PRAIRIE VIEW A. AND M. COLLEGE

SCHOOL OF ENGINEERING

PROGRESS REPORT OF RESEARCH

SAFETY ANALYSIS OF STRUCTURES SUBJECTED TO RANDOM VIBRATION

by

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Prairie View A. & M. College

School of Engineering

PROGRESS REPORT OF RESEARCH

"Safety Analysis of Structures Subjected to Randam Vibration"

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Amount: $15000/

Grant No: NGR 44-033-002

Principal Investigator: Dr. I. Kasiraj

Introduction

The project was authorized for operation in October 1969. Two students namely Mr. David C. Mosby (Senior) and Mr. James Marshall (Junior) were appointed as research assistants to this project. Another senior student Miss Mary L. Anderson was also attached later on to this project. When a structure is subjected to Random Vibration,many reversals of loadings are likely to be introduced and this may cause fatigue failure in the structure. For studying this fatigue damage, it is necessary to study the variation of strain at the critical section of the structure. Hence first step was taken to investigate the possible hysteresis loop of strain at a critical section and response of a structural system.

Procurement of minor equipments:

A portable Strain Indicator, Strain gage application kit, a tool box and a storage cabinet for the laboratory were procured under this grant.

Research conducted by the Student assistants:

Miss Mary L. Anderson was given opportunity to acquire experience of fixing strain gages in different directions on a steel plate. She practised by fixing many strain gages and carried out experiments on them. A copy of her investigative paper entitled "Measurement of Strain under Reversal of Loadings" which was carried out under this research is enclosed for reference. She has now graduated from the college in January 1970. Mr. David C. Mosby is making theoretical study on "The Application of Matrix in Structural Dynamics." So far, he has learnt to invert a matrix. The computer facility of IBM 1401 now available at Prairie View was fully utilised by him. Mr. James Marshall studied the response of a single-degree-freedom system and was also running computer program in IBM 1401.

Research conducted by the Principal Investigator:

The possibility of making use of the Analog Computer SD 3300 which is now available at Prairie View, for studying vibration problems under this project was investigated. It is now proposed to utilise this facility later on if this research can be extended for another year.

Theoretical study was made regarding the variations of deflection maximum strain in a rectangular section of a column. The relation between the extreme fibre strain e_x, which is a function of the variable x, and the response (deflection) y is given by the equation:

$$\frac{d^2y/dx^2}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{e_x}{d/2},$$

where d is the depth of cross section of the column and x is the variable representing the height of the column. If the deflection y is assumed to be small relative to the height of the column the above equation can be simplified to the one given below:

$$\frac{d^2y}{dx^2} \simeq \frac{e_x}{d/2} .$$

Considering a cantilever column fixed at A as shown in figure 1, the maximum deflection will occur at top when a side load P is applied.

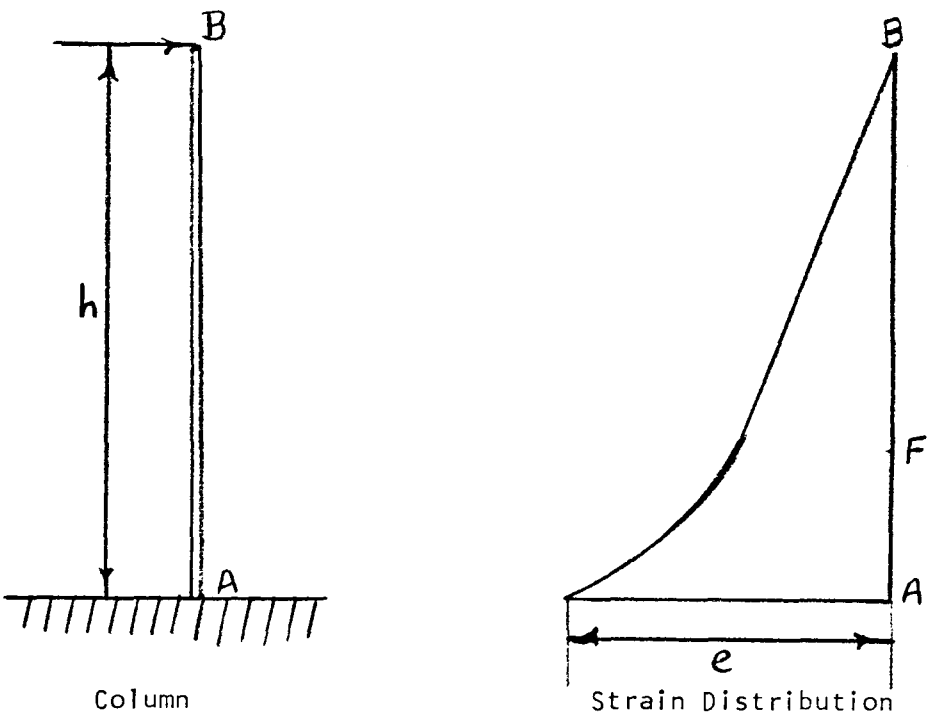


Figure 1

When the load is increased, the maximum strain **increases** at section A, while

the section B does not experience any strain since no bending moment is induced at that section.

In Figure 1, the non-linear deformation has extended from A to F. The distribution of bending moment along the column remains linear even after non-linear deformation is started. The value of bending moment M is maximum at the section A while it always remains zero at Section B. But the non-linear characteristics of the maximum strain slowly extends both inside the section and along the column starting from Section at A. The relation between the deflection y at the top of the frame and the maximum strain e at section A when the frame is subjected to non-linear strain the first time can be represented by the following equation:

$$y = \frac{2 e_y h^2}{d} \left\{ \frac{1}{3} \left[1 + \frac{\left\{1 - \left(\frac{1}{\alpha}\right)^2\right\}}{\left\{3 - \left(\frac{1}{\alpha}\right)^2\right\}} - \frac{\left\{1 - \left(\frac{1}{\alpha}\right)^2\right\}^2}{2\left\{3 - \left(\frac{1}{\alpha}\right)^2\right\}^2} \right] + \frac{1}{2}(\alpha - 1) - \frac{27\alpha + \frac{18}{\alpha} - 14 - \left(\frac{1}{\alpha}\right)^2}{6\left\{3 - \left(\frac{1}{\alpha}\right)^2\right\}^2} \right\}$$

where α = e/ey,

and e_y is yield strain.

When e becomes e_y; $y = \frac{2 e_y h^2}{3d}$,

which is a linear relationship. When e is less then e_y the linear relationship is given by the equation:

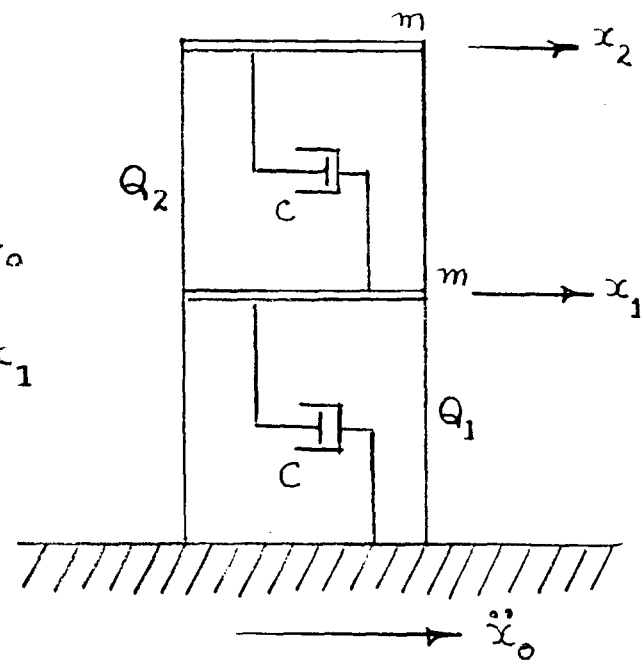
$$Y = \frac{2eh^2}{3d} .$$

By utilising the above equation it may be possible to make a reasonable assumption of hysteresis loop between the deflection and the maximum strain at the critical section. Hence if the time history of the deflection of a system subjected to random vibration is obtained, it may be possible to get the time history of the strain at the critical section. The fatigue damage factor can be calculated from the time history of the strain and we can study the safety of the structure.

A two-degree-freedom system as shown in figure 2 was considered for investigation.

$$Y_1 = x_1 - x_0$$

$$Y_2 = x_2 - x_1$$



Two-Degree-Freedom System

Figure 2

The equations of motion are given below:

$$\ddot{Y}_1 + \frac{c}{m}\dot{Y}_1 - \frac{c}{m}\dot{Y}_2 + \frac{Q_1}{m}Y_1 - \frac{Q_2}{m}Y_2 = -\ddot{x}_0$$

$$\ddot{Y}_2 + \ddot{Y}_1 + \frac{c}{m}\dot{Y}_2 + \frac{Q_2}{m}Y_2 = -\ddot{x}_0$$

Where C is coefficient of friction,

m is the mass,

y_1 is the relative deflection of first story with reference to ground,

Y_2 is the relative deflection of second story with reference to the first story,

$\dot{Y}$ is the first derivative of y,

$\ddot{Y}$ is the second derivative of y,

and $\ddot{X}_0$ is the ground random vibration.

By adopting the constant-velocity numerical integration method, if the response y(s) at time station S and y(S-1) at the preceding time station S-1 have been previously determined, the velocity y(s) at the time station S is approximately expressed as follows:

$$\dot{Y}(S) = \frac{Y(S) - Y(S-1)}{\Delta T} + \ddot{Y}(S) \cdot \frac{\Delta T}{2}$$

Where ΔT is the time internal choosen and $\ddot{y}(s)$ is the acceleration.

The response y(s+1) at the next time station is assumed as follows:

$$y(s+1) = 2 \cdot y(s) - y(s-1) + \ddot{y}(s) \cdot \Delta T^2 .$$

Incorporating the above two equations in the general equations of motion the following expanded equations are derived.

$$\ddot{y}_1(s) = \left[\frac{Q_2}{m} y_2(s) - \frac{Q_1}{m} y_1(s) - \frac{\omega n \Delta T \frac{Q_2}{m} y_2(s)}{1 + \omega n \Delta T} + 2\omega n \left\{ \frac{y_1(s-1) - y_1(s)}{\Delta T} + \frac{y_2(s) - y_2(s-1)}{\Delta T} \right\} - \ddot{x}_o(s) - \omega n \Delta T \left\{ 2\omega n \left(\frac{y_2(s) - y_2(s-1)}{\Delta T} \right) + \ddot{x}_o(s) \right\} \Big/ (1 + \omega n \Delta T) \right] \Big/ \left[1 + \omega n \Delta T + \frac{\omega n \Delta T}{1 + \omega n \Delta T} \right]$$

$$\ddot{y}_2(s) = \left\{ - \ddot{y}_1(s) \Big/ (1 + \omega n \Delta T) \right\} - \left[2\omega n \left\{ \frac{y_2(s) - y_2(s-1)}{\Delta T} \right\} + \frac{Q_2}{m} y_2(s) + \ddot{x}_o(s) \right] \Big/ \left[1 + \omega n \Delta T \right]$$

ω is the natural frequency of the system.

These expanded equations are utilised in writing the Computer Programe for determining the time history of responses of a two-degree-freedom system for a random input function. A portion of NS Component of El Centro 1940 Earthquake record is taken as the random input function. The degital Computer IBM 360 now available at College Station is being utilised for this research. A copy of the computer result obtained so far (as on January 28, 1970) is produced for reference. This program will be slowly expanded for determining the time history of critical strain and then the fatigue damage which can occur in the system. After analysing the effect of various possible improvements to the structural system, the results are proposed to be submitted for publication. Extension of this research beyond two-degree freedom systems will also be investigated.

2.11.70

Principal Investigator

School of Engineering

Prairie View A. & M. College

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$JOB X01300,TIME=001,PAGES=020      I.KASIRAJ
C    VIBRATION PROBLEM NASA RESEARCH
      DIMENSION T(110) , AC(110) , XDD (725) , Y1 (725) , Y2 (725)
      COMMON DEL,AN,OM,Q,QME,YDD1,QM2MAX,QM2MIN,Y2MAX,Y2MIN,XDD,Y1,Y2,NT
     1,YELD
      READ (5,100) (T(I),AC(I),I=1,104)
  100 FORMAT(4(F6.2,F12.7))
      DEL = .01
      I = 1
      NT = 1
      XDD (NT) = AC (I)
   47 F = (T( I+1) - T(I))/ DEL
   41 NT = NT + 1
      F = F - 1.
      XDD(NT) = XDD(NT-1) +((AC(I+1)-AC(I))* (DEL/(T(I+1)-T(I))))
      IF (F- .1) 43, 43, 41
   43 XDD(NT) = AC(I+1)
      I = I +1
      IF (I-104) 47, 45, 45
   45 FACTOR = .0321725
      DO 40 NT = 1 , 721
   40 XDD (NT) = FACTOR * XDD (NT)
      THETA = .1
   31 WRITE (6 , 500) THETA
  500 FORMAT (18X, 'THETA =' , F8.5)
      AN=.05
   83 OM = 30.
    2 NT = 1
      DO 101 N = 1 , 725
      Y1 (N) = 0
  101 Y2 (N) = 0
      Q = .0
      QM1MAX=0.
      QM1MIN=0.
      Y1MAX=0.
      Y1MIN=0.
      QM2MAX=0.
      QM2MIN=0.
      Y2MAX=0.
      Y2MIN=0.
      QME = THETA * 32.1725
      YELD = ( THETA * 32.1725) / ( OM **2 )
      WRITE (6, 1000) YELD
 1000 FORMAT(F15.6)
      YDD1 = - XDD (NT)
      YDD2 = - XDD (NT) - YDD1
      Y1 (NT+1) = (YDD1/2.) * (DEL **2 )
      Y2 (NT +1) = (YDD2/2.) * (DEL **2)
      NT = NT +1
    1 IF ((ABS (Y1(NT))) - YELD) 20, 20, 30
   20 QM1 = (OM **2) * Y1(NT)
      R=0.
      YDD1 = (QM1* (Y2 (NT) - Y1 (NT)) - (OM*AN * DEL * QM1* Y2(NT) )      1
     1/(1. + OM*AN*DEL) + 2. * OM * AN * ((Y1 (NT-1) - Y1(NT))/            2
```

```
     1DEL + (Y2(NT) - Y2 (NT-1))/DEL)- XDD(NT) - CM*AN * DEL *(2. *
     1OM *AN * ( Y2(NT)-Y2(NT-1))/DEL +XDD(NT))/(1. + OM*AN * DEL))/
     1(1. + OM *AN * DEL + (CM * AN * DEL)/(1. + CM * AN * DEL))
      CALL SMAIN(YDD2)
      Y1 (NT+1) = 2* Y1(NT) - Y1(NT-1) + YDD1 * (DEL**2)
      Y2 (NT+1) = 2 * Y2(NT) - Y2(NT-1) + YDD2* (DEL**2)
      NT = NT + 1
      WRITE(6,200) NT,R,Y1(NT),Q,Y2(NT)
  200 FORMAT(I15,4F15.5)
      IF ( NT - 721 ) 1 , 1 , 70
   30 IF(Y1 (NT) - 0.) 9, 3, 3
    3 Y 1 MAX = Y 1 (NT)
      R=1.
      QM1 = QME
      QM1 MAX = QM1
      YDD1 = (QM1* (Y2 (NT) - Y1 (NT)) - (OM*AN * DEL * QM1* Y2(NT) )
     1/(1. + CM*AN*DEL) + 2. * CM * AN * ((Y1 (NT-1) - Y1(NT))/
     1DEL + (Y2(NT) - Y2 (NT-1))/DEL)- XDD(NT) - OM*AN * DEL *(2. *
     1OM *AN * ( Y2(NT)-Y2(NT-1))/DEL +XDD(NT))/(1. + CM*AN * DEL))/
     1(1. + OM *AN * DEL + (CM * AN * DEL)/(1. + CM * AN * DEL))
      CALL SMAIN(YDD2)
      Y1 (NT+1) = 2* Y1(NT) - Y1(NT-1) + YDD1 * (DEL**2)
      Y2 (NT+1) = 2 * Y2(NT) - Y2(NT-1) + YDD2* (DEL**2)
      NT = NT +1
      WRITE(6,200) NT,R,Y1(NT),Q,Y2(NT)
      IF ( NT - 721) 4, 4, 70
    4 IF (Y1(NT) - Y1(NT-1)) 7, 3, 3
    7 IF ((Y1MAX -Y1(NT))-(2. * YELD)) 5, 9, 9
    5 QM1 = QM1 MAX - ( Y1 MAX - Y1(NT)) * ( CM** 2)
      R=-.5
      YDD1 = (QM1* (Y2 (NT) - Y1 (NT)) - (OM*AN * DEL * QM1* Y2(NT) )
     1/(1. + CM*AN*DEL) + 2. * CM * AN * ((Y1 (NT-1) - Y1(NT))/
     1DEL + (Y2(NT) - Y2 (NT-1))/DEL)- XDD(NT) - CM*AN * DEL *(2. *
     1OM *AN * ( Y2(NT)-Y2(NT-1))/DEL +XDD(NT))/(1. + OM*AN * DEL))/
     1(1. + OM *AN * DEL + (CM * AN * DEL)/(1. + CM * AN * DEL))
      CALL SMAIN(YDD2)
      Y1 (NT+1) = 2* Y1(NT) - Y1(NT-1) + YDD1 * (DEL**2)
      Y2 (NT+1) = 2 * Y2(NT) - Y2(NT-1) + YDD2* (DEL**2)
      NT = NT + 1
      WRITE(6,200) NT,R,Y1(NT),Q,Y2(NT)
      IF (NT-721) 6, 6, 70
    6 IF (Y1(NT) - Y1MAX) 7, 3, 3
    9 Y1MIN = Y1(NT)
      R=-1.
      QM1 = - QME
      QM1 MIN = QM1
      YDD1 = (QM1* (Y2 (NT) - Y1 (NT)) - (OM*AN * DEL * QM1* Y2(NT) )
     1/(1. + CM*AN*DEL) + 2. * CM * AN * ((Y1 (NT-1) - Y1(NT))/
     1DEL + (Y2(NT) - Y2 (NT-1))/DEL)- XDD(NT) - OM*AN * DEL *(2. *
     1OM *AN * ( Y2(NT)-Y2(NT-1))/DEL +XDD(NT))/(1. + OM*AN * DEL))/
     1(1. + OM *AN * DEL + (CM * AN * DEL)/(1. + CM * AN * DEL))
      CALL SMAIN(YDD2)
      Y1 (NT+1) = 2* Y1(NT) - Y1(NT-1) + YDD1 * (DEL**2)
      Y2 (NT+1) = 2 * Y2(NT) - Y2(NT-1) + YDD2* (DEL**2)
      NT = NT +1
      WRITE(6,200) NT,R,Y1(NT),Q,Y2(NT)
      IF (NT- 721) 8, 8, 70
    8 IF ( Y1(NT) - Y1 (NT-1)) 9, 9, 11
   11 IF (( Y1(NT) - Y1 MIN) - 2. * YELD) 13, 13,3
   13 QM1 = QM1 MIN - ( Y1 MIN-Y1(NT)) * (OM**2)
```

```
      R=.5
      YDD1 = (QM1* (Y2 (NT) - Y1 (NT)) - (OM*AN * DEL * QM1* Y2(NT) )
     1/(1. + OM*AN*DEL) + 2. * OM * AN * ((Y1 (NT-1) - Y1(NT))/
     1DEL + (Y2(NT) - Y2 (NT-1))/DEL)- XDD(NT) - OM*AN * DEL *(2. *
     1OM *AN * ( Y2(NT)-Y2(NT-1))/DEL +XDD(NT))/(1. + OM*AN * DEL))/
     1(1. + OM *AN * DEL + (OM * AN * DEL)/(1. + OM * AN * DEL))
      CALL SMAIN(YDD2)
      Y1 (NT+1) = 2* Y1(NT) - Y1(NT-1) + YDD1 * (DEL**2)
      Y2 (NT+1) = 2 * Y2(NT) - Y2(NT-1) + YDD2* (DEL**2)
      NT = NT + 1
      WRITE(6,200) NT,R,Y1(NT),Q,Y2(NT)
      IF ( NT - 721) 10, 10, 70
   10 IF (Y1 (NT) - Y1 (NT-1)) 15, 11, 11
   15 IF(Y1(NT)-Y1MIN) 9,13,13
   70 STOP
      END

      SUBROUTINE SMAIN(YDD2)
      DIMENSION XDD(725),Y1(725),Y2(725)
      COMMON DEL,AN,OM,Q,QME,YDD1,QM2MAX,QM2MIN,Y2MAX,Y2MIN,XDD,Y1,Y2,NT
     1,YELD
      IF(Q) 211,203,205
  203 IF((ABS(Y2(NT)))-YELD) 207,207,208
  207 QM2=(OM**2)*Y2(NT)
      YDD2 = - YDD1 / (1. + OM *AN * DEL) - (2. * OM * AN * ((Y2(NT)
     2-Y2(NT-1))/DEL) +QM2 * Y2(NT) + XDD(NT))/ (1. +OM *AN * DEL)
      GO TO 209
  205 IF(Y2(NT)-Y2MAX) 214,213,213
  208 IF(Y2(NT)-0.) 212,213,213
  214 IF((Y2MAX-Y2(NT))-(2.*YELD)) 222,212,212
  211 IF(Y2(NT)-Y2MIN) 212,212,217
  217 IF((Y2(NT)-Y2MIN)-(2.*YELD)) 221,213,213
  213 CALL TOP(YDD2)
      GO TO 209
  212 CALL BOTTOM(YDD2)
      GO TO 209
  221 CALL UP(YDD2)
      GO TO 209
  222 CALL DOWN(YDD2)
      GO TO 209
  209 RETURN
      END

      SUBROUTINE DOWN(YDD2)
      DIMENSION XDD(725),Y1(725),Y2(725)
      COMMON DEL,AN,OM,Q,QME,YDD1,QM2MAX,QM2MIN,Y2MAX,Y2MIN,XDD,Y1,Y2,NT
     1,YELD
      Q=-.5
      QM2=QM2MAX-(Y2MAX-Y2(NT))*(OM**2)
      YDD2 = - YDD1 / (1. + OM *AN * DEL) - (2. * OM * AN * ((Y2(NT)
     2-Y2(NT-1))/DEL) +QM2 * Y2(NT) + XDD(NT))/ (1. +OM *AN * DEL)
      RETURN
      END

      SUBROUTINE UP(YDD2)
      DIMENSION XDD(725),Y1(725),Y2(725)
      COMMON DEL,AN,OM,Q,QME,YDD1,QM2MAX,QM2MIN,Y2MAX,Y2MIN,XDD,Y1,Y2,NT
     1,YELD
      Q=.5
      QM2=QM2MIN-(Y2MIN-Y2(NT))*(OM**2)
```

```
      YDD2 = - YDD1 / (1. + CM *AN * DEL) - (2. * OM * AN * ((Y2(NT)
     2-Y2(NT-1))/DEL) +QM2 * Y2(NT) + XDD(NT))/ (1. +CM *AN * DEL)
      RETURN
      END

      SUBROUTINE BOTTOM(YDD2)
      DIMENSION XDD(725),Y1(725),Y2(725)
      COMMON DEL,AN,CM,Q,QME,YDD1,QM2MAX,QM2MIN,Y2MAX,Y2MIN,XDD,Y1,Y2,NT
     1,YELD
      Q=-1.
      WRITE(6,400) Q
  400 FORMAT (10X,F10.5)
      QM2=-QME
      Q2MIN=QM2
      Y2MIN=Y2(NT)
      YDD2 = - YDD1 / (1. + CM *AN * DEL) - (2. * OM * AN * ((Y2(NT)
     2-Y2(NT-1))/DEL) +QM2 * Y2(NT) + XDD(NT))/ (1. +CM *AN * DEL)
      RETURN
      END

      SUBROUTINE TOP(YDD2)
      DIMENSION XDD(725),Y1(725),Y2(725)
      COMMON DEL,AN,CM,Q,QME,YDD1,QM2MAX,QM2MIN,Y2MAX,Y2MIN,XDD,Y1,Y2,NT
     1,YELD
      Q=1.
      WRITE(6,400) Q
  400 FORMAT (10X,F10.5)
      QM2=QME
      Q2MAX=QM2
      Y2MAX=Y2(NT)
      YDD2 = - YDD1 / (1. + CM *AN * DEL) - (2. * OM * AN * ((Y2(NT)
     2-Y2(NT-1))/DEL) +QM2 * Y2(NT) + XDD(NT))/ (1. +CM *AN * DEL)
      RETURN
      END

$DATA
      THETA = 0.10000
```

YELD = 0.003575

time station

	R	$Y_1(s)$	Q	$Y_2(s)$
3	0.00000	-0.00003	0.00000	-0.00000
4	0.00000	-0.00007	0.00000	-0.00000
5	0.00000	-0.00011	0.00000	-0.00000
6	0.00000	-0.00015	0.00000	-0.00001
7	0.00000	-0.00021	0.00000	-0.00001
8	0.00000	-0.00027	0.00000	-0.00002
9	0.00000	-0.00035	0.00000	-0.00003
10	0.00000	-0.00045	0.00000	-0.00004
11	0.00000	-0.00057	0.00000	-0.00005
12	0.00000	-0.00073	0.00000	-0.00006
13	0.00000	-0.00091	0.00000	-0.00008
14	0.00000	-0.00110	0.00000	-0.00011
15	0.00000	-0.00131	0.00000	-0.00014
16	0.00000	-0.00152	0.00000	-0.00017
17	0.00000	-0.00173	0.00000	-0.00021
18	0.00000	-0.00192	0.00000	-0.00025
19	0.00000	-0.00212	0.00000	-0.00029
20	0.00000	-0.00232	0.00000	-0.00034
21	0.00000	-0.00254	0.00000	-0.00039
22	0.00000	-0.00278	0.00000	-0.00044
23	0.00000	-0.00306	0.00000	-0.00049
24	0.00000	-0.00336	0.00000	-0.00056