Innsbruck, 8. August 1967

F I N A L R E P O R T

Period: August 1, 1966 - June 30, 1967

RESEARCH GRANT NGR 52-046-001, Suppl. 1

Contractor and Principal Investigator: Prof. Dr. Ferdinand CAP

LIE SERIES FOR CELESTIAL MECHANICS, ACCELERATORS, SATELLITE STABILIZATION AND OPTIMIZATION

INSTITUTE OF THEORETICAL PHYSICS,
UNIVERSITY OF INNSBRUCK, TYROL,
AUSTRIA

FACILITY FORM 602

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(A) Scientific Work

(1) General Survey

Our scientific work in this period of contract covered the following areas:
separation of the Laplace equation in many different coordinate systems and solving the differential equations thus obtained; solution of the Eulerian equations of the heavy asymmetric gyroscope; deriving the equations of motion of satellites in circular and, later, in elliptic orbits ($\varphi = -\frac{\alpha}{r}$); theoretical and numerical treatment of a satellite with a circular orbit; making use of the recurrence formulae method to the same problems; optimization problems (soft landing with fuel minimization); and, finally, deriving the general equations of motion of a satellite ($\varphi = -\frac{\alpha}{r} + \frac{\beta - \gamma\cos^2\varphi}{r^3}$).

(2) Scientific Reports

Report No.9: On the Solution of the Differential Equations Resulting from the Separation of Laplace Equation in Various Coordinate Systems by Means of Lie Series, August 1966, by F.CAP and A.SCHETT: Weber-, Bessel-, Baer-, Mathieu-, Legendre-, Lamé-, Wangerin- and Heine functions are given in a presentation as Lie series.

Report No.10: Numerical Computation of Satellite Orbits Using Lie Series; Comparison with Other Methods, September 1966, by H.KNAPP: A survey on the application and the advantages of the Lie series method in celestial mechanics is given.

Report No.11: On the Solution of the Heavy Asymmetric Gyroscope Using the Properties of the Lie Operator, October 1966, by F.CAP and A.SCHETT: The solution of this problem is splitted up in three different ways, each one with another physical meaning.

Report No.12: The Solution of a System of n-th- Order Differential Equations Using Lie Series, November 1966, by F.CAP and D.FLORIANI: Some properties of Lie series are discussed and then a system of ordinary n-th-order differential equations is solved by means of Lie series.

Report No.13: The Forces and Torques Acting on Satellites and the Dynamical Equations Describing their Motion About a Fixed Point of the Body, December 1966, by A.SCHETT and J.WEIL: After a survey of forces and torques acting on a satellite the dynamical equations are derived. Then papers dealing with the attitude of satellites and with gravity gradient stabilization are discussed.

Report No.14: The Solution of the Eulerian Gyroscope Equations by Means of Lie Series Making Use of Recurrence Formulas, February 1967, by A.SCHETT and J.WEIL: The Euler equations derived in Rep.13 are solved by help of Lie series using recurrence formulas.

Report No.15: Lie Series Solution of Gravity-Gradient Stabilized Motion of a Satellite in Elliptic Orbits, April 1967, by A.SCHETT and J.WEIL: The equations from Rep.13 (ci- orbit) are generalized to an elliptic orbit; of differential equations is solved wi-

Report No.16: Optimization Problems solved by Lie Series: Soft Landing on the Moon with Fuel Minimization, May 1967, by F.CAP, W.GROEBNER and J.WEIL: The more dimensional problem of soft landing on a surface with fuel minimization as additional requirement is solved using Lie series.

Report No.17: Application of the Method of Lie Series to a Calculation on Particle Orbits in Accelerators, May 1967, by F.EHLOTZKY: Lie Series are used for the treatment of an isochronous AVF-Cyclotron, particularly for stability problems.

Report No.18: On the Equations of Motion of Earth Satellites, June 1967, by F.CAP, D.FLORIANI and A.SCHETT: Only considering gravitational forces the general equations of motion are derived in two different systems of reference. Then some ways to solve them by means of Lie series are shown.

Under the title "Lie Series for Celestial Mechanics, Accelerators, Satellite Stabilization and Optimization" a summary review of our investigations mentioned above is prepared for publication in the NASA CR-Series.

(3) Personal data and a short survey on the work done by the collaborators

The team consisted of:

Prof.Dr.Ferdinand CAP: Principal investigator

Dietmar FLORIANI: 1.Oct.1966 to 30.June 1967

Dr.Alois SCHETT: 1.Aug.1966 to 30.June 1967

Dr.Juergen WEIL: 1.Aug.1966 to 30.June 1967

The work in detail:

Prof.Dr.F.CAP: Being the principal investigator he was responsible for the coordination and general supervision of the work, comprising general investigations of the applicability of Lie series to various problems and a lot of discussions with the collaborators.

D.FLORIANI: During the rest of 1966 he dealt with the general n-th-order ordinary differential equation and optimisation problems. The next work was to check the calculation of the circular orbit satellites and to code the Lie series solution of this problem. Then he derived higher approximations of a more general formula of gravitational forces acting on satellites.

Dr.A.SCHETT: Initially he was concerned with the application of Lie series to solve the Laplace equation in various coordinate systems. Then he turned to solve the equations of the heavy asymmetric gyroscope. After an extended study of literature together with Dr.Weil he made calculations to a satellite in circular and elliptic orbits and solved both problems using recurrence formulas. Finally, he gave a comprehensive survey of solving the Eulerian equations with very general torques using Lie series in some different manners.

Dr.J.WEIL: At the beginning of this contract period he made some investigations to the general use of recurrence formulas for Lie series. Then he read a lot of papers to the gravity gradient stabilization etc., together with Dr.Schett. After some assistance in deriving the equations of satellites in

circular orbits. Then he turned with Dr.Schett to the problem of recurrence formulas for the mentioned case. Finally, he dealt with optimization problems (soft landing with fuel minimization). During the whole time he translated our reports into English.

(4) Inventory of purchased books

Number	Author	Title	Amount AS
1	Abrahamowitz, Milton, etc.	Handbook of Mathematical Functions	254,60
2	Campbell	Théorie générale de l'équation de Mathieu	251,95
3	Miller, Willard Jr.	On Lie Algebras and Some Special Functions of Mathematical Physics	51,30
4	Brainerd, Gray, Merwin	Solutions of the Mathieu Equation (Cop.)	24,--
5	Brainerd, Weygandt	" " " " " "	20,--
6	Bouwkamp, v.d.Pol	A note on Mathieu Functions (Cop.)	4,--
7	Weinstein	Characteristic Values of the Mathieu Equation (Cop.)	8,--
8	Tamir	Characteristic Exponents of Mathieu Functions (Cop.)	8,--
9	Kirckpatrick	Tables of Values of the Modified Mathieu Functions (Cop.)	12,--
10	Mc Lacklan	Theory and Application of Math.F. (Cop.)	83,90
11	Bickley, Mc Lacklan	The Tabulation of Mathieu Functions (Cop.)	161,50
14	Maass Gerhard	Verfahren zur Bestimmung periodischer Loesungen autonomer nicht linearer Dglen.	86,50
15	Hahn Wolfgang	Theorie und Anwendung der direkten Methode von Ljapunov	196,80
16	Ince	Integration gewoehnlicher Dglen.	36,50

Number	Author	Title	Amount
17	Zubov	Methods of Ljapunov	284,10
18	Zubov	Tables relating to Mathieu Functions	311,--
19) 20	Blanch G. and Clemm D.S.	Tables relating to the radial Mathieu Functions, Vol.1 and 2	235,10
21	Roy	Dynamics of Satellites	407,70
22	Forbat	Analytische Mechanik der Schwingungen	239,--
23	Bohrmann	Bahnen kuenstlicher Satelliten	26,70
27	Singer	Torques and Attitude Sensing in Earth Satellites	305,70
39	Bucerius	Himmelsmechanik I	47,90
55	div.	Copies from papers to the problem of stability of satellite's motion	178,60

We got the other 45 numbers gratis: 12, 13, 24-26, 28-38, 40-54, 56-69.