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COMPUTER PROGRAMS FOR THE MIXING AND COMBUSTION OF HYDROGEN IN AIR STREAMS

TECHNICAL REPORT 618

By

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Prepared for

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ABSTRACT

This report describes a series of computer programs developed for the mixing and combustion of hydrogen in air streams. These programs were developed under the sponsorship of the National Aeronautics and Space Administration.

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INTRODUCTION

The advent of hydrogen fueled vehicles has necessitated a thorough investigation of the mixing and combustion of hydrogen in air streams. Quantitative understanding of the hydrogen-air mixing and combustion processes is essential in the following areas of practical interest:

a. Combustion chamber design, which is closely related to the jet mixing problem.

b. The venting of cryogenic hydrogen, which involves the problems of two-phase flow and boundary layer flow.

c. The plume problem involving the afterburning of a jet of hot exhaust gases in an air stream.

The numerous analytical studies of the hydrogen-air mixing problem which have been completed are summarized in GASL TR-529

(Ref. 9 of Section A). In general, these studies were based upon progressively more sophisticated mixing and chemical reaction models. Many laminar and turbulent mixing models have been considered. Chemical models considered range from the simple frozen and complete combustion models to the more sophisticated equilibrium and finite-rate chemistry models. The special problem of multiphase flow has also been treated.

The purpose of this report is to describe the various computer programs that have been developed from the previously mentioned analytical studies. Each such program will now be fully discussed as a separate entity.

A. A FINITE DIFFERENCE SOLUTION FOR JETS

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List of Symbols

a	Defined by Eq. 15
C_p	Specific heat
Le	Lewis number
m	ψ index in grid network
n	x index in grid network
P	Pressure
R_o	Universal gas constant
T	Temperature
u	x component of velocity
v	y component of velocity
$\hat{W}_i$	Net rate of generation of Specie i
X	Stream-wise coordinate
y	Radial Coordinate
α	Mass fraction
σ	Prandtl number
μ	Viscosity
ρ	Density
ψ	Stream function

Subscripts

o	Evaluated at stagnation point
e	Evaluated at upper edge of jet
i	Evaluated at lower edge of jet

Superscripts

i	Refers to specie i
~	Refers to element, not specie

INTRODUCTION

The purpose of this section is to describe a finite difference method of solution of the free jet problem for plane two-dimensional and axisymmetric configurations (cf Fig. 1). A specified variation of pressure with axial distance may be prescribed. The presence of shock waves are not considered in the mixing region. Options are available for frozen, equilibrium, or finite-rate chemistry, and a variety of turbulent viscosity models are also available. Lewis and Prandtl numbers, while restricted to constant values, may be non-unity. This large number of options gives the current program considerable versatility. With relatively small additional programming time, any desired set of species can be considered, providing that the necessary chemical and thermodynamic properties are obtainable, and also provided that the increased running time associated with a large number of species is not excessively expensive. To date, the hydrogen-air, methane-air and overall hydrocarbon-air reaction systems have been programmed, and work is in progress with some higher hydrocarbon-air systems, such as kerosene, and with the hydrogen-fluorine system. However, the present discussion deals specifically with the hydrogen-air system, comprised of the seven species: H_2, O_2, H_2O, O, H, OH, and N_2 (inert dilutent).

The solution of the fluid dynamic and chemical kinetic system is obtained by the application of the numerical analysis of Refs. 2 and 3.

II. DISCUSSION OF SPECIFIC FEATURES

a. Input Data Requirements

1.) Initialization - The initial station location and initial profiles of velocity, static temperature, and species mass fractions (also element mass fractions for the case of equilibrium chemistry) are required. Input profiles may be given in either physical or streamfunction coordinates. In addition, the problem must be classified as either two-dimensional or axisymmetric. Simple initial step function profiles are found to be adequate in most cases. The prescribed pressure variation must be given as a polynomial of the form:

$$P(x) = P_0 + P_1 x + \ldots + P_6 x^6$$

The initial number of grid points desired must be specified, as well as the total axial running distance required (Xmax). Also, the desired constant values of the Lewis and Prandtl numbers must be input. These numbers are not required to be unity.

2.) Specification of Options - At this point, the selection of the required chemistry and viscosity models is necessary. The chemistry options available are:

a. Frozen Chemistry

b. Equilibrium Chemistry

c. Finite-Rate Chemistry

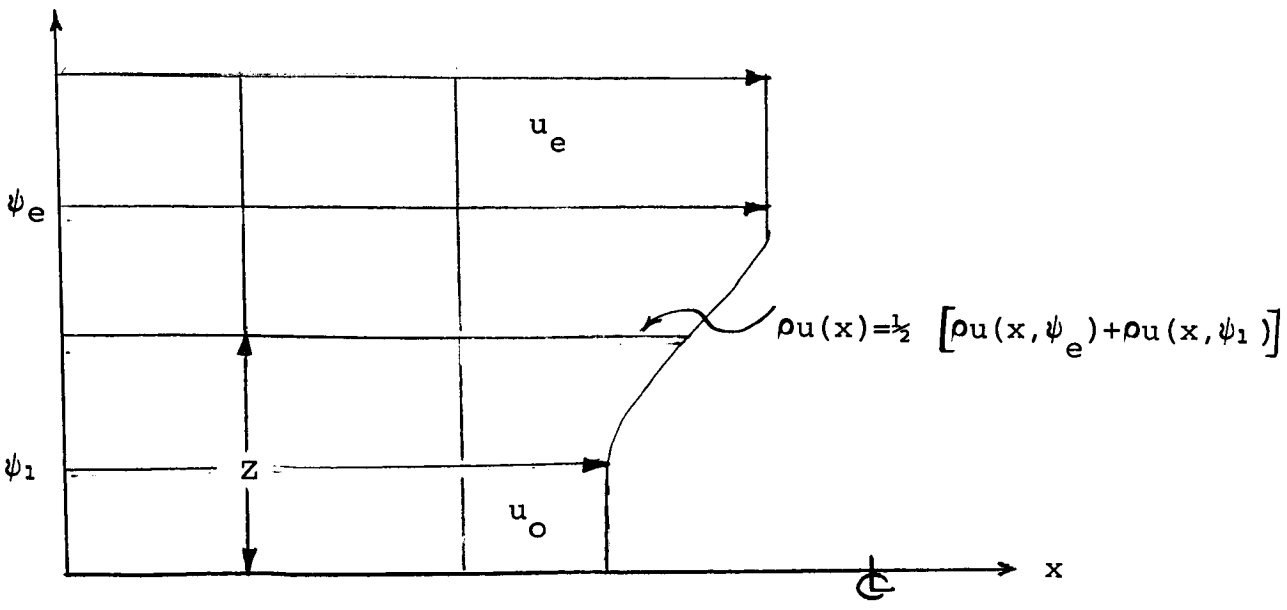
The selection of a viscosity model is not restricted to the choices given below - any desired model may be programmed. At present, there is no completely satisfactory theory of turbulent flow available; thus the models available are semi-empirical in origin. The turbulent viscosity models presented below will each be discussed briefly since they are

of current interest. Unless it is otherwise stated, the models apply to fully developed turbulent flows, or to flows where the potential core distance is small compared to the distance to any axial station of interest. It should be noted that not every model is presently available in each program version.

1. $\mu_{t_1} = 10^{-6}$ at $x = 0$, then $\mu_{t_1} = 10^{-3}\,x$, until $\mu_{t_1} = 6.211 \times 10^{-4}$ $\frac{\text{slugs}}{\text{ft-sec}}$, has been suggested to account for the transitional effects which occur in the initial mixing region of hydrogen jets diffusing into air.

2. $\mu_{t_2} = 6.211 \times 10^{-4}\ \frac{\text{slugs}}{\text{ft-sec}}$ has been experimentally determined by V. Zakkay for fully turbulent hydrogen jets diffusing into air.

3. $\mu_{t_3} = kz \left| (\rho u)_{max} - (\rho u)_{min} \right|$ is based upon Prandtl theory. This form was suggested by A. Ferry to account for the effects of compressibility. The non-dimensional constant, k, must be experimentally determined for the flow configurations and gases being studied. The half-radius, z, is defined in the following figure:



4. $\mu_{t_4} = kz(\rho u)_{\psi_1}$ has been suggested by Zakkay as an alternate to model number three above. The value of the constant, k, is in the range $0.025 \leq k \leq 0.04$ for hydrogen-air systems.

5. $\mu_{t_5} = kx\rho_e \left| u_e - u_1 \right| + 10^{-4}$ is the "potential core" model. For hydrogen jets diffusing into air, $k = 0.50 \times 10^{-3}$. Mixing is assumed to have reached the axis when a certain stipulated percentage change in either H(x,o) or $P_o(x,o)$ is attained. Then, the program automatically switches to a pre-selected fully developed turbulent model.

6. μ_{t_6} = Constant, where the appropriate constant must be obtained experimentally for the flow configuration and gases being studied.

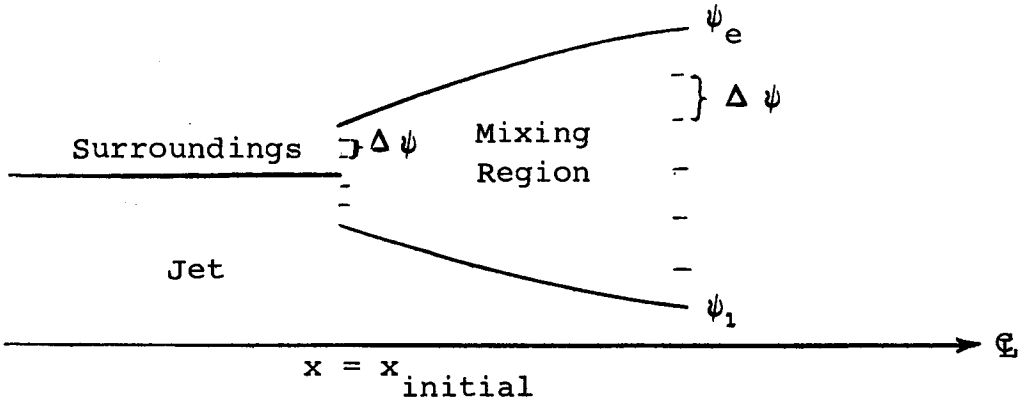
7. $\mu_{t_7} = \frac{x+\delta^*_{initial}}{k} \left[(\rho u)_{\psi_e} + (\rho u)_{\psi_1} \right] + 10^{-4}$ was applied to plumes, and takes into account the existence of an initial turbulent boundary layer. The various terms are described in the previous figure, except for $\delta^*_{initial}$, which is the initial boundary layer displacement thickness.

8. $\mu_{t_8} = k(z+\delta^*_{initial}) \left| (\rho u)_{\psi_e} - (\rho u)_{\psi_1} \right| + 10^{-4}$ has been proposed as an alternate to model number seven. It is applicable to cases where the mass flux varies significantly across the plume.

9. $\mu_{t_{9_i}} = \frac{\rho(x,\psi_r)}{\rho(x,\psi_i)} (\mu_{t_7})$ was devised to account for local density variation. The subscript r denotes a reference grid point, and the running subscript i denotes any grid point of interest. This model, therefore, supplies a viscosity which varies in the radial direction with the density ratio.

10. $\mu_{t_{10}} = \frac{\rho(x,\psi_r)}{\rho(x,\psi_i)} (\mu_{t_8})$ is similar to model number nine. It is used in cases where model number eight rather than model number seven applies.

Another very useful option is available for the fine resolution of the flow field downstream of the flow interface. To use this option, the physical axis is replaced by a boundary near the interface. The region between the boundary and the interface can now be divided into a reasonably large number of grid points for fine resolution of the local flow field, as shown below:



No loss of accuracy occurs provided that the lower edge of the mixing region does not intersect the shifted boundary. When intersections occur, the boundary is successively lowered until it finally coincides with the physical axis. From this point on the calculation proceeds in the usual manner.

3.) Preliminary Calculations - Before attempting to march downstream, the input data is tested for negative temperature, pressure, velocity, or species mass fraction. If an error of this type is found, the program is halted.

b. Calculations Performed at "Known" Stations

A "Known" station is a station at which the profiles of static temperature, velocity, and species mass fraction have already been determined. With these properties known, all other flow variables of interest are computed. The pressure and pressure gradient are determined by evaluation of the given polynominal.

The mass fraction profiles are used to calculate the density and mixture molecular weight profiles. The following equations apply:

(Eq. 1) $$\frac{1}{W\ (x,\psi)} = \sum_{j=1}^{n} \frac{\alpha^j(x,\psi)}{W_j}$$

(Eq. 2) $$\rho(x,\psi) = \frac{P(x)\ W(x,\psi)}{Ro\ \ T(x,\psi)}$$

The species specific heat, mixture specific heat, and static enthalpy profiles are then determined by use of the following relations:

(Eq. 3) $$C_p^{\ j}\ (x,\psi) = f_j\ [T\ (x,\psi)]$$

(Eq. 4) $$C_p\ (x,\psi) = \sum_{j=1}^{n} \alpha^j(x,\psi)\ C_p^{\ j}\ (x,\psi)$$

(Eq. 5) $$h(x,\psi) = \sum_{j=1}^{n} \alpha^j(x,\psi)\ h^j(x,\psi) = \sum_{j=1}^{n} \alpha^j(x,\psi)\ g_j\ [T(x,\psi)]$$

Where the functions f_j (T) and g_j (T) which describe the physical properties of the species are curvefits of thermodynamic data obtained from the current literature (Ref. 6,7).

The viscosity profile is also calculated for the model selected by the appropriate methods.

c. Calculations Performed to Determine Flow Properties at Subsequent Stations

1) Chemistry - Since the finite difference formulation uncouples the chemistry and diffusion processes over each axial step, the chemistry model selected is used to up-date the temperature and the species production term at the (n,m) grid point. For frozen chemistry this term is zero. For equilibrium chemistry the production term is indeterminate, forcing one to work with element mass fractions and total enthalpy rather than static temperature. For the case of finite-rate chemistry the species generation is obtained from the equation which results when the linearization technique (Ref. 2) is applied. The species generation, so determined, is used in the difference equations to be discussed below.

2) Finite Difference Formulation - Assuming that boundary layer technique $\left(\frac{\partial p}{\partial y} = 0 \right)$ is applicable to the chemically reacting jet problem under consideration, one obtains the following set of governing equations (Ref. 1):

Continuity

(Eq. 6) $$\frac{\partial}{\partial x} (\rho u y^N) + \frac{\partial}{\partial y} (\rho v y^N) = 0$$

Momentum

(Eq. 7) $$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{dp}{dx} + \frac{1}{y^N} \frac{\partial}{\partial y} (\mu y^N \frac{\partial u}{\partial y})$$

*Energy $$Cp\ \rho u \frac{\partial T}{\partial x} + Cp\ \rho v \frac{\partial T}{\partial y} = u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y}\right)^2 - \rho \sum_{i=1}^{N} h^i \dot{W}^i$$

(Eq.8) $$+\frac{1}{y^N}\frac{\partial}{\partial y}\left[\frac{C_p}{\sigma}\mu y^N \frac{\partial T}{\partial y}\right] + \frac{\mu Le}{\sigma}\frac{\partial T}{\partial y}\sum_{i=1}^{N} C_p{}^i \frac{\partial \alpha^i}{\partial y}$$

*<u>Diffusion</u>

(Eq.9) $$\rho u \frac{\partial \alpha^i}{\partial x} + \rho v \frac{\partial \alpha^i}{\partial y} = \frac{1}{y^N}\frac{\partial}{\partial y}\left[\frac{Le}{\sigma}\mu y^N \frac{\partial \alpha^i}{\partial y}\right] + \rho \dot{W}^i$$

Where N=0, 1 for two dimensional or axisymmetric flows respectively.

The boundary and initial conditions are:

$x=x_{initial}$: $u=u(y)$, $\alpha^i=\alpha^i(y)$, $T=T(y)$

$y=0$: $\frac{\partial u}{\partial y}(x)=\frac{\partial \alpha^i}{\partial y}(x)=\frac{\partial T}{\partial y}(x)=0$

$y=y_e$: $u=u_e(X)$, $\alpha^i=\alpha_e{}^i(X)$, $T=T_e(X)$

Since it is convenient to use the stream function as the radial coordinate, we apply the von-Mises transformation defined by:

(Eq. 10) $$\psi^N \frac{\partial \psi}{\partial y} = \rho u y^N$$

$$\psi^N \frac{\partial \psi}{\partial x} = \rho v y^N$$

Then $$\left.\frac{\partial}{\partial x}\right)_y = \left.\frac{\partial}{\partial x}\right)_\psi - \frac{\rho v y^N}{\psi^N}\left.\frac{\partial}{\partial \psi}\right)_x$$

(Eq. 11) $$\left.\frac{\partial}{\partial y}\right)_x = \frac{\rho u y^N}{\psi^N}\left.\frac{\partial}{\partial \psi}\right)_x$$

---

* For the case of equilibrium chemistry, the energy and diffusion equations should be reformulated in terms of total enthalpy and element mass fractions respectively. Note that $\dot{\tilde{W}}^i$ will then be zero.

The streamfunction identically satisfies the continuity equation. The other equations become:

Momentum

(Eq. 12a) $$\frac{\partial u}{\partial x} = -\frac{1}{\rho u}\frac{dp}{dx} + \frac{1}{\psi^N}\frac{\partial}{\partial \psi}\left(a\frac{\partial u}{\partial \psi}\right)$$

Energy

(Eq. 13a) $$C_p\frac{\partial T}{\partial x} = \frac{1}{\rho}\frac{dp}{dx} + \frac{1}{\psi^N}\frac{\partial}{\partial \psi}\left[\left(\frac{C_p}{\sigma}\right) a\frac{\partial T}{\partial \psi}\right] - \frac{1}{u}\sum_{i=1}^{N} h^i \dot{W}^i + \frac{a}{\psi^N}\left[\left(\frac{\partial u}{\partial \psi}\right)^2 + \frac{Le}{\sigma}\frac{\partial T}{\partial \psi}\sum_{i=1}^{N} C_p{}^i \frac{\partial \alpha^i}{\partial \psi}\right]$$

Diffusion

(Eq. 14a) $$\frac{\partial \alpha^i}{\partial x} = \frac{1}{\psi^N}\frac{\partial}{\partial \psi}\left[\left(\frac{Le}{\sigma}\right) a\frac{\partial \alpha^i}{\partial \psi}\right] + \frac{1}{u}\dot{W}^i$$

Where:

(Eq. 15) $$a = \frac{\mu\rho u y^{2N}}{\psi^N}$$

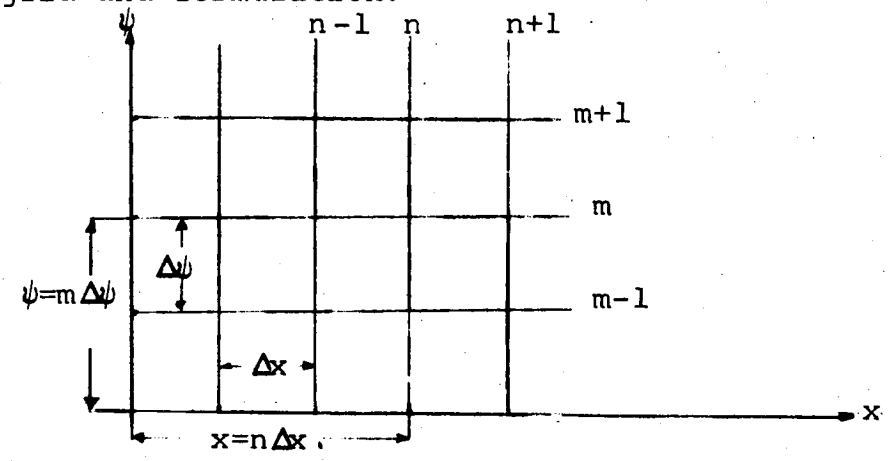
On the symmetry axis the above equations reduce to:

(Eq. 12b) $$\frac{\partial u}{\partial x} = 2^N (\rho u)^{1-N}\mu\frac{\partial^2 u}{\partial \psi^2} - \frac{1}{\rho u}\frac{dp}{dx}$$

(Eq. 13b) $$C_p\frac{\partial T}{\partial x} = \frac{1}{\rho}\frac{dp}{dx} + 2^N (\rho u)^{1-N}\frac{C_p}{\sigma}\mu\frac{\partial^2 T}{\partial \psi^2} - \frac{1}{u}\sum_{i=1}^{N} h^i \dot{W}^i$$

(Eq. 14b) $$\frac{\partial \alpha^i}{\partial x} = 2^N (\rho u)^{1-N} \frac{Le}{\sigma} \mu \frac{\partial^2 \alpha^i}{\partial \psi^2} + \frac{1}{u} \dot{W}^i$$

We now introduce the following finite difference grid and formulation.



Using backward differences for axial derivatives and central differences for radial derivatives one obtains the difference expressions:

(Eq. 16)

$$\left(\frac{\partial F}{\partial x}\right)_{n+1,m} = \frac{F_{n+1,m} - F_{n,m}}{\Delta x}$$

$$\left(\frac{\partial F}{\partial \psi}\right)_{n,m} = \frac{F_{n,m+1} - F_{n,m-1}}{2\Delta\psi}$$

$$\left[\frac{\partial}{\partial \psi}\left(a \frac{\partial F}{\partial \psi}\right)\right]_{n,m} = \frac{a_{n,m+\frac{1}{2}}(F_{n,m+1}-F_{n,m}) - a_{n,m-\frac{1}{2}}(F_{n,m}-F_{n,m-1})}{(\Delta\psi)^2}$$

in which

$$a_{n,m\pm\frac{1}{2}} = \frac{a_{n,m}+a_{n,m\pm1}}{2}$$

By using the representations given above, the governing

equations are now obtained in difference form as the linear, uncoupled, algebraic equations listed below:

(Eq. 18a)

$$u_{n+1,m} = -\frac{\Delta x}{(\rho u)_{n,m}} \left(\frac{dp}{dx}\right)_{n+1} + \frac{\Delta x}{\psi^N (\Delta\psi)^2} \left\{ a_{n,m+\frac{1}{2}} u_{n,m+1} + \left[\frac{\psi^N (\Delta\psi)^2}{\Delta x} - a_{n,m+\frac{1}{2}} - a_{n,m-\frac{1}{2}} \right] u_{n,m} + a_{n,m-\frac{1}{2}} u_{n,m-1} \right\}$$

(Eq. 18b)

$$u_{n+1,0} = -\frac{\Delta x}{(\rho u)_{n,0}} \left(\frac{dp}{dx}\right)_{n+1} + \frac{2^{N+1} (\rho u)_{n,0}^{1-N} \mu_{n,0} \Delta x}{(\Delta\psi)^2} \left\{ u_{n,1} + \left[\frac{(\Delta\psi)^2}{2^{N+1} (\rho u)_{n,0}^{1-N} \mu_{n,0} \Delta x} - 1 \right] u_{n,0} \right\}$$

(Eq. 19a)

$$T_{n+1,m} = \frac{\Delta x}{(\rho C_p)_{n,m}} \left(\frac{dp}{dx}\right)_{n+1} - \frac{\Delta x}{(u C_p)_{n,m}} \sum_{i=1}^{n} h^i_{n,m} \dot{w}^i_{n,m}$$

$$+ \left(\frac{a}{C_p}\right)_{n,m} \frac{\Delta x}{4\psi^N (\Delta\psi)^2} (u_{n,m+1} - u_{n,m-1})^2 + \frac{\Delta x}{C_{p_{n,m}}} \frac{1}{\psi^N (\Delta\psi)^2}$$

$$\left\{ \left[\left(\frac{C_p a}{\sigma}\right)_{n,m+\frac{1}{2}} + \frac{1}{4} \left(\frac{Le\ a}{\sigma}\right)_{n,m} \sum_{i=1}^{n} C^i_{p_{n,m}} (\alpha^i_{n,m+1} - \alpha^i_{n,m-1}) \right] T_{n,m+1} \right.$$

$$+ \left[\frac{C_{p_{n,m}} \psi^N (\Delta\psi)^2}{\Delta x} - \left(\frac{C_p a}{\sigma}\right)_{n,m+\frac{1}{2}} - \left(\frac{C_p a}{\sigma}\right)_{n,m-\frac{1}{2}} \right] T_{n,m}$$

$$+ \left[\left(\frac{C_p a}{\sigma}\right)_{n,m-\frac{1}{2}} - \frac{1}{4} \left(\frac{Le\ a}{\sigma}\right)_{n,m} \sum_{i=1}^{n} C_{p_{n,m}}^{i} \left(\alpha^{i}_{n,m+1} - \alpha^{i}_{n,m-1} \right) T_{n,m-1} \right] \Bigg\}$$

(Eq. 19b)

$$T_{n+1,0} = \frac{\Delta x}{(\rho C_p)_{n,o}} \left(\frac{dp}{dx}\right)_{n+1} - \frac{\Delta x}{(uC_p)_{n,o}} \sum_{i=1}^{n} h^{i}_{n,o} (\dot{W}^{i})_{n,o}$$

$$+ \frac{2^{N+1} (\rho u)^{1-N}_{n,o} \Delta x}{(\Delta\psi)^2} \cdot \left(\frac{\mu}{\sigma}\right)_{n,o} \left\{ T_{n,1} + \left[\frac{(\Delta\psi)^2}{2^{N+1} (\rho u)^{1-N}_{n,o} \Delta x} \left(\frac{\sigma}{\mu}\right)_{n,o} - 1 \right] T_{n,o} \right\}$$

(Eq. 20a)

$$\alpha^{i}_{n+1,m} = \Delta x \left(\frac{1}{u} \dot{W}^{i}\right)_{n,m} + \frac{\Delta x}{\psi^N (\Delta\psi)^2} \left\{ \left(\frac{Le\ a}{\sigma}\right)_{n,m+\frac{1}{2}} \alpha^{i}_{n,m+1} \right.$$

$$+ \left[\frac{\psi^N (\Delta\psi)^2}{\Delta x} - \left(\frac{Le\ a}{\sigma}\right)_{n,m+\frac{1}{2}} - \left(\frac{Le\ a}{\sigma}\right)_{n,m-\frac{1}{2}} \right] \alpha^{i}_{n,m}$$

$$+ \left(\frac{Le\ a}{\sigma}\right)_{n,m-\frac{1}{2}} \alpha^{i}_{n,m-1} \Big\}$$

(Eq. 20b)

$$\alpha^{i}_{n+1,0} = \Delta x \left(\frac{1}{u}\dot{W}^{i}\right)_{n,o} + \frac{2^{N+1}(\rho u)^{1-N}_{n,o}\ \Delta x}{(\Delta\psi)^{2}} \frac{(Le\mu)}{\sigma_{n,o}} \Big\{\alpha^{i}_{n,1}$$

$$+ \left[\frac{(\Delta\psi)^{2}}{2^{N+1}(\rho u)^{1-N}_{n,o}\ \Delta x} \left(\frac{\sigma}{Le\mu}\right)_{n,o} - 1\right] \alpha^{i}_{n,o} \Big\}$$

3) Grid Size Controls - As the mixing progresses, the jet dimensions change, and consequently the grid must be adjusted. The procedure for the addition of grid points is to test the values of velocity, temperature and species at the grid points which are adjacent to the upper and lower jet boundaries (for $\psi_1 \neq 0$ cases) respectively. If the values differ by 0.1%, or more, a grid point is added. The new grid point has the appropriate edge values of the flow variables at that axial station. If we denote by j the initial number of grid points, then $\Delta\psi$ is doubled (by discarding even numbered grid points) when the number of grid points reaches 2j-1.

The axial step size at any station must be carefully monitored to insure stable results. Stability analysis (Ref. 1,8) leads to the conclusion that the axial step size must be less than or equal to the smallest of the following:

1. $$\frac{\psi^N (\Delta\psi)^2}{3\left[\left(\frac{Le\ a}{\sigma}\right)_{n,m+\frac{1}{2}} + \left(\frac{Le\ a}{\sigma}\right)_{n,m-\frac{1}{2}}\right]}$$

2. $$\frac{(\Delta\psi)^2}{3\left[2^{N+1}(\rho u)_{n,0}^{1-N}\right]} \qquad \left(\frac{\sigma}{Le\,\mu}\right)_{n,0}$$

3. The minimum physical radial separation between any two grid points at the station of interest.

Having completed the calculations described in the preceeding section (c) we return to section (b) and recycle to continue downstream.

III. Input - Output Information

It has been previously mentioned that there are a wide variety of options available for this program. These options were added at different stages of development. Therefore, a variety of binary decks exists. Some decks are in Fortran II language, while others are in Fortran IV. Since there are slight differences in the input formats of these various decks, several formats are necessary. It is helpful, therefore, to first list the general type of data required, and also to provide a chart showing the current status of each program version.

General Type of Input Data Required

1. Identification.
2. Number of ψ grid points input initially, or
   Number of y grid points input initially.
3. Chemistry option desired.
4. Print out step increment desired (Δx) p.o. - ft.
5. Maximum running distance X_{max} - ft.
6. Initial station location X_i - ft.
7. Viscosity option desired.
8. Constant values of Lewis and Prandtl numbers desired.
9. Stream function step size $\Delta\psi$ $\left(\begin{array}{ll} \sqrt{\frac{\text{slugs}}{\text{sec}}} & \text{for axisym.} \\ \frac{\text{slugs}}{\text{ft.-sec.}} & \text{for 2-D} \end{array} \right)$
10. Value of constant in viscosity model.
11. Location of lower jet boundary ψ_1.
12. Coefficients of pressure polynomial.
13. Temperature (^{o}K), velocity ($\frac{\text{ft.}}{\text{sec.}}$), and species mass fraction (as well as element mass fraction for equilibrium chemistry) profiles at initial station.
14. Various other tolerances for the numerical procedure.

INPUT FORMAT FOR AXISYMMETRIC FINITE-RATE OR FROZEN CHEMISTRY VERSION

Card	Column	Description	Format
1	1-72	Title Information	12 A6
2	1-5	Number of ψ grid points input (N)	I5
	6-10	Number of y grid points input (it is recommended that ψ input be used)	"
	11-15	0 for finite-rate chemistry or 1 for frozen chemistry	"
	16-20	Control number of viscosity option - 1 to 10 (see Pg. 8 for models and their corresponding control numbers)	"
	21-25	Blank	"
	26-30	20	"
	31-35	10	"
	36-40	10	"
	41-45	Blank	"
	46-50	Control number of viscosity model to be switched to after mixing has reached the potential core	"
	51-60	Blank	"
	61-65	Control number of the streamline to be traced. For example: "11" would mean that the streamline starting at the eleventh ψ grid point is to be traced. In other words, the value of the eleventh ψ grid point is to be preserved at all axial stations.	"
	66-72	Blank	"
3	1-10	Print out step size, $\Delta x_{p.o.}$ (ft.)	E10.8
	11-20	Maximum axial distance, $x_{max.}$ (ft.)	"
	21-30	Initial station location, $x_{initial}$ (ft.)	"

Card	Column	Description	Format
	31-40	1.0-6 (1.0×10^{-6})	E10.8
	41-50	1.0	"
	51-60	Blank	
	61-70	Applies to fine resolution option ($\psi_{initial} > 0$) Blank - when $\Delta\psi > \psi_1$-freeze ψ_1 1.0 - when $\Delta\psi > \psi_1$, use curvefitting routine to restart with $\psi_1 = 0$.	"
4	1-10	Lewis number	E10.8
	11-20	Prandtl number	"
	21-30	ψ step size at input station, $(\Delta\psi)_{initial}$ $\left(\frac{slugs}{sec}\right)$	"
	31-40	Factor by which to reduce the axial step size determined from stability criteria (1.0 is suggested)	"
	41-50	Value of constant in viscosity model	"
	51-60	$(\psi_1)_{initial}$ $\left(\frac{slugs}{sec}\right)$	"
5	1-20	Blank	E10.8
	21-30	1.0	"
	31-40	Blank	"
	41-50	x_{min}(ft.) in potential core model (for $x < x_{min}$, $\mu = 10^{-4}$)	"
	51-60	Initial value of the displacement thickness (ft.) to be used in viscosity models 7 and 8.	"
5	61-70	x_{max} in potential core model (for $x > x_{max}$, switch to the model specified in columns 46-50 of Card 2)	"
6	1-10	p_0	"
	11-20	p_1 coefficients in prescribed pressure polynomial	"
	21-30	p_2 $p\,\frac{lbf}{ft^2} = p_0 + p_1 x + \ldots p_6 x^6$	"
	31-40	p_3	"
	41-50	p_4	"
	51-60	p_5	"
	61-70	p_6	"

Card	Column	Description		Format
7	1-10	1.0 + 6 ($1.0 \times 10^{+6}$)		E10.8
	11-72	Blank		"
8a	1-10	$T(\psi_1)$	Static temperature (^{o}K)	"
	11-20	$T(\psi_2)$	at each input grid point	"
	21-30	$T(\psi_3)$		"
	61-70	$T(\psi_7)$		"
8b,c	1-10	$T(\psi_8)$	continuation of temperature profile	"
		$T(\psi_N)$		"
9a	1-10	$u(\psi_1)$	Velocity (FPS) at each input	E10.8
	11-20	$u(\psi_2)$	grid point	"
	61-70	$u(\psi_7)$		"
9b,c	1-10	$u(\psi_8)$	continuation of velocity profile	"
		$u(\psi_N)$		"
10a	1-10	$\alpha_H(\psi_1)$	values of the species mass	"
	11-20	$\alpha_O(\psi_1)$	fractions input at ψ_1	"
	21-30	$\alpha_{H_2O}(\psi_1)$		"
	31-40	$\alpha_{H_2}(\psi_1)$		"
	41-50	$\alpha_{O_2}(\psi_1)$		"
	51-60	$\alpha_{OH}(\psi_1)$		"
	61-70	$\alpha_{N_2}(\psi_1)$		"
10b,c	N	Same as 10a, but for grid points $\psi_2 \ldots \psi_N$		E10.8
11a	1-10	y_1	Values of y (ft) needed	E10.8
	11-20	y_2	only if y input is given	"
	61-70	y_7		"
11b,c	1-10	y_8		E10.8
	"	"	y values continues	"
	"	y_{11}		"

INPUT FORMAT FOR AXISYMMETRIC EQUILIBRIUM CHEMISTRY VERSION

Card	Column	Description	Format
1	1-72	title information	12 A 6
2	1-5	Number at ψ grid points input (N)	I5
	6-10	Number of y grid points input (it is recommended that ψ input be used)	"
	11-15	Blank	
	16-20	Control number of viscosity option (models 1-4 are available)	"
	21-72	Blank	"
3	1-10	Print-out step size, $\Delta x_{p.o.}$ (ft).	E10.8
	11-20	Maximum axial distance, x_{max} (ft.).	"
	21-30	Initial station location, $x_{initial}$ (ft.).	"
	31-40	1.0 + 6 $(1.0 \times 10^{+6})$	"
	41-50	Blank	"
	51-60	Applies to fine resolution option ($\psi_{initial}>0$) Blank-when $\Delta\psi>\psi_1$, freeze ψ_1 1.0 - when $\Delta\psi>\psi_1$, use curvefitting routine to restart with $\psi_1=0$.	"
4	1-10	Lewis number	E10.8
	11-20	Prandtl number	"
	21-30	ψ step size at input station, $(\Delta\psi)_{initial}$ $\sqrt{\frac{slugs}{sec}}$	
	31-40	Factor by which to reduce the axial step size determined from stability criteria (1.0 is suggested)	"
	41-50	Value of constant in viscosity model	"

Card	Column	Description		Format
4	51-60	Base value of ψ at input station, ψ_1) initial ($\sqrt{\frac{\text{slugs}}{\text{sec}}}$ (for fine resolution option use ψ) initial >0)		E10.8
5	1-10	p_o	coefficients in prescribed	"
	11-20	p_1	pressure polynomial:	
	21-30	p_2	$p(\frac{LB}{ft^2}) = p_o + p_1 x + p_2 x^2$	
	31-72	Blank		"
6a	1-10	$T(\psi_1)$	static temperature (^oK)	E10.8
	11-20	$T(\psi_2)$	at each input grid	"
	"		point	"
	61-70	$T(\psi_7)$		"
6b,c	1-10	$T(\psi_8)$	continuation of	E10.8
			temperature profile	"
		$T(\psi_N)$		"
7a	1-10	$u(\psi_1)$	velocity (fps) at	E10.8
	11-20	$u(\psi_2)$	each input grid	"
	"	"	point	"
	61-70	$u(\psi_7)$		"
7b,c	1-10	$u(\psi_8)$	continuation of	E10.8
		"	velocity profile	"
		$u(\psi_N)$		"
8a	1-30	Blank	values of the	E10.8
	31-40	$\tilde{\alpha}_{H_2}$ (ψ_1)		"
	41-50	$\tilde{\alpha}_{O_2}$ (ψ_1)	ELEMENT mass fractions	"
	51-60	Blank	input at ψ_1	"
	61-70	$\tilde{\alpha}_{N_2}$ (ψ_1)		"
8b,c,	N	Same as 8a, but for grid points ψ_2---ψ_N		E10.8

Card	Column	Description		Format
9a.	1-10	y_1	Values of y (ft), needed only if y input is given	E10.8
	11-20	y_2		"
	"	"		"
	61-70	y_7		"
9b,c	1-10	y_8	y values continued	E10.8
		"		"
		y_N		"

INPUT FORMAT FOR TWO-DIMENSIONAL FINITE-RATE OR FROZEN CHEMISTRY VERSION

Card	Column	Description	Format
1	1-72	Title information	Hollerith
2	1-5	Number of ψ grid points input (N)	I 5
	6-10	Blank	"
	11-15	0 for finite-rate chemistry or 1 for frozen chemistry	" "
	16-20	Control number for viscosity option 0 - for $\mu_t = 6.211 \times 10^{-4} \frac{\text{slugs}}{\text{ft-sec}}$ 1 - for $\mu_t = kz \left\lvert (\rho u)_{max} - (\rho u)_{min} \right.$ 2 - for $\mu_t = kz\ (\rho u)_{\mathfrak{C}}$	"
	21-72	Blank	
3	1-10	Print-out step size, $\Delta x_{p.o.}$ (ft.)	E10.8
	11-20	Maximum axial distance, x_{max} (ft.)	"
	21-30	Initial station location, $x_{initial}$ (ft.)	"
	31-72	Blank	
4	1-10	Lewis number	E10.8
	11-20	Prandtl number	"
	21-30	ψ step size at input station, $(\Delta\psi)_{initial}$ $\left(\frac{\text{slugs}}{\text{ft-sec}}\right)$	"
	31-40	Factor by which to reduce the axial step size determined from stability criteria (1.0 is suggested)	"
	41-50	Value of constant in viscosity model	"
	51-72	Blank	
5	1-10	p_o coefficients in prescribed	E10.8
	11-20	p_1 pressure polynomial:	"
	21-30	p_2 $p\left(\frac{LB}{ft^2}\right) = p_o + p_1 x + p_2 x^2$	"
	31-72	Blank	

Card	Column		Description	Format
6a	1-10	$T(\psi_1)$	static temperature (^{o}K)	E10.8
	11-20	$T(\psi_2)$	at each input grid point	"
	"	"		
	61-70	$T(\psi_7)$		"
6b,c	1-10	$T(\psi_8)$	continuation of	E10.8
		"	temperature profile	"
		$T(\psi_N)$		"
7a	1-10	$u(\psi_1)$	velocity (fps) at each	E10.8
	11-20	$u(\psi_2)$	input grid point	"
	"	"		"
	61-70	$u(\psi_7)$		"
7b,c	1-10	$u(\psi_8)$	continuation of velocity	E10.8
		"	profile	"
		$u(\psi_N)$		
8a	1-10	$\alpha_H(\psi_1)$		E10.8
	11-20	$\alpha_O(\psi_1)$	values of the species	"
	21-30	$\alpha_{H_2O}(\psi_1)$	mass fractions input	"
	31-40	$\alpha_{H_2}(\psi_1)$	at ψ_1	"
	41-50	$\alpha_{O_2}(\psi_1)$		"
	51-60	$\alpha_{OH}(\psi_1)$		"
	61-70	$\alpha_{N_2}(\psi_1)$		"
8b,c	N	Same as 8a but for grid points $\psi_2 \ldots \psi_N$		E10.8

INPUT FORMAT FOR TWO-DIMENSIONAL EQUILIBRIUM CHEMISTRY VERSION

Card	Column	Description	Format
1	1-72	Title information	Hollerith
2	1-5	Number of ψ grid points input (N)	I 5
	6-15	Blank	"
	16-20	Control number for viscosity option 0- for $\mu_t = 6.211 \times 10^{-4} \frac{\text{slugs}}{\text{ft-sec}}$ 1- for $\mu_t = kz \left\lvert (\rho u)_{max} - (\rho u)_{max} \right\rvert$ 2- for $\mu_t = kz\ (\rho u)_{\mathbb{C}}$	"
	21-72	Blank	
3	1-10	Print-out step size, $\Delta x_{p.o.}$ (ft.)	E10.8
	11-20	Maximum axial distance, x_{max} (ft.)	"
	21-30	Initial station location, $x_{initial}$ (ft.)	"
	31-72	Blank	
4	1-10	Lewis number	E10.8
	11-20	Prandtl number	"
	21-30	ψ step size at input station $(\Delta\psi)_{initial}$ $(\frac{\text{slugs}}{\text{ft-sec}})$	"
	31-40	Factor by which to reduce the axial step size determined from stability criteria (1.0 is suggested)	"
	41-50	Value of constant in viscosity model	"
	51-72	Blank	
5	1-10	p_o coefficients in prescribed	E10.8
	11-20	p_1 pressure polynomial	"
	21-30	p_2 $p(\frac{LB}{ft^2}) = p_o + p_1 x + px^2$	"
	31-72	Blank	"

Card	Column	Description		Format
6a	1-10	$T(\psi_1)$	static temperature (^{o}k)	E10.8
	11-20	$T(\psi_2)$	at each input grid	"
	"	"	point	"
	61-70	$T(\psi_7)$		"
6b, c,	1-10	$T(\psi_8)$	continuation of temperature	E10.8
	"		profile	"
		$T(\psi_n)$		"
7a	1-10	$u(\psi_1)$	velocity (fps) at each	E10.8
	11-20	$u(\psi_2)$	input grid point	"
	"	"		"
	61-70	$u(\psi_7)$		"
7b, c,	1-10	$u(\psi_8)$		E10.8
		"	continuation of velocity profile	"
		$u(\psi_n)$		"
8a	1-30	Blank	values of the	E10.8
	31-40	$\tilde{\alpha}_{H_2}(\alpha_1)$	<u>ELEMENT</u> mass	"
	41-50	$\tilde{\alpha}_{O_2}(\alpha_1)$	fractions input	"
	51-60	Blank	at ψ_1	"
	61-70	$\tilde{\alpha}_{n_2}(\psi_1)$		"
8b, c,	N	Same as 8a, $\psi_2 \ldots\ldots \psi_n$	but for grid points	E10.8
9a	1-10	$\alpha_H(\psi_1)$		"
	11-20	$\alpha_O(\psi_1)$	values of the	"
	21-30	$\alpha_{H_2O}(\psi_1)$	species mass fractions	"
9a	31-40	$\alpha_{H_2}(\psi_1)$	input at	"
	41-50	$\alpha_{O_2}(\psi_1)$	ψ_1	"

Card	Column	Description	Format
	51-60	$\alpha_{OH}(\psi_1)$	E10.8
	61-70	$\alpha_{N_a}(\psi_1)$	"
9b,c,	N	Same as 9a, but for grid points $\psi_a \ldots \psi_N$	E10.8

Program Version Status Chart:

Program Version	Axisymmetric finite-rate or frozen chemistry	Two-Dimensional finite-rate or frozen chemistry	Axisymmetric Equilibium chemistry	Two-Dimensional Equilibrium Chemistry
Viscosity Models Available	All Models	2,3,4	1-4	2,3,4
Fortran Language	F IV	F II	F II	F II
Stability Criteria	1-3	1,2	1,2	1,2
L_e, P_r	Constant, though may be non-unity	→	→	→
$\frac{dp}{dx}$	May be non-zero (Prescribed)	→	→	→
Provision for input in y coordinates	Yes - but direct ψ input is recommended	No	Yes	No
Fine Resolution option	Yes	No	Yes	No

It can be seen from the above chart that the axisymmetric finite-rate or frozen chemistry version is in the most advanced stage of development. However, the other versions can be modernized without great difficulty.

Sample input and output may be found at the rear of this section.

IV. Summary:

A finite difference method solution for jet problems has been presented. Finite-rate chemistry calculations have been linearized, and no longer restrict the numerical technique to impractical step sizes. Viscosity models which cover a wide range of application have been made available to inhance the program's versitility.

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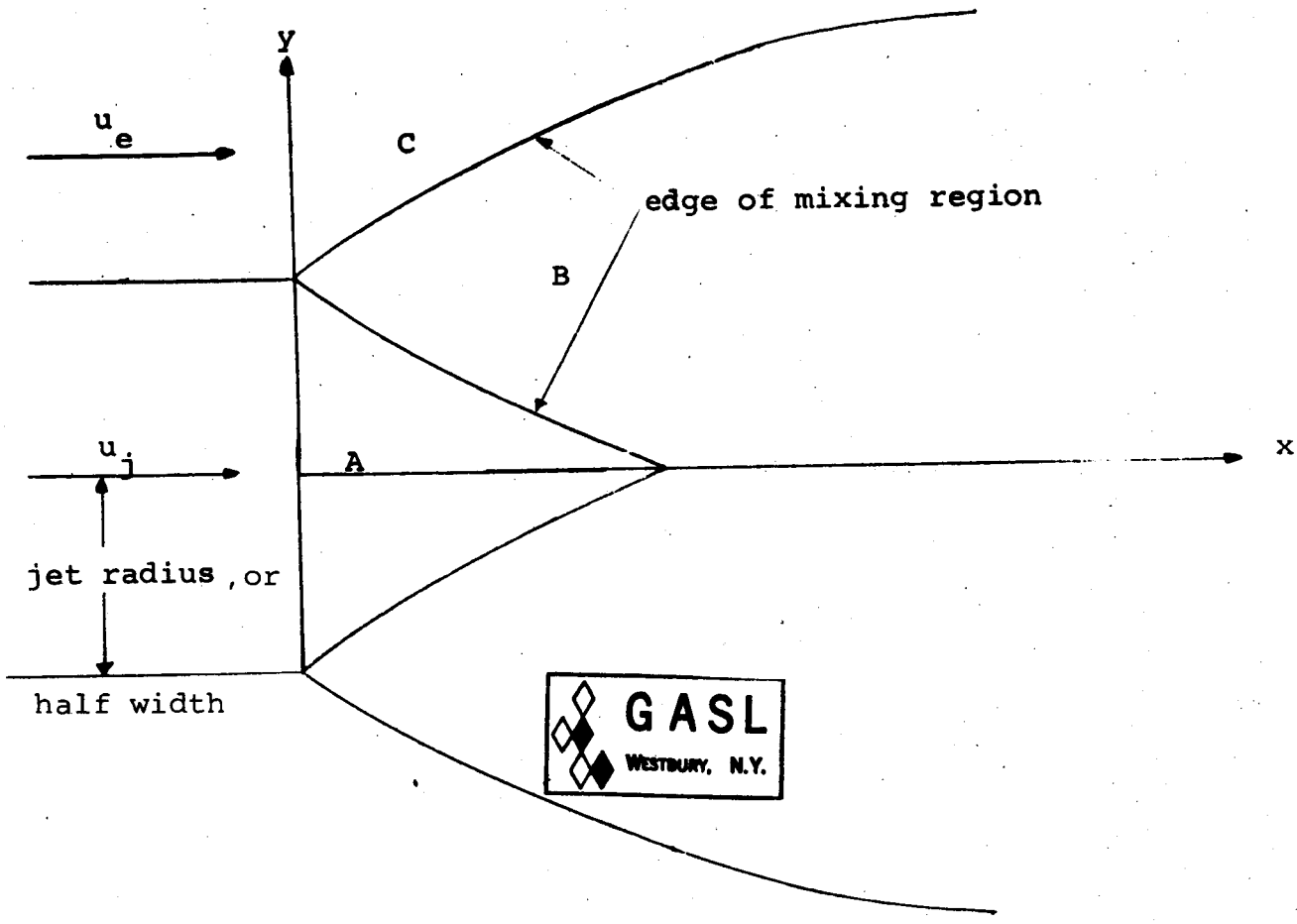

A = Pure Jet Properties Maintained

B = Mixing Region

C = Pure Edge Properties Maintained

FIG. 1 - SCHEMATIC OF FREE JET FLOW FIELD

INPUT LISTING

```
EQUILIBRIUM HYDROGEN-AIR AXISYMMETRIC      7/19/66
13   -0   -0    4   -0
# ψ pts input      visc. model
-9.99999988E-03   2.00000000E 00   -0.                1.00000000E 03   -0.              -0.
ΔX print out       Xmax              X initial
1.00000000E 00    1.00000000E 00    1.53299998E-03    1.00000000E 00   3.99999994E-02   0.
Lewis No.          Prandtl No.       Δψ                                 constant in
2.11600000E 03    -0.               -0.                                 visc. model
        coefficients in Pressure polynomial
1.00000000E 03    1.00000000E 03    1.00000000E 03    1.00000000E 03   1.00000000E 03   1.00000000E 03   1.00000000E 03
1.00000000E 03    1.00000000E 03    1.00000000E 03    1.00000000E 03   1.50000000E 03   1.50000000E 03
        Values of Static Temperature (°K) at each input grid pt.
7.88000000E 03    7.88000000E 03    7.88000000E 03    7.88000000E 03   7.88000000E 03   7.88000000E 03   7.88000000E 03
7.88000000E 03    7.88000000E 03    7.88000000E 03    7.88000000E 03   7.64999998E 03   7.64999998E 03
        Values of Velocity (fps) at each input grid pt.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                1.00000000E 00   -0.              -0.              -0.
0.                0.                0.                -0.              2.31999996E-01   -0.              7.67999995E-01
0.                0.                0.                -0.              2.31999996E-01   -0.              7.67999995E-01

1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
1.15791032E-09    0.                0.                1.00000004E 00   0.               0.               -0.
0.                1.20802660E-06    0.                0.               2.31998780E-01   0.               7.67999995E-01
0.                1.20802660E-06    0.                0.               2.31998780E-01   0.               7.67999995E-01
```

Values of element mass fractions of H_2, O_2, and N_2 at each input grid pt.

Computer calculated species mass fractions for H, O, H_2O, H_2, O_2, OH, and N_2 at each grid pt.

AXISYMMETRIC HYDROGEN-AIR ANNULAR JET WITH PRESSURE GRADIENT---EQUILIBRIUM CHEMISTRY

X= 4.4005621E-02 FEET EQUILIBRIUM HYDROGEN-AIR AXISYMMETRIC 7/19/66 PAGE 6

PRESSURE	LEWIS NUMBER	PRANDTL NUMBER	VISCOSITY	STEP SIZE	STEPS
2.1160000E 03	1.0000000E 00	1.0000000E 00	3.2513957E-04	1.6897714E-03	79

POINT	PSI	VELOCITY	TEMPERATURE	CP BAR	DENSITY	ENTHALPY	Y	POINT
1	0.	7.8739679E 03	1.0077461E 03	6.3290746E 00	4.8488440E-05	5.5444719E 03	0.	1
2	3.0660000E-03	7.8706006E 03	1.0121490E 03	6.2433326E 00	4.8971995E-05	5.4870910E 03	4.9384914E-03	2
3	6.1319999E-03	7.8584784E 03	1.0289284E 03	5.9346174E 00	5.0804324E-05	5.2805247E 03	9.7912252E-03	3
4	9.1979998E-03	7.8327115E 03	1.0703918E 03	5.2781813E 00	5.5250214E-05	4.8414502E 03	1.4426522E-02	4
5	1.2264000E-02	7.7929886E 03	1.1609025E 03	4.2677178E 00	6.3875316E-05	4.1645647E 03	1.8676954E-02	5
6	1.5330000E-02	7.7486133E 03	1.3336590E 03	3.1419397E 00	7.7611350E-05	3.4083999E 03	2.2417033E-02	6
7	1.8396000E-02	7.7107385E 03	1.6279884E 03	2.1864806E 00	9.6024650E-05	2.7630052E 03	2.5629195E-02	7
8	2.1462000E-02	7.6839458E 03	2.0756716E 03	1.5183582E 00	1.1789609E-04	2.3064498E 03	2.8384300E-02	8
9	2.4528000E-02	7.6674535E 03	2.6033186E 03	1.0991020E 00	1.4437758E-04	2.0254163E 03	3.0760998E-02	9
10	2.7594000E-02	7.6584339E 03	2.8837382E 03	8.3877856E-01	1.7995444E-04	1.8717161E 03	3.2795739E-02	10
11	3.0660000E-02	7.6539578E 03	2.6274443E 03	6.9342501E-01	2.3132043E-04	1.7954388E 03	3.4496992E-02	11
12	3.3725999E-02	7.6518446E 03	2.1793653E 03	6.0217912E-01	2.9757056E-04	1.7594255E 03	3.5896370E-02	12
13	3.6791999E-02	7.6508422E 03	1.8415165E 03	5.4969739E-01	3.6199266E-04	1.7423407E 03	3.7076848E-02	13
14	3.9857999E-02	7.6503580E 03	1.6510573E 03	5.2403126E-01	4.0903577E-04	1.7340870E 03	3.8137579E-02	14
15	4.2923999E-02	7.6501245E 03	1.5535240E 03	5.1170371E-01	4.3745729E-04	1.7301052E 03	3.9151653E-02	15
16	4.5989999E-02	7.6500166E 03	1.5071024E 03	5.0601323E-01	4.5225057E-04	1.7282614E 03	4.0161338E-02	16
17	4.9056000E-02	7.6500000E 03	1.4999727E 03	5.0514451E-01	4.5460519E-04	1.7279777E 03	4.1193901E-02	17

AXISYMMETRIC HYDROGEN-AIR ANNULAR JET WITH PRESSURE GRADIENT---EQUILIBRIUM CHEMISTRY

X= 4.4005621E-02 FEET EQUILIBRIUM HYDROGEN-AIR AXISYMMETRIC 7/19/66 PAGE 6

PT	SIGMA	ALPHA(H)	ALPHA(O)	ALPHA(H2O)	ALPHA(H2)	ALPHA(O2)	ALPHA(OH)	ALPHA(N2)		
1	1.00002	4.70830E-10	1.45150E-22	6.87259E-03	9.73016E-01	8.93029E-26	2.81754E-14	2.01333E-02	2.067196E 00	1
2	1.00004	5.21183E-10	2.94367E-22	1.07108E-02	9.57948E-01	2.86816E-25	5.11503E-14	3.13773E-02	2.096933E 00	2
3	1.00008	7.61584E-10	1.80178E-21	2.45281E-02	9.03701E-01	4.26850E-24	2.07069E-13	7.18552E-02	2.211455E 00	3
4	1.00019	1.84814E-09	3.94235E-20	5.38979E-02	7.88395E-01	2.36131E-22	1.72343E-12	1.57894E-01	2.501895E 00	4
5	1.00035	1.03783E-08	6.21169E-18	9.91750E-02	6.10637E-01	8.90352E-20	4.17773E-11	2.90534E-01	3.137047E 00	5
6	1.00053	1.46428E-07	8.65422E-15	1.49755E-01	4.12063E-01	2.80686E-16	3.29056E-09	4.38709E-01	4.378873E 00	6
7	1.00067	3.52696E-06	4.62048E-11	1.92925E-01	2.42566E-01	3.32426E-12	5.25952E-07	5.65178E-01	6.613424E 00	7
8	1.00065	7.24201E-05	2.15518E-07	2.23387E-01	1.22472E-01	3.60350E-08	7.36288E-05	6.54643E-01	1.035262E 01	8
9	0.99984	5.13886E-04	1.47859E-04	2.38612E-01	4.75810E-02	6.86393E-05	3.20256E-03	7.09714E-01	1.590081E 01	9
10	0.99970	5.45087E-04	4.51746E-03	2.15612E-01	9.83214E-03	9.20456E-03	2.01560E-02	7.39832E-01	2.195385E 01	10
11	1.00039	5.25006E-05	4.55647E-03	1.39565E-01	6.77340E-04	8.69385E-02	1.38216E-02	7.54779E-01	2.571223E 01	11
12	1.00025	1.11234E-06	5.66770E-04	7.04804E-02	2.22477E-05	1.64970E-01	2.37009E-03	7.61837E-01	2.743546E 01	12
13	1.00012	2.02321E-08	4.82753E-05	3.26946E-02	7.12331E-07	2.01873E-01	3.13216E-04	7.65185E-01	2.820121E 01	13
14	1.00005	9.11762E-10	7.49092E-06	1.39489E-02	4.31531E-08	2.19231E-01	5.94755E-05	7.66802E-01	2.857036E 01	14
15	1.00002	1.06640E-10	2.40359E-06	4.86026E-03	4.63044E-09	2.27555E-01	1.65372E-05	7.67583E-01	2.875053E 01	15
16	1.00000	1.67472E-11	1.32728E-06	6.47147E-04	3.34131E-10	2.31405E-01	4.07305E-06	7.67944E-01	2.883462E 01	16
17	1.00000	0.	1.20742E-06	0.	0.	2.31999E-01	0.	7.68000E-01	2.884762E 01	17

INTERMEDIATE DUMP OF SMALLH AND ELEMENTS

4.3063568E 03	0.	0.	0.	9.7378261E-01	6.0819391E-03	-0.	2.0133314E-02
4.2500346E 03	0.	0.	0.	9.5914187E-01	9.4785624E-03	-0.	3.1377306E-02
4.0472761E 03	0.	0.	0.	9.0643631E-01	2.1706250E-02	-0.	7.1855164E-02
3.6162756E 03	0.	0.	0.	7.9440606E-01	4.7697280E-02	-0.	1.5789442E-01
2.9517854E 03	0.	0.	0.	6.2169815E-01	8.7765523E-02	-0.	2.9053410E-01
2.2093930E 03	0.	0.	0.	4.2876201E-01	1.3252673E-01	-0.	4.3870911E-01
1.5756910E 03	0.	0.	0.	2.6408898E-01	1.7073091E-01	-0.	5.6517813E-01
1.1273724E 03	0.	0.	0.	1.4759839E-01	1.9775678E-01	-0.	6.5464306E-01
8.5139483E 02	0.	0.	0.	7.5892339E-02	2.1439264E-01	-0.	7.0971352E-01
7.0045523E 02	0.	0.	0.	3.6675518E-02	2.2349099E-01	-0.	7.3983223E-01
6.2554665E 02	0.	0.	0.	1.7213206E-02	2.2800630E-01	-0.	7.5477946E-01
5.9017921E 02	0.	0.	0.	8.0242718E-03	2.3013819E-01	-0.	7.6183671E-01
5.7340074E 02	0.	0.	0.	3.6649745E-03	2.3114959E-01	-0.	7.6518483E-01
5.6529496E 02	0.	0.	0.	1.5589551E-03	2.3163823E-01	-0.	7.6680240E-01
5.6138452E 02	0.	0.	0.	5.4292755E-04	2.3187399E-01	-0.	7.6758285E-01
5.5957367E 02	0.	0.	0.	7.2401530E-05	2.3198319E-01	-0.	7.6794434E-01
5.5929509E 02	0.	0.	0.	-0.	2.3200000E-01	-0.	7.6799999E-01

Dump of element mass fractions (obtained at last axial station for possible restart)

B. Multicomponent - Multiphase Viscous Flow with Chemical Reactions

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MULTICOMPONENT - MULTIPHASE VISCOUS FLOW WITH CHEMICAL REACTIONS

I. Introduction:

This section describes a finite difference method of solution for laminar axisymmetric two phase free mixing with hydrogen-air chemistry. The analysis (ref. 1-3) assumes that the liquid, which is composed of condensed O_2, H_2, N_2, and H_2O, forms a dilute suspension of minute spherical droplets which diffuse with the gas. It is further assumed that molecular transport and chemical reactions occur in the gas phase only. This assumption is applicable since the temperatures at which chemical reactions occur are very high in comparison with those at which the condensed phase may exist. The phases are assumed to be in dynamic and thermodynamic equilibrium.

Options are available for either frozen or finite rate chemistry. Lewis and Prandtl numbers, while restricted to constant values, may be non-unity. Axial pressure variation must be specified. The mixture laminar viscosity is determined by the usual method of mass averaging the individual species viscosities (ref.7). Local humidity conditions have been simulated by the inclusion of water vapor in the free-stream air.

II. Discussion of Specific Features:

a) Method of Analysis:

Details of the analysis and finite difference formulation are given in references 1-3. The analysis follows the procedures used in the single phase jet considerations. Specifically, this means that the governing equations are transformed into the Von-Mises plane and then put into difference form. Additional equations are obtained by imposing the condition of thermodynamic equilibrium between the phases, and from various constitutive relationships. Use is made of the vapor pressure curve in determining whether a grid point lies within the two phase region or not. If the grid point is within the two phase region, the chemistry is frozen, if it is in the gas phase, one dimensional chemistry calculations are coupled to the diffusion equations (ref.4-5). Finite-rate chemistry calculations are performed by the linear technique (ref.6).

b) Phase Separation:

The assumption that viscous transport is strictly a gas phase process leads to the phenomenon of phase separation. To understand this, consider a grid point in the two phase region. For laminar flows, or flows where the droplet size is larger than the microscale of the gas phase turbulence, the condensed species move along streamlines since no diffusion will occur in the condensed phase. The species in the gaseous state, however, may diffuse normal to the streamline, and so "separate" from the species in the condensed phase.

The assumption of phase equilibrium means that gaseous species condense instantaneously. Therefore, for extremely adverse conditions of low temperature and high pressure, severe condensation of O_2 and N_2 may be encountered. The rate of change of gas phase concentrations becomes exceedingly large, giving rise to a sheet of condensed air. This sheet has a shock-like behavior which halts further calculation. To resolve this problem, special analytical treatment in the form of "jump" conditions would be required to cross this sheet of condensed air. At present, no such effort has been initiated since, for most cases of interest this problem will not arise.

III. INPUT-OUTPUT FORMATS

CARD	COLUMN	DESCRIPTION	FORMAT
1	1-72	Title information	12A6
2	1-10	Input control number	I10
		= 0 Input supplied on cards	
		> 0 Input supplied on tape	
	11-20	Control number for tape	I10
		≤ 0 No binary restart tape record	
		> 0 Write record on restart tape	
	21-30	Total number of initial grid points (≤ 50)	
3	1-10	Tolerance for negative β_{H_2} mass fraction in Chemistry	E10.5
	11-20	" " " β_{O_2} " "	"
	21-30	" " " β_{N_2} " "	"
	31-40	" " " β_{H_2O} " "	"
	41-50	" " " β_{O} " "	"
	51-60	" " " β_{H} " "	"
	61-70	" " " β_{OH} " "	"
4	1-10	Tolerance for Sum of Species	E10.5
	11-20	Factor by which to multiply the Δx given by stability criterion	"
	21-30	Tolerance for addition of a grid point at outer edge	"
	31-40	Tolerance for conservation of mass flow	"
	41-50	Tolerance for conservation of enthalpy in temperature convergence	"
	51-60	% of T for ΔT in temperature iteration. If zero is used, the program sets this number.	"
	61-70	Tolerance below which ΔB_{H_2} is considered zero and no diffusion takes place in two phase ragion.	"

CARD	COLUMN	DESCRIPTION	FORMAT
5	1-10	Value of temperature (^{o}k) below which two phase calculation is entered and above which chemistry calculation is entered.	E10.5
	11-20	Δx (ft) for print out of results	E10.5
	21-30	Δx (ft) for writing binary record on tape	E10.5
	31-40	Δx (ft) to continue calculation after flow field has become completely gaseous	E10.5
	41-50	Limiting value of x (ft)	E10.5
6	1-10	Lewis number	E10.5
	11-20	Prandtl number	E10.5
	21-30	P_o } coefficients in prescribed	E10.5
	32-40	P_1 } pressure poly nominal	"
	41-50	P_2 } $P_e(\frac{lb}{ft^2}) = P_o + P_1 x + P_2 x^2 + P_3 x^3 + P_4 x^4$	"
	51-60	P_3 }	"
	61-70	P_4 }	"
7	1-10	β_{H_2} } Values of species mass fractions in the jet	E10.5
	11-20	β_{O_2} }	"
	21-30	β_{N_2} }	"
	31-40	β_{H_2O} }	"
	41-50	β_{H_2} gas }	"
	51-60	β_{O_2} gas }	"
	61-70	β_{N_2} gas }	"
8	1-10	β_{H_2O} gas }	E10.5
	11-20	$z = \frac{\rho gas}{\rho tot}$ } jet values	"
	21-30	$u_j(\frac{ft}{sec})$ }	"
	31-40	jet radius (ft.)	E10.5
	41-50	Φ = relative humidity	E10.5
	51-60	Temperature (^{o}k) } free stream	E10.5
	61-70	Velocity $\left(\frac{ft}{sec}\right)$ } free stream	"
9	1-10	Temperature (^{o}k) of jet. If set to zero the temperature is calculated.	E10.5

NOTE: Cards 1-5 must be supplied for all input options. Since reading input from a binary record on tape B6 is basically a restart option, the parameters given on cards 2-5 are contained in the binary record. Therefore, cards 2-5 should be blank except for the correct input control number. If a change of any parameter supplied by cards 2-5 is desired, the new value should be punched in the proper position, and will then be picked up by the program.

Sample output may be found at the rear of this section.

* This input format is for the existing program which is written in FortranII. Slight modifications will appear in Fortran IV revisions.

IV. Summary:

A computer program for the calcuation of laminar axisymmetric two phase free mixing in the presence of chemical reaction has been presented. The consequences of the assumptions made concerning transport processes in the condensed state have been discussed. A detailed input format has been provided. It should be noted that the current method forms a basis for the consideration of more complex models.

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4. Ferri, A., Moretti, G., and Slutsky, S., Mixing Processes in Supersonic Combustion, J.Soc. Indust. Appl. Math., Vol. 13, No.1, March 1965.

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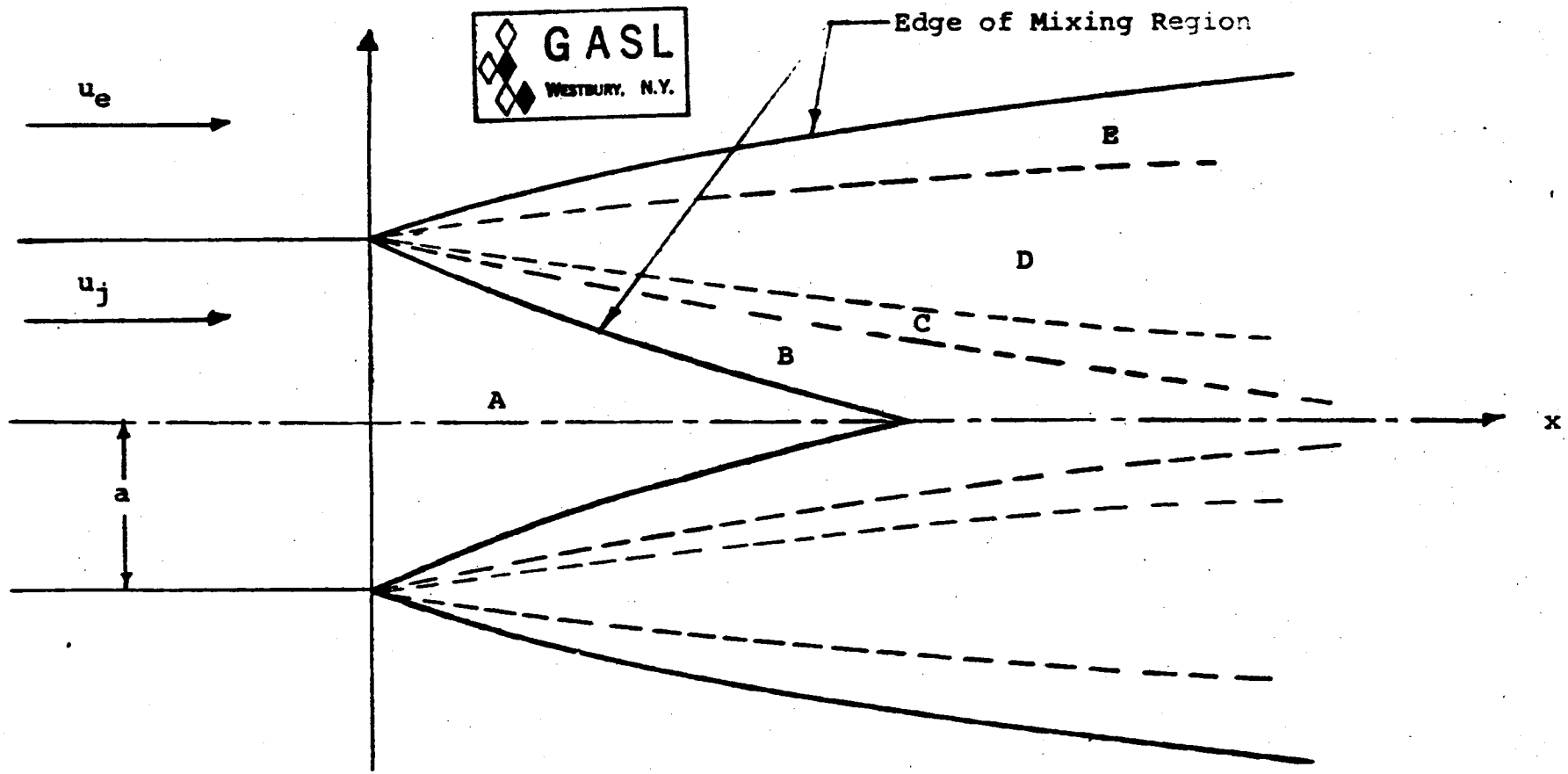

A = Hydrogen core
B = Two phase mixture
C = Low temperature gas mixture
D = High temperature gas mixture with chemistry
E = Gas

FIGURE 1 - SCHEMATIC OF FLOW REGION

GASL

VISCOUS TWO-PHASE JET WITH CHEMISTRY

TWO PHASE AND CHEMISTRY 2/13/65

INITIAL INPUT PARAMETERS

LEWIS NUMBER= 1.0000000E 00 PRANDTL NUMBER= 6.9999999E-01

COEFFICIENTS OF PRESSURE EQUATION P=P(0)+P(1)*X+P(2)*X**2+P(3)*X**3+P(4)*X**4 WHERE (P)=LB/FT**2
P(0)= 1.1400000E 02 P(1)= 0. P(2)= 0. P(3)= 0. P(4)= 0.

INPUT PARAMETERS FOR FREE STREAM REGION

RELATIVE HUMIDITY= 1.0000000E-05 TEMPERATURE= 1.5000000E 03 DEG-KELVIN VELOCITY= 1.0000000E 03 FT/SEC

INPUT PARAMETERS FOR JET REGION

BETA (H2) = 1.0000000E 00 BETA-G (H2) = 1.0000000E 00 Z= 1.0000000E 00
BETA (O2) = 1.0000000E-12 BETA-G (O2) = 1.0000000E-12 VELOCITY= 4.0000000E 02 FT/SEC
BETA (N2) = 1.0000000E-12 BETA-G (N2) = 1.0000000E-12 Y AT EDGE OF JET= 1.6666666E-01 FT
BETA (H2O)= 1.0000000E-12 BETA-G (H2O)= 1.0000000E-12 TEMPERATURE= 4.0000000E 01 DEG-KELVIN

LIMITING VALUE OF TEMPERATURE FOR TWO-PHASE REGION= 4.0000000E 02 DEG-KELVIN
TERMINATING VALUE OF X= 1.0000000E 00 FEET
STEP SIZE IN X FOR PRINT OUT= 0. FEET
STEP SIZE IN X FOR WRITING BINARY RECORD ON TAPE 86= 0. FEET
STEP SIZE IN X TO CARRY CALCULATION AFTER ALL GRID POINTS IN GAS= 1.0000000E 00 FEET

TOLERANCES USED IN CALCULATION

BETA (H2)	BETA (O2)	BETA (N2)	BETA (H2O)	BETA (O)	BETA (H)	BETA (OH)
9.9999999E-05	9.9999999E-05	9.9999999E-05	9.9999999E-05	9.9999999E-05	9.9999999E-05	9.9999999E-05

SUMMATIONS= 1.0000000E 00 EDGE CHECKS= 1.0000000E-05 M(DOT)/2*PI= 1.0000000E 02

TEMPERATURE CONVERGENCE= 9.9999999E-05 RATIO FOR DELTA T= 4.9999999E-04RATIO FOR STABILITY= 5.0000000E-01

DIMENSION OF ALL OUTPUT PARAMETERS

ENTHALPIES (H(M),H(G))..... (FT/SEC)**2
DENSITIES (RHO(M),RHO(G))..... (SLUGS/FT**3)
TEMPERATURES..... (DEGREES KELVIN)
VISCOSITIES (MU(G))..... (LB*SEC/FT**2)
VELOCITIES..... (FT/SEC)
PRESSURES..... (LB/FT**2)
LENGTHS..... (FEET)
PSI.... ((LB*SEC/FT)**1/2)

BINARY RECORD NO 1 WRITTEN ON TAPE 86 FOR X= 0.

X= 4.1122705E-01 TWO PHASE AND CHEMISTRY 2/19/65 CONTINUED STEP NO 88

PRESSURE= 1.1400000E 02 M(DOT)= 3.6746573E-04

VELOCITY	TEMPERATURE	Z	H(M)	H(G)	RHO(M)	RHO(G)	MU(G)	Y	PSI	
4.0000E 02	4.0000E 01	1.0000E 00	-3.9044E 07	-3.9044E 07	6.4139E-05	6.4139E-05	4.3053E-06	0.	0.	0
4.0000E 02	4.0000E 01	1.0000E 00	-3.9044E 07	-3.9044E 07	6.4139E-05	6.4139E-05	4.3053E-06	1.5152E-02	2.4269E-03	1
4.0000E 02	4.0000E 01	1.0000E 00	-3.9044E 07	-3.9044E 07	6.4139E-05	6.4139E-05	4.3053E-06	3.0303E-02	4.8537E-03	2
4.0000E 02	4.0000E 01	1.0000E 00	-3.9044E 07	-3.9044E 07	6.4139E-05	6.4139E-05	4.3053E-06	4.5455E-02	7.2806E-03	3
4.0000E 02	4.0000E 01	1.0000E 00	-3.9044E 07	-3.9044E 07	6.4139E-05	6.4139E-05	4.3053E-06	6.0606E-02	9.7075E-03	4
4.0000E 02	4.0000E 01	1.0000E 00	-3.9044E 07	-3.9044E 07	6.4139E-05	6.4139E-05	4.3053E-06	7.5758E-02	1.2134E-02	5
4.0000E 02	4.0000E 01	1.0000E 00	-3.9044E 07	-3.9044E 07	6.4139E-05	6.4139E-05	4.3053E-06	9.0909E-02	1.4561E-02	6
4.0000E 02	4.0000E 01	1.0000E 00	-3.9044E 07	-3.9044E 07	6.4139E-05	6.4139E-05	4.3053E-06	1.0606E-01	1.6988E-02	7
4.0178E 02	4.1482E 01	9.9906E-01	-3.8538E 07	-3.8572E 07	6.2298E-05	6.2239E-05	4.4465E-06	1.2140E-01	1.9415E-02	8
4.2358E 02	5.4155E 01	10.0000E-01	-3.4462E 07	-3.4462E 07	5.0673E-05	5.0673E-05	5.6223E-06	1.3817E-01	2.1842E-02	9
5.5299E 02	1.2300E 02	9.7350E-01	-2.2597E 07	-1.8428E 07	3.0460E-05	2.9653E-05	1.1544E-05	1.5829E-01	2.4269E-02	10
7.4235E 02	2.4448E 02	8.4036E-01	-3.5447E 07	-9.2847E 06	2.8041E-05	2.3564E-05	2.2240E-05	1.7795E-01	2.6696E-02	11
8.5888E 02	3.9860E 02	1.0000E 00	-5.8300E 06	-5.8300E 06	2.1839E-05	2.1839E-05	3.5945E-05	1.9633E-01	2.9122E-02	12
9.2247E 02	5.8018E 02	1.0000E 00	-4.1975E 06	-4.1975E 06	2.1170E-05	2.1170E-05	5.1111E-05	2.1502E-01	3.1549E-02	13
9.5623E 02	7.6856E 02	1.0000E 00	-3.2962E 06	-3.2962E 06	2.0904E-05	2.0904E-05	6.5270E-05	2.3298E-01	3.3976E-02	14
9.7502E 02	9.5222E 02	1.0000E 00	-2.7860E 06	-2.7860E 06	2.0789E-05	2.0789E-05	7.7453E-05	2.5053E-01	3.6403E-02	15
9.8579E 02	1.1248E 03	1.0000E 00	-2.6082E 06	-2.6082E 06	2.0663E-05	2.0663E-05	8.7433E-05	2.6786E-01	3.8830E-02	16
9.9203E 02	1.2696E 03	1.0000E 00	-2.6469E 06	-2.6469E 06	2.0664E-05	2.0664E-05	9.4963E-05	2.8506E-01	4.1257E-02	17
9.9563E 02	1.3568E 03	1.0000E 00	-2.4774E 06	-2.4774E 06	2.1122E-05	2.1122E-05	9.9613E-05	3.0197E-01	4.3684E-02	18
9.9767E 02	1.4100E 03	1.0000E 00	-2.3233E 06	-2.3233E 06	2.1625E-05	2.1625E-05	1.0267E-04	3.1845E-01	4.6111E-02	19
9.9880E 02	1.4441E 03	1.0000E 00	-2.2623E 06	-2.2623E 06	2.2021E-05	2.2021E-05	1.0474E-04	3.3460E-01	4.8537E-02	20
9.9941E 02	1.4658E 03	1.0000E 00	-2.2663E 06	-2.2663E 06	2.2304E-05	2.2304E-05	1.0610E-04	3.5051E-01	5.0964E-02	21
9.9972E 02	1.4789E 03	1.0000E 00	-2.3029E 06	-2.3029E 06	2.2496E-05	2.2496E-05	1.0696E-04	3.6629E-01	5.3391E-02	22
9.9987E 02	1.4865E 03	1.0000E 00	-2.3499E 06	-2.3499E 06	2.2621E-05	2.2621E-05	1.0749E-04	3.8199E-01	5.5818E-02	23
9.9995E 02	1.4907E 03	1.0000E 00	-2.3947E 06	-2.3947E 06	2.2698E-05	2.2698E-05	1.0780E-04	3.9766E-01	5.8245E-02	24
9.9998E 02	1.4929E 03	1.0000E 00	-2.4311E 06	-2.4311E 06	2.2743E-05	2.2743E-05	1.0797E-04	4.1333E-01	6.0672E-02	25
9.9999E 02	1.4939E 03	1.0000E 00	-2.4571E 06	-2.4571E 06	2.2770E-05	2.2770E-05	1.0805E-04	4.2900E-01	6.3099E-02	26
10.0000E 02	1.4942E 03	1.0000E 00	-2.4739E 06	-2.4739E 06	2.2786E-05	2.2786E-05	1.0807E-04	4.4468E-01	6.5526E-02	27
10.0000E 02	1.4942E 03	1.0000E 00	-2.4844E 06	-2.4844E 06	2.2797E-05	2.2797E-05	1.0808E-04	4.6039E-01	6.7952E-02	28
10.0000E 02	1.4941E 03	1.0000E 00	-2.4905E 06	-2.4905E 06	2.2804E-05	2.2804E-05	1.0807E-04	4.7611E-01	7.0379E-02	29
10.0000E 02	1.4940E 03	1.0000E 00	-2.4932E 06	-2.4932E 06	2.2807E-05	2.2807E-05	1.0807E-04	4.9185E-01	7.2806E-02	30
10.0000E 02	1.4941E 03	1.0000E 00	-2.4928E 06	-2.4928E 06	2.2807E-05	2.2807E-05	1.0807E-04	5.0761E-01	7.5233E-02	31
10.0000E 02	1.4943E 03	1.0000E 00	-2.4896E 06	-2.4896E 06	2.2804E-05	2.2804E-05	1.0808E-04	5.2339E-01	7.7660E-02	32
10.0000E 02	1.4947E 03	1.0000E 00	-2.4835E 06	-2.4835E 06	2.2797E-05	2.2797E-05	1.0810E-04	5.3919E-01	8.0087E-02	33
10.0000E 02	1.4954E 03	1.0000E 00	-2.4745E 06	-2.4745E 06	2.2788E-05	2.2788E-05	1.0813E-04	5.5502E-01	8.2514E-02	34
10.0000E 02	1.4962E 03	1.0000E 00	-2.4622E 06	-2.4622E 06	2.2775E-05	2.2775E-05	1.0817E-04	5.7086E-01	8.4941E-02	35
10.0000E 02	1.4973E 03	1.0000E 00	-2.4464E 06	-2.4464E 06	2.2758E-05	2.2758E-05	1.0822E-04	5.8673E-01	8.7367E-02	36
10.0000E 02	1.4987E 03	1.0000E 00	-2.4265E 06	-2.4265E 06	2.2738E-05	2.2738E-05	1.0828E-04	6.0262E-01	8.9794E-02	37
1.0000E 03	1.5000E 03	1.0000E 00	-2.4071E 06	-2.4071E 06	2.2717E-05	2.2717E-05	1.0834E-04	6.1854E-01	9.2221E-02	38

X= 4.1122705E-01 TWO PHASE AND CHEMISTRY 2/19/65 CONTINUED STEP NO 88

BETA (H2)	BETA (O2)	BETA (N2)	BETA (H2O)	BETA (O)	BETA (H)	BETA (OH)	SUMMATION	
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	0
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	1
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	2
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	3
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	4
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	5
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	6
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	7
9.9228932E-01	1.8546328E-03	5.8559210E-03	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	9.9999986E-01	8
9.3000633E-01	1.6318560E-02	5.3674850E-02	6.4842551E-13	1.0000000E-12	1.0000000E-12	1.0000000E-12	9.9999973E-01	9
6.6308709E-01	7.2233358E-02	2.3817946E-01	2.6499790E-02	1.0000000E-12	1.0000000E-12	1.0000000E-12	9.9999969E-01	10
3.3761410E-01	1.0879002E-01	3.5858907E-01	1.9500534E-01	1.0000000E-12	1.0000000E-12	1.0000000E-12	9.9999852E-01	11
2.3833148E-01	1.5969214E-01	5.2654819E-01	7.5420481E-02	3.0331044E-08	3.3891371E-06	1.9695913E-08	9.9999571E-01	12
1.4496420E-01	1.7596166E-01	5.8221446E-01	9.6809386E-02	4.9095619E-07	4.9145094E-05	5.3298756E-07	9.9999985E-01	13
9.0970180E-02	1.8261405E-01	6.1383408E-01	1.1233389E-01	6.3557722E-06	2.3478806E-04	6.6356667E-06	9.9999998E-01	14
5.6371116E-02	1.7820005E-01	6.3312173E-01	1.3131540E-01	6.7911627E-05	8.6781505E-04	5.5979826E-05	9.9999998E-01	15
3.0785305E-02	1.5592043E-01	6.4544342E-01	1.6440516E-01	5.9541503E-04	2.4020741E-03	4.4818527E-04	9.9999998E-01	16
1.1691464E-02	1.2357791E-01	6.5352730E-01	2.0075497E-01	4.0398913E-03	4.0142162E-03	2.3942265E-03	9.9999996E-01	17
4.3742750E-03	1.2356311E-01	6.5886398E-01	1.9464550E-01	1.0727226E-02	3.1309091E-03	4.6949902E-03	9.9999998E-01	18
2.1065785E-03	1.4153207E-01	6.6236545E-01	1.7475859E-01	1.2129507E-02	1.8741159E-03	5.2336842E-03	9.9999998E-01	19
1.1712951E-03	1.6015539E-01	6.6463751E-01	1.5789160E-01	1.0237177E-02	1.0225628E-03	4.8844640E-03	9.9999998E-01	20
6.8419847E-04	1.7496856E-01	6.6608556E-01	1.4611489E-01	7.4419169E-03	5.2450424E-04	4.1803612E-03	9.9999998E-01	21
4.0626351E-04	1.8567467E-01	6.6698531E-01	1.3849483E-01	4.8258835E-03	2.5324773E-04	3.3597955E-03	9.9999999E-01	22
2.4882224E-04	1.9300738E-01	6.6752694E-01	1.3381318E-01	2.7653474E-03	1.1469617E-04	2.5236244E-03	9.9999998E-01	23
1.6564198E-04	1.9780875E-01	6.6784132E-01	1.3108954E-01	1.3350388E-03	4.8289766E-05	1.7114290E-03	9.9999998E-01	24
1.2087983E-04	2.0071154E-01	6.6801660E-01	1.2965536E-01	5.0126566E-04	1.8070810E-05	9.7627765E-04	9.9999998E-01	25
8.5516350E-05	2.0218264E-01	6.6811026E-01	1.2904623E-01	1.3854315E-04	5.4156492E-06	4.3138720E-04	9.9999998E-01	26
5.1874784E-05	2.0275456E-01	6.6815815E-01	1.2886163E-01	2.9287666E-05	1.2142743E-06	1.4327433E-04	9.9999995E-01	27
2.6704479E-05	2.0292601E-01	6.6818155E-01	1.2882317E-01	5.1610270E-06	2.1071422E-07	3.7189655E-05	9.9999998E-01	28
1.2104047E-05	2.0296849E-01	6.6819248E-01	1.2881805E-01	8.0782109E-07	3.0473517E-08	8.0341531E-06	9.9999998E-01	29
4.9518182E-06	2.0297803E-01	6.6819735E-01	1.2881802E-01	1.1629508E-07	3.9213321E-09	1.5252641E-06	9.9999998E-01	30
1.8165030E-06	2.0298024E-01	6.6819944E-01	1.2881821E-01	1.5742180E-08	4.7147393E-10	2.6506494E-07	9.9999998E-01	31
5.6275256E-07	2.0298094E-01	6.6820030E-01	1.2881815E-01	2.0416690E-09	5.5085070E-11	4.3585888E-08	9.9999998E-01	32
1.1907273E-07	2.0298137E-01	6.6820063E-01	1.2881787E-01	2.6053783E-10	6.6065080E-12	7.1172349E-09	9.9999998E-01	33
5.9471705E-10	2.0298186E-01	6.6820077E-01	1.2881737E-01	3.6102360E-11	9.9549963E-13	1.3508928E-09	9.9999998E-01	34
1.0000000E-12	2.0298253E-01	6.6820083E-01	1.2881664E-01	8.1966652E-12	3.2944223E-13	4.3288696E-10	1.0000000E 00	35
1.0000000E-12	2.0298350E-01	6.6820089E-01	1.2881563E-01	4.5927043E-12	2.5588412E-13	2.4106699E-10	1.0000000E 00	36
1.0000000E-12	2.0298486E-01	6.6820092E-01	1.2881421E-01	3.8108352E-12	3.5038320E-13	1.3400689E-10	1.0000000E 00	37
1.0000000E-12	2.0298674E-01	6.6820097E-01	1.2881229E-01	1.0000000E-12	1.0000000E-12	1.0000000E-12	9.9999999E-01	38

X= 4.1122705E-01 TWO PHASE AND CHEMISTRY 2/19/65 CONTINUED STEP NO 88

Y (H2)	Y (O2)	Y (N2)	Y (H2O)	Y (O)	Y (H)	Y (OH)	SUMMATION	
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	0
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	1
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	2
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	3
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	4
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	5
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	6
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	7
9.9322785E-01	1.0545881E-03	5.7175552E-03	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	9.9999999E-01	8
9.3000675E-01	1.6318446E-02	5.3674809E-02	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	9
6.8113789E-01	7.4199640E-02	2.4466247E-01	2.9631701E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	9.9999999E-01	10
4.0175169E-01	1.2945718E-01	4.2671079E-01	4.2080349E-02	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E 00	11
2.3833148E-01	1.5969214E-01	5.2654819E-01	7.5420481E-02	3.0331044E-08	3.3891371E-06	1.9695913E-08	9.9999571E-01	12
1.4496420E-01	1.7596166E-01	5.8221446E-01	9.6809386E-02	4.9095619E-07	4.9145094E-05	5.3298756E-07	9.9999985E-01	13
9.0970180E-02	1.8261405E-01	6.1383408E-01	1.1233389E-01	6.3557722E-06	2.3478806E-04	6.6356667E-06	9.9999998E-01	14
5.6371116E-02	1.7820005E-01	6.3312173E-01	1.3131540E-01	6.7911627E-05	8.6781505E-04	5.5979826E-05	9.9999998E-01	15
3.0785305E-02	1.5592043E-01	6.4544342E-01	1.6440516E-01	5.9541503E-04	2.4020741E-03	4.4818527E-04	9.9999998E-01	16
1.1691464E-02	1.2357791E-01	6.5352730E-01	2.0075497E-01	4.0398913E-03	4.0142162E-03	2.3942265E-03	9.9999996E-01	17
4.3742750E-03	1.2356311E-01	6.5886398E-01	1.9464550E-01	1.0727226E-02	3.1309091E-03	4.6949902E-03	9.9999998E-01	18
2.1065785E-03	1.4153207E-01	6.6236545E-01	1.7475859E-01	1.2129507E-02	1.8741159E-03	5.2336842E-03	9.9999998E-01	19
1.1712951E-03	1.6015539E-01	6.6463751E-01	1.5789160E-01	1.0237177E-02	1.0225628E-03	4.8844640E-03	9.9999998E-01	20
6.8419847E-04	1.7496856E-01	6.6608556E-01	1.4611489E-01	7.4419169E-03	5.2450424E-04	4.1803612E-03	9.9999998E-01	21
4.0626351E-04	1.8567467E-01	6.6698531E-01	1.3849483E-01	4.8258835E-03	2.5324773E-04	3.3597955E-03	9.9999999E-01	22
2.4882224E-04	1.9300738E-01	6.6752694E-01	1.3381318E-01	2.7653474E-03	1.1469617E-04	2.5236244E-03	9.9999998E-01	23
1.6564198E-04	1.9780875E-01	6.6784132E-01	1.3108954E-01	1.3350388E-03	4.8289766E-05	1.7114290E-03	9.9999998E-01	24
1.2087983E-04	2.0071154E-01	6.6801660E-01	1.2965536E-01	5.0126566E-04	1.8070810E-05	9.7627765E-04	9.9999998E-01	25
8.5516350E-05	2.0218264E-01	6.6811026E-01	1.2904623E-01	1.3854315E-04	5.4156492E-06	4.3138720E-04	9.9999998E-01	26
5.1874784E-05	2.0275456E-01	6.6815815E-01	1.2886163E-01	2.9287666E-05	1.2142743E-06	1.4327433E-04	9.9999995E-01	27
2.6704479E-05	2.0292601E-01	6.6818155E-01	1.2882317E-01	5.1610270E-06	2.1071422E-07	3.7189655E-05	9.9999998E-01	28
1.2104047E-05	2.0296849E-01	6.6819248E-01	1.2881805E-01	8.0782109E-07	3.0473517E-08	8.0341531E-06	9.9999998E-01	29
4.9518182E-06	2.0297803E-01	6.6819735E-01	1.2881802E-01	1.1629508E-07	3.9213321E-09	1.5252641E-06	9.9999998E-01	30
1.8165030E-06	2.0298024E-01	6.6819944E-01	1.2881821E-01	1.5742180E-08	4.7147393E-10	2.6506494E-07	9.9999998E-01	31
5.6275256E-07	2.0298094E-01	6.6820030E-01	1.2881815E-01	2.0416690E-09	5.5085070E-11	4.3585888E-08	9.9999998E-01	32
1.1907273E-07	2.0298137E-01	6.6820063E-01	1.2881787E-01	2.6053783E-10	6.6065080E-12	7.1172349E-09	9.9999998E-01	33
5.9471705E-10	2.0298186E-01	6.6820077E-01	1.2881737E-01	3.6102360E-11	9.9549963E-13	1.3508928E-09	9.9999998E-01	34
1.0000000E-12	2.0298253E-01	6.6820083E-01	1.2881664E-01	8.1966652E-12	3.2944223E-13	4.3288696E-10	1.0000000E 00	35
1.0000000E-12	2.0298350E-01	6.6820089E-01	1.2881563E-01	4.5927043E-12	2.5588412E-13	2.4106699E-10	1.0000000E 00	36
1.0000000E-12	2.0298486E-01	6.6820092E-01	1.2881421E-01	3.8108352E-12	3.5038320E-13	1.3400689E-10	1.0000000E 00	37
1.0000000E-12	2.0298674E-01	6.6820097E-01	1.2881229E-01	1.0000000E-12	1.0000000E-12	1.0000000E-12	9.9999999E-01	38

X= 4.1122705E-01 TWO PHASE AND CHEMISTRY 2/19/65 CONTINUED STEP NO 88

X (H2)	X (O2)	X (N2)	X (H2O)	X (O)	X (H)	X (OH)	SUMMATION	
9.9999999E-01	1.8045530E-12	1.5797734E-12	1.0159633E-12	0.	0.	0.	9.9999999E-01	0
9.9999999E-01	1.8045530E-12	1.5797734E-12	1.0159633E-12	0.	0.	0.	9.9999999E-01	1
9.9999999E-01	1.8045530E-12	1.5797734E-12	1.0159633E-12	0.	0.	0.	9.9999999E-01	2
9.9999999E-01	1.8045530E-12	1.5797734E-12	1.0159633E-12	0.	0.	0.	9.9999999E-01	3
9.9999999E-01	1.8045530E-12	1.5797734E-12	1.0159633E-12	0.	0.	0.	9.9999999E-01	4
9.9999999E-01	1.8045530E-12	1.5797734E-12	1.0159633E-12	0.	0.	0.	9.9999999E-01	5
9.9999999E-01	1.8045530E-12	1.5797734E-12	1.0159633E-12	0.	0.	0.	9.9999999E-01	6
9.9999999E-01	1.8045530E-12	1.5797734E-12	1.0159633E-12	0.	0.	0.	9.9999999E-01	7
1.5658232E-05	8.4782006E-01	1.5216428E-01	1.0583974E-09	0.	0.	0.	9.9999999E-01	8
3.5883408E-02	6.2520077E-01	3.3891249E-01	3.3404107E-06	0.	0.	0.	1.0000000E 00	9
1.9335705E-07	2.9972262E-06	2.9387503E-05	9.9996742E-01	0.	0.	0.	1.0000000E 00	10
3.6598225E-08	9.9846350E-08	2.8924512E-06	9.9999697E-01	0.	0.	0.	1.0000000E 00	11
0.	0.	0.	0.	0.	0.	0.	0.	12
0.	0.	0.	0.	0.	0.	0.	0.	13
0.	0.	0.	0.	0.	0.	0.	0.	14
0.	0.	0.	0.	0.	0.	0.	0.	15
0.	0.	0.	0.	0.	0.	0.	0.	16
0.	0.	0.	0.	0.	0.	0.	0.	17
0.	0.	0.	0.	0.	0.	0.	0.	18
0.	0.	0.	0.	0.	0.	0.	0.	19
0.	0.	0.	0.	0.	0.	0.	0.	20
0.	0.	0.	0.	0.	0.	0.	0.	21
0.	0.	0.	0.	0.	0.	0.	0.	22
0.	0.	0.	0.	0.	0.	0.	0.	23
0.	0.	0.	0.	0.	0.	0.	0.	24
0.	0.	0.	0.	0.	0.	0.	0.	25
0.	0.	0.	0.	0.	0.	0.	0.	26
0.	0.	0.	0.	0.	0.	0.	0.	27
0.	0.	0.	0.	0.	0.	0.	0.	28
0.	0.	0.	0.	0.	0.	0.	0.	29
0.	0.	0.	0.	0.	0.	0.	0.	30
0.	0.	0.	0.	0.	0.	0.	0.	31
0.	0.	0.	0.	0.	0.	0.	0.	32
0.	0.	0.	0.	0.	0.	0.	0.	33
0.	0.	0.	0.	0.	0.	0.	0.	34
0.	0.	0.	0.	0.	0.	0.	0.	35
0.	0.	0.	0.	0.	0.	0.	0.	36
0.	0.	0.	0.	0.	0.	0.	0.	37
0.	0.	0.	0.	0.	0.	0.	0.	38

X= 4.1122705E-01 TWO PHASE AND CHEMISTRY 2/19/65 CONTINUED STEP NO 88

BETA-G (H2)	BETA-G (O2)	BETA-G (N2)	BETA-G (H2O)	BETA-G (O)	BETA-G (H)	BETA-G (OH)	
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	0
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	2
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	3
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	4
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	5
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	6
1.0000000E 00	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	7
9.9228928E-01	1.0535915E-03	5.7121522E-03	9.9905500E-13	1.0000000E-12	1.0000000E-12	1.0000000E-12	8
9.3000629E-01	1.6318438E-02	5.3674783E-02	9.9999949E-13	1.0000000E-12	1.0000000E-12	1.0000000E-12	9
6.6308707E-01	7.2233277E-02	2.3817867E-01	2.8846433E-12	1.0000000E-12	1.0000000E-12	1.0000000E-12	10
3.3761409E-01	1.0879001E-01	3.5858860E-01	3.5362436E-02	1.0000000E-12	1.0000000E-12	1.0000000E-12	11
2.3833148E-01	1.5969214E-01	5.2654819E-01	7.5420481E-02	3.0331044E-08	3.3891371E-06	1.9695913E-08	12
1.4496420E-01	1.7596166E-01	5.8221446E-01	9.6809386E-02	4.9095619E-07	4.9145094E-05	5.3298756E-07	13
9.0970180E-02	1.8261405E-01	6.1383408E-01	1.1233389E-01	6.3557722E-06	2.3478806E-04	6.6356667E-06	14
5.6371116E-02	1.7820005E-01	6.3312173E-01	1.3131540E-01	6.7911627E-05	8.6781505E-04	5.5979826E-05	15
3.0785305E-02	1.5592043E-01	6.4544342E-01	1.6440516E-01	5.9541503E-04	2.4020741E-03	4.4818527E-04	16
1.1691464E-02	1.2357791E-01	6.5352730E-01	2.0075497E-01	4.0398913E-03	4.0142162E-03	2.3942265E-03	17
4.3742750E-03	1.2356311E-01	6.5886398E-01	1.9464550E-01	1.0727226E-02	3.1309091E-03	4.6949902E-03	18
2.1065785E-03	1.4153207E-01	6.6236545E-01	1.7475859E-01	1.2129507E-02	1.8741159E-03	5.2336842E-03	19
1.1712951E-03	1.6015539E-01	6.6463751E-01	1.5789160E-01	1.0237177E-02	1.0225628E-03	4.8844640E-03	20
6.8419847E-04	1.7496856E-01	6.6608556E-01	1.4611489E-01	7.4419169E-03	5.2450424E-04	4.1803612E-03	21
4.0626351E-04	1.8567467E-01	6.6698531E-01	1.3849483E-01	4.8258835E-03	2.5324773E-04	3.3597955E-03	22
2.4882224E-04	1.9300738E-01	6.6752694E-01	1.3381318E-01	2.7653474E-03	1.1469617E-04	2.5236244E-03	23
1.6564198E-04	1.9780875E-01	6.6784132E-01	1.3108954E-01	1.3350388E-03	4.8289766E-05	1.7114290E-03	24
1.2087983E-04	2.0071154E-01	6.6801660E-01	1.2965536E-01	5.0126566E-04	1.8070810E-05	9.7627765E-04	25
8.5516350E-05	2.0218264E-01	6.6811026E-01	1.2904623E-01	1.3854315E-04	5.4156492E-06	4.3138720E-04	26
5.1874784E-05	2.0275456E-01	6.6815815E-01	1.2886163E-01	2.9287666E-05	1.2142743E-06	1.4327433E-04	27
2.6704479E-05	2.0292601E-01	6.6818155E-01	1.2882317E-01	5.1610270E-06	2.1071422E-07	3.7189655E-05	28
1.2104047E-05	2.0296849E-01	6.6819248E-01	1.2881805E-01	8.0782109E-07	3.0473517E-08	8.0341531E-06	29
4.9518182E-06	2.0297803E-01	6.6819735E-01	1.2881802E-01	1.1629508E-07	3.9213321E-09	1.5252641E-06	30
1.8165030E-06	2.0298024E-01	6.6819944E-01	1.2881821E-01	1.5742180E-08	4.7147393E-10	2.6506494E-07	31
5.6275256E-07	2.0298094E-01	6.6820030E-01	1.2881815E-01	2.0416690E-09	5.5085070E-11	4.3585888E-08	32
1.1907273E-07	2.0298137E-01	6.6820063E-01	1.2881787E-01	2.6053783E-10	6.6065080E-12	7.1172349E-09	33
5.9471705E-10	2.0298186E-01	6.6820077E-01	1.2881737E-01	3.6102360E-11	9.9549963E-13	1.3508928E-09	34
1.0000000E-12	2.0298253E-01	6.6820083E-01	1.2881664E-01	8.1966652E-12	3.2944223E-13	4.3288696E-10	35
1.0000000E-12	2.0298350E-01	6.6820089E-01	1.2881563E-01	4.5927043E-12	2.5588412E-13	2.4106699E-10	36
1.0000000E-12	2.0298486E-01	6.6820092E-01	1.2881421E-01	3.8108352E-12	3.5038320E-13	1.3400689E-10	37
1.0000000E-12	2.0298674E-01	6.6820097E-01	1.2881229E-01	1.0000000E-12	1.0000000E-12	1.0000000E-12	38

X= 4.1122705E-01 TWO PHASE AND CHEMISTRY 2/19/65 CONTINUED STEP NO 88

BETA-P (H2)	BETA-P (O2)	BETA-P (N2)	BETA-P (H2O)	BETA-P (O)	BETA-P (H)	BETA-P (OH)	
0.	0.	0.	0.	0.	0.	0.	0
0.	0.	0.	0.	0.	0.	0.	1
0.	0.	0.	0.	0.	0.	0.	2
0.	0.	0.	0.	0.	0.	0.	3
0.	0.	0.	0.	0.	0.	0.	4
0.	0.	0.	0.	0.	0.	0.	5
0.	0.	0.	0.	0.	0.	0.	6
0.	0.	0.	0.	0.	0.	0.	7
3.7252903E-08	8.0104131E-04	1.4376873E-04	9.4497706E-16	0.	0.	0.	8
2.9802322E-08	1.2153760E-07	6.7520887E-08	-3.5157397E-13	0.	0.	0.	9
2.2351742E-08	8.1025064E-08	7.8417361E-07	2.6499790E-02	0.	0.	0.	10
1.4901161E-08	1.6763806E-08	4.6938658E-07	1.5964290E-01	0.	0.	0.	11
0.	0.	0.	0.	0.	0.	0.	12
0.	0.	0.	0.	0.	0.	0.	13
0.	0.	0.	0.	0.	0.	0.	14
0.	0.	0.	0.	0.	0.	0.	15
0.	0.	0.	0.	0.	0.	0.	16
0.	0.	0.	0.	0.	0.	0.	17
0.	0.	0.	0.	0.	0.	0.	18
0.	0.	0.	0.	0.	0.	0.	19
0.	0.	0.	0.	0.	0.	0.	20
0.	0.	0.	0.	0.	0.	0.	21
0.	0.	0.	0.	0.	0.	0.	22
0.	0.	0.	0.	0.	0.	0.	23
0.	0.	0.	0.	0.	0.	0.	24
0.	0.	0.	0.	0.	0.	0.	25
0.	0.	0.	0.	0.	0.	0.	26
0.	0.	0.	0.	0.	0.	0.	27
0.	0.	0.	0.	0.	0.	0.	28
0.	0.	0.	0.	0.	0.	0.	29
0.	0.	0.	0.	0.	0.	0.	30
0.	0.	0.	0.	0.	0.	0.	31
0.	0.	0.	0.	0.	0.	0.	32
0.	0.	0.	0.	0.	0.	0.	33
0.	0.	0.	0.	0.	0.	0.	34
0.	0.	0.	0.	0.	0.	0.	35
0.	0.	0.	0.	0.	0.	0.	36
0.	0.	0.	0.	0.	0.	0.	37
0.	0.	0.	0.	0.	0.	0.	38

BINARY RECORD NO 49 WRITTEN ON TAPE 86 FOR X= 4.2929907E-01

C. FINITE RATE EVAPORATION OF CRYOGENIC HYDROGEN IN TWO-PHASE AIR

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FINITE-RATE EVAPORATION OF CYROGENIC HYDROGEN IN TWO-PHASE AIR

I. INTRODUCTION:

This section describes a finite difference method of solution for the finite rate evaporation of cryogenic hydrogen in two-phase air. The analysis (ref.1-2) assumes one-dimensional, globally-inviscid two-phase continuum flow at uniform pressure. The finite rate evaporation of condensed hydrogen is assumed to occur at uniform particle temperature, and a choice of either kinetic or film type heat and mass transfer is provided. The air is assumed to condense in phase equilibrium. It is further assumed that the condensed phase forms a dilute suspension of minute spherical droplets which travel at the mean gas phase velocity. Since the temperatures at which chemical reaction and dissociation may occur are significantly higher than those at which the condensed phase may exist, the flow is chemically frozen, and we need only consider the pure species of hydrogen, oxygen, and nitrogen.

It should be noted that the present method is applicable to streamline calculations providing that accelerations are small. Thus, it may be used for the determination of variable outer edge conditions in a number of two-phase flow problems. Furthermore, the analysis may be extended to account for the effect of having rarefied air relative to the hydrogen droplets at such high altitudes that the continuum flow assumption is invalid.

II. Discussion of Specific Features:

a) Method of Analysis:

Details of the analysis are given in references 1-2. The analysis commences by formulating the governing differential equations for two-phase flow. These equations are slightly modified from their usual single phase form due to the transfer of mass, momentum, and energy between the phases. These equations are then particularized to the one-dimensional, inviscid, constant pressure flow under consideration here. Constitutive relations are obtained from the most current literature available (refs. 4-7). Phase equilibrium relations are used in the conservation equations for nitrogen and oxygen. The hydrogen mass transfer from the droplets is formulated in terms of a kinetic model, or of one of two "film" type models (ref. 3). The governing equation for the hydrogen particle radius is deduced from the conservation of hydrogen mass for a single droplet.

b) Method of Numerical Solution:

The solution of the system of governing equations was obtained by converting the algebraic integrals to ordinary differential equations. These equations were integrated simultaneously by a standard predictor-corrector package after inverting the coefficient matrix by Gaussian reduction.

The program is presently available in Fortran II language, and could be converted to Fortran IV.

III. INPUT-OUTPUT FORMATS:

CARD	COLUMN	DESCRIPTION	FORMAT
1	1-10	Maximum time (Tmax-sec)	E10.4
	11-20	Time stepsize (ΔT-sec)	" "
	21-30	Minimum allowable particle radius ratio (r_p/r_{p_o})	" "
	31-40	Pressure (Po-atm.)	" "
	41-50	Total mass fraction of O_2	" "
	51-60	Total mass fraction of N_2	" "
	61-70	Initial particle radius (r_{p_o}- cm)	" "
2	1-10	Initial degree of vaporization	" "
	11-20	X_{O_2} } initial condensed species mass fractions	" "
	21-30	X_{N_2}	" "
	31-40	X_{H_2}	" "
	41-50	Y_{O_2} } initial gaseous species mass fractions	" "
	51-60	Y_{N_2}	" "
	61-70	Y_{H_2}	" "
3	1-10	Initial gas phase temperature (Tg-°K)	E10.4
	11-20	Initial condensed phase temperature (Tp-°K)	" "
4	1-10	Printout interval (prints out every input number of steps)	I10
	11-20	Trace amount of condensed air necessary to allow further air condensation	E10.4
	21-30	α } α=1; β=1 Standard film model	E10.4
		α=0; β=1 Film model with counter diffusion	
	31-40	β } α=0; β=0 Kinetic model	" "

Sample output may be found at the rear of this section. All parameters of interest are given in nondimensional form, having been normalized by their respective initial values, except for the time, which is given in seconds.

IV. Summary:

A computer program for the calculation of finite rate evaporation of cryogenic hydrogen in two-phase air has been presented. The specific purpose of this program is to obtain a detailed time history of the evaporation process under prescribed conditions. It has been noted that this program is applicable to a number of two-phase flow problems.

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7. Andrussow, L., Diffusion, viscosity and conductivity of gases, Prog. in International Research on Thermodynamics and transport Properties, Academic Press Inc., New York 1962, PP. 279-287.

TIME	RADIUS	Z	X(O2)	X(N2)	X(H2)	Y(O2)	Y(N2)	Y(H2)	T(G)	T(P)
0.00289537	0.6948138	0.9591444	-0.0000011	-0.0000003	1.0000014	0.2123386	0.7032691	0.0843922	0.386592	1.112466
0.00295937	0.6894281	0.9600870	-0.0000001	-0.0000000	1.0000002	0.2121291	0.7025796	0.0852913	0.383038	1.112693
0.00302337	0.6840621	0.9610118	-0.0000019	-0.0000006	1.0000025	0.2119242	0.7019045	0.0861714	0.379590	1.112930
0.00308737	0.6787143	0.9619191	-0.0000016	-0.0000005	1.0000021	0.2117233	0.7012434	0.0870333	0.376244	1.113175
0.00315137	0.6733831	0.9628094	-0.0000010	-0.0000003	1.0000013	0.2115266	0.7005958	0.0878775	0.372996	1.113420
0.00321537	0.6680673	0.9636832	-0.0000010	-0.0000003	1.0000012	0.2113339	0.6999614	0.0887046	0.369842	1.113669
0.00327937	0.6627659	0.9645409	-0.0000008	-0.0000002	1.0000011	0.2111451	0.6993399	0.0895150	0.366776	1.113931
0.00334337	0.6574782	0.9653829	-0.0000007	-0.0000002	1.0000009	0.2109601	0.6987308	0.0903091	0.363797	1.114212
0.00340737	0.6522024	0.9662095	-0.0000006	-0.0000002	1.0000008	0.2107788	0.6981338	0.0910874	0.360901	1.114506
0.00347137	0.6469361	0.9670215	-0.0000006	-0.0000001	1.0000007	0.2106010	0.6975484	0.0918505	0.358083	1.114786
0.00353537	0.6416773	0.9678192	-0.0000005	-0.0000001	1.0000006	0.2104266	0.6969743	0.0925991	0.355341	1.115031
0.00356737	0.6390521	0.9682125	-0.0000005	-0.0000001	1.0000006	0.2103407	0.6966916	0.0929677	0.353998	1.115180
0.00359937	0.6364286	0.9686024	-0.0000005	-0.0000001	1.0000006	0.2102556	0.6964115	0.0933328	0.352673	1.115325
0.00363137	0.6338066	0.9689889	-0.0000005	-0.0000001	1.0000005	0.2101714	0.6961342	0.0936944	0.351365	1.115470
0.00366337	0.6311863	0.9693719	-0.0000004	-0.0000001	1.0000005	0.2100880	0.6958595	0.0940526	0.350074	1.115615
0.00369537	0.6285673	0.9697516	-0.0000004	-0.0000001	1.0000005	0.2100053	0.6955874	0.0944073	0.348801	1.115760
0.00372737	0.6259496	0.9701279	-0.0000004	-0.0000000	1.0000004	0.2099235	0.6953179	0.0947586	0.347544	1.115906
0.00379137	0.6207174	0.9708708	-0.0000004	-0.0000000	1.0000004	0.2097621	0.6947867	0.0954512	0.345079	1.116201
0.00385537	0.6154885	0.9716008	-0.0000004	-0.0000000	1.0000004	0.2096038	0.6942654	0.0961308	0.342677	1.116498
0.00391937	0.6102618	0.9723181	-0.0000004	-0.0000000	1.0000004	0.2094484	0.6937539	0.0967977	0.340335	1.116796
0.00398337	0.6050361	0.9730232	-0.0000003	-0.0000000	1.0000003	0.2092959	0.6932519	0.0974521	0.338052	1.117096
0.00404737	0.5998103	0.9737162	-0.0000003	0.0000000	1.0000003	0.2091463	0.6927592	0.0980945	0.335826	1.117397
0.00411137	0.5945832	0.9743973	-0.0000003	0.0000000	1.0000003	0.2089994	0.6922756	0.0987250	0.333654	1.117697
0.00417537	0.5893537	0.9750670	-0.0000003	0.0000000	1.0000003	0.2088552	0.6918009	0.0993439	0.331536	1.117998
0.00423937	0.5841206	0.9757253	-0.0000005	-0.0000003	1.0000008	0.2087137	0.6913348	0.0999515	0.329469	1.118297
0.00430337	0.5788827	0.9763725	-0.0000005	-0.0000003	1.0000008	0.2085747	0.6908772	0.1005481	0.327451	1.118595
0.00436737	0.5736387	0.9770088	-0.0000005	-0.0000002	1.0000007	0.2084382	0.6904279	0.1011339	0.325481	1.118884
0.00443137	0.5683868	0.9776345	-0.0000005	-0.0000002	1.0000006	0.2083042	0.6899866	0.1017092	0.323557	1.119149
0.00446337	0.5657603	0.9779431	-0.0000005	-0.0000002	1.0000006	0.2082381	0.6897692	0.1019927	0.322613	1.119345
0.00449537	0.5631304	0.9782493	-0.0000004	-0.0000001	1.0000006	0.2081726	0.6895536	0.1022738	0.321680	1.119509
0.00452737	0.5604978	0.9785529	-0.0000004	-0.0000001	1.0000006	0.2081077	0.6893399	0.1025523	0.320757	1.119663
0.00455937	0.5578627	0.9788540	-0.0000004	-0.0000001	1.0000006	0.2080434	0.6891282	0.1028283	0.319846	1.119820
0.00459137	0.5552250	0.9791525	-0.0000004	-0.0000001	1.0000005	0.2079797	0.6889184	0.1031019	0.318944	1.119974
0.00462337	0.5525840	0.9794486	-0.0000004	-0.0000001	1.0000005	0.2079165	0.6887104	0.1033730	0.318054	1.120118
0.00465537	0.5499400	0.9797422	-0.0000004	-0.0000001	1.0000005	0.2078539	0.6885044	0.1036417	0.317173	1.120260
0.00468737	0.5472935	0.9800332	-0.0000004	-0.0000000	1.0000005	0.2077919	0.6883002	0.1039079	0.316303	1.120416
0.00471937	0.5446439	0.9803218	-0.0000004	-0.0000000	1.0000004	0.2077304	0.6880978	0.1041717	0.315443	1.120578
0.00475137	0.5419903	0.9806080	-0.0000004	-0.0000000	1.0000004	0.2076695	0.6878973	0.1044332	0.314593	1.120725
0.00478337	0.5393325	0.9808919	-0.0000004	0.0000000	1.0000004	0.2076091	0.6876985	0.1046924	0.313752	1.120856
0.00481537	0.5366716	0.9811734	-0.0000004	0.0000000	1.0000004	0.2075493	0.6875015	0.1049492	0.312921	1.121001
0.00484737	0.5340078	0.9814523	-0.0000004	0.0000000	1.0000004	0.2074900	0.6873064	0.1052036	0.312100	1.121175
0.00487937	0.5313397	0.9817289	-0.0000004	0.0000000	1.0000004	0.2074313	0.6871130	0.1054557	0.311288	1.121343
0.00491137	0.5286657	0.9820034	-0.0000005	0.0000001	1.0000004	0.2073730	0.6869212	0.1057058	0.310485	1.121464
0.00494337	0.5259866	0.9822756	-0.0000005	0.0000001	1.0000004	0.2073153	0.6867311	0.1059536	0.309691	1.121567
0.00497537	0.5233051	0.9825453	-0.0000005	0.0000001	1.0000004	0.2072581	0.6865429	0.1061990	0.308907	1.121730
0.00500736	0.5206212	0.9828125	-0.0000005	0.0000001	1.0000004	0.2072015	0.6863565	0.1064420	0.308131	1.121956
0.00502337	0.5192749	0.9829455	-0.0000005	0.0000001	1.0000004	0.2071733	0.6862638	0.1065629	0.307747	1.122000
0.00503936	0.5179279	0.9830779	-0.0000005	0.0000001	1.0000004	0.2071453	0.6861715	0.1066832	0.307364	1.122061
0.00505537	0.5165799	0.9832097	-0.0000005	0.0000001	1.0000004	0.2071174	0.6860796	0.1068029	0.306984	1.122129
0.00507136	0.5152307	0.9833409	-0.0000005	0.0000001	1.0000003	0.2070896	0.6859882	0.1069221	0.306606	1.122199
0.00508737	0.5138803	0.9834715	-0.0000005	0.0000001	1.0000003	0.2070620	0.6858972	0.1070408	0.306230	1.122271

D. COMPRESSIBLE TURBULENT BOUNDARY LAYER PROGRAM

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SYMBOLS

μ	Laminar viscosity coefficient
ζ	Transformed variable defined by Eq.25 of ref.1
$\tilde{\psi}$	Stream function defined by Eq.47 of ref.1

Subscripts

ℓ	edge of laminar sublayer
s	average over laminar sublayer

Superscripts

-	incompressible state

COMPRESSIBLE TURBULENT BOUNDARY LAYER PROGRAM

I. INTRODUCTION

This section describes a finite difference method solution for two dimensional turbulent compressible boundary layers in the absence of pressure gradients. Options are available for frozen, equilibrium, or finite-rate chemistry. Three turbulent boundary layer models are provided. Lewis and Prandtl numbers, while restricted to constant values, may be non-unity.

The hydrogen-air and pure air reaction systems have been programmed. The wall may be specified as adiabatic, or a set of linear temperature variations along the axial coordinate may be prescribed.

II. Discussion of Specific Features:

a. Method of Analysis:

The present analysis is based upon Coles' transformation theory (ref. 1-5). The essential feature of this type of treatment is to transform the compressible flow problem to an equivalent incompressible flow problem, and then to utilize the existing descriptions of the incompressible turbulent velocity field in the numerical solution of the transformed energy and diffusion equations. The solution is then transformed back to the physical plane.

b. Turbulent Boundary Layer Models:

The calculations depend upon the turbulent boundary layer selected through the transformation variable σ (ref.1, 6-7). This variable is defined differently for each model. The reference state model assumes that σ is constant. This constant is evaluated at a reference state given by the mean substructure values of viscosity, temperature, and species mass fraction. Explicitly:

Eq. (1) $$\sigma = \bar{\mu}/\mu_s = \text{constant}$$

The substructure hypothesis (ref.2,4) differs from the reference state model by allowing the mean value of viscosity in the logarithmic region, μ_s, to vary with axial position. The sublayer hypothesis (ref.5) states that the Reynolds number based upon the height of the laminar sublayer is an invarient of the compressibility transformation. This assumption yields:

$$\frac{\sigma}{\bar{\mu}} = \frac{1}{\zeta_\ell} \int_0^{\tilde{\psi}_\ell} \frac{\rho_e}{\rho} \frac{d\tilde{\psi}}{\sqrt{2\,\tilde{\psi}}} \qquad \text{(Eq.2)}$$

Due to the complex nature of the substructure and sublayer models, numerical difficulties are sometimes encountered when they are used. These difficulties have not been encountered in operation of the reference method.

c. Chemistry:

The three chemistry options available are frozen, equilibrium, or finite-rate chemistry. One-dimensional chemistry calculations are coupled to the two-dimensional diffusion equations in the finite difference formulation by the usual methods (Refs. 8,9). The finite-rate chemistry calculations are performed using the linear technique of Ref. 10 to achieve practical step sizes.

d. Finite Difference Grid Size:

The normal step size of the finite difference grid is not constant (Ref. 6). This is necessary since the boundary layer equations have a removable singularity at $\psi = 0$ in the von Mises plane. The mesh is therefore divided into three regions. The region near the wall is handled analytically by the methods of Ref. 11. The region above this region is spanned by a fine mesh to provide numerical accuracy. An outer region of coarse mesh is also provided.

III. Input-Output Formats:

Detailed input-output information is given in the existing programmer's reports (ref.6-7). Therefore, all that is necessary here is the following brief list of the data necessary to use this program.

1. Freestream Conditions (U_e, $\rho_e, \mu_e, T_e \left| Y_k \right|_e$).
2. Wall conditions (adiabatic or specified temperature variation with axial coordinate).
3. Either the initial compressible skin friction coefficient or the initial compressible Reynolds number based upon momentum thickness.
4. Initial temperature and species mass fraction profiles as functions of velocity ratio.

IV. Summary:

A program for the calculation of turbulent compressible boundary layers on flat plates has been presented. Transformation theory has been applied so that existing incompressible turbulent flow theories could be applicable. The energy and diffusion equations in von Mises coordinates have been solved numerically. Chemistry options have been coupled to the diffusion calculations.

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E. FLAME SHEET MIXING PROGRAM

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FLAME SHEET MIXING PROGRAM

I. Introduction:

This section describes an approximate method of solution for the combustion of a uniform axisymmetric jet of pure gaseous hydrogen mixing with a parallel air stream.The mixing process may be prescribed as either laminar or fully turbulent. The differences between laminar and turbulent flow are accounted for by the form of the viscosity coefficient used. Lewis and Prandtl numbers are taken to be unity, allowing the solution of the energy and mass fraction conservation equations by Crocco integrals. Options are available for frozen or flame sheet chemistry. Flame sheet chemistry is an approximation which assumes that since the equilibrium constant is large for the temperatures of interest here, complete combustion occurs along the surface of stoichiometric ratio. For complete combustion, only the species H_2, O_2 and H_2O enter into consideration. N_2 is considered to be an inert diluent. The mixing and combustion processes are assumed to occur at constant pressure ($\frac{dp}{dx} = 0$).

II. Method of Analysis:

Details of the analysis are given in reference 1. The analysis commences by considering the governing equations for a constant pressure axisymmetric viscous flow with Le=Pr=1. The system is then transformed to the Von-Mises plane, where the momentum equation is linearized by the methods presented in references 2 and 3. The linearized momentum equation is obtained in the same form for both laminar and turbulent flow. However, the inverse transformations for the axial coordinate are dependent upon the viscosity model selected. The form of the linearized momentum equation is well known in the theory of heat conduction, and so the solution for the velocity field is obtained immediately as the offset circular probability function (ref.6). The stagnation enthalpy and species mass fraction (for frozen flow) profiles are then obtained as Crocco integrals. For flame-sheet flow, element mass fractions are introduced, and it is found that the element mass fraction profiles are also given by Crocco integrals. The additional equation necessary for the determination of the species mass fractions is obtained from the flame sheet approximation to the equilibrium condition. The laminar viscosity coefficient is obtained by the method of mass averaging (ref.4). The turbulent viscosity model used is that suggested by Ferri, et al (ref.5).

III. INPUT-OUTPUT FORMATS:

CARD	COLUMN	DESCRIPTION	FORMAT
1	1-10	Number of Axial Stations ($MK \leq 9$)	I 10
2	1-10 η_1	Value of the transformed	E 10.4
	11-20 η_2	axial coordinate at each	"
	21-30 η_3	station of interest	"
	31-40 '	(Will need a 2B card if	"
	41-50 '	8 or 9 pts. used)	"
3	1-10 NN_1	Number of terms to retain at each	I 10
	11-20 NN_2	station of interest in the expansion	"
	'	of the offset circular probability	"
	'	fnct. (will need a 3B card if 8 or 9	"
	'	pts. used) ($NN_i \leq 70$)	"
4	AA 1-10,---61-70 Pi_1 i=1,---7	Numerical values of	E 10.4
	AB 1-10,---61-70 Pi_1 i=8,---14	the appropriate terms	"
	ANN_1 1-10,---61-70 Pi_1 i=--$NN_1 \leq 70$	in the expansion of	"
	BA 1-10,---61-70 Pi_2 i=1, ---7	the P fnct. at each	"
	⋮	station of interest	"
	MKA 1-10,---61-70 Pimk i=1, ---7		"
5.	1-10	Uj Velocity of jet $(\frac{cal}{gm})^{\frac{1}{2}}$	E 10.5
	11-20	Y_{1E} Mass fraction of O_2 in air stream	"
	21-30	Y_{4E} Mass fraction of N_2 in air stream	"
	31-40	Tr(^{o}k) Reference temp. in enthalpy fits	"
	41-50	T_E (^{o}k) static temp. of air stream	"
	51-60	H_E $(\frac{cal}{gm})$ stagnation enthalpy of air stm.	"
	61-70	Hj $(\frac{cal}{gm})$ stagnation enthalpy of hydrogen jet	"

CARD	COLUMN	DESCRIPTION	FORMAT
6	1-10	Ue $(\frac{cal}{gm})^{\frac{1}{2}}$ Velocity of air stream	E10.5
	11-20	σ Reciprocal of air Molecular Wt.	"
	21-30	Δ^1 Coefficients in enthalpy	"
	31-40	Δ^2 fits for O_2, H_2, H_2O, N_2	"
	41-50	Δ^3 respectively. Form of fit is:	"
	51-60	Δ^4 $h^i = \Delta^i + c_p^{\,i}\,(T-Tr)$	"
	61-70	$C_p^{\,1}$ $\Delta^i = \frac{cal}{gm}$	"
7	1-10	$C_p^{\,2}$ $C_p^{\,i} = \frac{cal}{gm^{\circ}k}$	"
	11-20	$C_p^{\,3}$	"
	21-30	$C_p^{\,4}$	"
	31-40	W^1 Molecular Wts.	"
	41-50	W^2 of species	"
	51-60	W^3	"
	61-70	W^4	"
8	1-10	$\frac{\rho_j}{\rho_e}$ density ratio of jet and stream	"
9	1-10	Br=0 flame sheet combustion model	F10.5
		Br>0 frozen chemistry	

The program is presently available in Fortran II language, and could be converted to Fortran IV.

Sample output may be found at the rear of this section. The output is in terms of the transformed axial and radial coordinates:

$$\eta = \frac{\psi j}{\sqrt{2\zeta\psi j}}$$

$$s = \frac{\psi}{\sqrt{2\zeta\psi j}}$$

At each station of interest profiles are supplied in the transformed plane. At present, the program calculates the corresponding physical coordinates for the particular case where the eddy viscosity is based upon Ferri's model. For other viscosity models, simple hand calcuations must be performed.

IV. Summary:

An approximate method of solution for the combustion of a uniform axisymmetric jet of pure gaseous hydrogen mixing with a parallel air stream has been presented. It has been noted that the analysis is based upon linearization technique and the flame sheet combustion approximation. A detailed input format has been provided.

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	Y_1	Y_2	Y_3	Y_4	$\frac{(\rho u)_d}{(\rho u)_e}$	$\frac{1+\frac{(\rho u)_d}{(\rho u)_e}}{2}$
	.1398839E 00	.0000000E-32	.1007100E 00	.7594060E 00	.2684484E 00	.6342242E 00
S=0	.9902093E 00	.9961125E 00	.3498721E 01	.3710628E 01	.1500000E 00	
	$\frac{u_f}{u_e}$	$\frac{u}{u_e}$	$\frac{T}{T_e}$	$\frac{\rho_e}{\rho}$	η	

S =	η/ψ_j	Y_1	Y_2	Y_3	Y_4
.1000	.1284563E 01	.1403449E 00	.0000000E-32	.1002060E 00	.7594490E 00
	.9961320E 00	.3488160E 01	.3698370E 01	.2693435E 00	
	u/u_e	T/T_e	ρ_e/ρ	$\rho u/\rho_e u_e$	
.2000	.2562682E 01	.1417278E 00	.0000000E-32	.9869400E-01	.7595781E 00
	.9961904E 00	.3456378E 01	.3661527E 01	.2720696E 00	
.3000	.3828443E 01	.1439011E 00	.0000000E-32	.9631800E-01	.7597808E 00
	.9962821E 00	.3406132E 01	.3603428E 01	.2764817E 00	
.4000	.5076041E 01	.1468728E 00	.0000000E-32	.9306900E-01	.7600581E 00
	.9964075E 00	.3336818E 01	.3523575E 01	.2827830E 00	
.5000	.6299994E 01	.1505229E 00	.0000000E-32	.8907840E-01	.7603986E 00
	.9965615E 00	.3250710E 01	.3424841E 01	.2909803E 00	
.6000	.7494804E 01	.1548760E 00	.0000000E-32	.8431920E-01	.7608047E 00
	.9967452E 00	.3146590E 01	.3306131E 01	.3014838E 00	
.7000	.8655982E 01	.1596308E 00	.0000000E-32	.7912080E-01	.7612483E 00
	.9969459E 00	.3031045E 01	.3175245E 01	.3139744E 00	
.8000	.9779716E 01	.1648120E 00	.0000000E-32	.7345620E-01	.7617317E 00
	.9971645E 00	.2902925E 01	.3031134E 01	.3289740E 00	
.9000	.1086267E 02	.1702130E 00	.0000000E-32	.6755130E-01	.7622356E 00
	.9973925E 00	.2766846E 01	.2879213E 01	.3464114E 00	
1.0000	.1190274E 02	.1757054E 00	.0000000E-32	.6154650E-01	.7627480E 00
	.9976243E 00	.2625747E 01	.2722894E 01	.3663837E 00	
1.1000	.1289818E 02	.1812554E 00	.0000000E-32	.5547870E-01	.7632658E 00
	.9978585E 00	.2480299E 01	.2563008E 01	.3893309E 00	
1.2000	.1384825E 02	.1866556E 00	.0000000E-32	.4957470E-01	.7637696E 00
	.9980864E 00	.2335926E 01	.2405521E 01	.4149147E 00	
1.3000	.1475307E 02	.1919066E 00	.0000000E-32	.4383360E-01	.7642595E 00
	.9983080E 00	.2192759E 01	.2250513E 01	.4435913E 00	
1.4000	.1561299E 02	.1969382E 00	.0000000E-32	.3833280E-01	.7647289E 00
	.9985203E 00	.2052939E 01	.2100214E 01	.4754373E 00	
1.5000	.1642953E 02	.2016123E 00	.0000000E-32	.3322260E-01	.7651650E 00
	.9987176E 00	.1920663E 01	.1958985E 01	.5098137E 00	
1.6000	.1720540E 02	.2058732E 00	.0000000E-32	.2856420E-01	.7655625E 00
	.9988974E 00	.1798026E 01	.1828860E 01	.5461855E 00	
1.7000	.1794273E 02	.2098797E 00	.0000000E-32	.2418390E-01	.7659363E 00
	.9990665E 00	.1680874E 01	.1705273E 01	.5858689E 00	
1.8000	.1864499E 02	.2133117E 00	.0000000E-32	.2043140E-01	.7662564E 00
	.9992113E 00	.1579087E 01	.1598439E 01	.6251168E 00	

1.9000	.1931604E 02	.2164431E 00	.0000000E-32	.1700820E-01	.7665486E 00
	.9993434E 00	.1485022E 01	.1500161E 01	.6661573E 00	
2.0000	.1995889E 02	.2191835E 00	.0000000E-32	.1401210E-01	.7668043E 00
	.9994591E 00	.1401756E 01	.1413519E 01	.7070715E 00	
2.1000	.2057673E 02	.2216252E 00	.0000000E-32	.1134270E-01	.7670320E 00
	.9995621E 00	.1326814E 01	.1335816E 01	.7482779E 00	
2.2000	.2117444E 02	.2234905E 00	.0000000E-32	.9303300E-02	.7672061E 00
	.9996408E 00	.1269072E 01	.1276125E 01	.7833406E 00	
2.3000	.2175592E 02	.2252174E 00	.0000000E-32	.7415280E-02	.7673672E 00
	.9997137E 00	.1215236E 01	.1220609E 01	.8190284E 00	
2.4000	.2232371E 02	.2265992E 00	.0000000E-32	.5904630E-02	.7674961E 00
	.9997720E 00	.1171894E 01	.1176010E 01	.8501385E 00	
2.5000	.2284748E 02	.2320000E 00	.0000000E-32	.0000000E-32	.7680000E 00
	.1000000E 01	.1000181E 01	.1000140E 01	.9998600E 00	

INT= .1879390E 02 .1795017E 01
FLAME .8675857E 00

$\frac{r_{1/2}}{r_j}$ (value of half-radius in viscosity model)

$\frac{x_{FL}}{r_j}$ (value of axial coordinate where flame sheet hits the axis)

$\frac{x_n}{r_j}$ (value of axial coordinate where n is the constant in the viscosity model)

III. Summary:

A series of computer programs for the mixing and combustion of hydrogen in supersonic air streams have been presented. The assumptions and analytical techniques upon which these programs are based have been discussed. Detailed input formats have been provided where necessary.