NASA TT F-10,481

GPO PRICE $ _____

CFSTI PRICE(S) $ _____

Hard copy (HC) 3.00

Microfiche (MF) .65

ff 653 July 65

NASA TT F-10,481

NUMERICAL SOLUTION OF THE PROBLEM OF SUPERSONIC FLOW AT AN ANGLE OF ATTACK PAST CONICAL BODIES

N. A. Makhin and V. F. Syagayev

Translation of "K chislennomu resheniyu zadachi o sverkhzvukovom obtekanii konicheskikh tel pod uglom ataki"
Izvestiya Akademii Nauk SSSR, Mekhanika Zhidkosti i Gaza,
No. 1, pp. 140-142, 1966

FACILITY FORM 602

N67-15693 (ACCESSION NUMBER)

(THRU)

6 (PAGES)

1 (CODE)

(NASA CR OR TMX OR AD NUMBER)

01 (CATEGORY)

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON DECEMBER 1966

NUMERICAL SOLUTION OF THE PROBLEM OF SUPERSONIC FLOW AT AN ANGLE OF ATTACK PAST CONICAL BODIES

N. A. Makhin and V. F. Syagayev

Analysis introducing an improvement to a method proposed by Syagayev for calculating the flow past an arbitrary smooth cone--e.g., an elliptical cone. The use of the original method is associated with difficulties due to unknown position of the critical point on a cone and the necessity of formulating additional singularities. Another drawback is the assumption (in no way obvious) that on a solid body the entropy function has its maximum value. It is shown that the potentialities of the method can be substantially increased by substituting ξ, φ coordinates for the spherical coordinates θ, φ, and to consider not the entropy function but rather density and pressure as the quantities to be determined.

If the method proposed in reference 1 is used to compute the flow past an arbitrary smooth conical body, for example an elliptical cone at an angle of attack, severe difficulties arise due to the formation of additional singularities and due to the unknown position of the critical point on the body. /140*

In addition to this the proposition used in reference 1 which states that the entropy function on a solid body assumes a maximum value, is not obvious in the general case.

The potentialities of the method are substantially expanded if, in place of spherical coordinates θ, φ, we introduce new coordinates ξ, φ and if in place of the entropy function s we consider density and pressure as the unknown quantities.

1. The system of equations which describes conic flows of homogeneous nonconducting gas in terms of the variables φ, ξ may be represented in the following form:

$$\left.\begin{aligned}
&\frac{\partial u}{\partial \xi} = -\frac{\varepsilon}{K_2 \sin\theta}\left[w\frac{\partial u}{\partial \varphi} - (v^2 + w^2)\sin\theta\right] \qquad \frac{\partial w}{\partial \xi} = -\frac{1}{K_2 \sin\theta}\left(\varepsilon K_3 + K_1 K_2 \frac{\partial v}{\partial \xi}\right)\\
&\frac{\partial v}{\partial \xi} = -\frac{\varepsilon}{K_2(\sin\theta + K_1 K_5)}\left[\frac{K_2 K_4 \sin\theta}{(a^2 - v^2)\sin\theta + K_1 v w} + K_3 K_5\right]\\
&\frac{\partial \rho}{\partial \xi} = -\frac{\rho}{K_2 \sin\theta}\left[\varepsilon\left(\frac{w}{\rho}\frac{\partial \rho}{\partial \varphi} + \frac{\partial w}{\partial \varphi} + 2u\sin\theta + v\cos\theta\right) + \frac{\partial v}{\partial \xi}\sin\theta - K_1\frac{\partial w}{\partial \xi}\right]\\
&\frac{\partial p}{\partial \xi} = -\rho\left[K_2\frac{\partial v}{\partial \xi} + \frac{\varepsilon}{\sin\theta}\left(w\frac{\partial v}{\partial \varphi} + uv\sin\theta - w^2\cos\theta\right)\right]
\end{aligned}\right\} \qquad (1.1)$$

*Numbers given in margin indicate pagination in original foreign text.

$$K_1 = \xi\varepsilon'(\varphi) + \theta^{\circ\prime}(\varphi), \qquad K_2 = v - \frac{w}{\sin\theta} K_1$$
$$\varepsilon(\varphi) = \theta_*(\varphi) - \theta^\circ(\varphi),$$
$$K_3 = w\left(\frac{\partial w}{\partial\varphi} + \frac{K_1}{\sin\theta}\frac{\partial v}{\partial\varphi} + u\sin\theta + K_2\cos\theta\right) + \frac{1}{\rho}\frac{\partial p}{\partial\varphi} + K_1 uv$$
$$K_4 = (a^2 - w^2)\frac{\partial w}{\partial\varphi} - vw\frac{\partial v}{\partial\varphi} + u\,(2a^2 - v^2 - w^2)\sin\theta + va^2\cos\theta$$
$$K_5 = \frac{vw\sin\theta + (a^2 - w^2)K_1}{(a^2 - v^2)\sin\theta + vwK_1} \qquad \xi = \frac{\theta - \theta^\circ}{\theta_* - \theta^\circ}$$

Here u, v, w, are dimensionless velocities along the axes of the spherical coordinates referred to V_∞; ρ is the density referred to ρ_∞; p is the pressure referred to $\varkappa p_\infty M_\infty^2$; M_∞ is the Mach number of the unperturbed flow; $\theta^\circ = \theta^\circ(\varphi)$ is the equation of the assigned body, $\theta_* = \theta_*(\varphi)$ is the equation of the assigned shock wave; the superscript * means that the quantities pertain to the body while the subscript * indicates that they pertain to the shock wave. The apostrophe ' indicates that differentiation is carried out with respect to the argument.

The system of equations (1.1) is integrated numerically from the assigned /141 shock wave $\theta_* = \theta_*(\varphi)$ ($\xi=1$) to the assigned body $\theta^\circ = \theta^\circ(\varphi)$ ($\xi=0$) under the following boundary conditions on the shock wave (α is the angle of attack)

$$u_* = u, \qquad w_* = w + (v - v_*)\operatorname{tg}\beta_*$$
$$v_* = \frac{\varkappa - 1}{\varkappa + 1}\,\frac{1 + 2(\varkappa - 1)^{-1}M_\infty^{-2} - u^2 - (w + v\operatorname{tg}\beta_*)^2\cos^2\beta_*}{v - w\operatorname{tg}\beta_*} + (w + v\operatorname{tg}\beta_*)\operatorname{tg}\beta_*\cos^2\beta_*$$
$$\rho_* = \frac{\varkappa + 1}{\varkappa - 1}\left[1 + \frac{2}{(\varkappa - 1)M_\infty^2(v - w\operatorname{tg}\beta_*)^2\cos^2\beta_*}\right]^{-1}$$
$$p_* = \frac{2}{\varkappa - 1}(v - w\operatorname{tg}\beta_*)^2\cos^2\beta_* - \frac{\varkappa - 1}{\varkappa M_\infty^2(\varkappa + 1)} \tag{1.2}$$
$$\operatorname{tg}\beta_* = \frac{1}{\sin\theta_*}\frac{d\theta_*}{d\varphi}, \quad u = (\cos\alpha\operatorname{ctg}\theta_* + \sin\alpha\cos\varphi)\sin\theta_*, \quad v = (\sin\alpha\operatorname{ctg}\theta_*\cos\varphi - \cos\alpha)\sin\theta_*$$
$$w = -\sin\alpha\sin\varphi$$

If the quantities $\varepsilon(\varphi)$ are selected with the required degree of accuracy the boundary condition on the assigned body will be ($\xi=0$)

$$v_n = v\cos\beta^\circ - w\sin\beta^\circ = 0 \tag{1.3}$$

The derivatives with respect to the angle φ which are contained in equation (1.1) were determined numerically in the course of the calculations by using the Sterling formula (ref. 2).

2. As an example, figures 1-4 show the computation results for flow around an elliptic cone with parameters $\lambda=1.66$; $K_9=1.39$ at $M_\infty=6$.

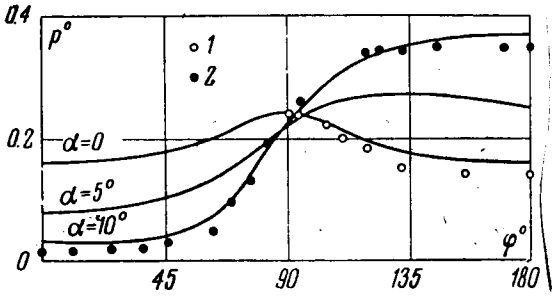


Figure 1.

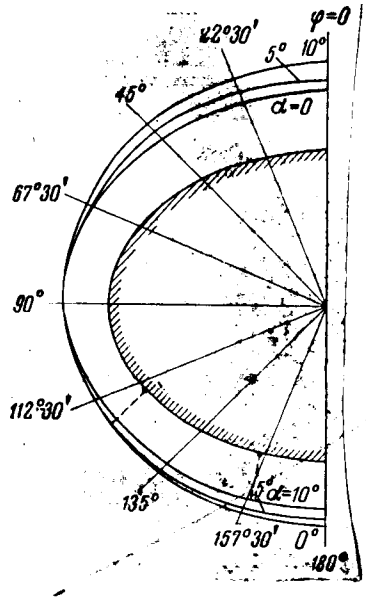


Figure 2.

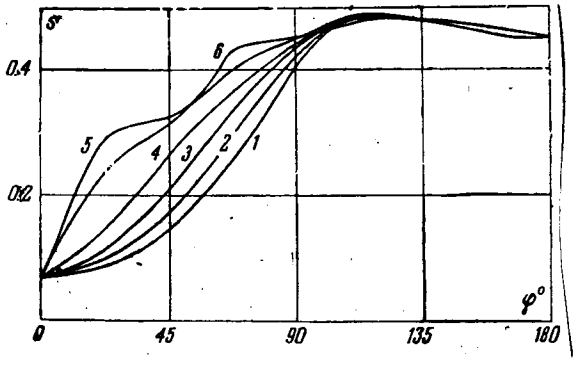


Figure 3.

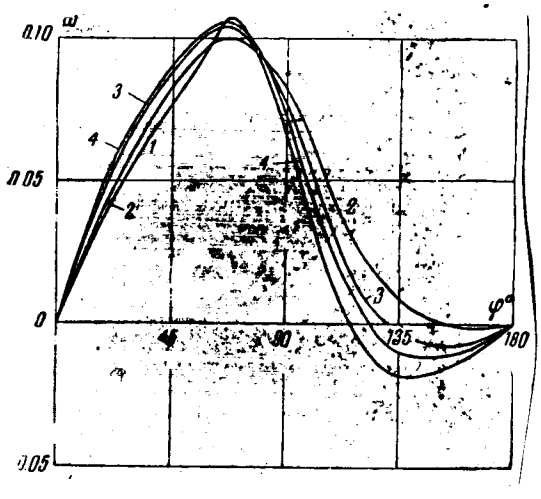


Figure 4.

The system of equations (1.1) in this case was integrated in the first approximation using the Euler method with an integration step of $\Delta\xi=1/64$ and $\Delta\varphi=\pi/8$. The computations were carried out for angles of attack $\alpha=0.5°$, $10°$.

The results of computing the pressure coefficients $p°$ on the cone for $\alpha=0$ and $\alpha=10°$ are compared in figure 1 with experimental data for a cone which is close to an oval cone (ref. 3).

The cross sections of the plane normal to the axis of the cone represented in figure 2 illustrate the shape and position of the attached conical shock wave and also the position of the critical points on the body for the investigated angles of attack

Figure 3 shows the graph for the variation of the entropy function for the case when the angle of attack is $\alpha=5^\circ$

$$S=\frac{1}{\varkappa(\varkappa-1)}\ln\frac{\varkappa M_\infty^2 p}{\rho^\varkappa}$$

The curves 1-6 correspond to the parameter ξ=1, 0.5, 0.25, 0.125, 0.0325, 0.

We can see that as the body is approached (ξ=0) the values of the entropy function are smoothed out; however they do not become constant on the body apparently due to the errors in the computations which occur close to the body ($\xi<0.03125$) in the neighborhood of the singularity ($\varphi=22.5^\circ$ and $\varphi=45^\circ$). For this same angle of attack $\alpha=5^\circ$, figure 4 shows the variation in the velocity /142 component which is normal to the meridional plane φ=const. The curves 1-4 correspond to the parameter ξ=0.1, 0.5, 0.25. The error noted above when $\varphi=22.5^\circ$ and $\varphi=45^\circ$ when ξ=0 also exists in the relationship $w=w(\varphi)$.

3. The calculations carried out by means of the above scheme also turned out to be rather complex because unlike the one discussed in reference 1, this scheme makes it possible to pass through the singularity when computing the intermediate inverse problems. This could create certain difficulties in selecting the aperture angles of the shock in the planes φ=const, which adjoin the plane of the singular ray. It should be pointed out that from the practical standpoint these difficulties were not too great because the aperture angles of the shock wave could be selected in the specified planes without any additional efforts compared with the method of selecting the shock aperture angles in other planes φ=const, when the requirements for the accuracy in the fulfillment of the boundary conditions on the assigned body were moderate.

However, as a whole, calculations of this type showed that when the flow is around cones which are different from circular cones, for example, elliptical cones, in this system of computations there are also additional difficulties which are manifested by the fact that the accuracy which is achieved for the boundary conditions on the body substantially decreases. Thus the computations of flows around circular and elliptical cones using the same computation method and using the simplest method of integrating the system of equations (1.1) made it possible to satisfy the boundary conditions on the circular cone with an accuracy up to $|\Delta v_n|=0.001$ and on the elliptical cone only with an accuracy up to $|\Delta v_n|\approx 0.01$. Obviously this is explained by the effect of the additional singularity and also by the more complex boundary conditions on the elliptical cone (in the case of a circular cone, condition (1.3) has the form: $v_n=v^\circ=0$), which makes it more difficult to select the aperture angles of the shock wave.

REFERENCES

1. Syagayev, V. F. Metod chislennogo resheniya zadachi ob obtekanii konicheskikh tel sverkhzvukovym potokom gaza (The Method of the Numerical Solution

of Problems on the Supersonic Gas Flow around Conical Bodies). Zh. Vychislit. matem. i matem. fiz., Vol. 3, No. 3, 1963.

2. Panov, D. Yu. Spravochnik po chislennomy resheniyu differentsial'nykh uravneniy v chastnykh proizvodnykh (Handbook on the Numerical Solution of Differential Equations with Partial Derivatives). Gostekhizdat, 1951.
3. Ferri, A. Preprint of paper to read at the First International Congress in the Aeronautical Sciences, Madrid, Spain, 13 Sept 1958.

Translated for the National Aeronautics and Space Administration
by John F. Holman and Co. Inc.

11 A

808 words

TTF 10,481

1495-38

(11 A)

A66-24441 #

NUMERICAL SOLUTION OF THE PROBLEM OF SUPERSONIC FLOW AT AN ANGLE OF ATTACK PAST CONICAL BODIES [K CHISLENNOMU RESHENIIU ZADACHI O SVERKHZVUKOVOM OBTEKANII KONICHESKIKH TEL POD UGLOM ATAKI].

N. A. Makhin and V. F. Siagaev.

Akademiia Nauk SSSR, Izvestiia, Mekhanika Zhidkosti i Gaza, Jan.-Feb. 1966, p. 140-142. In Russian

Analysis introducing an improvement to a method proposed by Siagaev for calculating the flow past an arbitrary smooth cone, e.g., an elliptical cone. The use of the original method is associated with difficulties due to unknown position of the [illegible] point on a cone and the necessity of formulating additional [illegible]. Another drawback is the assumption (in no way obvious) [illegible] body the entropy function has its maximum value. It is shown [illegible] potentialities of the method can be substantially increased by substituting ξ, φ coordinates for the spherical coordinates θ, [illegible] and to consider not the entropy function but rather density and pressure as the quantities to be determined. V. P.

This Abstract Published in
INTERNATIONAL AEROSPACE ABSTRACTS
Issued by
TECHNICAL INFORMATION SERVICE
AMERICAN INSTITUTE OF AERONAUTICS AND ASTRONAUTICS, INC.
750 THIRD AVENUE, NEW YORK, N.Y. 10017

A66 2444

К ЧИСЛЕННОМУ РЕШЕНИЮ ЗАДАЧИ О СВЕРХЗВУКОВОМ ОБТЕКАНИИ КОНИЧЕСКИХ ТЕЛ ПОД УГЛОМ АТАКИ

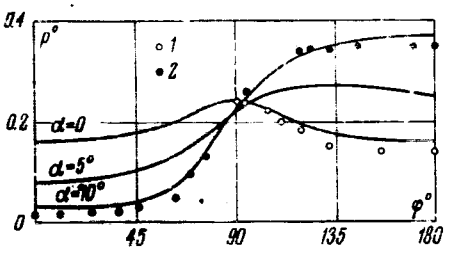
Н. А. МАХИН, В. Ф. СЯГАЕВ

(Москва)

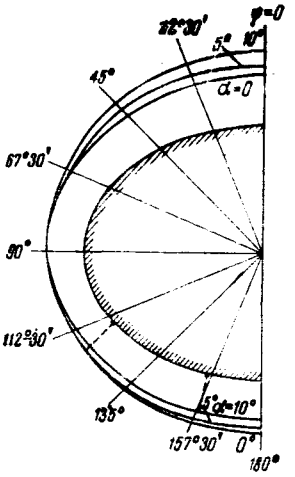
При расчете обтекания произвольного гладкого конического тела, например, эллиптического конуса под углом атаки, методом, предложенным в [1], возникают большие затруднения, связанные с образованием дополнительных особенностей и неизвестным положением критической точки на теле.

Кроме того, высказанное в [1] предположение о том, что на твердом теле энтропийная функция принимает максимальное значение, в общем случае не является очевидным.

Возможности метода существенно расширяются, если вместо сферических координат θ, φ ввести новые координаты ξ, φ и вместо энтропийной функции s в качестве искомых рассматривать плотность и давление.



Фиг. 1



Фиг. 2

1. Система уравнений, описывающая конические течения однородного нетеплопроводного газа в переменных φ, ξ, может быть представлена в следующем виде:

$$\frac{\partial u}{\partial \xi} = -\frac{\varepsilon}{K_2 \sin\theta}\left[w\frac{\partial u}{\partial \varphi} - (v^2+w^2)\sin\theta\right] \qquad \frac{\partial w}{\partial \xi} = -\frac{1}{K_2\sin\theta}\left(\varepsilon K_3 + K_1K_2\frac{\partial v}{\partial \xi}\right)$$

$$\frac{\partial v}{\partial \xi} = -\frac{\varepsilon}{K_2(\sin\theta + K_1K_2)}\left[\frac{K_2K_4\sin\theta}{(a^2-v^2)\sin\theta + K_1vw} + K_2K_5\right] \tag{1.1}$$

$$\frac{\partial \rho}{\partial \xi} = -\frac{\rho}{K_2\sin\theta}\left[\varepsilon\left(\frac{w}{\rho}\frac{\partial \rho}{\partial \varphi} + \frac{\partial w}{\partial \varphi} + 2u\sin\theta + v\cos\theta\right) + \frac{\partial v}{\partial \xi}\sin\theta - K_1\frac{\partial w}{\partial \xi}\right]$$

$$\frac{\partial p}{\partial \xi} = -\rho\left[K_3\frac{\partial v}{\partial \xi} + \frac{\varepsilon}{\sin\theta}\left(w\frac{\partial v}{\partial \varphi} + uv\sin\theta - w^2\cos\theta\right)\right]$$

$$K_1 = \xi_0 s'(\varphi) + \theta^{0\prime}(\varphi), \qquad K_3 = v - \frac{w}{\sin\theta}K_1$$

$$\varepsilon(\varphi) = \theta_s(\varphi) - \theta^0(\varphi).$$

$$K_3 = w\left(\frac{\partial w}{\partial \varphi} + \frac{K_1}{\sin\theta}\frac{\partial v}{\partial \varphi} + u\sin\theta + K_2\cos\theta\right) + \frac{1}{\rho}\frac{\partial p}{\partial \varphi} + K_1uv$$

$$K_4 = (a^2-v^2)\frac{\partial w}{\partial \varphi} - vw\frac{\partial v}{\partial \varphi} + u(2a^2-v^2-w^2)\sin\theta + va^2\cos\theta$$

$$K_5 = \frac{w\sin\theta + (a^2-w^2)K_1}{(a^2-v^2)\sin\theta + vwK_1} \qquad \zeta = \frac{\theta-\theta^0}{\theta_s-\theta^0}$$

Здесь u, v, w — безразмерные скорости по осям сферических координат, отнесенные к V_∞; ρ — плотность, отнесенная к ρ_∞; p — давление, отнесенное к $\varkappa p_\infty M_\infty^2$; M_∞ — число M невозмущенного потока; $\theta^0 = \theta^0(\varphi)$ — уравнение заданного тела, $\theta_s = \theta_s(\varphi)$ — уравнение задаваемой ударной волны; верхний индекс ^{0} обозначает величины, относящиеся к телу, нижний индекс $_s$ к ударной волне, штрих $'$ дифференцирование по [illegible]

Система уравнений (1.1) интегрируется численно от задаваемой ударной волны $\theta_* = \theta_*(\varphi)$ ($\xi = 1$) до заданного тела $\theta^\circ = \theta^\circ(\varphi)$ ($\xi = 0$) при следующих граничных условиях на ударной волне (α — угол атаки)

$$u_* = u, \qquad w_* = w + (v - v_*)\operatorname{tg}\beta_*$$

$$v_* = \frac{\varkappa - 1}{\varkappa + 1}\,\frac{1 + 2(\varkappa - 1)^{-1} M_\infty^{-2} - u^2 - (w + v\operatorname{tg}\beta_*)^2\cos^2\beta_*}{v - w\operatorname{tg}\beta_*} + (w + v\operatorname{tg}\beta_*)\operatorname{tg}\beta_*\cos^2\beta_*$$

$$\rho_* = \frac{\varkappa + 1}{\varkappa - 1}\left[1 + \frac{2}{(\varkappa - 1) M_\infty^2 (v - w\operatorname{tg}\beta_*)^2\cos^2\beta_*}\right]^{-1} \tag{1.2}$$

$$p_* = \frac{2}{\varkappa - 1}(v - w\operatorname{tg}\beta_*)^2\cos^2\beta_* - \frac{\varkappa - 1}{\varkappa M_\infty^2(\varkappa + 1)}$$

$$\operatorname{tg}\beta_* = \frac{1}{\sin\theta_*}\frac{d\theta_*}{d\varphi}, \quad u = (\cos\alpha\operatorname{ctg}\theta_* + \sin\alpha\cos\varphi)\sin\theta_*, \quad v = (\sin\alpha\operatorname{ctg}\theta_*\cos\varphi - \cos\alpha)\sin\theta_*$$

$$w = -\sin\alpha\sin\varphi$$

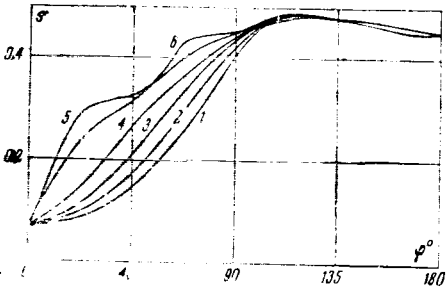
В результате подбора величин $\varepsilon(\varphi)$ должно быть удовлетворено с требуемой точностью граничное условие на заданном теле ($\xi = 0$)

$$v_n = v\cos\beta^\circ - w\sin\beta^\circ = 0 \tag{1.3}$$

Входящие в уравнения (1.1) производные по углу φ в проведенных расчетах определялись численно по формуле Стирлинга [2].

2. В качестве примера на фиг. 1÷4 приведены результаты расчетов обтекания эллиптического конуса с параметрами $\lambda = 1.66$; $K_2 = 2.09$ при числе $M_\infty = 6$.

Система уравнений (1.1) интегрировалась при этом в первом приближении методом Эйлера с шагом интегрирования $\Delta\xi = 1/61$ в шагах $\Delta\varphi = \pi/8$. Расчеты приведены при значениях углов атаки $\alpha = 0, 5^\circ, 10^\circ$.



Фиг. 3

[illegible] расчетов [illegible] давления [illegible] конусе при $\alpha = 0$ и $\alpha = 10^\circ$ [illegible] на фиг. 1 с экспери[illegible] данными для близкого [illegible] конуса [3].

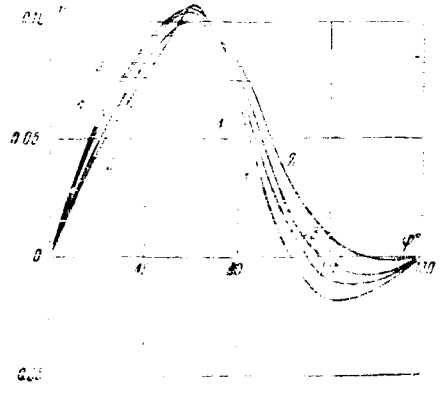
[illegible] на фиг. 2 сечения [illegible] нормальной к оси конуса [illegible] о форме и [illegible] конической [illegible] а также о по[illegible] точек на теле [illegible] углах атаки.

[illegible] случая [illegible] атаки α [illegible] графики изменения [illegible] функции

$$S = \frac{\varkappa}{\varkappa(\varkappa - 1)} \cdot \frac{\rho_\infty^2 p}{\rho^\varkappa}$$

кривые [illegible] параметру ξ = [illegible] 0.0325, 0.

[illegible] приближении к телу ([illegible] энтропийной функции [illegible] однако не [illegible] на теле, [illegible] поверхностей расчета, возникающих вблизи тела ([illegible] $\alpha = 22.5^\circ$ и $\varphi = 45^\circ$). Для этого же угла атаки $\alpha = 5^\circ$ на фиг. [illegible] составляю-



Фиг. 4

щей скорости, нормальной к меридиональной плоскости $\varphi = \text{const}$. Кривые *1—4* соответствуют параметру $\xi = 0{,}1, 0{,}5, 0{,}25$. Отмеченная выше погрешность расчета при $\varphi = 22{,}5°$ и $\varphi = 45°$ при $\xi = 0$ проявляется также в зависимости $w = w(\varphi)$.

3. Расчеты по указанной выше схеме так же представлялись достаточно сложными, поскольку в отличие от изложенной в [1] данная схема допускает переход через особенность при расчете промежуточных обратных задач. Это могло вызвать некоторые затруднения при подборе углов раствора скачка в плоскостях $\varphi = \text{const}$, примыкающих к плоскости особого луча. Надо заметить, что в практическом отношении эти затруднения оказались не столь велики, поскольку углы растворов скачка и в названных плоскостях удавалось подбирать без каких-либо дополнительных условий по сравнению с методикой подбора углов раствора скачка в других плоскостях $\varphi = \text{const}$ при умеренных требованиях к точности удовлетворения граничных условий на заданном теле.

Однако в целом пробные расчеты показали, что при расчете обтекания конусов, отличных от круглых (например, эллиптических), и при этой схеме расчета возникают дополнительные трудности, которые выражаются в том, что достигаемая при расчете точность удовлетворения граничного условия на теле при этом существенно падает. Так, расчеты обтекания круглых и эллиптических конусов по одной и той же расчетной схеме с использованием простейшей методики интегрирования системы уравнений (1.1) позволили удовлетворить граничные условия на круглом конусе с точностью до $|\Delta v_n| \approx 0{,}001$, а на эллиптическом — лишь с точностью до $|\Delta v_n| \approx 0{,}01$. Очевидно, это объясняется влиянием дополнительной особенности, а также более сложными граничными условиями на эллиптическом конусе (в случае круглого конуса условие (1.3) имеет вид: $v_n = v^2 = 0$), усложняющими подбор углов раствора скачка.

Поступило 17 III 1965

ЛИТЕРАТУРА

1. Сягаев Н. Ф. Метод численного решения задачи об обтекании конических тел сверхзвуковым потоком газа. Ж. вычислит. матем. и матем. физ., 1963, т. 3, № 3.
2. Панов Д. Ю. Справочник по численному решению дифференциальных уравнений в частных производных. Гостехиздат, 1951.
3. A. Ferri. Preprint of paper to read of the First International Congress in the Aeronautical Sciences. Madrid, Spain, 1958, 13. Sept.