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THE SUBJECT OF CONSTRAINT EQUATIONS RELATING STRESSES AND STRAINS IN FIBER GLASS

G. A. Wang Fou Feng

Based on models of a nonhomogeneous body, constraint equations are derived relating stresses and strains in a singly-reinforced strip of fiber glass, with proper consideration for the rheological properties of the binder. It has been established that, to describe the creep properties of a unidirectionally reinforced strip, it is necessary to take into consideration the time variation in the Poisson ratio and the modulus of elasticity of the binder. /110*

A comparison was made between the derived theoretical formulas and the results of experimental investigations of fiber glass in the elastic deformation range of the binder.

The high ultimate strength of thin fibers encourages widespread use of synthetic materials in new manufacturing techniques. For the utilization of this strength in products, fibers are bound into monolithic materials by means

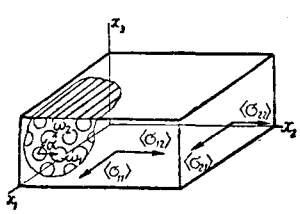

Figure 1.

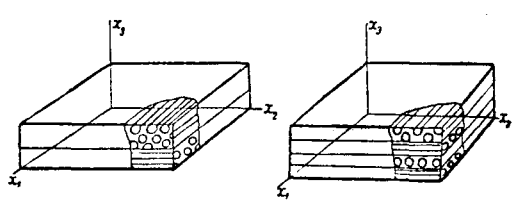

Figure 2.

of polymers or cements possessing sufficiently high adhesion to the fibers.

The resulting nonhomogeneous material, fiber glass or fiber-reinforced cement, is characterized by anisotropic mechanical and structural properties, imparted by the individual physico-mechanical properties of the filler and binder and by the manufacturing process. Of greatest practical interest are fiber glasses comprised of straightened fibers having the highest possible tensile properties at specific levels.

*Numbers in the margin indicate pagination in the original foreign text.

In this work, the mechanical properties of such fiber glasses are determined based on models, using the known properties of fillers and binders.

1. Let us consider a body comprised of a system of individual glass fibers and a viscoelastic homogeneous medium. Let the fibers be located parallel to each other at the lattice points of a regular biperiodic lattice while the spaces between the fibers are filled with a binder polymer (Figure 1).

Models of bodies reinforced in mutually perpendicular directions are shown in Figure 2. The manufacturing process for fiber glass also makes it possible to get other types or structures of the material. For finding the mechanical /111 properties of materials with arbitrary orientation of the filler, let us investigate the properties of a unidirectionally reinforced strip formed by a large number of individual fibers in a complex state of stress.

2. Results of investigations of the mechanical properties of glass fibers carried out by several authors [4] show that in the temperature range below 100°C, glass is a perfectly elastic material for which Hooke's law is valid.

$$\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33} = 3K_a\varepsilon_{kk},$$
$$\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} = 2G_a\left(\varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}\right). \tag{1}$$

Mechanical properties of polymer binders are significantly more complex than properties of glasses, and even cross-linked polymers exhibit rheological properties at ordinary temperatures. When subjected to a constant load for an extended period, polymers exhibit creep, only to return to their original state within a fixed time period after the load is removed. The stress-strain diagram is nonlinear; for epoxy-based binders, the distortion of the diagram, even at high stresses, is insignificant. Therefore, in the chosen approximation, we confine ourselves to investigation of the inelastic deformations of the binders based on the theory of elastic memory.

The total strain of a cross-linked polymer consists of elastic and hyperelastic constituents

$$\varepsilon = e + e_\infty.$$

Residual strain for epoxy resin binders is infinitely small and, therefore, is ignored. The elastic part e is related to the stress through the instantaneous modulus of elasticity

$$\sigma = E_0 e.$$

Hyperelastic strains are cumulative with time. Therefore, for treatment of experimental data, the equation of state should be plotted on the basis of

those models leading to the simplest kernels in the formulas of the theory of memory-possessing media, and allowing an approximate description of the fundamentals governing the deformation of the binder. An equation of the form

$$\sigma = E_\infty e_\infty + \mu D^\lambda e_\infty \qquad (D = \partial/\partial t; \quad 0 < \lambda < 1) \tag{2}$$

is proposed for describing the principal phenomena of hyperelastic deformation of cross-linked polymers. In particular, if we set

$$\lambda = 1; \qquad \mu = \mu_0 \exp\left\{ \frac{U - a\,|\,\sigma - E_\infty e_\infty\,|}{kT} \right\},$$

then (2) is transformed into the well-known equation of Maxwell-~~Gurevich~~ Hurewicz, used in [3] for treating experimental data from tests of BF-4.

The solution of linearized equation (2) leads to the well-known relations for memory-possessing elastic media. In the case of a unidimensional problem, the relation between strains and stresses for a homogeneous polymer is expressed as follows:

$$\varepsilon(t) = \frac{1}{E_0}\left[\sigma(t) + \varkappa_0 \int_0^t \sigma(t')\,\vartheta_{1-\lambda}(-\varkappa_\infty, t - t')\,dt'\right], \tag{3}$$

where /112

$$\varkappa_0 = \frac{E_0}{\mu}; \qquad \varkappa_\infty = \frac{E_\infty}{\mu}.$$

Here, E_∞ is the modulus of hyperelastic strain;

$$\vartheta_{1-\lambda}(-\varkappa_\infty, t - t') = (t - t')^{\lambda-1} \sum_{k=0}^{\infty} \frac{(-\varkappa_\infty)^k (t - t')^{k\lambda}}{\Gamma[\lambda(k+1)]}.$$

As is evident, the function of Yu. Rabotnov [5] appears as the aftereffect kernel. In the derivation of formula (3), the following relation was utilized:

$$D^{-\nu} f(t) = \int_0^t f(t') \frac{(t - t')^{\nu-1}}{\Gamma(\nu)}\,dt'.$$

The solution of (3) makes it possible to find stresses as a function of strain

$$\sigma(t) = E_0 \left[\varepsilon(t) - \varkappa_0 \int_0^t \varepsilon(t') \mathcal{Э}_{1-\lambda} (-\varkappa_0 - \varkappa_\infty, t - t') \, dt' \right] \tag{4}$$

or

$$\sigma(t) = E^* \varepsilon(t) = E_0 [1 - \varkappa_0 \mathcal{Э}^*_{1-\lambda} (-\varkappa_0 - \varkappa_\infty)] \varepsilon(t). \tag{5}$$

Here $\mathcal{Э}^*_{1-\lambda}(-\varkappa_0 - \varkappa_\infty, t - t')$ is the relaxation kernel.

During simple creep, when $\sigma = \sigma_0$, using the approximation of M. Rozovsky [6] we find

$$\varepsilon(t) \simeq \frac{\sigma_0}{E_0} + \frac{\sigma_0}{E_0} \{1 - \exp[-\varkappa_\infty (\lambda t)^\lambda]\}.$$

When $t \to \infty$ the continuous modulus will be

$$E_{эл}^{-1} = E_0^{-1} + E_\infty^{-1}.$$

In the case of simple relaxation, $\varepsilon = \varepsilon_0$, it follows from (4) that

$$\sigma(t) \simeq E_0 \varepsilon_0 = \frac{\varkappa_0 E_0 \varepsilon_0}{\varkappa_0 + \varkappa_\infty} \{1 - \exp[-(\varkappa_0 + \varkappa_\infty)(\lambda t)^\lambda]\}.$$

3. Let us consider a composite material in which the elements are located as shown in Figure 1. Let there be a large number of cross sections of individual fibers per unit of cross-sectional area. When investigating the stressed state of this body, tensor σ_{ik} can be usefully split into a deduced mean stress tensor and a tensor of lattice (fluctuating) stresses

$$\sigma_{ik} = \langle \sigma_{ik} \rangle + \sigma''_{ik}.$$

The components of σ''_{ik} characterize the nonuniform stressed state and at the average levels they do not increase, i.e., they are self-balancing. Similarly, let us assume

$$\varepsilon_{ik} = \langle \varepsilon_{ik} \rangle + \varepsilon''_{ik}.$$

In use, a constraint is set up only between $\langle\sigma_{ik}\rangle$ and $\langle\varepsilon_{ik}\rangle$.

Let us assume that the elementary rectangle (Figure 1) is under tensile stress $\langle\sigma_{11}\rangle$. In this case, the total solution of the problem is comprised of two separate ones: one for the case of tension without considering the interaction between filler and binder, and the second taking this interaction into account. /113

In order to consider the rheological properties of the binder we will make use of the principle of V. Volterra, according to which E*, ν*, G* for the binder are considered as integral operators. Then equations (1) will also be valid for polymer binders, but in these equations it is necessary to replace the modulus values by integral operators. Taking into account the high value of the Poisson's ratio for epoxy binders, we will ignore the inelastic deformations under bulk compression in the following. This assumption leads to the formula

$$\nu^* = \nu_0\left[1 + \frac{1-2\nu_0}{2\nu_0}\varkappa_0 \vartheta^*_{1-\lambda}(-\varkappa_0 - \varkappa_\infty)\right].$$

The stress adduced for the strip will be

$$\langle\sigma_{11}\rangle = (E_a\xi + E^*\eta)\langle\varepsilon_{11}\rangle + \langle\sigma'_{11}\rangle. \tag{6}$$

Here, ξ, η are the corresponding relative volume contents of filler and binder;

$$E^* f(t) = E_0\left[f(t) - \varkappa_0\int_0^t f(t')\vartheta_{1-\lambda}(-\varkappa_0 - \varkappa_\infty, t-t')\,dt'\right].$$

$\langle\sigma'_{11}\rangle$ represents the mean stresses found from the solution by considering the interaction between the reinforcement material and the binder. Analogous relations will occur for other operators as well.

When a bar is pulled by stresses $\langle\sigma_{22}\rangle$, due to the adhesion of the filler and the binder, the reinforcement will also take a corresponding part of the stress. We shall present the solution of this problem as the sum of two solutions: the first corresponding to the plane strained state when $\langle\varepsilon_{11}\rangle = 0$, and the second corresponding to the problem of a bar under tension along the x_1 axis.

For solving plane problems, let us utilize the methods of Kolosov-Muskhelishvili. In our case, all the functions will depend of the volume variable z and on time. The basic relations will be

$$\sigma_{rr} - i\sigma_{r\vartheta} = \Phi(z, t) + \overline{\Phi(z, t)} - e^{2i\vartheta}[\bar{z}\Phi'(z, t) + \Psi(z, t)];$$

$$2G(u + iv) = \varkappa\varphi(z, t) - \bar{z}\varphi'(z, t) - \psi(z, t); \qquad \sigma'_{11} = 4\nu \operatorname{Re}\Phi(z, t). \tag{7}$$

Here

$$\varphi'(z, t) = \Phi(z, t); \qquad \psi'(z, t) = \Psi(z, t).$$

Using the method of separation of variables for the region occupied by the reinforcing material, we will get

$$\varphi_a(\zeta, t) = iD \sum_{k=1,3}^{\infty} a_k(t)\zeta^k; \quad \psi_a(\zeta, t) = -iD \sum_{k=1,3}^{\infty} b_k(t)\zeta^k; \quad \zeta = \frac{z}{\omega_1}. \tag{8}$$

For the external region, we assume

$$\Phi_s(\zeta, t) = c_0(t) + \sum_{k=0}^{\infty} c_{2k+2}(t)\frac{\lambda^{2k+2}\mathfrak{P}^{(2k)}(\zeta)}{(2k+1)!};$$

$$\Psi_s(\zeta, t) = d_0(t) + \sum_{k=0}^{\infty} d_{2k+2}(t)\frac{\lambda^{2k+2}\mathfrak{P}^{(2k)}(\zeta)}{(2k+1)!} - \sum_{k=0}^{\infty} c_{2k+2}(t)\frac{\lambda^{2k+2}Q^{(2k+1)}(\zeta)}{(2k+1)!}, \tag{9}$$

where $\mathfrak{P}(\zeta)$, $Q(\zeta)$ are elliptic functions [7]; λ is a dimensionless radius. /114

$$\mathfrak{P}(\zeta) = \frac{1}{\zeta^2} + \sum_{m,n}{}' \left[\frac{1}{(\zeta - P)^2} - \frac{1}{P^2}\right]; \qquad P = m\omega_1 + n\omega_1 e^{\pi i/3};$$

$$Q(\zeta) = \sum_{m,n}{}' \left[\frac{\bar{P}}{(\zeta - P)^2} - \frac{2\zeta\bar{P}}{P^3} - \frac{\bar{P}}{P^2}\right].$$

ω_1, ω_2 are the basic periods in the structure.

From the conditions of equal displacement and equal stress at the glass-resin boundary (when $\xi = \tau$) we arrive at a system of equations which allows us to evaluate functions $a_n(t)$, $b_n(t)$, $c_n(t)$, $d_n(t)$,

$$\varkappa^*\overline{\Phi_s(\tau, t)} - \Phi_s(\tau, t) + e^{2i\vartheta}[\bar{\tau}\Phi_s'(\tau, t) + \Psi_s(\tau, t)] -$$
$$- G^*G_a^{-1}\{\varkappa_a\overline{\Phi_a(\tau, t)} - \Phi_a(\tau, t) + e^{2i\vartheta}[\bar{\tau}\Phi_a'(\tau, t) + \Psi_a(\tau, t)]\} =$$
$$= -G^*(\nu_a - \nu^*)\langle\varepsilon_{11}\rangle;$$

$$\Phi_a(\tau, t) + \overline{\Phi_a(\tau, t)} - e^{2i\vartheta}[\bar{\tau}\Phi_a'(\tau, t) + \Psi_a(\tau, t)] =$$
$$= \Phi_s(\tau, t) + \overline{\Phi_s(\tau, t)} - e^{2i}[\bar{\tau}\Phi_s'(\tau, t) + \Psi_s(\tau, t)]. \tag{10}$$

The given model allows us to get an approximate evaluation of the stresses of the bonding areas between the fibers and the resin.

4. During shear in plane (x_1, x_2) the interaction between the fibers occurs through the binder. If we assume that only the shear strains between the fibers differ from zero, the solution of the problem resolves itself into the determination of an analytical function u_1, characterizing normal displacements along the x_1 axis, whereas

$$\sigma_{x_1r} - i\sigma_{x_1\vartheta} = 2e^{i\vartheta}G\frac{\partial u_1}{\partial z}.$$

The boundary conditions reduce to equations of equivalent displacement and equality of tangential stress at the region of contact between the fibers and the resin when $z = \tau$

$$u_1(\tau, t) = u_1'(\tau, t); \qquad \sigma_{x_1r}(\tau, t) = \sigma_{x_1r}'(\tau, t), \tag{11}$$

where u_1', σ_{x_1r}' are the displacement and stress of the binder.

The solution of the given problem results in a formula of the form

$$\langle\varepsilon_{12}\rangle = X\langle\sigma_{12}\rangle, \tag{12}$$

where

$$X = \frac{\eta + (1+\xi)G^*/G_a}{1+\xi+\eta G^*/G_a}\cdot\frac{1}{G^*} - \frac{2\xi(1-G^*/G_a)}{1+\xi+\eta G^*/G_a}\cdot\frac{1}{G^*}\sum_{k=1}^{\infty} c_{2k+2}\lambda^{2k+2}a_{0,k}. \tag{13}$$

5. The relation between the mean normal stresses and strains is set up based on the system of equations (10, 11) and on the following averaging of stresses and strains. For describing the creep phenomena, operator relations have been obtained which have the form

$$\langle\varepsilon_{11}\rangle = X_{11}\langle\sigma_{11}\rangle + X_{12}\langle\sigma_{12}\rangle,$$
$$\langle\varepsilon_{22}\rangle = X_{21}\langle\sigma_{11}\rangle + X_{22}\langle\sigma_{22}\rangle. \tag{14}$$

Operators X_{ik} are generally large enough to be used in the theory of /115 shells. For fiber glass, the derived formulas can be simplified without allowing significant errors. These simplifications lead to the following relations, suitable for structuring a theory for creep in thin-walled shells:

$$X_{11}^{-1} \simeq \xi E_a + \eta E^*; \qquad X \simeq \frac{\eta + (1+\xi) G^*/G_a}{1 + \xi + \eta G^*/G_a} \cdot \frac{1}{G^*};$$

$$X_{22} \simeq \frac{1 - \nu^{*2}}{E^*} \left[\frac{2 + (\varkappa_a - 1) G^*/G_a}{1 + \eta + \xi\varkappa^* + \eta G^*/G_a (\varkappa_a - 1)} + \frac{2\xi (1 - G^*/G_a)}{\xi + \varkappa^* + \eta G^*/G_a} \right];$$

$$X_{12} = X_{21} \simeq - X_{22} \left[\nu^* - \frac{4\xi (1 - \nu^*)(\nu^* - \nu_a)}{1 + \eta + \xi\varkappa^* + \eta (\varkappa_a - 1) G^*/G_a} \right]. \qquad (15)$$

As is evident, for a unidirectionally reinforced strip it is essential to take the creep of the material into account during shear and tension in the direction normal to the reinforcement.

The values of the moduli determined for instantaneous elastic deformation of the binder were compared with the results of experimental investigations of the mechanical properties of a unidirectionally reinforced strip. For calculation, the values used were

$$E_a = 7 \times 10^5; \qquad E_0 = 3.5 \times 10^4; \qquad \nu_a = 0.2; \qquad \nu_0 = 0.4.$$

The results of the calculations and their comparison with experimental data* are shown in the table.

	G_1	E_1	E_2	ν_1^0	ν_2^0
Theory	6.05×10^4	5×10^5	1.6×10^5	0.24	0.07
Experiment	8×10^4	4.5×10^5	1.2×10^5	0.23	0.06

*Experimental data was obtained by G. V. Bilida -- staff member of the Institute of Mechanics of the Academy of Sciences, Ukranian SSR.

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