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CASE FILE
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SPATIAL DENSITY OF THE KNOWN ASTEROIDS IN THE ECLIPTIC PLANE

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ABSTRACT

The spatial density of the known asteroids is presented graphically as a function of absolute magnitude for heliocentric distances within the range of 1 to 4.5 astronomical units in 0.1 astronomical-unit increments. Two sets of graphs are presented. The spatial density averaged over all ecliptic longitudes is presented in one set, and the spatial density for eight ecliptic longitude intervals at each heliocentric distance is presented in the other set. A partial correction for observational selection effects is shown on each graph.

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SPATIAL DENSITY OF THE KNOWN ASTEROIDS IN THE ECLIPTIC PLANE

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SUMMARY

The spatial density of asteroids is studied as a function of absolute magnitude, distance from the Sun, and heliocentric longitude and latitude. The probability of finding each asteroid at a particular distance from the Sun is computed, along with the heliocentric longitude and latitude of the asteroid at that distance.

A series of graphs has been prepared which illustrates the spatial density of asteroids as a function of absolute magnitude for various distances from the Sun, and longitude intervals. Each graph includes a correction for observational selection effects and a tabular latitude distribution.

INTRODUCTION

Asteroids are usually observed from Earth as bodies in orbit around the Sun between the orbits of Mars and Jupiter. The asteroids discovered to date are within the range of a few kilometers to 700 kilometers in diameter. The orbits for 1684 asteroids are listed in reference 1. Less than 40 of these asteroids cross the orbit of Mars; only three cross the orbit of Earth.

The purpose of this document is to present graphically the spatial density of the cataloged asteroids as a function of the absolute magnitude for various distances from the Sun, and the heliocentric longitude intervals. The graphs were derived by computing the probability of finding each asteroid at a specific heliocentric distance and then computing the sum of these probabilities to obtain the spatial density. The upper curve in each graph has been partially corrected for selection effects by adopting the procedure outlined in reference 2.

From these graphs, the radial distribution of asteroids can be studied as a function of heliocentric longitude and limiting size. However, because of the remaining observational selection effects, it may be necessary, in many cases, to extrapolate the distribution curves to the desired size. This approach differs from that used by Narin (ref. 3) who calculated the position of the numbered asteroids for a series of days. His resulting distributions were not corrected to a limiting asteroid size.

The programing necessary to produce this document was prepared by Jesse L. Gamble, Computation and Analysis Division, Manned Spacecraft Center, Houston, Texas.

SYMBOLS

A	surface area of a spherical shell around the Sun, km^2
a	semimajor axis of an asteroid orbit, AU
C	factor to correct for observational selection effects
e	eccentricity of an asteroid orbit
F	fraction of the total number of asteroids found within $\pm 2^\circ$ of the ecliptic at any one time (The F associated with the table in figs. 2 to 277 has been redefined to be the fraction of asteroids found in the ecliptic latitude interval noted in the table.)
g	absolute photographic magnitude of an asteroid (that is, the apparent photographic magnitude of an asteroid located 1 AU from the Sun and viewed from the Sun)
i	inclination of an asteroid orbit
N	number of asteroids with apparent photographic magnitudes between $p - \frac{1}{2}$ and $p + \frac{1}{2}$
P	probability of finding an asteroid at a distance of $R \pm \frac{1}{2} \Delta R$ from the Sun
p	apparent photographic magnitude of an asteroid when it is at a distance from the Sun equal to its semimajor axis
q	perihelion distance of an asteroid orbit, AU
q'	aphelion distance of an asteroid orbit, AU
R	distance from the Sun, AU
S	spatial density caused by one asteroid, assuming it can be found at any heliocentric latitude, number/km^3
S_2	spatial density caused by one asteroid, assuming it is found within $\pm 2^\circ$ of the ecliptic, number/km^3
$S_2(g)$	spatial density of asteroids with absolute magnitude g or brighter, assuming that all the asteroids are found within $\pm 2^\circ$ of the ecliptic, number/km^3

S_e average spatial density in the ecliptic plane as the result of one asteroid $S_e = FS_2$, number/km^3

$S_e(g)$ average spatial density of asteroids with absolute magnitude g or brighter, located in the ecliptic plane $S_e(g) = FS_2(g)$, number/km^3

T period of revolution of an asteroid around the Sun, sec

t time required for an asteroid to transverse a distance ΔR, sec

V_h heliocentric velocity of an asteroid at R, km/sec

V_r radial velocity component of an asteroid at R, km/sec

β heliocentric latitude (noted as B on figs. 2 to 277)

ΔR increment of R, km

λ heliocentric longitude

μ angle between the heliocentric velocity vector of an asteroid and the tangent to a spherical shell around the Sun

ν true anomaly of an asteroid at R

Ω longitude of ascending node of an asteroid orbit

ω argument of perihelion of an asteroid orbit

ANALYSIS

Average Spatial Density as a Function of Distance from the Sun

Consider a spherical shell around the Sun which has a radius of R astronomical units (AU) and a thickness of ΔR kilometers. The probability of finding an asteroid in this shell is

$$P = \frac{2t}{T} \tag{1}$$

where t is the time required for the asteroid to transverse ΔR and where T is the period of revolution of the asteroid around the Sun. The factor 2 appears because the asteroid will penetrate the shell twice for each revolution around the Sun.

The period of revolution of an asteroid around the Sun is determined from standard orbital mechanics as follows.

$$T = a^{3/2}\left(3.156 \times 10^{7}\right) \tag{2}$$

Since ΔR is small, the trajectory of the asteroid may be considered as rectilinear between R and $R + \Delta R$. Thus, the time required for the asteroid to transverse ΔR is $t = \frac{\Delta R}{V_r}$ where V_r is the radial velocity component of the asteroid at R. The radial velocity is given by $V_r = V_h \sin \mu$ where V_h is the heliocentric velocity of the asteroid and μ is the angle between V_h and the surface of the imaginary spherical shell. From standard orbital mechanics

$$V_h = 29.76(2/R - 1/a)^{1/2} \tag{3}$$

and

$$\cos^2\mu = \frac{a^2\left(1 - e^2\right)}{R(2a - R)} \tag{4}$$

where a represents the orbital semimajor axis of the asteroid and e represents the orbital eccentricity of the asteroid. The average spatial density at R, caused by a single asteroid, is $S = \frac{P}{A\Delta R}$ where A (the area of the shell) is given by

$$A = 4\pi R^2\left(1.495 \times 10^{8}\right)^2 \tag{5}$$

By combining and then rearranging the preceding equations and by using the well-known expressions for the perihelion q and aphelion q' distances, then

$$S = \frac{7.58 \times 10^{-27}}{R^2 a\left[\left(1 - \frac{q}{R}\right)\left(\frac{q'}{R} - 1\right)\right]^{1/2}} \tag{6}$$

The average density over the entire sphere is presented in equation (6). However, the asteroids are known to be concentrated close to the ecliptic plane, with approximately one-fourth of them located within ±2° of the ecliptic plane. If F is the fraction of asteroids within ±2° of the ecliptic plane, then the spatial density in the ecliptic plane is $S_e = S_2F$ where S_2 is the spatial density found by assuming that all asteroids are located within ±2° of the ecliptic plane. This procedure was chosen since it is possible that the value of F is a function of R. Thus

$$S_2 = \frac{2.1725 \times 10^{-25}}{R^2 a\left[\left(1 - \frac{q}{R}\right)\left(\frac{q'}{R} - 1\right)\right]^{1/2}} \tag{7}$$

The heliocentric latitude β of the asteroid at R can be computed from

$$\sin \beta = \sin i \sin (\omega + \nu) \tag{8}$$

where i is the orbital inclination of the asteroid, ω is the argument of perihelion of the orbit, and ν is the true anomaly at R. The true anomaly is computed from

$$\cos \nu = \frac{a\left(1 - e^2\right)}{eR} - \frac{1}{e} \tag{9}$$

and has two values for each R.

Spatial Density as a Function of Heliocentric Longitude

The heliocentric longitude λ of an asteroid can be computed from

$$\lambda = \Omega + \tan^{-1}[\cos i \tan (\nu + \omega)] \tag{10}$$

where Ω is the longitude of the ascending node of the asteroid orbit. Since each value of R has two values of ν, there will also be two values for λ at each R. Thus, the spatial density as a function of R can be found for any interval of λ by determining if the asteroid passes through that interval at that R. Since each asteroid will have two values for λ at each R (one value as the asteroid approaches the Sun and another value as it departs), each case must be considered individually.

Thus, the factor 2 must be removed from equation (1). The factor $2\pi/\Delta\lambda$ is introduced to modify equation (7). Thus, equation (7) becomes

$$S_2 = \frac{2\pi}{\Delta\lambda} \frac{1.0862 \times 10^{-25}}{R^2 a\left[\left(1 - \frac{q}{R}\right)\left(\frac{q'}{R} - 1\right)\right]^{1/2}} \tag{11}$$

Selection Effects

The probability of an asteroid being detected depends on many variables: the location of the observer, the size of his telescope, the absolute magnitude of the asteroid, and the length of time the asteroid is bright enough to be observed.

Since the asteroids listed in reference 1 are compiled from the results of many astronomer's observations, it is nearly impossible to correct for every possible selection effect. However, Kuiper (ref. 2) concludes that the probability of an asteroid being detected is very nearly a function of its average opposition magnitude (that is, its apparent magnitude if it were placed at a distance from the Sun equal to its semimajor axis and observed from Earth which is located between the Sun and the asteroid). Kuiper's procedure for correcting the cataloged asteroids will be adopted in this document. The number of asteroids (ref. 2) of average photographic magnitude p between $p - \frac{1}{2}$ and $p + \frac{1}{2}$ is given by

$$N(p) = -2.38 + 0.35p \tag{12}$$

Thus, the correction factor as a function of p can be computed by comparing equation (12) with the distribution of p of the cataloged asteroids (fig. 1). Hence, the logarithm of the correction factor is the difference between the two curves. This difference is expressed by

$$\log_{10} C(p) = 0.02(p - 13)^2 \tag{13}$$

where C is the correction factor needed and where $p \geq 13$ and

$$\log_{10} C = 0 \tag{14}$$

where $p < 13$. Since the correction factor becomes very large and hence uncertain for $p > 18$, all 14 asteroids which are greater than this limit are eliminated from the analysis. If the selection effects were only a function of p, this would not introduce new bias; the net effect is to make the "plate limit" at p equal 18.

PROCEDURE

The graphs presented in this document are the results of a computer program which was written to compute the spatial density S_2, the two ecliptic longitudes λ, and the two corresponding ecliptic latitudes β of each of the cataloged asteroids (ref. 1) for different values of R.

The spatial density of asteroids in the ecliptic plane with absolute magnitude g or brighter, $S_e(g)$, at each radial distance R, was computed by adding the values of S_2 for all asteroids of absolute magnitude g or brighter and then by multiplying by an adopted value $F = 0.24$. This spatial density is plotted as the lower curve on each graph.

The upper curve of each graph includes a correction for observational selection effects as described in the analytical section of this document. This curve was obtained in the same manner as the previously discussed lower curve except that each value of S_2 was weighted by the correction factor C. For absolute magnitudes of one or two less than the maximum value of g on each graph, this correction factor becomes incomplete because of the nature of the correction factor used; that is, two asteroids with equal values of g probably will have different values for p. As g approaches its maximum value on each graph, there will be increasingly more values of p that are larger than 18. Since p is a function of the semimajor axis of the asteroid, a second correction factor is needed that reflects the distribution of the semimajor axis. This second correction factor is not determined in this document.

The value $F = 0.24$ was adopted to facilitate the computation for each graph. The validity of this approach was confirmed by the results which indicate that F is roughly independent of R and λ, if the statistical fluctuations are smoothed out. The actual value of F at R and λ is indicated on each graph.

The latitude distribution of asteroids at each position in space is given in the table at the right of each graph. The table was constructed by assuming the latitude distributions are symmetrical about the ecliptic plane. For purposes of the table, F has been redefined to be the fraction of asteroids located within the interval of heliocentric latitude (noted as B) given in the table. Thus, the average spatial density at other latitudes can be found by multiplying $S_e(g)$ by the product of two ratios: (1) The appropriate value in the table to 0.24 and (2) the volume of space within 2° of the ecliptic to the volume of space within the desired latitude (for example, for $B = 8^\circ$ to 10°, this second ratio is 1.0123).

Two sets of graphs were produced.

1. The first set of graphs (figs. 2 to 37) is for values of R from 1.0 to 4.5 AU for every 0.1-AU increment, with the spatial density averaged over all longitudes (eq. (7)).

2. The second set of graphs (figs. 38 to 277) was produced by dividing each R into eight intervals of heliocentric longitude. The two values of heliocentric longitude of each asteroid at R were computed (eq. (10)), and the asteroid was then considered in its appropriate heliocentric longitude interval. Within that heliocentric longitude interval, spatial densities were computed (eq. (11)) for values of R from 1.0 to 4.5 AU at every 0.1-AU increment. When less than three asteroids were found in a particular $\Delta\lambda$, no graph was produced.

CONCLUDING REMARKS

The spatial density of asteroids in the ecliptic plane is presented graphically as a function of absolute magnitude for various distances from the Sun and heliocentric longitude intervals. A tabular latitude distribution is presented on each graph.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, March 5, 1969
124-09-14-01-72

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3. Narin, Francis: Spatial Distribution and Motion of the Known Asteroids. J. Spacecraft Rockets, vol. 3, no. 9, Sept. 1966, pp. 1438-1440.

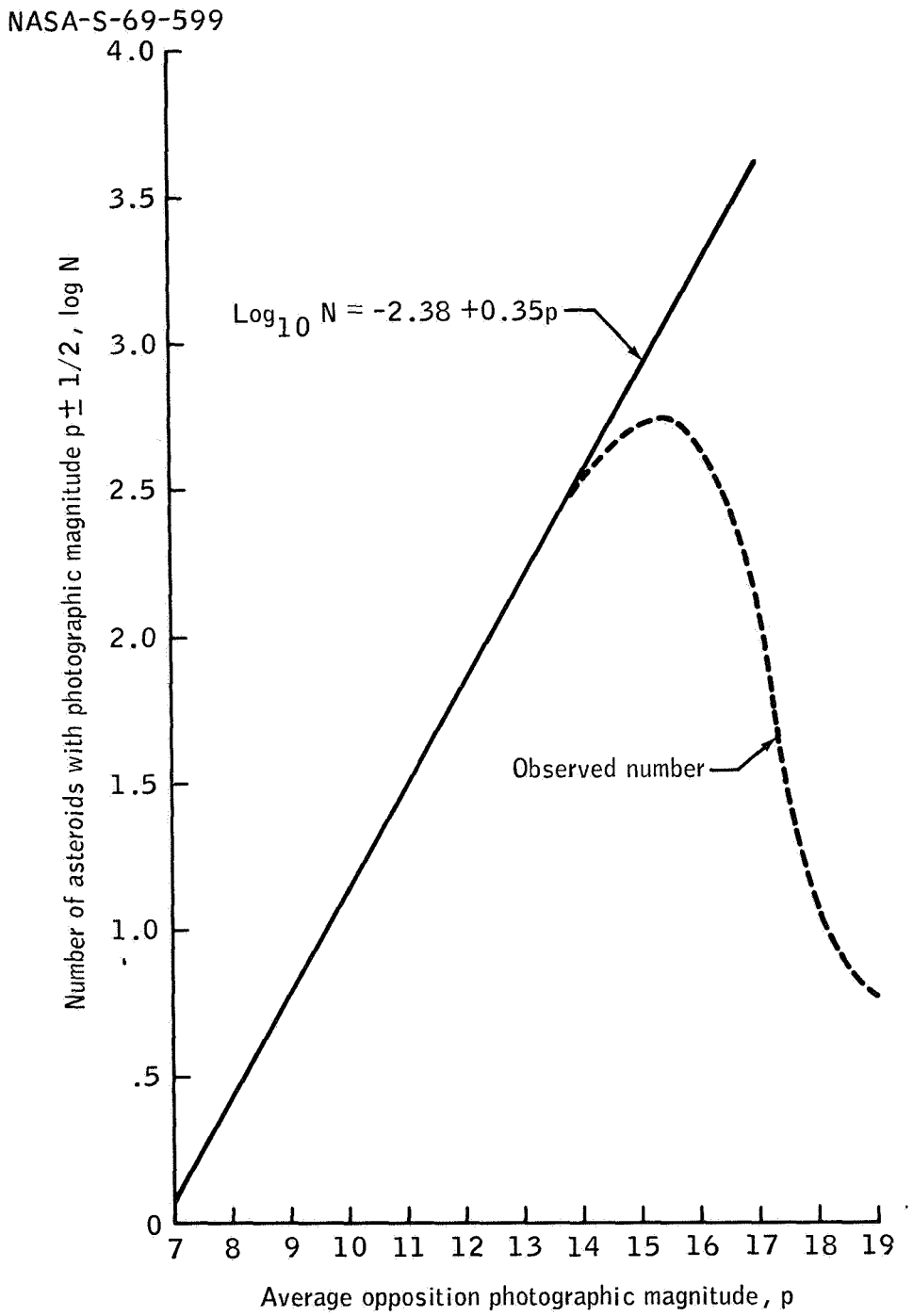


Figure 1. - Number distribution of asteroids as a function of opposition magnitude (refs. 1 and 2).

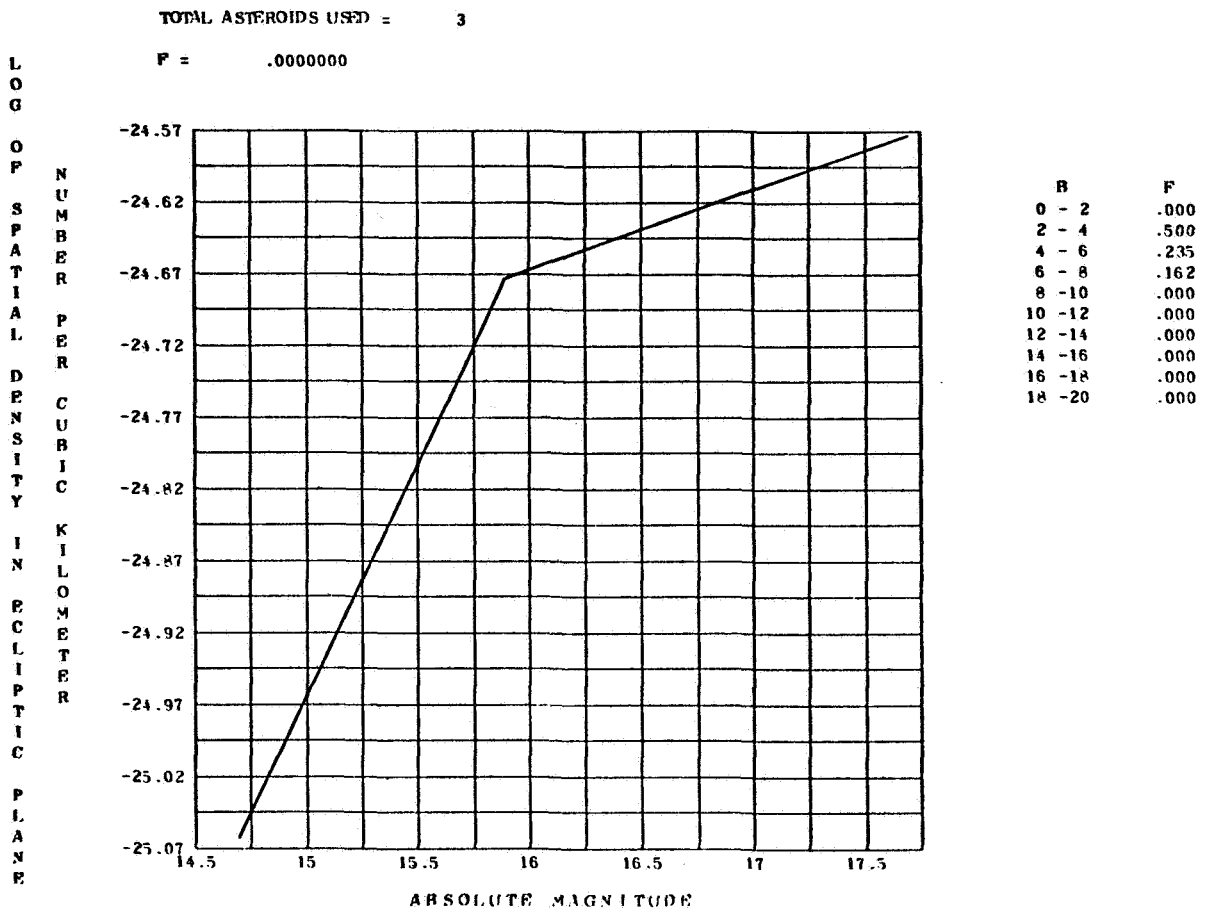


Figure 2. - Spatial density at R = 1.00 averaged over all ecliptic longitudes.

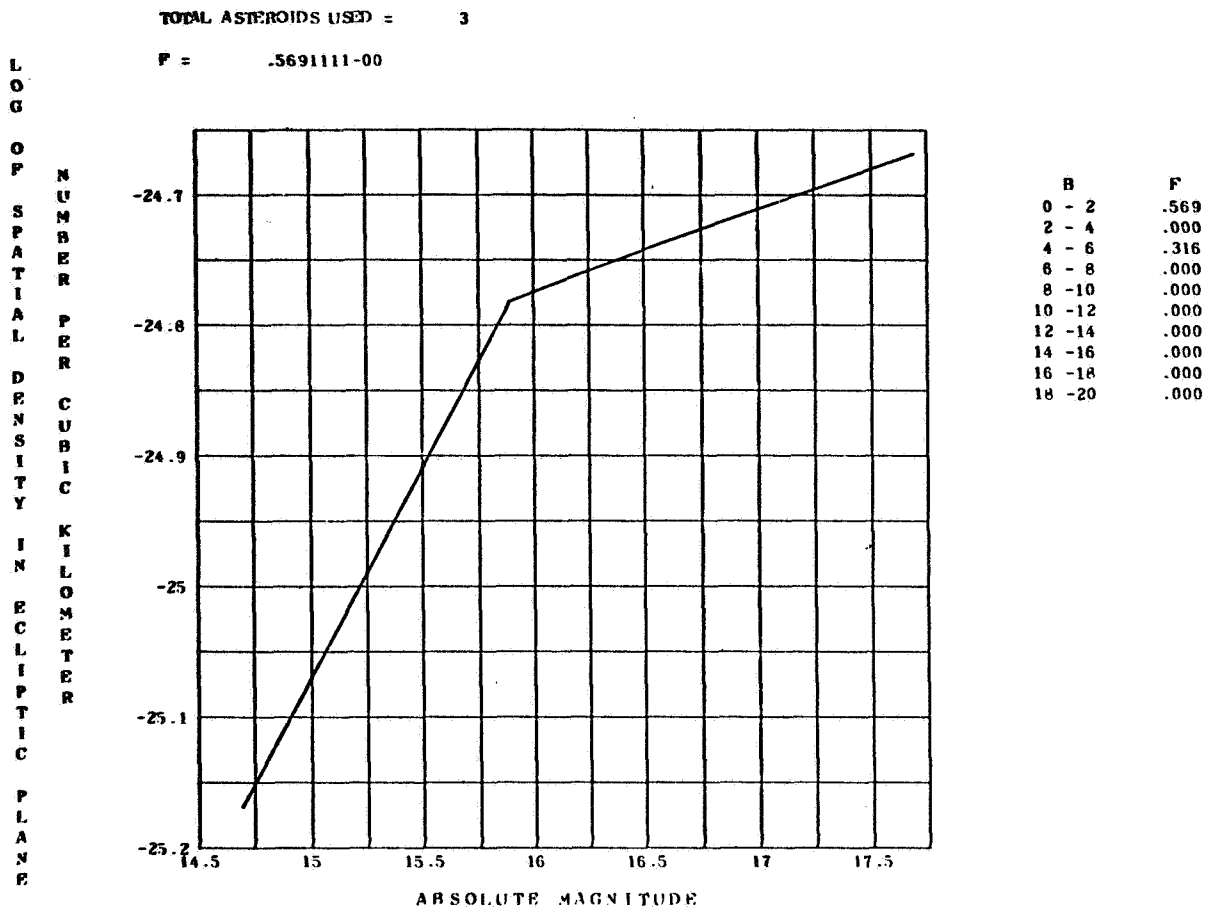


Figure 3. - Spatial density at R = 1.10 averaged over all ecliptic longitudes.

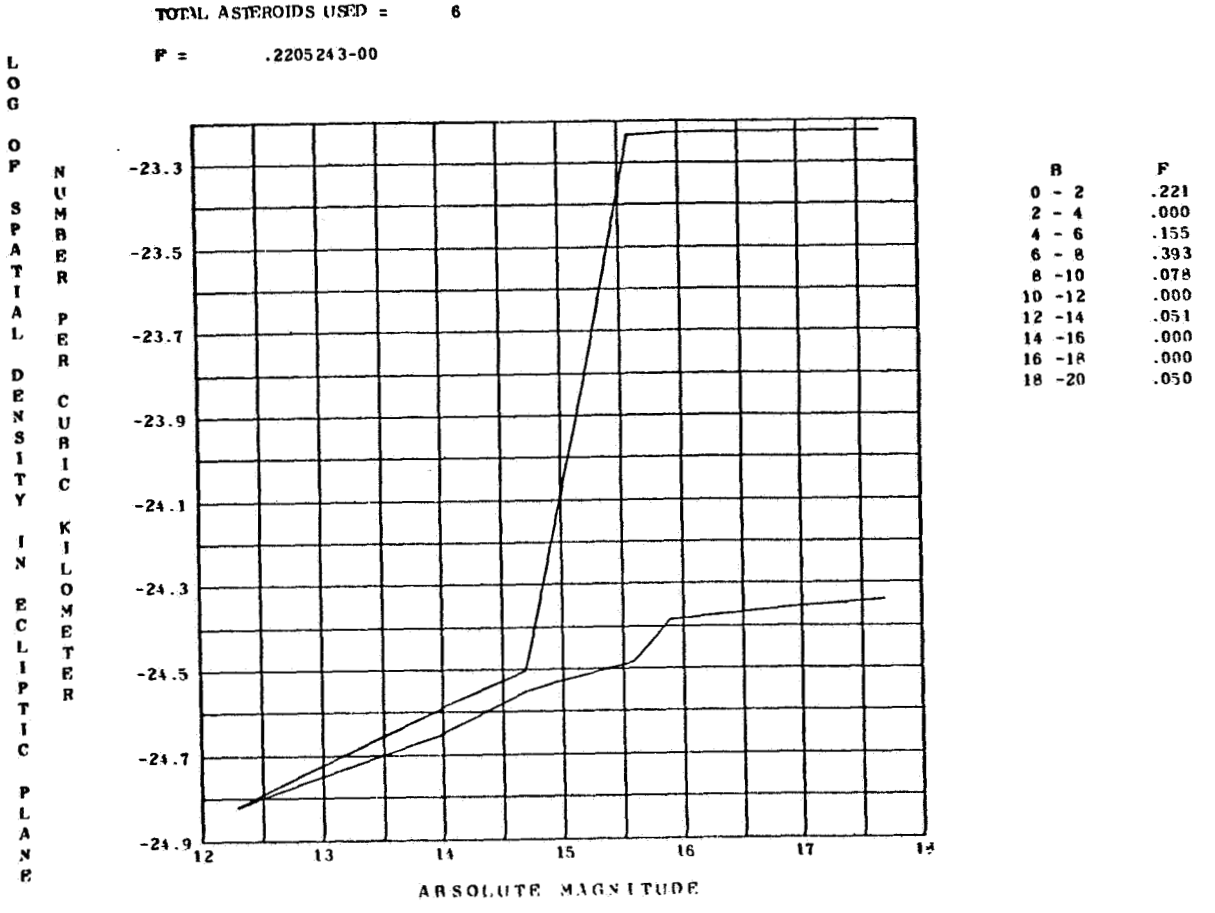


Figure 4. - Spatial density at R = 1.20 averaged over all ecliptic longitudes.

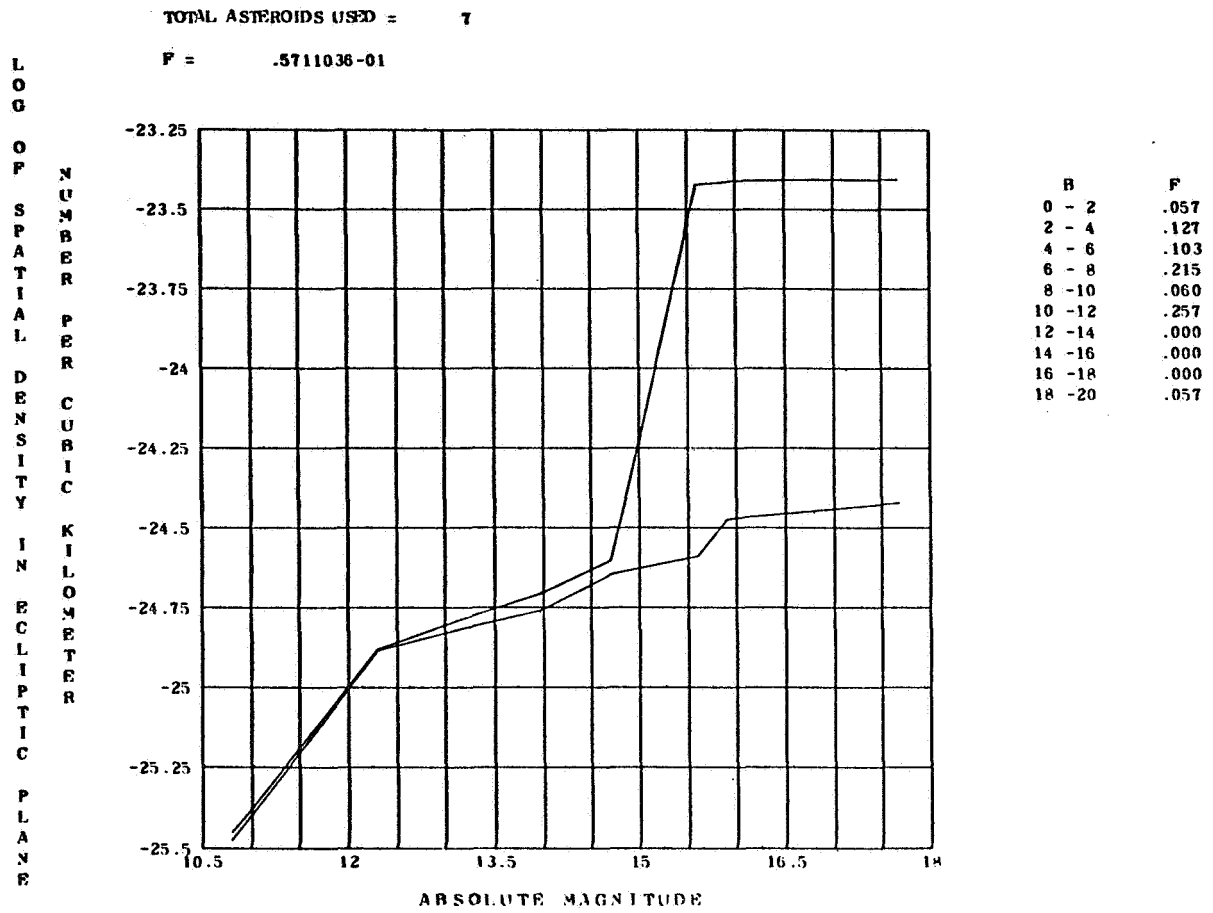


Figure 5. - Spatial density at R = 1.30 averaged over all ecliptic longitudes.

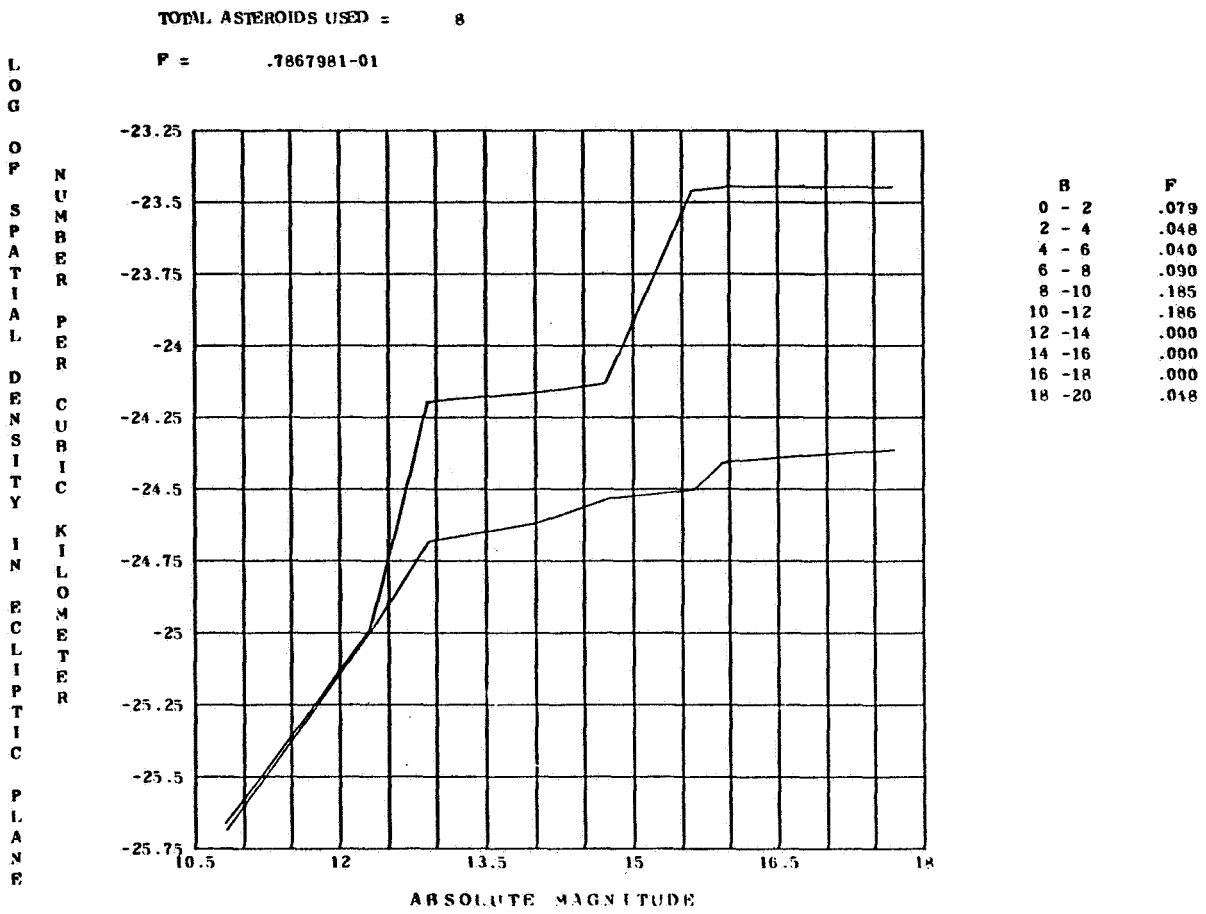


Figure 6. - Spatial density at R = 1.40 averaged over all ecliptic longitudes.

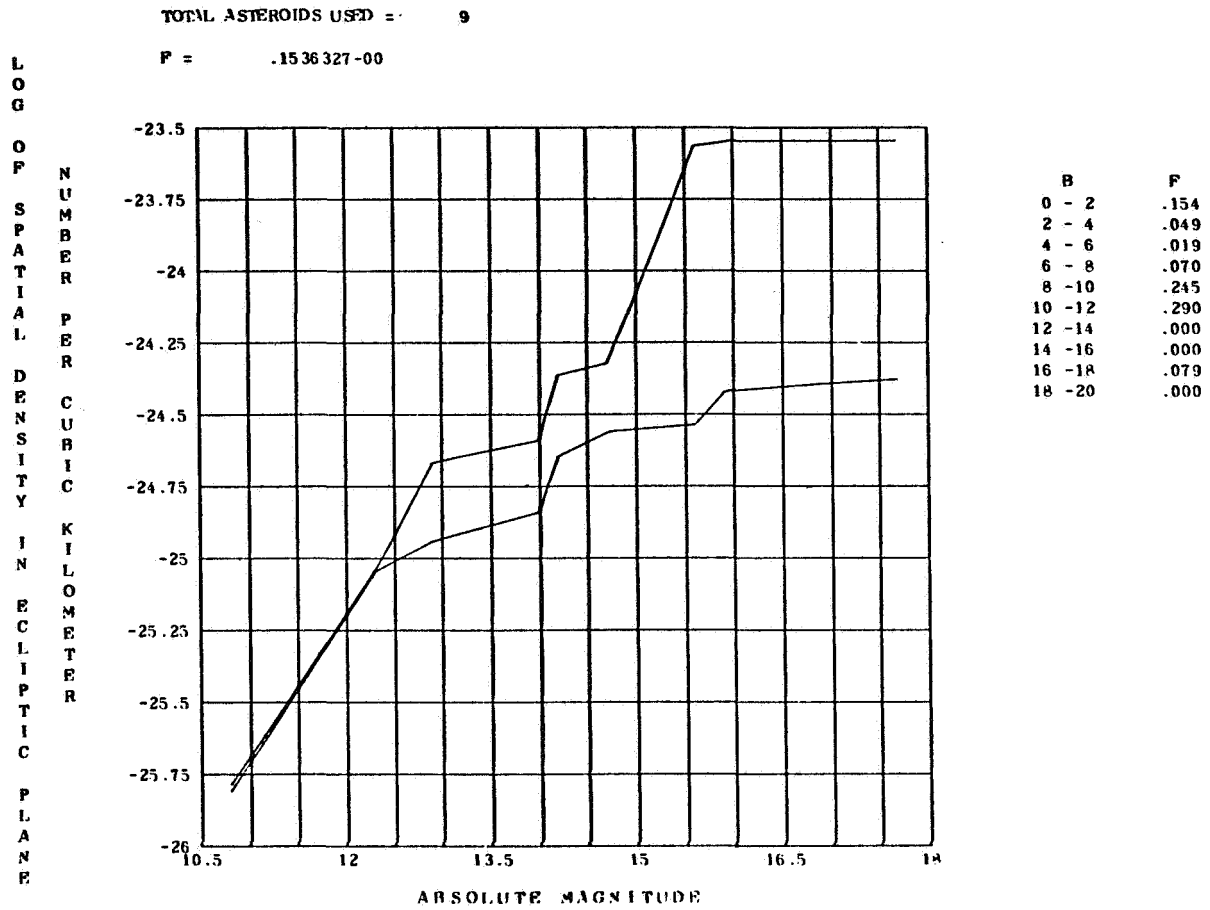


Figure 7. - Spatial density at R = 1.50 averaged over all ecliptic longitudes.

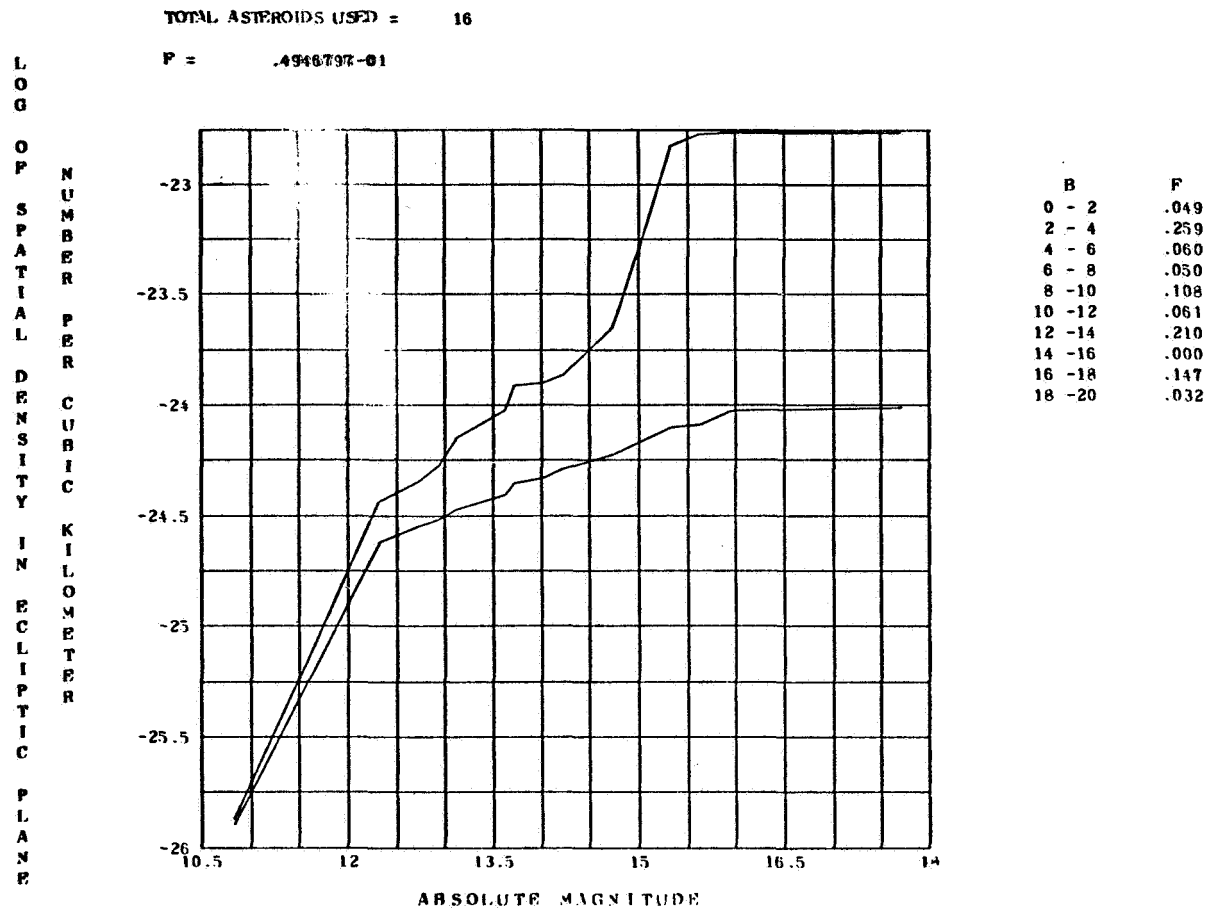


Figure 8. - Spatial density at R = 1.60 averaged over all ecliptic longitudes.

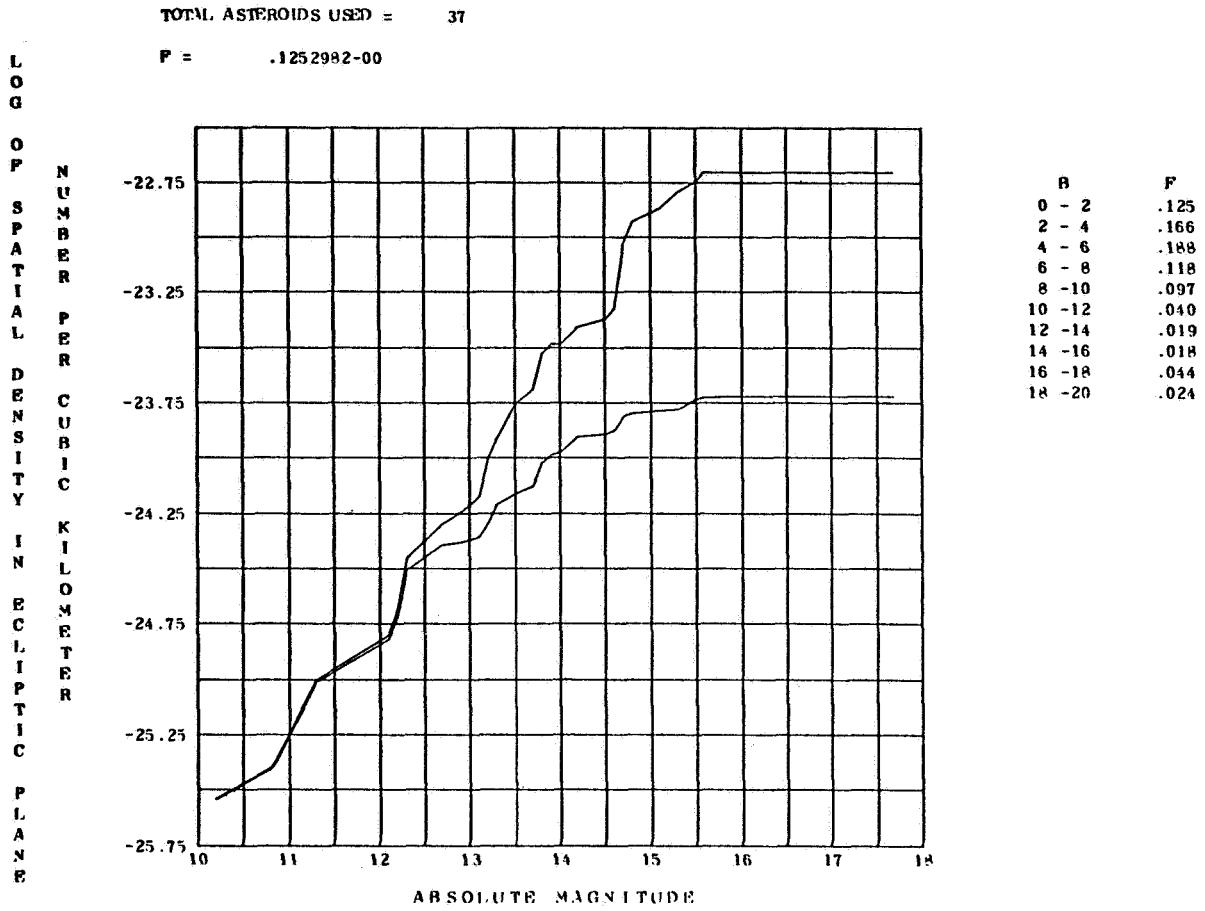


Figure 9. - Spatial density at R = 1.70 averaged over all ecliptic longitudes.

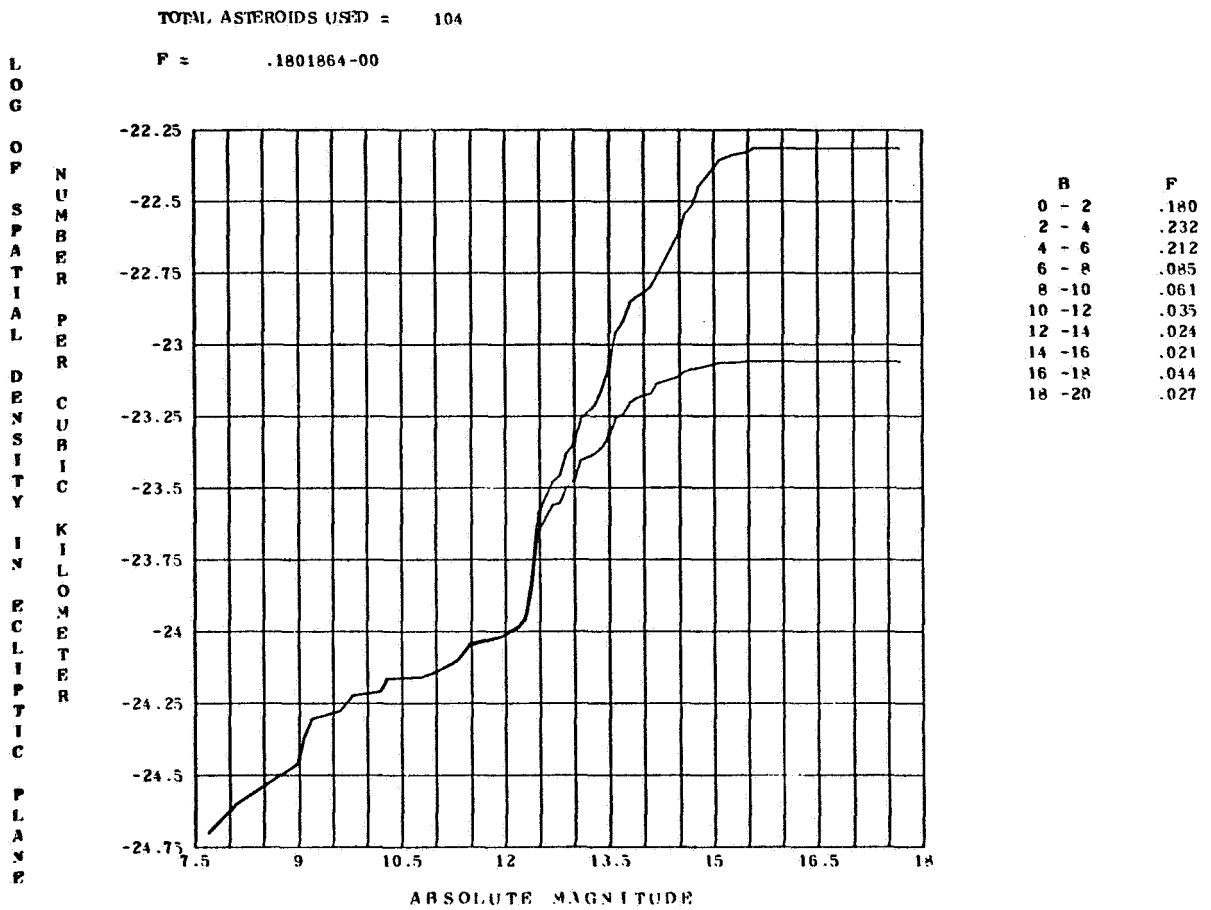


Figure 10. - Spatial density at R = 1.80 averaged over all ecliptic longitudes.

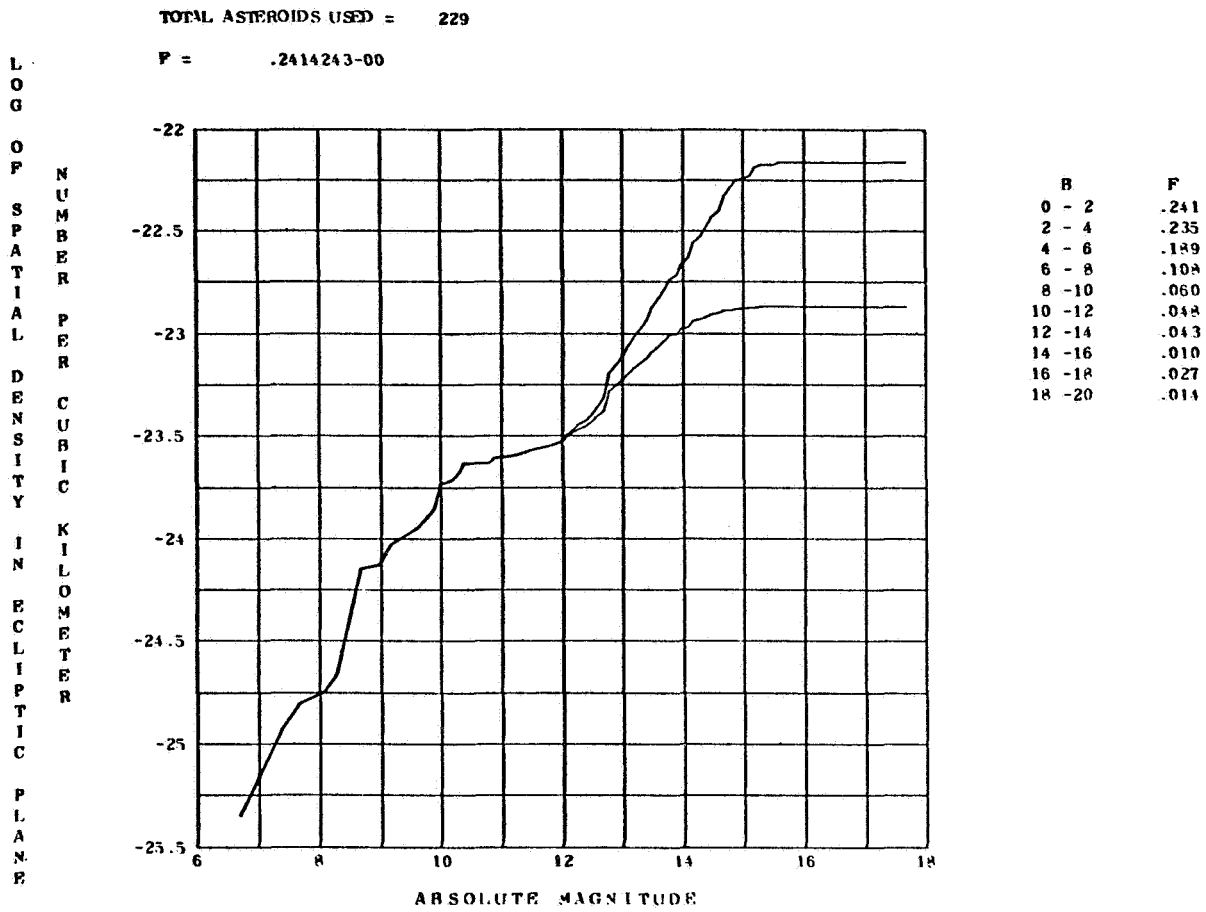


Figure 11. - Spatial density at R = 1.90 averaged over all ecliptic longitudes.

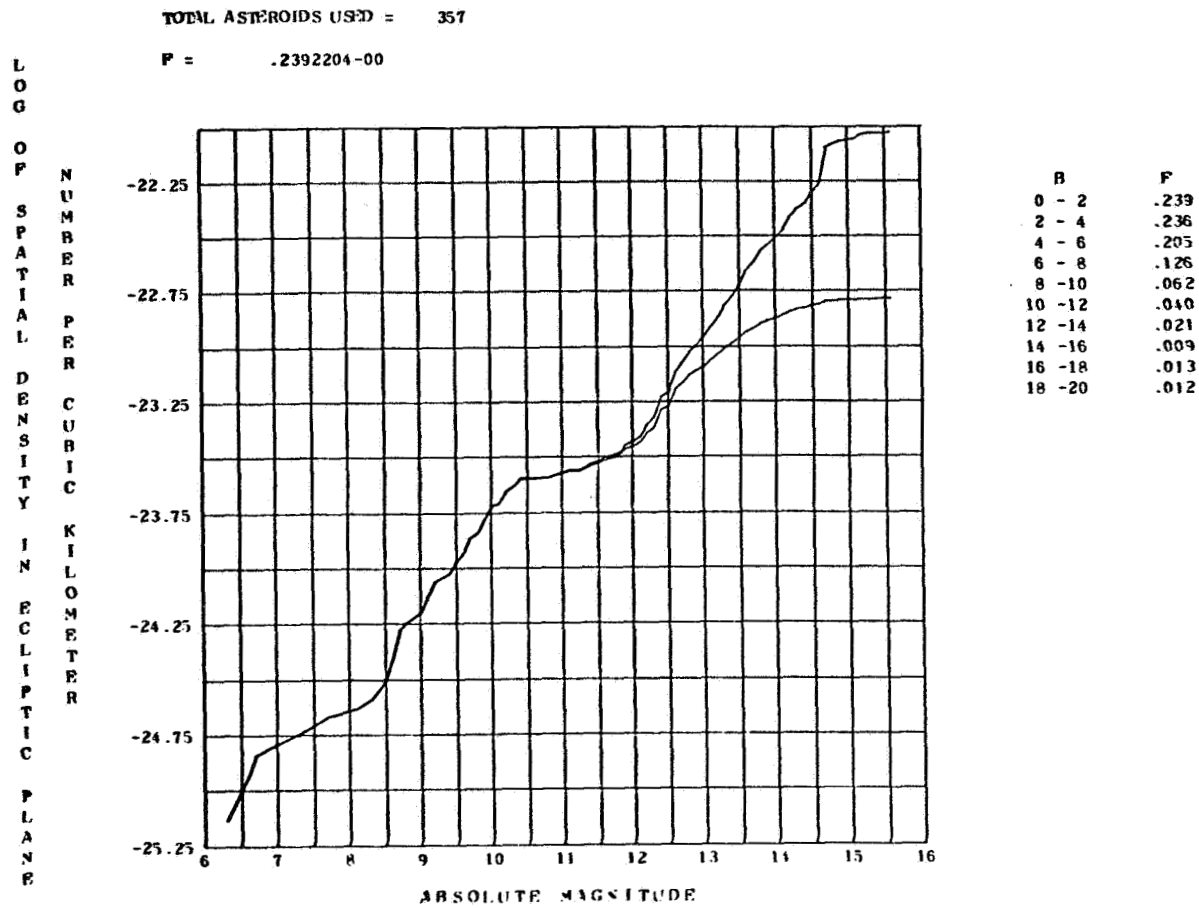


Figure 12. - Spatial density at R = 2.00 averaged over all ecliptic longitudes.

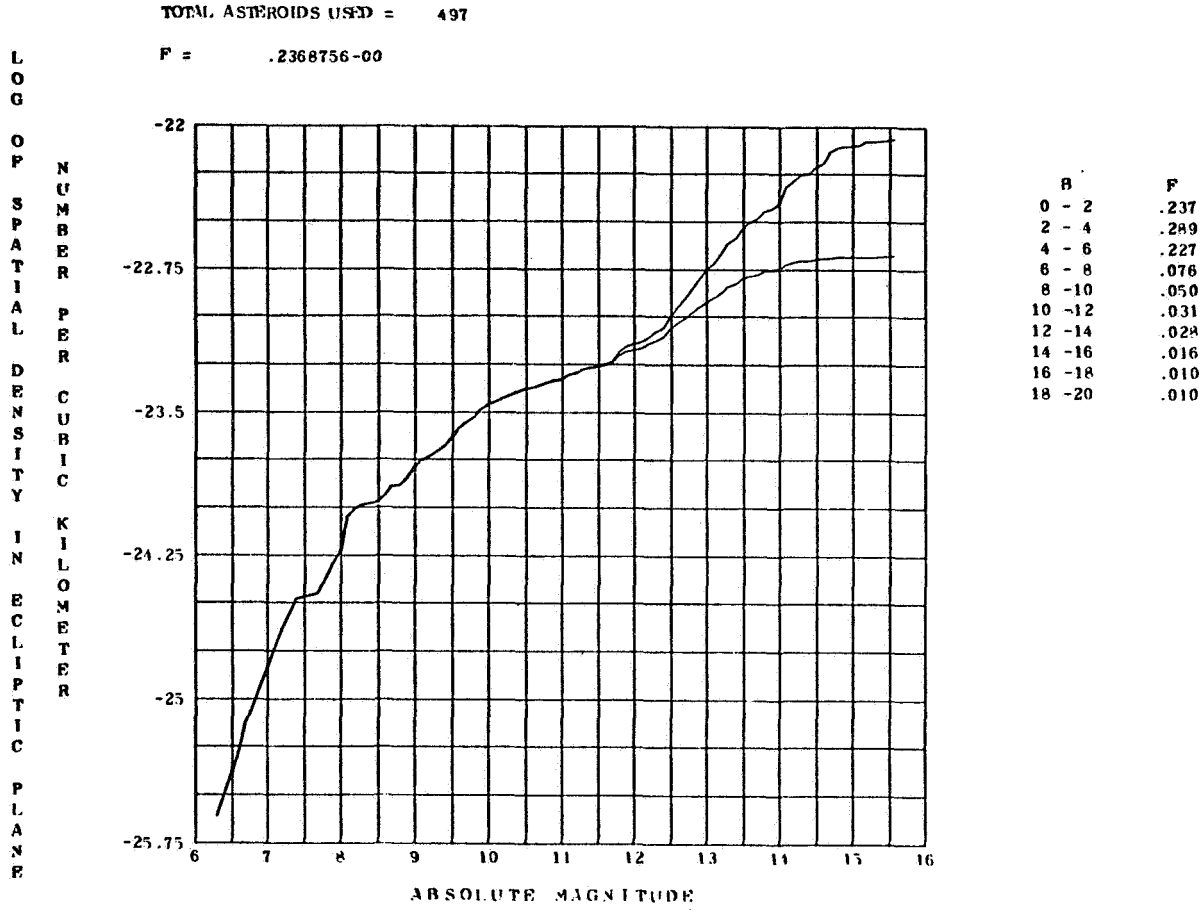


Figure 13. - Spatial density at R = 2.10 averaged over all ecliptic longitudes.

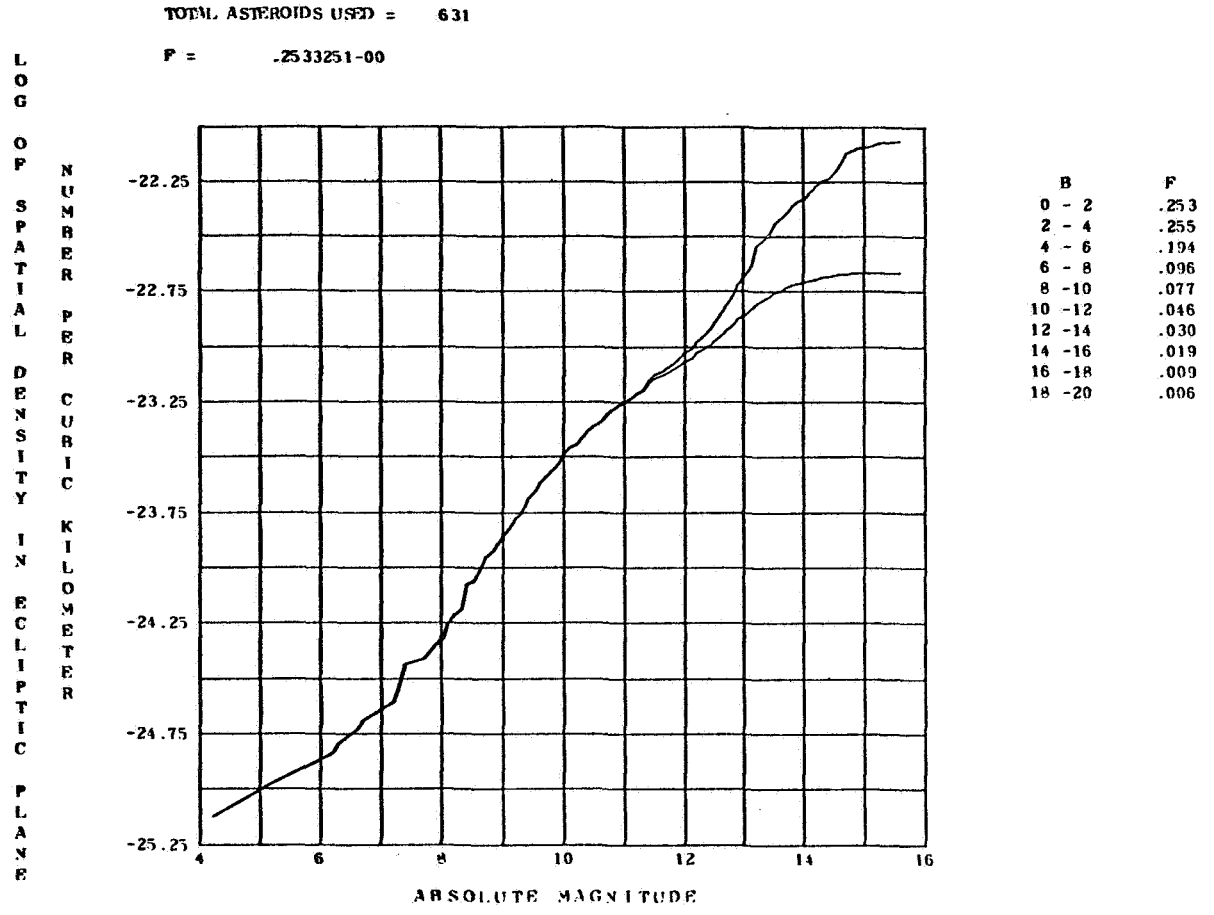


Figure 14. - Spatial density at R = 2.20 averaged over all ecliptic longitudes.

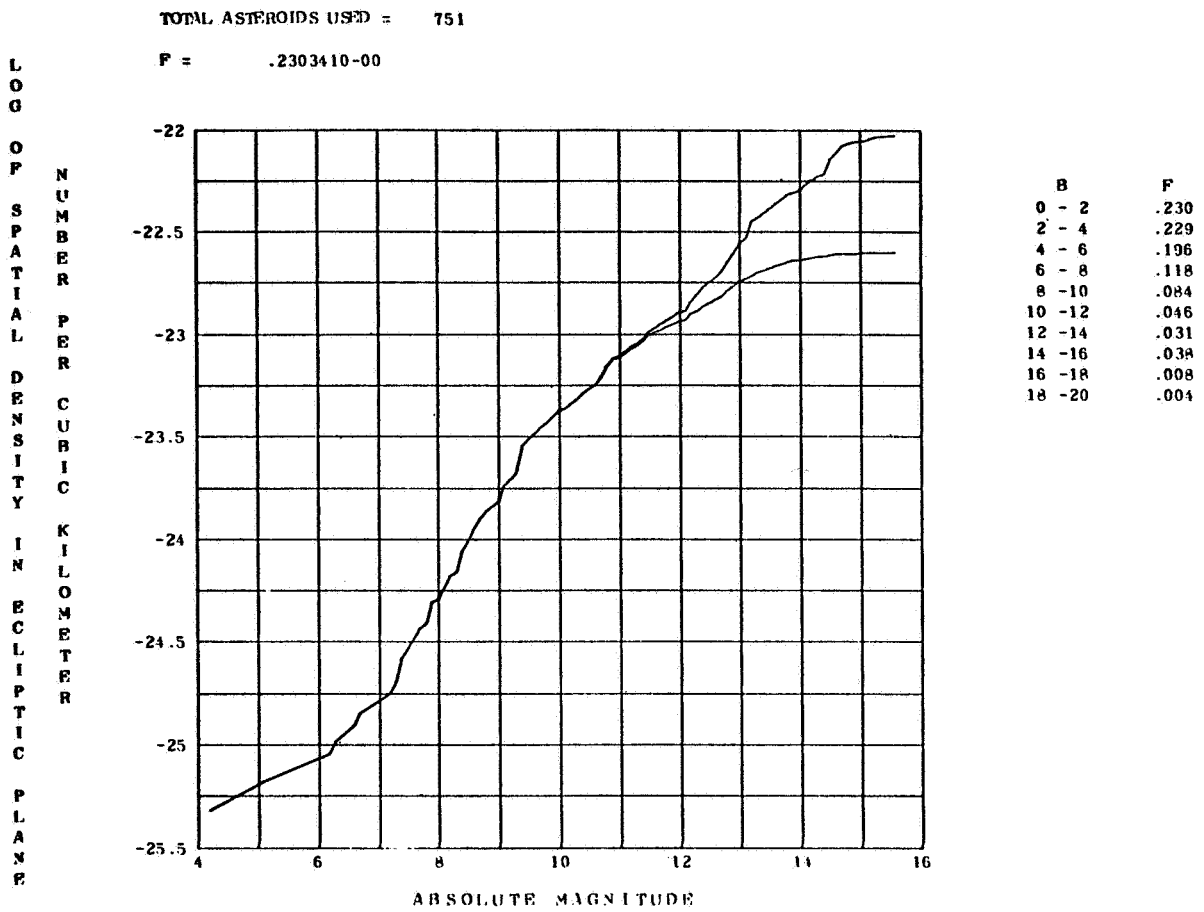


Figure 15. - Spatial density at R = 2.30 averaged over all ecliptic longitudes.

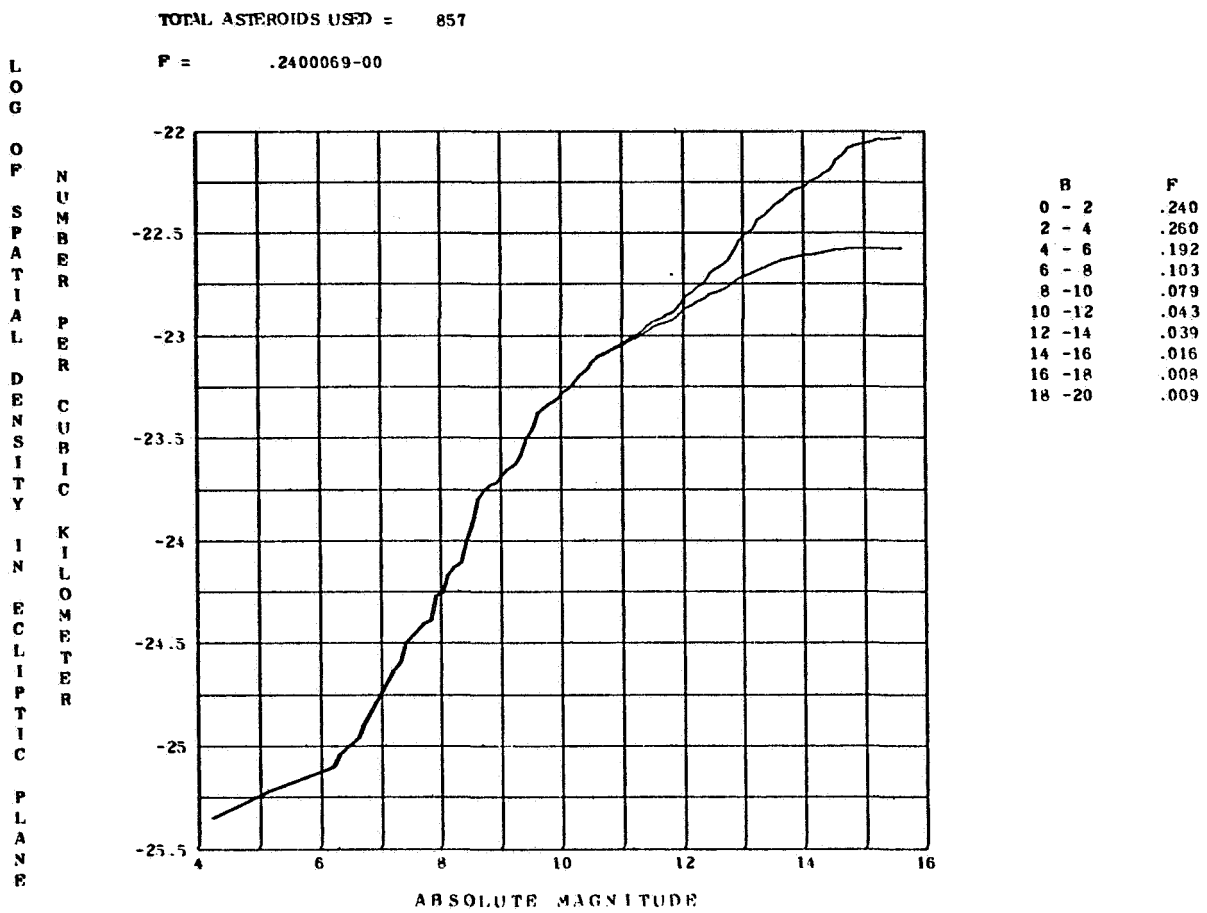


Figure 16. - Spatial density at R = 2.40 averaged over all ecliptic longitudes.

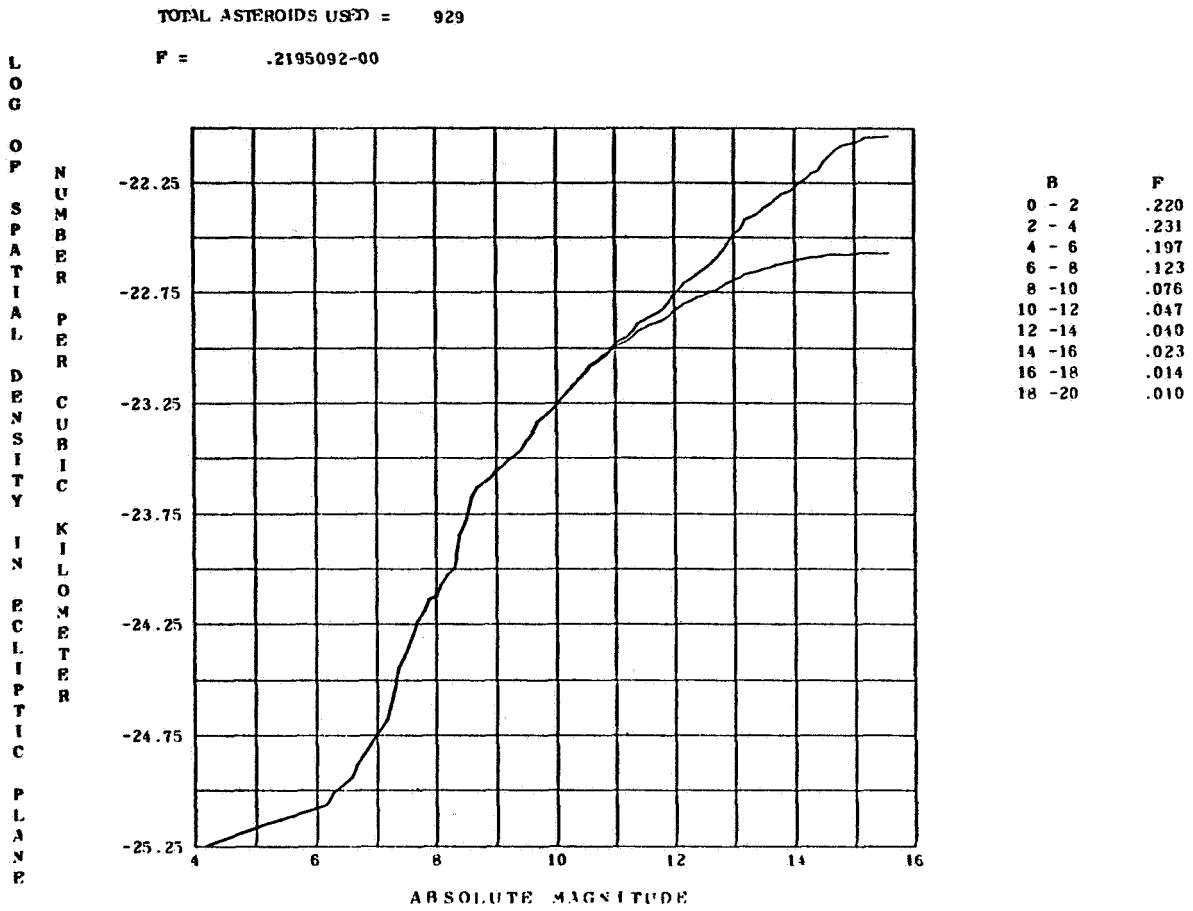


Figure 17. - Spatial density at R = 2.50 averaged over all ecliptic longitudes.

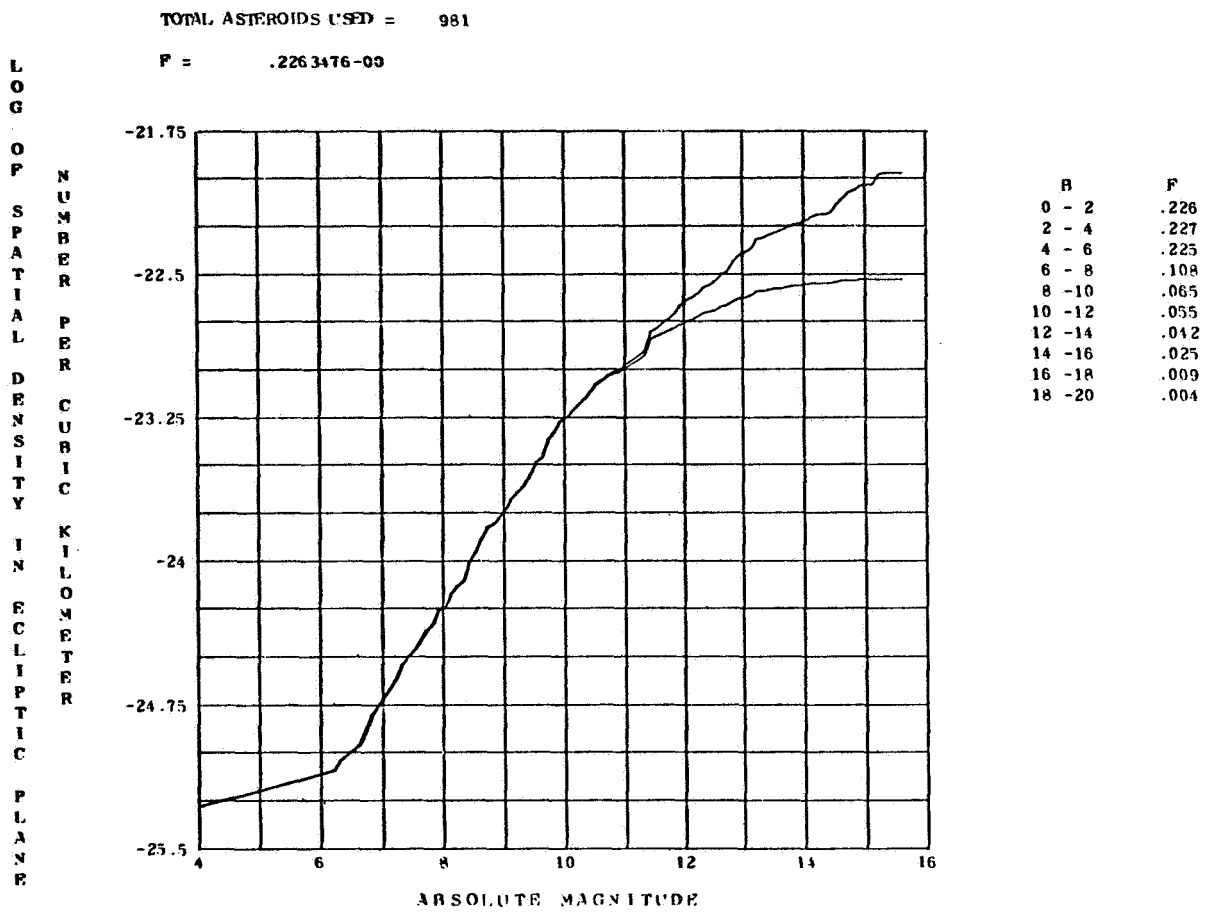


Figure 18. - Spatial density at R = 2.60 averaged over all ecliptic longitudes.

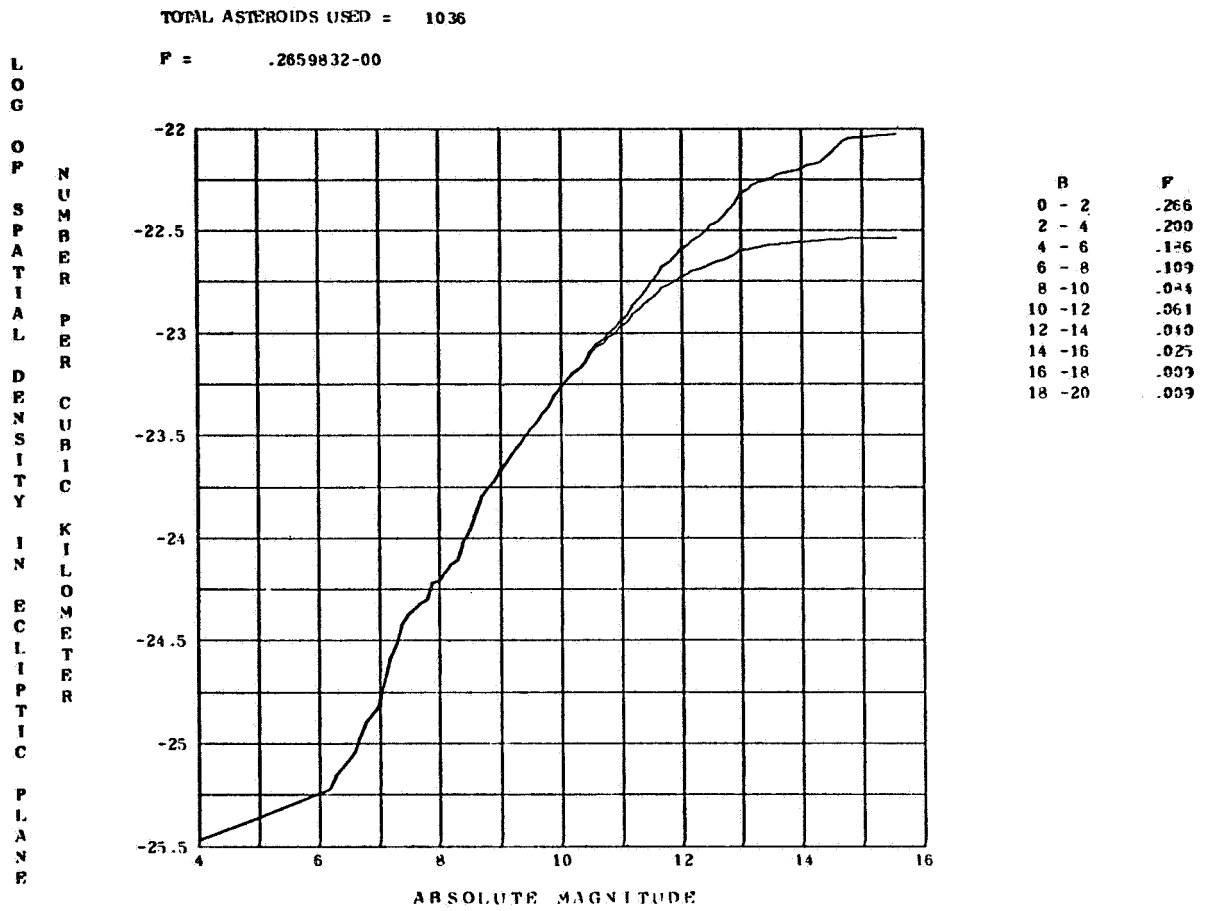


Figure 19. - Spatial density at R = 2.70 averaged over all ecliptic longitudes.

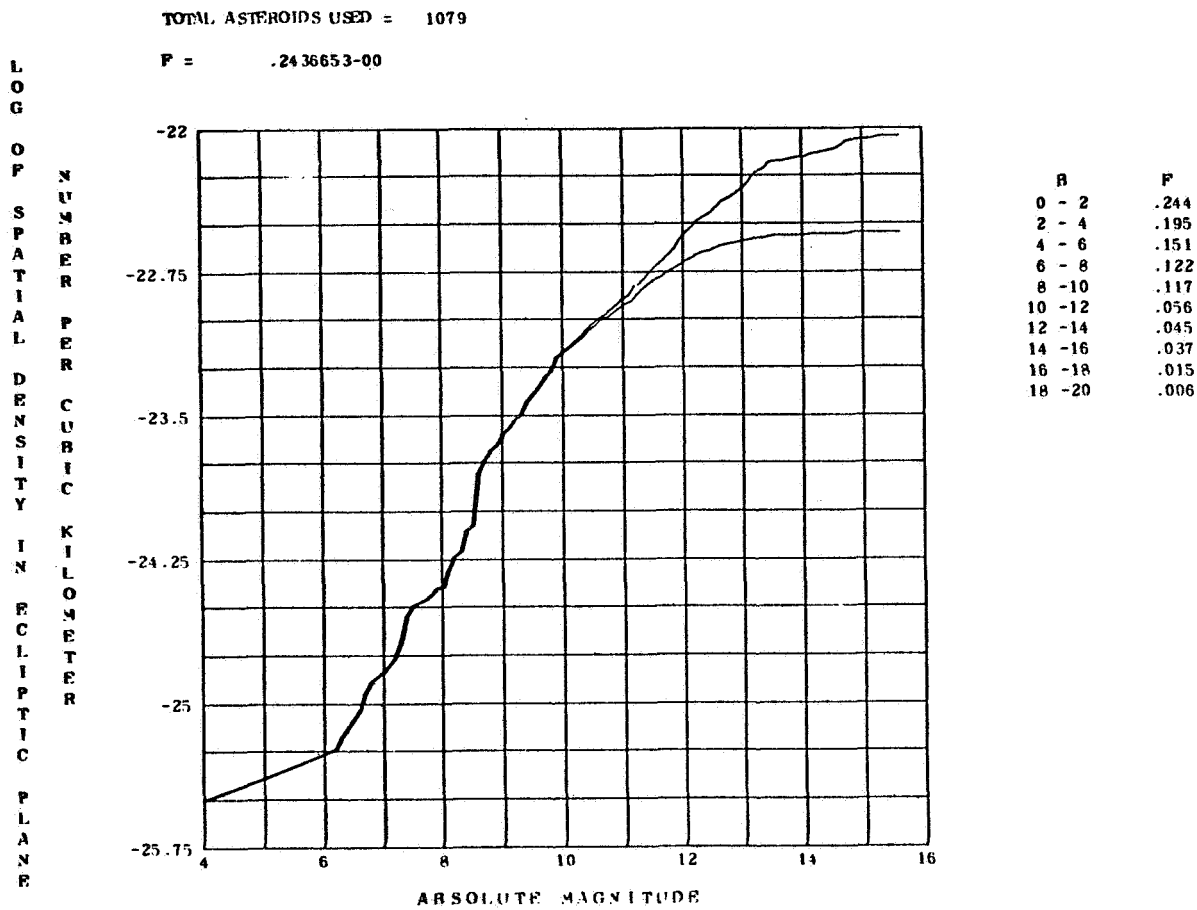


Figure 20. - Spatial density at R = 2.80 averaged over all ecliptic longitudes.

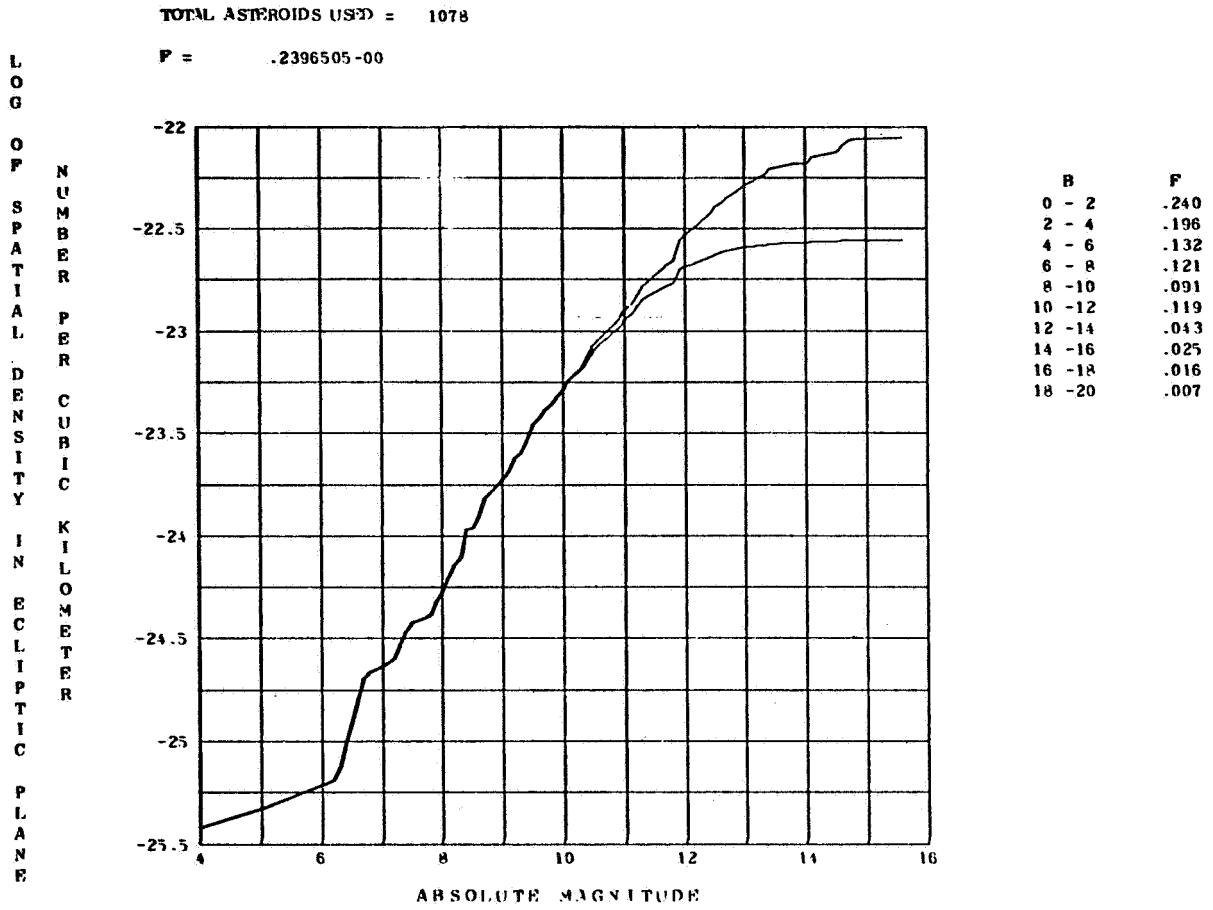


Figure 21. - Spatial density at R = 2.90 averaged over all ecliptic longitudes.

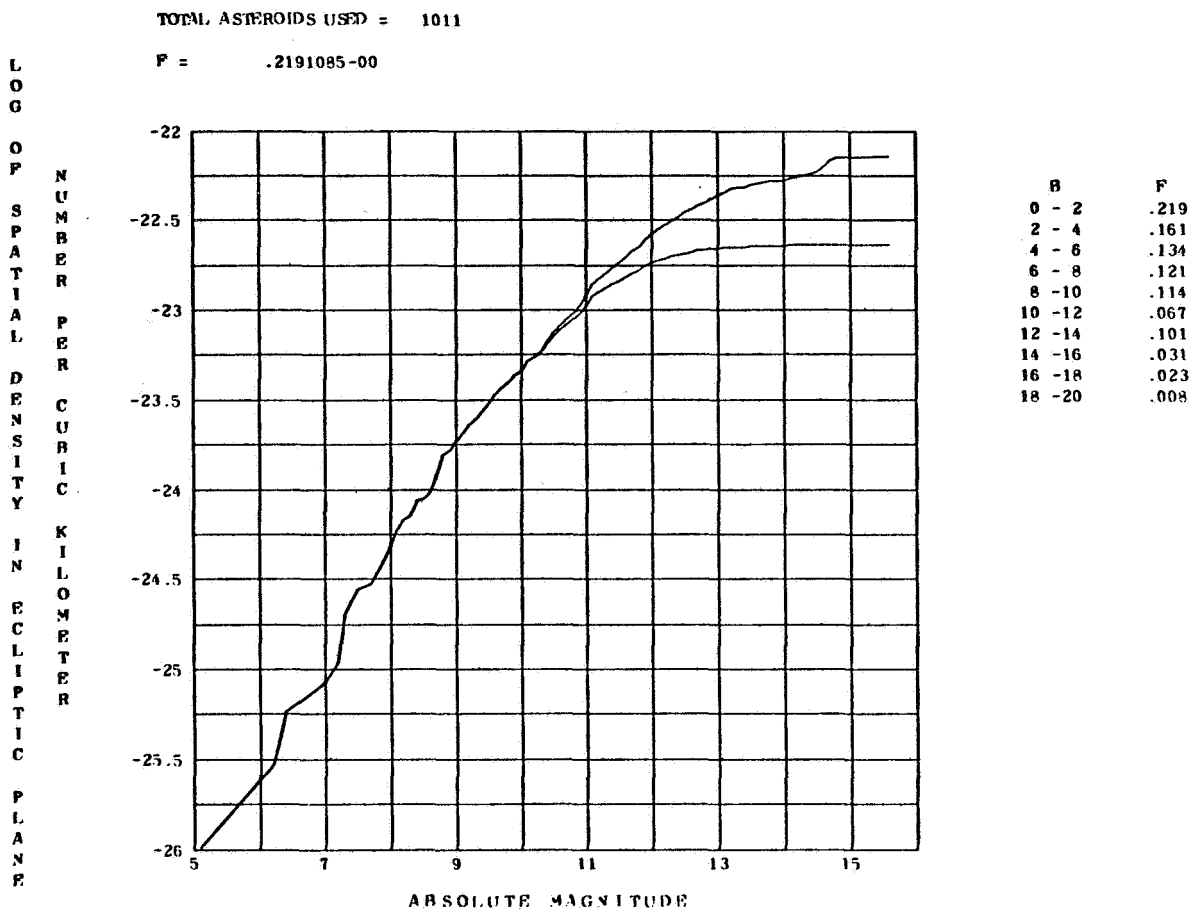


Figure 22. - Spatial density at R = 3.00 averaged over all ecliptic longitudes.

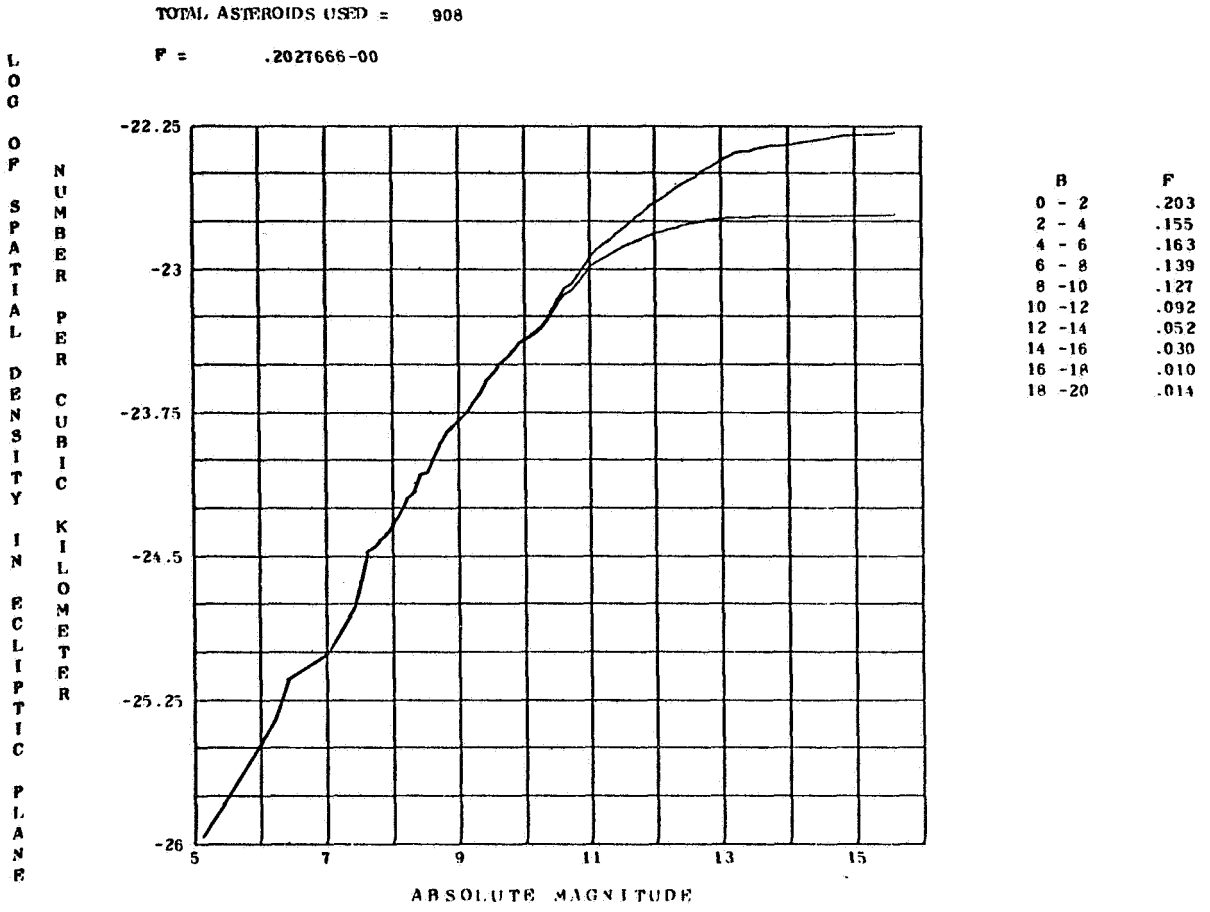


Figure 23. - Spatial density at R = 3.10 averaged over all ecliptic longitudes.

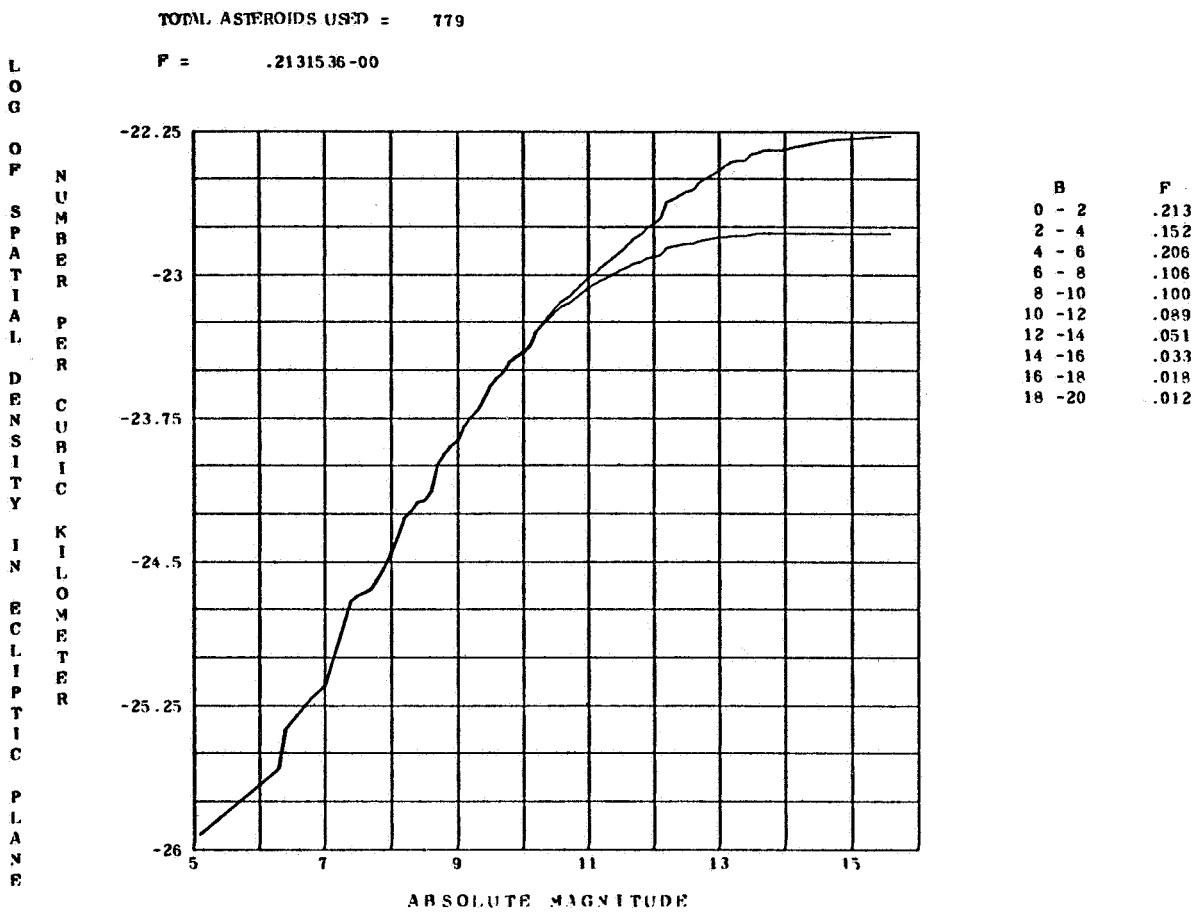


Figure 24. - Spatial density at R = 3.20 averaged over all ecliptic longitudes.

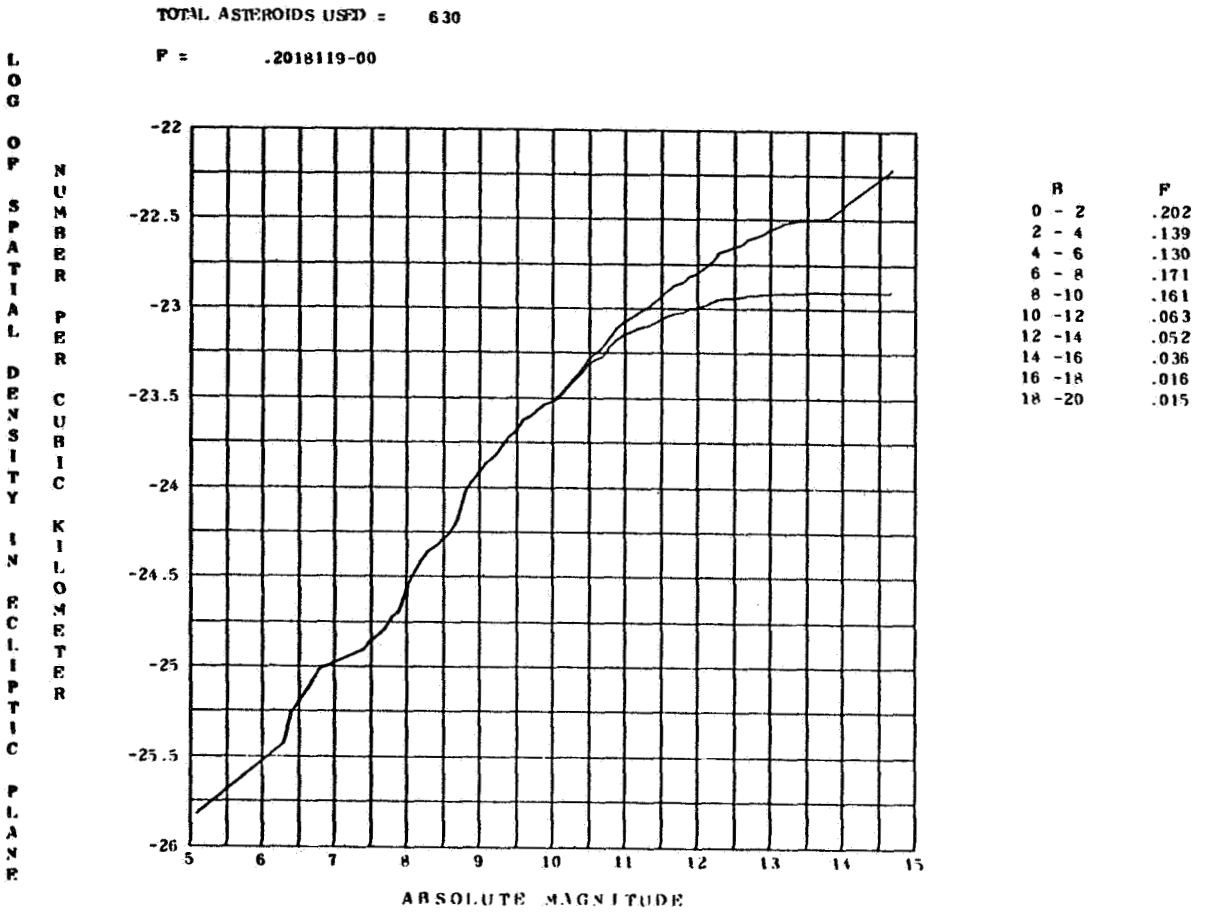


Figure 25. - Spatial density at R = 3.30 averaged over all ecliptic longitudes.

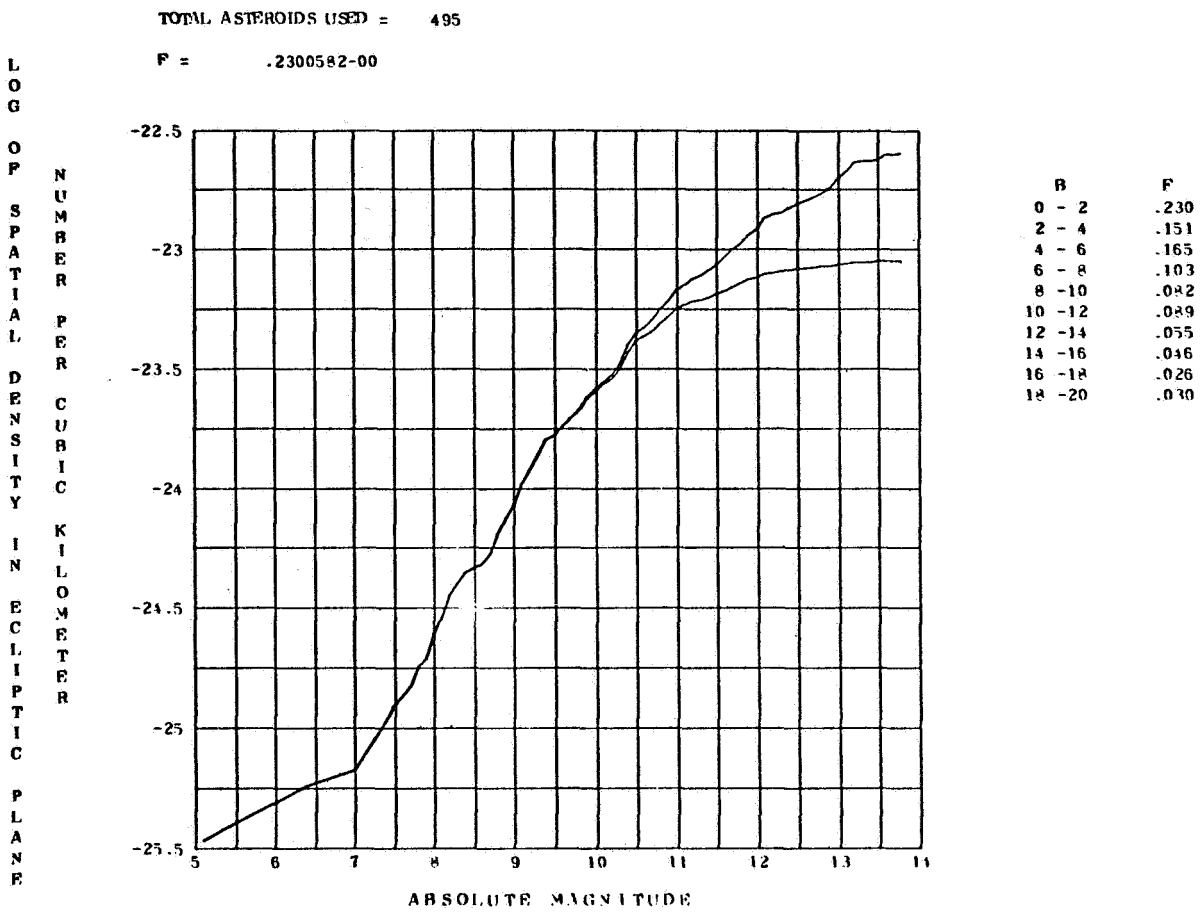


Figure 26. - Spatial density at R = 3.40 averaged over all ecliptic longitudes.

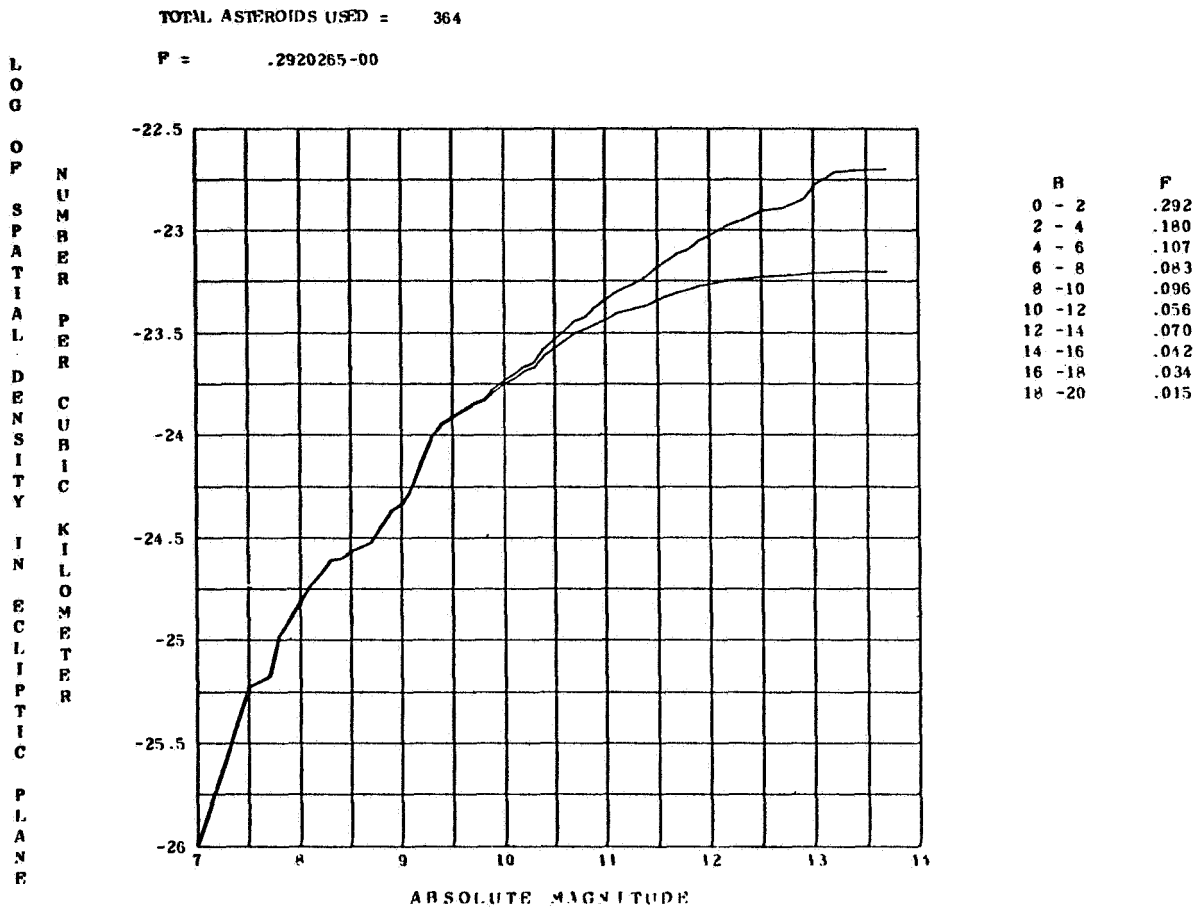


Figure 27. - Spatial density at R = 3.50 averaged over all ecliptic longitudes.

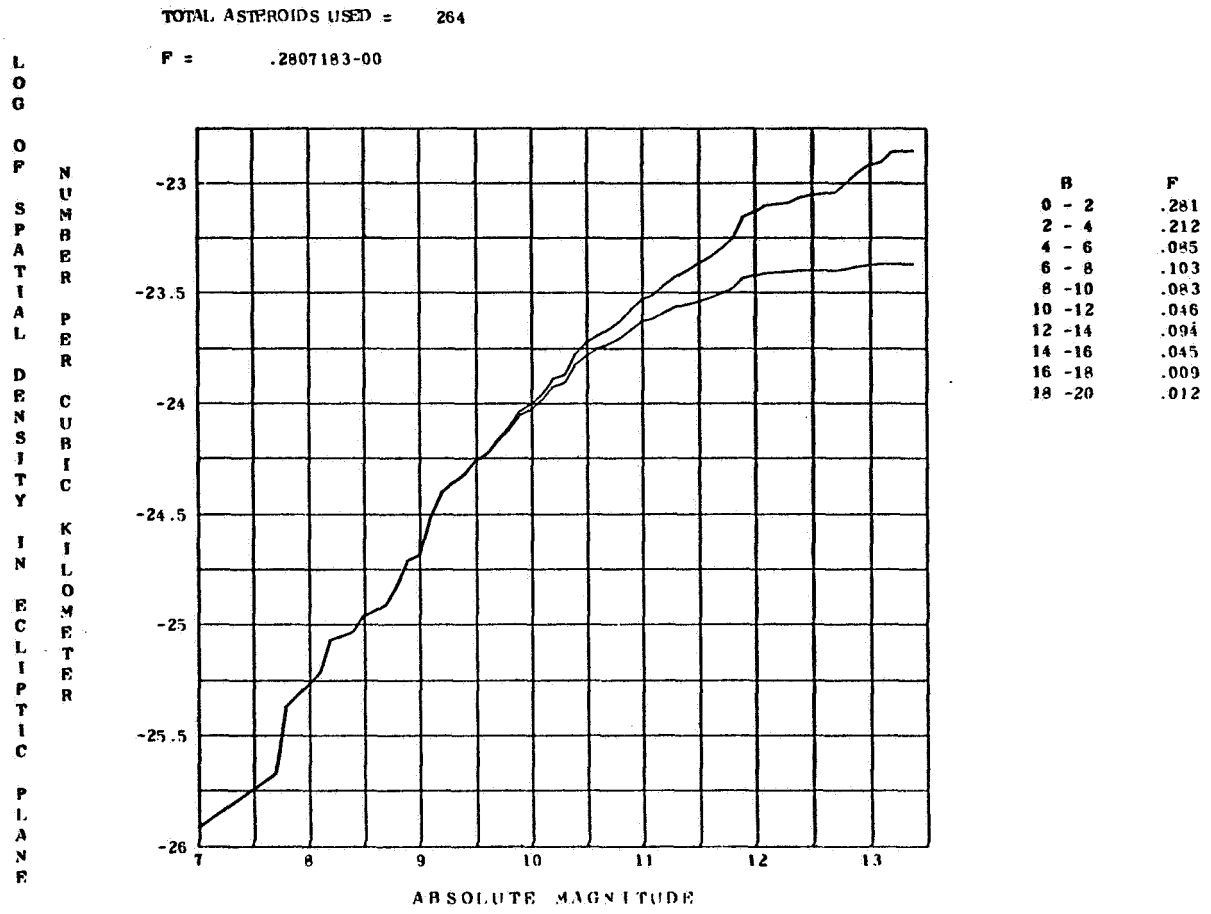


Figure 28. - Spatial density at R = 3.60 averaged over all ecliptic longitudes.

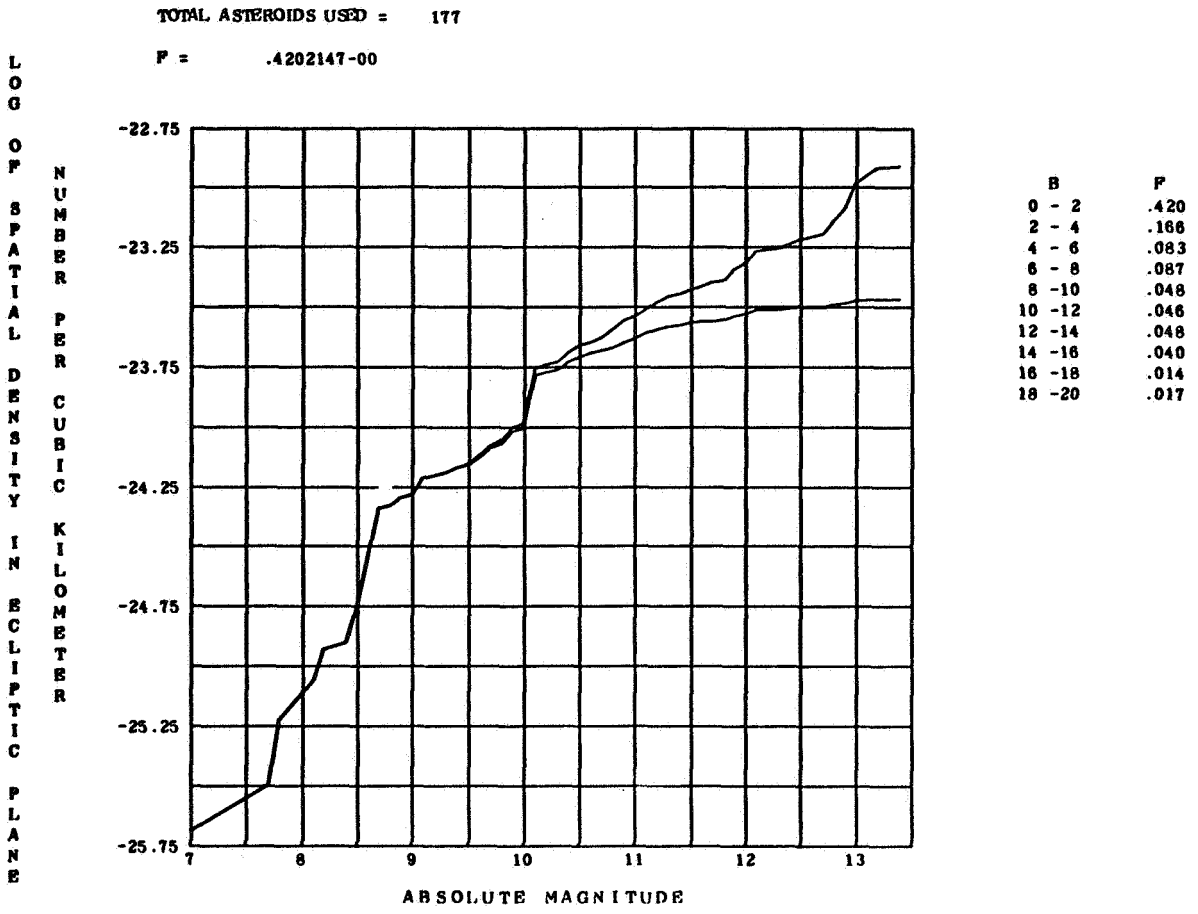


Figure 29. - Spatial density at R = 3.70 averaged over all ecliptic longitudes.

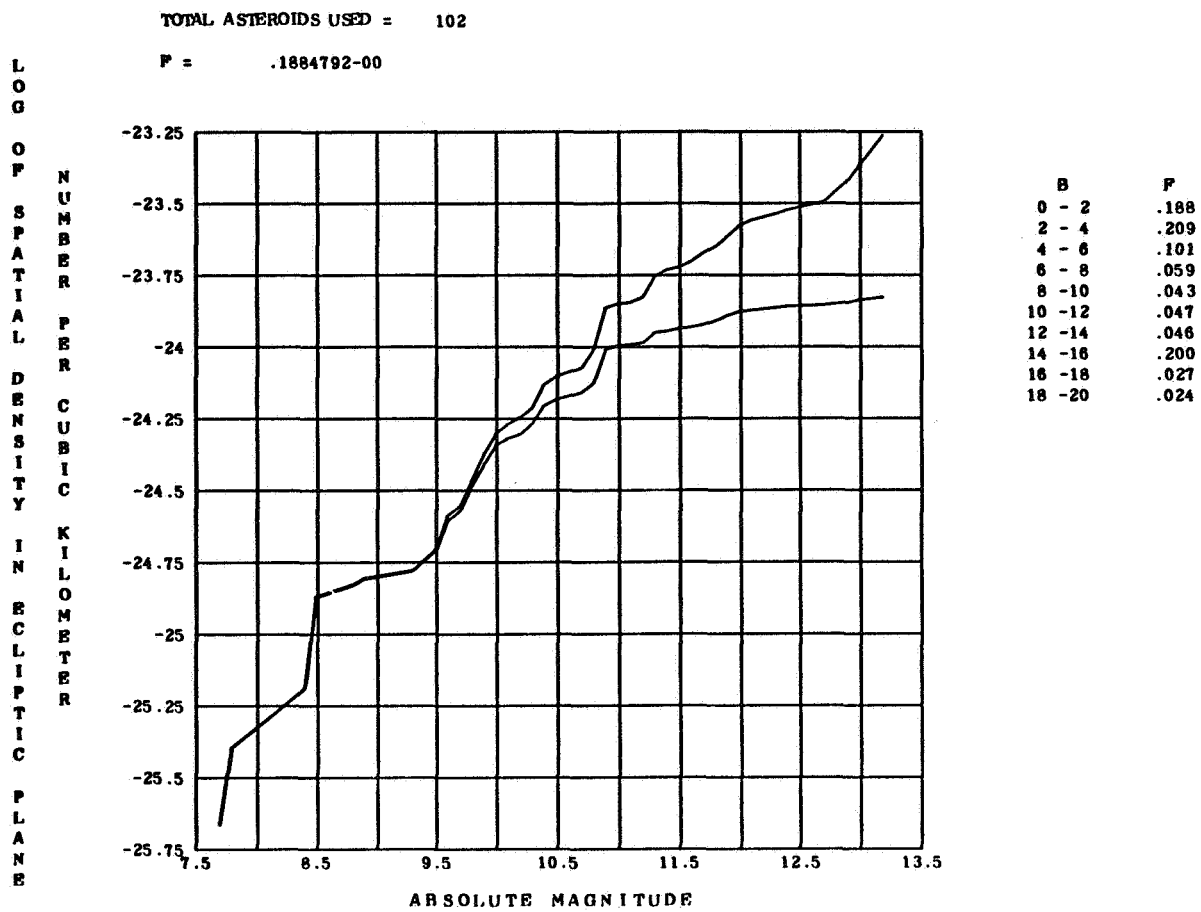


Figure 30. - Spatial density at R = 3.80 averaged over all ecliptic longitudes.

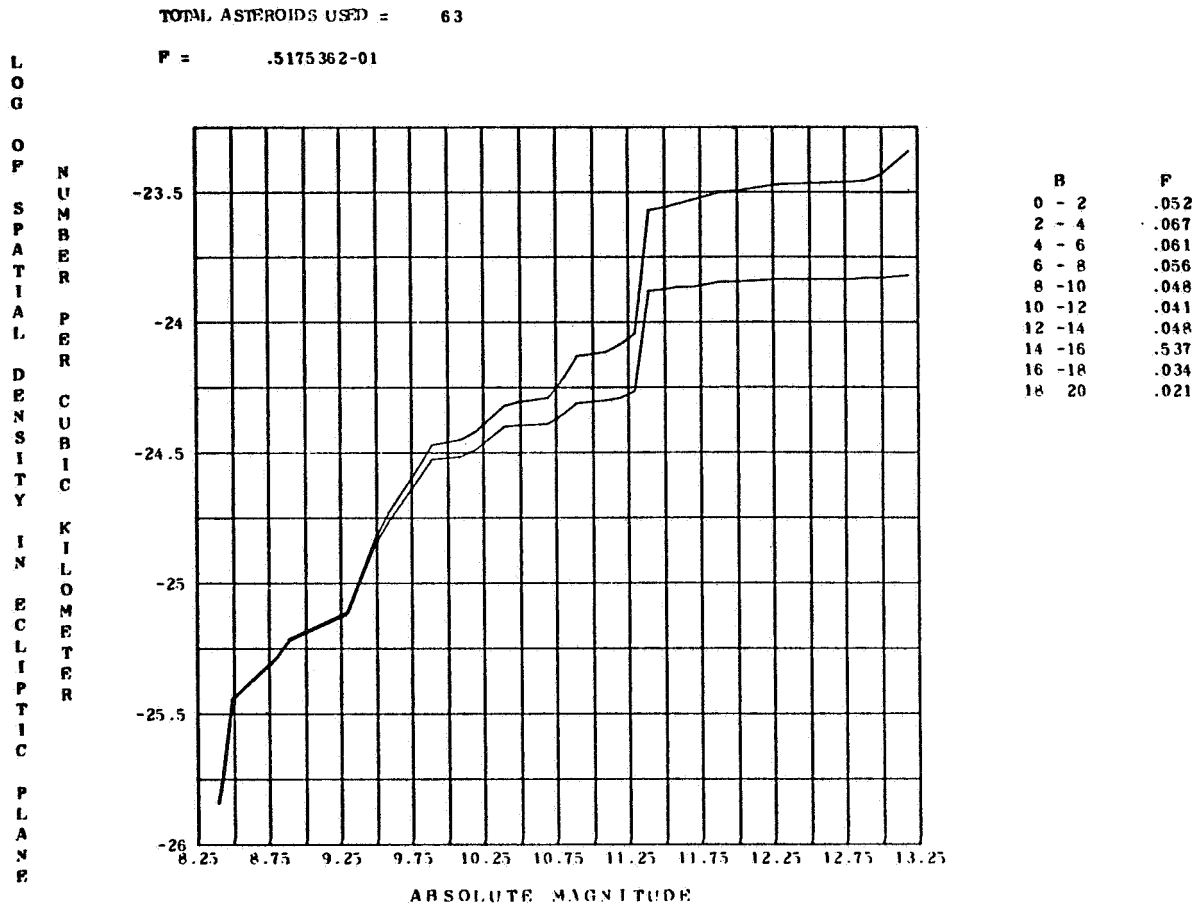


Figure 31. - Spatial density at R = 3.90 averaged over all ecliptic longitudes.

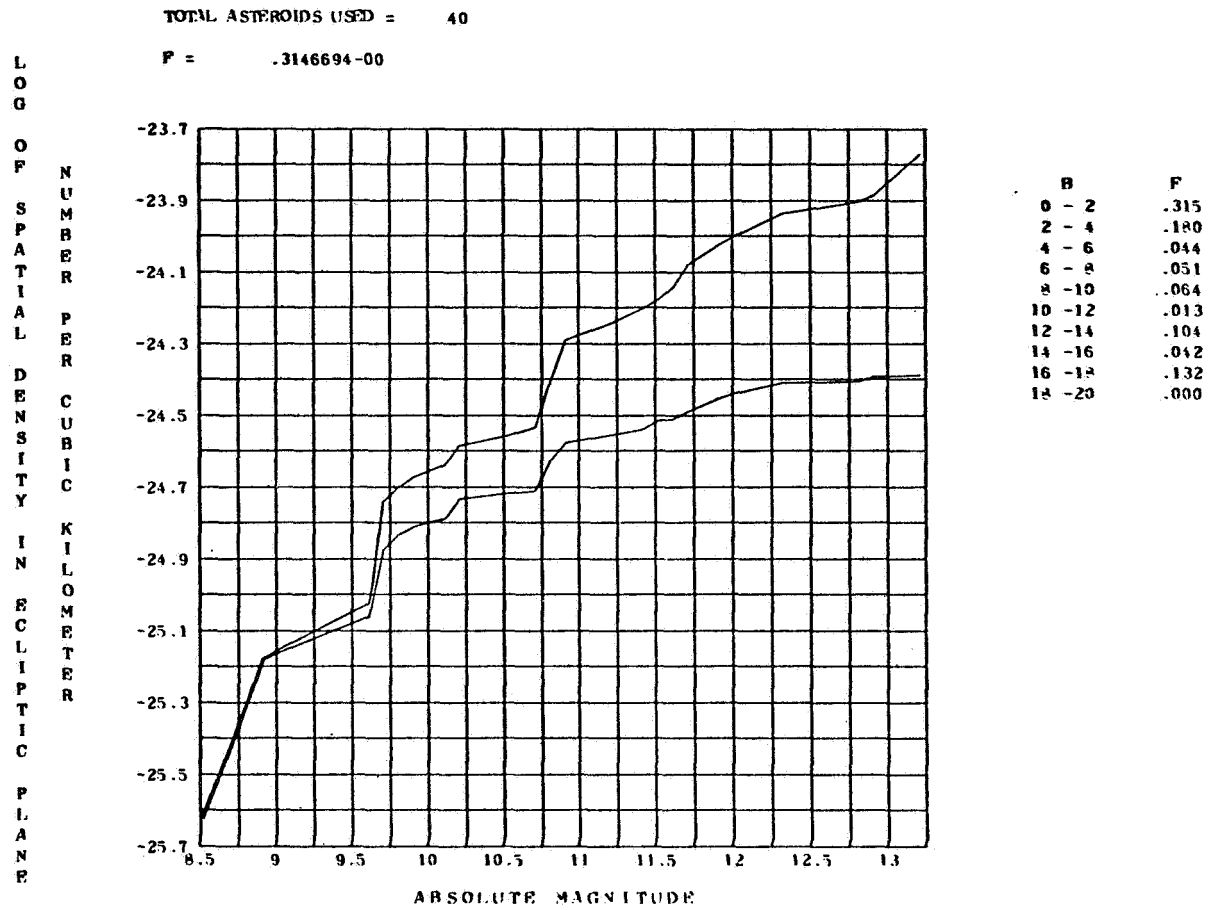


Figure 32. - Spatial density at R = 4.00 averaged over all ecliptic longitudes.

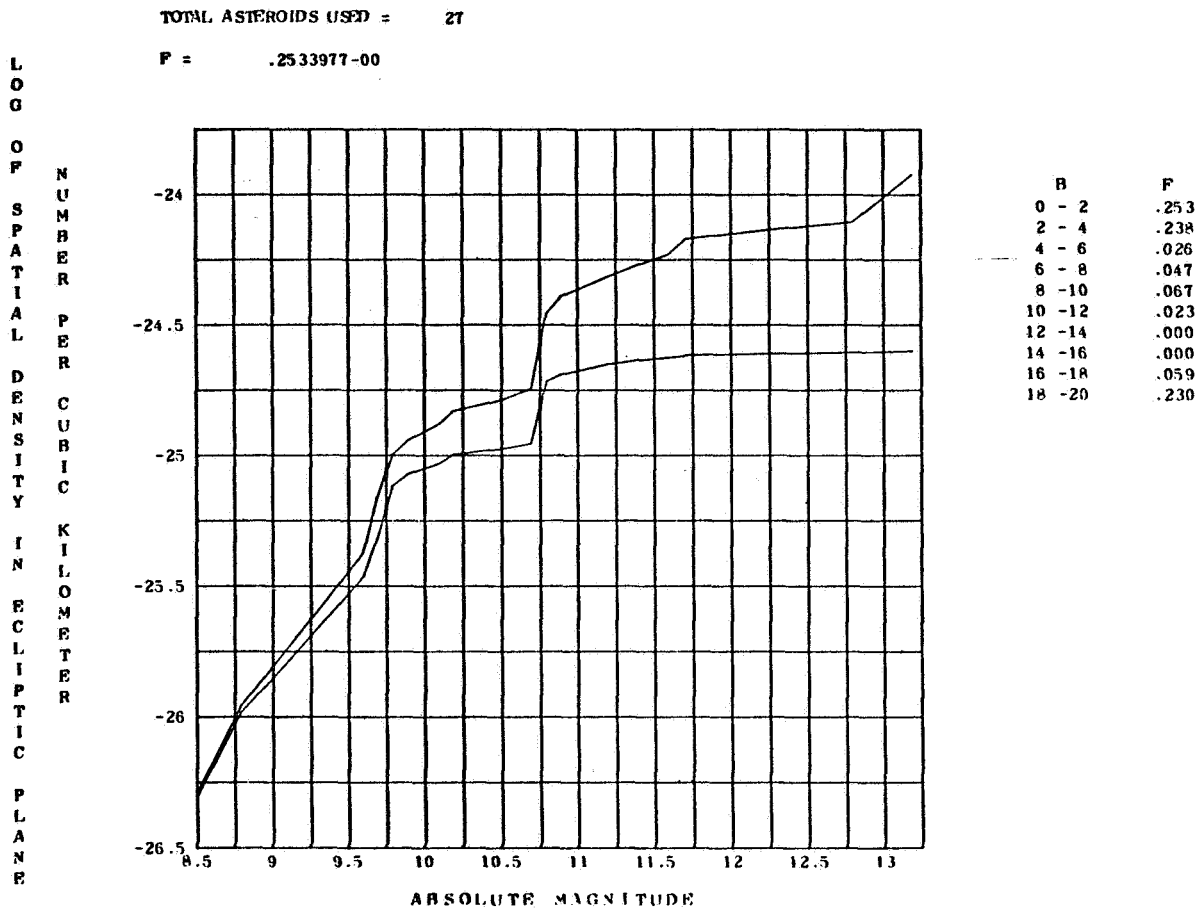


Figure 33. - Spatial density at R = 4.10 averaged over all ecliptic longitudes.

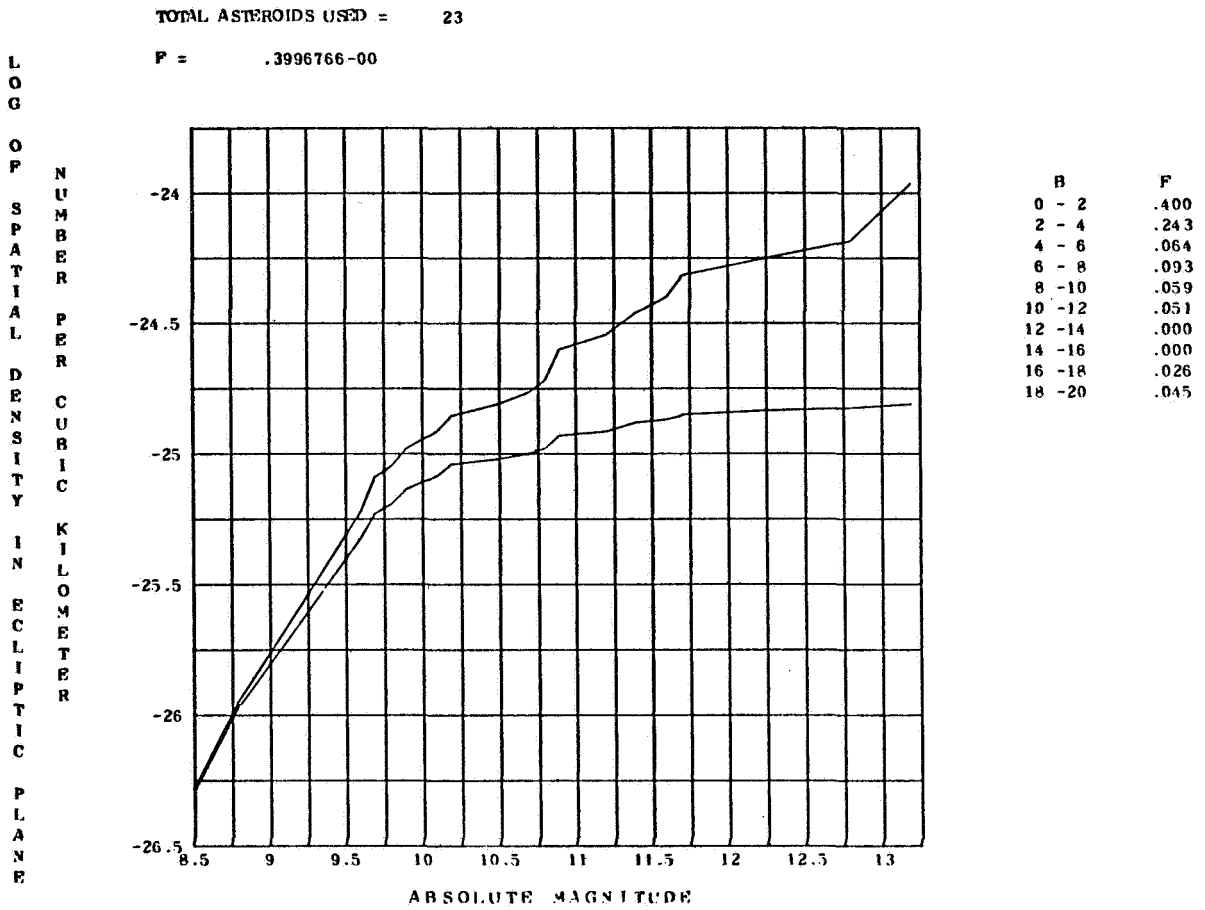


Figure 34. - Spatial density at R = 4.20 averaged over all ecliptic longitudes.

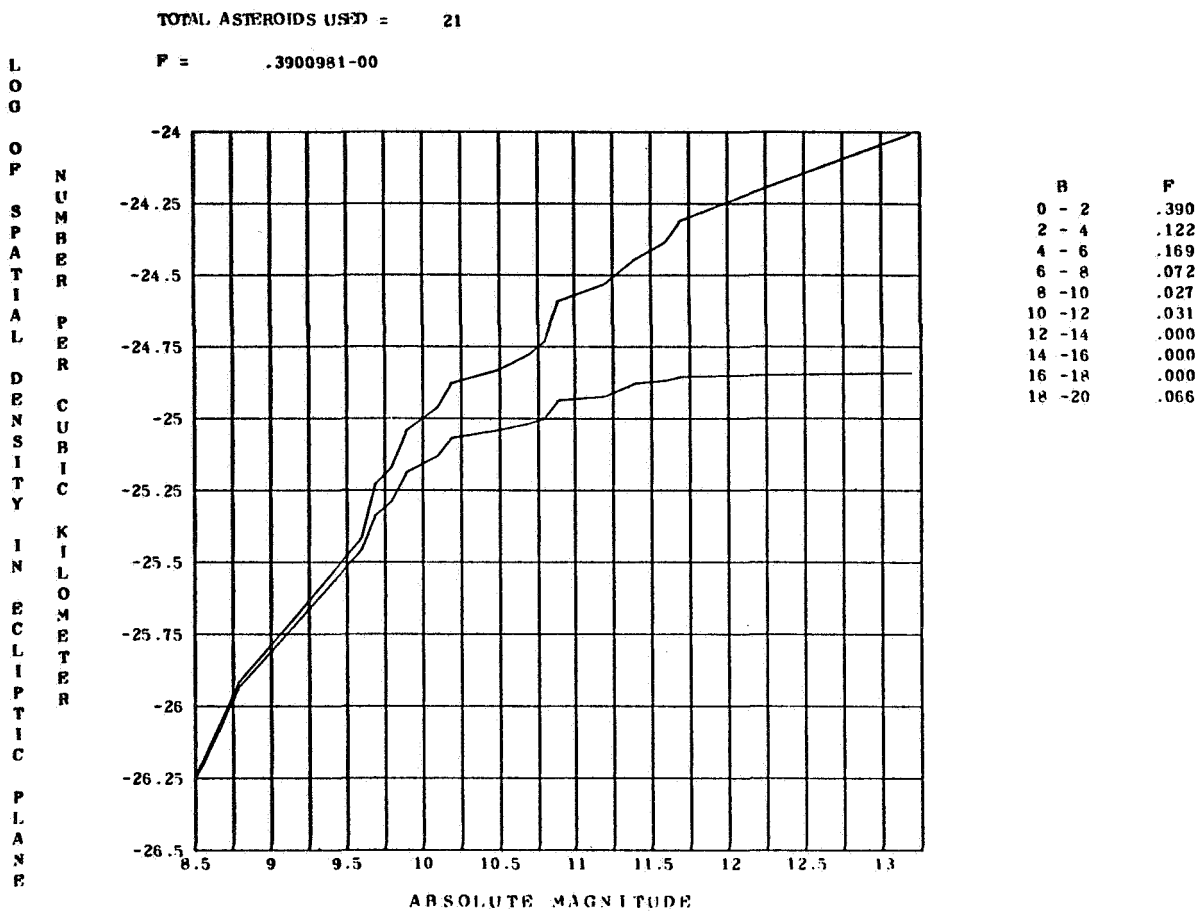


Figure 35. - Spatial density at R = 4.30 averaged over all ecliptic longitudes.

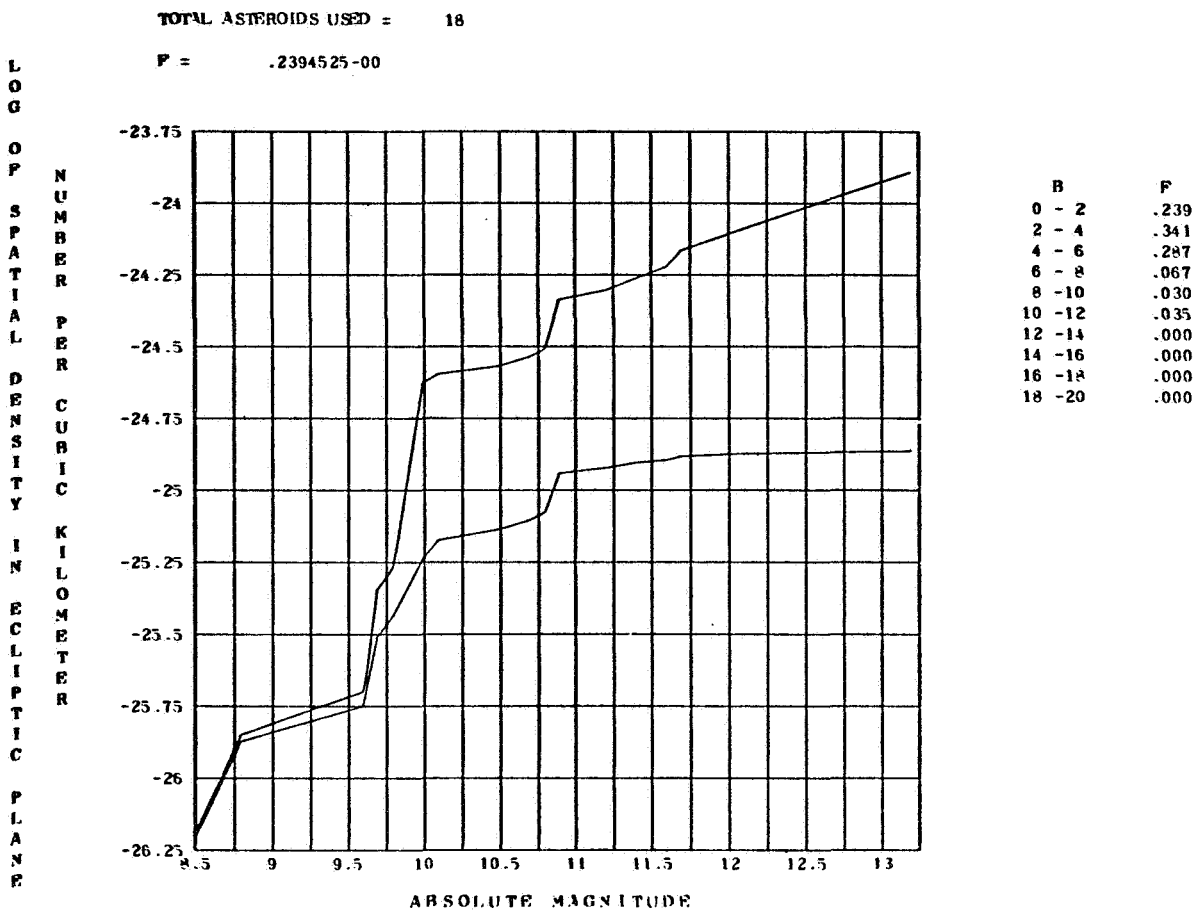


Figure 36. - Spatial density at R = 4.40 averaged over all ecliptic longitudes.

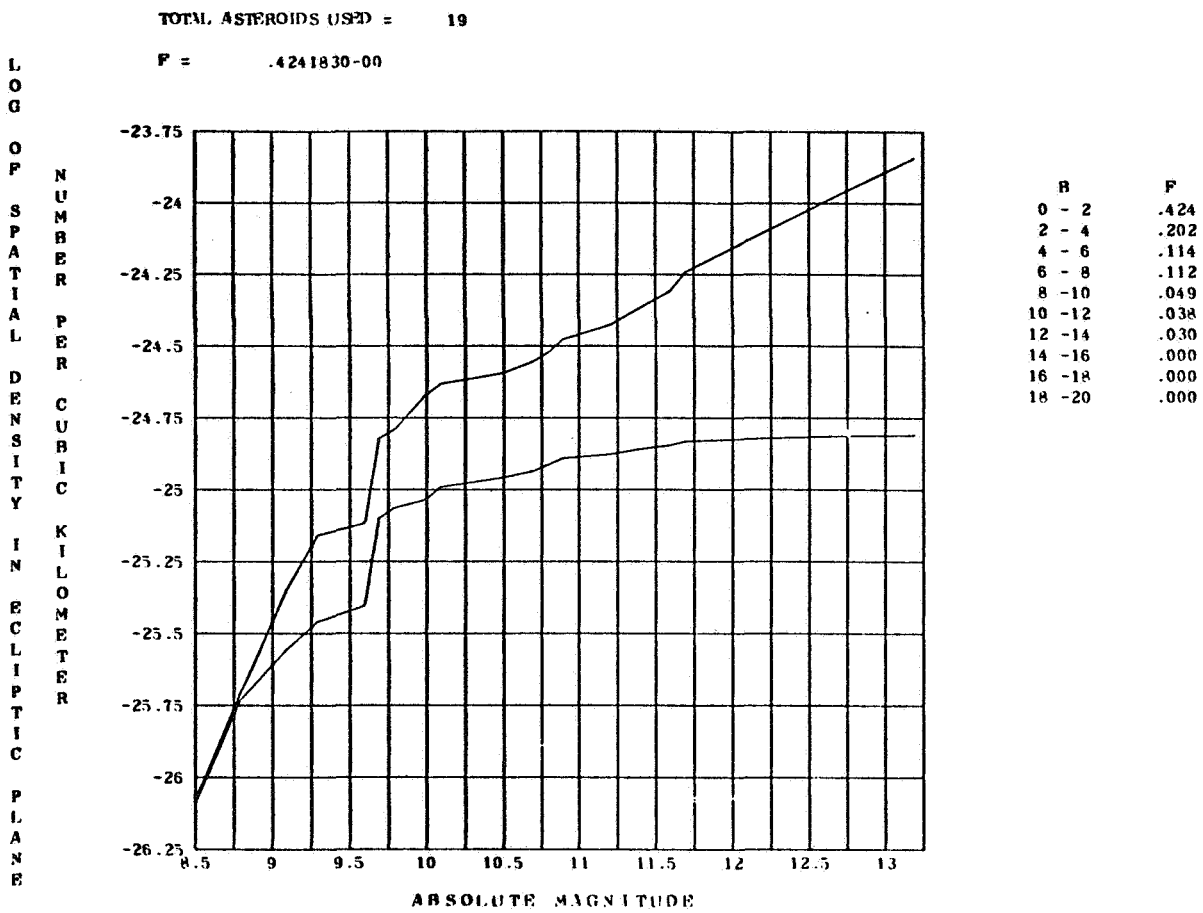
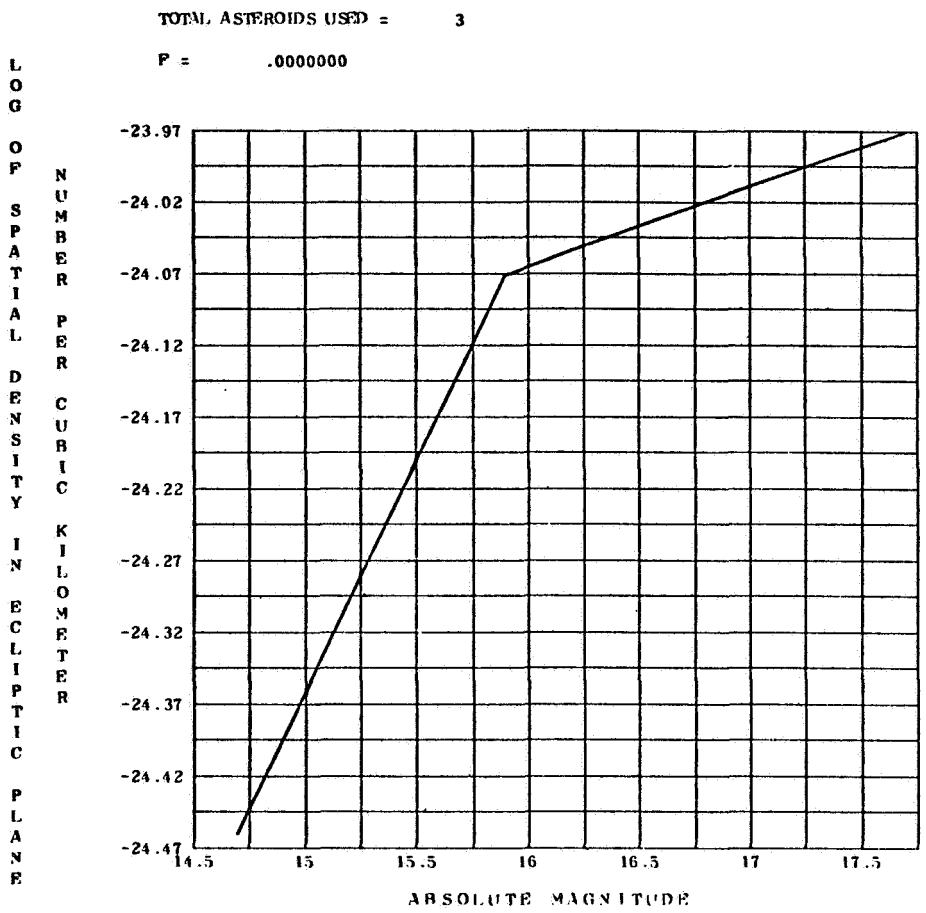


Figure 37. - Spatial density at R = 4.50 averaged over all ecliptic longitudes.



B	F
0 - 2	.000
2 - 4	.469
4 - 6	.000
6 - 8	.324
8 -10	.000
10 -12	.000
12 -14	.000
14 -16	.000
16 -18	.000
18 -20	.000

Figure 38. - Spatial density at R = 1.00 and at longitudes between 315.0 and 360.0.

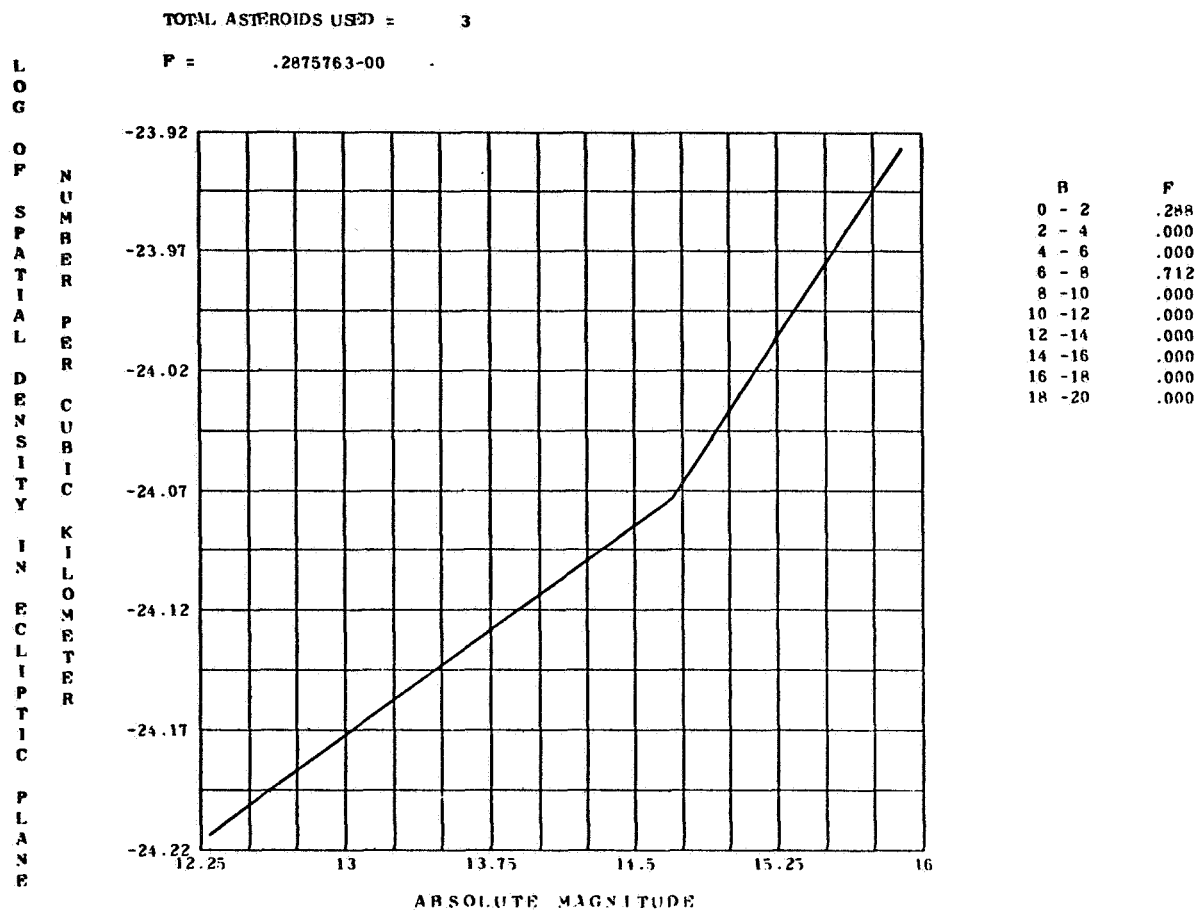


Figure 39. - Spatial density at R = 1.20 and at longitudes between 135.0 and 180.0.

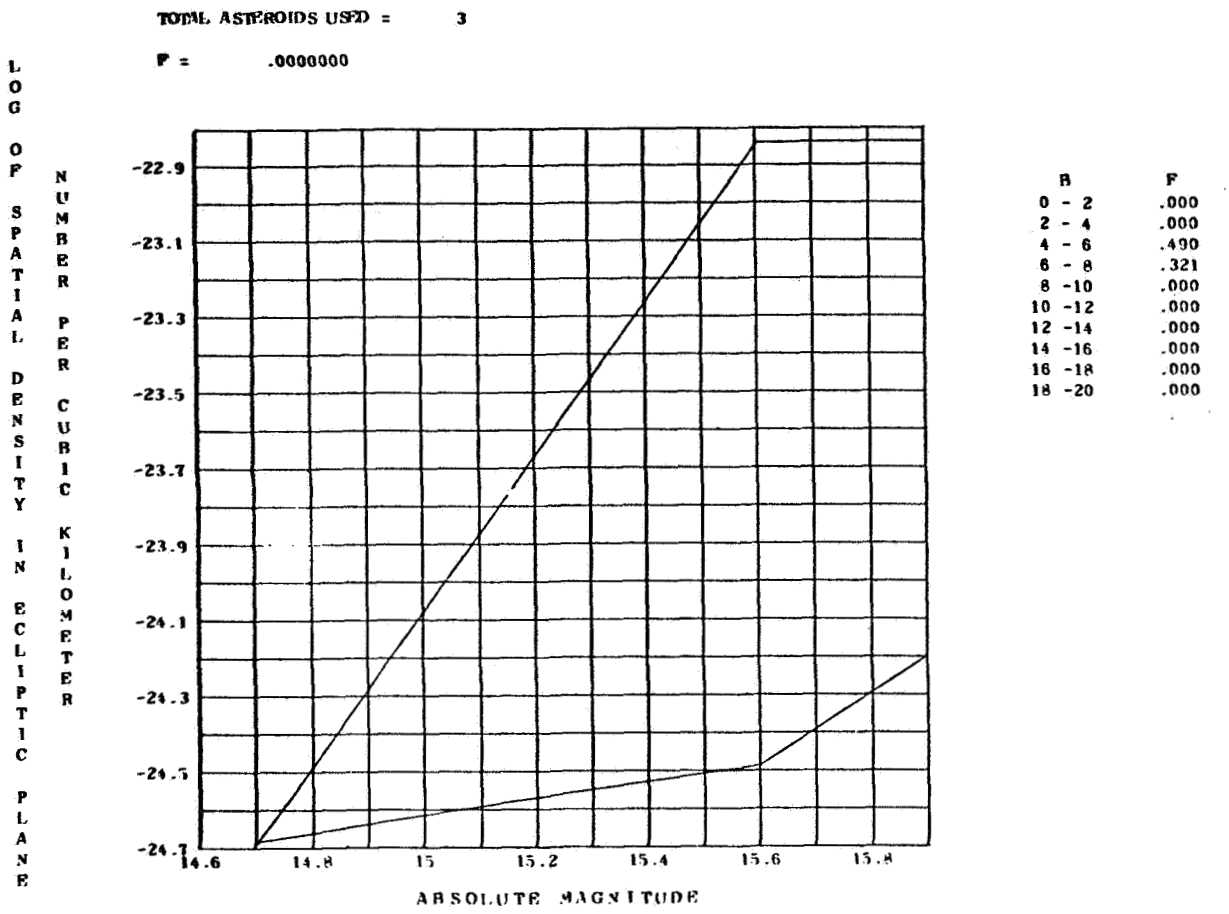


Figure 40. - Spatial density at R = 1.30 and at longitudes between 135.0 and 180.0.

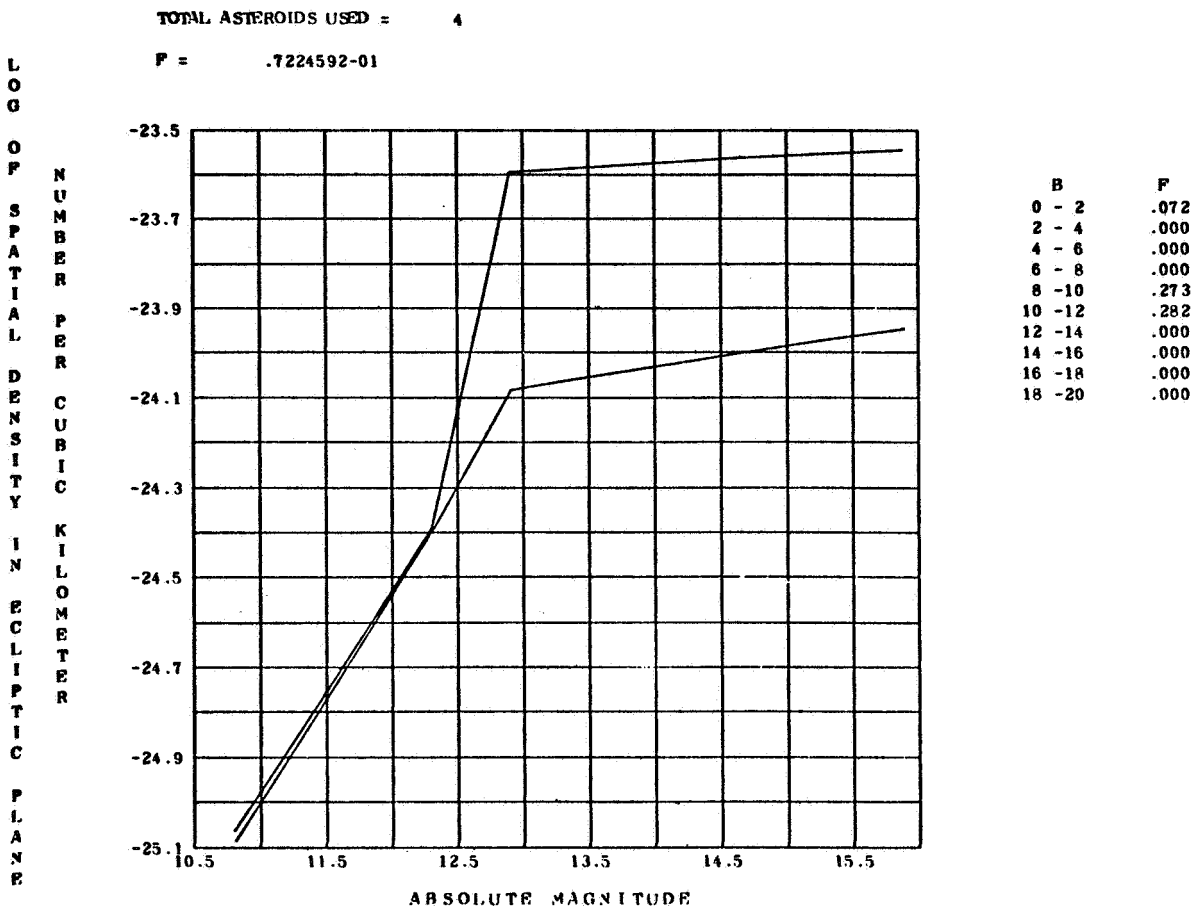


Figure 41. - Spatial density at R = 1.40 and at longitudes between 0 and 45.0.

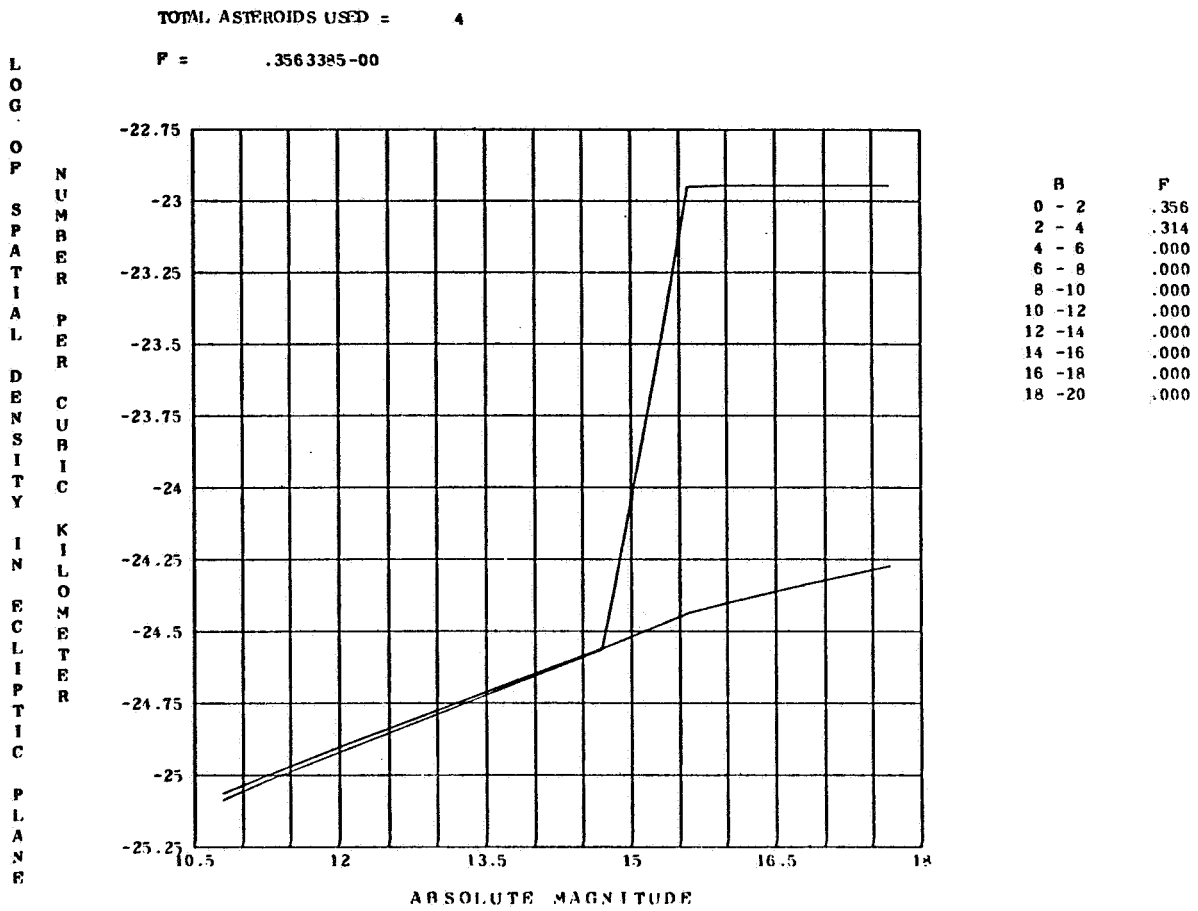


Figure 42. - Spatial density at R = 1.40 and at longitudes between 270.0 and 315.0.

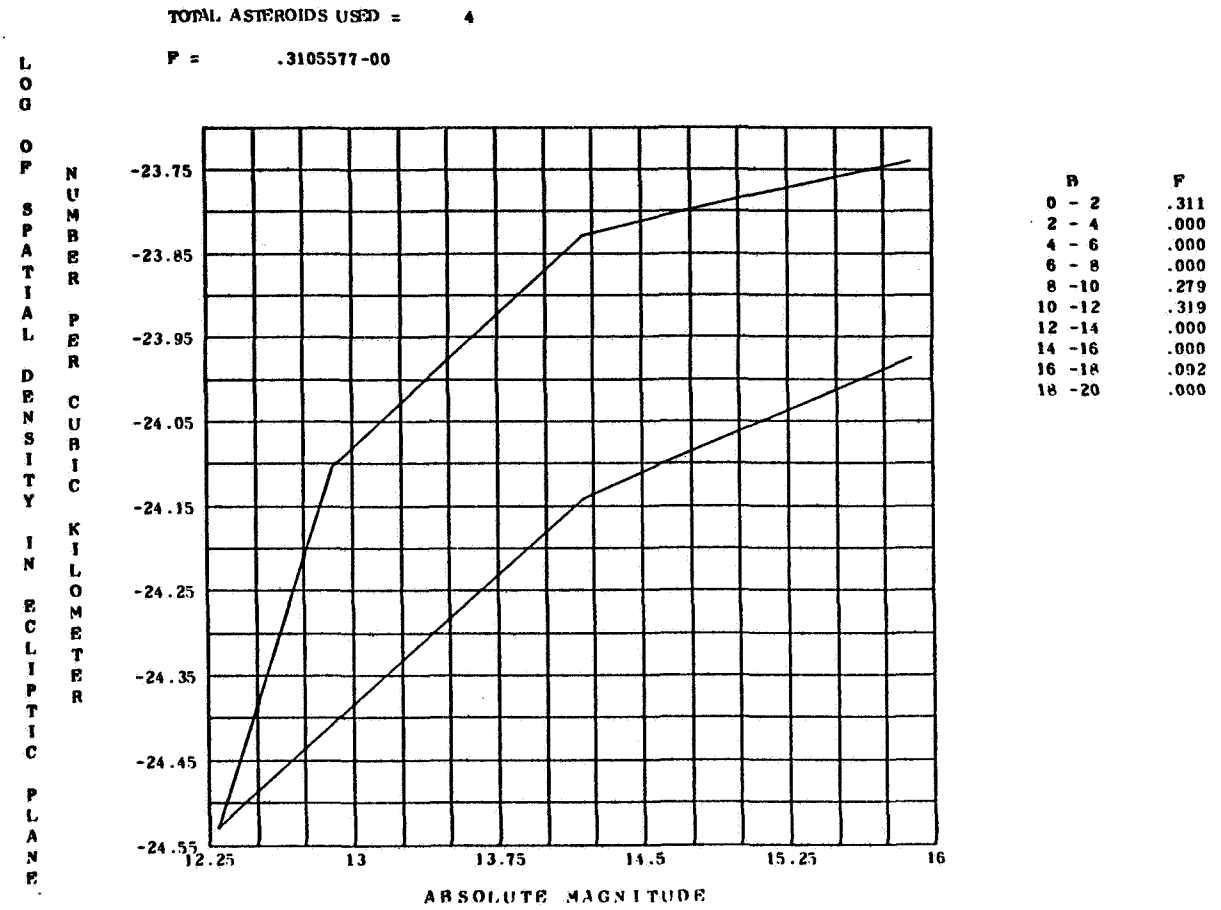


Figure 43. - Spatial density at R = 1.50 and at longitudes between 0 and 45.0.

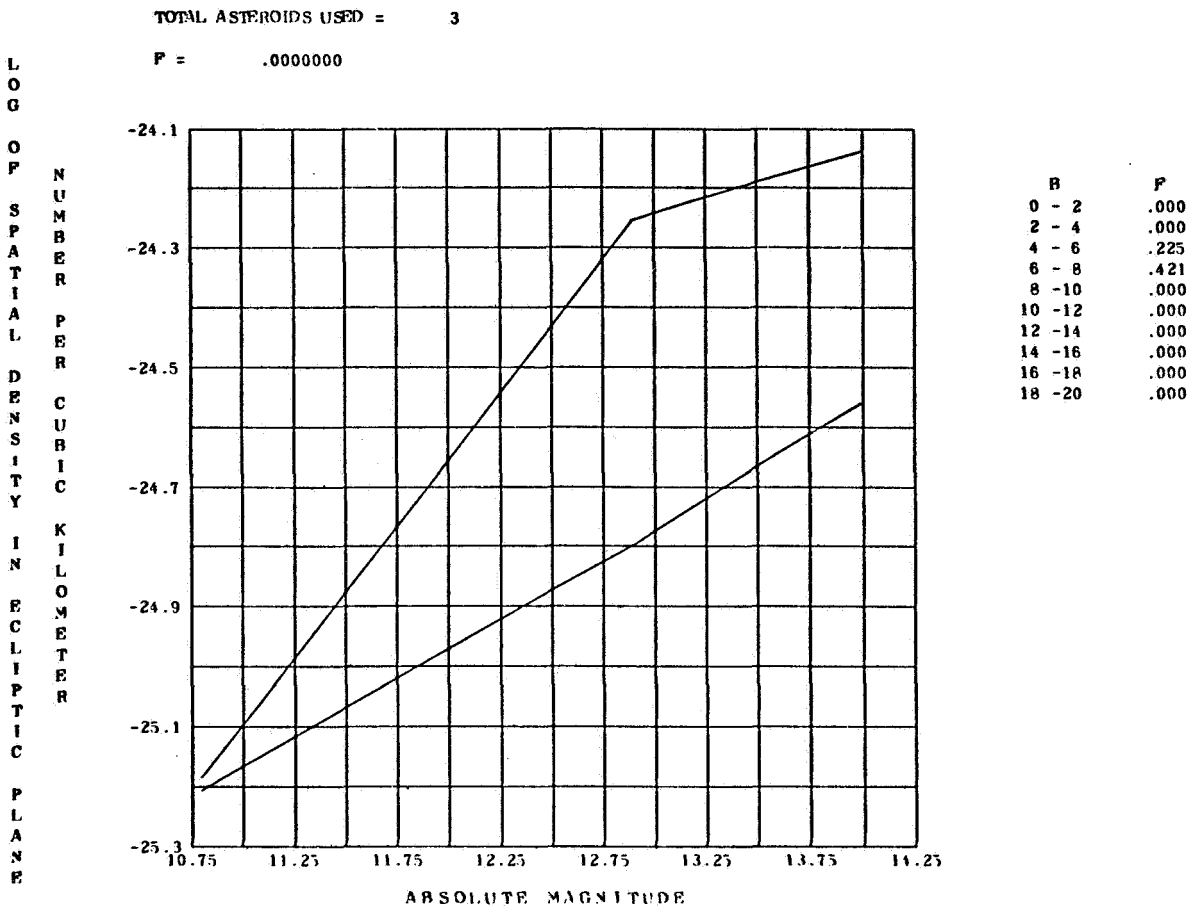


Figure 44. - Spatial density at R = 1.50 and at longitudes between 45.0 and 90.0.

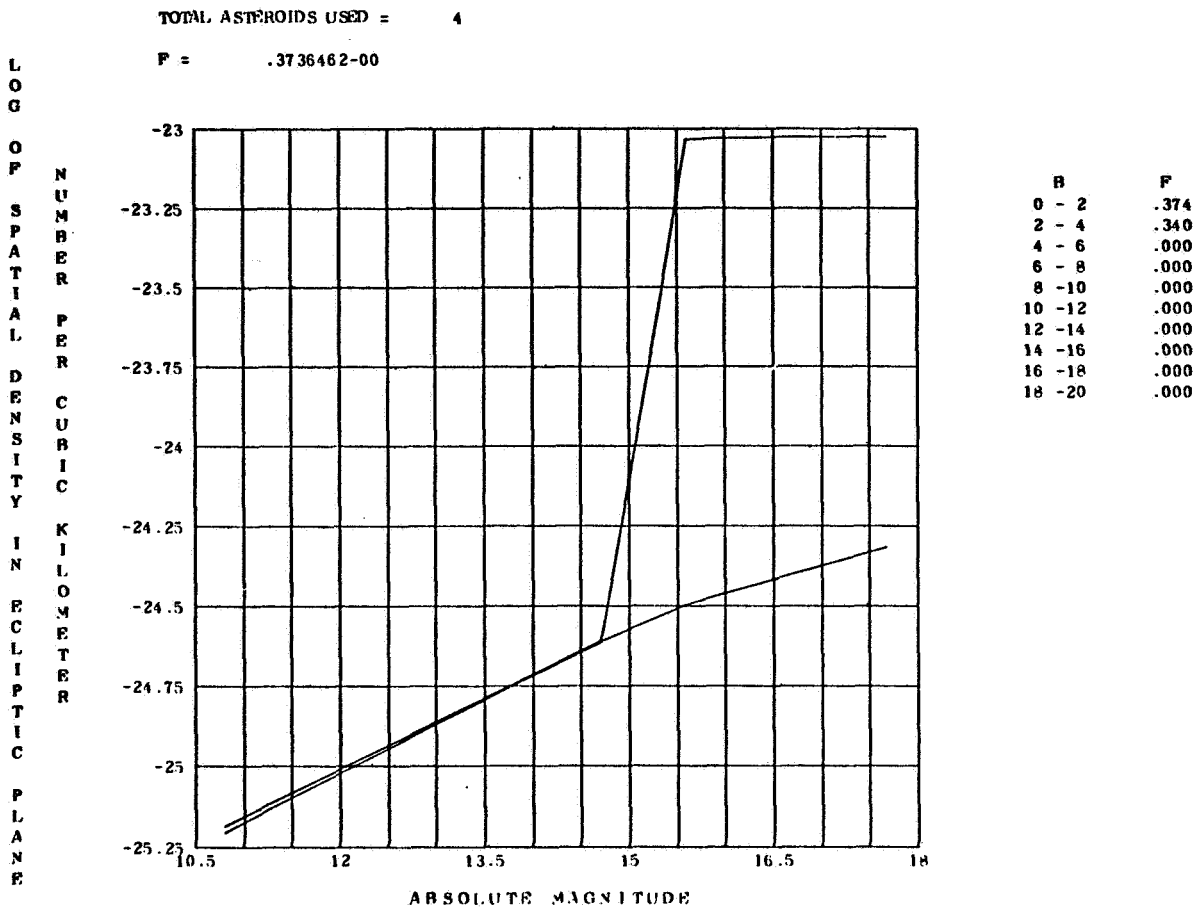


Figure 45.- Spatial density at R = 1.50 and at longitudes between 270.0 and 315.0.

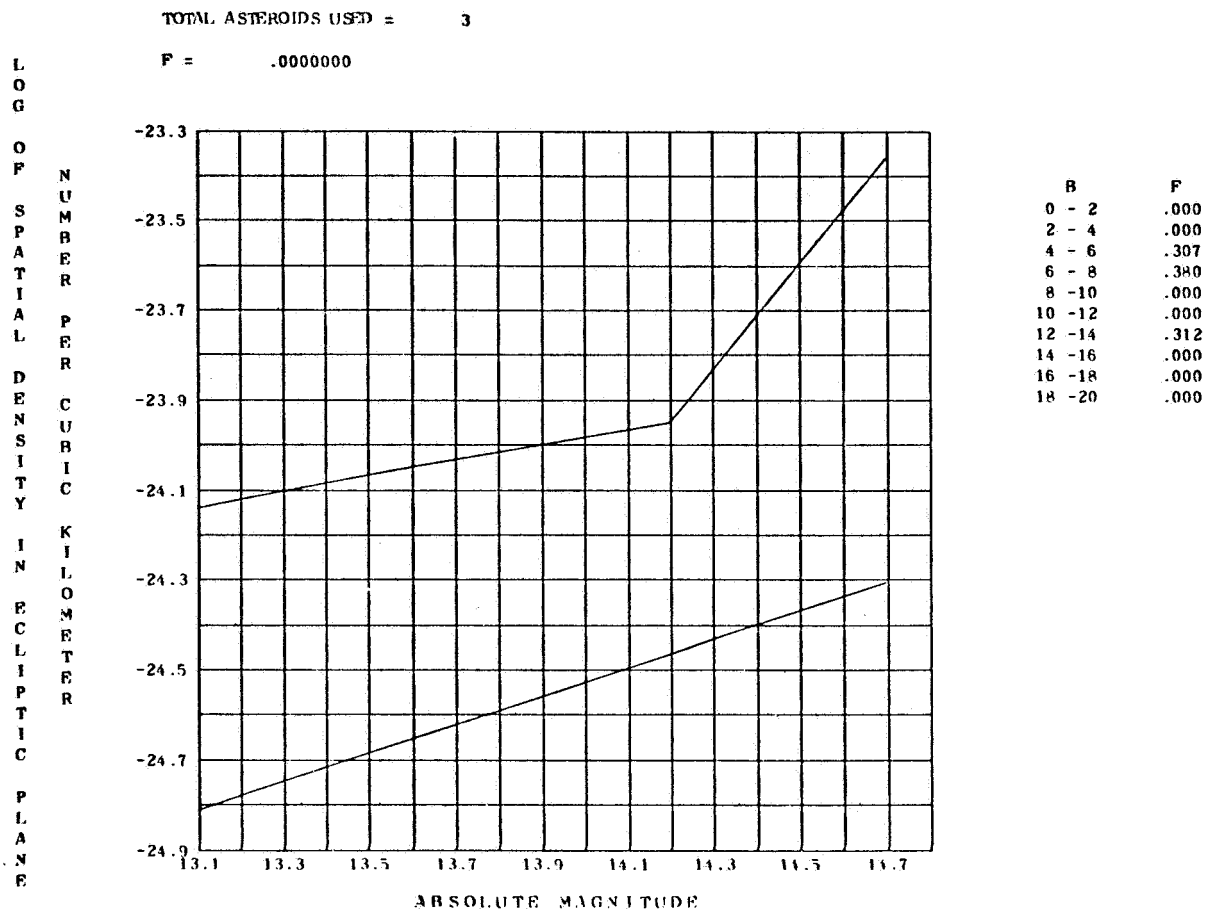


Figure 46. - Spatial density at R = 1.60 and at longitudes between 0 and 45.0.

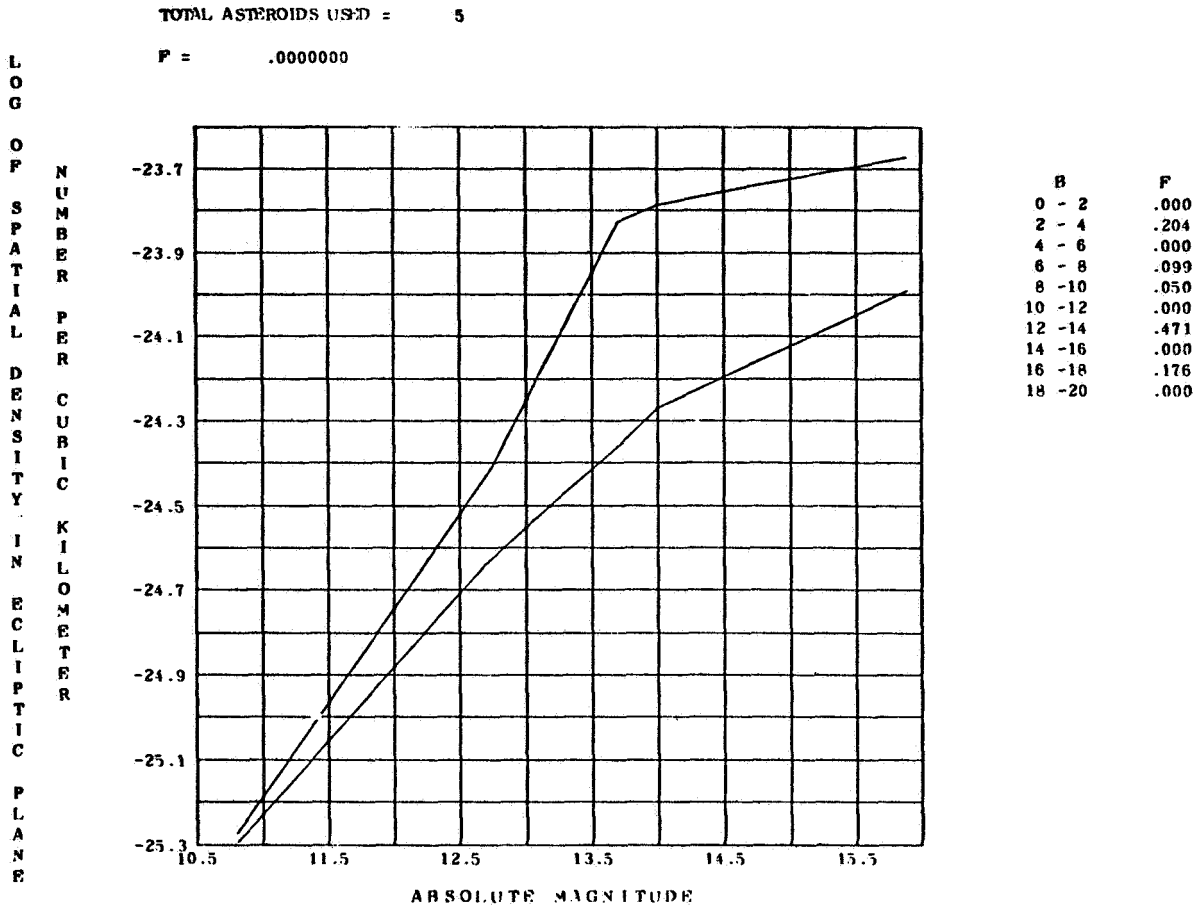


Figure 47. - Spatial density at R = 1.60 and at longitudes between 45.0 and 90.0.

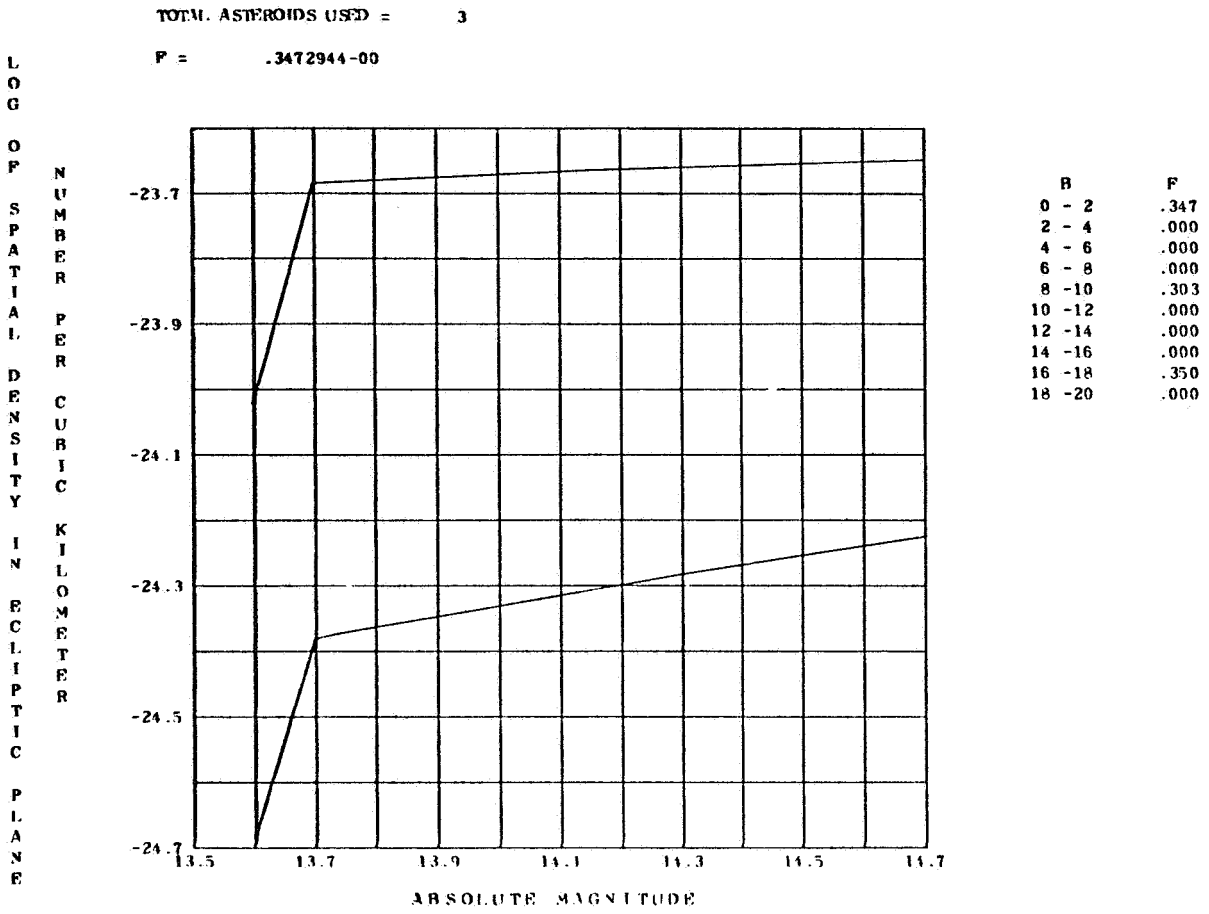


Figure 48. - Spatial density at R = 1.60 and at longitudes between 90.0 and 135.0.

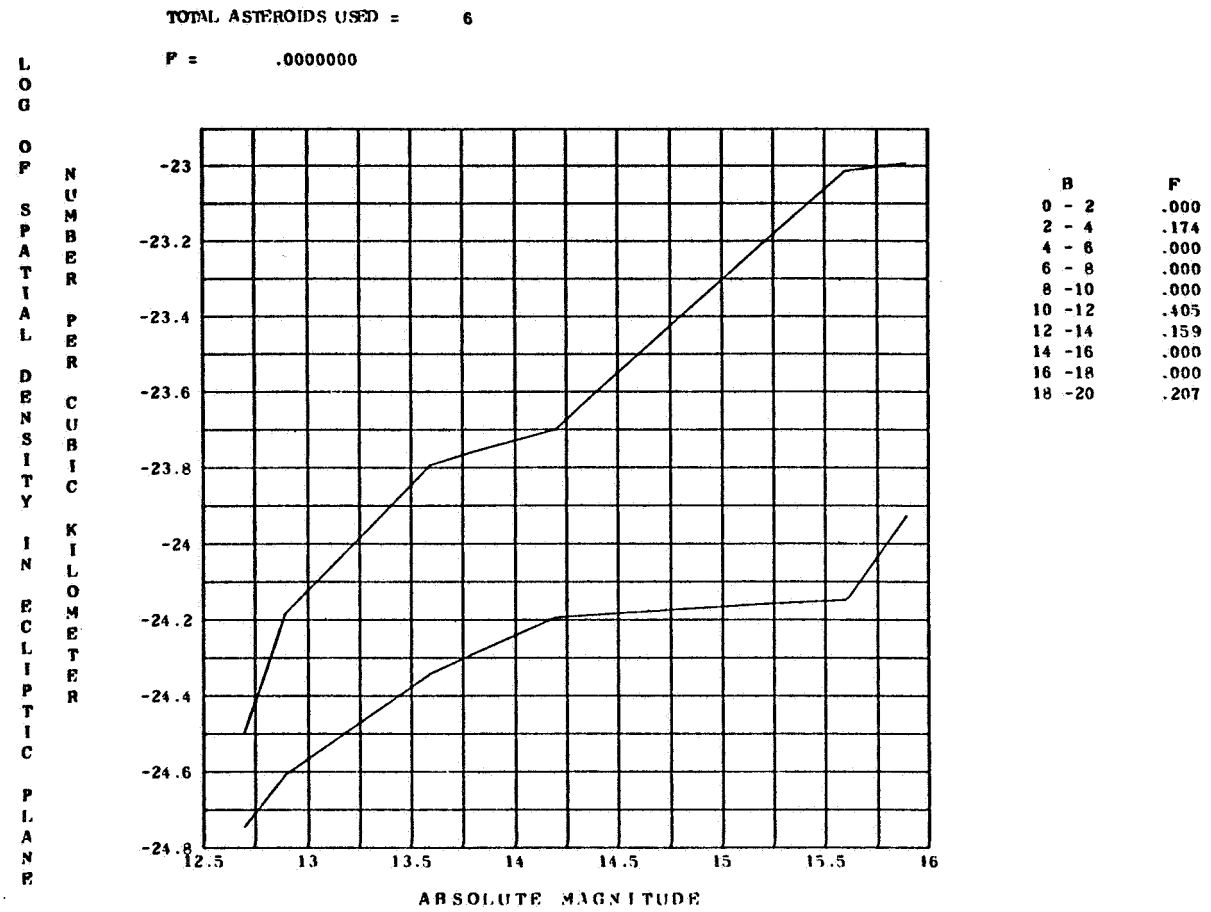


Figure 49. - Spatial density at R = 1.60 and at longitudes between 135.0 and 180.0.

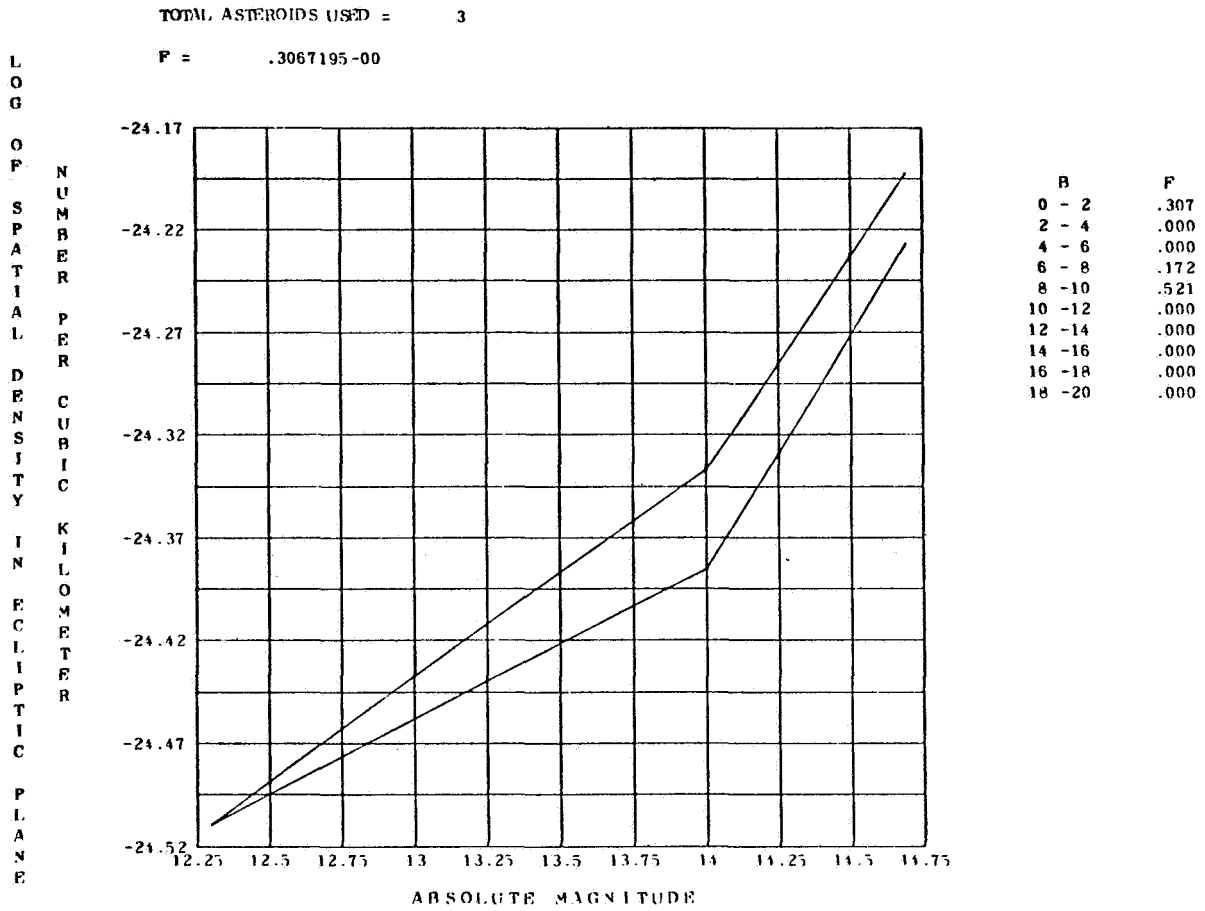


Figure 50.- Spatial density at R = 1.60 and at longitudes between 225.0 and 270.0.

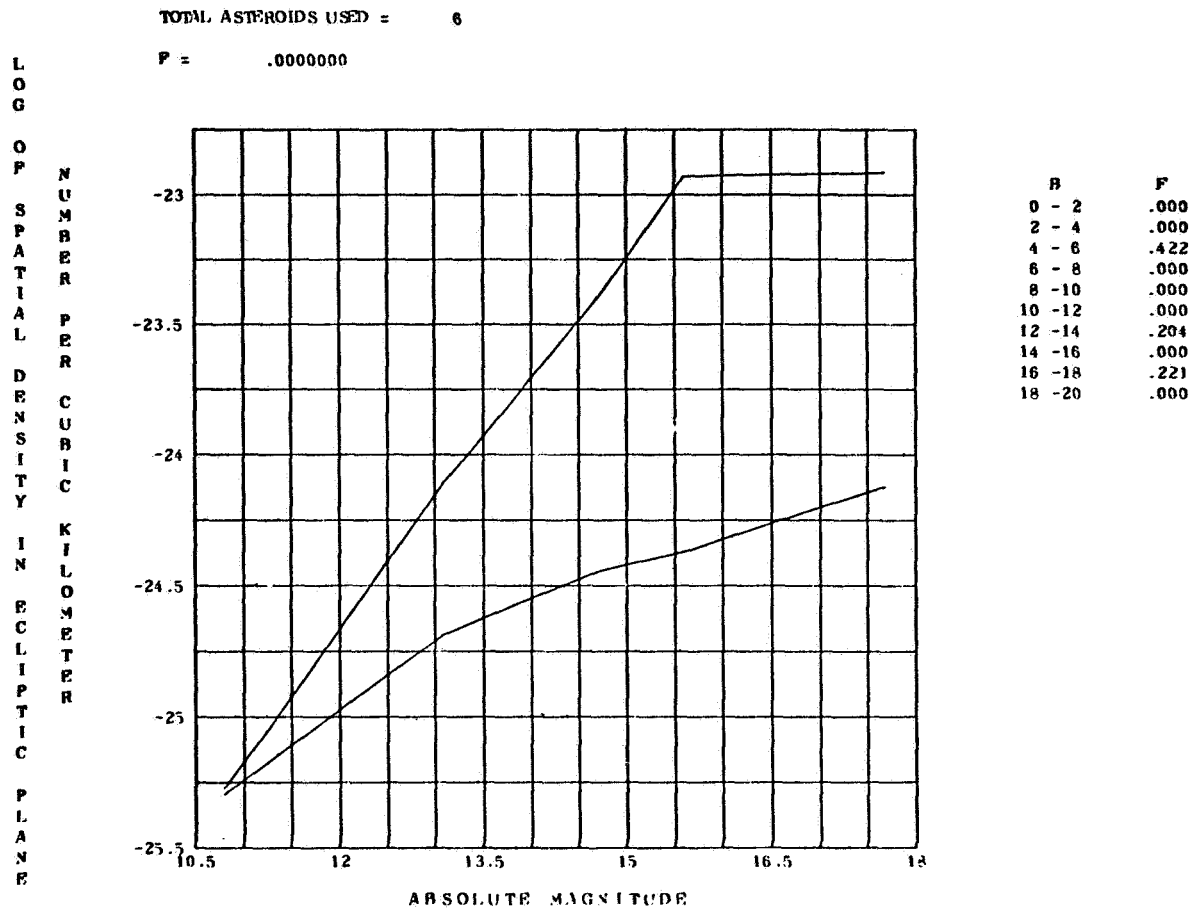


Figure 51. - Spatial density at R = 1.60 and at longitudes between 270.0 and 315.0.

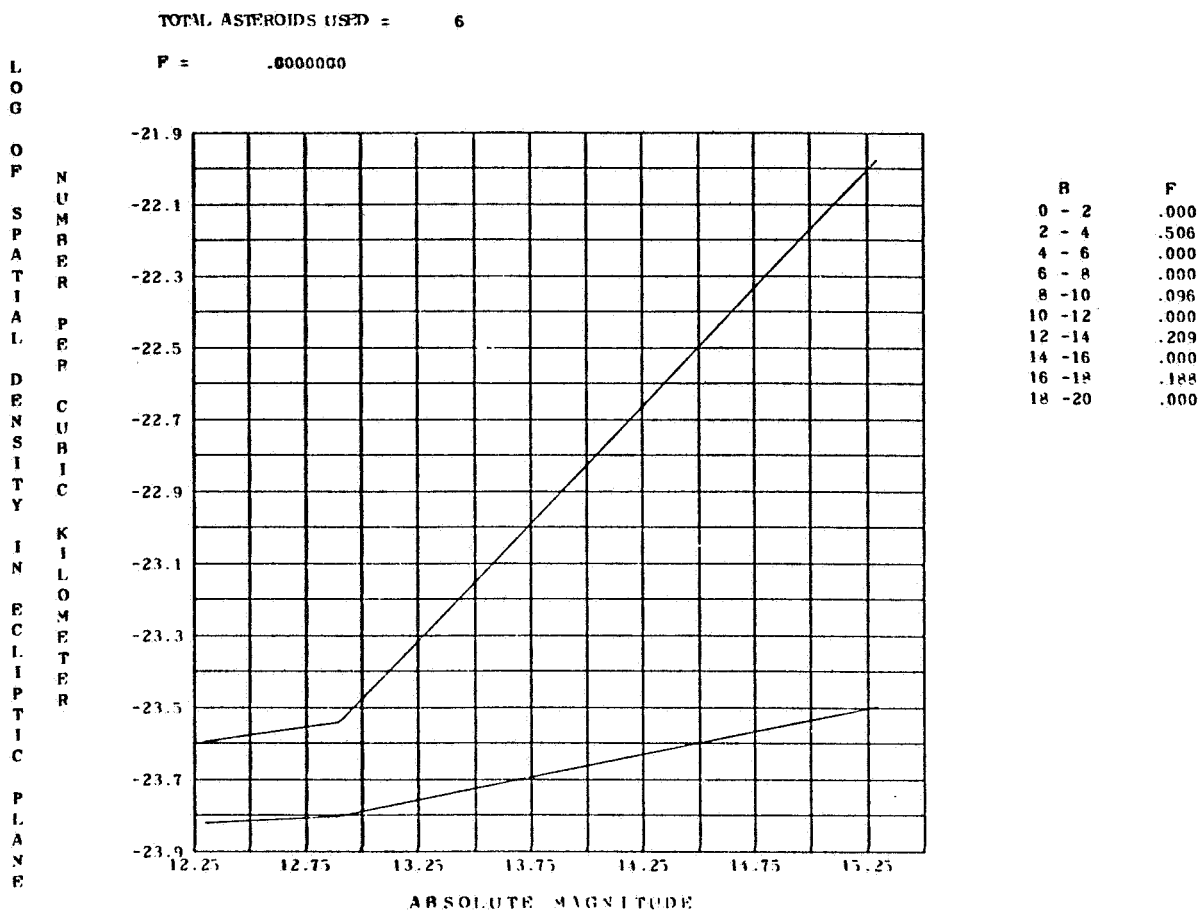


Figure 52. - Spatial density at R = 1.60 and at longitudes between 315.0 and 360.0.

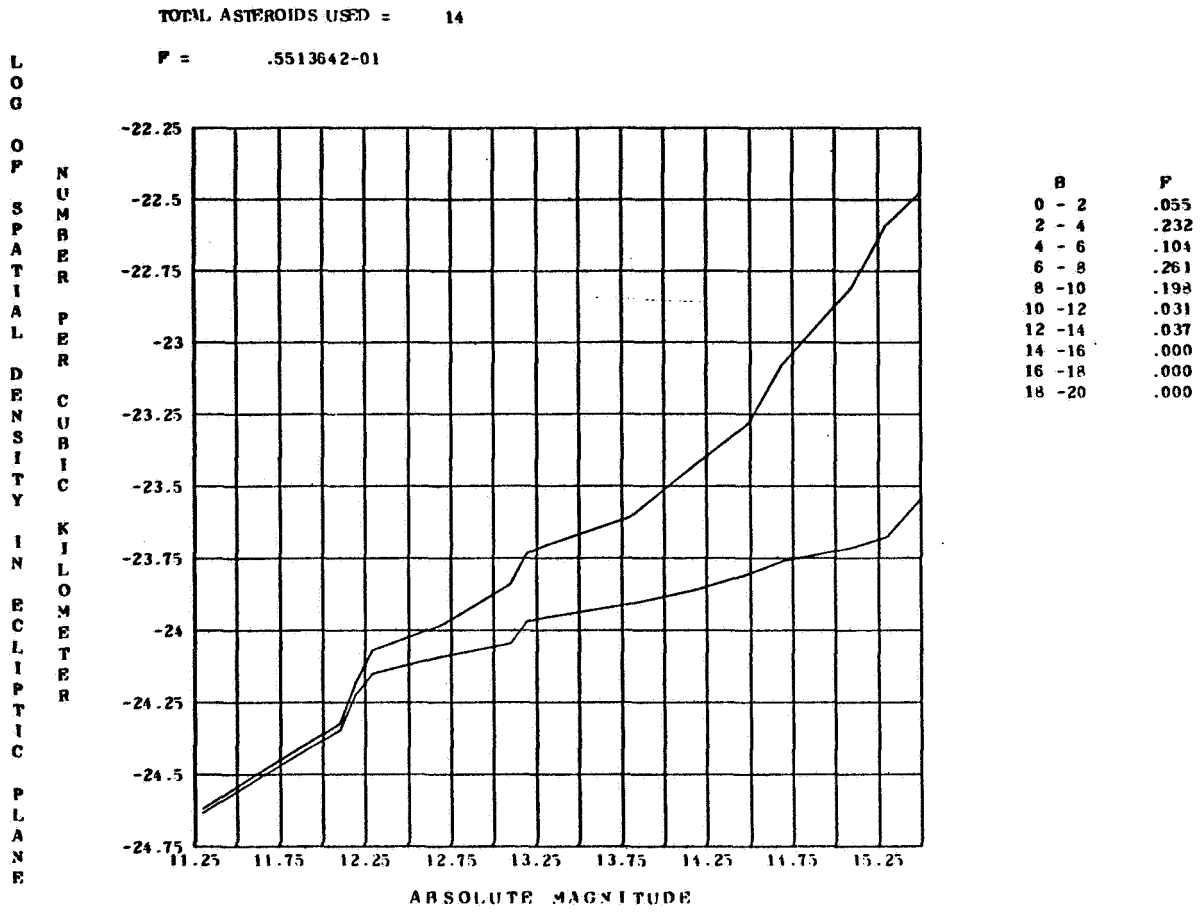


Figure 53. - Spatial density at R = 1.70 and at longitudes between 0 and 45.0.

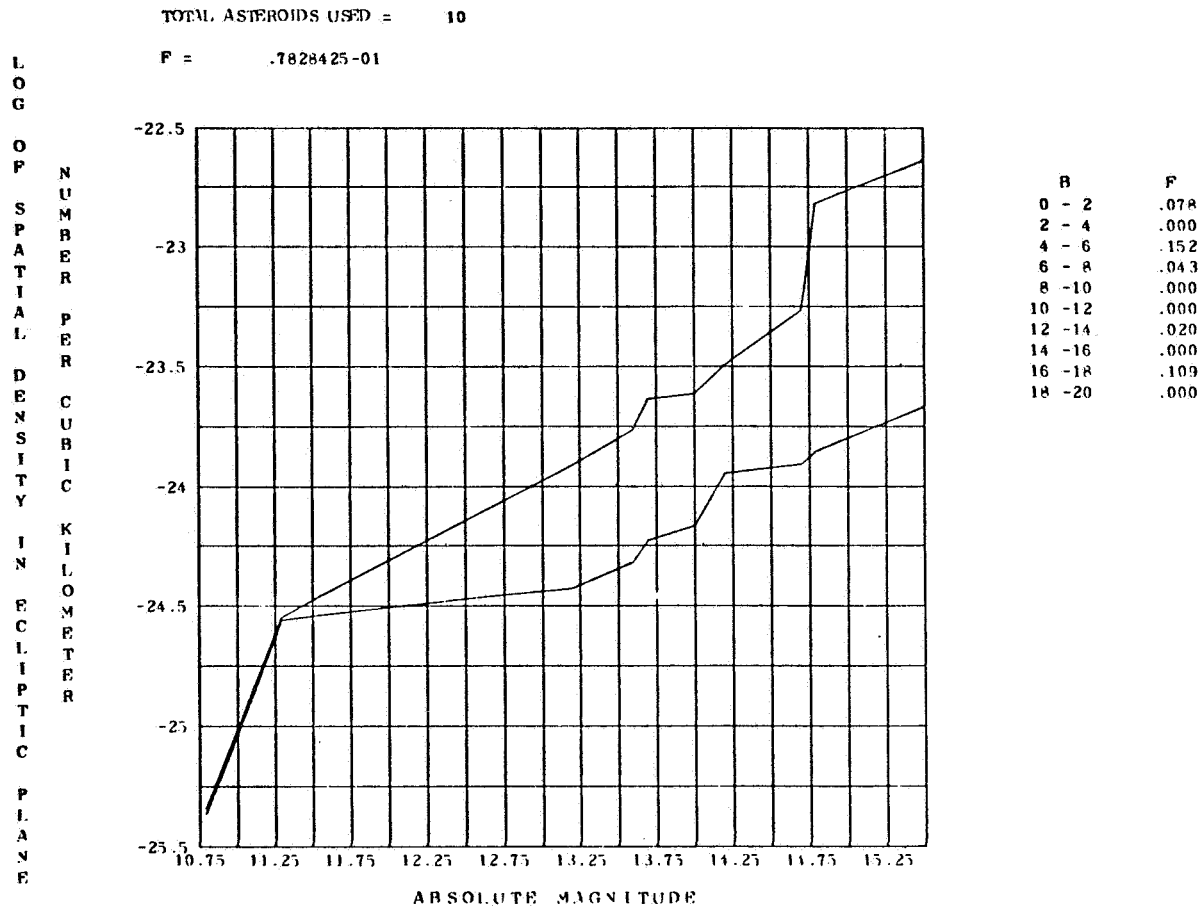


Figure 54. - Spatial density at R = 1.70 and at longitudes between 45.0 and 90.0.

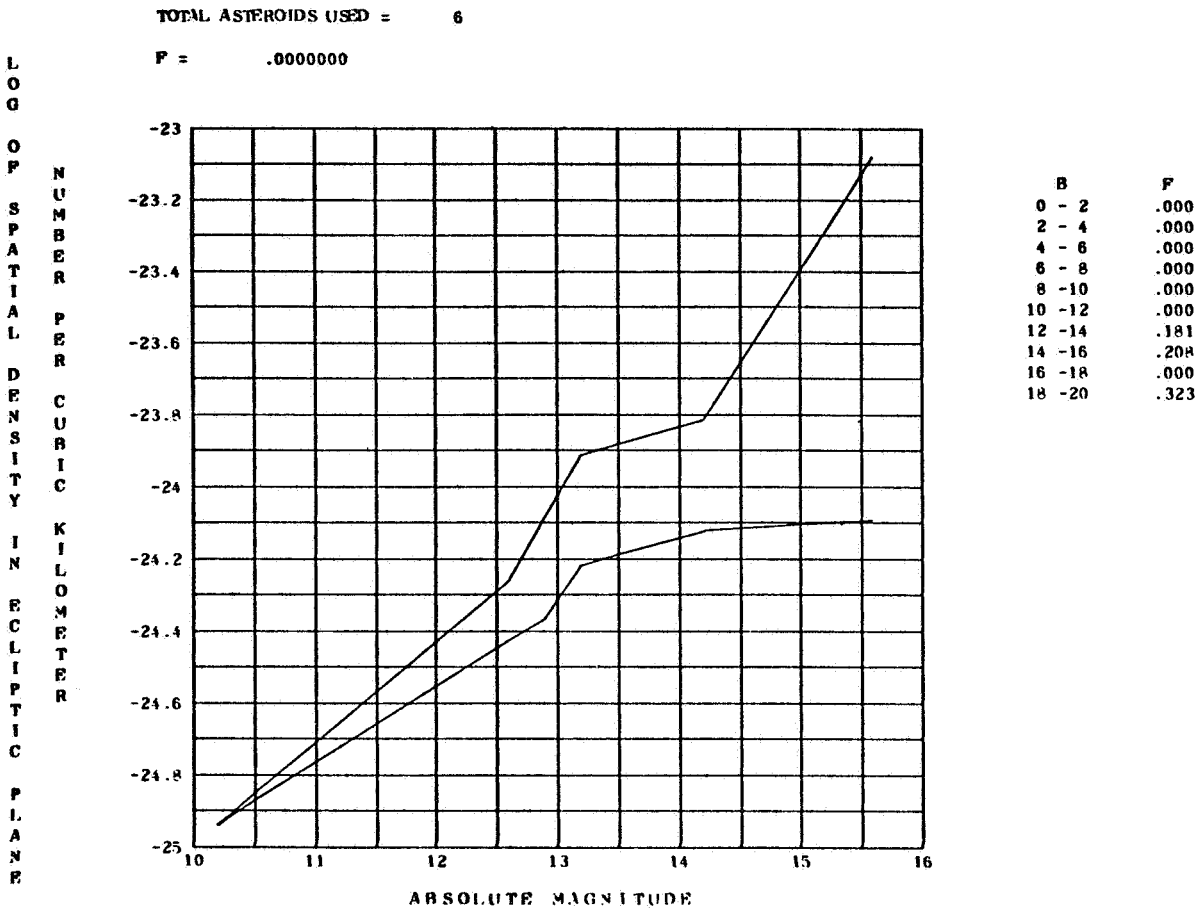


Figure 55. - Spatial density at R = 1.70 and at longitudes between 90.0 and 135.0.

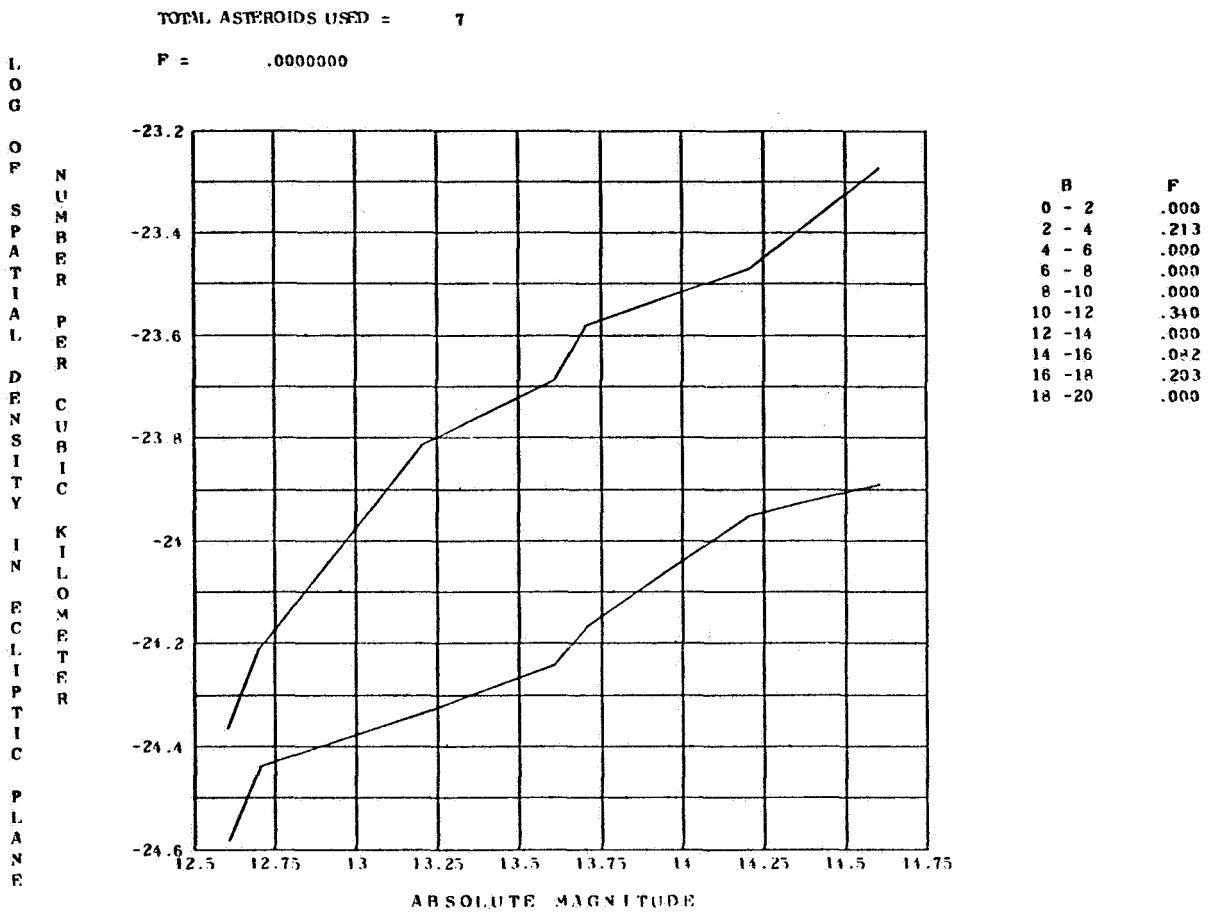


Figure 56. - Spatial density at R = 1.70 and at longitudes between 135.0 and 180.0.

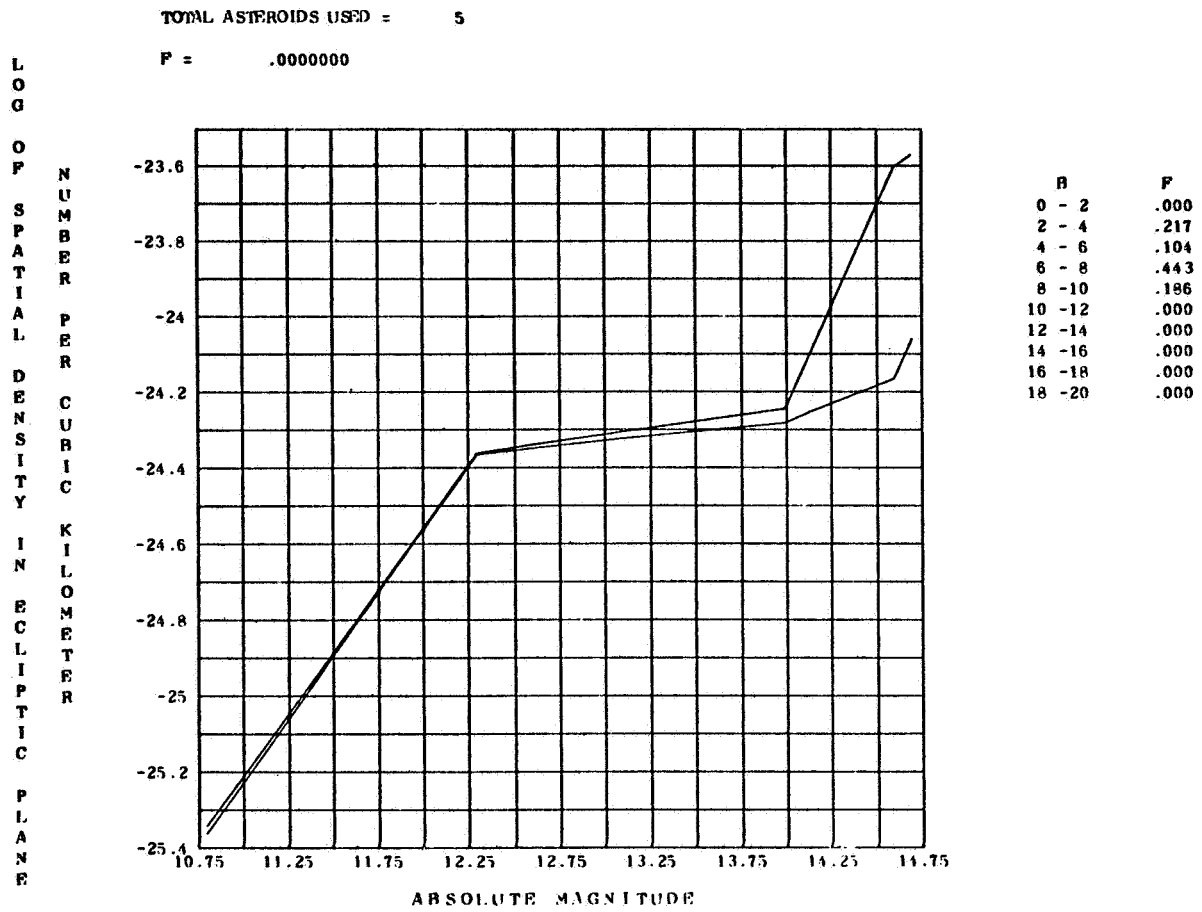


Figure 57. - Spatial density at R = 1.70 and at longitudes between 225.0 and 270.0.

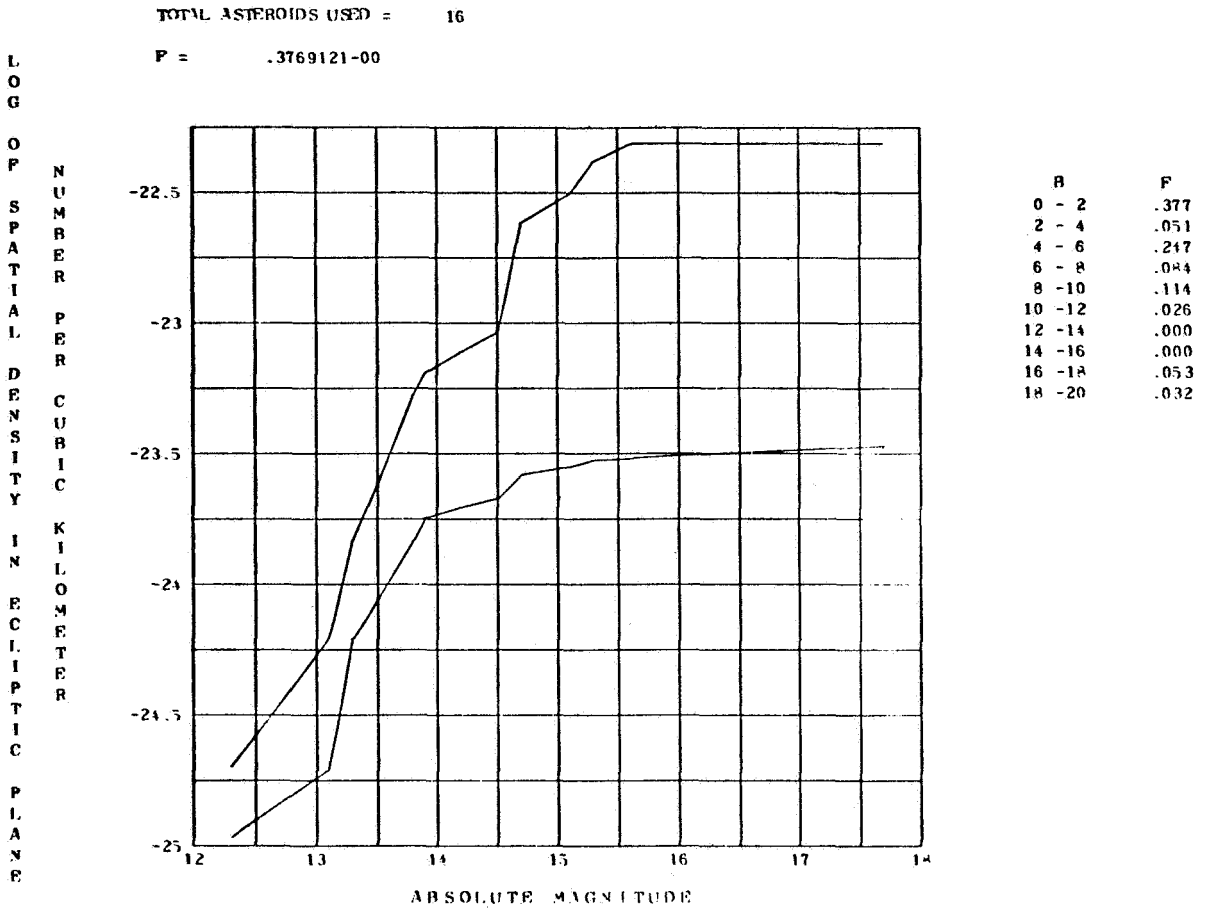


Figure 58. - Spatial density at R = 1.70 and at longitudes between 270.0 and 315.0.

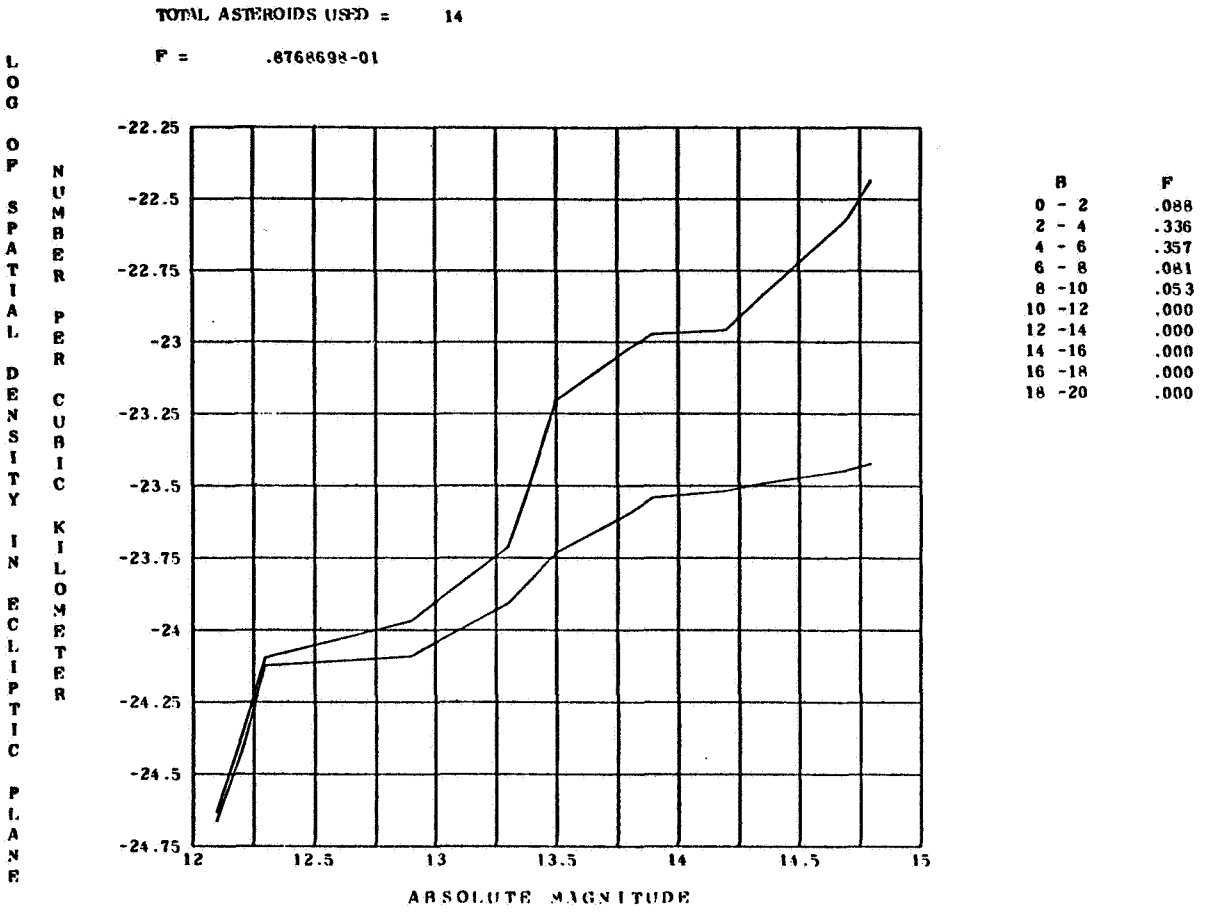


Figure 59.- Spatial density at R = 1.70 and at longitudes between 315.0 and 360.0.

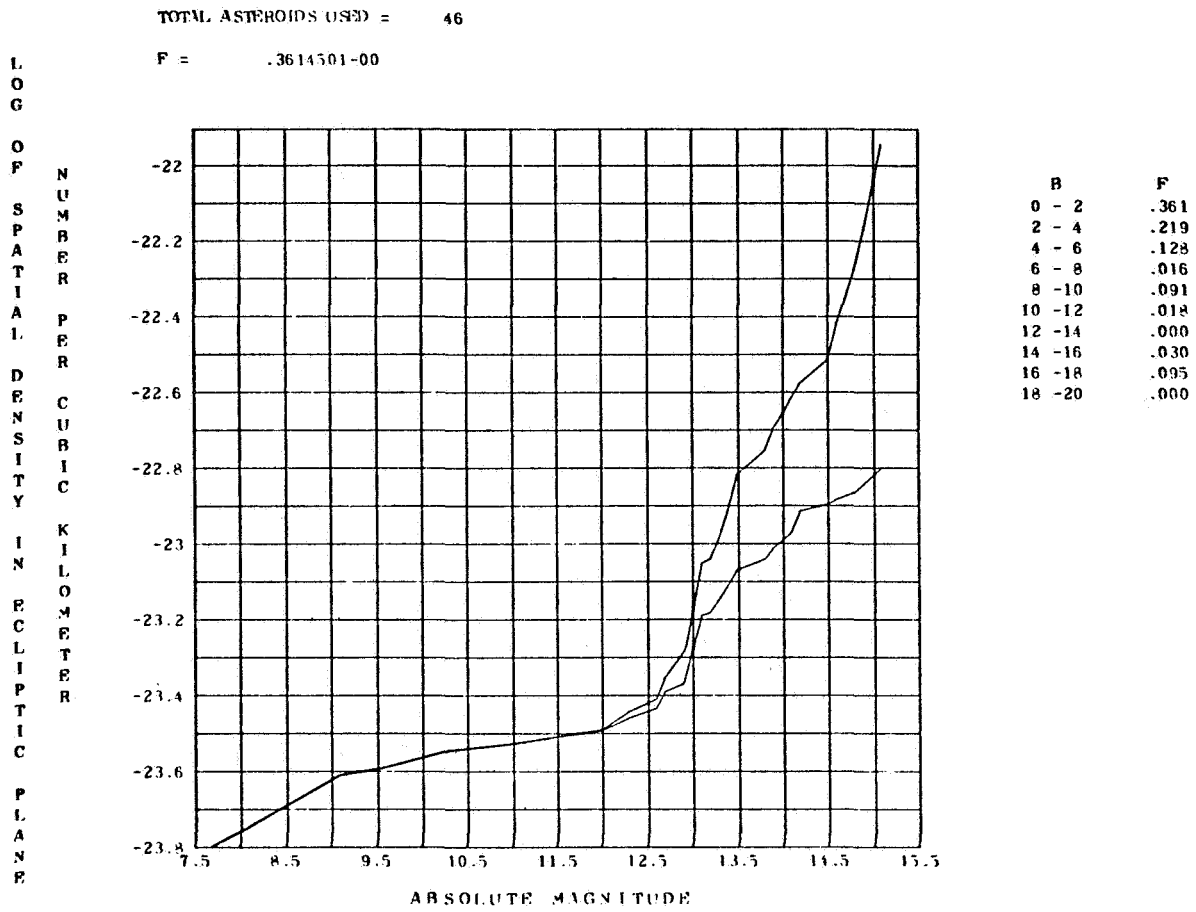


Figure 60. - Spatial density at R = 1.80 and at longitudes between 0 and 45.0.

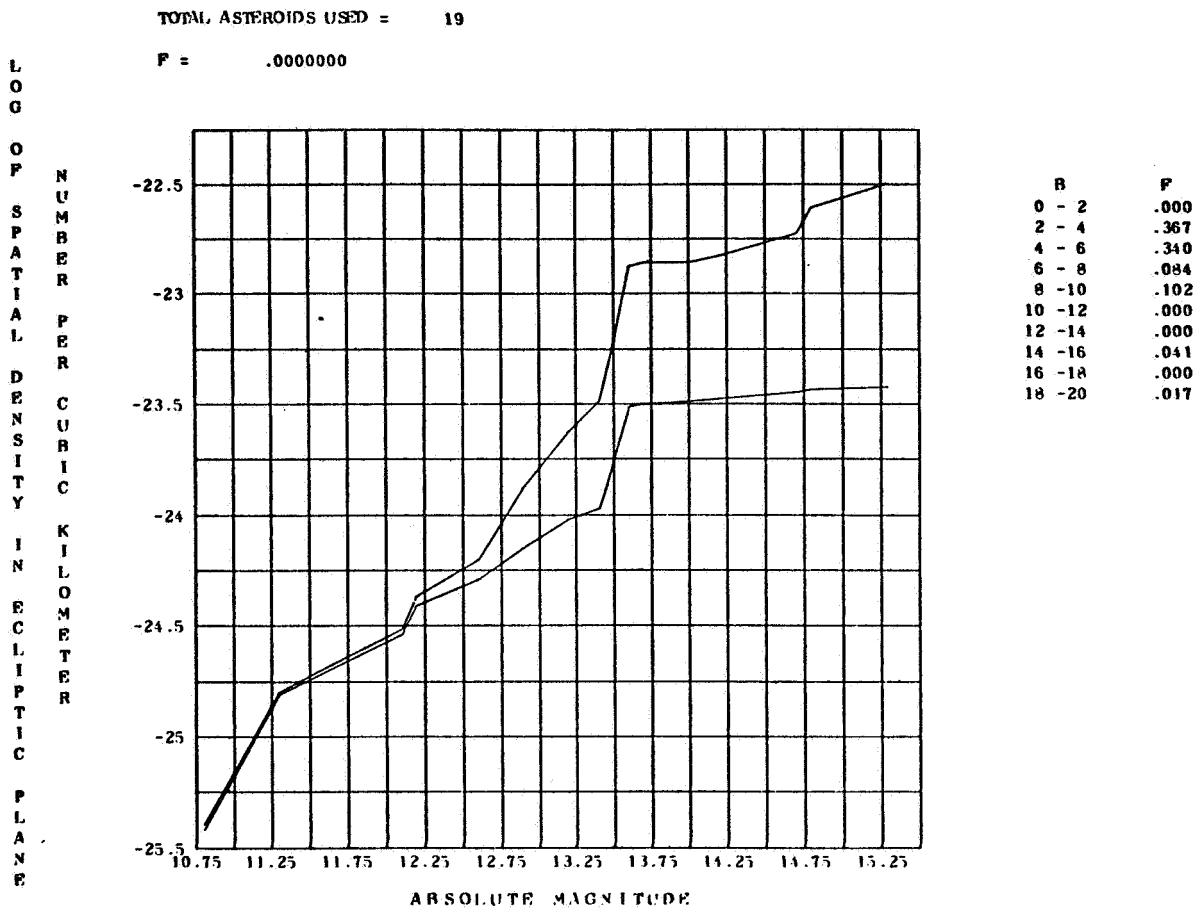


Figure 61. - Spatial density at R = 1.80 and at longitudes between 45.0 and 90.0.

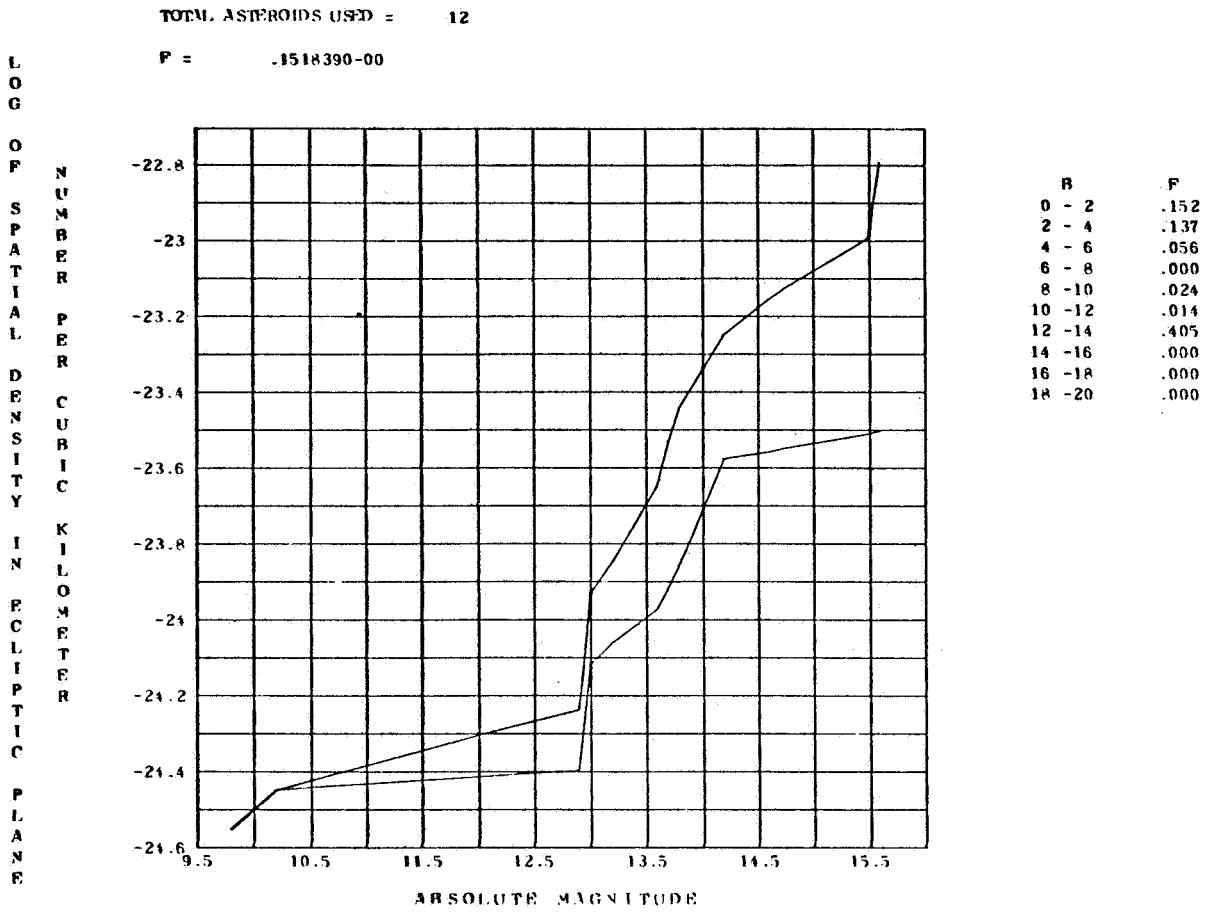


Figure 62. - Spatial density at R = 1.80 and at longitudes between 90.0 and 135.0.

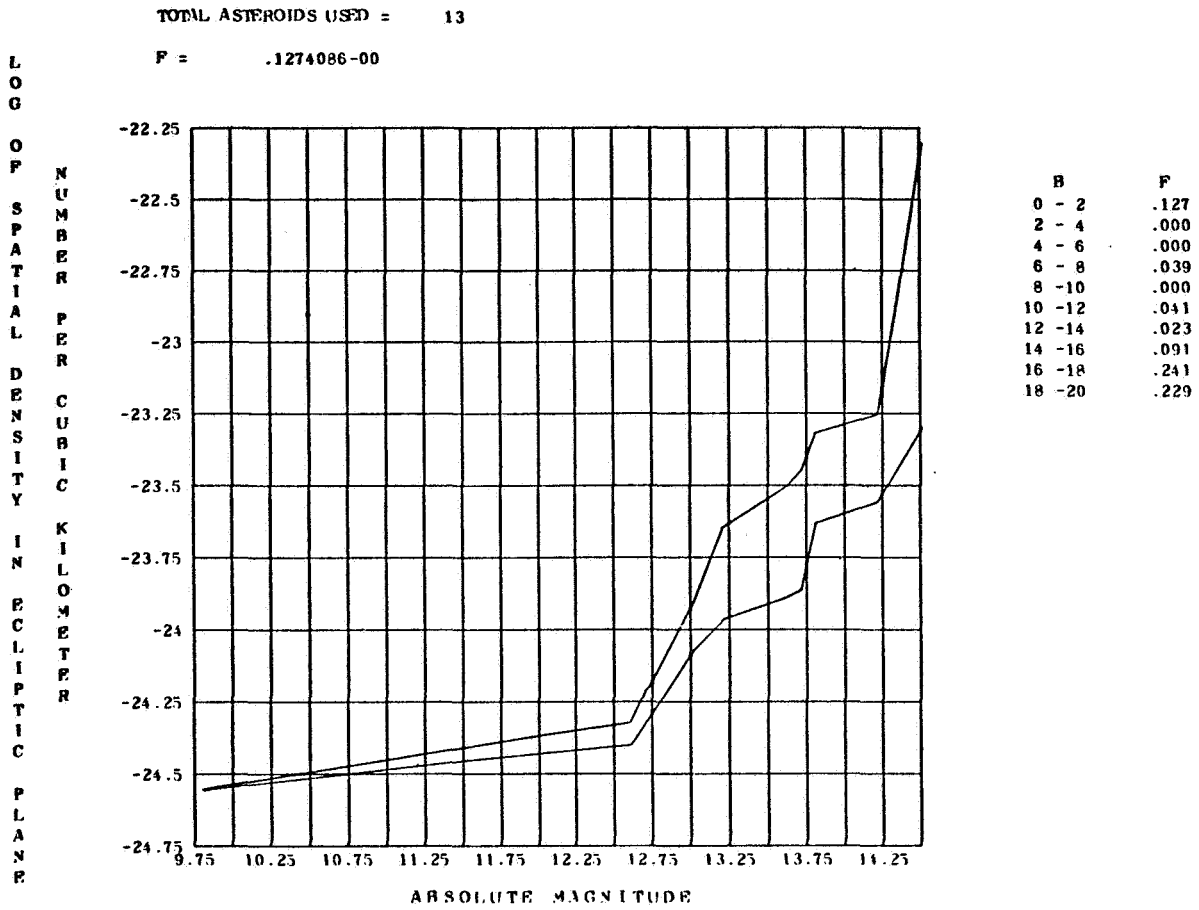


Figure 63.- Spatial density at R = 1.80 and at longitudes between 135.0 and 180.0.

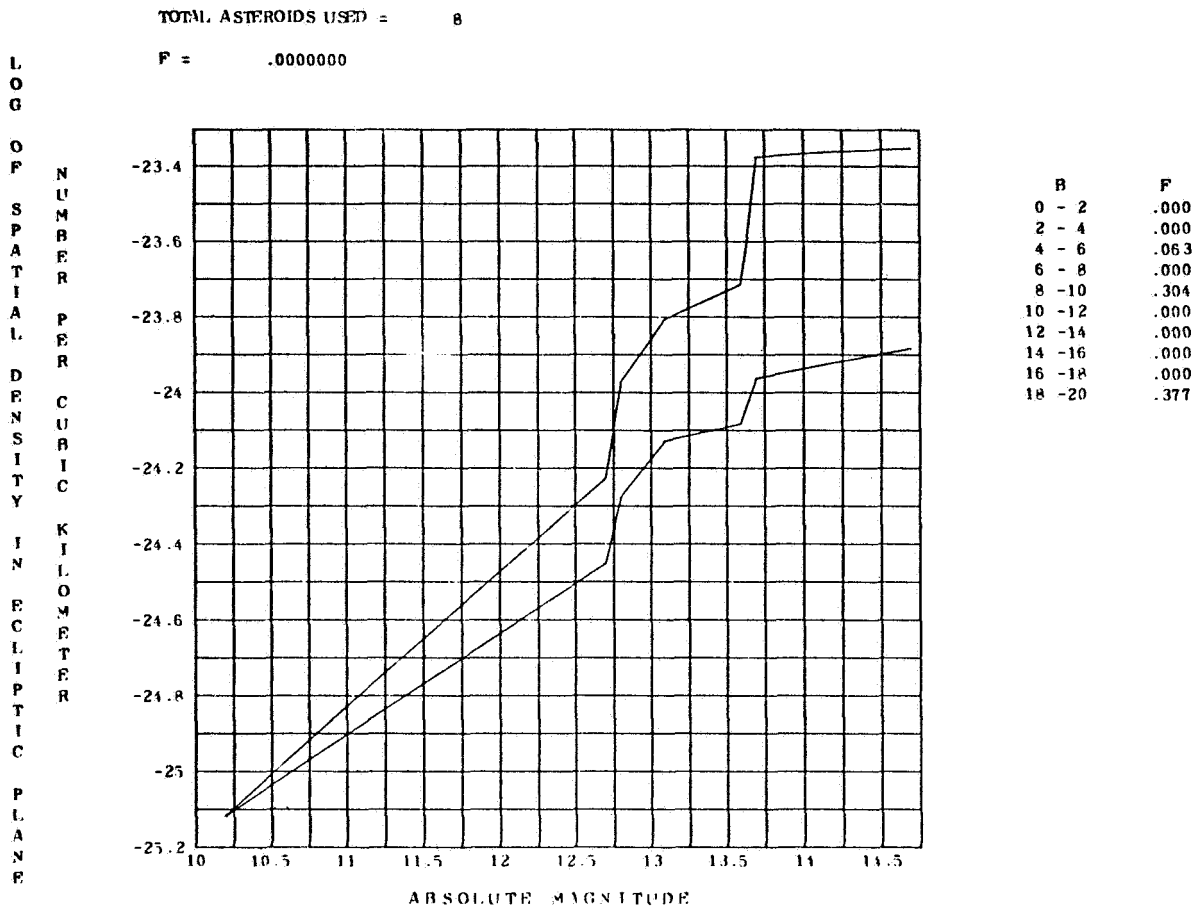


Figure 64. - Spatial density at R = 1.80 and at longitudes between 180.0 and 225.0.

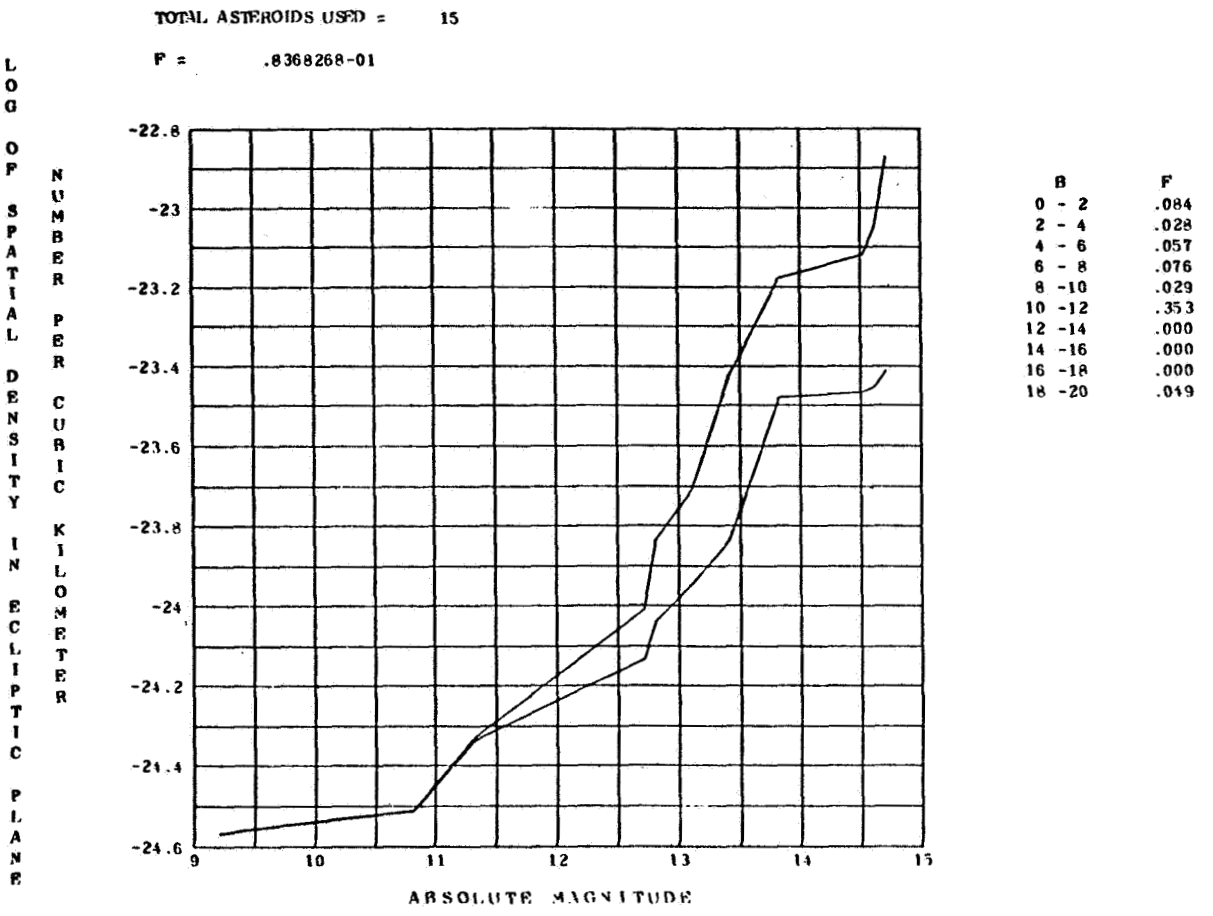


Figure 65.- Spatial density at R = 1.80 and at longitudes between 225.0 and 270.0.

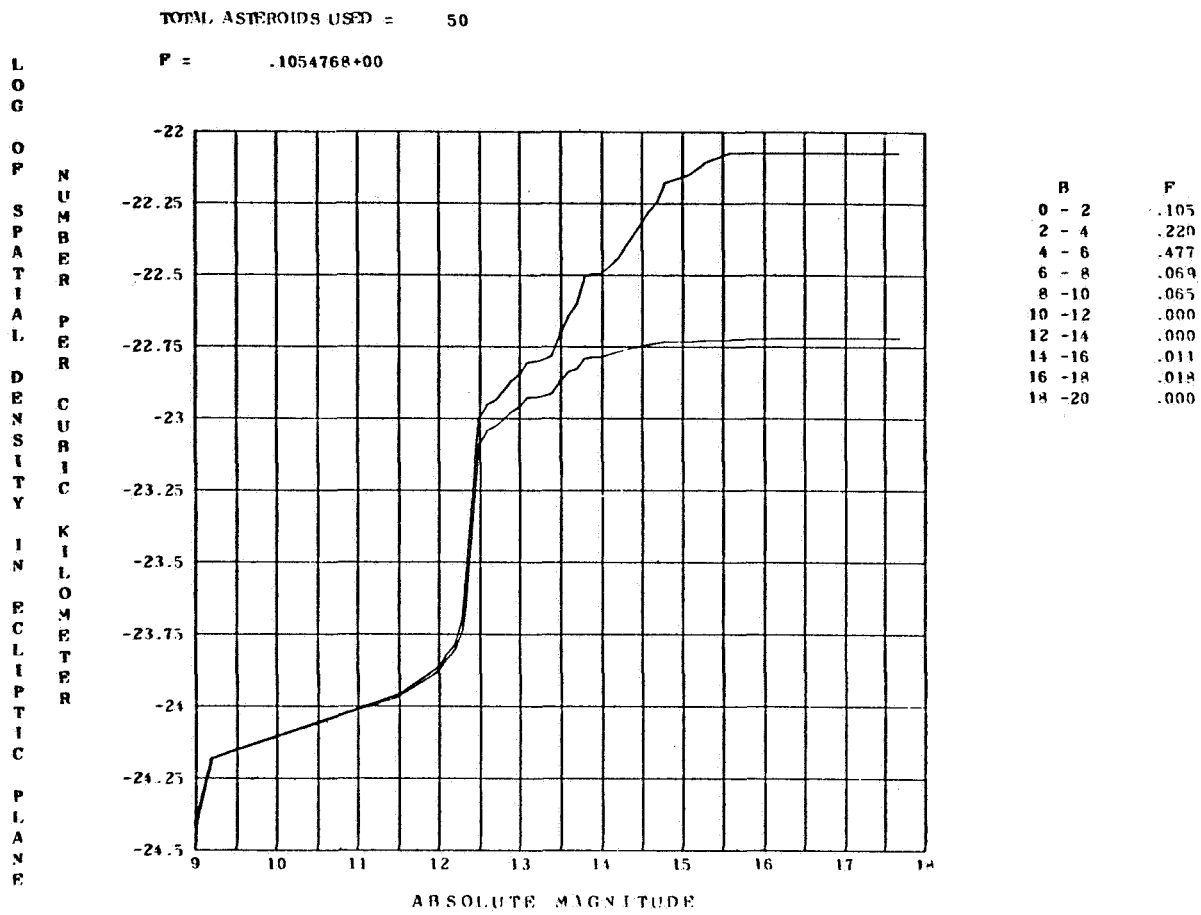


Figure 66. - Spatial density at R = 1.80 and at longitudes between 270.0 and 315.0.

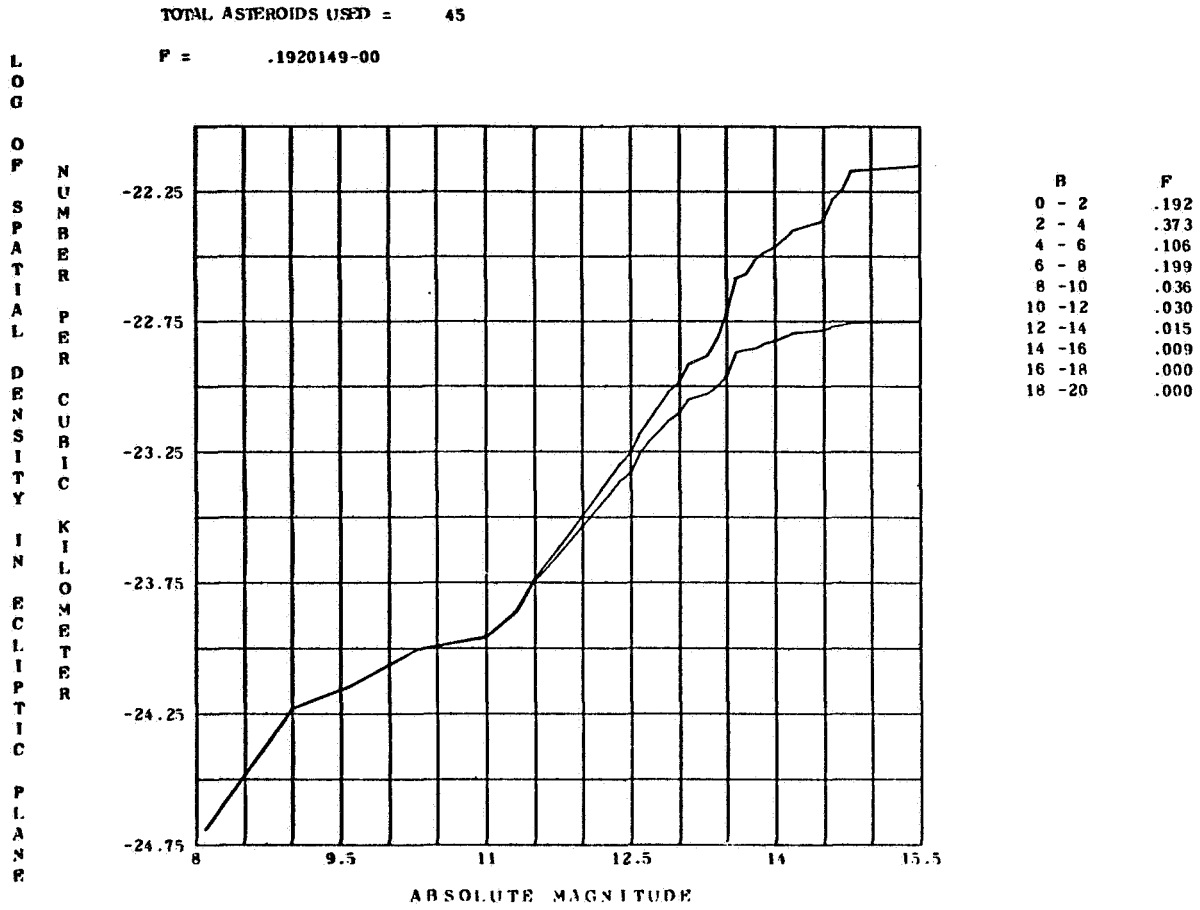


Figure 67.- Spatial density at R = 1.80 and at longitudes between 315.0 and 360.0.

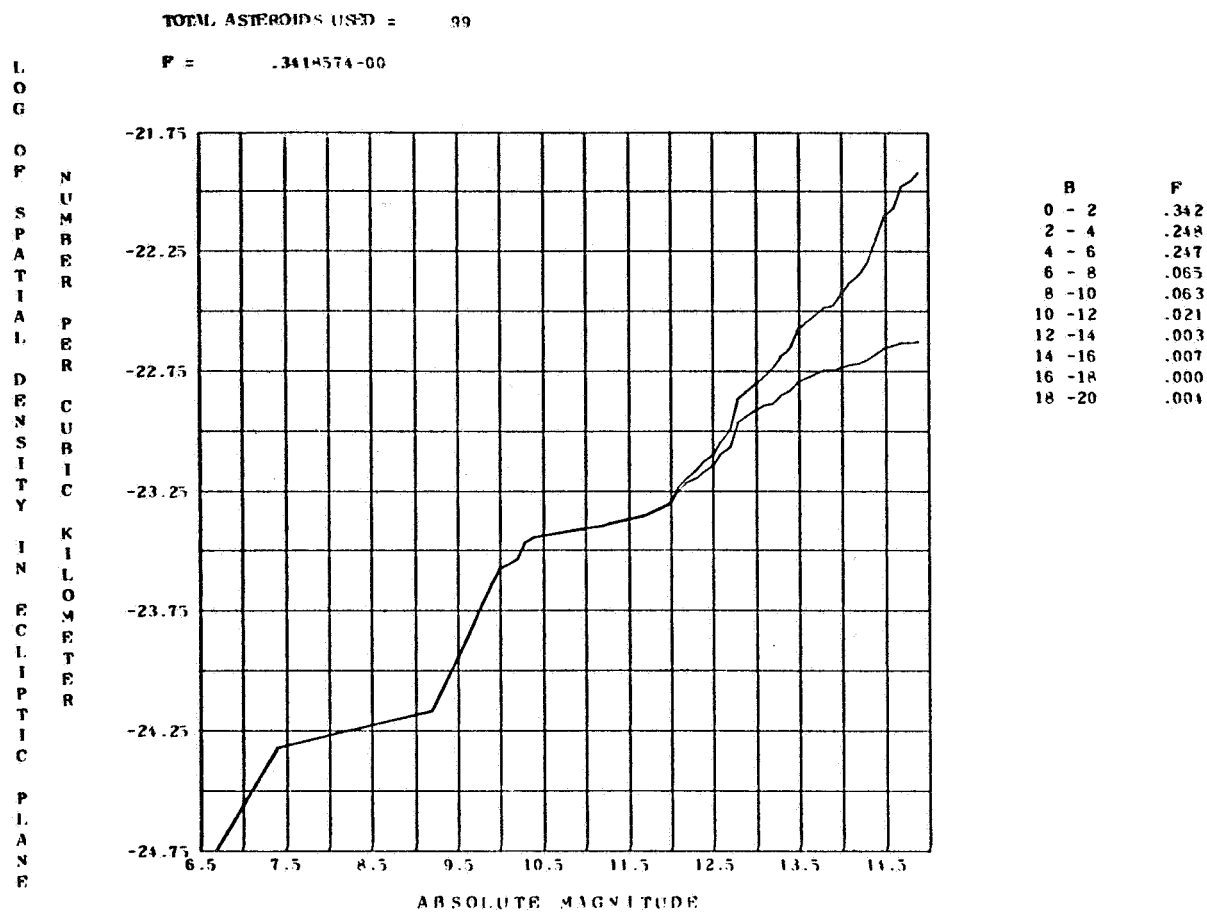


Figure 68. - Spatial density at R = 1.90 and at longitudes between 0 and 45.0.

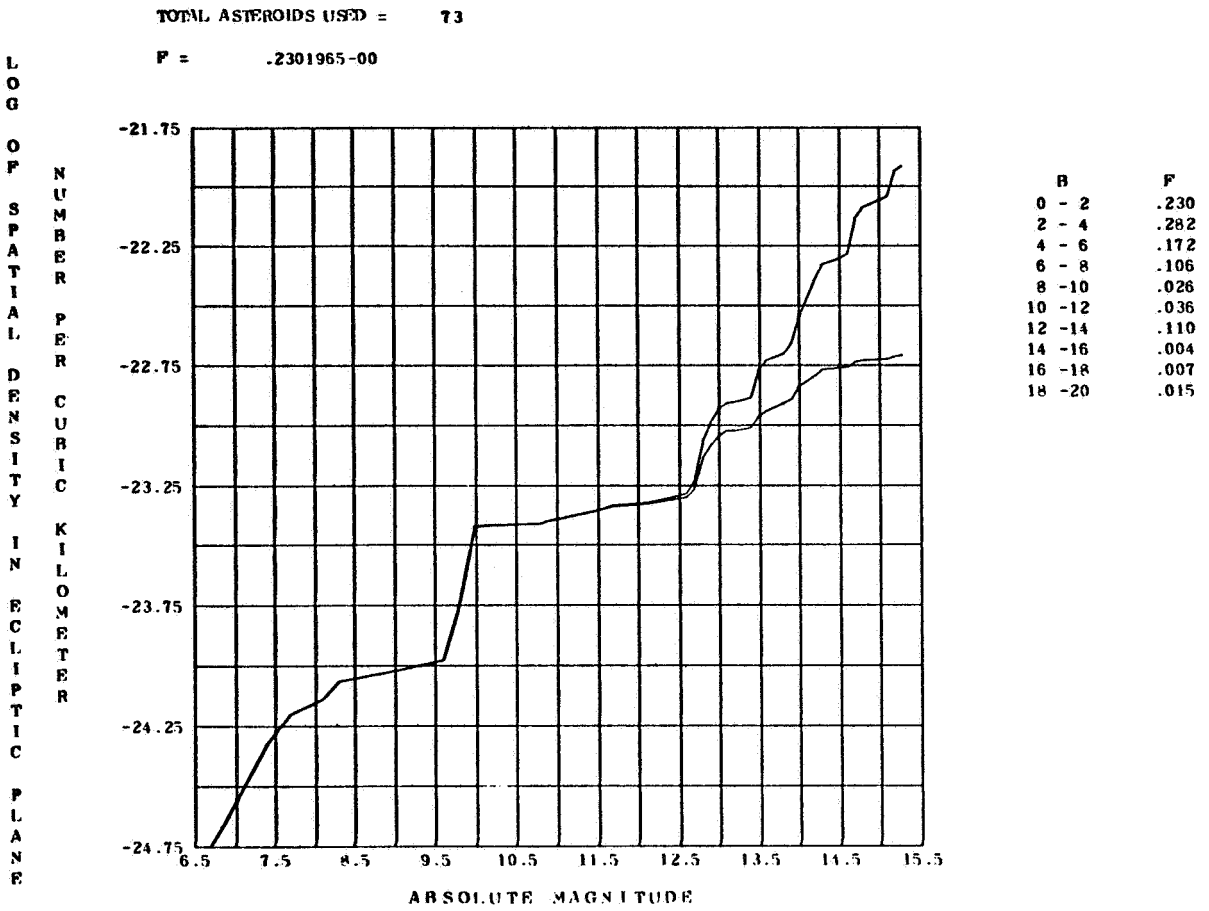


Figure 69. - Spatial density at R = 1.90 and at longitudes between 45.0 and 90.0.

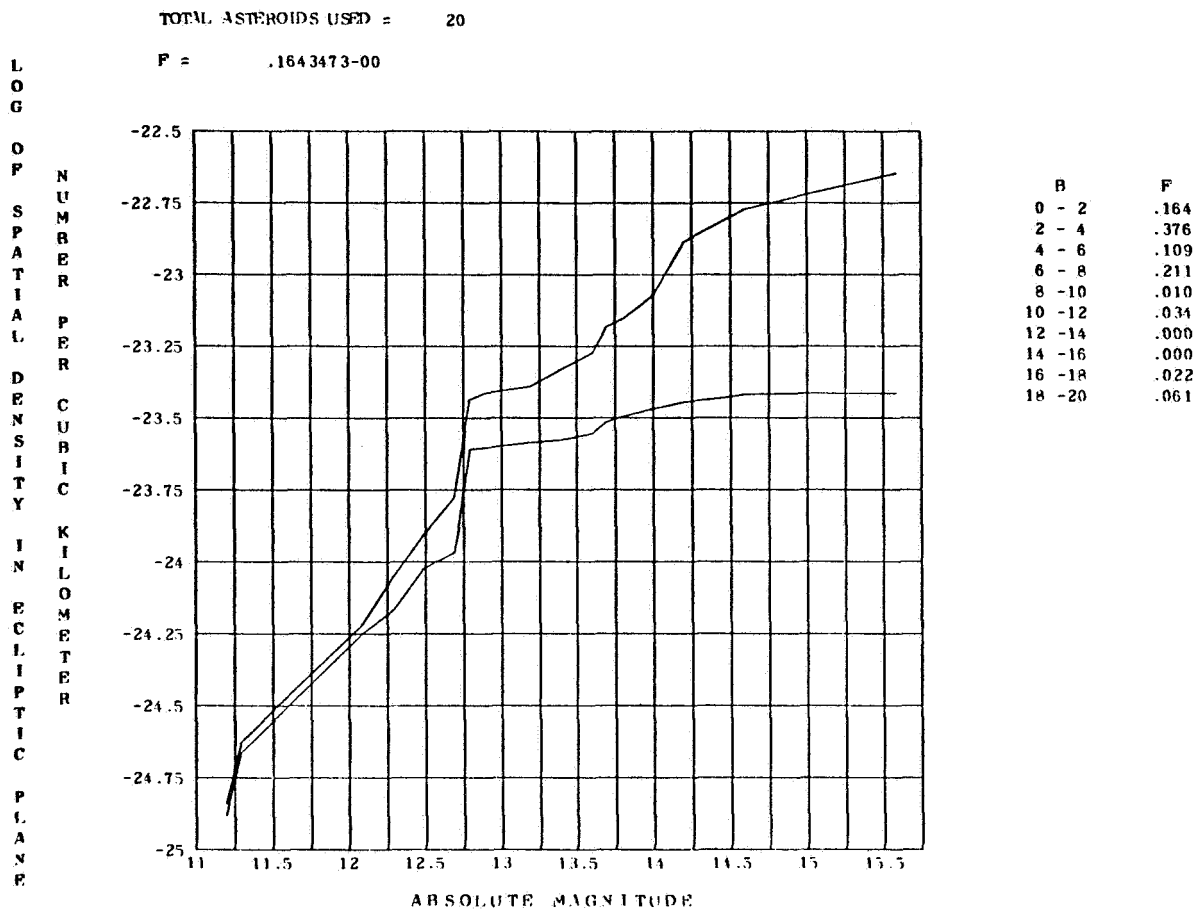


Figure 70. - Spatial density at R = 1.90 and at longitudes between 90.0 and 135.0.

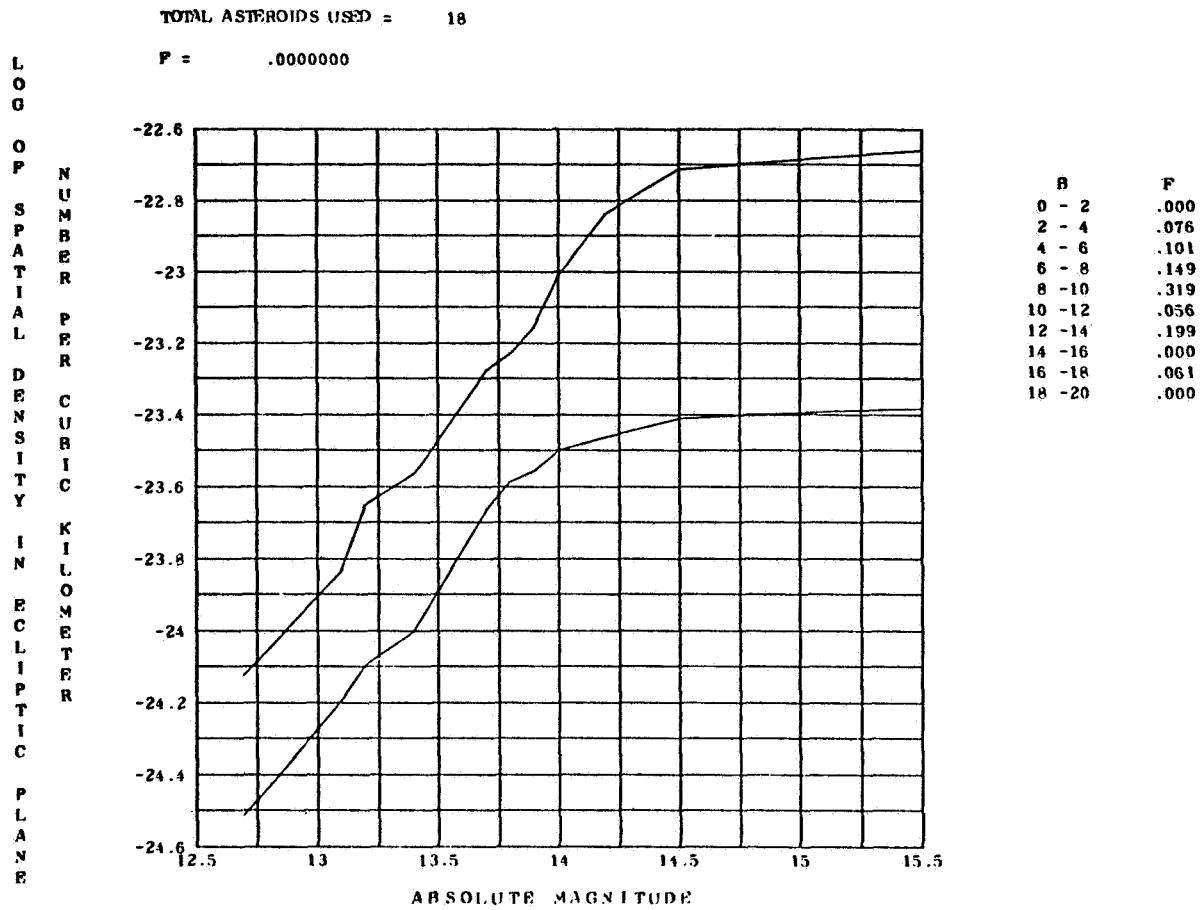


Figure 71. - Spatial density at R = 1.90 and at longitudes between 135.0 and 180.0.

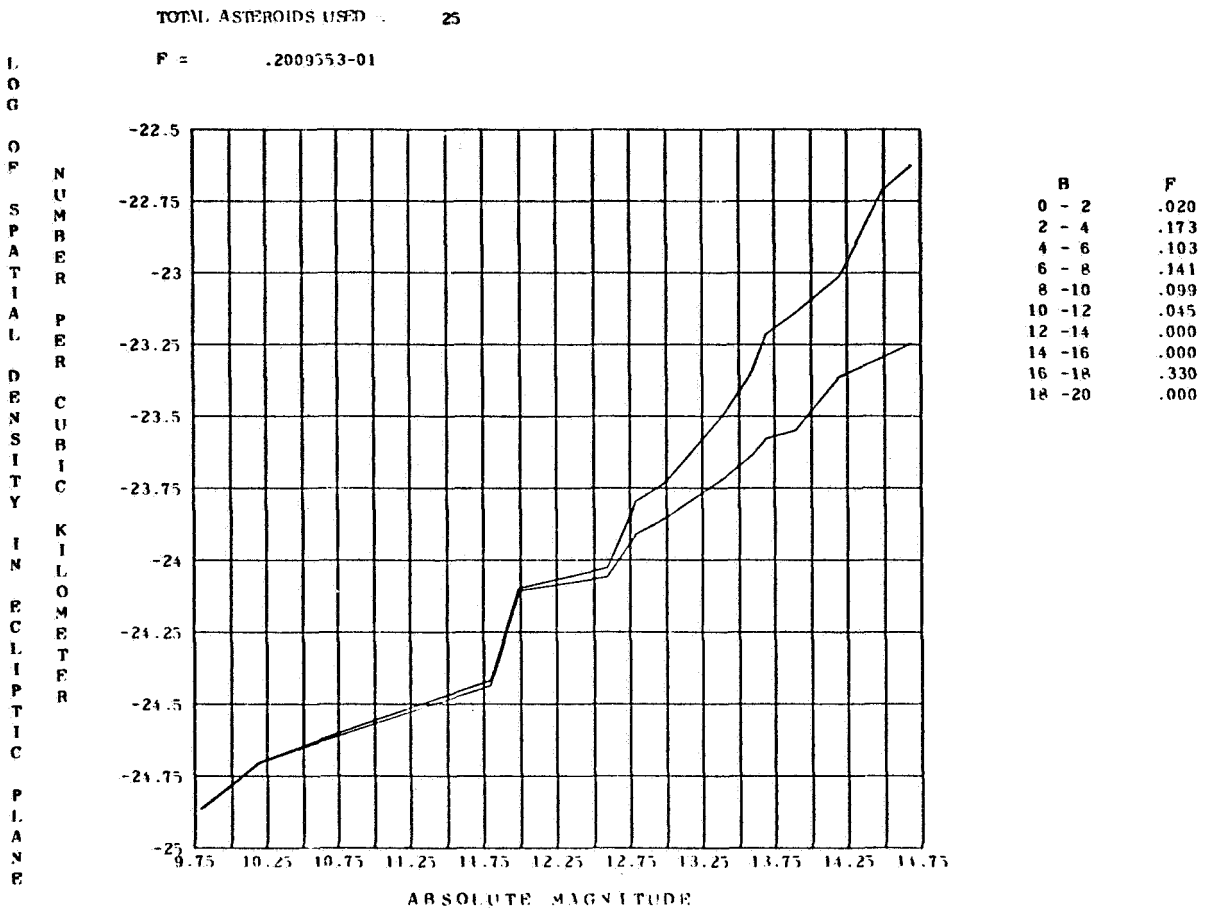


Figure 72. - Spatial density at R = 1.90 and at longitudes between 180.0 and 225.0.

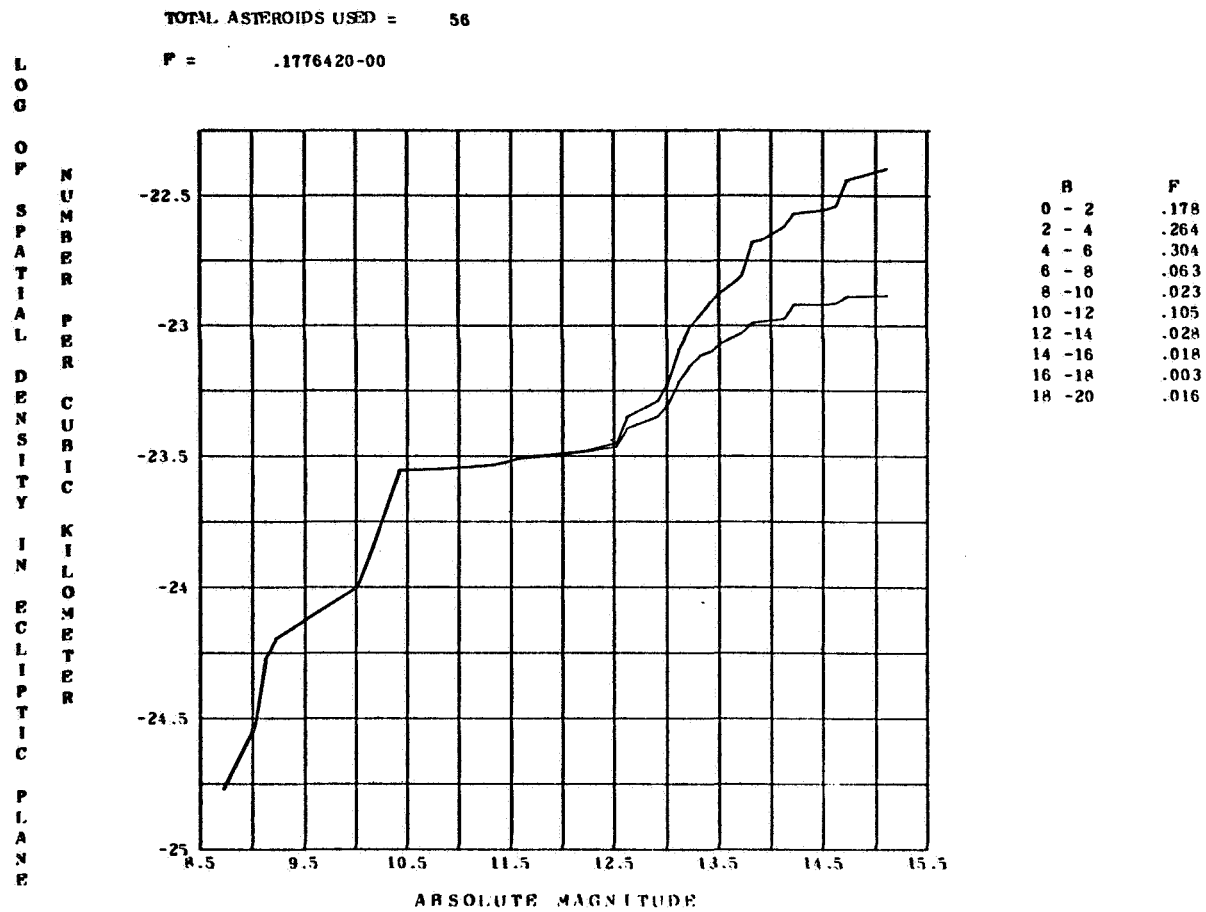


Figure 73. - Spatial density at R = 1.90 and at longitudes between 225.0 and 270.0.

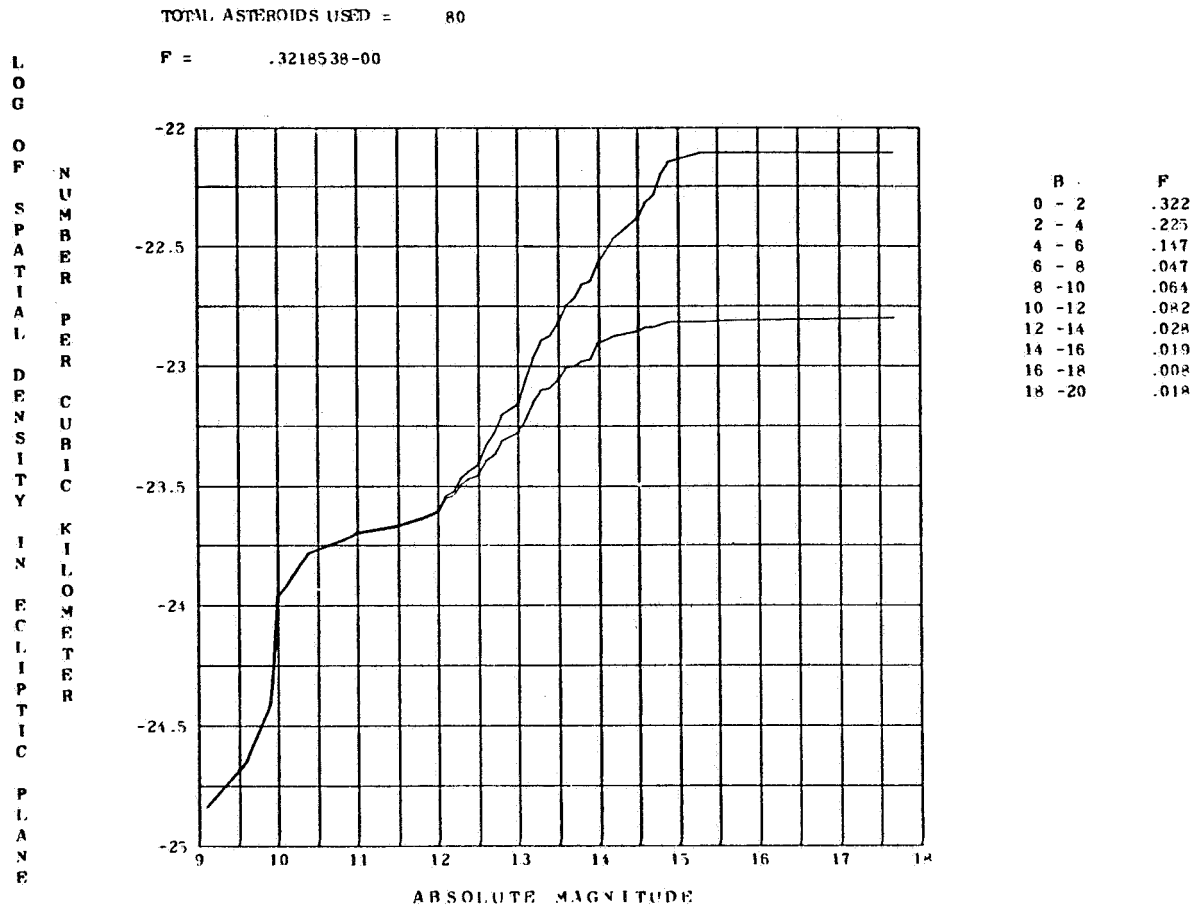


Figure 74. - Spatial density at R = 1.90 and at longitudes between 270.0 and 315.0.

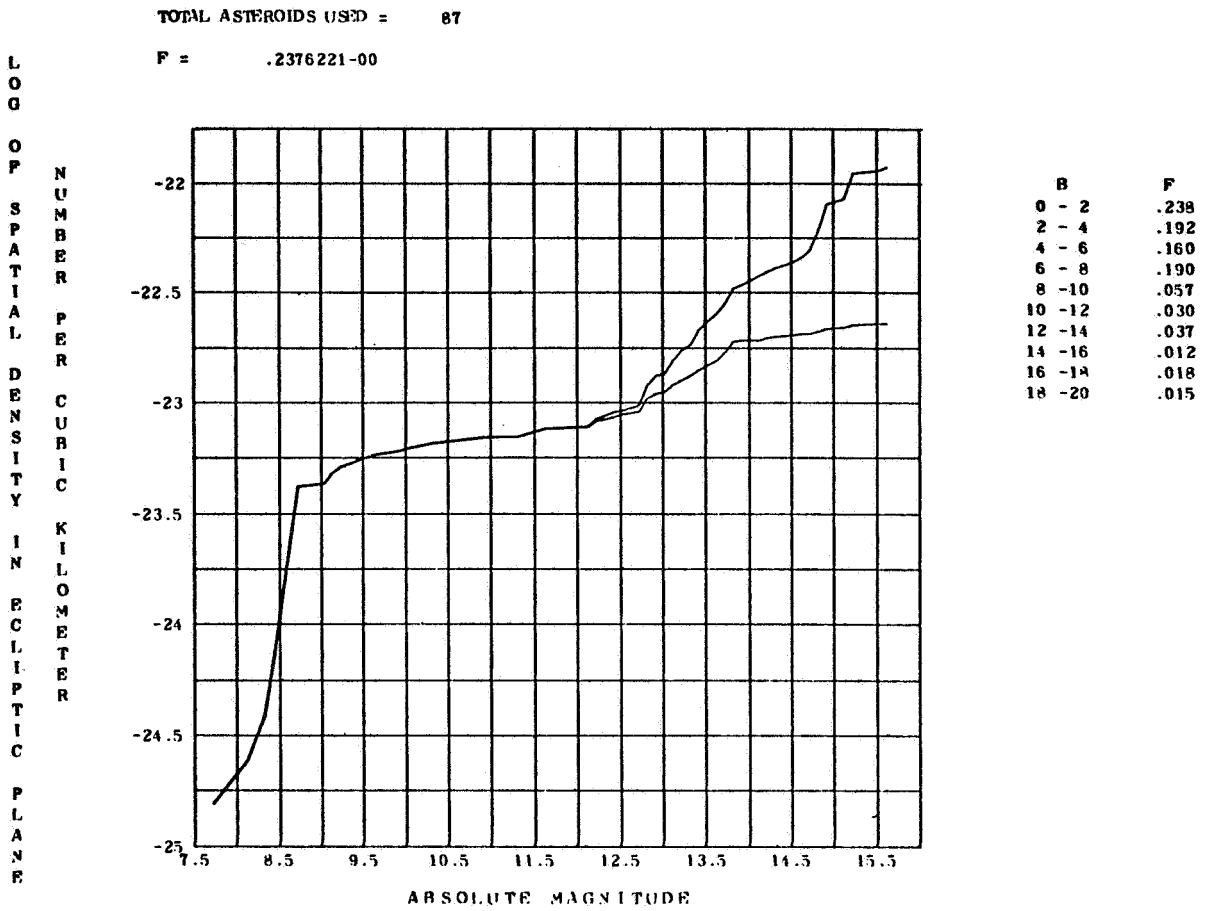


Figure 75. - Spatial density at R = 1.90 and at longitudes between 315.0 and 360.0.

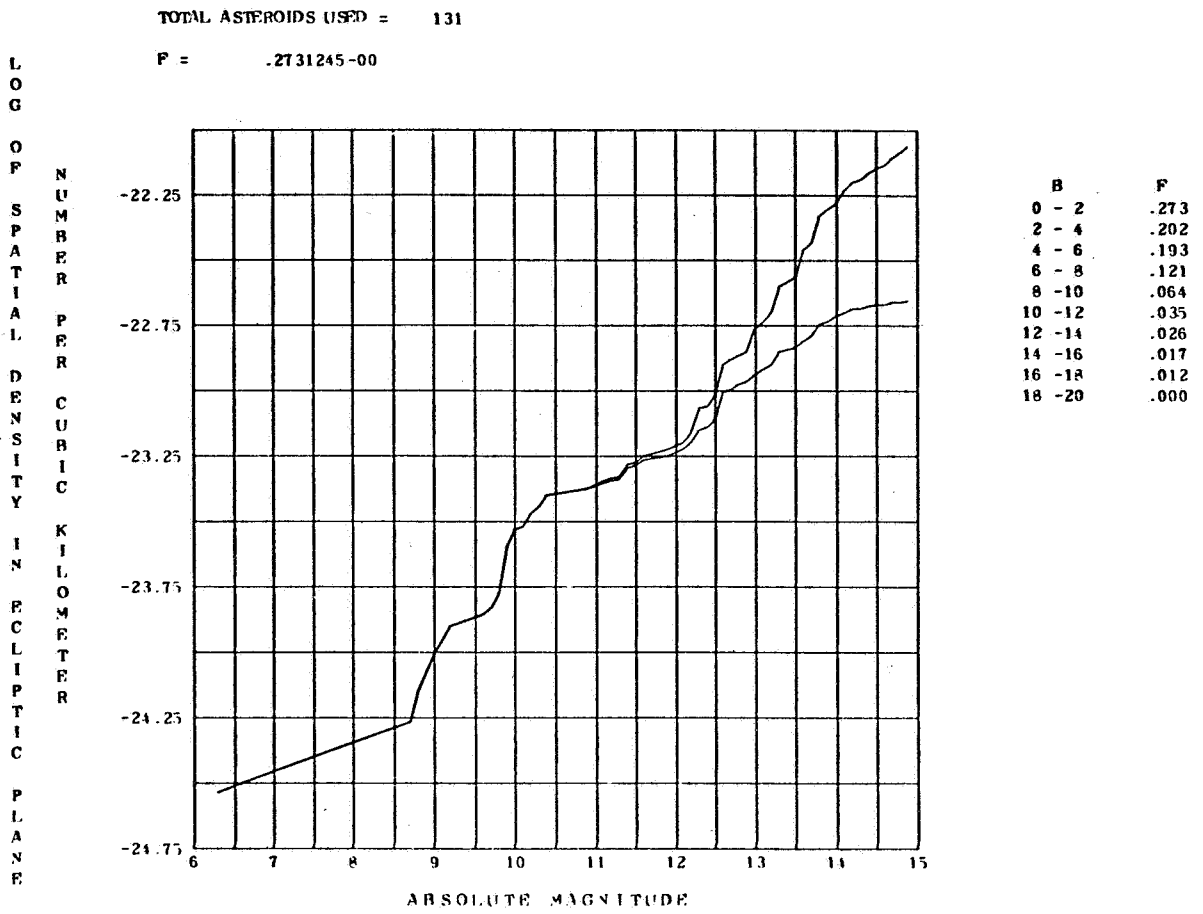


Figure 76. - Spatial density at R = 2.00 and at longitudes between 0 and 45.0.

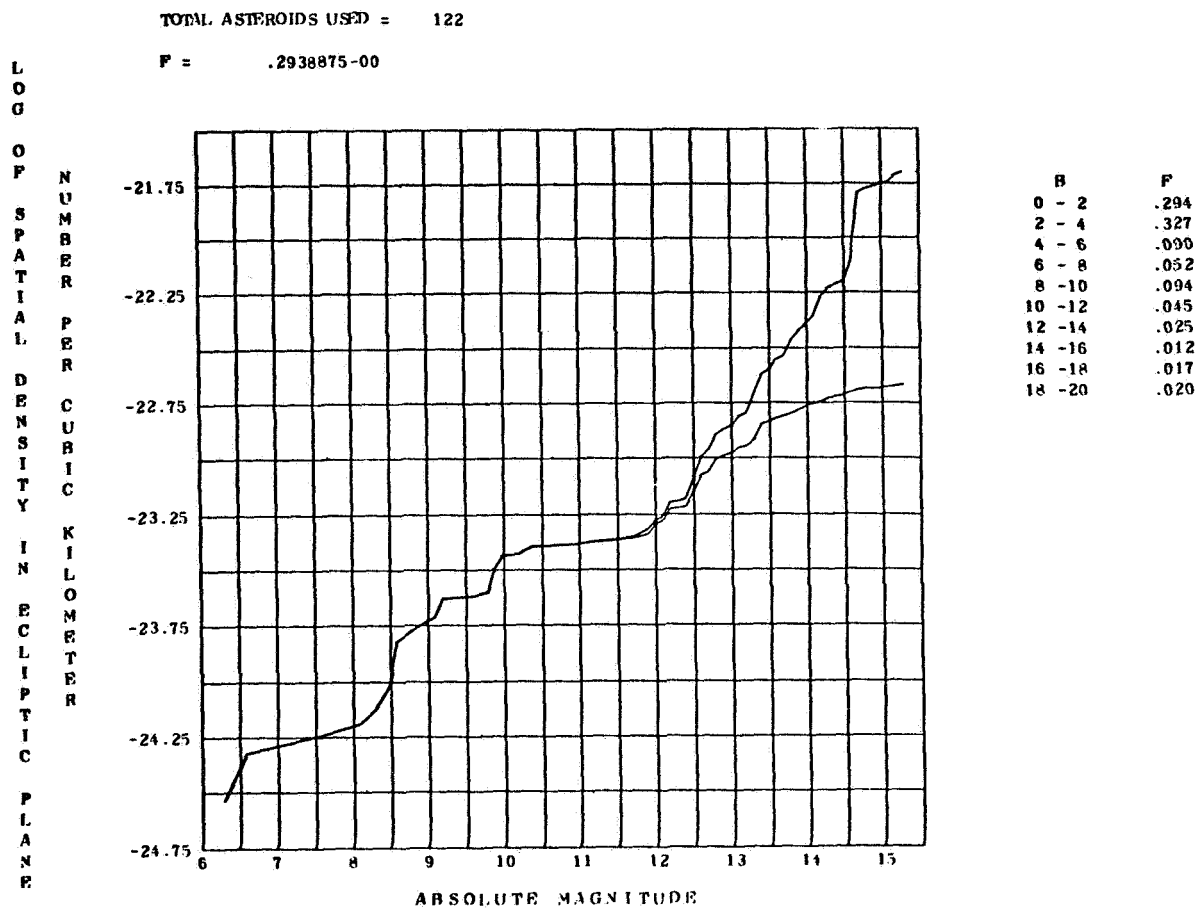


Figure 77. - Spatial density at R = 2.00 and at longitudes between 45.0 and 90.0.

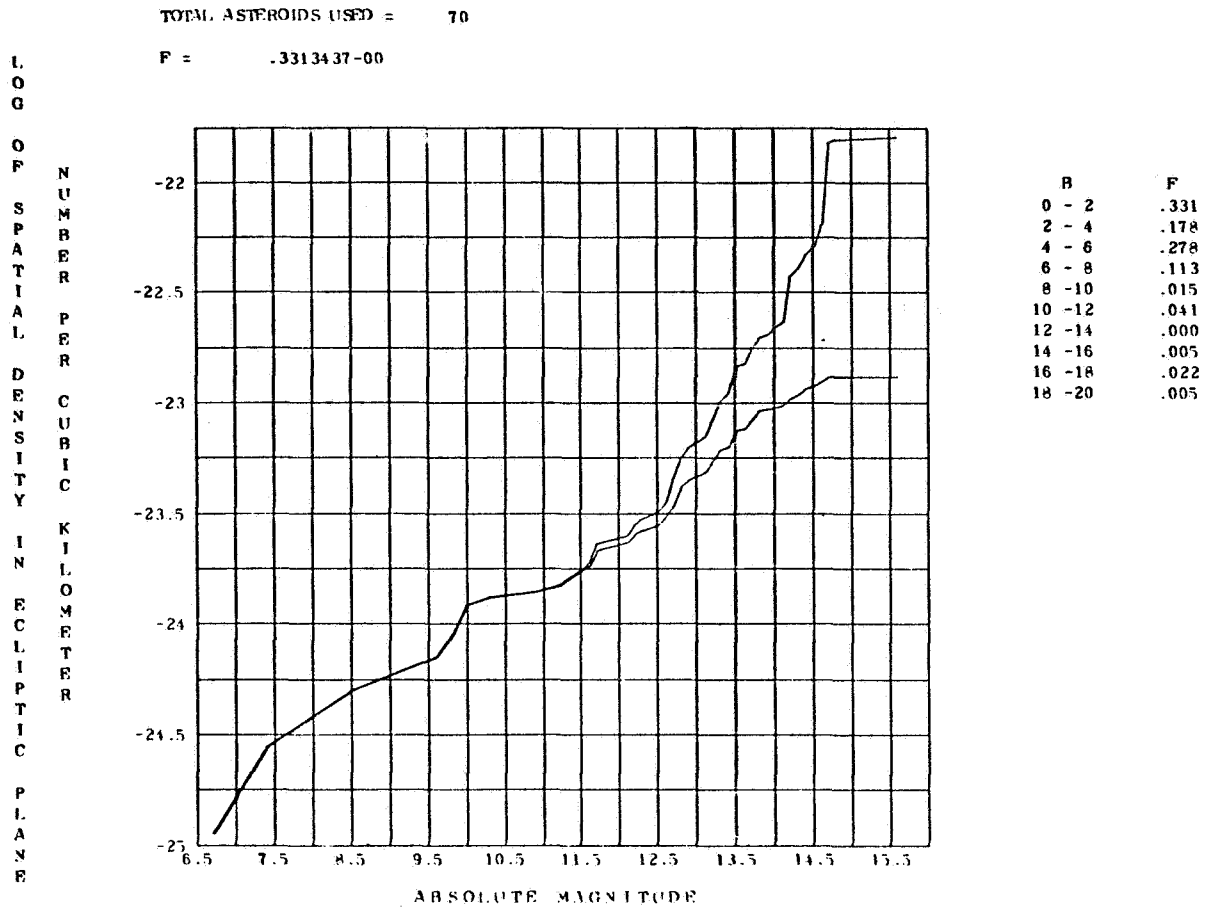


Figure 78. - Spatial density at R = 2.00 and at longitudes between 90.0 and 135.0.

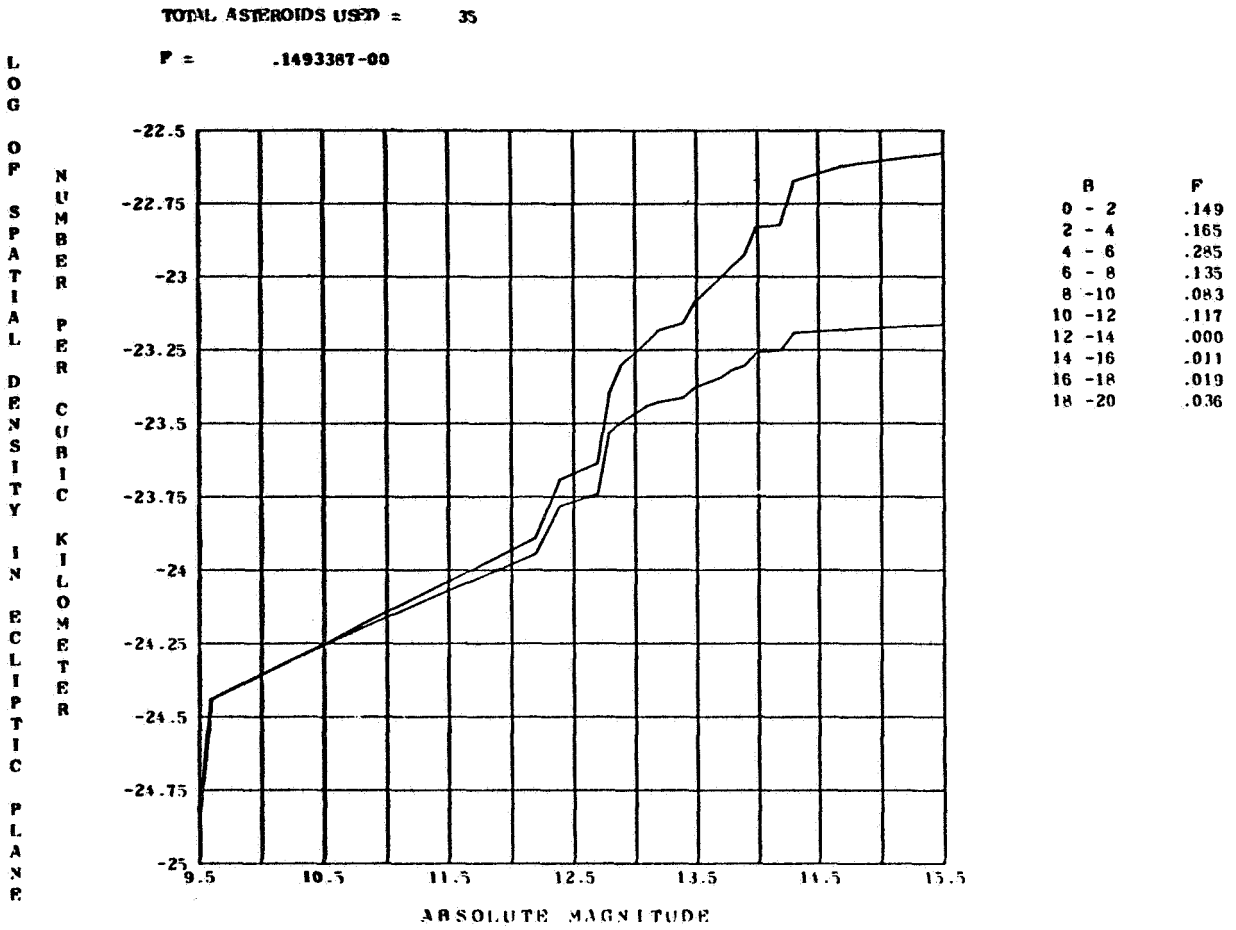


Figure 79. - Spatial density at R = 2.00 and at longitudes between 135.0 and 180.0.

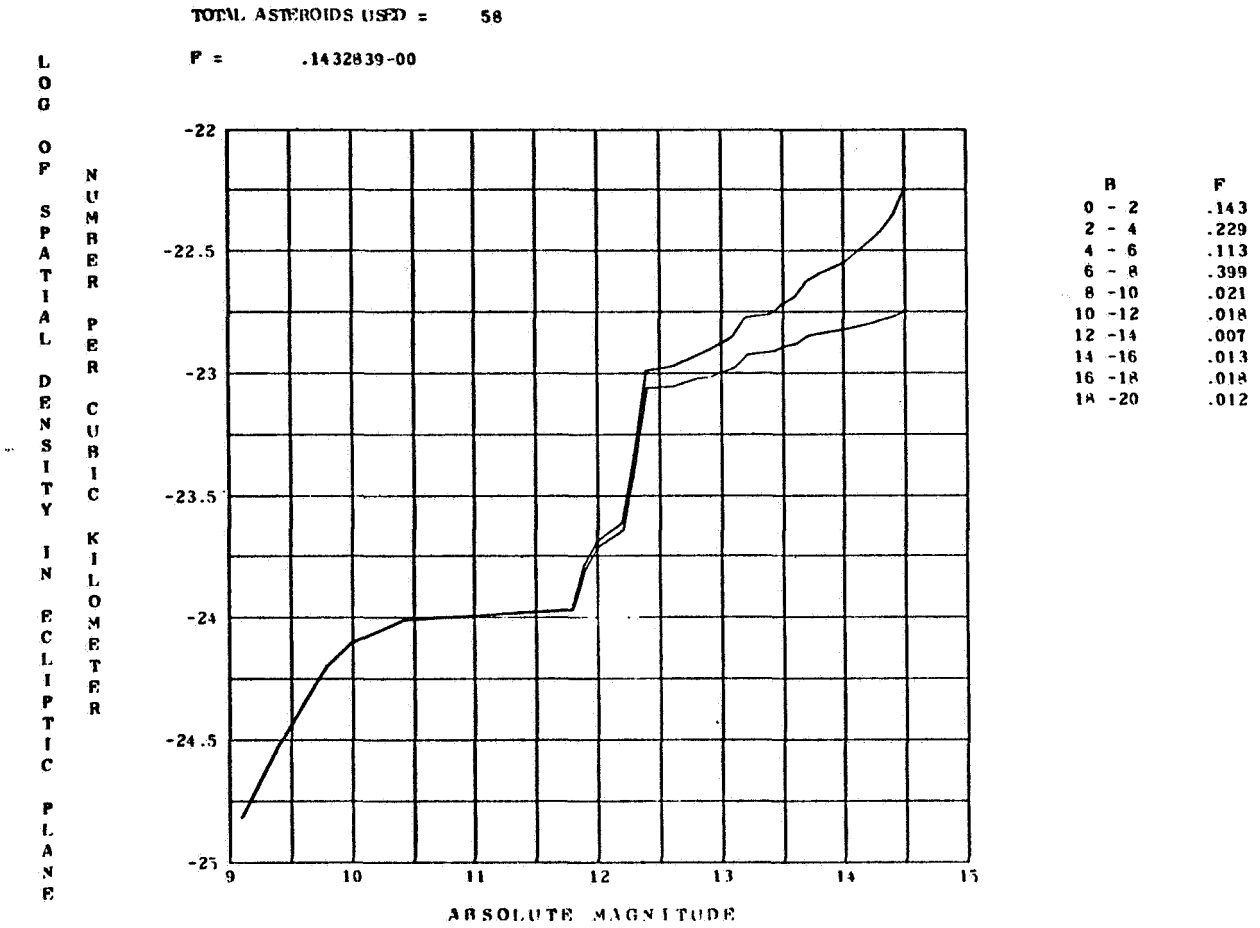


Figure 80. - Spatial density at R = 2.00 and at longitudes between 180.0 and 225.0.

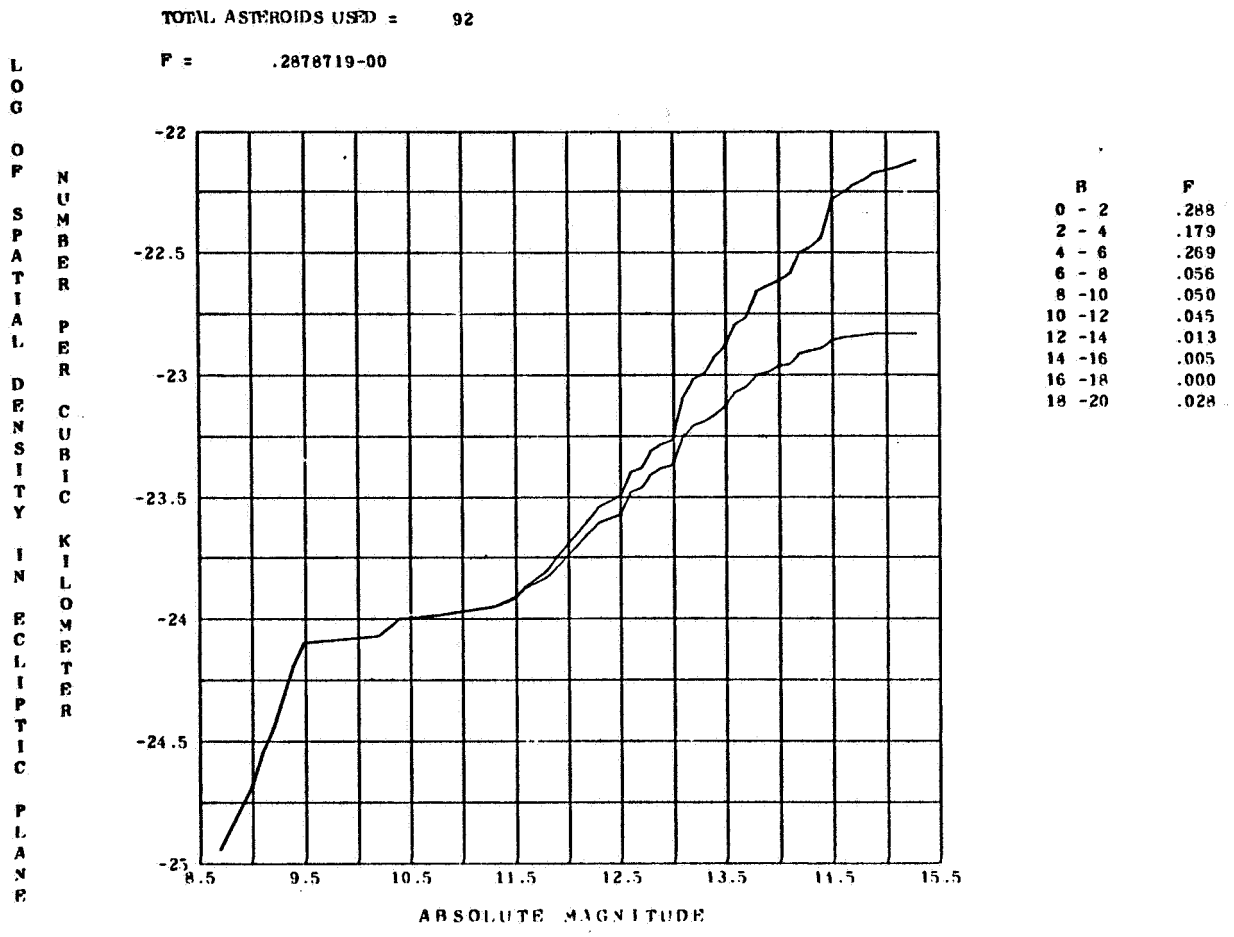


Figure 81. - Spatial density at R = 2.00 and at longitudes between 225.0 and 270.0.

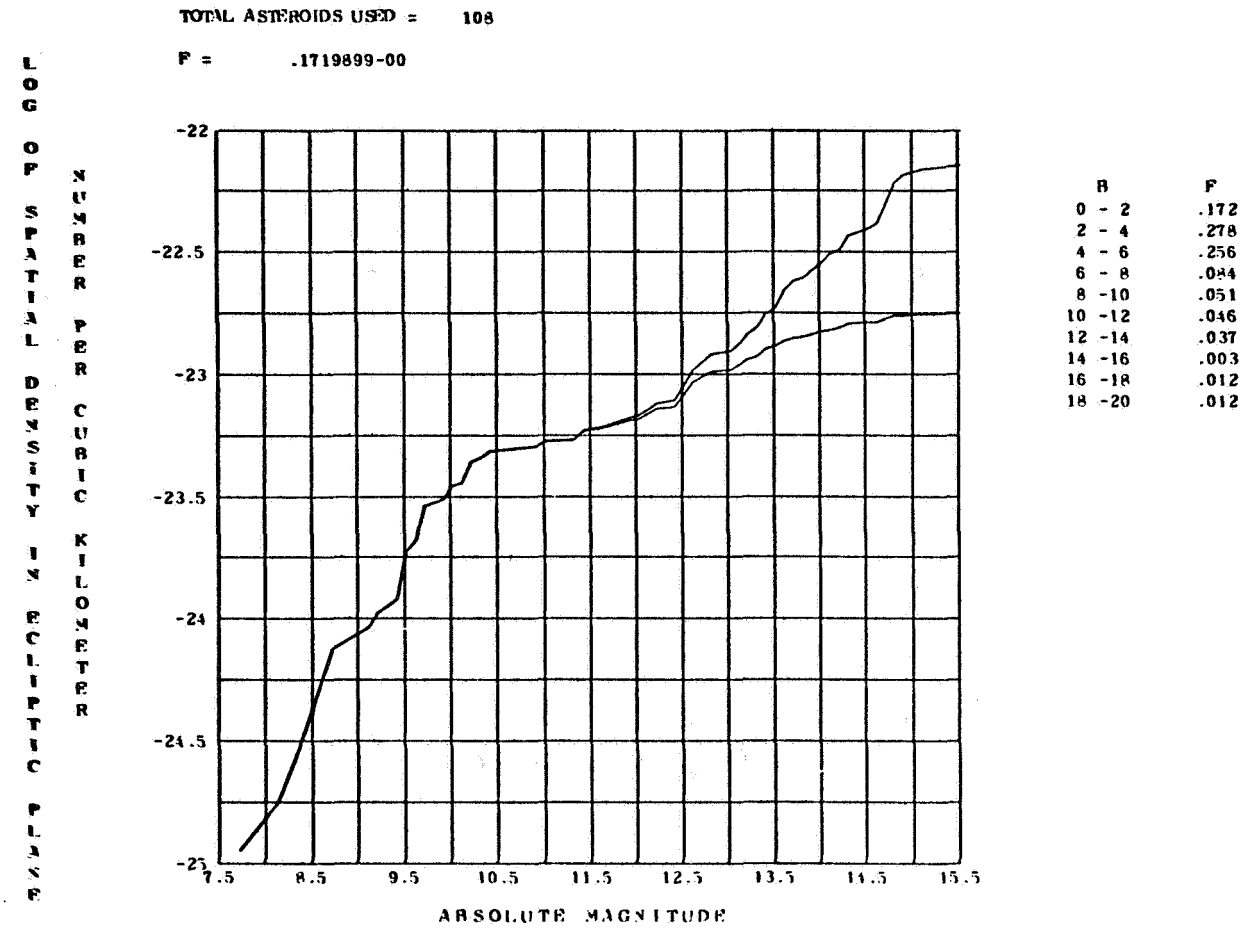


Figure 82. - Spatial density at R = 2.00 and at longitudes between 270.0 and 315.0.

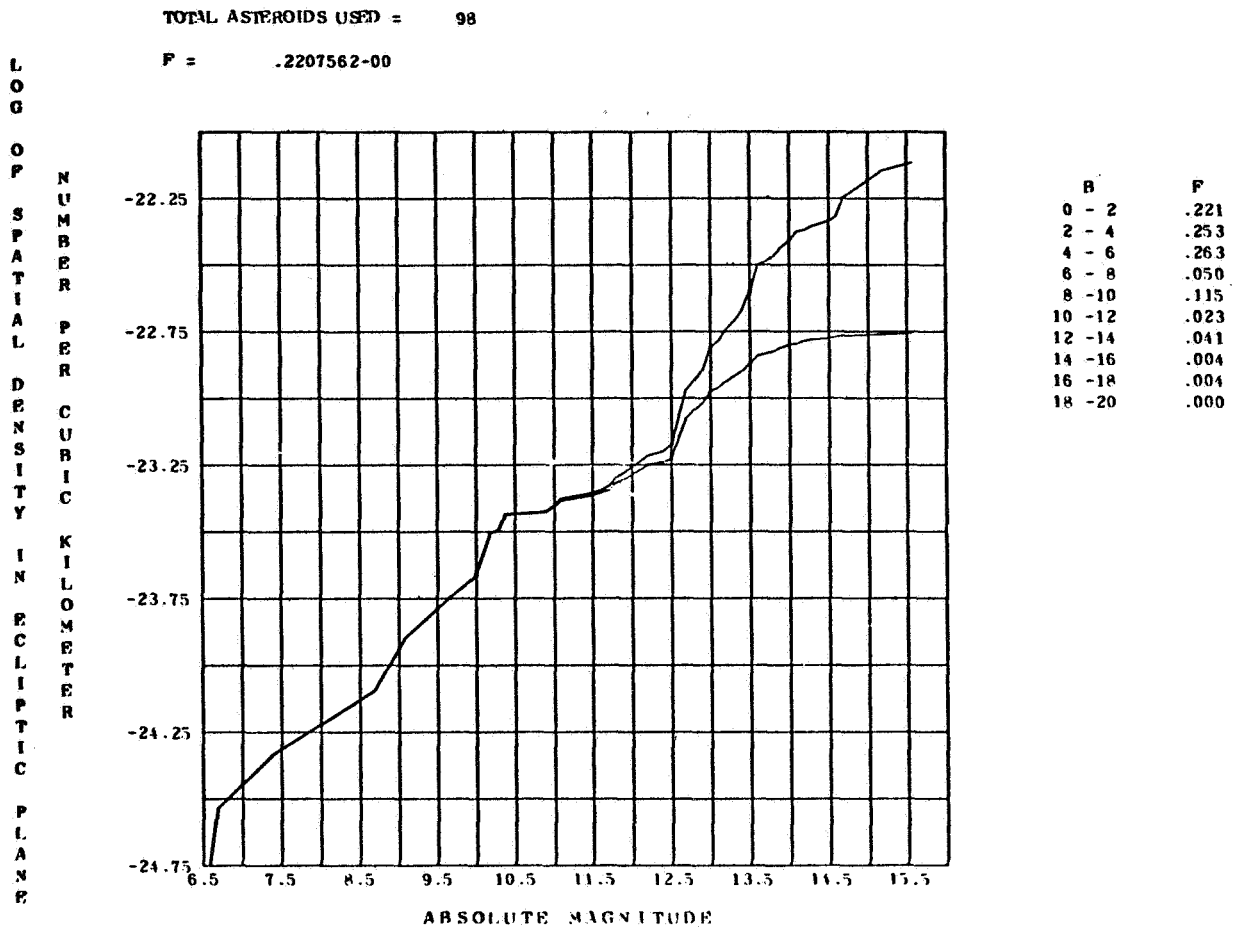
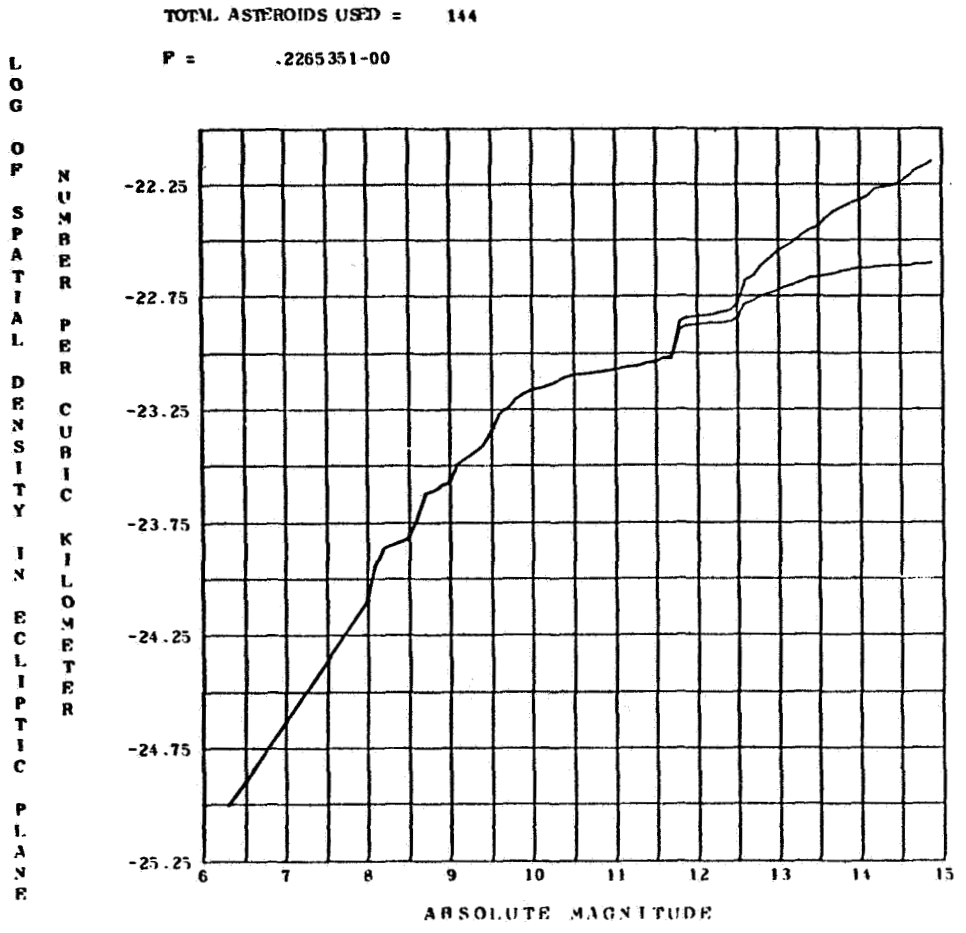


Figure 83. - Spatial density at R = 2.00 and at longitudes between 315.0 and 360.0.



B	F
0 - 2	.227
2 - 4	.243
4 - 6	.269
6 - 8	.084
8 -10	.082
10 -12	.030
12 -14	.047
14 -16	.012
16 -18	.002
18 -20	.000

Figure 84. - Spatial density at R = 2.10 and at longitudes between 0 and 45.0.

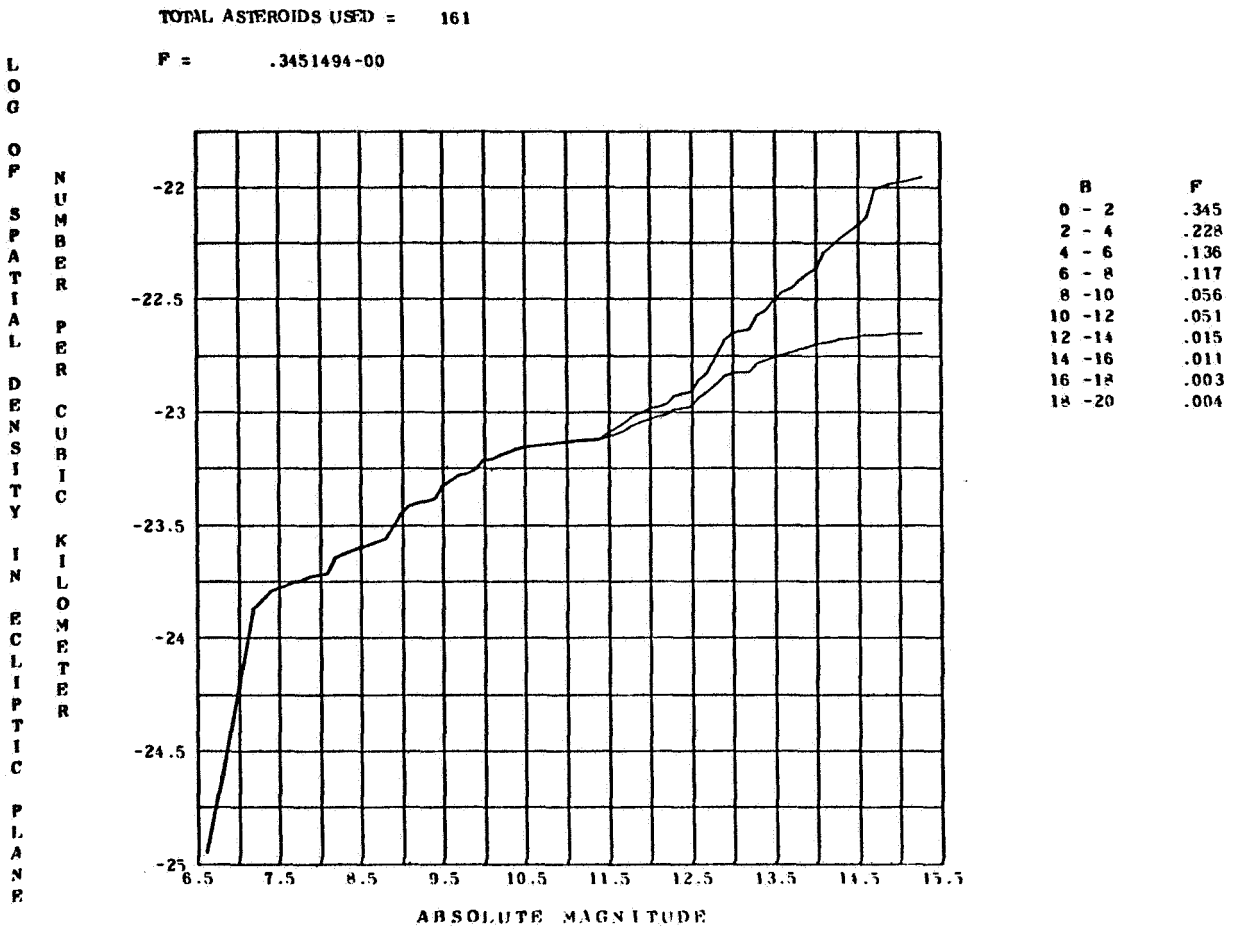


Figure 85. - Spatial density at R = 2.10 and at longitudes between 45.0 and 90.0.

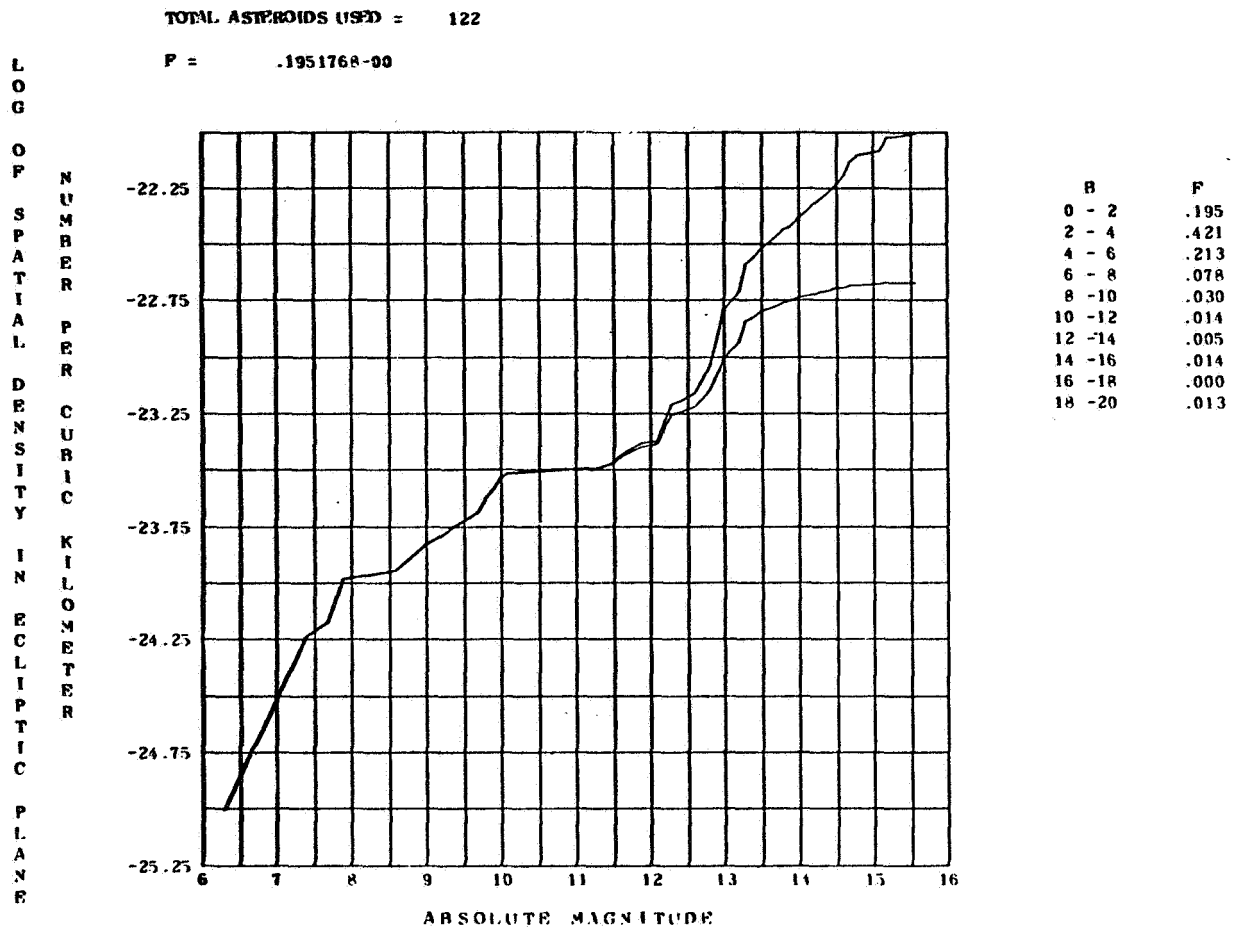


Figure 86. - Spatial density at R = 2.10 and at longitudes between 90.0 and 135.0.

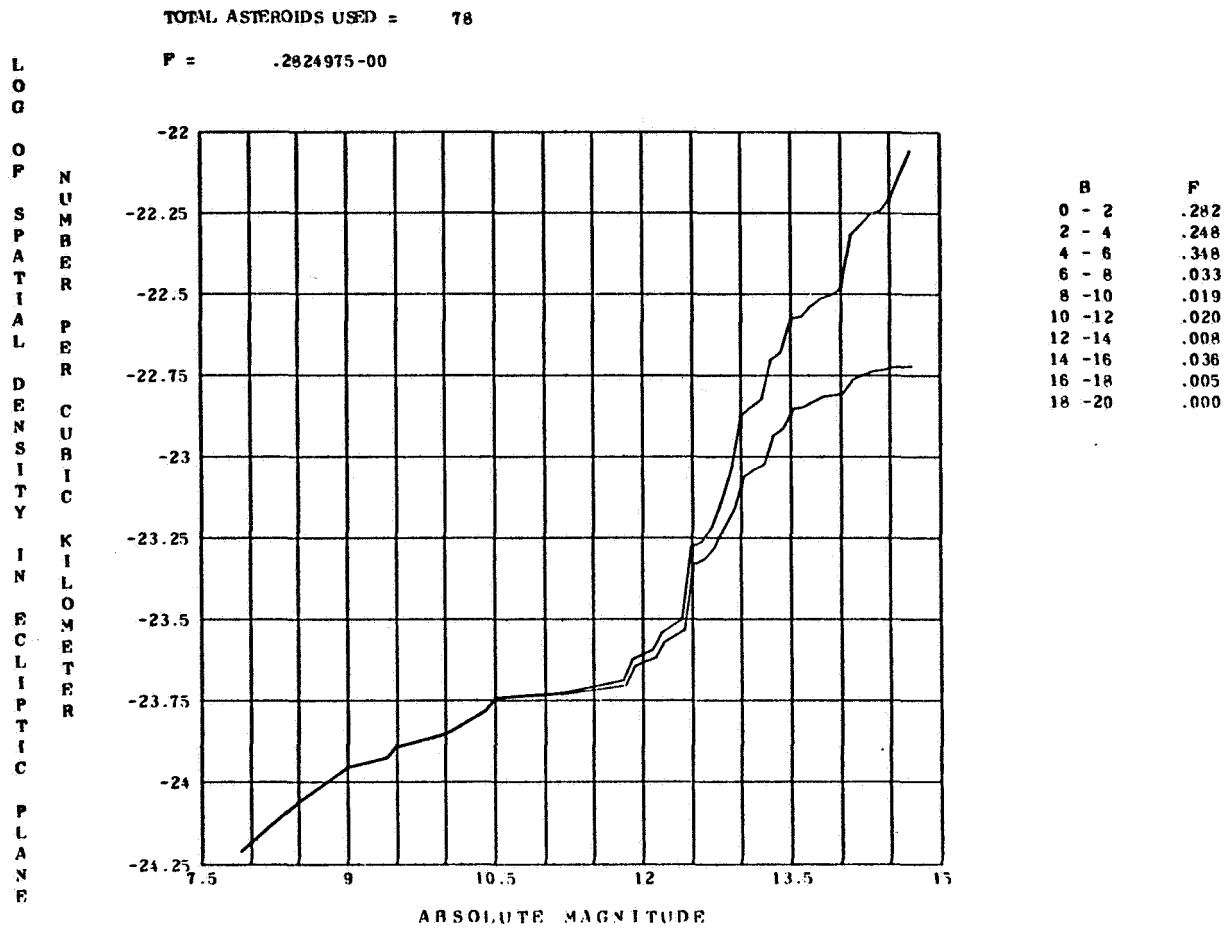


Figure 87. - Spatial density at R = 2.10 and at longitudes between 135.0 and 180.0.

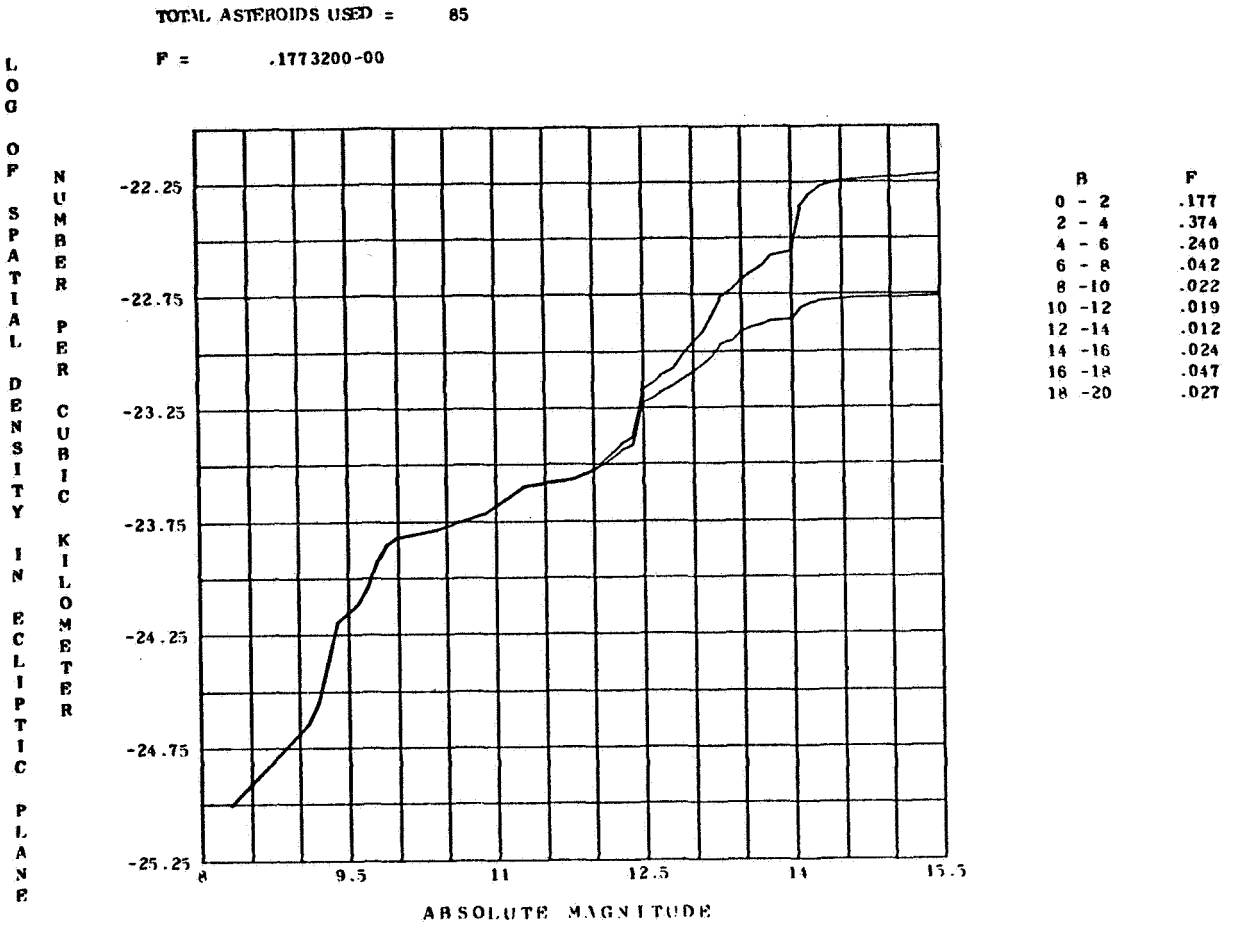


Figure 88. - Spatial density at R = 2.10 and at longitudes between 180.0 and 225.0.

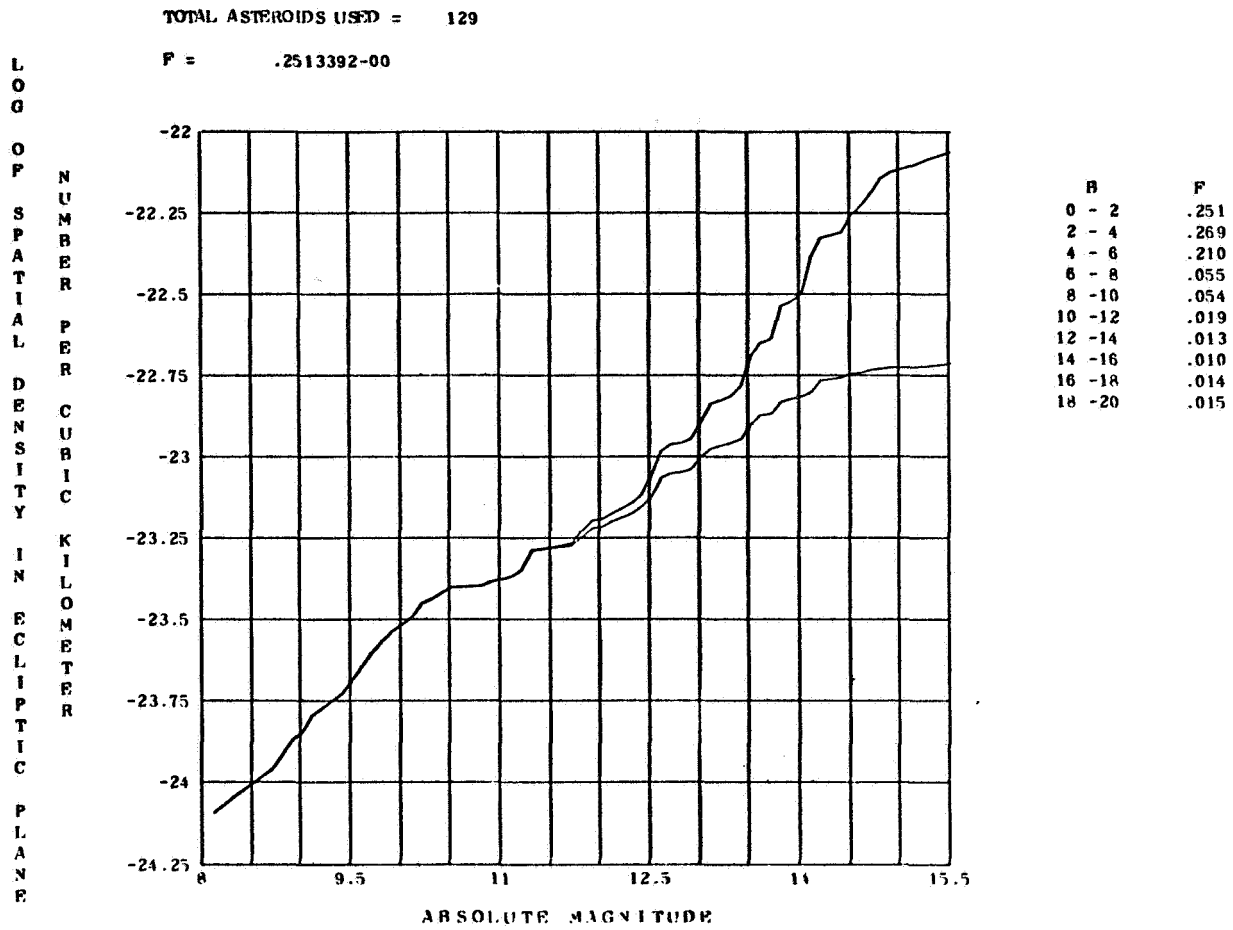


Figure 89. - Spatial density at R = 2.10 and at longitudes between 225.0 and 270.0.

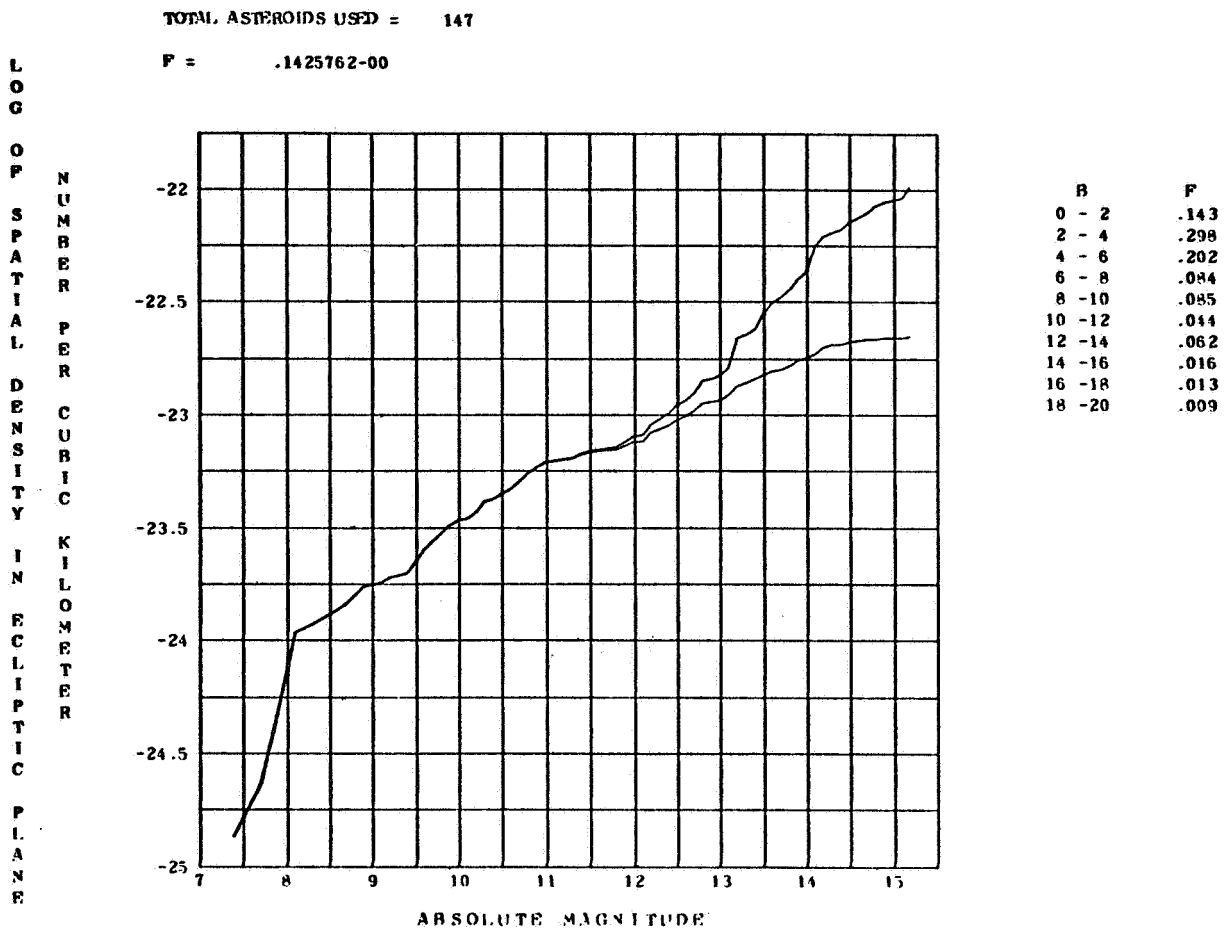


Figure 90. - Spatial density at R = 2.10 and at longitudes between 270.0 and 315.0.

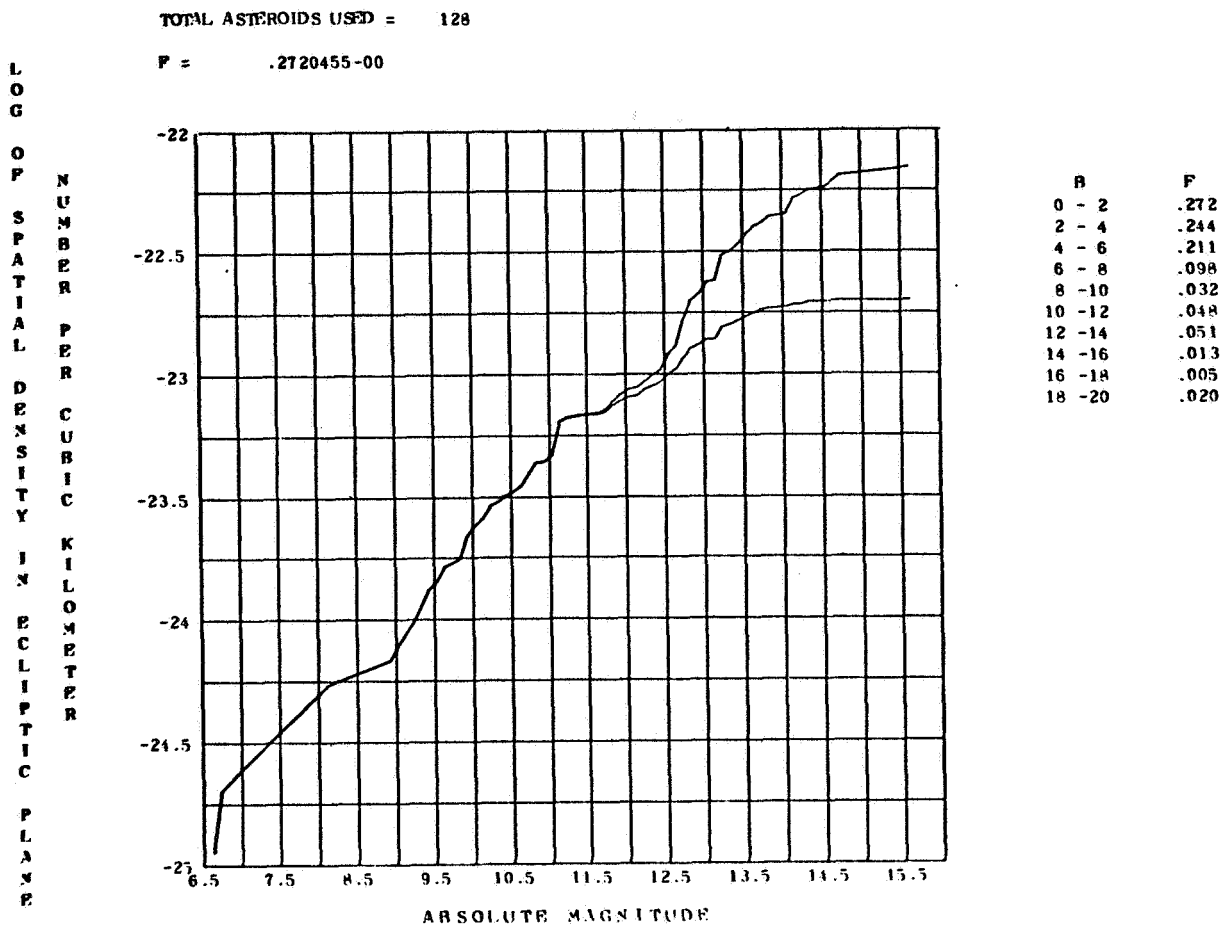


Figure 91. - Spatial density at R = 2.10 and at longitudes between 315.0 and 360.0.

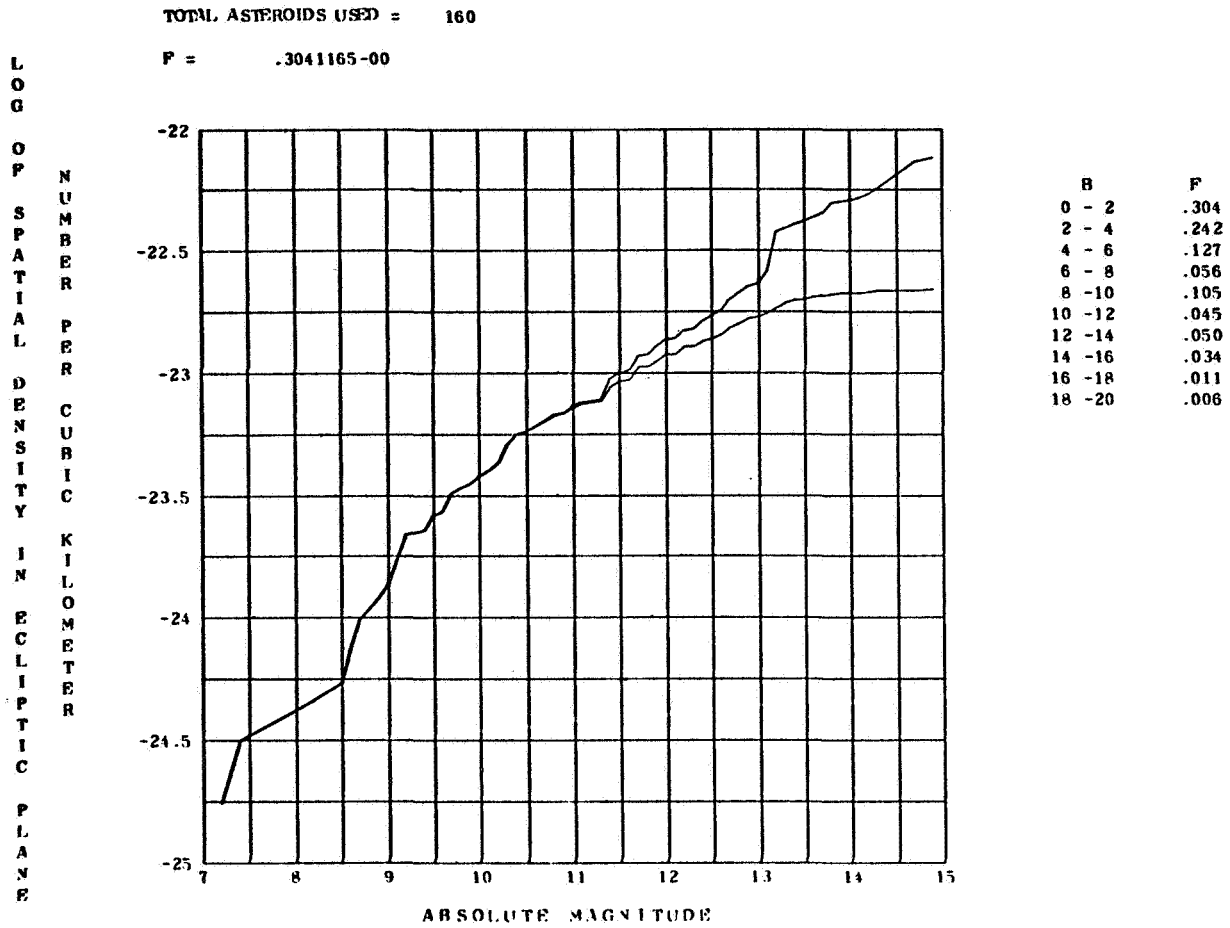


Figure 92. - Spatial density at R = 2.20 and at longitudes between 0 and 45.0.

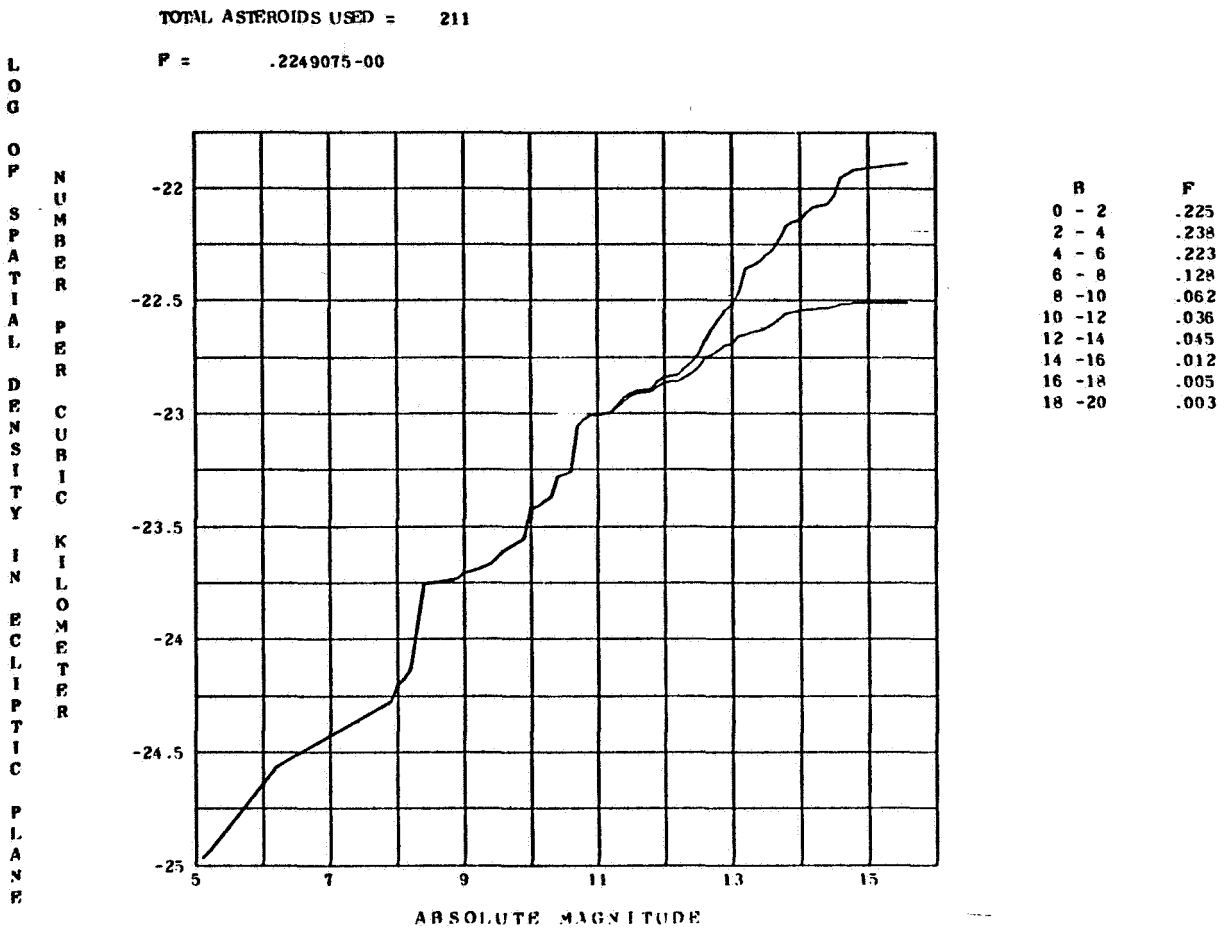


Figure 93. - Spatial density at R = 2.20 and at longitudes between 45.0 and 90.0.

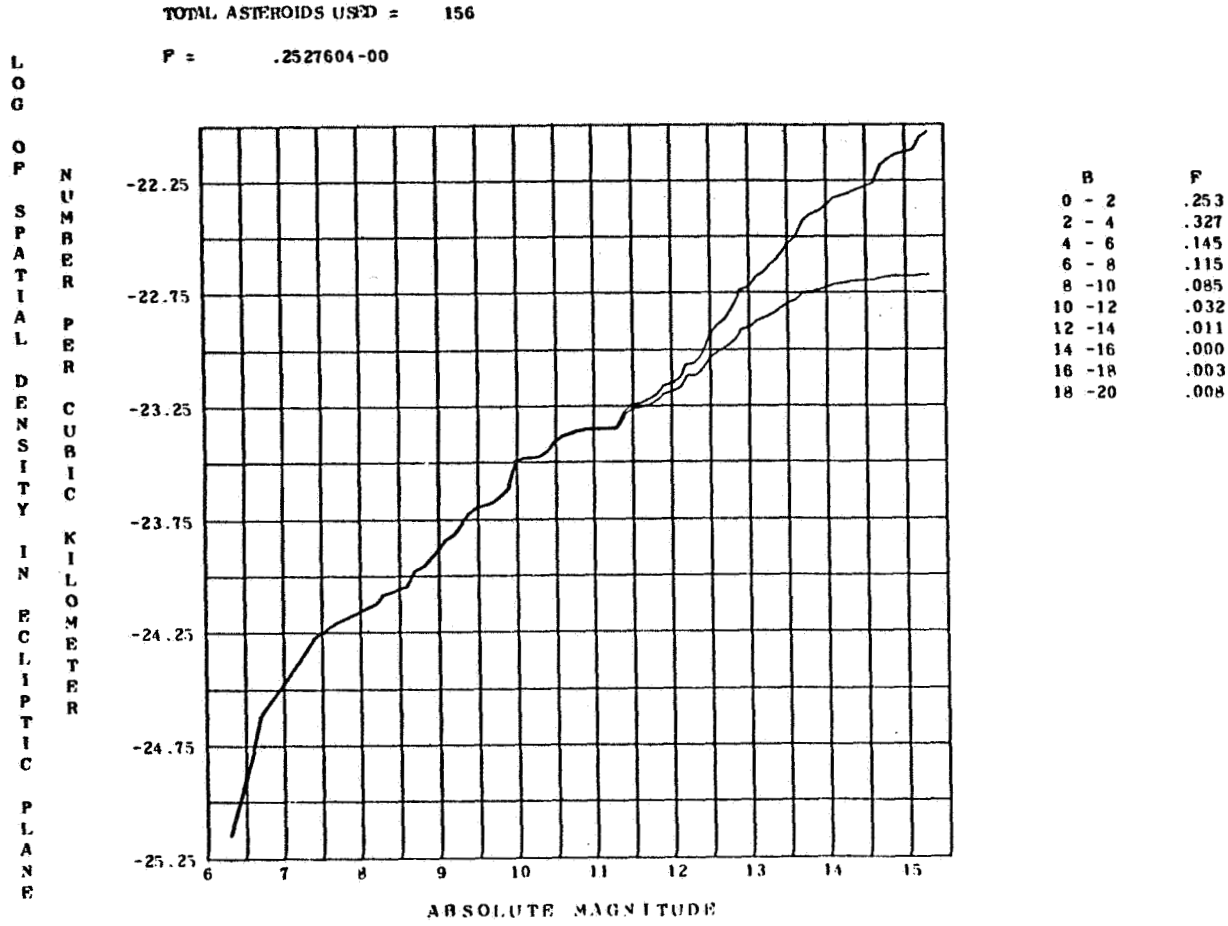


Figure 94. - Spatial density at R = 2.20 and at longitudes between 90.0 and 135.0.

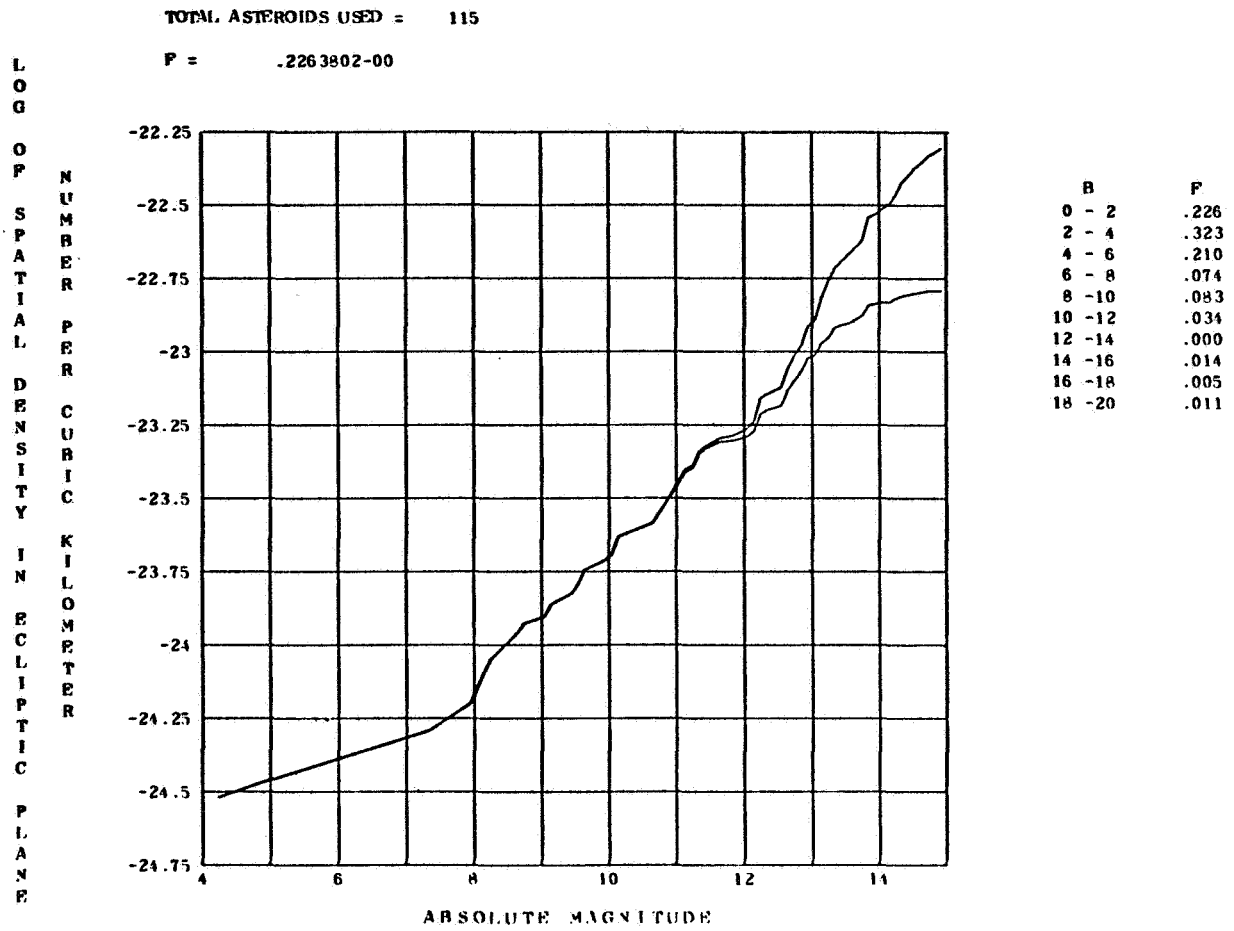


Figure 95. - Spatial density at R = 2.20 and at longitudes between 135.0 and 180.0.

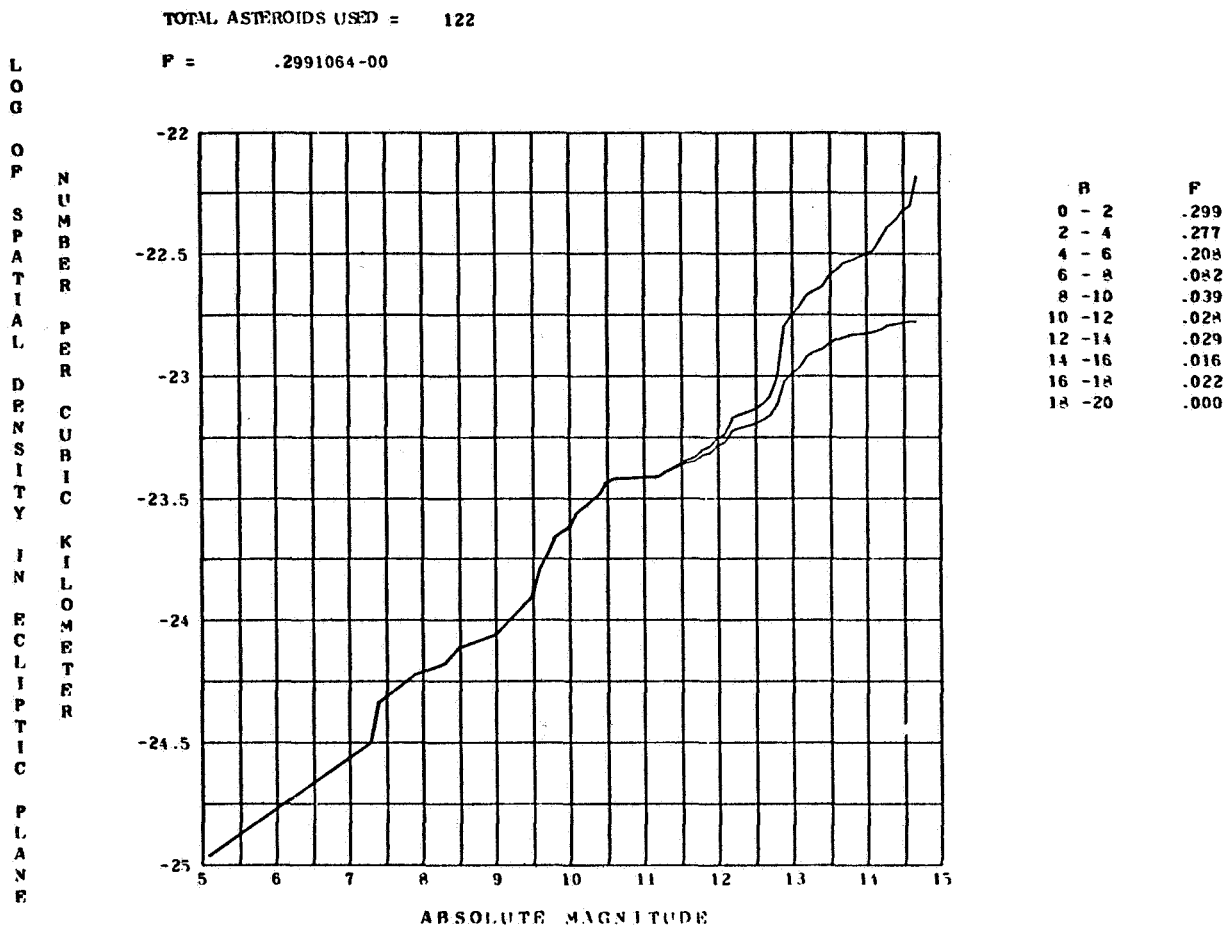


Figure 96. - Spatial density at R = 2.20 and at longitudes between 180.0 and 225.0.

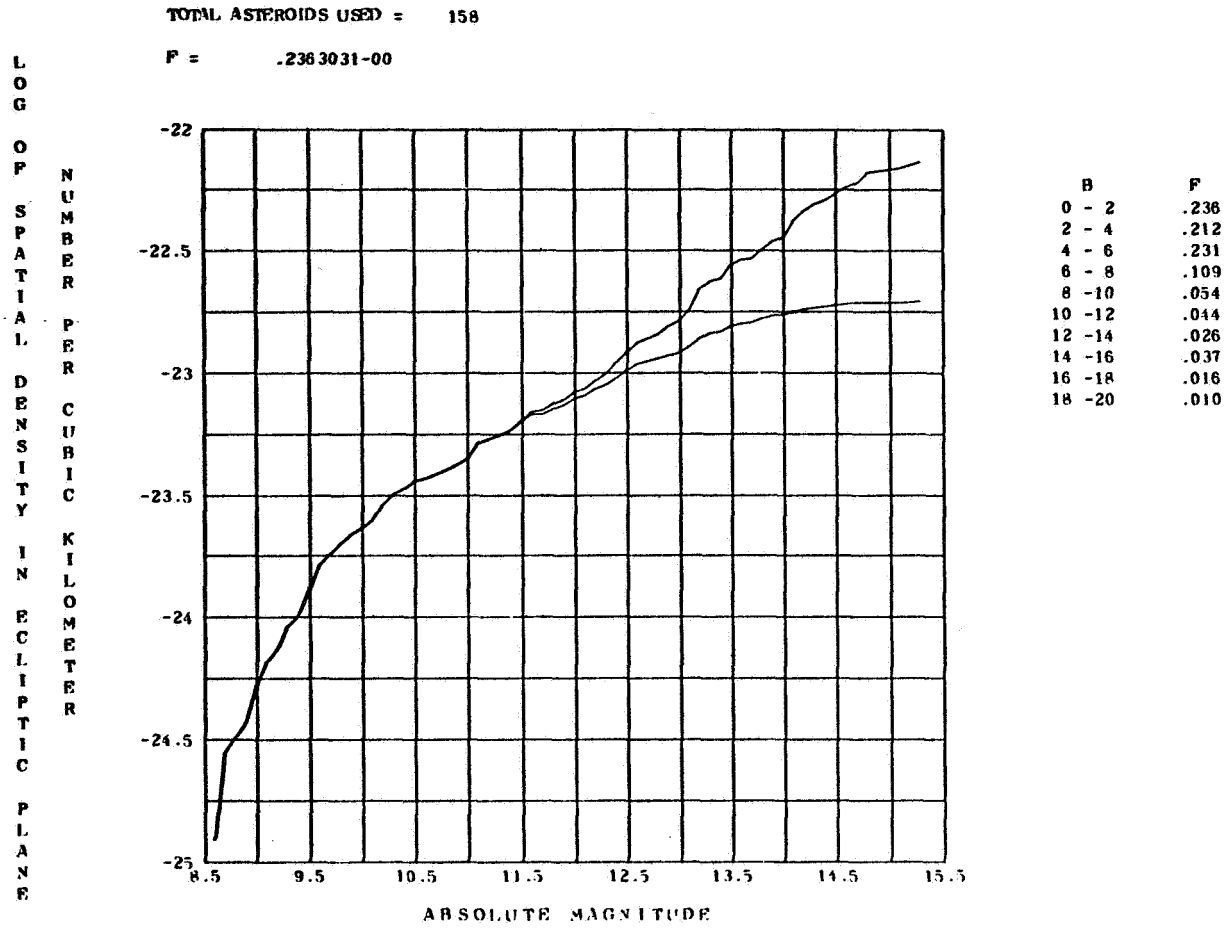


Figure 97. - Spatial density at R = 2.20 and at longitudes between 225.0 and 270.0.

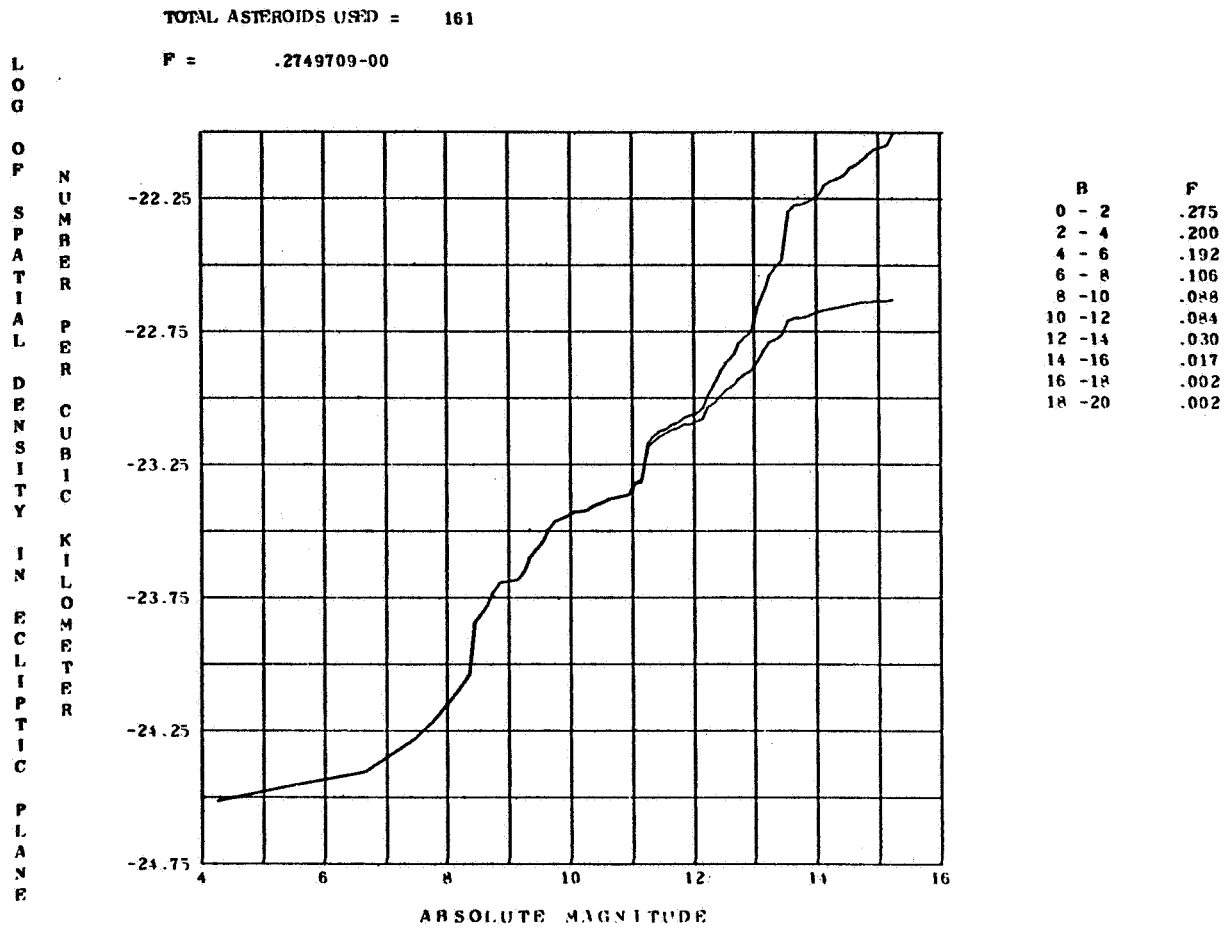


Figure 98. - Spatial density at R = 2.20 and at longitudes between 270.0 and 315.0.

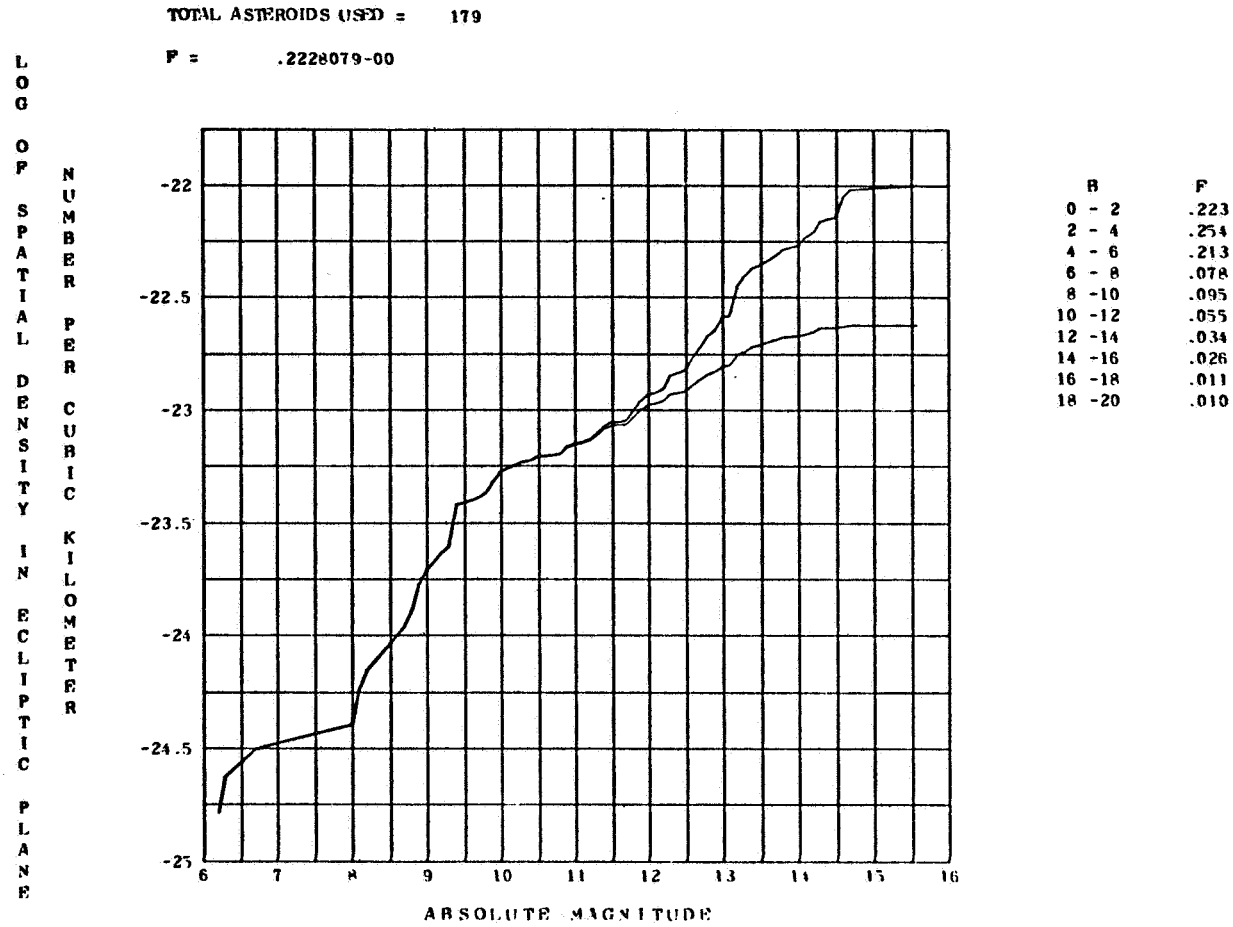


Figure 99. - Spatial density at R = 2.20 and at longitudes between 315.0 and 360.0.

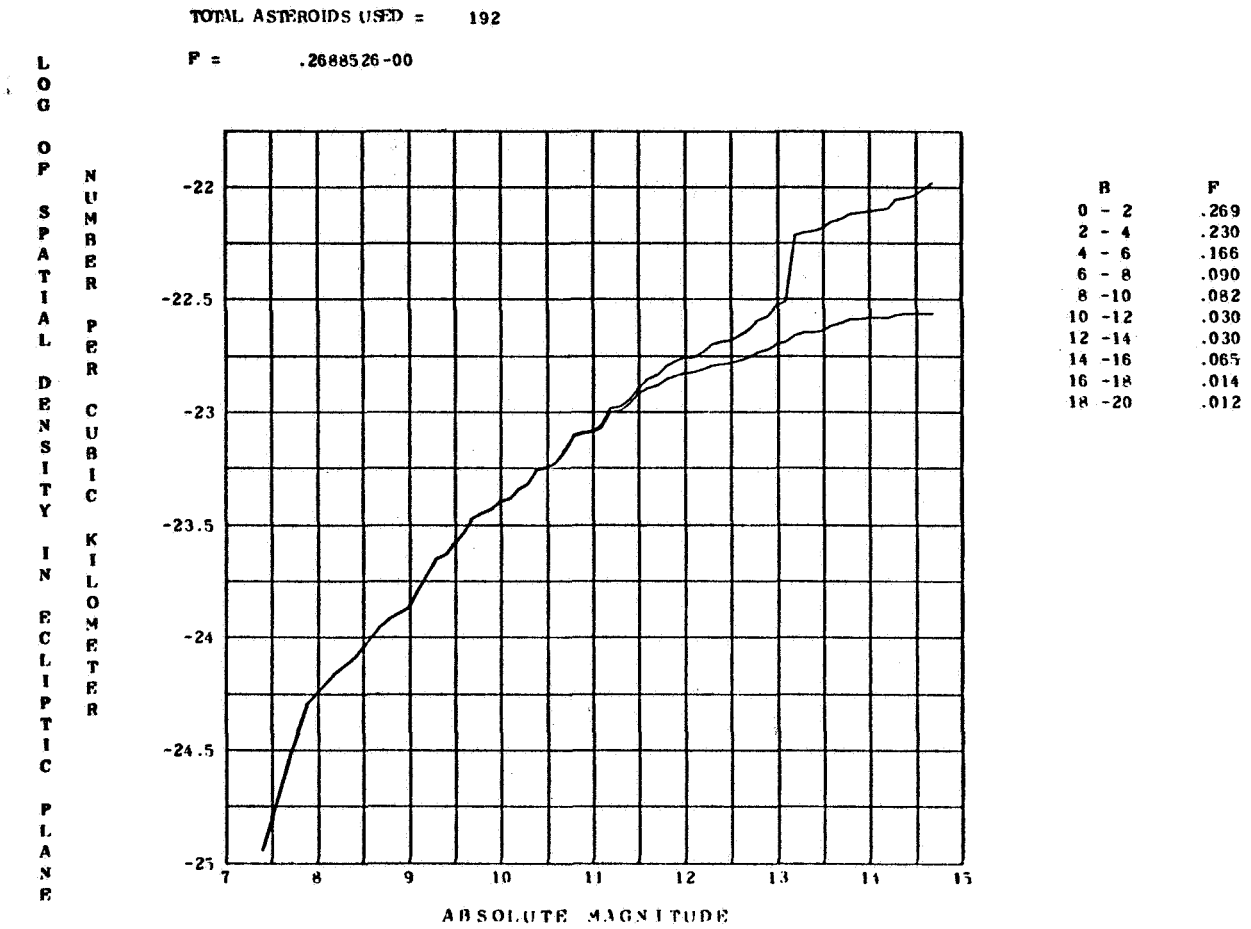


Figure 100. - Spatial density at R = 2.30 and at longitudes between 0 and 45.0.

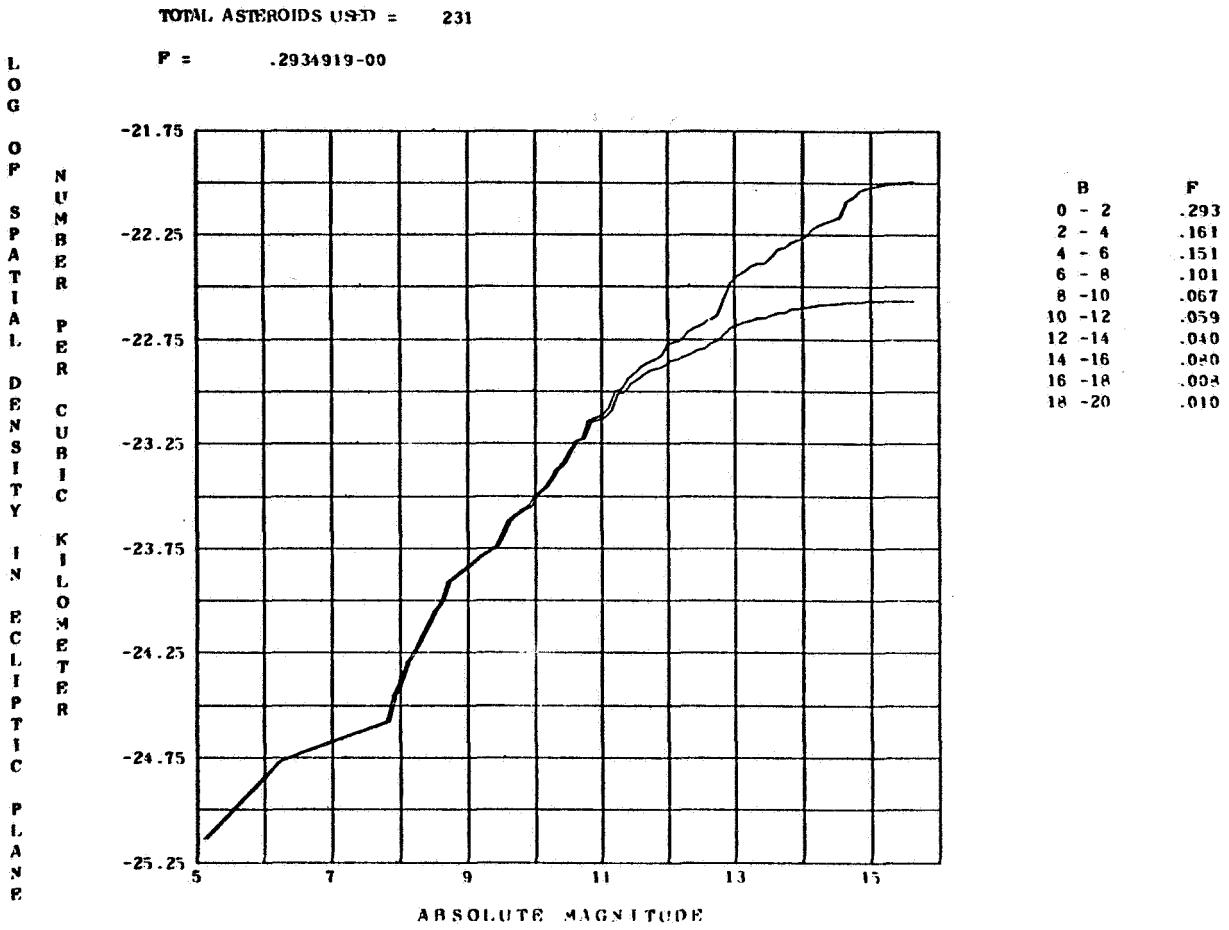


Figure 101. - Spatial density at R = 2.30 and at longitudes between 45.0 and 90.0.

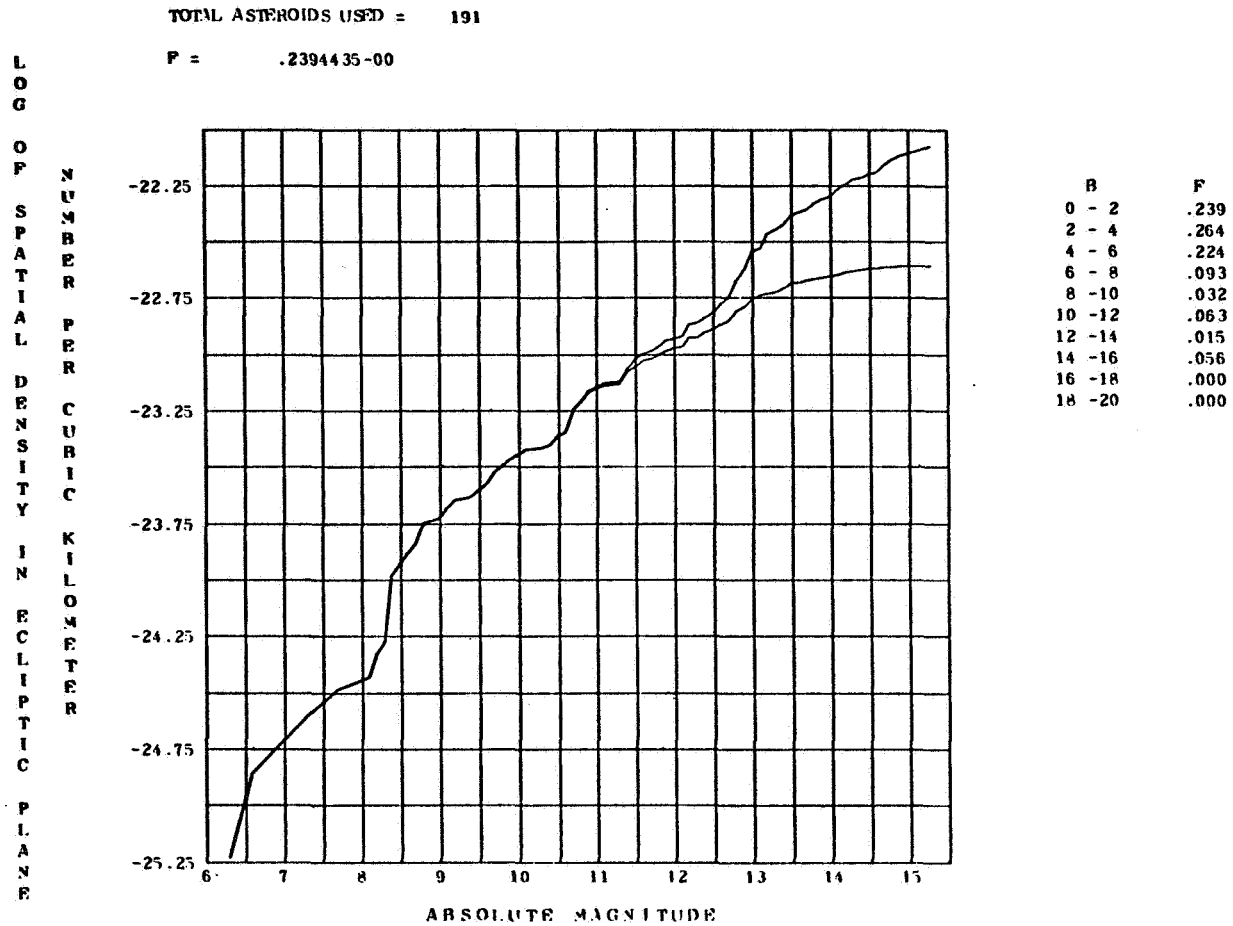


Figure 102. - Spatial density at R = 2.30 and at longitudes between 90.0 and 135.0.

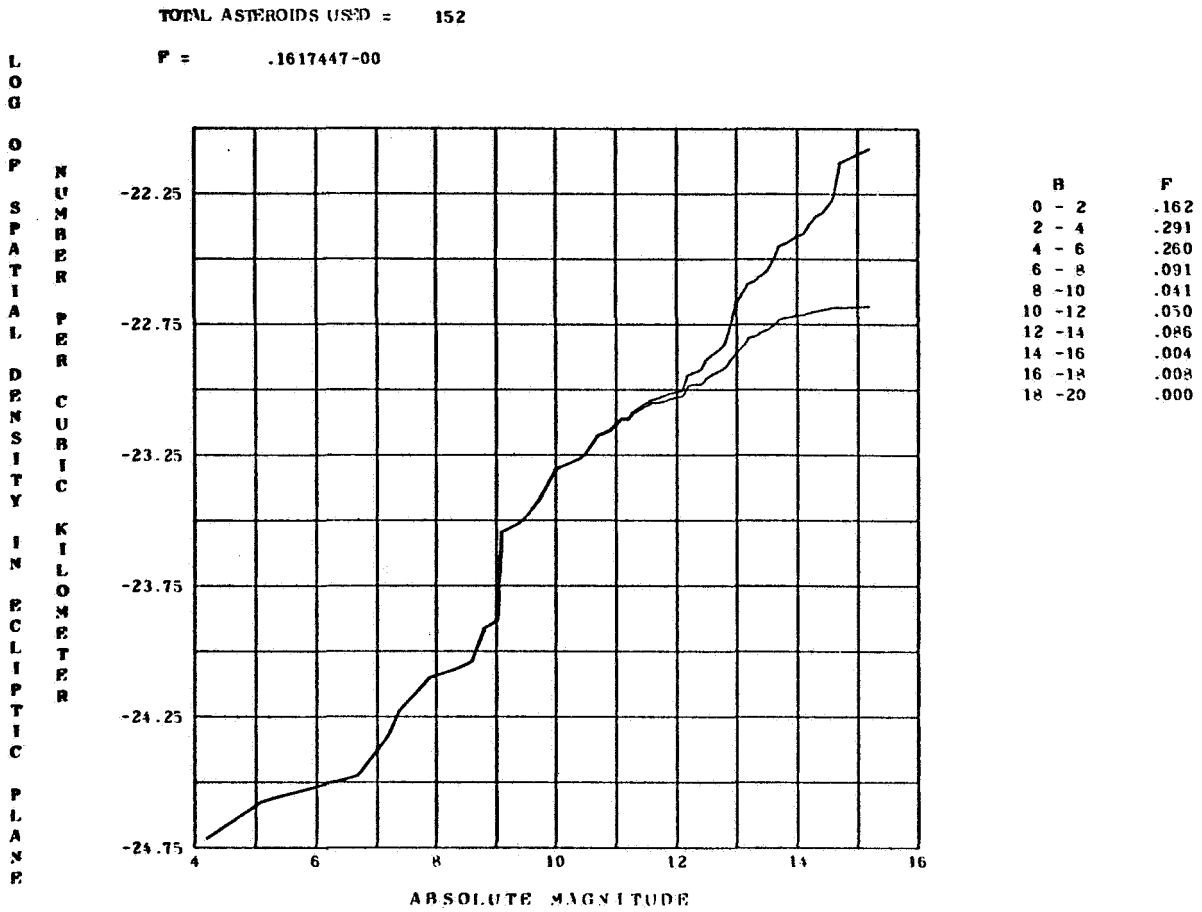


Figure 103. - Spatial density at R = 2.30 and at longitudes between 135.0 and 180.0.

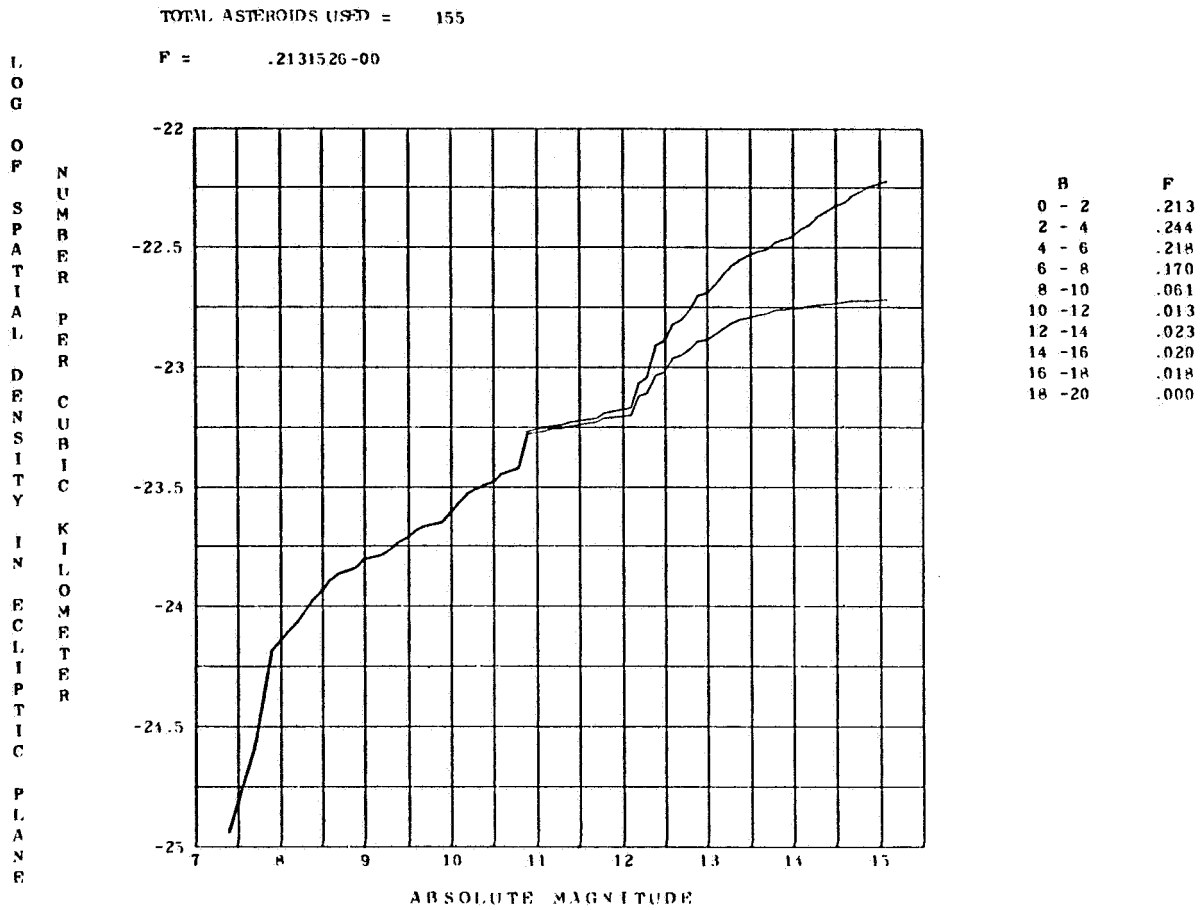


Figure 104. - Spatial density at R = 2.30 and at longitudes between 180.0 and 225.0.

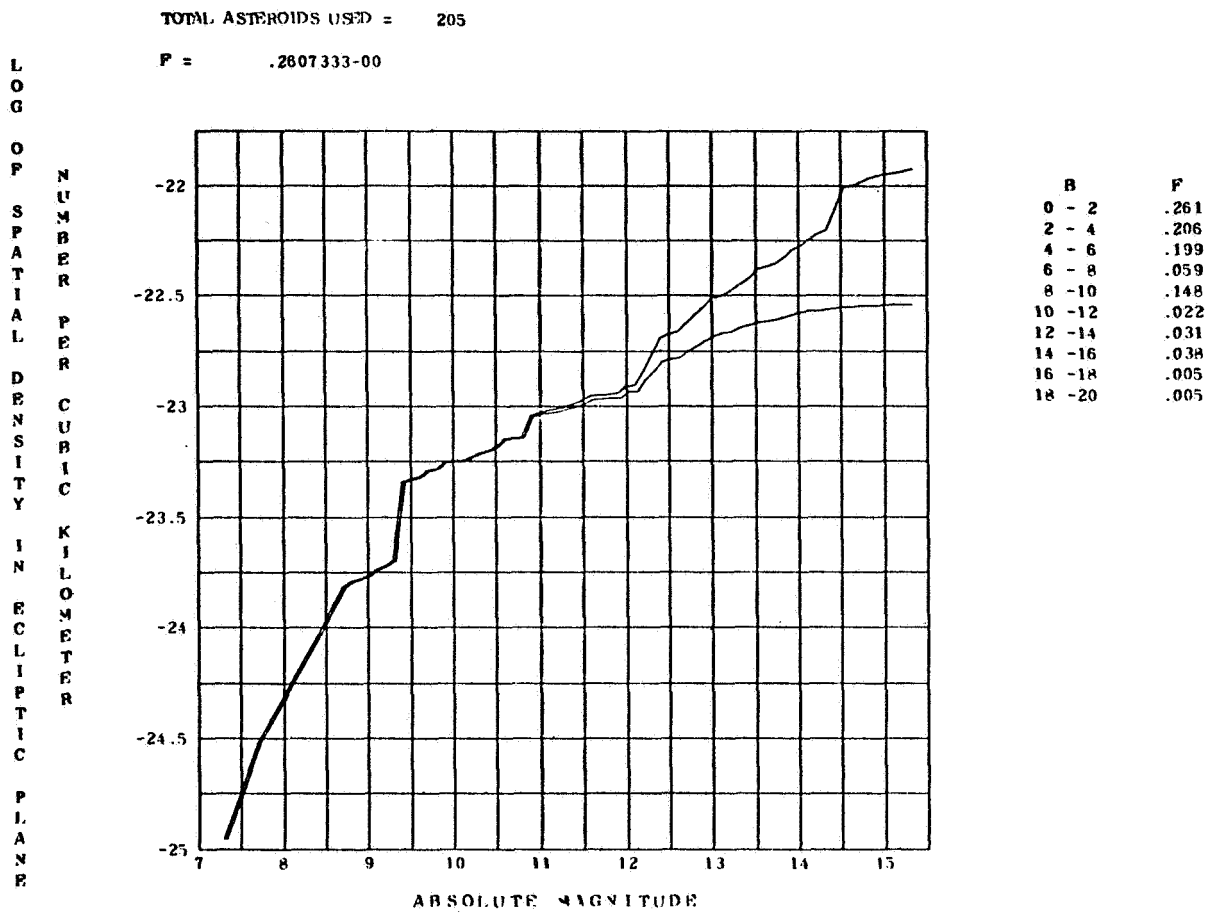


Figure 105. - Spatial density at R = 2.30 and at longitudes between 225.0 and 270.0.

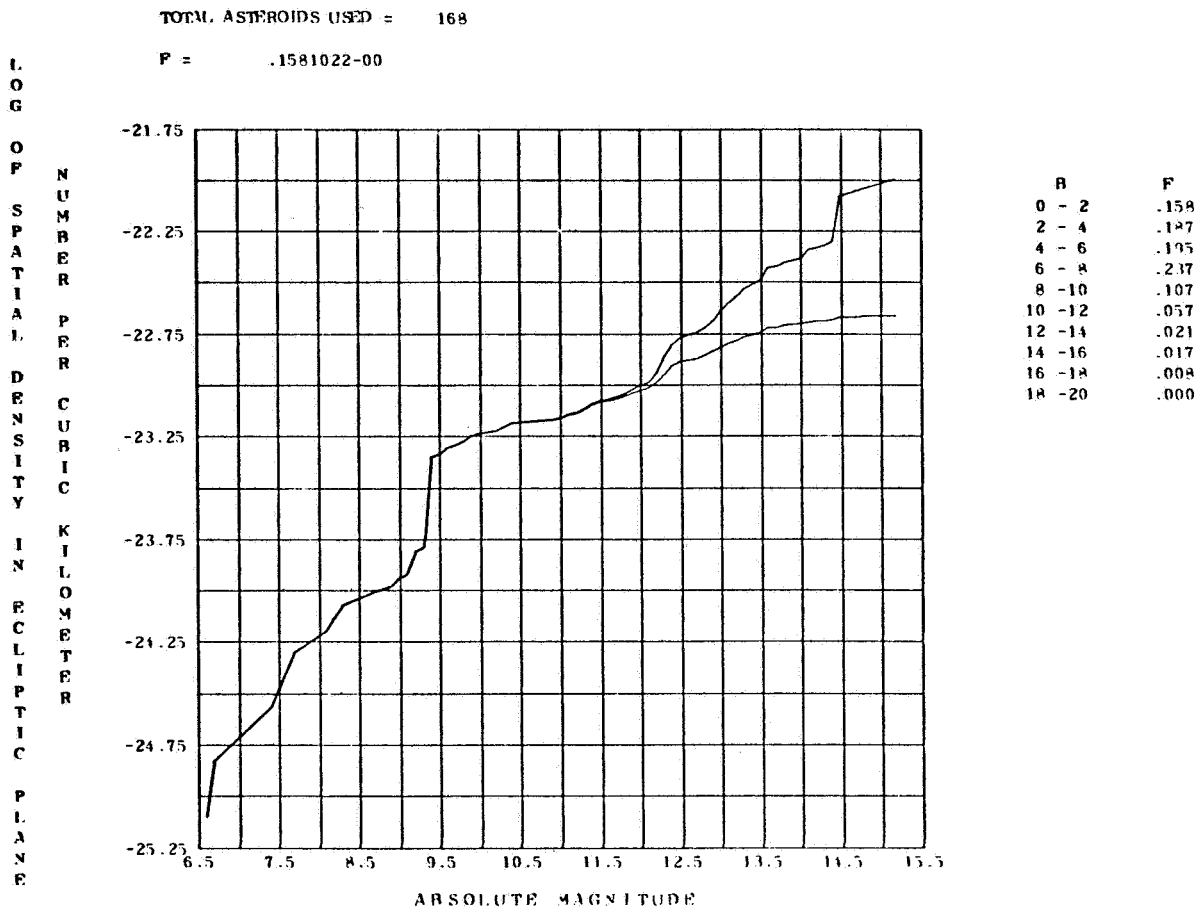


Figure 106. - Spatial density at R = 2.30 and at longitudes between 270.0 and 315.0.

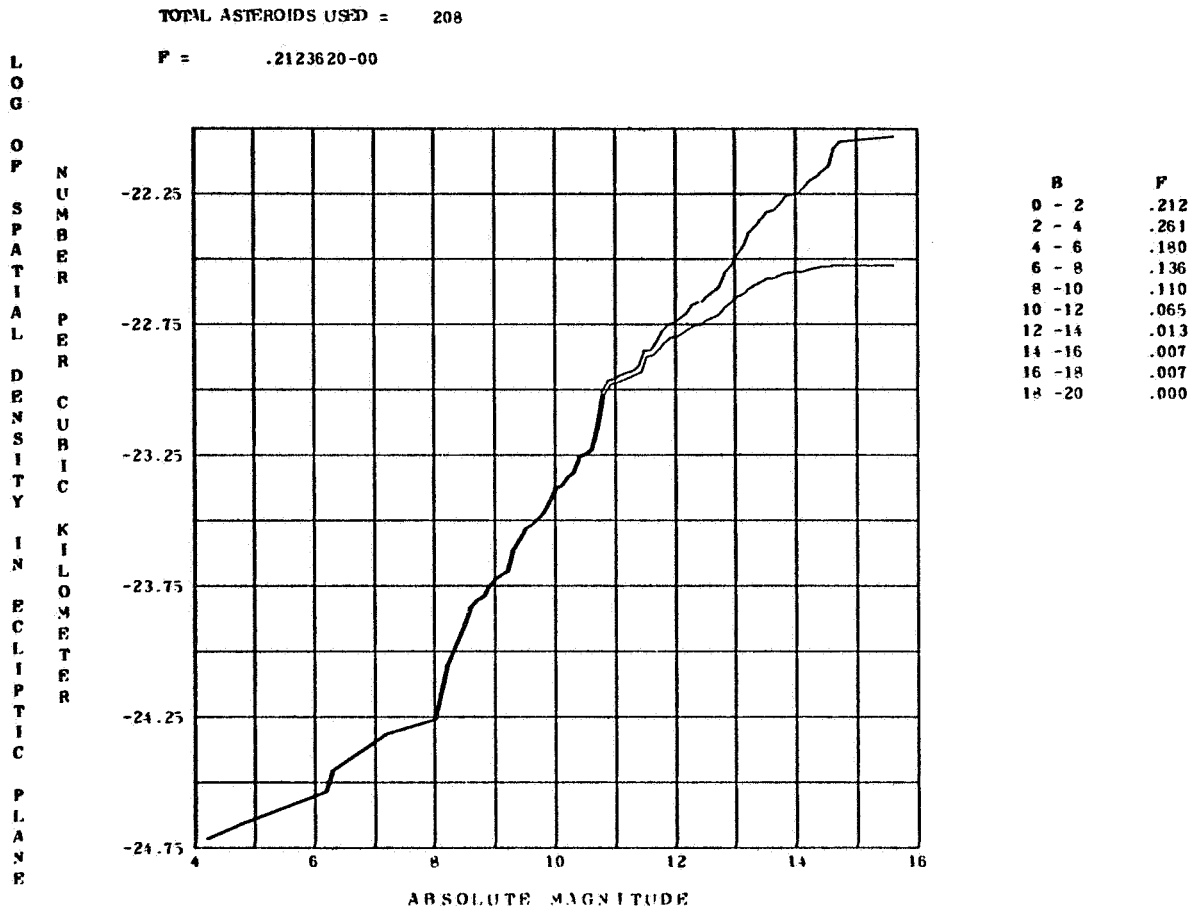


Figure 107. - Spatial density at R = 2.30 and at longitudes between 315.0 and 360.0.

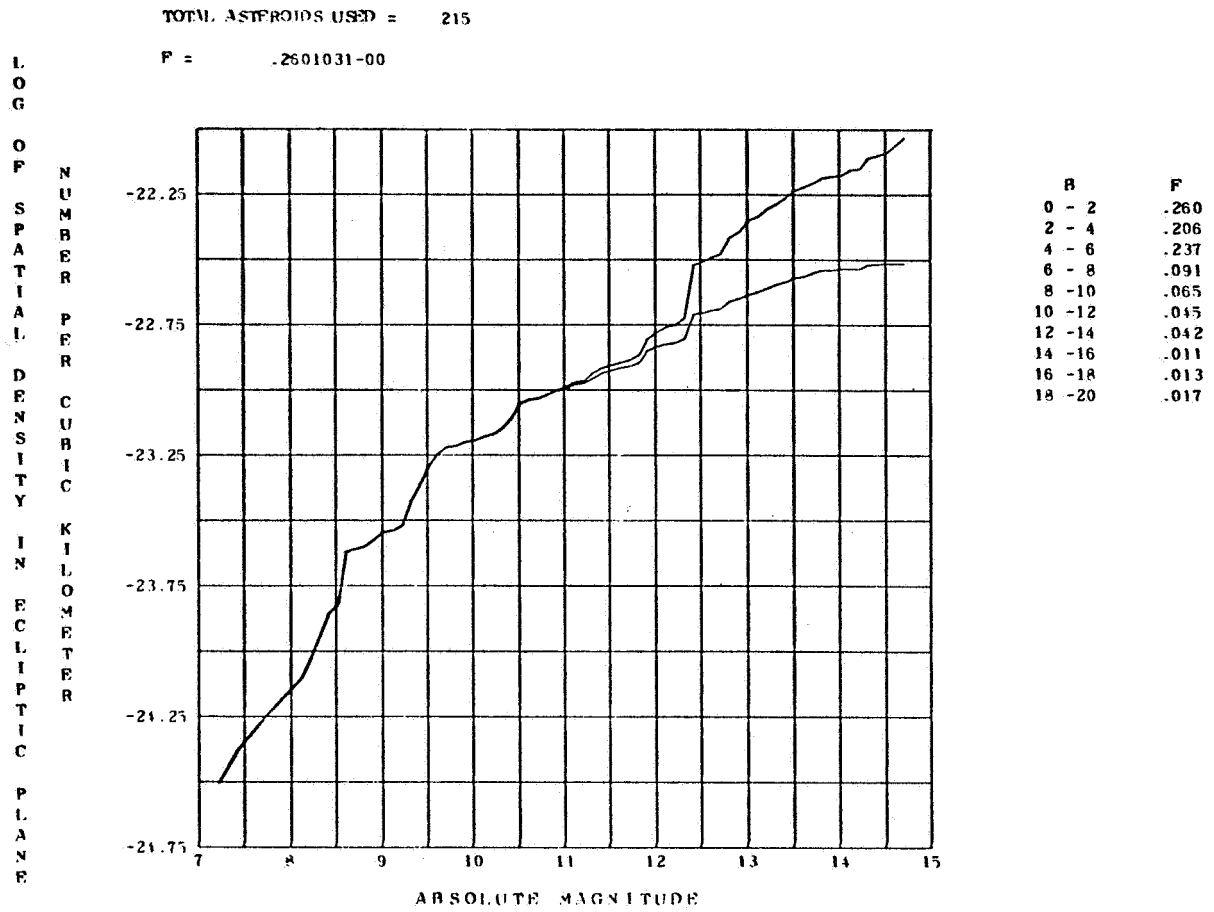


Figure 108. - Spatial density at R = 2.40 and at longitudes between 0 and 45.0.

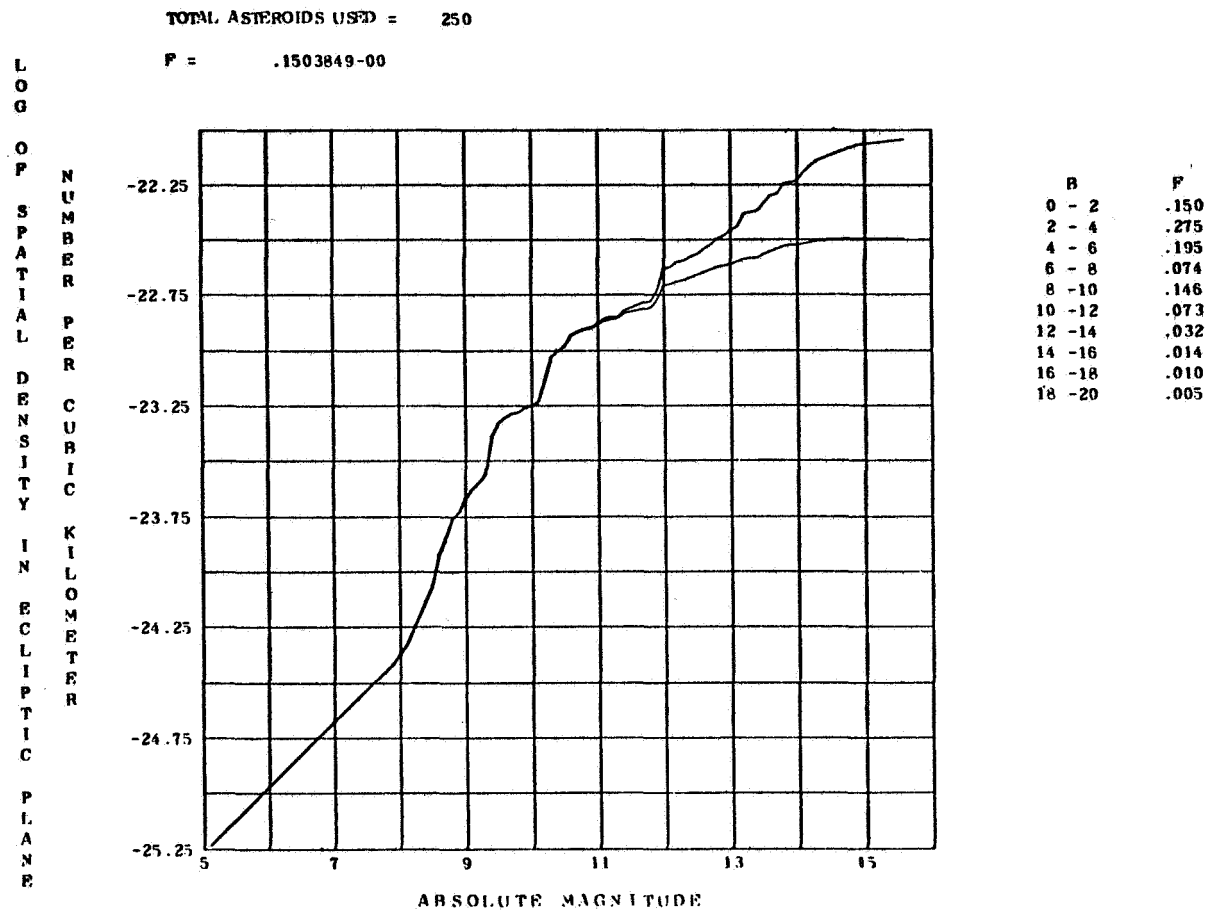


Figure 109. - Spatial density at R = 2.40 and at longitudes between 45.0 and 90.0.

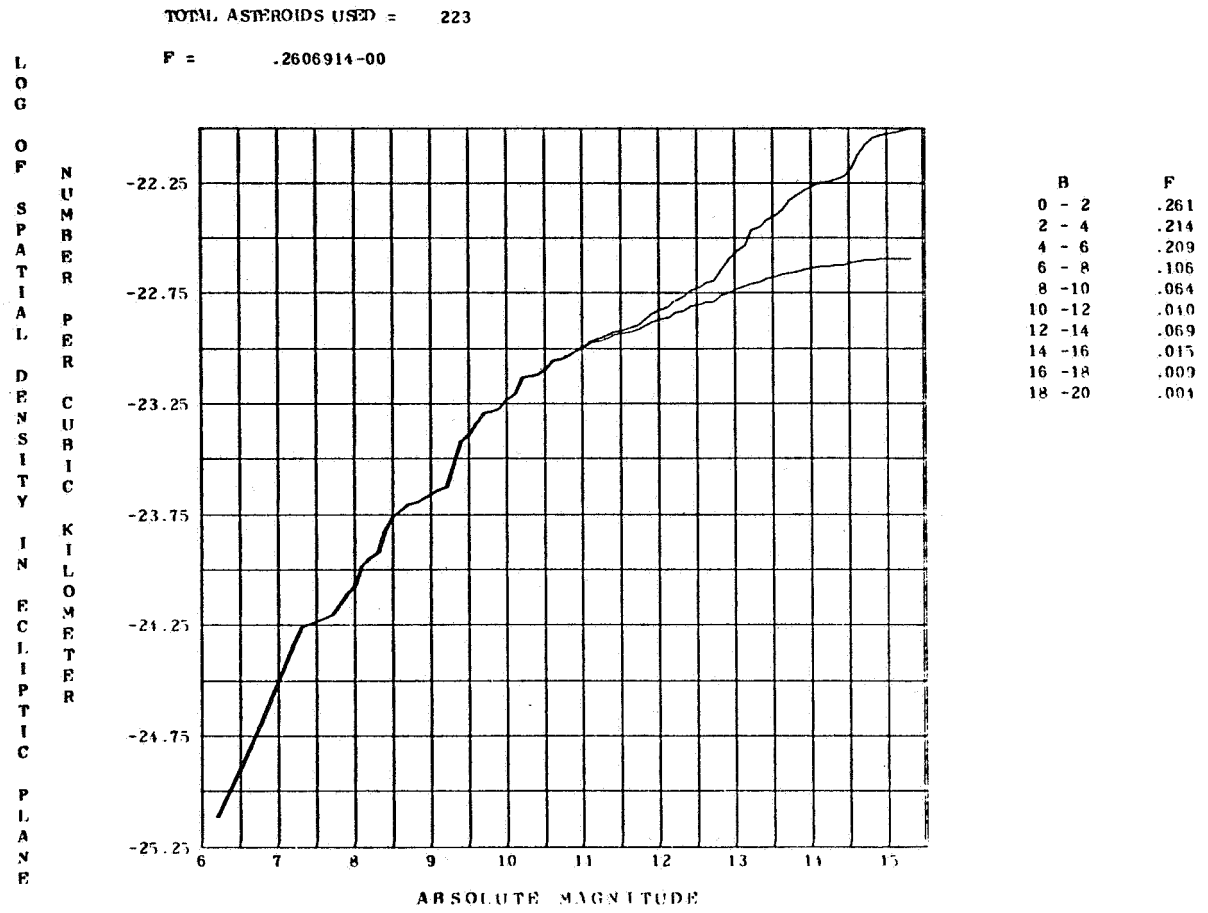


Figure 110. - Spatial density at R = 2.40 and at longitudes between 90.0 and 135.0.

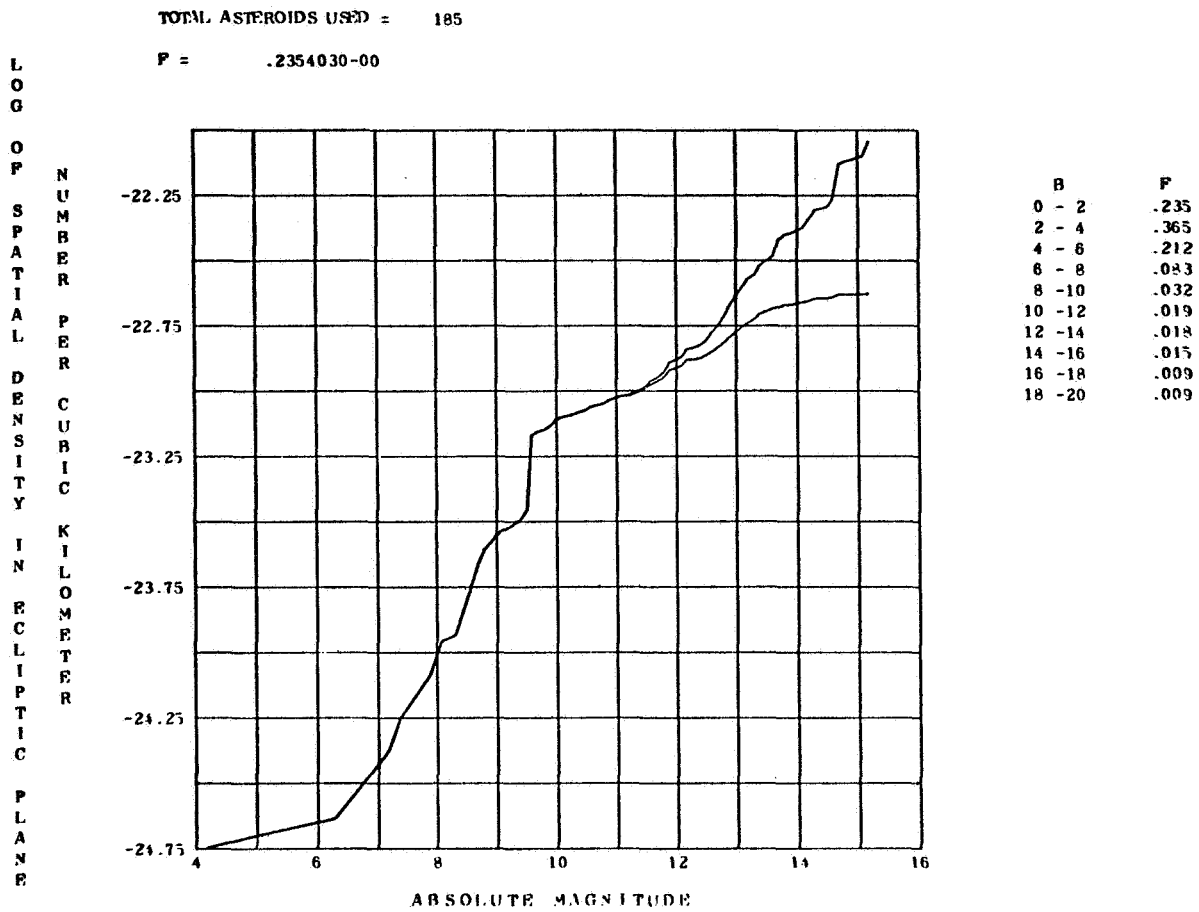


Figure 111. - Spatial density at R = 2.40 and at longitudes between 135.0 and 180.0.

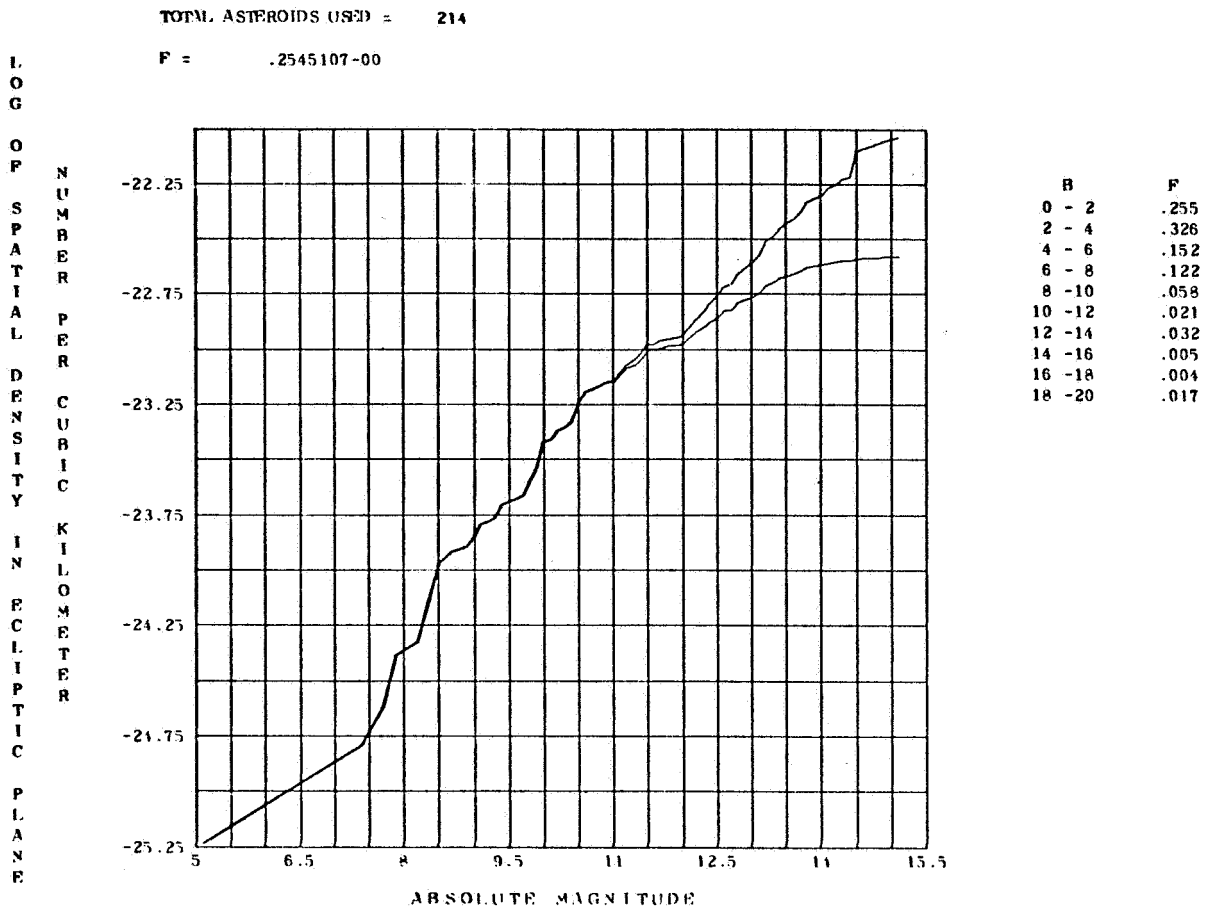


Figure 112. - Spatial density at R = 2.40 and at longitudes between 180.0 and 225.0.

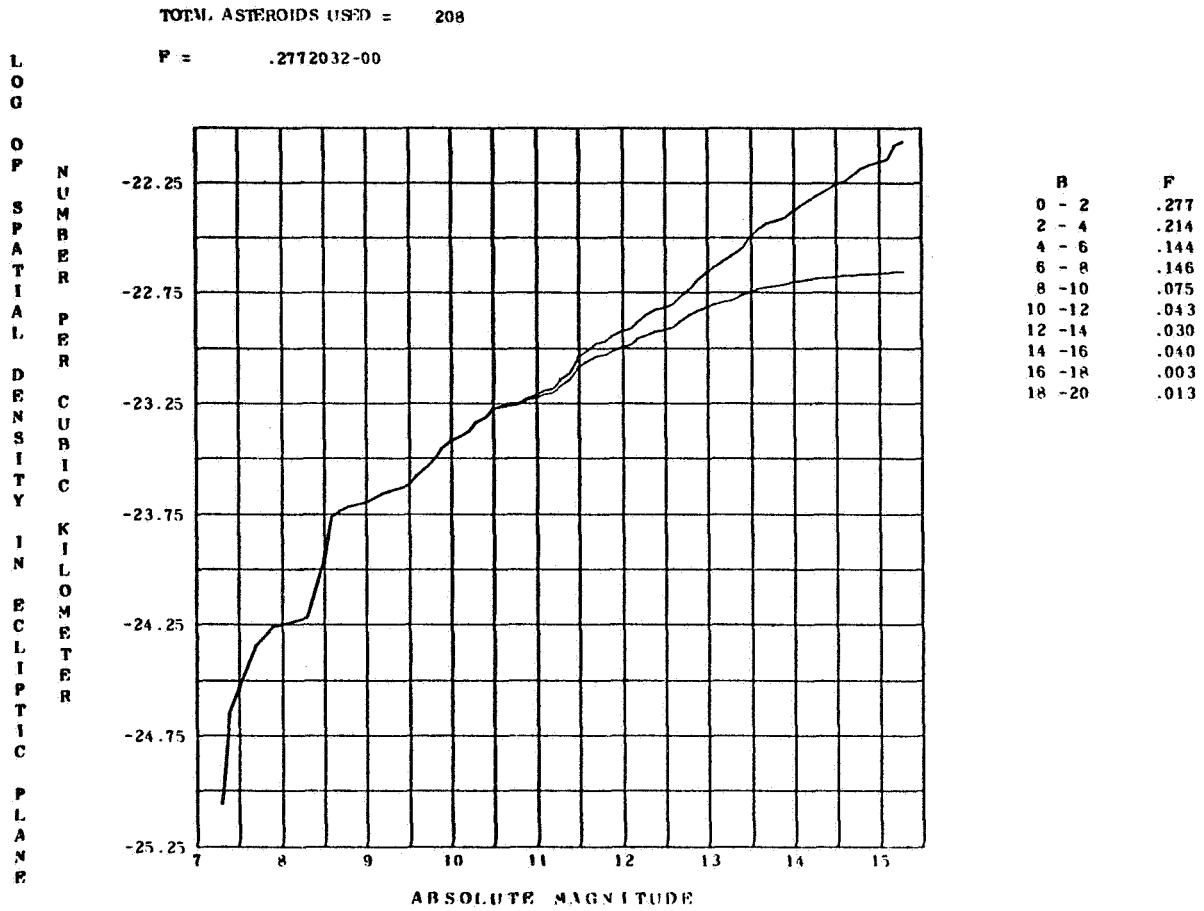


Figure 113. - Spatial density at R = 2.40 and at longitudes between 225.0 and 270.0.

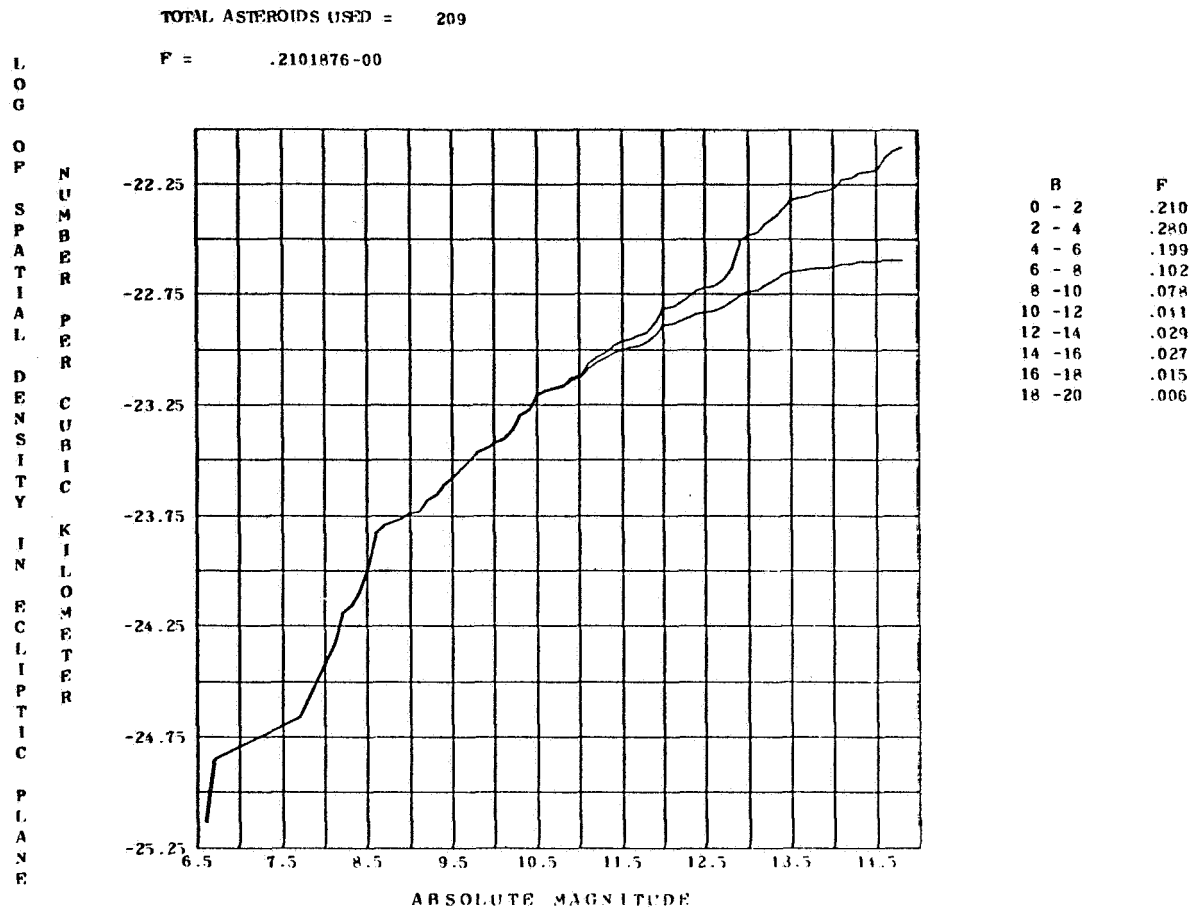


Figure 114. - Spatial density at R = 2.40 and at longitudes between 270.0 and 315.0.

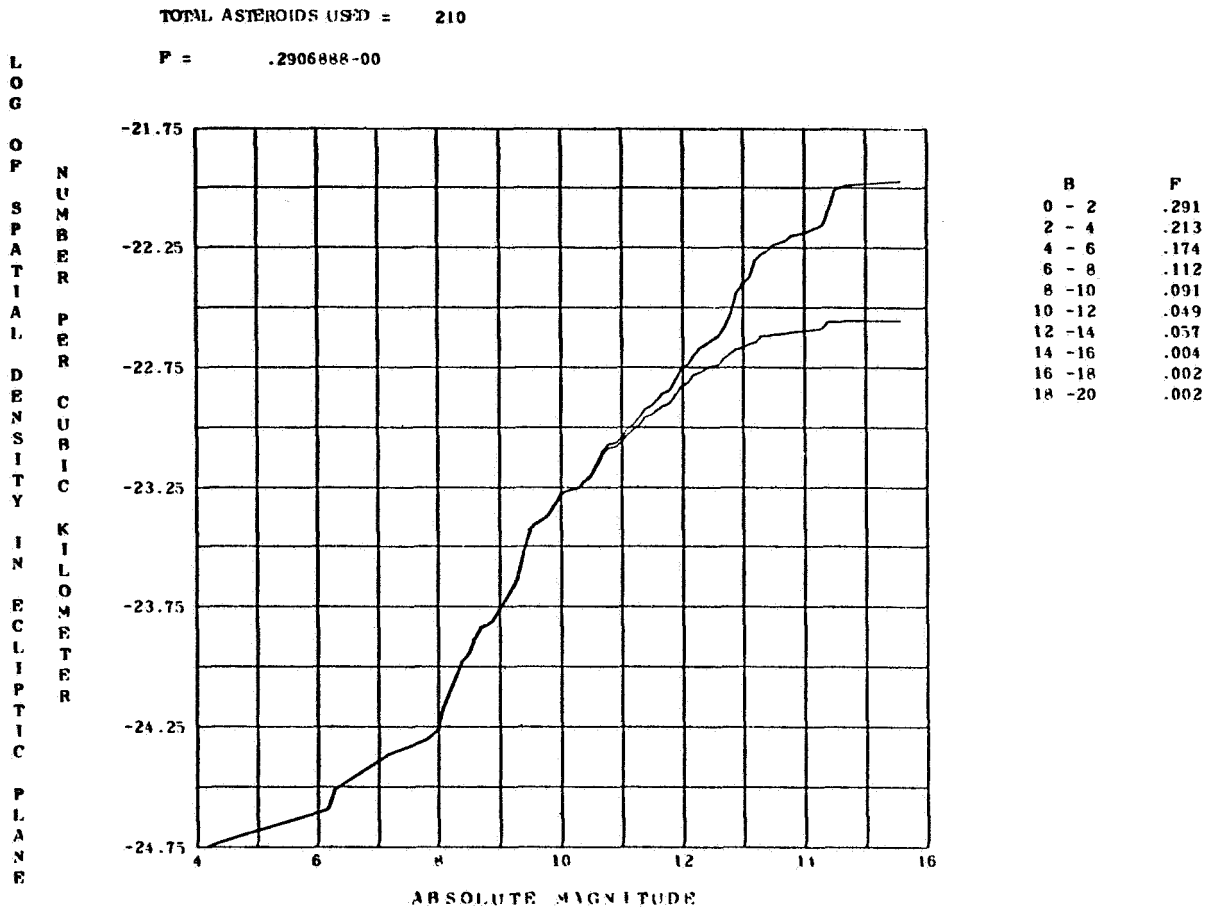


Figure 115. - Spatial density at R = 2.40 and at longitudes between 315.0 and 360.0.

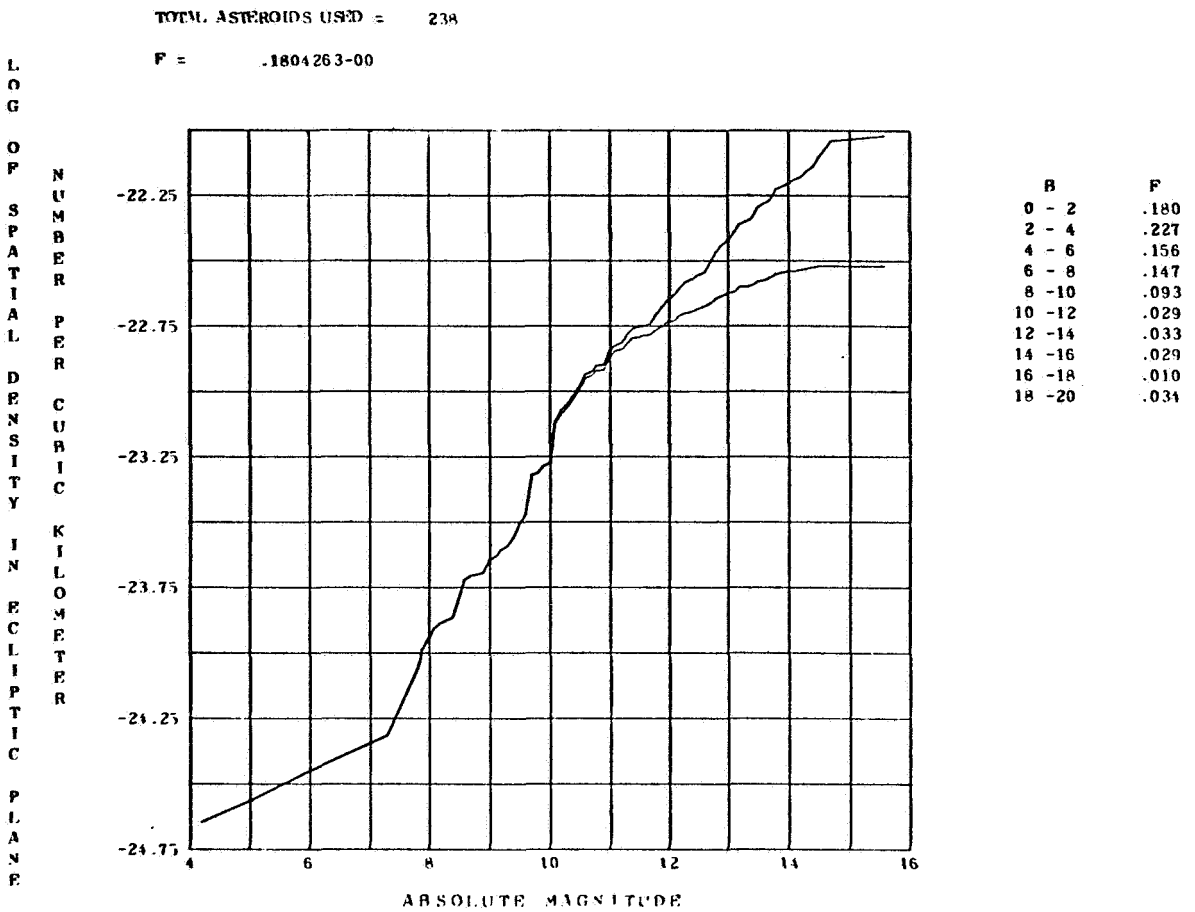


Figure 116. - Spatial density at R = 2.50 and at longitudes between 0 and 45.0.

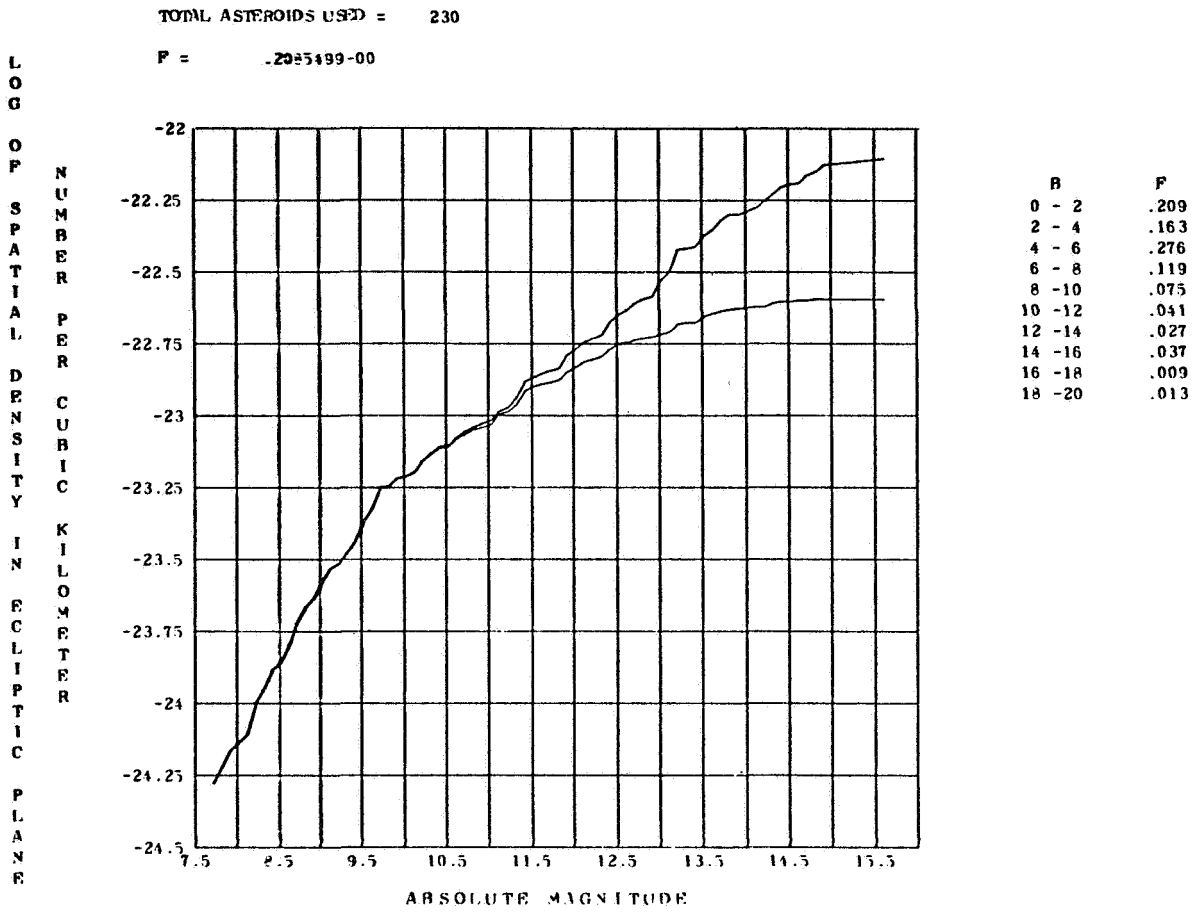


Figure 117. - Spatial density at R = 2.50 and at longitudes between 45.0 and 90.0.

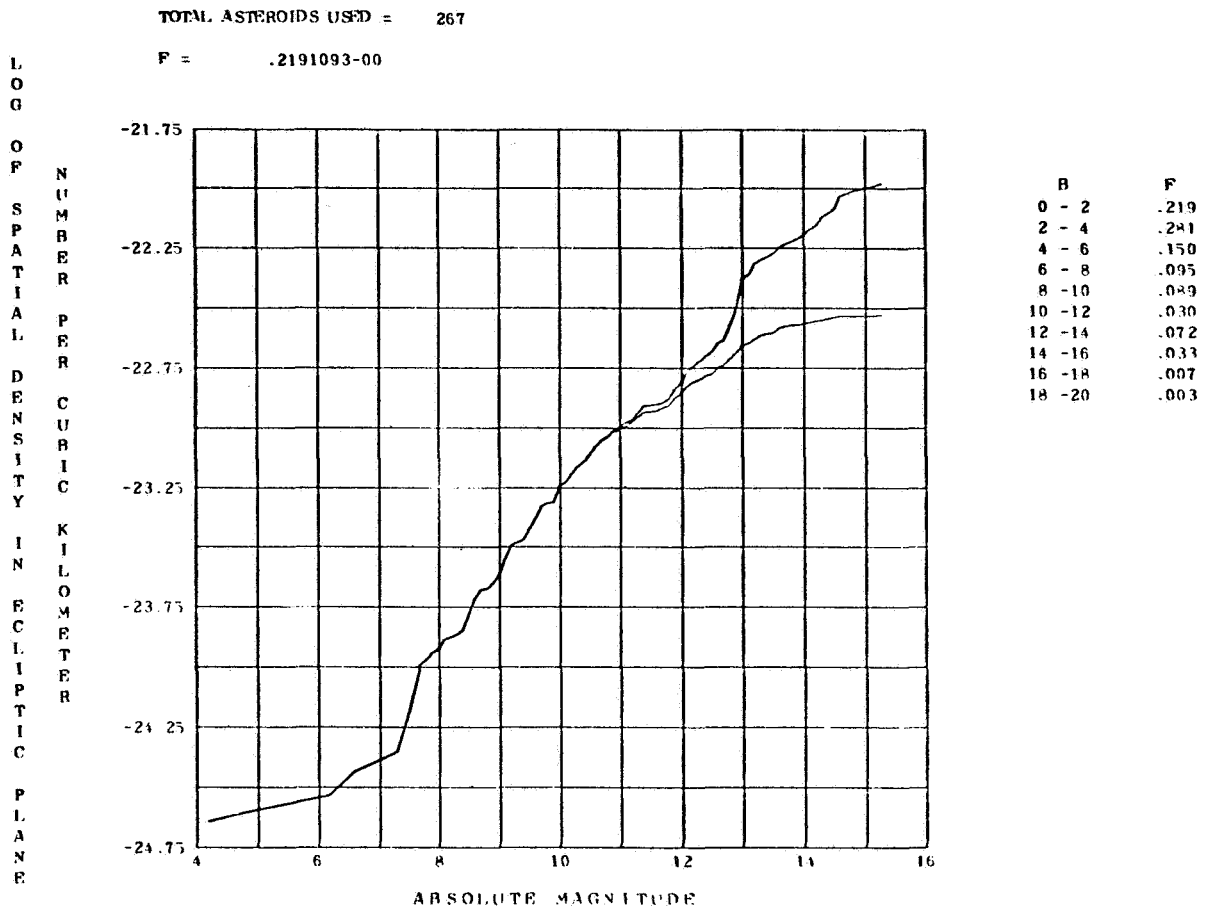


Figure 118. - Spatial density at R = 2.50 and at longitudes between 90.0 and 135.0.

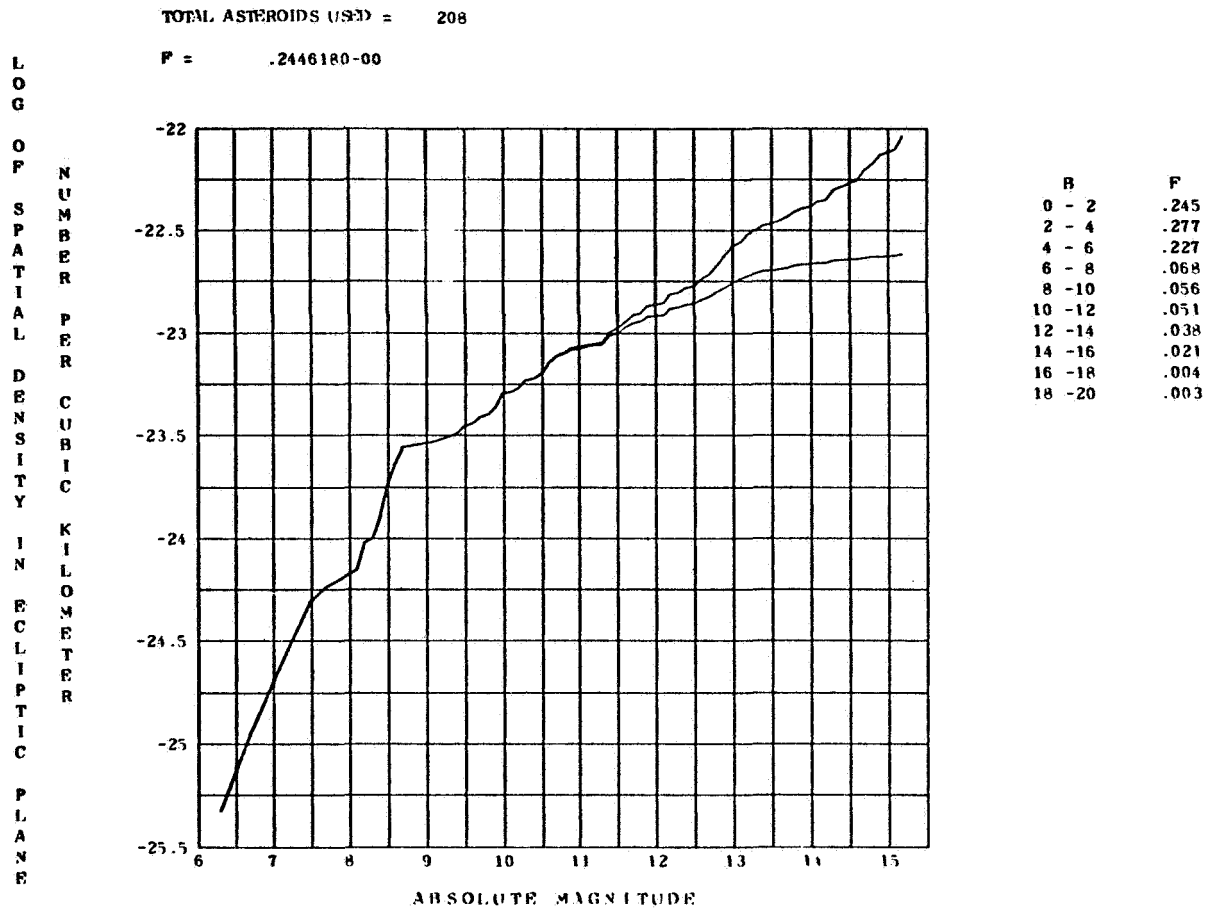


Figure 119. - Spatial density at R = 2.50 and at longitudes between 135.0 and 180.0.

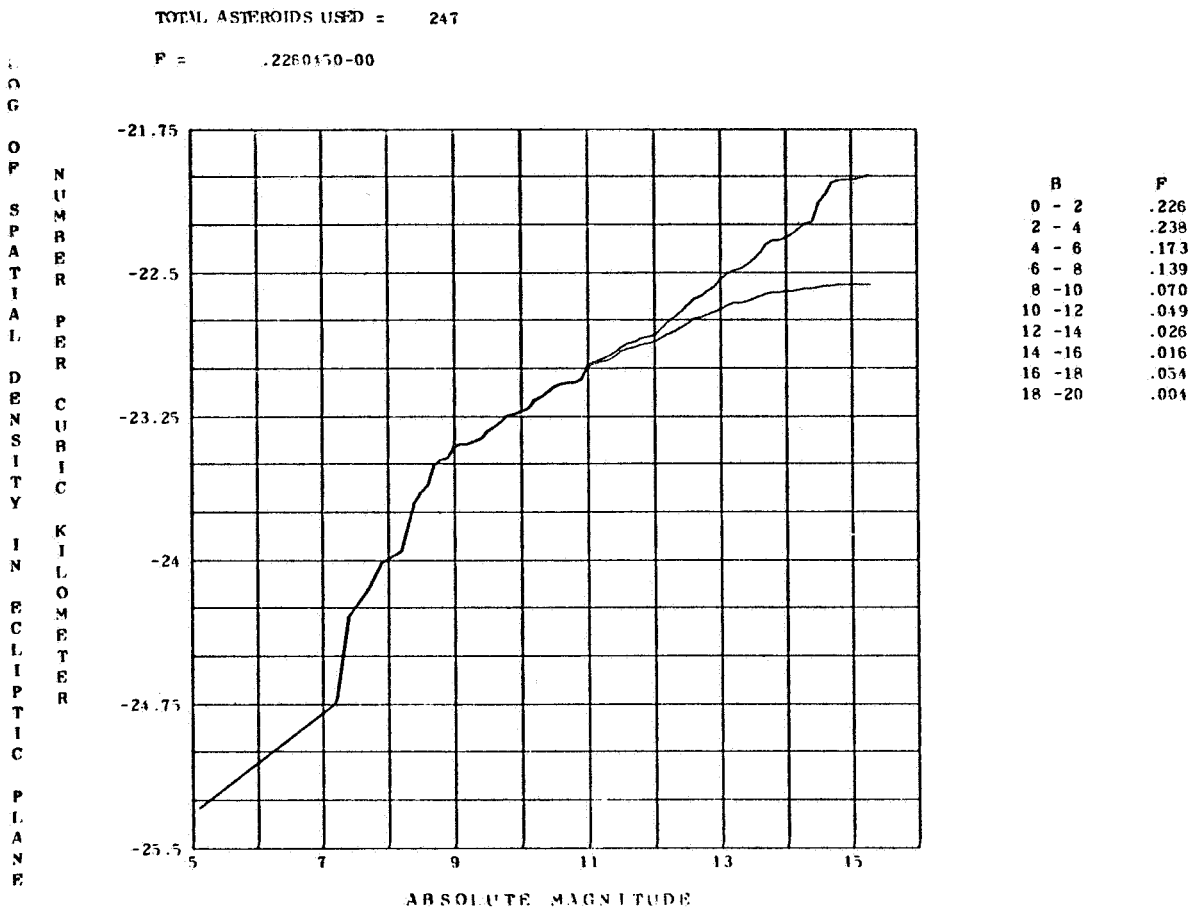


Figure 120. - Spatial density at R = 2.50 and at longitudes between 180.0 and 225.0.

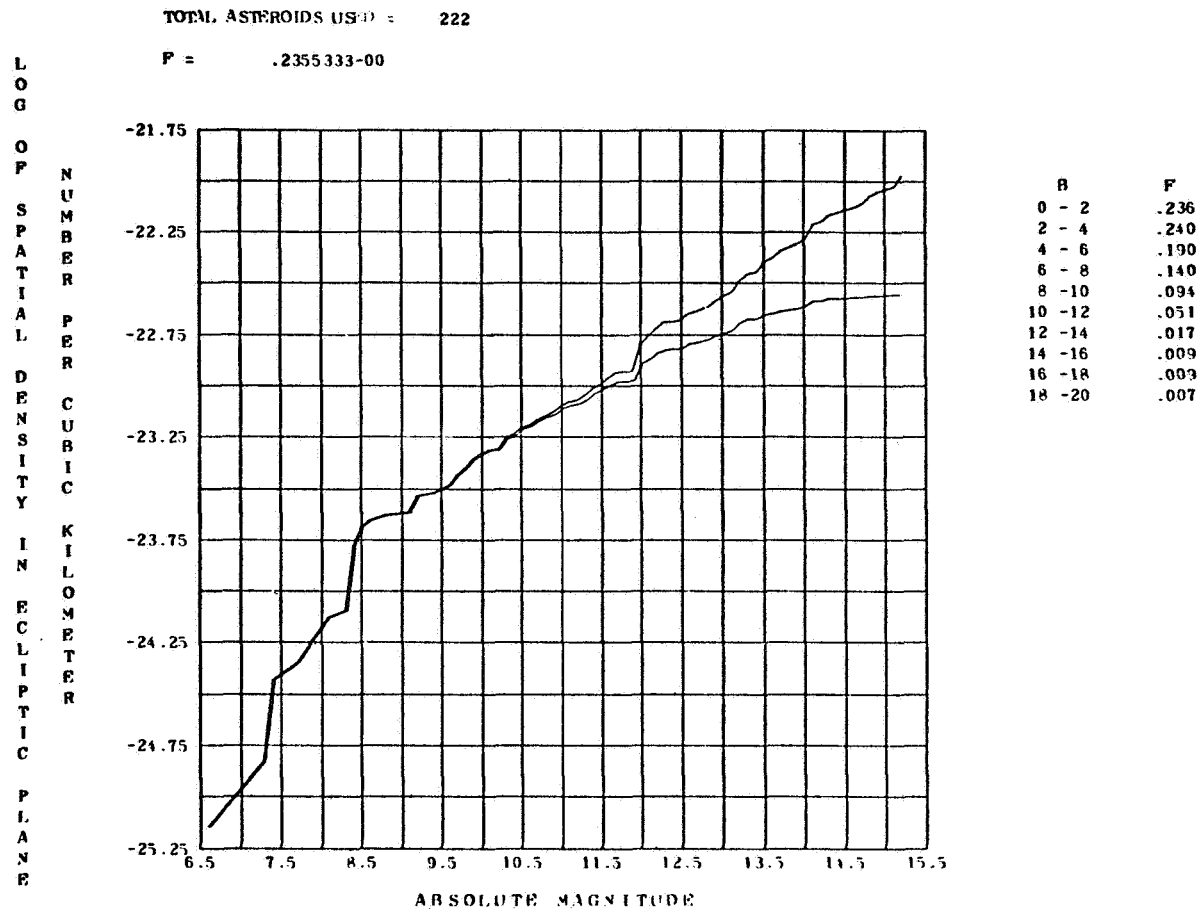


Figure 121. - Spatial density at R = 2.50 and at longitudes between 225.0 and 270.0.

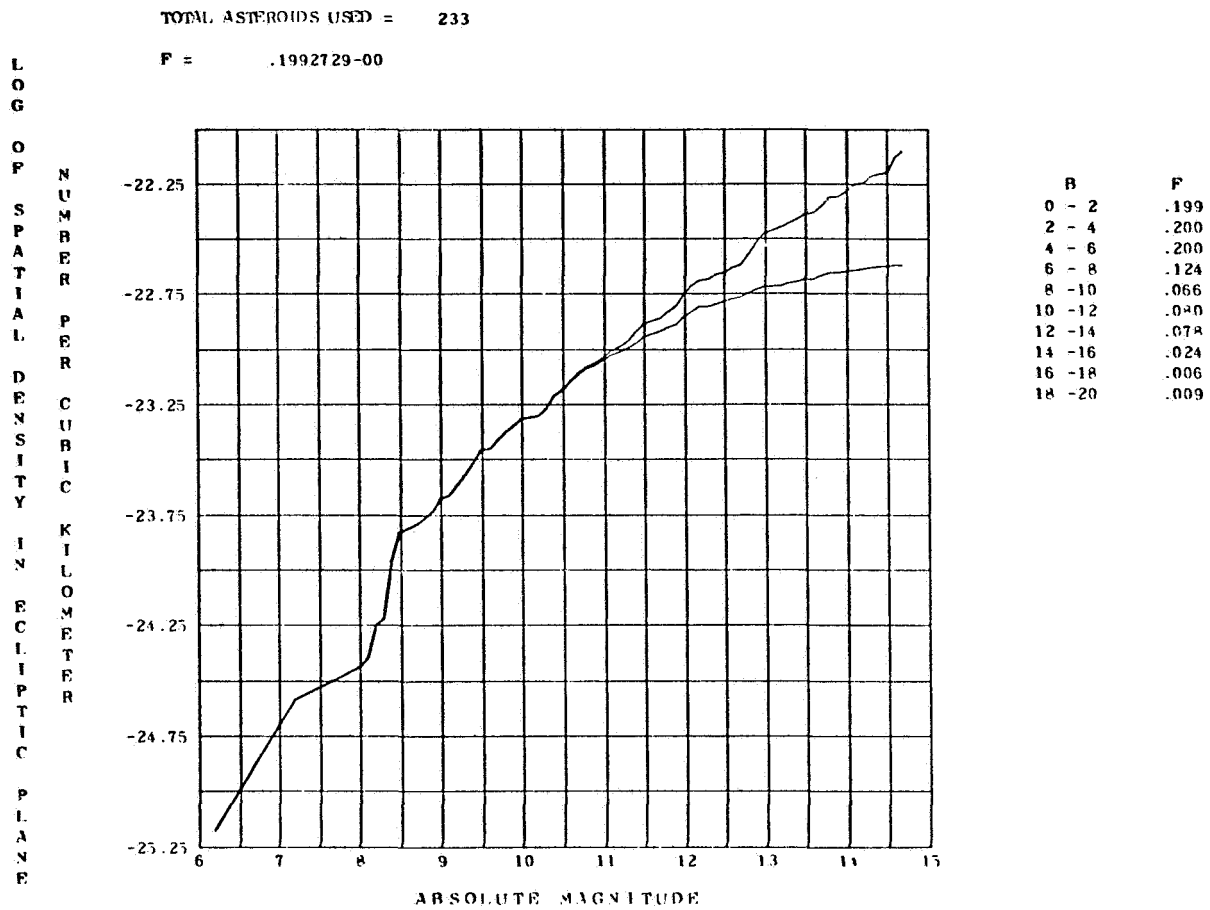


Figure 122. - Spatial density at R = 2.50 and at longitudes between 270.0 and 315.0.

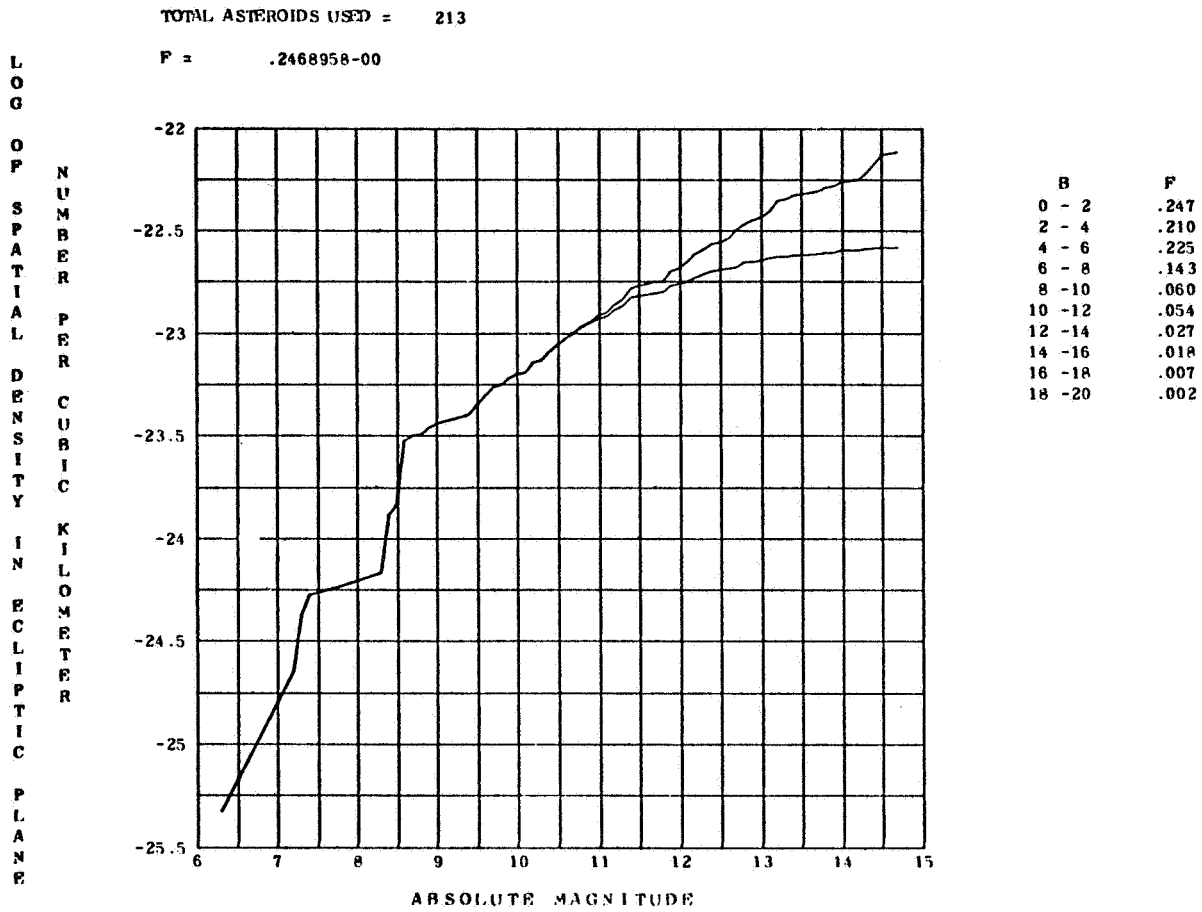


Figure 123. - Spatial density at R = 2.50 and at longitudes between 315.0 and 360.0.

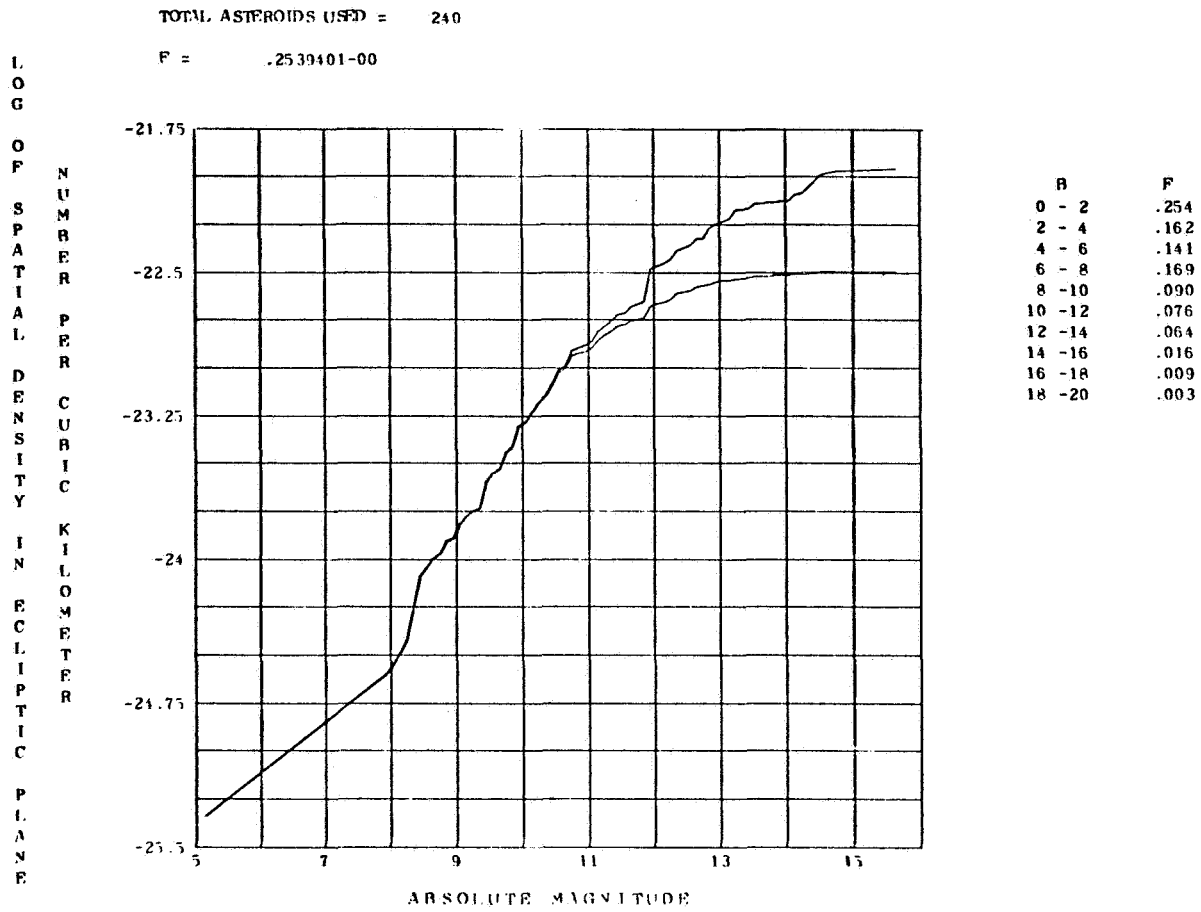


Figure 124. - Spatial density at R = 2.60 and at longitudes between 0 and 45.0.

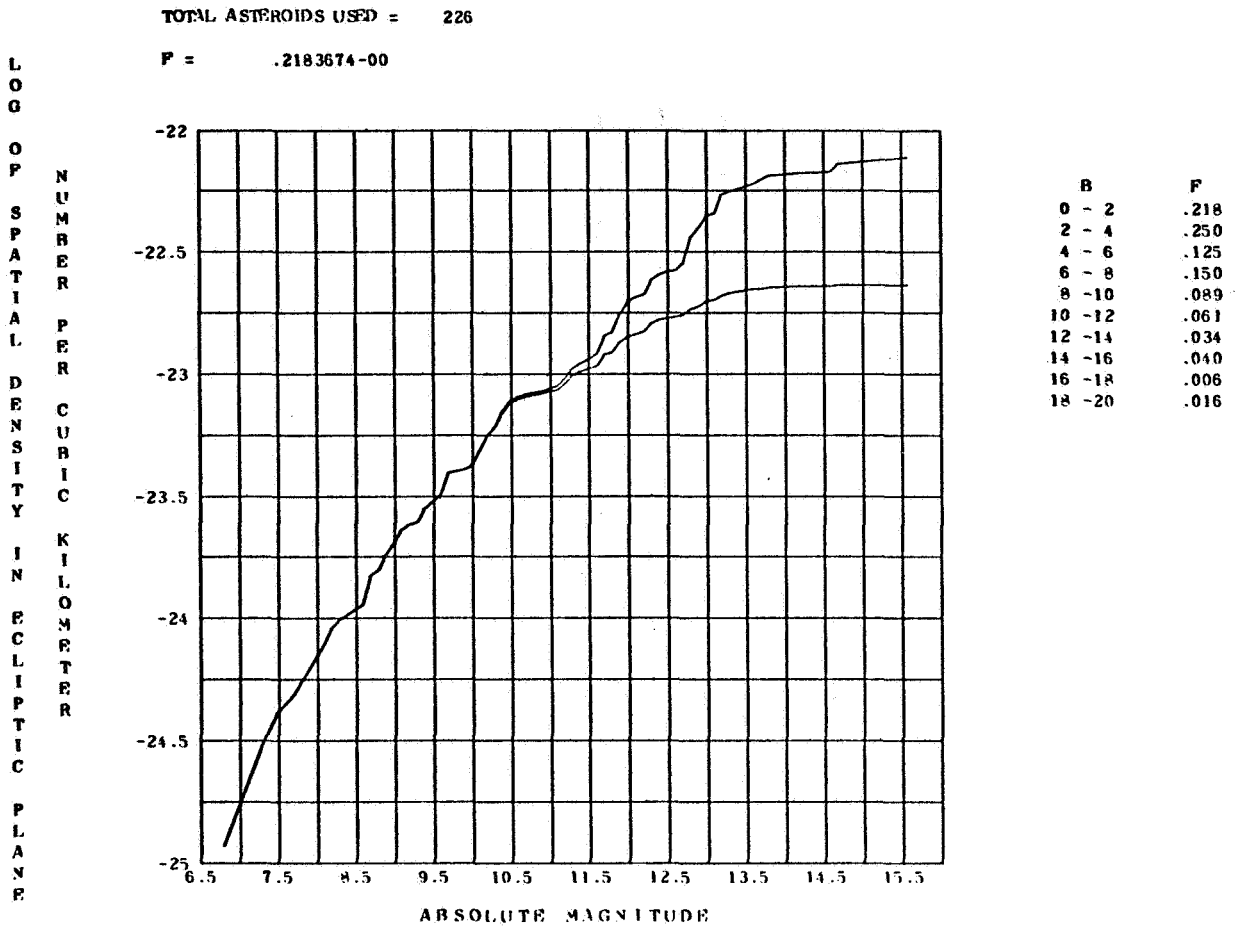


Figure 125. - Spatial density at R = 2.60 and at longitudes between 45.0 and 90.0.

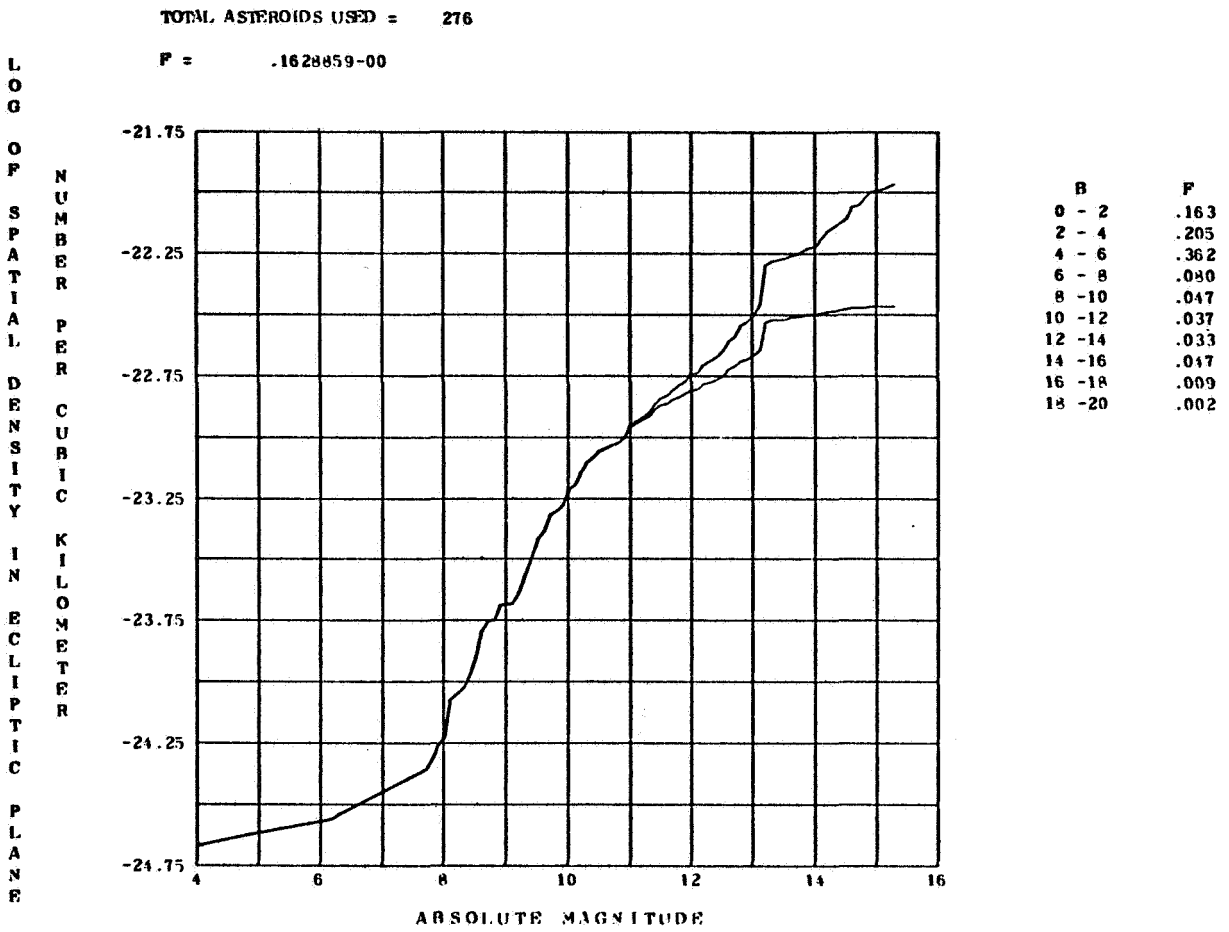


Figure 126. - Spatial density at R = 2.60 and at longitudes between 90.0 and 135.0.

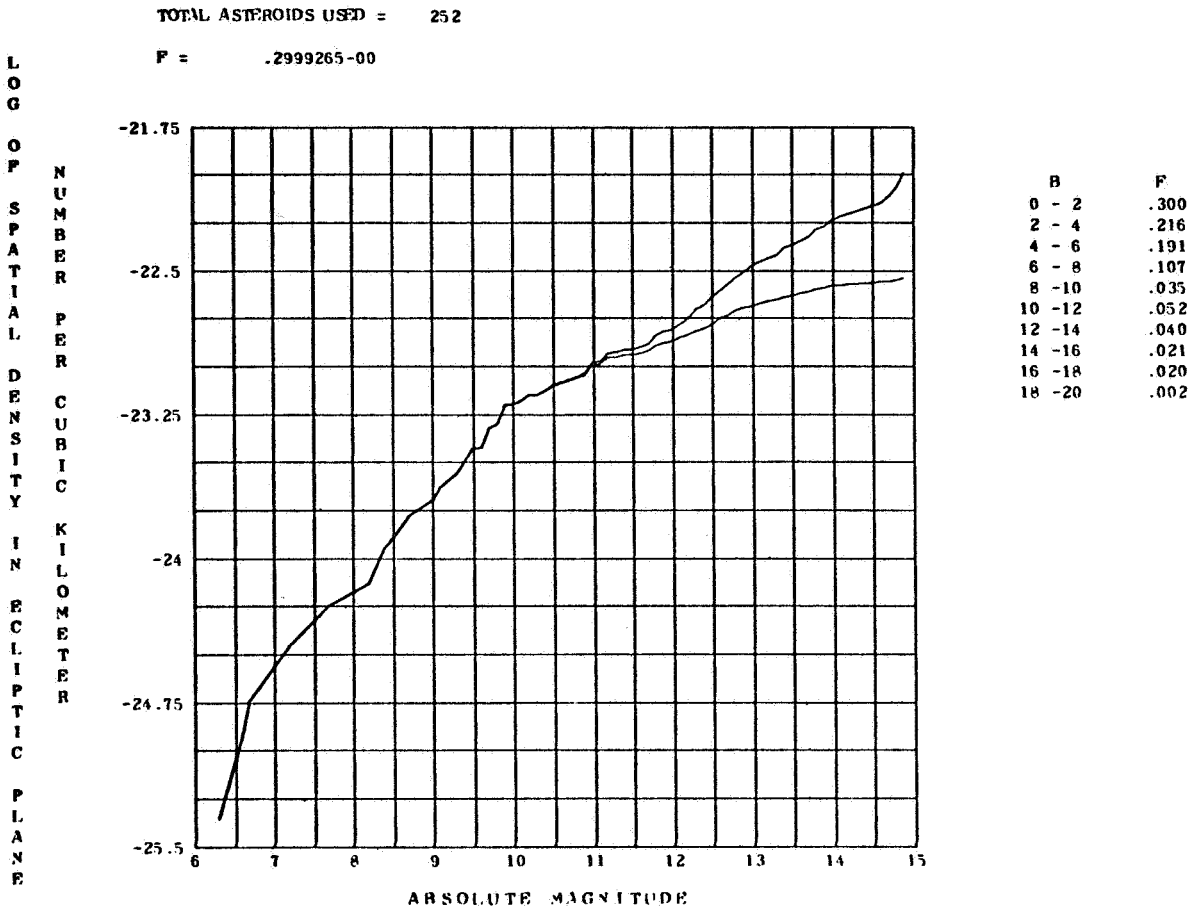


Figure 127. - Spatial density at R = 2.60 and at longitudes between 135.0 and 180.0.

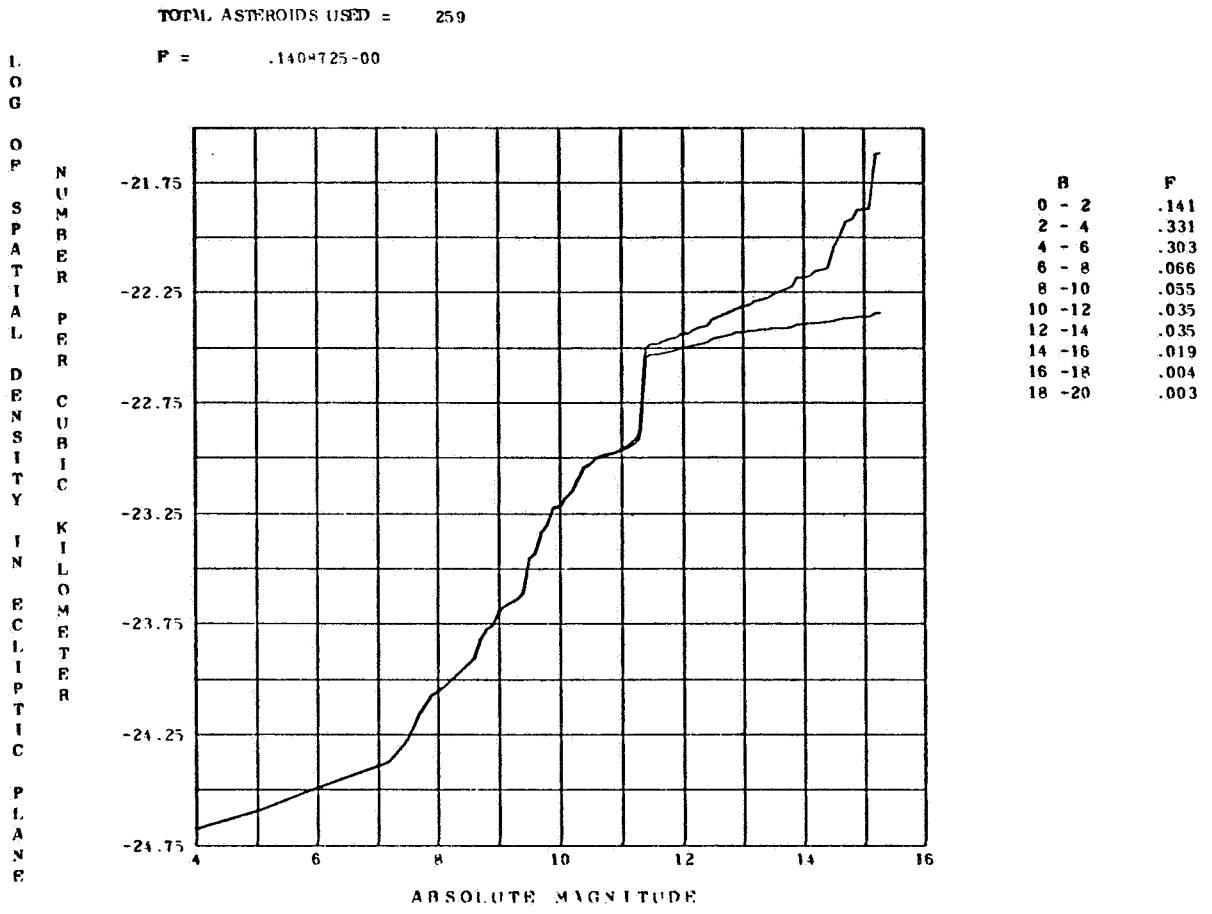


Figure 128. - Spatial density at R = 2.60 and at longitudes between 180.0 and 225.0.

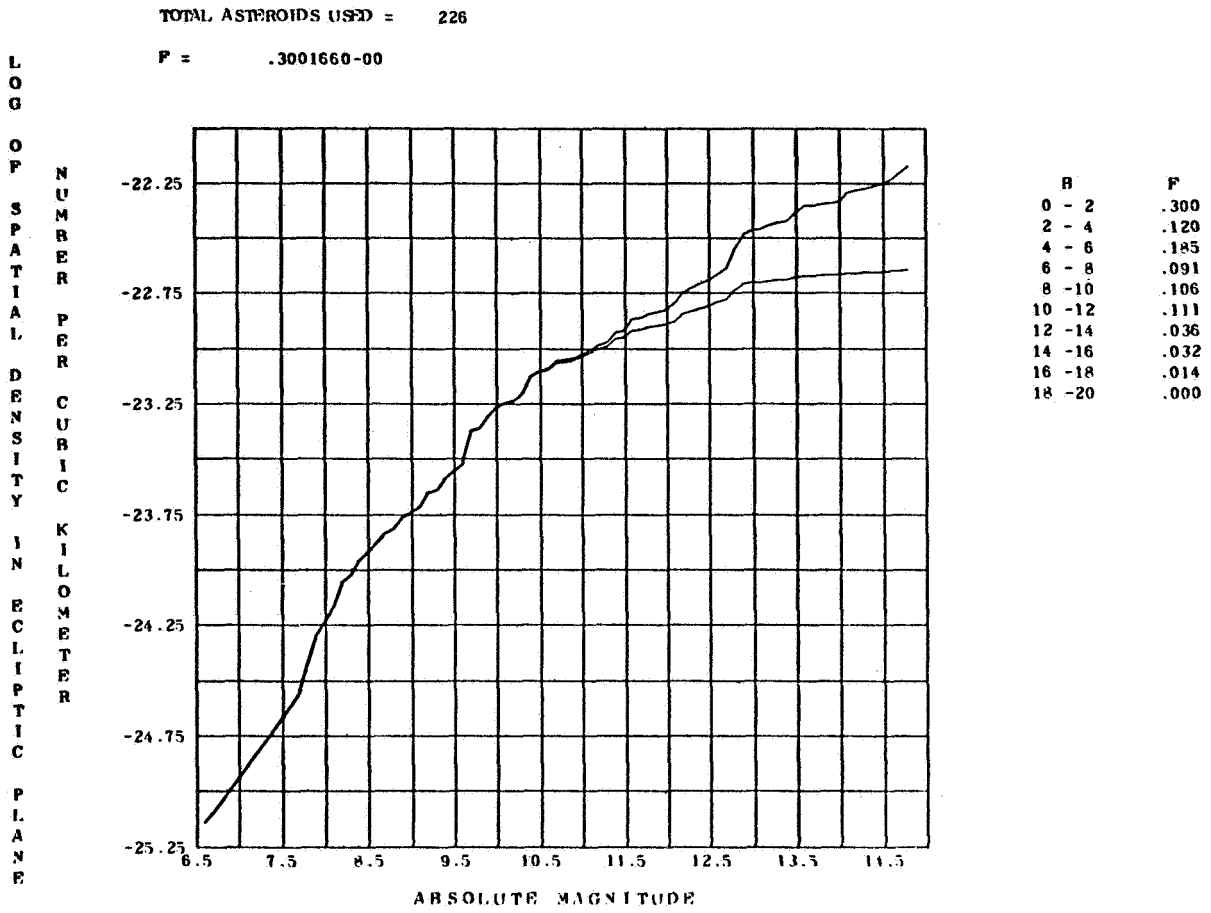


Figure 129. - Spatial density at R = 2.60 and at longitudes between 225.0 and 270.0.

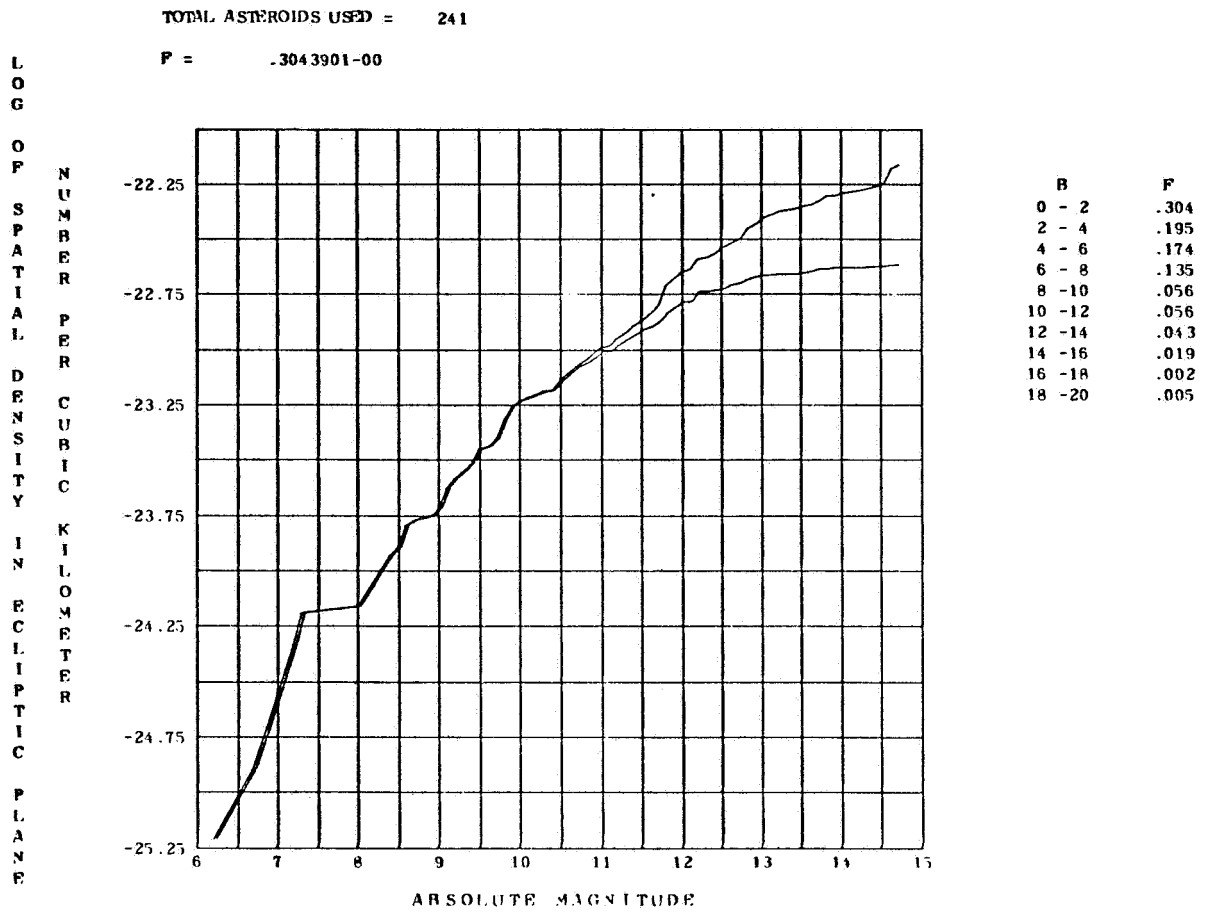


Figure 130. - Spatial density at R = 2.60 and at longitudes between 270.0 and 315.0.

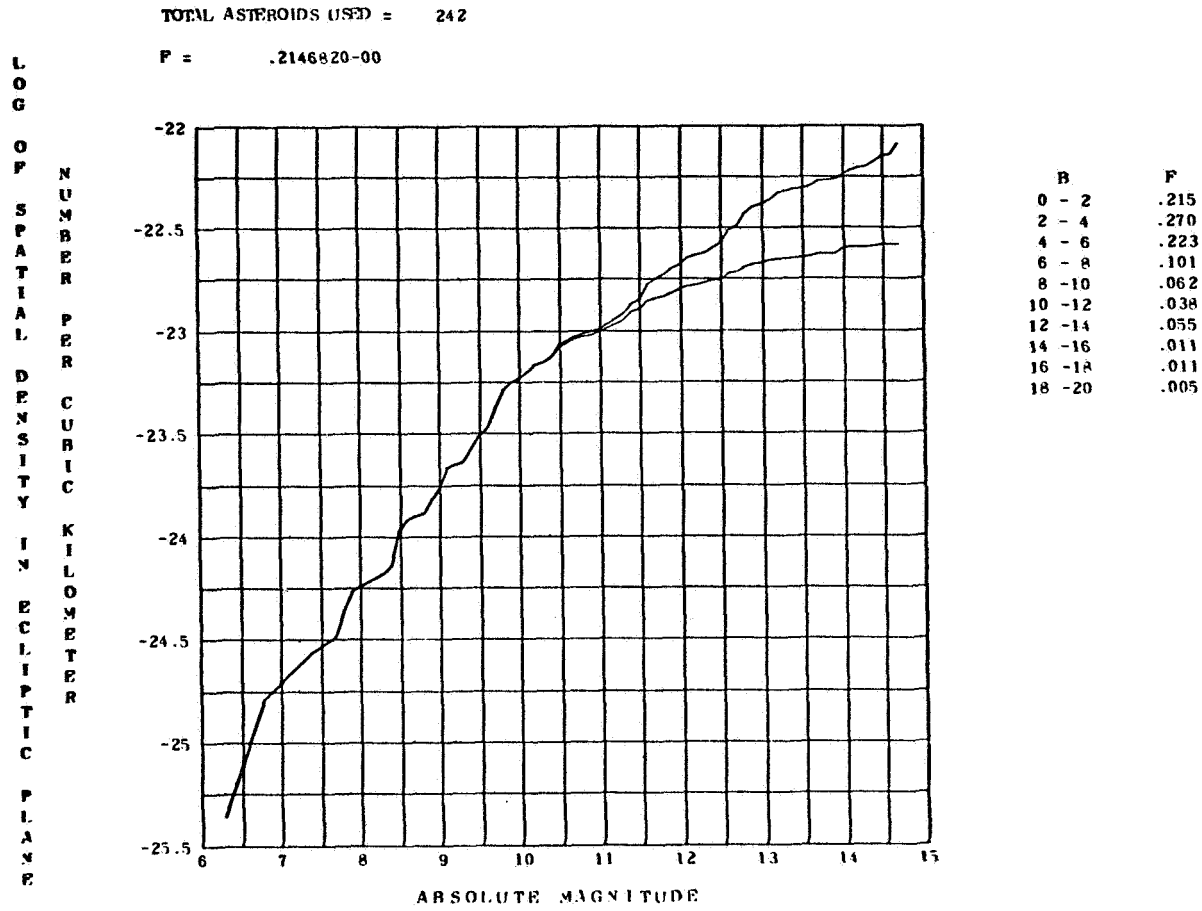


Figure 131. - Spatial density at R = 2.60 and at longitudes between 315.0 and 360.0.

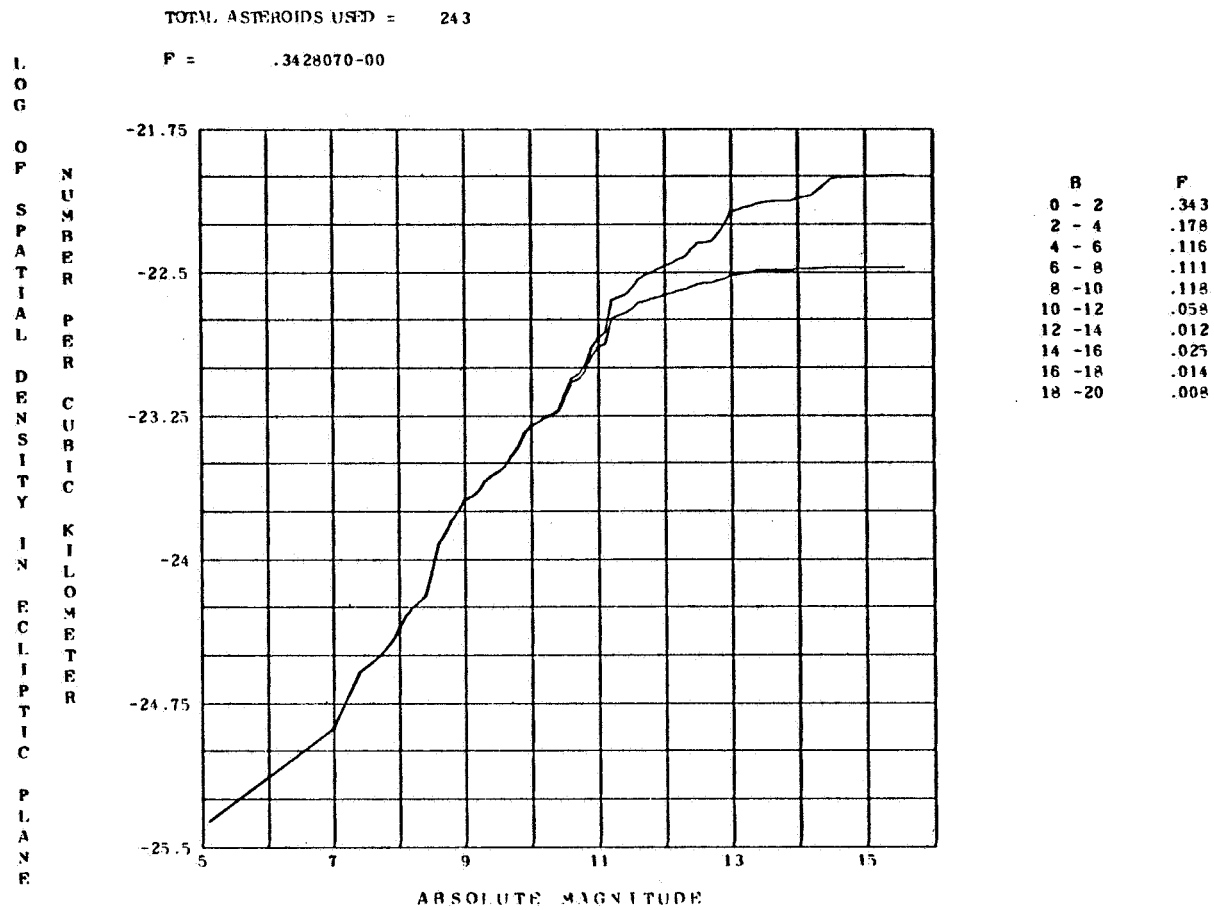


Figure 132.- Spatial density at R = 2.70 and at longitudes between 0 and 45.0.

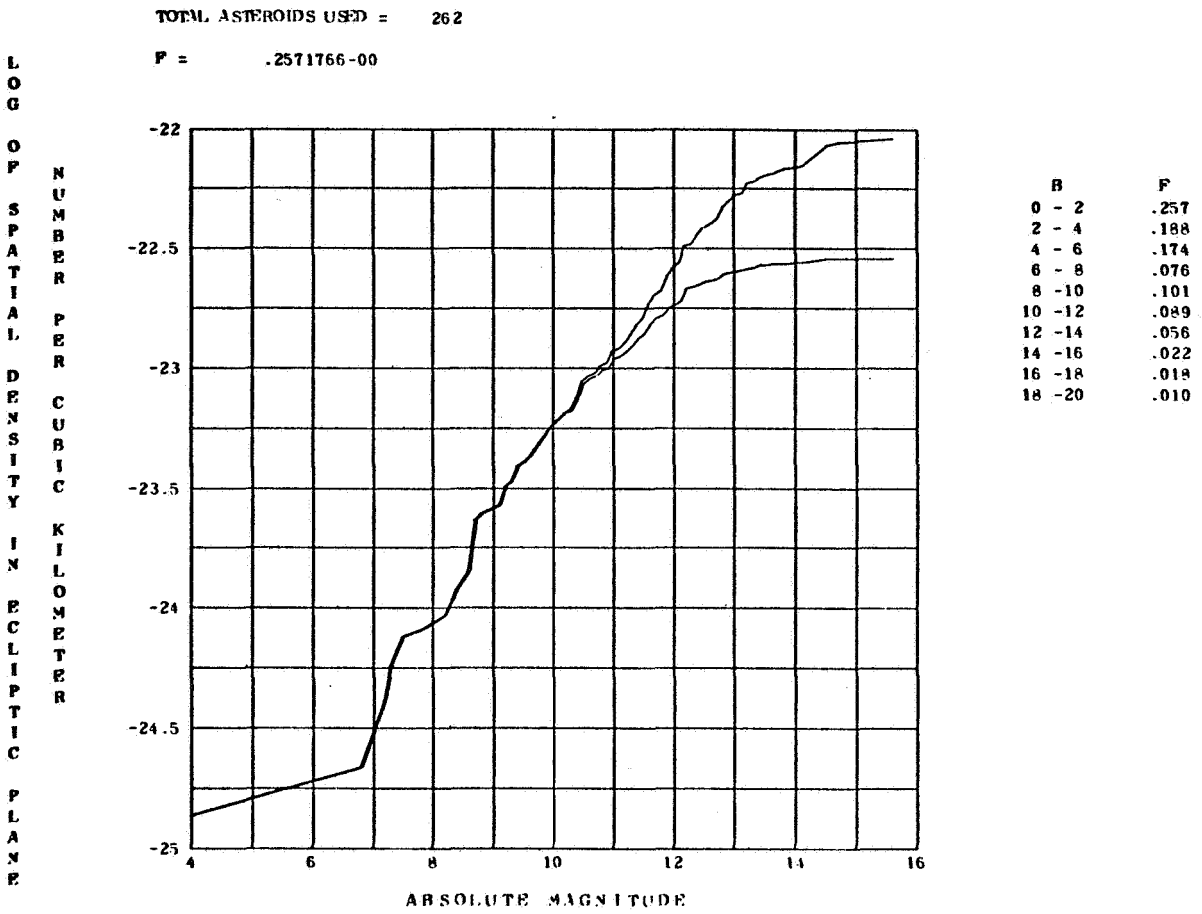


Figure 133. - Spatial density at R = 2.70 and at longitudes between 45.0 and 90.0.

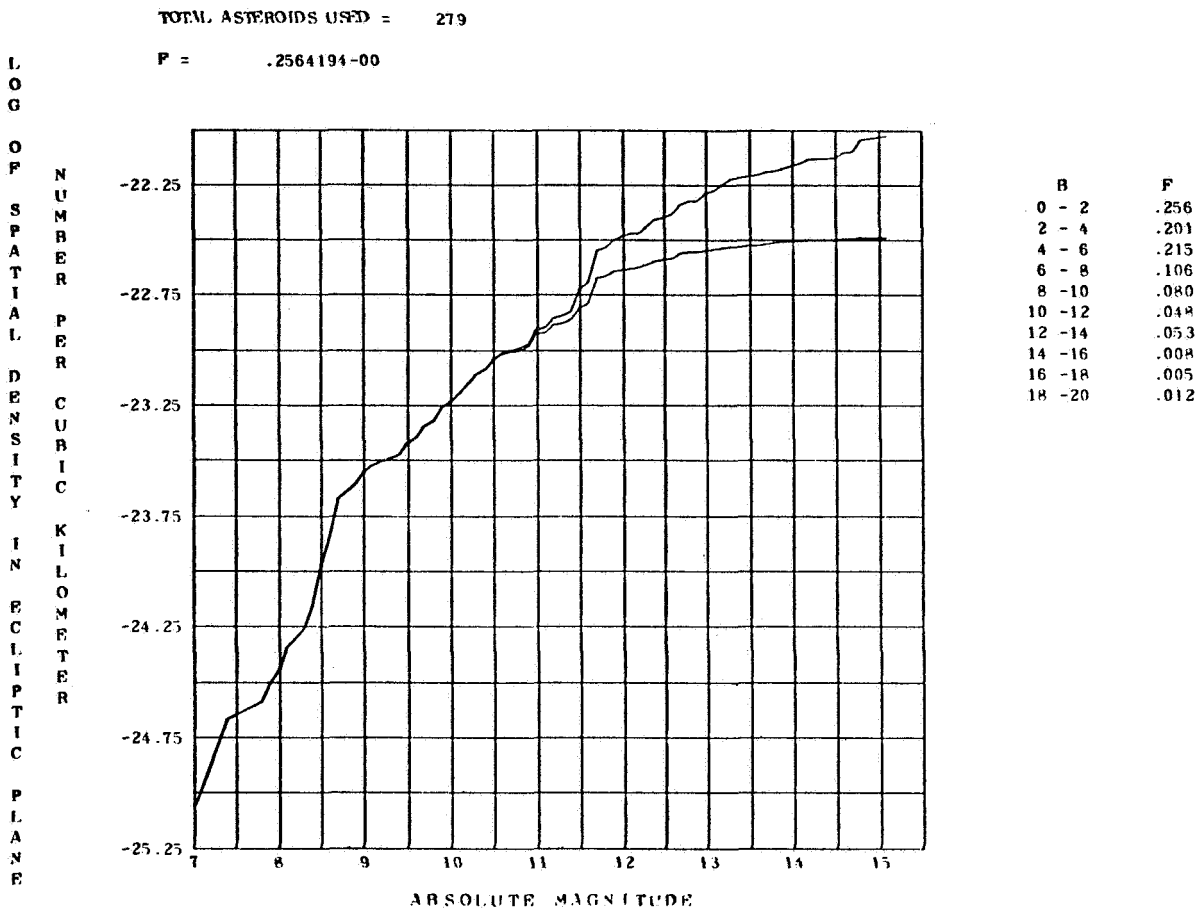


Figure 134. - Spatial density at R = 2.70 and at longitudes between 90.0 and 135.0.

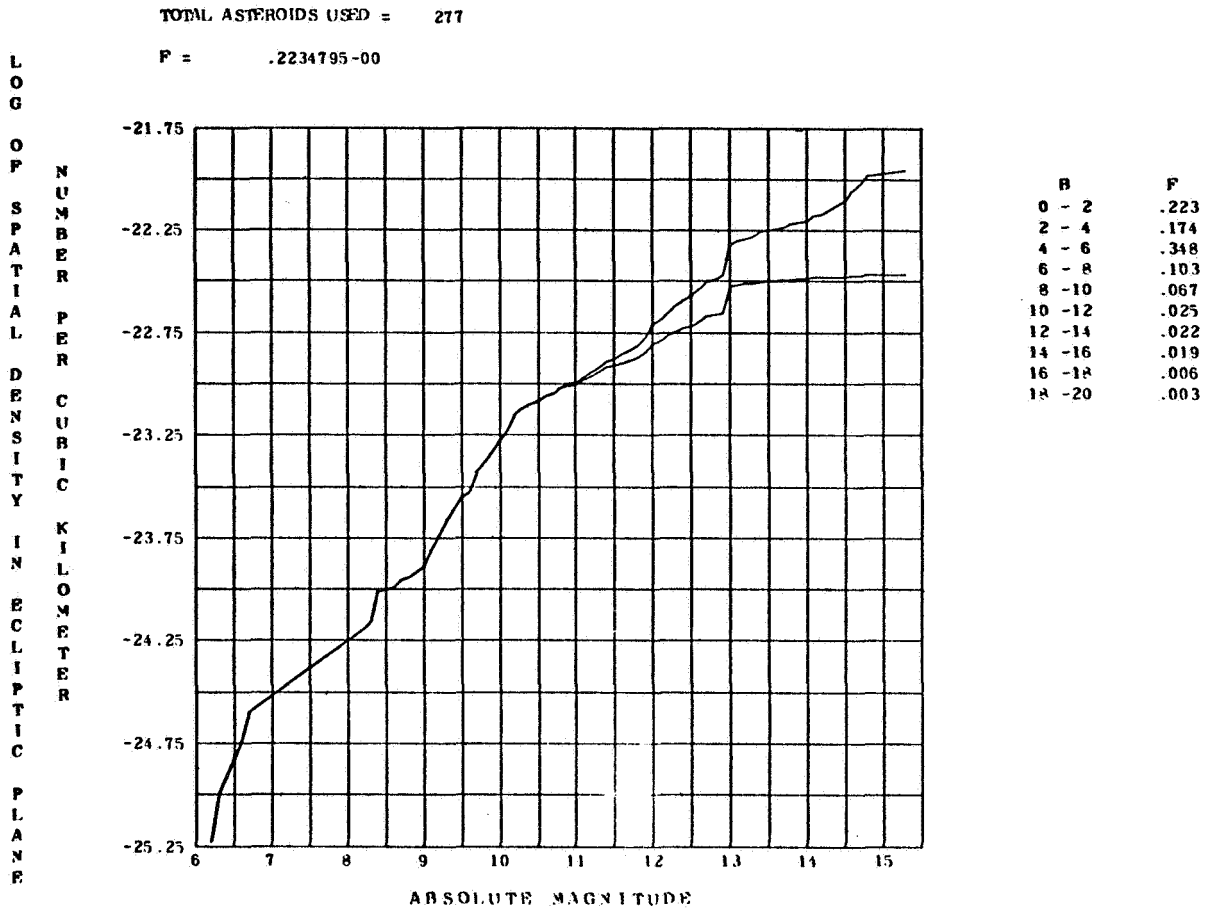
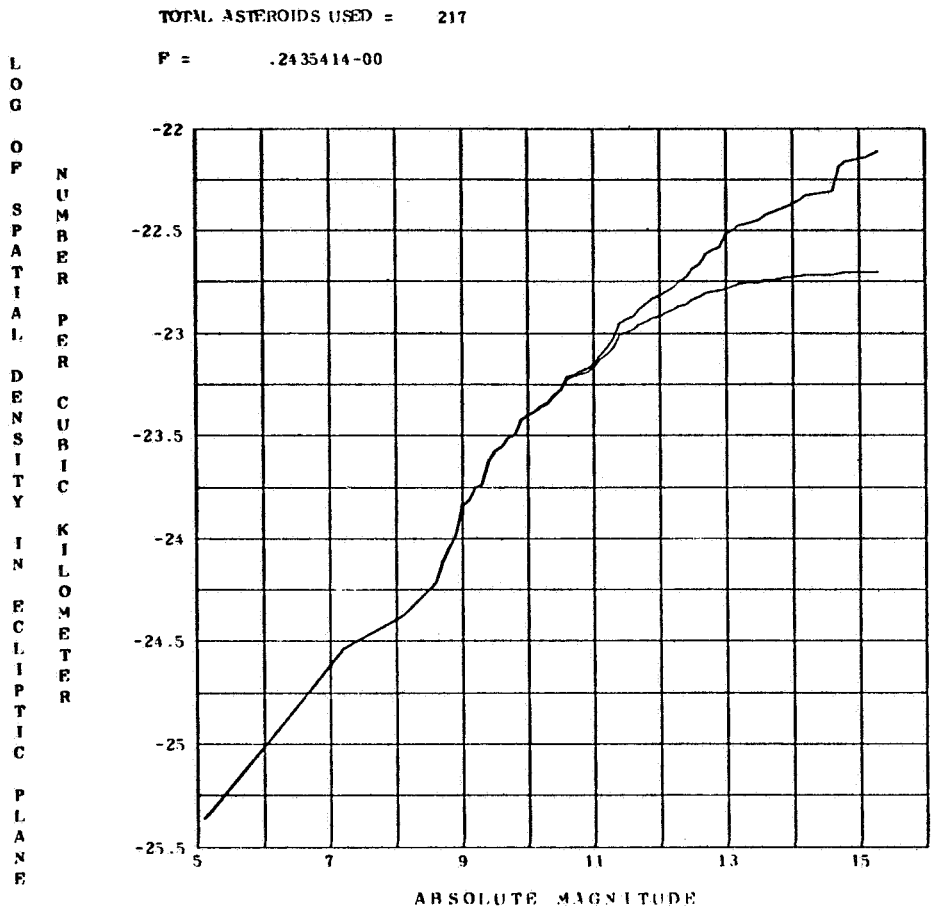


Figure 135. - Spatial density at R = 2.70 and at longitudes between 135.0 and 180.0.



B	F
0 - 2	.244
2 - 4	.252
4 - 6	.134
6 - 8	.118
8 -10	.086
10 -12	.055
12 -14	.064
14 -16	.029
16 -18	.004
18 -20	.009

Figure 136. - Spatial density at R = 2.70 and at longitudes between 180.0 and 225.0.

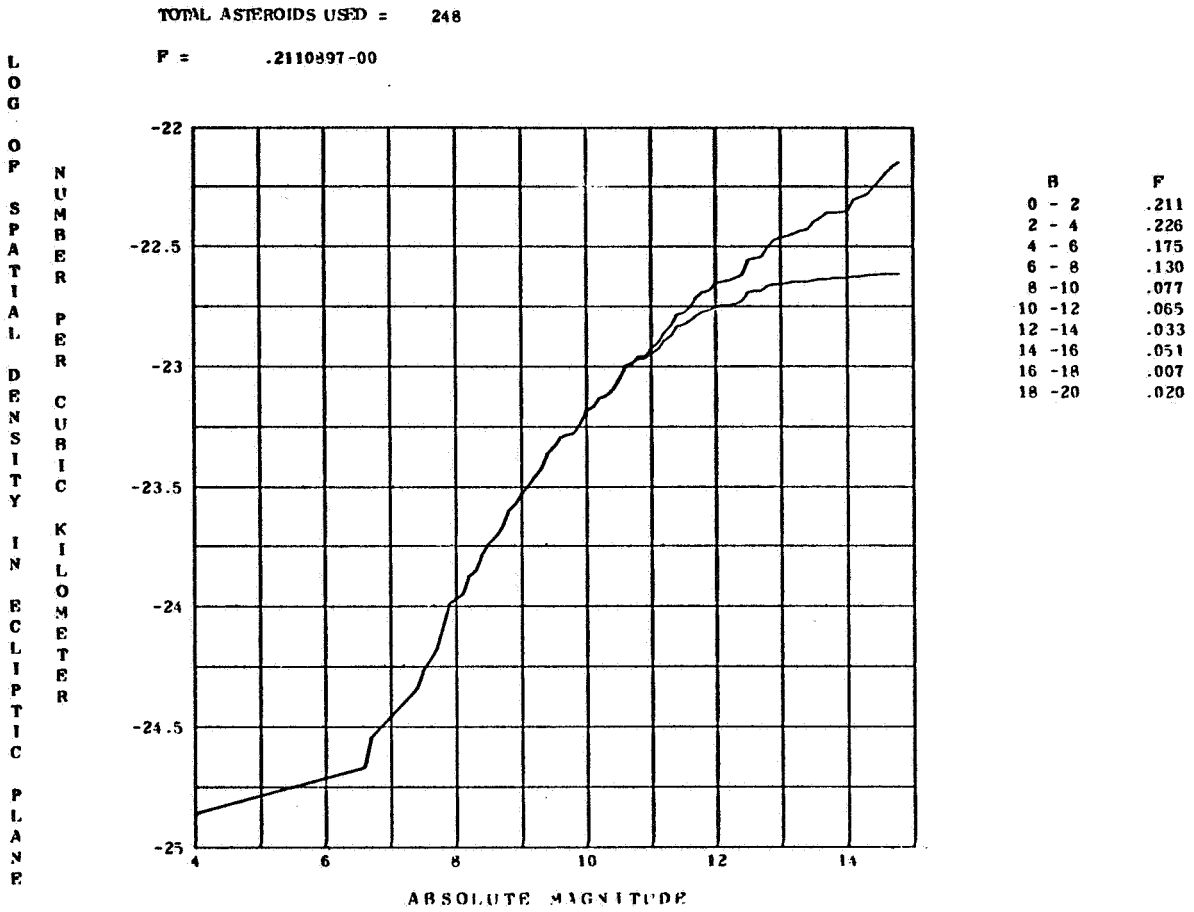


Figure 137. - Spatial density at R = 2.70 and at longitudes between 225.0 and 270.0.

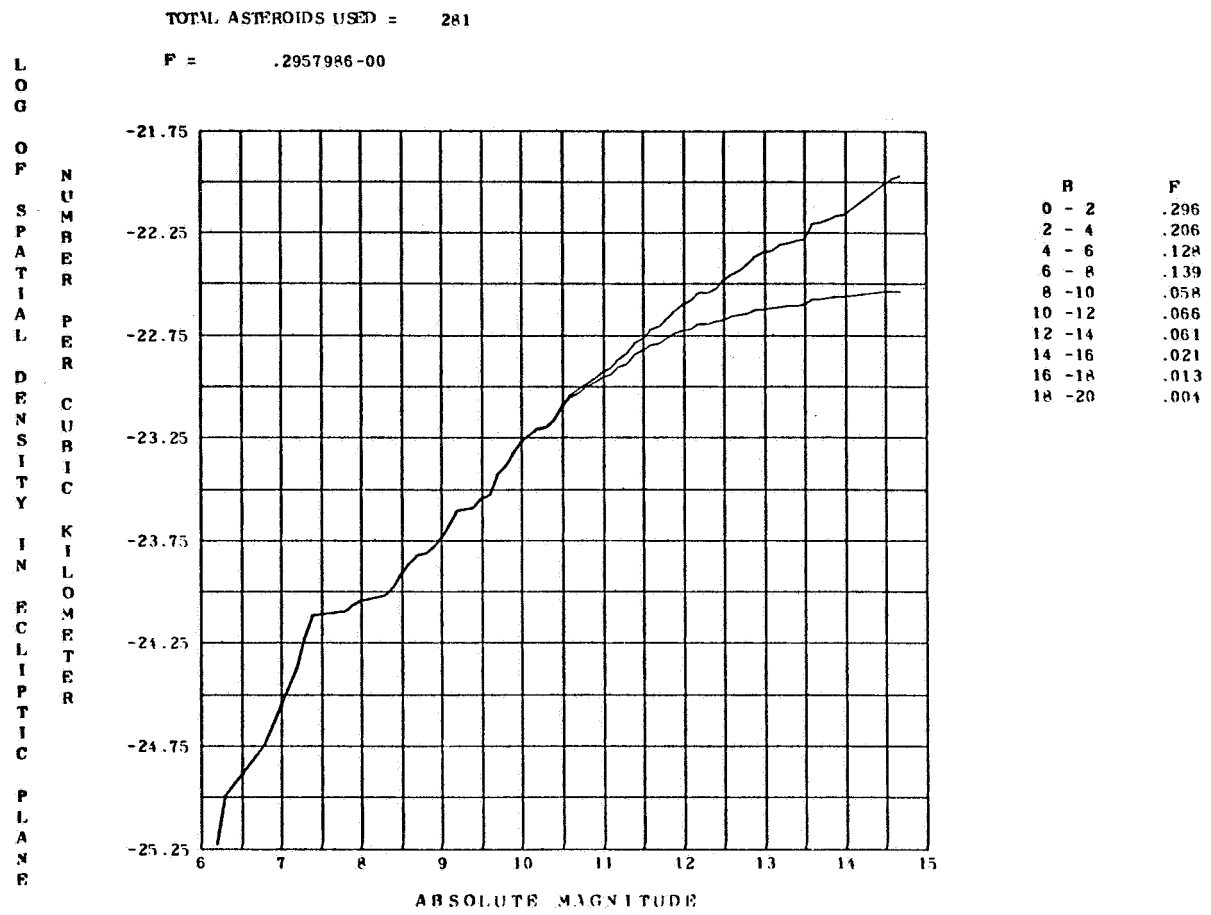


Figure 138. - Spatial density at R = 2.70 and at longitudes between 270.0 and 315.0.

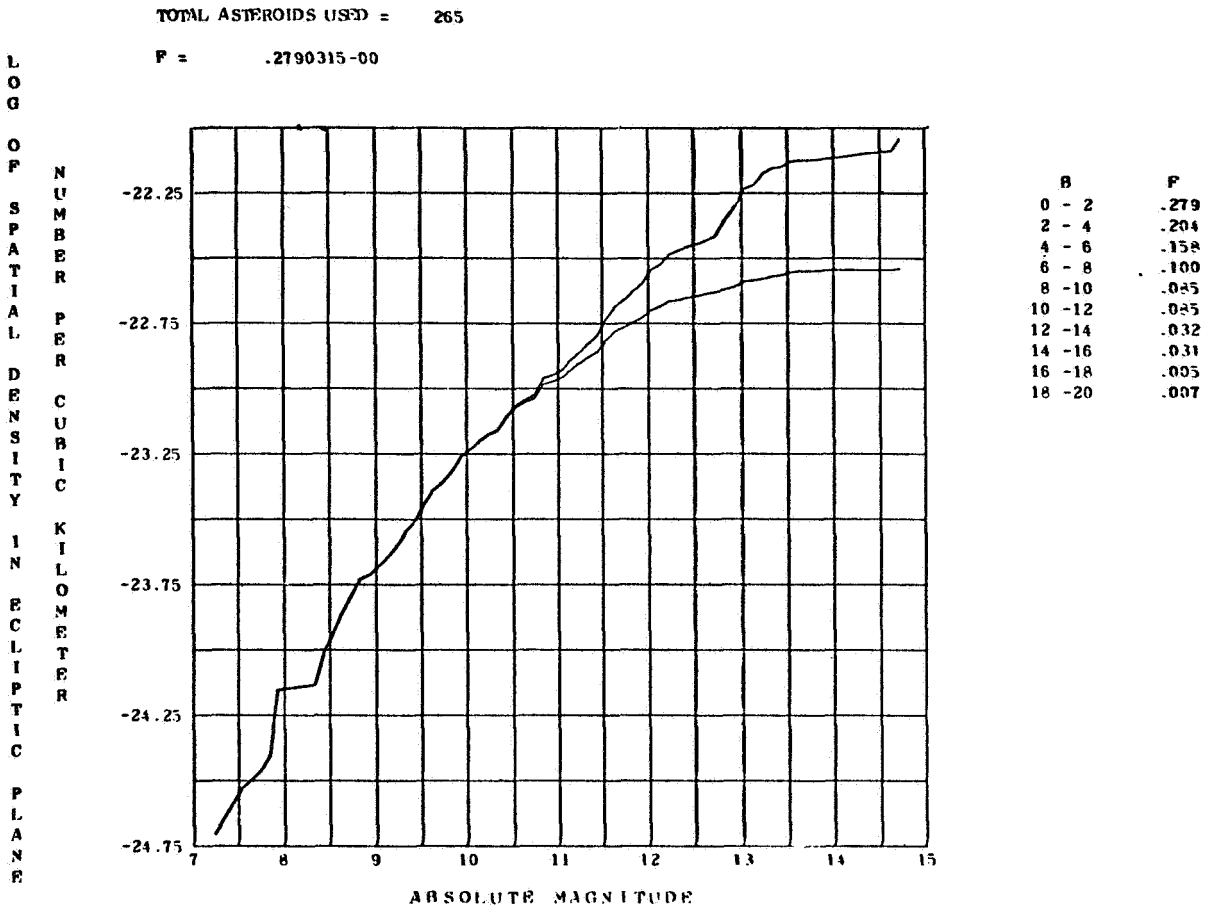


Figure 139. - Spatial density at R = 2.70 and at longitudes between 315.0 and 360.0.

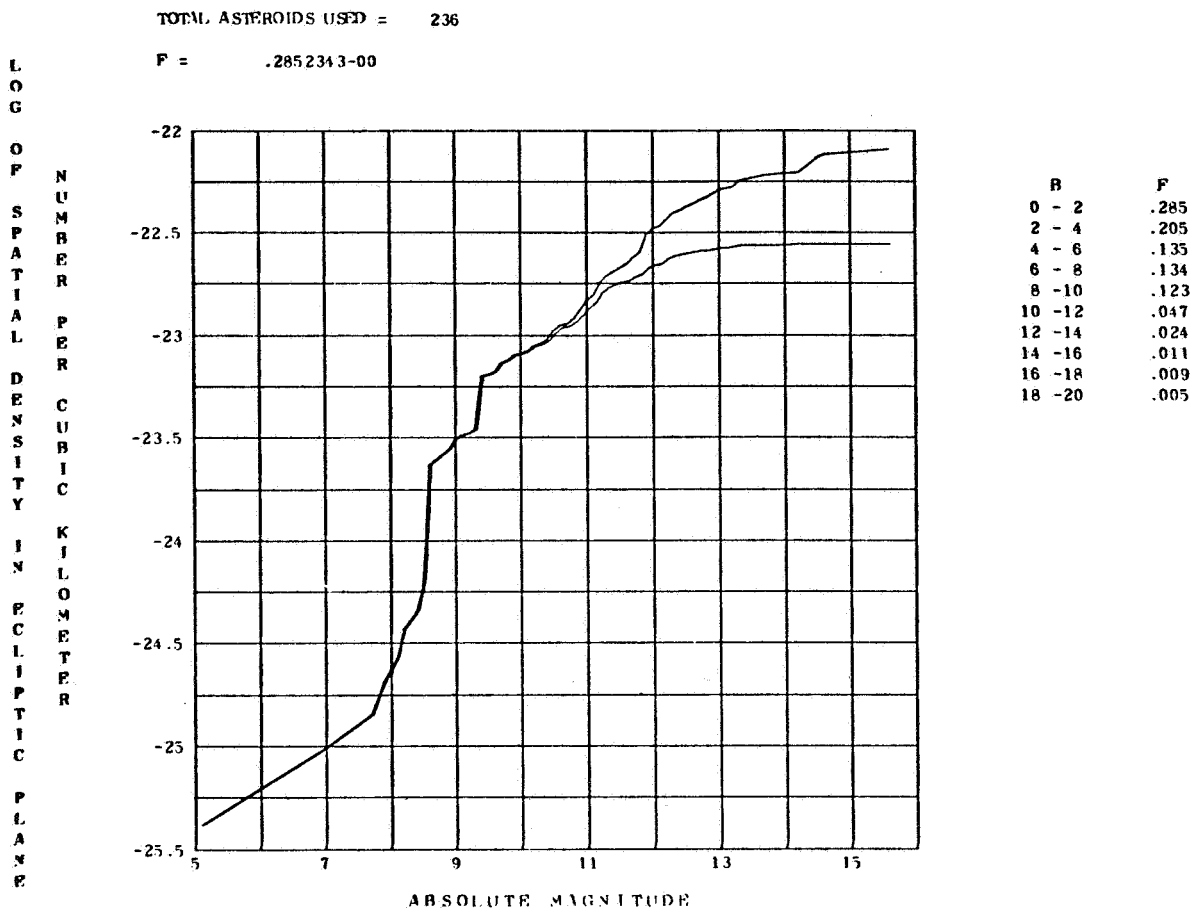


Figure 140.- Spatial density at R = 2.80 and at longitudes between 0 and 45.0.

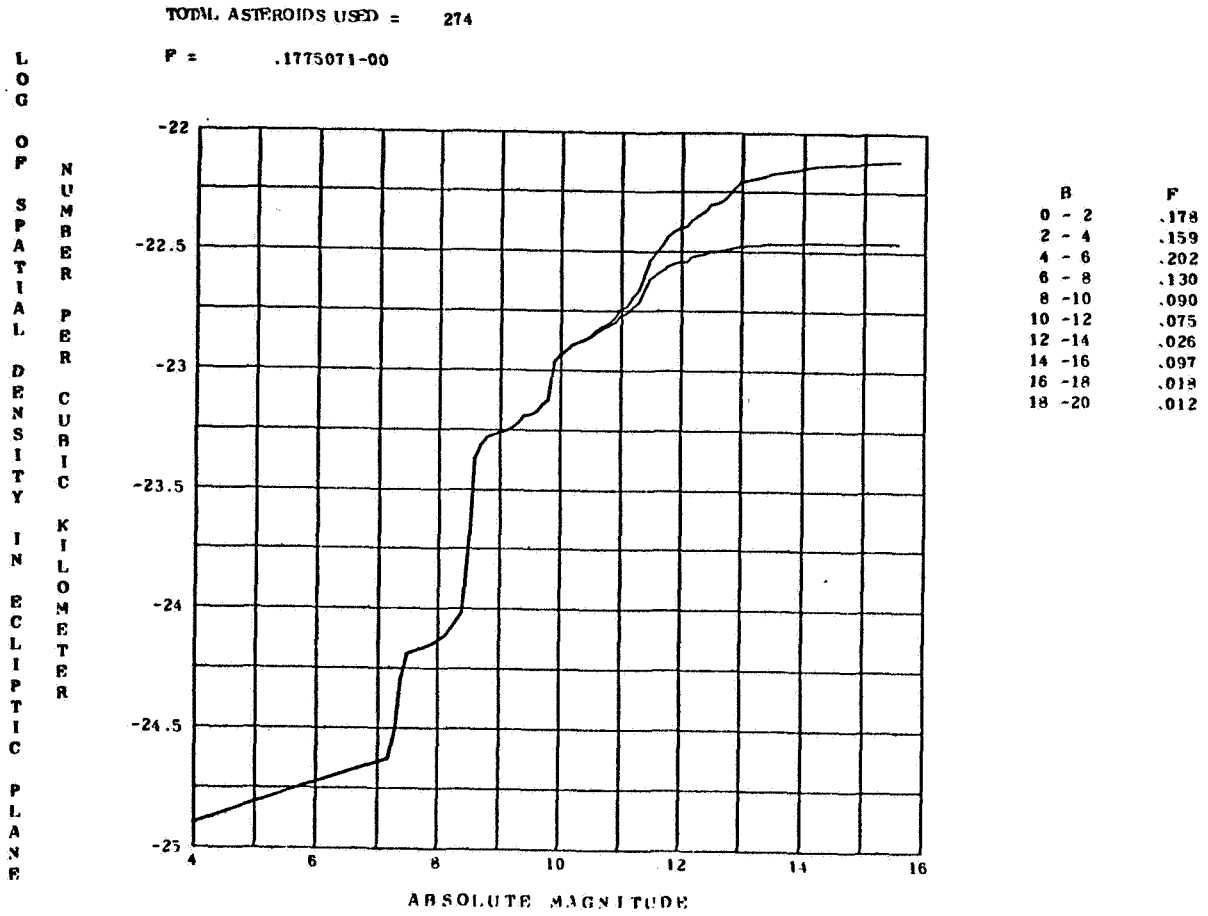


Figure 141. - Spatial density at R = 2.80 and at longitudes between 45.0 and 90.0.

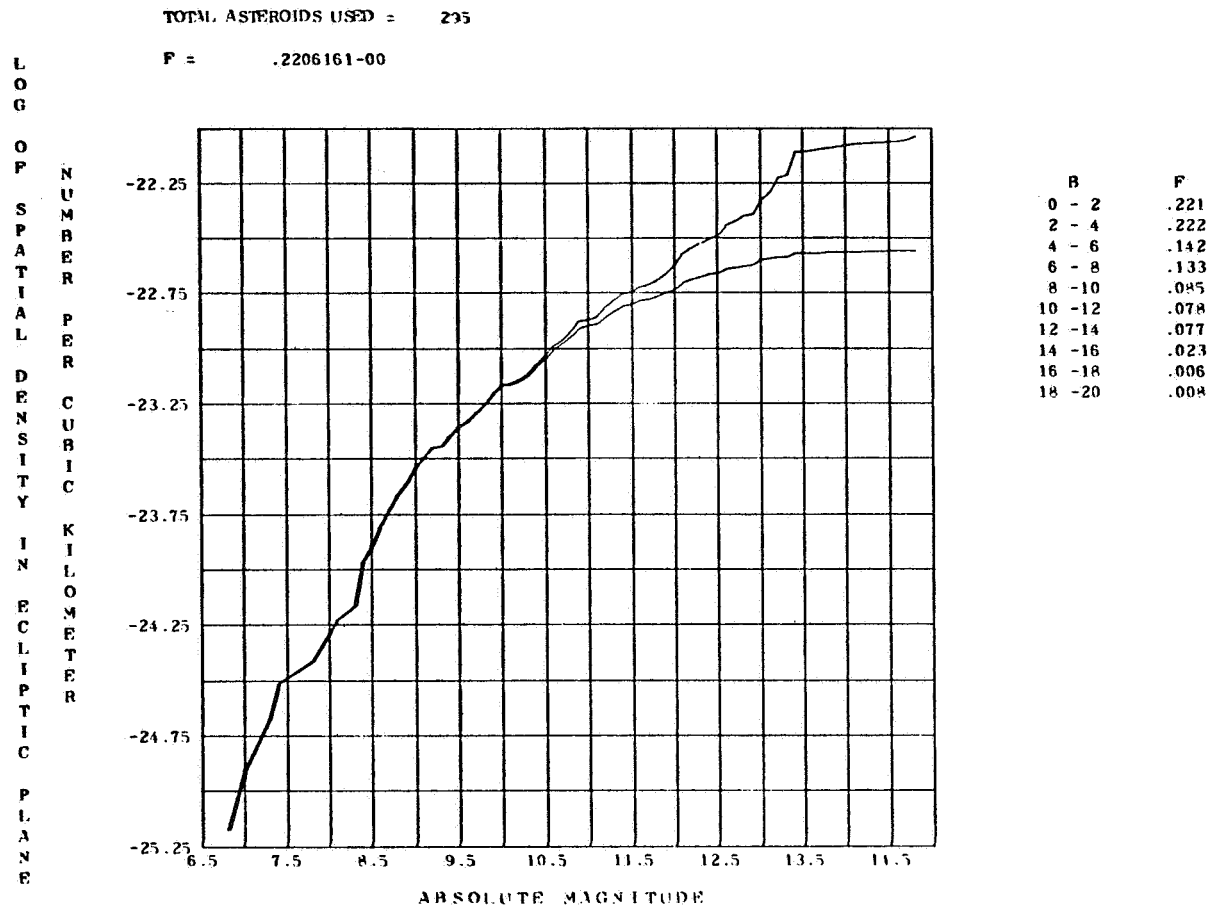


Figure 142. - Spatial density at R = 2.80 and at longitudes between 90.0 and 135.0.

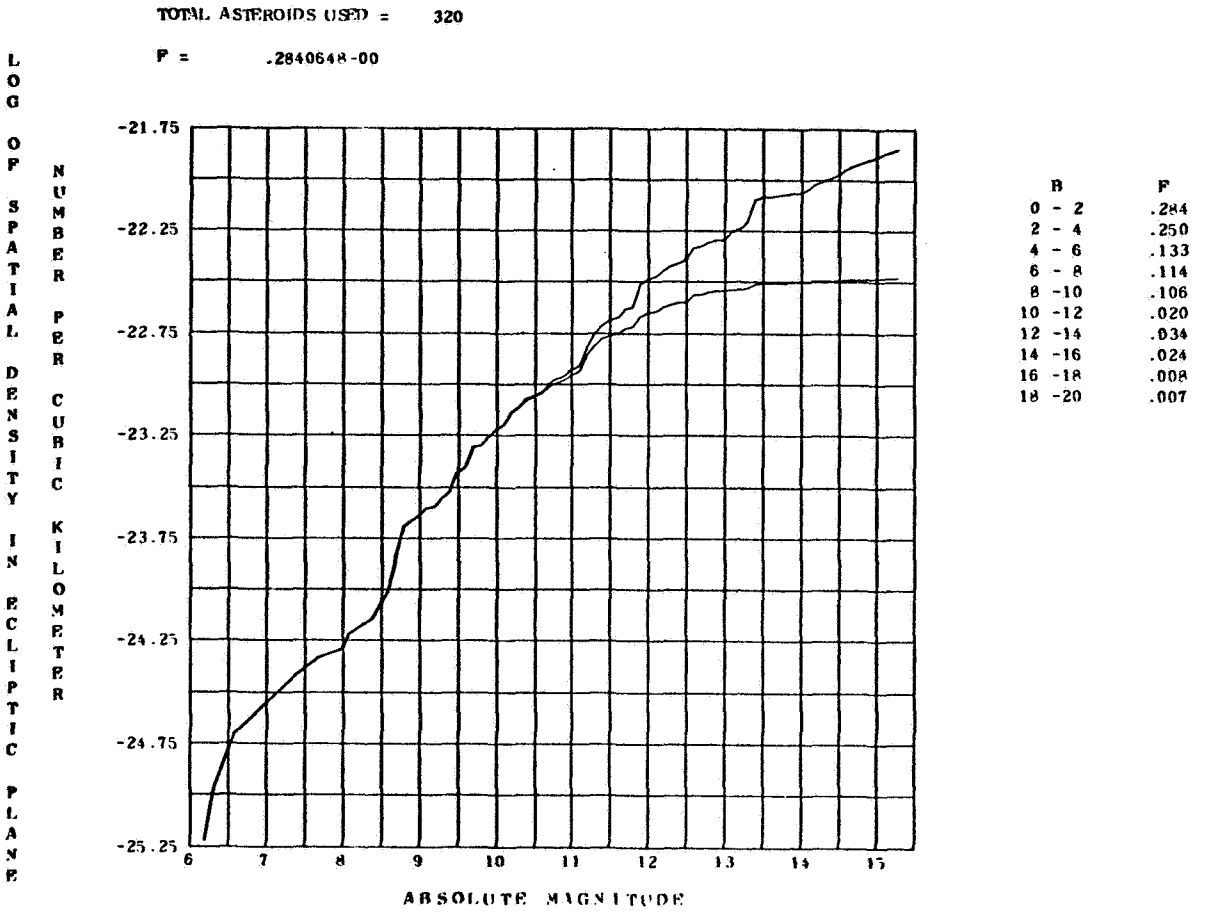


Figure 143. - Spatial density at R = 2.80 and at longitudes between 135.0 and 180.0.

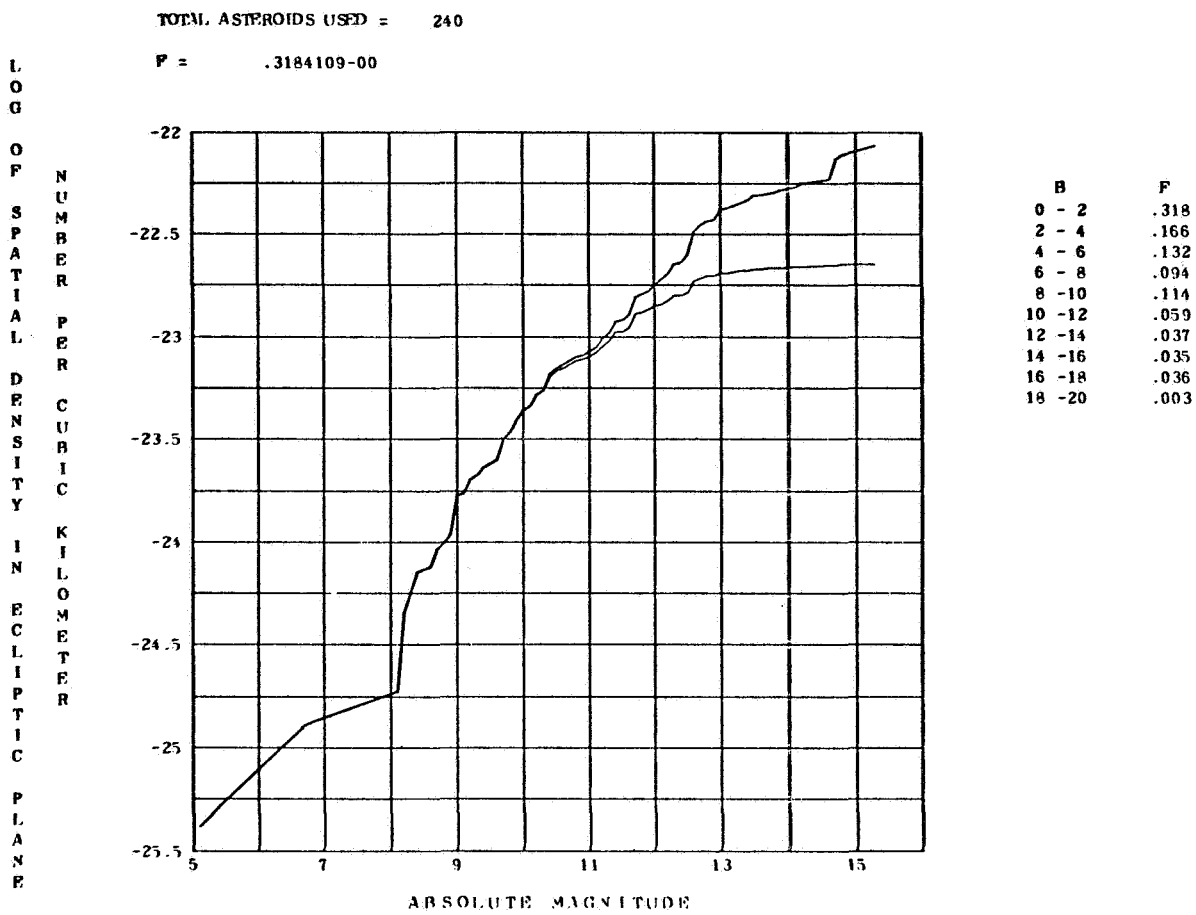


Figure 144. - Spatial density at R = 2.80 and at longitudes between 180.0 and 225.0.

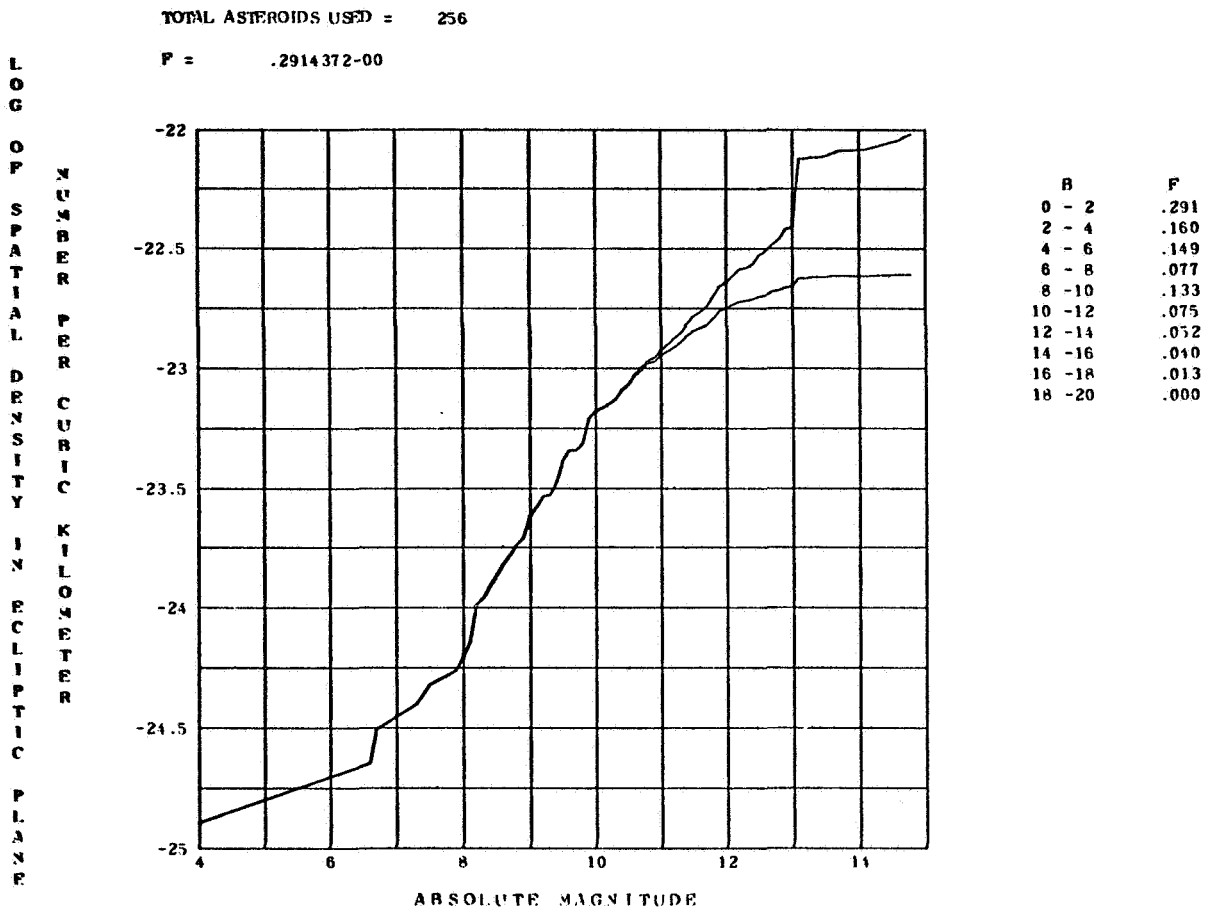


Figure 145. - Spatial density at R = 2.80 and at longitudes between 225.0 and 270.0.

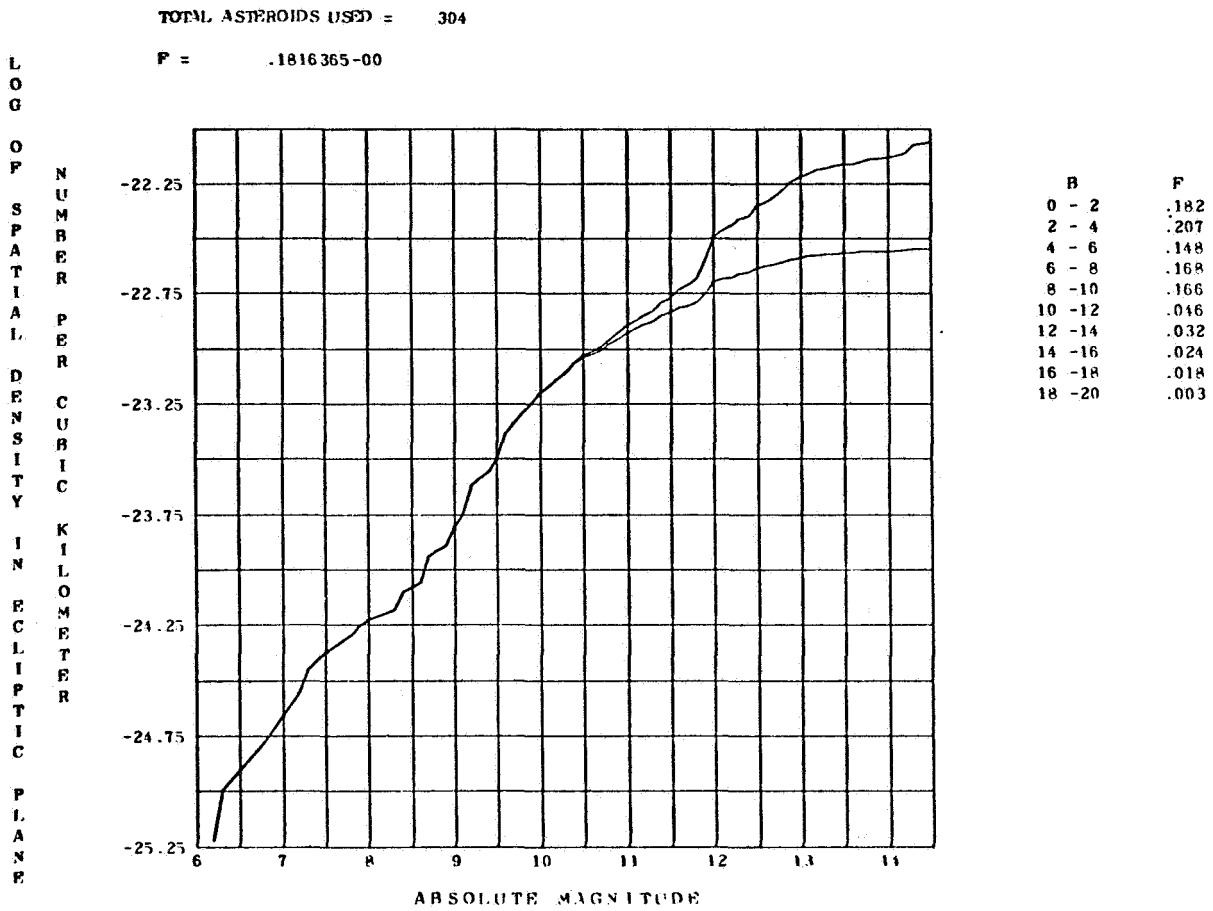


Figure 146. - Spatial density at R = 2.80 and at longitudes between 270.0 and 315.0.

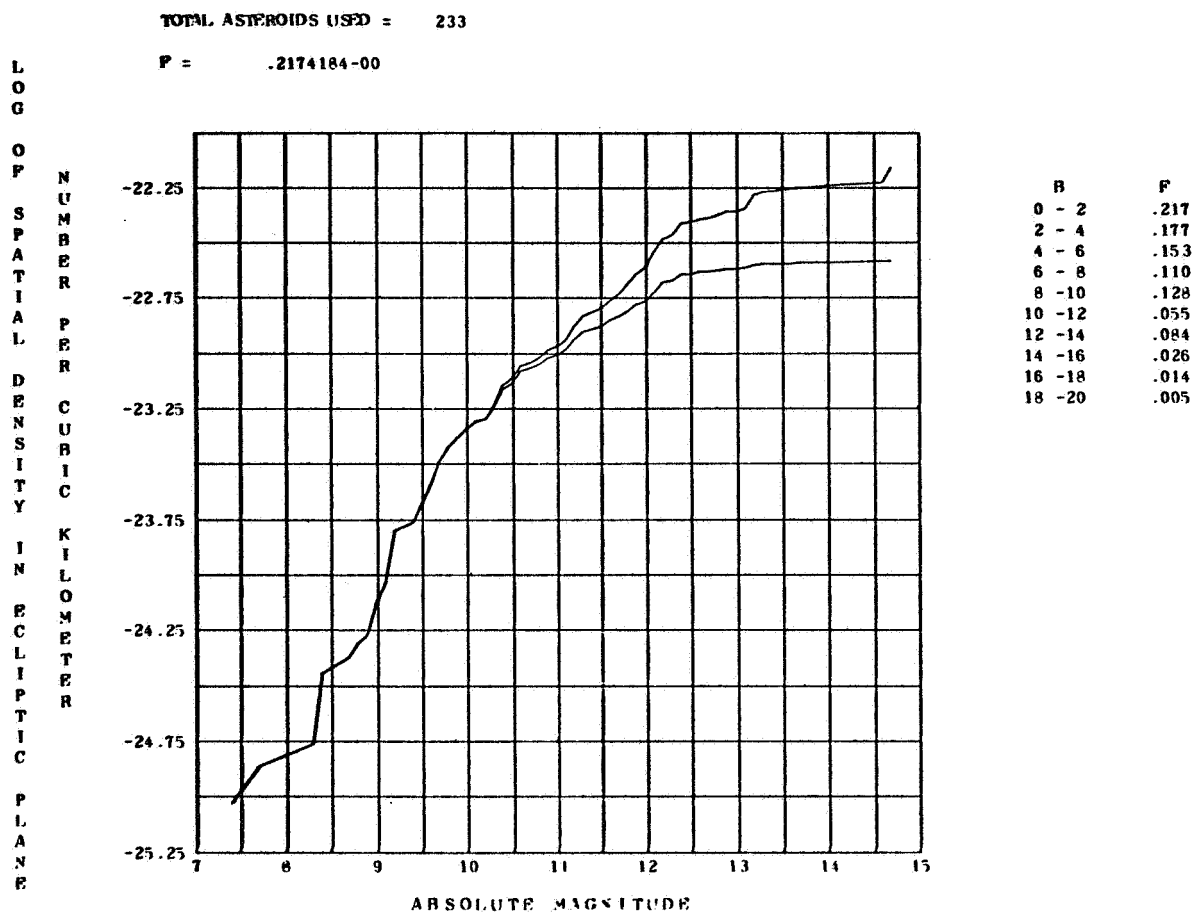


Figure 147. - Spatial density at R = 2.80 and at longitudes between 315.0 and 360.0.

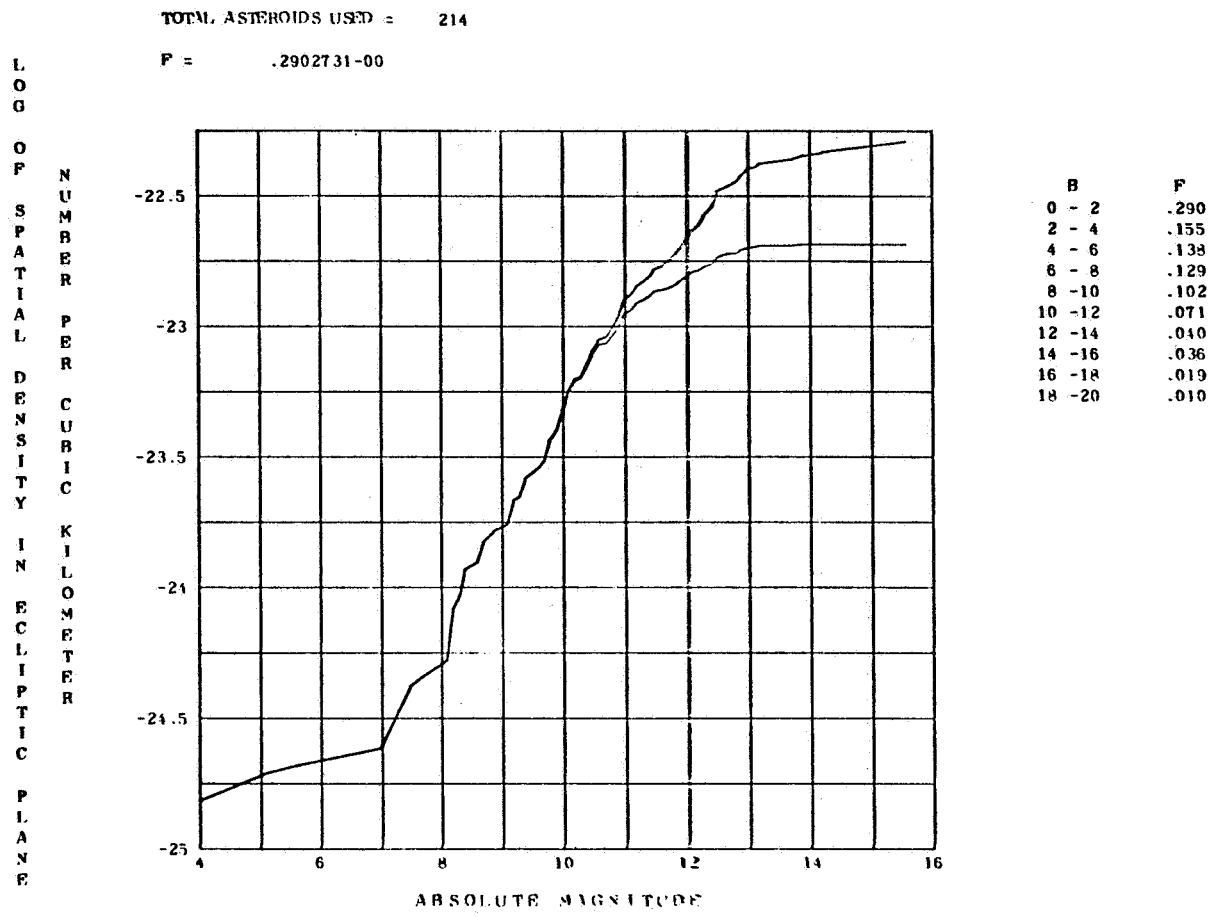


Figure 148.- Spatial density at R = 2.90 and at longitudes between 0 and 45.0.

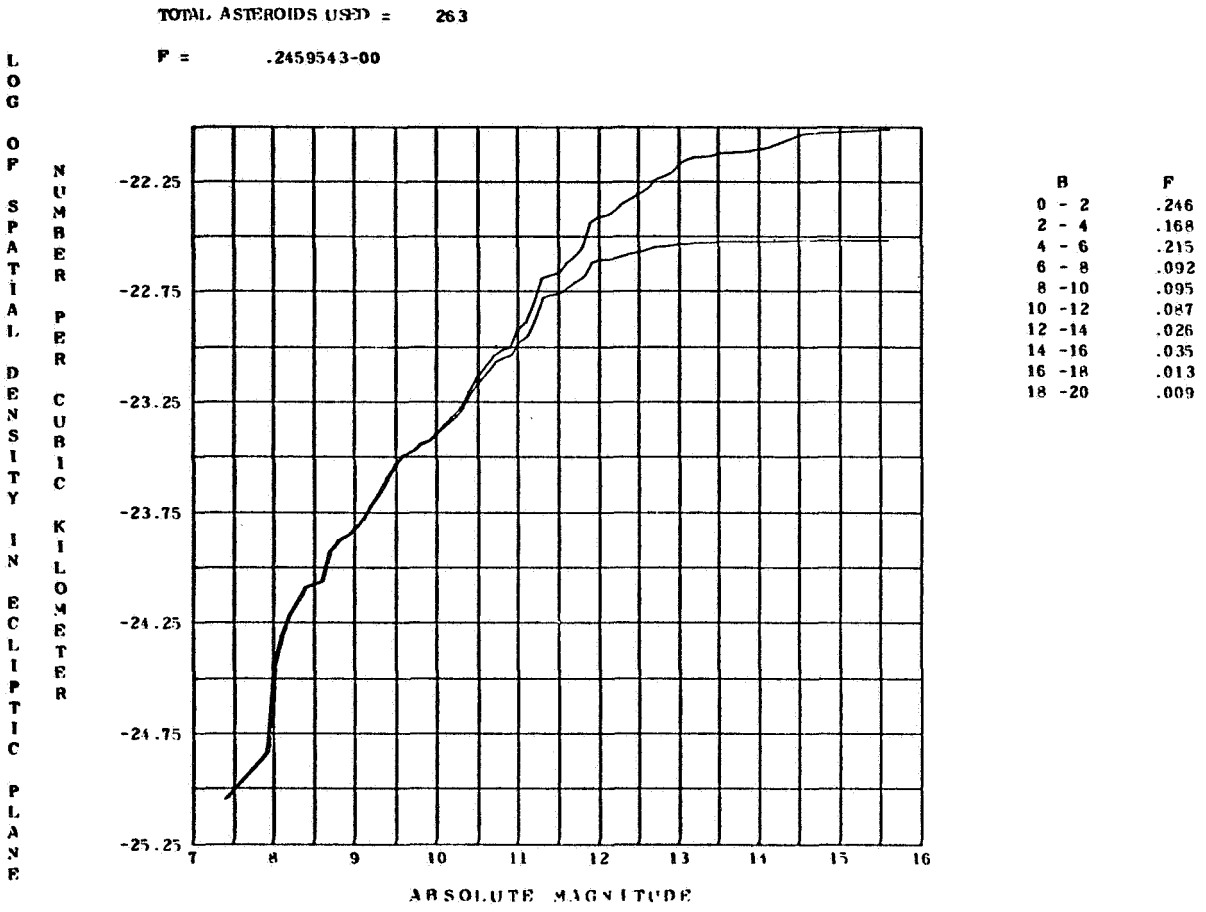


Figure 149. - Spatial density at R = 2.90 and at longitudes between 45.0 and 90.0.

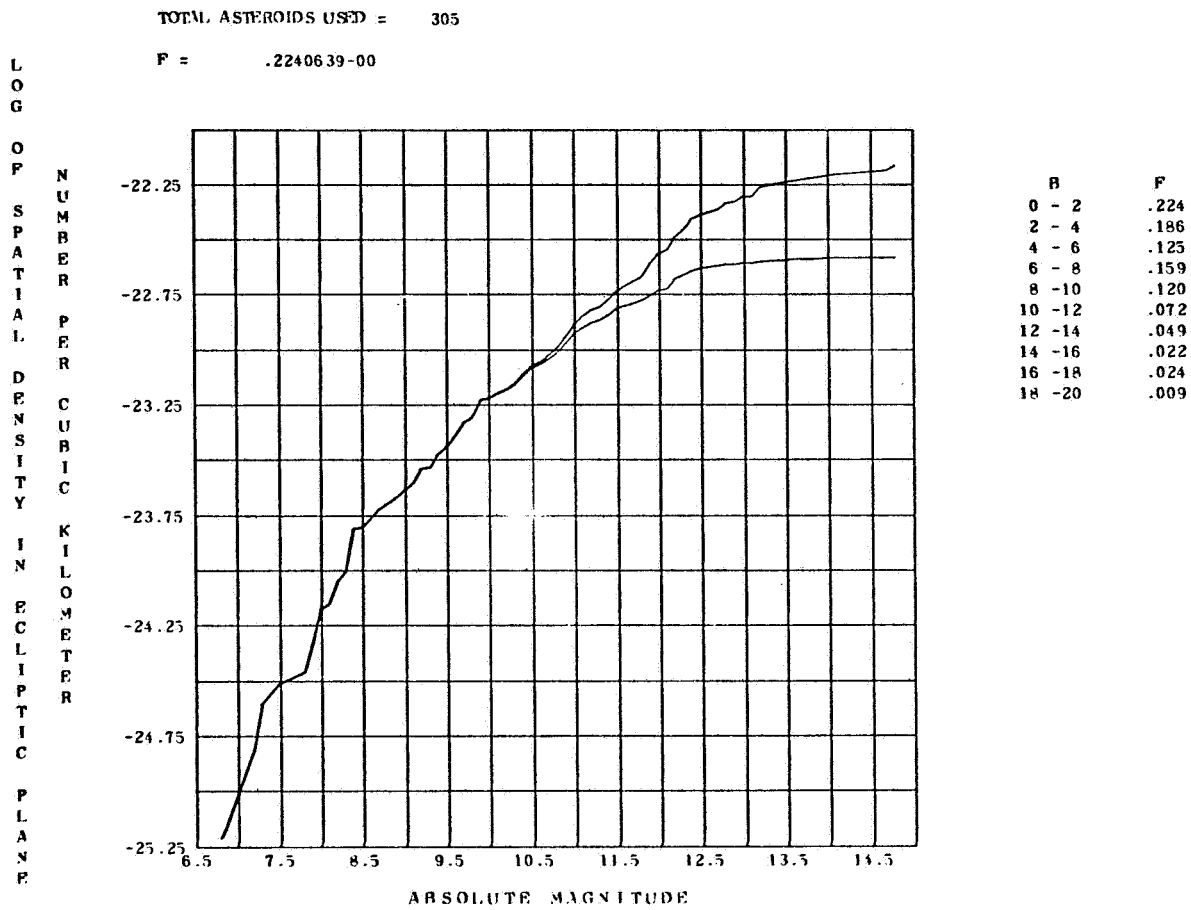


Figure 150. - Spatial density at R = 2.90 and at longitudes between 90.0 and 135.0.

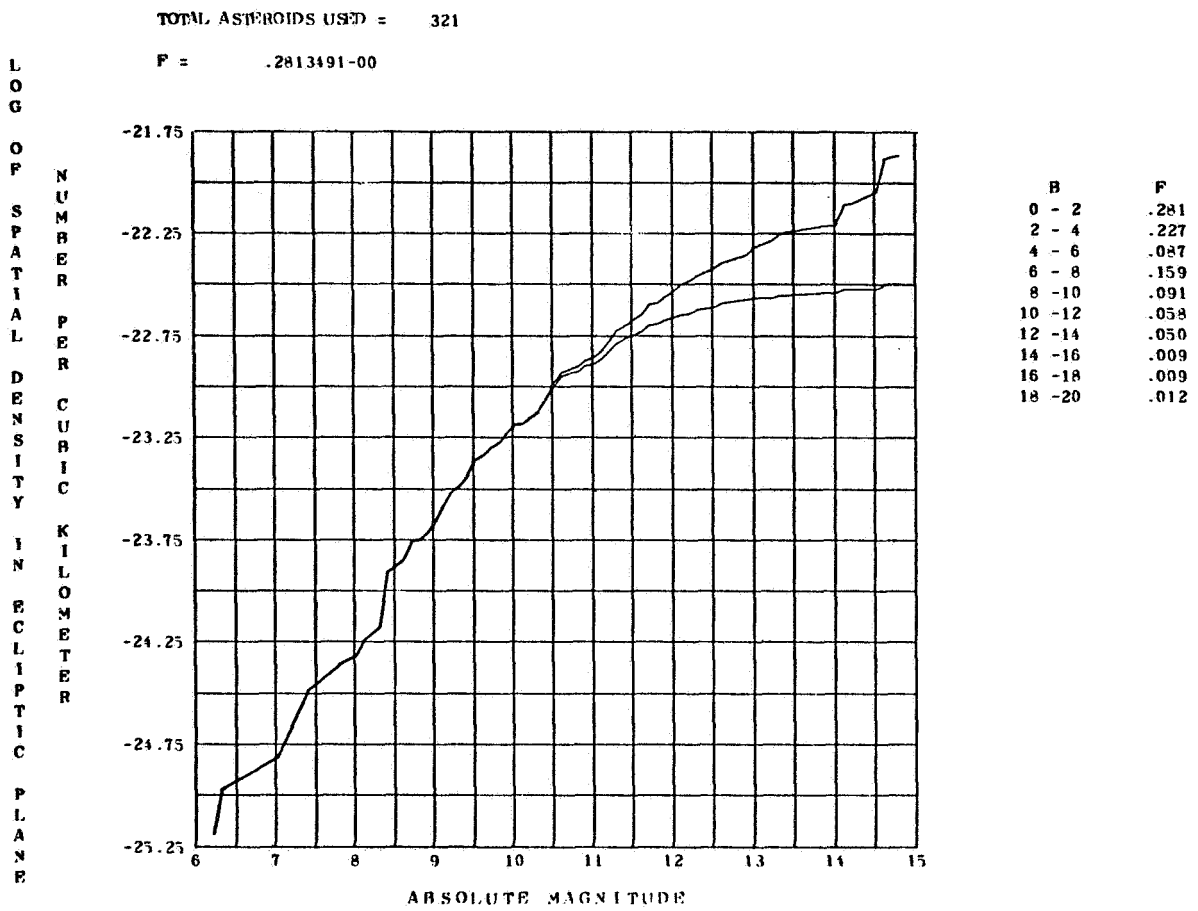


Figure 151. - Spatial density at R = 2.90 and at longitudes between 135.0 and 180.0.

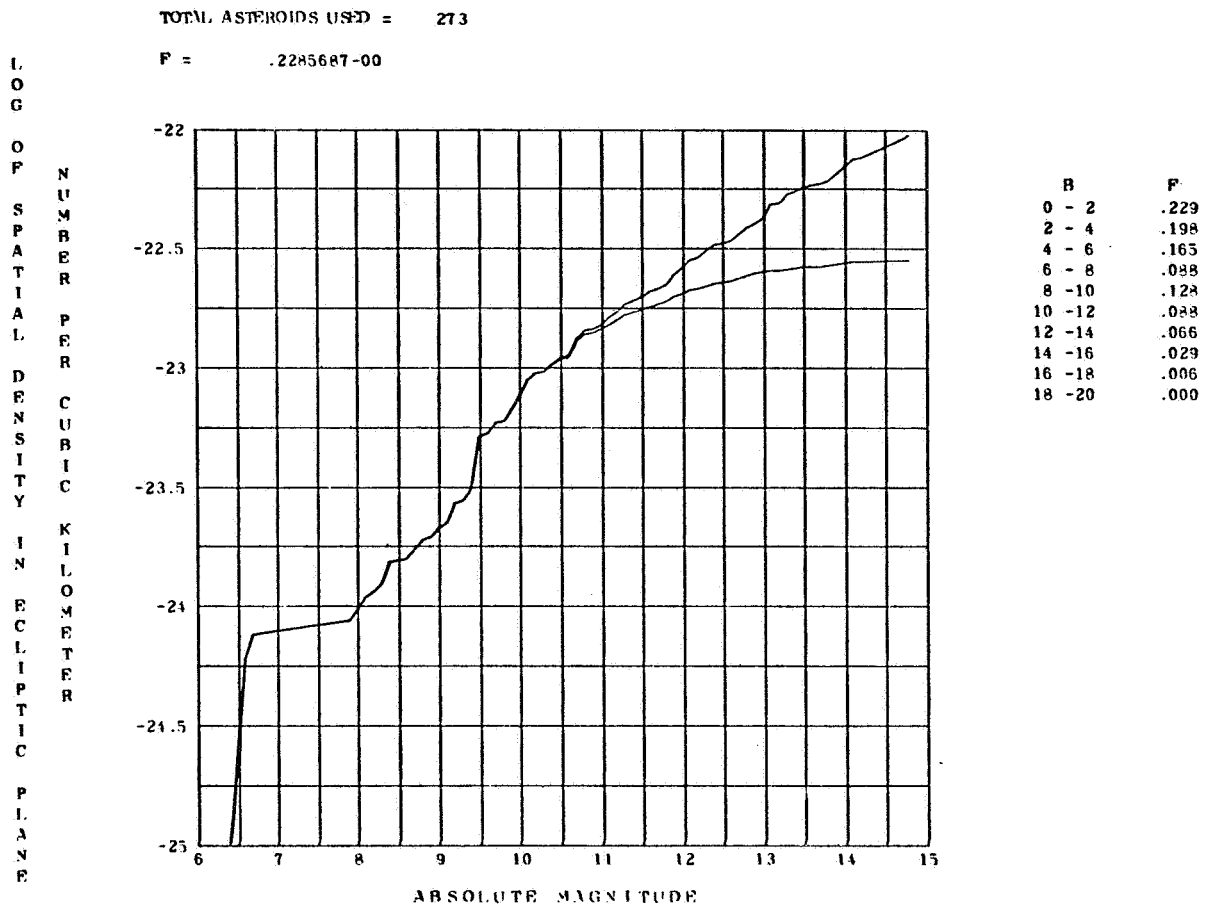


Figure 152. - Spatial density at R = 2.90 and at longitudes between 180.0 and 225.0.

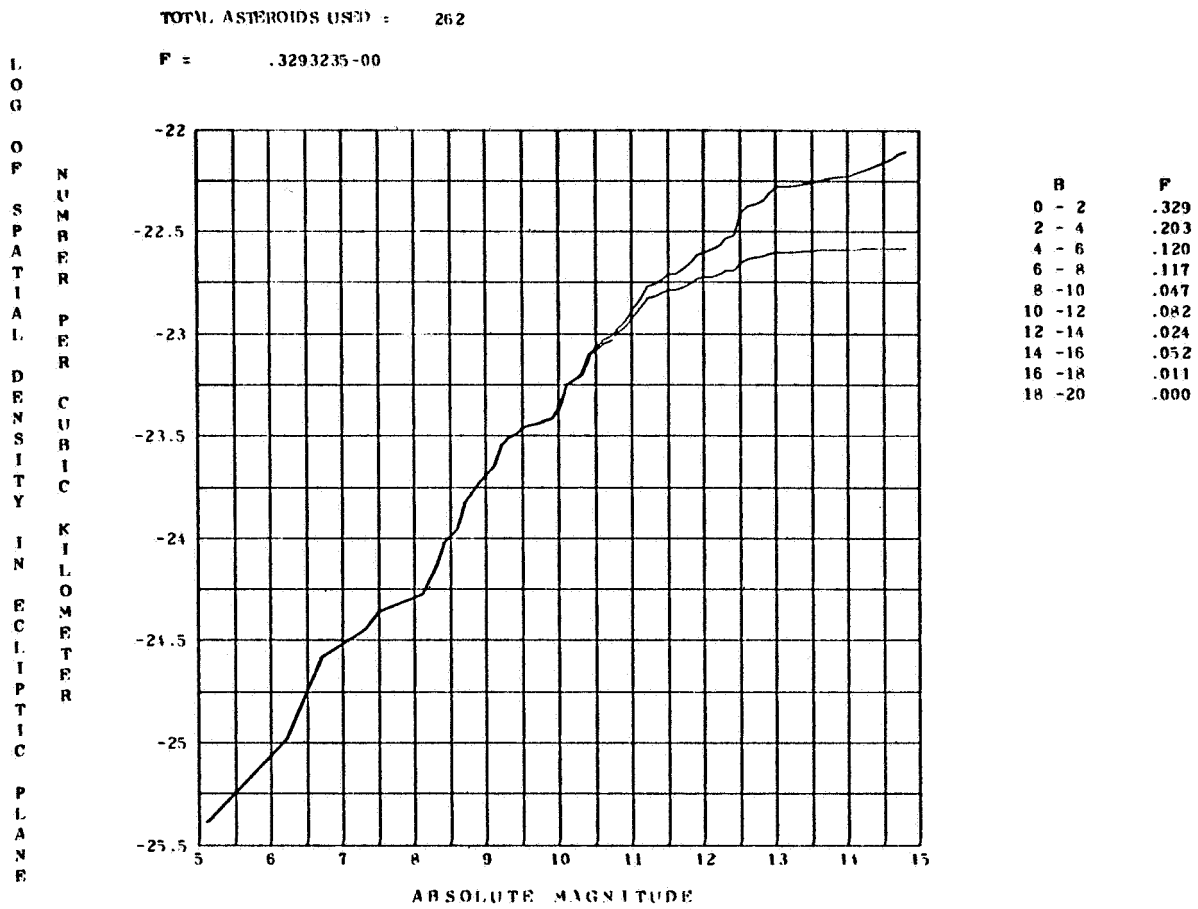


Figure 153. - Spatial density at R = 2.90 and at longitudes between 225.0 and 270.0.

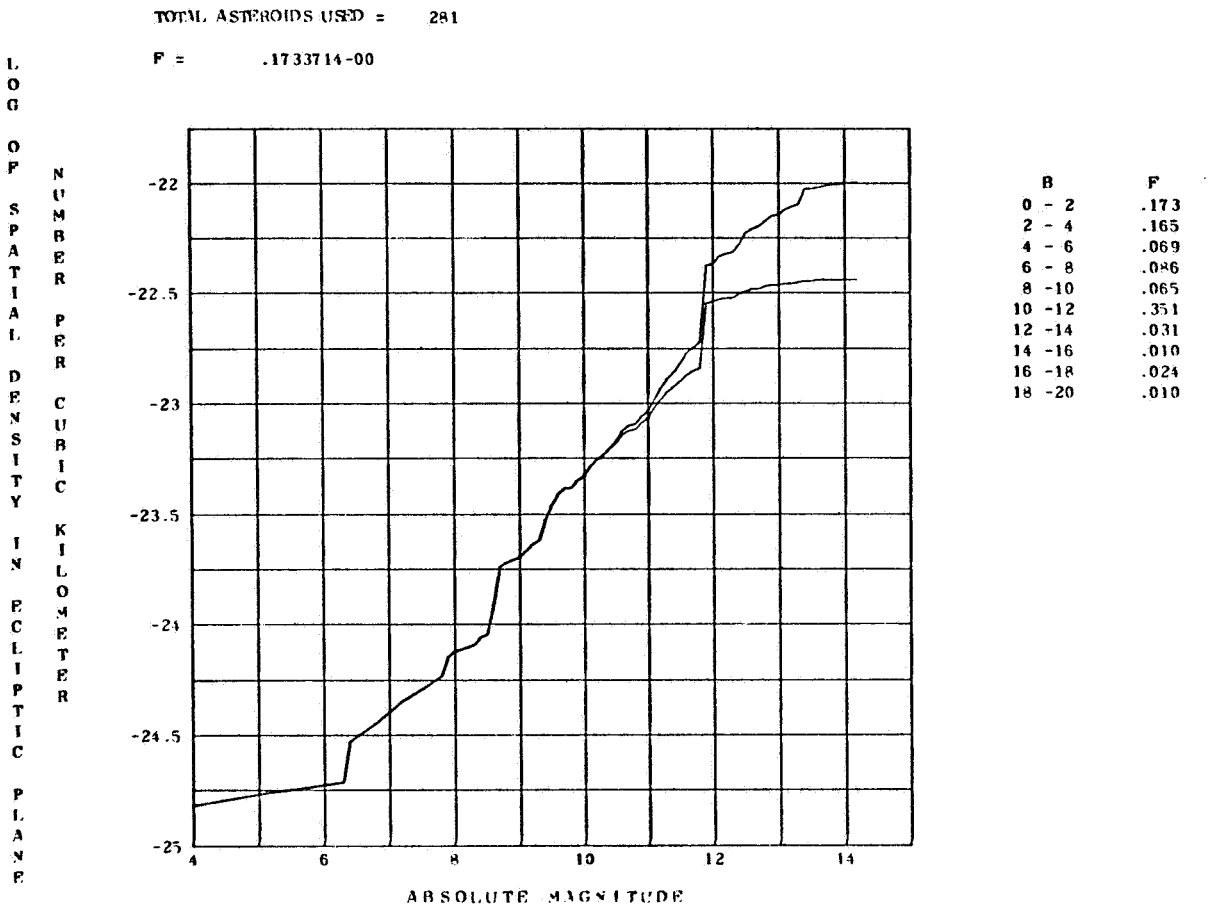


Figure 154. - Spatial density at R = 2.90 and at longitudes between 270.0 and 315.0.

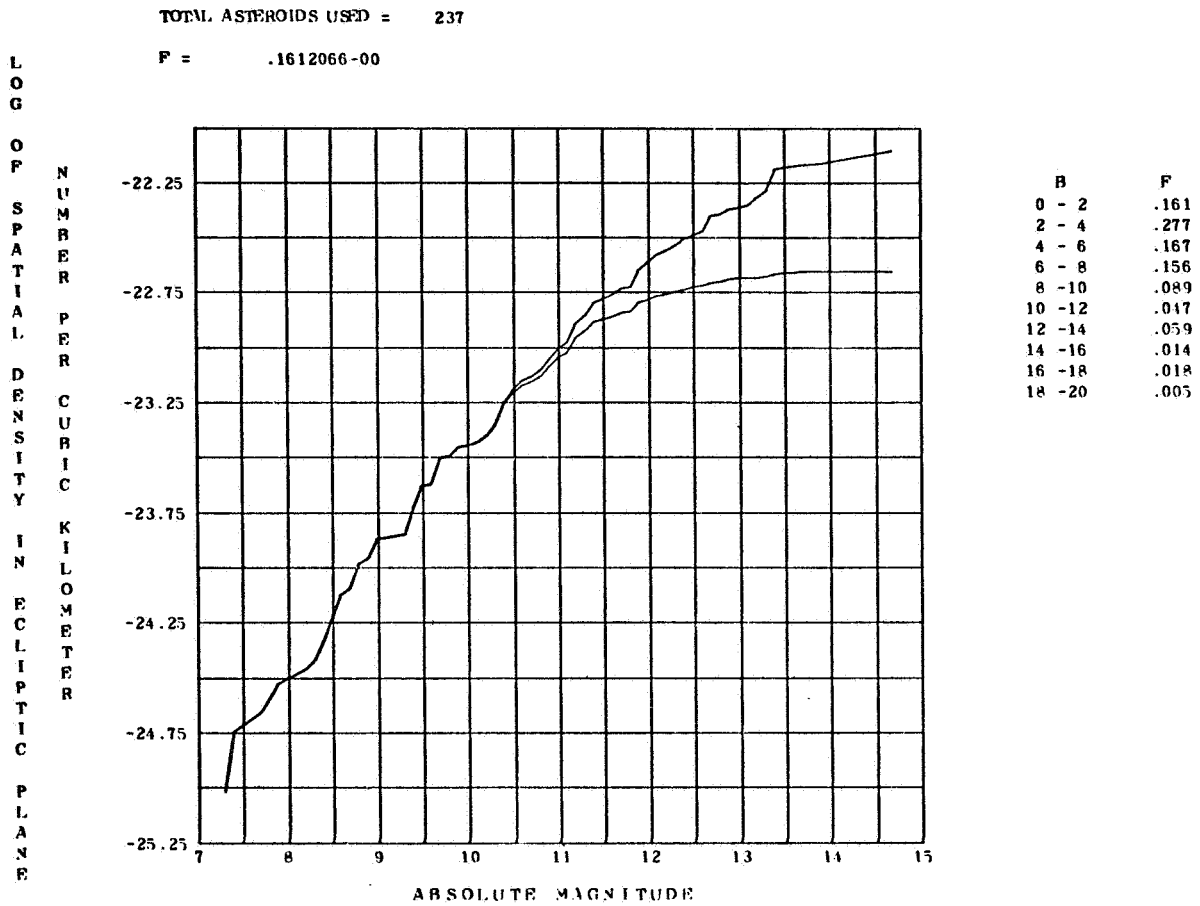


Figure 155. - Spatial density at R = 2.90 and at longitudes between 315.0 and 360.0.

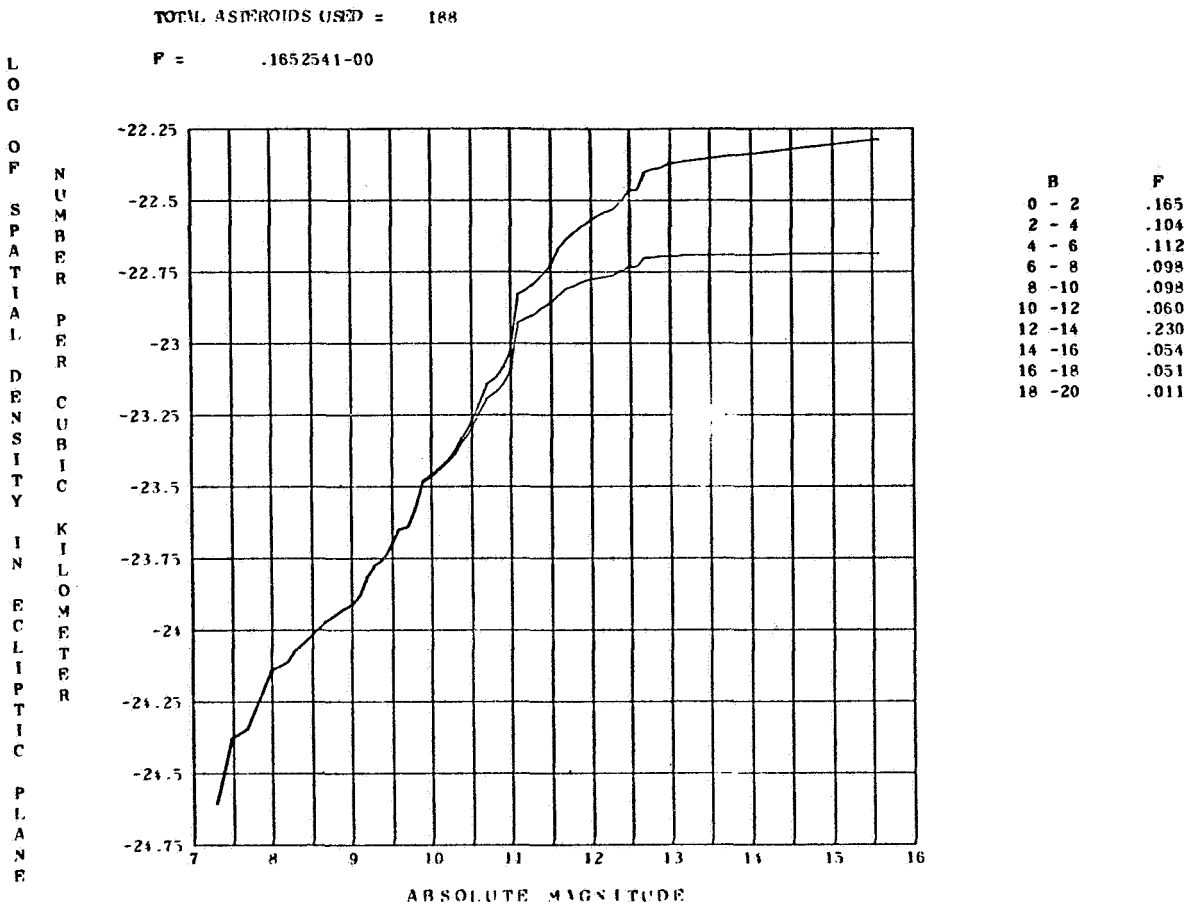


Figure 156.- Spatial density at R = 3.00 and at longitudes between 0 and 45.0.

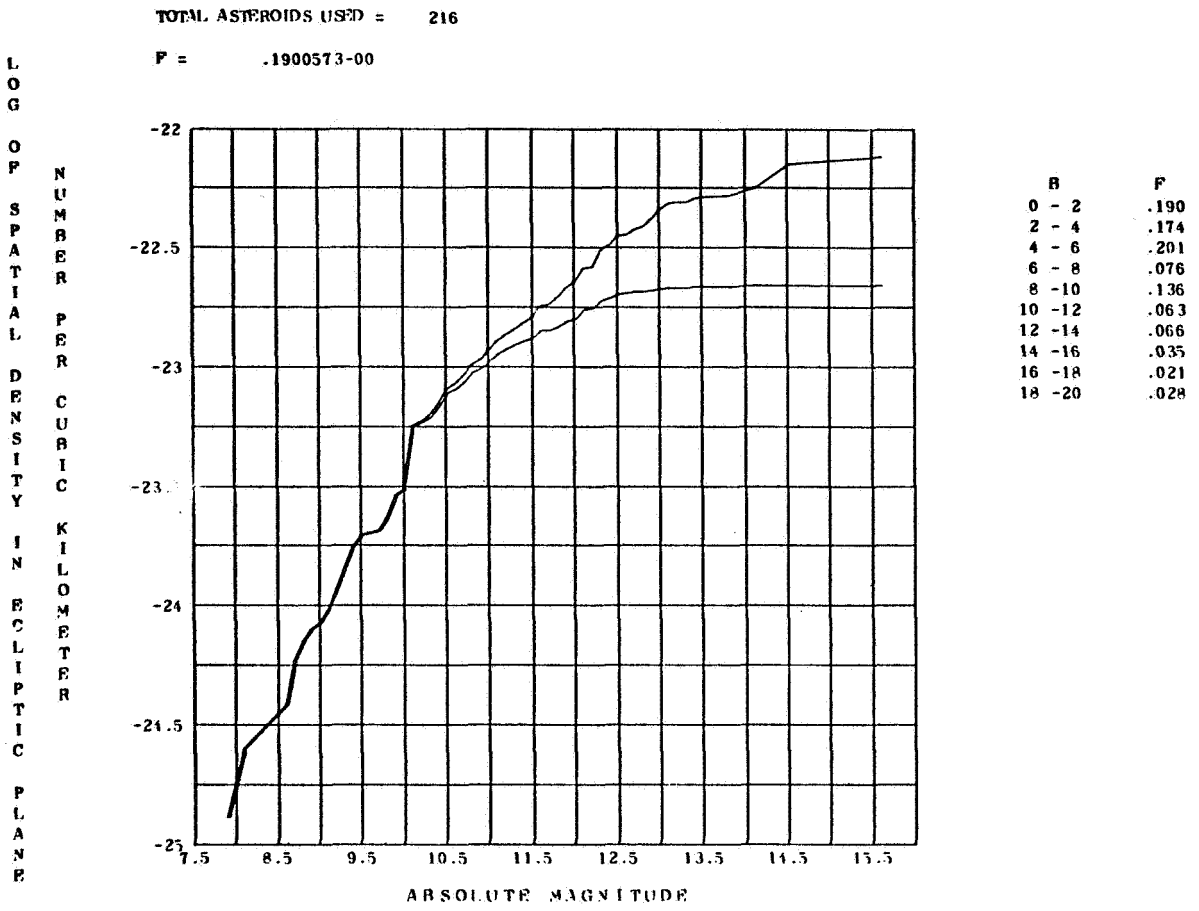


Figure 157. - Spatial density at R = 3.00 and at longitudes between 45.0 and 90.0.

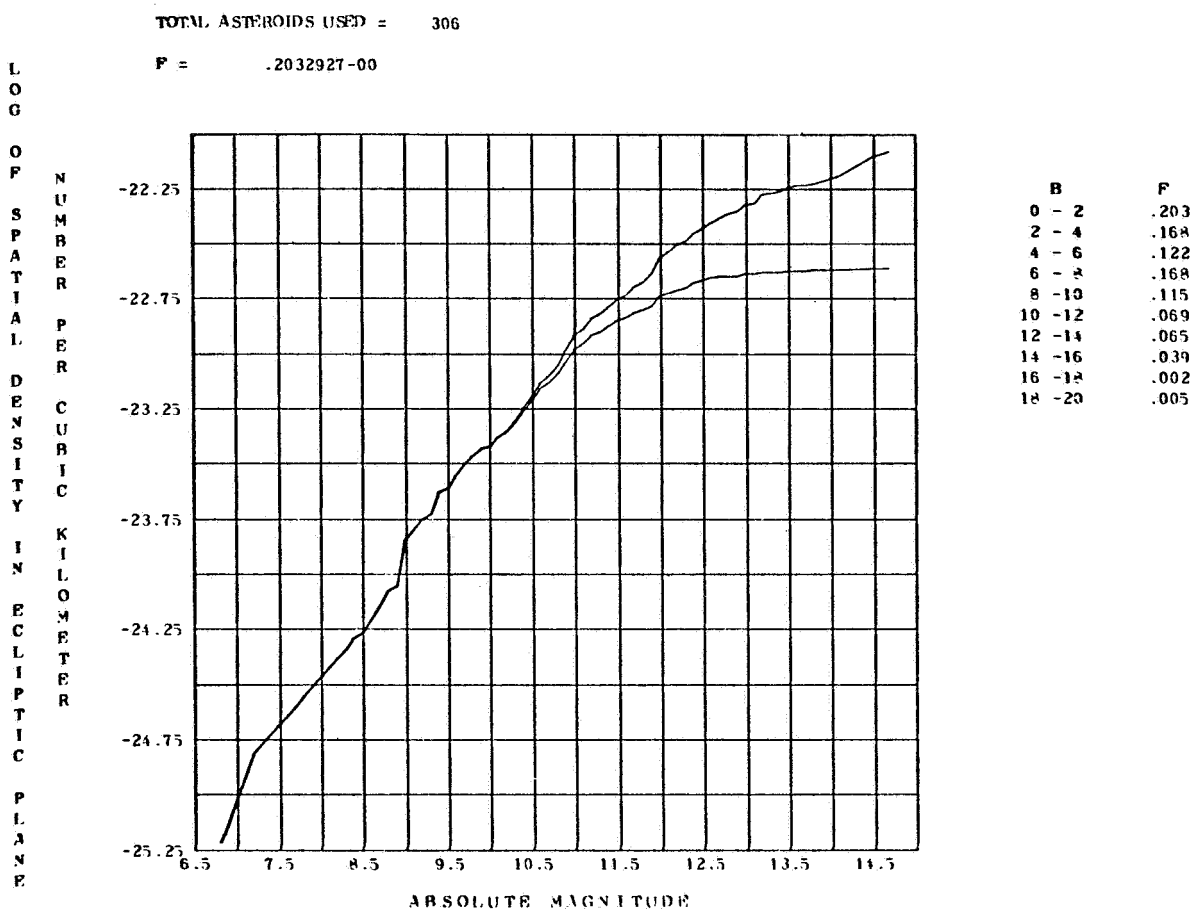


Figure 158. - Spatial density at R = 3.00 and at longitudes between 90.0 and 135.0.

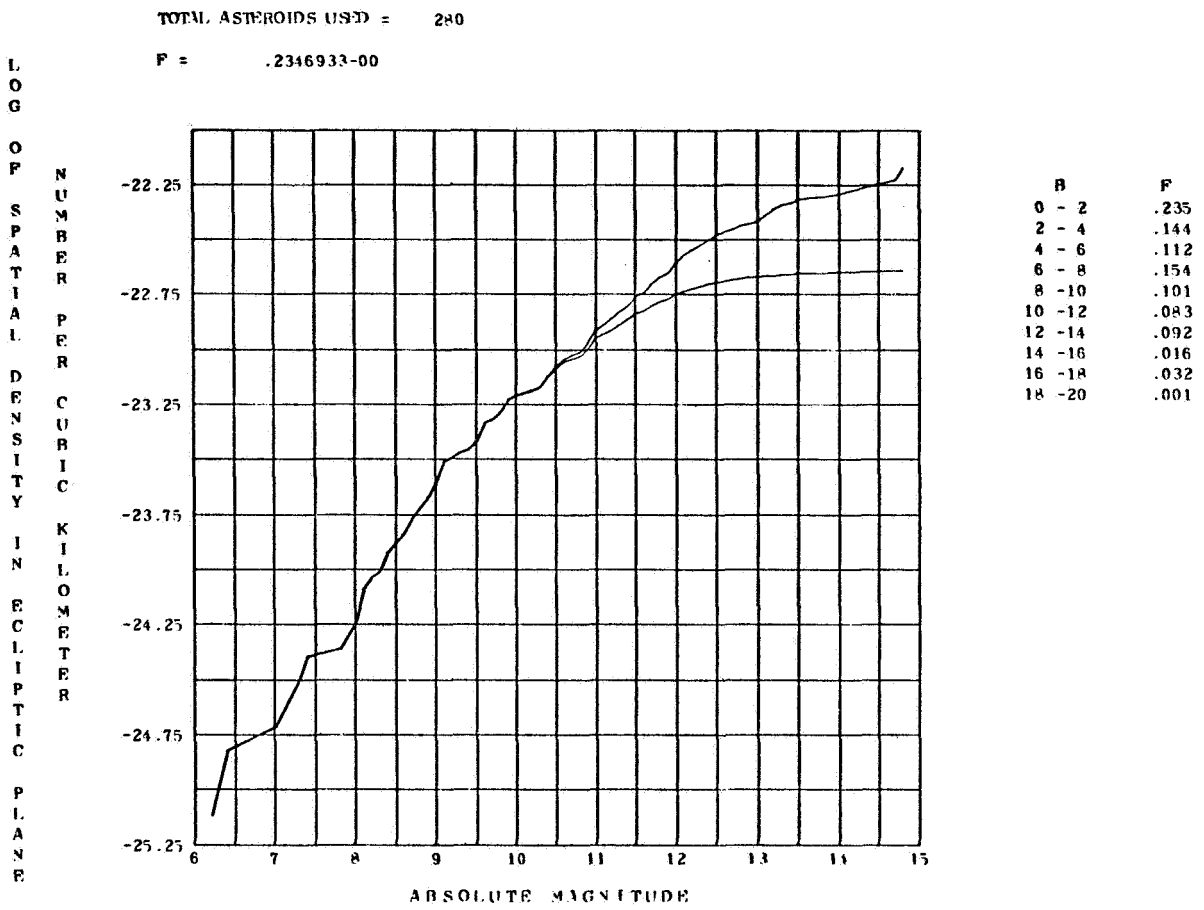


Figure 159. - Spatial density at R = 3.00 and at longitudes between 135.0 and 180.0.

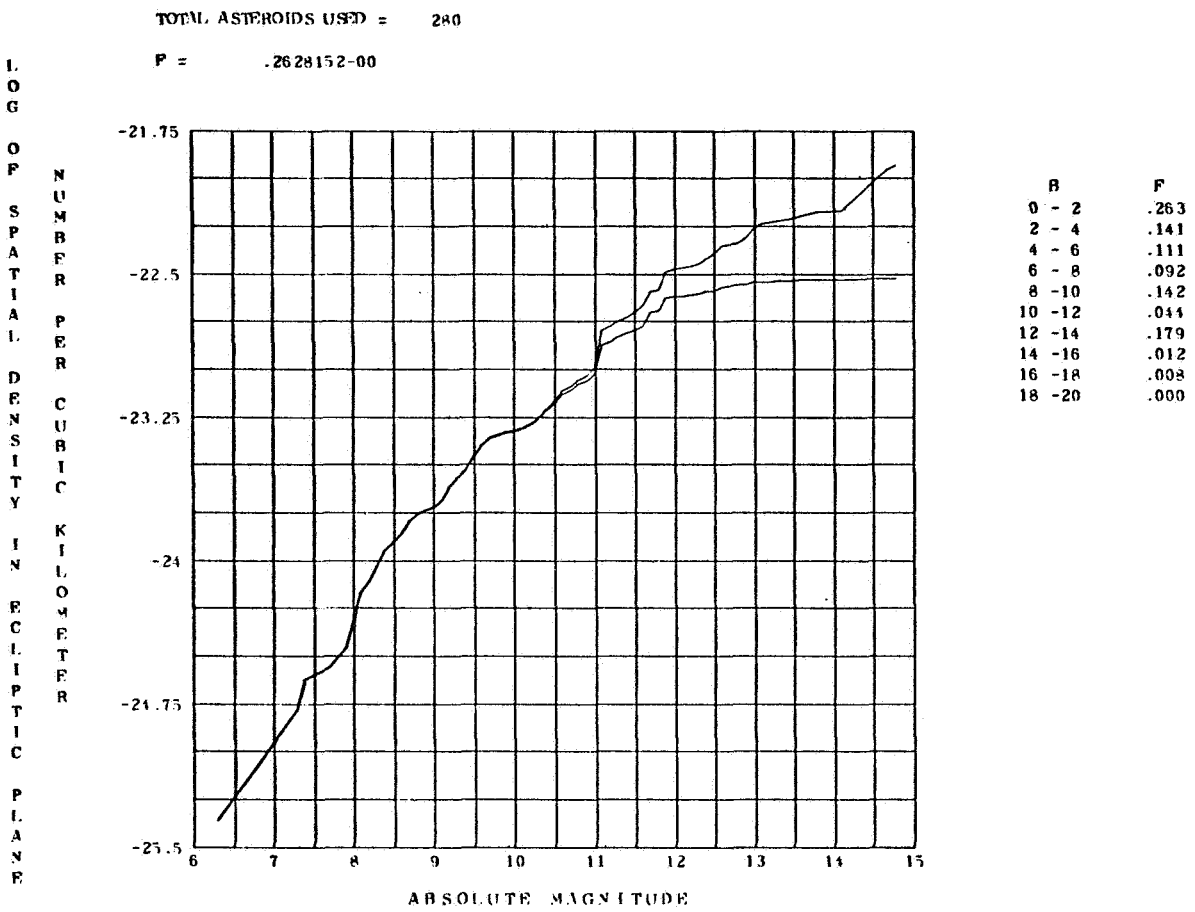


Figure 160. - Spatial density at R = 3.00 and at longitudes between 180.0 and 225.0.

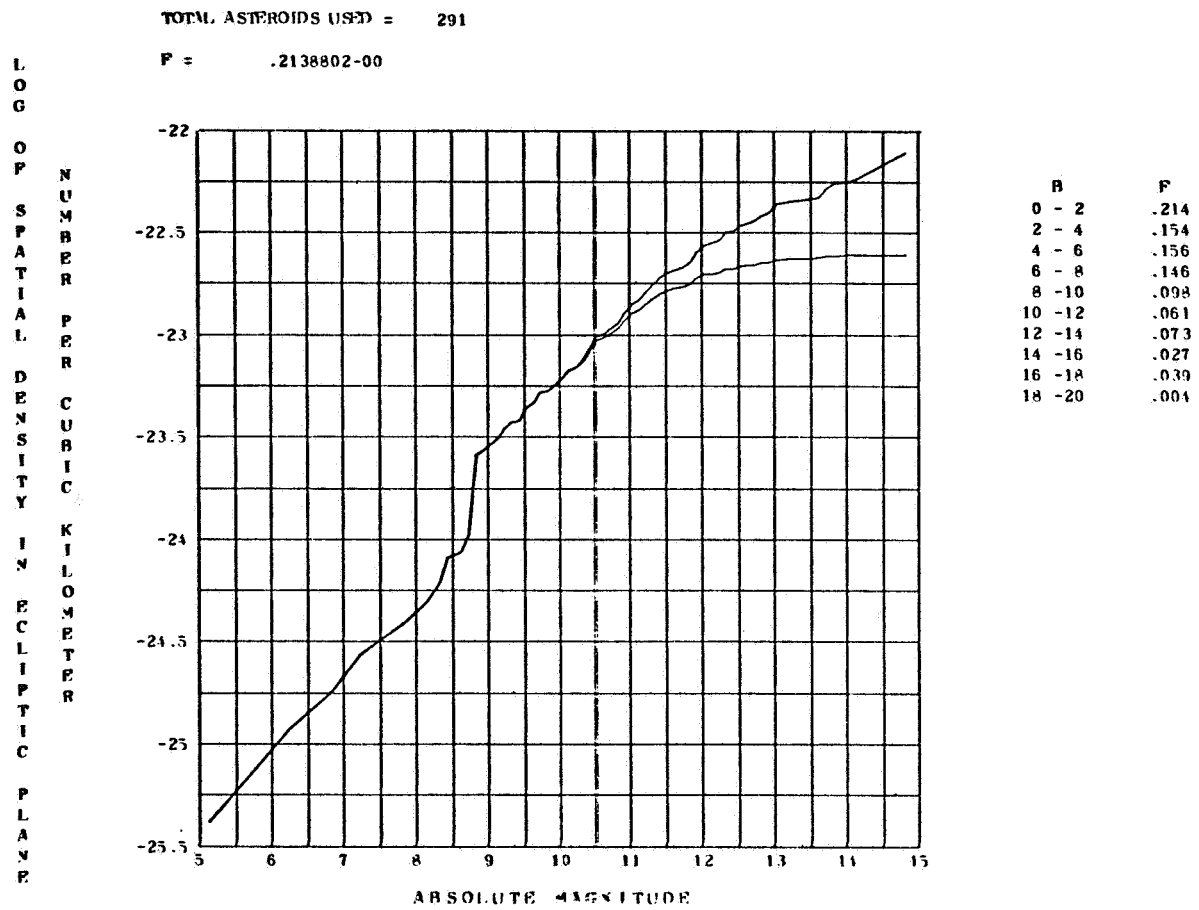


Figure 161. - Spatial density at R = 3.00 and at longitudes between 225.0 and 270.0.

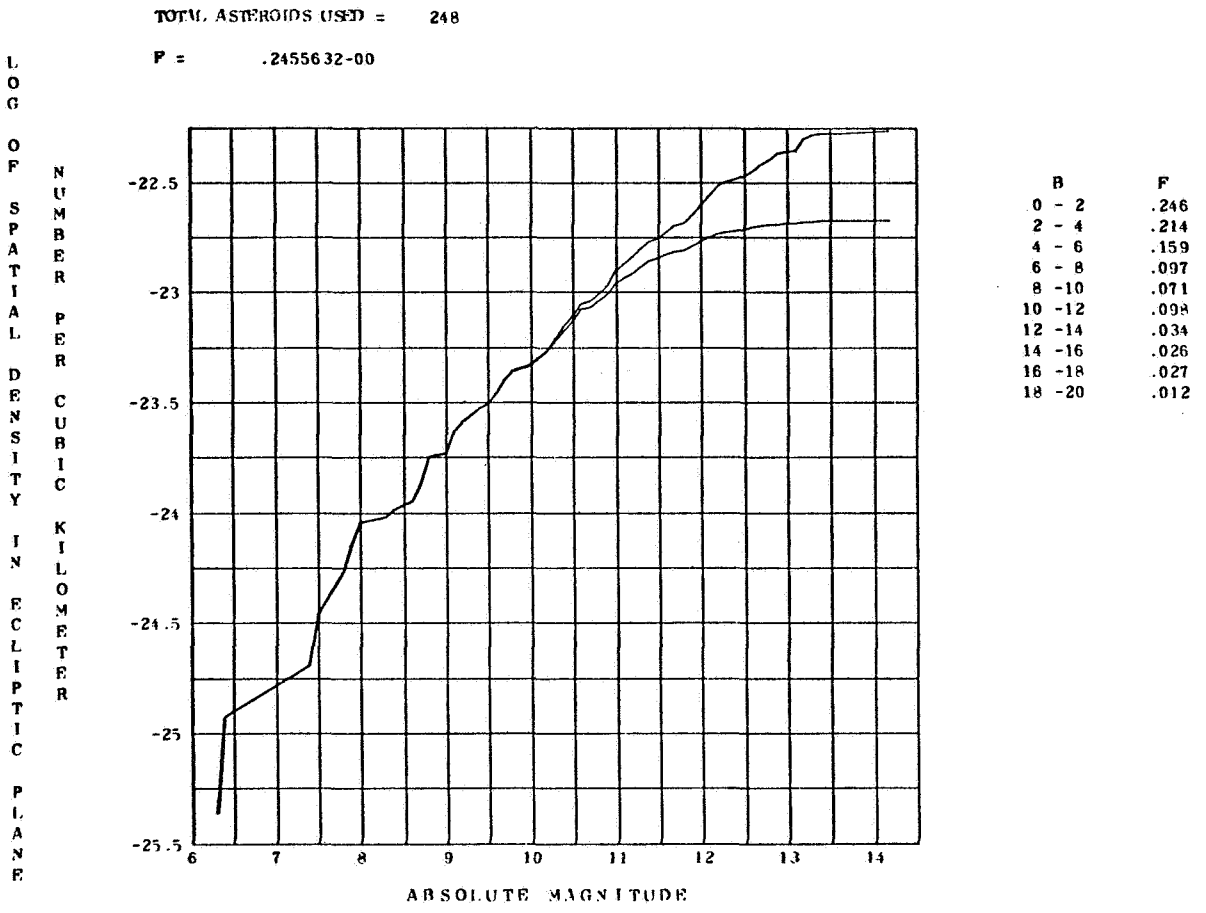


Figure 162. - Spatial density at R = 3.00 and at longitudes between 270.0 and 315.0.

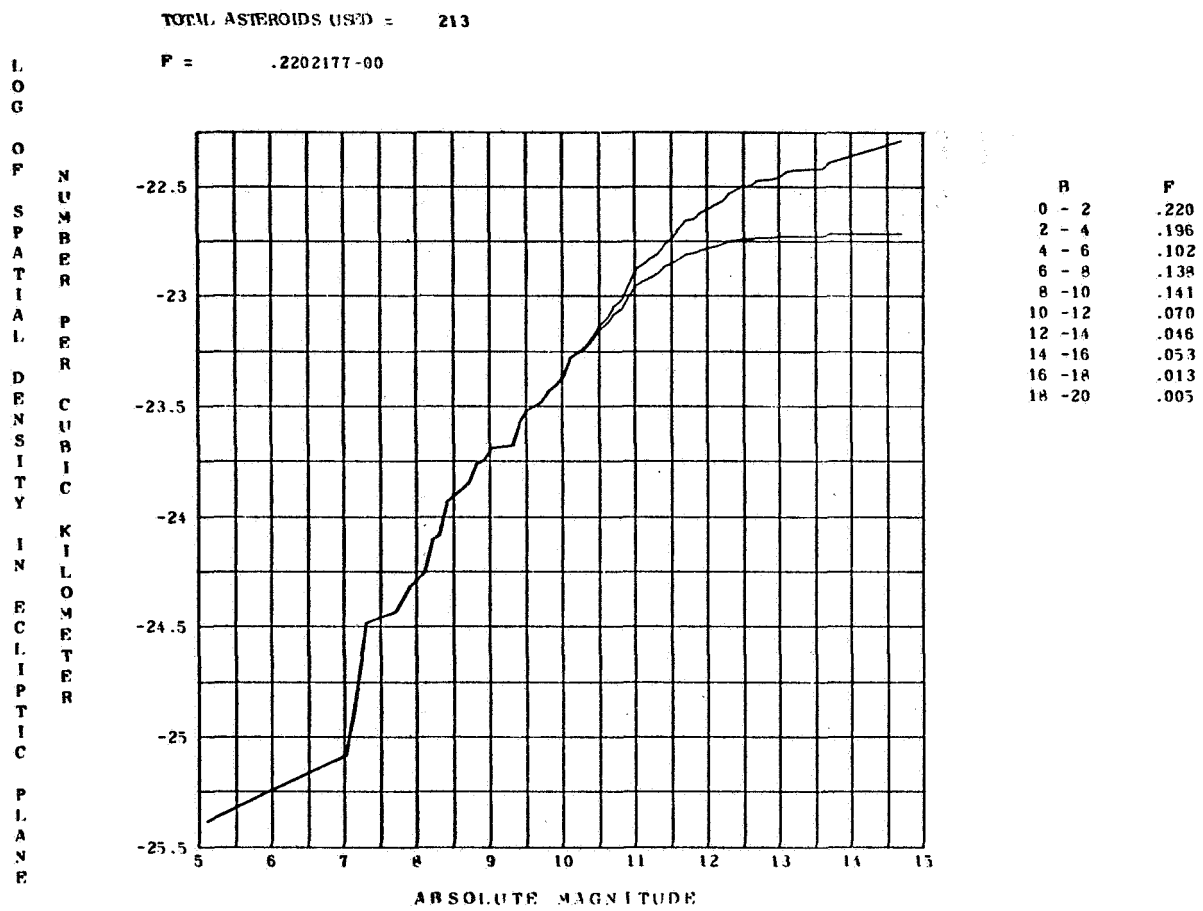


Figure 163. - Spatial density at R = 3.00 and at longitudes between 315.0 and 360.0.

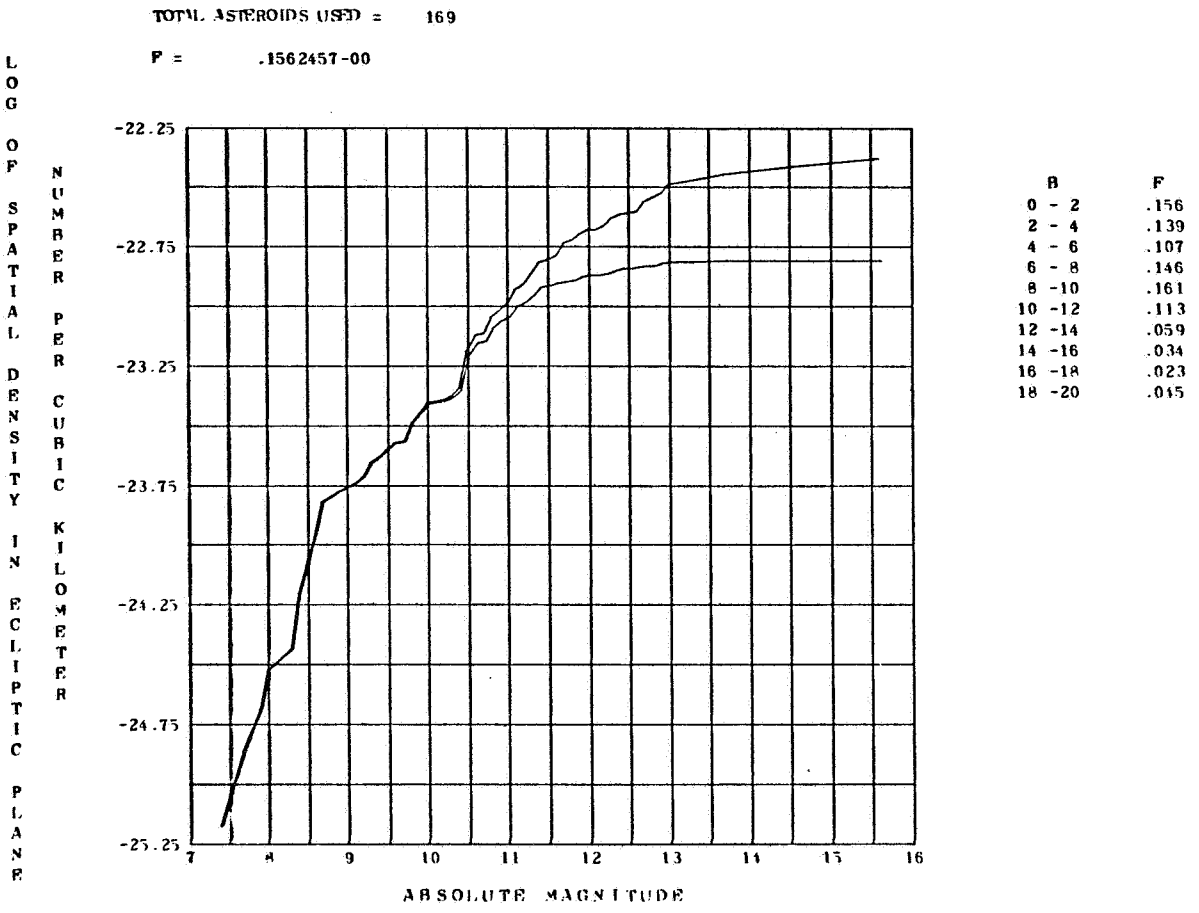


Figure 164. - Spatial density at R = 3.10 and at longitudes between 0 and 45.0.

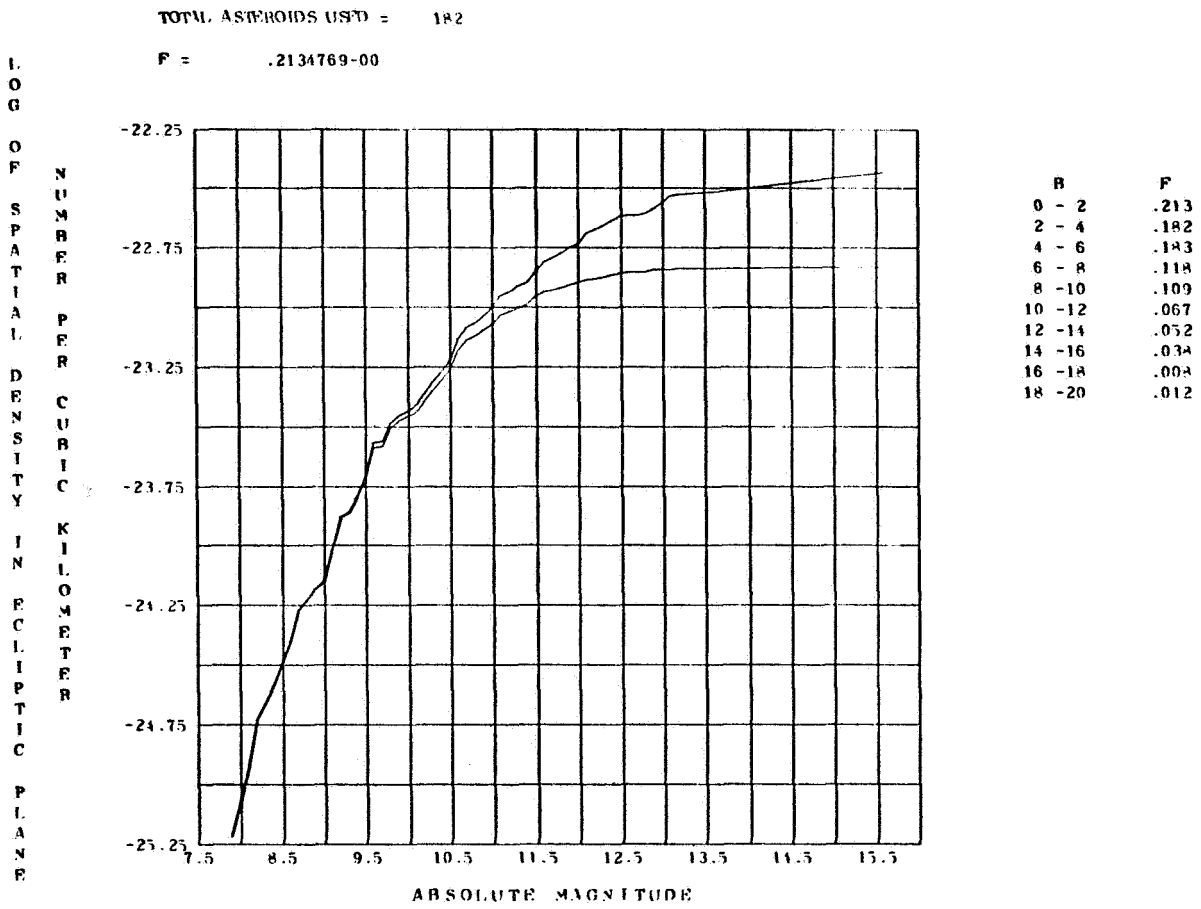


Figure 165. - Spatial density at R = 3. 10 and at longitudes between 45. 0 and 90. 0.

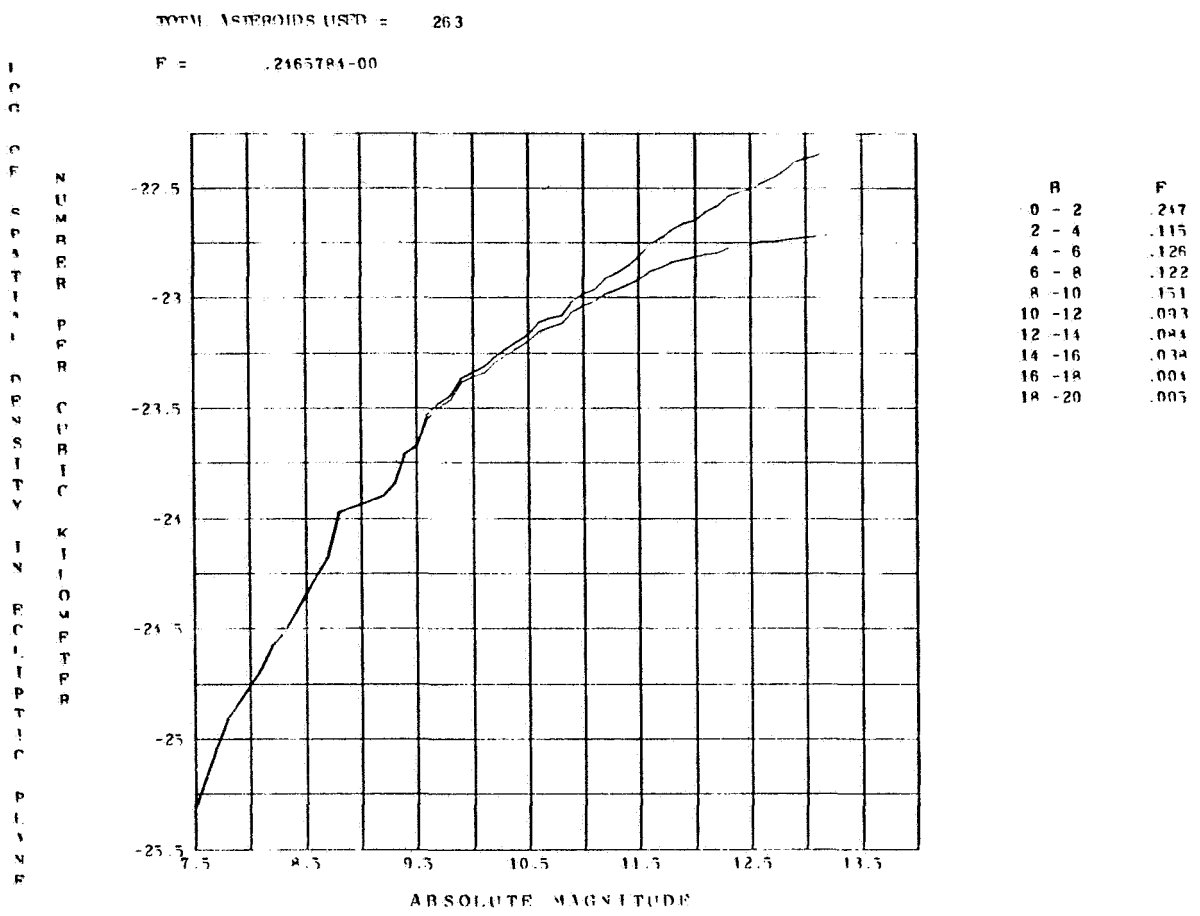


Figure 166. - Spatial density at R = 3.10 and at longitudes between 90.0 and 135.0.

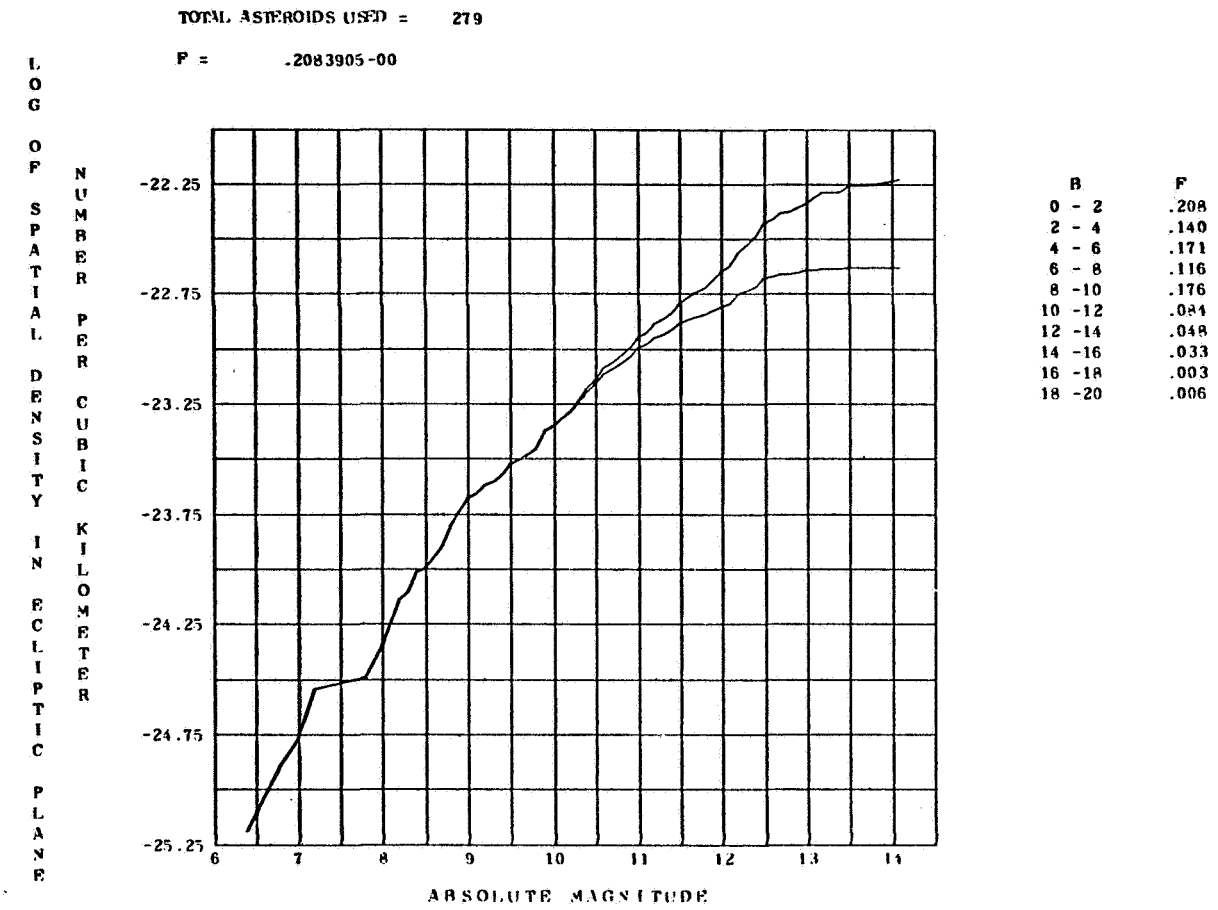


Figure 167. - Spatial density at R = 3.10 and at longitudes between 135.0 and 180.0.

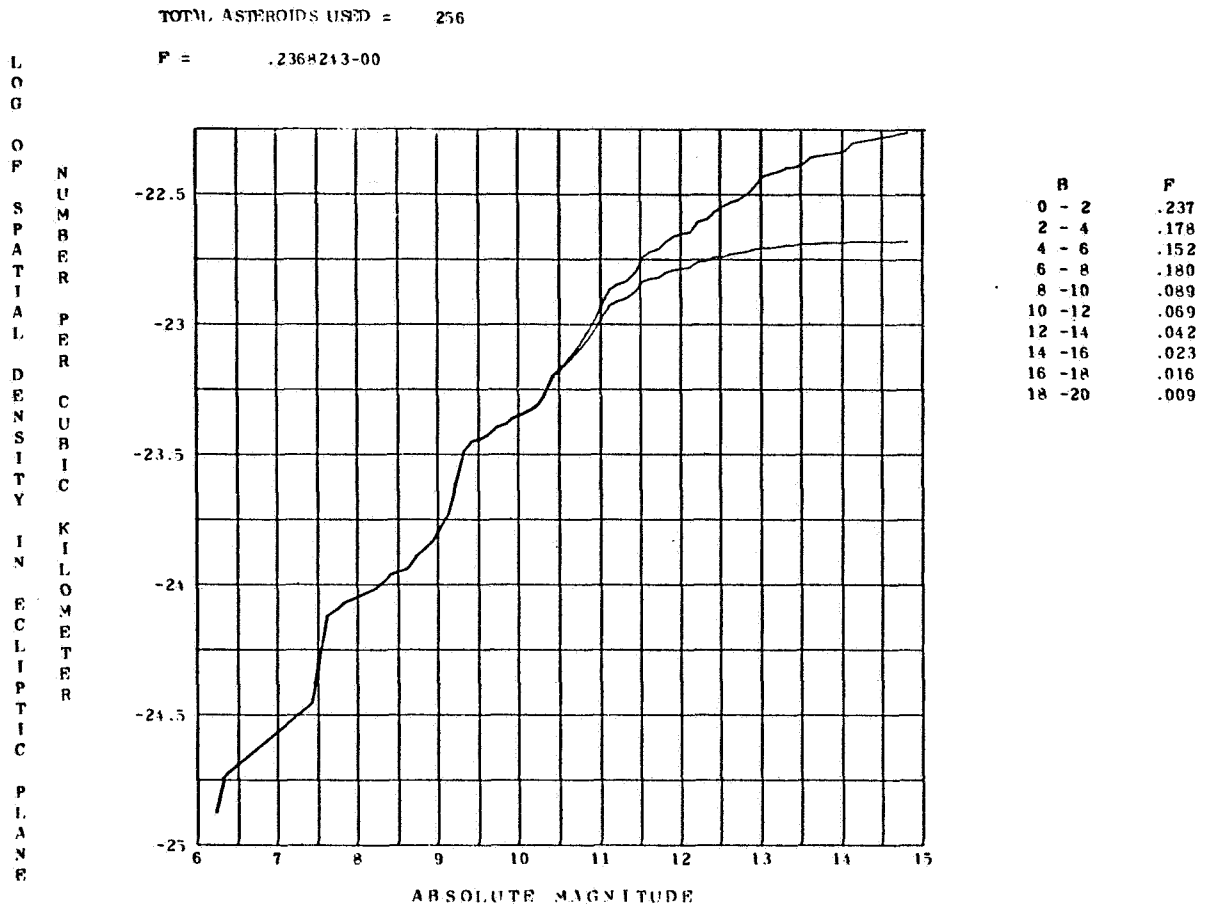


Figure 168. - Spatial density at R = 3.10 and at longitudes between 180.0 and 225.0.

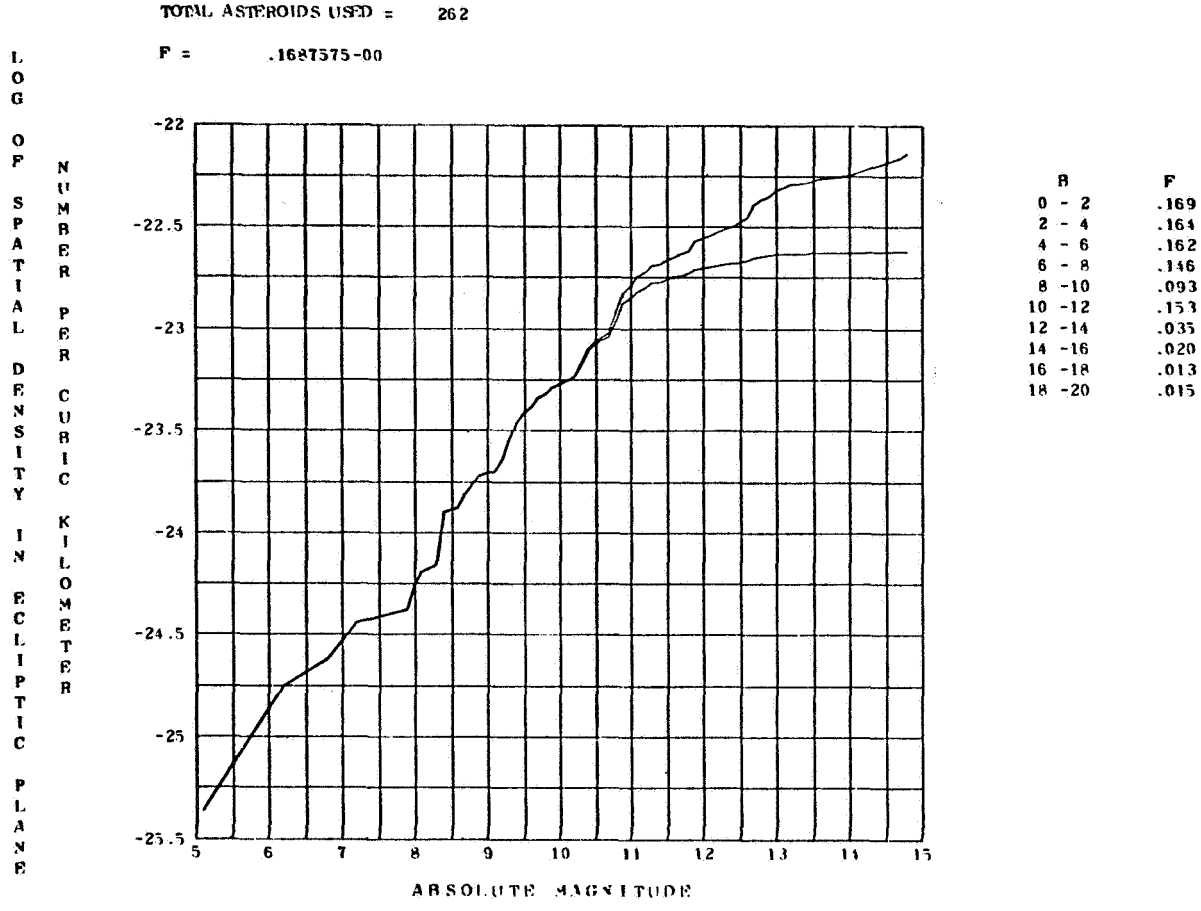


Figure 169. - Spatial density at R = 3.10 and at longitudes between 225.0 and 270.0.

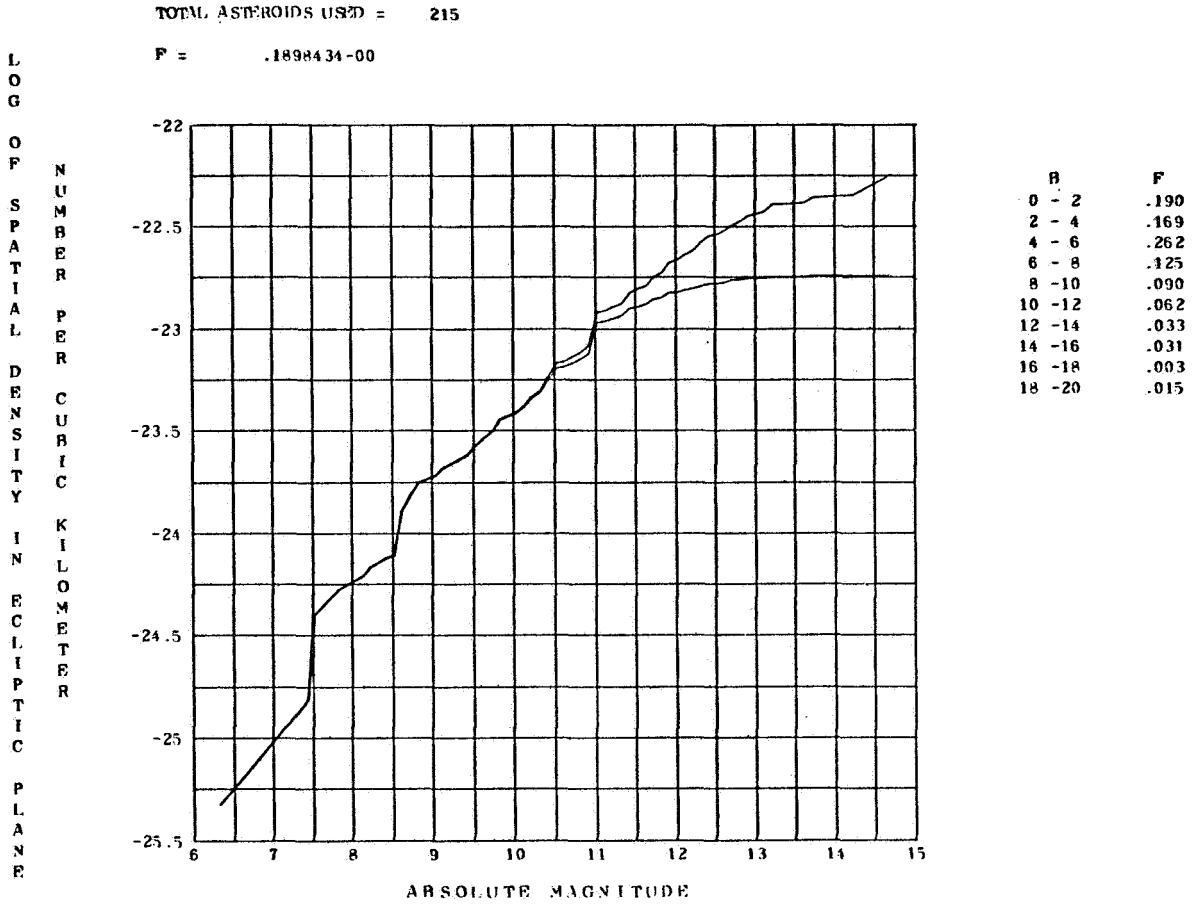


Figure 170. - Spatial density at R = 3.10 and at longitudes between 270.0 and 315.0.

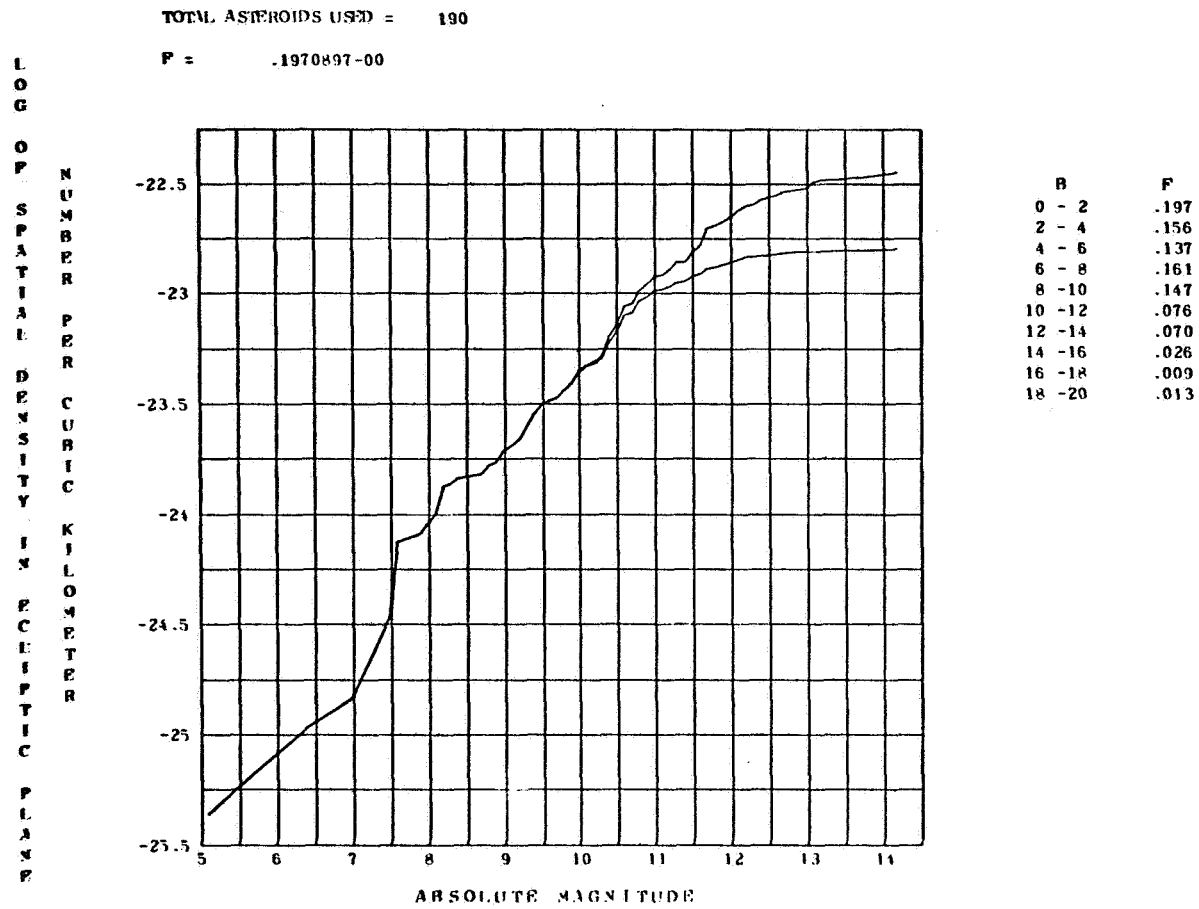


Figure 171. - Spatial density at R = 3.10 and at longitudes between 315.0 and 360.0.

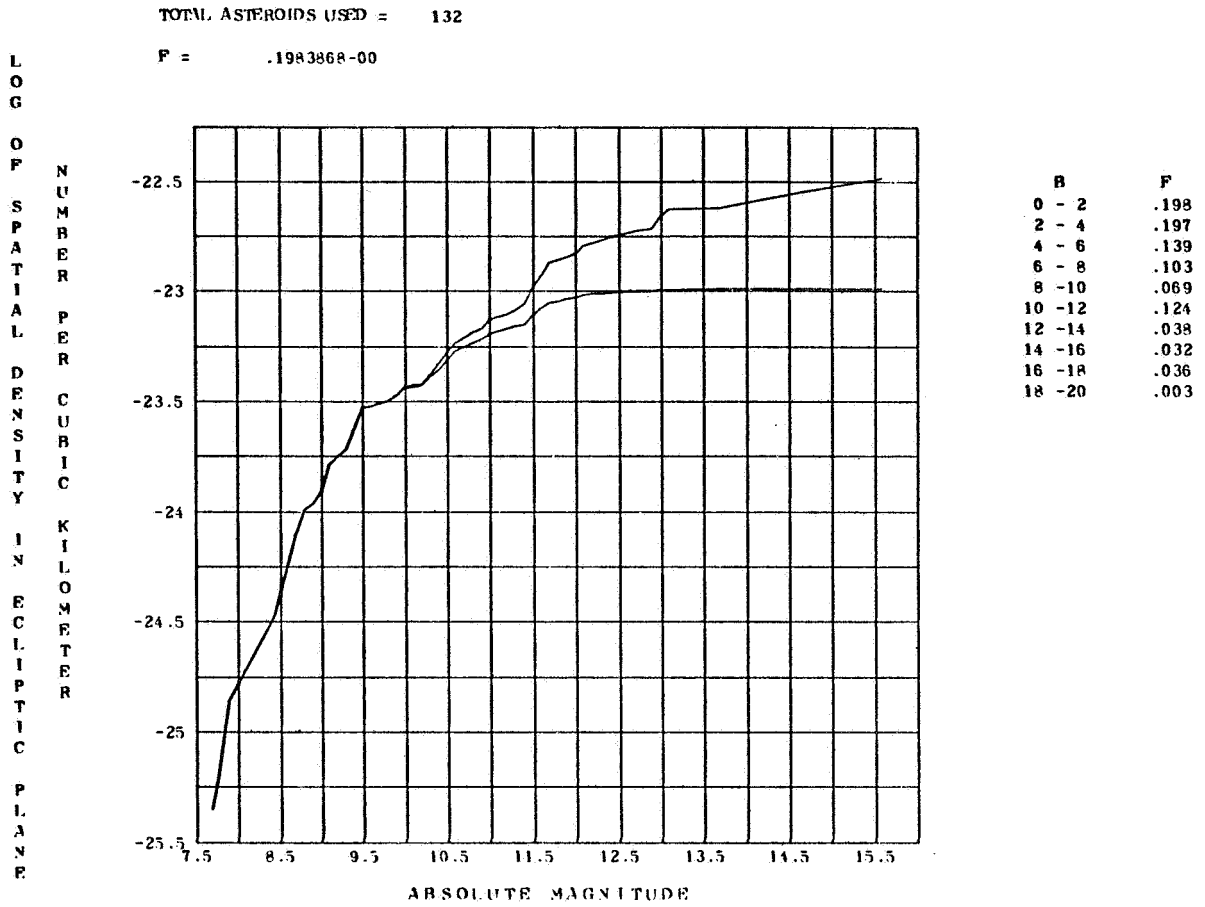


Figure 172. - Spatial density at R = 3.20 and at longitudes between 0 and 45.0.

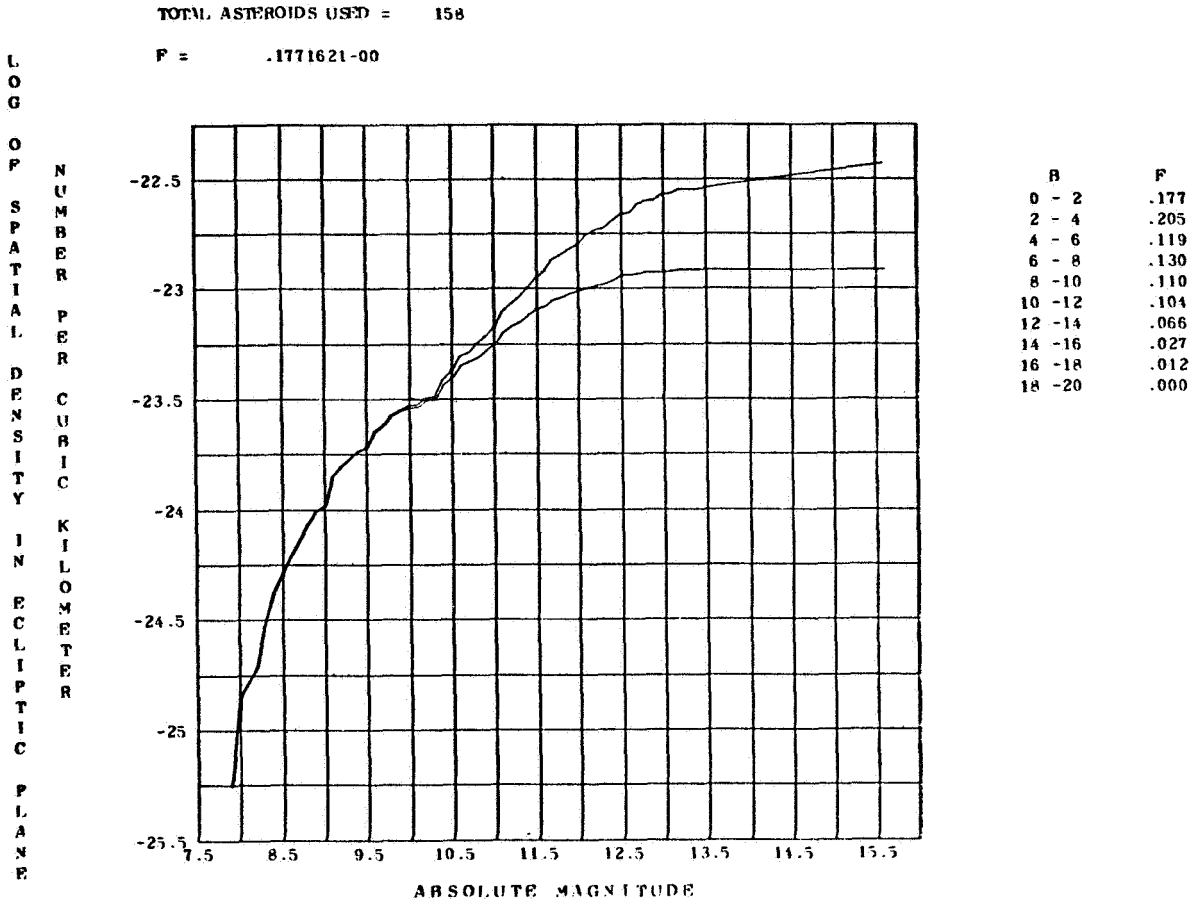


Figure 173. - Spatial density at R = 3.20 and at longitudes between 45.0 and 90.0.

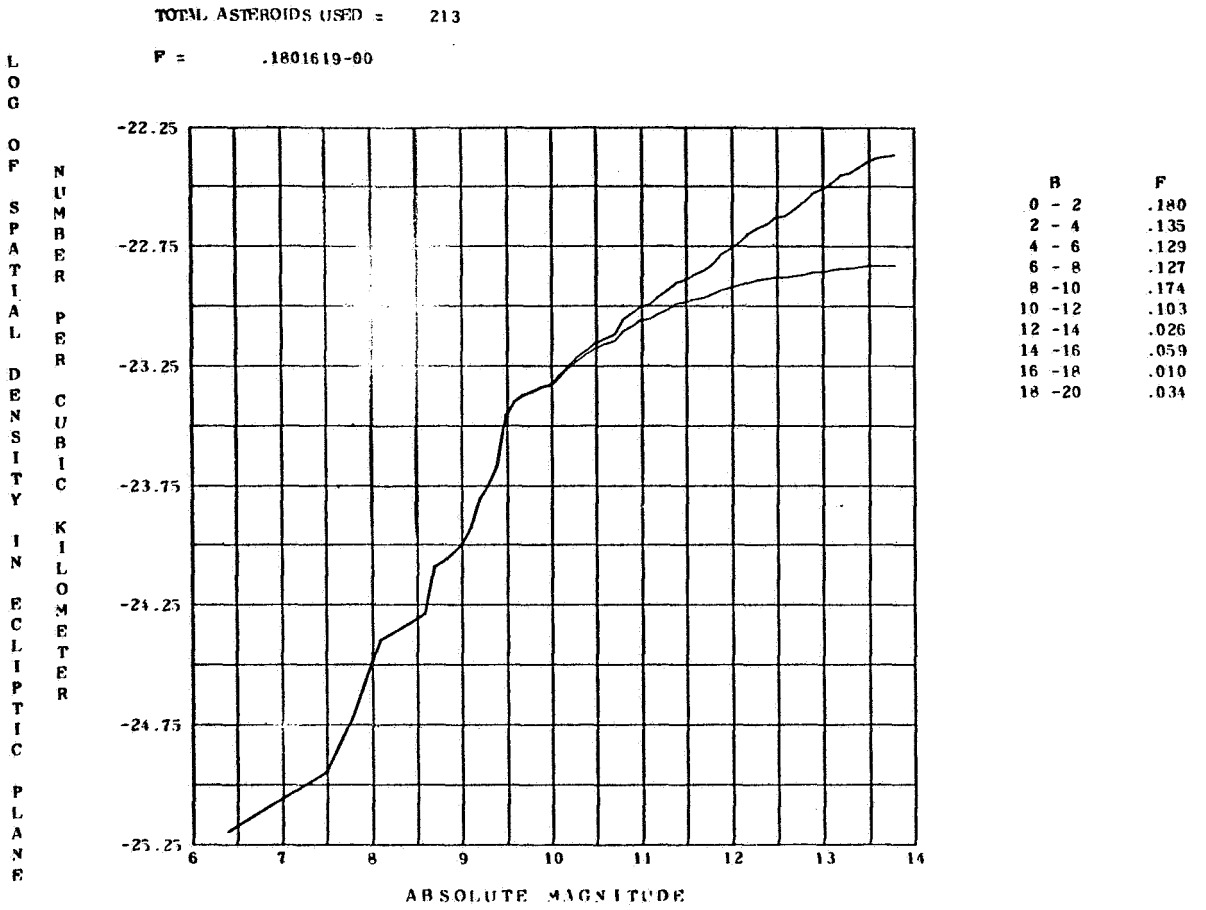


Figure 174. - Spatial density at R = 3.20 and at longitudes between 90.0 and 135.0.

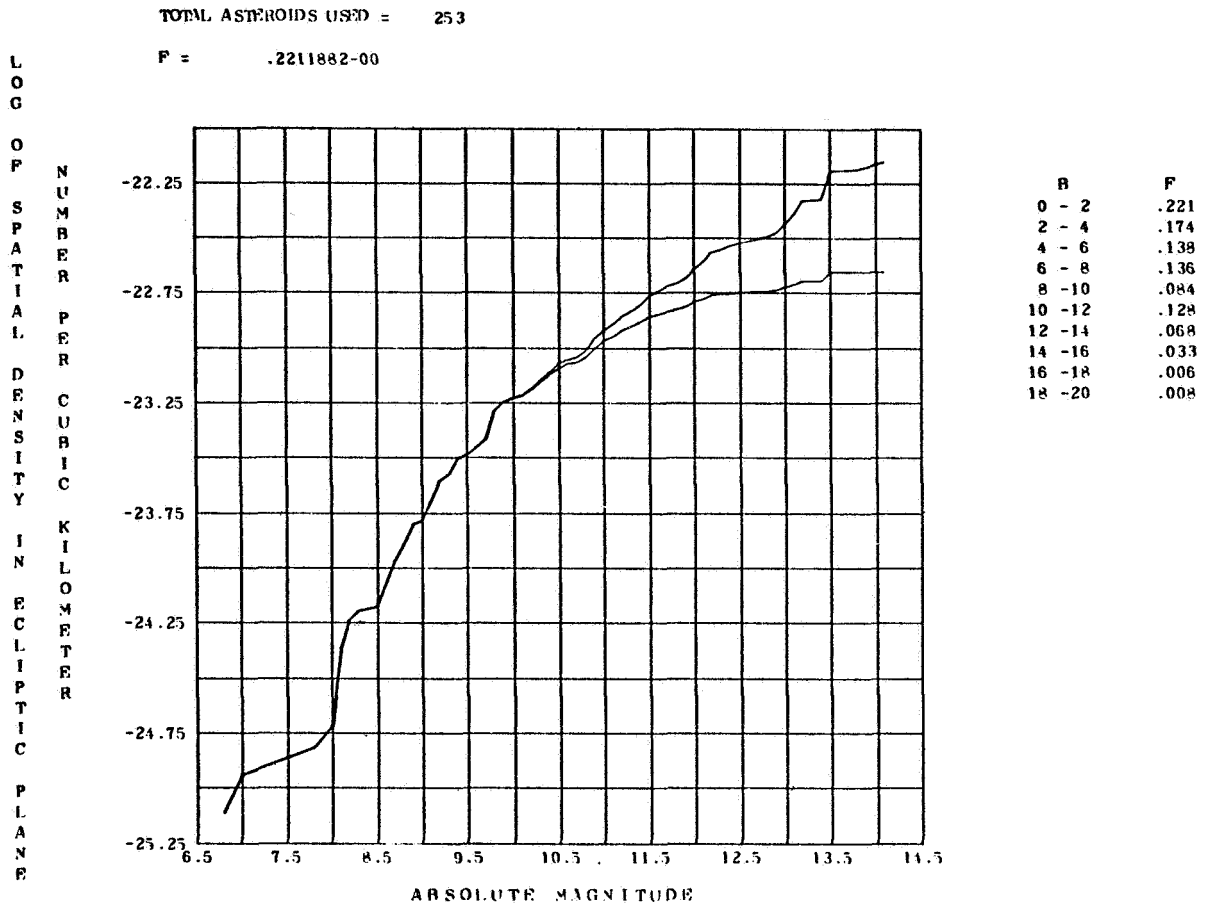


Figure 175. - Spatial density at R = 3.20 and at longitudes between 135.0 and 180.0.

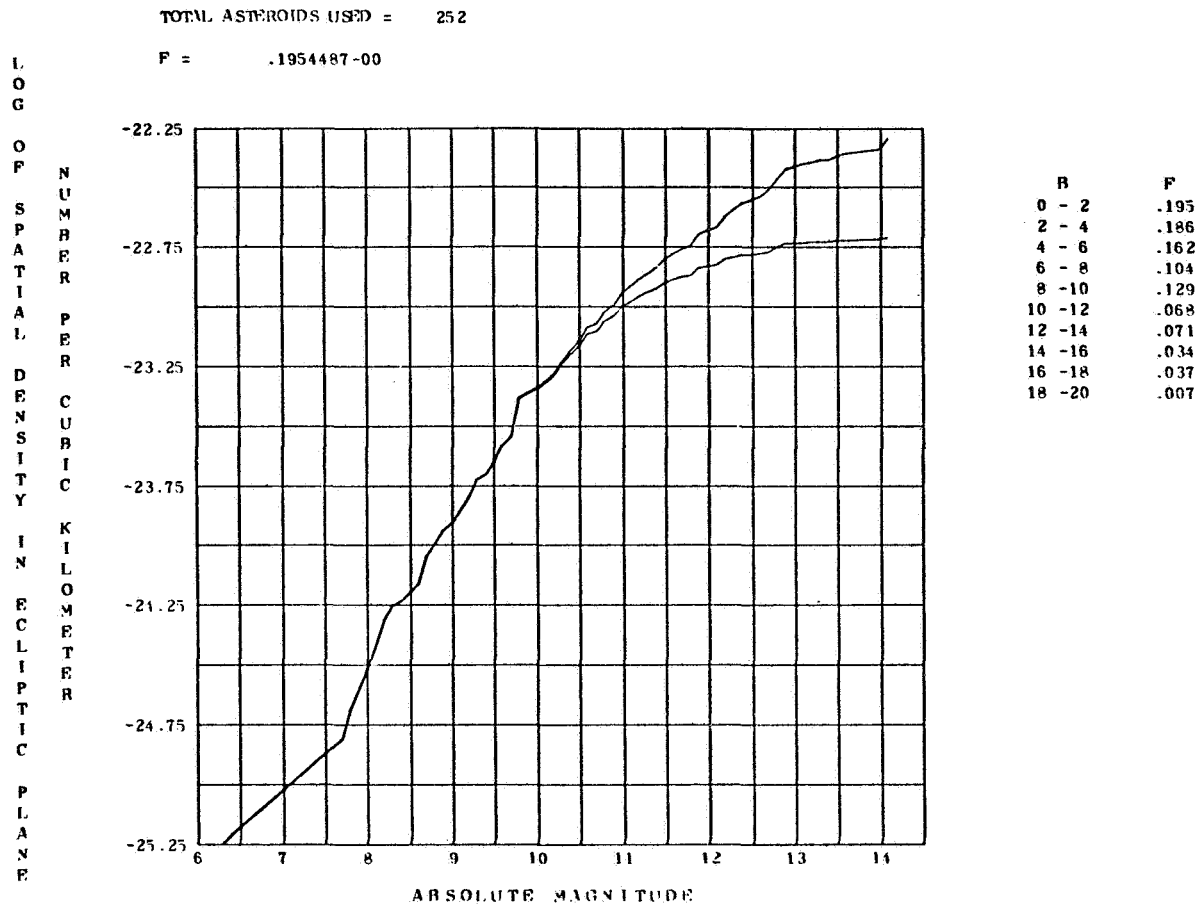


Figure 176. - Spatial density at R = 3.20 and at longitudes between 180.0 and 225.0.

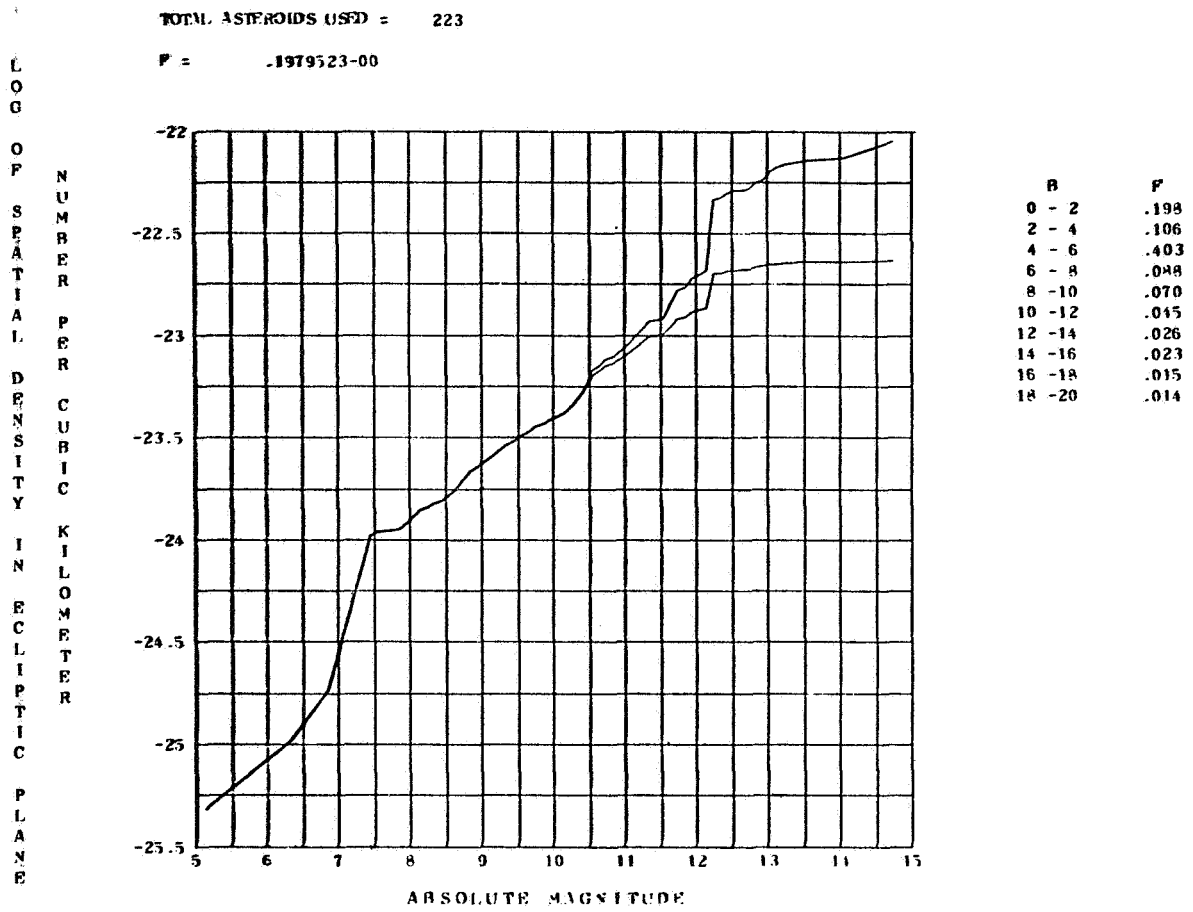


Figure 177. - Spatial density at R = 3.20 and at longitudes between 225.0 and 270.0.

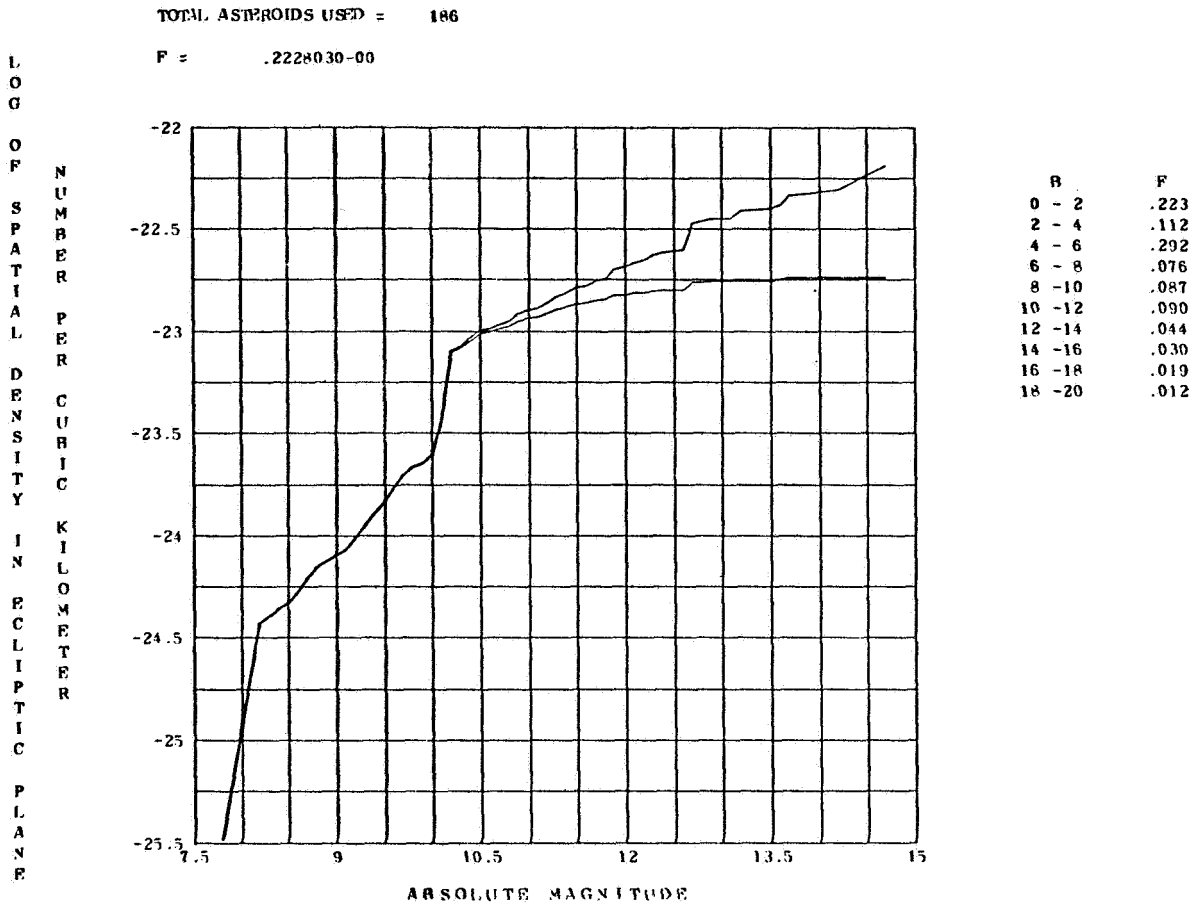


Figure 178. - Spatial density at R = 3.20 and at longitudes between 270.0 and 315.0.

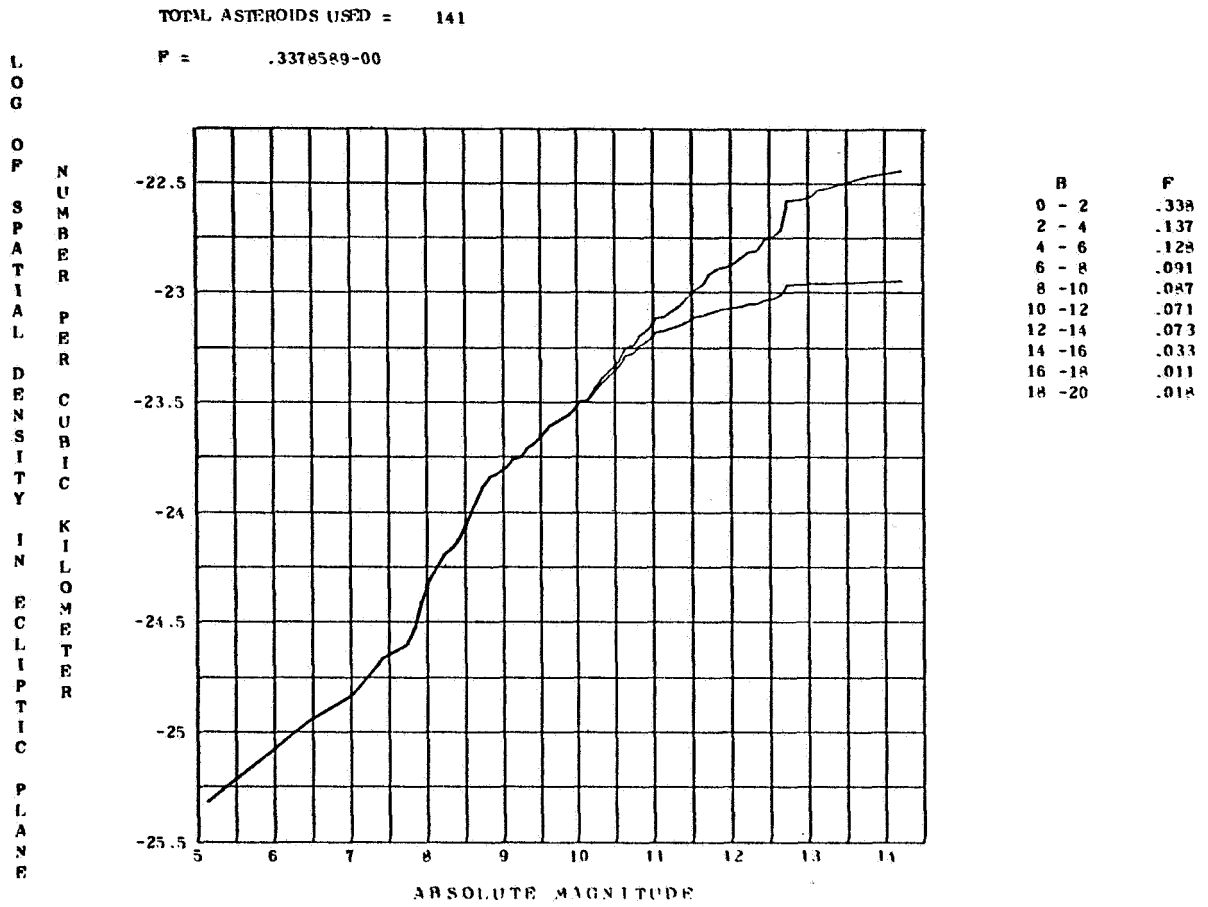


Figure 179. - Spatial density at R = 3.20 and at longitudes between 315.0 and 360.0.

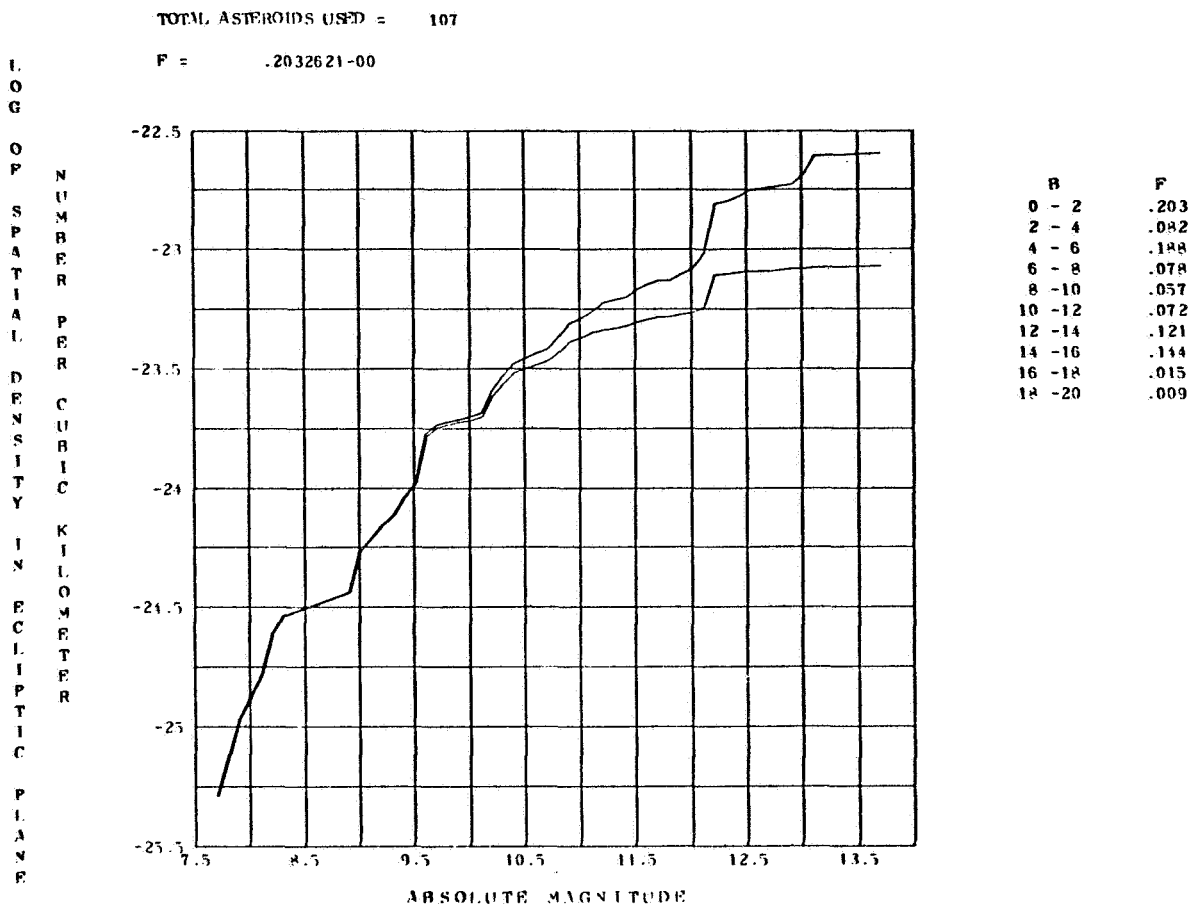


Figure 180.- Spatial density at R = 3.30 and at longitudes between 0 and 45.0.

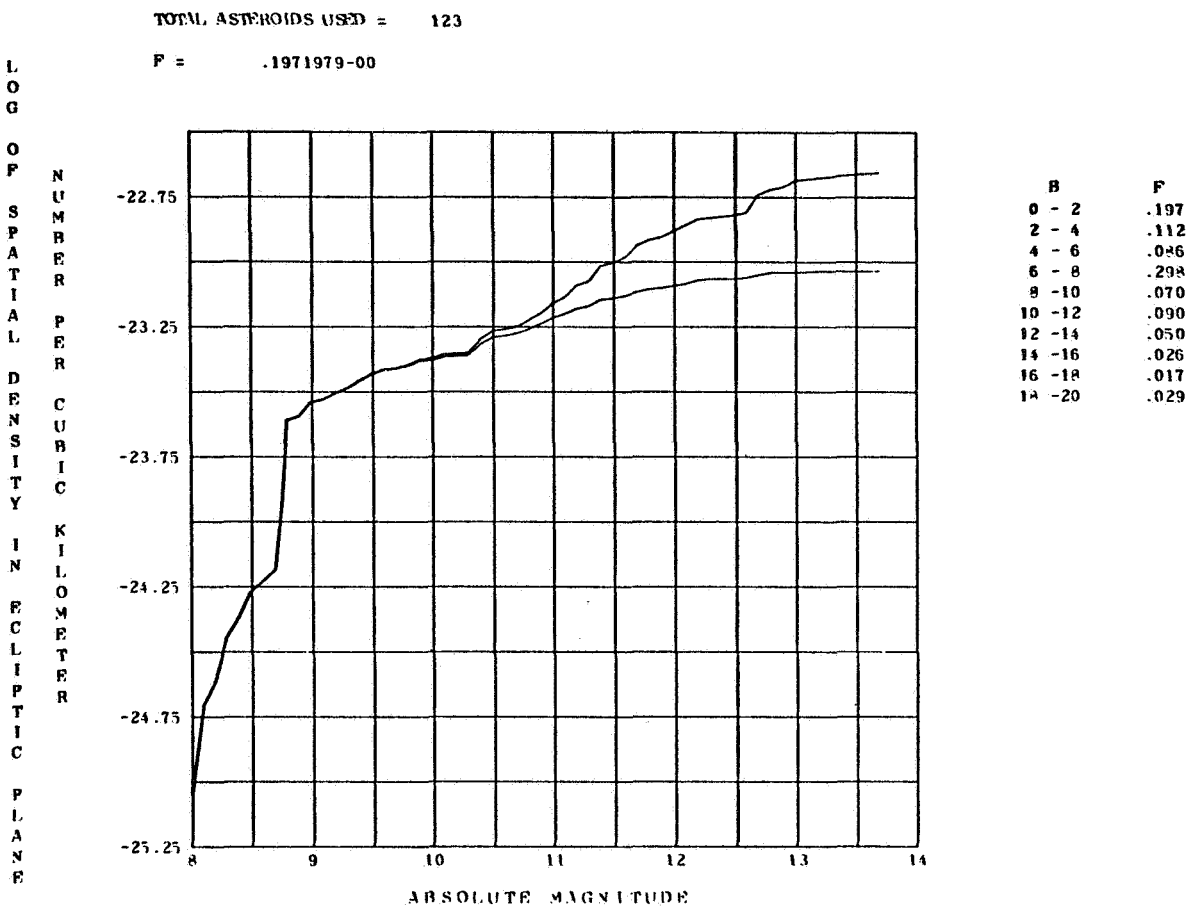
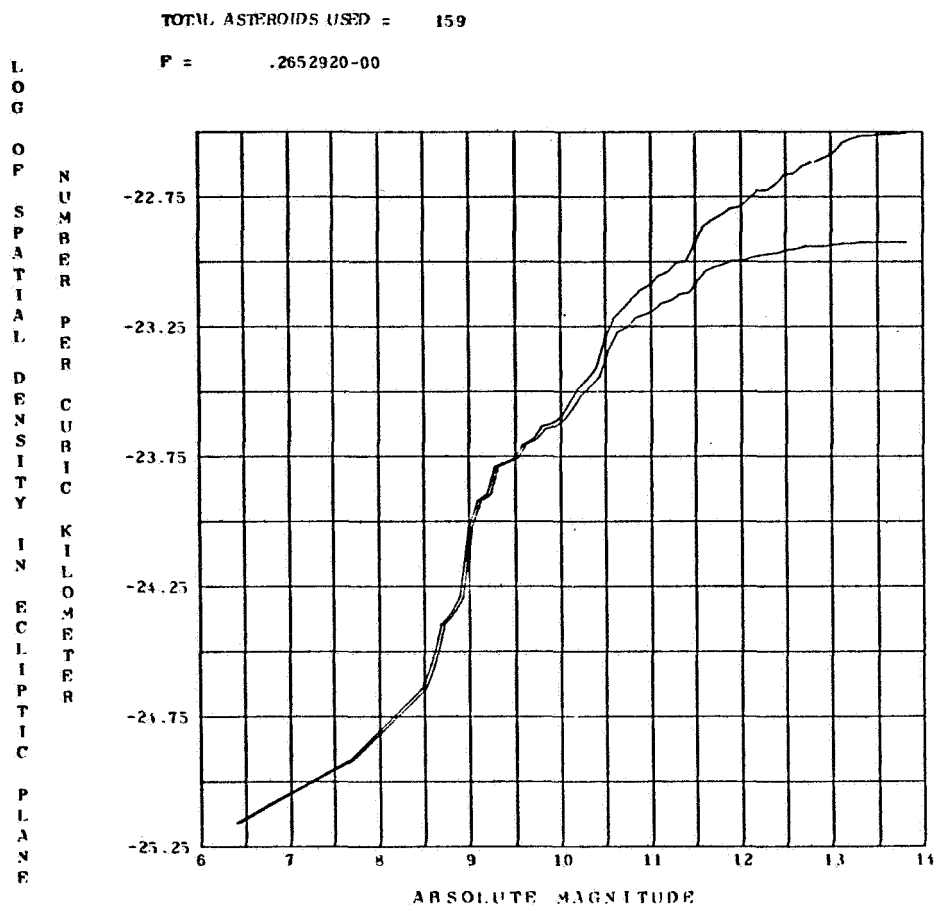


Figure 181.- Spatial density at R = 3.30 and at longitudes between 45.0 and 90.0.



B	F
0 - 2	.265
2 - 4	.114
4 - 6	.103
6 - 8	.152
8 -10	.205
10 -12	.021
12 -14	.082
14 -16	.027
16 -18	.010
18 -20	.019

Figure 182. - Spatial density at R = 3.30 and at longitudes between 90.0 and 135.0.

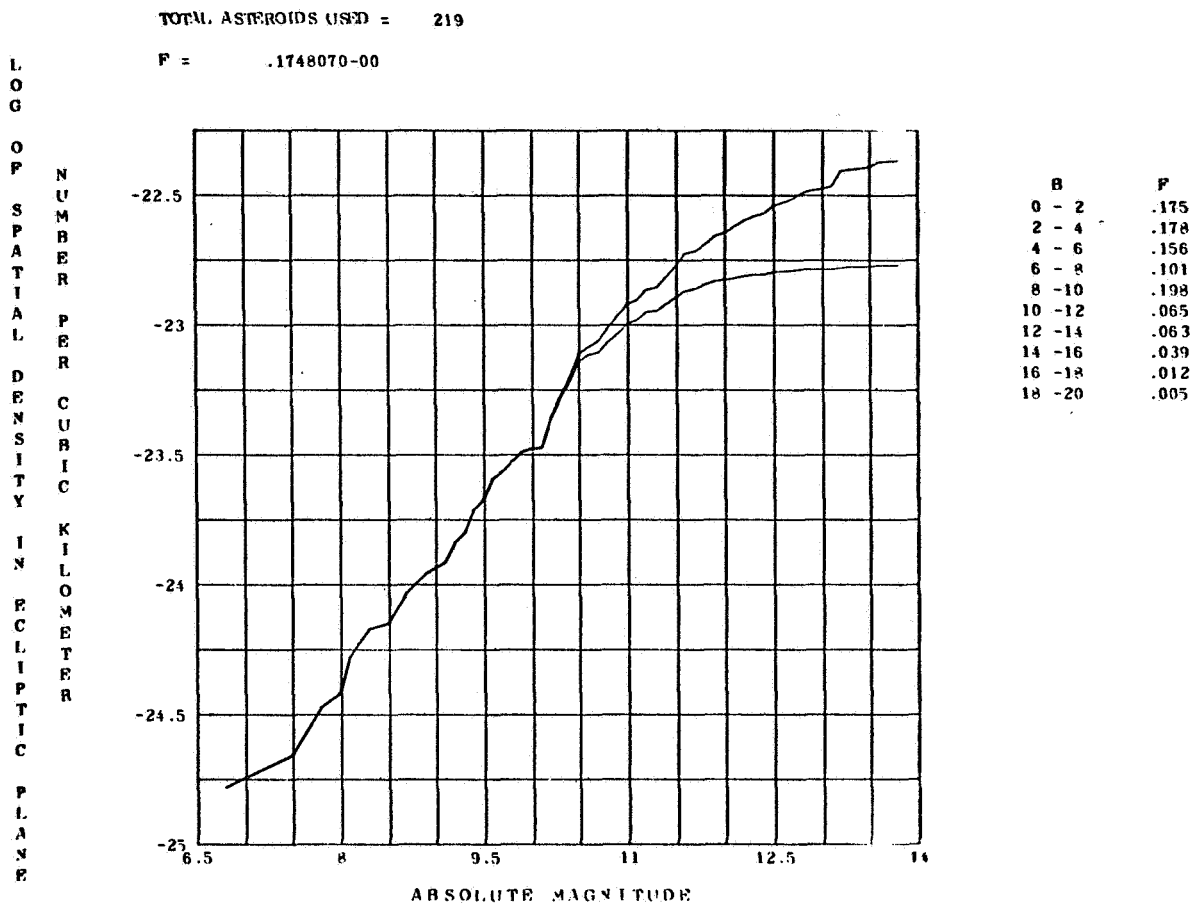


Figure 183. - Spatial density at R = 3.30 and at longitudes between 135.0 and 180.0.

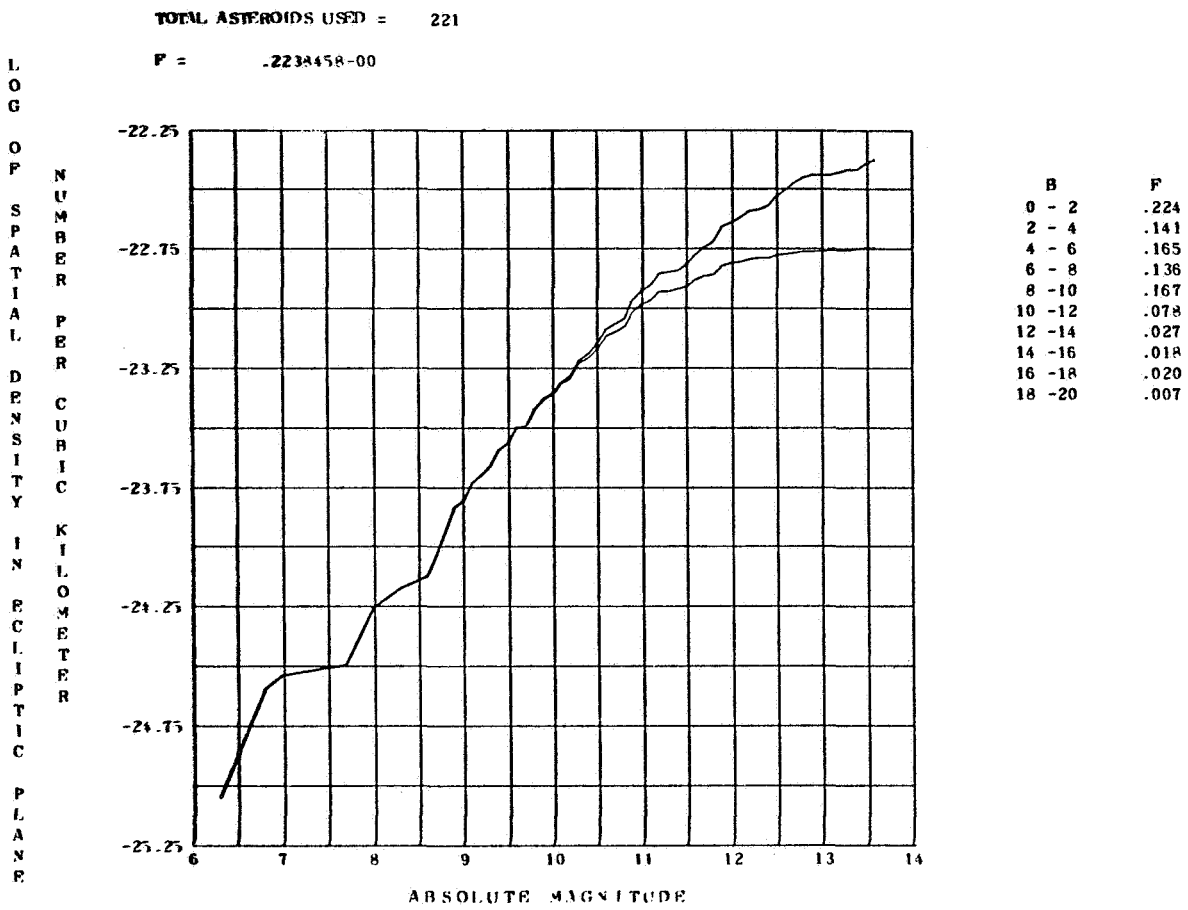


Figure 184. - Spatial density at R = 3.30 and at longitudes between 180.0 and 225.0.

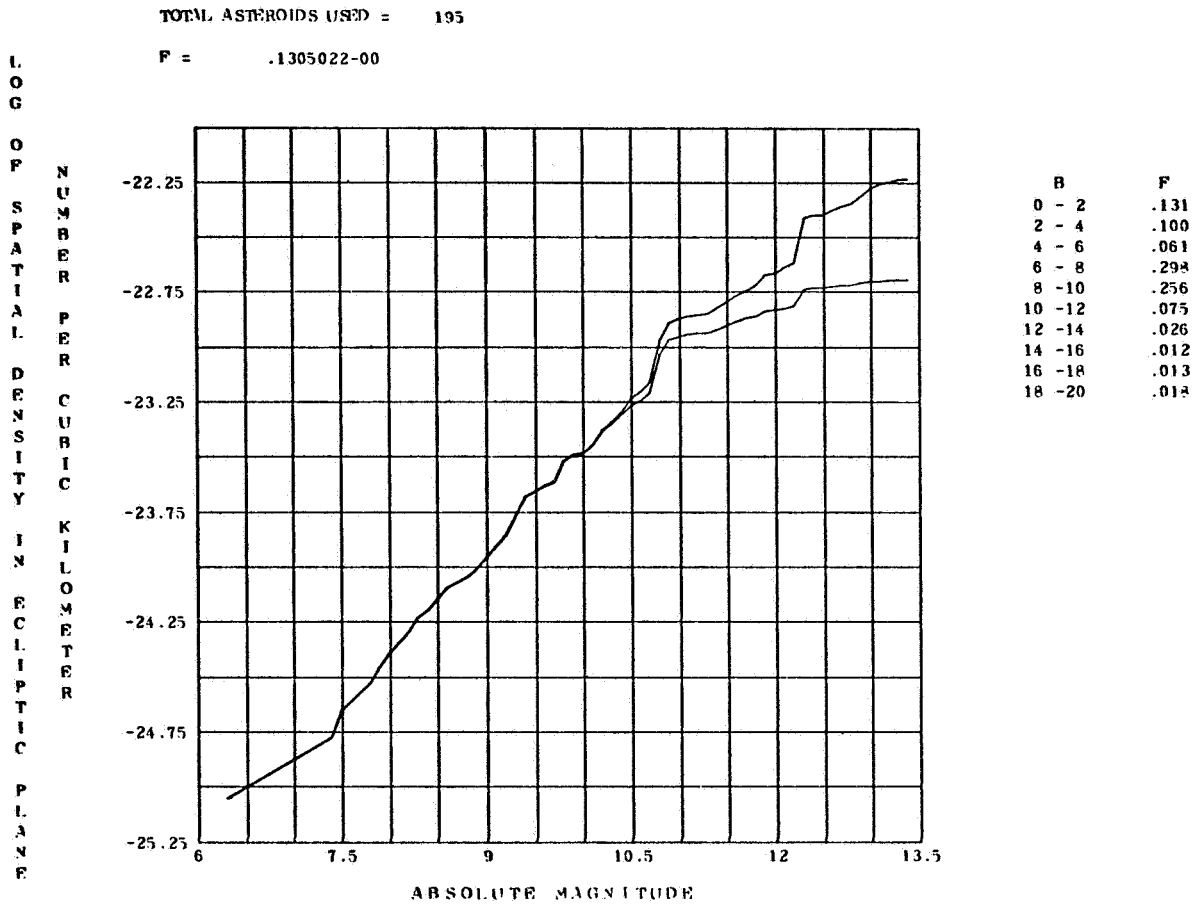


Figure 185. - Spatial density at R = 3.30 and at longitudes between 225.0 and 270.0.

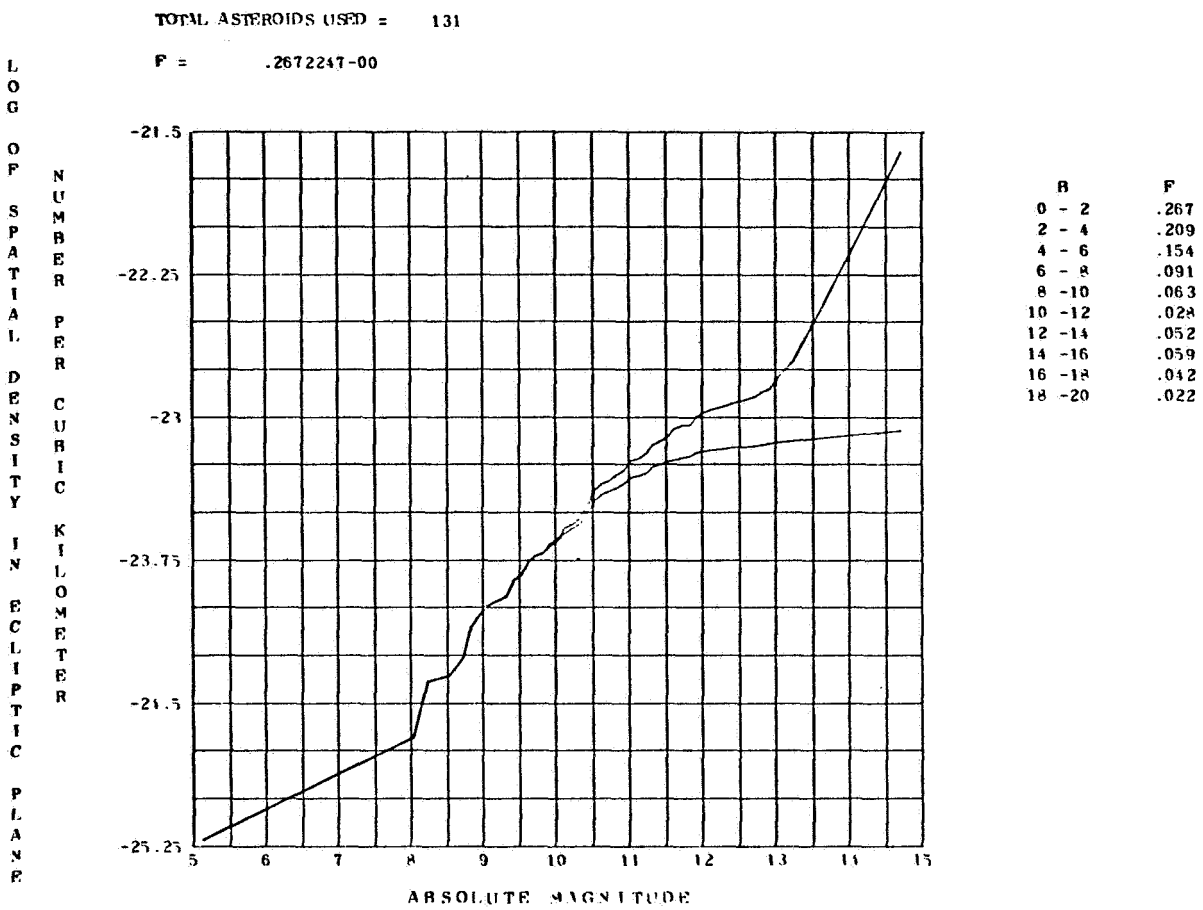


Figure 186. - Spatial density at R = 3.30 and at longitudes between 270.0 and 315.0.

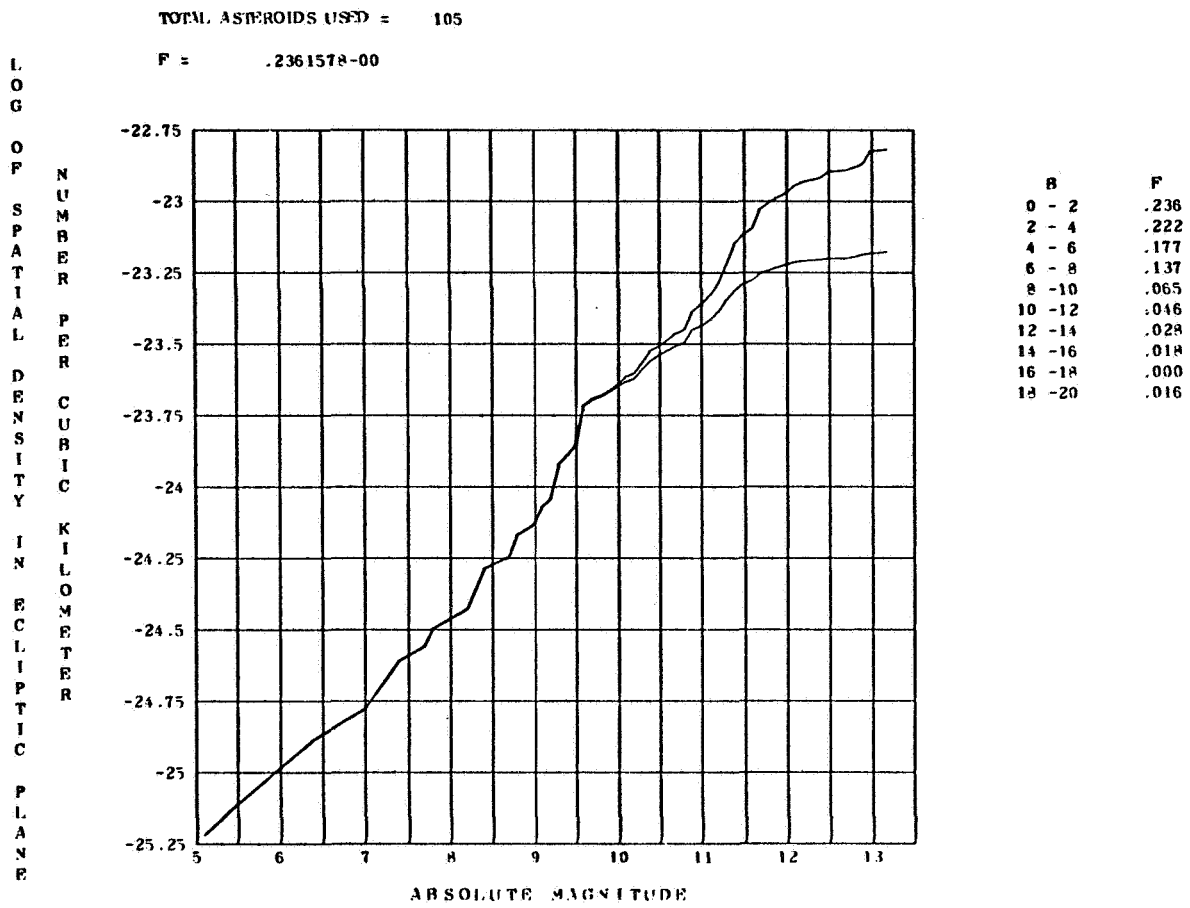


Figure 187. - Spatial density at R = 3.30 and at longitudes between 315.0 and 360.0.

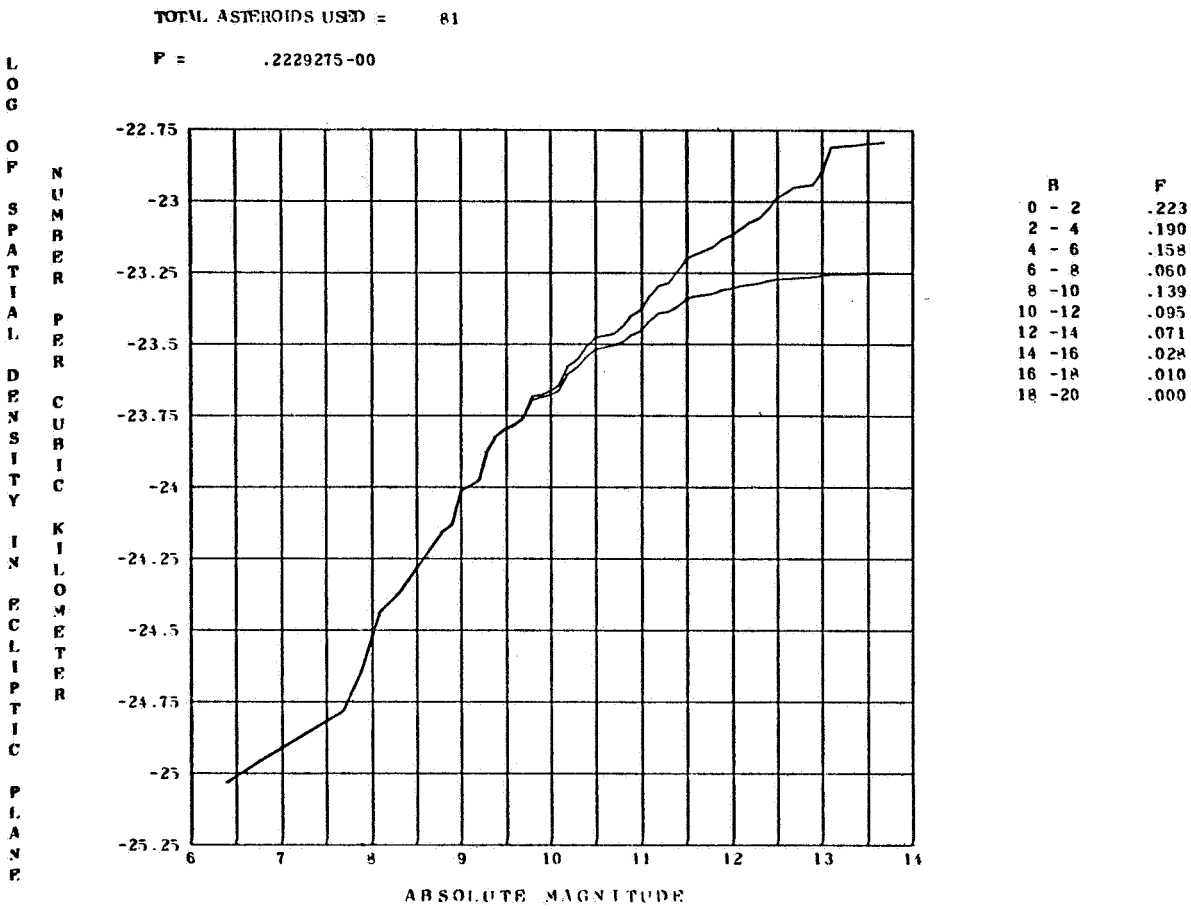


Figure 188. - Spatial density at R = 3.40 and at longitudes between 0 and 45.0.

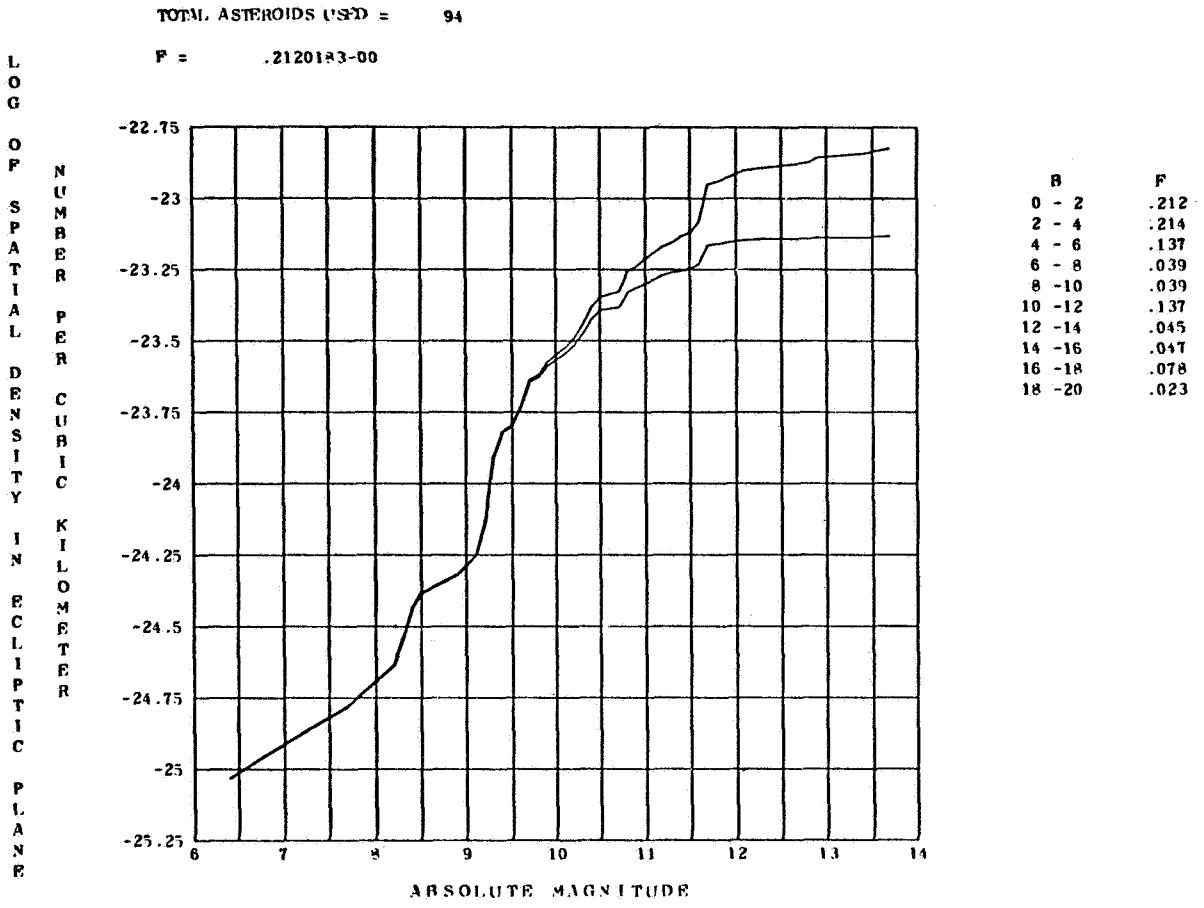


Figure 189.- Spatial density at R = 3.40 and at longitudes between 45.0 and 90.0.

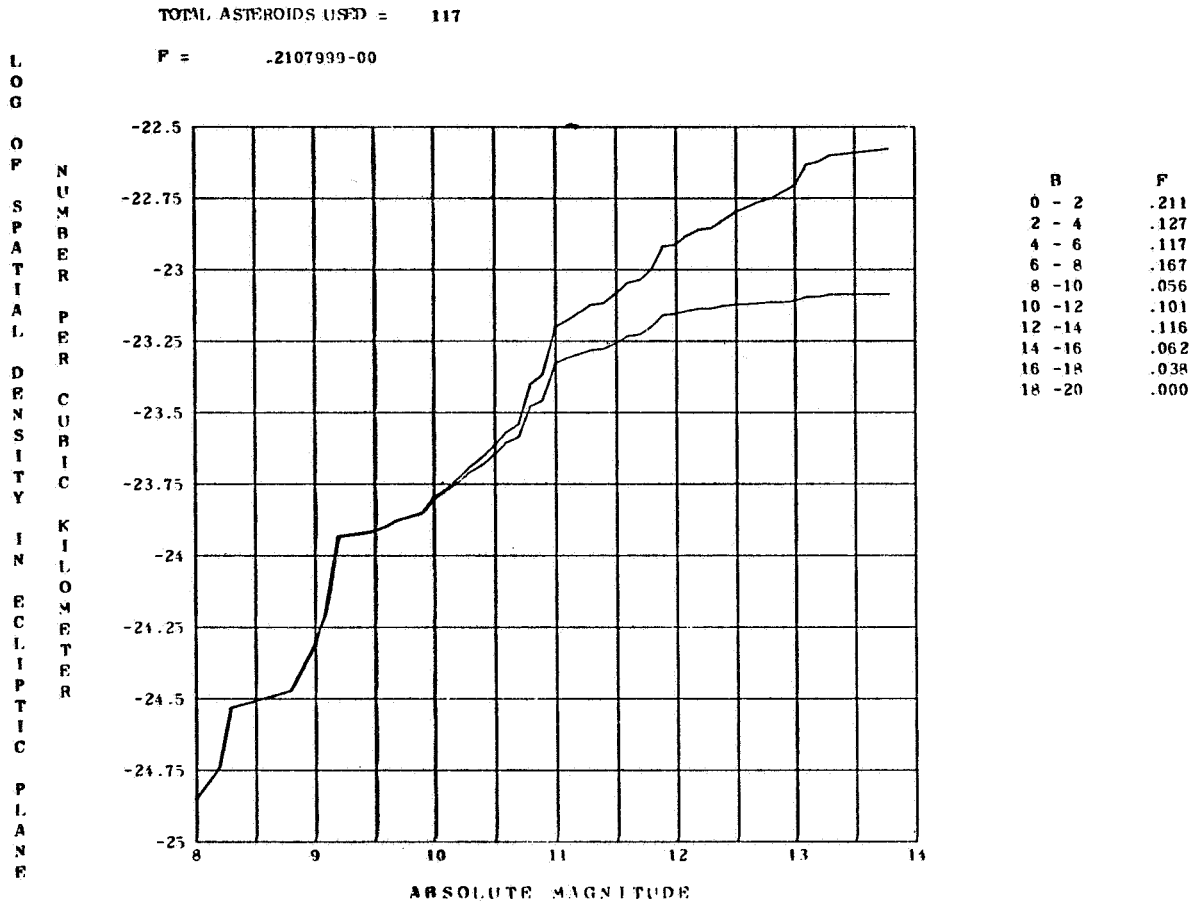


Figure 190. - Spatial density at R = 3.40 and at longitudes between 90.0 and 135.0.

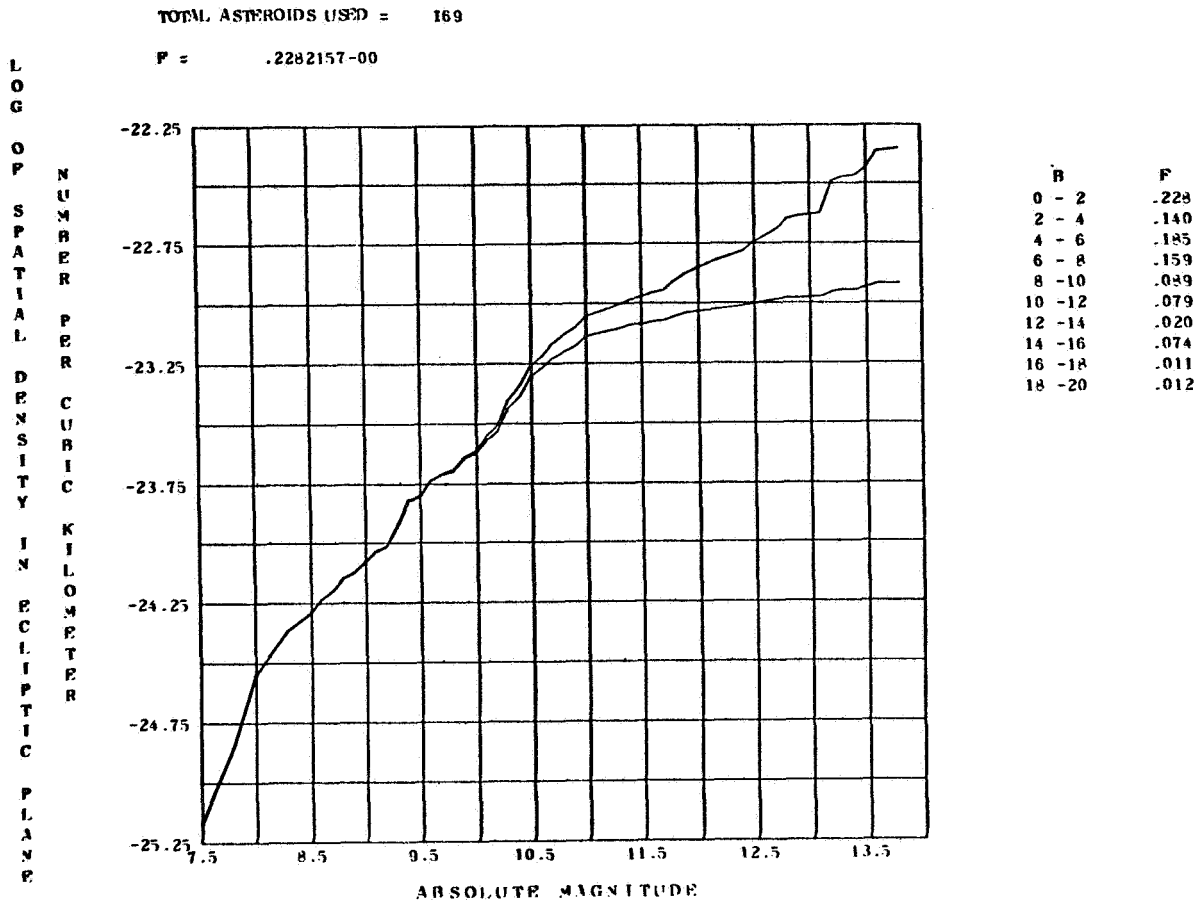


Figure 191. - Spatial density at R = 3.40 and at longitudes between 135.0 and 180.0.

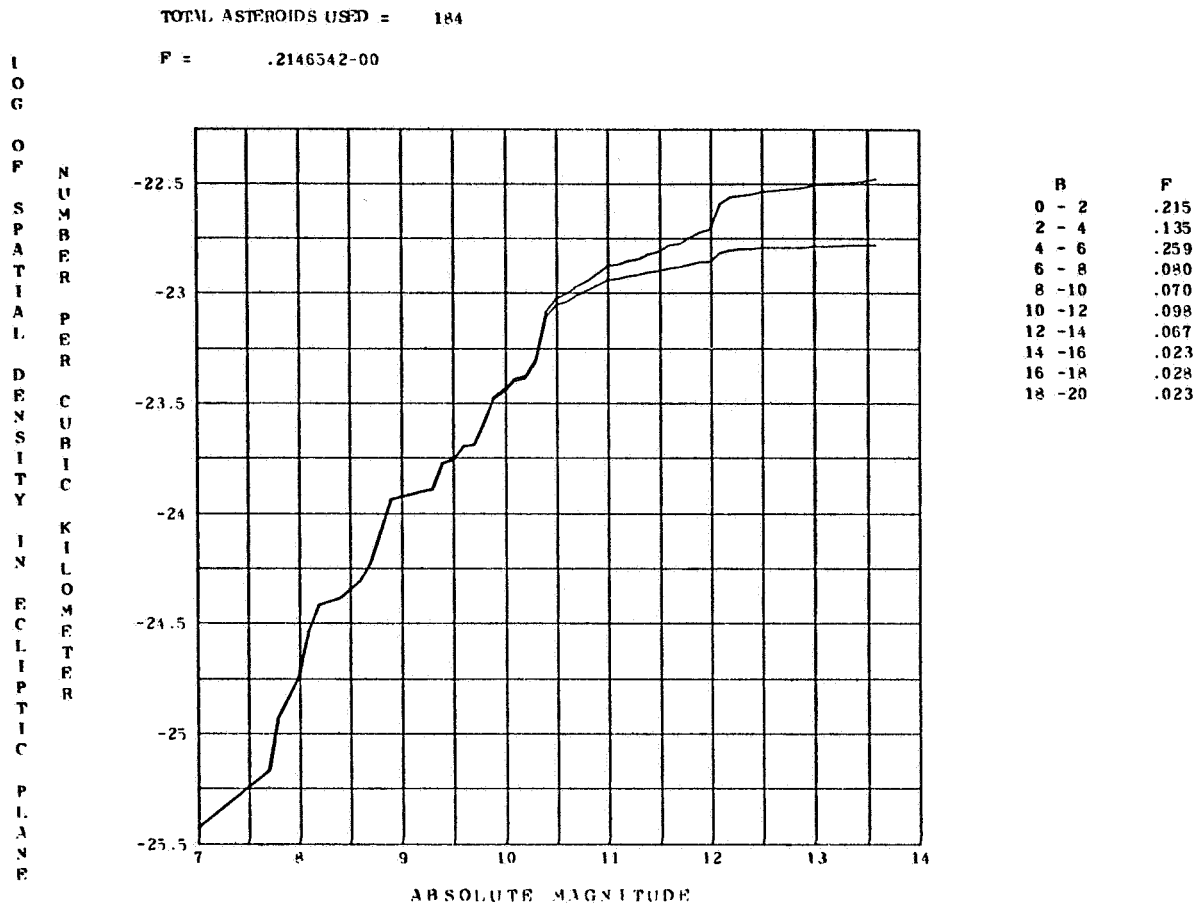


Figure 192. - Spatial density at R = 3.40 and at longitudes between 180.0 and 225.0.

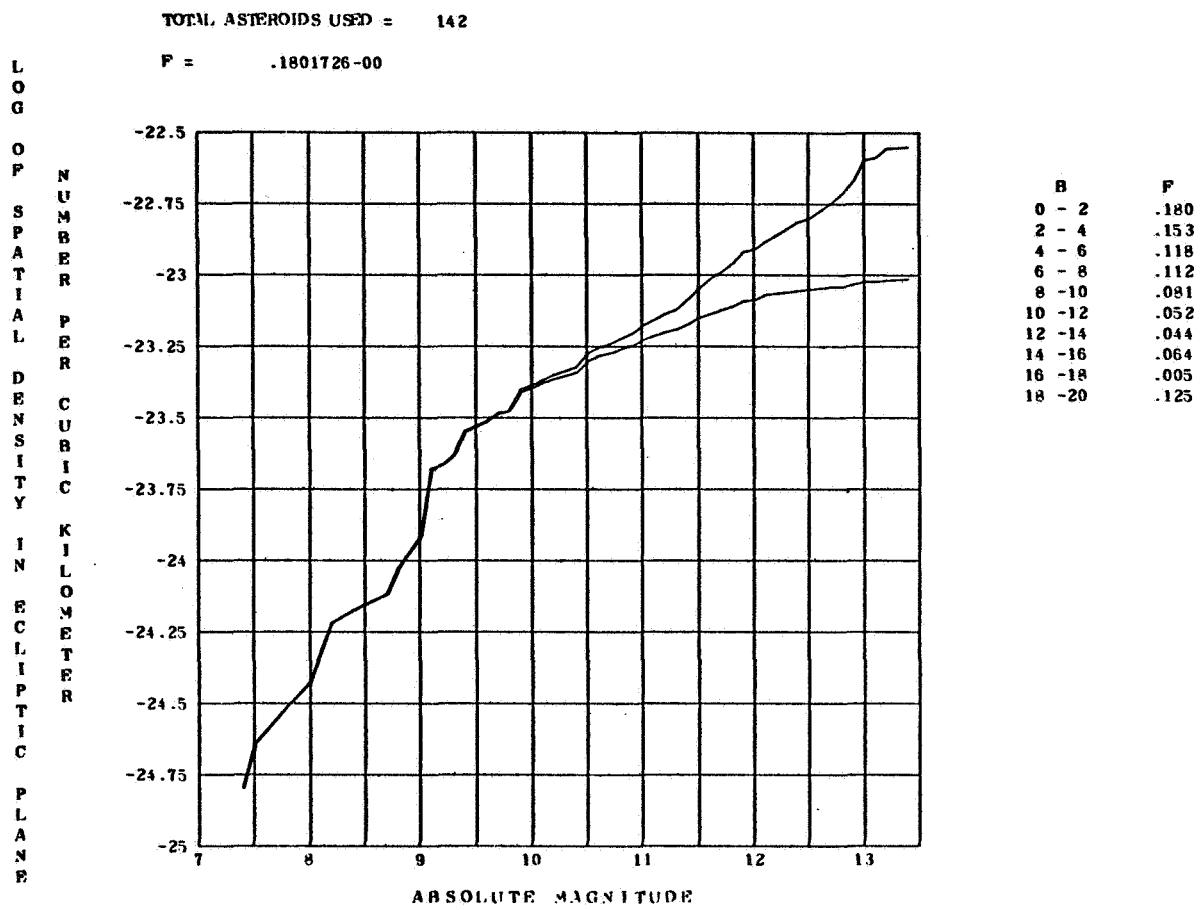


Figure 193. - Spatial density at R = 3.40 and at longitudes between 225.0 and 270.0.

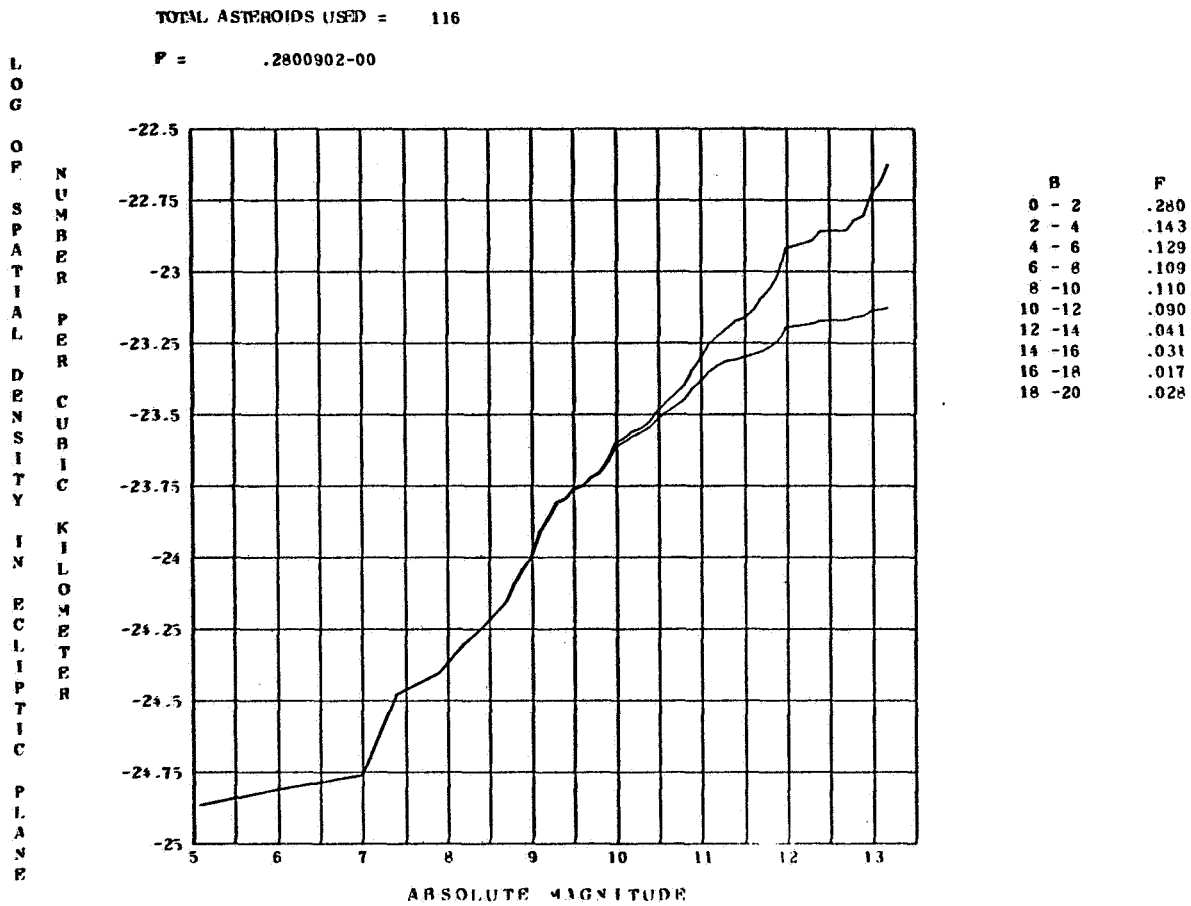


Figure 194. - Spatial density at R = 3.40 and at longitudes between 270.0 and 315.0.

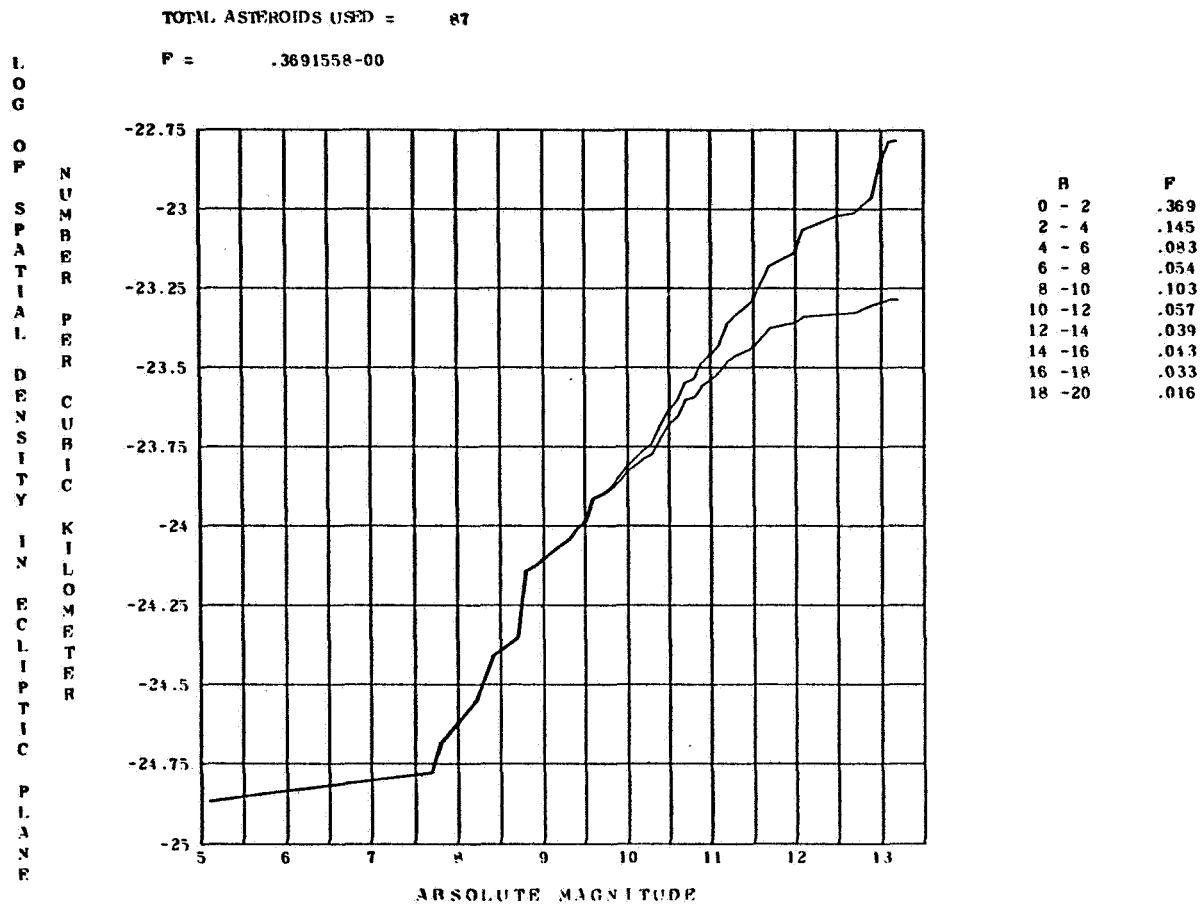


Figure 195. - Spatial density at R = 3.40 and at longitudes between 315.0 and 360.0.

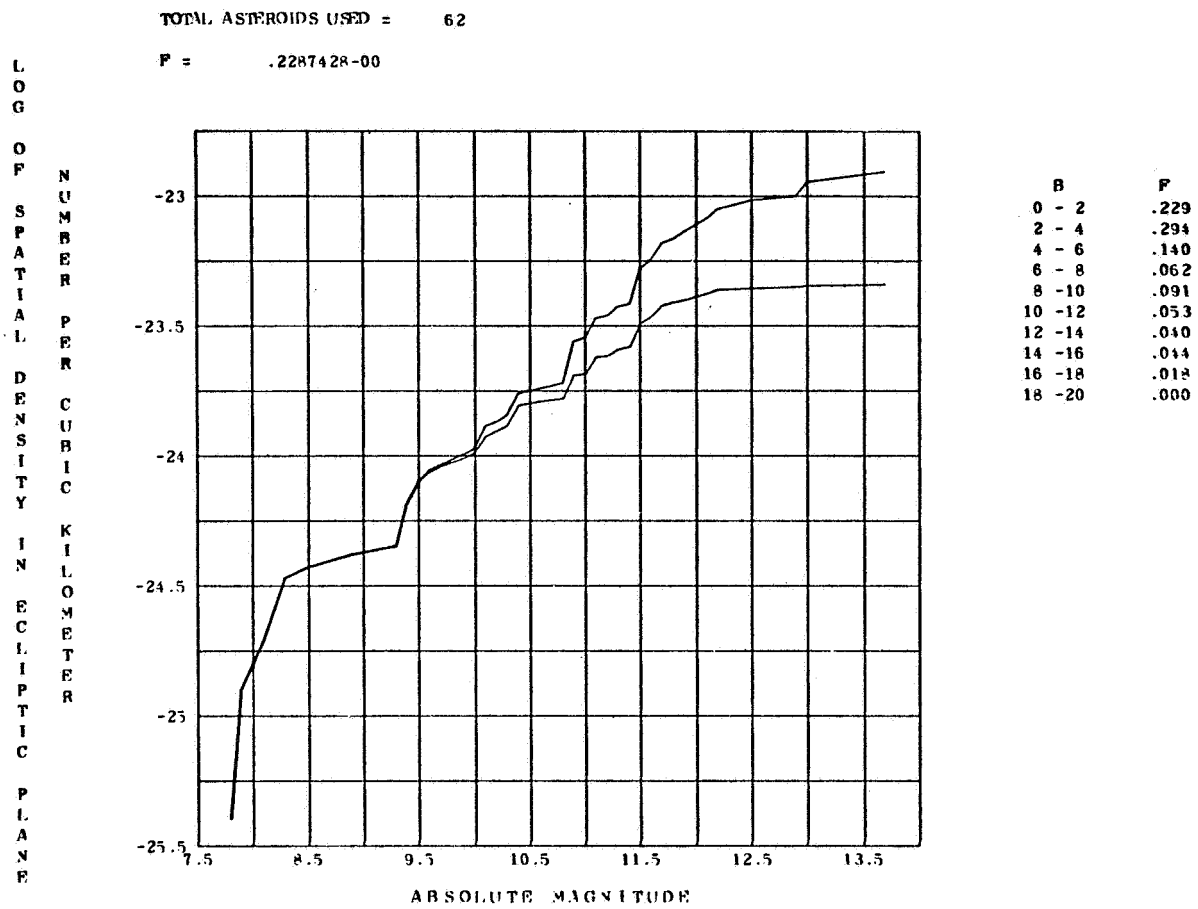


Figure 196.- Spatial density at R = 3.50 and at longitudes between 0 and 45.0.

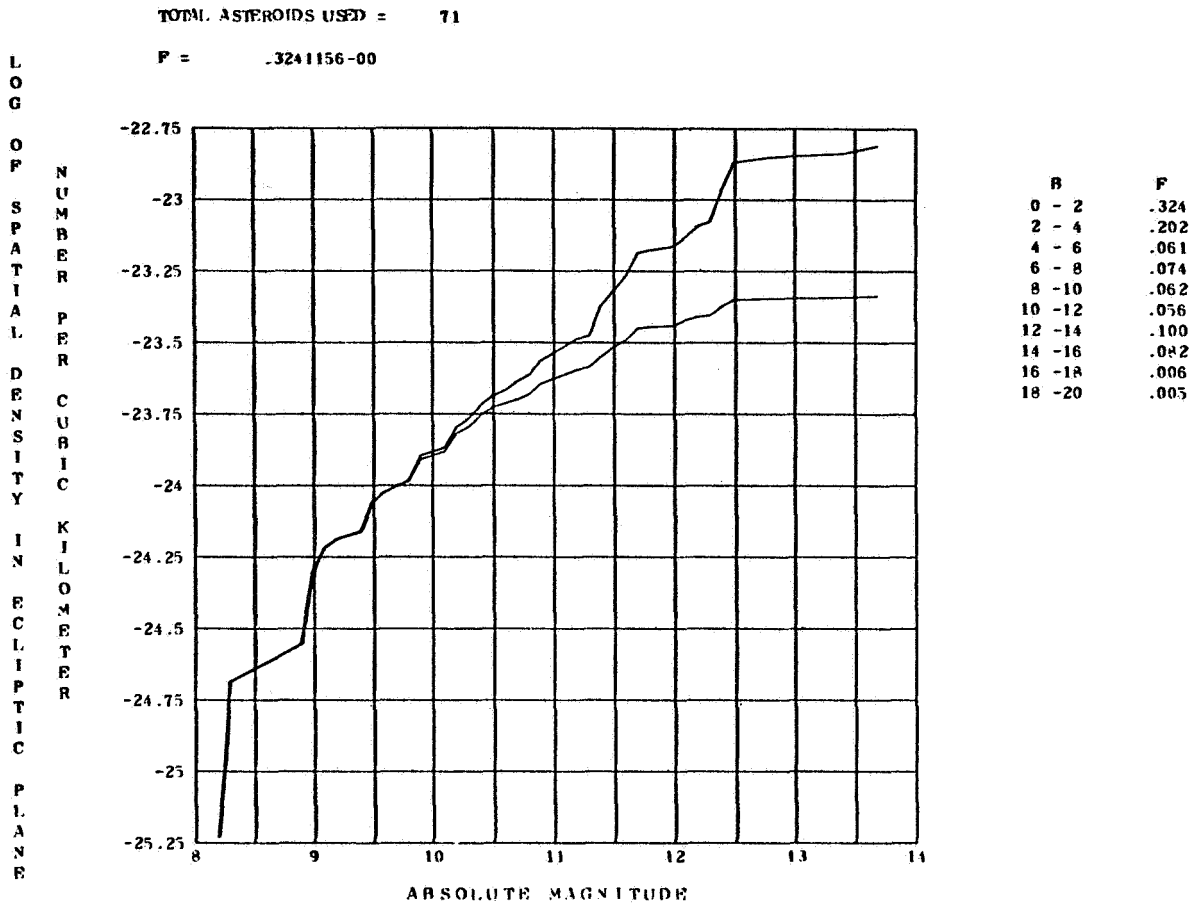


Figure 197. - Spatial density at R = 3.50 and at longitudes between 45.0 and 90.0.

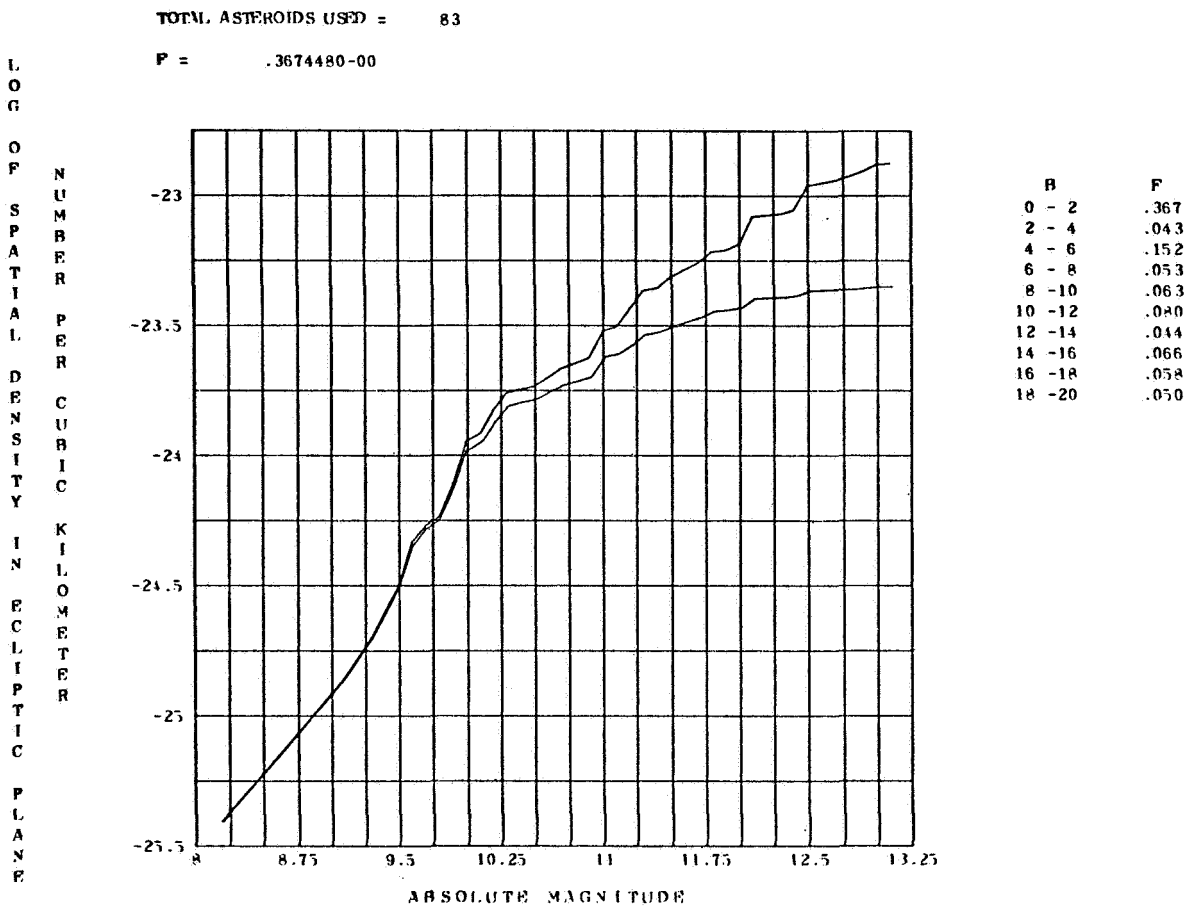


Figure 198. - Spatial density at R = 3.50 and at longitudes between 90.0 and 135.0.

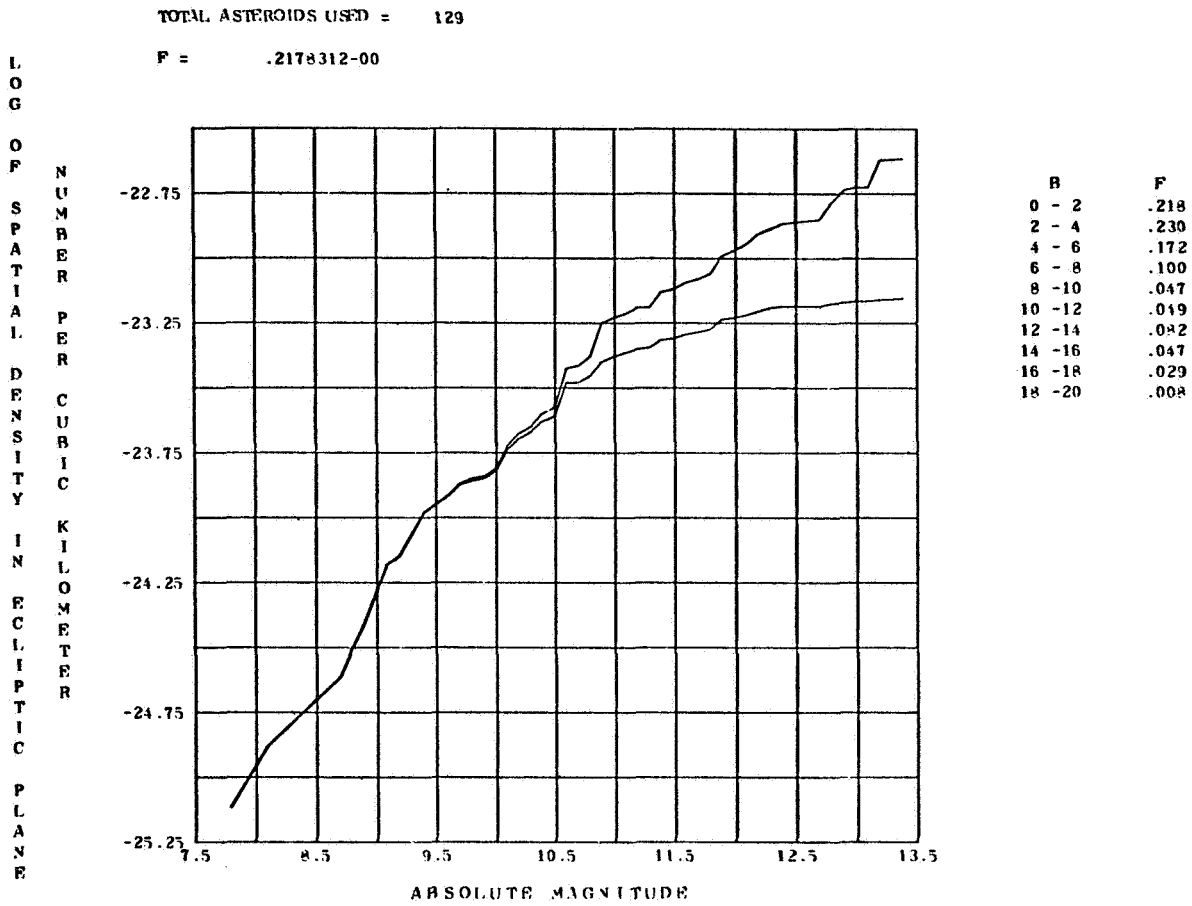


Figure 199. - Spatial density at R = 3.50 and at longitudes between 135.0 and 180.0.

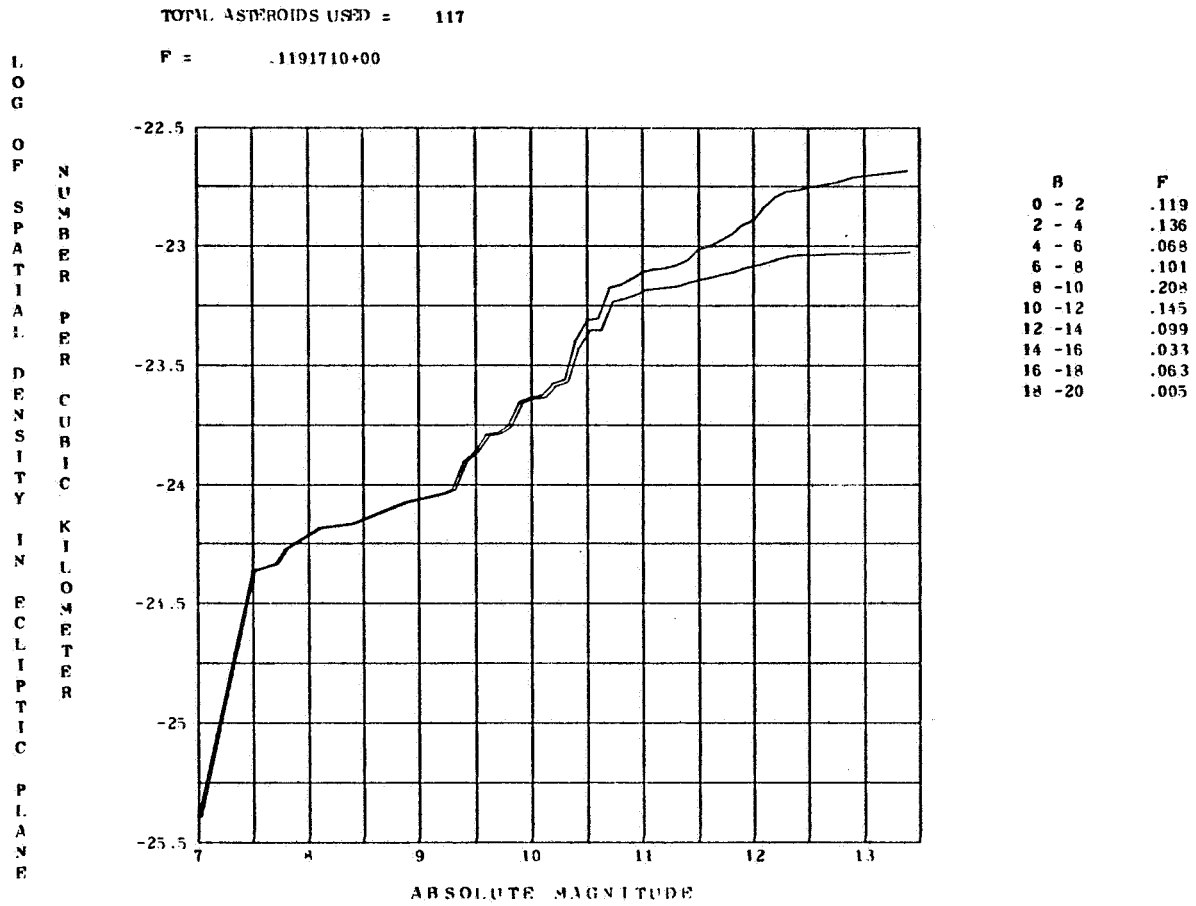


Figure 200. - Spatial density at R = 3.50 and at longitudes between 180.0 and 225.0.

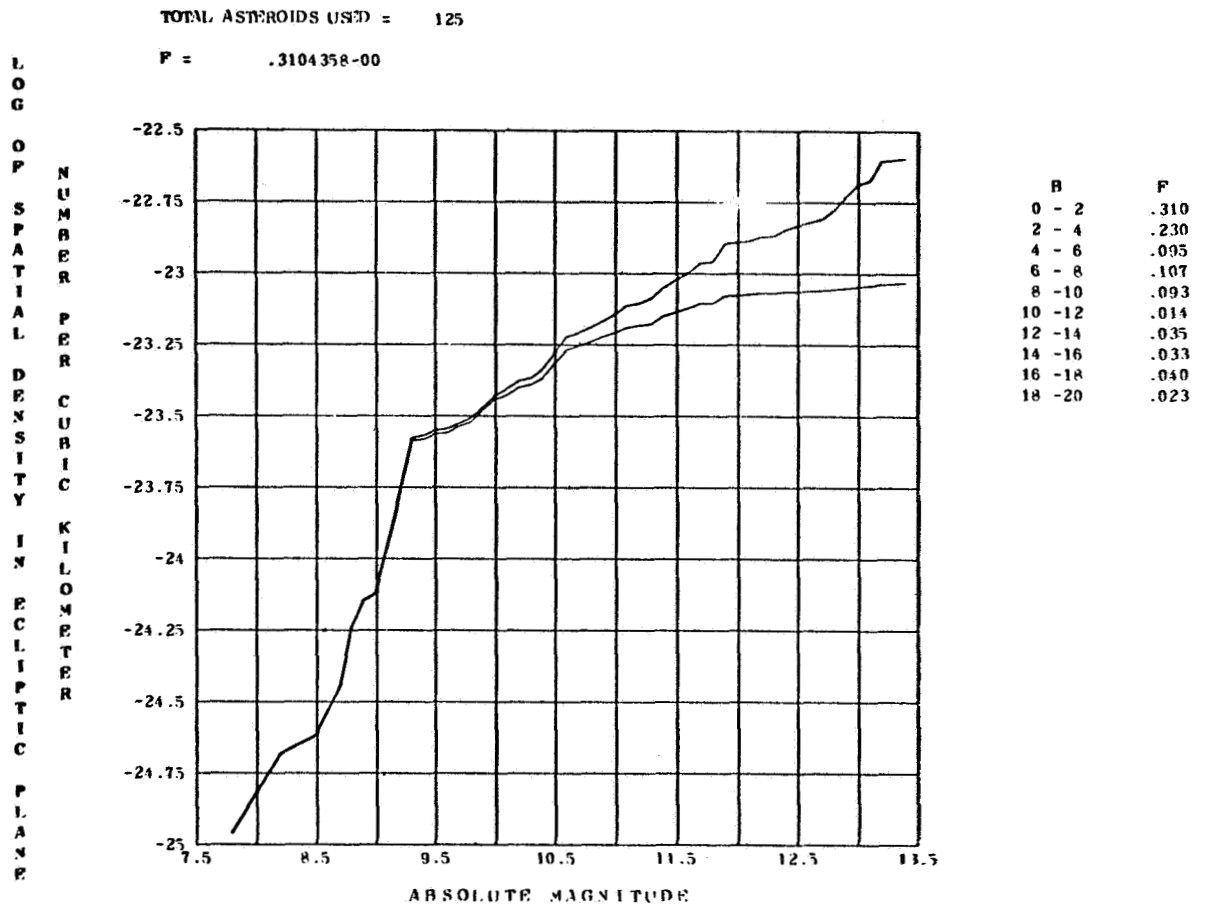


Figure 201. - Spatial density at R = 3.50 and at longitudes between 225.0 and 270.0.

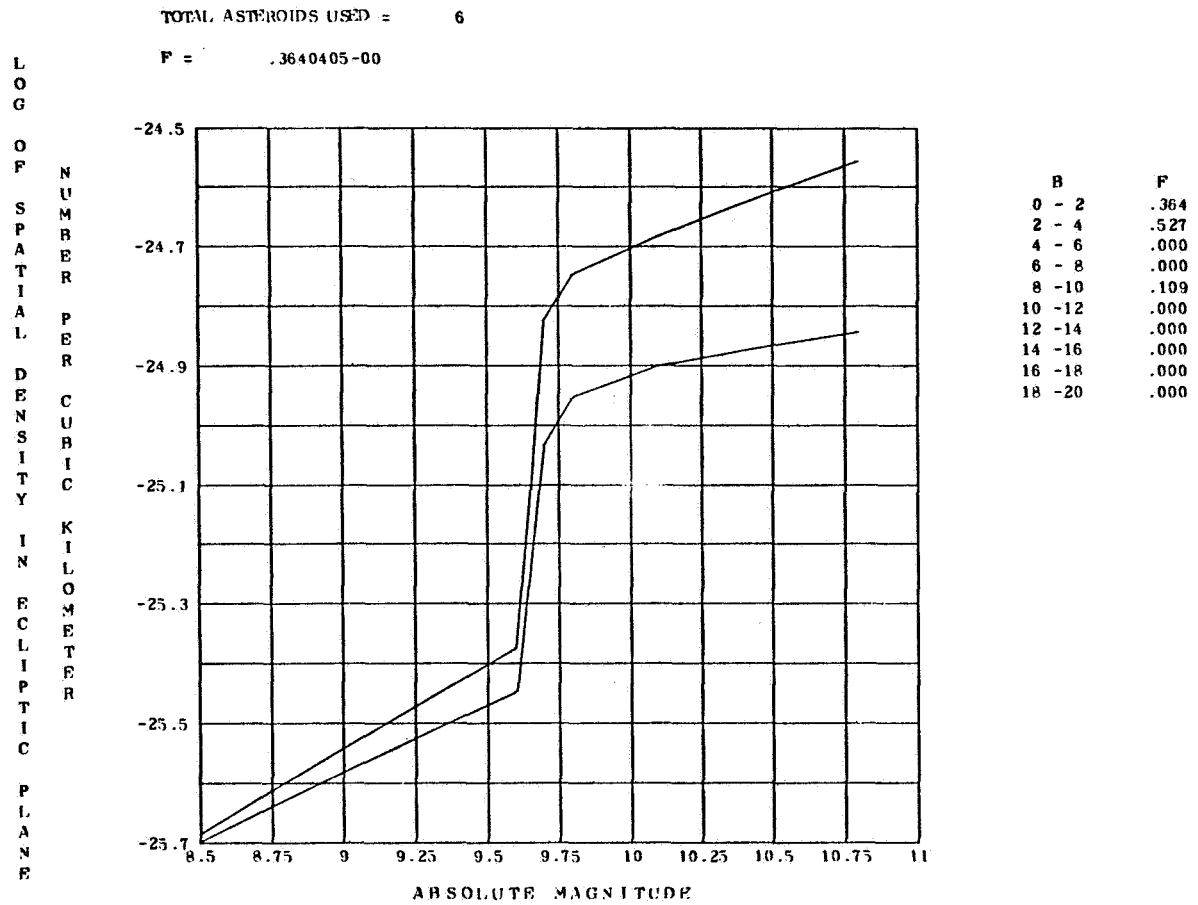


Figure 202. - Spatial density at R = 4.10 and at longitudes between 315.0 and 360.0.

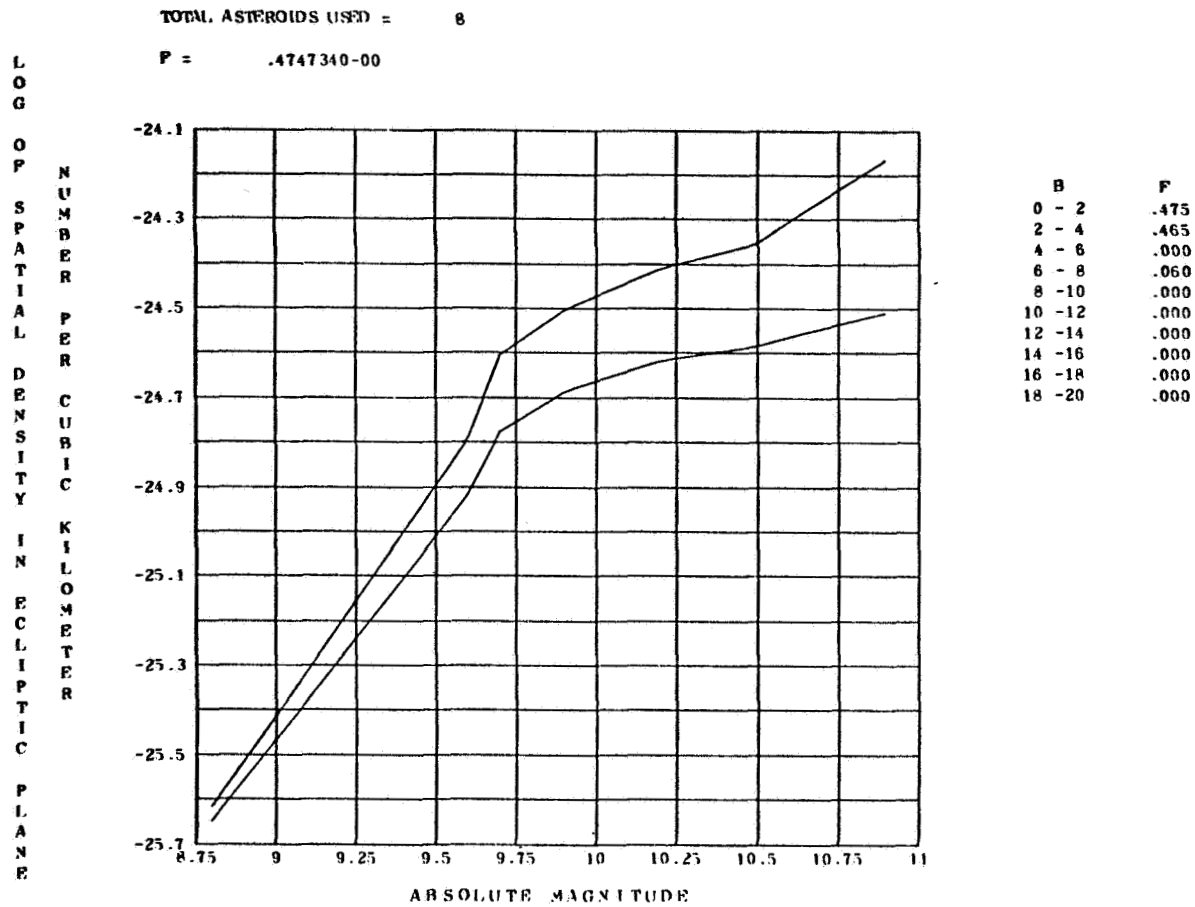


Figure 203. - Spatial density at R = 4.20 and at longitudes between 0 and 45.0.

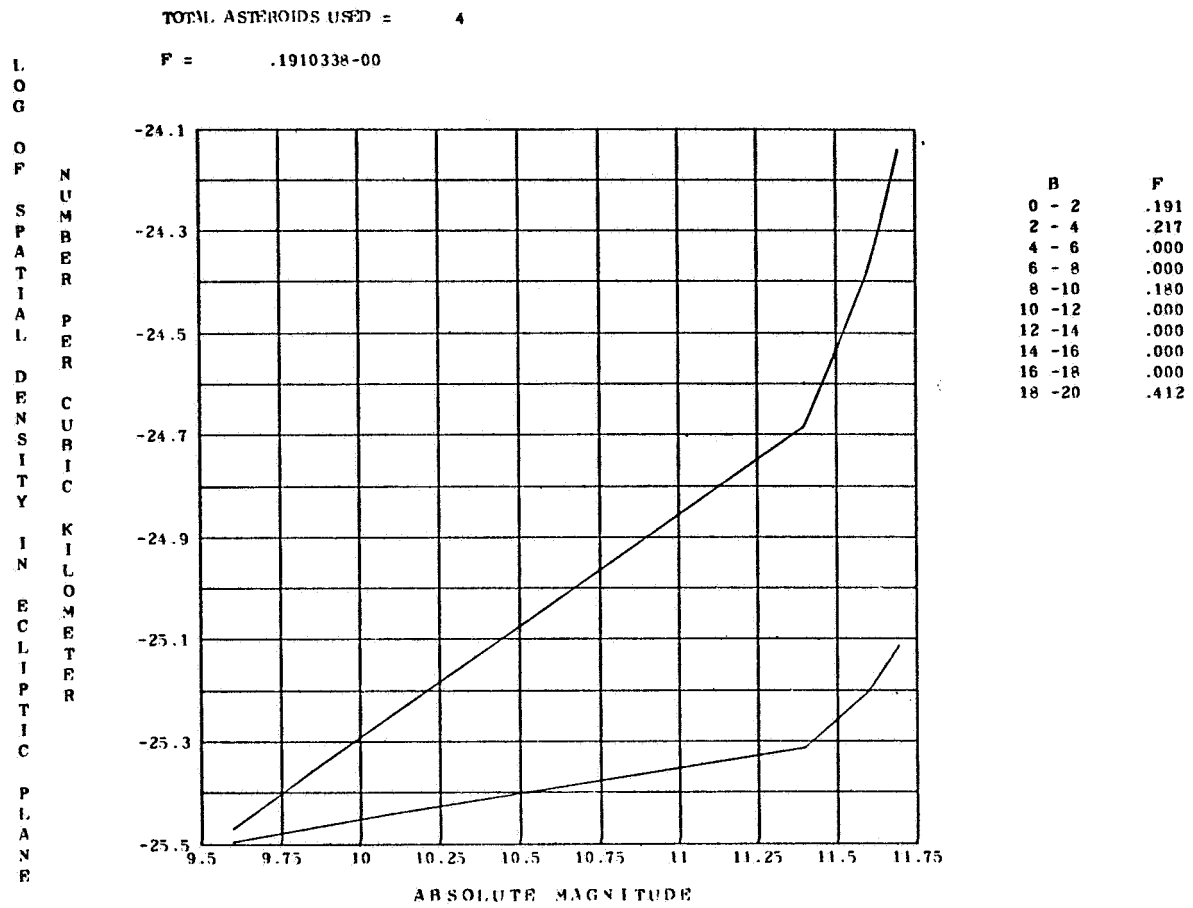


Figure 204. - Spatial density at R = 4.20 and at longitudes between 90.0 and 135.0.

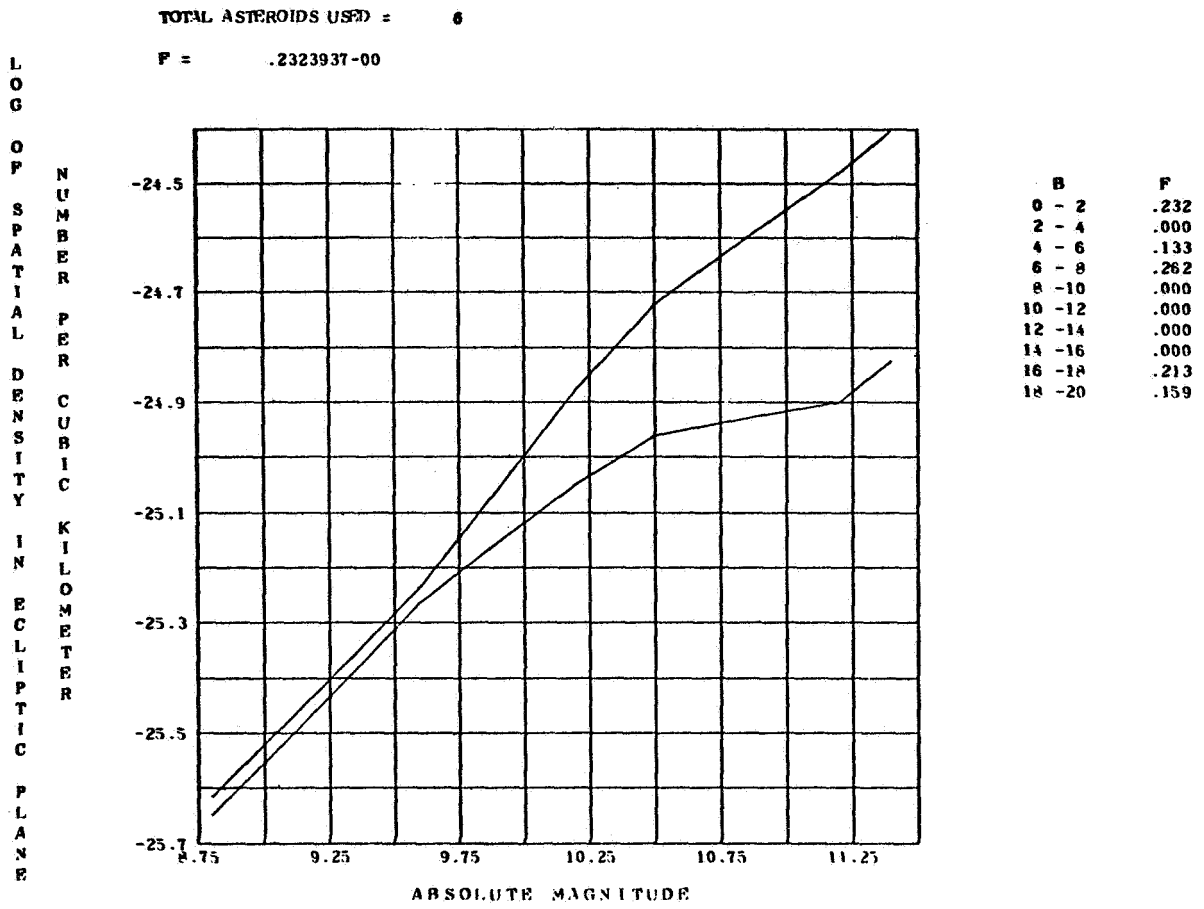


Figure 205. - Spatial density at R = 4.20 and at longitudes between 135.0 and 180.0.

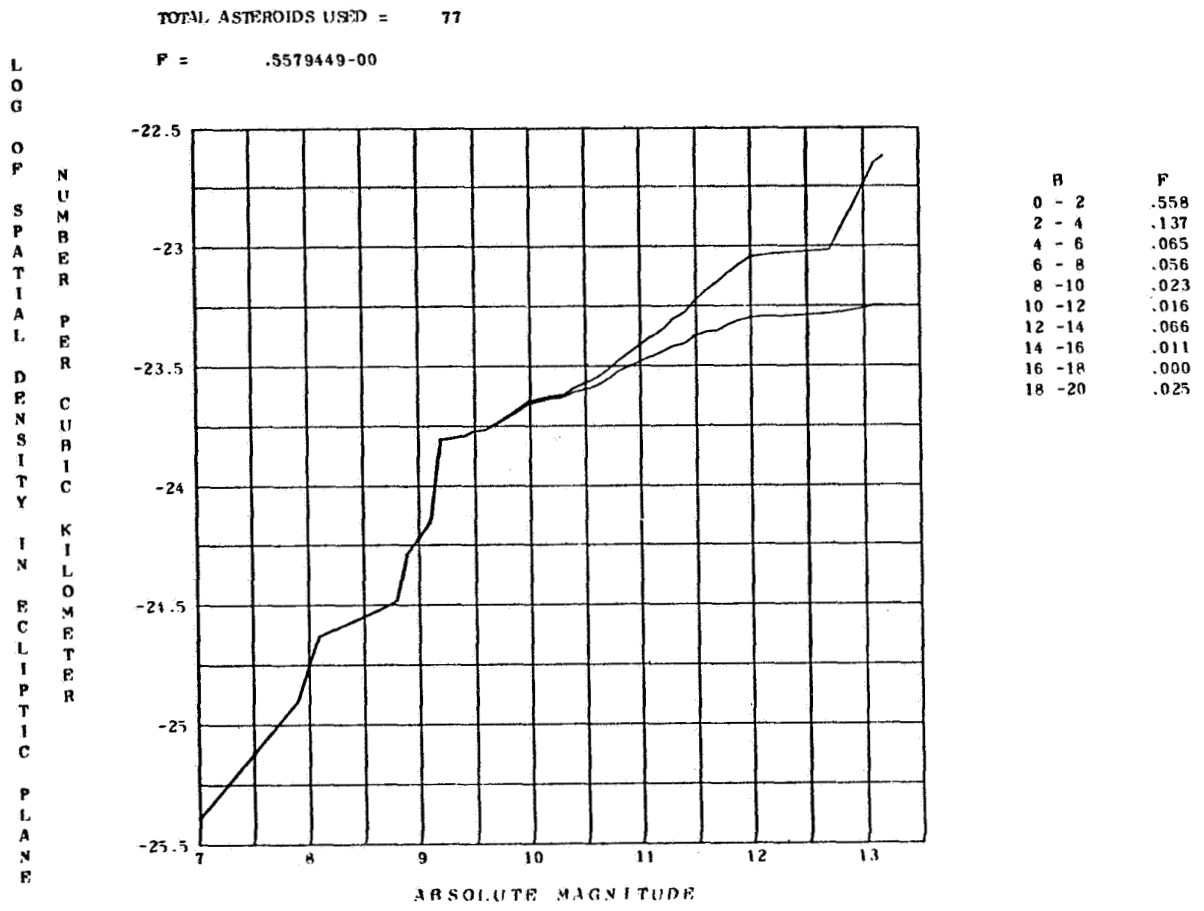


Figure 206. - Spatial density at R = 3.50 and at longitudes between 270.0 and 315.0.

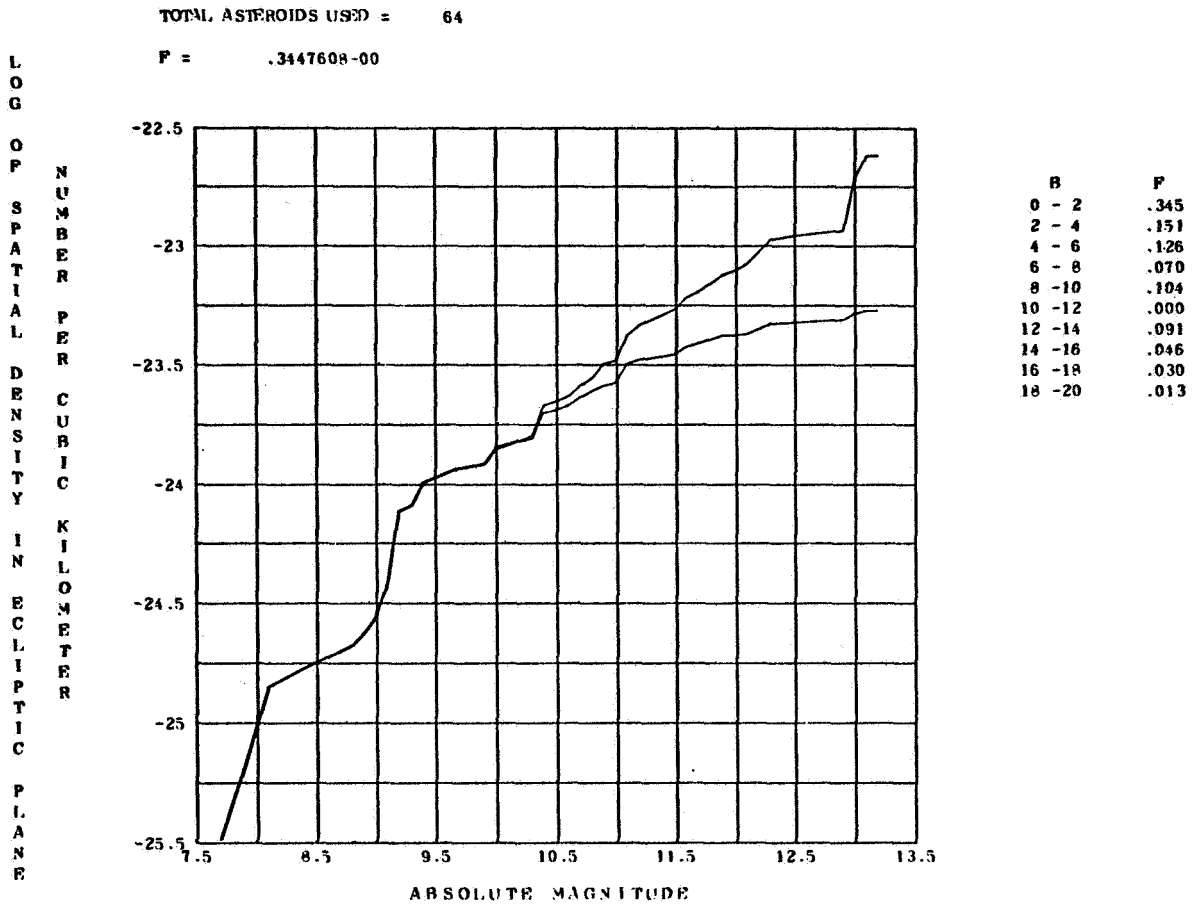


Figure 207. - Spatial density at R = 3.50 and at longitudes between 315.0 and 360.0.

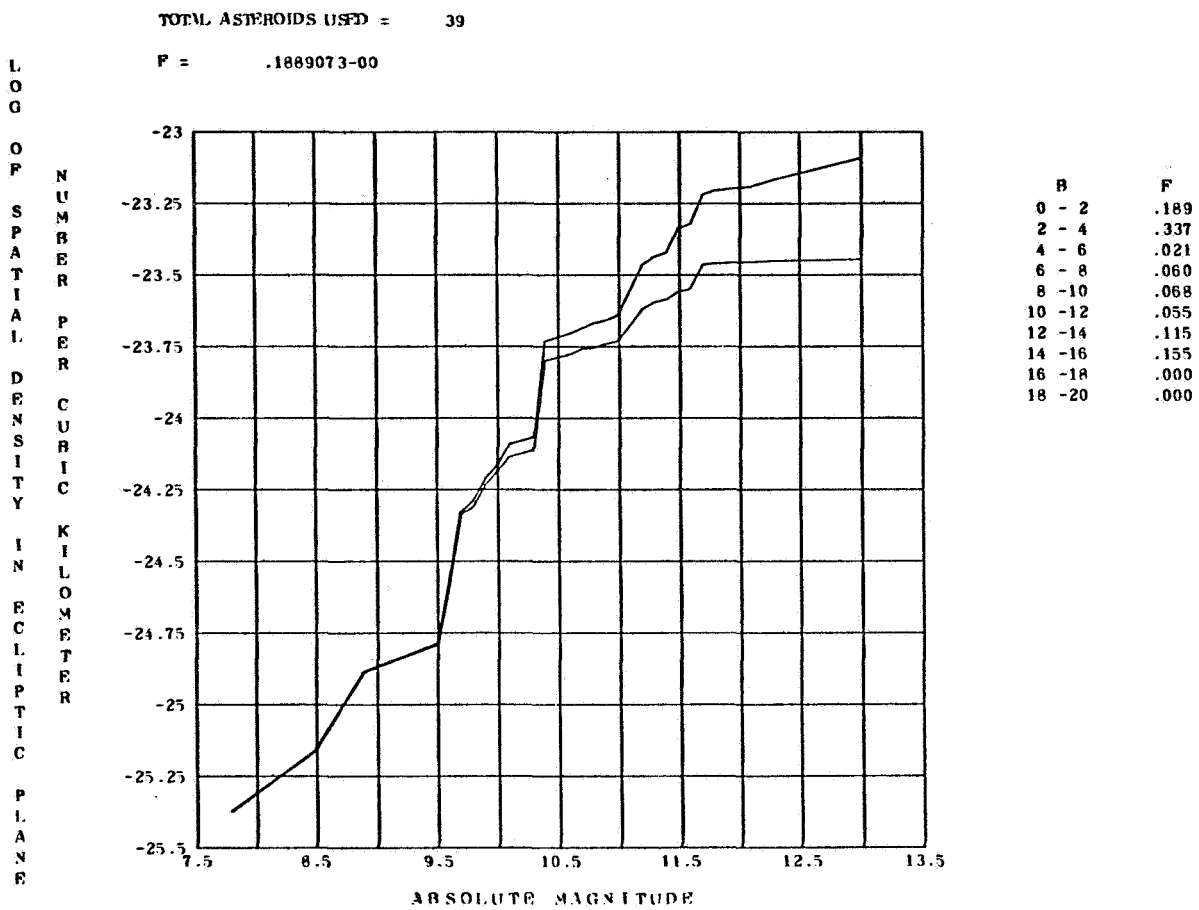


Figure 208. - Spatial density at R = 360.0 and at longitudes between 0 and 45.0.

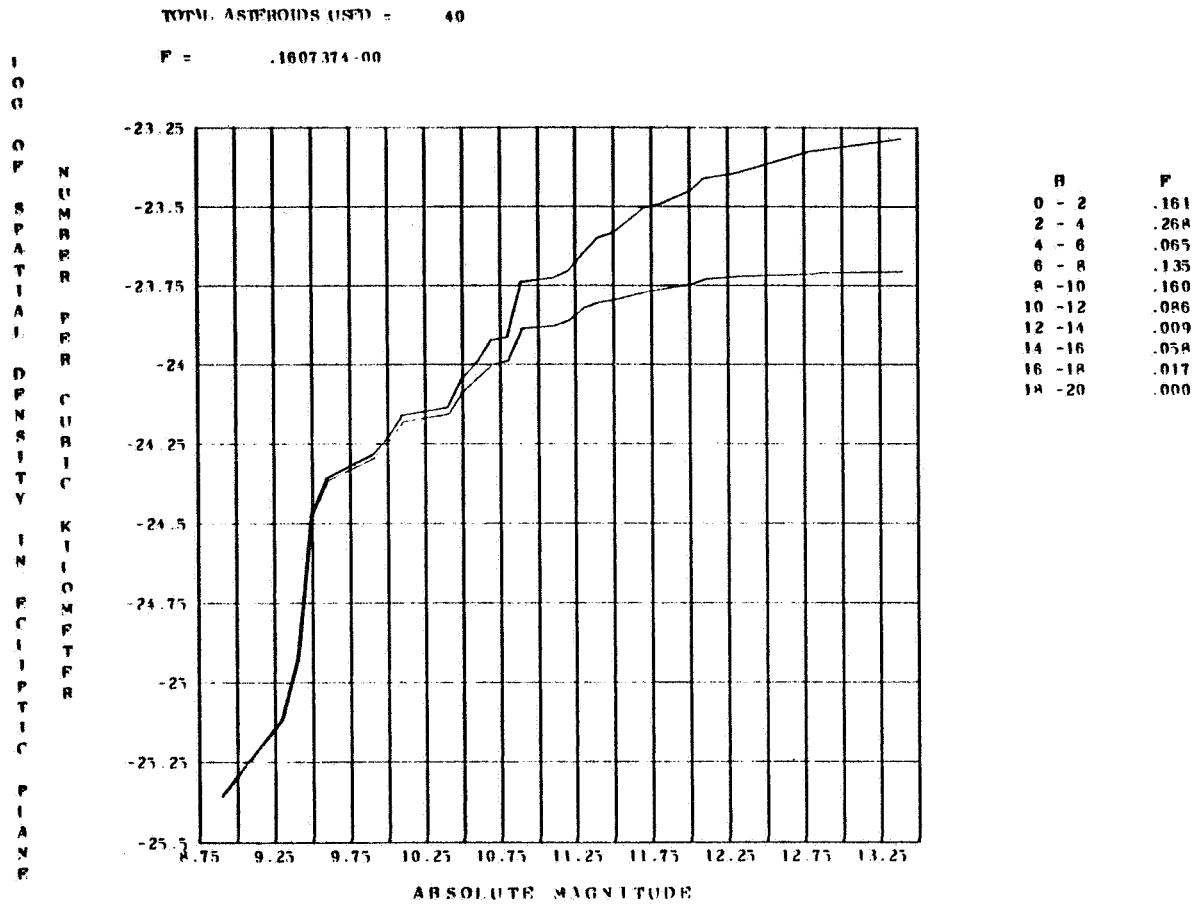


Figure 209. - Spatial density at R = 3.60 and at longitudes between 45.0 and 90.0.

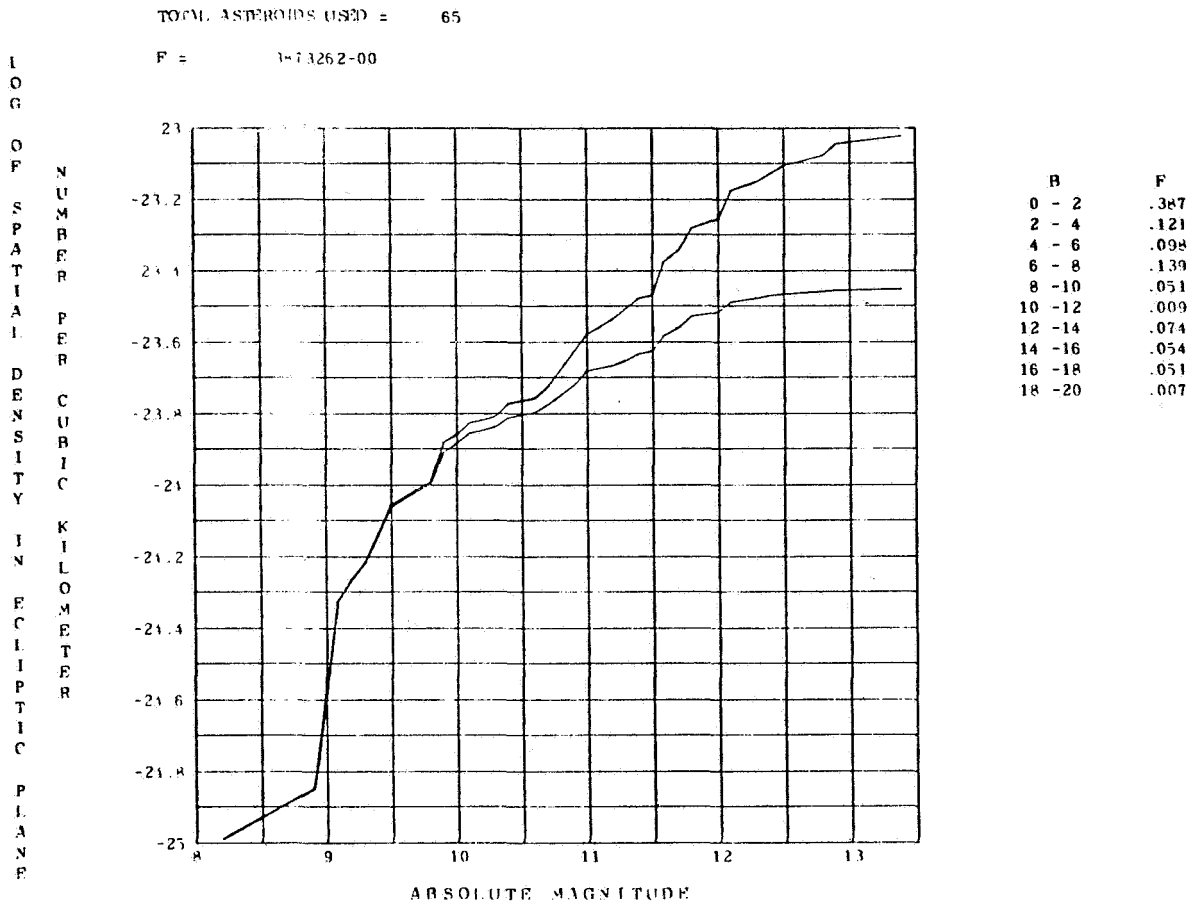


Figure 210. - Spatial density at R = 3.60 and at longitudes between 90.0 and 135.0.

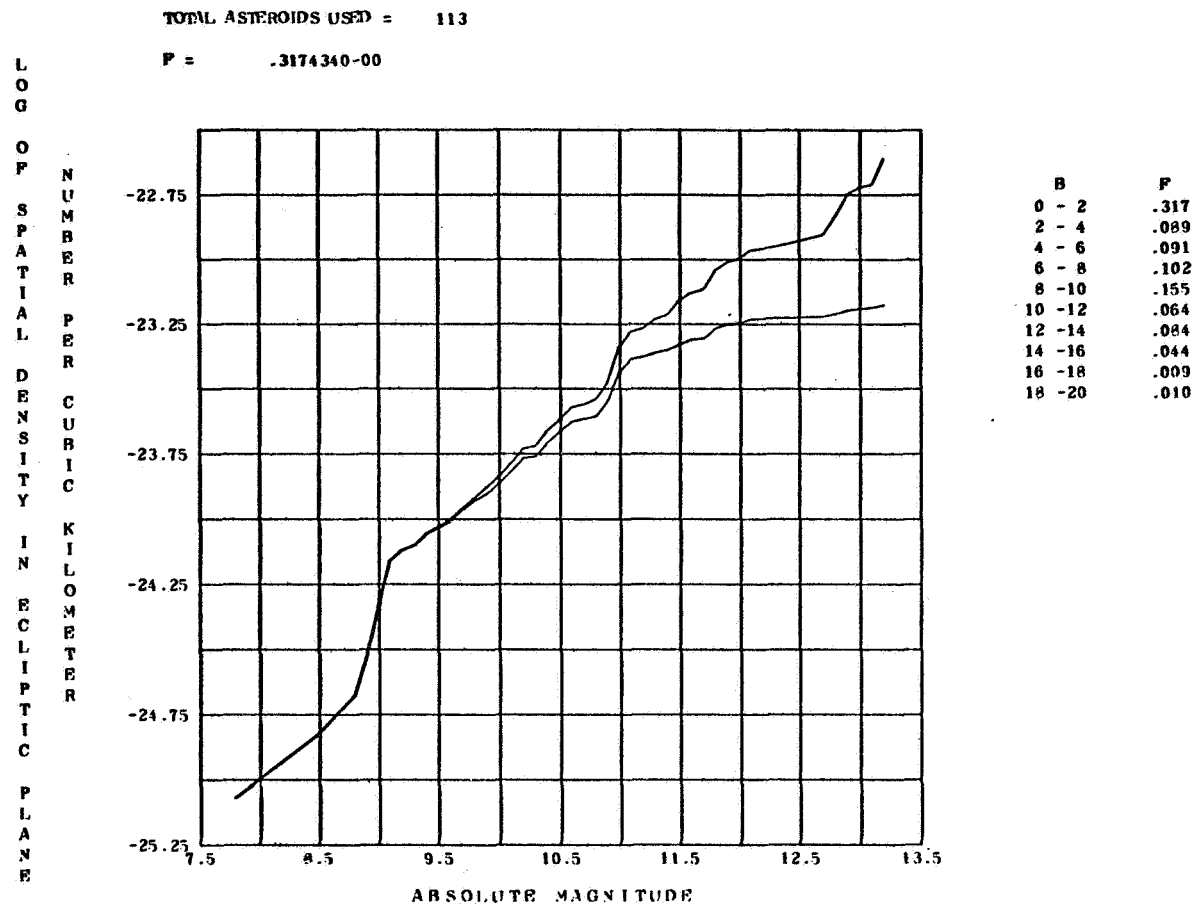


Figure 211. - Spatial density at R = 3.60 and at longitudes between 135.0 and 180.0.

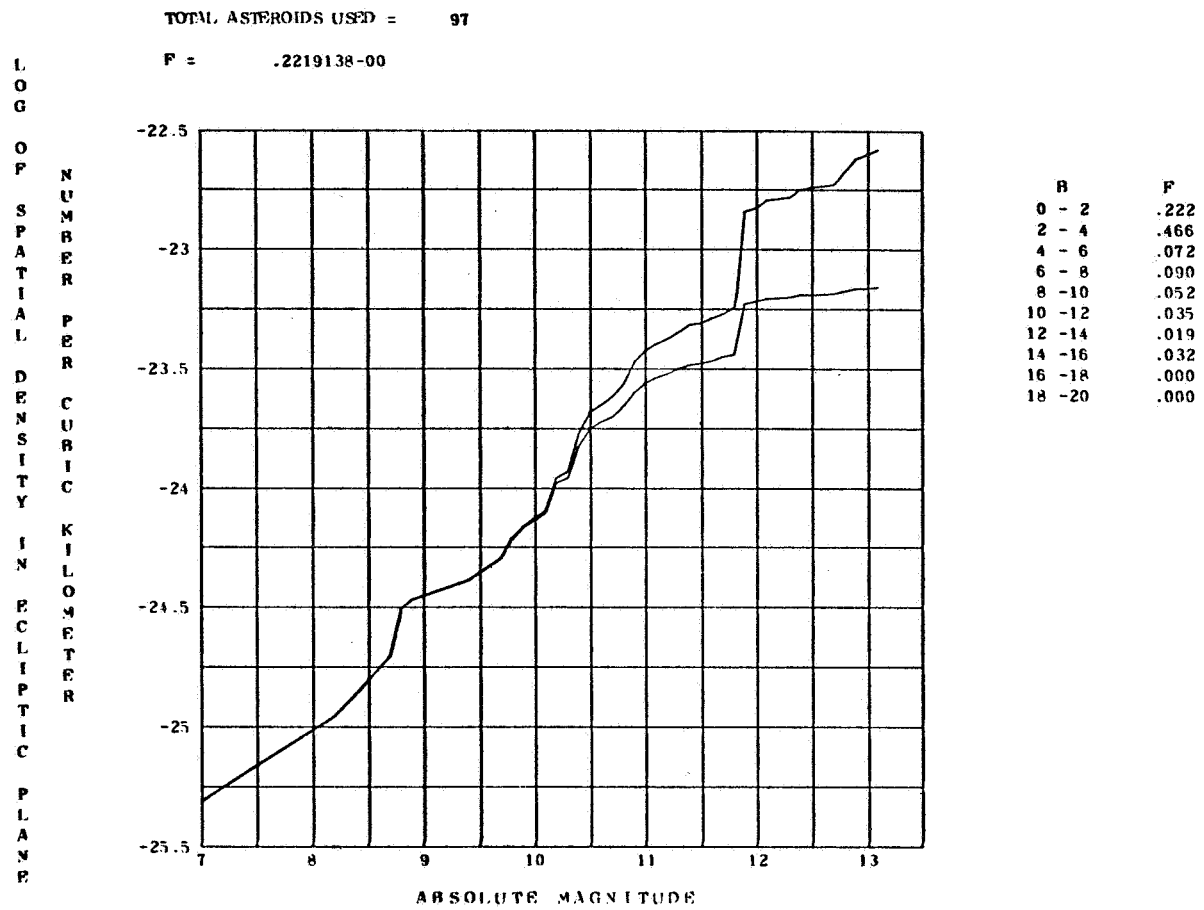


Figure 212. - Spatial density at R = 3.60 and at longitudes between 180.0 and 225.0.

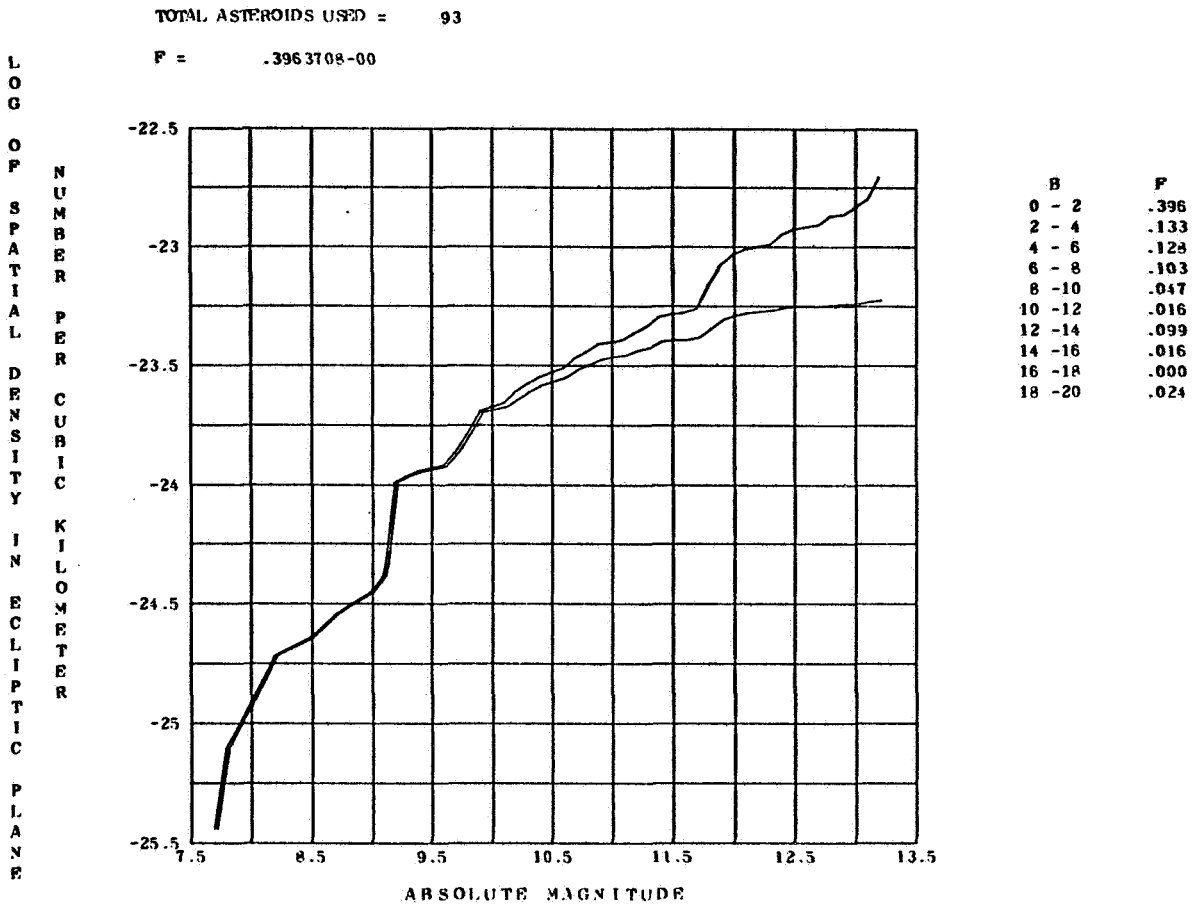


Figure 213. - Spatial density at R = 3.60 and at longitudes between 225.0 and 270.0.

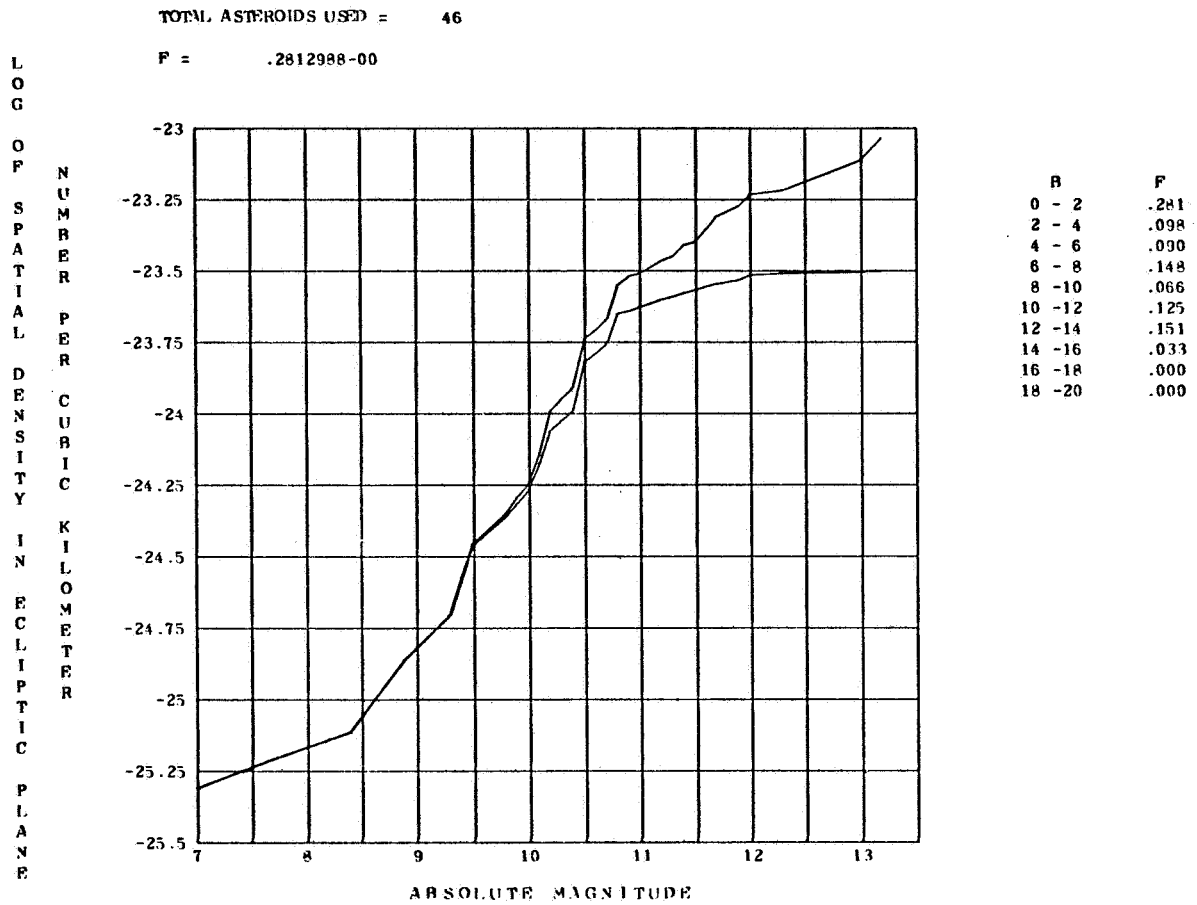


Figure 214. - Spatial density at R = 3.60 and at longitudes between 270.0 and 315.0.

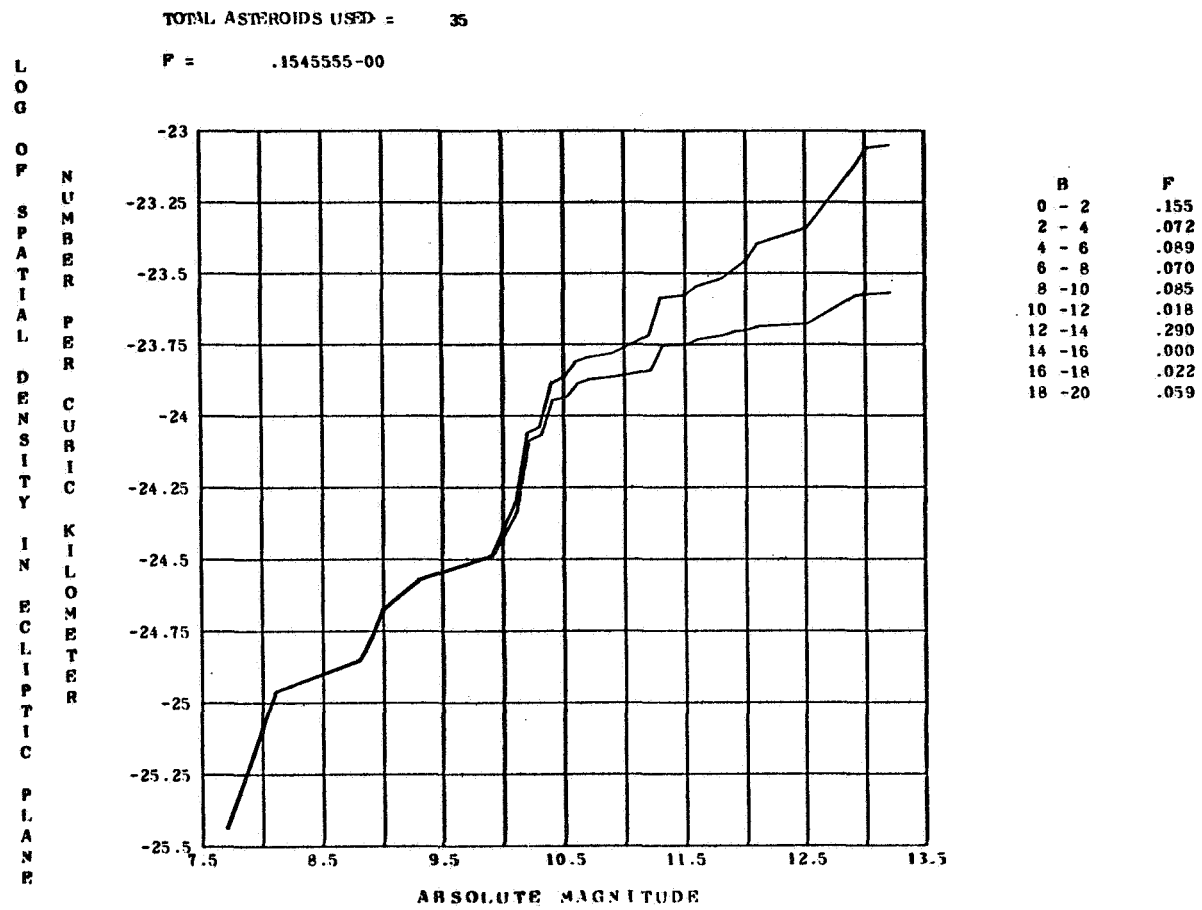


Figure 215. - Spatial density at R = 3.60 and at longitudes between 315.0 and 360.0.

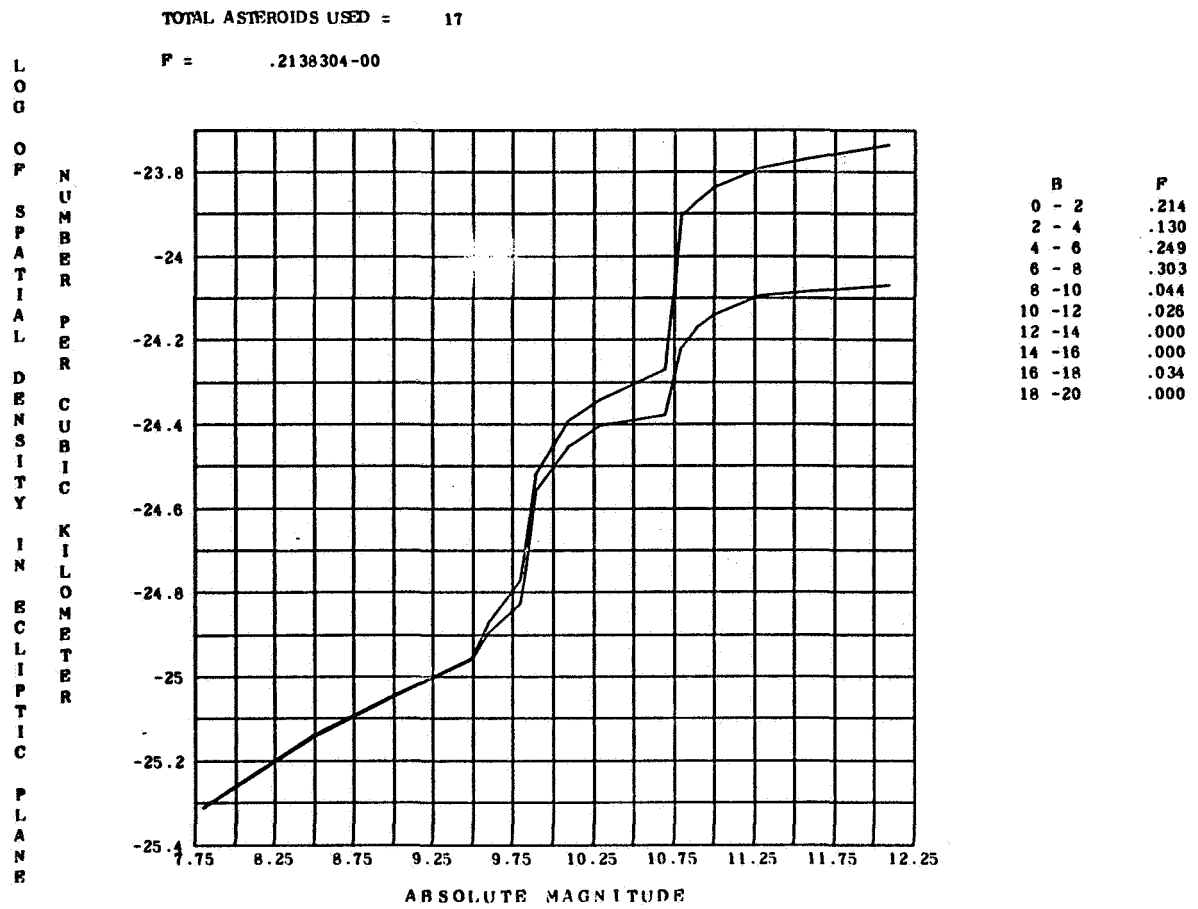


Figure 216. - Spatial density at R = 3.70 and at longitudes between 0 and 45.0.

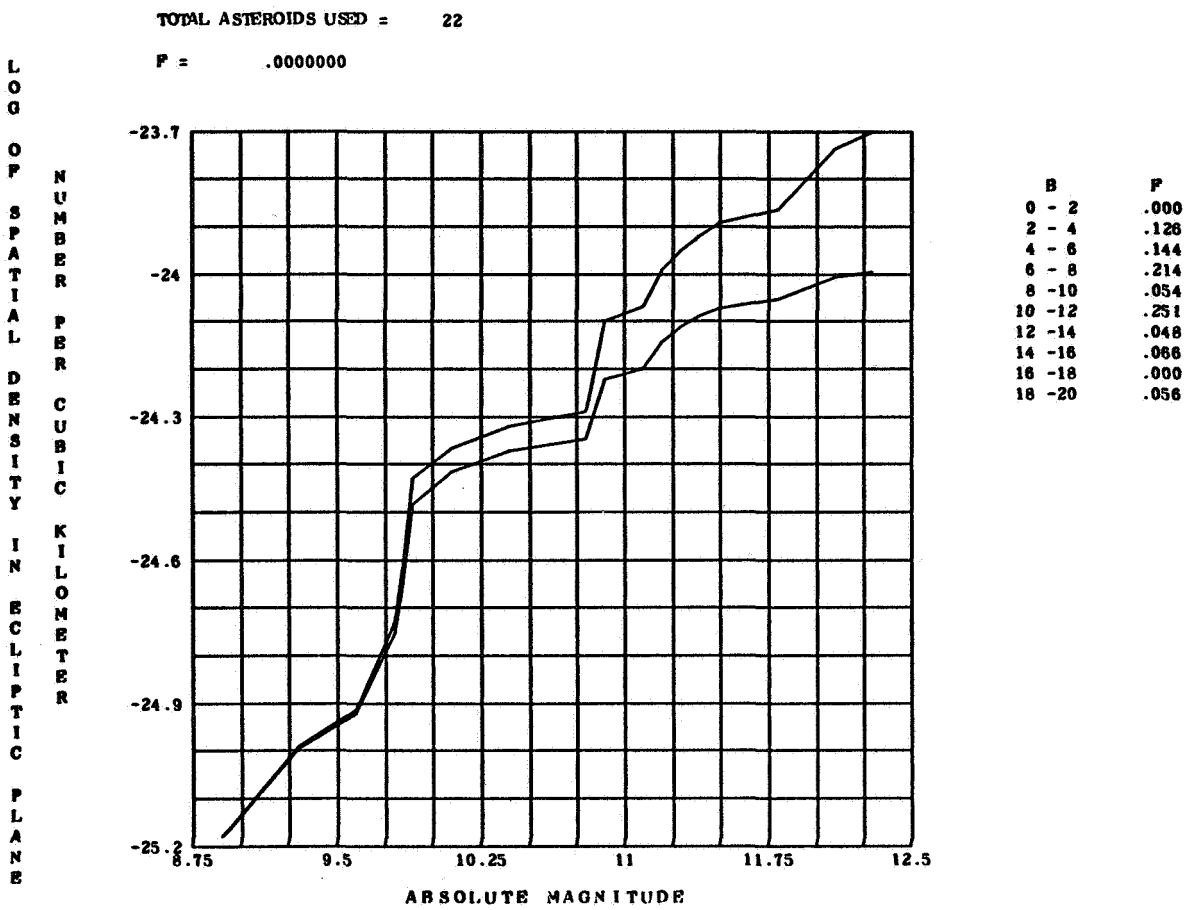


Figure 217. - Spatial density at R = 3.70 and at longitudes between 45.0 and 90.0.

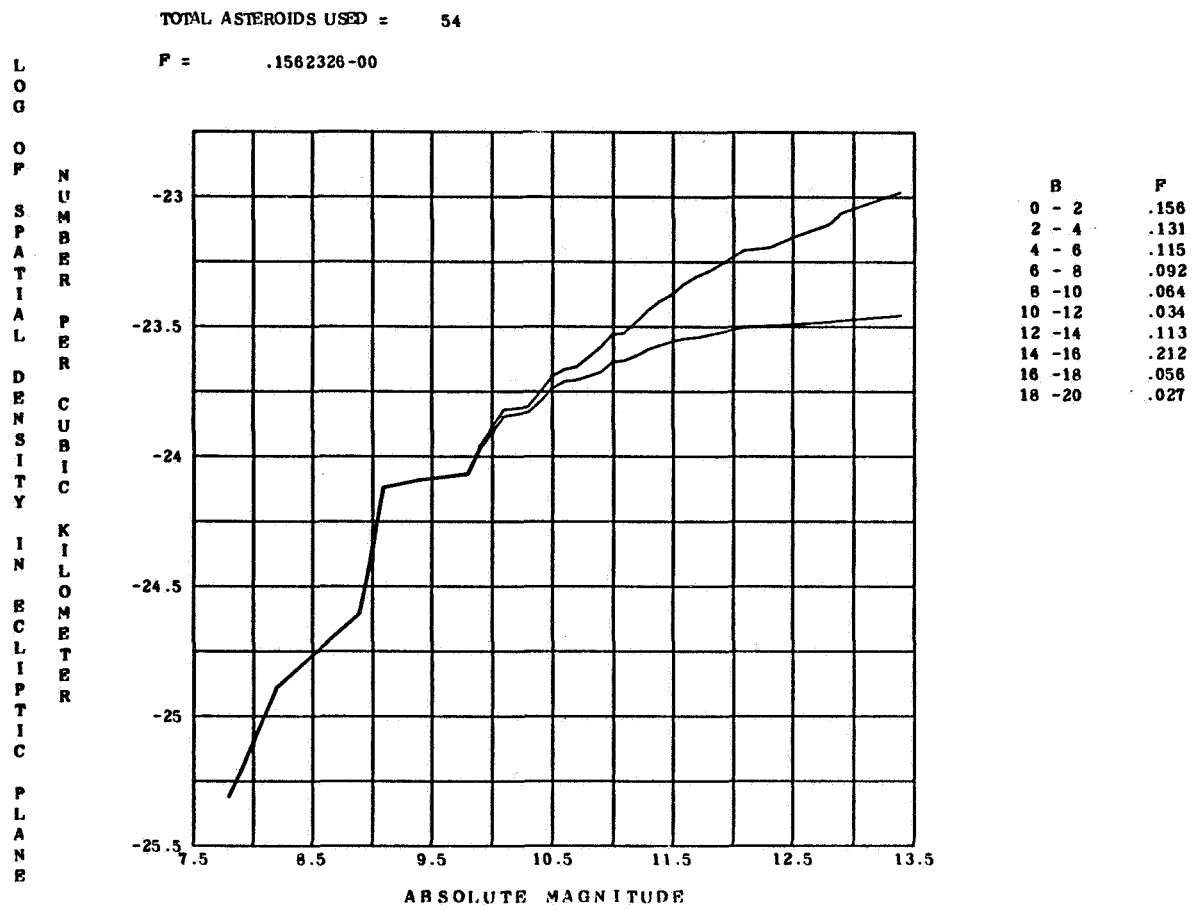


Figure 218. - Spatial density at R = 3.70 and at longitudes between 90.0 and 135.0.

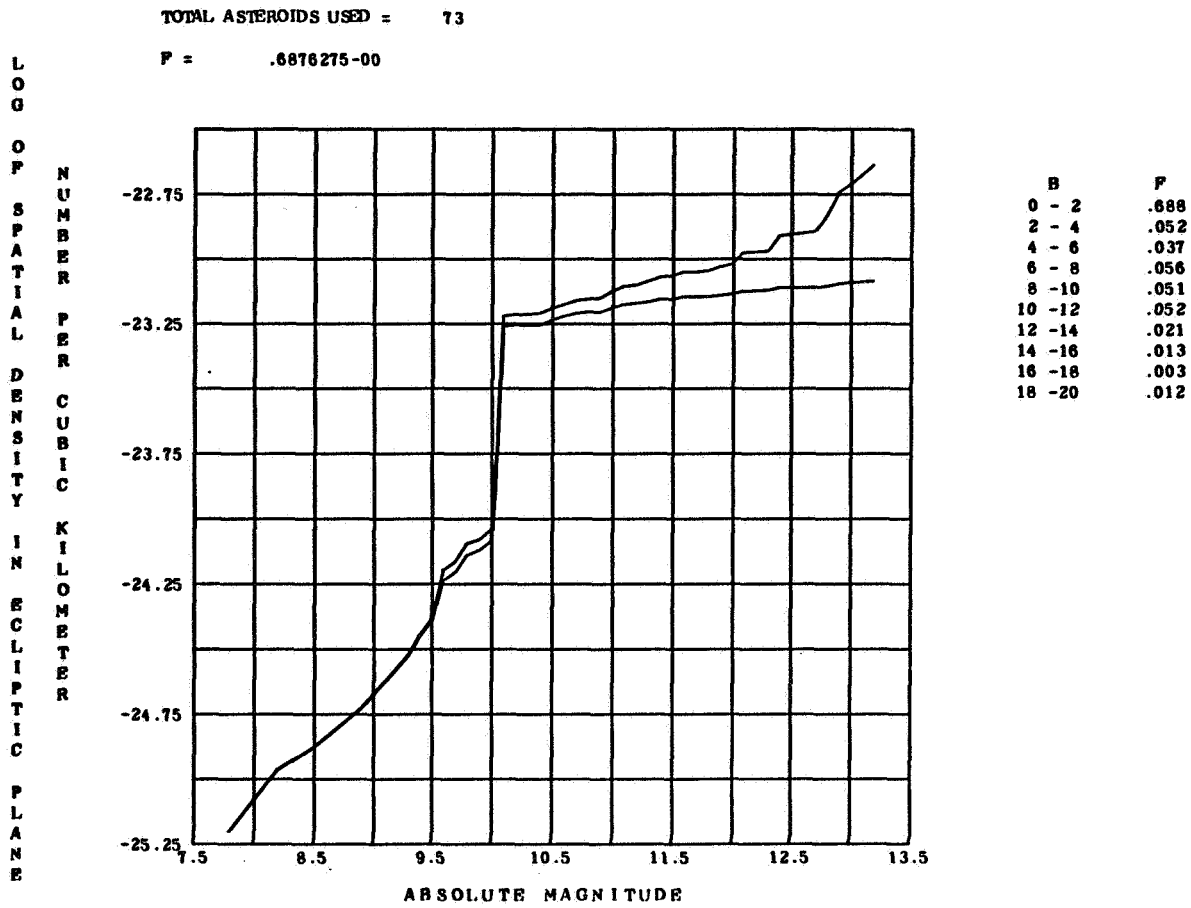


Figure 219. - Spatial density at R = 3.70 and at longitudes between 135.0 and 180.0.

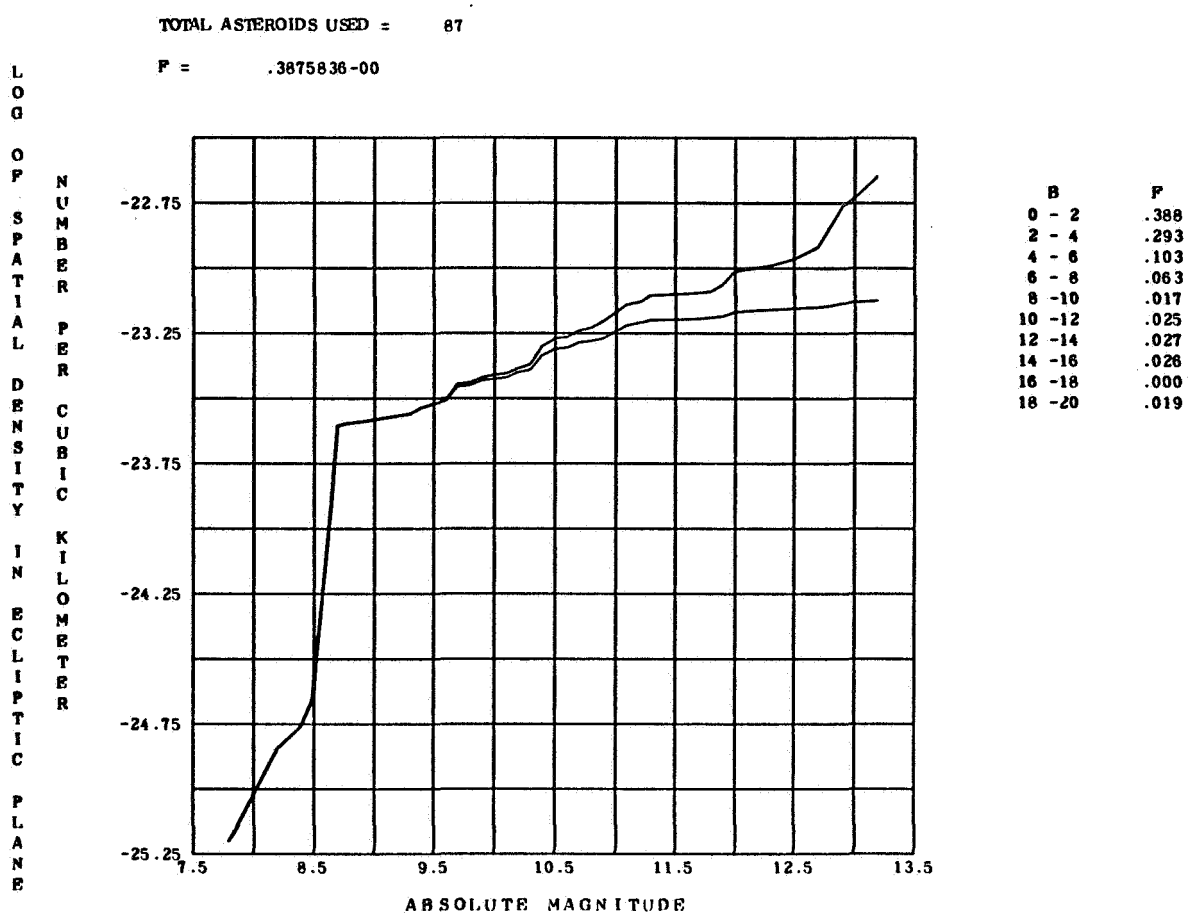


Figure 220. - Spatial density at R = 3.70 and at longitudes between 180.0 and 225.0.

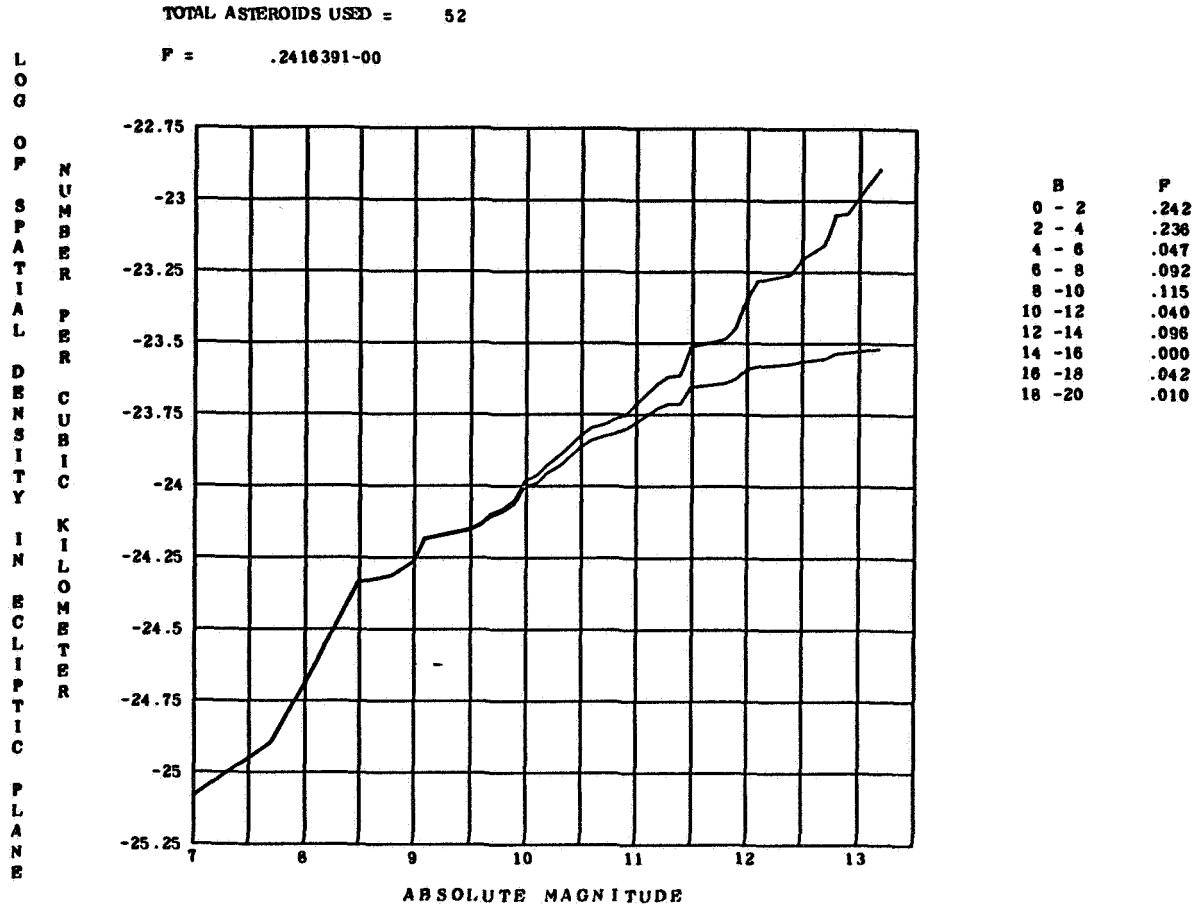


Figure 221. - Spatial density at R = 3.70 and at longitudes between 225.0 and 270.0.

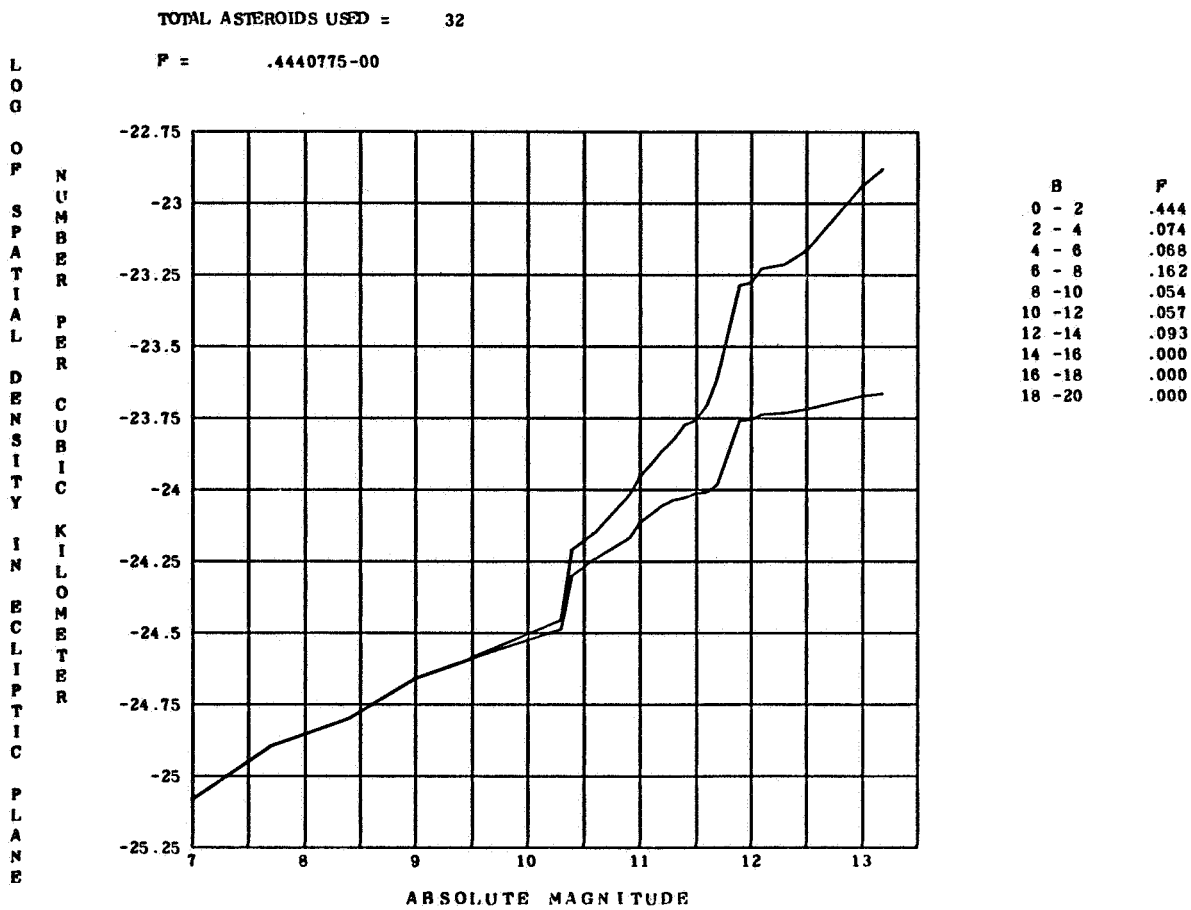


Figure 222. - Spatial density at R = 3.70 and at longitudes between 270.0 and 315.0.

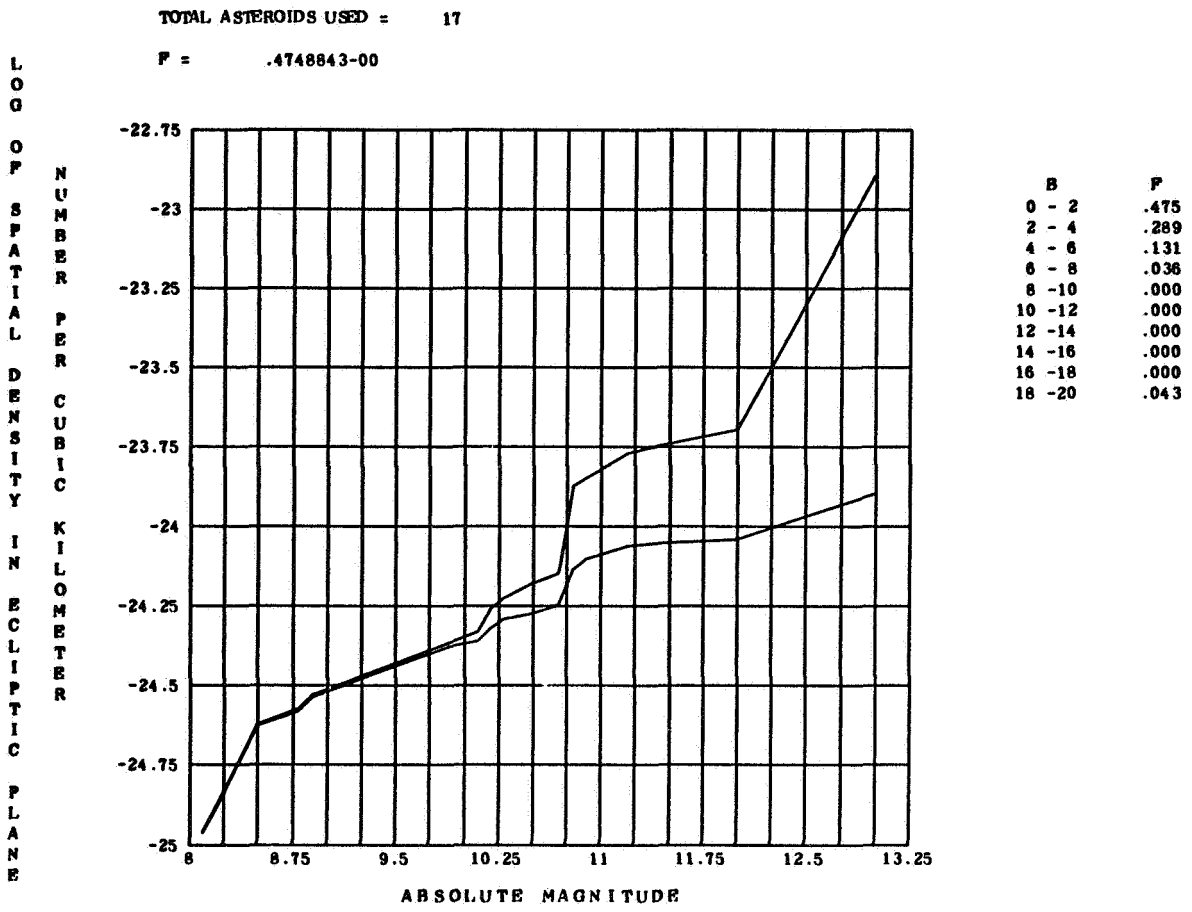


Figure 223. - Spatial density at R = 3.70 and at longitudes between 315.0 and 360.0.

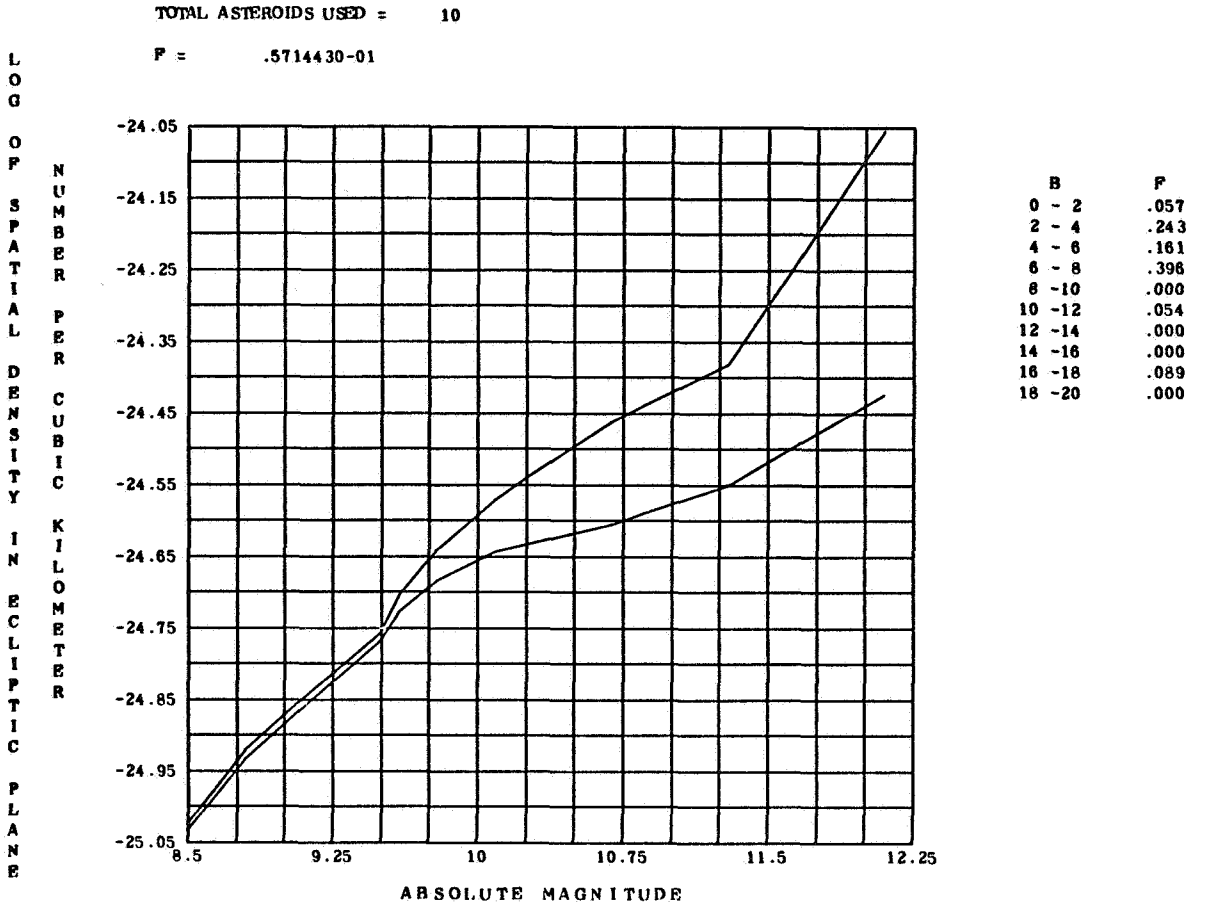


Figure 224. - Spatial density at R = 3.80 and at longitudes between 0 and 45.0.

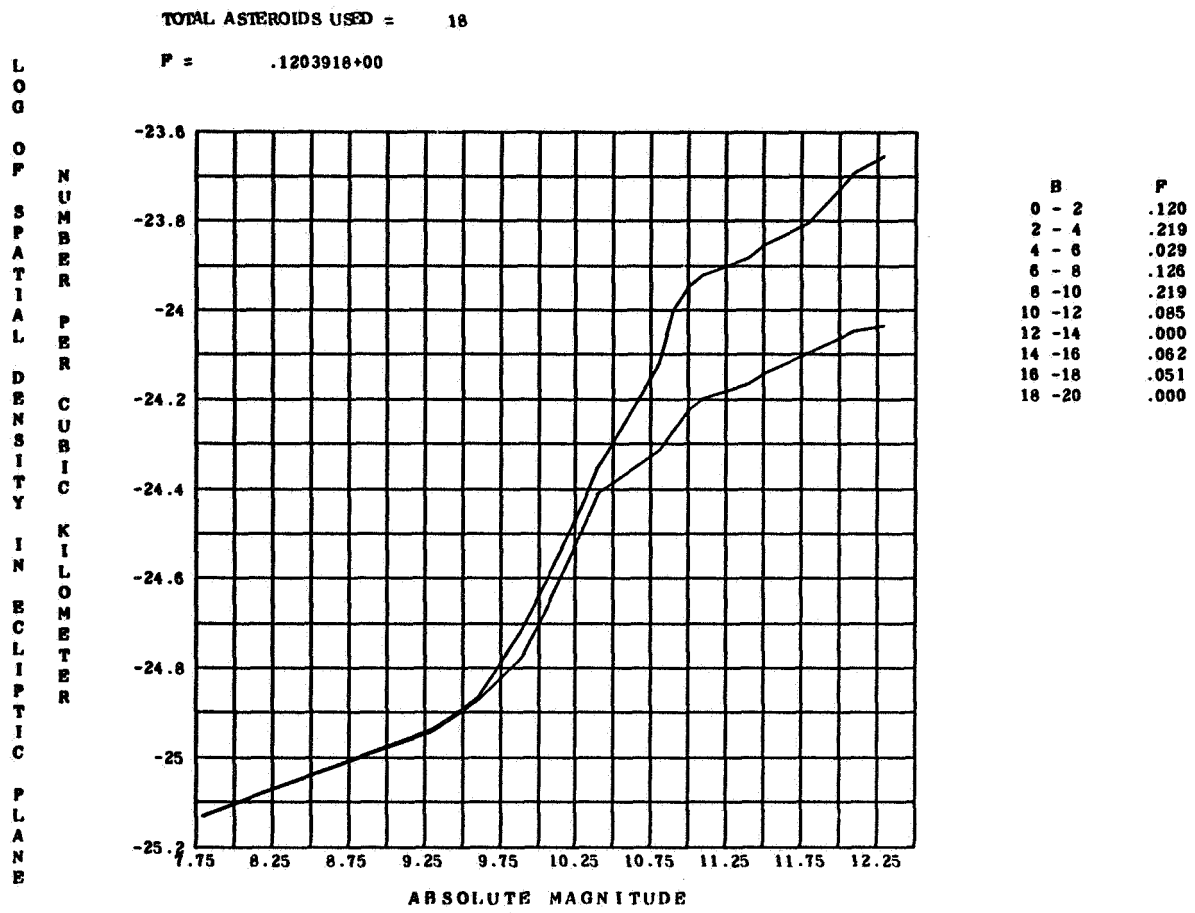


Figure 225. - Spatial density at R = 3.80 and at longitudes between 45.0 and 90.0.

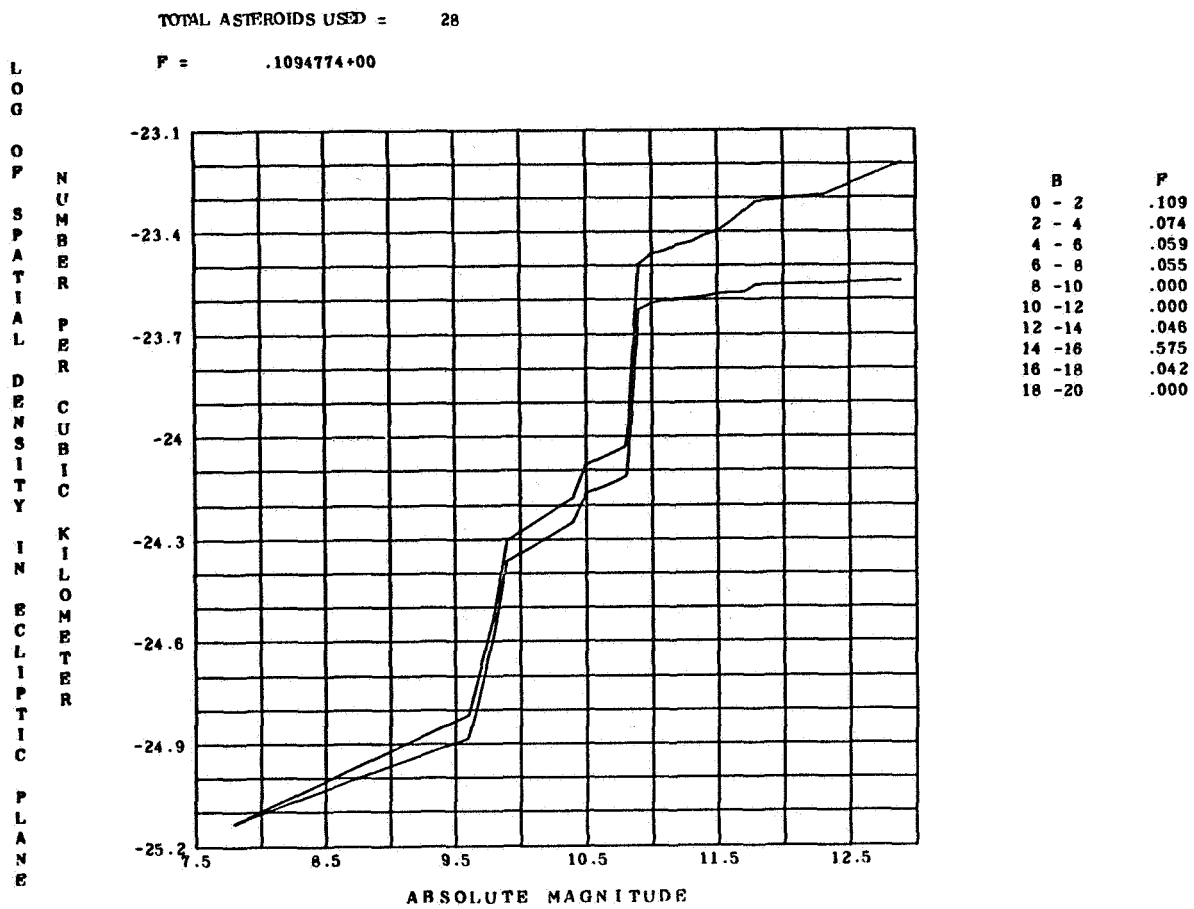


Figure 226. - Spatial density at R = 3.80 and at longitudes between 90.0 and 135.0.

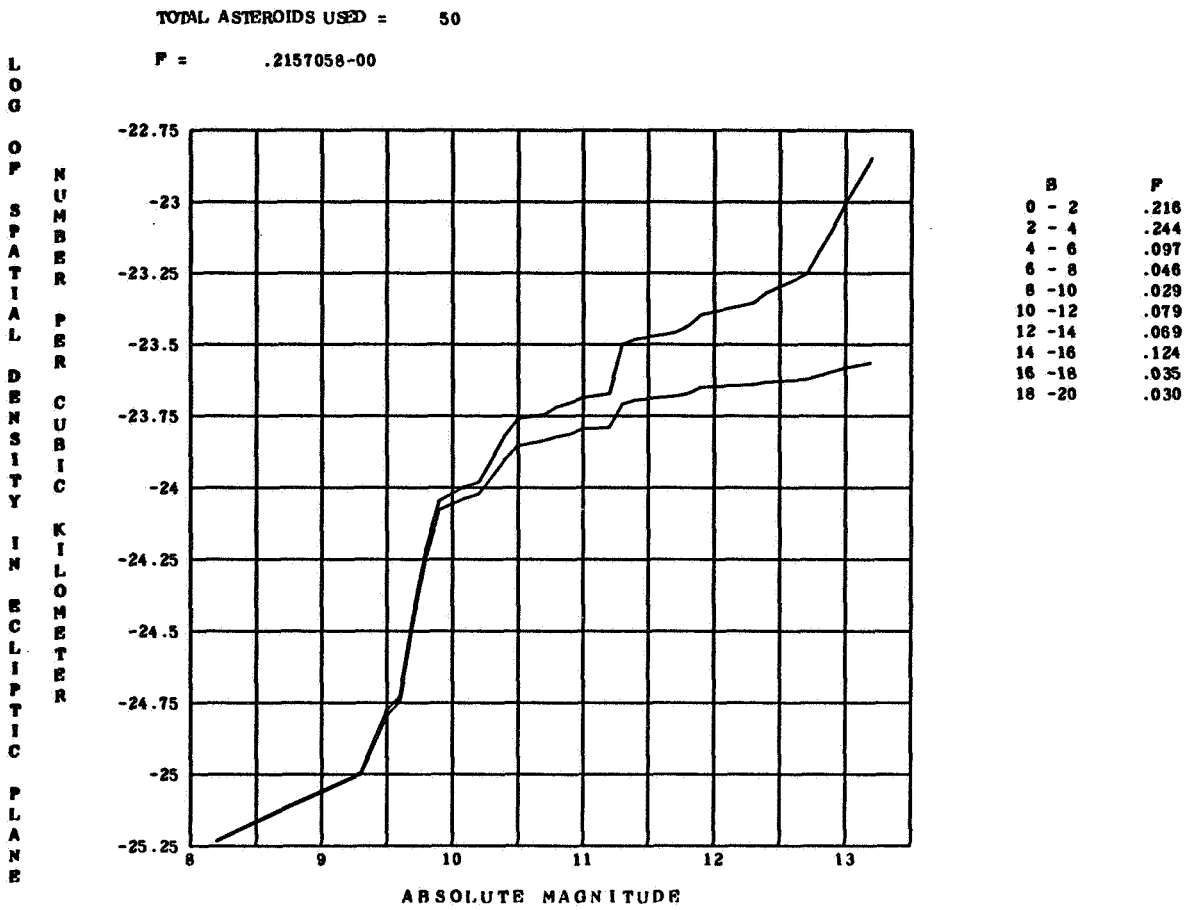


Figure 227. - Spatial density at R = 3.80 and at longitudes between 135.0 and 180.0.

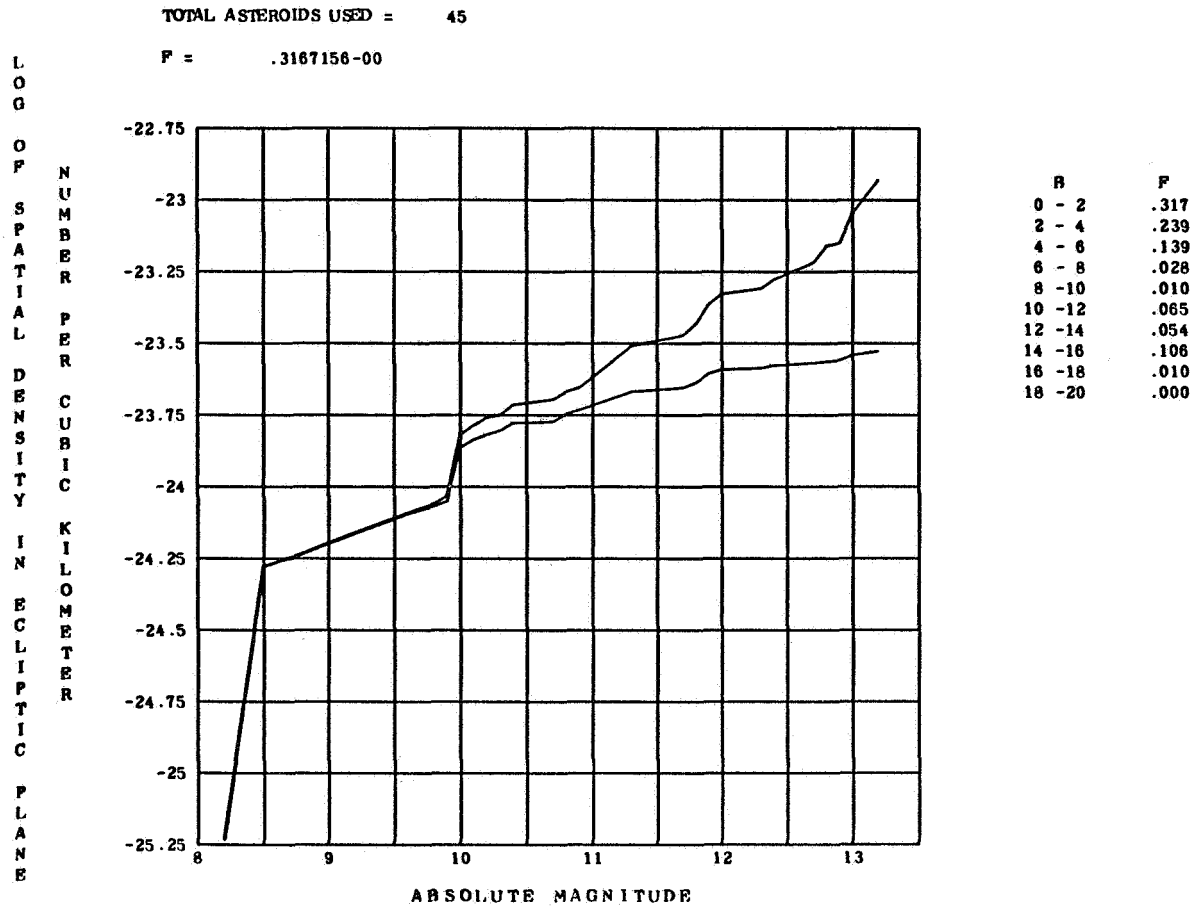


Figure 228. - Spatial density at R = 3.80 and at longitudes between 180.0 and 225.0.

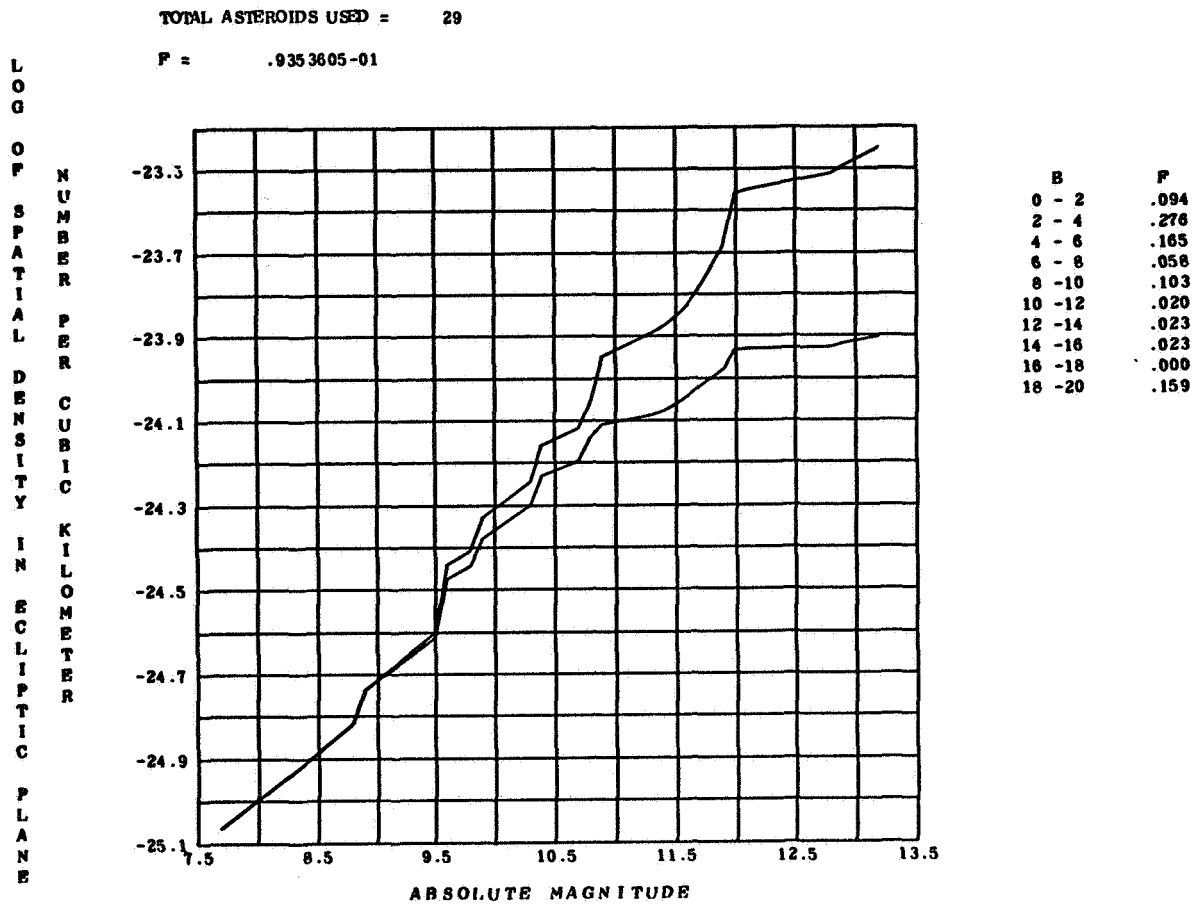


Figure 229. - Spatial density at R = 3.80 and at longitudes between 225.0 and 270.0.

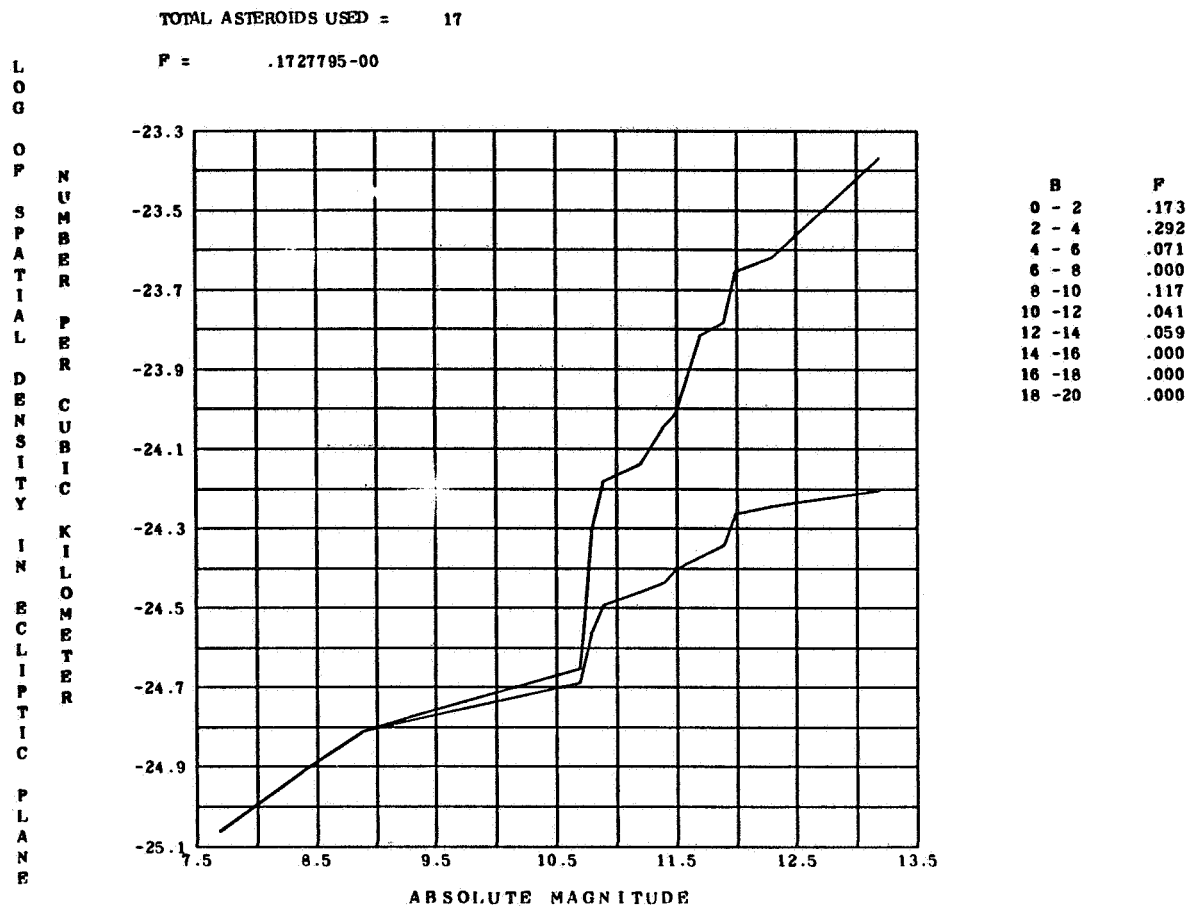


Figure 230. - Spatial density at R = 3.80 and at longitudes between 270.0 and 315.0.

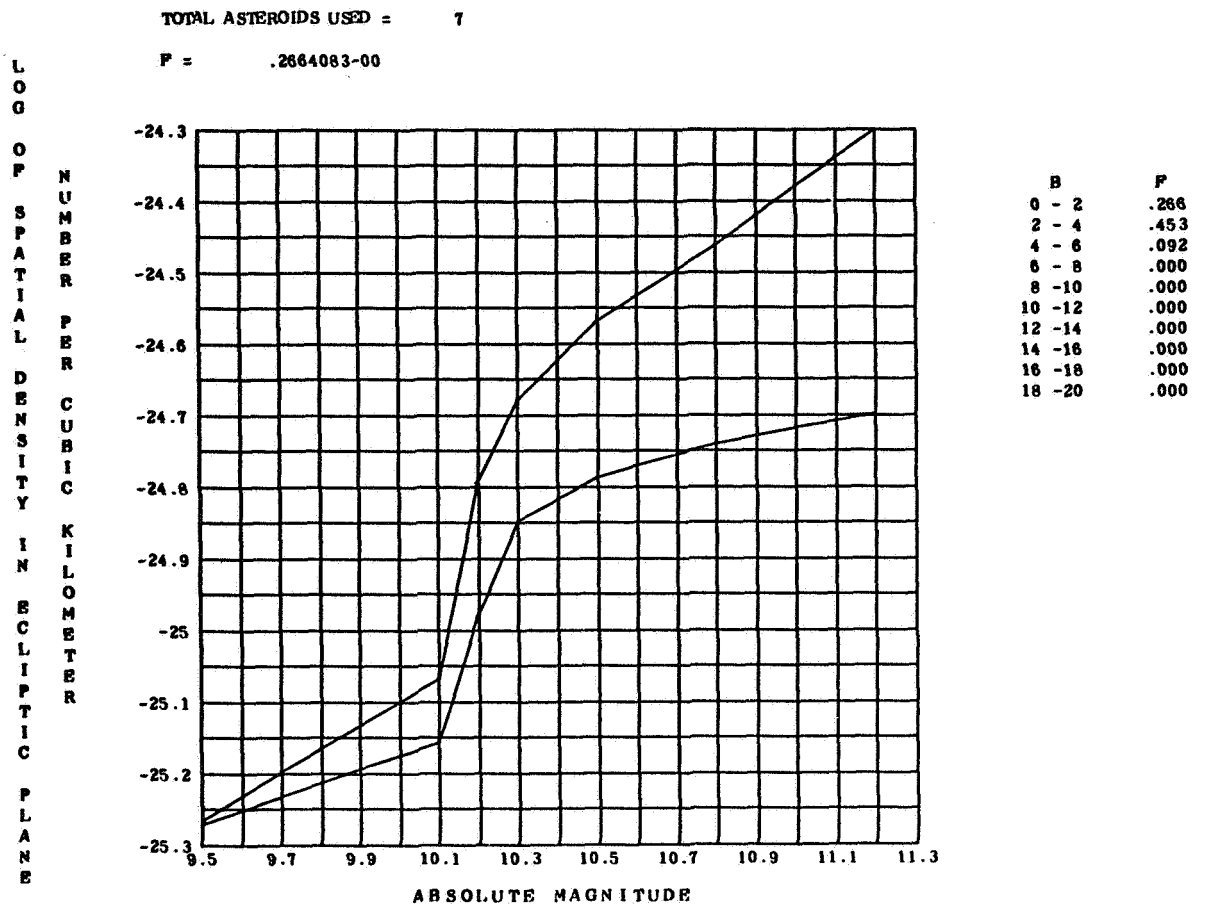


Figure 231. - Spatial density at R = 3.80 and at longitudes between 315.0 and 360.0.

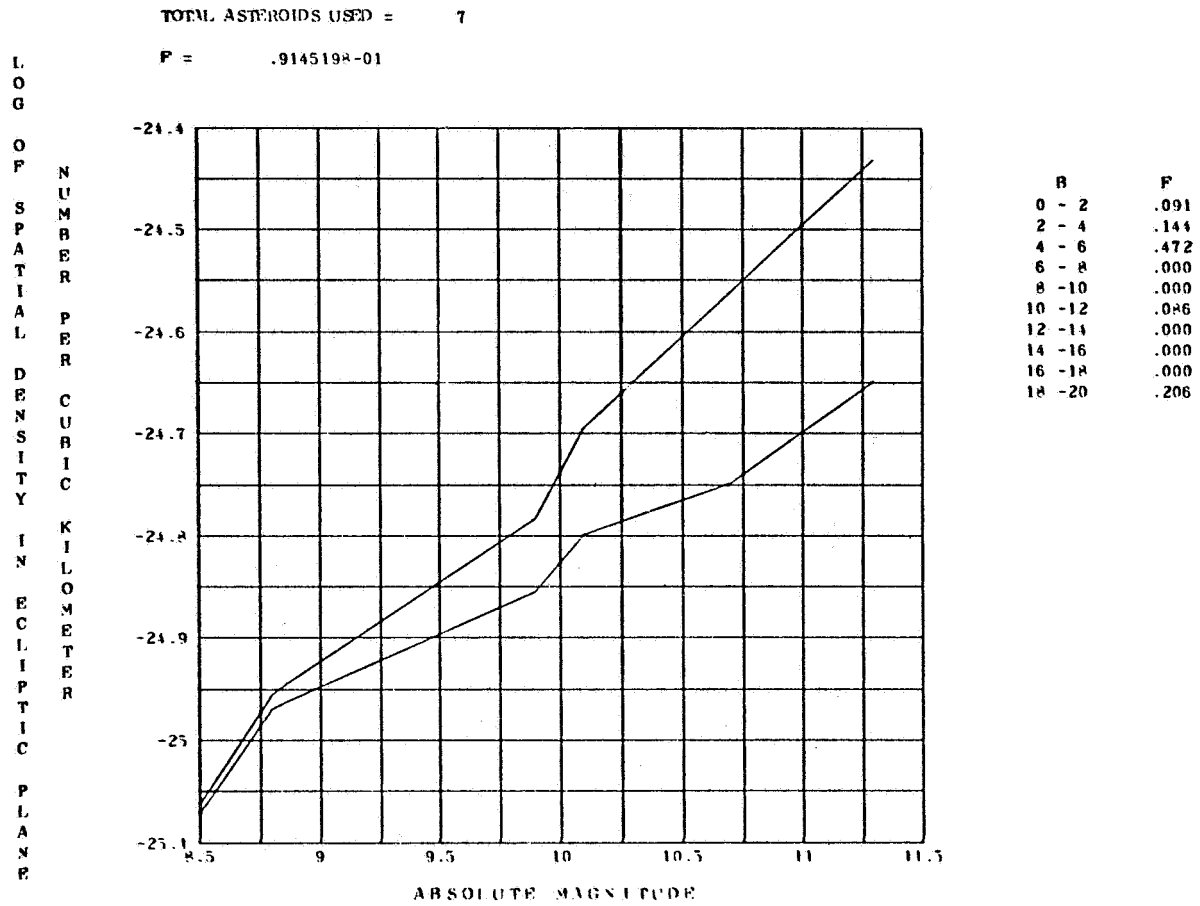


Figure 232. - Spatial density at R = 3.90 and at longitudes between 0 and 45.0.

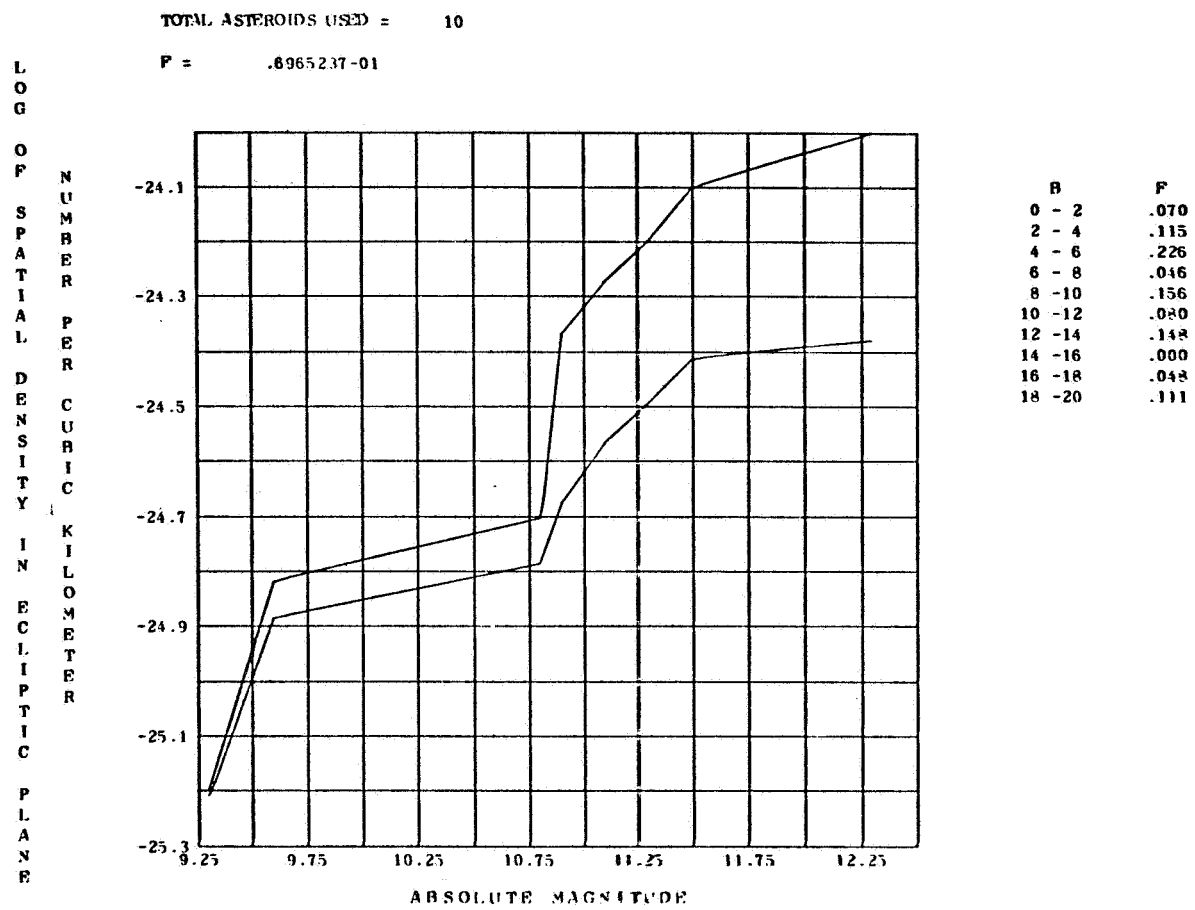


Figure 233. - Spatial density at R = 3.90 and at longitudes between 45.0 and 90.0.

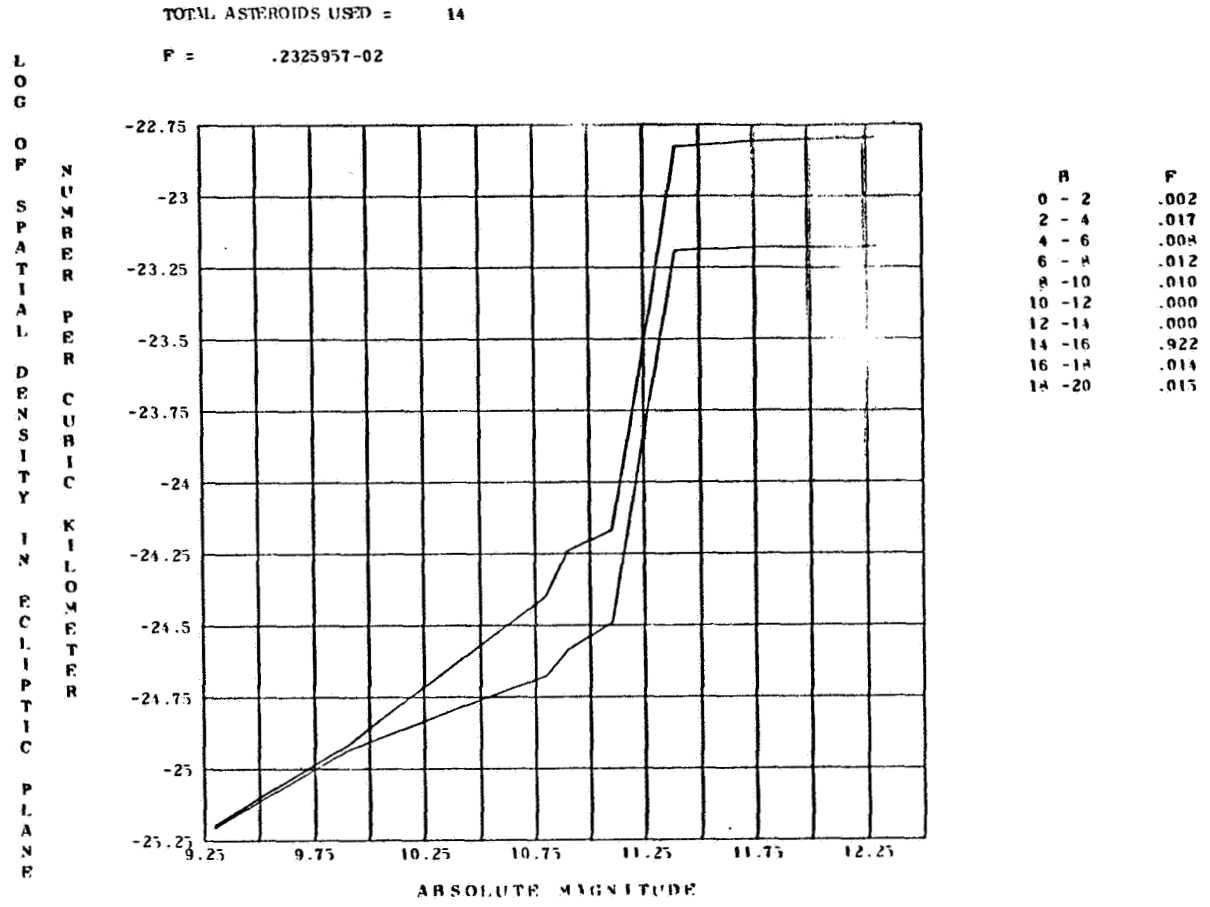


Figure 234. - Spatial density at R = 3.90 and at longitudes between 90.0 and 135.0.

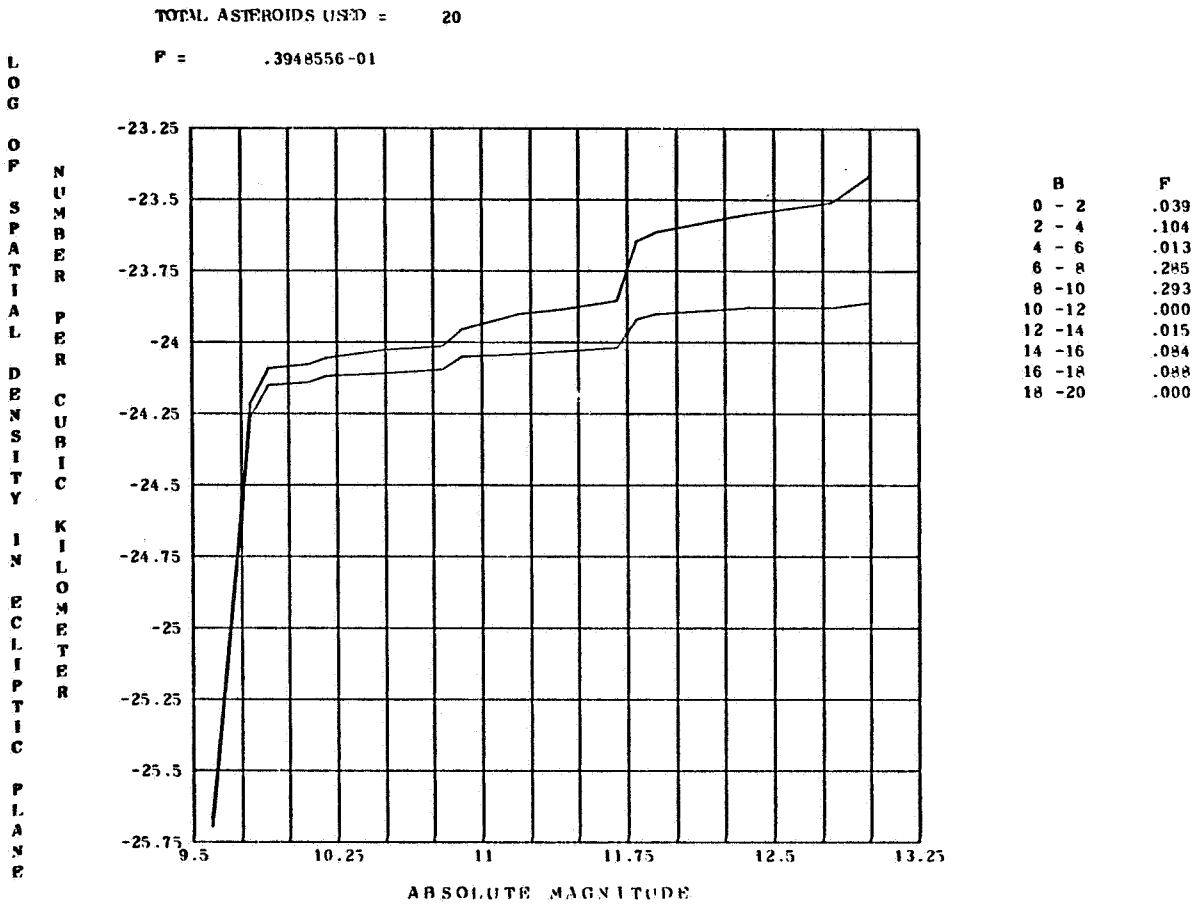


Figure 235. - Spatial density at R = 3.90 and at longitudes between 135.0 and 180.0.

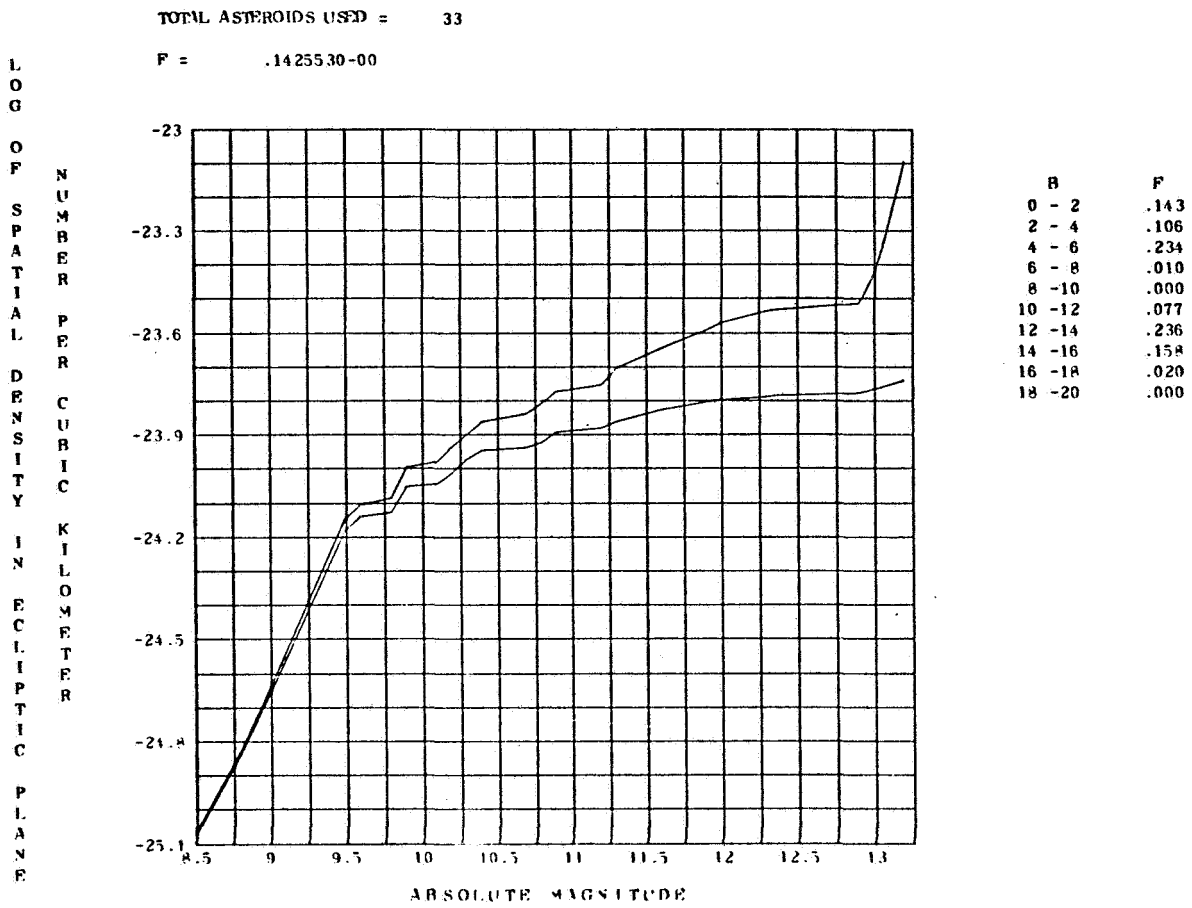


Figure 236. - Spatial density at R = 3.90 and at longitudes between 180.0 and 225.0.

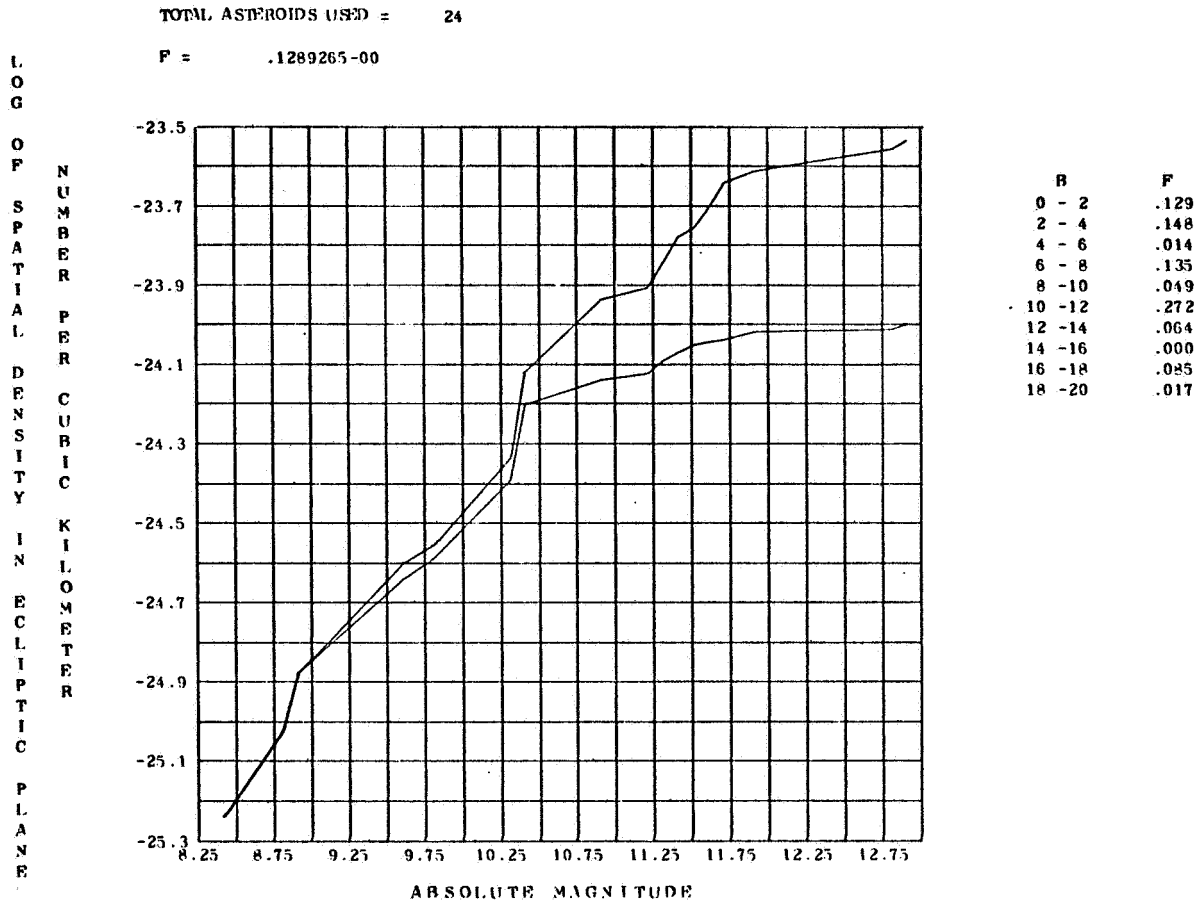


Figure 237. - Spatial density at R = 3.90 and at longitudes between 225.0 and 270.0.

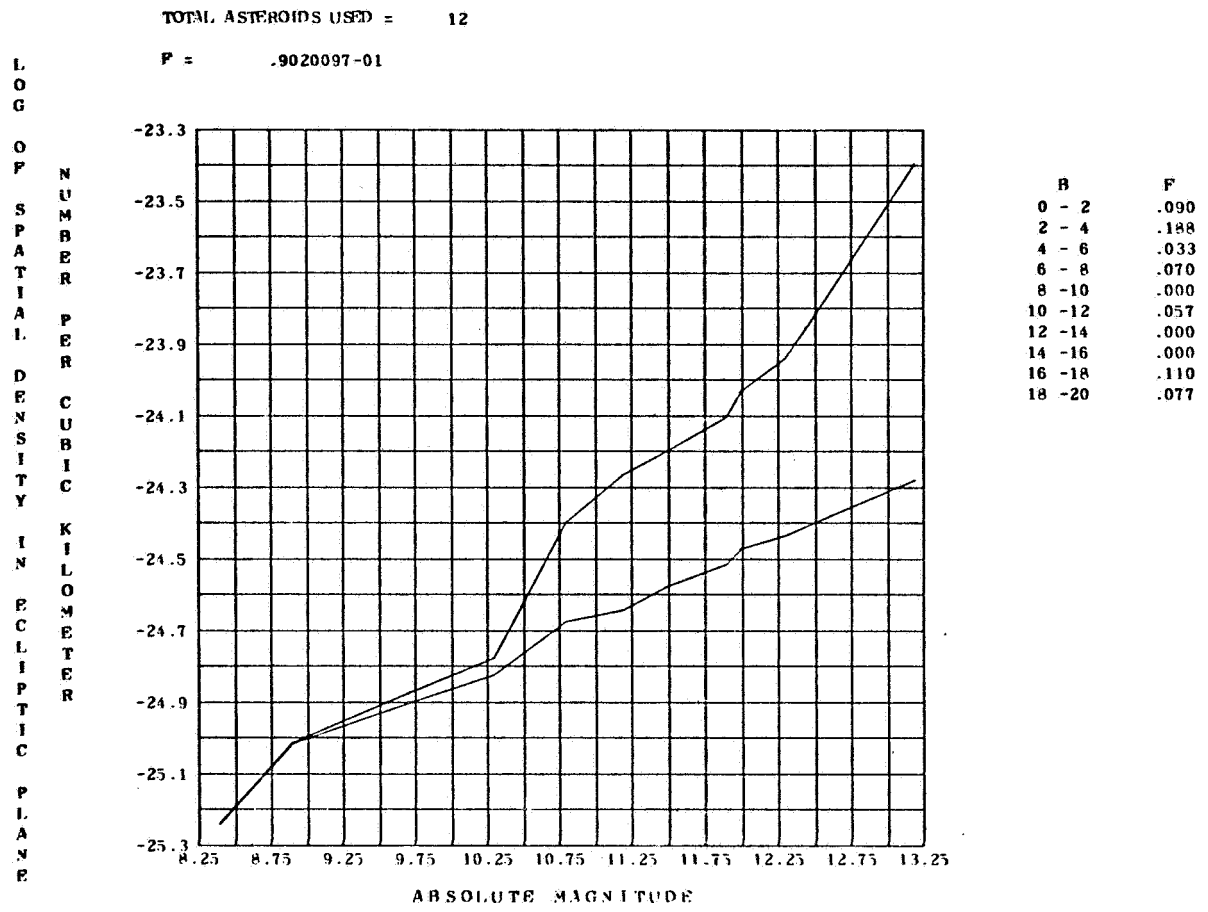


Figure 238. - Spatial density at R = 3.90 and at longitudes between 270.0 and 315.0.

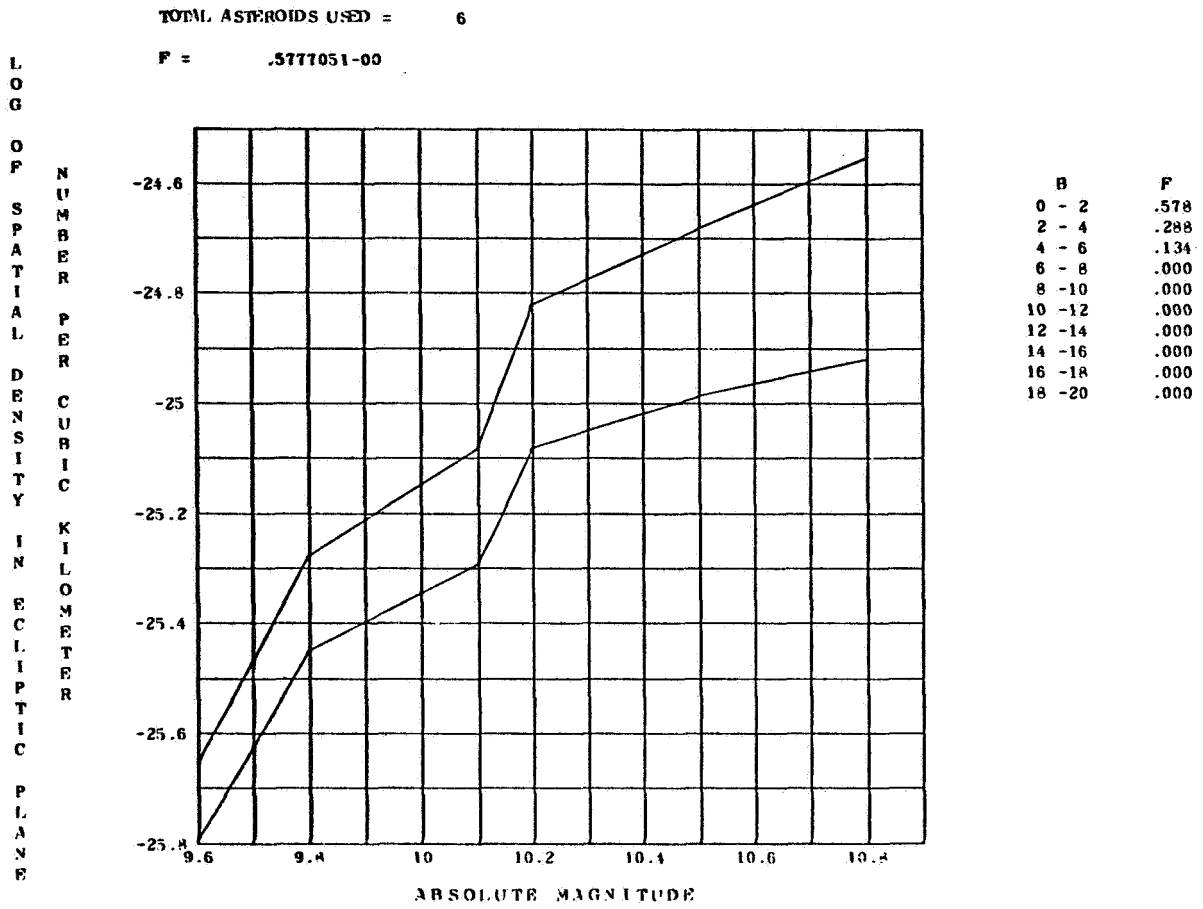


Figure 239. - Spatial density at R = 3.90 and at longitudes between 315.0 and 360.0.

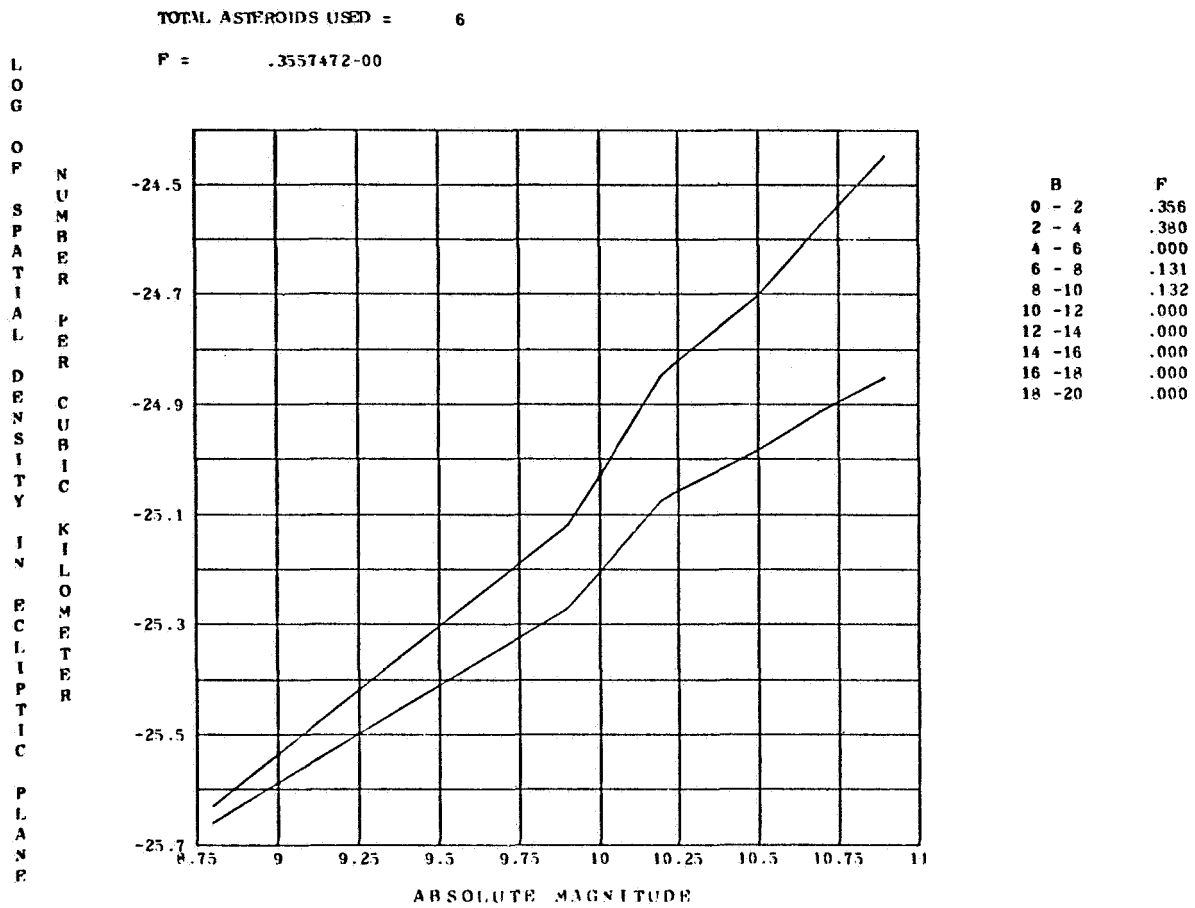


Figure 240. - Spatial density at R = 4.00 and at longitudes between 0 and 45.0.

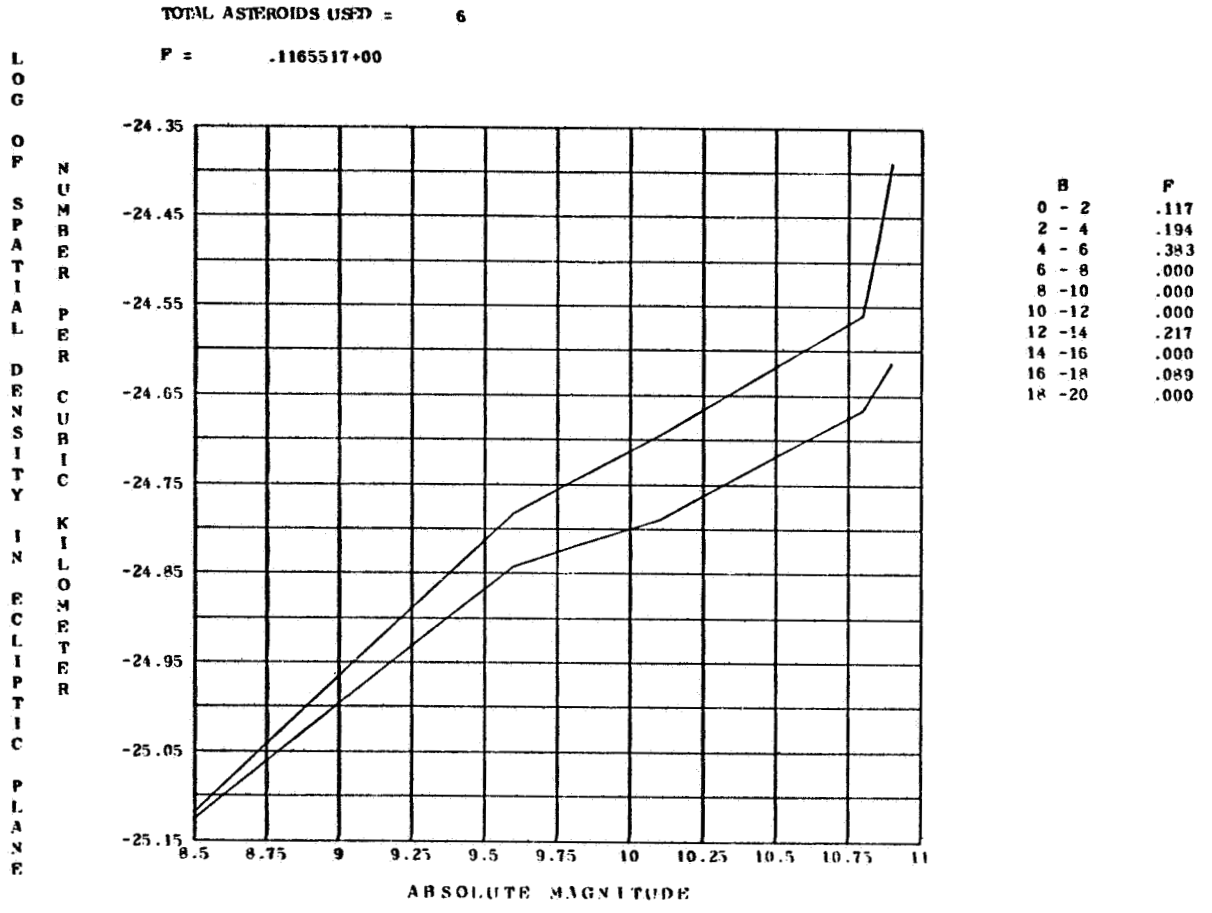


Figure 241. - Spatial density at R = 4.00 and at longitudes between 45.0 and 90.0.

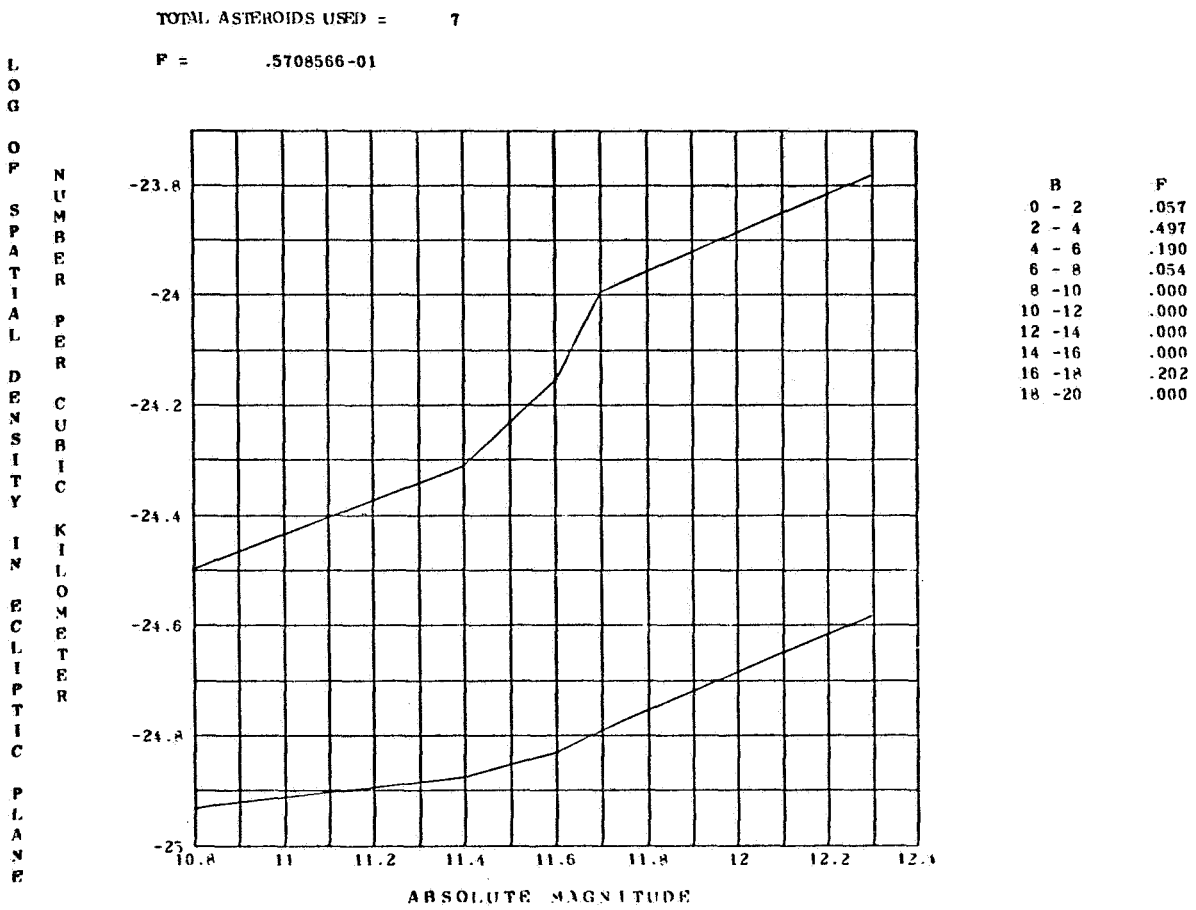


Figure 242. - Spatial density at R = 4.00 and at longitudes between 90.0 and 135.0.

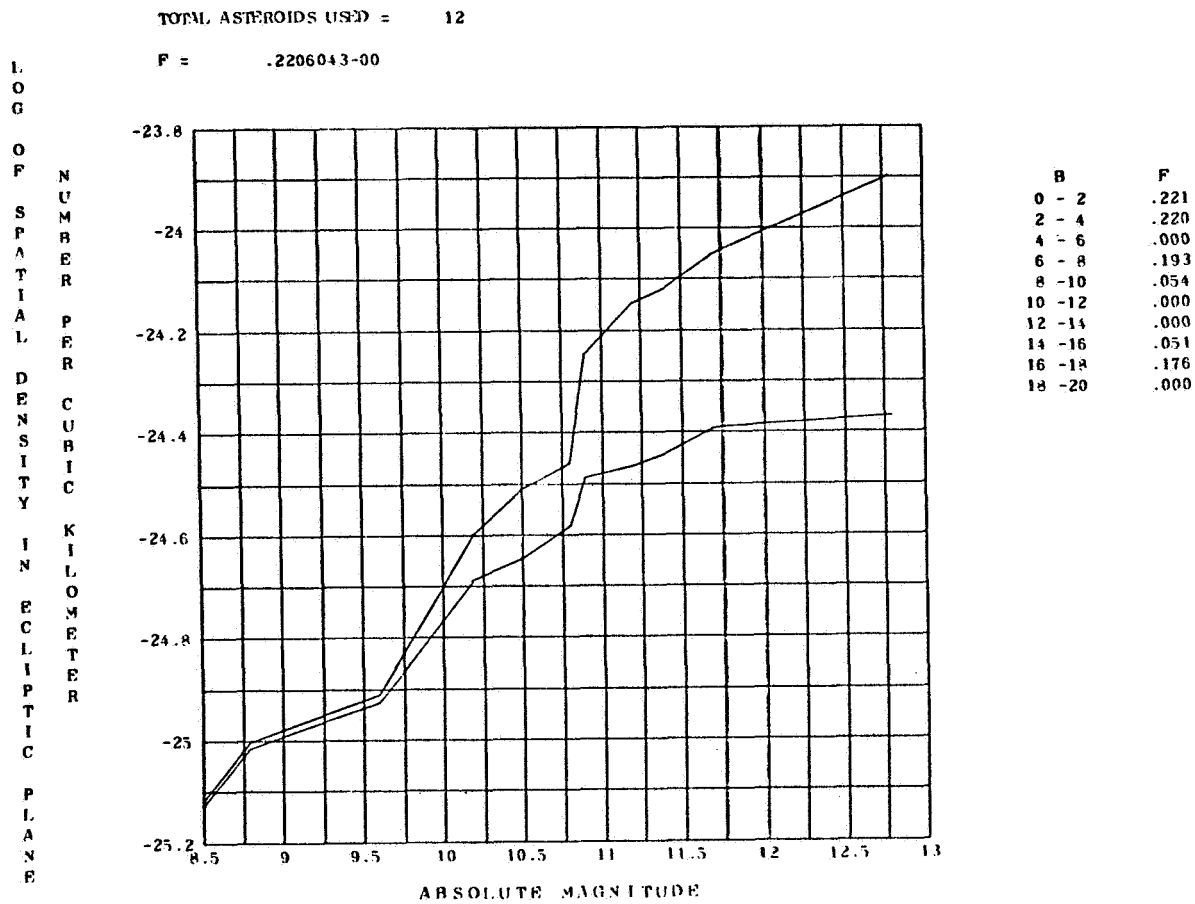


Figure 243. - Spatial density at R = 4.00 and at longitudes between 135.0 and 180.0.

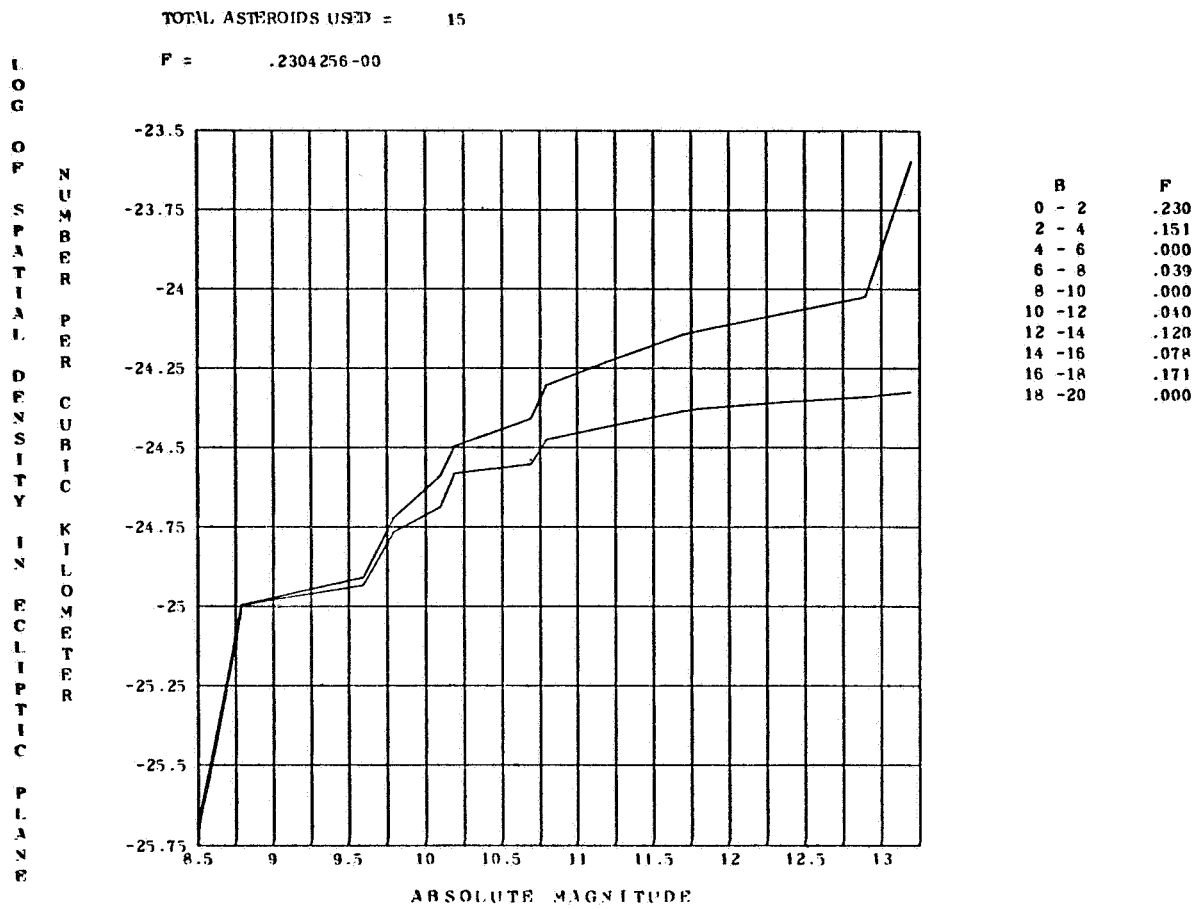


Figure 244. - Spatial density at R = 4.00 and at longitudes between 180.0 and 225.0.

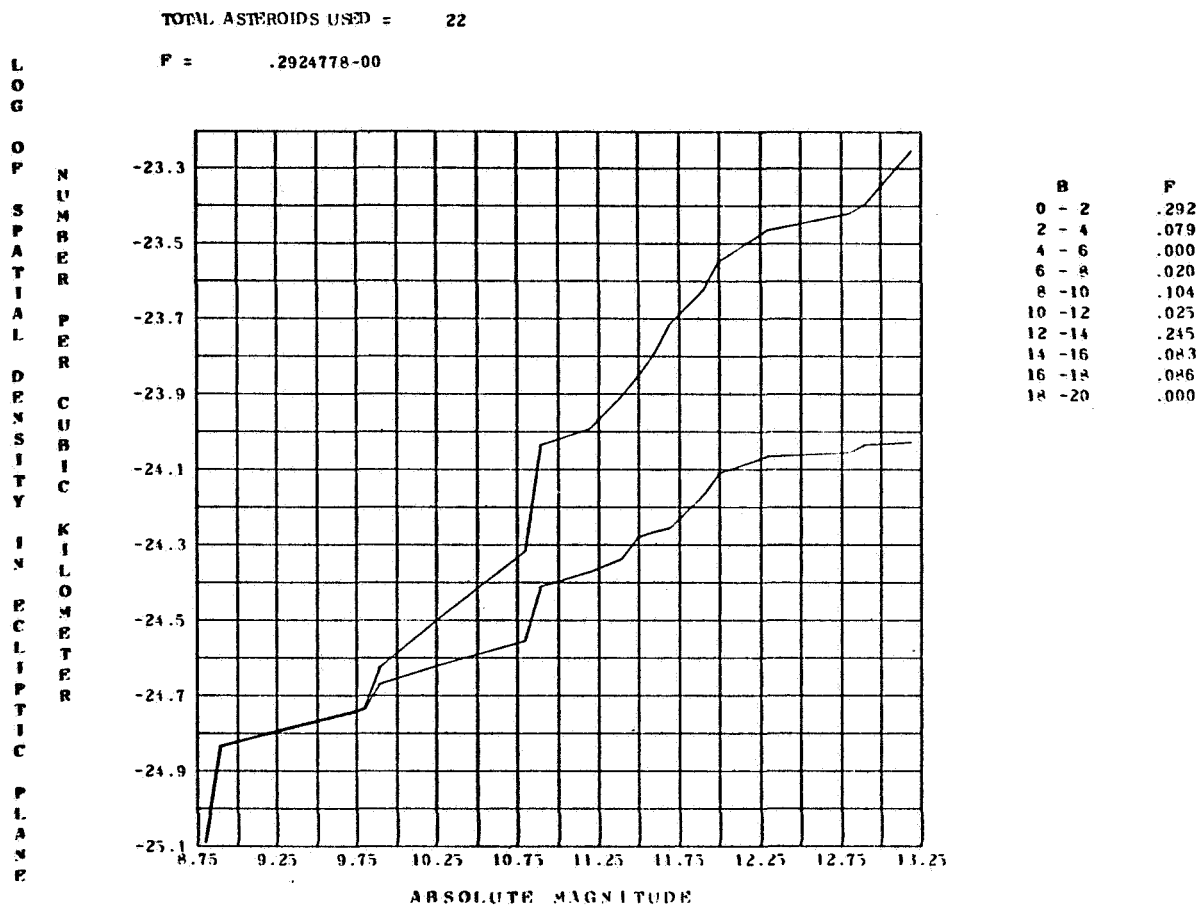


Figure 245. - Spatial density at R = 4.00 and at longitudes between 225.0 and 270.0.

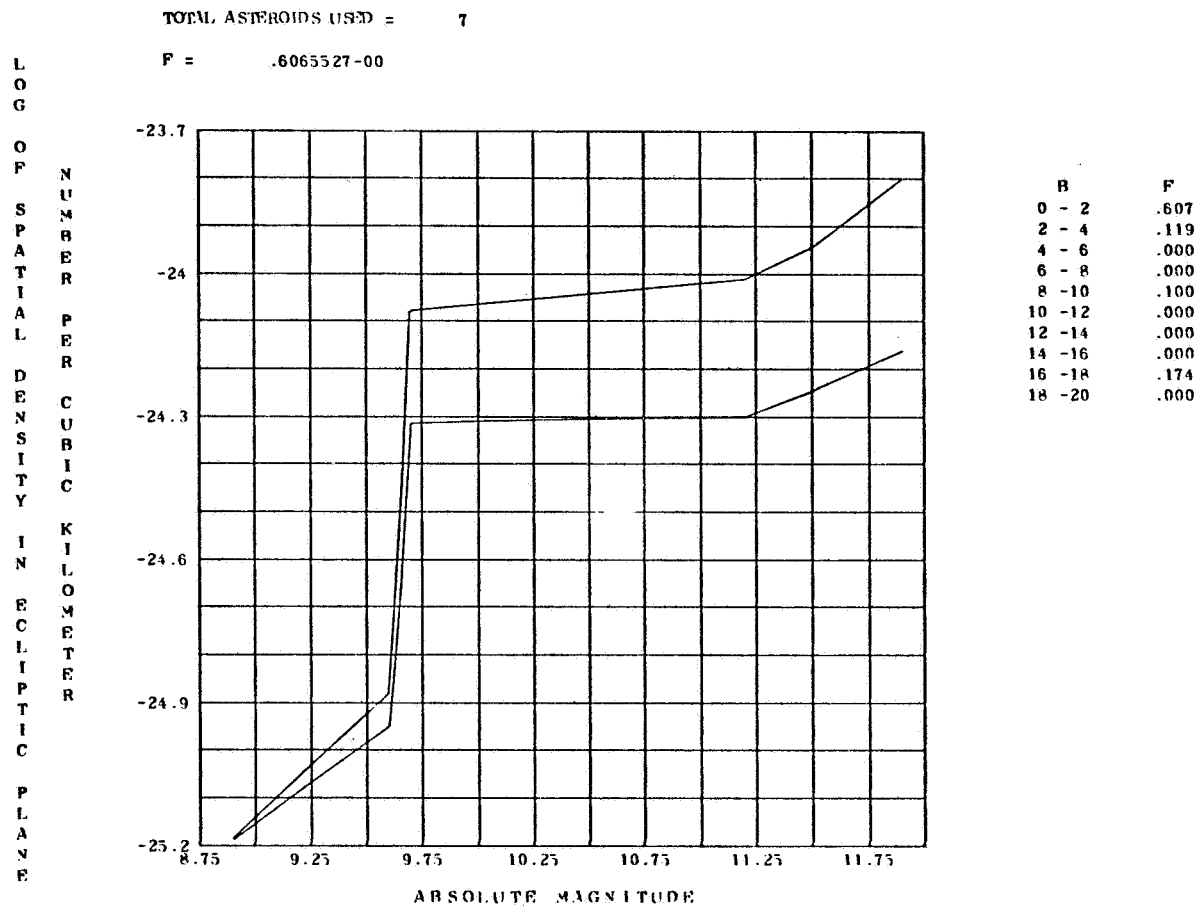


Figure 246. - Spatial density at R = 4.00 and at longitudes between 270.0 and 315.0.

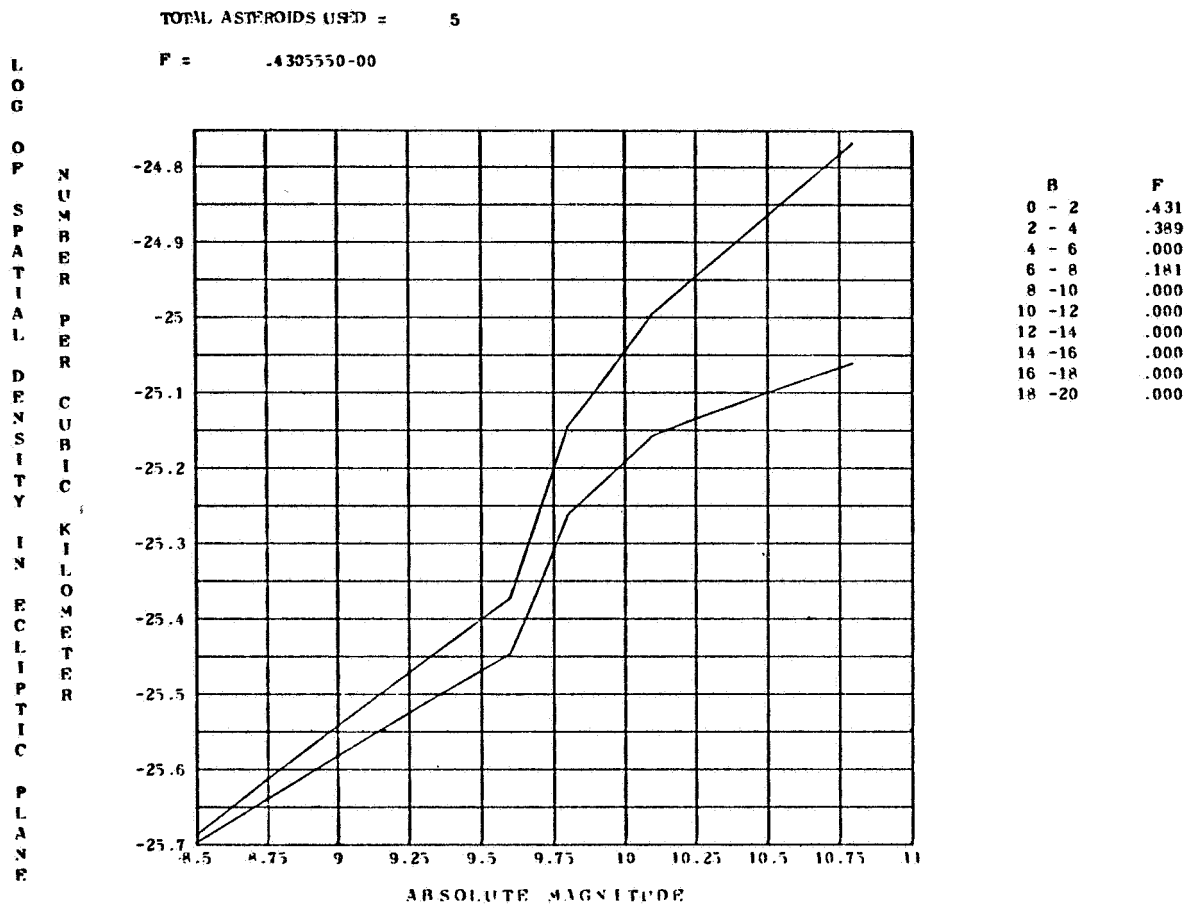


Figure 247. - Spatial density at R = 4.00 and at longitudes between 315.0 and 360.0.

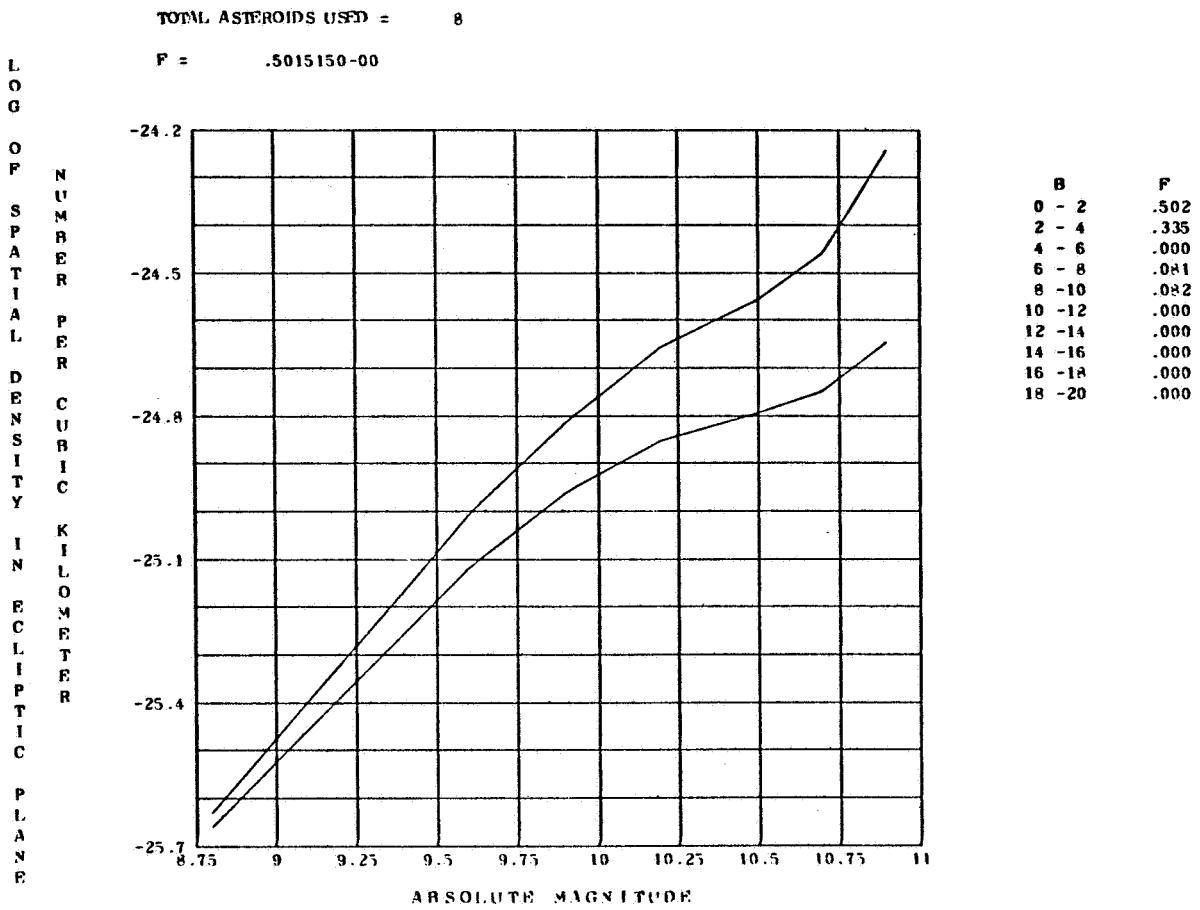


Figure 248. - Spatial density at R = 4.10 and at longitudes between 0 and 45.0.

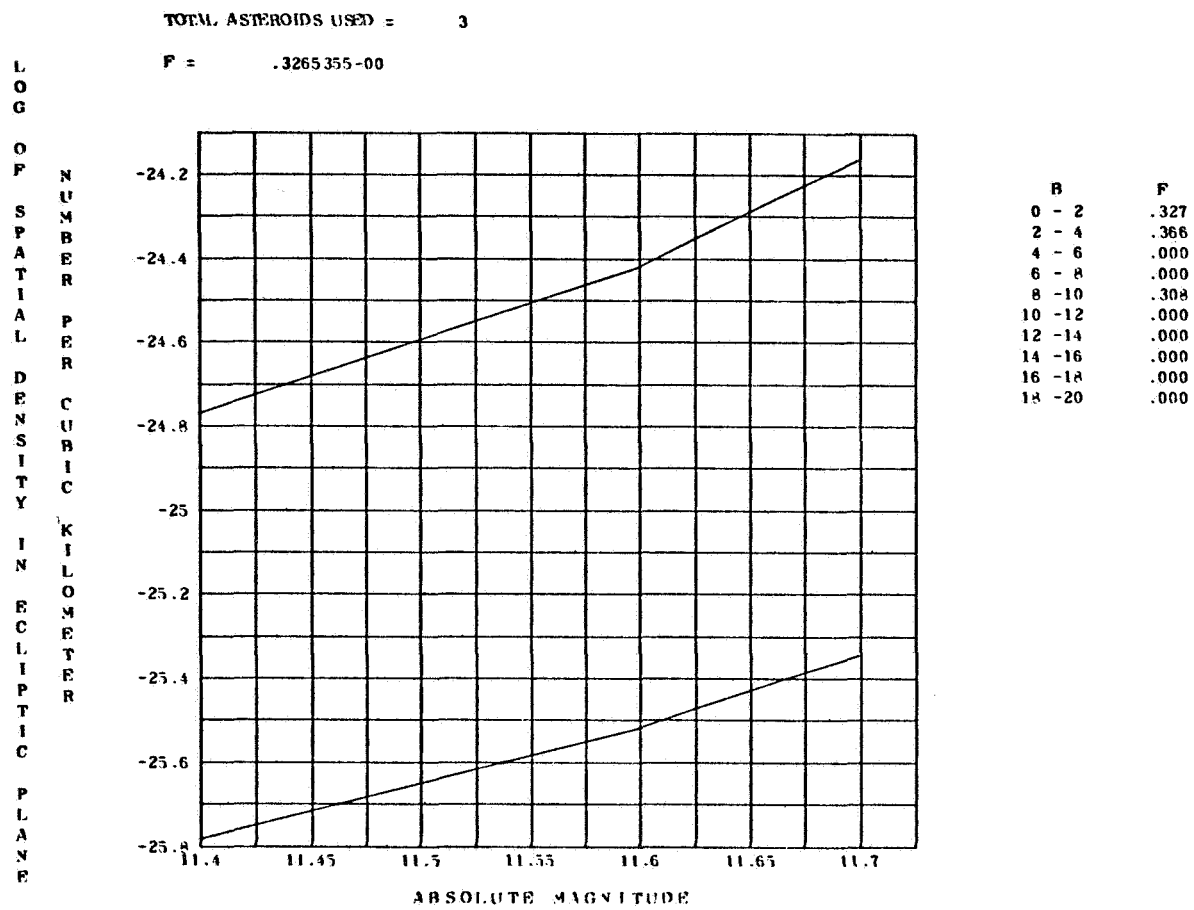


Figure 249. - Spatial density at R = 4.10 and at longitudes between 90.0 and 135.0.

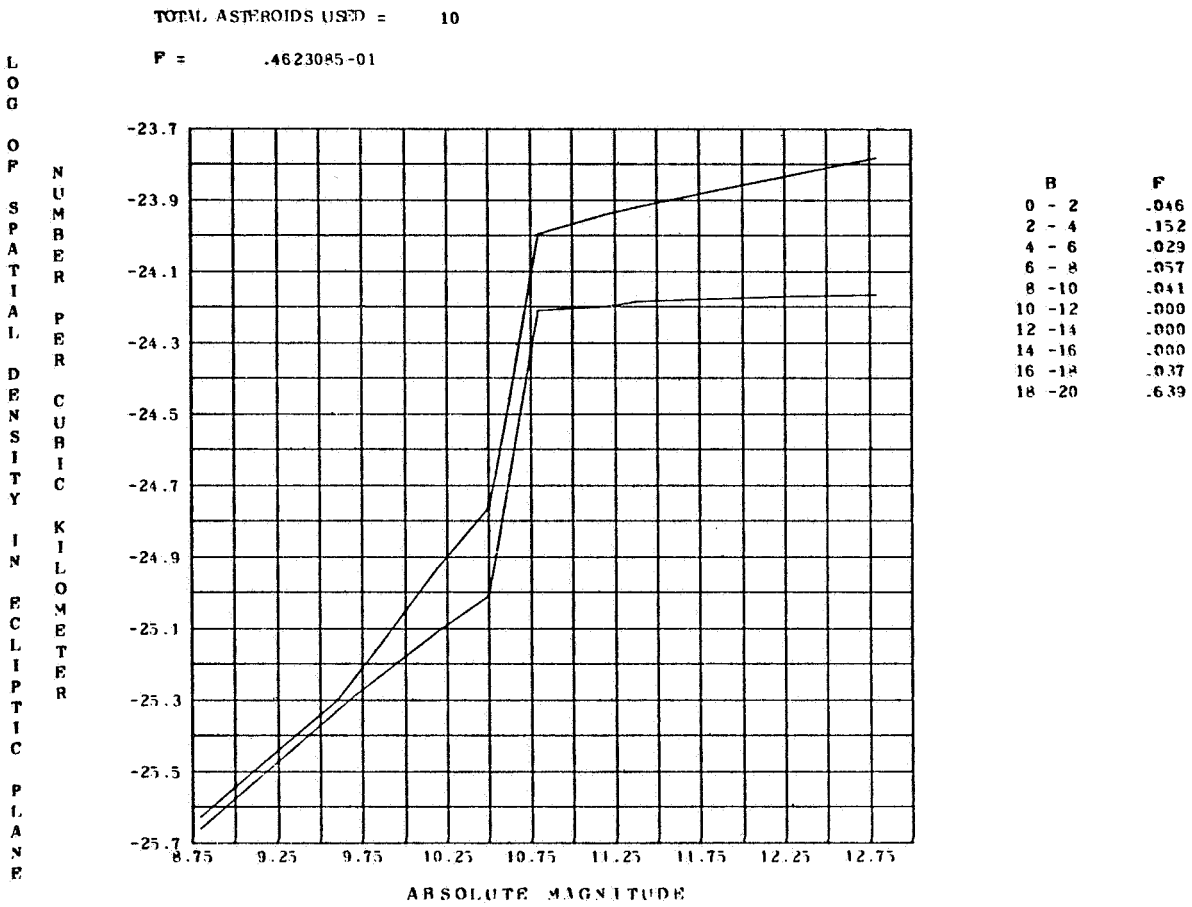


Figure 250. - Spatial density at R = 4.10 and at longitudes between 135.0 and 180.0.

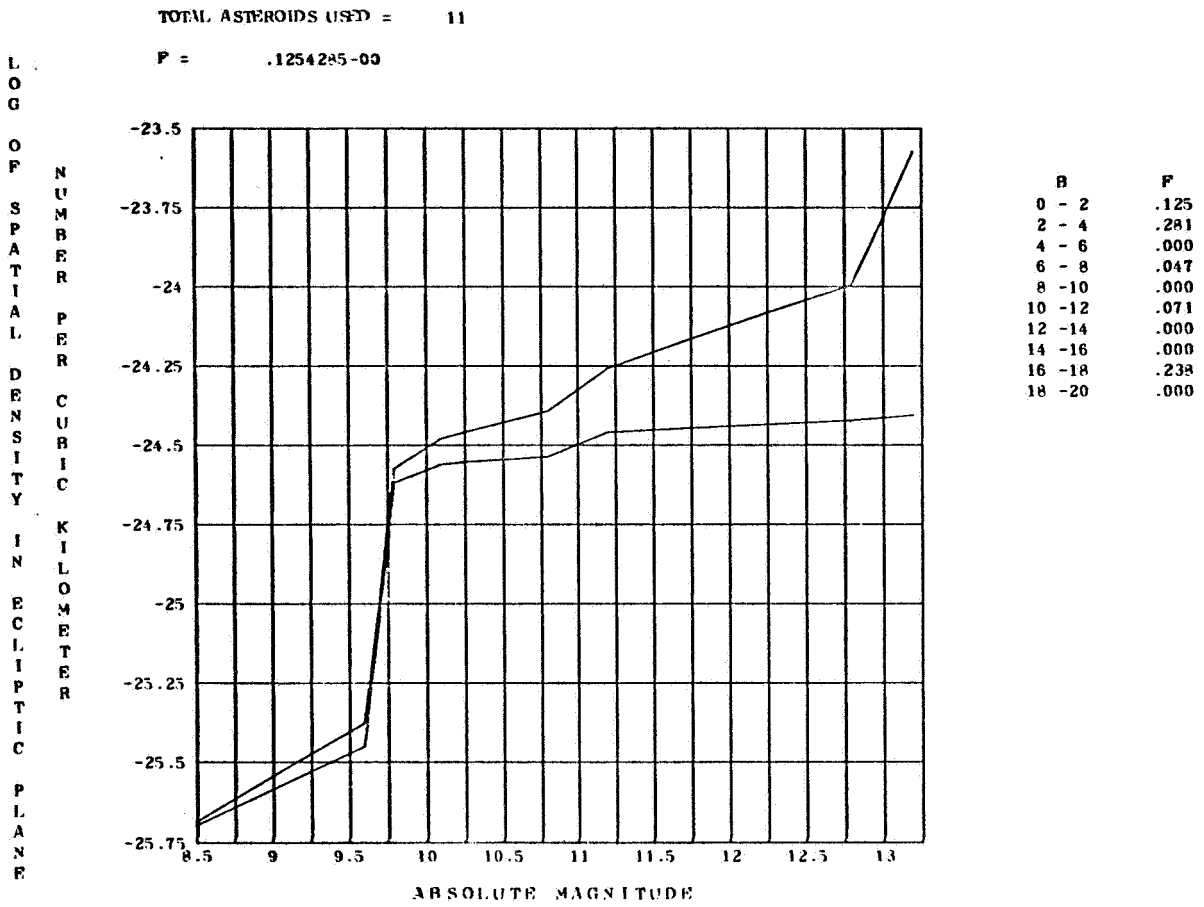


Figure 251. - Spatial density at R = 4.10 and at longitudes between 180.0 and 225.0.

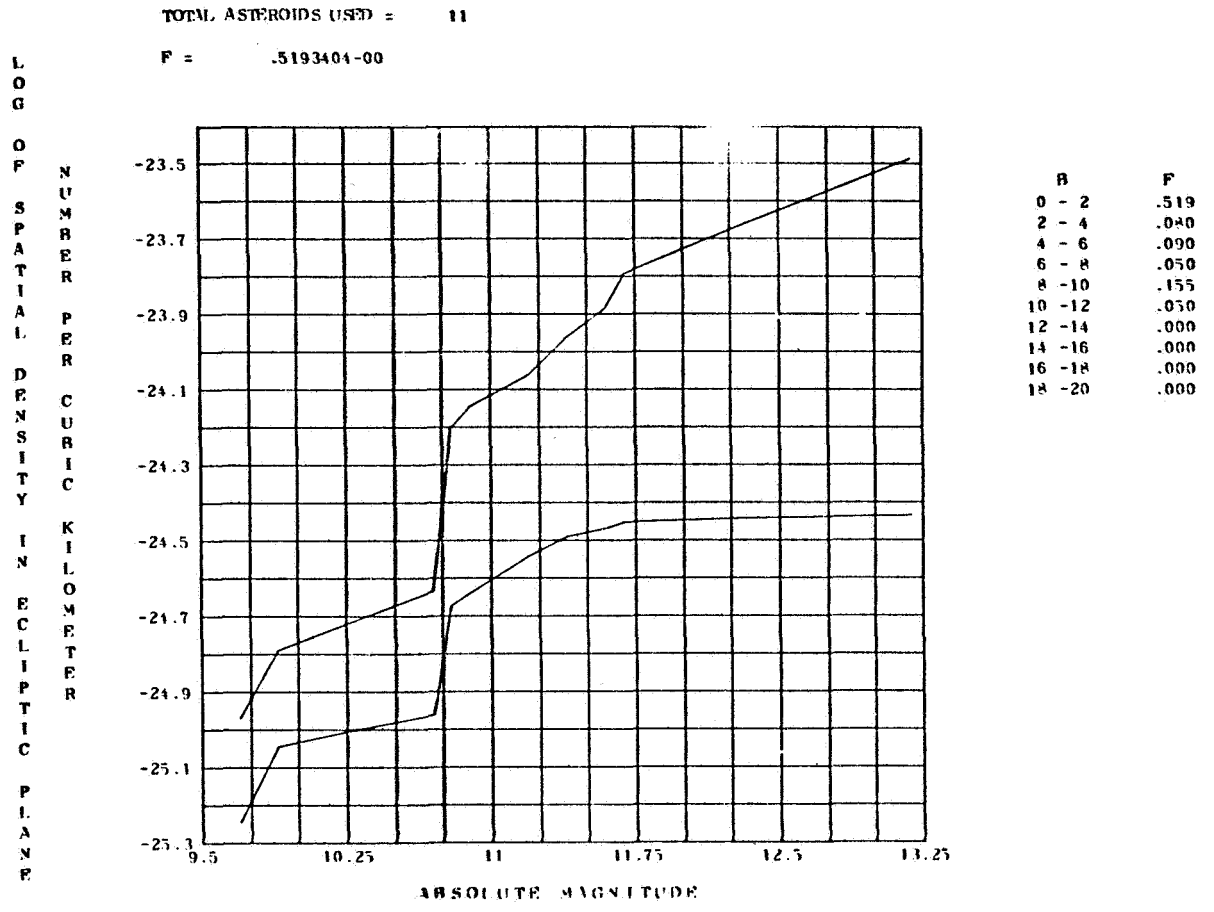


Figure 252. - Spatial density at R = 4.10 and at longitudes between 225.0 and 270.0.

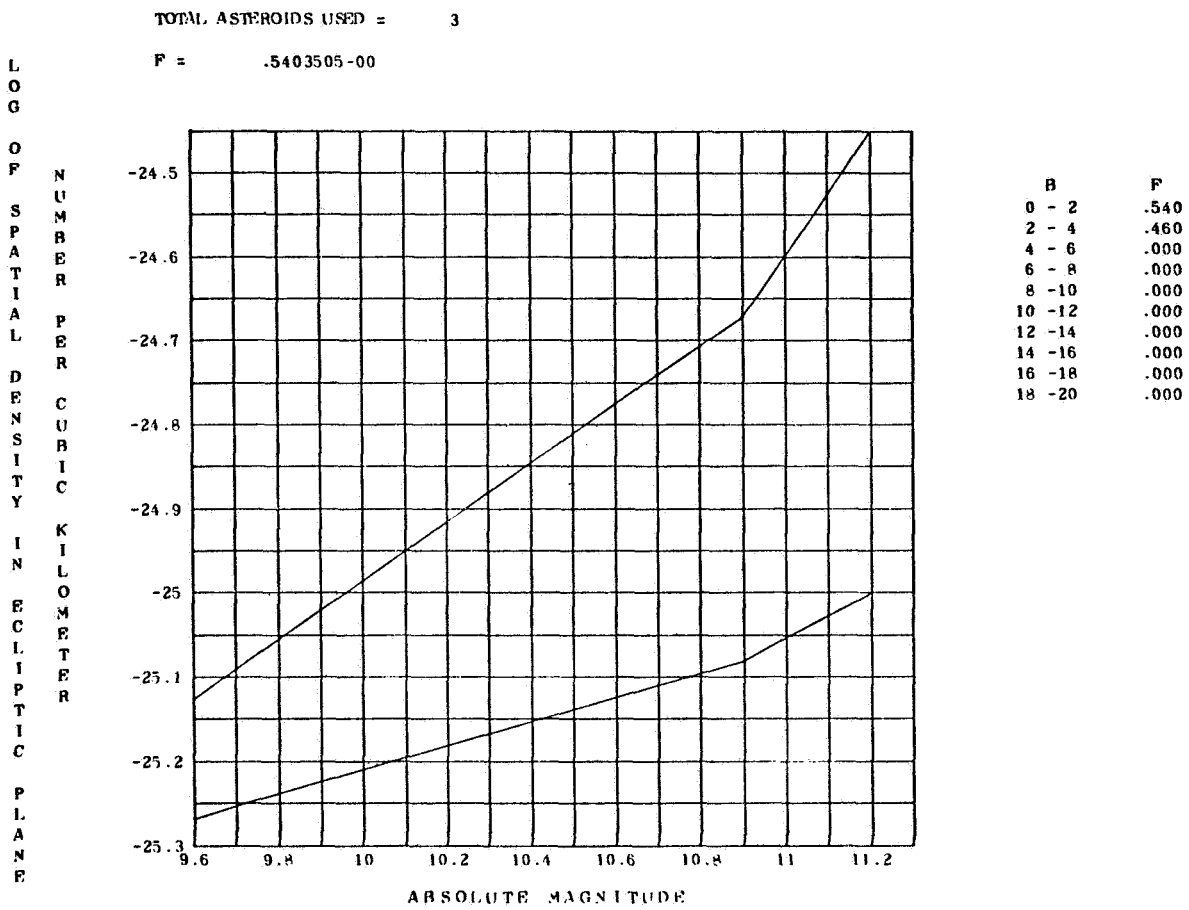


Figure 253. - Spatial density at R = 4.10 and at longitudes between 270.0 and 315.0.

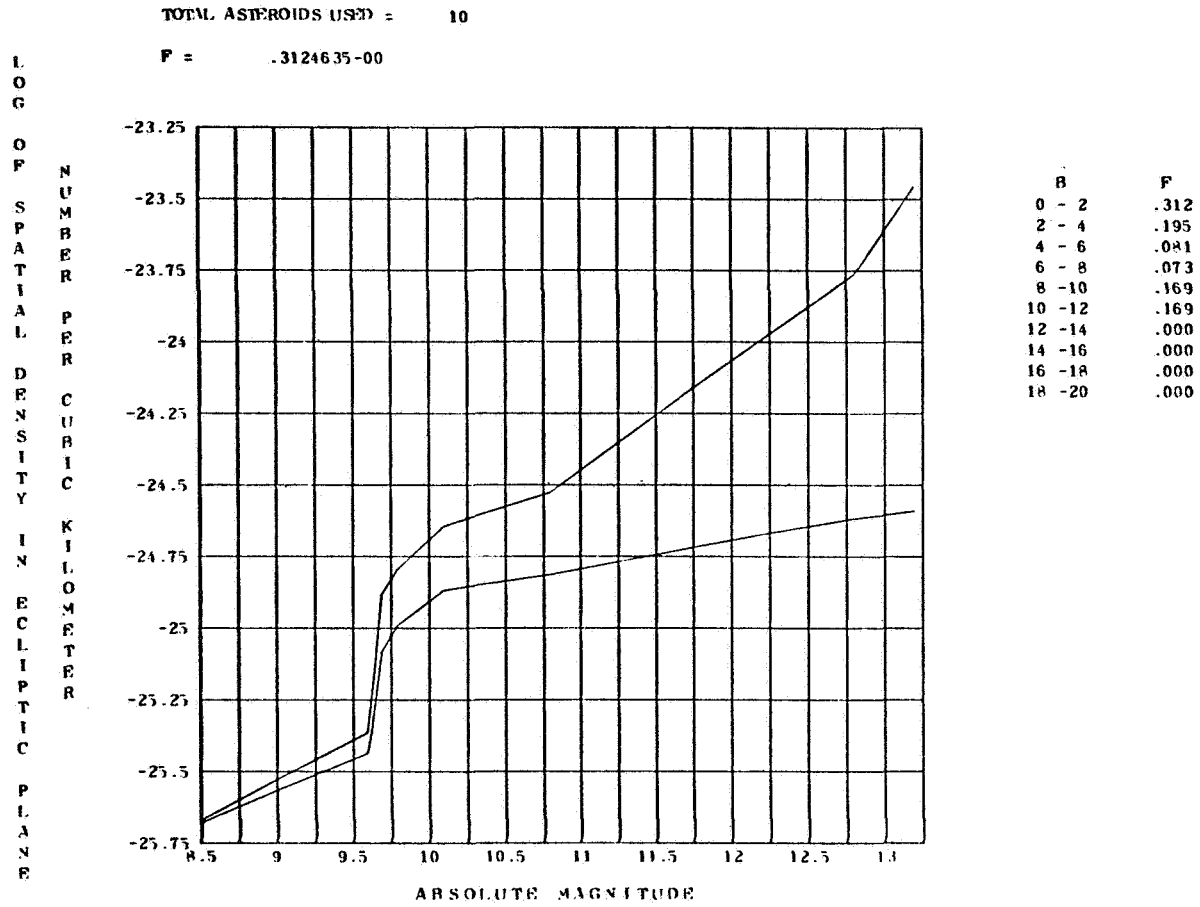


Figure 254. - Spatial density at R = 4.20 and at longitudes between 225.0 and 270.0.

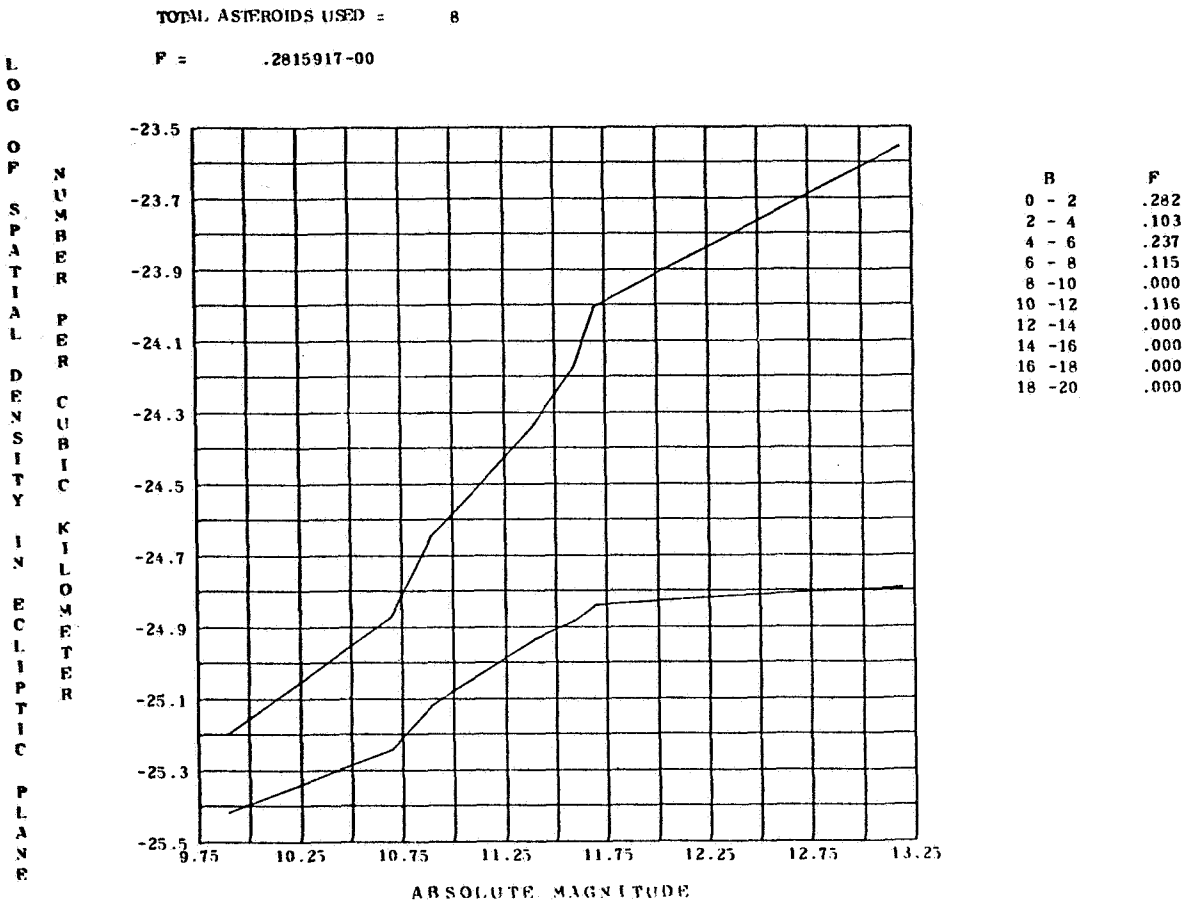


Figure 255. - Spatial density at R = 4.20 and at longitudes between 225.0 and 240.0.

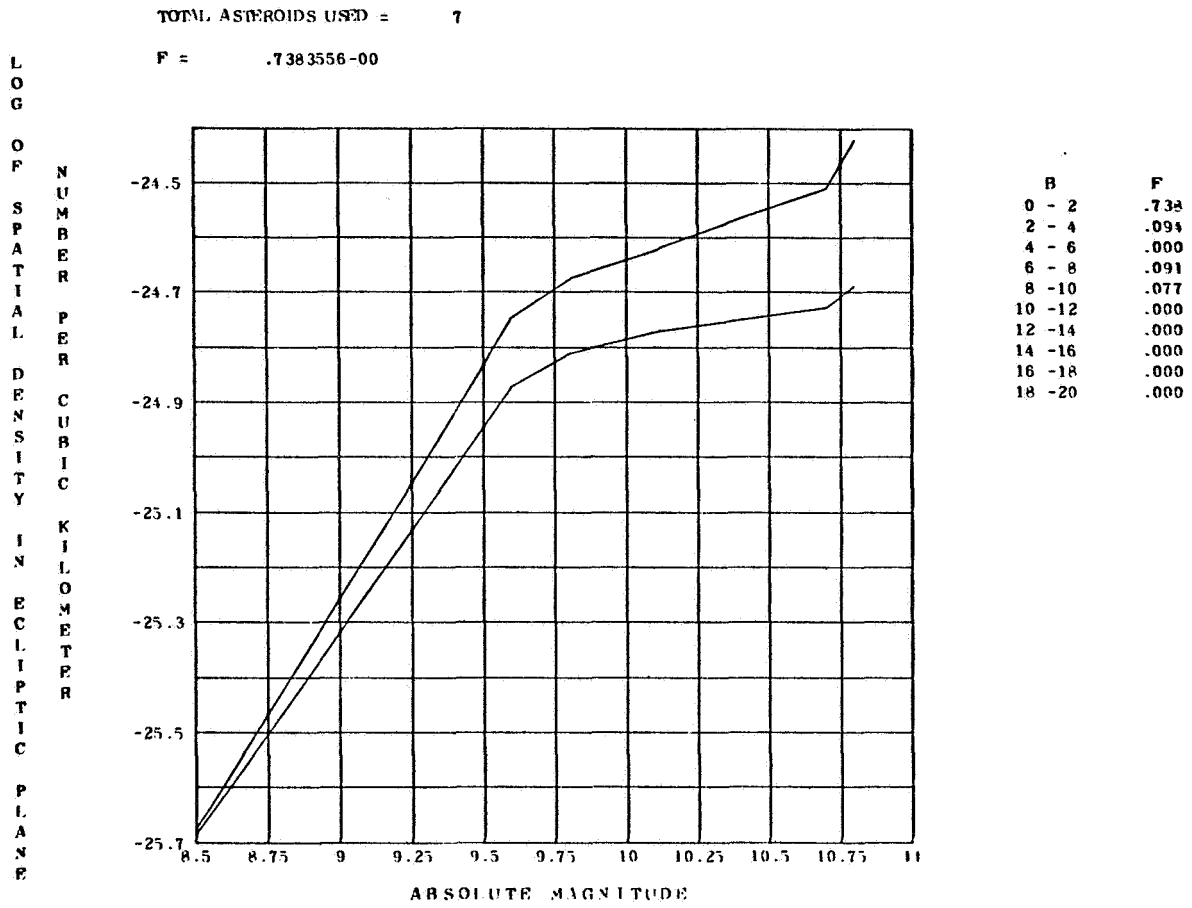


Figure 256. - Spatial density at R = 4.20 and at longitudes between 315.0 and 360.0.

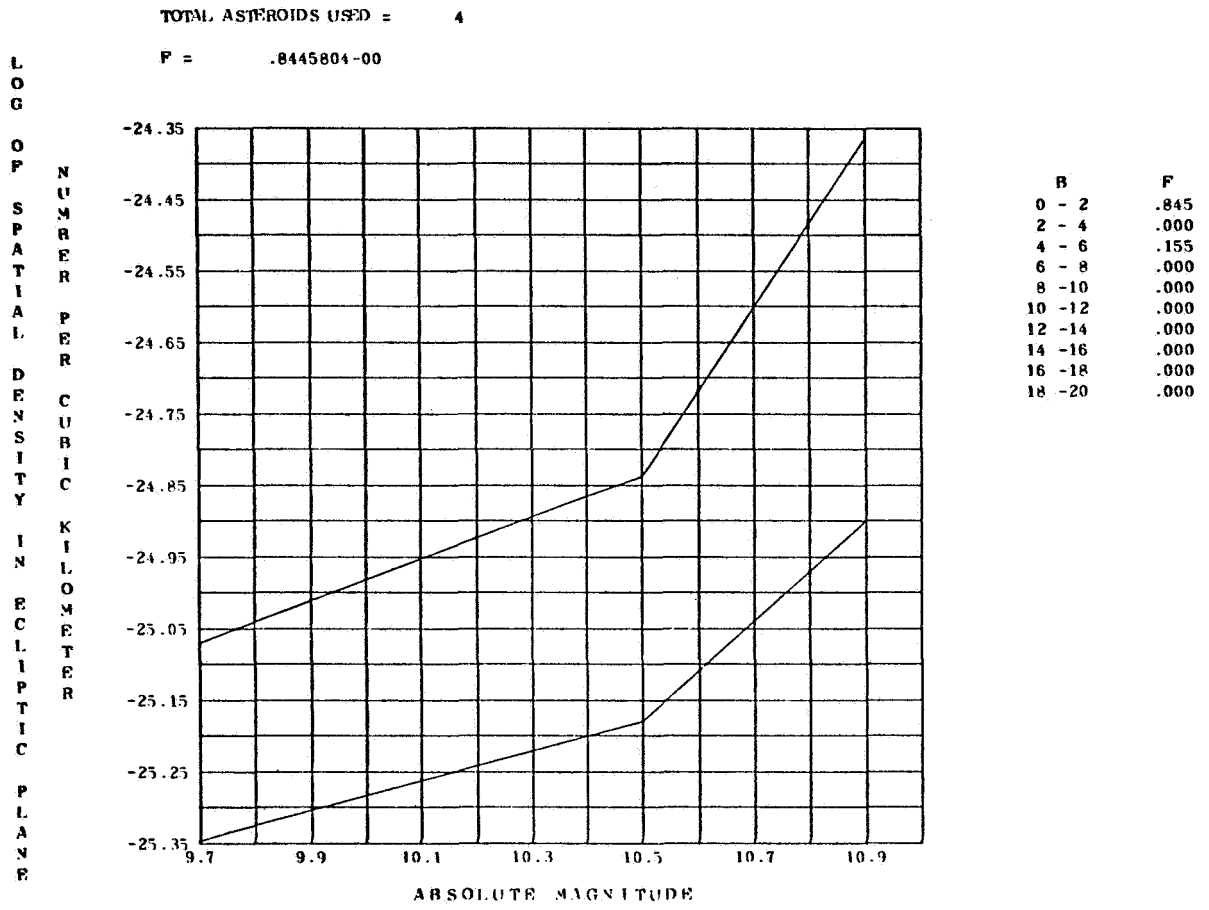


Figure 257. - Spatial density at R = 4.30 and at longitudes between 0 and 45.0.

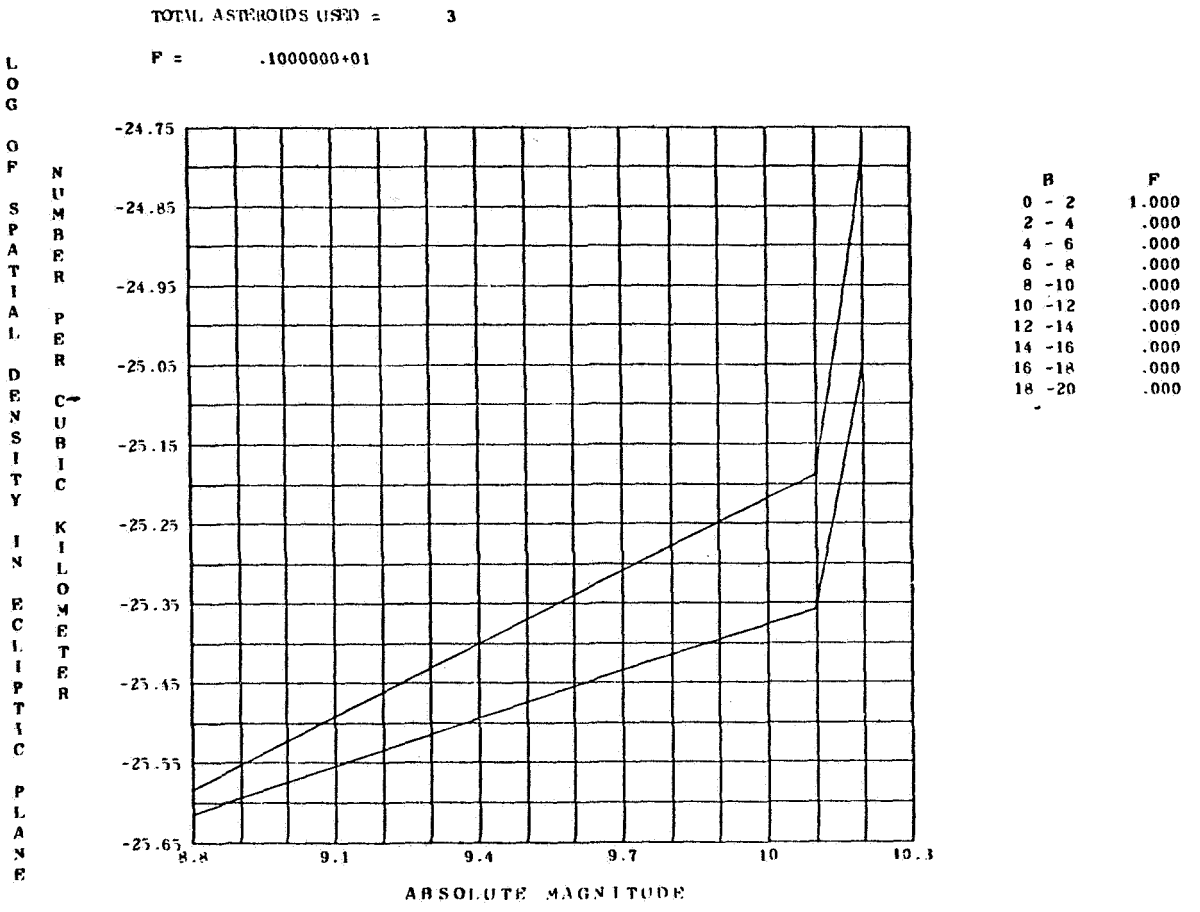


Figure 258. - Spatial density at R = 4.30 and at longitudes between 45.0 and 90.0.

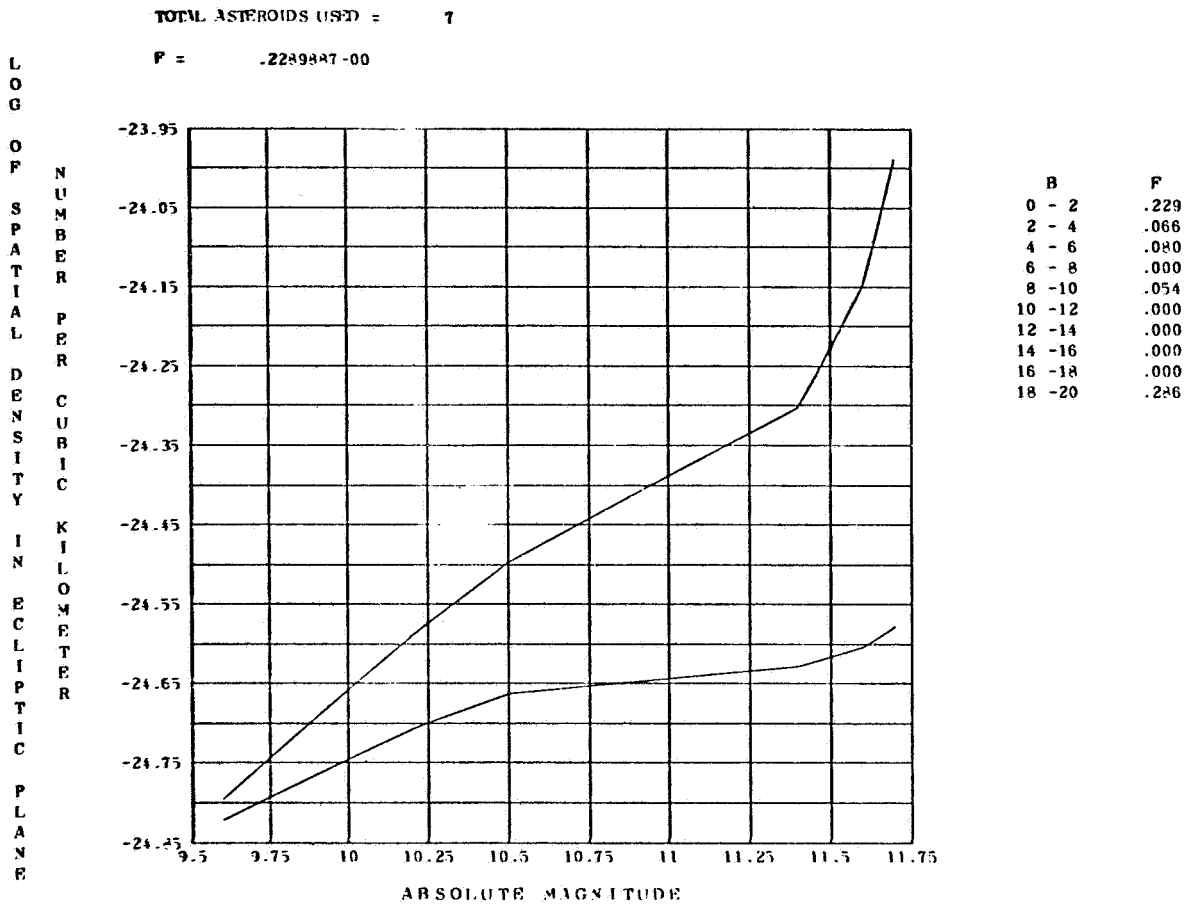


Figure 259. - Spatial density at R = 4.30 and at longitudes between 90.0 and 135.0.

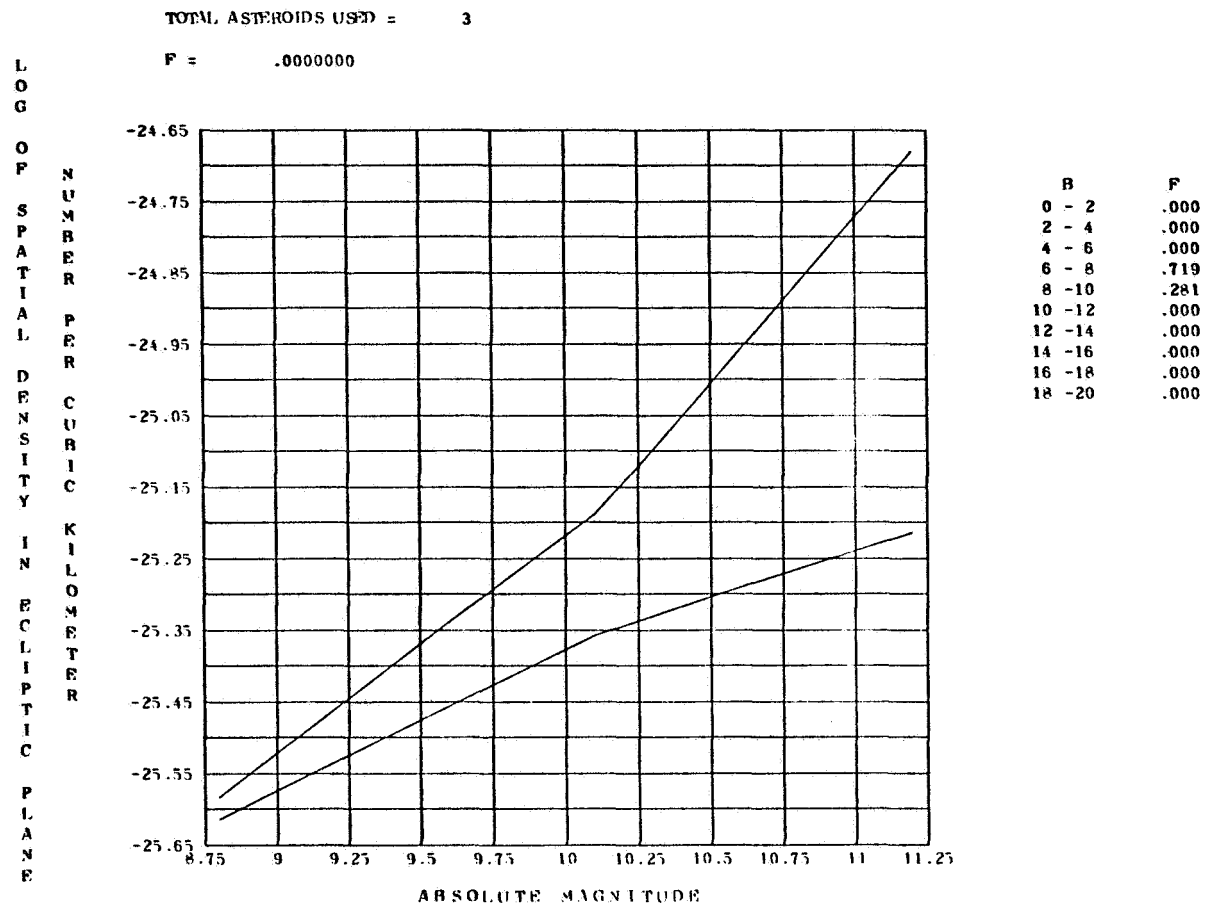


Figure 260. - Spatial density at R = 4.30 and at longitudes between 135.0 and 180.0.

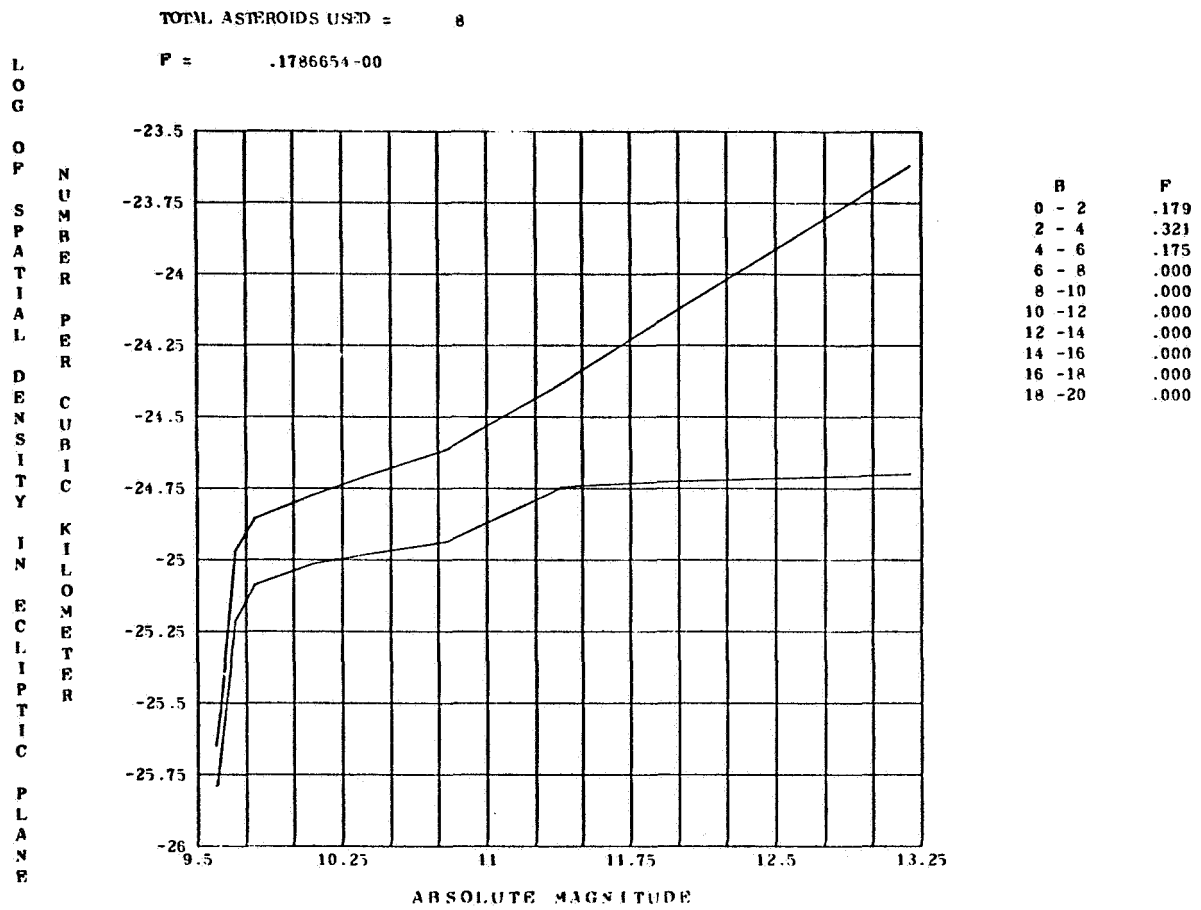


Figure 261. - Spatial density at R = 4.30 and at longitudes between 180.0 and 225.0.

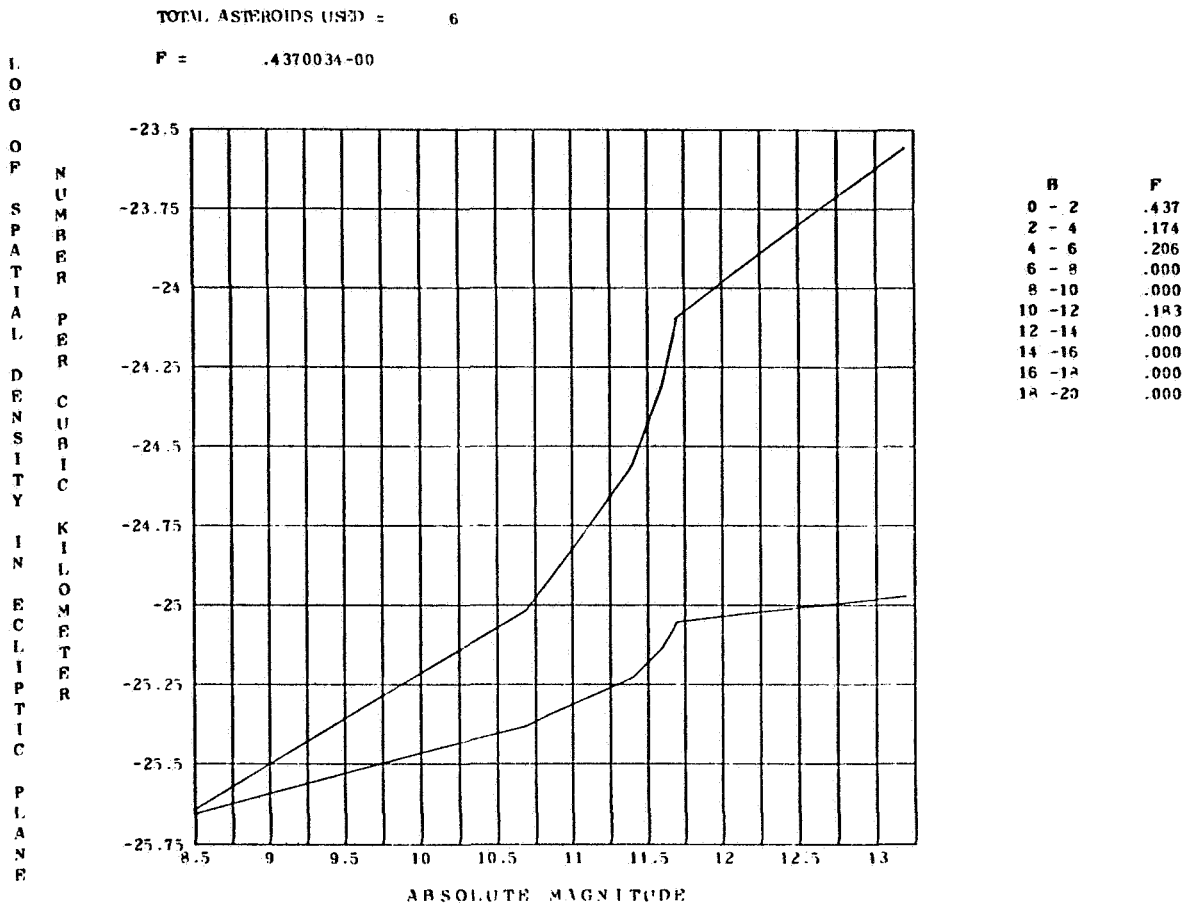


Figure 262. - Spatial density at R = 4.30 and at longitudes between 225.0 and 270.0.

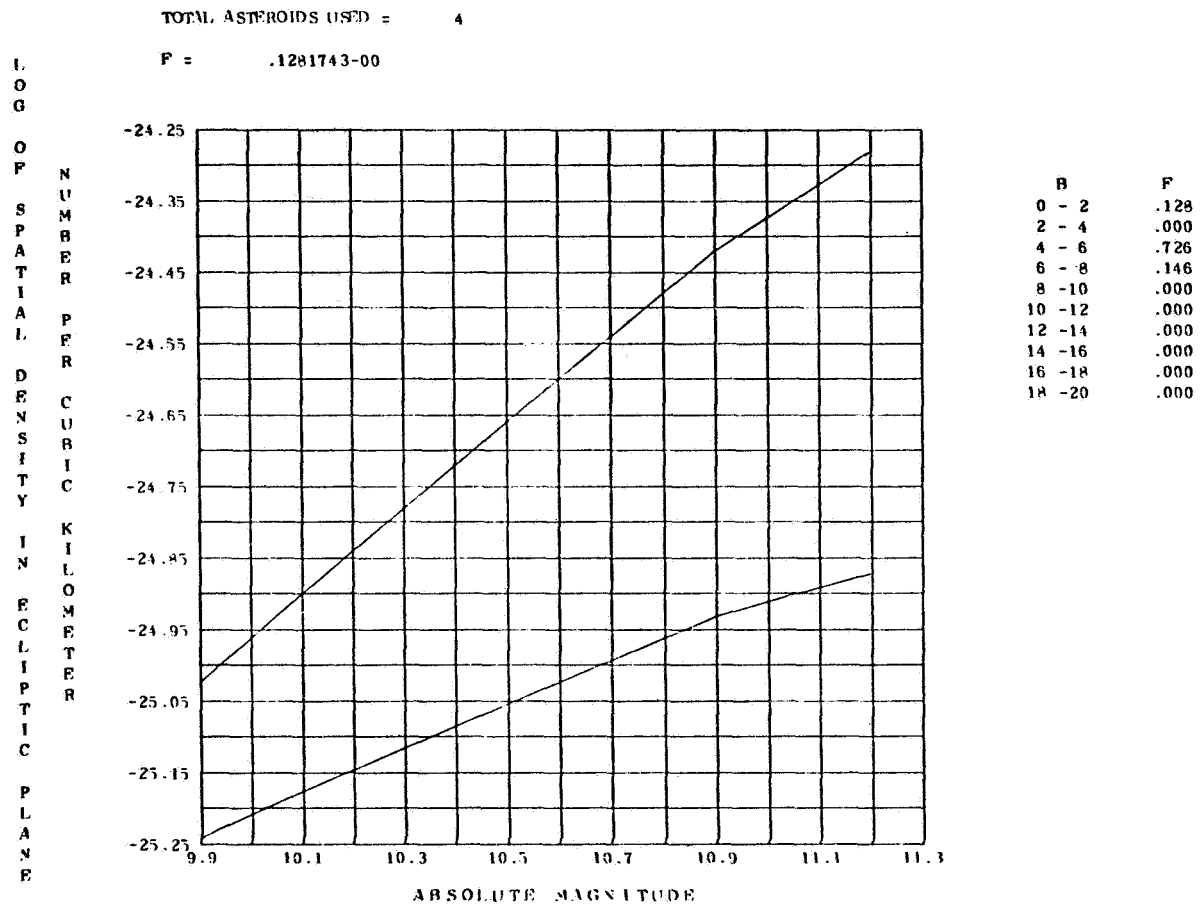


Figure 263. - Spatial density at R = 4.30 and at longitudes between 270.0 and 315.0.

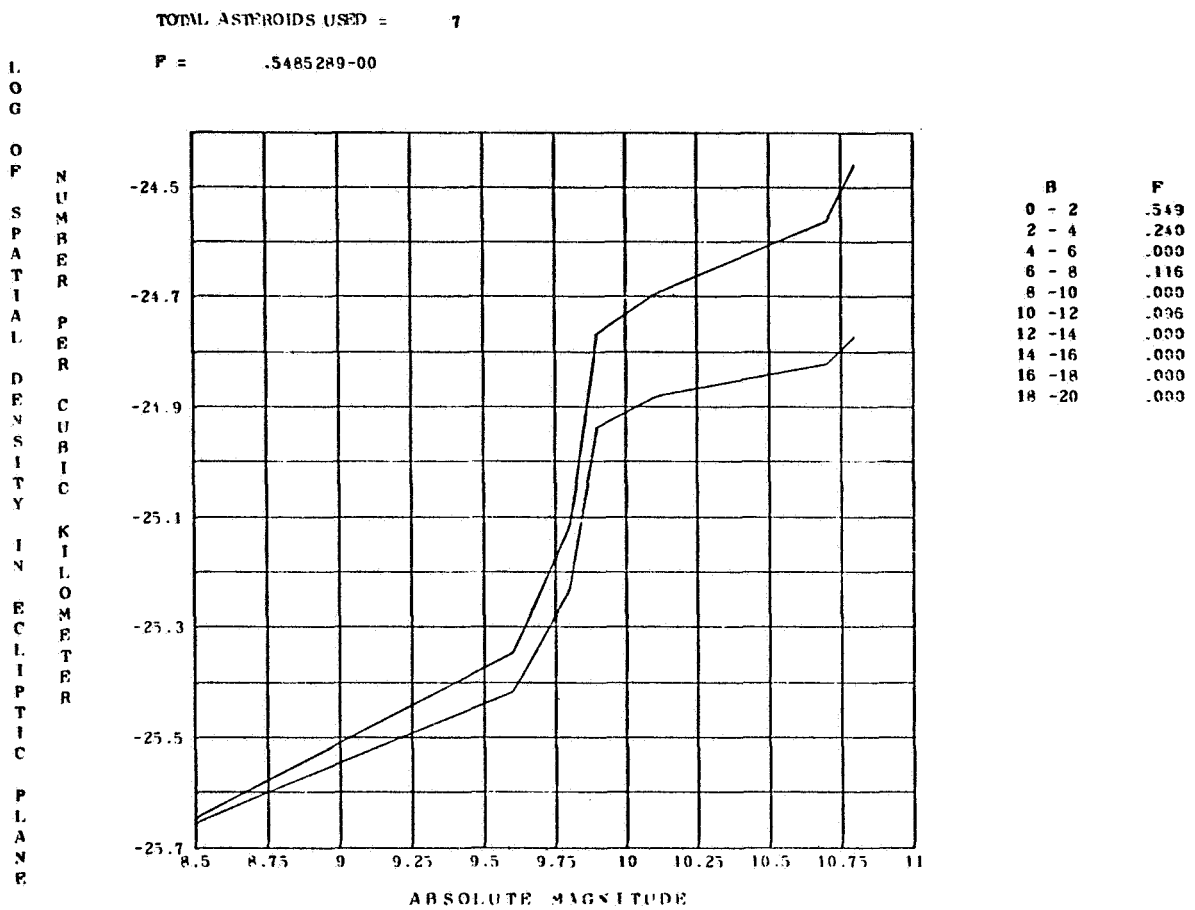


Figure 264. - Spatial density at R = 4.30 and at longitudes between 315.0 and 360.0.

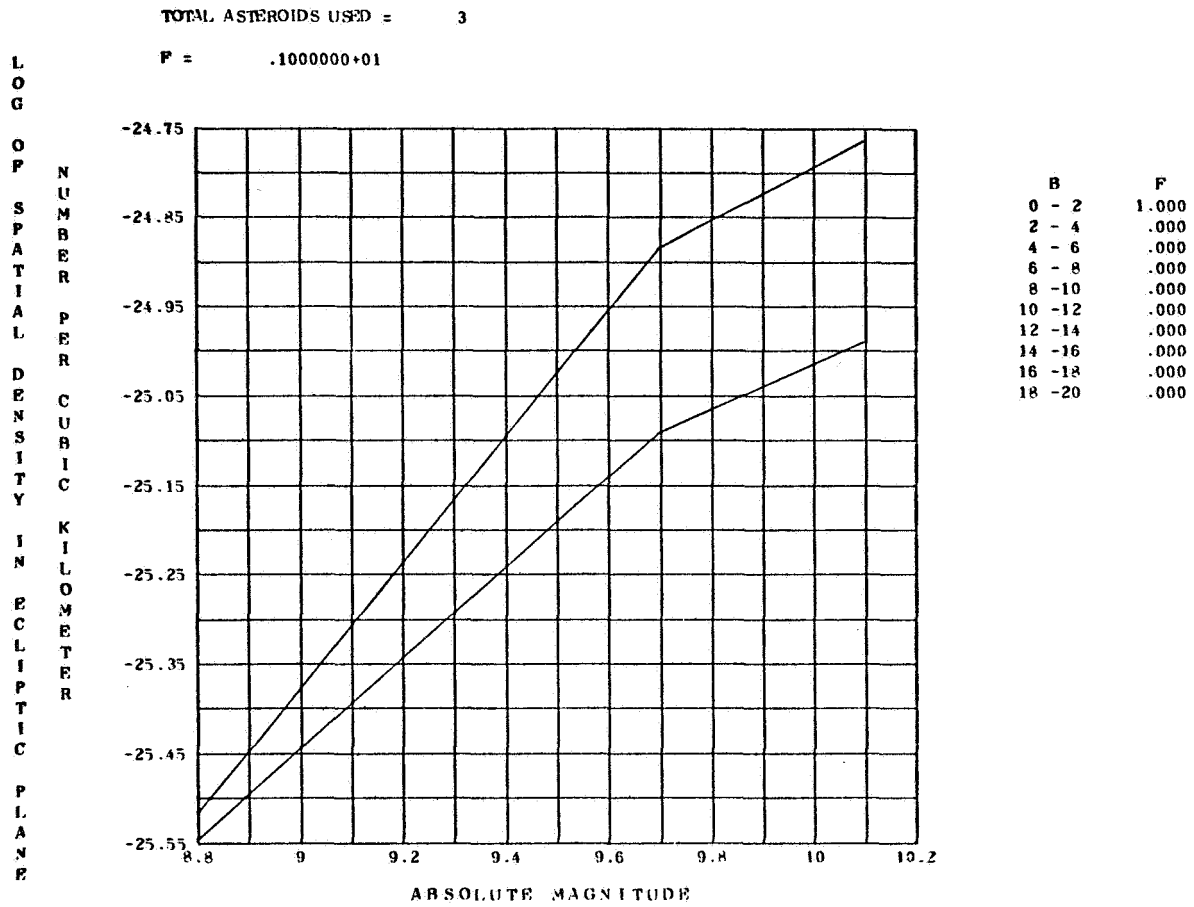


Figure 265. - Spatial density at R = 4.40 and at longitudes between 45.0 and 90.0.

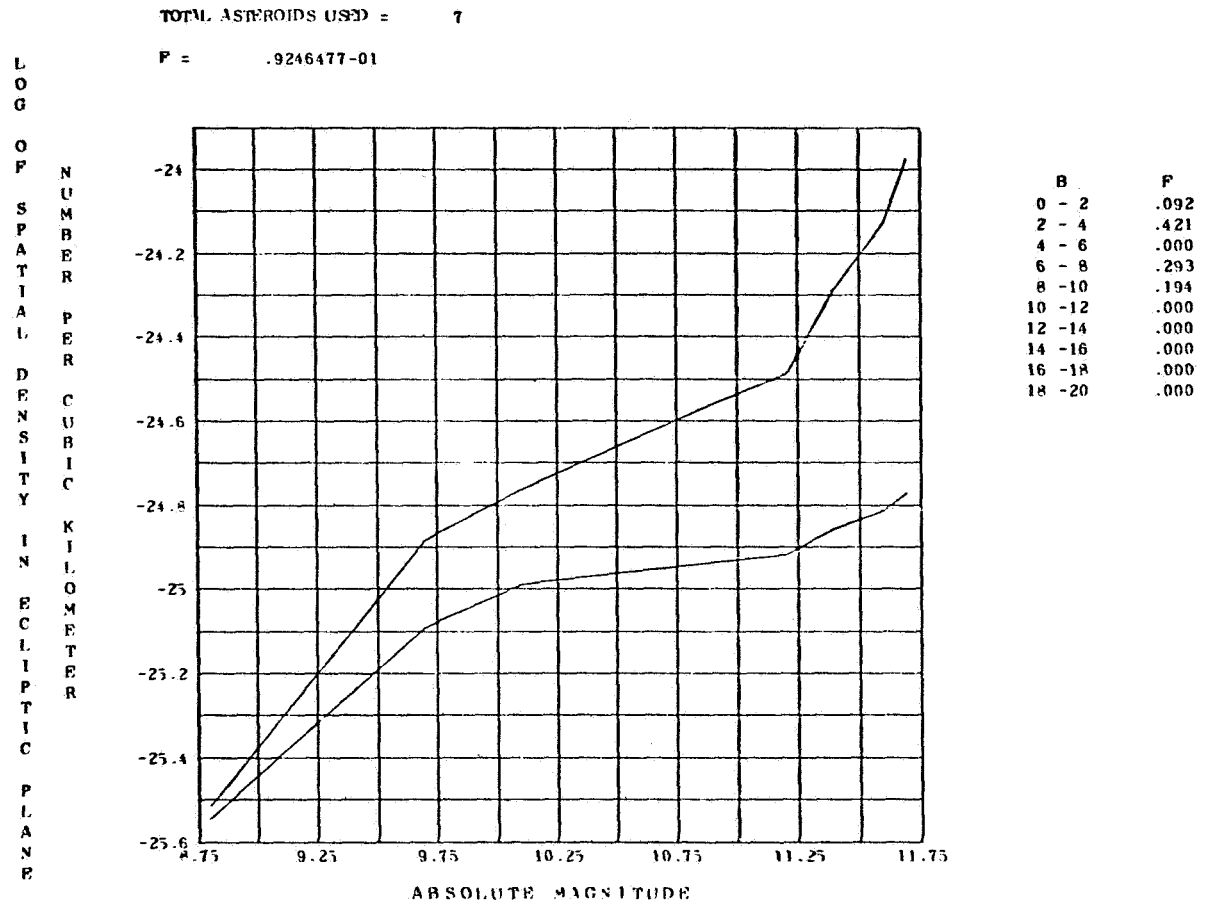


Figure 266. - Spatial density at R = 4.40 and at longitudes between 135.0 and 180.0.

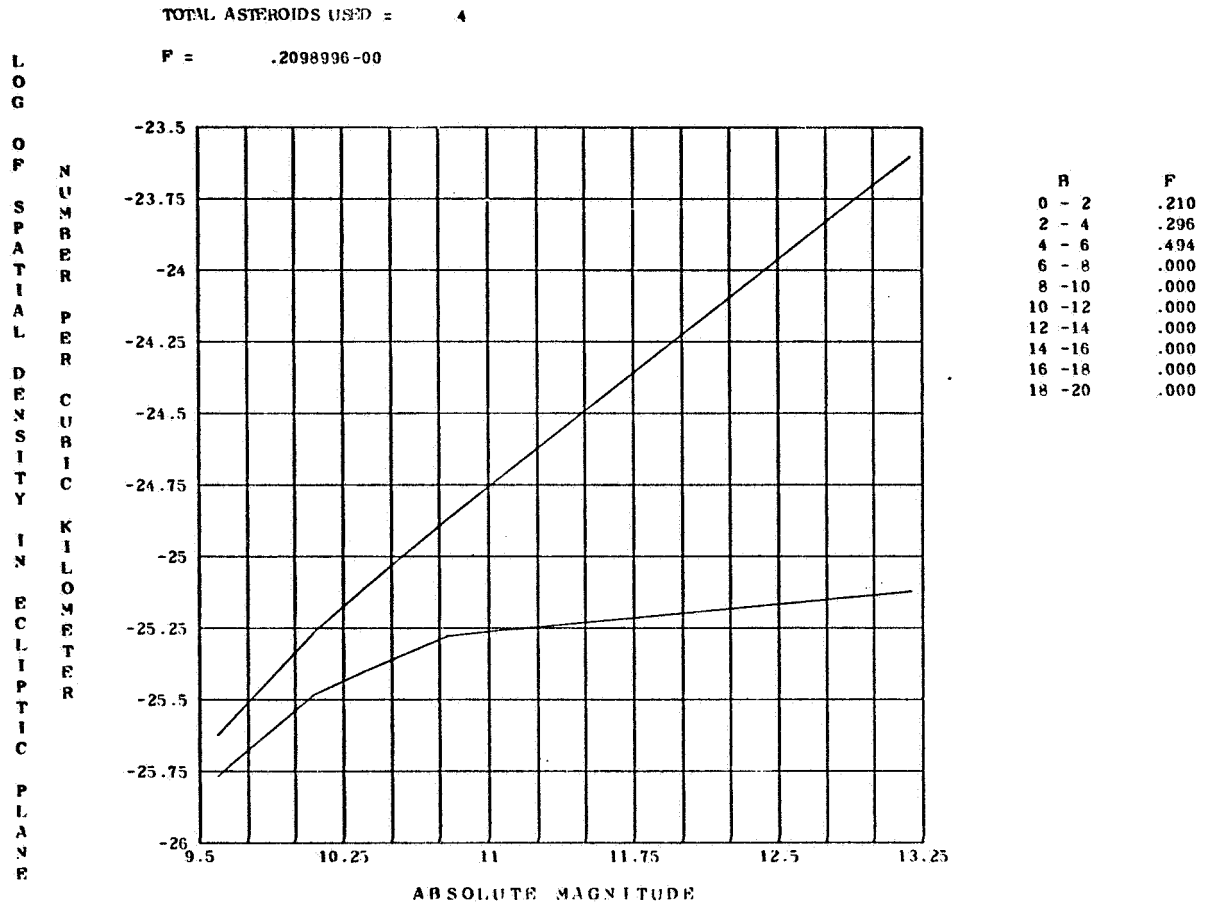


Figure 267. - Spatial density at R = 4.40 and at longitudes between 180.0 and 225.0.

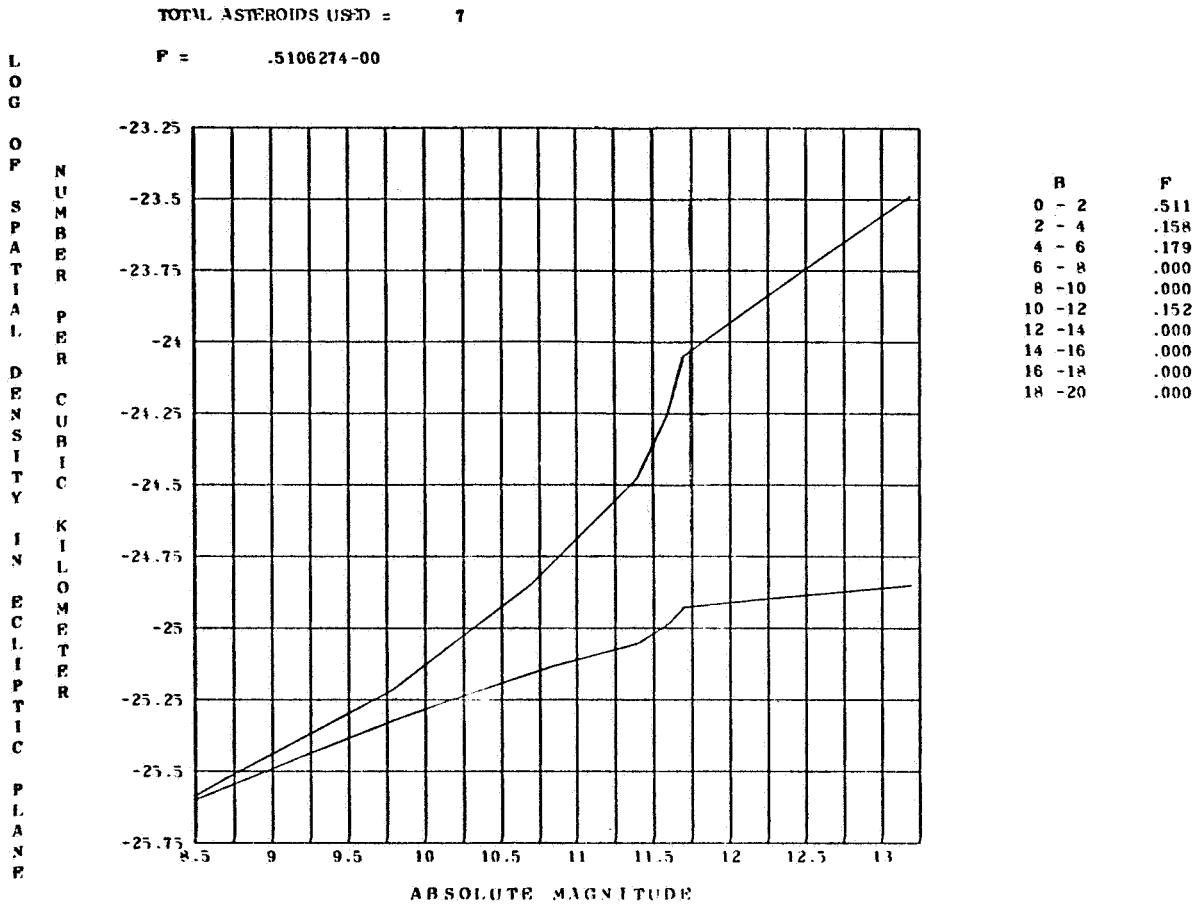


Figure 268. - Spatial density at R = 4.40 and at longitudes between 225.0 and 270.0.

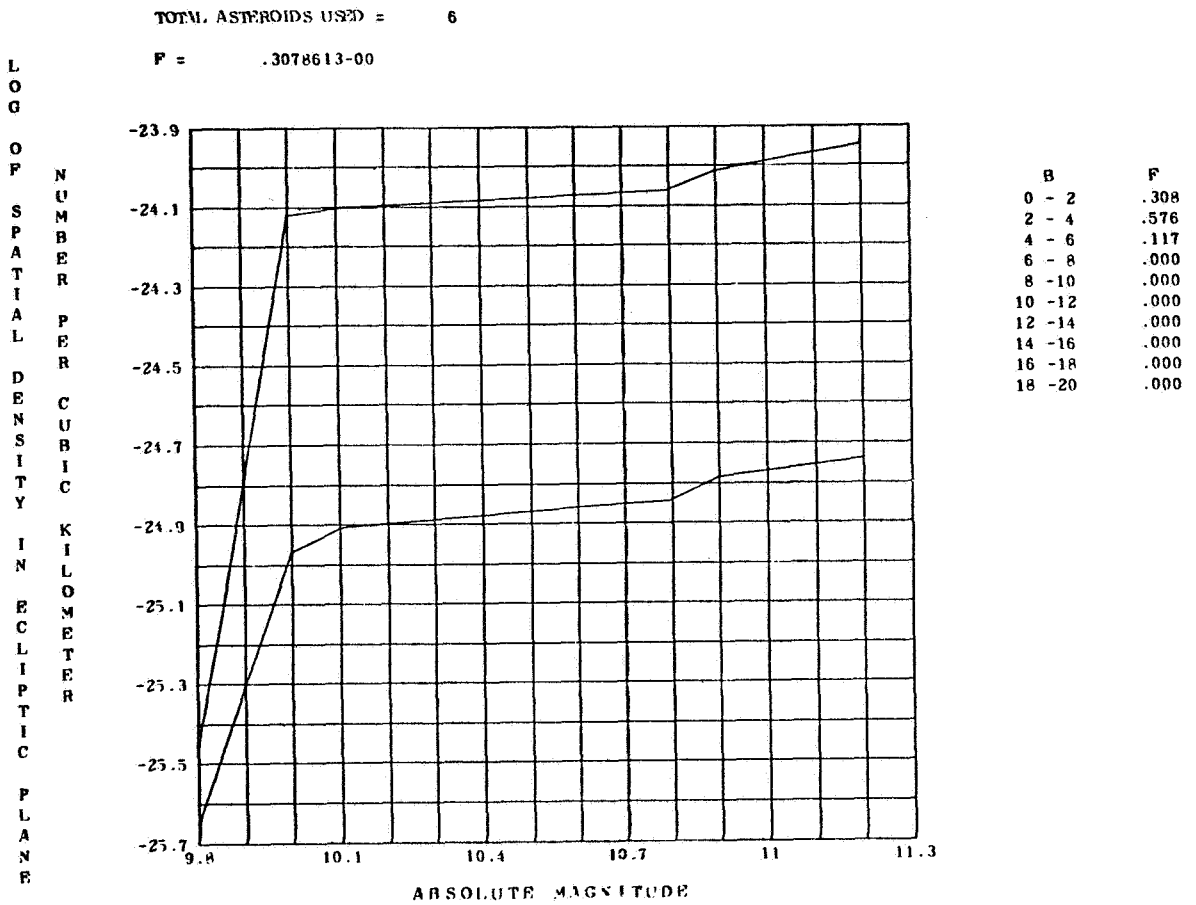


Figure 269. - Spatial density at R = 4.40 and at longitudes between 270.0 and 315.0.

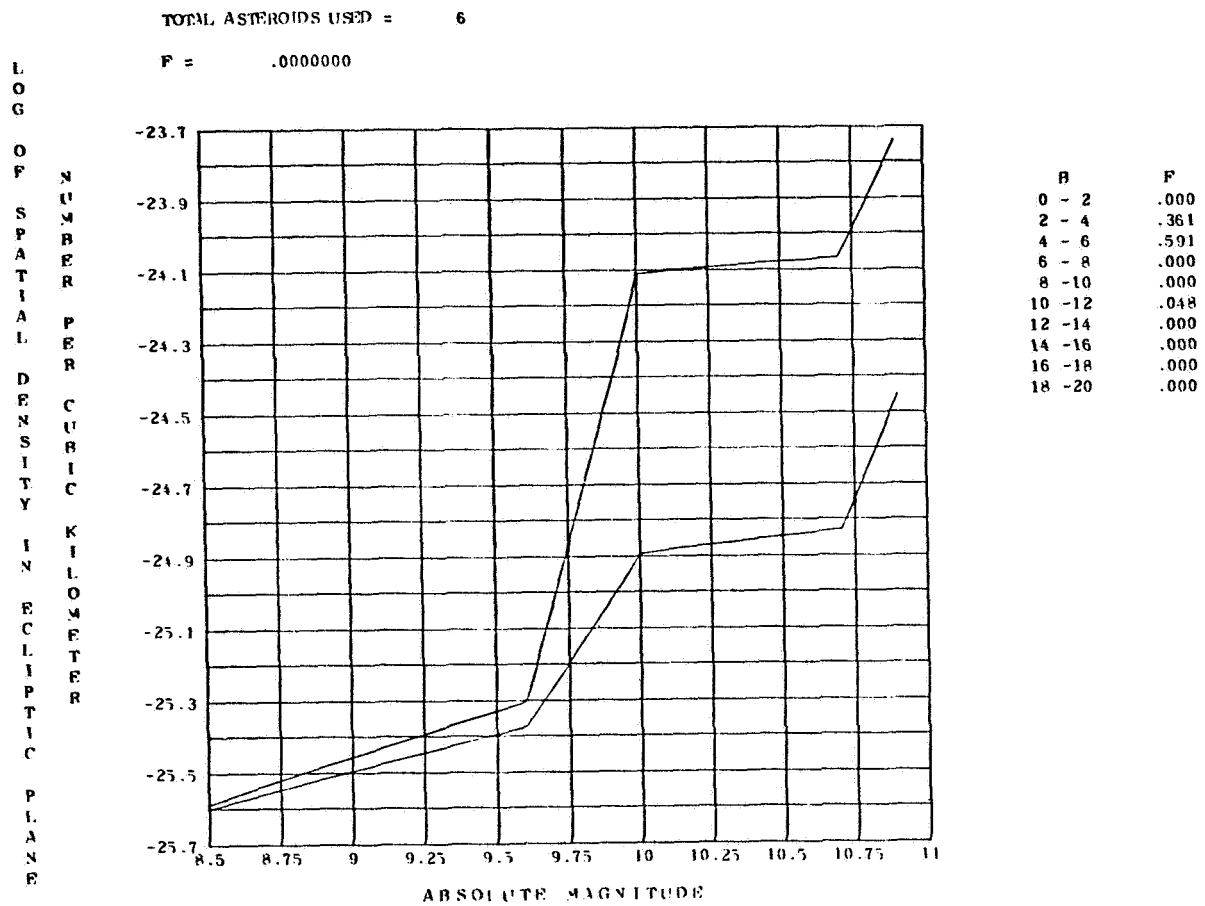


Figure 270. - Spatial density at R = 4.40 and at longitudes between 315.0 and 360.0.

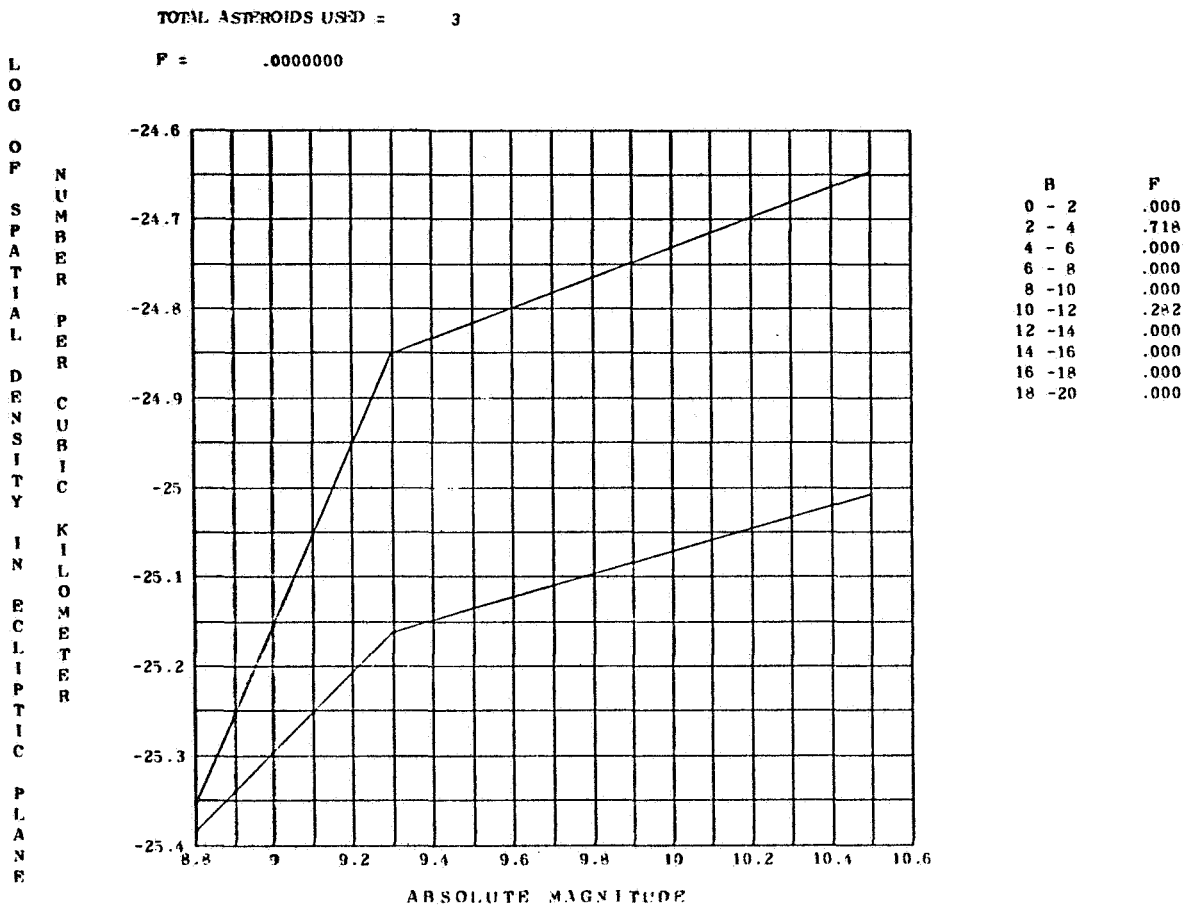


Figure 271. - Spatial density at R = 4.50 and at longitudes between 45.0 and 90.0.

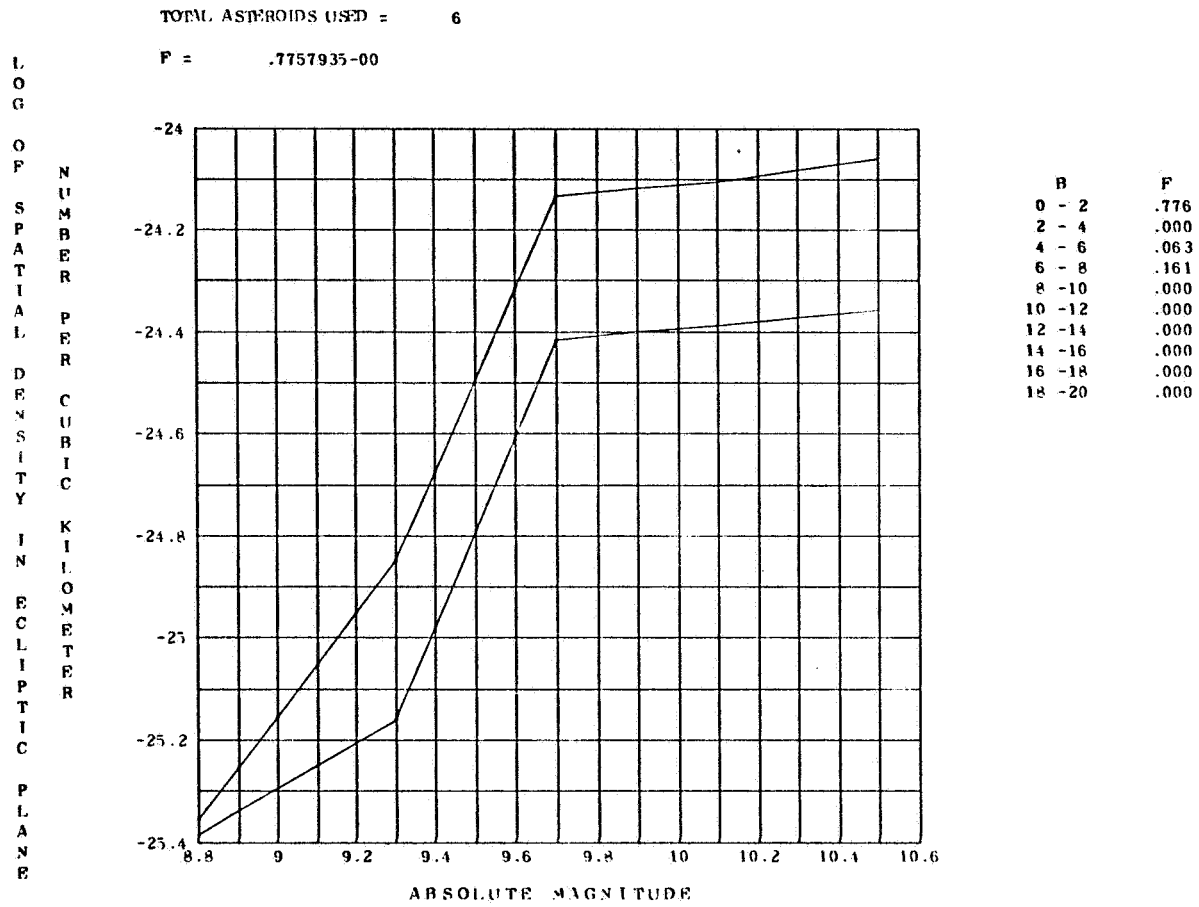


Figure 272. - Spatial density at R = 4.50 and at longitudes between 90.0 and 135.0.

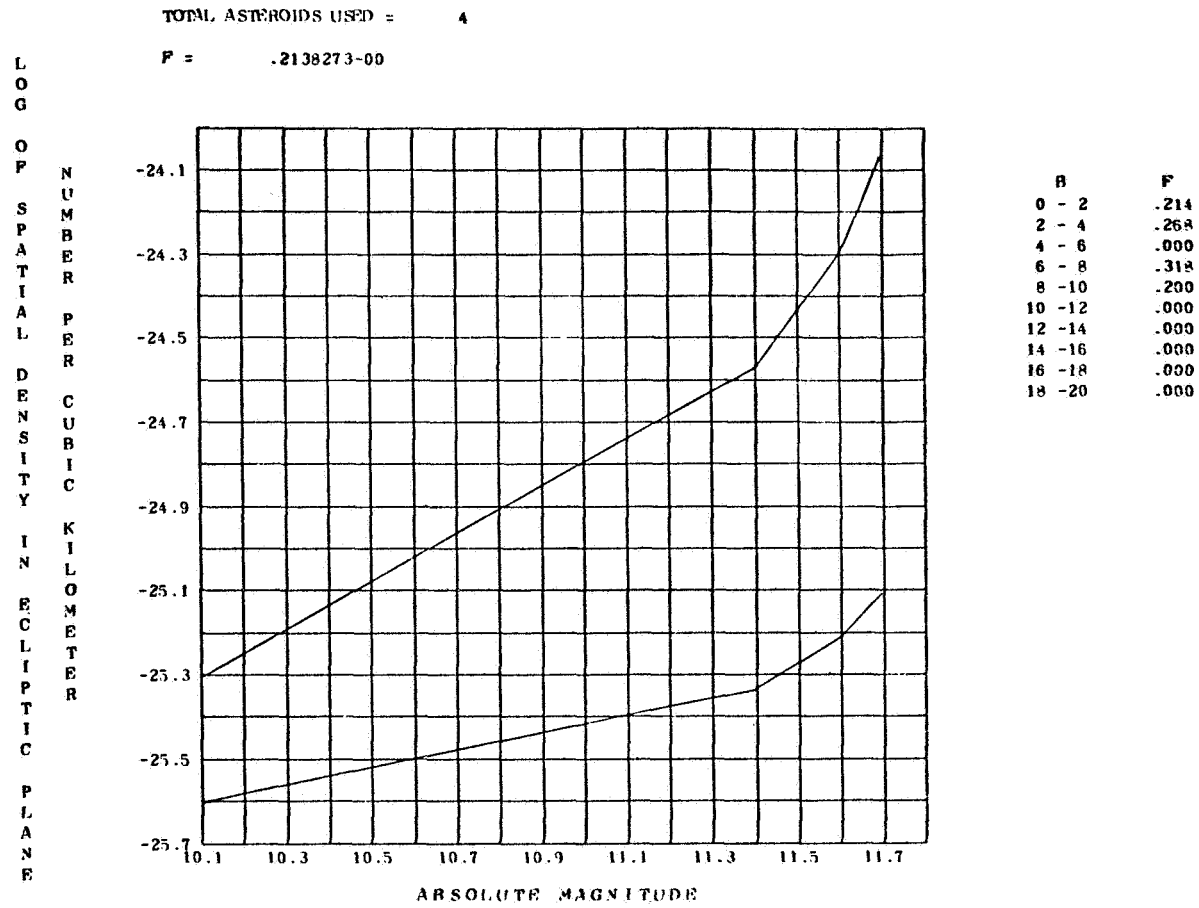


Figure 273. - Spatial density at R = 4.50 and at longitudes between 135.0 and 180.0.

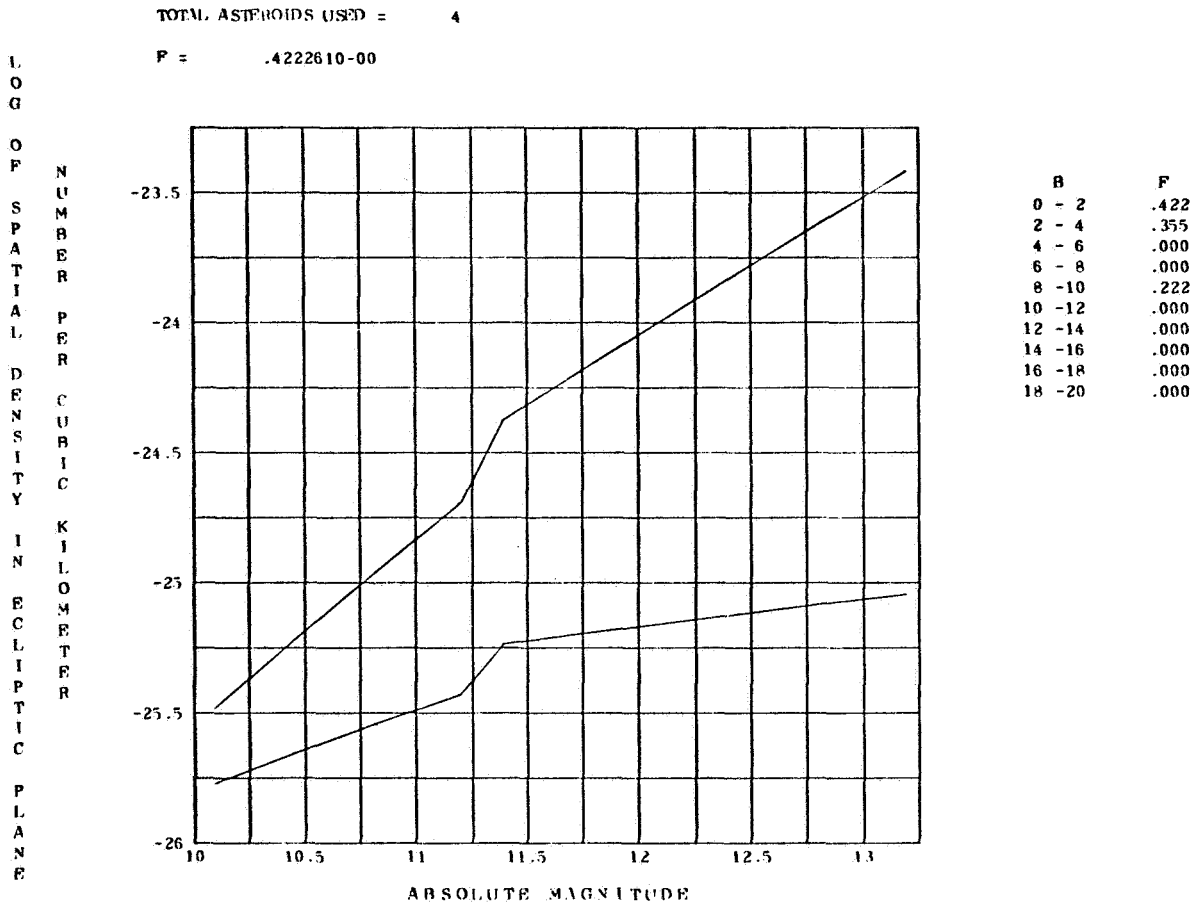


Figure 274. - Spatial density at R = 4.50 and at longitudes between 180.0 and 225.0.

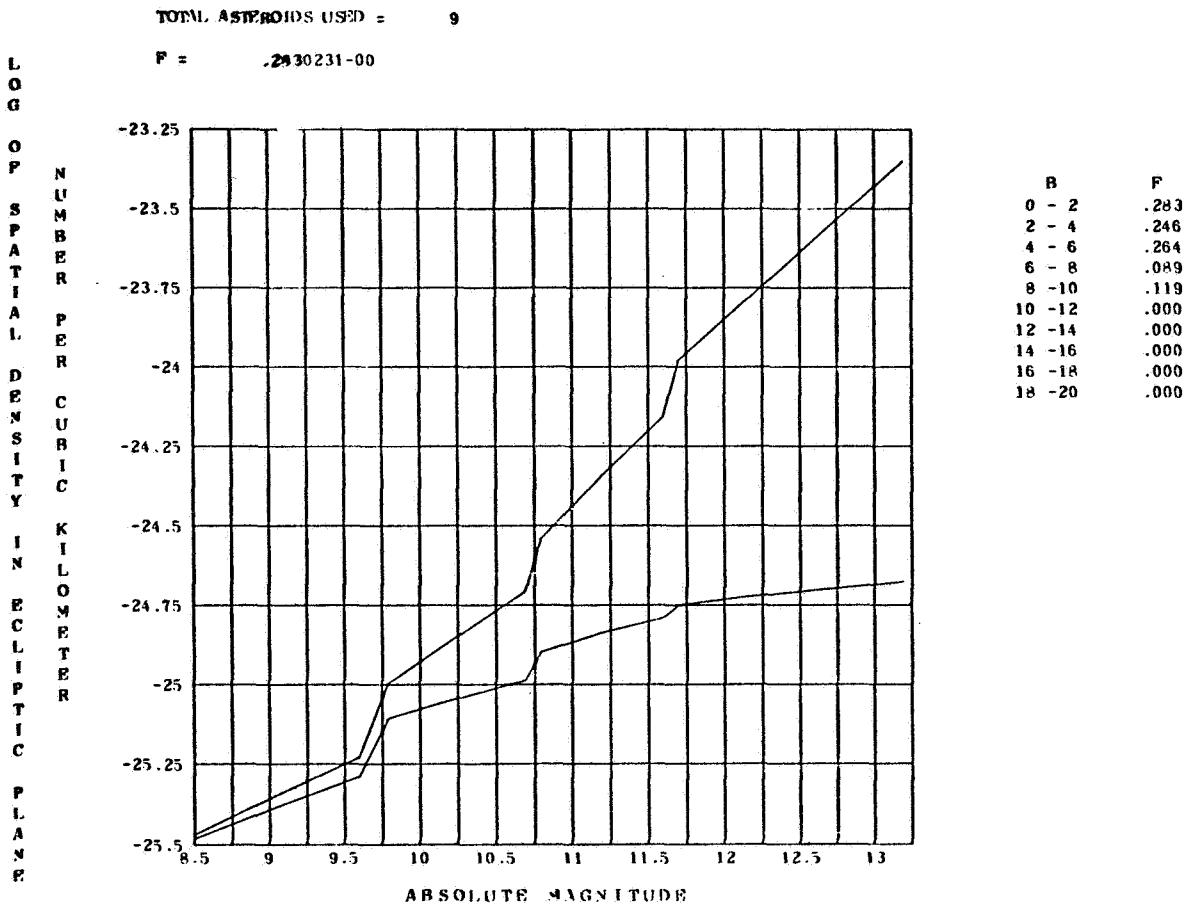


Figure 275. - Spatial density at R = 4.50 and at longitudes between 225.0 and 270.0.

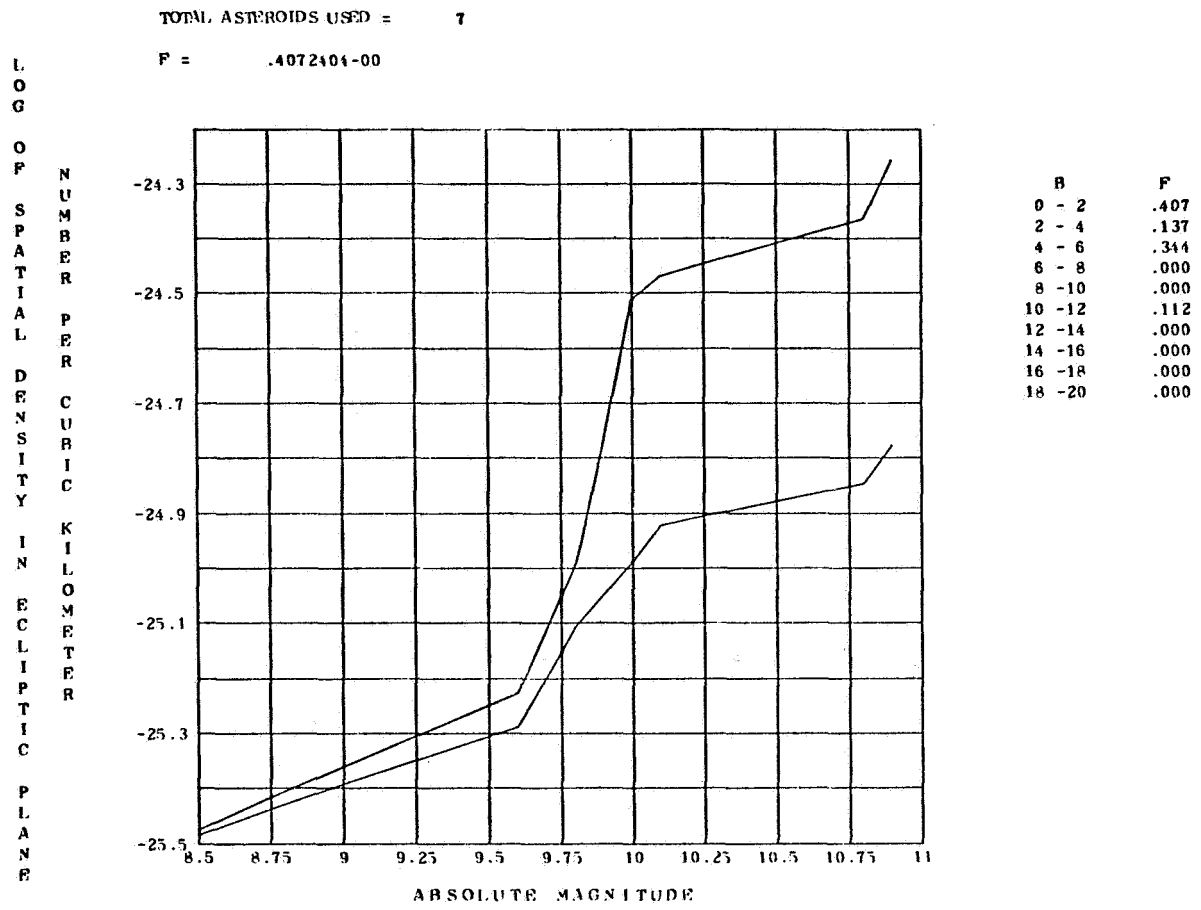


Figure 276. - Spatial density at R = 4.50 and at longitudes between 270.0 and 315.0.

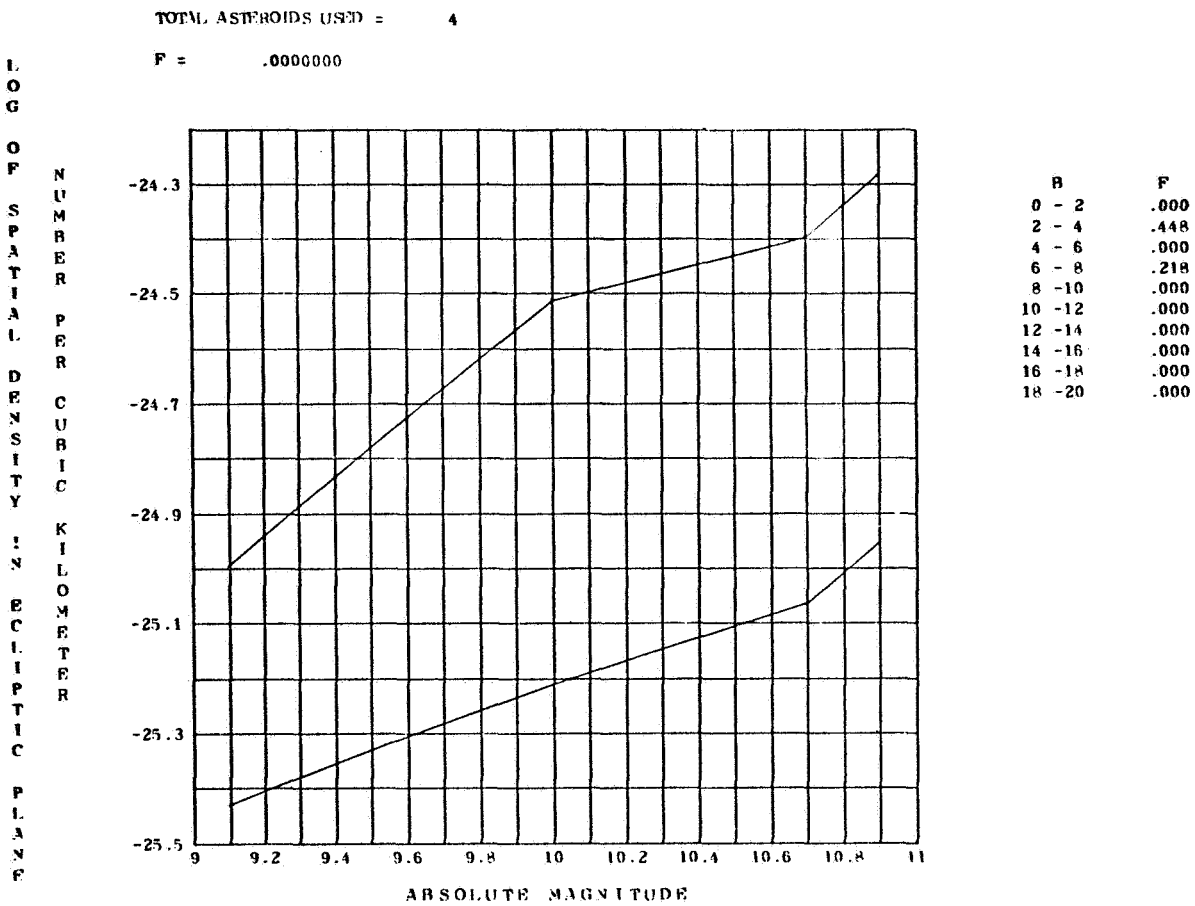


Figure 277. - Spatial density at R = 4.50 and at longitudes between 315.0 and 360.0.