5TH QUARTERLY PROGRESS REPORT

Contract No. NSR 05-264-002

October 1967

STUDY OF SHAPE AND INTERNAL STRUCTURE OF MOON, UTILIZING LUNAR ORBITER DATA

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LUNAR ORBITER PROGRAM OFFICE

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PREFACE

This report summarizes progress to date on a study of the shape and internal structure of the moon utilizing data from the Lunar Orbiter Program under NASA Contract No. NSR 05-264-002. This report includes the results summarized in quarterly progress reports numbers 1 through 4 and thus supersedes these previous reports. In this report the nomenclature and form of the potential function are modified to be consistent with recommendations of the International Astronomical Union.

SUMMARY

Comparison of the terms in a spherical harmonic expansion of the lunar gravity field derived from analyses of Lunar Orbiter data with similar terms in a lunar continentality function reveal that excess mass must underlie the maria. Density variations and internal structures which would lead to the observed gravity field are calculated for a rigid moon with a random lumpiness and a moon in isostatic equilibrium and thermal convection. It is found that either model of internal structure is consistent with the data. The density contrasts and dimensions for each model were found to be remarkably consistent for the $C_{2,0}$, $C_{2,2}$ and $C_{3,0}$ terms. The overall moment of inertia as determined from the orbiter data is still too poorly known to determine whether the moon is homogeneous or differentiated.

INTRODUCTION

The principal objective of the present study is to determine the relationship between the gravitational field of the moon and the shape, internal structure and gross continent-maria distribution. Analysis of tracking data from the U.S. Lunar Orbiter series of spacecraft and Luna-10 have provided values for the coefficients of a spherical harmonic expansion of the external lunar gravity field. Lunar orbiter photographs of the backside provide almost complete information on the distribution of continents and maria so that it has been possible to calculate coefficients of a spherical harmonic expansion of a lunar continentality function for comparison with similar terms in the gravity field.

EFFECTS OF LUNAR CONTINENTALITY FUNCTION ON GRAVITY FIELD

Utilizing the form of the potential recommended by the International Astronomical Union and adopted in analyses of the Lunar Orbiter tracking data (Michael et al., 1967; Tolson and Gapcynski, 1967)

$$U = \frac{\mu}{r}\left[1 + \sum_{n=2}^{\infty}\sum_{m=o}^{n}\left(\frac{a}{r}\right)^{n} P_{n,m}(\sin\phi)(C_{n,m}\cos m\lambda + S_{n,m}\sin m\lambda)\right]$$

where μ is the product of the gravitational constant and the mass of the moon, r is the radial distance from the center of the moon, a is the mean radius of the moon, $P_{n,m}$ are associated Legendre polynomials, ϕ is the latitude, and λ is the longitude.

In order to compare the gravity field and the distribution of continents and maria the values of the coefficients of a lunar continentality function were calculated utilizing a computer program which coded the equations presented by Munk and MacDonald (1960). Table 1 indicates the proportion of continental areas on the lunar surface in squares 10° on each side utilized in the program. The distribution was compiled from the USAF Lunar Reference Mosaic (1960), the USAF Project Apollo Lunar Planning Chart (1962), and Lunar Farside Charts LFC-1 and LFC-2 (1967). Because of the different nature of the expansions used by Munk and MacDonald (1960) and in the above form of the potential, each term in the continentality function was multiplied by $(n - m)!/n!$ to permit direct comparison with similar terms in the gravity field. The values of the coefficients in the continentality function and in the gravity field presented by Tolson and Gapcynski (1967) and Akim (1966) with the correction in sign of the terms with odd value of m, suggested by Tolson and Gapcynski (1967), are presented in Tables 2 and 3.

A Cartesian set of coordinate axes fixed in the moon are defined with the X and Y axes in the equatorial plane, the X axis directed toward the mean direction to the earth, and the Z axis directed along the axis of rotation. The second degree terms in the gravity field are related to the moments and products of inertia as follows:

East Longitude (Degrees)

Colatitude (Degrees)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
10	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
20	0.9	1.0	1.0	0.9	1.0	1.0	1.0	1.0	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
30	0.3	0.4	0.1	0.1	0.8	0.8	1.0	0.7	0.6	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
40	0.9	1.0	0.4	0.4	0.9	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
50	0.4	0.3	0.1	0.2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
60	0.6	0.0	0.0	0.7	1.0	0.6	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.1	1.0	1.0	1.0	
70	0.2	0.4	0.0	0.1	0.8	0.1	0.4	1.0	0.3	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
80	0.4	0.6	0.0	0.1	0.4	0.5	1.0	1.0	0.5	0.8	1.0	1.0	1.0	0.7	0.6	1.0	1.0	1.0	
90	0.9	1.0	0.5	0.9	1.0	0.0	0.9	0.9	0.3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
100	1.0	1.0	0.8	0.1	0.9	0.1	0.0	0.4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
110	1.0	1.0	1.0	0.9	1.0	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
120	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.3	0.2	0.7	1.0	1.0	1.0	1.0	0.9	0.1	1.0	
130	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.1	0.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0	0.6	1.0	
140	1.0	1.0	1.0	1.0	1.0	1.0	0.2	0.2	0.3	1.0	1.0	1.0	0.6	0.4	1.0	0.4	0.6	1.0	
150	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.5	0.4	0.7	1.0	1.0	1.0	
160	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.4	0.4	0.8	1.0	1.0	1.0	
170	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
180																			

Table 1 — Distribution of Continental Areas on the Moon

East Longitude (Degrees)

Colatitude (Degrees)	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320	230	340	350	360
0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
10	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
20	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.8	0.7	0.9	
30	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.8	0.6	0.8	0.2	0.3	0.5	0.6	0.5	
40	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.4	0.0	0.0	0.8	0.5	0.2	0.0	0.3	
50	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.2	0.0	0.0	0.2	0.1	0.0	0.0	0.0	
60	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	
70	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.8	0.1	0.0	0.0	0.0	0.3	0.4	0.5	
80	1.0	1.0	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.0	0.0	0.0	0.2	0.4	0.7	
90	1.0	0.7	0.1	1.0	1.0	1.0	1.0	0.8	0.7	0.8	1.0	0.9	0.1	0.0	0.0	0.1	0.5	1.0	
100	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.6	0.5	1.0	1.0	0.9	0.8	0.2	0.1	0.1	0.2	
110	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.4	0.5	1.0	1.0	1.0	0.6	0.4	0.3	0.1	0.8	
120	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.8	0.7	0.9	1.0	1.0	1.0	1.0	1.0	0.7	1.0	1.0	
130	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	
140	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
150	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
160	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
170	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
180																			

Table 1 — (Continued)

		$C_{n,m} \times 10^{-4}$		$S_{n,m} \times 10^{-4}$	
n	m	Tolson and Gapcynski (1967)	Akim (1966)	Tolson and Gapcynski (1967)	Akim (1966)
2	0	-2.0596 ± .141	-2.06 ± .22	-	-
	1	-.1661 ± .051	-.157 ± .059	.0080 ± .039	-.0361 ± .0358
	2	.2042 ± .029	.140 ± .012	-.0342 ± .025	-.0139 ± .0145
3	0	-.3773 ± .180	-.363 ± .099	-	-
	1	.3012 ± .048	.568 ± .026	.1762 ± .053	.178 ± .032
	2	.1294 ± .028	.118 ± .047	-.0147 ± .033	-.00702 ± .046
	3	.0317 ± .015	-	-.0043 ± .018	-
4	0	.0798 ± .128	.333 ± .270	-	-
	1	-.1560 ± .036	-	.0391 ± .028	-
	2	.0011 ± .010	-	.0072 ± .013	-
	3	-.0082 ± .008	-	-.0001 ± .006	-
	4	-.0007 ± .003	-	.0011 ± .003	-
5	0	-.5505 ± .171	-	-	-
	1	-.0385 ± .037	-	.0829 ± .031	-
	2	.0342 ± .009	-	-.0203 ± .008	-
	3	-.0071 ± .002	-	-.0078 ± .002	-
	4	-.0008 ± .001	-	-.0013 ± .001	-
	5	-.0003 ± .0002	-	.0003 ± .0002	-

Table 2 — Gravity Field of Moon Determined from Lunar Orbiters I, III and IV (Tolson and Gapcynski, 1967) and Luna-10 (Akim, 1966).

n	m	$C'_{n,m}$	H(km)	$S'_{n,m}$	H(km)
2	0	.159	-3.75	-	-
	1	-.121	.40	.0549	.04
	2	-.0230	-2.58	.0321	-.31
3	0	.115	-1.34	-	-
	1	.0331	3.69	-.0189	-3.77
	2	-.0172	3.05	.0102	-.59
	3	.00536	2.40	.00485	-.36
4	0	-.00574	-7.25	-	-
	1	.0147	-5.53	.00822	2.48
	2	.00096	.60	.00425	.88
	3	.000354	-12.22	-.000627	.08
	4	.000713	-.51	.000388	1.48
5	0	-.00946	37.08	-	-
	1	.0101	-2.47	-.0174	-3.03
	2	-.000633	-34.43	-.00209	6.19
	3	-.000212	21.34	-.0000658	75.54
	4	.000262	-1.95	-.000365	2.27
	5	.0000446	-4.29	.0000055	34.76

Table 3 — Values of Coefficients in Lunar Continentality Function and Height (H) of Continents above Maria Required to Produce Coefficients in Gravity Field if the Moon is Homogeneous.

$$I_{xx} = A \qquad I_{yy} = B = A + 4Ma^2C_{2,2} \qquad I_{zz} = C = A + Ma^2(2C_{2,2} - C_{2,0})$$

$$I_{xy} = 2Ma^2S_{2,2} \qquad I_{xz} = Ma^2C_{2,1} \qquad I_{yz} = Ma^2S_{2,1}$$

It is interesting to note that in Tolson and Gapcynski's (1967) analysis the $C_{2,1}$, $S_{2,1}$, and $S_{2,2}$ terms depart significantly from zero. In Akim's (1966) analysis only the $C_{2,1}$ term departs from zero by much more than the standard error. Thus for both analyses the principal axes of the moon do not correspond to the rotational axis and the direction to the earth. As pointed out by Jeffreys (1962, p. 141), the lack of a wobble of the moon's axis of rotation indicates that the $C_{2,1}$ and $S_{2,1}$ terms in the gravity field must be very nearly zero or no more than 0.001 of the $C_{2,2}$ and $S_{2,2}$ terms. If the $S_{2,2}$ term is not very close to zero, then the resulting torque should cause the moon to rotate until the line pointing toward the earth is a principal axis and this term vanishes. William H. Michael (personal communication, 1967) has suggested that the mass distribution for a stable orientation should include higher order terms and that consideration of these terms could lead to a stable configuration in which the $C_{2,1}$, $S_{2,1}$, and $S_{2,2}$ terms are non-zero. Until such an extension of Euler's equations is accomplished the possibility should also be considered that the standard errors in these and the rest of the terms are too small. Thus it is premature to draw firm conclusions as to the internal structure of the moon from existing analyses of the gravity field.

The relationship between the terms in the continentality function and the gravity field is derived from Jeffrey's (1962, p. 182) equation for the gravitational potential of a surface distribution over a sphere of radius $R(R \leq a)$

$$U = \frac{4\pi G\sigma R}{(2n+1)} \sum_{n=2}^{\infty} \sum_{m=o}^{n} \left(\frac{R}{r}\right)^{n+1} P_{n,m}(\sin \phi)(C'_{n,m} \cos m\lambda + S'_{n,m} \sin m\lambda)$$

where G is the gravitational constant, σ is the surface area density over the sphere expressed in a spherical harmonic representation with coefficients $C'_{n,m}$ and $S'_{n,m}$. Thus the effects on individual coefficients of the gravity field of the surface density is

$$\begin{matrix} C_{n,m} \\ S_{n,m} \end{matrix} = \begin{matrix} C'_{n,m} \\ S'_{n,m} \end{matrix} \times \frac{4\pi\sigma R^{n+2}}{(2n+1)Ma^n}$$

where M is the mass of the moon.

Thus if the moon is homogeneous and the gravity field is the result of a surface effect due to the shape of the moon and substituting the expression for the mass of a homogeneous sphere

$$\begin{matrix} C_{n,m} \\ S_{n,m} \end{matrix} = \begin{matrix} C'_{n,m} \\ S'_{n,m} \end{matrix} \times \frac{3H\rho_c}{(2n+1)a\rho_a}$$

where ρ_a is the average density of the moon, H is the height of a surface layer having a volume density ρ_c expressed in a spherical harmonic representation with coefficients $C'_{n,m}$ and $S'_{n,m}$. The values of H required to produce the gravity field on the assumptions that $\rho_c = \rho_a$ and the surface layer is related to the distribution of continents and maria as expressed by the continentality function are indicated on table 3.

Assuming that the general nature of the gravity field as reflected in the oblateness ($C_{2,0}$), ellipticity of the equator ($C_{2,2}$), and north-south asymmetry ($C_{3,0}$) is correct, these results indicate that if the moon is homogeneous, then the elevation of the maria with respect to the center of mass must be 1 to 4 km greater than the elevation of the continents. For the higher order terms there is no consistent relationship between the coefficients in the continentality function and the gravity field. However for most of the higher degree terms in the gravity field the standard errors are comparable in size to the value of the coefficient.

With regard to possible excess mass beneath the maria it is of interest that Nash (1963) has also suggested such a relationship as an explanation for the relative lack of maria on the moon's farside. Nash hypothesizes that the moon was locked into synchronous rotation with excess mass beneath the maria facing the earth in a stable orientation. He also suggests that the maria formed as the result of impact of objects denser than the moon and that the apparent concentration of maria in the plane of the ecliptic may be explained by an asteroidal origin for the impacting objects. Actually any process of maria formation that is random and that leads to an excess of mass beneath the maria surface could lead to the present distribution of maria with respect to the moon's principal axes and the direction to the earth. The present distribution may be the result of wander of the moon's axes to the position of stable equilibrium regardless of the original cause of the density distribution.

In order to estimate simple mass distributions which will produce the values of the higher order terms in the gravity field it is necessary to derive general equations analogous to those presented in an earlier paper for the second order terms (Lamar and McGann, 1966). In integral form the effect on individual coefficients in the gravity field is

$$\frac{C_{n,m}}{S_{n,m}} = \frac{C'_{n,m}}{S'_{n,m}} \times \frac{3}{(2n+1)\rho_a a^{n+3}} \int_0^{a+H} r^{n+2} \, \Delta\rho(r) \, dr$$

where $\Delta\rho(r)$ is the difference in density between material underlying continents and maria.

For the rigid lumpy model we assume a linear decrease in the density contrast with depth, or $\Delta\rho(r) = \Delta\rho(a)(r/a)$ where $\Delta\rho(a)$ is the density contrast at the surface; by integration

$$\Delta\rho(a) = \rho_a \left[\frac{-d_{n,m}}{3} - \frac{H}{a} \right] (n+4)$$

where $d_{n,m}$ is (2n + 1) times the ratio of the coefficients in the gravity field to those in the continentality function, and it is assumed that the density of material near the surface is equal to the average density of the moon.

For the extreme case of a rigid, lumpy model we assume that the maria are directly underlain by a layer of nickel-iron. The thickness (t) of the layer is found to be

$$t = \frac{\rho_a}{(\rho_i - \rho_a)} \left[H - \frac{d_{n,m} a}{3} \right]$$

where ρ_i is the density of nickel iron.

In the case where topographic irregularities are assumed to be compensated by variations in the thickness of a floating crust, we assume zero thickness for the crust beneath the maria, compensation at a depth, D, beneath the continents and a density contrast between continents and maria which decreases linearly with depth; by integration

$$\Delta\rho(a) = \frac{(n + 4)}{\left[1 - (n + 4) \frac{D}{a}\right]} \left[\frac{\rho_a d_{n,m}}{3} - (\rho_c - \rho_{mm}) \frac{D}{a} - \rho_c \frac{H}{a} \right]$$

where ρ_c is the density of crustal material and ρ_{mm} is the density of material directly beneath the maria. We assume a condition of zero stress at the moon's center; from Lamar and McGann (1966)

$$\Delta\rho(a) = \frac{3}{\left(1 - 3 \frac{D}{a}\right)} \left[\rho_c \frac{H}{a} - (\rho_{mm} - \rho_c) \frac{D}{a} \right]$$

Note that for the second order terms and the relations

$$C_{2,0} = -J_2^o = \frac{1}{Ma^2} \left[\frac{A + B}{2} - C \right], \quad C'_{2,0} = a_2^o$$

$$C_{2,2} = -J_2^2 = \frac{1}{4Ma^2}(B - A), \quad C'_{2,2} = \frac{1}{2} a_2^2$$

$$\beta = \frac{C - A}{B} \simeq \frac{C - A}{C}, \quad \gamma = \frac{B - A}{C}, \quad C \simeq \frac{2}{5} Ma^2$$

where C, B, and A are the principal moments of inertia of the moon, the above equations reduce to those presented in Lamar and McGann (1966).

In our earlier paper (Lamar and McGann, 1966), it was suggested that there is a systematic difference in elevation between continents and maria such that the average elevation of the continents is 3 km (quantity H in the above equations) greater than that of the maria relative to the center of mass. Goudas (1966) has questioned the validity of this assumption and points out that recent stereoscopic height determinations reveal no such relationship. However the origin of coordinates for these systems is a mean sphere which best fits the points or the center of volume. Thus it is possible that the continents are systematically higher relative to the center of mass and some maria surfaces relative to the center of figure are higher than some continental areas. Similarly if the earth lacked oceans, which provide a convenient level surface, and the origin of coordinates were the center of volume, then the center of the Pacific basin would have about the same elevation as continental areas on the opposite side of the earth. It is suggested that until the coordinate systems based on earth-based stereoscopic observations are transformed so that the origin of coordinates is the center of mass, any systematic difference in elevation relative to the center of mass will be obscured.

Values of the parameters for the models of internal structure calculated from the above equations for Tolson and Gapcynski's (1967) values for the coefficients of the gravity field are as follows:

	$\Delta\rho(a)$, Rigid Lumpy Model (gm/cm^3)	Thickness of Nickel-Iron Layer (km)	Isostatic Model $\Delta\rho(a)$ (gm/cm^3)	Isostatic Model Depth of Compensation (km)
$C_{2,0}$, H = 0	-0.043	2.8	--	--
$C_{2,0}$, H = 3 km	-0.078	5.0	-0.016	42
$C_{2,2}$, H = 0	-0.029	1.9	--	--
$C_{2,2}$, H = 3 km	-0.064	4.2	-0.010	35
$C_{3,0}$, H = 0	-0.038	2.1	--	--
$C_{3,0}$, H = 3 km	-0.079	4.4	-0.012	38

The values are remarkably consistent and suggest that the assumption of a systematic difference in density of the material underlying continents and maria may be correct. This is true whether or not there is a systematic difference in elevation between continents and maria. The above values are similar to those calculated from the pre-Lunar Orbiter second order terms and are equally favorable to the rigid lumpy and isostatic models of internal structure (Lamar and McGann, 1966).

MOMENT OF INERTIA

Tolson and Gapcynski (1967) have calculated the value of the quantity $q = 3C/2Ma^2 = 0.5895 \pm 0.05$ where C is the moment of inertia about the axis of rotation from their values of $C_{2,0}$ and $C_{2,2}$. According to Jeffreys (1962), for a chemically homogeneous moon with increase in density with depth due to internal pressure, $q = 0.596$. For a linear increase in density from 3.2 gm/cm^3 below a thin crust to 3.76 gm/cm^3 at the center, $q = 0.592$, and for a more extreme differentiated model in which the density increases linearly from 2.8 gm/cm^3 at the surface to 5.0 gm/cm^3 at the center, $q = 0.568$ (Lamar and McGann, 1966). The problem of the overall moment of inertia has been considerably improved by the orbiter data and it is apparently no longer necessary to consider the possibility that the density of the moon decreases with depth. However the data are still insufficient to decide whether the moon is differentiated or chemically homogeneous.

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