BUBBLE FORMATION AND GROWTH

Study of the Boundary Conditions at a Liquid-Vapor Interface through Irreversible Thermodynamics

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ABSTRACT

During the past year an experiment which involved the steady-state evaporation of mercury was performed to measure the principal transport coefficient σ appearing in the mass-rate equation and the coupling transport coefficient U appearing in both the mass-rate equation and the energy-rate equation. As reported in the February 1966-1967 final report under Contract Number NAS 8-20013, the experimental value of the coupling coefficient U did not agree with the value predicted from Schrage's kinetic analysis of the phase change. Presented here is a modified kinetic analysis in which the Onsager reciprocal law and the conservation laws are invoked. This analysis removes the discrepancy which previously existed between experiment and theory for the value of U.

In an attempt to obtain another experimental measurement for σ during evaporation, preliminary design work and experiments have been performed to develop a technique to measure bubble growth rates at low pressures.

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LIST OF SYMBOLS

Symbols

f	velocity distribution function - see Equation (I-2-1)
h	specific enthalpy
h_{fg}	$h_{fg} \equiv h_g - h_f$
J_i	mass flux
J_m	momentum flux
J_s	$J_s \equiv P/\sqrt{2\pi RT}$
J_u	energy flux
K	transport coefficient
k	thermal conductivity
L_{ii}	transport coefficient
L_k	transport coefficient
P	pressure
q/A	heat transfer per unit area per unit time
Q	fluxes over and above that calculated from integrating half maxwellian velocity distributions - see Section I
R	gas constant
T	temperature
U	see Equation (I-3-11)
X	distance
φ	energy accommodation coefficient
δP	$\delta P \equiv P_g - P_s\ (T_{fi})$
δT	$\delta T \equiv T_{gi} - T_{fi}$
δT_k	δT as defined by Kennard
γ	specific heat ratio c_p/c_v
σ	condensation or evaporation coefficient

Subscripts

f	liquid
g	vapor
i	liquid-vapor interface
s	saturation condition
ss	steady state
w	wall
q', q''	see Figure 3

I. ANALYTICAL CONSIDERATION OF THE KINETICS AND THERMODYNAMICS OF THE INTERFACE

I-1. Introduction

In this section the kinetic analysis of the phase change process is studied. A method of analysis is presented which is consistent with the conservation laws, Onsager's reciprocal law, and experiment.

Before proceeding with the analysis, it is necessary to explain two different methods of describing the kinetics in the vapor adjacent to the interface. In the first method, termed method A, velocity distributions are assigned to the molecules crossing a control surface. The net fluxes of mass, momentum, and energy across the control surface are calculated by integrating the distribution functions. This method thus requires that the velocity distributions employed be exact. This is the method of description used by Schrage.

In the second method, termed method B, we do not attempt to assign the exact distributions; instead we use approximate distributions. In general, the fluxes obtained from integration of these distributions are in error. To account for this error, we allow for additional fluxes of mass, momentum, and energy. We term these additional fluxes Q_i, Q_m, and Q_u. Method B is best explained by considering the familiar situation of a temperature gradient dT/dx in a semiperfect monatomic gas moving at a low Mach number. At a control surface in the gas, we consider the molecules to be described by a maxwellian distribution with a superimposed bulk flow velocity. Integration of this distribution yields the enthalpy flux $h_g J_i$. The net energy flux is greater than this, however, by the amount

$$Q_u = - k \frac{dT}{dx}$$

where k is the thermal conductivity of the gas. This example illustrates that we do not attempt to guess the exact velocity distribution. Instead we assume an approximate distribution and account for the difference between it and the real distribution by means of a conductivity term.

I-2. Conservation of Momentum Considerations

Relationship Between the Parameters Defining the Velocity Distribution

In this section the kinetics of the vapor adjacent to the liquid surface is described by method A. Let the velocity distribution of the

molecules incident on the liquid surface be denoted by f_2; the molecules emitted from the liquid surface, by f_3; and the molecules reflected from the surface, by f_4. Let the distribution functions be half maxwellian characterized by the parameters P_j and T_j

$$f_j = \frac{P_j}{m\,RT_j(2\pi RT_j^{3/2})}\, e^{-\frac{1}{2\,RT_j}(u^2+v^2+w^2)} \quad , \qquad \text{(I-2-1)}$$

where m is the weight of a molecule, u is the molecular velocity perpendicular to the interface, and v and w are the mutually perpendicular velocities in a plane perpendicular to u (Figure 1). By a half maxwellian it is meant that for the distribution f_2, $-\infty \leq u \leq 0$ and for f_3 and f_4, $0 \leq u \leq \infty$. For all distributions $-\infty \leq v \leq \infty$ and $-\infty \leq w \leq \infty$. Assume that P_3 and T_3 are given by

$$T_3 = T_{fi}$$

$$P_3 = P_s(T_{fi}) \quad . \qquad \text{(I-2-2)}$$

With the above assumptions and by invoking the conservation of momentum equation, we shall proceed to find a relationship between the parameters P_j and T_j.

Let J_{mj} represent the momentum flux of the distribution f_j. The conservation of momentum requires that

$$J_{m2} + J_{m3} + J_{m4} = J_{m1} \qquad \text{(I-2-3)}$$

where the subscript 1 denotes the bulk flow region of the vapor. For a half maxwellian velocity distribution, it can be shown that

$$J_{mj} = J_{ij}\sqrt{\frac{\pi RT_j}{2}} \qquad \text{(I-2-4)}$$

and

$$J_{ij} = \frac{P_j}{\sqrt{2\pi RT_j}} \quad . \qquad \text{(I-2-5)}$$

The temperature T_4 of the reflected velocity distribution is related to the temperature of the incident distribution T_2 and the liquid temperature T_{fi} by the energy accommodation coefficient α

$$\frac{T_4}{T_2} = (1 - \alpha) + \alpha \left(\frac{T_{fi}}{T_2}\right) \quad . \tag{I-2-6}$$

Substituting (I-2-4), (I-2-5), and (I-2-6) into (I-2-3) and using σ to relate J_{i4} to J_{i2}

$$J_{i4} = (1 - \sigma)\, J_{i2} \quad , \tag{I-2-7}$$

the following is obtained

$$\frac{1}{2} P_2 + \frac{\sigma}{2} P_s + \frac{1}{2} (1 - \sigma)\, P_2 \left[(1 - \alpha) + \left(\frac{T_{fi}}{T_2}\right) \right]^{1/2} = J_{m1} \quad . \tag{I-2-8}$$

The quantities δP^* and δT^* are defined as

$$\delta P^* \equiv P_2 - P_s \tag{I-2-9}$$

$$\delta T^* \equiv T_2 - T_s \quad . \tag{I-2-10}$$

For situations where a linear analysis is valid, that is where terms such as

$$\left(\frac{\delta P}{P}\right)^2 << \left(\frac{\delta P}{P}\right) \tag{I-2-11}$$

and

$$\left(\frac{\delta P^*}{P}\right) \left(\frac{\delta P}{P}\right) << \left(\frac{\delta P}{P}\right) \quad , \tag{I-2-12}$$

J_{m1} is given by

$$J_{m1} = P_2 \quad . \tag{I-2-13}$$

Also Equation (I-2-3) reduces to

$$\frac{\delta P^*}{P} = \frac{2}{2-\sigma}\ \frac{\delta P}{P} - \left(\frac{\sigma-1}{2-\sigma}\right)\alpha\ \frac{\delta T^*}{2T} \quad . \qquad \text{(I-2-14)}$$

Experimental values of σ equal to unity have been observed, and for such a case Equation (I-2-14) reduces to

$$\frac{\delta P^*}{P} = 2\,\frac{\delta P}{P} \qquad \text{(I-2-15)}$$

Momentum Considerations in Schrage's Model

We shall now proceed to show that the velocity distributions assumed by Schrage do not, in general, satisfy the momentum equation. Schrage incorporates a bulk velocity V in the distribution f_2. When $J_i = 0$ the bulk velocity V is also zero, and his distributions are all half maxwellian. Equation (I-2-15) must, therefore, be satisfied. This requires that

$$P_2 = P_g + \delta P \ . \qquad \text{(I-2-16)}$$

Schrage, however, assigns the pressure P_g to the distribution f_2.

I-3. Use of Half Maxwellian Velocity Distributions to Completely Describe the Interface

In this section method A is used to describe the kinetics of the interface. The velocity distributions used in this section are the same as those of Section I-2. The parameters T_2 and P_2 are determined by requiring the conservation laws and the Onsager relation to be satisfied. For simplicity, and without loss of generality, σ is taken to be unity. Thus there are no reflected molecules (Figure 1). The linear assumptions of Equations (I-2-11 and 12) are made. It will be shown that this method of describing the interface in terms of half maxwellian velocity distributions yields a solution which is not in accord with experiment.

Conservation of Momentum

Since the distribution functions are assumed to be half maxwellian, the results of Section I-2 may be used. For $\sigma = 1$ Equation (I-2-15) requires that

$$P_2 = P_g + 2(\delta P) \ . \qquad \text{(I-3-1)}$$

Conservation of Energy

Let J_{uj} represent the energy flux of the distribution function f_j. The conservation of energy applied to Figure 1 requires that

$$J_{u3} - J_{u2} = J_{u1} \quad . \tag{I-3-2}$$

For a half maxwellian distribution, the energy flux is given by

$$J_{uj} = 2\, RT_j\, J_{ij} \quad . \tag{I-3-3}$$

The energy flux J_{u1} in the bulk-flow region of the vapor is given by

$$J_{u1} = h_g J_i - (q/A)_g \quad . \tag{I-3-4}$$

On using

$$h_g = \frac{5}{2}\, RT_g \, , \tag{I-3-5}$$

Equation (I-3-4) becomes

$$J_{u1} = \frac{5}{2}\, RT\, J_i - (q/A)_g \quad . \tag{I-3-6}$$

Substitution of Equations (I-3-6) and (I-3-3) into Equation (I-3-2) yields

$$2\, RT_{fi}\, J_{i3} - 2RT_2\, J_{i2} = \frac{5}{2}\, RT\, J_i - (q/A)_g \tag{I-3-7}$$

The expression for the heat transfer in the vapor in terms of the transport coefficients K and L_k is given by (see Equation (I-5-7) in final report for February 1966-1967)

$$(q/A)_g = \frac{K}{K + 1}\, h_{fg}\, J_i - L_k\, \delta T_k \frac{1}{T^2} \quad . \tag{I-3-8}$$

By using Equations (I-3-7) and (I-3-8), an expression relating T_2 to J_i and δT_k may be obtained

$$\frac{\Delta T_2}{2T} = \frac{\delta T_k}{T} - \frac{(1 - U)}{8} \frac{J_i}{J_s} \quad , \tag{I-3-9}$$

where ΔT_2 is related to T_2 by

$$\Delta T_2 = T_2 - T_{fi} \quad , \tag{I-3-10}$$

and where

$$U \equiv \frac{K}{K + 1} \frac{2 h_{fg}}{RT} \tag{I-3-11}$$

is a property which is determined from the steady-state evaporation experiment.

Kinetic Expression for J_i

The conservation of mass as applied to Figure 1 requires that

$$J_i = J_{i3} - J_{i2} \tag{I-3-12}$$

which may be rewritten as

$$\frac{J_i}{J_s} = - 2 \frac{\delta P}{P} + \frac{\Delta T_2}{2T} \quad . \tag{I-3-13}$$

This is accomplished by substituting Equation (I-2-5) for J_{ij} and replacing T_2 by ΔT_2. On using Equation (I-3-9) to eliminate ΔT_2 from (I-3-13), the following is obtained

$$J_i = J_s \frac{16}{9 - U} \left[- \frac{\delta P}{P} + \frac{\delta T_k}{2T} \right] \quad . \tag{I-3-14}$$

Thermodynamic Expression for J_i

The thermodynamic equation for J_i may be rewritten in terms of U as

$$J_i = RL_{ii} \left[- \frac{\delta P}{P} + U \frac{\delta T_k}{2T} \right] \quad . \tag{I-3-15}$$

The coefficient RL_{ii} resulting from the above kinetic model is found by considering the special case when $\delta T_k = 0$.

$$RL_{ii} = J_s \frac{16}{9 - U} \quad . \qquad \text{(I-3-16)}$$

Discussion of the Analysis

If the above analysis is to be consistent, then the value of U must be unity as can be readily seen by comparing the coefficients of $\frac{\delta T_R}{2T}$ in Equations (I-3-14) and (I-3-15). This analysis is thus not general enough to allow for the value of U not equal to unity as has been observed in the steady-state evaporation experiment. Conversely, if the experimentally determined values of U other than unity are correct, then the above method of modeling the interface is not correct. In summary we conclude that the interface kinetics cannot be modeled with half maxwellian velocity distributions if the pressure and temperature of the emitted distribution f_3 are assumed to be T_{fi} and $P_s(T_{fi})$.

It is possible, however, to model the interface with half maxwellian distributions if in addition to allowing P_2 and T_2 to be free parameters, the pressure P_3 is also allowed to be a free parameter (different than P_s). When this is done the experimental results require that P_2 be greater than P_s. This requires that the rate of emission during evaporation be greater than that at equilibrium. This solution is rejected since we expect the emitted flux to be less for nonequilibrium conditions than for equilibrium conditions for the following reason: At equilibrium molecules emitted from the liquid are replenished from the vapor. During nonequilibrium evaporation the replenishment rate is less. This results in a decrease in concentration of molecules capable of being emitted. Thus, if anything, the emission rate is expected to be lower.

I-4. Improved Method of Modeling the Interface

In this section method B is used to describe the kinetics in the vapor adjacent to the interface. The net flux across α' in Figure 1 is given by the superimposed fluxes $Q_{\alpha'}$ in addition to that calculated from integrating the distribution functions f_2, f_3, and f_4. The basic assumption in this analysis is that by altering the boundary conditions $Q_{\alpha'}$ can be forced to be zero and that for this condition the distributions f_2, f_3, and f_4 are half maxwellian at the common temperature T_{fi}. The parameter P_3 is taken to be P_s, and P_2 is determined by requiring that the momentum equation be satisfied. In the following analysis the

subscript $Q_{\alpha'} = 0$ is used to denote this condition. As before, all equations will be linearized.

At the $Q_{\alpha'} = 0$ condition the mass flux $(J_i)_{Q_{\alpha'}=0}$ can be calculated from kinetics since, consistent with the above assumptions, the distribution functions are exactly half maxwellian at this condition. At any other condition ($Q_{\alpha'} \neq 0$), we cannot calculate the mass flux $(J_i)_{Q_{\alpha'}\neq 0}$ because the distribution functions will be perturbed to allow for the $Q_{\alpha'}$. The general relationship for the mass flux may be determined, however, by employing the Onsager reciprocal law. In essence, through the formalism of irreversible thermodynamics, this law relates the effect of the Q perturbation on the mass flux to the Q perturbation on the energy flux. The condition that must be satisfied to have $Q_{\alpha'} = 0$ is related to the experimentally measured quantities as is shown below.

Conservation of Momentum

For the special case of $Q_{\alpha'} = 0$, the momentum equation (I-2-14) requires that

$$(P_2)_{Q_{\alpha'}=0} = P_s + \frac{2}{2-\sigma} (\delta P)_{Q_{\alpha'}=0} \quad . \tag{I-4-1}$$

Kinetic Expression for $(J_i)_{Q_{\alpha'}=0}$

The continuity equation requires that

$$J_i = J_{i3} + J_{i4} - J_{i2} \quad , \tag{I-4-2}$$

and for the assumed distributions, this reduces to

$$\left(\frac{J_i}{J_s}\right)_{Q_{\alpha'}=0} = -\frac{2\sigma}{2-\sigma} \left(\frac{\delta P}{P}\right)_{Q_{\alpha'}=0} \quad . \tag{I-4-3}$$

Thermodynamic Expression for $(J_i)_{Q_{\alpha'}=0}$

The thermodynamic equation for J_i may be rewritten in terms of U for $Q_{\alpha'} = 0$ as

$$(J_i)_{Q_{\alpha'}=0} = RL_{ii} \left[-\left(\frac{\delta P}{P}\right)_{Q_{\alpha'}=0} + U \left(\frac{\delta T_1}{2T}\right)_{Q_{\alpha'}=0} \right] . \tag{I-4-4}$$

This expression for $(J_i)_{Q_{\alpha'}=0}$ must be consistent with the kinetic expression given by Equation (I-4-3); however, before equating them we shall determine $(\delta T_k)_{Q_{\alpha'}=0}$ from energy considerations.

Conservation of Energy

The conservation of energy applied to Figure 1 requires that

$$J_{u4} + J_{u3} - J_{u2} = J_{u1} \qquad \text{(I-4-5)}$$

which for $Q_{\alpha'} = 0$ becomes

$$2\,RT_{fi}(1 - \sigma)\,J_{i2} + 2\,RT_{fi}\,\sigma J_s - 2\,RT_{fi}\,J_{i2} = \frac{5}{2}\,RT_g(J_i)_{Q_{\alpha'}=0} - \left[(q/A)_g\right]_{Q_{\alpha'}=0} . \qquad \text{(I-4-6)}$$

This equation may be linearized to yield

$$\left[(q/A)_g\right]_{Q_{\alpha'}=0} = \frac{1}{2}\,RT\,(J_i)_{Q_{\alpha'}=0} \quad . \qquad \text{(I-4-7)}$$

The heat transfer in the vapor $\left[(Q/A)_g\right]_{Q_{\alpha'}=0}$ is related to the transport coefficients K and L_k by Equation (I-3-8).

$$\left[(q/A)_g\right]_{Q_{\alpha'}=0} = \frac{K}{K+1}\,h_{fg}\,(J_i)_{Q_{\alpha'}=0} + \frac{L_k}{T^2}\,(\delta T_k)_{Q_{\alpha'}=0} \; . \qquad \text{(I-4-8)}$$

Eliminating $\left[(q/A)_g\right]_{Q_{\alpha'}=0}$ from Equations (I-4-7) and (I-4-8) results in the expression for $(\delta T_k)_{Q_{\alpha'}=0}$

$$(\delta T_k)_{Q_{\alpha'}=0} = \frac{1}{2L_k}\;RT^3(J_i)_{Q_{\alpha'}=0}\,(1 - U) \quad . \qquad \text{(I-4-9)}$$

Thus for the perturbation $Q_{\alpha'}$ to vanish, (δT_k) must be related to J_i by Equation (I-4-9). This relationship is in terms of experimentally determined quantities.

Calculation of L_{ii}

Substitution of $(\delta T_k)_{Q_{\alpha'}=0}$ from Equation (I-4-9) into Equation (I-4-4) yields

$$(J_i)_{Q_{\alpha'}=0} = - \left[\frac{1}{RL_{ii}} - \frac{RU(1-U)}{4\,L_k} \right]^{-1} \left(\frac{\delta P}{P}\right)_{Q_{\alpha'}=0} . \qquad \text{(I-4-10)}$$

The coefficient RL_{ii} is obtained by substituting in the equation (see Equation (I-5-5) in final report 1966-1967) for L_k with $\gamma = 1.667$ (monatomic gas) and by equating the expressions for J_i given by Equations (I-4-3) and (I-4-10). The result is

$$RL_{ii} = J_s\, C_\sigma \left[\frac{1}{1 + \frac{C_\sigma U(1-U)}{16}} \right] , \qquad \text{(I-4-11)}$$

where

$$C_\sigma = \frac{2\sigma}{2-\sigma} , \qquad \text{(I-4-12)}$$

Since the experimental value of U is approximately 0.25 and since $0 \leq \sigma \leq 1$, Equation (I-4-11) may be rewritten with sufficient accuracy as

$$RL_{ii} \approx \frac{2\sigma}{2-\sigma}\, J_s , \qquad \text{(I-4-13)}$$

which is the same relationship obtained previously from Schrage's theory.

General Expression for J_i

The general expression for J_i is given by the thermodynamic equation with RL_{ii} given by (I-4-11)

$$J_i = J_s\, C_\sigma \left[\frac{1}{1 + \frac{C_\sigma U(1-U)}{16}} \right] \left[- \frac{\delta P}{P} + U \frac{\delta T_k}{2T} \right] . \qquad \text{(I-4-14)}$$

Discussion of Analysis

It should be pointed out that we do not attempt to calculate the general equation for J_i from kinetic theory; instead we use the thermodynamic equation for J_i. The reason for this is that in the general case when $Q_{\alpha'} \neq 0$, we do not know the exact distribution at α'. The kinetic theory model is used to obtain a relation for L_{ii} (Equation (I-4-11)).

This is obtained by considering the special case $Q_{\alpha'} = 0$. We do not predict a value for U as this would require a knowledge of the exact distribution at α'. Instead we let U take on values consistent with experiment.

It is important to note that the coefficient σ appearing in Schrage's equation is, for all practical purposes, the same as that found in the present analysis; therefore, even though Schrage's model does not yield the correct value of U, it does result in an equation which, in conjunction with experimental results, yields sufficiently accurate values of the condensation or evaporation coefficient.

Alternative Description

In the preceding analysis we have taken δT to be the difference between the temperature defined by the intercept of the vapor temperature profile with the liquid surface and the temperature T_{fi} as shown in Figure 1. This is the same as the temperature difference δT_k used by Kennard. This choice is, however, arbitrary. We could just as well, for example, have chosen the difference between T_{fi} and the intercept at a distance λ from the interface. Though the choice of δT is arbitrary, it is important to be consistent throughout the analysis. When the definition of δT is changed so also do the expressions for L_k and K (or U).

It is of advantage to define δT to be equal to δT_c such that when $Q_{\alpha'} = 0$, δT_c is also equal to zero. With this definition the $Q_{\alpha'} = 0$ experiment is also the isothermal experiment, and as is shown in Appendix A, the general equation for J_i becomes

$$J_i = J_s \frac{2\sigma}{2-\sigma} \left[-\frac{\delta P}{P} + \frac{\delta T_c}{2T} \right] . \qquad \text{(I-4-15)}$$

This equation is identical with the linearized form of Schrage's equation except that the definition, and therefore the value of the δT term, is different. The quantity δT_c as shown in Figure 3 is given by

$$\delta T_c = T_c - T_{fi} \qquad \text{(I-4-16)}$$

where T_c is the value of the intercept of the vapor temperature profile extrapolated into the liquid a distance $c\lambda$ from the interface (λ is

the mean free path in the vapor). The quantity c is related to U by the following expression which is derived in Appendix A.

$$c = \frac{1 - U}{2} \quad . \qquad \text{(I-4-17)}$$

Using the experimental value of U = 0.27, we see that c = 0.365.

II. BUBBLE GROWTH RATES INVESTIGATION

II-1. Introduction

As explained in the final report for 1965-1966, we were able to show that, in evaluating the theoretical bubble growth rates at low pressures, significant errors could arise if one were to retain the conventional assumption of thermodynamic equilibrium at the bubble wall. In other words, for such conditions the nonequilibrium effect must be taken into consideration.

To determine a value of σ and to check the validity of the theory, this work has been initiated to generate the necessary experimental data. It is intended to measure the rate at which bubbles grow in the bulk of superheated water held at pressures of the order of one-tenth of an atmosphere. One apparent advantage of this type of experiment to measure σ is that a clean liquid-vapor interface can be attained.

II-2. Experimental Methods

It is decided to adopt the method used by Dergarabedian, as described in Reference (1), to measure the bubble growth rates. This method has proved successful in producing data which are in excellent agreement with theoretical calculations for bubbles growing at one atmosphere. A special feature of the method is that uniformity of the temperature field in the liquid phase can be approximately attained. This condition is vital to the analytical work.

Experimental Apparatus

It is also decided that the apparatus to be employed for the present investigation will consist of the following major elements (see Fig. 2).

1. TEST SECTION: The test section will be a pyrex glass beaker about 2-1/2 inches in diameter and 6 inches in height. This is to hold the test water which will be degassed prior to its introduction into the test section.

2. VACUUM SYSTEM: A pyrex bell jar, together with a vacuum pump and necessary piping can conveniently be used to provide the required vacuum environment.
3. HEATING ELEMENT: The test section, which is to be positioned inside the bell jar, will be placed between two (2) 250-watt reflector-type infrared lamps. The lamps are to heat the water to the desired superheat temperature. This type of heating process causes the water temperature to rise very slowly and hence eliminates large temperature gradients in the liquid.
4. CAMERA: In order to record rapidly the growing bubble, a Fastex type, or equal, camera capable of taking about 5000 frames per second will be used.

Part of the equipment has already been purchased, and arrangement has been made so that a specialist in film making is to take care of the photographic part of this work. It is believed more economical of time and money to make use of an expert's experience and equipment in doing the job well.

Although the set-up just mentioned is relatively simple, a satisfactory way to initiate the bubble has yet to be found. In the superheated water, the bubbles probably will form at random. In order to minimize wasted effort, it is necessary to have positive control over the time and the place where the bubble is to form. Efforts have been made in searching for the method. The idea is to give the liquid a small localized disturbance at a spot in the bulk of water.

Ultrasound Method of Bubble Generation

The feasibility of using the ultrasound focusing method (2), which is used in neurosurgery research, was investigated. This method, developed partly at the Massachusetts General Hospital, can focus precisely controlled bursts of high intensity ultrasound at specific points in the brain in such a manner that there are minimal or zero effects in surrounding regions. The principle involved is as follows: Radio-frequency power developed by a tuned-plate triode oscillator is applied to a quartz crystal transducer. By the piezoelectric effect of the crystal, pressure waves are generated which then pass through a focusing lens located adjacent to the transducer and concentrate at the desired spot.

A trip was made to the Massachusetts General Hospital to study the existing equipment. Unfortunately, it was found that this method is not suitable for operating conditions involving high temperatures. A mechanical version of the acoustic focusing method then came to mind. A beaker made of a section of pyrex glass tubing with an acoustic focusing lens attached at the bottom was constructed. The lens was placed in contact with the end of an inverted conical wave guide, the apex of which was subjected to an impulsive force. This force generates pressure waves which, after passing through the wave guide, are concentrated by the lens at a particular spot in the beaker. In principle this concentration should disturb the superheated liquid sufficiently to trigger bubble formation. The apparatus was constructed and tested and found to be unsuccessful. In all the test runs a great number of bubbles emerged at places other than the expected one when the external impulsive load was suddenly applied. The idea was thus abandoned. It appears that the high density of the nucleation sites on the container walls more than compensated for the less severe pressure disturbances occurring away from the point at which the sound was focused.

II-3. Planned Method for Initiating Bubbles

Presently another possible means of bubble-triggering is under consideration. This is to use an arrangement similar to a spark plug which is to be immersed in the water. It is hoped that bubbles will form when an electric current is forced through the narrow gap between the two electrodes. The apparatus has been set up, and test runs are currently underway.

III. CONCLUSIONS AND RECOMMENDATIONS

III-1. Concerning the Transport Coefficients

We conclude that the experimental values previously obtained by us for the transport coefficient U are consistent with our theoretical understanding of the kinetics at the phase interface.

For the next quarter we plan to make modifications on the experimental apparatus so that the coefficients U and σ can be measured at higher temperatures.

III-2. Concerning Bubble Growth Experiments

We feel that bubble growth measurements at low pressures will help to give a better understanding of phase changes and in particular give the influence of the evaporation coefficient on bubble growth rates.

For the next quarter we plan to continue the experiment work as described above.

BIBLIOGRAPHY

1. Dergarabedian, P., J. of Appl. Mech., 75, 537-545 (1953).

2. Cosman, B. J. and Hueter, T. F., Electronics, pp. 53-56, issue of May 15, 1959.

3. Kennard, E. H., Kinetic Theory of Gases, McGraw-Hill Book Co., Inc., New York, 1938.

4. Jeans, J., An Introduction to the Kinetic Theory of Gases, Cambridge Press (1962).

5. Hatsopoulos, G. N. and Keenan, J. H., Principles of General Thermodynamics, John Wiley and Sons, Inc., New York, 1965.

APPENDIX A - DEVELOPMENT OF THE ALTERNATIVE DESCRIPTION (SECTION I)

It was pointed out in Section I-4 that there is freedom of choice in the definition of δT. The δT_k chosen in Section I is the same as that used by Kennard.[3] In this Appendix we shall choose δT to be equal to δT_c where δT_c is such that when $Q_{\alpha'}=0$ (Section I-4) δT_c is also equal to zero.

With this choice of $\delta T = \delta T_c$, we proceed to obtain the general equation for J_i in terms of δT_c. This relationship is found by transforming Equation (I-4-14) which is reproduced here.

$$J_i = J_s \; C_\sigma \left[\frac{1}{1 + \frac{C_\sigma U(1 - U)}{16}} \right] \left[- \frac{\delta P}{P} + U \frac{\delta T_k}{2T} \right] . \qquad \text{(A-1)}$$

The transformation is accomplished by relating δT_c to δT_k. Referring to Figure 3 we obtain the following relationship:

$$\delta T_c = \delta T_k - \frac{(q/A)_g}{k_g} \; c\lambda \qquad \text{(A-2)}$$

where λ is the mean free path in the gas, and $c\lambda$ locates the distance from the interface where the extrapolated vapor temperature equals T_c. From the thermodynamic Equation (see Equation (I-5-2) in the final report for 1966-1967), the energy flux is given by

$$J_u = \left(h_g - \frac{K}{K+1} h_{fg}\right) J_i - \frac{L_k}{T^2} \delta T_k . \qquad \text{(A-3)}$$

The energy flux at α'' (Figure 1) is also given by the usual bulk flow relation

$$J_u = h_g J_i - (q/A)_{\alpha''} . \qquad \text{(A-4)}$$

Equations (A-3) and (A-4) may be combined to yield the following for $(q/A)_{\alpha''}$

$$(q/A)_{\alpha''} = \frac{K}{K+1} h_{fg} J_i + L_k \frac{\delta T_k}{T^2} . \qquad \text{(A-5)}$$

Substitution of $(q/A)_{q''}$ from Equation (A-5) into (A-2) and using the definition

$$U \equiv K/(K + 1)\ 2\ h_{fg}/RT$$

to relate K to U results in the following expenses for δT_k

$$\delta T_k = \frac{T_c + \frac{c\lambda}{k_g} U \frac{RT}{2} J_i}{1 - c\left(\frac{L_k/T^2}{k_g/\lambda}\right)} \quad . \tag{A-6}$$

The ratio $(L_k/T^2)/(k_g/\lambda)$ is found to be a constant

$$(L_k/T^2)/(k_g/\lambda) = 2 \quad . \tag{A-7}$$

This result evolves when L_k given by Equation (I-5-5) in the final report 1966-1967 with $\alpha = 1$ and $\gamma = 1.667$ (monatomic gas) and when the conductivity of the vapor phase k_g is given by the kinetic expression for thermal conductivity.[4]

$$k_g = \frac{1}{2} \rho \bar{v} \lambda C_v \quad , \tag{A-8}$$

where $\bar{v}$ is the mean molecular speed, and C_v is the specific heat at constant volume. For the maxwellian velocity distribution, $\bar{v}$ is given by[4]

$$\bar{v} = \sqrt{\frac{8RT}{\pi}} \quad . \tag{A-9}$$

Substitution of Equations (A-6) and (A-7) into (A-1) yields

$$J_i = \Theta J_s C_\sigma \left[- \frac{\delta P}{P} + \frac{U}{1 - 2c} \frac{\delta T_c}{2T} \right] \tag{A-10}$$

where

$$\Theta = \frac{Z}{1 - Z U^2 \left(\frac{c}{1 - 2c}\right) \frac{\lambda}{k} \frac{R}{h} J_s C_\sigma} \tag{A-11}$$

and

$$Z = \frac{1}{1 + \frac{U(1 - U)}{16} C_{\sigma}} \quad . \tag{A-12}$$

The relationship between c and U is found as follows: We first find the relationship between L_k and L_c where L_c is defined as

$$L_c = \frac{(J_u)_{J_i=0} \; T^2}{(\delta T_c)_{J_i=0}} \quad . \tag{A-13}$$

The coefficient L_c is the equivalent of L_k when δT_c is chosen for δT. Since the energy flux

$$(J_u)_{J_i=0} = (q/A)_g \tag{A-14}$$

must be independent of the choice of definition of δT, we may write

$$L_c(\delta T_c)_{J_i=0} = L_k(\delta T_k)_{J_i=0} \quad . \tag{A-15}$$

From Equations (A-15), (A-14), (A-7), and (A-2), the following relationship between L_c and L_k is obtained

$$\frac{L_c}{L_k} = \frac{1}{1 - 2c} \quad . \tag{A-16}$$

The quantity U_c, which is the equivalent of U when δT_c is chosen for δT, is given by

$$U_c = - \frac{2 L_c}{RT^3} \left(\frac{\delta T_c}{J_i}\right)_{ss} \tag{A-17}$$

while the expression for U is given by

$$U = - \frac{2 L_k}{RT^3} \left(\frac{\delta T_k}{J_i}\right)_{ss} \quad . \tag{A-18}$$

By combining Equations (A-17), (A-18), and (A-16) and noting that

$$(\delta T_c)_{ss} = (\delta T_k)_{ss} \quad , \tag{A-19}$$

we obtain

$$\frac{U}{1 - 2c} = U_c \quad . \tag{A-20}$$

Finally we obtain the value of U_c. To do this we consider the $Q_{\alpha'} = 0$ situation which is also the $\delta T_c = 0$ case. The conservation of energy applied to Figure 1 for $Q_{\alpha'} = 0$ requires that

$$\left[(q/A)_g\right]_{\delta T_c=0} = \frac{1}{2} RT(J_i)_{\delta T_c=0} \quad . \tag{A-21}$$

The heat transfer in the vapor for $\delta T_c = 0$ is also given by Equation (1-5-7) in the final report 1966-1967.

$$\left[(q/A)_g\right]_{\delta T_c=0} = \frac{K_c}{K_c + 1} \; h_{fg}(J_i)_{\delta T_c=0} \quad . \tag{A-22}$$

Combining Equations (A-21) and (A-22) yields

$$\frac{K_c}{K_c + 1} \; \frac{2h_{fg}}{RT} = 1 \quad . \tag{A-23}$$

This, however, is the definition of U_c; thus

$$U_c = 1 \quad , \tag{A-24}$$

and from Equation (A-20), we obtain the desired relation between U and c

$$U = 1 - 2c \quad . \tag{A-25}$$

On substituting (A-25) into Equations (A-12), (A-11), and (A-10), we obtain the desired relation for J_i in terms of δT_c

$$J_i = J_s \, e^{\left[-\frac{\delta P}{P} + \frac{\delta T_c}{2T}\right]} \quad . \tag{A-26}$$

It should be pointed out that for the $\delta T_c = 0$ situation, Equation (A-26) reduces to the kinetic Equation (I-4-9) for J_i as it must.

It is noteworthy that the heat of transport q*, which for $\delta T_c = 0$ is given by $\left[(q/A)_g/J_i\right]_{\delta T_c=0}$, has the value $\frac{1}{2}$ RT. This is the same as the heat of transport for a perfect gas flowing through a Knudsen plug.[5]

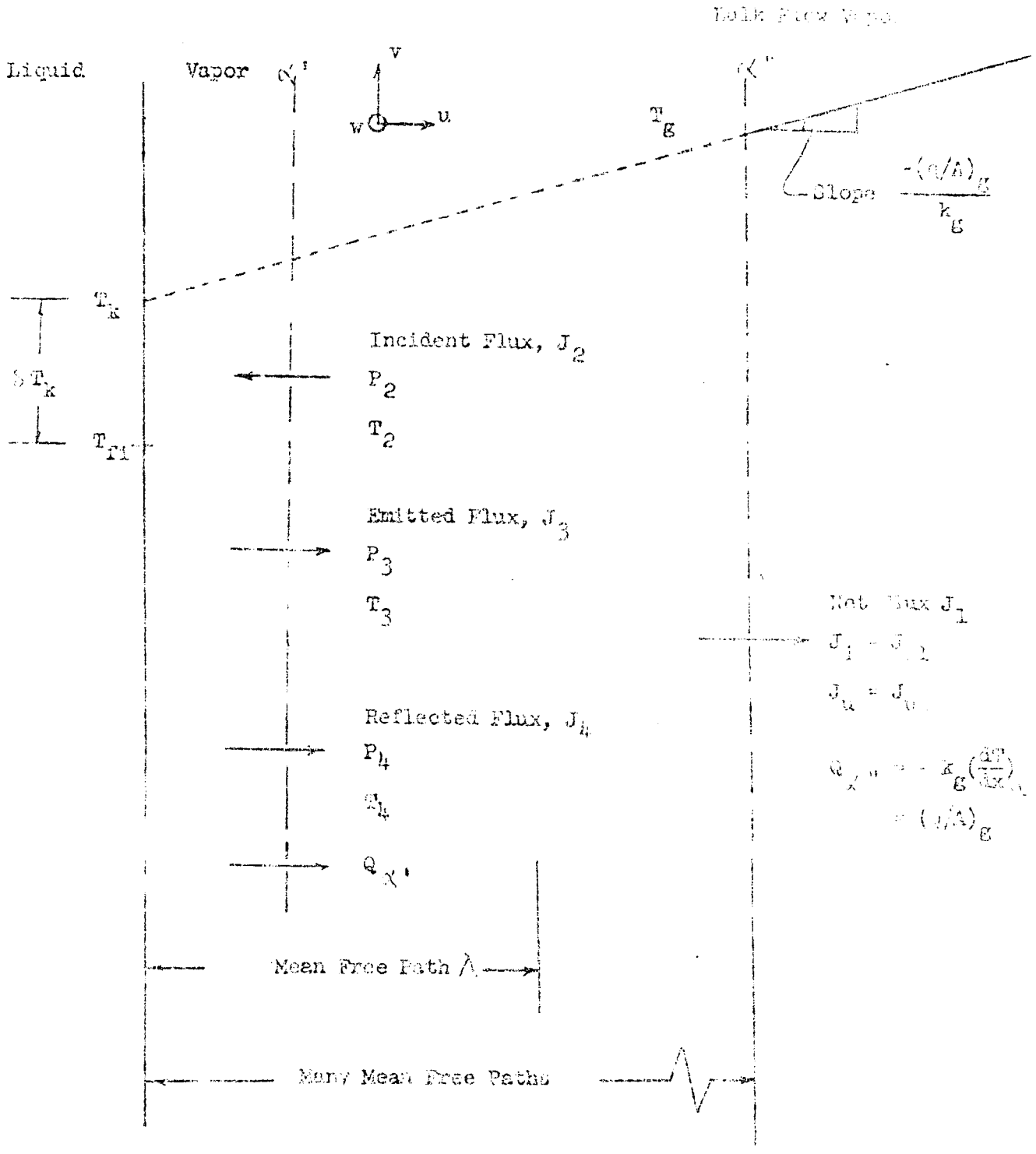

Figure 2 - General Kinetic Model of the Interface

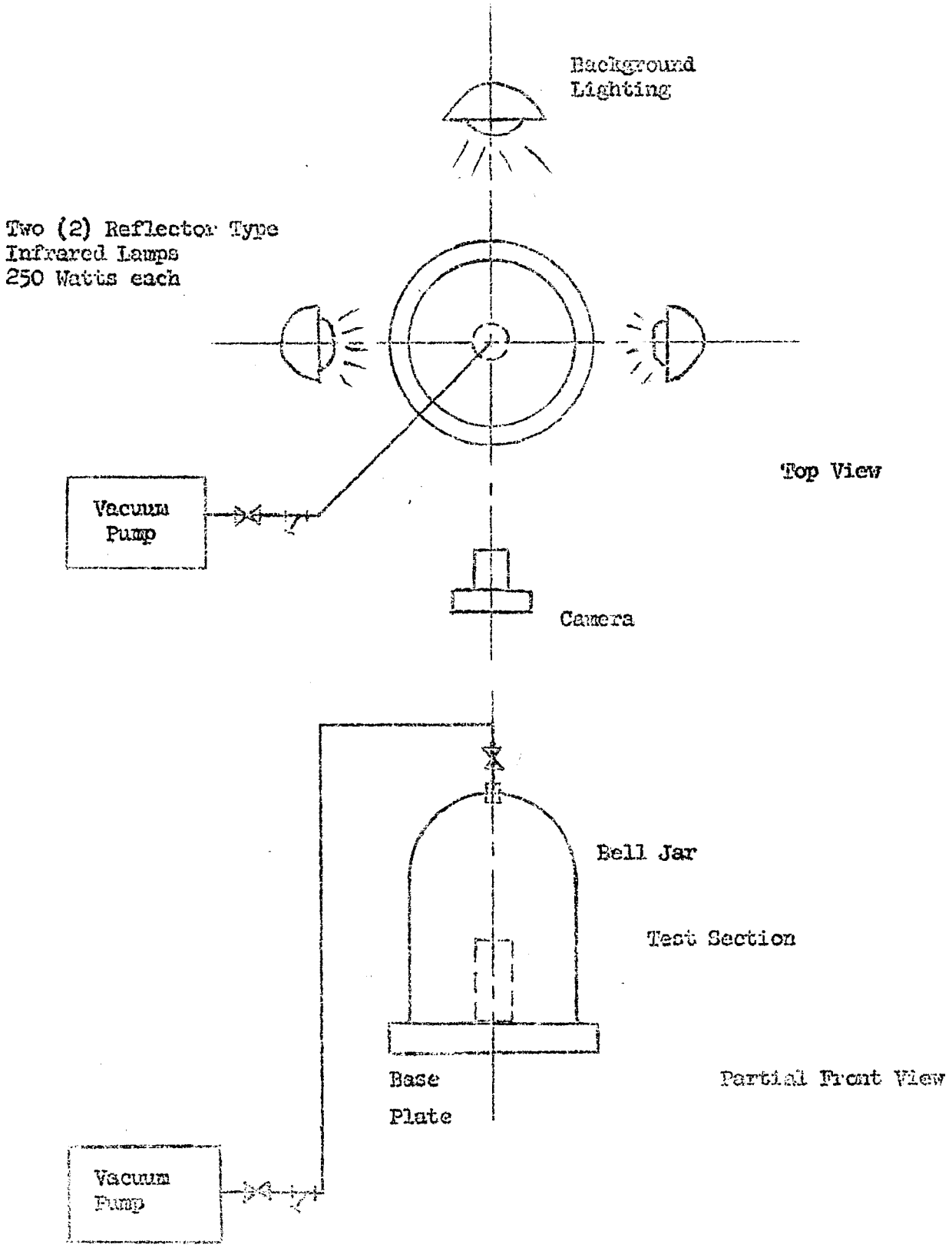

Figure 2 - Preliminary Sketch of the Experimental Set-up
for Bubble Growth Rate Measurements

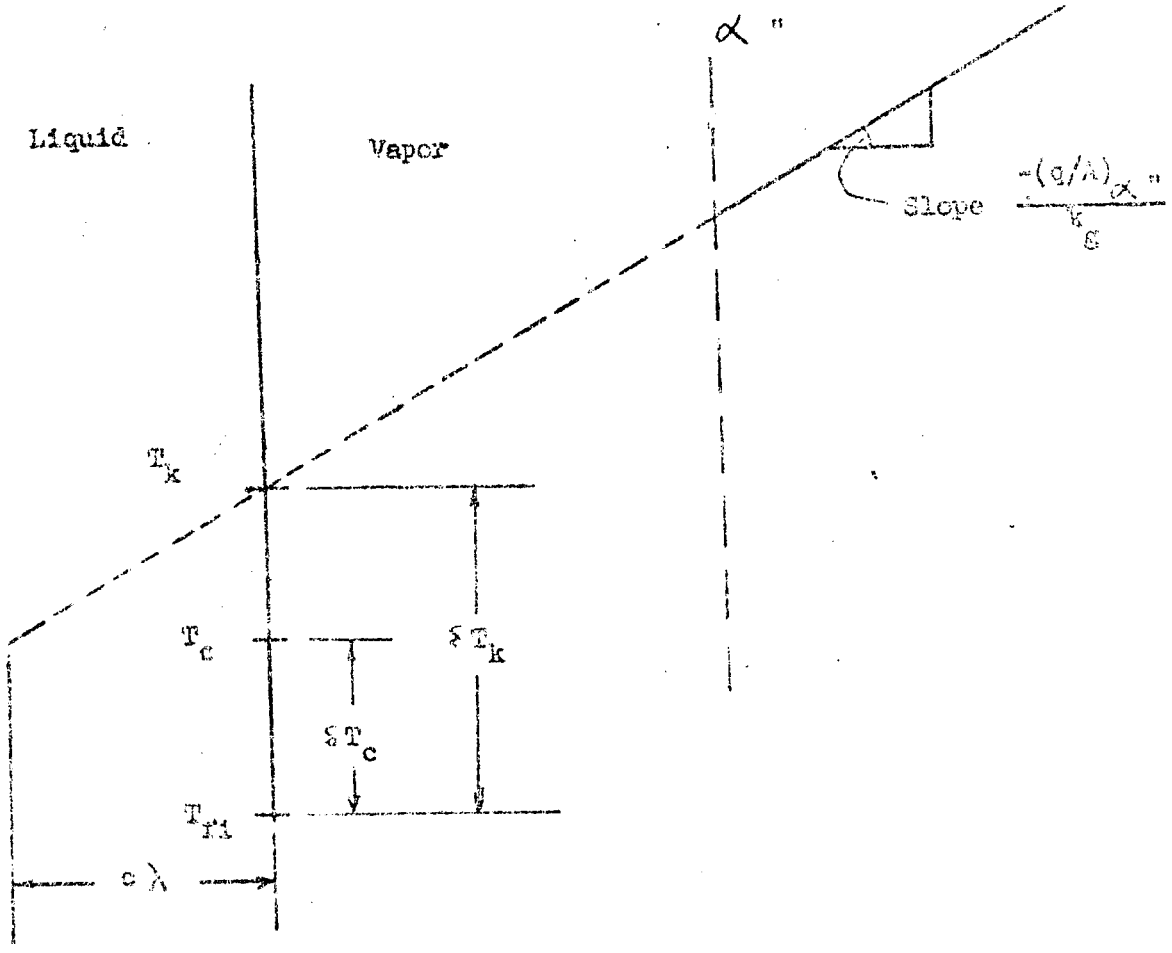

Figure 3 - Illustration of the Relationship Between δT_k and δT_c