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Abstract

This paper presents a model of the perceptual process through which an observer decides whether two stimuli are the same or different, with emphasis on the role of perceptual memory in such tasks. The results of two experiments, one visual and one auditory, are presented which illustrate the model's application and suggest its validity.

Detection, Recognition and Perceptual Memory

To distinguish a difference between two stimuli an observer must have a receptor or input process which will respond differently to the two stimuli. Furthermore, to compare two stimuli presented at different points in time, he must maintain a memory of the first stimulus until the second one occurs. For example, suppose an observer were asked to judge the similarity in loudness of two serially presented tones. A minimal requirement for accurate discrimination would be a receptor process which reacted differently to the two tone amplitudes. Furthermore, if the second tone occurred with some delay following cessation of the first tone, the observer would have to maintain a memory of the initial tone's loudness until he heard the second tone. Thus one might say that discrimination of serially observed stimuli is limited by at least two factors: one, the resolving power of the receptor or input process; and two, the efficiency of the memory process through which the observer retains information about one stimulus until the comparison stimulus occurs.

Recent theoretical work on the psychophysics of detection has emphasized the role of the input process in determining sensory limits. The most prominent of these models is the psychophysical Theory of Signal Detection (see Green

and Swets, 1966, for a comprehensive development of this theory). This is a two-process theory in that detection is represented as the product of two sub-processes, an input process and a decision or response process. The input process specifies how the stimulus event evokes a hypothetical sensory state in the observer; the decision process specifies how this sensory state determines the observer's overt response. The particular sensory state evoked on each trial is treated as a value of a random variable whose distribution depends on the stimulus value. If two stimulus values evoke similar distributions of sensory states, it is difficult for the observer to distinguish which stimulus has given rise to a particular sensory state. His decision is a statistical inference about the stimulus value based on his "sensory sample", the sensory state.

A two-process model like the Theory of Signal Detection does not specify how an observer, judging the similarity of two temporally separated stimulus events, remembers the initial sensory state until the second (comparison) state occurs. We shall refer to a memory process of this sort as perceptual memory. Specification of this process in discrimination is the unique feature of the model presented in this paper.

Note that the burden on perceptual memory is minimized in certain forms of discrimination; specifically, in cases where the stimuli to be discriminated occur in immediate temporal succession. If, for example, in a loudness discrimination task, the on-set of the comparison amplitude occurs right at the off-set of the initial amplitude, the problem of perceptual memory would be minimal. It is argued here that the term detection is most usefully employed to denote the discrimination of such temporally contiguous stimuli; whereas recognition should denote the discrimination of non-contiguous stimuli. Further elaboration of this distinction can best be made later in the paper after the model has been developed and applied to some actual data.

The discrimination experiments we shall consider consist of a series of trials. On each trial the observer is presented with two stimuli, one after another, which may or may not differ along some particular stimulus dimension. The observer's task is to report a stimulus difference when one exists. (We restrict our discussion to single dimension comparisons because this is the simplest case; consideration of the multi-dimensional case is interesting but beyond the scope of this paper.) We shall denote the relevant stimulus variable by S, its initial value terminating at time zero by s_o, and the comparison value commencing at time t by s_t. Note that the duration of each stimulus value is not specified, simply the interstimulus interval (t) between the off-set of the first value and the on-set of the comparison value. While stimulus duration is certainly important, it is a constant in the experiments presented here and will therefore be ignored in the present development of the model.

Performance of the discrimination tasks we consider can be summarized by two proportions: the proportion of hits and the proportion of false alarms. The <u>proportion of hits</u> equals the number of trials on which the observer correctly reported a stimulus difference, divided by the total number of trials on which s_o and s_t were different. The <u>proportion of false alarms</u> equals the number of trials on which the observer incorrectly reported a stimulus difference, divided by the total number of trials on which s_o equalled s_t. These proportions are normally treated as estimates of corresponding conditional probabilities: respectively, the probability of a hit, Pr(H), and the probability of a false alarm, Pr(FA).

The purpose of the model is to account for changes in the hit and false-alarm rates produced by variations of the stimulus difference $(s_t - s_o)$ and the interstimulus delay (t).

The Model

The basic structure of the model is shown schematically in Fig. 1. Each time some value of the stimulus value S initiates the input process,

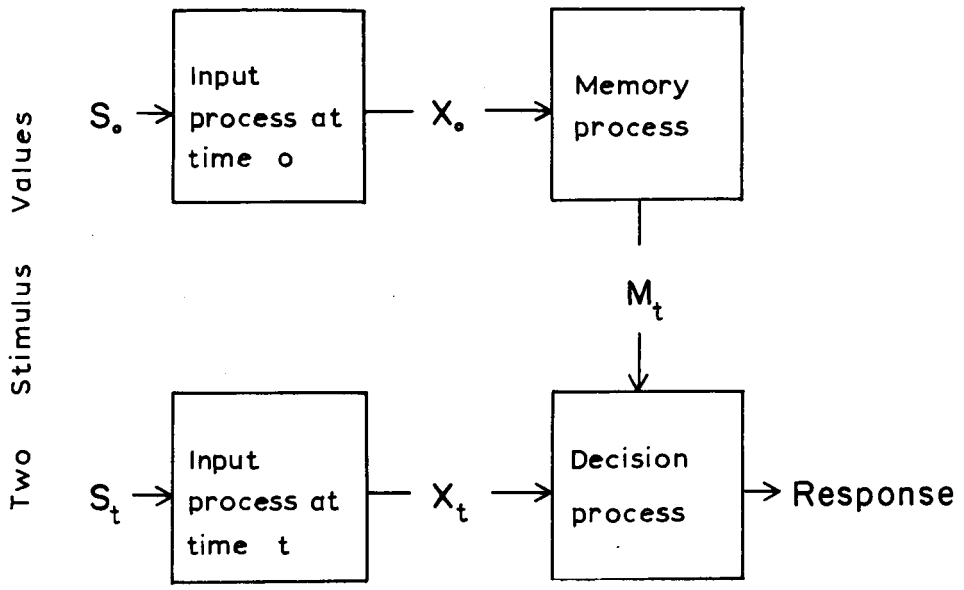

Fig. 1. Schematic of model

it evokes some value of the sensory variable X. The values of the stimulus variable at time o and at time t are denoted, respectively, by s_o and s_t. Similarly, the values of the sensory variable evoked by s_o and s_t are denoted by x_o and x_t. Since x_t occurs later in time than x_o, the observer stores x_o in memory until time t. He then makes a similarity decision regarding s_o and s_t on the basis of the discrepancy between m_t, his memory of x_o at time t, and x_t. Thus, three processes interact to determine the relationship between stimulus and response: input, memory, and decision. While these three processes can be defined in a rigorous, axiomatic manner, a simple, verbal presentation should suffice here.

The Input Process

Repeated inputs of the same stimulus value, s, do not necessarily evoke the same sensory value; however, the distribution of the evoked values will be Gaussian with an expected value equal to the actual stimulus value. (Thus x can be expressed in the same units as s.)

The Memory Process

Once the sensory value x_o is stored in memory at time 0, it is "diffused" or modified through a random walk process until it is read into the decision process at time t as the memory m_t. One step in the random walk process occurs every $1/\rho$ seconds when the value in memory is increased by the amount ω with probability p, or decreased by the same amount with probability 1-p. (We shall assume the unit parameter ω is chosen so that m_t is in the same units as s.)

The Decision Process

The observer has some response criterion at time t, which we shall denote by C_t, and only reports a stimulus difference if the discrepancy between x_t and m_t exceeds C_t.

Some Properties of the Model

We shall now consider some properties of the model which are important for our experimental analysis. First, note that the memory of x_o at time t (m_t) depends only on the initial value in memory (x_o) and the cumulative effect of the random walk at time t. This cumulative effect, which will be denoted by d_t, is simply the sum of incremental steps minus the sum of all the decremental steps; specifically,

$$d_t = k\omega - (N-k)\omega \tag{1}$$

where N denotes the total number of steps occurring by time t, k denotes the number of incremental steps, and ω is the step size. The value of the memory at time t can be written as

$$m_t = x_o + d_t \tag{2}$$

Since the decision process operates on the difference between x_t and m_t, it will be useful to denote this discrepancy by y_t where

$$y_t = x_t - m_t \tag{3a}$$

or by Eq. 2,

$$y_t = x_t - x_o - d_t \tag{3b}$$

Suppose the same stimulus values s_o and s_t, were presented on every discrimination trial. The values x_o, x_t, d_t, and y_t on any particular trial could be treated as values of 5 corresponding random variables - variables which we shall denote, respectively, by X_o, X_t, D_t, M_t and Y_t. According to the definition of the input process, the expected values of the sensory variables X_o and X_t are the actual stimulus values; specifically,

$$E(X_o) = s_o , \tag{4}$$

$$E(X_t) = s_t . \tag{5}$$

By definition of the memory process, and Eq. 2,

$$E(M_t) = E(X_o) + E(D_t) . \tag{6}$$

Considering the additivity of variance,

$$\mathrm{Var}\,(M_t) = \mathrm{Var}\,(X_o) + \mathrm{Var}\,(D_t) . \tag{7}$$

Similar arguments imply that

$$E(Y_t) = E(X_t) - E(M_t) \tag{8}$$

or, by substitution according to Eq. 6,

$$E(Y_t) = E(X_t) - E(X_o) - E(D_t) \tag{9}$$

Again considering the additivity of variance,

$$\mathrm{Var}(Y_t) = \mathrm{Var}(X_o) + \mathrm{Var}(X_t) + \mathrm{Var}(D_t) \tag{10}$$

The expected value and variance of D_t can be derived with standard methods for analyzing random walks (see Feller, 1957, on random walks and diffusion processes):

$$E(D_t) = \rho\, t\, \omega\, (2p - 1) \tag{11}$$

and

$$\mathrm{Var}(D_t) = 4\, \rho\, t\, \omega^2 p\, (1 - p)\,. \tag{12}$$

To simplify our subsequent arguments we shall consider the distribution of D_t (actually binomial) to be Gaussian. This approximation is accurate so long as p is close to 1/2 and the product ρ t is not too small. As will be shown, these assumptions seem reasonable for the experiments considered here. Thus D_t can be considered to have a normal distribution with the following mean and variance (substituting p equals 1/2 in Eqs. 11 and 12):

$$E(D_t) = 0 \tag{13}$$

and

$$\mathrm{Var}(D_t) = \varphi\, t \tag{14}$$

where φ equals $\rho\, \omega^2$. Thus φ, which we shall refer to as the <u>diffusion rate</u>,

is the rate at which the variance of D_t increases as a linear function of the interstimulus delay (t).

Since Y_t is defined as a linear combination of three Gaussian random variables (Eq. 9), it too will have a Gaussian distribution. The expected value of Y_t depends on the actual pair of stimulus values; i.e., substituting in Eq. 9 on the basis of Eqs. 4, 5, and 13 yields,

$$E\ (Y_t) = s_t - s_o \tag{15}$$

Furthermore, substituting in Eq. 10 according to Eq. 14 yields,

$$\mathrm{Var}\ (Y_t) = \varphi\ t + \mathrm{Var}\ (X_o) + \mathrm{Var}\ (X_t) \tag{16}$$

Thus the variance of Y_t is the simple sum of the input variance plus the diffusion variance accrued in memory prior to time t. It will be convenient to denote Var (Y_t) by the symbol σ_t^2. Thus σ_o^2 denotes the variance of Y_o, which is simply the total input variance, Var (X_o) plus Var (X_t), according to Eq. 16.

The observer in an actual discrimination experiment must decide whether the discrepancy (y_t) on a particular trial was produced by an actual stimulus difference or not. For example, suppose s_t equalled s_o on a randomly determined 50 percent of the trials, and equalled s_o plus Δ s on the remaining trials. If the distributions of Y_t were similar for both types of trials, it would be difficult for the observer to decide which stimulus pattern had actually occurred on any one trial. His decision problem is illustrated in Fig. 2a which presents two overlapping probability density functions of Y_t. The mean zero distribution applies on the equal stimuli trials, whereas, the mean Δ s

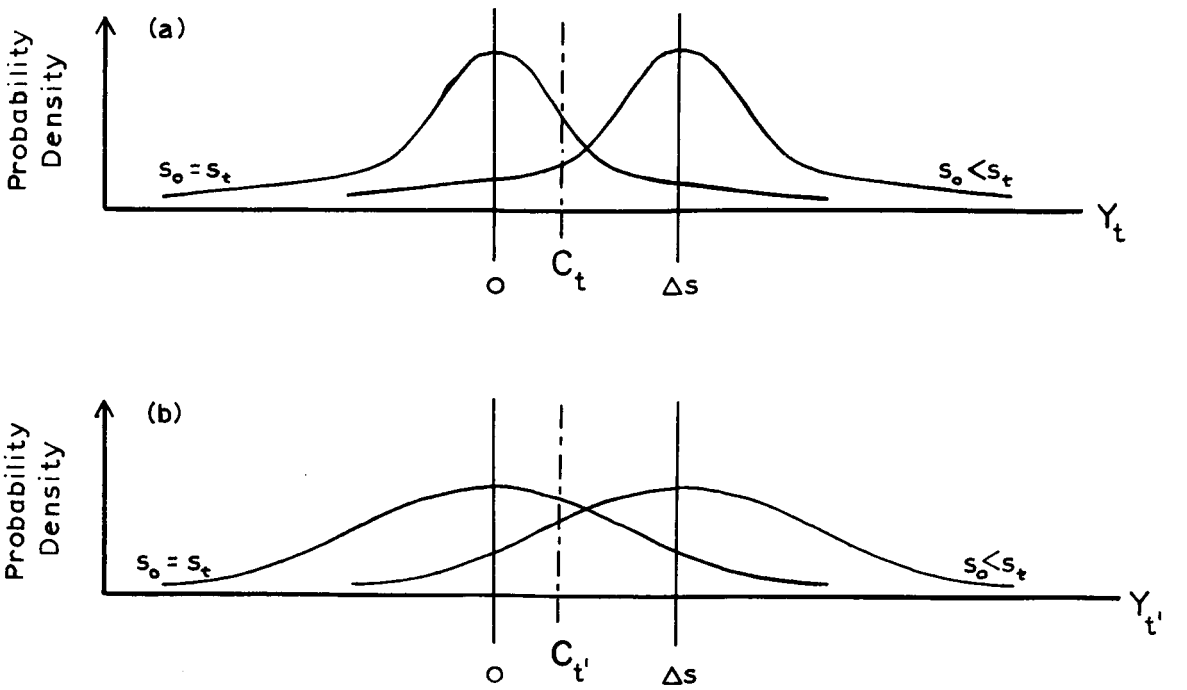

Fig. 2. Distributions of Y_t conditional on the difference between s_o and s_t and the interstimulus interval ($t' > t$).

distribution applies on the unequal stimuli trials (Eq. 15). If he adopted the decision criterion C_t shown in the diagram, the area to the right of C_t under the equal stimuli distribution would represent his false-alarm rate, while the corresponding area under the other distribution would be his hit rate. There are many other combinations of Pr(H) and Pr(FA) he can produce by adopting some other decision criterion, but he can never change Pr(H) without simultaneously changing Pr(FA).

The distributions in Fig. 2b correspond to the same two stimulus conditions as those in Fig. 2a but with a longer interstimulus interval ($t' > t$). The means of the two distributions at time t' are the same as those at time t, since they depend solely on the stimulus difference (Eq. 15). However, Eq. 16 implies that Var ($Y_{t'}$) will exceed Var (Y_t) by the amount φ ($t' - t$). The net effect of this increased variance is that the observer must now accept a higher false-alarm rate in order to attain any given hit rate.

The possible pairs of hit and false-alarm rates available to the observer through variation in the decision criterion are his <u>operating characteristic</u>. They can often be specified with a single number, the <u>sensitivity measure</u> δ_t defined as follows:

$$\delta_t = \frac{\left| E_1 (Y_t) - E_o (Y_t) \right|}{\sqrt{Var_o (Y_t)}} \tag{17}$$

where $E_o (Y_t)$ and $Var_o (Y_t)$ are the expected value and variance of Y_t on trials where s_o equals s_t, and $E_1 (Y_t)$ is the expected value of Y_t when s_t exceeds s_o by the amount Δs. Thus δ_t is simply the absolute separation between the means of the two distributions of Y_t expressed in standard deviation units of the equal stimuli distribution. If the variance of Y_t on unequal stimuli trials, $Var_1 (Y_t)$, equals $Var_o (Y_t)$, the observer's operating characteristic is specified by δ_t. If $Var_o (Y_t)$ does not equal $Var_1 (Y_t)$, both the ratio of these variances and δ_t would be required to specify the observer's operating characteristic. In either case, the greater the value of δ_t, the smaller will be the false-alarm rate associated with any particular hit rate.

Most readers will recognize that the present model is equivalent to the psychophysical Theory of Signal Detection when the interstimulus delay is zero, so that δ_o and d' (the sensitivity measure in that theory) are equivalent. The present model is a more general representation of discrimination in that it applies to both detection ($t = o$) and recognition ($t > 0$). The manner in which the sensitivity measure for detection (δ_o) is modified by the interstimulus delay in recognition is apparent if we rewrite Eq. 17 according to Eqs. 15 and 16 to obtain the following:

$$\delta_t = \frac{\Delta s}{\sqrt{\varphi t + Var(X_o) + Var(X_t)}} \tag{18}$$

Note that in detection δ_o is simply the absolute value of the stimulus difference, Δs, divided by the square root of the total input variance (a kind of signal-to-noise ratio). In recognition, the variance introduced by perceptual memory (φt) is added to the input variance and reduces δ_t as the interstimulus delay is increased.

We shall now consider two experiments which provide a concrete basis for further discussion of the model.

Experiment One: Visual Position Discrimination

Each observer in this experiment sat in complete darkness and tried to discriminate a lateral difference in position between two successively presented points of light. The basic experimental variables were the spatial separation between the two lights, and the temporal interval between their occurrences.

Apparatus and Procedure

The stimulus display consisted of a horizontal array of 3 circular, white lights. Each light (Dialco No. 39, 28V, .04 amp operated at 20V) was 5 mm in diameter, and there was a 1 cm separation between the midpoints of adjacent lamps. The display was placed at eye level, 1.5 meters in front of the seated observer. Each discrimination trial began with a 100 msec. illumination of the light on the right of the display; then, following a delay of t seconds this same light, or one of the other two, was illuminated for another 100 msec. Finally, the observer was given 2 seconds to indicate (by pressing an appropriate pushbutton) one of two decisions: both lights occurred in the same position; or, the second light was to the left of the first.

In terms of the model, the stimulus variable, S, corresponds to the horizontal position of each light. It seems most reasonable to specify values of S in degrees visual angle displacement from the midpoint of the lamp on the right end of the array. Thus the initial value of S on each trial, s_o, would always be zero degrees. The comparison value, s_t, could equal 0, .38, or .76 degrees (1 cm on the display subtends .38 degrees of visual angle at 1.5 meters).

The stimulus sequence was determined in blocks of 50 trials. Within all such blocks s_t equaled s_o (0 degrees) on a randomly selected 25 trials; on the remaining 25 trials s_t equaled .38 degrees in some blocks and .76 degrees in other blocks. The interstimulus interval (t) was constant for all trials within a particular block. Thus each of the 8 experimental conditions employed in the experiment corresponds to some combination of t (.5, 1.0, 1.5, or 2.0 seconds) and Δs (.38 or .76 degrees visual angle).

Each of the 4 observers in this experiment performed under all 8 experimental conditions (8 blocks of 50 trials in randomly determined order) during 11 daily 55 minute sessions (including 10 minutes of pre-session dark adaptation). Three prelininary practice sessions were not recorded. The observers were completely familiar with the stimulus display and the procedure for generating the stimulus sequences. They also knew when each block of trials began, since they were allowed a 3 minute rest break (in the dark) between blocks. They were not told the particular sequence of conditions for the 8 blocks of trials, nor the accuracy of their decisions.

Results

The performance of each of the four observers can be summarized by the hit and false-alarm proportions averaged over the 11 daily sessions for each

of the 8 experimental conditions. These proportions are presented in Table 1. A Chi-square test on each observer's data indicated that both the inter-stimulus interval (t) and the stimulus difference (Δs) had a statistically significant ($p > .01$) effect on performance. A deeper analysis of these results is provided by the model.

Table 1

Average Hit and False-Alarm Proportions for Experiment One

		Obs 1		Obs 2		Obs 3		Obs 4	
Δs	t	H	FA	H	FA	H	FA	H	FA
.38°	0.5 sec.	.83	.27	.80	.43	.92	.53	.59	.45
.38°	1.0 sec.	.64	.23	.64	.34	.74	.49	.51	.43
.38°	1.5 sec.	.50	.26	.53	.29	.70	.50	.56	.43
.38°	2.0 sec.	.49	.29	.51	.29	.72	.51	.53	.44
.76°	0.5 sec.	.88	.12	.97	.35	.94	.48	.74	.43
.76°	1.0 sec.	.83	.17	.86	.25	.90	.42	.65	.41
.76°	1.5 sec.	.74	.24	.76	.28	.85	.51	.62	.46
.76°	2.0 sec.	.72	.24	.71	.27	.87	.56	.56	.35

Theoretical Analysis

If the hit and false-alarm proportions in Table 1 are interpreted as estimates of Pr(H) and Pr(FA) it is possible to deduce an estimate of δ_t for each experimental condition. (The manner in which a table of normal deviates is consulted to obtain these estimates follows in an obvious fashion from the definition of δ_t and C_t). These estimates, denoted $\hat{\delta}_t$, are presented as data points in Fig. 3 and numerically in Table 2.

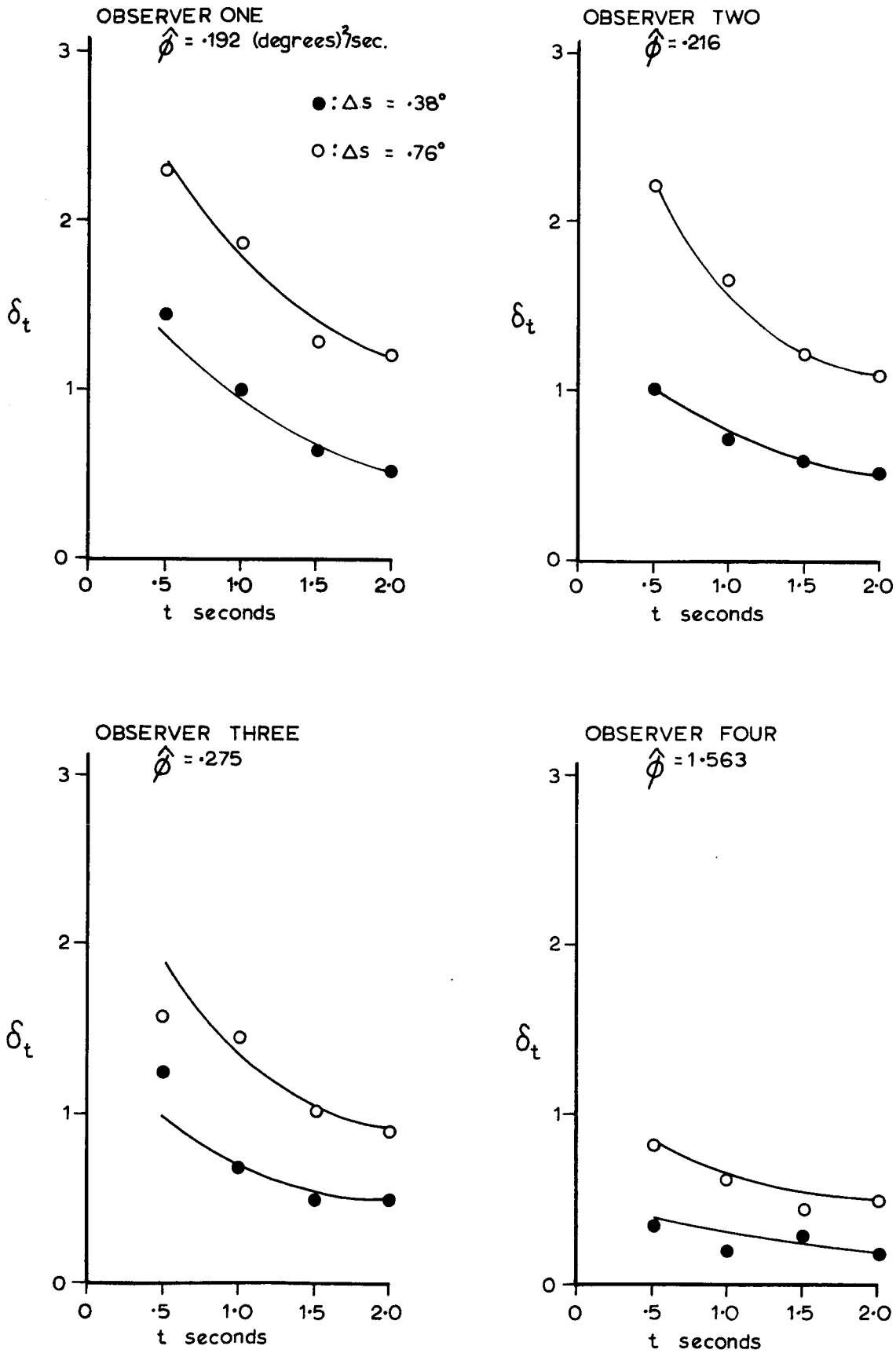

Fig. 3. Estimated (points) and predicted (lines) values of δ_t given $\hat{\varphi}$ for Experiment One.

Table 2

Estimated Values of δ_t and C_t with Predicted Values of δ_t

Based on a Single Estimate of φ

		Obs 1			Obs 2			Obs 3			Obs 4		
		($\hat{\varphi}$ = .192)			($\hat{\varphi}$ = .216)			($\hat{\varphi}$ = .275)			($\hat{\varphi}$ = 1.563)		
Δs	t	$\hat{\delta}_t$	(Pred δ_t)	$\hat{C}_t$	$\hat{\delta}_t$	(Pred δ_t)	$\hat{C}_t$	$\hat{\delta}_t$	(Pred δ_t)	C_t	$\hat{\delta}_t$	(Pred δ_t)	$\hat{C}_t$
.38°	0.5 sec.	1.56	(1.23)	.14	1.02	(1.16)	.09	1.32	(1.02)	-.10	.35	(.43)	.14
.38°	1.0 sec.	1.10	(.87)	.27	.77	(.82)	.22	.66	(.72)	.01	.20	(.30)	.17
.38°	1.5 sec.	.64	(.71)	.35	.63	(.67)	.33	.54	(.59)	.02	.33	(.25)	.39
.38°	2.0 sec.	.53	(.61)	.35	.58	(.58)	.38	.56	(.51)	-.04	.22	(.21)	.27
.76°	0.5 sec.	2.35	(2.45)	.40	2.26	(2.31)	.13	1.60	(2.04)	.10	.81	(.86)	.17
.76°	1.0 sec.	1.90	(1.74)	.38	1.76	(1.63)	.30	1.56	(1.45)	.11	.62	(.61)	.32
.76°	1.5 sec.	1.34	(1.42)	.39	1.28	(1.33)	.37	1.02	(1.18)	.03	.40	(.50)	.23
.76°	2.0 sec.	1.28	(1.22)	.41	1.16	(1.15)	.42	.98	(1.02)	-.11	.54	(.43)	.59

The degree to which these estimates of δ_t are consistent with the model requires an estimate of the diffusion parameter φ, and of the total input variance, σ_o^2. These were obtained separately for each observer in the following manner. First, note that Eq. 18 may be rearranged to read

$$\text{Var}\ (Y_t) = (\Delta s/\delta_t)^2 \tag{19}$$

Inserting the proper value of Δs and substituting $\hat{\delta}_t$ for δ_t allows us to estimate Var (Y_t) for each experimental condition. These estimates are plotted as points in Fig. 4.

The linear theoretical curves in Fig. 4 are based on Eq. 16 which indicates that Var (Y_t) is a linear function of t with slope φ and an intercept equal to the total input variance (σ_o^2). However, in fitting the theoretical curves to the points in Fig. 4 it became apparent that the input variance was negligible. This suggested an interesting simplification of the model: let the input variance equal zero by assumption and fit the data with a single parameter φ. Thus the linear functions in Fig. 4 represent the least squares fit for this single parameter form of the general model. Note that the estimates of Var (Y_t) obtained from the Δs equal .38 degree and .86 degree conditions are generally consistent. Furthermore the linear fit is quite good for all except observer 4; although even his data shows no systematic deviation from linearity, simply considerable variance. The slope of each function in Fig. 4 represents an estimate of φ. Substituting $\hat{\varphi}$ for φ in Eq. 18 (with the input variance equal to zero) leads to a predicted value of δ_t for each experimental condition. The estimates of φ and the predicted values of δ_t for each observer are listed in Table 2 and presented graphically (solid lines) in Fig. 3. It is apparent that the single parameter φ allows a generally good prediction of δ_t under the various experimental conditions.

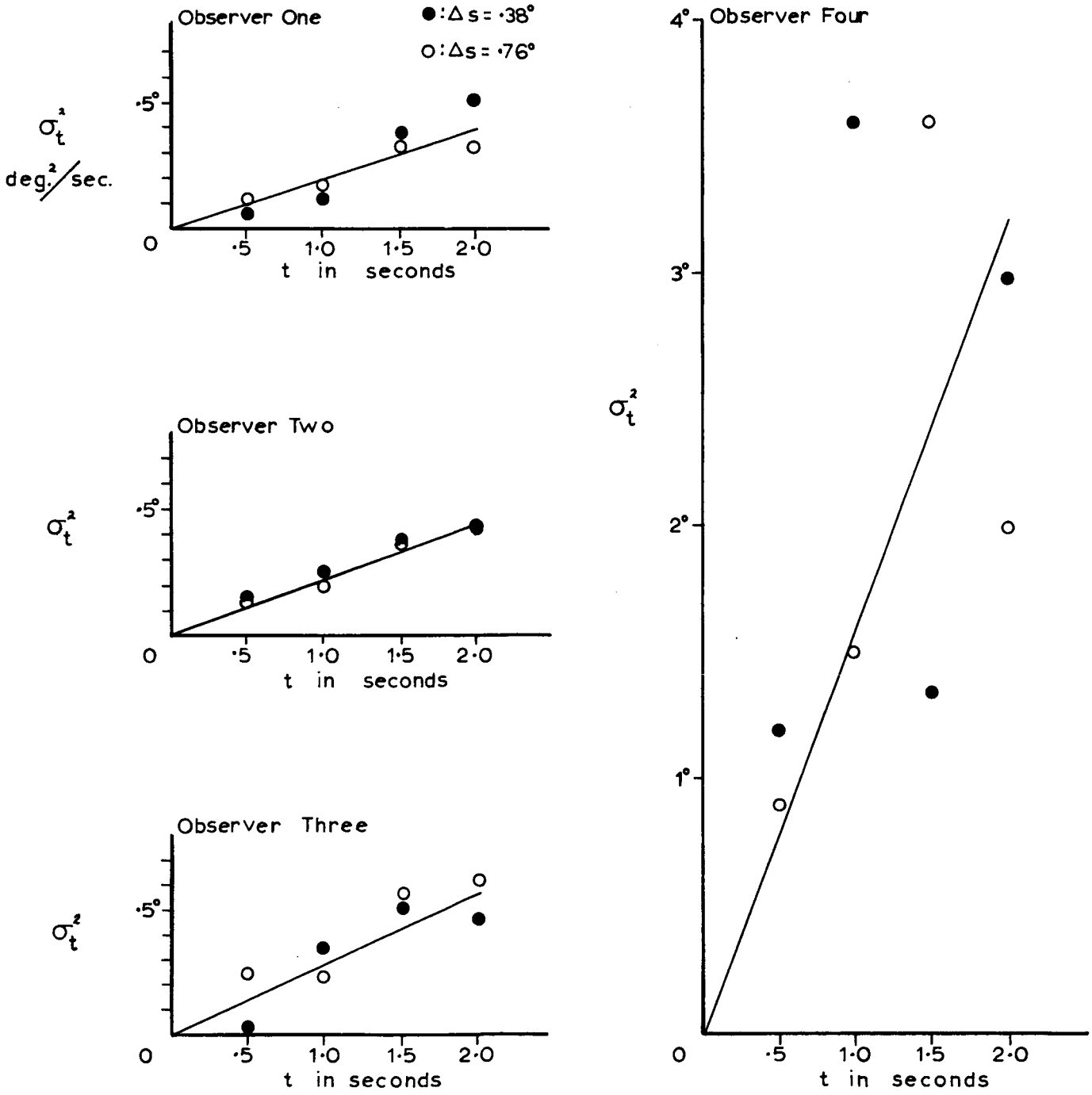

Fig. 4. Estimated (points) and predicted (lines) values of σ_t^2 given $\hat{\varphi}$ for Experiment One.

Even the performance of observer 4 appears reasonably consistent with the model; he simply appears to have an unusually poor memory process with most of the decrement in his performance occurring prior to t equal .5 seconds.

Another theoretical question of interest is the degree to which an observer's decision criterion, C_t, appears to depend on Δs and t. Estimates of C_t for each experimental condition were obtained as follows. If the model is correct, the difference between any observed pair of hit and false-alarm rates and the predicted operating characteristic must be attributed to sampling variance. Thus the best estimate of an observer's theoretical P(H) and P(FA) would be that pair of probabilities on the predicted operating characteristic most similar to his observed performance. For example, when Δs was .76 degree and t was .5 sec, Observer One's hit proportion was .88 and false-alarm proportion .12 (Table 1). A P(H) of .90 and P(FA) of .10 are the most similar pair of values consistent with the sensitivity measure (δ_t) of 2.45 predicted for this condition (Table 2). A table of normal deviates indicates that C_t would have to be 1.28 standard deviations from the mean of the equal stimuli ($s_o = s_t$) distribution of Y_t to yield a false-alarm rate of .10. Since the theoretical standard error (σ_t) for t equal .5 sec was .316 for Observer One, the estimate of his criterion, denoted $\hat{C}_t$, would be 1.28 times .316 or .40. Estimates of C_t obtained in this manner from all 8 experimental conditions for each observer are presented in Table 2.

While there is some suggestion that $\hat{C}_t$ might be positively correlated with both t and Δs for Observers One and Two, the overall variability of $\hat{C}_t$ relative to these possible systematic effects precludes any firm conclusions. These estimates are presented primarily to illustrate the model's application and to allow the reader to draw his own conclusions regarding the stability of C_t. This issue is considered in more detail in Experiment Two.

Experiment Two: Auditory Amplitude Discrimination

Each observer in this experiment was asked to discriminate an amplitude (loudness) difference between two successively presented bursts of a clearly audible pure tone. He had to decide whether the second signal had the same amplitude as the first or a lower amplitude. The object of the experiment was to assess the effect on performance of various temporal separations between the two signals.

Apparatus and Procedure

Each signal was a 500 msec duration (including a 10 msec rise and decay period), 1000 cps tone presented binaurally through earphones (Permoflux PDR-8, 600 ohm impedance). The stimulus variable, S, was the signal amplitude. In terms of the RMS voltage at the earphones (from a 600 ohm source), s_o was always 100 mv while s_t was either 100 mv or 80 mv. The observer was aware of this procedure and responded "same" or "lower" to indicate his decision regarding s_t on each trial. The particular amplitudes employed were chosen to be both comforably audible and sufficiently similar to produce about 80 percent correct responses during three days of prliminary testing with the interstimulus delay equal to zero.

Stimulus sequences were defined in blocks of 50 trials just as in Experiment One. On a randomly chosen 25 of each 50 trials, s_o equaled s_t, and t was fixed throughout each block. The 5 experimental conditions corresponded to a particular value of t: 0,0.5, 1.0, 1.5, or 2.0 seconds.

Each of 3 observers performed for 10 daily sessions. Each session consisted of 10 randomly ordered blocks of 50 trials, two blocks under each of the 5 experimental conditions (there was a 3 minute rest period between blocks to that the total session consumed about 60 minutes). The observers in this

experiment had the same knowledge concerning stimulus schedules as those in Experiment One. They were given examples of the high and low amplitudes prior to each experimental session, but obtained no information concerning the accuracy of their responses.

Results

The hit and false-alarm proportions averaged over all ten sessions for each experimental condition are presented in Table 3. Since this represents a total of 1000 trials per condition, and the two stimulus patterns were equally frequent under all conditions, each proportion in Table 2 is based on 500 trials. A Chi-square test indicated a statistically significant ($p > .01$) effect of t on each observer's performance. Further analysis was done in terms of the model.

Table 3

Average Hit and False-Alarm Proportions for Experiment Two

	Obs 1		Obs 2		Obs 3	
t	H	FA	H	FA	H	FA
0.0	.84	.05	.87	.10	.65	.07
0.5	.81	.06	.86	.10	.66	.07
1.0	.79	.07	.82	.11	.60	.06
1.5	.72	.07	.78	.09	.50	.06
2.0	.69	.05	.75	.07	.35	.03

Theoretical Analysis

As in Experiment One, the statistics in Table 3 are sufficient to estimate δ_t for each experimental condition. These estimates, $\hat{\delta}_t$, are presented as data points in Fig. 5(b) and listed in Table 4. Substituting the appropriate $\hat{\delta}_t$ for

for δ_t in Eq. 19 and letting Δs equal 1 (an arbitrary choice of unit for the stimulus variable which simplifies calculation) yields estimates of the variance of Y_t ($\hat{\sigma}_t^2$). These estimates are plotted as points in Fig. 5(a). The theoretical curves in that figure are based in Eq. 16 and were fitted by a least squares criterion to provide an estimate of φ (the slope) and the total input variance, σ_o^2, (the intercept at t equal 0). These estimates were then used in Eq. 18 (along with the appropriate value of t and Δs equal to 1) to yield the predicted values of δ_t; these predicted values are listed in Table 4 and graphed as solid lines in Fig. 5(a).

Table 4

Estimated Values of δ_t and C_t and Predicted Values of δ_t Based on Estimates of φ and σ_o^2.

	Obs 1 ($\hat{\varphi}$=.055, $\hat{\sigma}_o^2$=.133)			Obs 2 ($\hat{\varphi}$=.031, $\hat{\sigma}_o^2$=.174)			Obs 3 ($\hat{\varphi}$=.071, $\hat{\sigma}_o^2$=.279)		
t	$\hat{\delta}_t$	(Pred δ_t)	$\hat{C}_t$	$\hat{\delta}_t$	(Pred δ_t)	$\hat{C}_t$	$\hat{\delta}_t$	(Pred δ_t)	$\hat{C}_t$
0.0	2.66	(2.74)	.470	2.41	(2.40)	.467	1.86	(1.89)	.206
0.5	2.49	(2.49)	.362	2.37	(2.30)	.470	1.89	(1.78)	.195
1.0	2.27	(2.31)	.370	2.13	(2.21)	.417	1.79	(1.69)	.101
1.5	2.06	(2.16)	.297	2.12	(2.13)	.362	1.56	(1.61)	.019
2.0	2.13	(2.03)	.226	2.08	(2.06)	.296	1.49	(1.54)	-.247

It seems apparent that the model provides a reasonable account of the performance decrement produced by an interstimulus delay. The systematically superior performance in detection (t = 0) is consistent with a decrement in δ_t produced by the memory process in recognition (t > 0).

Again as in Experiment One, it is interesting to consider the relation

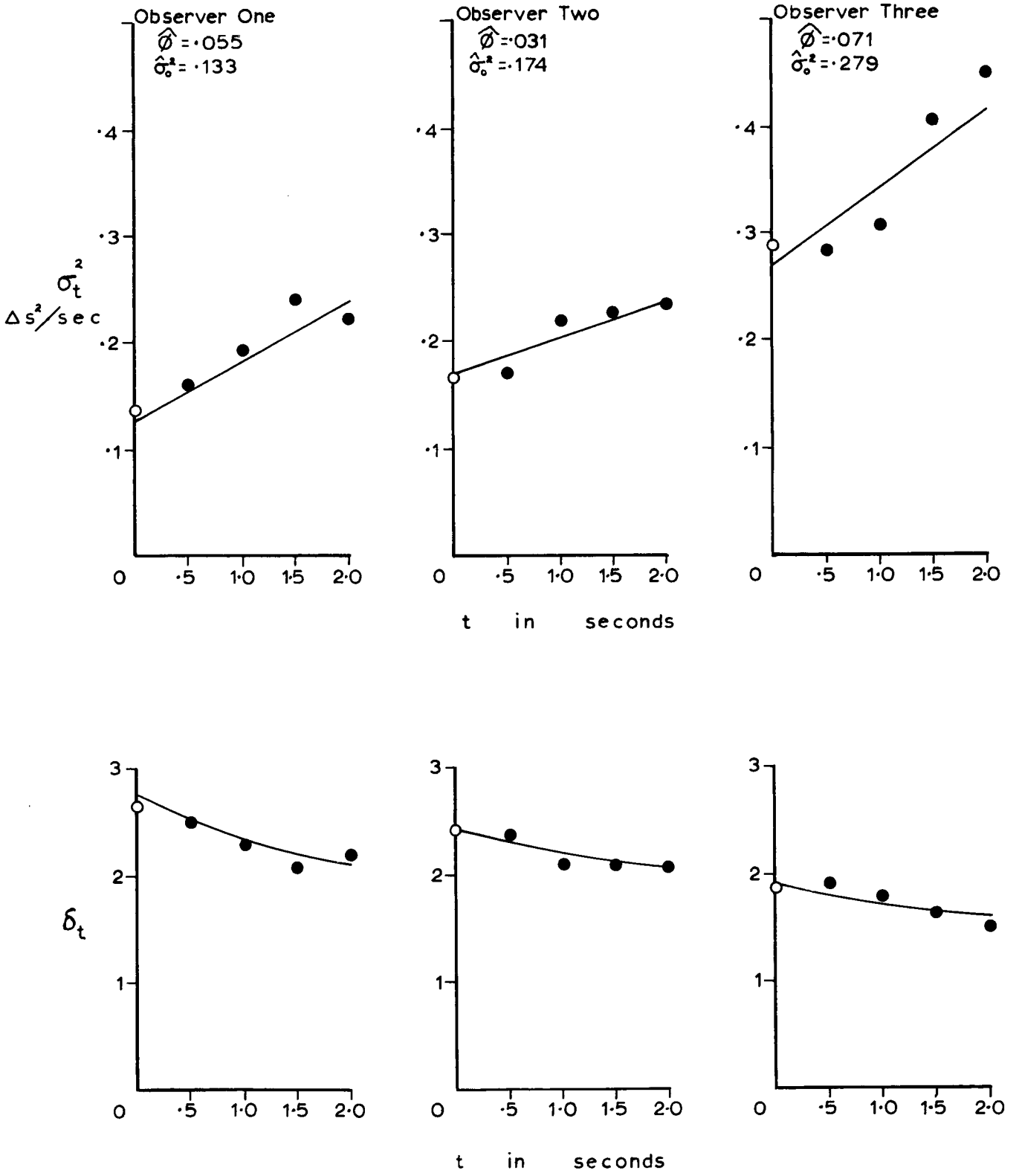

Fig. 5. Estimated (points) and predicted (lines) values of σ_t^2 and δ_t given $\hat{\phi}$ and σ_o^2 for Experiment Two.

between the estimates of C_t and t. These estimates, $\hat{C}_t$, were calculated in the same manner as those in the first experiment and are presented in Table 4. In interpreting these estimates it is important to note that the expected variance of Y_t ($\hat{\sigma}_t$) is minus one on trials when the second amplitude is 80 mv. This follows from Eq. 15 and the choice of Δs as our unit of S. Thus the unequal stimuli distribution (mean minus one) would be to the left of the equal stimuli (mean zero) distribution in Fig. 2 for this particular task. An observer's decision rule would be to respond "lower" whenever Y_t was <u>less</u> than C_t and "equal" otherwise. The apparent negative relation between the proportion of "lower" responses and t (see Table 3) is, therefore, consistent with the similar relation between $\hat{C}_t$ and t (see Table 4).

Most readers will recognize the observer's progressive reticence to respond "lower" as t increases as the classic time order error in psychophysics: the second presentation of the same stimulus amplitude tends to sound louder than the first. There is an interesting alternative to the interpretation that C_t is a function of t, one which is similar to the so called "fading trace" interpretation of time order error (see Woodworth and Schlosberg 1954 on this "fading trace" concept, which was first introduced by Fechner in 1860). Specifically, instead of assuming that the response criterion can vary and the random walk in memory is symmetrical, assume a constant criterion (denoted simply by C) and an asymmetrical random walk. Since the value of p in Eq. 11 would not equal 1/2 the expected value of D_t would equal zero only when t was zero. If p was greater than 1/2, E (D_t) would become progressively positive as t increased; if p was less than 1/2, E (D_t) would become progressively negative as t increased. It is the latter case which could be interpreted as an explicit representation of a "fading trace", since m_t would have a systematic negative drift during the interstimulus interval. So long as p was not too

far from 1/2, we could consider the distribution of Y_t to be Gaussian (the approximation employed earlier). However, the mean value of Y_t would bear a negative relation to E (D_t) (see Eq. 9). Therefore, even though the response criterion was constant, its position relative to the overlapping distributions of Y_t (the stimulus contingent distributions illustrated in Fig. 2) would be a function of t.

It is possible to interpret the values $\hat{C}_t$ presented in Table 4 on the basis of a fixed criterion and an asymmetrical random walk. Note that $\hat{C}_t$ was calculated on the assumption that the expected value of Y_t on equal stimuli trials, E_o (Y_t), was zero. Since this assumption would be inappropriate if the random walk were asymmetrical, $\hat{C}_t$ should be interpreted as an estimate of C, the fixed criterion, plus the actual value of E_o (Y_t); specifically,

$$\hat{C}_t = \hat{C} + E_o \ (Y_t).$$

Substituting for E_o (Y_t) on the basis of Eqs. 9 and 11 yields,

$$\hat{C}_t = \hat{C} - \rho\omega \ (2 \ p - 1) \ t. \tag{20}$$

Thus the estimates of C_t in Table 3 should bear a linear relation to t with a value at t equal zero of $\hat{C}$. The degree to which this relation obtains is indicated by Fig. 6 which presents the best fitting linear function for the values $\hat{C}_t$ from each observer.

The data conform reasonably well to the linear relation specified by Eq. 20. The intercept of each linear function at t equal zero (.44, .51, and .25, respectively) can be interpreted as an estimate of the observer's fixed response criterion, $\hat{C}$. The slope of each function (-.095, - .100, and -.200, respectively) can be interpreted as the rate at which the expected value of m_t changes with t (the quantity $\rho\omega$ (2p - 1) in Eqs. 11 and 20). The fact that all three slopes are negative suggests a systematic negative

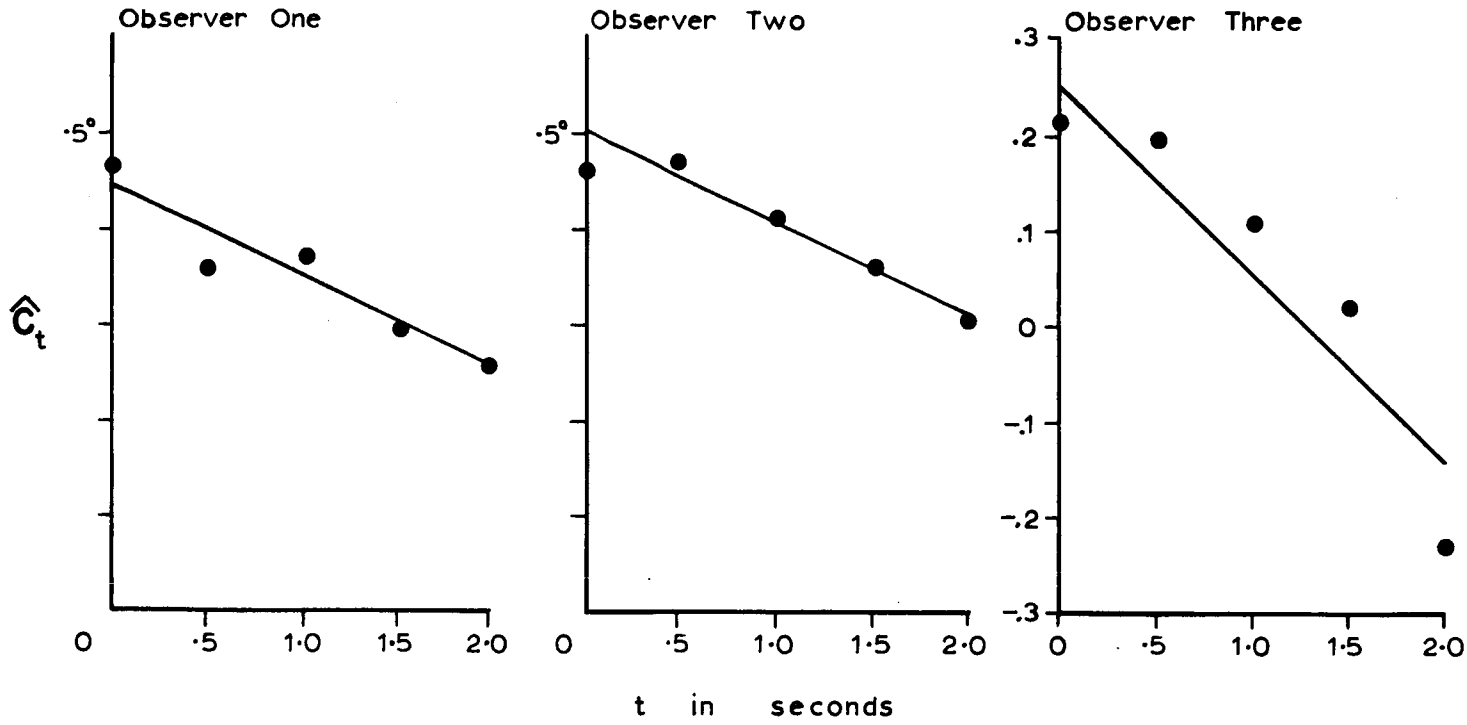

Fig. 6. Estimated values of $\hat{C}_t$ and best fitting linear function for Experiment Two.

drift of m_t during the interstimulus interval ($p < 1/2$); i.e., a "fading trace".

Discussion

Experiment One

The results of the visual experiment seem relevant to some general issues in visual psychophysics; in particular, autokinetic phenomena and the measurement of involuntary eye movements. The perception of illusionary movement when one observes a dim point of light in the dark (autokinesis) has been studied for two basically different reasons. First, autokinesis has been used as an apparently sensitive index of an observer's susceptibility to suggestion or social pressure (e.g., the generic work of Sherif, 1935). Second, it has been studied as a source of information about the stability of the visual system (e.g., Gregory and Zangwill, 1963); Matin and MacKinnon, 1964). A recent review of both types of studies emphasized the serious methodological problems encountered in the measurement of autokinesis and the need for more satisfactory techniques (Royce, et al., 1966). It is proposed here that the visual task employed in Experiment One might be considered a discrete analogue to the usual autokinetic situation; e.g., a false-alarm could be considered the perception of illusionary movement. Of course there are important differences to be considered such as the use of transient rather than continuous points of lights. Nevertheless, $\hat{C}_t$ might prove to be a precise and sensitive measure of susceptibility to suggestion in the Sherif type of experiment, since it appears to be a relatively pure measure of the decision making aspects of motion perception. Similarly, $\hat{\varphi}$ might be a purer measure of visual stability than could ordinarily be obtained from conventional autokinetic data. The degree to which $\hat{\varphi}$ is a measure of involuntary eye movements is particularly intriguing. It is quite possible that the "random walk"

during the interstimulus interval may to a large extent reflect actual eye movements (although it would also include any other source of instability such as involuntary head movements and "internal noise"). While Cornsweet reported in 1956 that direct measurement of eye movements in the dark suggests a random walk process, a more complete analysis of this issue has recently been conducted by Matin, Matin, and Kibler[2], who have kindly provided us with a prepublication description of their results. Their experimental situation differed from ours in only two respects: one, the duration of the original stimulus (s_o) was 4 seconds with t fixed at 3 seconds; and two, the observer's head was stabilized with a biting block. Measurement of eye movements during the 3 second interstimulus interval suggests a random walk process although there was evidence that the parameters of this random walk were non-stationary. While their cumulative variance was very nearly additive (linear) over the range of t values used in our Experiment One, there was some evidence of a negative acceleration during the later stages of the interstimulus interval (2-3 sec.). The rate at which variance was accumulated was approximately .05 degrees^2/sec. This is in the order of 1/4 the typical value of $\hat{\varphi}$ computed in Experiment One. These results are quite encouraging as far as the model is concerned. It seems reasonable to suppose that simply stabilizing the observer's head (e.g., providing him with a biting block) could reduce the estimates of $\hat{\varphi}$ by as much as 50 percent. This would put the measures of visual stability obtained from purely psychophysical data quite close to those obtained through direct measurement of eye movement, a considerable accomplishment considering the degree of precision achieved in previous analysis of autokinetic phenomena.

It should also be noted that Matin <u>et al</u>., found systematic components of the random walk much like the negative drifts suggested by the auditory

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[2] Matin, L., Matin, E., and Kibler, G., Columbia University. (personal communication)

data from Experiment Two (those represented in Fig. 6). However the direction of these shifts sometimes changed during the interstimulus period. These effects seem consistent with the values of $\hat{C}_t$ presented in Table 2. The apparent shifts in these values could be interpreted as evidence of asymmetry in the random walk (just as the values of $\hat{C}_t$ presented in Fig. 6); while variation in the rate of shift would suggest a non-stationary value of p.

Experiment Two

Subsequent to the development of the material presented in this paper, the authors learned of a paper by Tanner (1961) in which he discusses various memory factors relevant to psychophysical measures. One of the experiments he describes differs from Experiment Two in only three respects: he employed 100 rather than 500 msec. signals; there was a constant background of white noise; and his observer learned whether his response was correct at the conclusion of each trial. While Tanner only presents data for a single representative observer[3], his results are generally consistent with ours except for very short interstimulus intervals (between 0 and .5 seconds) where there was a sharp drop in apparent sensitivity (δ_t). Tanner's interpretation of these results is essentially the same as our own except for the short intervals where he proposes the drop in sensitivity is caused by the inability of the input process to generate independent sensory states (stating his comments in terms of our model). Since Tanner did not collect data for the t equal zero condition, his results do not directly conflict with ours. The drop in sensitivity may occur between our data points at t equal zero and t equal .5 seconds. Furthermore, the differences in signal duration, noise background, and information feedback may make a direct comparison misleading.

[3] A complete report of these data and additional data from similar experiments on memory for pitch is currently being prepared for publication by C.D. Creelman, University of Toronto and W.P. Tanner, University of Michigan (personal communication).

Although Tanner does not report measures of the response criterion, his use of feedback undoubtedly influenced the pattern of these results precluding a direct comparison with our data. The reader is referred to Tanner's paper as an excellent treatment of the problem of memory in psychophysics.

General Comments

The approach to discrimination represented in this paper suggests that detection tasks can be viewed as a special form of discrimination; specifically, one in which the stimulus values to be discriminated are temporally contiguous. For example, the conventional "yes-no" auditory detection task requires the observer to judge the similarity of the stimulus value presented just prior to an observation interval (s_o) and the value presented during the interval (s_t). This is true whether s_o is a zero amplitude tone and s_t greater than zero (an "absolute" detection task), or s_o and s_t are both positive amplitudes (an "incremental" or "quantal" detection task). Whenever s_o and s_t are not presented in immediate temporal succession one speaks of a recognition problem. For example, a simple recognition task requires the observer to decide which of two possible frequencies of tone is presented to him. The crucial feature of this task is that the two possible frequencies are not presented in immediate temporal succession. Thus, the recognition response is essentially a similarity judgement between the pitch he hears and his memory of that pitch. Viewed in this way, conventional recognition tasks are highly complicated paradigms, since the observer is usually required to maintain "memories" of several possible stimulus alternatives simultaneously. The simple recognition paradigm employed in this paper seems to be a more fundamental approach to the role of perceptual memory in discrimination. Most of the basic issues in the study of memory could be approached with this simple paradigm. For example, variation in the stimulus value just prior to s_o or the value during the interstimulus

interval would be similar to studying the role of pro-active and retro-active interference in conventional memory experiments.

It should be emphasized that the type of memory process we have considered is only one kind of memory. It is clear that certain forms of perceptual memory depend to a large extent on some kind of verbal coding; i.e., a person might remember the amplitude of a sound by remembering "it was as loud as a pistol shot"; or, he might remember a visual pattern by remembering "it was a picture of a dog". However, the simple stimuli, small stimulus differences, and short temporal intervals employed in the present paradigm may allow one to deal with a more fundamental form of memory uncomplicated by such verbal coding.

Finally, it is worth noting that we have only considered the discrimination of serially presented stimulus values. However, it may well be the case that many "simultaneously presented" stimuli are really only simultaneously available to an observer and he actually observes them in sequence. Such ad libitum observing would be difficult to control, let alone assess, and this may be a very poor experimental design. Nevertheless it is interesting to consider the degree to which spatial and temporal separation of stimuli may be equivalent. For example, suppose an observer were asked to judge the similarity of two simultaneously presented patches of grey. He may actually scan them sequentially so that the greater the spatial separation between them the longer the temporal delay between their successive observation. Thus the discrimination of spatially contiguous patches of grey would be the spatial equivalent of auditory detection. This interpretation is consistent with an observer's loss of efficiency in comparing two patches of grey as their spatial separation increases (a phenomenon which can easily be domonstrated).

SUMMARY AND CONCLUSION

We have shown how three measures may be extracted from an observer's performance of a discrimination task: an estimate of input variance, $\hat{\sigma}_o^2$; of diffusion rate, $\hat{\varphi}$; and of response criterion, $\hat{C}_t$. Furthermore, in some cases it is possible to obtain a measure of systematic components of the diffusion process: our estimate of the drift rate, the quantity $\rho\omega$ (2p - 1) in Eqs. 11 and 20.

The emphasis in our analysis has been on the role of perceptual memory in the discrimination of stimulus values which are not temporally continuous. It is argued that this temporal aspect of discrimination is the most basic feature of recognition tasks. Discrimination in detection tasks is more efficient because the temporal contiguity of the stimuli minimizes the degrading (diffusing) effect of imperfect perceptual memory.

Further research bearing on the memory aspects of simple discrimination tasks should provide an interesting bridge between the classical issues of memory and the psychophysics of perception.

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