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A SIMPLE TEST FOR CERTAIN CASES OF ADHESION

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ABSTRACT

The peel test is commonly used to evaluate bonding efficiency. It is particularly valuable for a qualitative comparison among various adhesives. On the other hand, it is not particularly adaptable for predicting the degree of adhesion in multiaxial stress states or in complicated geometric configurations. A modified pressurized blister test has been found useful because the data from the test can be used not only for comparative evaluations, but also for other design situations for which the bonding techniques (cleaning technique, adhesive thickness, adherend rigidity, etc.) is similar to that used in preparing the blister specimen. The correlation is accomplished using recent advances in fracture mechanics which are described in the paper, along with applications to adhesive coatings.

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A SIMPLE TEST FOR CERTAIN CASES OF ADHESION*

INTRODUCTION

Peel tests have been popular for qualitative comparison of adhesive bonds. However, results from these tests have not always been useful to designers as failure criteria. A simple test with interpretable results has been needed. In 1961, Dannenberg[1] reported on the measurement of the work necessary to unbond a blister of organic coating from a metal substrate. Malyshev and Salganik,[2] using fracture mechanics relations, applied an energy balance to bonded circular plates with concentrated center loads. More recently, Williams[3] has shown the applicability of fracture mechanics to adhesion and has suggested the use of a bonded circular plate ("Blister test"). The analysis of the bonded circular plate has been extended to include large deflections and into the membrane region.

THEORY

Fracture mechanics comes into use for geometric situations such as sharp reintrant corners, notches and cracks, where a standard stress analysis predicts stresses that are infinite. These are called stress singularities. Basically the idea developed by fracture mechanics is that although the stress may tend to infinity, the integral of the stress-strain product (i.e. the strain energy) for certain "well behaved" singularities will remain finite. Then a thermodynamic energy balance may be made with a postulated surface energy term included.[4] Then, impending fracture, or criticality for an elastic material may be defined when the rate of decrease in stored energy is equal to the rate of increase of surface energy. For elastoplastic and viscoelastic materials, additional energy dissipation terms must be included.

In the study of bimaterial half planes, Williams[5] found that the singularity for a crack following the bond line between the two materials is well behaved for the conditions that one material is incompressible ($\nu = 0.5$) while the other is infinitely stiff ($E \to \infty$).

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Let us then consider an axisymmetric sheet of low modulus, but incompressible material bonded to a sheet of very high modulus material. By then introducing a pressure P_i between the sheets, the upper sheet can be caused to deflect and to peel away from the rigid lower plate.

By considering the upper sheet to be a plate with small deflections, fixed at the edges, from plate bending theory,[6] it is found that

$$w(r) = \frac{PR^4}{64D}\left(1 - \left(\frac{r}{R}\right)^2\right)^2 \tag{1}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{Eh^3}{9} \tag{2}$$

An energy balance gives

$$\dot{I} = \dot{F} + 2D + \dot{SE} \tag{3}$$

where

I - energy input
F - energy stored
2D - energy dissipation rate
SE - surface energy

and the dot indicates differentiation with respect to time.

But for an elastic material (zero energy dissipation) and taking the variation with respect to fracture surface area, eqn (3) reduces to

$$\delta(u - A)_{eq} = 2\pi R \gamma_a \tag{4}$$

where

$$A_{eq} = \int_0^{Area} P\, w(r)\, d\,Area \tag{5}$$

And from Clapyron's Theorem

$$U_{eq} = \frac{1}{2} A_{eq} \tag{6}$$

Then from eqn (4),

$$P_{cr}\, w_o = 2\, \gamma_a \tag{7}$$

where

$$w_o = w(r)\Big|_{r=0} = \frac{R^4}{64D} P$$

or

$$w_o = K\, P \tag{8}$$

Now if the upper sheet is considered to have very large deflections such that the bending stresses are negligibly small compared to the mid-plane stretching or membrane stresses, it is found that from eqn (4)*

$$P_{cr}\, w_o = 2.4\, \gamma_a \tag{9}$$

For an analysis where the contribution of both plate bending and membrane stresses, the derivation is somewhat more complicated. This has been approximated by Berger,[7] who reduces the two field equations in the in-plane deformation, u(r), and bending deflection, w(r), to only one equation

$$\nabla^4 w - \alpha^2 \nabla^2 w = \frac{P}{D} \tag{10}$$

where a relation for evaluation of α is derived from considering the displacement boundary condition u(R) = 0. Then it is found that

---

* Note: For the membrane, the relation for w_o is $0.652\sqrt[3]{(PR^4/Dh)}$ or $w_o = K_1 P^{1/3}$. It can be shown that $u_{eq} = \frac{1}{4}A_{eq}$ for this situation and the energy balance becomes $\delta(-3A_{eq}) = 2\pi R\gamma_a$ (again where $A_{eq} = \int_0^{Area} pw(r)\,dArea$).

$$\left(\frac{PR^4}{Dh}\right)^2 = \frac{\frac{1}{3}(\alpha R)^6}{\frac{3}{4} + \frac{4}{(\alpha R)^2} - \frac{1}{\alpha R}\frac{I_0(\alpha R)}{I_1(\alpha R)} - \frac{1}{2}\left(\frac{I_0(\alpha R)}{I_1(\alpha R)}\right)^2} \tag{11}$$

where $I_0(\alpha R)$ and $I_1(\alpha R)$ are modified Bessel functions of the zeroth and first order respectively.

The deflection is found to be

$$w(r) = \frac{PR^2}{4D\alpha^2}\left[1 - \frac{2\left(I_0(\alpha R) - I_0(\alpha r)\right)}{\alpha R\ I_1(\alpha R)} - \left(\frac{r}{R}\right)^2\right] \tag{12}$$

The strain energy u, is found to be

$$U = \pi D \int_0^R \left\{ (\nabla^2 w)^2 + \frac{12}{h^2} e^2 - 2(1-\nu)\left[\frac{12}{h^2} e_2 + \frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r^2}\right]\right\} r\,dr \tag{13}$$

where e_2 is considered to be negligibly small, and

$$A_{eq} = \frac{2\pi P^2 R^4}{16 D \alpha^2}\left[1 - \frac{4}{\alpha R}\frac{I_0(\alpha R)}{I_1(\alpha R)} + \frac{8}{(\alpha R)^2}\right] \tag{14}$$

Then criticality can be evaluated from eqn (4)

$$\delta(U - A)_{eq} = 2\pi R\,\gamma_a \tag{4}$$

The relation is not easily evaluated, thus has been programmed for evaluation on the computer. It should be noted from eqns (11), (15) and (16) that the variables h and R appear with exponentials as high as 8. Thus any error in measurement of these variables will strongly affect the result. This derivation is summarized in Appendix I.

EXPERIMENTAL VERIFICATION

To verify the analysis and to determine values for the adhesive surface energy density, thin disks of polyurethane rubber were cast bonded to glass plates. Care was taken so that the glass surface was clean prior to casting. Pressurized air was then introduced through a hole in the glass plate to

cause the rubber to blister up and peel away from the glass as shown in Figures 2 and 3. Pressure was measured by a diaphragm type transducer and the deflection in the center of the blister was measured using an LVDT. Data were recorded directly by an X-Y Plotter. Typical test results appear as shown in Figure 4.[8]

Fracture was defined when the deflection began to grow with little or no increase in pressure. Shown in Figure 5 is a plot of the pressure and deflection at fracture. It is noted from eqn (7) that the locus of fracture points would fall along a hyperbola of parametric value $2\gamma_a$ from plate theory. And indeed, while the deflections are small the data points seem to follow this type curve. Note also that the value γ_a is determined from this curve, so at least one data point would coincide anyway. Then when the deflections become large the data points should tend toward a hyperbola of parametric value $2.4\gamma_a$. This corresponds to the membrane solution of the plate. Again, this does happen. In the transition between the two bounding curves, γ_a is determined using eqns (4), (15) and (16) where α is determined from eqn (11). From these, it appears $\gamma_a = 0.22$ best fits the data.

DISCUSSION

A simple adhesion test has been described, analyzed and verified experimentally. Results are interpretable from small deflections continuously through large deflections in terms of an adhesive surface energy density, γ_a. This constant can now be viewed as a property of the bond joint. So that if the joint is poorly made, the measured γ_a value will be low, or if the surfaces are clean and "good adhesion" is attained, measured γ_a will have higher values. And since γ_a reflects the quality of the bond, this simple blister test can be used for process control. But more important, once γ_a is found, it is applicable to any other design condition, provided the associated stress distribution can be analyzed, and if the surface preparation is the same as for the test specimen.

An example of this application may be found in reference 9, where the problem was to predict unbonding at the end of solid rocket motors. Other immediate applications may be made to such problems, as elastomeric coatings to metal surfaces and bonded prosthetic devices.

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APPENDIX

The analysis for large deflections of plates has been greatly simplified by Berger. The simplification comes about from the observation that in eqn (1) the contribution of the second strain invariant is very small in comparison with the other terms.

$$\mathcal{U} = \pi D \int_0^R \left\{ (\nabla^2 w)^2 + \frac{12}{h^2} e^2 - 2(1-\nu)\left[\frac{12}{h^2} e_2 + \frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r^2} \right] \right\} r\,dr \qquad (1)$$

From the application of the principle of virtual work, the field equation is found to be

$$\nabla^4 w - \alpha^2 \nabla^2 w = \frac{P}{D} \qquad (2)$$

where

$$e \equiv \frac{\alpha^2 h^2}{12} \qquad (3)$$

The α term is determined using the boundary condition on the radial deflection u, i.e.

$$\left(\frac{PR^4}{Dh}\right)^2 = \frac{\frac{1}{3}(\alpha R)^6}{\frac{3}{4} + \frac{4}{(\alpha R)^2} - \frac{I_0(\alpha R)}{\alpha R\, I_1(\alpha R)} - \frac{1}{2}\left(\frac{I_0(\alpha R)}{I_1(\alpha R)}\right)^2} \qquad (4)$$

Using the above equations, the deflection is calculated to be

$$w(r) = \frac{PR^2}{4D\alpha^2}\left[1 - \left(\frac{r}{R}\right)^2 - \frac{2}{\alpha R}\,\frac{I_0(\alpha R) - I_0(\alpha r)}{I_1(\alpha R)} \right] \qquad (5)$$

For a circularly symmetric incompressible plate, fixed at the edges and loaded with a uniformly distributed load, P, eqn (1) becomes

$$\mathcal{U} = \pi D \int_0^R \left\{ r\left(\frac{\partial^2 w}{\partial r^2}\right)^2 + \left(\frac{\partial w}{\partial r}\right)\left(\frac{\partial^2 w}{\partial r^2}\right) + \frac{1}{r}\left(\frac{\partial w}{\partial r}\right)^2 + \frac{\alpha^4 h^2}{12} r \right\} dr \qquad (6)$$

Now applying the energy balance, but taking variations with respect to the variable αR,

$$\delta \left[U - A \right]_{eq} = 2\pi R \gamma_a \delta R \tag{7}$$

where δR is determined from eqn (4)

$$\delta R = \frac{2\left(\frac{Dh}{P}\right)^2 (\alpha R)^5 - R^8 \left[\left(\frac{I_0(\alpha R)}{I_1(\alpha R)}\right)^3 - \frac{8}{(\alpha R)^3} - \frac{1}{\alpha R} - \frac{I_0(\alpha R)}{I_1(\alpha R)} \right]}{8 R^7 \left[\frac{3}{4} + \frac{4}{(\alpha R)^2} - \frac{1}{\alpha R} \frac{I_0(\alpha R)}{I_1(\alpha R)} - \frac{1}{2}\left(\frac{I_0(\alpha R)}{I_1(\alpha R)}\right)^2 \right]} \tag{8}$$

Using eqns (5) and (6) and developing the relation for "applied work"

$$(U-A)_{eq} = 2\pi \left(\frac{P^2 R^2}{8D\alpha^4}\right) \left[\left(\frac{I_0}{I_1}\right)^2 \left(\frac{(\alpha R)^2}{2} - 1\right) - (\alpha R)^2 - 4 - \alpha R \frac{I_0}{I_1} + \frac{1}{I_1^2} + \frac{\alpha^8 h^2 D^2}{6 P^2} \right] \tag{9}$$

where $$A_{eq} = 2\pi \int_0^R P w(r)\, r\, dr$$

After the variation of eqn (9) is taken with respect to αR, the surface energy density can be evaluated to be

$$\gamma_a = -\frac{P^2}{8D} \left\{ \frac{R^5}{\delta R} \left[\frac{I_0}{I_1}\left(\frac{1}{(\alpha R)^2} - \frac{4}{(\alpha R)^4} - \frac{1}{(\alpha R)^4 I_1}\right) + \left(\frac{I_0}{I_1}\right)^3 \left(\frac{2}{(\alpha R)^4} - \frac{1}{(\alpha R)^2}\right) \right.\right.$$

$$+ \left(\frac{I_0}{I_1}\right)^2 \left(\frac{2}{(\alpha R)^5} - \frac{1}{(\alpha R)^3}\right) + \frac{1}{(\alpha R)^5}\left(\frac{1}{I_1} - \frac{4}{I_1^2}\right) + \frac{3}{(\alpha R)^3} \tag{10}$$

$$\left. + \frac{16}{(\alpha R)^5} \right] + 6 R^4 \left[\left(\frac{I_0}{I_1}\right)^2 \left(\frac{1}{2(\alpha R)^2} - \frac{1}{(\alpha R)^4}\right) - \frac{1}{(\alpha R)^2} - \frac{4}{(\alpha R)^4} + \frac{I_0}{(\alpha R)^3 I_1} + \frac{1}{(\alpha R)^4 I_1^2} \right] \Bigg\}$$

$$- \frac{Dh^2}{24} \left[\frac{2\alpha^3}{\delta R} - \alpha^4 \right]$$

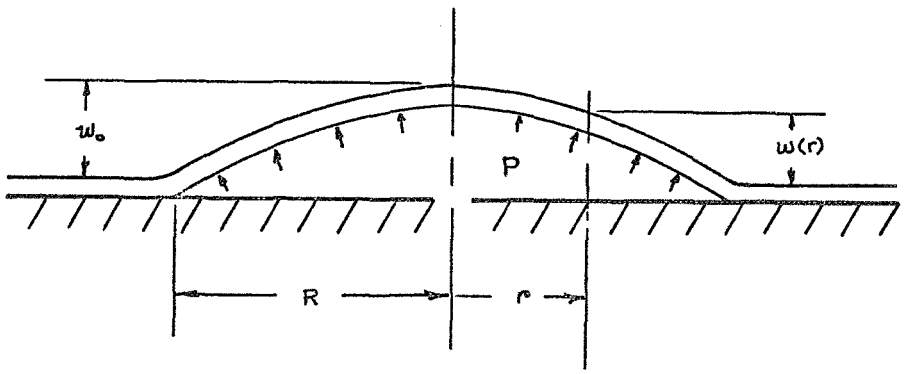

Fig. 1 Sketch of the deflected plate.

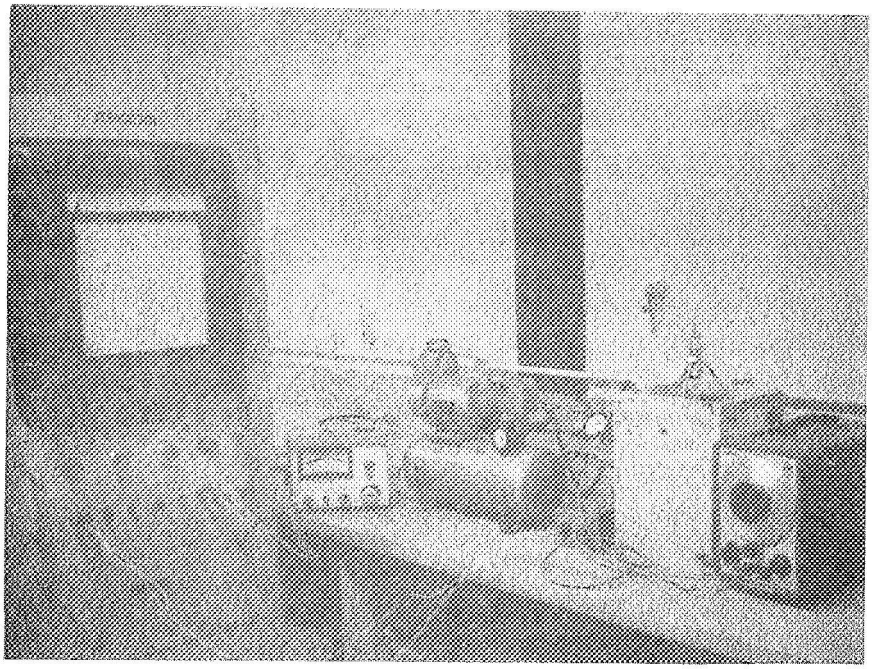

Fig. 2 Photograph of blister test apparatus.

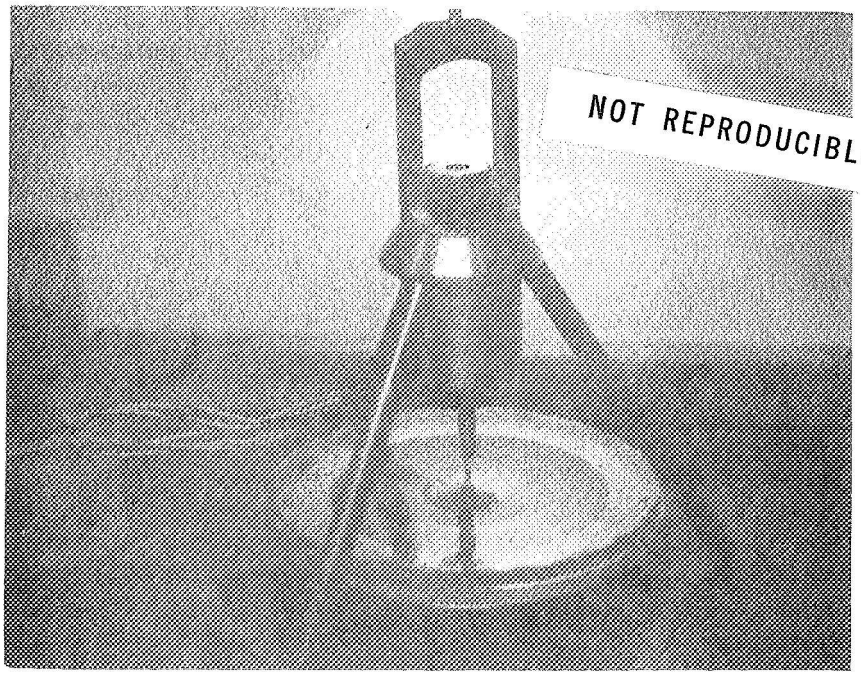

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Fig. 3 A blister test in progress.

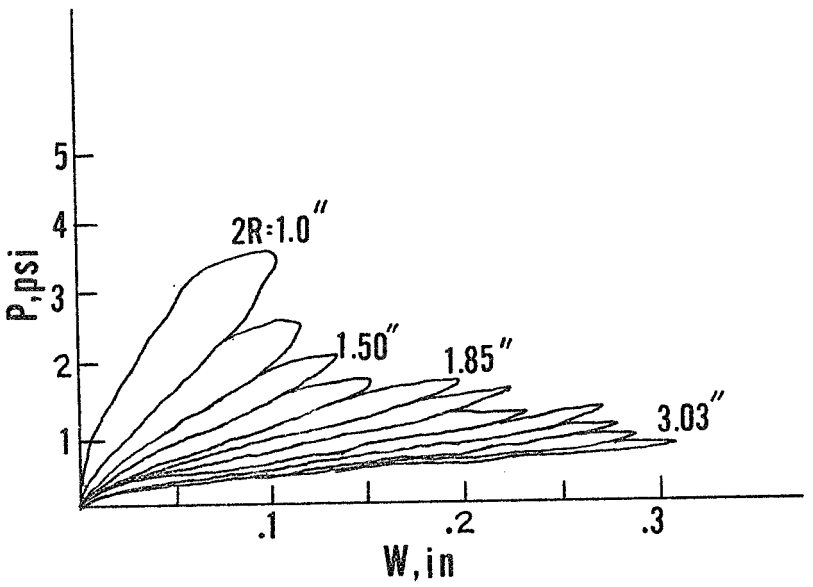

Fig. 4 Typical pressure versus deflection test data.

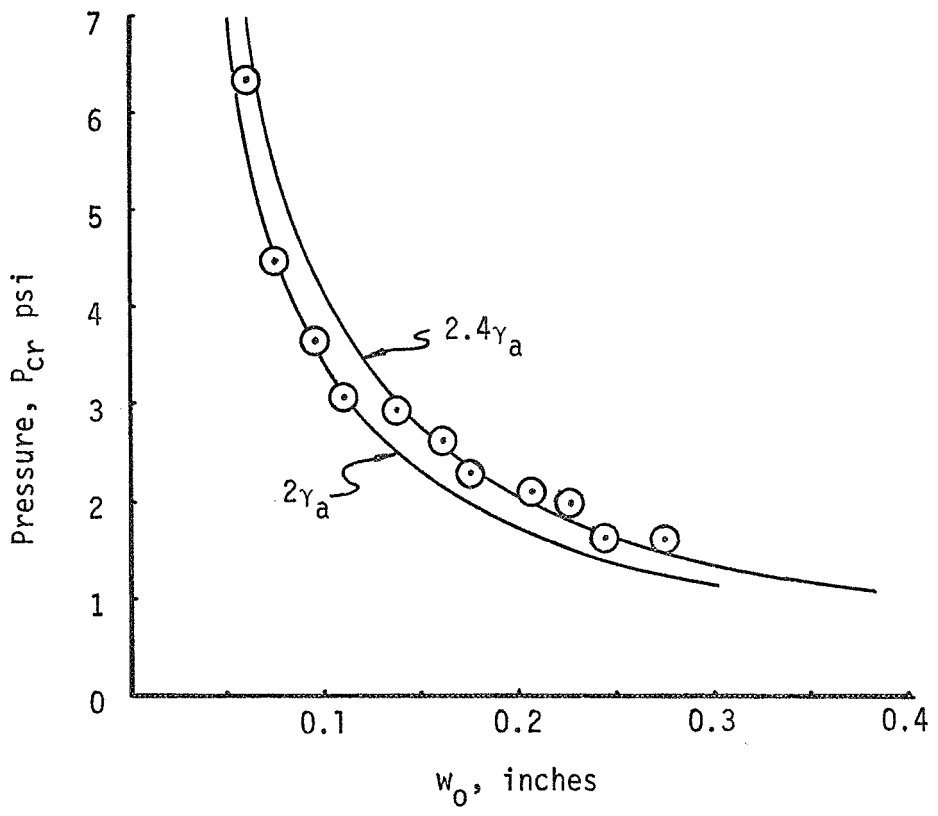

Fig. 5 Plot of the critical pressure versus center deflection from a blister test.

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