assumed, the implications of these assumptions are discussed in an appendix, where it is concluded that future investigation should be conducted in this area to study both the effects of considering a more exact prestress deformation and the effects of eccentric loading.

APPENDIX

PRESTRESS DEFORMATIONS

In this appendix some of the implications of the solutions assumed in this paper for the prestress deformations, i.e., w_A equals a constant for the cylinder and equals zero for the flat plate, are discussed. Prebuckling solutions of this type have been employed in the past to formulate classical theories for vibration and buckling of unstiffened plates and shells. Recent studies (refs. 9 and 10), however, have demonstrated that shell buckling predictions can be sensitive to prebuckling deformations introduced by restraint of the shell boundaries prior to buckling. Hence the comparable effects of shell boundary restraints on natural frequencies and buckling behavior of stiffened shells deserve to be investigated. Moreover, in the stiffened shell or plate, the stiffening elements themselves introduce restraints which have no counterpart in unstiffened structures. For example, a rectangular flat plate, stiffened in one direction and loaded by in-plane loads perpendicular to the stiffening experiences lateral displacements from the inception of loading. The mechanism which causes the lateral deflection is the restriction of the transverse in-plane displacements (resulting from the Poisson effect) by the eccentric stiffening. If, however, the stiffeners are in the same direction as the loading and the load is applied at the neutral axis of the stiffener skin combination, no lateral deflection arises.

Similar situations also arise in eccentrically stiffened cylinders. For instance, an unsupported axially stiffened cylinder subjected to a lateral pressure suffers nonconstant lateral radial deformations. The reason again is the restriction of the axial displacement by the eccentric stiffeners. However, for the case of the axial loading of a ring and stringer stiffened cylinder w_A = Constant is valid for an unsupported cylinder when the line of application of the load $\overline{N}_x$ is located such that

$$\bar{e} = \frac{\dfrac{\bar{z}_s E_s A_s}{d}}{\dfrac{Et}{1 - \mu^2} + \dfrac{E_s A_s}{d} - \dfrac{\left(\dfrac{Et\mu}{1 - \mu^2}\right)^2}{\dfrac{Et}{1 - \mu^2} + \dfrac{E_r A_r}{l}}} \tag{A1}$$

Equation (A1) is obtained by a consideration of equations (22), (23), and (24). Note that for the case of no rings, equation (A1) locates the neutral surface of the stiffened cylinder. If the ring and stringer-stiffened cylinder is not loaded on the line defined by equation (A1), w_A = Constant is not a valid solution for an unsupported cylinder. In reference 3 the nonconstant prebuckling solution is considered in obtaining the solution for the buckling loads of a stringer stiffened cylinder loaded in axial compression. Results are presented in reference 3 which show that if the axial load is not applied at the neutral axis of the cylinder-stiffener combination, significant changes in the critical compressive load may result.

In the present paper the exact prestress state is not considered, but it is believed that the essential effects of stiffener eccentricities are illustrated. Future investigations should be conducted in this area in order to study both the effects of considering more exact prestress deformations, and the effects of eccentric loading.

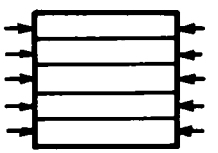
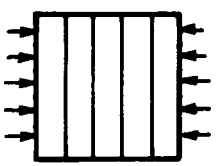
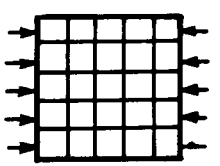
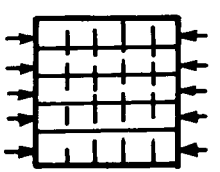
REFERENCES

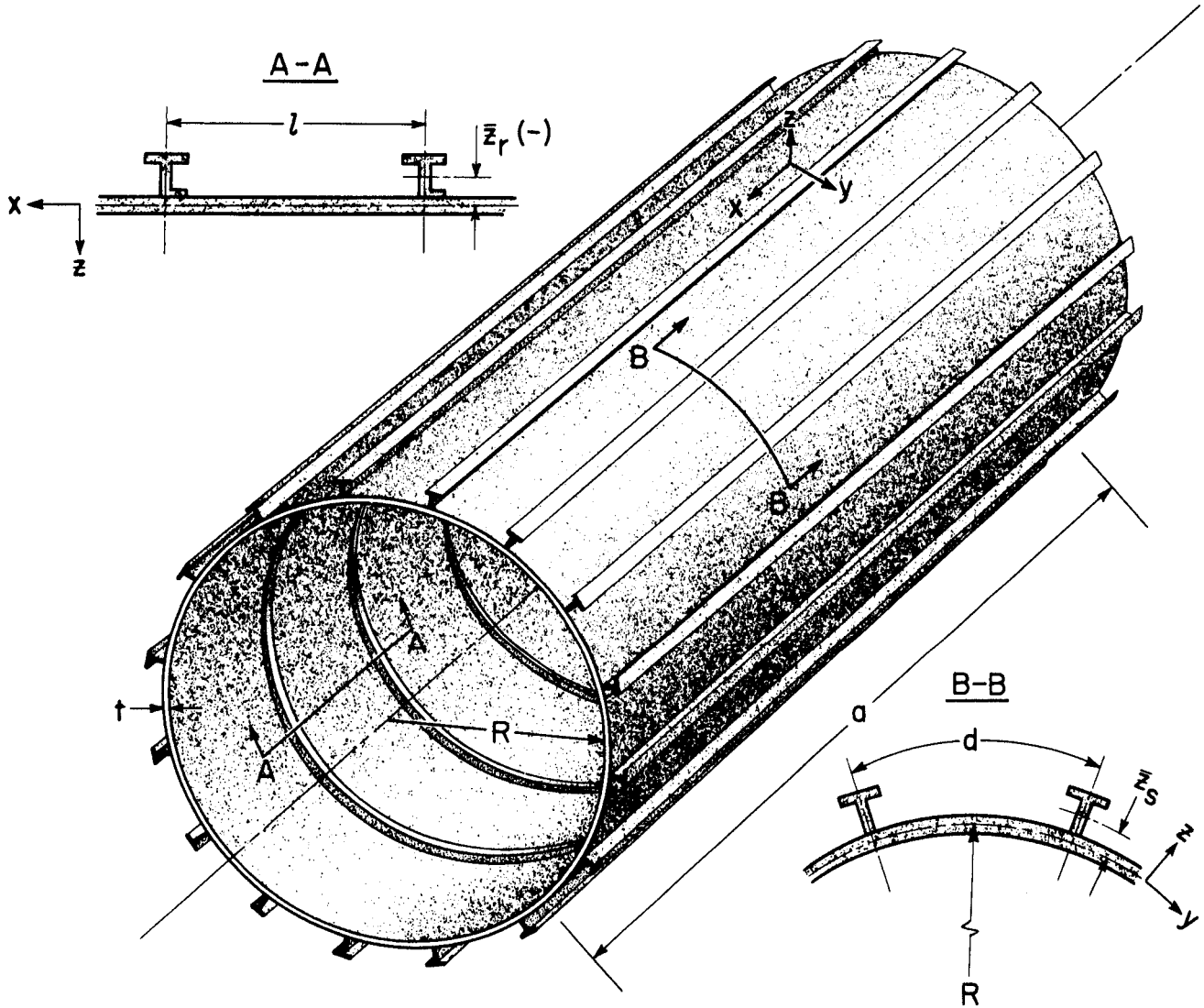
1. van der Neut, A.: The General Instability of Stiffened Cylindrical Shells Under Axial Compression. Rep. 5. 314, Nat. Aero. Res. Inst. (Amsterdam), 1947.
2. Baruch, M.; and Singer, J.: Effect of Eccentricity of Stiffeners on the General Instability of Stiffened Cylindrical Shells Under Hydrostatic Pressure. Jour. Mech. Eng. Sci., vol. 5, no. 1, 1963, pp. 23-27.
3. Anon.: Theoretical and Experimental Analysis of Orthotropic-Shell Stability. LSMC-A 701014 prepared under Contract NAS8-9500. Lockheed Missile and Space Co., Lockheed Aircraft Corp., Sunnyvale, Calif., 1964.
4. Hedgepeth, John M.; and Hall, David B.: Stability of Stiffened Cylinders. AIAA Paper No. 65-79 presented at Second Aerospace Sciences Meeting, New York, N.Y., Jan. 25-27, 1965.
5. Block, David L.; Card, Michael F.; and Mikulas, Martin M., Jr.: Buckling of Eccentrically Stiffened Orthotropic Cylinders. Proposed NASA TN.
6. Card, Michael F.: Preliminary Results of Compression Tests on Cylinders With Eccentric Longitudinal Stiffeners. NASA TM X-1004, 1964.
7. Bohon, Herman L.: Flutter of Flat Rectangular Orthotropic Panels With Biaxial Loading and Arbitrary Flow Direction. NASA TN D-1949, 1963.
8. Card, Michael F.: Bending Tests of Large Diameter Stiffened Cylinders Susceptible to General Instability. NASA TN D-2200, 1964.
9. Stein, Manuel: The Influence of Prebuckling Deformations and Stresses on the Buckling of Perfect Cylinders. NASA TR R-190, 1964.
10. Almroth, B. O.: Influence of Edge Conditions on the Stability of Axially Compressed Cylindrical Shells. NASA CR-161, 1965.

TABLE I. - HYDROSTATIC BUCKLING PRESSURES (PSI) FOR CYLINDERS WITH VARIOUS STIFFENING CONFIGURATIONS

LENGTH - RADIUS RATIO	1.26	2.51	3.77	6.28
RINGS - EXTERNAL STRINGERS - EXTERNAL	291	147	96	75
RINGS - INTERNAL STRINGERS - EXTERNAL	251	151	104	83
RINGS - EXTERNAL STRINGERS - INTERNAL	221	130	86	72
RINGS - INTERNAL STRINGERS - INTERNAL	221	141	96	81
RINGS - EXTERNAL NO STRINGERS	207	121	75	67
RINGS - INTERNAL NO STRINGERS	215	139	90	78
NO RINGS STRINGERS - EXTERNAL	11.1	3.7	2.1	1.4
NO RINGS STRINGERS - INTERNAL	8.0	2.7	1.6	1.2
UNSTIFFENED ISOTROPIC CYLINDER	3.7	1.9	1.2	0.9

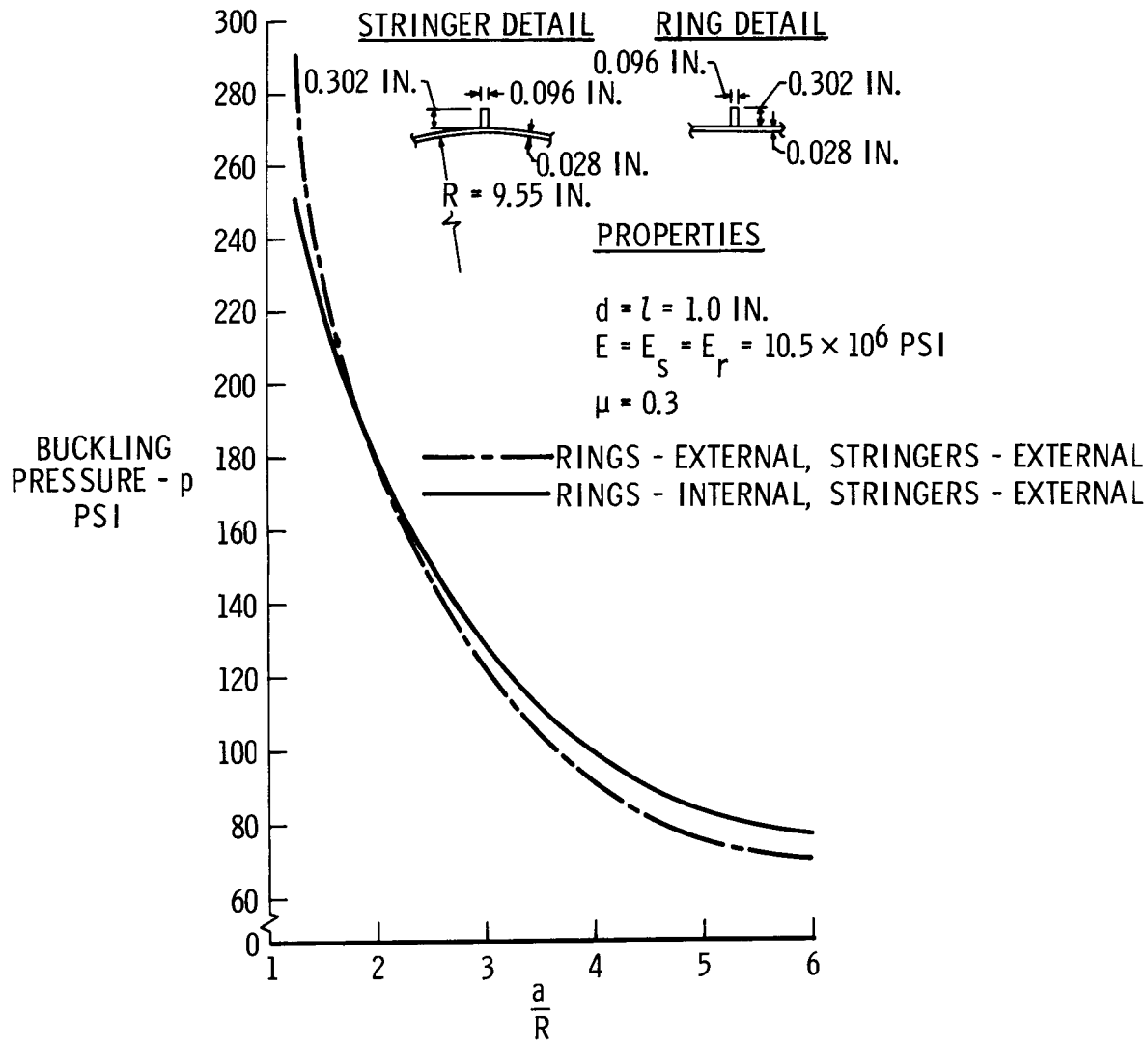
TABLE 2 - COMPARISON BETWEEN ORTHOTROPIC PLATE THEORY AND PRESENT THEORY FOR THE BUCKLING OF THE ECCENTRICALLY STIFFENED PLATE DETAILED IN FIGURE 5

CONFIGURATION	ORTHOTROPIC THEORY - $\bar{N}_x$(LB/IN.)	PRESENT THEORY - $\bar{N}_x$(LB/IN.)
$\bar{N}_x$  $\bar{N}_x$	120	125
	20	23
	240	294
	240	220



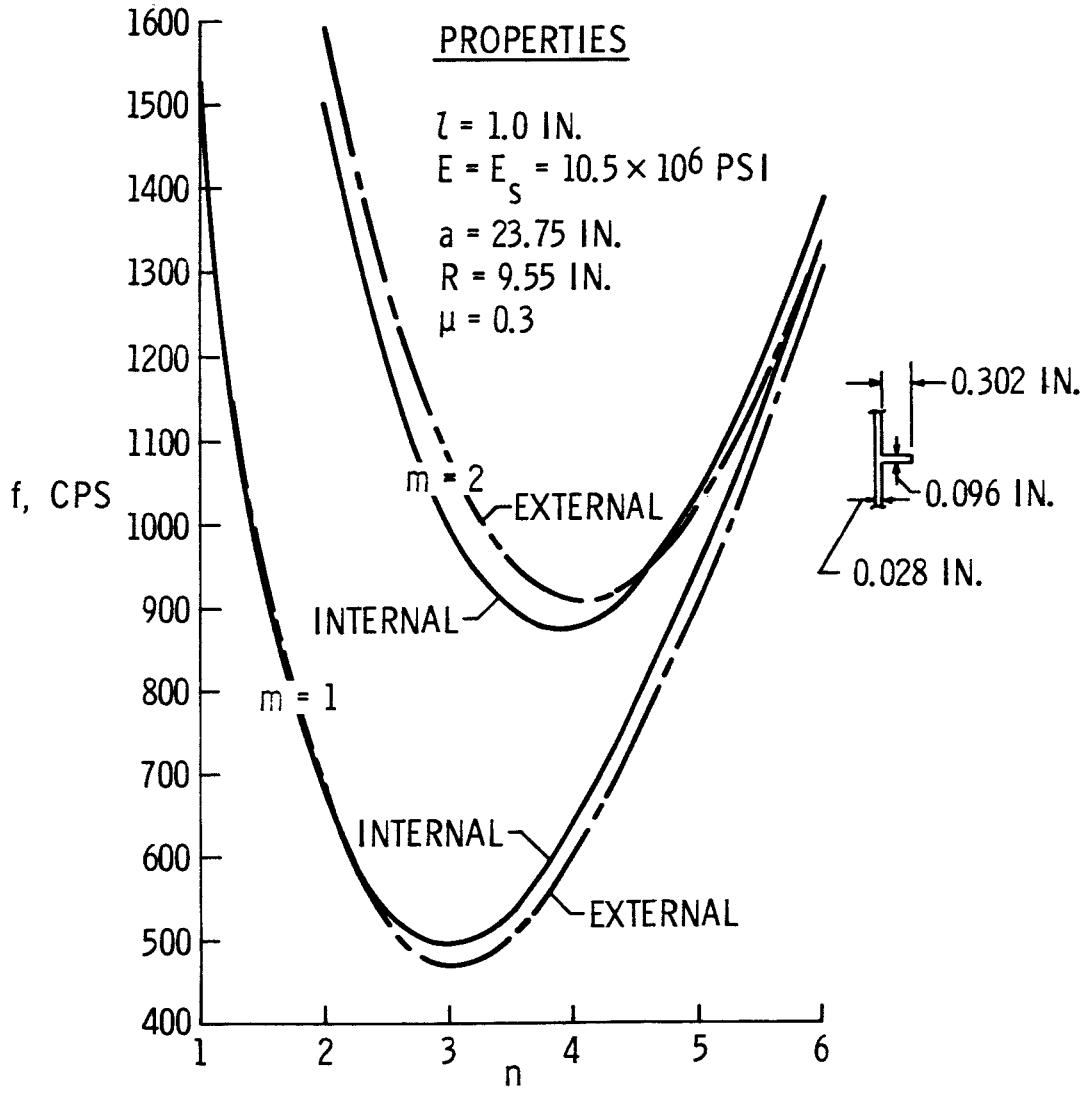
NASA

Figure 1.- Geometry of eccentrically stiffened cylinder.



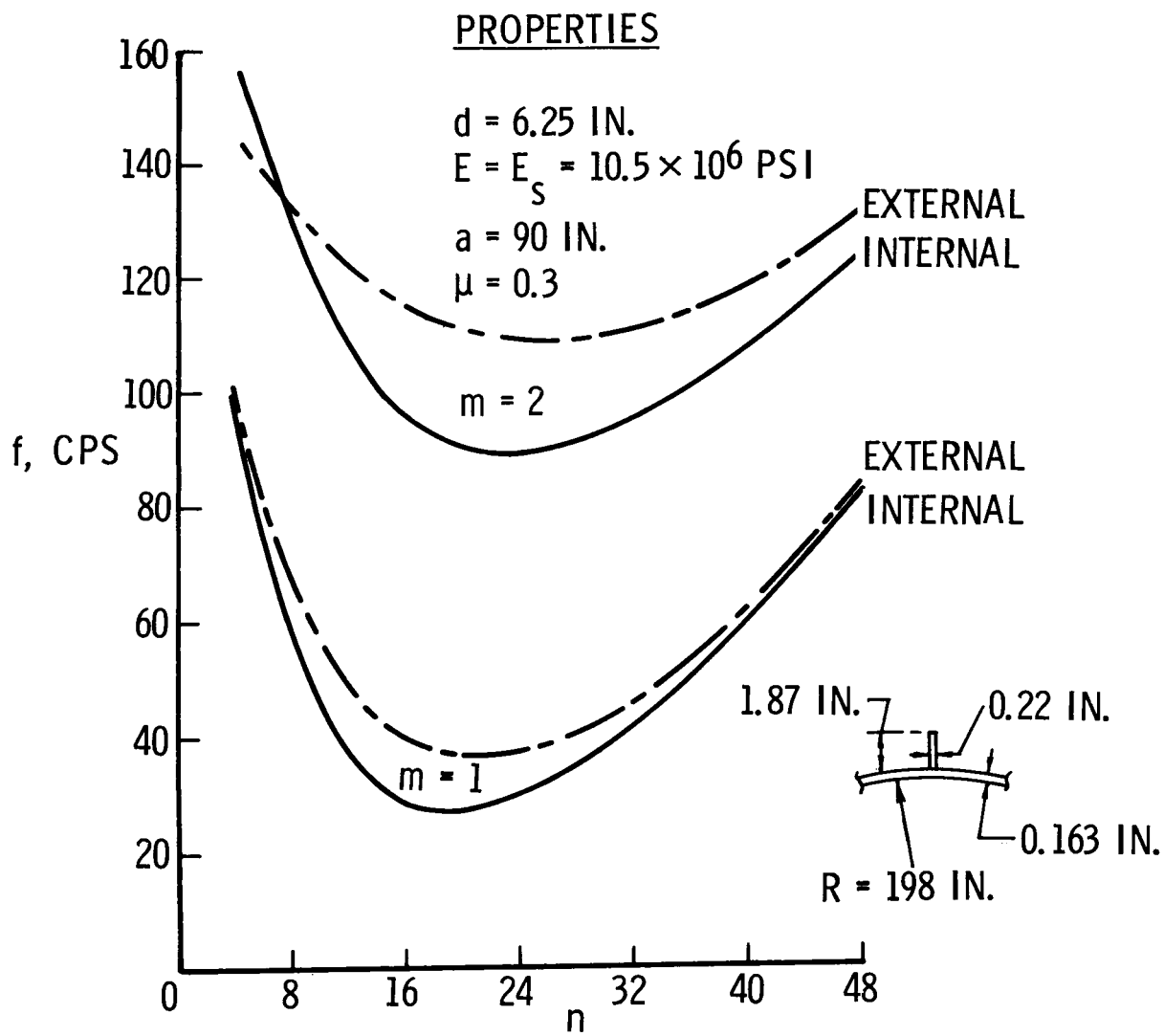
NASA

Figure 2.- Hydrostatic pressure buckling of a ring and stringer stiffened cylinder.



NASA

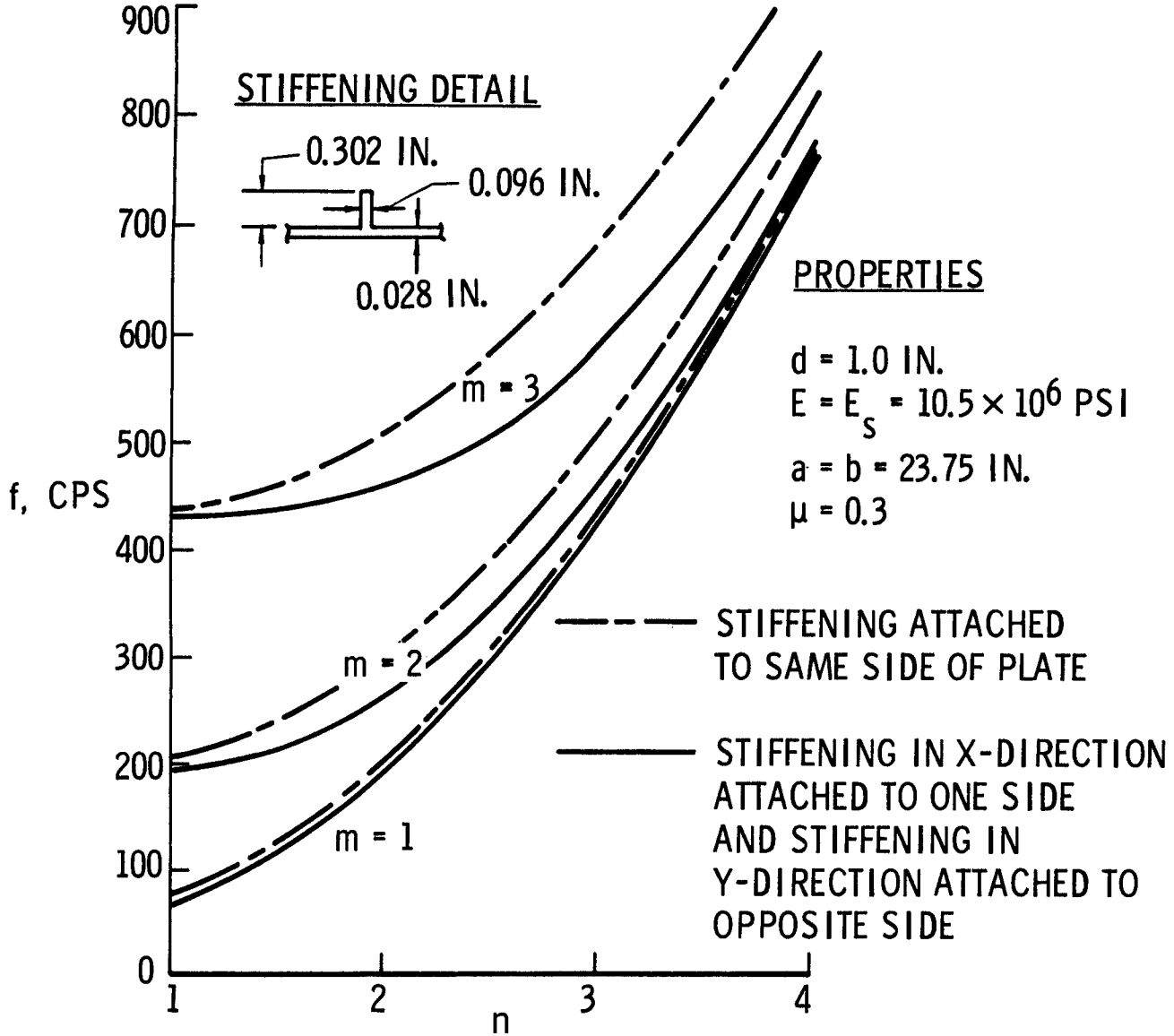
Figure 3.- Natural frequencies of a ring stiffened cylindrical shell.



NASA

Figure 4.- Natural frequencies of stringer stiffened cylindrical shell.

NASA-Langley, 1965



NASA

Figure 5.- Natural frequencies for a square plate stiffened in x and y directions.