3 EDUCATIONAL MONOGRAPH

CRITICAL FLOW OF REAL GASES THROUGH NOZZLES

prepared from

Real-Gas Effects in Critical-Flow Nozzles and Tabulated Thermodynamic Properties 6

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FOREWORD

This Monograph was produced in a pilot program at Oklahoma State University in Stillwater, Oklahoma, under contract to the NASA Office of Technology Utilization. The program was organized to determine the feasibility of presenting the results of recent research in NASA Laboratories and under NASA contract in an educational format suitable as supplementary material in classwork at engineering colleges. The Monograph may result from editing single technical reports or synthesizing several technical reports resulting from NASA's research efforts.

Following the preparation of the Monographs, the program includes their evaluation as educational material in a number of universities throughout the country. The results of these individual evaluations in the classroom situation will be used to help determine if this procedure is a satisfactory way of speeding research results into engineering education.

ABSTRACT

Methods for calculating the mass flow of real gases through critical-flow nozzles are presented by: (1) equation derivations for the entropy-enthalpy balances and the isentropic-exponent methods, (2) tabulations of thermodynamic properties for critical-flow conditions of steam, (3) problem on application of tabulated data in thrust calculation, and (4) problem on evaluation of thermodynamic critical-flow properties of fluids represented by the Redlich-Kwong Equation of State.

INSTRUCTOR'S GUIDE FOR MONOGRAPHS

1. Educational level of the Monograph--Senior or beginning graduate level.

2. Prerequisite course material--The course material required for this Monograph consists of Introductory Material in Thermodynamics, Introductory Material in Differential Equations and Introductory Material in Fluid Mechanics.

3. Estimated number of lecture periods required--One hour of lecture.

4. Technical significance of the Monograph--The material presented in this Monograph represents the most convenient rigorous technique for the solution of real-gas critical-flow problems. Thermodynamic methods have been applied to the developments of the equations and procedures for calculating the acoustic velocity obtained at the throat of a critical-flow nozzle after isentropic expansion of the gas. A tabulation of thermodynamic properties of steam at critical-flow is included. In addition to critical-flow velocities, isentropic exponents for the expansion process are included for steam. By using the same techniques described herein and illustrated for steam, similar information can be obtained for other gases.

5. New concepts or unusual concepts illustrated--This Monograph presents useful engineering methods and data that have applications in real-gas metering calculations.

6. How Monographs can best be used--

   a) It is suggested that approximately a one hour lecture be given over the Monograph material.

   b) It is suggested that the class be assigned a home problem similar to the two indicated at the end of the Monograph, one of which involves the use of a digital computer while the other can be solved easily with a slide rule.

   c) If possible, the students should be allowed to compare the results of their critical-flow solution to a problem solved by the exponent technique.

7. Other literature, Briefs or Monographs of interest--"Functions for the Calculation of Entropy, Enthalpy, and Internal Energy for Real Fluids Using Equations of State and Specific Heats", J. G. Hust and A. L. Gosman; "Advances in Cryogenic Engineering", Vol. 9 (1963); "Equations, Tables and Charts for Compressible Flow", by Ames Research

7. Continued--
Staff, NACA Report 1135 (1953); "Tables of Thermodynamic and Transport Properties of Air, Argon, Carbon Dioxide, Carbon Monoxide, Hydrogen, Nitrogen, Oxygen and Steam", by Joseph Hilsenrath, et al., Pergamon Press (1960); "The Dynamics and Thermodynamics of Compressible Fluid Flow", A. H. Shapiro, Ronald Press (1954); "Thermodynamics", Gordon J. Van Wylen, John Wiley and Sons (1959); "Applied Hydrocarbon Thermodynamics", W. C. Edmister, Gulf Publishing Company (1961).

8. Other reports reviewed by the editor in preparing this Monograph--"Critical Flow Meters for the Measurement of Mass Flow Rates of Gases", B. T. Arnberg, University of Colorado, Contract 57-10194, AF 04(645)-56; "Review of Critical Flowmeters for Gas Flow Measurements", B. T. Arnberg, Paper 62-WA-177, ASME (1962); "Maximum Steam Flow Through Pipes to the Atmosphere", Russell James, The Institution of Mechanical Engineers, Vol. 178, Part 1, No. 18, pp. 473-482 (1963-64).

9. Who to contact for further information--Technology Utilization Officer, Lewis Research Center, Cleveland, Ohio.

10. Note to Instructor: All uncolored pages of the instructors Monograph are in the copies intended for student use.

CRITICAL FLOW THROUGH NOZZLES

In flowing from a plenum chamber through a nozzle with negligible back pressure, gases will reach a critical-flow condition of acoustic velocity at the throat of the nozzle. The calculation of flow conditions in the nozzle throat, from the gas properties and the plenum conditions, is an important and interesting problem in thermodynamics, as is evidenced by previous investigations (1,2,3,4,5).

For perfect gases, the calculation of the critical flow pressure, temperature, density, velocity, and mass-flow function is a simple problem involving the use of the heat-capacity ratio, $k = C_p/C_v$, for the perfect gas. For real gases, these conditions are found by calculations made with real-gas equations of state, as well as the ideal-gas-state heat capacities.

The thermodynamic calculations are made for one-dimensional, isentropic expansion of the gas from the stagnation conditions of the plenum to the critical-flow point, which is the point at which the gas velocity is equal to the velocity of sound, i.e., a Mach Number of unity. For the reversible and adiabatic conditions of this initial velocity, the final velocity is calculated from the kinetic energy produced by the decrease in enthalpy during the isentropic expansion, assuming no change in chemical composition and no potential-energy change. The velocity of sound in the gas at the throat conditions is calculated from the first density derivative of pressure for the expansion isentrope, evaluated at the throat conditions.

Two functions of the mass velocity at the nozzle throat have been defined as "critical-flow functions". Murdock and Bauman (3) used the following.

$$\phi = \frac{G\,T_o^{1/2}}{P_o} \qquad (1)$$

where: G = mass-flow per unit area, lb_m/ft^2sec

T_o = plenum temperature, °R.

P_o = plenum pressure, lb_f/ft^2, psf

Johnson (5) used the following "critical-flow function"

$$C^* = \frac{G}{P_o}\left(\frac{RT_o}{g_c}\right)^{1/2} = \phi\left(\frac{R}{g_c}\right)^{1/2} \tag{2}$$

These two critical-flow functions differ by a constant factor, $(R/g_c)^{1/2}$. C^* is dimensionless.

THERMODYNAMIC EQUATIONS FOR CRITICAL FLOW

The velocity increase during one-dimensional flow of a constant-composition gas mixture through an adiabatic nozzle, for which the work and potential energy terms are zero, may be calculated from the enthalpy change, as follows

$$\Delta H = -\frac{\Delta(u^2)}{2g_c} \tag{3}$$

Equation 3 applies to both isentropic and polytropic (i.e., reversible and irreversible) expansion processes. It is applicable to the calculation of critical-flow velocities.

From the first law of thermodynamics for a reversible and adiabatic process, for which the work and potential energy terms are zero, ΔH may be found as follows.

$$\Delta H = \int_{P_o}^{P_*} VdP = \int_{P_o}^{P_*} \frac{dP}{\rho} \tag{4}$$

where: P_* = throat pressure; P_o = plenum or stagnation pressure

This expression for ΔH can be substituted into Equation 3 to give

$$\frac{\Delta(u^2)}{2g_c} = \int_{P_o}^{P_*} VdP = -\int_{P_o}^{P_*} \frac{dP}{\rho} \tag{5}$$

Equation 5 is limited to reversible and adiabatic (i.e., isentropic) expansion processes only.

The flow velocity in a reversible and adiabatic nozzle may be found by applying Equation 3 or 5 to an isentropic expansion. This isentropic path is determined by additional calculations, using heat capacities and an equation of state to follow a constant-entropy path.

Enthalpy differences for Equation 3 are evaluated from heat capacities and an equation of state for this constant-entropy path, using the same heat capacities and equation of state that were used in making the entropy-balance calculations.

The reversible-expansion-work calculation in Equation 5 is made by using the isentropic exponent for this constant-entropy expansion path, i.e., the same exponent used to find the nozzle throat conditions.

These alternate calculation procedures, i.e., via entropy and enthalpy balances and via the isentropic exponents, will give a nozzle-throat velocity that is at critical or below. Critical-flow conditions in the nozzle throat are determined by an additional calculation based upon the relationship for the velocity of sound in a gas and the requirement that a Mach-One condition exist, namely

$$u_* = c_* = [g_c(\partial P/\partial \rho)_s]^{1/2} \tag{6}$$

where: c = velocity of sound.

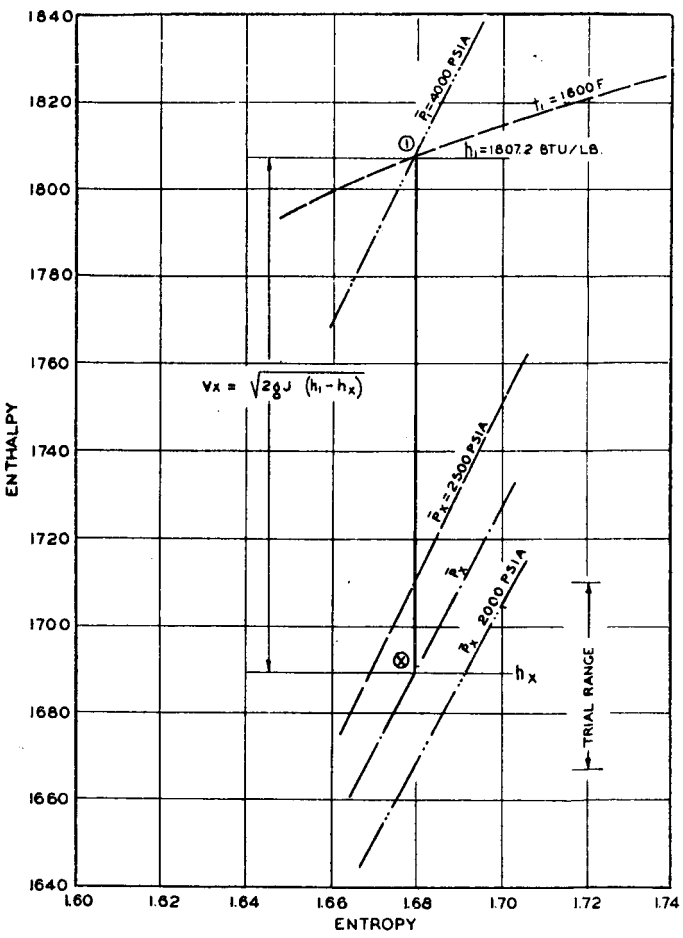
Equation 6 condition must be satisfied in either calculation method, i.e., by the enthalpy-entropy procedure or by the isentropic-exponent procedure. For application in these two methods, Equation 6 is transformed into appropriate forms.

The basic principles of these two methods and the equations are outlined next. Basic relationships for both volume-explicit and pressure-explicit equations of state are included. Interrelationships between the two methods are shown.

ENTHALPY-ENTROPY METHOD

In the enthalpy-entropy method of predicting the critical flow velocity and properties, an isentropic expansion path is followed, in the calculations, until a point of maximum flux (at acoustic or critical velocity) is reached. This can be done numerically from data given in tables of thermodynamic properties, such as the steam tables, or it can be done analytically with heat capacities and an equation of state.

Murdock and Bauman (3) obtained values of the critical-flow function (see Equation 1) for steam by a "flow maximization procedure", i.e., a trial method using thermodynamic properties from the steam tables. This procedure is illustrated by a sample calculation in Figure 1, using Keenan and Keyes steam tables (6).



CALCULATION PROCEDURE

① SELECT ARBITRARY INLET STATE $\bar{P}_1$ = 4000 PSIA t_1 = 1600F

② OBTAIN h_1 = 1807.2 AND S_1 = 1.6795

③ ASSUME VALUES OF $\bar{P}_X / \bar{P}_1 >$ = 0.50

ASSUMED $\bar{P}_X$		2000	2100	2200	2300	2400	2500
INTERPOLATE FROM TABLES AT $\bar{P}_X$ & S_1	t_X	1322.5	1340.7	1358.1	1374.7	1390.6	1406.5
	V_X	0.5087	0.4894	0.4717	0.4553	0.4402	0.4261
	h_X	1668.4	1677.4	1686.0	1694.2	1702.1	1710.0
$h_1 - h_X$	Δh_s	138.8	129.8	121.2	113.0	105.1	97.1
$\sqrt{2gJ\Delta h_s}$	u_X	2636.4	2549.5	2463.6	2378.8	2294.2	2205.1
u_X / V_X	G_X	5182.7	5209.5	5222.8	5224.8	5211.7	5175.1
$G_X \sqrt{T_1} / P_1$	ϕ_X	0.4083	0.4105	0.4115	0.4117	0.4106	0.4077
$\bar{P}_X / \bar{P}_1$	r_{CX}	0.500	0.525	0.550	0.575	0.600	0.625
T_X / T_1		0.865	0.874	0.883	0.891	0.898	0.900

④ PLOT ϕ_X VS. $\bar{P}_X$ TO DETERMINE MAXIMUM ϕ_X

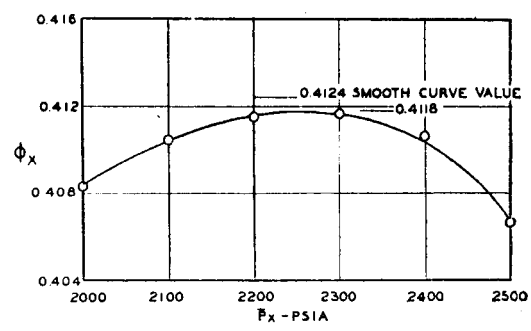


Figure 1: FLOW MAXIMIZATION PROCEDURE OF MURDOCK AND BAUMAN (3)

The flow maximization, as illustrated in Figure 1, is limited to fluids, such as steam, for which tabulations of thermodynamic properties are available. For other fluids it is necessary to use analytical procedures, employing heat capacities and an equation of state.

The equation-of-state calculation technique depends upon the type of equation of state available, i.e., volume-explicit or pressure-explicit. For the volume-explicit case, the following expressions are integrated to determine the isothermal effects of pressure on the entropy and enthalpy.

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \tag{7}$$

and

$$\left(\frac{\partial H}{\partial P}\right)_T = \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right] \tag{8}$$

With the pressure-explicit equation of state, the following expressions are integrated to determine the isothermal effects of volume, or density, on entropy and enthalpy

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \tag{9}$$

$$\left(\frac{\partial H}{\partial V}\right)_T = \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] + \left(\frac{\partial (PV)}{\partial V}\right)_T \tag{10a}$$

or

$$\left(\frac{\partial H}{\partial V}\right)_T = \left[T\left(\frac{\partial P}{\partial T}\right)_V + V\left(\frac{\partial P}{\partial V}\right)_T\right] \tag{10b}$$

Equations 10a and 10b are identities. Both may be evaluated by a pressure-explicit equation of state and the same results obtained. Equation 10a is more convenient, however, and is recommended as the preference.

The calculation of the effects of temperature on the entropy and enthalpy require heat capacity information. If the paths are chosen correctly, ideal-gas-state heat capacities, C_P^o, can be used in calculating the effects of temperature on entropy and enthalpy with either type of isothermal calculations. This is a convenience.

For calculational purposes, the lower pressure terminus of the isentropic expansion path is reached by an alternate route, a combination of three paths. Since entropy and enthalpy are exact functions, their values depend upon point conditions and are independent of the route followed. The paths followed in these calculations are shown in Figure 2.

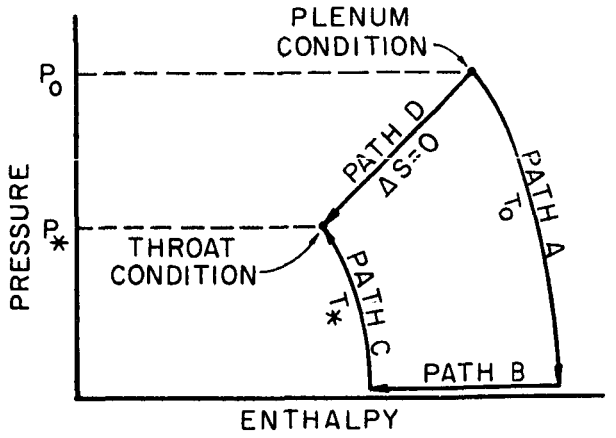


Fig. 2. PATHS FOR ENTROPY AND ENTHALPY CALCULATIONS.

Path A is an isothermal expansion process between the plenum conditions and the ideal gas state (i.e., zero density or pressure).

Path B is a cooling process from T_o to T_* at the ideal-gas state.

Path C is an isothermal compression process between the ideal-gas state and the throat conditions.

Path D is the isentropic expansion process between plenum and throat conditions.

For the processes represented on Figure 2, the following entropy and enthalpy balances may be written

$$\Delta S_D = \Delta S_A + \Delta S_B + \Delta S_C = 0 \tag{11}$$

and

$$\Delta H_D = \Delta H_A + \Delta H_B + \Delta H_C = -\frac{u_*^2}{2g_c} \tag{12}$$

Equations 11 and 12 apply to both types of equation of state calculations. With Path B at the ideal-gas state, the effects of temperature can be found by integrating the following

$$\left(\frac{\partial S}{\partial T}\right)_{P^o} = \frac{C_P^o}{T} \tag{13}$$

and

$$\left(\frac{\partial H}{\partial T}\right)_{P^o} = C_P^o \tag{14}$$

The entropy balance calculations, made with Equations 11 and 13 and either Equation 7 or 9, involve finding the lower pressure or density reached after an isentropic expansion for trial values of the final temperature, T_*. A separate calculation, to be discussed later, is required to determine which of these trial temperatures leads to a critical flow throat velocity.

The enthalpy calculations, made with Equations 12 and 14 and either Equation 8 or 10, give the ΔH values for the isentropic expansions to the trial final temperatures. With these ΔH values, the gas velocities are calculated by Equation 12. The velocity of sound in the gas at these flow conditions is calculated by applying Equation 6 with the same heat capacity and equation of state information. Velocities by Equations 3 and 6 are compared. When these two velocities are equal, the critical flow condition has been found.

The solution of Equation 6 for the sonic velocity requires evaluating the first density derivative of pressure for the expansion isentrope. There are also two ways to do this, one via a volume-explicit equation of state and one via a pressure-explicit equation of state. Both methods are presented for completeness.

For the volume-explicit case, Equation 6 is written

$$\frac{1}{c^2} = \frac{1}{g_c}\left(\frac{\partial \rho}{\partial P}\right)_S = -\frac{\rho^2}{g_c}\left(\frac{\partial V}{\partial P}\right)_S = -\frac{1}{g_c V^2}\left(\frac{\partial V}{\partial P}\right)_S \tag{15}$$

The pressure derivative of volume at constant entropy is transformed

and combined with the definition of the constant pressure heat capacity, $C_P = T(\partial S/\partial T)_P$, and the Maxwell relation $(\partial S/\partial P)_T = -(\partial V/\partial T)_P$ to give

$$\left(\frac{\partial V}{\partial P}\right)_S = \left(\frac{\partial V}{\partial P}\right)_T + \frac{T}{C_P}\left(\frac{\partial V}{\partial T}\right)_P^2 \tag{16}$$

combining Equations 15 and 16 gives

$$\frac{1}{c^2} = -\frac{\rho^2}{g_c}\left[\left(\frac{\partial V}{\partial P}\right)_T + \frac{T}{C_P}\left(\frac{\partial V}{\partial T}\right)_P^2\right] \tag{17}$$

The pressure and temperature derivatives of volume can be computed from a volume-explicit equation of state. The heat capacity, C_P, is found by evaluating $\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P$ at constant temperature to obtain

$$C_P = C_P^o - T\int_{P_o}^{P_*}\left(\frac{\partial^2 V}{\partial T^2}\right)_P dP \tag{18}$$

Equation 18 gives the effects of pressure on the heat capacity at constant pressure and is a necessary step in solving Equation 17. By a convenient choice of the intermediate conditions for the alternate route shown on Figure 2, the pressure effects on heat capacity are not involved in the entropy and enthalpy calculations, i.e., Equations 7 and 8.

For the pressure-explicit case, the solution of Equation 6 involves evaluating the density derivative of pressure at constant entropy. This is first transformed and combined with the definition of the constant-volume heat capacity, $C_V = T(\partial S/\partial T)_V$, and the Maxwell relation, $(\partial S/\partial V)_T = (\partial P/\partial T)_V$ to give

$$\left(\frac{\partial P}{\partial \rho}\right)_S = \left(\frac{\partial P}{\partial \rho}\right)_T + \frac{T}{\rho^2 C_V}\left(\frac{\partial P}{\partial T}\right)_\rho^2 \tag{19}$$

combining Equations 6 and 19 gives

$$c^2 = -\frac{g_c}{\rho^2}\left[\left(\frac{\partial P}{\partial V}\right)_T - \frac{T}{C_V}\left(\frac{\partial P}{\partial T}\right)_V^2\right] \tag{20}$$

The temperature and volume derivatives of pressure are evaluated from a pressure-explicit equation of state. The heat capacity, C_V, is found by evaluating $\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$ at constant temperature to obtain

$$C_V = C_V^o - T\int_{\rho_o}^{\rho} \frac{1}{\rho^2}\left(\frac{\partial^2 P}{\partial T^2}\right)_\rho d\rho \tag{21}$$

Equation 21 gives the effects of fluid density on the heat capacity at constant volume and is a necessity in solving Equation 20 for the velocity of sound. By a convenient choice of the intermediate conditions for the alternate route shown on Figure 2, the density effects on heat capacity, i.e., Equation 21, are not needed in the entropy and enthalpy calculations via Equations 9 and 10.

The similarities and differences between Equations 17 and 20 should be noted. In applying Equations 7, 8, 17 and 18, the same volume-explicit equation of state should be used to obtain the volume and the derivatives of volume. Likewise, the same pressure-explicit equation of state should be used in obtaining the pressure and the derivatives of pressure in applying Equations 9, 10, 20 and 21.

After finding the critical-flow velocity, mass velocity may be calculated as

$$G_* = \frac{u_*}{V_*} = \rho_* u_*$$

and the critical-flow function, C^*, can be evaluated by Equation 2. The critical-flow density, ρ_*, is required in this computation and can be found by the same equation of state used in making the entropy and enthalpy calculations.

ISENTROPIC EXPONENT METHOD

In the isentropic-exponent method of predicting the critical flow velocity and properties, the pressure, density, and temperature conditions for the isentropic expansion process are formulated in terms of the isentropic exponents. These relationships are used in solving Equation 5 for the velocity and Equation 6 for the acoustic velocity. The combination gives the critical flow function.

For most fluids and conditions, the isentropic changes of specific volume (or density) can be defined by PV^n = constant. For a point on the isentrope,

$$n = -\left(\frac{\partial \ln P}{\partial \ln V}\right)_S = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_S \tag{22}$$

For an isentropic process of finite length, the exponent may be defined in terms of the terminal conditions as follows:

$$n = \left[\frac{\ln(P_*/P_o)}{\ln(\rho_*/\rho_o)}\right]_S \tag{23}$$

When these two exponents are the same, as they frequently are, the following relationships may be applied to the solution of the critical-flow-through-nozzle problem.

First, Equation 22 is transformed, combined with the definitions of the heat capacities C_P and C_V, and rearranged to give

$$n = \frac{C_P}{C_V}\left[-\frac{V}{P}\left(\frac{\partial P}{\partial V}\right)_T\right] = \frac{C_P}{C_V}\left[\frac{1}{PV}\left(\frac{\partial P}{\partial \rho}\right)_T\right] \tag{24}$$

The next step is to transform Equation 6, combine with the definitions of C_P and C_V, and rearrange to give

$$c = \left[g_c \frac{C_P}{C_V}\left(\frac{\partial P}{\partial \rho}\right)_T\right]^{1/2} \tag{25}$$

Equations 24 and 25 are next combined to eliminate $\frac{C_P}{C_V}\left(\frac{\partial P}{\partial \rho}\right)_T$ giving

$$c = \left(g_c\ nZRT\right)^{1/2} \tag{26}$$

Equation 26 can also be derived from the differential form of Equation 5,

$$VdP = -\frac{udu}{g_c} \tag{27}$$

The velocity, u, may be replaced by the product of specific volume and mass velocity. From this relationship, i.e., u = GV, we can write

$$du = GdV + VdG \tag{28}$$

At the constant area of the nozzle throat, the mass velocity is constant and the second right-hand member of Equation 28 is zero. Combining these u(G,V) relationships with Equation 27 and rearranging gives

$$\frac{dP}{dV} = -\frac{G^2}{g_c} \tag{29}$$

For isentropic expansion, Equation 22 gives

$$\frac{dP}{dV} = -n\frac{P}{V} \tag{30}$$

Combining Equations 29 and 30 gives

$$G^2 = g_c\, n\, \frac{P}{V} = g_c\, n\, P\rho \tag{31}$$

Now replace G^2 by u^2/V^2 and to obtain PV by ZRT

$$u = (g_c\, n\, PV)^{1/2} = (g_c\, n\, ZRT)^{1/2} \tag{32}$$

In the derivation of Equation 32, it was assumed that the PV energy was converted to kinetic energy. If the nozzle is not choked, critical flow will result. In the derivation of Equation 26, the conditions required for critical flow, i.e., Equation 6, were assumed. It is interesting to note that these different starting points give the same results.

At the throat of an isentropic-expansion nozzle, in critical flow, Equations 26 and 32 give the same velocity, i.e., $c_* = u_*$, and the mass velocity is

$$G = \left(g_c \; n \; \frac{P_*}{V_*}\right)^{1/2} = (g_c \; n \; P_* \rho_*)^{1/2} \tag{33}$$

Equation 33 is an expression for the mass velocity of a gas at critical flow through a reversible and adiabatic nozzle in terms of the isentropic exponent and the throat conditions, designated by the subscript *.

In terms of the PV^n = constant relationship,

$$\int_{P_o}^{P_*} VdP = \frac{n}{n-1} \; P_o V_o \left[\left(\frac{P_*}{P_o}\right)^{\frac{n-1}{n}} - 1\right] \tag{34}$$

Combining Equations 5 and 34, and assuming that $u_o = 0$, gives

$$\frac{u_*^2}{g_c \; n \; P_o V_o} = \frac{2}{n-1}\left[1 - \left(\frac{P_*}{P_o}\right)^{\frac{n-1}{n}}\right] \tag{35}$$

Combining Equations 35 with $u_*^2 = c_*^2 = g_c \; n \; P_* V_*$ from Equations 26 and 32 gives

$$\frac{P_* V_*}{P_o V_o} = \frac{2}{n-1}\left[1 - \left(\frac{P_*}{P_o}\right)^{\frac{n-1}{n}}\right] \tag{36}$$

combining Equation 36 with Equation 23 gives

$$\frac{P_*}{P_o} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \tag{37}$$

Equation 37 gives the critical pressure ratio as a function of the isentropic exponent only.

Combining Equation 37 with $P_* V_*^{\,n} = P_o V_o^{\,n}$ gives

$$\frac{P_*}{V_*} = \frac{P_o}{V_o}\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}} \tag{38}$$

Combining Equations 33 and 38 gives

$$G^2 = g_c \frac{P_o}{V_o} \left[n \left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}} \right] \tag{39}$$

Equation 39 gives the mass velocity at the nozzle throat in terms of the isentropic exponent n and the plenum conditions.

From Equation 39, the critical-flow function C*, as defined by Equation 2, can be expressed as a function of the isentropic exponent n and the plenum conditions, as follows:

$$C^* = \left(\frac{RT_o}{g_c}\right)^{1/2} \frac{G}{P_o} = Z_o^{-1/2} \left[n\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}} \right]^{1/2} \tag{40}$$

As can be seen by Equation 40, the critical-flow function C* is dimensionless and its evaluation by this method requires the compressibility factor in the plenum as well as the isentropic exponent.

The critical pressure ratio is shown by Equation 37 to be a single-valued function of the isentropic exponent. From the critical pressure ratio, the critical density ratio may be found, using the same isentropic exponent, by

$$\frac{\rho_*}{\rho_o} = \left(\frac{P_*}{P_o}\right)^{1/n} \tag{41}$$

For the calculation of the critical temperature ratio from P_*/P_o, a different exponent is involved, however. This temperature-change exponent is defined as follows for an isentropic path.

$$\frac{T_*}{T_o} = \left(\frac{P_*}{P_o}\right)^{\frac{n'-1}{n'}} \tag{42}$$

In differential form,

$$\frac{n'-1}{n'} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_S = \frac{P}{T}\left(\frac{\partial T}{\partial P}\right)_S \tag{43}$$

Transforming gives a form of Equation 43 that is convenient for

evaluation with heat-capacity and equation-of-state information

$$\frac{n'-1}{n'} = -\frac{P}{C_P}\frac{\left(\frac{\partial P}{\partial T}\right)_V}{\left(\frac{\partial P}{\partial V}\right)_T} \tag{44}$$

The temperature-change exponent, n', is very nearly constant for the isentropic-expansion paths of interest, as is the volume- and density-change exponent, n. Where this constancy exists, the values of n' and n evaluated at the plenum conditions, can be used in calculating the critical-flow function for the isentropic-expansion path.

In the isentropic-exponent method, it is not necessary to calculate the throat conditions and velocity in finding the value of the critical-flow function.

One of the steps in evaluating the isentropic exponent, n, is to find the heat capacity ratio, C_P/C_V, at the same conditions. This ratio could be obtained from the results of Equations 18 and 21, if both can be evaluated. One or the other of these heat capacities, but not both, is required in the velocity of sound calculations. It is convenient, therefore, to apply the following expressions for the heat capacity difference

$$C_P - C_V = -T\frac{\left(\frac{\partial V}{\partial T}\right)_P^2}{\left(\frac{\partial V}{\partial P}\right)_T} = -T\frac{\left(\frac{\partial P}{\partial T}\right)_V^2}{\left(\frac{\partial P}{\partial V}\right)_T} \tag{45}$$

Equation 45 is written in two forms, one for use with the volume-explicit equation of state and one for the pressure-explicit form.

Evaluation of the isentropic exponents involves the solution of Equations 24 and 44 for n and n' and Equations 18 or 21, and 45 for heat capacities. These exponent evaluations require the use of an equation of state in calculations that are about as lengthy as the entropy-enthalpy calculations. Without simple generalized correlations for the exponents, this method offers no advantage for the final calculations.

The isentropic exponents for ideal gases, i.e., C_P^o/C_V^o, can be used to advantage, however, in developing a computer program that will converge on a correct solution for the critical-flow velocity by the entropy-enthalpy method.

VAN DER WAALS FLUID

A previous treatment (1) of the isentropic-flow characteristics of the van der Waals fluid assumed that the differential expression for the constant entropy process is

$$C_V \, dT - \frac{P}{\rho^2} \cdot d\rho = 0 \tag{46}$$

rather than

$$C_V \, dT - \frac{T}{\rho^2} \left(\frac{\partial P}{\partial T}\right)_\rho d\rho = 0 \tag{47}$$

Equation 47 is the correct relationship. Equation 46 may be obtained by assuming that $PV = RT$, from which $T(\partial P/\partial T)_V = RT/V = P$. In the solution of Equation 46, the van der Waals equation was used (1), so the results partly represent the van der Waals fluid and partly the perfect gas.

From the van der Waals equation,

$$P = \frac{RT}{V-b} - \frac{a}{V^2} \tag{48}$$

the following are obtained for the isothermal effects of density on the entropy and enthalpy by integrating Equations 9 and 10 at constant temperature and choosing a reference state of ideal gas at unity pressure as the lower limit at which the entropy is S^o and the enthalpy is H^o (see Appendix for details of derivation);

$$\frac{S-S^o}{R} + \ln P = \ln (1-b\rho) + \ln Z \tag{49}$$

and

$$\frac{H-H^o}{RT} = \frac{b\rho}{1-b\rho} - 2\frac{a\rho}{RT} \tag{50}$$

The left side of Equation 49 is the residual entropy over R and is dimensionless. The left-hand member of Equation 50 is dimensionless also. The solution of Equation 20 for the velocity of sound with the derivatives of pressure from the van der Waals equation is

$$c^2 = g_c\left[\frac{RT}{(1-b\rho)^2}\left(1 + \frac{R}{C_V}\right) - 2a\rho\right] \tag{51}$$

The solution of Equation 21 for C_V involves the second temperature derivation of pressure, which is zero by the van der Waals equation. Therefore, $C_V = C_V^o$ in this case.

Equations 48, 49, 50 and 51 and heat capacities at the ideal gas state provide all the necessary information for making critical-flow calculations for gases represented by the van der Waals equation.

The isentropic exponents could also be evaluated from expressions based on the van der Waals equation, if desired. These exponent expressions are not included, however, because of their lesser utility.

REDLICH-KWONG EQUATION OF STATE

A two-constant equation of state that represents real-gas behavior more realistically is the Redlich-Kwong (7) equation, of which the pressure form is

$$P = \frac{RT}{V-b} - \frac{a}{T^{0.5}V(V+b)} \tag{52}$$

where: $a = 0.4278 \quad R^2T_c^{2.5}/P_c$

$b = 0.0867 \quad RT_c/P_c$

The compressibility-factor form of the Redlich-Kwong equation of

state is

$$Z = \left(\frac{1}{1-h}\right) - \frac{A^2}{B}\left(\frac{h}{1+h}\right) \tag{53}$$

where: $A^2 = \dfrac{a}{R^2T^{2.5}}$

$$B = \frac{b}{RT}$$

$$h = \frac{BP}{Z} = \frac{b}{V} = b\rho$$

The two parameters, A^2/B and BP, are sufficient to define the RK thermodynamic state. The third parameter, h, appears in the expressions for thermodynamic properties.

In terms of the Redlich-Kwong equation of state, the isothermal effects of density on entropy and enthalpy are obtained by integrating Equations 9 and 10, using Equation 52, and obtaining the following.

$$\frac{S-S^o}{R} + \ln P = \ln (Z-BP) - \frac{1}{2}\frac{A^2}{B}\ln (1+h) \tag{54}$$

and

$$\frac{H-H^o}{RT} = -\frac{3}{2}\frac{A^2}{B}\ln (1+h) + Z - 1 \tag{55}$$

The detailed derivations of Equations 54 and 55, showing the definitions of the reference states, are given in the Appendix. The solution of Equations 54 and 55 for the effects of density on the entropy and enthalpy involves first finding the value of the parameter, h, and the compressibility factor, Z, by a trial-and-error solution of Equations 52 or 53.

In a similar manner, an expression for the velocity of sound is found by solving Equation 20, with derivatives obtained from the Redlich-Kwong equation, obtaining

$$c^2 = g_c\left[\frac{RT\left(\frac{R}{C_V} + 1\right)}{(1-h)^2} + \frac{A^2\ RT\ h}{B(1+h)}\left(\frac{R}{C_V(1-h)} + \frac{1}{4}\ \frac{A^2\ R\ h}{B\ C_V(1+h)} - \frac{(2+h)}{(1+h)}\right)\right] \tag{56}$$

The solution of Equation 56 to obtain the velocity of sound at the nozzle throat requires evaluating the parameter h, via Equations 52 or 53, and the heat capacity C_V.

The isothermal effect of density on the heat capacity, C_V, is obtained by solving Equation 21 with the second temperature derivative of pressure obtained from Equation 52 and then integrating with the ideal gas state as the lower limit. The result is

$$\frac{C_V - C_V^o}{R} = \frac{3}{4}\frac{A^2}{B} \ln (1+h) \tag{57}$$

The same value of h is used in solving both Equations 56 and 57. The value of C_V from Equation 57 is used in Equation 56.

PROPERTIES OF REAL GASES AT CRITICAL FLOW

Thermodynamic properties of six real gases (air, nitrogen, oxygen, normal hydrogen, para-hydrogen, and steam) at critical-flow conditions have been computed by the enthalpy-entropy method (4, 5). The results of these calculations are reported (5) in comprehensive tabulations, giving the following nine properties for each gas, as a function of the plenum temperature and pressure conditions: (1) critical-flow factor, (2) nozzle-throat velocity, (3) critical pressure ratio, (4) critical temperature ratio, (5) critical density ratio, (6) compressibility factor, (7) normalized specific heat, C_P/R, (8) specific-heat ratio, C_P/C_V, and (9) speed of sound. The last four of these properties are at the plenum or stagnation conditions.

Pressure-explicit equations of state were used in the calculations for air and steam while volume-explicit equations of state were used for the nitrogen, oxygen and hydrogen. The same heat-capacity and equation-of-state information was used in the velocity-of-sound calculations.

Convergence on the correct throat temperature to give a critical-flow throat velocity, or Mach One, was accelerated in the calculations by using the following temperature correction to be added to the previous temperature estimate (5).

$$\Delta T = \left[\left(\frac{\partial T}{\partial M}\right)_S\right]_i \Delta M \tag{58}$$

where $\left[\left(\frac{\partial T}{\partial M}\right)_S\right]_i$ is estimated from isentropic, ideal-gas relations and ΔM is the difference between the desired and actual Mach Number, i.e., $\Delta M = M - M'$, $\left[M' = (u/c)_{calc}\right]$. An expression for $\left[(\partial T/\partial M)_S\right]_i$ will be derived next.

Applying Equation 3 to the perfect gas, for which $\Delta H = C_P^o \Delta T$, gives the following equation for the velocity of a perfect gas after adiabatic expansion

$$u = \left[2g_c C_P^o (T_o - T)\right]^{1/2} \tag{3a}$$

In like manner, Equation 26 is applied to a perfect gas, for which $n = k = C_P^o/C_V^o$ and $Z = PV/RT = 1$, to obtain the following expression for the velocity of sound in a perfect gas

$$c = (g_c\ k\ RT)^{1/2} \tag{26a}$$

From the definition of Mach Number, i.e., $M = u/c$, an expression relating the temperature ratio and M is obtained for the perfect gas, for which $C_P^o - C_V^o = R$, by combining Equations 3a and 26a; giving the following

$$\frac{T_o}{T} = 1 + \frac{k-1}{2} M^2 \tag{59}$$

Solving for T and differentiating with respect to M gives

$$\left[\left(\frac{\partial T}{\partial M}\right)_S\right]_i = -\frac{T_o(k-1)M}{(1 + \frac{k-1}{2} M^2)^2} \tag{60}$$

combining Equations 59 and 60, to eliminate the denominator of the right hand member of Equation 60, gives

$$\left[\left(\frac{\partial T}{\partial M}\right)_S\right]_i = -T\left(\frac{T}{T_o}\right)(k-1)\ M \tag{61}$$

As written, Equations 58 and 61 may be applied at any Mach Number. For the critical-flow nozzle calculations, $M = 1$ and $T = T_*$.

Another useful perfect gas relationship is found by combining Equations 37 and 42, using the exponent k instead of n and n', to obtain

$$\frac{T_*}{T_o} = \frac{2}{k+1} \tag{62}$$

Equation 62 can be used to estimate T_* at the beginning of these calculations.

THERMODYNAMIC PROPERTIES OF STEAM IN CRITICAL-FLOW

In his Table V, pp. 106-131, Johnson (5) presents the thermodynamic properties of steam at critical-flow conditions, covering plenum temperatures from 700 to 1500°R by 10° intervals and plenum pressures from 0 to 300 atm by 1 to 5 atm intervals. An abridged and supplemented version of this table was prepared by taking values from Johnson's Table V at less frequent intervals of T_o and P_o, adding isentropic exponents and eliminating plenum specific heats.

This reconstituted table of the thermodynamic properties of steam in critical-flow is included herewith for use in solving problems to illustrate the computation of these properties and their application.

Although heat-capacity ratios are included in the original tables, no actual isentropic exponents were included. From the information given, it is a simple matter to evaluate the values of n, the isentropic-density- or volume-change exponent, however. Values of n

at both plenum and throat conditions can be found by inverting Equation 26 using values of c, Z and T given in the tabulations for both the plenum and critical-flow nozzle throat.

The following example illustrates this calculation for steam, for which the following readings are made from the tables

$$T_o = 1500°R$$

$$P_o = 300 \text{ atm}$$

$$Z_o = 0.8083$$

$$c_o = 2088.7 \text{ ft/sec}$$

$$n = \frac{c_o^2}{g_c Z_o R T_o} = \frac{(2088.7)^2 \ (18)}{(32.2)(0.8083)(1543)(1500)}$$

$$n = 1.303$$

The value of n may also be calculated at the throat conditions, by using other data given in the tabulations

$$P_* = (300)(0.5462) = 163.9 \text{ atm}$$

$$T_* = (1500)(0.8691) = 1304°R$$

$$Z_* = (0.8083)(0.5462)\left(\frac{1}{0.8691}\right)\left(\frac{1}{0.6296}\right) = 0.810$$

$$c_* = 1942.1$$

$$n = \frac{(1942.1)^2 \ (18)}{(32.2)(0.810)(1544)(1304)} = 1.293$$

The isentropic exponent for the paths may be found from the critical rate values; $P_*/P_o = 0.5462$ and $\rho_*/\rho_o = 0.6296$, with Equation 23, giving

$$n = \frac{\ln 0.5462}{\ln 0.6296} = 1.304$$

For these conditions, the three values of n are nearly equal. These values of the isentropic-volume-change exponent were calculated and included in the new table, as can be seen.

The isentropic-temperature-change exponent, n', could also be derived from the Johnson Table V data, using Equation 42 and the critical temperature and critical pressure ratios, which are: $(T_*/T_o) = 0.8691$ and $(P_*/P_o) = 0.5462$. Solving Equation 42 for n' and substituting, gives

$$n' = \frac{1}{1 - \left[\frac{\ln (T_*/T_o)}{\ln (P_*/P_o)}\right]} = \frac{1}{1 - \left[\frac{\ln 0.8691}{\ln 0.5462}\right]} = 1.302$$

In this case, n' is almost identical to the values of n, which is a coincidence; n' will not always have the same numerical value as n.

The value of the heat-capacity ratio C_P/C_V for steam at the same plenum conditions is 1.6222, which differs greatly from the above values of n. For the ideal gas state at 1500°R

$$k = \frac{C_P^o}{C_P^o - R} = \frac{9.33}{9.33 - 1.987} = 1.274$$

Being closer to n than is the C_P/C_V ratio, k is a better exponent than C_P/C_V for the approximation calculations of Equations 61 and 62.

NOMENCLATURE

A, B	-	constants in one arrangement of the Redlich-Kwong Equation of State
a, b	-	constants in the van der Waals or in the Redlich-Kwong Equation of State
C_P	-	constant-pressure heat capacity of gas, Btu/lb_m°R
C_V	-	constant-volume heat capacity of gas, Btu/lb_m°R
C*	-	critical-flow function of Johnson (5), dimensionless
c	-	speed of sound in gas, ft/sec
c_*	-	speed of sound in gas at throat, ft/sec
G	-	mass-flow rate, lb_m/ft^2sec
g_c	-	gravitational conversion factor, equals 32.174 $lb_m ft/sec^2 lb_f$
H	-	enthalpy of gas, Btu/lb_m
h	-	constant in one arrangement of the Redlich-Kwong Equation of State
k	-	heat-capacity ratio, equals C_P/C_V (dimensionless)
M	-	Mach number, i.e., ratio of the flow velocity to the speed of sound in the gas
n	-	isentropic-volume-change exponent (dimensionless)
n'	-	isentropic-temperature-change exponent (dimensionless)
P	-	pressure (absolute), atm (or psf)
R	-	gas-law constant, ft^3atm/lb_m°R (or ft^3psf/lb_m°R)
S	-	entropy of gas, Btu/lb_m°R
T	-	temperature (absolute), °R
u	-	velocity of gas, ft/sec
V	-	specific volume of gas, ft^3/lb_m
Z	-	compressibility factor of gas, equals PV/RT

GREEK LETTERS

Δ - change of

ρ - density of gas, lb_m/ft^3

ϕ - critical-flow function of Murdock and Bauman (3), $lb_m(°R)^{1/2}/sec\text{-}lb_f$

SUBSCRIPTS

o - plenum

* - throat

SUPERSCRIPT

o - reference state

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THERMODYNAMIC PROPERTIES OF STEAM AT CRITICAL FLOW CONDITIONS (1 of 6 pages)

Plenum Conditions			Critical Ratios			Isentropic Exponents				Velocity of Sound ft/sec		Critical Flow Function	Plenum	
						Plenum		Path						
T_o °R	P_o atm	$Z = P_oV_o/RT_o$	P_*/P_o	ρ_*/ρ_o	T_*/T_o	C_p/C_v	n	n	n'	c_o	$c_* = u_*$	C^*	P_o atm	T_o °R
850	0	1.0000	0.5421	0.6280	0.8632	1.3125	1.3125	1.3122	1.3162	1754.7	1634.7	0.6703	0	850
900	0	1.0000	0.5425	0.6277	0.8643	1.3092	1.3092	1.3132	1.3131	1803.3	1681.5	0.6697	0	900
	10	0.9553	0.5455	0.6275	0.8693	1.3640	1.2983	1.3005	1.3006	1755.2	1638.2	0.6828	10	
950	0	1.0000	0.5430	0.6275	0.8654	1.3059	1.3059	1.3103	1.3101	1850.3	1726.8	0.6692	0	950
	10	0.9645	0.5449	0.6272	0.8679	1.3487	1.2982	1.3015	1.3044	1811.9	1692.6	0.6798	10	
	20	0.9248	0.5474	0.6269	0.8716	1.4065	1.2893	1.2904	1.2954	1768.1	1653.0	0.6921	20	
1000	0	1.0000	0.5435	0.6272	0.8666	1.3025	1.3025	1.3071	1.3069	1895.9	1770.9	0.6686	0	1000
	10	0.9712	0.5448	0.6270	0.8677	1.3369	1.2968	1.3010	1.3049	1864.4	1743.0	0.6773	10	
	20	0.9400	0.5464	0.6268	0.8696	1.3801	1.2909	1.2939	1.3007	1830.0	1711.9	0.6871	20	
	30	0.9061	0.5483	0.6266	0.8720	1.4352	1.2841	1.2856	1.2952	1792.0	1677.3	0.6982	30	
1050	0	1.0000	0.5440	0.6270	0.8677	1.2991	1.2991	1.3042	1.3039	1940.2	1813.8	0.6681	0	1050
	10	0.9762	0.5450	0.6267	0.8682	1.3274	1.2949	1.2989	1.3035	1913.9	1790.5	0.6753	10	
	20	0.9510	0.5461	0.6266	0.8690	1.3611	1.2906	1.2942	1.3023	1885.9	1765.2	0.6832	20	
	30	0.9242	0.5473	0.6265	0.8702	1.4017	1.2864	1.2890	1.2998	1856.1	1737.9	0.6920	30	
	40	0.8957	0.5487	0.6264	0.8716	1.4511	1.2819	1.2831	1.2969	1824.1	1708.3	0.7018	40	
	50	0.8651	0.5503	0.6263	0.8733	1.5124	1.2775	1.2764	1.2934	1789.6	1675.9	0.7128	50	
1100	0	1.0000	0.5446	0.6267	0.8689	1.2957	1.2957	1.3005	1.3008	1983.3	1855.7	0.6675	0	1100
	10	0.9801	0.5453	0.6265	0.8689	1.3194	1.2924	1.2968	1.3020	1961.0	1835.8	0.6736	10	
	20	0.9593	0.5460	0.6264	0.8692	1.3466	1.2894	1.2937	1.3015	1937.8	1814.8	0.6802	20	
	30	0.9376	0.5469	0.6263	0.8696	1.3779	1.2866	1.2897	1.3013	1913.7	1792.6	0.6873	30	
	40	0.9149	0.5478	0.6263	0.8703	1.4143	1.2839	1.2862	1.3001	1888.4	1768.9	0.6950	40	
	50	0.8911	0.5489	0.6263	0.8711	1.4570	1.2813	1.2819	1.2988	1861.8	1743.7	0.7034	50	
	60	0.8660	0.5500	0.6263	0.8722	1.5077	1.2792	1.2776	1.2965	1833.9	1716.9	0.7127	60	
	70	0.8395	0.5512	0.6264	0.8733	1.5687	1.2774	1.2733	1.2944	1804.3	1688.0	0.7230	70	
	80	0.8112	0.5525	0.6266	0.8746	1.6435	1.2760	1.2692	1.2917	1772.7	1656.8	0.7345	80	
1150	0	1.0000	0.5451	0.6265	0.8701	1.2924	1.2924	1.2976	1.2976	2025.3	1896.5	0.6669	0	1150
	10	0.9831	0.5456	0.6263	0.8699	1.3125	1.2899	1.2948	1.2988	2006.2	1879.4	0.6721	10	
	20	0.9657	0.5462	0.6262	0.8698	1.3348	1.2878	1.2920	1.2998	1986.7	1861.6	0.6778	20	

THERMODYNAMIC PROPERTIES OF STEAM AT CRITICAL FLOW CONDITIONS (2 of 6 pages)

Plenum Conditions			Critical Ratios			Isentropic Exponents				Velocity of Sound ft/sec		Critical Flow Function	Plenum	
						Plenum		Path						
T_o °R	P_o atm	$Z = P_oV_o/RT_o$	P_*/P_o	ρ_*/ρ_o	T_*/T_o	C_P/C_V	n	n	n'	c_o	$c_* = u_*$	C^*	P_o atm	T_o °R
1150	30	0.9478	0.5468	0.6262	0.8698	1.3599	1.2858	1.2896	1.3005	1966.7	1843.1	0.6835	30	1150
	40	0.9293	0.5475	0.6262	0.8700	1.3882	1.2841	1.2869	1.3007	1946.1	1823.7	0.6898	40	
	50	0.9102	0.5482	0.6262	0.8703	1.4200	1.2826	1.2842	1.3006	1924.9	1803.4	0.6965	50	
	60	0.8903	0.5489	0.6263	0.8707	1.4563	1.2817	1.2819	1.3001	1903.1	1782.1	0.7037	60	
	70	0.8697	0.5497	0.6264	0.8713	1.4977	1.2813	1.2792	1.2991	1880.6	1759.7	0.7114	70	
	80	0.8483	0.5505	0.6266	0.8720	1.5455	1.2811	1.2765	1.2978	1857.2	1736.2	0.7199	80	
	90	0.8260	0.5513	0.6269	0.8727	1.6011	1.2816	1.2752	1.2965	1833.0	1711.3	0.7291	90	
	100	0.8025	0.5522	0.6273	0.8736	1.6667	1.2831	1.2735	1.2946	1807.8	1684.8	0.7392	100	
	120	0.7519	0.5541	0.6284	0.8758	1.8408	1.2890	1.2709	1.2897	1753.9	1626.5	0.7631	120	
1200	0	1.0000	0.5456	0.6262	0.8713	1.2891	1.2891	1.2944	1.2943	2066.2	1936.3	0.6663	0	1200
	10	0.9856	0.5461	0.6261	0.8710	1.3063	1.2872	1.2919	1.2958	2049.8	1921.6	0.6708	10	
	20	0.9708	0.5465	0.6260	0.8707	1.3251	1.2856	1.2899	1.2973	2033.1	1906.4	0.6755	20	
	30	0.9557	0.5469	0.6258	0.8706	1.3458	1.2842	1.2875	1.2981	2016.1	1890.7	0.6803	30	
	40	0.9403	0.5474	0.6257	0.8706	1.3685	1.2829	1.2852	1.2986	1998.8	1874.6	0.6854	40	
	50	0.9244	0.5478	0.6255	0.8707	1.3934	1.2823	1.2827	1.2988	1981.3	1858.0	0.6908	50	
	60	0.9082	0.5483	0.6252	0.8709	1.4210	1.2815	1.2794	1.2987	1963.3	1840.9	0.6964	60	
	70	0.8915	0.5487	0.6250	0.8712	1.4516	1.2813	1.2770	1.2982	1945.0	1823.3	0.7024	70	
	80	0.8743	0.5492	0.6247	0.8716	1.4856	1.2816	1.2738	1.2975	1926.4	1805.0	0.7087	80	
	90	0.8567	0.5496	0.6244	0.8721	1.5237	1.2820	1.2709	1.2964	1907.2	1786.2	0.7154	90	
	100	0.8385	0.5501	0.6240	0.8727	1.5663	1.2832	1.2666	1.2951	1887.7	1766.6	0.7225	100	
	120	0.8002	0.5509	0.6231	0.8741	1.6694	1.2871	1.2604	1.2915	1846.9	1725.2	0.7382	120	
	140	0.7591	0.5515	0.6219	0.8759	1.8049	1.2941	1.2529	1.2865	1803.7	1680.2	0.7565	140	
	160	0.7143	0.5520	0.6204	0.8779	1.9907	1.3057	1.2447	1.2806	1757.5	1630.8	0.7782	160	
1250	0	1.0000	0.5462	0.6260	0.8725	1.2859	1.2859	1.2911	1.2912	2106.1	1975.3	0.6658	0	1250
	10	0.9876	0.5465	0.6259	0.8721	1.3008	1.2845	1.2895	1.2928	2091.9	1962.4	0.6696	10	
	20	0.9750	0.5468	0.6258	0.8717	1.3168	1.2832	1.2879	1.2944	2077.5	1949.2	0.6736	20	
	30	0.9621	0.5472	0.6257	0.8714	1.3342	1.2824	1.2859	1.2958	2063.1	1935.7	0.6778	30	
	40	0.9491	0.5475	0.6256	0.8711	1.3530	1.2817	1.2843	1.2971	2048.5	1921.9	0.6821	40	

THERMODYNAMIC PROPERTIES OF STEAM AT CRITICAL FLOW CONDITIONS (3 of 6 pages)

Plenum Conditions			Critical Ratios			Isentropic Exponents				Velocity of Sound ft/sec		Critical Flow Function	Plenum	
						Plenum		Path						
T_o °R	P_o atm	$Z = P_oV_o/RT_o$	P_*/P_o	ρ_*/ρ_o	T_*/T_o	C_P/C_V	n	n	n'	c_o	$c_* = u_*$	C^*	P_o atm	T_o °R
1250	50	0.9357	0.5478	0.6255	0.8709	1.3733	1.2813	1.2827	1.2981	2033.7	1907.8	0.6866	50	1250
	60	0.9222	0.5480	0.6253	0.8708	1.3953	1.2811	1.2811	1.2987	2018.8	1893.4	0.6913	60	
	70	0.9084	0.5483	0.6252	0.8708	1.4191	1.2813	1.2794	1.2991	2003.8	1878.7	0.6961	70	
	80	0.8943	0.5486	0.6250	0.8708	1.4451	1.2818	1.2774	1.2994	1988.6	1863.6	0.7012	80	
	90	0.8799	0.5488	0.6248	0.8709	1.4733	1.2826	1.2758	1.2993	1973.1	1848.1	0.7065	90	
	100	0.8652	0.5490	0.6245	0.8711	1.5042	1.2838	1.2737	1.2989	1957.5	1832.3	0.7121	100	
	120	0.8348	0.5493	0.6239	0.8716	1.5750	1.2876	1.2699	1.2976	1925.6	1799.4	0.7241	120	
	140	0.8030	0.5495	0.6232	0.8723	1.6607	1.2932	1.2662	1.2956	1892.7	1764.7	0.7374	140	
	160	0.7696	0.5494	0.6222	0.8732	1.7662	1.3013	1.2622	1.2927	1858.7	1728.0	0.7523	160	
	180	0.7342	0.5490	0.6210	0.8743	1.8984	1.3124	1.2587	1.2887	1823.2	1689.1	0.7692	180	
	200	0.6966	0.5480	0.6193	0.8755	2.0677	1.3275	1.2553	1.2838	1786.1	1647.4	0.7886	200	
	220	0.6564	0.5464	0.6172	0.8769	2.2908	1.3481	1.2525	1.2777	1747.2	1602.7	0.8114	220	
	240	0.6130	0.5434	0.6142	0.8785	2.5943	1.3765	1.2513	1.2697	1706.1	1554.2	0.8384	240	
1300	0	1.0000	0.5467	0.6258	0.8737	1.2826	1.2826	1.2883	1.2880	2145.1	2013.4	0.6652	0	1300
	10	0.9893	0.5470	0.6257	0.8732	1.2957	1.2816	1.2867	1.2899	2132.8	2002.1	0.6686	10	
	20	0.9784	0.5472	0.6256	0.8727	1.3096	1.2809	1.2855	1.2917	2120.4	1990.5	0.6720	20	
	30	0.9674	0.5475	0.6256	0.8723	1.3244	1.2803	1.2843	1.2933	2108.0	1978.8	0.6756	30	
	40	0.9562	0.5477	0.6255	0.8719	1.3402	1.2800	1.2831	1.2948	2095.5	1966.9	0.6793	40	
	50	0.9449	0.5479	0.6254	0.8716	1.3570	1.2799	1.2819	1.2960	2083.0	1954.8	0.6831	50	
	60	0.9334	0.5481	0.6253	0.8713	1.3750	1.2802	1.2807	1.2972	2070.5	1942.5	0.6870	60	
	70	0.9218	0.5482	0.6252	0.8711	1.3943	1.2807	1.2798	1.2980	2058.0	1930.0	0.6911	70	
	80	0.9101	0.5484	0.6251	0.8709	1.4148	1.2813	1.2786	1.2988	2045.4	1917.3	0.6953	80	
	90	0.8981	0.5485	0.6250	0.8707	1.4368	1.2823	1.2778	1.2996	2032.7	1904.4	0.6997	90	
	100	0.8860	0.5485	0.6248	0.8706	1.4603	1.2835	1.2769	1.2999	2019.9	1891.3	0.7042	100	
	120	0.8613	0.5486	0.6244	0.8705	1.5124	1.2870	1.2741	1.3004	1994.2	1864.5	0.7137	120	
	140	0.8357	0.5485	0.6239	0.8705	1.5726	1.2920	1.2730	1.3002	1968.2	1836.7	0.7239	140	
	160	0.8094	0.5481	0.6233	0.8707	1.6421	1.2982	1.2720	1.2991	1941.6	1807.9	0.7350	160	
	180	0.7821	0.5475	0.6224	0.8711	1.7231	1.3063	1.2704	1.2971	1914.5	1778.2	0.7471	180	
	200	0.7539	0.5466	0.6214	0.8715	1.8179	1.3161	1.2696	1.2948	1886.7	1747.4	0.7603	200	
	220	0.7246	0.5452	0.6199	0.8721	1.9295	1.3279	1.2685	1.2913	1858.0	1715.5	0.7749	220	

THERMODYNAMIC PROPERTIES OF STEAM AT CRITICAL FLOW CONDITIONS (4 of 6 pages)

Plenum Conditons			Critical Ratios			Isentropic Exponents				Velocity of Sound ft/sec		Critical Flow Function	Plenum	
						Plenum		Path						
T_o °R	P_o atm	$Z_o = P_oV_o/RT_o$	P_*/P_o	ρ_*/ρ_o	T_*/T_o	C_p/C_v	n	n	n'	c_o	$c_* = u_*$	C^*	P_o atm	T_o °R
1300	240	0.6942	0.5432	0.6181	0.8728	2.0617	1.3423	1.2885	1.2869	1828.4	1682.6	0.7910	240	1300
	260	0.6626	0.5404	0.6158	0.8735	2.2188	1.3595	1.2894	1.2817	1797.7	1648.6	0.8089	260	
	280	0.6298	0.5367	0.6128	0.8743	2.4054	1.3800	1.2708	1.2753	1765.8	1613.6	0.8290	280	
	300	0.5958	0.5312	0.6086	0.8750	2.6248	1.4044	1.2739	1.2675	1732.6	1577.7	0.8509	300	
1350	0	1.0000	0.5473	0.6255	0.8749	1.2795	1.2795	1.2846	1.2849	2183.3	2050.8	0.6846	0	1350
	10	0.9907	0.5475	0.6255	0.8743	1.2909	1.2787	1.2839	1.2870	2172.5	2040.7	0.6875	10	
	20	0.9812	0.5477	0.6255	0.8738	1.3031	1.2784	1.2831	1.2888	2161.8	2030.5	0.6906	20	
	30	0.9717	0.5479	0.6255	0.8733	1.3158	1.2781	1.2823	1.2906	2151.1	2020.2	0.6938	30	
	40	0.9621	0.5481	0.6256	0.8729	1.3293	1.2781	1.2820	1.2921	2140.4	2009.8	0.6970	40	
	50	0.9524	0.5482	0.6256	0.8724	1.3436	1.2783	1.2812	1.2938	2129.8	1999.2	0.6804	50	
	60	0.9427	0.5484	0.6257	0.8719	1.3586	1.2787	1.2812	1.2956	2119.2	1988.4	0.6838	60	
	70	0.9328	0.5486	0.6258	0.8715	1.3745	1.2793	1.2809	1.2971	2108.6	1977.5	0.6874	70	
	80	0.9229	0.5487	0.6260	0.8711	1.3912	1.2801	1.2814	1.2985	2098.0	1966.5	0.6911	80	
	90	0.9128	0.5489	0.6261	0.8707	1.4088	1.2813	1.2810	1.3001	2087.5	1955.3	0.6949	90	
	100	0.9027	0.5490	0.6263	0.8704	1.4274	1.2825	1.2815	1.3012	2076.9	1943.9	0.6988	100	
	120	0.8821	0.5492	0.6268	0.8697	1.4677	1.2860	1.2829	1.3037	2055.8	1920.8	0.7071	120	
	140	0.8611	0.5493	0.6273	0.8692	1.5126	1.2903	1.2848	1.3055	2034.6	1897.0	0.7160	140	
	160	0.8397	0.5494	0.6280	0.8688	1.5626	1.2955	1.2874	1.3069	2013.2	1872.5	0.7256	160	
	180	0.8178	0.5493	0.6288	0.8685	1.6182	1.3017	1.2913	1.3078	1991.5	1847.4	0.7359	180	
	200	0.7955	0.5491	0.6297	0.8683	1.6801	1.3087	1.2961	1.3082	1969.5	1821.6	0.7470	200	
	220	0.7727	0.5487	0.6307	0.8683	1.7489	1.3166	1.3022	1.3077	1946.9	1795.2	0.7591	220	
	240	0.7494	0.5481	0.6318	0.8685	1.8253	1.3254	1.3095	1.3063	1923.7	1768.1	0.7723	240	
	260	0.7256	0.5473	0.6330	0.8688	1.9098	1.3349	1.3181	1.3044	1899.7	1740.5	0.7866	260	
	280	0.7013	0.5461	0.6342	0.8693	2.0030	1.3451	1.3285	1.3013	1874.7	1712.4	0.8023	280	
	300	0.6766	0.5446	0.6356	0.8699	2.1046	1.3555	1.3409	1.2976	1848.5	1683.9	0.8195	300	
1400	0	1.0000	0.5478	0.6253	0.8760	1.2763	1.2763	1.2818	1.2820	2220.6	2087.4	0.6841	0	1400
	10	0.9918	0.5480	0.6253	0.8755	1.2865	1.2759	1.2811	1.2837	2211.2	2078.5	0.6867	10	
	20	0.9836	0.5481	0.6253	0.8750	1.2971	1.2758	1.2807	1.2855	2201.9	2069.5	0.6894	20	
	30	0.9754	0.5483	0.6254	0.8745	1.3083	1.2758	1.2803	1.2873	2192.7	2060.4	0.6921	30	

THERMODYNAMIC PROPERTIES OF STEAM AT CRITICAL FLOW CONDITIONS (5 of 6 pages)

Plenum Conditions			Critical Ratios			Isentropic Exponents				Velocity of Sound ft/sec		Critical Flow Function	Plenum	
						Plenum		Path						
T_o °R	P_o atm	$Z = P_oV_o/RT_o$	P_*/P_o	ρ_*/ρ_o	T_*/T_o	C_p/C_v	n	n	n'	c_o	$c_* = u_*$	C^*	P_o atm	T_o °R
1400	40	0.9671	0.5484	0.6254	0.8740	1.3200	1.2759	1.2799	1.2890	2183.5	2051.3	0.6749	40	1400
	50	0.9587	0.5485	0.6255	0.8735	1.3322	1.2764	1.2800	1.2907	2174.4	2042.0	0.6778	50	
	60	0.9503	0.5487	0.6256	0.8731	1.3449	1.2771	1.2797	1.2922	2165.4	2032.7	0.6808	60	
	70	0.9419	0.5488	0.6257	0.8726	1.3582	1.2778	1.2797	1.2938	2156.4	2023.3	0.6838	70	
	80	0.9334	0.5489	0.6259	0.8722	1.3721	1.2788	1.2801	1.2952	2147.5	2013.8	0.6870	80	
	90	0.9248	0.5489	0.6260	0.8718	1.3866	1.2800	1.2806	1.2965	2138.6	2004.2	0.6902	90	
	100	0.9162	0.5490	0.6262	0.8714	1.4018	1.2813	1.2811	1.2979	2129.7	1994.5	0.6935	100	
	120	0.8989	0.5491	0.6266	0.8707	1.4340	1.2844	1.2824	1.3003	2112.1	1974.9	0.7004	120	
	140	0.8814	0.5492	0.6271	0.8700	1.4690	1.2883	1.2843	1.3027	2094.6	1955.0	0.7077	140	
	160	0.8636	0.5491	0.6276	0.8695	1.5069	1.2928	1.2868	1.3042	2076.9	1934.7	0.7154	160	
	180	0.8456	0.5490	0.6283	0.8691	1.5478	1.2979	1.2903	1.3054	2059.2	1914.2	0.7235	180	
	200	0.8274	0.5488	0.6290	0.8687	1.5919	1.3032	1.2942	1.3065	2041.1	1893.3	0.7322	200	
	220	0.8089	0.5484	0.6297	0.8685	1.6392	1.3089	1.2989	1.3067	2022.6	1872.2	0.7414	220	
	240	0.7903	0.5479	0.6304	0.8684	1.6898	1.3147	1.3040	1.3064	2003.6	1850.8	0.7511	240	
	260	0.7714	0.5473	0.6312	0.8685	1.7435	1.3204	1.3100	1.3053	1983.8	1829.2	0.7615	260	
	280	0.7523	0.5464	0.6320	0.8686	1.8003	1.3258	1.3172	1.3039	1963.1	1807.5	0.7725	280	
	300	0.7329	0.5454	0.6327	0.8689	1.8597	1.3309	1.3243	1.3018	1941.3	1785.6	0.7842	300	
1450	0	1.0000	0.5483	0.6251	0.8772	1.2732	1.2732	1.2790	1.2788	2257.2	2123.3	0.6635	0	1450
	10	0.9928	0.5485	0.6251	0.8767	1.2823	1.2731	1.2782	1.2806	2249.0	2115.4	0.6658	10	
	20	0.9857	0.5486	0.6252	0.8762	1.2917	1.2731	1.2783	1.2823	2240.9	2107.4	0.6682	20	
	30	0.9785	0.5487	0.6252	0.8757	1.3015	1.2733	1.2779	1.2839	2232.9	2099.4	0.6706	30	
	40	0.9712	0.5488	0.6253	0.8752	1.3117	1.2738	1.2780	1.2856	2225.0	2091.4	0.6731	40	
	50	0.9640	0.5489	0.6254	0.8747	1.3223	1.2742	1.2780	1.2873	2217.1	2083.3	0.6756	50	
	60	0.9567	0.5490	0.6255	0.8742	1.3332	1.2750	1.2781	1.2890	2209.4	2075.1	0.6782	60	
	70	0.9494	0.5490	0.6256	0.8738	1.3446	1.2760	1.2785	1.2903	2201.8	2066.9	0.6808	70	
	80	0.9421	0.5491	0.6257	0.8734	1.3564	1.2770	1.2785	1.2917	2194.2	2058.7	0.6835	80	
	90	0.9348	0.5491	0.6259	0.8729	1.3685	1.2782	1.2794	1.2933	2186.7	2050.4	0.6862	90	
	100	0.9275	0.5491	0.6260	0.8725	1.3811	1.2795	1.2798	1.2945	2179.2	2042.1	0.6891	100	
	120	0.9127	0.5492	0.6264	0.8718	1.4076	1.2826	1.2812	1.2969	2164.4	2025.3	0.6949	120	

THERMODYNAMIC PROPERTIES OF STEAM AT CRITICAL FLOW CONDITIONS (6 of 6 pages)

Plenum Conditions			Critical Ratios			Isentropic Exponents				Velocity of Sound ft/sec		Critical Flow Function	Plenum	
						Plenum		Path						
T_o °R	P_o atm	$Z_o = P_oV_o/RT_o$	P_*/P_o	ρ_*/ρ_o	T_*/T_o	C_P/C_V	n	n	n'	c_o	c_*	C^*	P_o atm	T_o °R
1450	140	0.8978	0.5491	0.6269	0.8711	1.4357	1.2863	1.2838	1.2990	2149.7	2008.4	0.7010	140	1450
	160	0.8829	0.5490	0.6273	0.8705	1.4656	1.2901	1.2859	1.3008	2135.0	1991.3	0.7073	160	
	180	0.8678	0.5489	0.6278	0.8699	1.4972	1.2943	1.2885	1.3027	2120.1	1974.1	0.7140	180	
	200	0.8527	0.5486	0.6284	0.8695	1.5304	1.2984	1.2923	1.3036	2104.9	1956.9	0.7209	200	
	220	0.8374	0.5483	0.6289	0.8692	1.5651	1.3027	1.2957	1.3042	2089.4	1939.5	0.7282	220	
	240	0.8220	0.5478	0.6295	0.8689	1.6013	1.3068	1.3004	1.3046	2073.3	1922.0	0.7358	240	
	260	0.8065	0.5473	0.6300	0.8687	1.6388	1.3103	1.3045	1.3047	2056.4	1904.6	0.7437	260	
	280	0.7909	0.5466	0.6305	0.8687	1.6773	1.3132	1.3096	1.3038	2038.7	1887.1	0.7520	280	
	300	0.7752	0.5458	0.6309	0.8687	1.7165	1.3151	1.3146	1.3029	2019.8	1869.6	0.7606	300	
1500	0	1.0000	0.5489	0.6249	0.8784	1.2701	1.2701	1.2758	1.2758	2293.0	2158.6	0.6630	0	1500
	10	0.9937	0.5490	0.6249	0.8778	1.2782	1.2702	1.2754	1.2777	2285.8	2151.5	0.6650	10	
	20	0.9874	0.5490	0.6250	0.8773	1.2866	1.2704	1.2759	1.2793	2278.7	2144.5	0.6671	20	
	30	0.9811	0.5491	0.6250	0.8769	1.2953	1.2708	1.2755	1.2806	2271.8	2137.4	0.6693	30	
	40	0.9748	0.5492	0.6251	0.8764	1.3043	1.2713	1.2755	1.2823	2265.0	2130.2	0.6714	40	
	50	0.9685	0.5492	0.6252	0.8759	1.3135	1.2721	1.2760	1.2839	2258.3	2123.1	0.6736	50	
	60	0.9622	0.5493	0.6253	0.8754	1.3231	1.2729	1.2760	1.2855	2251.7	2116.0	0.6759	60	
	70	0.9558	0.5493	0.6254	0.8750	1.3329	1.2740	1.2764	1.2868	2245.2	2108.8	0.6782	70	
	80	0.9495	0.5493	0.6256	0.8746	1.3430	1.2751	1.2773	1.2881	2238.7	2101.6	0.6805	80	
	90	0.9432	0.5494	0.6257	0.8741	1.3534	1.2763	1.2774	1.2898	2232.3	2094.4	0.6829	90	
	100	0.9369	0.5494	0.6259	0.8737	1.3640	1.2776	1.2782	1.2911	2226.0	2087.2	0.6853	100	
	120	0.9242	0.5493	0.6262	0.8729	1.3862	1.2807	1.2799	1.2935	2213.5	2072.7	0.6903	120	
	140	0.9115	0.5492	0.6266	0.8722	1.4094	1.2839	1.2821	1.2956	2201.0	2058.2	0.6954	140	
	160	0.8988	0.5491	0.6270	0.8716	1.4336	1.2874	1.2842	1.2974	2188.6	2043.7	0.7008	160	
	180	0.8860	0.5489	0.6275	0.8710	1.4587	1.2909	1.2872	1.2991	2175.9	2029.1	0.7063	180	
	200	0.8732	0.5486	0.6279	0.8705	1.4847	1.2943	1.2901	1.3004	2163.0	2014.6	0.7120	200	
	220	0.8603	0.5483	0.6283	0.8701	1.5115	1.2976	1.2931	1.3013	2149.7	2000.0	0.7179	220	
	240	0.8474	0.5479	0.6287	0.8697	1.5388	1.3003	1.2964	1.3021	2135.7	1985.5	0.7240	240	
	260	0.8344	0.5474	0.6291	0.8694	1.5665	1.3025	1.3002	1.3025	2121.1	1971.0	0.7303	260	
	280	0.8214	0.5468	0.6294	0.8692	1.5944	1.3036	1.3039	1.3024	2105.4	1956.5	0.7368	280	
	300	0.8083	0.5462	0.6296	0.8691	1.6222	1.3038	1.3071	1.3021	2088.7	1942.1	0.7435	300	

APPENDIX

Detailed derivations of Equations 49 (page 15) and 54 (page 17) for isothermal changes in the entropy are presented here, as are derivations of Equations 50 (page 16) and 55 (page 17) for isothermal changes in the enthalpy.

At constant temperature

$$S_2 - S_1 = \int_{V_1}^{V_2} (\partial P/\partial T)_V \, dV \qquad \text{(A-1)}$$

From the van der Waals Equation, i.e., Equation 48 (page 15)

$$(\partial P/\partial T)_V = R(V-b)^{-1} \qquad \text{(A-2)}$$

Combining Equations A-1 and A-2 and integrating

$$S_2 - S_1 = \int_{V_1}^{V_2} R \, (V-b)^{-1} \, dV = R \ln \frac{V_2-b}{V_1-b} \qquad \text{(A-3)}$$

Rearranging Equation A-3

$$S_2 - R \ln (V_2-b) = S_1 - R \ln (V_1-b) \qquad \text{(A-4)}$$

Let state "2" be the state of interest and state "1" be a low pressure ideal gas reference state, for which

$$\lim_{V \to \infty} \left[S_1 - R \ln (V_1-b)\right] = \left[S_1 - R \ln V_1\right] = \left[S_1 - R \ln \frac{RT}{P_1}\right]$$

$$= \left[S_1 + R \ln P_1 - R \ln RT\right] = \left[S^o - R \ln RT\right] \qquad \text{(A-5)}$$

where: $S^o = \lim_{P_1 \to 0} (S + R \ln P)$ = entropy of ideal gas at P = 1.

In view of Equation A-5, Equation A-4 becomes

$$\frac{S - S^o}{R} = \ln \frac{V-b}{RT} \qquad \text{(A-6)}$$

Adding ln P to both sides of Equation A-6 and rearranging by combining with $V = 1/\rho$ and $Z = PV/RT$ gives

$$\frac{S - S^o}{R} + \ln P = \ln (1-b\rho) + \ln Z \qquad (49)$$

The left hand side of Equation 49 is the residual entropy ratio.

For isothermal changes in the entropy, start with

$$H_2 - H_1 = \int_{V_1}^{V_2} \left[T(\partial P/\partial T)_V - P\right] dV + \Delta(PV) \qquad \text{(A-7)}$$

The integrand of Equation A-7 is found by combining Equations A-2 and 48, which gives

$$H_2 - H_1 = \int_{V_1}^{V_2} a\ V^{-2}\ dV + \Delta(PV) \qquad \text{(A-8)}$$

Integrating gives

$$H_2 - H_1 = a\left[\frac{1}{V_1} - \frac{1}{V_2}\right] + \Delta(PV) \qquad \text{(A-9)}$$

Let state "2" be the condition of interest and let state "1" be a zero pressure where $1/V_1 = 0$ and $P_1V_1 = RT$. Substituting these limits and dividing through by RT gives

$$\frac{H - H^o}{RT} = -\frac{a\rho}{RT} + Z - 1 \qquad \text{(A-10)}$$

where: H^o = ideal gas enthalpy at zero pressure.

Combining with Equation 48, to eliminate Z gives

$$\frac{H - H^o}{RT} = \frac{b\rho}{1 - b\rho} - \frac{2a\rho}{RT} \qquad (50)$$

Similar equations will next be derived for predicting the isothermal changes of entropy and enthalpy via the Redlich-Kwong equation (i.e., Equation 52 (page 16)).

From Equation 52, the following derivative is found

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V-b} + \frac{0.5a}{T^{1.5}\,V(V+b)} \tag{A-11}$$

Combining Equations A-1 and A-11 gives

$$S_2 - S_1 = R\int_{V_1}^{V_2} \frac{dV}{V-b} + \frac{0.5a}{T^{1.5}}\int_{V_1}^{V_2} \frac{dV}{V(V+b)} \tag{A-12}$$

Integration gives

$$S_2 - S_1 = R \ln \frac{V_2-b}{V_1-b} + \frac{0.5a}{bT^{1.5}} \ln \frac{V_2(V_1+b)}{V_1(V_2+b)} \tag{A-13}$$

Rearranging Equation A-13

$$\begin{aligned} S_2 - R \ln (V_2 - b) + \frac{0.5a}{bT^{1.5}} \ln \frac{V_2+b}{V_2} &= \\ S_1 - R \ln (V_1 - b) + \frac{0.5a}{bT^{1.5}} \ln \frac{V_1+b}{V_1} & \end{aligned} \tag{A-14}$$

Let state "2" be the state of interest and state "1" be a low pressure ideal gas reference state, for which

$$\begin{aligned} \lim_{V_1 \to \infty} \left[S_1 - R \ln (V_1-b) + \frac{0.5a}{bT^{1.5}} \ln \frac{V_1+b}{V_1}\right] &= \left[S_1 - R \ln V_1\right] = \\ \left[S_1 - R \ln \frac{RT}{P_1}\right] &= \left[S_1 + R \ln P_1 - R \ln RT\right] = \\ \left[S^o - R \ln RT\right] & \end{aligned} \tag{A-15}$$

where: $S^o = \lim_{P \to 0} (S + R \ln P)$

In view of Equation A-15, Equation A-14 may be written

$$S - R \ln (V-b) + \frac{0.5a}{bT^{1.5}} \ln \frac{V+b}{V} = S^o - R \ln RT \qquad \text{(A-16)}$$

Rearranging

$$\frac{S - S^o}{R} = \ln \frac{V-b}{RT} - \frac{0.5a}{bRT^{1.5}} \ln \frac{V+b}{V} \qquad \text{(A-17)}$$

Adding ln P to both sides, rearranging with the definitions $Z = PV/RT$, $Pb/RT = BP$, $b/V = h$, $a/bRT^{1.5} = A^2/B$ gives

$$\frac{S - S^o}{R} + \ln P = \ln (Z-BP) - 1/2 \frac{A^2}{B} \ln (1+h) \qquad \text{(54)}$$

The Redlich-Kwong equation for isothermal changes in enthalpy is found from Equation A-7, for which the integrand is obtained by combining Equations 52 and A-11, giving

$$H_2 - H_1 = \frac{1.5a}{T^{0.5}} \int_{V_1}^{V_2} \frac{dV}{V(V+b)} + \Delta(PV) \qquad \text{(A-18)}$$

Integrating

$$H_2 - H_1 = \frac{1.5a}{bRT^{1.5}} \left[\ln \frac{V_2}{V_2+b} - \ln \frac{V_1+b}{V_1} \right] + \Delta(PV) \qquad \text{(A-19)}$$

Let state "2" be the state of interest and state "1" be ideal gas at zero pressure, for which

$$\lim_{V_1 \to \infty} \ln \frac{V_1+b}{V_1} = 0$$

with these definitions, Equation A-19 may be rearranged and divided by RT to give

$$\frac{H - H^o}{RT} = \frac{1.5a}{bRT^{1.5}} \ln \frac{V}{V+b} + Z - 1 \qquad \text{(A-20)}$$

Substituting the equivalents $b/V = h$ and $a/bRT^{1.5} = A^2/B$, gives

$$\frac{H - H^o}{RT} = - \frac{3}{2} \frac{A^2}{B} \ln (1+h) + Z - 1 \qquad (55)$$

HOME PROBLEMS - STATEMENTS

Problem 1: A large steam-filled tank has a small conveying nozzle in one wall. The area of the nozzle opening is 2 square inches. Find the thrust on the tank if the steam is at 1200°R and 70 atm. Atmospheric conditions are 760 mn and 60°F.

Problem 2: Steam at 1450°R and 240 atm flows through an isentropic nozzle to the atmosphere. Using the Redlich-Kwong Equation of State and ideal gas state heat capacities, calculate the critical-flow velocity at the nozzle throat. Also find the value of the critical-flow function. Prepare a program for making these calculations on the digital computer.

HOME PROBLEMS - SOLUTIONS

<u>Problem 1</u>: From the table of thermodynamic properties of steam at critical flow conditions find $Z_o = 0.8915$.

$$\rho_o = \frac{(70)(14.7)(144)(18)}{(1544)(0.8915)(1200)} = 1.616 \text{ lb}_m/\text{ft}^3$$

From table $P_*/P_o = 0.5487$

$T_*/T_o = 0.8712$

$\rho_*/\rho_o = 0.6250$

$u_* = 1823.3$ ft/sec

$C^* = 0.7024$

In general (for quasi-steady conditions)

$$\Sigma F_x = \text{Momentum Out} - \text{Momentum In}$$

For the conditions of this problem

$$\Sigma F_x = Th + (P_{atm} - P_*)\, A_*$$

where: Th = Thrust

$$\text{Momentum Out} = \frac{\rho_*}{g_c}\, u_*^{\,2}\, A_*$$

Momentum In = 0

Combining terms and solving for the thrust gives

$$Th = (P_* - P_{atm})\, A_* + \frac{\rho_*}{g_c}\, u_*^2$$

Substituting

$$Th = [(0.5487)\ 1000 - 14.7]\ 2 + \frac{(1.616)(0.6250)}{32.174}\ (1823.3)^2\ 2/144$$

$$= 1068 + 1456 = 2524 \text{ lb}_f$$

Mass velocity

$$G = (1823.3)(1.616)(0.6250) = 1841 \text{ lb}_m/\text{ft}^2\text{sec}$$

Critical-flow function (via Equation 2, page 2)

$$C^* = \frac{G}{P_o}\left(\frac{RT_o}{g_c}\right)^{1/2} = \frac{1841}{144 \times 1030}\left(\frac{1544 \times 1200}{18 \times 32.174}\right)^{1/2}$$

$C^* = 0.70$ vs 0.7024 from tabulation

<u>Problem 2</u> Mr. Pin Techasiriwan prepared the manual solution.

Following is a step-by-step hand solution for the critical flow properties of steam at 1450°R and 240 atm, using the Redlich-Kwong equation of state (i.e,, Equation 52, page 16).

Solution: The plan-of-attack is to find the steam conditions along an isentropic expansion that gives a Mach-one velocity in the nozzle throat. This involves first finding a temperature that satisfies the $\Delta S = 0$ requirement and then calculating the actual velocity and the velocity of sound.

<u>Step 1</u>

The constants a and b of the R-K equations are found from the critical conditions

$$T_c = 1165.1°R; \; P_c = 3206 \text{ psia}$$

$$R = 10.74 \frac{\text{psia-ft}^3}{\text{lb-mole}°R} \; ; \; MW = 18.016$$

$$a = \frac{0.4278 \; R^2 T_c^{2.5}}{P_c}$$

$$= \frac{(0.4278)(10.74)^2(1165.1)^{2.5}(144)}{(18.016)^2(3206)}$$

$$= 3.158 \times 10^5 \; (\text{lb}_f/\text{ft}^2)(\text{ft}^3/\text{lb}_m)^2(°R)^{1/2}$$

$$b = 0.0867 \frac{RTc}{P_c}$$

$$= \frac{(0.0867)(10.74)(1165.1)}{(18.016)(3206)}$$

$$= 0.01877 \text{ ft}^3/\text{lb}_m$$

Step 2

Throat conditions (P,V,T) are found as follows: an estimated temperature, found by an ideal gas exponent calculation, is used as a starting point for successive approximation calculations to find the throat conditions that satisfy the two requirements, i.e., isentropic expansion and Mach-One.

The ideal gas exponent calculation is made by combining Equations 37 and 42, assuming n = n' = k, to give

$$T_* = T_o \left(\frac{2}{k+1}\right)$$

where: $k = C_p^o/C_v^o$ evaluated at the plenum

$$k = \frac{C_p^o}{C_p^o - R} = \frac{9.264}{9.264 - 1.987} = 1.273$$

$$T_* = 1450 \left(\frac{2}{1.273 + 1.0}\right) = 1276°R$$

This temperature is too high. The correct value is $T_* = 1247°R$ and the correct throat pressure is $P_* = 1945$ psia.

In the following are sample calculations for these conditions, which have been found by calculation trials that are omitted.

Step 3

An entropy balance calculation is made by using Equations 11 and 54 and the ideal gas state values of S^o found from separate compilations (8). These calculations are tabulated.

	Plenum	Throat
T, °R	1450	1247
P, psia	3527	1945
V, ft^3/lb_m	0.194	0.309
h = b/V	0.0968	0.0607

Continued

	Plenum	Throat
A^2/B	3.563	4.459
Z	0.793	0.809
BP	0.0766	0.0491
Z-BP	0.7164	0.7603
ln (Z-BP)	-0.336	-0.2703
ln (1+h)	0.0923	0.0589
$(1/2)(A^2/B)\ln(1+h)$	0.1644	0.1313
$(S-S^o)/R + \ln p$	-0.5004	-0.4016
ln p	8.17	7.59
$(S-S^o)/R$	-8.67	-7.972
$S-S^o$	-17.22	-15.82
S^o, Btu/lb-mole°R	53.5	52.1

Referring to Figure 2, page 6

$$\Delta S_D = \Delta S_A + \Delta S_B + \Delta S_C$$

$$= 17.22 + (52.1 - 53.5) - 15.82$$

$$= 0$$

Step 4

Calculate the enthalpy change for the same path by applying Equations 12 and 55 and using ideal gas state H° values from a separate source (8), as follows:

	Plenum	Throat
T, °R	1450	1247
A^2/B	3.563	4.459
ln (1+h)	0.0923	0.0589
$(3/2)(A^2/B)\ln(1+h)$	0.4928	0.3946
Z	0.793	0.809
$(H-H^o)/RT$	-0.7008	-0.5852
$H-H^o$, Btu/lb-mole	-2018	-1449
H^o, Btu/lb-mole	12075	10240

$$\Delta H_D = \Delta H_A + \Delta H_B + \Delta H_C$$

$$= 2018 + (10240 - 12075) - 1449$$

$$= -1266 \text{ Btu/lb-mole}$$

Step 5

Calculate the throat velocity U_* by applying Equation 12, as follows:

$$\frac{u_*^2}{2g_c} = \frac{Ho-H*}{18.016}$$

$$u_* = \sqrt{\frac{(2)(32.174)(778)(1266)}{18.016}} = 1874 \text{ ft/sec}$$

Step 6

Calculate velocity of sound in steam at these conditions by applying Equation 56, as follows:

$$\frac{C_*^2}{g_c} = \frac{RT\left(\frac{R}{C_v} + 1\right)}{(1 - b\rho)^2} + \frac{a\rho}{T^{1/2}(1 + b\rho)}\left[\frac{R}{C_v(1 - b\rho)} + \frac{a\rho}{4\,C_v\,T^{3/2}(1 + b\rho)} - \frac{2 + b\rho}{1 + b\rho}\right]$$

$$\frac{C_v - C_v^o}{R} = \frac{3}{4}\,\frac{A^2}{B}\ln(1 + h)$$

$$= \left(\frac{3}{4}\right)(4.459)(0.059)$$

$$= 0.1973$$

$$C_p^o = 8.865 \text{ Btu/lb-mole-°R at } 1247°R$$

$$\frac{c_*^2}{g_c} = \frac{(1.987)(1247)(1.987/7.271 + 1)}{(1 - 0.0607)^2} + \frac{(3.158 \times 10^5)(18.016)}{(778)(1247)^{1/2}(1+0.0607)(0.309)}\left[\frac{1.987}{7.271(1-0.067)} + \frac{(3.158 \times 10^5)(18.016)}{(4)(7.271)(1247)^{3/2}(778)(0.309)(1+0.0607)} - \frac{2 + 0.0607}{1 + 0.0607}\right]$$

$$= 2540.8 \text{ Btu/lb-mole}$$

$$c_* = \frac{(2540.8)(32.174)(778)}{18.016}$$

$$= 1879 \text{ ft/sec}$$

$$M = \frac{u_*}{c_*} = \frac{1874}{1879} = 0.9973$$

Step 7

Calculate the critical flow function from

$$C^* = \frac{G}{P_o}\left(\frac{RT_o}{g_c}\right)^{1/2}$$

where $G = (\rho u)_*$

$$C^* = \frac{1874}{(0.309)(3527)(144)}\left[\frac{(10.74)(144)(1450)}{(18.016)(32.174)}\right]^{1/2}$$

$$= 0.7418$$

This value of C*is slightly higher than the value of 0.7358 from the tabulations.

Computer Solution of Problem 2

The following computer program was written in FORTRAN IV language specifically for use with an IBM 7040 computer system. The program could be converted into FORTRAN II-D language by:

(a) Changing the READ and WRITE statements

READ (NRD, --) → READ --,
WRITE (NWR, --) → PUNCH --, or TYPE --, or PRINT --,

(b) Shortening the FORMAT lengths.

(c) Changing to the FORTRAN II-D library-function names.

ALOG → LOGF
ABS → ABSF
SQRT → SQRTF

(d) Replacing all Logical IF statements with Arithmetic IF statements.

This program takes as input the critical temperature and pressure of the gas and calculates from these the Redlich-Kwong equation a and b constants using the equations given by Redlich and Kwong. Equation of state and derivatives thereof, i.e., Equations 52 through 57 (pp. 16-18).

The ideal gas enthalpy and entropy differences, ΔH^o and ΔS^o, are obtained by integrating

$$\Delta H^o = \int C_p^o \, dT = H^o(T_o) - H^o(T_*)$$

and

$$\Delta S^o = \int \frac{C_p^o}{T} \, dT = S^o(T_o) - S^o(T_*)$$

The evaluation of $H^o(T_o)$ and of $H^o(T_*)$, and also of $S^o(T_o)$ and of $S^o(T_*)$, take place at separated places in the program. But since only the differences, ΔH^o and ΔS^o, are ever subsequently utilized, there is no need to ever introduce the constants of integration.

Similarly, only the difference between $(S-S^o)_o$ and $(S-S^o)_*$ is ever utilized. Therefore, the standard-state pressure need never be specified. (It could be taken to approach a zero limit.) The program takes this standard-state pressure to be the same at all points. The above equation for ΔS^o is correct only for a path between two points of equal pressure.

All intermediate results are printed out so that the convergence rates can be examined. If a large number of critical flows were to be calculated, then only the final results would need be printed. Also, if these calculations are all for the same gas, then the physical-property data need only be read in once and R, a, and b need only be calculated once.

Mr. H. G. Rackett prepared the following computer program and calculation.

```
C  PROGRAM TO CALCULATE CRITICAL THROAT VELOCITY USING THE
C   REDLICH-KWONG EQUATION OF STATE        (PAGE I OF 2)
C
      DIMENSION   GAS(3)
   11 FORMAT ( 3A6, F8.3, F8.2, F8.3, 3F8.5, F7.1, F7.2,                )
   13 FORMAT ( 1H1, ///// , 12X, 3A6, 30X, 11HPLENUM,  T=, F8.2,
     C   13H DEG.R.,    P=, F7.2, 5H ATM., ///, 12X, 7HPLENUM,, 4X, 2HTX,
     C   8X, 2HVX, 7X, 1HI, 7X, 2HPX, 8X, 2HZX,                        / )
   14 FORMAT ( // , 12X, 7HTHROAT,, 4X, 2HTY, 8X, 2HVY, 6X, 4HJ  K, 5X,
     C   2HPY, 8X, 2HZY, 9X, 1HM, 9X, 2HC*, 8X, 1HU, 7X, 4HDELS,      / )
   16 FORMAT (      F38.5, I5, F12.3,                                   )
   17 FORMAT (      F27.2, F11.5, I5, F12.3, F10.5,                   / )
   21 FORMAT (      F38.5, I7, F59.4,                                   )
   22 FORMAT (      F27.2, F11.5, I4, I3, F10.3, 2F10.5, 2F10.2, F9.4,/ )
         NRD = 5
         NWR = 6
C
C  DEFINITION OF FUNCTIONS USED IN PROGRAM
      CPO(T) = ( CO + C1*T/100. + C2*T*T/10000. )*RJ                    Cp°
      HO(T) = ( CO + C1*T/200. + C2*T*T/30000. )*T*RJ                   H° ∓
      SO(T) = (CO*ALOG(T/100.)+C1*T/100.+C2*T*T/20000.)*RJ              S° ∓
      ABVT(V,T) = A*ALOG(1.+B/V)/(B*SQRT(T)*T)                  a ln(1+b/v)/bT^1.5
      HP(V,T) = B*R*T/(V-B)-A/((V+B)*SQRT(T))-1.5*ABVT(V,T)*T           H-H°
      SP(V,T) = R*ALOG((V-B)/T) - 0.5*ABVT(V,T)                         S-S° ∓
      AKO(T) = CPO(T) / (CPO(T) - R)                                    k = Cp°/Cv°
C
C  INTRODUCTION OF INPUT DATA, SETTING OF CONSTANTS
      RJ  = 778.17                                                      ft lbf/Btu
      PJ  = 2116.2                                                      (lbf/ft²)/atm.
      GC  = 32.174                                                      gc, lbm ft/sec² lbf
   28 READ (NRD, 11)  (GAS(I),I=1,3), AMW, TC, PC, CO, C1, C2, TX, PP
      WRITE (NWR, 13)  (GAS(I),I=1,3), TX, PP
      R   = 1545.3 / AMW                                                ft lbf/lbm °R
      A = 0.4278*(R*TC)**2 *SQRT(TC)/(PC*PJ)                    R-K a, (ft⁴ lbf/lbm²)°R^0.5
      B   = 0.0867 * R * TC/(PC*PJ)                                     R-K b, ft³/lbm
      PX  = PP * PJ                                                     Po, Given, lbf/ft²
      RT  = R * TX                                                      RTo
      RTP = RT / PX                                                     RTo/Po
      AT  = A / SQRT(TX)                                                a/To^0.5
      APT = AT / PX                                                     a/PoTo^0.5
      AKOX = AKO(TX)                                                    ko= (Cp°/Cv°)o
C
C  DETERMINATION OF VOLUME IN PLENUM AT GIVEN TEMPERATURE AND PRESSURE
      VX  = RTP                                                 Initial Estimate Vo, ft³/lbm
      DO 35   I = 1, 15
      PX  = RT/(VX-B) - AT/(VX*(VX+B))                          calculated Po, lbf/ft²
      P   = PX / PJ                                             calculated Po, atm.
      IF (ABS(PP-P) .LT. 0.005)   GO TO 38                      Po(Given) - Po(calc.), atm
      WRITE (NWR, 16)   VX, I, P
   35 VX  = RTP * VX/(VX-B) - APT/(VX+B)                        Vo, successive estimates
   38 ZX  = VX * PX / RT                                                Zo
      WRITE (NWR, 17)   TX, VX, I, P, ZX
C
```

∓ plus undetermined integration constant

```
C  PROGRAM TO CALCULATE CRITICAL THROAT VELOCITY USING THE
C   REDLICH-KWONG EQUATION OF STATE          (PAGE 2 OF 2)
C
C  DETERMINATION OF TEMPERATURE IN THROAT THAT GIVES MACH NUMBER EQUAL UNITY
      HPOX = HP(VX,TX) + HO(TX)                          Ho, ft. lbf/lbm ∓
      SPOX = SP(VX,TX) + SO(TX)                          So, ft. lbf/lbm°R ∓
      WRITE (NWR, 14)
      TY  = TX / (1. + (AKOX-1.)/2.)                     T*, Initial Est. °R
      VY  = VX * ((AKOX+1.)/2.)**(1./(AKOX-1.))          V*, Initial Est. ft³/lbm
      DO 55   J = 1, 10
C
C     DETERMINATION OF VOLUME IN THROAT THAT GIVES ZERO ENTROPY DIFFERENCE
C      AT ASSUMED THROAT TEMPERATURE
        SPOXY = SPOX - SO(TY)                            So-S*°, ft.lbf/lbm°R ∓
        RT  = R * TY                                            RT*
        AT  = A / SQRT(TY)                                      a/T*^0.5
        A2T = AT / (2.*TY)                                      a/2T*^1.5
        DO 45   K = 1, 10
        DELS = SPOXY - SP(VY,TY)                         ΔS = (So-S*), ft lbf/lbm°R
        IF (ABS(DELS) .LT. 0.0005)   GO TO 48
        WRITE (NWR, 21)   VY, K, DELS
        DSDV = R/(VY-B) + A2T/(VY*(VY+B))                     (∂S*/∂V*)T*
   45   VY  = VY + DELS/DSDV                             V*, successive estimates
C
   48 DELH = HPOX - HP(VY,TY) - HO(TY)                   ΔH = (Ho-H*), ft lbf/lbm
      U2  = 2. * GC * DELH
      U   = SQRT(U2)                                     u, Throat Velocity, ft/sec
      CVY = CPO(TY) - R + 0.75*ABVT(VY,TY)               (Cv)*, ft lbf/lbm°R
      RTVB = RT * VY/(VY-B)                              RT*V*/(v*-b)
      AVBT = A / ((VY+B)*SQRT(TY))                            a/(v*+b)T*^0.5
      CY2 = GC * ( RTVB*VY/(VY-B) - (1.+VY/(VY+B))*AVBT
     C      + (1./(CVY*TY)) * (RTVB + AVBT/2.)**2 )
      CY  = SQRT(CY2)                                c*, speed of sound in throat, ft/sec
      AM  = U / CY                                       M, Mach Number
      PY  = RT/(VY-B) - AT/(VY*(VY+B))                   P*, calculated, lbf/ft²
      P   = PY / PJ                                      P*, calculated, Atm
      ZY  = VY * PY / RT                                      Z*
      WRITE (NWR, 22)   TY, VY, J, K, P, ZY, AM, CY, U, DELS
      DELM = 1. - AM                                          ΔM
      IF (ABS(DELM) .LT. 0.00001)   GO TO 28
      DTDM = - TY*(TY/TX) * (AKO(TY)-1.)                    (∂T*/∂M)s*°
   55 TY  = TY + DELM*DTDM                             T* = Successive Estimates
C
      END
```

∓ plus undetermined integration constant.

SAMPLE DATA INPUT CARD

STEAM	18.016	1165.1	218.17	0.4006 0.00638 0.0001	1450.	240.
GAS (I): Name of gas	AMW: Av. Mol. Wt.	TC: T_c, Crit. Temp., °R	PC: P_c, Crit. Press. Atm.	CO C1 C3 Constants in Equation C_p^o = CO + C1 T/100 + C2 T^2/10000 C_p^o = Heat capacity in ideal gas state, Btu/lb_m °R T = °R	TX: T_o	PP: P_o

PROBLEM 2 COMPUTER OUTPUT

STEAM PLENUM, T= 1450.00 DEG.R., P= 240.00 ATM.

PLENUM,	TX	VX	I	PX	ZX
		0.24488	1	199.205	
		0.20326	2	231.701	
		0.19623	3	238.265	
		0.19481	4	239.636	
		0.19451	5	239.923	
		0.19445	6	239.984	
	1450.00	0.19444	7	239.997	0.79400

THROAT,	TY	VY	J	K	PY	ZY	M	C*	U	DELS
		0.31070		1						-7.3868
		0.28815		2						0.3276
		0.28907		3						0.0006
	1275.91	0.28907	1	4	144.362	0.80694	0.92117	1896.85	1747.32	0.0000
		0.28907		1						6.3436
		0.30682		2						0.2171
	1250.77	0.30747	2	3	133.524	0.80982	0.99449	1879.32	1868.97	0.0003
		0.30747		1						0.4308
		0.30877		2						0.0011
	1249.07	0.30877	3	3	132.815	0.81002	0.99933	1878.15	1876.91	-0.0000
		0.30877		1						0.0519
	1248.87	0.30893	4	2	132.730	0.81005	0.99992	1878.01	1877.86	0.0000
		0.30893		1						0.0064
	1248.84	0.30894	5	2	132.720	0.81005	0.99999	1878.00	1877.98	0.0000
		0.30894		1						0.0008
	1248.84	0.30895	6	2	132.718	0.81005	1.00000	1877.99	1877.99	0.0000

T, °R	V, ft^3/lb_m	P, ATM	Z	M	c_*, ft/sec	u, ft/sec	$S_o - S_*$ $ft\ lb_f/lb_m\ ^oR$