Semiannual Status Report

for Period December 15, 1966, to June 15, 1967

NASA Grant ~~05-010-008~~ 80-NGR-05-010-008

I. Work accomplished under NASA Grant 05-010-008.

1. Certain modifications, principally notational, were made to the paper "Reflection Laws of Systems of Second Order Elliptic Differential Equations in Two Independent Variables with Constant Coefficients". Copies of the revised version are enclosed. This paper was accepted for publication in the Pacific Journal of Mathematics.

2. Copies of a translation from Russian of the paper "Integrating the Equations for Spherical Shells" by I. N. Vekua are also enclosed. This work was done by one of the research assistants on this contract.

3. As mentioned in the semiannual status report in December, 1966, we have been working with the Culler-Fried on-line computer to try to determine ways of numerically solving non-linear singular integral equations of the form investigated in [1], namely, equations of the form:

$$w(z) = B(z) + \lambda C(w) + \lambda F[z,w(z),\lambda] + \frac{\lambda}{2\pi i}\int_{\Gamma} \frac{N[t,z,w(t),\lambda]}{t-z}\,dt$$

where z belongs to the enclosed smooth contour Γ, the Cauchy principal value is meant by the integral, $C(w)$ is a bounded functional, B, F and N are complex-valued functions defined for z on Γ and w is in some closed convex set. We were led to an equation of this form when we were trying to determine the deformation of an elastic circular cylinder in a two dimensional ideal fluid flow. In this case the equation

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is of the form:

$$g'(t) = B + U^2C(g') + U^2F[t,g'(t)] + \frac{U^2}{2\pi i}\int_\Gamma \frac{N[\sigma,g'(\sigma)]}{\sigma-t}\,d\sigma$$

where $g(\sigma)$ measures deformation,

$$B = -\frac{K}{2\mu}\Pi$$

$$C(g') = -\frac{(1+K)}{8\mu}\,\frac{1}{2\pi i}\int_\Gamma \frac{\left|\frac{\sigma^2-1}{1+g'(\sigma)}\right|^2}{\sigma}\,d\sigma\,,$$

$$F[\sigma,g'(\sigma)] = \frac{\rho K}{4\mu}\left|\frac{\sigma^2-1}{1+g'(\sigma)}\right|^2 - \frac{K\rho}{4\mu}\,,$$

$$N[\sigma,g'(\sigma)] = \frac{1+K}{\mu}\,\frac{\rho}{2}\left|\frac{\sigma^2-1}{1+g'(\sigma)}\right|^2,$$

and where K, μ are elastic constants, ρ = density of the fluid, Π = pressure of fluid and U is the velocity of the fluid at infinity. In [1], it was shown that we get a solution provided U^2 is sufficiently small. An immediate question which arose was: is the necessity to have $|\lambda|$ small a restriction on the range of validity of the physical problem? or is it simply a necessity for the mathematical proof of existence and uniqueness? If the second question is the one to be asked, is it then possible to improve the proof or perhaps to invent another proof valid for larger $|\lambda|$?

The hydroelastic integral equation was put on the Culler-Fried on-line computer and observations indicated that U^2 need not be small to get a solution. This led me to conjecture that perhaps the smallness of $|\lambda|$ was simply a mathematical artifice necessary for the general

proof of existence and uniqueness. Working in this direction, I think I have been able to show that the solution can be continued as a function of λ into the λ plane beyond the original region of validity given by the theorem in [1]. I am writing these results down at present. This result will answer all of the above questions.

4. A graduate student and I have also begun work on setting up the problem for the determination of the distortion of a thin elastic spherical shell (e.g., a balloon) in an electrostatic field due to the pressure of the field on the balloon.

II. Work in progress.

1. By observing the numerical results (mentioned in 3 above) graphically on the oscilloscope, it seems that convergence can be much improved by averaging iterations. I have done some preliminary work on determining when such iteration averaging is advantageous and I intend to pursue this. If this can be worked out it will mean that much computer time can be saved in calculations.

2. Continuation of work mentioned in I. 3., viz. working out carefully the solution continuation as a function of λ.

3. Continuation of work mentioned in I. 4.

III. Work to be done.

1. I wish to consider problems of reflection for high order elliptic differential equations in more than two dimensions and reflection problems for systems of elliptic equations in more than two independent variables. I have an approach to this question, viz. that of expressing

the solution in terms of matrix Bessel and matrix Neumann functions.

2. I wish to consider questions of reflection for systems of elliptic equations in two independent variables in which the characteristics are not necessarily the same.

3. I wish to try to combine my results given in the papers [3] and [4] (which were done under this grant and sent in with the last semiannual report) with the ideas of J. Leray given in [2], and try to derive Green's functions for differential equations of the type treated in [3] and [4].

4. I wish to investigate what reflection across surfaces other than the plane or sphere might mean. As a start, I shall consider developable surfaces.

[1] Finnila, R., and Sloss, J.M., Existence and Uniqueness Theorems for a Class of Non-linear Singular Integral Equations with Applications to a Hydroelastic Problem, J. Math and Mechs., Vol. 16, No. 6, Dec. 1966, pp. 509-534.

[2] Leray, J., Calcul, par reflextions, des fonction M-harmoniques dans une bande plane verifiant aux bands M condition differentielles a coefficients constants, Archiavum Mechaniki Stosowaj 5, 16 (1964).

[3] Sloss, J.M., Reflection laws of systems of second order elliptic differential equations in two independent variables with constant coefficients, accepted for publication in Pac. Journal of Math.

[4] _____ , Reflection laws of high order elliptic differential equations in two independent variables with constant coefficients and unequal characteristics across analytic boundary conditions, submitted for publication.