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A THEORETICAL INVESTIGATION OF THE EFFECTS OF THE FUSELAGE ON THE SPANWISE LIFT DISTRIBUTION OF A WING

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A Thesis

Presented to

the Faculty of the Department of

Engineering

University of Virginia

In Partial Fulfillment

of the Requirements for the Degree

Master of Aeronautical Engineering

by

Martin Klotnick

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Author _____

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CHAPTER I

INTRODUCTION

I. THE PROBLEM OF WING-FUSELAGE INTERFERENCE

The importance of the problem. A knowledge of the aerodynamic loading on the wing of an airplane is required for the stress analysis of the wing and for the prediction of the flying qualities of the airplane. Experimental determination of this loading is very tedious and is resorted to in very few cases. Satisfactory theoretical methods for the calculation of the spanwise lift distribution on the wing alone have been developed, but the problem of predicting the effect of the fuselage on the spanwise lift distribution on the wing has not been solved. Experience has indicated that this effect tends to be small at low angles-of-attack, and, for the lack of any better information, it is usually neglected. However, refinements in the methods of stress analysis place a premium on the accurate prediction of the aerodynamic loading applied to the wing, and hence to the effects of the fuselage on that loading. Also, in the case of swept wings there are indications that the effects of the fuselage on the spanwise lift distribution

of the wing are not as small as in the case of unswept wings. Consequently, development of a theoretical method for predicting these effects is very desirable at this time. The analysis in this paper is for the case of incompressible flow, however the treatment of compressibility effect is indicated in Chapter IV.

The mechanism of fuselage-wing interference. The fuselage alters the flow about the wing on which it is mounted in the following ways:

(1) The fuselage causes an increase in the longitudinal velocity in the vicinity of the wing. This effect is hereinafter referred to as the "inflow effect."

(2) If the fuselage is at an angle-of-attack relative to the free stream it changes the flow about the wing in planes normal to the free stream. This effect is hereinafter referred to as the "crossflow effect."

(3) The fuselage has a blocking effect, since there can be no flow normal to its surface.

The inflow effect which is a result of the finite length of the fuselage causes an increase in the dynamic pressure in the vicinity of the wing and, hence, an increase in the lift on the wing. This increase is not large for a slender fuselage but may be important when the fuselage is relatively thick, so that it causes a substantial increase

in the local longitudinal velocity. For the same reason this effect may be important for a wing-nacelle combination. However, an increase in lift may be expected only if the configuration is aerodynamically clean; in the case of a poorly streamlined nacelle attached to a thick wing, for example, the lift near the juncture may be reduced due to the turbulence in that region.

The crossflow caused by the fuselage changes the component of free-stream velocity normal to the fuselage axis, and also affects the downwash flow produced by the wing. In calculating the crossflow effect it is assumed in this paper that the flow in planes normal to the fuselage axis is two-dimensional.

The blocking effect of the fuselage is always present even if the fuselage is an infinite cylinder aligned with the free stream in which case the other two effects are not present.

II. SCOPE OF THE PROBLEM CONSIDERED IN THIS PAPER

Review of the literature. Outlines are presented in Chapter II of the available theoretical methods for calculating the spanwise lift distribution on wings alone and the velocity distribution on bodies of revolution, since these methods form the basis for much of the work on wing-fuselage

combinations, including that presented herein. This background material is presented for the sake of completeness and may be omitted by the reader who is familiar with the methods described.

In addition to the outline of the methods for treating the wing alone and the fuselage alone, two important solutions for highly idealized wing-fuselage combinations are discussed in Chapter II:

(1) Lennerts' analysis[1] and Vandrey's correction[2] to Lennerts' analysis of the problem of a fuselage represented by a sphere and a wing represented by an infinite vortex of constant strength.

(2) Lennerts' analysis of a fuselage represented by an infinite cylinder with the axis aligned with the free stream and a wing represented by an infinite vortex of constant strength.

The relation of these idealized solutions to the solutions for real wing-fuselage combinations is discussed. These

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[1] J. Lennerts, Beitrag zur theoretischen Behandlung des gegenseitigen Einflusses von Tragflache und Rumpf. Z. f. a. M. M., Bd. 7, Aug. 1927, pp. 249-276.

[2] F. Vandrey, Zur theoretischen Behandlung des gegenseitigen Einflusses von Tragflügel und Rumpf. Luftfahrtforschung, Bd. 14, July 20, 1937, pp. 347-355.

idealized solutions are shown to be useful in judging what sort of interference effects are to be expected.

In the last part of Chapter II three previously described methods for solving practical wing-body problems, the limitations of these methods and their relation to the analysis of the present paper are discussed. In this connection Wieselberger's method[3] for calculating the spanwise lift distribution on a wing-nacelle combination is presented, and it is shown how neglect of the inflow effect in his analysis can lead to results that are not consistent with experimental data. It is noted also why Multhopp's method[4] of calculating the lift distribution on a wing attached to a fuselage of infinite length is suitable only to unswept wings of moderate aspect ratio and why the calculations made using Multhopp's method give poor agreement with experimental results. Finally, Spreiter's solution[5] for the wing-fuselage configuration having an

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[3] C. Wieselberger, Der Einfluss von eingebauten Motorgondeln auf die Luftkräfte eines Tragflügels. Paper No. 203 read before the Internat. Engineering Congress, Tokyo, 1929. (from W. F. Durand, Aerodynamic Theory, Vol. IV, p. 161).

[4] H. Multhopp, Zur Aerodynamik des Flugzeugrumpfes. Luftfahrtforschung, Bd. 18, March 29, 1941, pp. 52-56, (translation NACA T.M. No. 1036).

[5] John R. Spreiter, Aerodynamic Properties of Slender Wing-Body Combinations at Subsonic, Transonic, and Supersonic Speeds, NACA T. N. No. 1662.

aspect ratio approaching zero is presented and its application to the analysis of this paper is discussed.

A new method for calculating the spanwise lift distribution in the presence of a fuselage. In Chapter III a new method for calculating the effects of the fuselage on the spanwise lift distribution on a wing is derived in a somewhat intuitive fashion from Multhopp's method. This method consists of the application of a conformal mapping procedure to the simplified lifting surface theory. It is shown that at low angles-of-attack the method goes to the theoretically correct limit for the wing-fuselage configuration having a wing of very low aspect ratio and for the wing-fuselage configuration having a wing of infinite aspect ratio. The method is applicable to any wing-fuselage configuration, and a computing scheme is given for the general case. Only the blocking and crossflow effects are initially taken into account; however, the manner of making the small inflow correction separately is indicated. Some calculations have been made by this new method, and the results are presented in the last section of Chapter III.

Chapter IV consists of a discussion of the results and of the limitations of the method, and includes a statement of some of the general conclusions reached as a result of the investigation.

SYMBOLS

c_l	local lift coefficient
c_{l_α}	section lift curve slope
α	geometric angle of attack
α_i	induced angle of attack
l	local section lift
ℓ	length of body
ρ	mass density of air
V	free stream velocity
U	longitudinal component of free-stream velocity, $(V \approx U)$
u	local longitudinal velocity
Γ	circulation
y, η	lateral ordinates
x	longitudinal ordinate
z	vertical ordinate
S	wing area
b	wing span
A	aspect ratio, b^2/S
r	radial ordinate of cylindrical coordinates
a	radius of body
ψ	stream function
Λ	angle of sweep

$w_{z=c/4}$	downwash velocity at three-quarter chord line
w_n	component of velocity normal to plane of wing
γ	dimensionless circulation $4\Gamma/bV$
$\bar{\gamma}$	dimensionless circulation $4\Gamma/\bar{b}V$
$\bar{c}^*$	chord made dimensionless with respect to transformed semispan $\bar{c}^* = \frac{c}{\bar{b}/2}$
θ	polar ordinate for fuselage cross-section ($\theta = 0$ at top)
u	complex coordinate in the real plane. $u = z + iy$
$\bar{u}$	complex coordinate in the transformed plane $\bar{u} = \bar{z} + i\bar{y}$
$\mathcal{R}\left(\frac{du}{d\bar{u}}\right)$	real part of the conformal transformation factor

CHAPTER II

CRITICAL REVIEW OF THE LITERATURE

I. ANALYSIS OF WINGS ALONE AND BODIES ALONE

Theoretical calculation of the spanwise lift distribution on wings alone. The theoretical methods most commonly applied to the problem of calculating the lift distribution on a wing are based on either of two theories, the lifting-line theory or the lifting surface theory. In the lifting line theory the assumption is made that a correction for the three dimensional effects can be applied to the two-dimensional characteristics of the airfoil at each wing section, this correction for the three dimensional effects having the form of a decrease in the effective angle-of-attack at each wing section. In order to calculate this correction the wing is considered to be replaced by a single straight vortex in the plane of the wing and perpendicular to the free-stream velocity.

In the lifting-surface theory a solution to the problem is obtained by satisfying the boundary condition that the flow be tangent to the surface of the wing; the fundamental assumption in the lifting-surface theory being that the boundary conditions are satisfied in a plane parallel to

the free-stream velocity rather than in the plane of the wing. An exact formulation of the lifting surface problem is not mathematically treatable at present, so that certain simplifications must be resorted to; for instance, the wing may be represented by a finite number of discrete vortices or even a single lifting vortex.

Of the two theories for calculating the spanwise lift distribution, the lifting-surface theory gives a better approximation to experimental results in all cases and can, in addition, be used in calculations of the lift distribution on swept wings and wings of low aspect ratio for which lifting-line theory is invalid.

<u>Lifting-line theory</u>[6]. In the lifting line method, the familiar expression for the two dimensional lift coefficient is written

$$c_l = c_{l_\alpha} \alpha_{effective} \tag{1}$$

where

$$\alpha_{effective} = \alpha - \alpha_i \tag{2}$$

and

$$c_l = c_{l_\alpha} (\alpha - \alpha_i) \tag{3}$$

where α_i in turn is the angle of attack by which the

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[6] H. Glauert, <u>The Elements of Aerofoil and Airscrew Theory</u>, Cambridge, 1948, p. 157.

geometric angle of attack, α, is decreased because of three-dimensional effects. In order to calculate α_i the wing is replaced by a lifting vortex line, the so-called bound vortex, at the quarter-chord line of the wing.

The lift at each spanwise station varies with the strength of the bound vortex at that station according to the law of Kutta and Joukowski,

$$l(y) = \rho V \Gamma(y) \tag{4}$$

where $l(y)$ is the lift per unit span at each station and $\Gamma(y)$ is the circulation about each station. The circulation of the bound vortex must vary spanwise as the lift, but since the amount of circulation cannot be decreased in an inviscid fluid a vortex filament having a strength equal to the change in the strength of the bound vortex at each station is assumed to trail off into the free stream behind the wing, and it follows that a sheet of vortices is formed behind the wing as a result. By applying the law of Biot and Savart an expression can be derived for α_i in terms of the circulation $\Gamma(y)$, and an integral equation involving only $\Gamma(y)$ as an unknown can then be written at each spanwise station.

$$l(y) = \rho V \Gamma(y) = c_{l_\alpha} c \tfrac{1}{2} \rho V^2 (\alpha - \alpha_i) \tag{5}$$

$$\Gamma(y) = c_{l_\alpha} \frac{c}{2} V \cdot (\alpha - \alpha_i) \tag{6}$$

where
$$\alpha_i = \frac{1}{4\pi V} \int_{-b/2}^{b/2} \frac{d\Gamma(y)}{d\eta} \frac{d\eta}{y - \eta} \tag{7}$$

and η is a dummy variable representing the spanwise ordinate. Consequently,

$$\alpha = \frac{2\Gamma(y)}{c_{l_\alpha} c V} + \frac{1}{4\pi V} \int_{-b/2}^{b/2} \frac{d\Gamma}{d\eta} \frac{d\eta}{y - \eta} \tag{8}$$

This equation may be solved by the method indicated in Glauert's text[7] or in the papers of Multhopp[8] or Benscoter[9]. The problem of the lifting line has also been treated by Trefftz[10] who considered a plane normal to the free stream located infinitely far behind the wing and showed that the three-dimensional problem of the lift distribution on a wing of finite span is equivalent to a two-dimensional problem in this "Trefftz plane."

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[7] Ibid.

[8] H. Multhopp, Die Berechnung der Auftriebverteilung von Tragflügeln, Luftfahrtforschung, Bd. 15, Lft. 4, April 6, 1938.

[9] Stanley U. Benscoter, Matrix Development of Multhopp's Equations for Spanwise Air-Load Distribution. Journal of the Aeronautical Sciences, February 1943, Vol. 15, No. 2.

[10] E. Trefftz, Prandtlsche Tragfläche und Propeller-Theorie. Z. a. M. M., Vol. 1, p. 206.

Lifting-surface theory. The lifting-surface approach, first suggested in 1919 by Betz[11] has been used more recently by Falkner[12], Wieghardt[13] and Weissinger[14] who introduced certain simplifications which facilitated the use of this approach. Falkner reduced the mathematical difficulties inherent in the lifting surface approach by using a finite number of discrete horseshoe vortices to represent the wing rather than an infinite number of infinitesimal vortices. This procedure leads to a set of linear simultaneous equations in which the downwash angle induced by the vortex field representing the wing is equated to the geometric angle of attack at a number of discretely chosen points.

A simplified lifting surface method for calculating spanwise lift distributions was proposed by Wieghardt on the basis of a result obtained from two-dimensional

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[11] Albert Betz, Beitrage zur Tragflügel theorie mit besonderer Berücksichtigung des einfachen rechteckigen Flügels, Diss. Georg-August-Universitat (Gottingen), 1919.

[12] V. M. Falkner, The Calculation of the Aerodynamic Loading on Surfaces of any Shape, ARC 6997, R and M 1910.

[13] Karl Wieghardt, Chordwise Load Distribution of a Simple Rectangular Wing. NACA T. M. No. 963, 1940.

[14] J. Weissinger, The Lift Distribution of Swept-Back Wings, T. M. No. 1120, NACA, March 1947 (translation).

thin-airfoil theory which states that if all the lift carried by a flat plate or parabolically cambered airfoil were represented by a concentrated vortex at the quarter-chord line this vortex would induce a downwash angle equal to the slope of the airfoil at the three-quarter chord line. Consequently, Wieghardt reasoned that a solution for the lift distribution on a finite wing could be obtained by locating a vortex at the quarter-chord line of the wing and equating the induced downwash angle to the geometric angle-of-attack at the three-quarter chord line.

Weissinger developed a convenient method for making computations based on this simplified theory. The equation for the spanwise distribution of the circulation, Γ as given by Weissinger is

$$\frac{w_n}{V} = \frac{w_{3c/4}}{V} = \frac{1}{4\pi}\int_{-1}^{1} \frac{d\Gamma}{d\eta^*} \frac{d\eta^*}{y^*-\eta^*} + \frac{1}{8\pi\, c^*/2}\int_{-1}^{1} L(y^*,\eta^*)\frac{d\Gamma}{d\eta^*}\, d\eta^* \qquad (9$$

$$\text{where} \quad L(y^*,\eta^*) = \frac{c^*/2}{y^*-\eta^*}\left\{\sqrt{\left[1+\left(\frac{y^*-\eta^*}{c^*/2}\right)\tan\Lambda\right]^2 + \left(\frac{y^*-\eta^*}{c^*/2}\right)^2} - 1\right\};\ \eta^* \geq 0$$

$$= \frac{c^*/2}{y^*-\eta^*}\left\{\frac{\sqrt{\left[1+\left(\frac{y^*+\eta^*}{c^*/2}\right)\tan\Lambda\right]^2 + \left(\frac{y^*-\eta^*}{c^*/2}\right)^2}}{1+2\frac{y^*}{c^*/2}\tan\Lambda} - 1\right\};\ \eta^* \leq 0$$

$$+ 2\tan\Lambda\,\frac{\sqrt{\left(1+\frac{y^*}{c^*/2}\tan\Lambda\right)^2 + \left(\frac{y^*}{c^*/2}\right)^2}}{1+2\frac{y^*}{c^*/2}\tan\Lambda}\Bigg]$$

The integrations are performed by numerical methods, and a set of linear simultaneous equations is written in which the values of the circulation at several points along the span are the only unknowns. A step by step procedure for using the Weissinger method is given in a paper by Van Dorn and De Young[15].

Another approach using the Wieghardt quarter-chord, three-quarter chord concept but using discrete horseshoe vortices rather than a line vortex is also available. In this approach a number of horseshoe vortices are distributed on the wing quarter-chord line and the downwash angle induced by this vortex field is set equal to the geometric angle-of-attack at the three-quarter chord line. Convenient tables for applying this calculation scheme are presented in a paper by Diederich[16]. This technique is applicable to any planform, while Weissinger's method is restricted to wings which do not have curved quarter-chord lines.

The Weissinger method can be shown to give the theoretically correct result for the limiting cases of

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[15] Nicholas H. Van Dorn and John DeYoung, A Comparison of Three Theoretical Methods of Calculating Spanload Distribution on Swept-Wings, NACA TN No. 1491, 1947.

[16] Franklin W. Diederich, Charts and Tables for Use in Calculations of Downwash of Wings of Arbitrary Plan Form, (To be published as NACA TN No. 2353).

zero aspect ratio[17] and infinite aspect ratio. A comparison of experimental results with results calculated by the Weissinger method is presented in a paper by Jacobs[18].

Calculation of the velocity induced by bodies of revolution. In general it is possible to represent a body in a flow by introducing a series of sources and sinks in the stream. When the body is slender, the problem may be simplified by assuming that the disturbances are small, and are confined to the transverse section at which they take place. A technique using this assumption to treat the problem of flow about slender bodies is described in a paper by Young and Owen[19], and Figure 1 shows the longitudinal distribution of the velocity about a typical slender body of revolution calculated by Young and Owen. Another technique which assumes that the disturbances produced by a slender body are small and that the flow is

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[17] Franklin W. Diederich and Martin Zlotnick, Theoretical Spanwise Lift Distributions of Low-Aspect-Ratio Wings at Speeds Below and Above the Speed of Sound, NACA TN No. 1973, 1949.

[18] W. Jacobs, Pressure Distribution Measurement on Unraked Swept-Back Wings, NACA TN No. 1164.

[19] A. D. Young and P. R. Owen, A Simplified Theory for Streamline Bodies of Revolution and its Application to the Development of High-Speed Low-Drag Shapes. R & M No. 2071.

two dimensional at each transverse section is presented by Munk[20].

The lateral distribution of velocity induced by a body is of special interest since it affects the local velocity at each wing section and hence helps determine the lift at each section. The lateral velocity distribution is calculated below for a Rankine solid[21] along an axis passing through the midpoint of the body and normal to its axis of symmetry.

A Rankine solid is generated by placing a source and sink in a uniform rectilinear flow. In cylindrical coordinates, the stream function ψ is written[22]

$$\psi = \frac{U}{2}\left[\left(\frac{R}{a}\right)^2 + \frac{\sqrt{1+\left(\frac{R}{a}\right)^2}}{2m/a}\left\{\frac{(x/a - m/a)}{\sqrt{(R/a)^2 + (x/a - m/a)^2}} - \frac{(x/a + m/a)}{\sqrt{(R/a)^2 + (x/a + m/a)^2}}\right\}\right] \tag{10}$$

where the midpoint of the body is at the origin $x/a = 0$ and $(-m/a)$ and $(+m/a)$ are the distances of the source and sink from the origin, and

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[20] Max M. Munk, The Aerodynamic Forces on Airship Hulls, NACA TR No. 184, 1924.

[21] L. M. Milne-Thomson, Theoretical Hydrodynamics, The Macmillan Company, New York, 1950, p. 411.

[22] Ibid.

$$\frac{u}{U} = \frac{1}{UR/a} \frac{\partial \psi}{\partial (R/a)} \tag{11}$$

Taking the partial derivative of ψ with respect to R/a, dividing by UR/a, and setting $x = 0$ yields the desired expression,

$$\frac{u}{U} = 1 + \frac{\sqrt{1 + (m/a)^2}}{2\left[(R/a)^2 + (m/a)^2\right]^{3/2}} \tag{12}$$

The relation between m and ℓ is then

$$\ell/a \sqrt{1 + (m/a)^2} = \left[(\ell/a)^2 + (m/a)^2\right]^2 \tag{13}$$

Figure 2 shows $\frac{\Delta u}{U} = \frac{u}{U} - 1$ plotted at various values of R/a for two Rankine solids, one a relatively slender body $\ell/a = 7$, the other $\ell/a = 3.02$.

II. ANALYSIS OF THE WING FUSELAGE COMBINATION

<u>Solutions of idealized cases by Lennertz and Vandrey.</u> The wing-fuselage combinations most easily treated by theoretical methods is that of a spherical fuselage with an infinite wing, since this configuration can be represented by a single doublet located at the midpoint of an infinite vortex of constant strength in a uniform rectilinear flow. Using this representation of a wing-fuselage combination Lennertz[23] found that the lift distribution on the wing

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[23] J. Lennertz, <u>Beitrag zur theoretischen Behandlung des gegenseitigen Einflusses von Tragfläche und Rumpf</u>, Z. f. a. M. M., Bd. 7, August 1927, pp. 249-276.

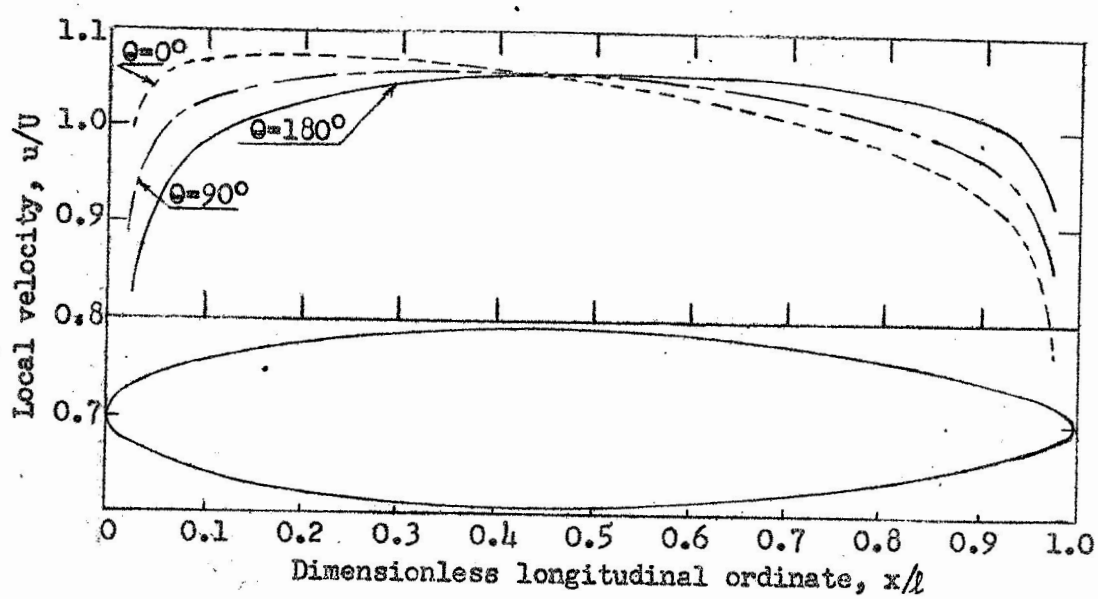

Figure 1.- Longitudinal velocity distribution on a typical slender body calculated by Young and Owen, $\alpha = 5^{\circ}$.

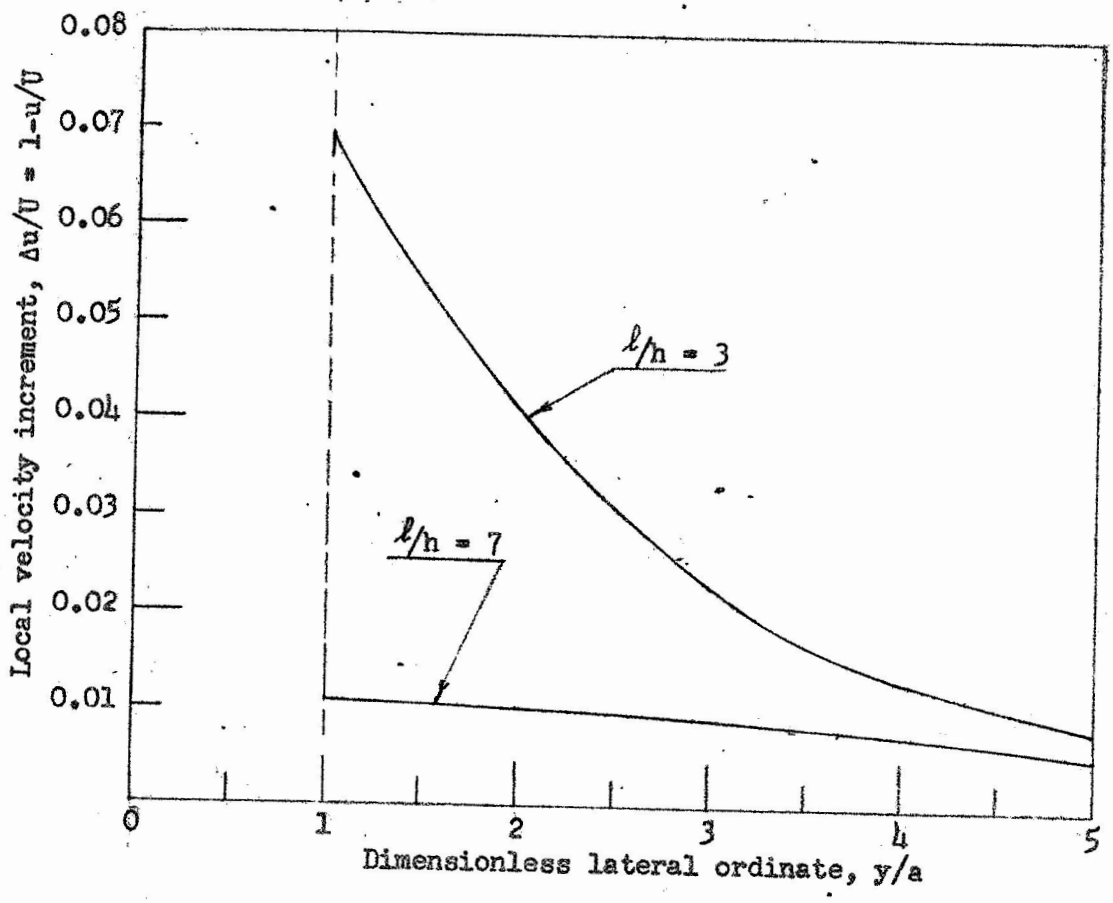

Figure 2.- Lateral velocity distribution for two Rankine solids.

is unaffected by the presence of the sphere and that the sphere carries a lift exactly equal to the lift carried by the part of the wing which it covers. However, as pointed out by Vandrey[24], this result is obtained only when the longitudinal velocity induced by the sphere is neglected. Vandrey shows that the lift on the wing is increased in the presence of the sphere, and that the lift on the sphere is reduced by an amount exactly equal to the increase in lift on the wing. The increase in lift on the wing can be ascribed only to the increase in the longitudinal velocity induced by the sphere since the presence of the sphere does not alter the effective angle-of-attack on the wing.

Lennertz also treats the case of a wing-fuselage combination consisting of a wing represented by an infinite vortex of constant strength which intersects an infinitely long cylindrical fuselage aligned with the free stream. For this case Lennertz finds that the lift on the cylinder is equal to the lift on the part of the wing which is covered by the cylinder and that the effective angle of attack on the wing is unchanged so that there is no change

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[24] F. Vandrey, Zur theoretischen Behandlung des gegenseitigen Einflusses von Tragflügel und Rumpf. Luftfahrtforschung, Bd. 14, July 20, 1937, pp. 347-355.

in the lift distribution on the wing in the presence of the fuselage.

The flow over a cylinder-vortex combination represents the flow over a wing at an angle of attack attached to a long fuselage at zero angle of attack. Similarly, the flow over the sphere-vortex combination is representative in a qualitative sense of the flow over a wing with a nacelle. However, the assumption of constant circulation is contrary to reality. Also, there is some objection to representing the wing of a wing-fuselage combination by a single concentrated vortex, inasmuch as this approximation neglects the variation of mutual inductance effects along the chord. Consequently, the results obtained for this idealized case are useful in a qualitative sense only.

Despite the fact that the sphere has a slenderness ratio of 1, whereas nacelles have slenderness ratios usually greater than 3, the same type of effect on the wing is to be expected if the nacelle is in the mid-wing position, although the magnitude of the effect is likely to depend on the slenderness-ratio of the nacelle.

<u>Method of Wieselberger</u>. In calculating the lift distribution on a wing-nacelle combination the nacelles can be considered as sections of the wing at which there

are discontinuities of the chord and of the section characteristics or effective angle-of-attack. This concept was used by Wieselsberger[25]; he used a lifting-line method and reduced the value of the local section lift-curve slope at the spanwise station which is at the center line of the nacelle. His results, reproduced in Figure 3, show a decrease in the lift extending over the portion of the wing adjoining the nacelle. This result is not in general agreement with experimental results, since Wieselsberger does not take the vertical and longitudinal location of the nacelle into account at all, and does not take its lateral location on the wing and its maximum width into account accurately. For example, the results from wind-tunnel tests by McLellan and Cangelosi[26] shown in Figure 4 on a nacelle in the mid-wing position indicate an increase in the lift (rather than a decrease calculated by Wieselsberger) on the portion of the wing adjoining the nacelle. This increase in lift can be attributed to the increase in longitudinal flow induced by the nacelle; unconservative estimates of the shear and bending moment,

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[25] C. Wieselsberger, Der Einfluss von eingebauten Motorgondeln auf die Luftkräfte eines Tragflügels. Paper No. 20[illegible] read before the Internat. Engineering Congress, Tokyo, 1929. (From W. F. Durand, Aerodynamic Theory, Vol. IV, p. 161)

[26] Charles H. McLellan and John I. Cangelosi, Effects of Nacelle Position on Wing-Nacelle Interference, NACA TN 1593.

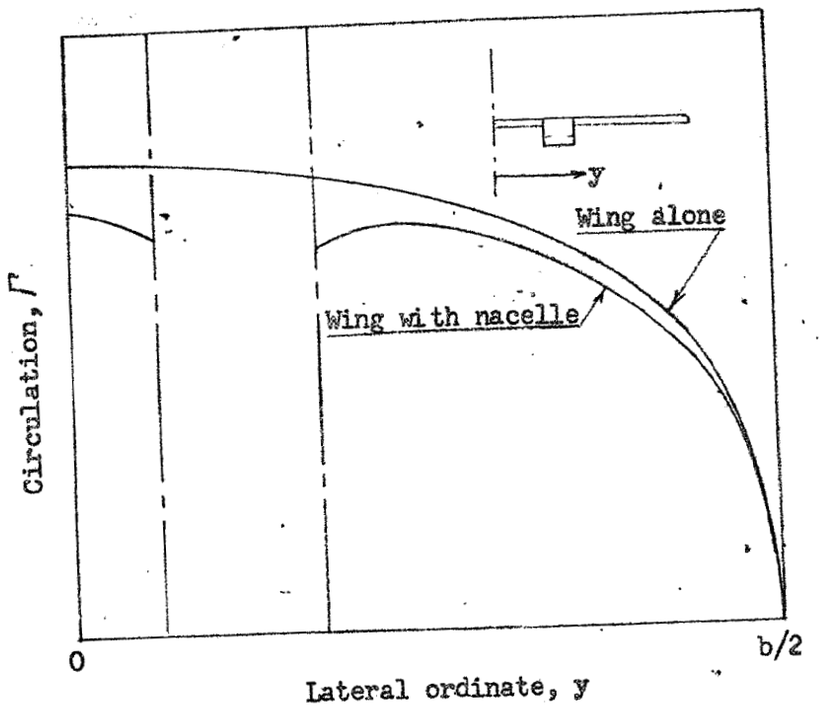

Figure 3.- Lift distribution on wing-nacelle combination calculated by Wieselsberger.

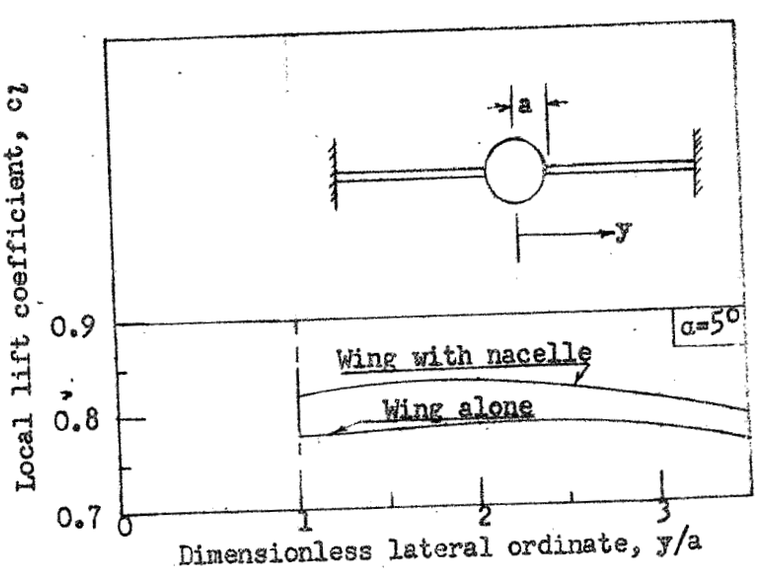

Figure 4.- Spanwise lift distribution on a wing-nacelle combination obtained in experiments by McLellan and Cangelosi.

especially at the outboard juncture of the wing and nacelle, may be made if this additional longitudinal flow is neglected. The effect of this additional longitudinal flow is likely to vary with the vertical position, of the nacelle the amount and type of filleting, and the amount of turbulence in the flow.

Method of Multhopp. In a paper by Multhopp[27] conformal mapping is used in conjunction with the lifting-line theory in order to calculate the lift distribution on a wing in the presence of a fuselage. Two significant effects of the fuselage on the spanwise lift distribution are considered:

(1) The blocking effect of the fuselage.

(2) The increased vertical flow on the wing due to the presence of the fuselage, which changes the wing effective angle of attack.

These two effects are for a fuselage which is assumed to be an infinite cylinder, so that no longitudinal flow is induced by the fuselage. A fuselage of finite length would have different effects on the longitudinal and vertical

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[27] H. Multhopp, Zur Aerodynamik des Flugzeugrumpfes Luftfahrtforschung, Bd. 18, March 29, 1941, pp. 52-56, (translation NACA TM No. 1056).

flows, but these differences are judged by Multhopp to be of second order for the usual configurations.

According to the concept of the Trefftz plane the lifting-line equation can be satisfied in a plane infinitely far downstream of the wing. In this Trefftz plane a complex coordinate $u = z + iy$ may be defined where y is the lateral and z the vertical ordinate. By means of a conformal transformation[28], the u-plane may be transformed into a $\bar{u}$-plane in which the fuselage cross-section at the intersection of the cylindrical fuselage and the u-plane is transformed into a vertical slit. The flow in the u plane has the y and z components of the actual flow; its "free-stream" velocity far away from the fuselage is an upward velocity of magnitude V_g, the vertical component of the actual free-stream velocity. This flow when transformed into a flow in the $\bar{u}$ plane is much simpler to analyze, since the vertical slit in the $\bar{u}$ plane poses no obstacle to the "free stream." Furthermore, in the $\bar{u}$ plane, the boundary condition of zero flow normal to the surface of the (transformed) fuselage is automatically satisfied if the flow in the u plane is

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[28] H. Glauert, The Elements of Aerofoil and Airscrew Theory, Cambridge, 1948, p. 64.

symmetrical about the z axis. The velocities in the $\overline{w}$-plane must be multiplied by $\mathcal{R}(\frac{dw}{du})$, the real part of the conformal-transformation factor in order to get the flow in the physical plane; the real part is used since only the vertical component of the flow is of interest.

The lift distribution for the wing-fuselage combination is calculated by lifting-line theory. In this case the local lift is written

$$\Gamma = c_{l_a} \rho/2 \, V_{a_{effective}} = c_{l_a} \rho/2 \, w_n \tag{16}$$

where $$w_n = V_a + w_{n_f} - w_i \tag{17}$$

where, in turn, V_a is the normal flow on the wing, w_{n_f} is the incremental normal flow induced by the fuselage and w_i is the downwash flow. Now, in the transformed plane infinitely far behind the wing $\overline{w}_{i_\infty}(\overline{y})$ is written

$$\overline{w}_{i_\infty}(\overline{y}) = -\frac{1}{2\pi}\int_{-b/2}^{b/2} \frac{d\Gamma}{d\overline{\eta}} \frac{d\overline{\eta}}{\overline{\eta} - \overline{y}} \tag{18}$$

so that in the real plane

$$w_{i_\infty}(y) = \mathcal{R}(\frac{dw}{du}) \, \overline{w}_{i_\infty}(\overline{y}) \tag{19}$$

where $w_{i_\infty}(y)$ the downwash infinitely far behind the

wing is twice the downwash at the quarter-chord line, $w_1(y)$. Similarly, the incremental vertical flow due to the fuselage in the real plane can be expressed as

$$w_{nf} = V\alpha_f(\bar{y})\,\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right) - V\alpha_f(\bar{y}) \tag{20}$$

where α_f is the angle of attack of the fuselage.

The lifting line equation for the wing in the physical plane can then be written in terms of the transformed ordinates as,

$$\Gamma(\bar{y}) = \frac{1}{2}\,c\ell_\alpha(\bar{y})c(\bar{y})\left\{V\alpha(\bar{y}) + V\alpha_f\left[\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)-1\right] + \mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{d\Gamma}{d\bar{\eta}}\frac{d\bar{\eta}}{\bar{\eta}-\bar{y}}\right\}$$

or

$$\Gamma(\bar{y}) = \frac{1}{2}c\ell_\alpha(\bar{y})c(\bar{y})\,\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)V\left\{\frac{\alpha(\bar{y})-\alpha_f(\bar{y})}{\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)} + \alpha_f + \frac{1}{4\pi V}\int_{-b/2}^{b/2}\frac{d\Gamma}{d\bar{\eta}}\frac{d\bar{\eta}}{\bar{\eta}-\bar{y}}\right\} \tag{21}$$

This equation may be solved in the same way as the original lifting-line equation, except that the chord of the wing is multiplied by the factor $\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)$ and that the angle of incidence between the wing and fuselage axis, $\alpha - \alpha_f$, is divided by the factor $\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)$. A calculated value of at

a given value of $\bar{y}$ is plotted at the station y in the real plane corresponding to the station $\bar{y}$ in the transformed plane.

The results of a calculation by Multhopp's method for a rectangular wing of aspect ratio 4.5 attached at the mid-wing position to a fuselage having a radius equal to 0.2 $b/2$ is compared in Figure 5 with experimental results on

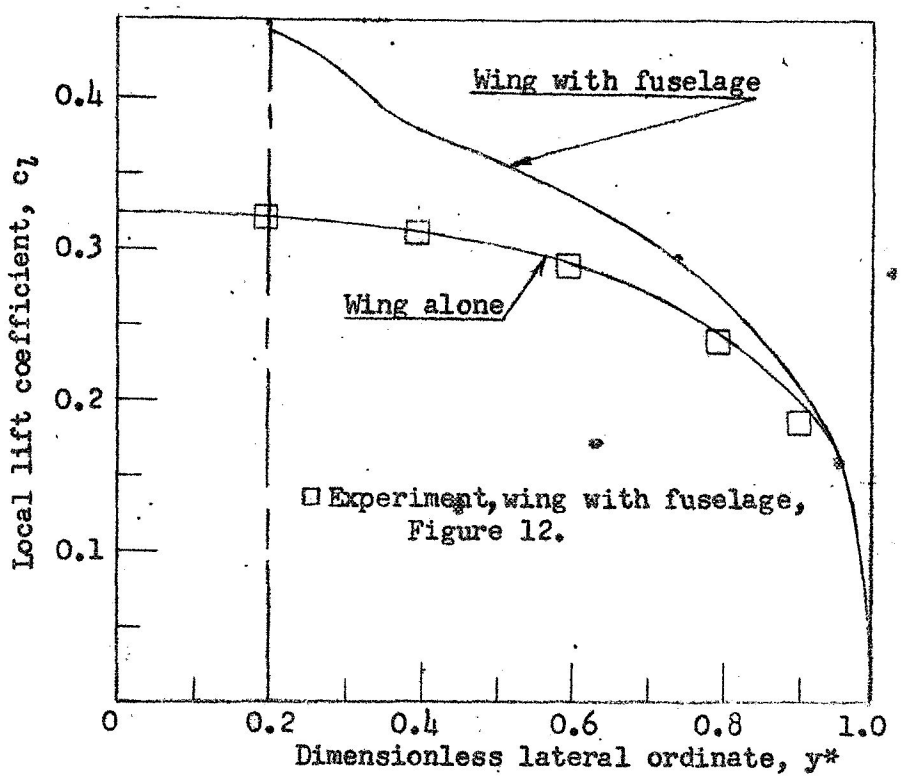

Figure 5.- Calculation for wing with and without fuselage by Multhopp's method, $A = 4.5$, $\lambda = 1$, $\Lambda = 0^\circ$, $a^* = 0.2$, $\alpha = 4^\circ$.

similar configuration. Although there is qualitative agreement, it is seen that the calculated lift distribution is too high at the root. Although he did not have any measurements with which to compare his results Multhopp carefully noted that his method for treating the wing-fuselage combination does not take into account the effect of the fuselage on the flow induced by the bound vortex and that it would be more correct to calculate the angles of attack at the three-quarter chord line rather than at the quarter-chord line as he does. Multhopp therefore indicates that the lift calculated by his method is likely to be too large near the wing root. Another limitation of Multhopp's method is that it is subject to the same restrictions as the lifting-line theory on which it is based and consequently cannot be expected to yield satisfactory results for wings of low aspect ratio or wings with large angles of sweepback.

<u>Spreiter's method</u>. When the aspect ratio of the wing approaches zero the spanwise lift distribution on a wing-fuselage combination can be calculated by making the assumption that the longitudinal flow-disturbances are small, as is done in the aforementioned slender-body theory so that the problem of calculating the lift distribution is essentially two-dimensional in planes normal

to the free-stream velocity. A solution for the lift distribution on a delta wing of low aspect ratio attached at the mid-wing position to a body having a circular cross section has been effected by Spreiter[29] and is

$$\gamma(y^*) = 2\sqrt{2}\left[-\left(1+\frac{a^{*4}}{y^{*4}}\right)y^{*2}+(1+a^{*4}) + \sqrt{y^{*4}\left(1+\frac{a^{*8}}{y^{*8}}\right)+2a^{*4}+(1+a^{*4})-2(1+a^{*4})\left(1+\frac{a^{*4}}{y^{*4}}\right)y^{*2}}\right]^{1/2} \tag{22}$$

where all the symbols are defined on page 7. The lift distribution for the case when the fuselage radius is 0.3 $b/2$ is shown in Figure 6.

Spreiter's solution is presented here because it is exact for a wing of vanishingly small aspect ratio attached to a slender fuselage, so that it is useful in judging the accuracy of results obtained by applying more general methods to the analysis of this limiting case.

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[29] John R. Spreiter, Aerodynamic Properties of Slender Wing-Body Combinations at Subsonic, Transonic, and Supersonic Speeds. NACA TN No. 1662.

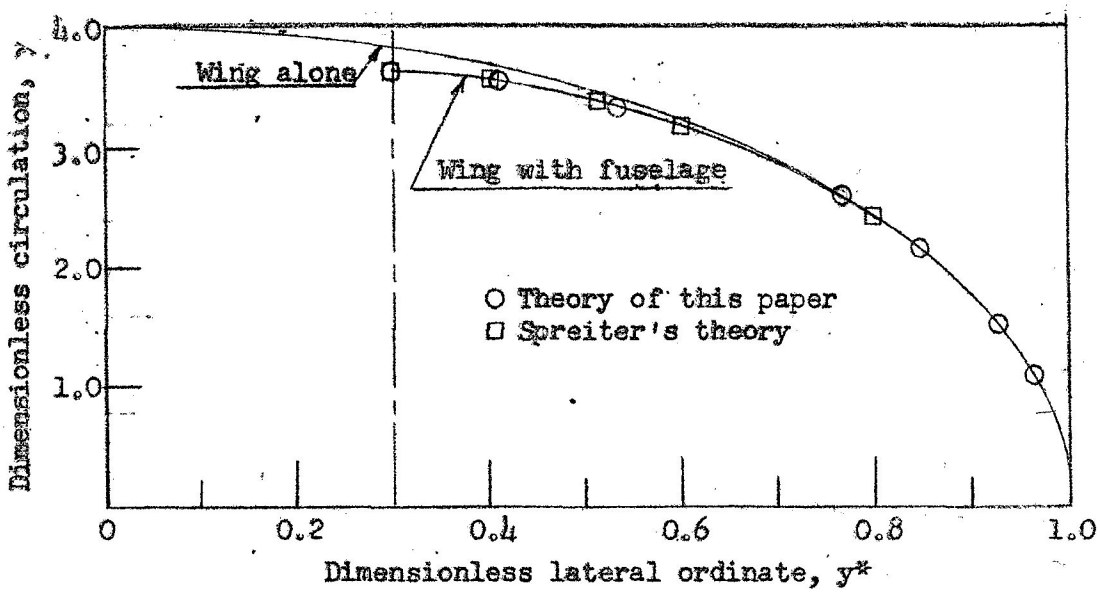

Figure 6.- Theoretical lift distribution on a combination of a fuselage with a wing of low aspect ratio, $a^* = 0.3$.

CHAPTER III

A NEW METHOD FOR CALCULATING THE EFFECT OF THE FUSELAGE ON THE SPANWISE LIFT DISTRIBUTION ON THE WING

I. DESCRIPTION OF THE NEW METHOD

<u>Analysis for the infinitely long fuselage</u>. It has been pointed out in the preceding chapter that by means of a simplified lifting-surface method, which consists of placing a concentrated vortex at the quarter-chord line and satisfying the boundary condition at the three-quarter chord line, accurate lift distributions can be calculated for wings of any aspect ratio with or without sweepback. This method is combined in this chapter with Multhopp's conformal mapping procedure[30] in an effort to obtain a method suitable for calculating lift distributions more accurately and for a wider variety of plan forms than possible by means of lifting-line theory.

Inasmuch as the concept of conformal mapping is based on two-dimensional considerations, and inasmuch as the

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[30] H. Multhopp, <u>Zur Aerodynamik des Flugzeugrumpfes</u>, Luftfahrtforschung, Bd. 18, March 29, 1941, pp. 52-56. (Translation NACA TM No. 1036).

simplified lifting-surface method cannot in general be reduced to a two-dimensional problem as can lifting-line theory, the method outlined herein is not rigorously justifiable. However, as is shown in Chapter IV it is justifiable for wings of very low aspect ratio, because in those cases the simplified lifting-surface method can be reduced to a two-dimensional problem. It will also be shown in Chapter IV that the results obtainable by the method under consideration constitute a very close approximation to the correct results in the case of wings of infinite aspect ratio. Therefore, there is good reason to believe that it will furnish useful answers for wings of intermediate aspect ratios.

The integro-differential equation of the simplified lifting-surface method is given in Chapter II as equation 9. Now in the presence of the fuselage the normal-flow component w_n is

$$w_n = V\alpha + w_{nf} \tag{23}$$

where α is the local angle of attack on the wing, and w_{nf} is the incremental normal flow on the wing caused by the fuselage. The lift distribution is now calculated for a wing-fuselage configuration obtained from the given one by a conformal transformation (in planes normal to the

free-stream velocity) in which the cross-section of the fuselage is a vertical slit, so that there is no fuselage interference. The velocity components normal to the free-stream velocity and parallel to the plane of symmetry are then multiplied by the conformal-transformation factors, and the simplified-lifting-surface equation is written for the wing in physical space, but in terms of the transformed ordinates. Thus in physical space

$$w_{n_f} = V\alpha_f(\bar{y})\,\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right) - V\alpha_f(\bar{y}) \tag{24}$$

where α_f is the angle of attack of the fuselage; also

$$w_{3c/4} = \mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)\bar{w}_{3c/4}(\bar{y}) \tag{25}$$

where $\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)$ is the real part of the conformal transformation factor as in Multhopp's treatment of the wing-fuselage combination.

$$\alpha(\bar{y}^*) + \alpha_f(\bar{y}^*)\left[\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right) - 1\right] = \mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)\left\{\frac{1}{4\pi}\int_{-1}^{1}\frac{d\bar{\Gamma}}{d\bar{\eta}^*}\,\frac{d\bar{\eta}^*}{\bar{y}^* - \bar{\eta}^*} + \frac{2}{\bar{c}^*\beta^*}\int_{-1}^{1} L(\bar{y}^*, \bar{\eta}^*)\,\frac{d\bar{\Gamma}}{d\bar{\eta}^*}\,d\bar{\eta}^*\right\} \tag{26}$$

$$\frac{\alpha(\bar{y}^*) - \alpha_f(\bar{y}^*)}{\mathcal{R}\left(\frac{d\bar{\eta}}{du}\right)} + \alpha_f(\bar{y}^*) = \frac{1}{4\pi}\int_{-1}^{1}\frac{d\bar{\Gamma}}{d\bar{\eta}^*}\,\frac{d\bar{\eta}^*}{\bar{y}^* - \bar{\eta}^*} + \frac{2}{\bar{c}^*\beta^*}\int_{-1}^{1} L(\bar{y}^*, \bar{\eta}^*)\,\frac{d\bar{\Gamma}}{d\bar{\eta}^*}\,d\bar{\eta}^* \tag{27}$$

Comparison of equation 27 with equation 9 shows that the procedure for calculating the lift distribution on the wing-fuselage combination is only a slightly modified form of that required for the wing alone.

<u>Step-by-step computing procedure for a mid-wing configuration.</u> The case of the cylindrical fuselage with a mid-wing is considered in this section for the sake of definiteness, but other fuselage cross-sections and wing locations may be treated in the same manner provided the correct transformation factor is used. (See also Multhopp's paper[31])

The lateral and vertical coordinates in physical space y and z, respectively, are combined into one complex coordinate

$$u = z + iy \tag{28}$$

Similarly, these coordinates in the space obtained by transforming each plane in physical space normal to the free-stream velocity are combined into a complex coordinate

$$\bar{u} = \bar{z} + i\bar{y} \tag{29}$$

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[31] H. Multhopp, <u>Zur Aerodynamik des Flugzeugrumpfes</u>. Luftfahrtforschung, Bd. 18, March 29, 1941, pp. 52-56, (Translation NACA TM No. 1036).

For the given configuration the relation between the two complex coordinates is

$$\bar{u} = u + \frac{a^2}{u} \tag{30}$$

where a is the radius of the fuselage. The calculation for the spanwise lift distribution is then made as follows:

(1) In the transformed plane, the spanwise ordinates of the wing $\bar{y}$ and $\bar{x}$ are

$$\bar{y} = y\left(1 - \frac{a^2}{y^2}\right) \tag{31}$$

so that when $y = a$, $\bar{y} = 0$, and when $y = b/2$, $\bar{y} = \bar{b}/2$, where $\frac{\bar{b}}{b} = \left[1 - \left(\frac{a}{b/2}\right)^2\right]$

and

$$\bar{x} = x$$

since the longitudinal ordinates are not changed by the transformation.

(2) Since

$$\frac{d\bar{u}}{du} = 1 - \frac{a^2}{u^2} \tag{32}$$

the real part of the conformal-transformation factor,

$$\mathcal{R}\left(\frac{d\bar{u}}{du}\right) = 1 + \frac{a^2}{y^2} \tag{33}$$

for $z = 0$, the vertical position of the wing.

(3) If the wing is unswept, equation 27 is solved in the usual manner for the transformed wing, except that $(\alpha - \alpha_f)$ is divided by $\mathcal{R}(\frac{dz}{dx})$. The difference $\alpha - \alpha_f$ is 0 when the wing is not set at an angle of incidence with the fuselage axis.

(4) If the wing is swept another method must be used for treating the transformed wing, since the transformed swept wing has a curved quarter-chord line (see Figure 7), a case for which Weissinger's method cannot be applied.

The simplest procedure is probably the use of horseshoe vortices, about 10 on each semispan appears to be a satisfactory number, centered on the quarter-chord line. By calculating the downwash induced by each vortex at 10 points on the three-quarter chord line of the semispan and equating the sum to the slope of the wing at the three-quarter chord line, a set of 10 simultaneous equations is obtained for the 10 unknown strengths of the vortices. In calculating the downwash due to the vortices,

the tables in a paper by Diederich[32] are very helpful.

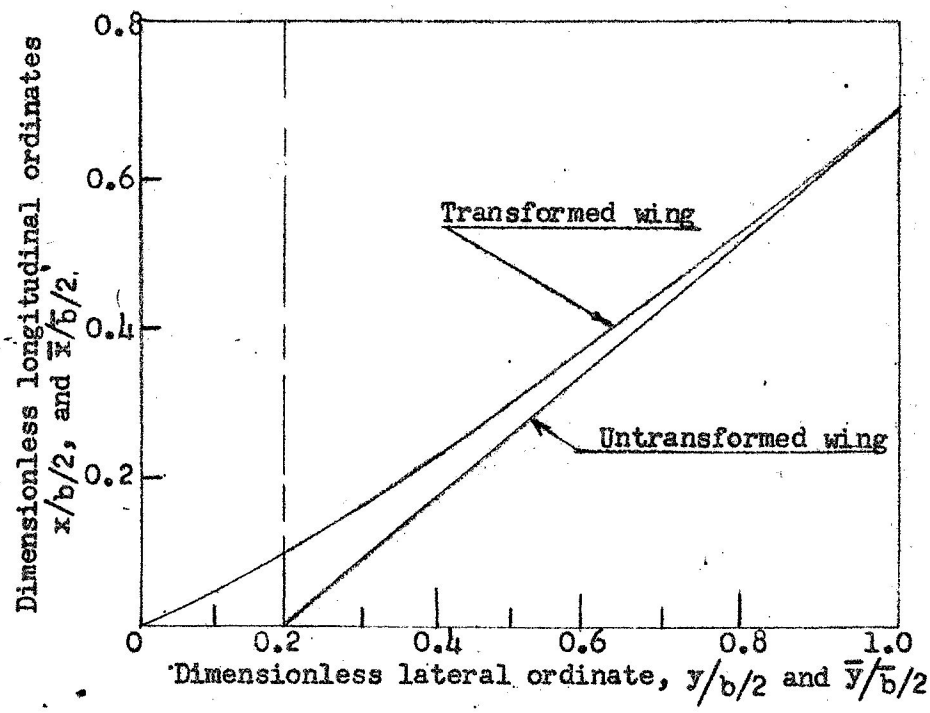

Figure 7.- Dimensionless coordinates of quarter-chord lines of transformed and untransformed wing, $\Lambda = 40^{\circ}$.

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[32] Franklin W. Diederich, Charts and Tables for Use in Calculations of Downwash of Wings of Arbitrary Plan Form (To be published as NACA TN No. 2355).

(5) The relation between the dimensionless circulation on the wings in the physical and transformed space, respectively, is

$$\gamma = \frac{\bar{b}}{b}\,\bar{\gamma} \tag{34}$$

(6) The dimensionless circulation γ is plotted at the ordinates in the physical plane; which, from step 2 is

$$y^* = \frac{\bar{b}}{b}\left[\frac{\bar{y}^* + \sqrt{\bar{y}^{*2} + 4\left(\frac{b}{\bar{b}}a^*\right)^2}}{2}\right] \tag{35}$$

where

$$\bar{y}^* = \frac{\bar{y}}{\bar{b}/2}$$

and

$$a^* = \frac{a}{\bar{b}/2}$$

If for instance the lift distribution is known on a rectangular wing alone, the lift distribution for the combination of this wing and a cylindrical fuselage can be obtained quite readily. The results of the calculation for the wing alone can be assumed to be those for the transformed wing except for a small correction which can be made by applying lifting line theory, provided the ratio of the

span to the fuselage diameter is large so that $\bar{b}/b$ (equation 31) is nearly unity. The transformation is then made back into the physical plane so that the lift is plotted as indicated in step (6). If γ_{w_f} is the dimensionless circulation on the wing-fuselage combination and $\bar{\gamma}_w$ is the dimensionless circulation on the wing alone then

$$\gamma_{w_f}(y^*) = \left[\bar{\gamma}_w(\bar{y}^*)\,\frac{\bar{b}}{b}\right]\left[\frac{\frac{\bar{b}}{b}A + 2}{A + 2}\right] \tag{36}$$

The correction applied in equation 36 is based on the lifting line solution for the elliptical wing[33]:

$$c_l = 2\pi\,\frac{A}{A + 2}\,\alpha \tag{37}$$

and the identity which holds in the case of a rectangular wing

$$\gamma = \frac{2}{A}\,c_l \tag{38}$$

<u>Correction for the finite length of the fuselage.</u> In representing the fuselage by an infinite cylinder the inflow effect is neglected. The purpose of this section is to indicate a correction for this effect in the case

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[33] H. Glauert, <u>The Elements of Aerofoil and Airscrew Theory</u>, Cambridge, 1948, p. 137.

of a mid-wing configuration. The lateral distribution of the increment in longitudinal velocity induced by the fuselage is shown in Figure 2 for two fairly typical bodies; the body with the slenderness ratio of 3 approximates a nacelle, and the body with the slenderness ratio of 7 approximates a slender fuselage. An increase of the effective free-stream dynamic pressure is the immediate result of the induced longitudinal velocity. If the increase in the velocity is small and does not have a large lateral gradient as, for example, in the case of the slender body in Figure 2 and the wing has a high aspect ratio, the effect on the spanwise lift distribution may be calculated as follows:

If the local lift at a given spanwise station is $U(1 + \delta)$ where $\delta << 1$ then the lift in the region of increased velocity is

$$l = c_l \, c \, \frac{1}{2} \rho U^2 (1 + \delta)^2 \tag{39}$$

$$\approx c_l \, c \, \frac{1}{2} \rho U^2 (1 + 2\delta) \tag{40}$$

but the lift in the uniform flow is

$$l_o = c_l \, c \, \frac{1}{2} \rho U^2 \tag{41}$$

so that

$$l = (1 + 2\delta)l_o \tag{42}$$

Consequently, the lift calculated by the method outlined in the preceding section can be corrected approximately for the inflow effect by multiplying it by the factor $(1 + 2\delta)$ appropriate to the given fuselage and any spanwise station.

The foregoing analysis applies only to the case of the mid-wing configuration, since in the case of the high wing or low-wing configurations there are additional effects which cannot be calculated easily. As shown in Figure 1, there is a larger induced velocity at the high-wing position than there is at the low-wing position, so that there is likely to be more lift on the high-wing configuration than there is on the low-wing configuration. However, since the wing is of finite thickness the longitudinal velocity induced by the fuselage may be different on the wing upper surface and on the lower surface; this difference tends to increase or decrease the circulation on the wing and hence to change its lift accordingly.

II. PRESENTATION OF RESULTS

The spanwise lift distributions have been calculated for two mid-wing configurations, one with an unswept wing and one with a swept wing; that for the swept wing has been performed by means of the horseshoe-vortex method described previously. The correction for the finite length of the fuselage was made by using the lateral velocity distribution shown in Figure 2. Calculated spanwise lift distributions for the two wing-fuselage combinations are compared with the calculated spanwise lift distributions for the wings alone in Figures 8 and 9.

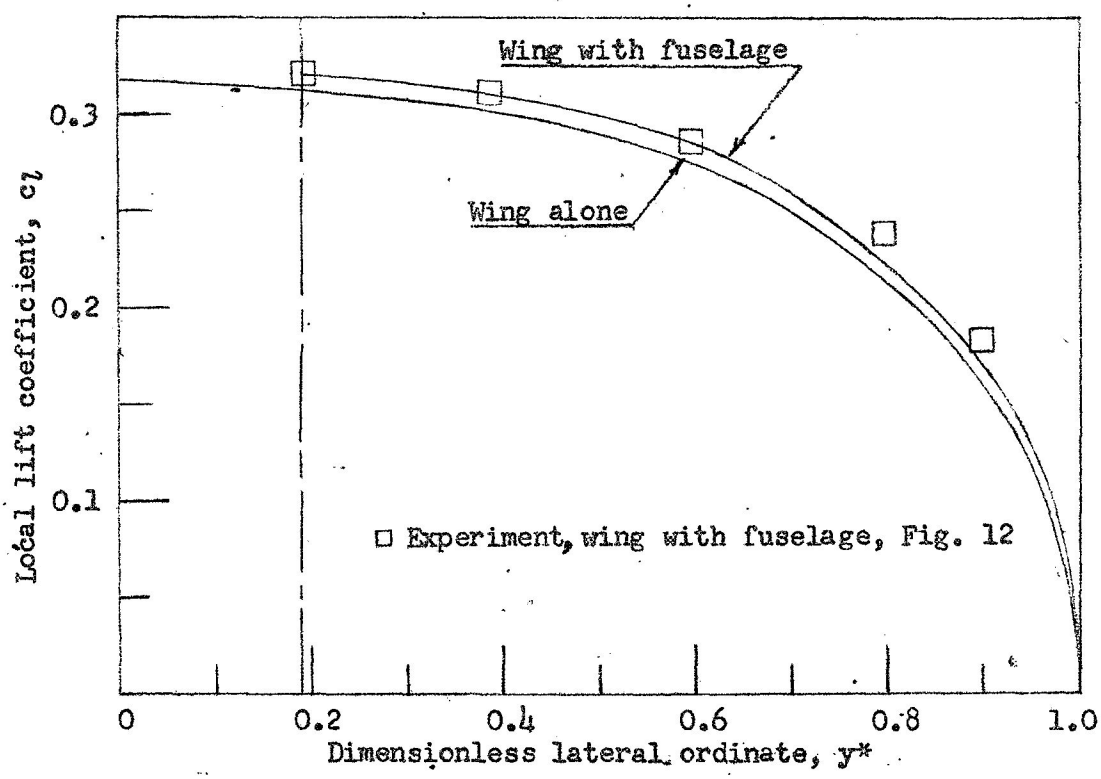

Figure 8.- Theoretical lift distribution on straight wing with and without fuselage, $A = 4.5$, $\lambda = 1$, $\Lambda = 0^\circ$, $a^* = 0.19$, $\alpha = 4^\circ$.

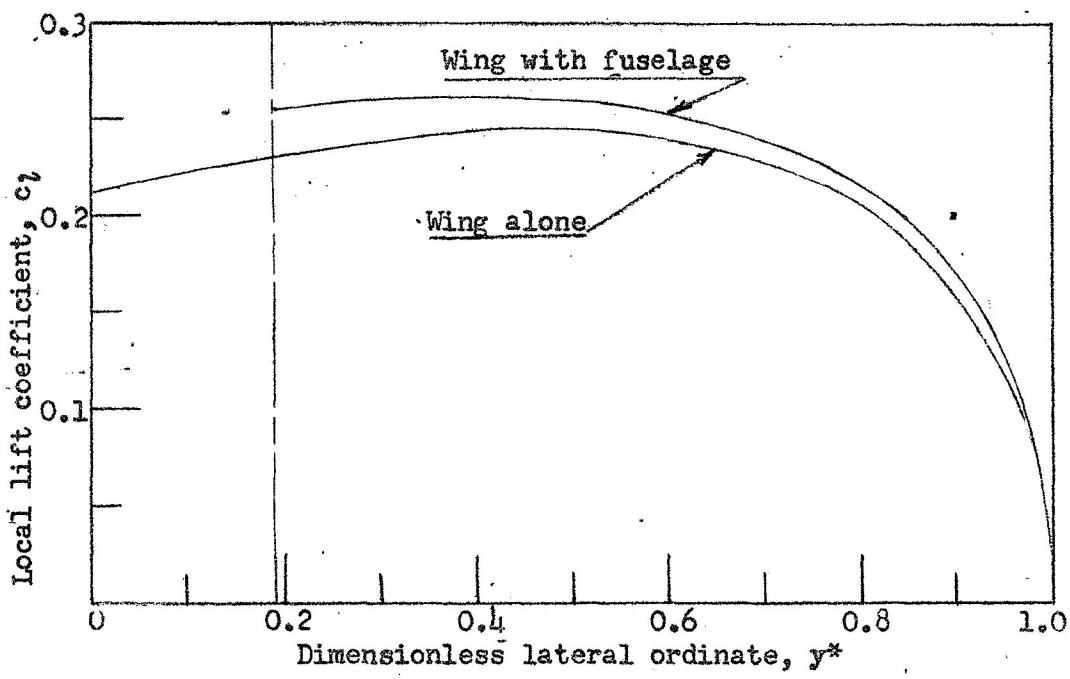

Figure 9.- Theoretical lift distribution on swept wing with and without fuselage, $A = 4.5$, $\lambda = 1$, $\Lambda = 40^\circ$, $a^* = 0.19$, $\alpha = 4^\circ$.

CHAPTER IV

DISCUSSION OF METHOD AND CALCULATED RESULTS

Comparison of the results of the new method for the limiting case of zero aspect ratio with the results of Spreiter's method. To obtain the spanwise lift distribution on the wing-fuselage combination for the limiting case of zero aspect ratio it is only necessary to multiply the value of $\bar{\gamma}(\bar{y}')$, the loading coefficient for the wing alone by the factor $\bar{b}/b$, and to plot the result γ at the points on the wing-fuselage in the physical plane which correspond to the points for the wing alone in the transformed plane. (See steps 5 and 6 of the computing procedure indicated in a previous section.) The relation between the ordinates in the physical plane and the ordinates in the transformed plane is given in equation 35. The solution for the wing alone of zero aspect ratio given in a paper by Diederich and Klotnick[34] is

$$\bar{\gamma}(\bar{y}')_{\text{wing alone}} = 4\sqrt{1 - \bar{y}'^2} \tag{43}$$

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[34] Franklin W. Diederich and Martin Klotnick, Theoretical Spanwise Lift Distributions of Low-Aspect-Ratio Wings at Speeds Below and Above the Speed of Sound, NACA TN No. 1975, 1949.

which can be used for the wing-fuselage combination by setting

$$\gamma(y^*)_{\text{wing-fuselage}} = \frac{\bar{b}}{b}\,\bar{\gamma}\,(\bar{y}^*)_{\text{wing alone}} \qquad (44)$$

Figure 6 shows a comparison of the results obtained by using the new method for the limiting case of zero aspect ratio with the results of Spreiter's method. No discrepancy is apparent, as would be expected because in this limiting case, the problem of calculating the lift distribution is essentially a two-dimensional one (in planes perpendicular to the stream), so that the procedure of transforming the physical space plane for plane into a distorted space is rigorously justifiable.

<u>Comparison of the results of the new method at the limiting case of infinite aspect ratio with the results obtained from considerations of an idealized case.</u> The calculation by Lennerts for the spanwise lift distribution on a combination of a constant-strength vortex with a cylinder aligned with the free stream can be modified slightly to give the lift distribution when the cylinder axis is placed at an angle-of-attack with the free stream. In this case an additional vertical velocity component w_n

normal to the axis of the cylinder must be considered in addition to the longitudinal velocity U considered by Lennertz so that the resultant free stream velocity V is

$$V = \sqrt{w_n^2 + U^2} \tag{45}$$

and the angle of attack of the cylinder is

$$\alpha_{cyl} = \tan^{-1} \frac{w_n}{U} \tag{46}$$

In a potential flow this additional upward velocity cannot change the lift on the cylinder, however, it can give rise to an additional lift on the part of the wing (represented by the vortex) adjacent to the fuselage.

The magnitude of this increase in the lift will be calculated by treating the components of the lift l_U and l_{w_n} due respectively to the components of the free-stream velocity U and w_n, as follows:

The component of the lift l_U which acts normal to the cylinder axis is

$$l_U = \rho U \Gamma \tag{47}$$

The component of the lift l_{w_n} which acts parallel to the cylinder axis and opposite in sense to the velocity U is

$$l_{w_n} = \rho \Gamma w_n \left(1 + \frac{a^2}{y^2}\right) \tag{48}$$

where $w_n\left(1 + \frac{a^2}{y^2}\right)$ is the velocity of the vertical flow at the vortex line in the presence of the cylinder[35]. But, as a result of equation 46

$$l_{w_n} = \rho U \Gamma \tan \alpha \left(1 + \frac{a^2}{y^2}\right) \tag{49}$$

The component of l_U normal to the free-stream velocity is

$$l'_U = l_U \cos \alpha \tag{50}$$

and the component of l_{w_n} normal to the free stream velocity is

$$l_{w_n}' = l_{w_n} \sin \alpha \tag{51}$$

The resultant lift l normal to the free-stream velocity is therefore

$$l = l_U \cos \alpha + l_{w_n} \sin \alpha \tag{52}$$

$$= \rho U \Gamma \cos \alpha + \rho U \Gamma \frac{\sin^2 \alpha}{\cos \alpha}\left(1 + \frac{a^2}{y^2}\right) \tag{53}$$

$$= \rho U \Gamma / \cos \alpha \left[1 + \sin^2 \alpha \left(\frac{a^2}{y^2}\right)\right] \tag{54}$$

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[35] H. Glauert, The Elements of Aerofoil and Airscrew Theory, Cambridge, 1948, p. 30.

$$l = \rho V \Gamma \left[1 + \sin^2 \alpha \left(\frac{a^2}{y^2}\right)\right] \tag{55}$$

since $\quad U = V \cos \alpha$

Now if the angle of attack α is small the term $\sin^2 \alpha \left(\frac{a^2}{y^2}\right)$ is negligible so that

$$l \approx \rho V \Gamma \tag{56}$$

which is the result for the wing alone. Consequently, as a first-order approximation, the lift distribution on the straight wing of infinite aspect ratio is unaffected by the presence of the fuselage. The same result is given by equation 36 for the limiting case of the wing-fuselage combination having a wing of infinite aspect ratio.

<u>Comparison of calculations with experiment</u>. The results of calculations made by the method presented in this paper, including the correction for the finite length of the fuselage, are shown in Figures 8 and 9. The experimental results may be seen to be in good agreement with the calculated results for the combination of a fuselage with an unswept wing.

A summary of the experimental results for the wings and wing-fuselage combinations treated in papers by

Holme[36] and Jacobs[37] is shown in Figure 10, and some experimental data obtained by Moller[38] for a similar case are shown in Figure 11. In contrast to the results for the unswept wing configurations considered in Figure 8, the experiments give much higher local lift coefficients than the calculations for the wing-fuselage combinations with swept wings. However, Figure 10 also shows that a swept wing has higher local lift coefficients in the presence of a fuselage than does an unswept wingin the presence of a fuselage. This trend appears to be incorrect and is, in all likelihood, the result of a false angle-of-attack measurement, or of longitudinal velocity gradients in the wind tunnel. Wind-tunnel tests at the Langley Laboratory on a swept wing in the presence of a fuselage which are not available for direct comparison at this writing show good qualitative agreement with the calculated results of Figure 9.

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[36] O. Holme, Comparative Wind Tunnel Tests of a Swept-Back and a Straight Wing Having Equal Aspect Ratios, Flygtekniska Forskningsanstalten, No. 31.

[37] Willi Jacobs, Lift and Moment Changes Due to the Fuselage for a Yawed Aeroplane with Unswept and Swept Wings, Flygtekniska Forskningsanstalten, No. 34.

[38] E. Moller, Systematische Druckverteilungsmessungen an Flügel/Rumpf-Anordnungen (Vlaf-Mittle-Hochdecker). Bericht Nr. 44/22 Aerodynamisches Institut der T. H. Braunschweig

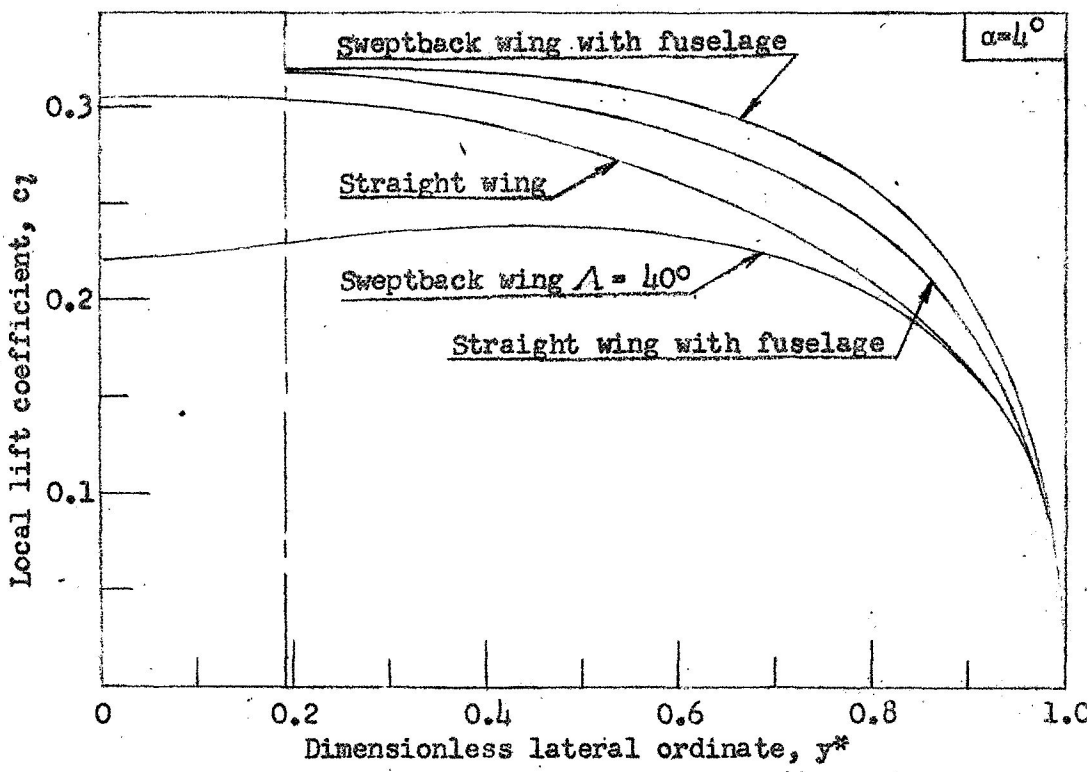

Figure 10.- Summary of experimental data obtained by Jacobs and Holme on a wing of A = 5, λ = 1 with and without fuselage, a*=0.19.

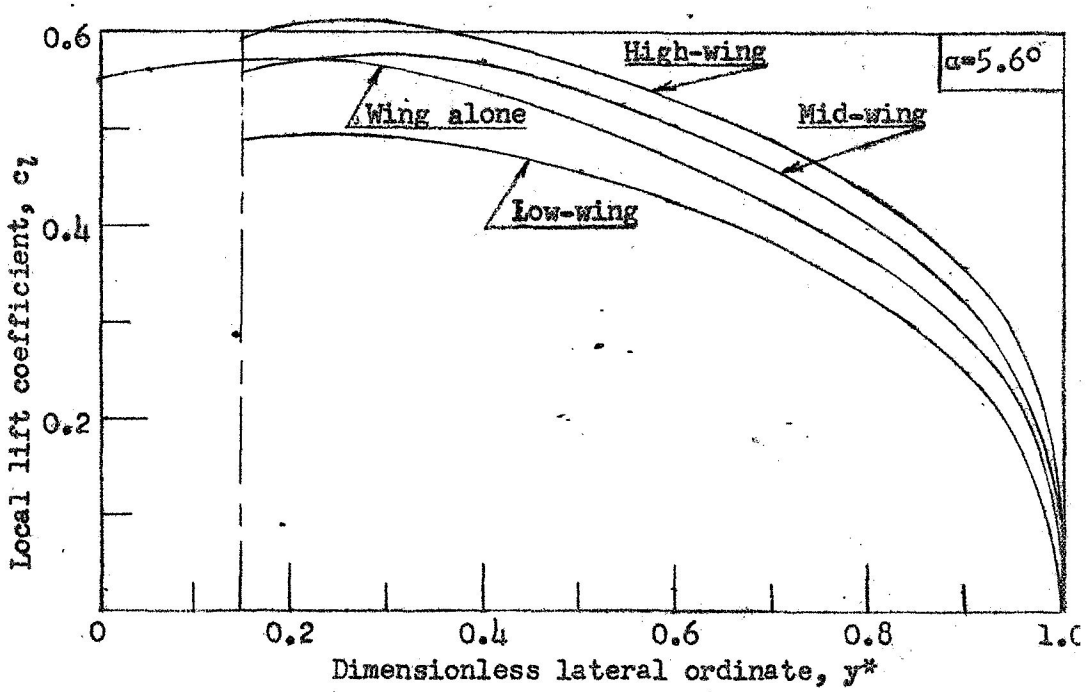

Figure 11.- Spanwise lift distribution on a rectangular wing, A = 5, in the presence of a fuselage, a* = 0.15, obtained experimentally by Möller.

CONCLUDING REMARKS

A method has been presented for calculating the lift distributions on swept or unswept wings with fuselages in incompressible flow. This method is more generally applicable than any method presented heretofore, and the calculations made by this method are in good agreement with experimental results. Although further comparison with experimental lift distributions is desirable before the method is applied generally, certain conclusions may be drawn from the available results.

It has been found that the presence of a slender fuselage does not have an important effect on the lift distribution on an unswept wing of moderate aspect ratio, but that a significant change in the lift distribution on a wing in the presence of a fuselage may be anticipated if the wing is swept or if the fuselage is relatively short.

Extension of the analysis of this paper to the treatment of flow at high subsonic speeds. The analysis in this paper treats only the case of a pure incompressible flow but it is possible to include the effects of compressibility which are encountered at high subsonic speeds by applying the

well known Prandtl-Glauert correction. A detailed discussion of the Prandtl-Glauert correction and its application is given in a paper by Hess and Gardner[39].

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[39] Robert V. Hess and Clifford S. Gardner, Study by the Prandtl Glauert Method of Compressibility Effects and Critical Mach Number for Airfoils of Various Aspect Ratios and Thickness Ratios, NACA TN No. 1792.

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