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LIE SERIES SOLUTION OF GRAVITY-GRADIENT STABILIZED MOTION OF A SATELLITE IN ELLIPTIC ORBITS.

by

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Abstract

In this report the equations of motion of a satellite spinning around its center of mass are solved with the help of Lie series technique which has proved its usefulness in a series of problems connected with the solution of differential equations in the past few years. The satellite is assumed to move on an elliptic orbit in which case the external torques have a more complex form than in the circular case. The neglect of several types of external forces is motivated.

1) Introduction.

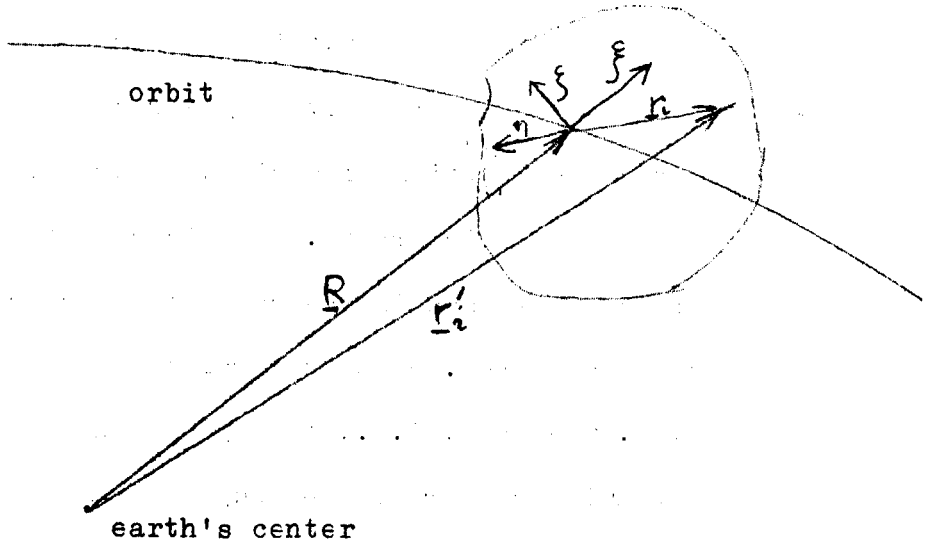
In Report No.13 we derived equations of motion describing the motion of a satellite about its center of mass. In Report No.14 these equations were solved by a recurrence formula technique under the assumption of a circular orbit. In this report we shall treat elliptic orbits. In this case the torques N_1, N_2, N_3 are more complex than in the case of the circular orbit.

As shown in Rep.13 the general expression for the torques in the (ξ, η, ζ)-frame, i.e., the system whose origin coincides with the spinning center of the body and whose ζ-axis lies in the direction of the radius vector from the center of earth to the satellite, is given by the following expression:

$$N = \sum_i \underline{r}_i \times \underline{F}_i = \sum_i \Big\{ \underline{r}_i \times F_{ei} - m_i \underline{r}_i \times \underline{a} + \\ + m_i \underline{r}_i \times (\underline{r}_i \times \dot{\underline{\omega}}) + 2m_i \underline{r}_i \times (\underline{v}_i \times \underline{\omega}) + \\ + m_i \underline{r}_i \times (\underline{\omega} \times (\underline{r}_i \times \underline{\omega})) \Big\} \qquad (1)$$

where the sum is extended over all mass points of the satellite. $\underline{\omega}$ is the angular velocity of the origin of the (ξ, η, ζ)-frame about the earth, $\dot{\underline{\omega}}$ the corresponding angular acceleration, and $\underline{a}$ the translatory acceleration of the (ξ, η, ζ)-frame. $\underline{v}_i$ is the translatory velocity of a mass point m_i with respect to the frame (ξ, η, ζ). Final, $\underline{F}_e$ means the entirety of external forces acting on the satellite; r_i is the vector point from the (ξ, η, ζ)-origin to the mass point m_i.

The following illustration will help to clarify the situation:



We shall now discuss the individual terms of Eq.(1):

a) $\underline{r}_i \times \underline{F}_{ei}$:

$\underline{F}_{ei}$ is the sum of all external forces, with

$$\underline{F}_{ei} = \underline{F}_{oi} + \underline{F}_{1i} + \underline{F}_{2i} + \underline{F}_{3i} + \underline{F}_{4i} + \underline{F}_{5i} + \underline{F}_{6i}$$

where

$$\underline{F}_{oi} + \underline{F}_{1i} = \underline{F}_{gi}$$

($\underline{F}_{oi}$ is the gravitational force due to an exactly spherical earth; $\underline{F}_{1i}$ is the correction due to deviations from spherical shape; g means gravitational).

The forces F_2(drag force), F_3(radiation pressure), F_4(gravity of sun), F_5(gravity of moon) and F_6(magnetic field, eddy current, etc.) are not considered in this report.

As was shown in Report 13, the term $\underline{r}_i \times \underline{F}_{gi}$ due to the

gravitational force reads for circular and elliptic orbits:

$$\underline{r}_i \times \underline{F}_{gi} = \underline{i}\,\frac{3\nu}{|R|^3}\, m_i \eta_i \zeta_i - \underline{j}\,\frac{3\nu}{|R|^3}\, m_i \xi_i \zeta_i$$

for both circular and elliptic orbits, in this relation we have:

$$\underline{r}_i' = \underline{R} + \underline{r}_i$$

(see Fig.1)

$$\bar{r}_i = (\xi, \eta, \zeta)$$

$$\nu = m_E f$$

where f is the gravitational constant and m_E the mass of the earth.

b) $\sum m_i \underline{r}_i \times \bar{a} = 0$ if we assume that the origin of the (ξ, η, ζ)-frame is the mass center of the body

c) $\sum_i m_i \underline{r}_i \times (\underline{r}_i \times \underline{\dot{\omega}}) = 0$ for a circular orbit

$$= \sum_i \{(\underline{i}\xi_i + \underline{j}\eta_i + \underline{k}\zeta_i)|\dot{\omega}|\eta_i - \dot{\omega}(\xi_i^2 + \eta_i^2 + \zeta_i^2)$$

for an elliptic orbit

d) $\sum_i 2m_i \underline{r}_i \times (\underline{v}_i \times \underline{\omega}) = 2\sum_i (m_i \underline{i}|\omega|\eta_i \dot{\zeta}_i + m_i \underline{j}|\omega|\eta_i \dot{\eta}_i + m_i \underline{k}|\omega|\eta_i \dot{\zeta}_i)$

for a circular orbit

$$= 2\sum_i (m_i \underline{i}|\omega|\eta_i \dot{\zeta}_i - m_i \underline{j}|\omega|(\xi_i \dot{\zeta}_i + \zeta_i \dot{\zeta}_i) + m_i \underline{k}|\omega|\eta_i \dot{\zeta}_i)$$

for an elliptic orbit.

e) $\sum_i m_i \underline{r}_i \times (\underline{\omega} \times (\underline{r}_i \times \underline{\omega})) = \sum_i (-m_i \underline{i} |\omega|^2 \eta_i \xi_i +$

$+ m_i \underline{k} |\omega|^2 \eta_i \xi_i$

for both circular and elliptic orbits (for all these relations see Report 13)

The equations to be studied have the following form:

$$\ddot{\varphi} = -\dot{\varphi}\dot{\chi}\,\bar{c}c(B_1 + B_2) - \dot{\varphi}\dot{\vartheta}\,\frac{(\bar{b} + B_2\bar{c}^2\bar{b} - c^2B_1\bar{b})}{b} - \frac{\dot{\vartheta}\dot{\chi}}{b}(-1 + B_2\bar{c}^2 - c^2B_1) - \dot{\varphi}^2 c\bar{c}\bar{b}\,(B_1 + B_2) + \frac{N_1}{I_1}\frac{c}{b} + \frac{N_2}{I_2}\frac{\bar{c}}{b} \tag{1a}$$

$$\ddot{\vartheta} = \dot{\varphi}^2 b\bar{b}(-B_1\bar{c}^2 + B_2c^2) + \dot{\varphi}\dot{\chi}c\bar{c}\,(B_1 + B_2) - b\dot{\varphi}\dot{\chi}\left[1 + B_1\bar{c}^2 - B_2c^2\right] + \dot{\vartheta}\dot{\chi}\,(B_1 + B_2)\,c\bar{c} + \frac{N_1}{I_1}\bar{c} - \frac{N_2}{I_2}c \tag{1b}$$

and

$$\ddot{\chi} = \dot{\vartheta}^2 B_3 c\bar{c} + \dot{\varphi}^2 c\bar{c}\,(-B_3b^2 + \bar{b}^2(B_1 + B_2)) + \frac{\dot{\varphi}\dot{\vartheta}}{b}(1 + b^2B_3(c^2 - \bar{c}^2)) + \bar{b}^2(B_2\bar{c}^2 - c^2B_1) + \dot{\varphi}\dot{\chi}b\bar{c}c\,(B_1 + B_2) + \dot{\vartheta}\dot{\chi}\,B_2\bar{c}^2 - B_1c^2 - 1)\cdot\frac{\bar{b}}{b} + \frac{N_3}{I_3} - \frac{N_1}{I_1}\cdot\frac{c\bar{b}}{b} - \frac{N_2}{I_2}\frac{\bar{c}}{b}\bar{b} \tag{1c}$$

where φ, χ, ϑ are the Eulerian angles, N_i the external torques, and the B_i abbreviations for:

$$B_1 = \frac{T_{23}}{I_1}, \quad B_2 = \frac{T_{31}}{I_2}, \quad B_3 = \frac{T_{12}}{I_3} \tag{2}$$

The I_i are the main moments of inertia, B_i, I_i, T_{ij} are constants and $T_{ij} = T_i - T_j$, where $T_l = X_l^2\, dm$, $l = 1,2,3$, X_l are the coordinates fixed with respect to the body.

As shown in Report 13, the a, b, c, $\bar{a}$, $\bar{b}$, $\bar{c}$ are transcendental functions of the Eulerian angles forming the components of matrix A which transforms from the body-fixed (X, Y, Z) to the rigid rotating (ξ, η, ζ)-system, i.e.:

$$A = \begin{pmatrix} \bar{c}\bar{a} - ca\bar{b}, & -\bar{c}a\bar{b} - c\bar{a}, & ab \\ \bar{c}a + c\bar{a}\bar{b}, & \bar{c}\bar{a}\bar{b} - ca, & -\bar{a}b \\ cb\ , & \bar{c}b\ , & \bar{b} \end{pmatrix} \tag{3}$$

with

$$\begin{aligned} a &= \sin\varphi, & b &= \sin\vartheta, & c &= \sin\chi \\ \bar{a} &= \cos\varphi, & \bar{b} &= \cos\vartheta, & \bar{c} &= \cos\chi \end{aligned} \tag{4}$$

The torques N_i are given by:

$$N_1 = \omega^2(3a_{32}a_{33} - a_{22}a_{33})\, T_{23} + 2\omega(T_3 a_{23}\dot{\theta}_2 - T_2 a_{22}\dot{\theta}_3) + N_{1e}$$

$$N_2 = \omega^2(3a_{31}a_{33} - a_{21}a_{23})\, T_{31} + 2\omega(T_1 a_{21}\dot{\theta}_3 - T_3 a_{23}\dot{\theta}_1) + N_{2e}$$

$$N_3 = \omega^2(3a_{31}a_{32} - a_{21}a_{22})\, T_{12} + 2\omega(-T_1 a_{21}\dot{\theta}_2 + T_2 a_{22}\dot{\theta}_1) + N_{3e}$$

where the $N_i e$ are the additional terms due to (5) orbit ellipticity, such that

$$\begin{aligned} N_1 &= N_{1c} + N_{1e} \\ N_2 &= N_{2c} + N_{2e} \\ N_3 &= N_{3c} + N_{3e} \end{aligned} \tag{5a}$$

where "c" means "circular" and "e" "elliptic".

The additional terms N_{ie} are given by:

$$N_{1e} = \Big[\dot{\omega}\,(a_{12}a_{22}T_2 + a_{13}a_{23}T_3)\,a_{11} + \Big[-\dot{\omega}\,(a_{12}^2T_2 + a_{13}^2T_3) -$$

$$- \dot{\omega}\,(a_{32}^2T_2 + a_{33}^2T_3) - 2\omega(a_{21}\dot{a}_{21}T_1 + a_{22}\dot{a}_{22}T_2 + a_{23}\dot{a}_{23}T_3) +$$

$$+ 2\omega\,(a_{11}\dot{a}_{11}T_1 + a_{12}\dot{a}_{12}T_2 + a_{13}\dot{a}_{13}T_3) +$$

$$+ 2\omega\,(a_{31}\dot{a}_{31}T_1 + a_{32}\dot{a}_{32}T_2 + a_{33}\dot{a}_{33}T_3)\Big]\,a_{21} +$$

$$+ \dot{\omega}\,(a_{32}a_{22}T_2 + a_{33}a_{23}T_3)\,a_{31}$$

$$N_{2e} = \Big[\dot{\omega}\,(a_{11}a_{21}T_1 + a_{13}a_{23}T_3)\,a_{12} + \Big[-\dot{\omega}\,(a_{11}^2T_1 + a_{13}^2T_3) -$$

$$- \dot{\omega}\,(a_{31}^2T_1 + a_{33}^2T_3) -$$

$$- 2\omega\,(a_{21}\dot{a}_{21}T_1 + a_{22}\dot{a}_{22}T_2 + a_{23}\dot{a}_{23}T_3) +$$

$$+ 2\omega\,(a_{11}\dot{a}_{11}T_1 + a_{12}\dot{a}_{12}T_2 + a_{13}\dot{a}_{13}T_3) +$$

$$+ 2\omega\,(a_{31}\dot{a}_{31}T_1 + a_{32}\dot{a}_{32}T_2 + a_{33}\dot{a}_{33}T_3)\Big]\,a_{22} +$$

$$+ \dot{\omega}\,\Big[(a_{31}a_{21}T_1 + a_{33}a_{23}T_3)\Big]\,a_{32}$$

and

$$N_{3e} = \Big[\dot{\omega}\,(a_{11}a_{21}T_1 + a_{12}a_{22}T_2)\,a_{13} + \Big[-\dot{\omega}\,(a_{11}^2T_1 + a_{12}^2T_2) -$$

$$- \dot{\omega}\,(a_{31}^2T_1 + a_{32}^2T_2) - 2\omega(a_{21}\dot{a}_{21}T_1 + a_{22}\dot{a}_{22}T_2 + a_{23}\dot{a}_{23}T_3) +$$

$$+ 2\omega\,(a_{11}\dot{a}_{11}T_1 + a_{12}\dot{a}_{12}T_2 + a_{13}\dot{a}_{13}T_3) + 2\omega\,(a_{31}\dot{a}_{31}T_1 +$$

$$+ a_{32}\dot{a}_{32}T_2 + a_{33}\dot{a}_{33}T_3)\Big]\,a_{23} + \dot{\omega}\Big[(a_{31}a_{21}T_1 + a_{32}a_{22}T_2)\Big]a_{33}$$

The time derivative of the transformation matrix is given by:

$$\dot{A} = \begin{pmatrix} -c\bar{a}\dot{\chi}-\bar{c}a\dot{\varphi}-\bar{c}a\bar{b}\dot{\chi}-c\bar{a}\bar{b}\dot{\varphi}+cab\dot{\vartheta}; & ca\bar{b}\dot{\chi}-\bar{c}\bar{a}\bar{b}\dot{\varphi}+\bar{c}ab\dot{\vartheta}-\bar{c}\bar{a}\dot{\chi}+ca\dot{\varphi}; & \bar{a}b\dot{\varphi}+a\bar{b}\dot{\vartheta} \\ -ca\dot{\chi}+\bar{c}\bar{a}\dot{\varphi}+\bar{c}\bar{a}\bar{b}\dot{\chi}-ca\bar{b}\dot{\varphi}-c\bar{a}b\dot{\vartheta}; & -c\bar{a}\bar{b}\dot{\chi}-\bar{c}a\bar{b}\dot{\varphi}-\bar{c}\bar{a}b\dot{\vartheta}-\bar{c}a\dot{\chi}-c\bar{a}\dot{\varphi}; & ab\dot{\varphi}-\bar{a}\bar{b}\dot{\vartheta} \\ \bar{c}b\dot{\chi}+c\bar{b}\dot{\vartheta}; & -cb\dot{\chi}-\bar{c}\bar{b}\dot{\vartheta}; & -b\dot{\vartheta} \end{pmatrix}$$

In order to apply Lie series formalism we now replace the system (1 a, b, c) of three second-order differential equations by the following system of six first-order differential equations:

$$\dot{Z}_j \equiv Z_{j+3}$$

$$\ddot{Z}_j \equiv \dot{Z}_{j+3} = f_j = \sum_{i=1}^{5} n_i d_{ji} + \sum_{i=6}^{8} \bar{n}_i d_{ji}$$

$$(j = 1, 2, 3)$$

where we have used the following designations and abbreviations:

$$\varphi = Z_1, \qquad \vartheta = Z_2, \qquad \chi = Z_3$$
$$\dot{\varphi} = Z_4, \qquad \dot{\vartheta} = Z_5, \qquad \dot{\chi} = Z_6 \tag{7}$$

as well as

$$n_1 = \dot{\varphi}\dot{\chi},\ n_2 = \dot{\varphi}\dot{\vartheta},\ n_3 = \dot{\vartheta}\dot{\chi},\ n_4 = \dot{\varphi}^2,$$
$$n_5 = \dot{\vartheta}^2,\ \bar{n}_6 = N_1,\ \bar{n}_7 = N_2,\ \bar{n}_8 = N_3 \tag{8}$$

The d_{ij} are given by:

$$d_{11} = -\,c\bar{c}\,(B_1 + B_2)$$
$$d_{12} = -\,\frac{\bar{b}}{b}\,(1 + B_2\bar{c}^2 - c^2B_1)$$
$$d_{13} = -\,\frac{1}{b}\,(B_2\bar{c}^2 - c^2B_1 - 1) \tag{9}$$
$$d_{14} = -\,c\bar{c}\bar{b}\,(B_1 + B_2)$$
$$d_{16} = \frac{1}{I_1}\ \frac{c}{b}$$
$$d_{17} = \frac{1}{I_2}\ \frac{c}{b}$$
$$d_{21} = -b\left[1 + B_1\bar{c}^2 - B_2c^2\right]$$
$$d_{22} = c\bar{c}\,(B_1 + B_2) = -d_{11}$$
$$d_{23} = -d_{11} \tag{10}$$
$$d_{24} = b\bar{b}\,(-B_1\bar{c}^2 + B_2c^2)$$
$$d_{26} = \frac{\bar{c}}{I_1}$$
$$d_{27} = -\,\frac{c}{I_2}$$
$$d_{31} = -\,d_{14}$$
$$d_{32} = \frac{1}{b}\,(1 + b^2B_3\,(c^2 - \bar{c}^2))$$
$$d_{33} = \frac{\bar{b}}{b}\,(B_2\bar{c}^2 - B_1c^2 - 1)$$

$$d_{34} = c\bar{c}(-B_3 b^2 + \bar{b}^2(B_1 + B_2))$$

$$d_{35} = B_3 c\bar{c} \tag{11}$$

$$d_{36} = \frac{1}{I_3},\ d_{37} = \frac{c\bar{b}}{I_1 b},\ d_{38} = -\frac{\bar{c}\bar{b}}{I_2 b}$$

The formal solution of the system is given by:

$$Z_\sigma = e^{tD} z_\sigma = \sum_{\varrho=0}^{\infty} \frac{t^\varrho}{\varrho!} D^\varrho z_\sigma \quad (\sigma = 1, 2, 3)$$

and

$$\hat{Z}_\sigma = \sum_{\varrho=0}^{\infty} \frac{t^\varrho}{\varrho!} D^{\varrho+1} z_\sigma \tag{12}$$

Now we attempt to derive recurrence formulas connecting higher-[illegible] powers of the D-operator with lower ones; for this purpose we write $D^{\varrho+2} z_\sigma$ in the following form:

$$D^{\varrho+2} z_i = D^\varrho (D^2 z_i) = D^\varrho f_i = D^\varrho \left(\sum_{i=1}^{5} n_i d_{ji} + \sum_{i=6}^{8} \bar{n}_i d_{ji} \right) =$$

$$= \sum_{j_1=0}^{\varrho} \binom{\varrho}{j_1} \left\{ D^{j_1} n_i D^{\varrho-j_1} d_{ji} + D^{j_1} \bar{n}_i D^{\varrho-j_1} d_{ji} \right\} =$$

$$= \sum_{j_1=0}^{\varrho} \binom{\varrho}{j_1} \left\{ \left[\sum_{j_2=0}^{j_1} \binom{j_1}{j_2} \left(D^{j_2} z_{\lambda_1} D^{j_1-j_2} z_{\lambda_2} \right) \right] \cdot \right.$$

$$\left. \cdot D^{\varrho-j_1} d_{ji} + D^{j_1} N_\sigma D^{\varrho-j_1} d_{ji} \right\} \tag{13}$$

where $j_1 \leq \varrho$, $j_2 \leq \varrho$

$\underline{D^{\varrho-j_1} d_{ji}}$:

$$\underline{D^{\varrho-j_1} d_{11}} = -(B_1 + B_2) D^{\varrho-j_1} c\bar{c} \tag{14}$$

$$D^{\varrho-j_1}c\bar{c} = D^{j_4}c\bar{c} = \sum_{j_5}^{j_4} \binom{j_4}{j_5} D^{j_5}cD^{j_4-j_5}\bar{c} \tag{15}$$

$$D^{j_5}c = +D^{j_5-1}(\bar{c}z_6) = \sum_{j_6}^{j_5-1} \binom{j_5-1}{j_6} D^{j_6}\bar{c}D^{j_5-1-j_6}z_6 \tag{16a}$$

$$D^{j_4-j_5}\bar{c} = D^{j_4-j_5-1}(cz_6) = \sum_{j_6}^{j_4-j_5-1} \binom{j_4-j_5-1}{j_6} D^{j_6}cD^{j_4-j_5-1-j_6}z_6 \tag{16b}$$

where $j_6 \leq j_5 - 1$, $j_4 - j_5 - 1 - j_6 \leq \varrho$, $j_5 - j_1 - j_6 \leq \varrho$ (17)

$D^{\varrho-j_1}d_{12}$:

$$D^{\varrho-j_1}d_{12} = -D^{\varrho-j_1}\frac{\bar{b}}{b} - B_2D^{\varrho-j_1}\left(\frac{\bar{b}\bar{c}}{b}\right)^2 + B_1D^{\varrho-j_1}\left(\frac{\bar{b}c^2}{b}\right) \tag{18}$$

$$D^{\varrho-j_1}\frac{\bar{b}}{b} = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3}\frac{1}{b}D^{\varrho-j_1-j_3}\bar{b} \tag{19}$$

$$D^{j_3}\frac{1}{b} = -D^{j_3-1}\left(\frac{\bar{b}}{b^2}z_5\right) = -D^{j_3-1}(\bar{b}b^{-1}b^{-1}z_5) =$$

$$= -\sum_{j_4}^{j_3-1} \binom{j_3-1}{j_4} D^{j_4}(\bar{b}b^{-1})D^{j_3-1-j_4}(b^{-1}z_5) \tag{20}$$

$$D^{j_4}(\bar{b}b^{-1}) = \sum_{j_5}^{j_4} \binom{j_4}{j_5} D^{j_5}\bar{b}D^{j_4-j_5}b^{-1} \tag{20a}$$

$$D^{j_3-1-j_4}(b^{-1}z_5) = \sum_{j_6}^{j_3-1-j_4} \binom{j_3-1-j_4}{j_6} D^{j_6}b^{-1}D^{j_3-1-j_4-j_6}z_5 \tag{20b}$$

$$D^{\varrho-j_1-j_3}\bar{b} = D^{l_1}\bar{b} = -D^{l_1-1}(bz_5) =$$

$$= -\sum_{l_2}^{l_1-1} \binom{l_1-1}{l_2} D^{l_2}bD^{l_1-1-l_2}z_5 \tag{21a}$$

$$D^{l_1} b = D^{l_1-1}(\bar{b}z_5) = \sum_{l_2}^{l_1-1} \binom{l_1-1}{l_2} D^{l_2}\bar{b} D^{l_1-1-l_2} z_5 \qquad (21b)$$

$$l_1 - 1 - l_2 \leq \varrho, \quad j_4 \leq \varrho, \quad j_3 - 1 - j_4 \leq \varrho \qquad (22)$$

$$D^{\varrho-j_1}(\frac{\bar{b}}{b}\bar{c}^2) = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3}\frac{\bar{b}}{b} D^{\varrho-j_1-j_3}\bar{c}^2 \qquad (23)$$

$D^{j_3}\frac{\bar{b}}{b}$: see (19)

$$D^{\varrho-j_1-j_3}\bar{c}^2 = D^{l_1}(\bar{c}\bar{c}) = \sum_{l_2}^{l_1} \binom{l_1}{l_2} D^{l_2}\bar{c}\, D^{l_1-l_2}\bar{c} \qquad (24)$$

$D^{l_2}\bar{c}$: see (16)

$$D^{\varrho-j_1}(\frac{\bar{b}c^2}{b}) = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3}\frac{\bar{b}}{b} D^{\varrho-j_1-j_3}c^2 \qquad (25)$$

$D^{j_3}\frac{\bar{b}}{b}$: see (19)

$$D^{\varrho-j_1-j_3}c^2 = D^{l_1}(cc) = \sum_{l_2}^{l_1} \binom{l_1}{l_2} D^{l_1} c\, D^{l_1-l_2} c \qquad (26)$$

$D^{l_1} c$: see (16)

$$\underline{D^{\varrho-j_1} d_{13}}: \quad -B_2 D^{\varrho-j_1}\frac{\bar{c}^2}{b} + B_1 D^{\varrho-j_1}\frac{c^2}{b} + D^{\varrho-j_1}\frac{1}{b} \qquad (27)$$

$$D^{\varrho-j_1}\frac{\bar{c}^2}{b} = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3}\frac{1}{b} D^{\varrho-j_1-j_3}\bar{c}^2 \qquad (28)$$

$D^{j_3}\frac{1}{b}$: see (20)

$D^{\varrho-j_1-j_3}\,\bar{c}^2$: see (24)

$$D^{\varrho-j_1}\,\frac{c^2}{b} = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3}\, D^{j_3}\,\frac{1}{b}\, D^{\varrho-j_1-j_3}\, c^2 \tag{29}$$

$D^{j_3}\,\frac{1}{b}$: see (20)

$D^{\varrho-j_1-j_3}\,c^2$: see (26)

$D^{\varrho-j_1}\,\frac{1}{b}$: see (20)

$D^{\varrho-j_1}\,d_{14}$:

$$-(B_1 + B_2)\, D^{\varrho-j_1}(\bar{b}c\bar{c}) = D^{\varrho-j_1}\, d_{14} \tag{30}$$

$$D^{\varrho-j_1}(\bar{b}c\bar{c}) = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3}\, D^{j_3}\,\bar{b}D^{\varrho-j_1}(c\bar{c}) \tag{31}$$

$D^{j_3}\,\bar{b}$: see (21)

$D^{\varrho-j_1}\,c\bar{c}$: see (15)

$D^{\varrho-j_1}\,d_{15}$:

$$D^{\varrho-j_1}\, d_{16} = \frac{1}{I_1}\, D^{\varrho-j_1}\,\frac{c}{b} \tag{32}$$

$$D^{\varrho-j_1}\,\frac{c}{b} = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3}\, D^{j_3}\, c\, D^{\varrho-j_1}\,\frac{1}{b} \tag{33}$$

$D^{j_3}\,c$: see (16)

$D^{\varrho-j_1}\,\frac{1}{b}$: see (20)

$D^{\varrho-j_1} d_{23}$: $$D^{\varrho-j_1} d_{23} = - D^{\varrho-j_1} d_{11} \tag{40}$$

$D^{\varrho-j_1} d_{24}$: $$D^{\varrho-j_1} d_{24} = - B_1 D^{\varrho-j_1}(b\bar{b}\bar{c}^2) + B_2 D^{\varrho-j_1}(b\bar{b}c^2) \tag{41}$$

$$D^{\varrho-j_1}(b\bar{b}\bar{c}^2) = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3}(b\bar{b}) D^{\varrho-j_1-j_3}\, \bar{c}^2 \tag{42}$$

$D^{1_1}\, \bar{c}^2$: see (24)

$$D^{j_3}(b\bar{b}) = \sum_{j_4}^{j_3} \binom{j_3}{j_4} D^{j_4} b\, D^{j_3-j_4}\, \bar{b} \tag{43}$$

$D^{j_4}\, b$: see (21)

$D^{j_3-j_4}\, \bar{b}$: see (21)

$D^{\varrho-j_1} d_{26}$: $$D^{\varrho-j_1} d_{26} = \frac{1}{I_1} D^{\varrho-j_1}\, \bar{c} \quad \text{see (16)} \tag{44}$$

$D^{\varrho-j_1} d_{27}$: $$- \frac{1}{I_2} D^{\varrho-j_1}\, c\text{: see (16)} \tag{45}$$

$D^{\varrho-j_1} d_{31}$: $$D^{\varrho-j_1} d_{31} = - D^{\varrho-j_1} d_{14} \tag{46}$$

$D^{\varrho-j_1} d_{32}$: $$D^{\varrho-j_1} \frac{1}{b} + B_3 D^{\varrho-j_1}(bc^2) - B_3 D^{\varrho-j_1}(b\bar{c}^2) \tag{47}$$

$D^{\varrho-j_1} \frac{1}{b}$: see (20)

$$D^{\varrho-j_1}(bc^2) = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3} b D^{\varrho-j_1-j_3}\, c^2 \tag{48}$$

$D^{\varrho-j_1} d_{17}$:

$$D^{\varrho-j_1} d_{17} = \frac{1}{I_2} D^{\varrho-j_1} \frac{\bar{c}}{b} \tag{34}$$

$$D^{\varrho-j_1} \frac{\bar{c}}{b} = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3} \bar{c} \, D^{\varrho-j_1-j_3} \frac{1}{b} \tag{35}$$

$D^{j_3} \bar{c}$: see (16)

$D^{\varrho-j_1-j_3} \frac{1}{b}$: see (20)

$D^{\varrho-j_1} d_{21}$:

$$D^{\varrho-j_1} d_{21} = - D^{\varrho-j_1} b - B_1 D^{\varrho-j_1} b\bar{c}^2 + B_2 D^{\varrho-j_1} bc^2 \tag{36}$$

$D^{\varrho-j_1} b$: see (21)

$$D^{\varrho-j_1} b\bar{c}^2 = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3} b \, D^{\varrho-j_1-j_3} \bar{c}^2 \tag{37}$$

$D^{j_3} b$: see (21)

$D^{\varrho-j_1-j_3} \bar{c}^2$: see (24)

$$D^{\varrho-j_1}(bc^2) = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3} b D^{\varrho-j_1-j_3} c^2 \tag{38}$$

$D^{j_3} b$: see (21)

$D^{\varrho-j_1-j_3} c^2$: see (26)

$D^{\varrho-j_1} d_{22}$:

$$D^{\varrho-j_1} d_{22} = - D^{\varrho-j_1} d_{11} \tag{39}$$

$D^{j_3}\, b$: see (21)

$D^{\varrho-j_1-j_3}\, c^2$: see (26)

$D^{\varrho-j_1}\, d_{33}$:

$$B_2 D^{\varrho-j_1}\, \frac{\bar{b}\bar{c}^2}{b} - B_1 D^{\varrho-j_1}\, \frac{\bar{b}c^2}{b} - D^{\varrho-j_1}\, \frac{\bar{b}}{b} = D^{\varrho-j_1}\, d_{33} \qquad (49)$$

$D^{\varrho-j_1}\, \frac{\bar{b}\bar{c}^2}{b}$: see (23)

$D^{\varrho-j_1}\, \frac{\bar{b}c^2}{b}$: see (25)

$D^{\varrho-j_1}\, \frac{\bar{b}}{b}$: see (19)

$D^{\varrho-j_1}\, d_{34}$:

$$D^{\varrho-j_1}\, d_{34} = -B_3 D^{\varrho-j_1}\, (c\bar{c}b^2) + B_1 D^{\varrho-j_1}(c\bar{c}\bar{b}^2) + B_2 D^{\varrho-j_1}(c\bar{c}\bar{b}^2) \qquad (50)$$

$$D^{\varrho-j_1}(c\bar{c}b^2) = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3}(c\bar{c}) D^{\varrho-j_1-j_3}\, b^2 \qquad (51)$$

$D^{j_3}(c\bar{c})$: see (16)

$$D^{l_1} b^2 = \sum_{l_2}^{l_1} \binom{l_1}{l_2} D^{l_2} b D^{l_1-l_2}\, b \qquad (52)$$

$D^{l_2}\, b$: see (21)

$$D^{\varrho-j_1}\, (c\bar{c}b^2): = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3}\, c\bar{c}\, D^{\varrho-j_1-j_3}\, b^2 \qquad (53)$$

$$D^{\varrho-j_1-j_3} b^2 = \sum_{j_4}^{\varrho-j_1-j_3} \binom{\varrho-j_1-j_3}{j_4} D^{j_4}\, b\, D^{\varrho-j_1-j_3}\, b \qquad (54)$$

$D^{j_4}\, b$: see (21)

$D^{j_3}\, c\bar{c}$: see (15)

$D^{\varrho-j_1} d_{35}$:

$$D^{\varrho-j_1} d_{35} = B_3 D^{\varrho-j_1}(c\bar{c}) \text{ see (15)} \tag{55}$$

$D^{\varrho-j_1} d_{36}$:

$$D^{\varrho-j_1} d_{36} = 0 \tag{56}$$

$D^{\varrho-j_1} d_{37}$:

$$D^{\varrho-j_1} d_{37} = -\frac{1}{I_1} D^{\varrho-j_1} \frac{c\bar{b}}{b} \tag{57}$$

$$D^{\varrho-j_1} \frac{c\bar{b}}{b} = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{j_3} c \, D^{\varrho-j_1-j_3} \frac{\bar{b}}{b} \tag{58}$$

$D^{j_3} c$: see (16)

$D^{\varrho-j_1-j_3} \frac{\bar{b}}{b}$: see (19)

$D^{\varrho-j_1} d_{38}$: $-\frac{1}{I_2} D^{\varrho-j_1} \frac{\bar{c}\bar{b}}{b}$ (59)

$$D^{\varrho-j_1} \frac{\bar{c}\bar{b}}{b} = \sum_{j_3}^{\varrho-j_1} \binom{\varrho-j_1}{j_3} D^{\varrho-j_1-j_3} \bar{c} \, D^{j_3} \frac{\bar{b}}{b} \tag{60}$$

$D^{j_3} \frac{\bar{b}}{b}$: see (19)

$D^{l_1} \bar{c}$: see (16)

The recurrence formulas for $D^{j_1} N_i$ (i=1,2,3) can be derived in analogy to the expressions presented above.

$D^{j_1} N_i$:

$D^{j_1} N_1$:

$$D^{j_1} N_1 = \omega^2 T_{23} D^{j_1}(3a_{32}a_{33} - a_{22}a_{23}) - 2\omega T_2 D^{j_1}(a_{22}\dot{\theta}_3) + \\ + 2\omega T_3 D^{j_1}(a_{23}\dot{\theta}_2) \tag{61}$$

$$D^{j_1}(a_{32}a_{33}) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3}a_{32}\, D^{j_1-j_3}\, a_{33} \tag{62}$$

$$D^{j_3}a_{32} = D^{j_3}(\bar{c}b) = \sum_{j_4}^{j_3} \binom{j_3}{j_4} D^{j_4}\, \bar{c}\, D^{j_3-j_4}\, b \tag{63}$$

$D^{j_4}\, \bar{c}$: see (16), $D^{j_3-j_4}\, b$: see (21)

$$D^{j_1-j_3}\, a_{33} = D^{j_1-j_3}\, \bar{b}\text{: see (21)} \tag{64}$$

$$D^{j_1}(a_{22}a_{23}) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3}a_{22}\, D^{j_1-j_3}\, a_{23} \tag{65}$$

$$D^{j_3}\, a_{22} = D^{j_3}(\bar{c}a + c\bar{a}\bar{b}) = D^{j_3}(\bar{c}a) + D^{j_3}(c\bar{a}\bar{b}) \tag{66}$$

$$D^{j_3}(\bar{c}a) = \sum_{j_4}^{j_3} \binom{j_3}{j_4} D^{j_4}\, \bar{c}\, D^{j_3-j_4}\, a \tag{67}$$

$D^{j_4}\, \bar{c}$: see (16), $D^{j_3-j_4}\, a = D^{l_1}\, a =$

$$= D^{l_1-1}(\bar{a}z_4) = \sum_{l_2}^{l_1-1} \binom{l_1-1}{l_2} D^{l_2}\, \bar{a} D^{l_1-1-l_2}\, z_4 \tag{68}$$

$$D^{l_1}\, \bar{a} = -\, D^{l_1-1}(az_4) = -\sum_{l_2}^{l_1-1} \binom{l_1-1}{l_2} D^{l_2} a D^{l_1-1-l_2} z_4 \tag{69}$$

$$D^{j_3}(\bar{a}\bar{b}c) = \sum_{j_4}^{j_3} \binom{j_3}{j_4} D^{j_4}(\bar{a}\bar{b}) D^{j_3-j_4}\, c \tag{70}$$

$D^{j_3-j_4}\, c$: see (16); $D^{j_4}(\bar{a}\bar{b}) = \sum_{j_5}^{j_4} \binom{j_4}{j_5} D^{j_5}\, \bar{a} D^{j_4-j_5} \bar{b}$ (71)

$D^{j_5}\, \bar{a}$: see (69,70), $D^{j_4-j_5}\, \bar{b}$: see (21) (72)

$$D^{j_2-j_3}\, a_{23} = -\, D^{j_2-j_3}\, \bar{a}b = -\sum_{j_4}^{j_2-j_3} \binom{j_2-j_3}{j_4} D^{j_4}\bar{a}\, D^{j_2-j_3-j_4} b \tag{73}$$

$D^{j_4}\, \bar{a}$: see (69,70); $D^{j_2-j_3-j_4}\, b$: see (21) (74)

$$D^{j_1}(a_{22}\theta_3) = \sum_{j_3}^{j_1} \binom{j_1}{j_3}\, D^{j_3}\, a_{22}\, D^{j_1-j_3}\, \theta_3 \tag{75}$$

$D^{j_3}\, a_{22}$: see (65) (76)

$$D^{j_1-j_3}\, \theta_3 = D^{j_1-j_3}(z_4\bar{b} + z_6) =$$

$$= \sum_{j_4}^{j_1-j_3} \binom{j_1-j_3}{j_4} D^{j_4} z_4 D^{j_1-j_3-j_4}\, \bar{b} + D^{j_1-j_3} z_6 \tag{77}$$

$D^{j_1-j_3-j_4}\bar{b}$: see (21); $j_4 \leq \varrho$, $j_1-j_3 \leq \varrho$ (78)

$$D^{j_1}(a_{23}\theta) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3} a_{23} D^{j_1-j_3}\, \theta_2 \tag{79}$$

$D^{j_3}\, a_{23}$: see (13) (80)

$$D^{j_1-j_3}\, \theta_2 = D^{j_1-j_3}(z_4\bar{c}b - z_5 c) =$$

$$= \sum_{j_4}^{j_1-j_3} \binom{j_1-j_3}{j_4} \left\{ D^{j_1-j_3}(z_4\bar{c}b) + D^{j_1-j_3}(z_5 c) \right\} \tag{81}$$

$$D^{j_1-j_3}(z_4\bar{c}b) = \sum_{j_5}^{j_1-j_3} \binom{j_1-j_3}{j_5} D^{j_5}(\bar{c}b) D^{j_1-j_3-j_5}\, z_4 \tag{82}$$

$j_1-j_3-j_5 \leq \varrho$

$$D^{j_5}(\bar{c}b) = \sum_{j_6}^{j_5} \binom{j_5}{j_6} D^{j_6}\, \bar{c}\, D^{j_5-j_6}\, b \tag{83}$$

$$D^{j_6}\,\bar{c}\text{: see (16); } D^{j_5-j_6}\,b\text{: see (21)} \tag{84}$$

$$D^{j_1-j_3}(z_5 c) = \sum_{j_4}^{j_1-j_3} \binom{j_1-j_3}{j_4} D^{j_4}\, z_5\, D^{j_1-j_3-j_4}\, c \tag{85}$$

$$D^{j_4}\, c\text{: see (16)} \tag{86}$$

$$j_4 \leq \varrho$$

$D^{j_1}N_2$:

$$D^{j_1}N_2 = D^{j_1}\Big\{(3a_{31}a_{33} - a_{21}a_{23})\,\omega^2 T_{31} + 2\omega(T_1 a_{21}\theta_1 - T_3 a_{23}\theta_3)\Big\} \tag{87}$$

$$D^{j_1}(a_{31}a_{33}) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3}\, a_{31}\, D^{j_1-j_3}\, a_{33} \tag{88}$$

$$D^{j_3}\, a_{31} = D^{j_3}(cb) = \sum_{j_4}^{j_3} \binom{j_3}{j_4} D^{j_4}\, c\, D^{j_3-j_4}\, b \tag{89}$$

$$D^{j_4}\, c\text{: see (16); } D^{j_3-j_4}\, b\text{: see (21)} \tag{90}$$

$$D^{j_1-j_3}\, a_{33}\text{: see (64)} \tag{91}$$

$$D^{j_1}(a_{22}a_{23}) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3}\, a_{22}\, D^{j_1-j_3}\, a_{23} \tag{92}$$

$$D^{j_1-j_3}\, a_{23}\text{: see (73)} \tag{93}$$

$$D^{j_3}\, a_{22}\text{: see (66)} \tag{94}$$

$$D^{j_3}(a_{21}\theta_3) = \sum_{j_4}^{j_3} \binom{j_3}{j_4} D^{j_4} a_{21} D^{j_3-j_4}\, \theta_3 \tag{95}$$

$$D^{j_3-j_4}\, \theta_3\text{: see (77)} \tag{96}$$

$$D^{j_4} a_{21} = D^{j_4}(\bar{c}a + c\bar{a}\bar{b}) =$$

$$= \sum_{j_5}^{j_4} \binom{j_4}{j_5} \left\{ D^{j_4} (\bar{c}a) + D^{j_4} (c\bar{a}\bar{b}) \right\} \tag{97}$$

$$D^{j_4}(\bar{c}a)\text{: see (67)}, \quad D^{j_4}(c\bar{a}\bar{b})\text{: see (68)} \tag{98}$$

$$D^{j_1}(a_{23}\theta_1) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3} a_{23} D^{j_1-j_3} \theta_1 \tag{99}$$

$$D^{j_3} a_{23}\text{: see (73)} \tag{100}$$

$$D^{j_1-j_3} \theta_1 = D^{j_1-j_3}(z_4 cb + z_5\bar{c}) =$$

$$= D^{j_1-j_3}(z_4 cb) + D^{j_1-j_3}(z_5\bar{c}) \tag{101}$$

$$D^{j_1-j_3}(z_4 cb) = \sum_{j_4}^{j_1-j_3} \binom{j_1-j_3}{j_4} D^{j_1-j_3} z_4 D^{j_1-j_3-j_4}(cb) \tag{102}$$

$$j_1-j_3 \leq \varrho$$

$$D^{j_1-j_3-j_4}(cb) = D^{l_1}(cb) = \sum_{l_2}^{l_1} \binom{l_1}{l_2} D^{l_2} c D^{l_1-l_2} b \tag{103}$$

$$D^{l_2} c\text{: see (16)}; \quad D^{l_1-l_2} b\text{: see (21)} \tag{104}$$

$$D^{j_1-j_3}(z_5\bar{c}) = \sum_{j_4}^{j_1-j_3} \binom{j_1-j_3}{j_4} D^{j_4} z_5 D^{j_1-j_3-j_4} \bar{c} \tag{105}$$

$$j_4 \leq \varrho$$

$$D^{j_1-j_3-j_4} \bar{c}\text{: see (16)} \tag{106}$$

$D^{j_1} N_3$:

$$D^{j_1} N_3 = D^{j_1} \Big\{ (3a_{31}a_{32} - a_{21}a_{22})\,\omega^2 T_{12} +$$

$$+ 2\omega(-T_1 a_{21}\dot{\theta}_2 + T_2 a_{22}\dot{\theta}_1) \Big\} \tag{107}$$

$$D^{j_1}(a_{31}a_{32}) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3} a_{31} D^{j_1-j_3} a_{32} \tag{108}$$

$$D^{j_3} a_{31}: \text{ see } (89); \quad D^{j_1-j_3} a_{32}: \text{ see } (63) \tag{109}$$

$$D^{j_1}(a_{21}a_{22}) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3} a_{21} D^{j_1-j_2} a_{22} \tag{110}$$

$$D^{j_3} a_{21}: \text{ see } (97); \quad D^{j_1-j_2} a_{22}: \text{ see } (65) \tag{111}$$

$$D^{j_1}(a_{21}\theta_2) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3} a_{21} D^{j_1-j_3} \theta_2 \tag{112}$$

$$D^{j_3} a_{21}: \text{ see } (97); \quad D^{j_1-j_3} \theta_2: \text{ see } (81) \tag{113}$$

$$D^{j_1} (a_{22} \theta_1) = \sum_{j_3}^{j_1} \binom{j_1}{j_3} D^{j_3} a_{22} D^{j_1-j_3} \theta_1 \tag{114}$$

$$D^{j_3} a_{22}: \text{ see } (66); \quad D^{j_1-j_3} \theta_1: \text{ see } (101) \tag{115}$$

The above recurrence formulas are identical with those of the circular orbit. In the case of elliptic orbits additional recurrence formulas have to be derived for the expression $\Gamma^{j_1} N_{i1}$:$(i = 1,2,3)$

In this case, essentially for the following expressions recurrence formulas have to be derived:

$$D^{j_1}(a_{ik}\mathring{a}_{ln}) \quad \text{and} \quad D^{j_1} a_{ik}^2,$$

where i, k, l, n = 1, 2, 3

$\underline{D^{j_1} a_{ik}^2}$:

$$D^{j_1} a_{ik}^2 = \sum_{j_2}^{j_1} \binom{j_1}{j_2} D^{j_2} a_{ik} \, D^{j_1-j_2} a_{ik};$$

for a_{ik}, however, recurrence formulas have already been derived.

$\underline{D^{j}(a_{ik}\mathring{a}_{ln})}$:

$$D^{j}(a_{ik}\mathring{a}_{ln}) = \sum_{j_2}^{j_1} \binom{j_1}{j_2} D^{j_2} a_{ik} \, D^{j_1-j_2} a_{ln}$$

$\underline{D^{j_3} a_{ln}}$:

$$\underline{D^{j_3} a_{11}} = D^{j_3}(-c\bar{a}z_6 - \bar{c}az_4 - \bar{c}a\bar{b}z_6 - c\bar{a}\bar{b}z_4 + cabz_5) =$$

$$= -D^{j_3}(c\bar{a}z_6) - D^{j_3}(\bar{c}az_4) - D^{j_3}(\bar{c}a\bar{b}z_6) -$$

$$- D^{j_3}(c\bar{a}\bar{b}z_4) + D^{j_3}(cabz_5) \tag{116}$$

$$D^{j_3}(c\bar{a}z_6) = \sum_{j_4}^{j_3} \binom{j_3}{j_4} D^{j_4}(c\bar{a}) \, D^{j_3-j_4} z_6 \tag{117}$$

$$D^{j_4}(c\bar{a}) = \sum_{j_5}^{j_4} \binom{j_4}{j_5} D^{j_5} c \, D^{j_4-j_5} \bar{a} \tag{118}$$

$D^{j_5} c$: see (16)

$D^{j_4-j_5}\,\bar{a}$: see (69)

$$D^{j_3}\,(\bar{c}az_4) = \sum_{j_4}^{j_3} \binom{j_3}{j_4}\, D^{j_4}\,(\bar{c}a)\, D^{j_3-j_4}\, z_4; \tag{119}$$

$$D^{j_4}\,(\bar{c}a) = \sum_{j_5}^{j_4} \binom{j_4}{j_5}\, D^{j_5}\,\bar{c}\, D^{j_4-j_5}\, a \tag{120}$$

$D^{j_5}\,\bar{c}$: see (16)

$D^{j_4-j_5}\,a$: see (68)

$$D^{j_3}(\bar{c}a\bar{b}z_6) = \sum_{j_4}^{j_3} \binom{j_3}{j_4}\, D^{j_4}\,(\bar{c}a)\, D^{j_3-j_4}(\bar{b}z_6) \tag{121}$$

$$D^{j_4}(\bar{c}a) = \sum_{j_5}^{j_4} \binom{j_4}{j_5}\, D^{j_5}\bar{c}\, D^{j_4-j_5}\, a \tag{122}$$

$D^{j_5}\bar{c}$: see (16)

$D^{j_4-j_5}\,a$: see (68)

$$D^{j_3-j_4}\,(\bar{b}z_6) = \sum_{j_5}^{j_3-j_4} \binom{j_3-j_4}{j_5}\, D^{j_5}\,\bar{b}\, D^{j_3-j_4-j_5}\, z_6 \tag{123}$$

$D^{j_5}\bar{b}$: see (21) a.s.o.

In analogy to $D^{j_3}\dot{a}_{11}$ recurrence formulas for $D^{j_3}\,a_{ln}$ $(l,\ n = 1,2,3)$ can be derived.

Conclusion.

In the present calculations we neglected the contributions of F_2 (drag force), F_3 (force caused by radiation pressure), F_4 and F_5 (forces caused by the gravitational field of the sun and the moon, respectively) and some additional forces, which may influence the motion of the satellite (magnetic field, eddy current, e.t.c.). If F_2 and F_3 are holomorphic, recurrence formulas can also be derived for these functions(under certain conditions at least). The forces F_4 and F_5 can be treated in analogy to the force F_{ge}.

As mentioned in Rep.14, possibly recurrence formulas are not the most useful tool for numerical computations. We have, however, successfully used such formulas for various problems /1-4, 6-7/, but there are hints that another iterative method (developed by H.Knapp /5/) is more advantageous if high accuracy is required.

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