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COMPRESSION OF THE MAGNETIC FIELD IN A COLLAPSING SPHERICAL CAVITY

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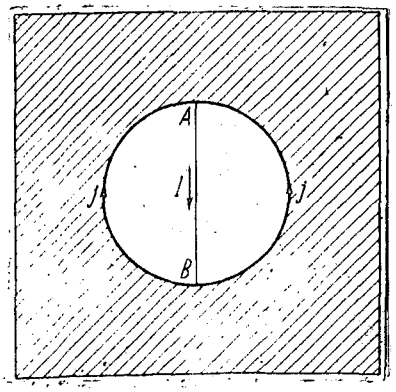
ABSTRACT. The characteristics method of analysis, employing integral transformations, used in this presentation, describes a primary dimension physical phenomenon; magnetic field compression in a collapsing spherical cavity.

Solutions are found for equations describing the compression of a magnetic field created by a system of meridional currents in a collapsing spherical cavity. /114*

The problem of the compression of a magnetic field between two approaching, ideally conductive planes was examined by the characteristics method in [1, 2]. In [2], the analogous problem for cylindrical geometry was examined. The problem was resolved by the method of integral transformations. The results of the solution of these problems may be suitable for the description of the physical phenomena which arise in systems having a primary dimension.

Often (for example, in the tests described in [3]) the most important end stage of the compression of the field takes place in a space which has no markedly expressed primary dimension. In this connection we examine below the problem of compression of a magnetic field in a spherical cavity. In contradistinction to the case of cylindrical geometry [2], the problem at issue makes possible simpler solutions which are more readily apprehended.



1. Let us examine the electrodynamic Problem of the compression of an initial electromagnetic field in a collapsing spherical cavity in an ideally conductive field. We shall limit ourselves to the case of linear dependence of the radius of the cavity upon time

*Numbers in the margin indicate pagination in the foreign text.

$$a(t) = a_0 - vt \tag{1.1}$$

where a_0 is the initial radius of the cavity.

We should note that from the experimental relationship a(t) introduced in [1] it follows that in practice the relation of (1.1) is sufficiently well sustained right up to a moment in time approaching the time of maximum compression.

Let us select as the initial field, a field created by an axially symmetrical system of currents as shown diagrammatically in the figure. A system of surface currents j' (sum current I) at point A flows into a diametral conductor AB and dissipates over the surface from point B.

From symmetry it follows that in a spherical system of coordinates having their origin at the center of the sphere the components H_ϕ, E_ϑ, and E_r can be other than zero. It follows directly from the Maxwell equations that at the initial moment the derivatives of the radial component in accordance with time, of all orders, are equal to zero, so that the radial component is not affected upon compression.

On this account we come to the following problem. We are to find a solution for a Maxwell equation having initial condition

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$$H_\varphi(r, \vartheta, 0) = \begin{cases} Ir^{-1}(\sin\vartheta)^{-1} & (r \leqslant a_0) \\ 0 & (r > a_0) \end{cases} \qquad E_\vartheta(r, \vartheta, 0) = 0 \tag{1.2}$$

and boundary condition

$$E_\vartheta(a(t), \vartheta, t) - a^{\cdot}c^{-1} H_\varphi(a(t), \vartheta, t) = 0 \tag{1.3}$$

The latter condition must be met upon an ideally conductive surface moving with a velocity da/dt [4].

The equations as well as the boundary and the initial conditions permit separation of the angular variable if we assume that

$$H_\varphi(r, \vartheta, t) = (\sin\vartheta)^{-1} h(r, t), \quad E_\vartheta(r, \vartheta, t) = (\sin\vartheta)^{-1} e(r, t) \tag{1.4}$$

2. Separating the angular relationship and excluding the function e(r, t) from the equations for the boundary and initial conditions, we come to the problem

$$\frac{\partial^2 h}{\partial r^2}+\frac{2}{r}\frac{\partial h}{\partial r}-\frac{1}{c^2}\frac{\partial^2 h}{\partial t^2}=0,\quad h(r,0)=\frac{I}{r},\quad \left(\frac{\partial h}{\partial t}\right)_{t=0}=0$$

$$\left(\frac{\partial h}{\partial r}\right)_{r=a}+\frac{a^{\cdot}}{c^2}\left(\frac{\partial h}{\partial t}\right)_{r=a}+\frac{1}{c^2}\frac{d}{dt}[a^{\cdot}h(a,t)]=-(1+\beta^2)\frac{h(a,t)}{a} \tag{2.1}$$

$$(\beta=v/c=-a^{\cdot}/c)$$

We solve (2.1) with the help of the Fourier-Bessel transform

$$\Phi(k,t)=\int_0^{a(t)} h(r,t)\,n_0(kr)\,r^2dr \qquad \left(n_0(kr)=-\frac{\cos kr}{kr}\right)$$

$$h(r,t)=2\pi^{-1}\int_0^{\infty}\Phi(k,t)\,n_0(kr)\,k^2dk \tag{2.2}$$

Here n_0 is a spherical Neumann function of zero order. The choice of n_0 $n_0(kr)$ as an eigenfunction is occasioned by the singularity of the initial distribution of (1.2) when r = 0.

After transformation of (2.2), (2.1) is reduced to the solution of equations for $\Phi(k, t)$

$$d^2\Phi/dt^2+k^2c^2\Phi=-c^2(1-\beta^2)\,ka^2\,j_0^{\cdot}(ka)\,h(a,t) \tag{2.3}$$

having initial conditions

$$\Phi(k,0)=-Ik^{-2}\sin ka_0,\quad (d\Phi/dt)_{t=0}=0 \tag{2.4}$$

The solution satisfying (2.3) and (2.4) has the form

$$\Phi(k,t)=-Ik^{-2}\sin ka_0\cos kct-$$
$$-c(1-\beta^2)\int_0^t a^2(\tau)\,h^*(\tau)\,j_0^{\cdot}(ka(\tau))\sin kc(t-\tau)\,d\tau \tag{2.5}$$
$$(h^*(t)=h(a(t),t))$$

An unknown field at the limit of the area forms part of (2.5). In order to define it we make use of expression (2.2), letting r = a(t) and doubling the integral in the right hand term, inasmuch as

$$h^*(t)=2^{-1}[h(a(t)+0,t)+h(a(t)-0,t)],\quad h(a(t)+0,t)=0$$

Then for $H^*(t)$ we get the following functional equation:

$$h^*(t)=\frac{I}{(1-\beta)\,a(t)}\qquad \left(0<t<\frac{2\beta T}{1+\beta}=t_1,\ T=\frac{a_0}{v}\right) \tag{2.6}$$

$$h^*(t)=\gamma^2h^*(\gamma(t-t_1))\qquad (t_1<t<T,\ \gamma=(1+\beta)/(1-\beta))$$

We determine the sequence of times $\{t_n\}$ in the following manner:

$$c\,(t_n - t_{n-1}) = a\,(t_n) + a\,(t_{n-1}) \tag{2.7}$$

From (2.7) we get

$$t_n = T\,(1 - \gamma^{-n}) \qquad (n = 0, 1, 2, \ldots)$$

The sequence $\{t_n\}$ is reduced to T as $n \to \infty$. The difference $t_n - t_{n-1}$ corresponds to the time over which the wave travels the path wall-center-wall. Hence, the solution of (2.6 has the form

$$h^*(t) = \gamma^{n-1}\,(1 - \beta)^{-1} I\,[a\,(t)]^{-1} \qquad (t_{n-1} < t < t_n) \tag{2.8}$$

Substituting (2.8) in (2.5), and then in (2.2), we find the desired solution:

$$h\,(r, t) = \begin{cases} I r^{-1}, & 0 < t < c^{-1}\,(a_0 - r) \\ {}^1/_2 I r^{-1} \gamma^n\,(\gamma + 1), & t_n + c^{-1}\,[a\,(t_n) - r] < t < t_n + c^{-1}\,[a\,(t_n) + r] \\ I r^{-1} \gamma^{n+1}, & t_n + c^{-1}\,[a\,(t_n) + r] < t < t_{n+1} + c^{-1}\,[a\,(t_{n+1}) - r] \end{cases}$$

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