MASSACHUSETTS INSTITUTE OF TECHNOLOGY

# APOLLO

## GUIDANCE, NAVIGATION AND CONTROL

CMC

GUIDANCE & NAVIGATION EQUATIONS

LUNAR LANDING MISSION GSOP

PRELIMINARY SECTION 5

JUNE 1967

[redacted]

INSTRUMENTATION LABORATORY

CAMBRIDGE 39, MASSACHUSETTS

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS
INSTRUMENTATION LABORATORY
CAMBRIDGE, MASS. 02139

C. S. DRAPER
DIRECTOR

June 19, 1967

THROUGH: National Aeronautics and Space Administration
Resident Apollo Spacecraft Program Office at MIT
Instrumentation Laboratory
75 Cambridge Parkway
Cambridge, Massachusetts 02142

TO: National Aeronautics and Space Administration
Manned Spacecraft Center
Houston, Texas 77058

Attn: Mr. T. F. Gibson - FS

FROM: N. E. Sears

SUBJECT: Preliminary GSOP Chapter 5 Guidance Equations for CMC Lunar Landing Mission

Gentlemen:

This document is the fourth submittal of GSOP Chapter 5 guidance and navigation equations for MSC review. It represents the first complete Chapter 5 for the CMC lunar landing mission, and will later be incorporated in the GSOP for this mission.

With the exception of introductory and general comment sections, the following sections are being submitted for MSC review for the first time.

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The other sections of this document have been submitted and reviewed previously. These remaining sections have been revised on the basis of the following three reviews:

| MIT Partial Submittal Date | MSC-MIT Review Date |
|---|---|
| Feb. 6, 1967 | Feb. 28, 1967 |
| March 20, 1967 | April 12, 1967 |
| April 17, 1967 | May 9, 1967 |

# SECTION 5

# GUIDANCE EQUATIONS

## 5.1 INTRODUCTION

### 5.1.1 GENERAL COMMENTS

The purpose of this section is to present the Guidance and Navigation Computation Routines associated with the Apollo Lunar Landing Mission. These Routines are utilized by the Programs outlined in Section 4 which consider astronaut and other subsystem interface and operational requirements. The guidance and navigation equations and procedures presented in Section 5 are primarily concerned with mission type programs representing the current CSM GNCS Computer (CMC) capability. A restricted number of CMC service type program operations which involve computation requirements are also included.

The CSM GNCS Computer (CMC) guidance and navigation equations for the lunar landing mission are presented in the following six catagories:

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| Section 5.2 | Coasting Flight Navigation Routines |
| Section 5.3 | Powered Flight Navigation and Guidance Routines |
| Section 5.4 | Targeting Routines |
| Section 5.5 | Basic Subroutines |
| Section 5.6 | General Service Routines |
| Section 5.7 | Entry Guidance |

A complete table of contents for Section 5 is presented in the following section 5.1.2.1. A cross reference between the CMC program and callable routine numbers of Section 4 that are described in Section 5 is listed in Section 5.1.2.2.

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## 5.1.3 GENERAL PROGRAM UTILIZATION

The following outline is a brief summary of the major CMC programs that would be used in the principal phases of a lunar landing mission. This outline reflects the CMC capability for the nominal and abort cases of such a mission.

I Earth Orbit Injection (EOI) Phase

A) Nominal

P - 11 Earth Orbit Insertion (EOI) Monitor Program

B) Abort to Entry

P - 61 Maneuver to CM/SM Separation Attitude Prior to Entry

P - 62 CM / SM Separation Maneuver Program

P - 63 Entry Initialization Program

P - 64 Post 0.05 G Entry Program

P - 67 Final Entry Phase Program

II. Earth Orbit Phase

A) Nominal

P - 15 TLI Program (Pre-Thrust Mode)

R - 30 Orbit Parameter Display Routine

B) Aborts to Entry

1. RTCC Targeted Abort

P - 30 External $\Delta V$ Maneuver Program (De-orbit)

(or)

P - 31 Lambert Aim Point Maneuver Program (De-orbit)

P - 40, P - 41 } SPS or RCS Thrust Program

P - 61, P - 62, P - 63, P - 64, P - 67 } Entry Programs

II Earth Orbit Phase (cont)

B. Aborts to Entry

2. CMC Targeted Abort

| | |
|---|---|
| P - 22 | Lunar Orbit Navigation Program (Earth Mode) |
| P - 21 | Ground Track Program |
| P - 37 | Return to Earth Program (Targeting and Pre-Thrust) |
| P - 40, P - 41 | SPS or RCS Thrust Program |
| P - 61, P - 62, P - 63, P - 64, P - 67 | Entry Programs |

C) Service Programs (Nominal and Abort Cases)

| | |
|---|---|
| P - 27 | CMC Update Program |
| P - 52 | IMU Realignment Program |
| P - 00 | CMC Idling Program |
| R - 30 | Orbit Parameter Display Routine |

## III Trans-Lunar Injection (TLI) Phase

### A) Nominal

P - 15 TLI Program - Active Mode (CMC Controlled Maneuver)

(or)

P - 15 TLI Program - Monitor Mode (Saturn Controlled Maneuver)

### B) Aborts to Earth Entry

1. RTCC Targeted Abort

P - 27 CMC Update Program

P - 30 External $\Delta V$ Pre-Thrust Program

P - 31 Lambert Aim Point Pre-Thrust Program

P - 40, P - 41 } SPS or RCS Thrust Program

P - 61 to P - 67 } Entry Programs

III Trans-Lunar Injection (TLI) Phase (cont)

B) Aborts to Entry

2. CMC Targeted Abort

P - 23 Cislunar Navigation Program

P - 37 Return to Earth Program (Targeting and Pre-Thrust)

P - 40, P - 41 } SPS or RCS Thrust Programs

P - 61 to P - 67 } Entry Programs

C) Service Programs Used in Abort Cases

P - 51 IMU Orientation Determination

P - 52 IMU Realignment Program

P - 06 GNCS Power Down

P - 05 GNCS Start Up

P - 00 CMC Idling

IV Trans-Lunar Phase

A) Nominal (Midcourse Correction Maneuvers)

P - 27 CMC Update Program

P - 31 Lambert Aim Point Pre-Thrust Program

P - 40, P - 41 } SPS or RCS Thrust Program

P - 47 Thrust Monitor Program (Manual Transposition and Docking Maneuver)

B) Aborts to Earth Entry

1. RTCC Targeted Abort

P - 27 CMC Update Program

P - 31 Lambert Aim Point Pre-Thrust Program

P - 40, P - 41 } SPS or RCS Thrust Program

IV Trans-Lunar Phase (cont)

B) Aborts to Earth Entry

2. CMC Targeted Abort (Limited Capability)

| | |
|---|---|
| P - 23 | Cislunar Navigation Program |
| P - 70 | Safe Perilune Program (If Required) |
| P - 37 | Return to Earth Program Targeting and Pre-Thrust (Capability limited to outside lunar sphere of influence) |
| P - 40 to P - 41 | SPS or RCS Thrust Programs |
| P - 61 to P - 67 | Entry Programs |

C) Service Programs for Nominal and Abort Cases

| | |
|---|---|
| P - 05 | GNCS Start Up |
| P - 06 | GNCS Power Down |
| P - 51 | IMU Orientation Determination |
| P - 52 | IMU Realignment Program |
| P - 00 | CMC Idling Program |
| R - 05 | S-Band Antenna Routine |

V Lunar Orbit Insertion (LOI) Phase

A) Nominal

P - 27 CMC Update Program

P - 32 Lunar Orbit Insertion Pre-Thrust Program

P - 40 SPS Thrust Program

B) Aborts to Return to Earth Trajectory

1. RTCC Targeted Abort

P - 27 CMC Update Program

P - 30 External $\Delta V$ Pre-Thrust Program (To establish safe lunar orbit if required)

P - 31 Lambert Aim Point Pre-Thrust Program (TEI)

P - 40 SPS Thrust Program

V Lunar Orbit Insertion (LOI) Phase (cont)

B) Aborts to Return to Earth Trajectory

2. CMC Semi-Controlled Abort (Limited Capability)

| | |
|---|---|
| P - 22 | Lunar Orbit Navigation Program |
| P - 21 | Ground Track Program |
| R - 35 | Lunar Landmark Selection Routine |
| P -70 | Safe Perilune Program (If Required) |
| P -27 | CMC Update Program |
| P -31 | Lambert Aim Point Pre-Thrust Program (TEI) |
| P - 40 | SPS Thrust Program |

C) Service Programs for Nominal and Abort Cases

| | |
|---|---|
| P - 52 | IMU Realignment Program |
| R - 05 | S-Band Antenna Routine |

## VI Lunar Orbit Phase Prior to LM Descent Orbit Injection

### A) Nominal

| | |
|---|---|
| P - 22 | Lunar Orbit Navigation Program |
| P - 21 | Ground Track Program |
| R - 35 | Lunar Landmark Selection Routine |
| P - 25 | Predicted Lunar Landing Time Program |
| R - 32 | Target $\Delta V$ Routine (LM Separation) |
| P - 20 | Rendezvous Navigation Program (Tracking Mode only for LM RR Check-Out) |

### B) Aborts to Return Earth Trajectory

1. RTCC Targeted Abort

| | |
|---|---|
| P - 27 | CMC Update Program |
| P - 31 | Lambert Aim Point Pre-Thrust Program (TEI) |
| P - 40 | SPS Thrust Program |

VI Lunar Orbit Phase Prior to LM Descent Orbit Injection (cont)

B) Aborts to Return Earth Trajectory

2. CMC Semi-Controlled Abort (Limited Capability)

| | |
|---|---|
| P - 22 | Lunar Orbit Navigation Program |
| P - 21 | Ground Track Program |
| R - 35 | Lunar Landmark Selection Routine |
| P - 27 | CMC Update Program |
| P - 31 | Lambert Aim Point Pre-Thrust Program (TEI) |
| P - 40 | SPS Thrust Program |
| P - 47 | Thrust Monitor Program (TEI by DPS Case) |

C) Service Programs for Nominal and Abort Cases

| | |
|---|---|
| P - 05 | GNCS Start Up |
| P - 06 | GNCS Power Down |
| P - 51 | IMU Orientation Determination |
| P - 52 | IMU Realignment Program |
| P - 00 | CMC Idling Program |
| R - 05 | S-Band Antenna Routine |
| R - 30 | Orbit Parameter Display Routine |

## VII LM Descent Orbit Injection (DOI), Descent Coast and Landing Phases

### A) Nominal

P - 20 Rendezvous Navigation Program (Tracking attitude and monitoring only)

### B) Aborts to Rendezvous Condition

#### 1. LM Active Vehicle

P - 20 Rendezvous Navigation Program

R - 32 Target $\Delta V$ Routine

P - 74 LM TPI Targeting Program (Monitoring or commands via voice link)

P - 75 LM TPM Targeting Program (Rendezvous MCC monitoring or commands via voice link)

P - 77 TPI Search Program (LM mode)

P - 78 LM Stable Orbit Rendezvous Targeting Program

P - 79 LM Stable Orbit Rendezvous Midcourse Targeting Program

P - 21 Ground Track Program

VII LM Descent Orbit Injection (DOI), Descent Coast and Landing Phases (cont)

B) Aborts to Rendezvous Condition

2. CSM Active Retrieval

| | |
|---|---|
| P - 27 | CMC Update Program |
| P - 20 | Rendezvous Navigation Program |
| P - 30 | External $\Delta V$ Pre-Thrust Program |
| P - 17 | TPI Search Program |
| P - 21 | Ground Track Program |
| P - 34 | TPI Pre-Thrust Program |
| P - 35 | TPM Pre-Thrust Program |
| P - 38 | Stable Orbit Rendezvous Program |
| P - 39 | Stable Orbit Rendezvous Midcourse Program |
| P - 40, P - 41 | SPS or RCS Thrust Programs |
| P - 47 | Thrust Monitor Program (Manual terminal rendezvous maneuver) |

C) Serivice Programs

| | |
|---|---|
| P - 52 | IMU Realignment Program |
| R - 31 | Rendezvous Parameter Display Routine No. 1 |
| R - 34 | Rendezvous Parameter Display Routine No. 2 |
| R - 63 | Rendezvous Final Attitude Routine |
| P - 00 | CMC Idling |
| R - 05 | S-Band Antenna Routine |
| R - 30 | Orbit Parameter Display Routine |

## VIII Lunar Stay Phase to LM Ascent Launch

### A) Nominal

| | |
|---|---|
| P - 22 | Lunar Orbit Navigation Program (for landing site surveillance and CMC update if desired) |
| P - 21 | Ground Track Program |
| R - 35 | Lunar Landmark Selection Routine |
| P - 27 | CMC Update Program (LM launch time update) |
| P - 33 | Lunar Orbit Plane Change (LOPC) Pre-Thrust Program |
| P - 40 | SPS Thrust Program |
| P - 20 | Rendezvous Navigation Program (Tracking attitude and monitoring only) |

### B) Service Programs

| | |
|---|---|
| P - 05 | GNCS Start Up |
| P - 51 | IMU Orientation Determination |
| P - 52 | IMU Realignment Program |
| P - 00 | CMC Idling Program |
| R - 05 | S-Band Antenna Routine |

## IX LM Ascent and Rendezvous Phase

### A) Nominal

| | |
|---|---|
| P - 20 | Rendezvous Navigation Program |
| R - 32 | Target ΔV Routine |
| P - 74 | LM TPI Targeting Program (Monitoring) |
| P - 75 | LM TPM Targeting Program (Monitoring) |
| P - 78, P - 79 | LM Stable Orbit Rendezvous Programs (Monitoring) |
| P - 21 | Ground Track Program |

### B) Aborts to Rendezvous Condition

#### 1. LM Active Vehicle

| | |
|---|---|
| P - 20 | Rendezvous Navigation Program |
| R - 32 | Target ΔV Routine |
| P - 74 | LM TPI Targeting Program (Monitoring) |
| P - 75 | LM TPM Targeting Program (Monitoring) |
| P - 78, P - 79 | LM Stable Orbit Rendezvous Programs (Monitoring) |
| P - 21 | Ground Track Program |
| P - 77 | LM TPI Search Program |

IX LM Ascent and Rendezvous Phase (cont)

B) Aborts to Rendezvous Conditions

2. CSM Active Retrieval

| | |
|---|---|
| P - 27 | CMC Update Program |
| P - 20 | Rendezvous Navigation Program |
| P - 30 | External ΔV Pre-Thrust Program (Targeted from RTCC or LGC) |
| P - 17 | TPI Search Program |
| P - 38 | Stable Orbit Rendezvous Program |
| P - 39 | Stable Orbit Rendezvous Midcourse Program |
| P - 21 | Ground Track Program |
| P - 34 | TPI Pre-Thrust Program |
| P - 35 | TPM Pre-Thrust Program |
| P - 40, P - 41 | SPS or RCS Thrust Programs |
| P - 47 | Thrust Monitor Program (Manual terminal rendezvous maneuver) |

C) Service Programs for Nominal and Abort Cases

| | |
|---|---|
| P - 52 | IMU Realignment Program |
| R - 05 | S-Band Antenna Routine |
| R - 31 | Rendezvous Parameter Display Routine No. 1 |
| R - 34 | Rendezvous Parameter Display Routine No. 2 |
| R - 63 | Rendezvous Final Attitude Routine |
| P - 30 | Orbit Parameter Display Routine |
| P - 00 | CMC Idling Program |

X Lunar Orbit Phase Prior to TEI

A) Nominal

| | |
|---|---|
| P - 27 | CMC Update Program |
| (or) | |
| P - 22 | Lunar Orbit Navigation Program |
| P - 21 | Ground Track Program |
| R - 35 | Lunar Landmark Selection Routine |
| P - 52 | IMU Realignment Program |
| P - 00 | CMC Idling Program |
| P - 47 | Thrust Monitor (Manual CSM-LM Separation Maneuver) |

B) Aborts Prior to Nominal TEI

| | |
|---|---|
| P - 27 | CMC Update Program |

## XI Trans-Earth Injection (TEI) Phase

### A) Nominal

P - 27 CMC Update Program

P - 31 Lambert Aim Point Pre-Thrust Program (TEI maneuver targeted by RTCC)

P - 40 SPS Thrust Program

### B) Aborts to Earth Return Trajectory

Same as nominal case
No CMC targeting capability

## XII Trans-Earth Phase Midcourse Correction Maneuvers

### A) Nominal

| | |
|---|---|
| P - 27 | CMC Update Program |
| P - 31 | Lambert Aim Point Pre-Thrust Program |
| P - 40, P - 41 | SPS or RCS Thrust Program |

### B) Aborts to Maintain Earth Return Trajectory (CMC Limited Capability)

| | |
|---|---|
| P - 23 | Cislunar Navigation Program |
| P - 37 | Return to Earth Program (Outside lunar sphere of influence only) |
| P - 40, P - 41 | SPS or RCS Thrust Programs |

### C) Service Programs for Nominal and Abort Cases

| | |
|---|---|
| P - 05 | GNCS Start Up Program |
| P - 06 | GNCS Power Down |
| P - 51 | IMU Orientation Determination |
| P - 52 | IMU Realignment Program |
| R - 05 | S-Band Antenna Routine |
| P - 00 | CMC Idling Program |

XIII Entry Phase

A) Nominal

| | |
|---|---|
| P - 61 | Maneuver to Separation prior to Entry |
| P - 62 | CM / SM Separation Maneuver |
| P - 63 | Entry Initialization |
| P - 64 | Post 0.05 G Entry Phase |
| P - 65 | Entry Up Control Phase |
| P - 66 | Entry Ballistic Phase |
| P - 67 | Final Entry Phase |

## 5.1.4 COORDINATE SYSTEMS

There are five major coordinate systems used in the navigation and guidance programs. These five coordinate systems are defined individually in the following descriptions. Any additional coordinate systems used in particular CMC programs are defined in the individual section describing that program.

### 5.1.4.1 Basic Reference Coordinate System

The Basic Reference Coordinate System is an orthogonal inertial coordinate system whose origin is centered at either the earth or the moon. The orientation of this coordinate system is defined by the line of intersection of the mean earth equatorial plane and the mean orbit of the earth (the ecliptic) at the beginning of the Besselian year which starts January 0.525, 1969. The X-axis ($\underline{u}_{XI}$) is along this intersection with the positive sense in the direction of the ascending node of the ecliptic on the equator (the equinox), the Z-axis ($\underline{u}_{ZI}$) is along the mean earth north pole, and the Y-axis ($\underline{u}_{YI}$) completes the right-handed triad. This coordinate system is shifted from earth-centered to moon-centered when the estimated vehicle position from the moon first falls below a specified value $r_{SPH}$, and is likewise shifted from moon-centered to earth-centered when the estimated vehicle position from the moon first exceeds $r_{SPH}$. This procedure is described in Section 5.2.2.6 and Fig. 2.2-3. All navigation stars and lunar-solar ephemerides are referenced to this coordinate system. All vehicle state vectors are referenced to this system during coasting or free fall phases of the mission

### 5.1.4.2 IMU Stable Member or Platform Coordinates

The orthogonal inertial coordinate system defined by the GNCS inertial measurement unit (IMU) is dependent upon the current

IMU alignment. There are many possible alignments during a mission, but the primary IMU alignment orientations described in Section 5. 6. 3. 3 are summarized below and designated by the subscript SM:

1. <u>Preferred Alignment (Thrusting Maneuvers)</u>

$$\underline{u}_{XSM} = \text{UNIT}\,(\underline{x}_B)$$

$$\underline{u}_{YSM} = \underline{u}_{XSM} \times \text{UNIT}\,(\underline{r}) \qquad (5.1.1)$$

$$\underline{u}_{ZSM} = \underline{u}_{XSM} \times \underline{u}_{YSM}$$

where:

$\underline{u}_{XSM}$, $\underline{u}_{YSM}$, $\underline{u}_{ZSM}$ = IMU stable member coordinate unit vectors referenced to the Basic Reference Coordinate System

$\underline{x}_B$ = vehicle or body X-axis

$\underline{r}$ = current position vector in the Basic Reference Coordinate System

2. <u>Nominal Alignment (Local Vertical)</u>

$$\underline{u}_{ZSM} = -\,\text{UNIT}\,(\underline{r}) \text{ at } t_{align}$$

$$\underline{u}_{YSM} = \text{UNIT}\,(\underline{r} \times \underline{v}) \qquad (5.1.2)$$

$$\underline{u}_{XSM} = \underline{u}_{YSM} \times \underline{u}_{ZSM}$$

where $\underline{r}$ and $\underline{v}$ represent the vehicle state vector at the alignment time, $t_{align}$.

3. Lunar Landing Alignment

$$\underline{u}_{XSM} = \text{UNIT}\ (\underline{r}_{LS})\ \text{at}\ t_L$$

$$\underline{u}_{ZSM} = \text{UNIT}\ (\underline{r}_C \times \underline{v}_C) \times \underline{u}_{XSM} \qquad (5.1.3)$$

$$\underline{u}_{YSM} = \underline{u}_{ZSM} \times \underline{u}_{XSM}$$

where $\underline{r}_{LS}$ is the lunar landing site vector at the predicted landing time $t_L$, and $\underline{r}_C$ and $\underline{v}_C$ are the CSM position and velocity vectors.

4. Lunar Launch Alignment

The same as that defined in Eq. (5. 1. 3) except that $\underline{r}_{LS}$ is the landing or launch site at the predicted launch time $t_L$.

The IMU coordinate system is assumed to be centered at the same point that the Basic Reference Coordinate System is currently centered at.

5. 1. 4. 3 Vehicle or Body Coordinate System

The vehicle or body coordinates (subscript B) are the general CM coordinates. The origin of this coordinate frame is at the GNCS Navigation Base. The X-axis ($\underline{x}_B$) is parallel to the spacecraft longitudinal axis directed towards the CM apex, the Z-axis ($\underline{z}_B$) is normal to $\underline{x}_B$ and in the direction of the foot rest of the center couch. The Y-axis is orthogonal to the X and Z axes so as to form a right-handed system. The nominal CM center of gravity is in the plane formed by the $\underline{x}_B$ and $\underline{z}_B$ vectors.

### 5.1.4.4 Earth-Fixed Coordinate System

The Earth-fixed Coordinate System is an orthogonal rotating coordinate system centered at the earth. The Z-axis is along the true polar axis of the earth, the X-axis is along the intersection of the meridian of Greenwich and the equatorial plane of the earth, and the Y-axis is in the equatorial plane and completes the right-handed triad.

### 5.1.4.5 Moon-Fixed Coordinate System

The moon-fixed Coordinate System is an orthogonal rotating coordinate system centered at the moon. The Z-axis is along the true polar axis of the moon, the X axis is along the intersection of the meridian of $0^{o}$ longitude and the equatorial plane of the moon, and the Y-axis is in the equatorial plane and completes the right-handed triad.

## 5.2 COASTING FLIGHT NAVIGATION

### 5.2.1 GENERAL COMMENTS

The CMC Coasting Flight Navigation Routines which are presented in Sections 5.2.2 through 5.2.6 are used during free fall phases of the Apollo mission. The basic objective of these navigation routines is to maintain estimates of the position and velocity vectors of both the CSM and the LM. Let $\underline{r}$ and $\underline{v}$ be the estimates of a vehicle's position and velocity vectors, respectively. Then, the six-dimensional state vector, $\underline{x}$, of the spacecraft is defined by

$$\underline{x} = \begin{pmatrix} \underline{r} \\ \underline{v} \end{pmatrix}$$

Coasting Flight Navigation is accomplished by extrapolating the state vector, $\underline{x}$, by means of the Coasting Integration Routine (Section 5.2.2), and updating or modifying this estimated state using optical tracking data by the recursive method of navigation (Sections 5.2.3-5.2.6).

The Coasting Integration Routine (Section 5.2.2) is used by other navigation and targeting routines to extrapolate the following:

1) Present estimated CSM state vector

2) Present estimated LM state vector

3) An arbitrary specified state vector, such as the predicted result of a maneuver

The CMC idling program (Section 5.6.12) periodically uses the Coasting Integration Routine to advance the estimated CSM state vector (and the estimated LM state vector when the LM is active and not on the surface of the moon) to approximately current time. This procedure has the lowest priority of all programs, and is performed only when no other program is active. This periodic state vector extrapolation is not necessary from a theoretical point of view, but does have two practical purposes. First, it is advisable to maintain current (or at least nearly current) state vector estimates in case an emergency situation arises. Second, a significant amount of computation time is transferred from a period of high computer activity (navigation measurement processing, targeting, etc.) to a period of low activity.

State vector extrapolation is accomplished by means of Encke's method of differential accelerations. The motion of a spacecraft is dominated by the conic orbit which would result if the spacecraft were in a central force field. In Encke's method the differential equations for the deviations from conic motion are integrated numerically. This technique is in contrast to a numerical integration of the differential equations for the total motion, and it provides a more accurate orbit extrapolation. The numerical integration is accomplished by means of Nystrom's method which gives fourth-order accuracy while requiring only three computations of the derivatives per time step. The usual fourth-order Runga-Kutta integration methods require four derivative computations per time step.

Regardless of the accuracy of the state vector extrapolation, errors in the initial conditions will propagate and soon grow to intolerable size. Thus, it is necessary periodically to obtain additional data in the form of either new state vector estimates or modifications to the current state vector estimates. These state vector modifications are computed from navigation data obtained by means of navigation measurements.

The CSM GNCS uses optical angle data from the scanning telescope (SCT) and the sextant (SXT) to compute state vector changes, while the LM PGNCS uses rendezvous radar (RR) tracking data. Navigation measurement data are used to update state vector estimates during orbit navigation, rendezvous navigation, and cislunar-midcourse navigation procedures. These three navigation procedures will be used normally during the lunar-orbit navigation phase, all LM-CSM lunar-orbit rendezvous phases, and CSM return-to-earth aborts, respectively, in the lunar landing mission. However, in order to provide for alternate mission capability, the orbit and rendezvous navigation procedures can be used near the moon or the earth.

Although the state vector of the CSM is six-dimensional, it is not necessary that the quantities estimated during a particular navigation procedure be the position and velocity vectors of the CSM. A variety of "estimated state vectors", not necessarily of six-dimensions, are used.

In order to achieve desired landing objectives, it is necessary to expand the lunar-orbit navigation procedure to nine dimensions, and to include in the estimation the position vector of the landmark being tracked. The estimated state vector that is used in orbit navigation is given by

$$\underline{x} = \begin{pmatrix} \underline{r}_C \\ \underline{v}_C \\ \underline{r}_\ell \end{pmatrix}$$

where $\underline{r}_C$ and $\underline{v}_C$ are the estimated CSM position and velocity vectors and $\underline{r}_\ell$ is the estimated landmark position vector.

During the rendezvous phase, the six-dimensional state vector of either the CSM or the LM can be updated from the measurement data obtained with the CSM-based optics. The selection of the update mode is an astronaut option and will be based primarily upon which vehicle's state vector is most accurately known initially, and which vehicle is controlling the rendezvous maneuvers.

The standard six-dimensional CSM state vector is used during cislunar-midcourse navigation.

Navigation data is incorporated into the state vector estimates by means of the Measurement Incorporation Routine (Section 5. 2. 3) which has both six- and nine-dimensional modes. The Measurement Incorporation Routine is a subroutine of the following CMC navigation routines:

1) Orbit Navigation Routine (Section 5. 2. 4)

2) Rendezvous Navigation Routine (Section 5. 2. 5)

3) Cislunar-Midcourse Navigation Routine (Section 5. 2. 6)

Simplified functional diagrams of the navigation programs which use these routines are given in Figs. 2. 1-1, 2. 1-2, and 2. 1-3, respectively.

In all three navigation programs, estimated position and velocity vectors are obtained at required times by means of the Coasting Integration Routine (Section 5. 2. 2). The Measurement Incorporation Routine (Section 5. 2. 3) is used to incorporate the measurement data into the state vector estimates.

The updating procedure, which is illustrated in simplified

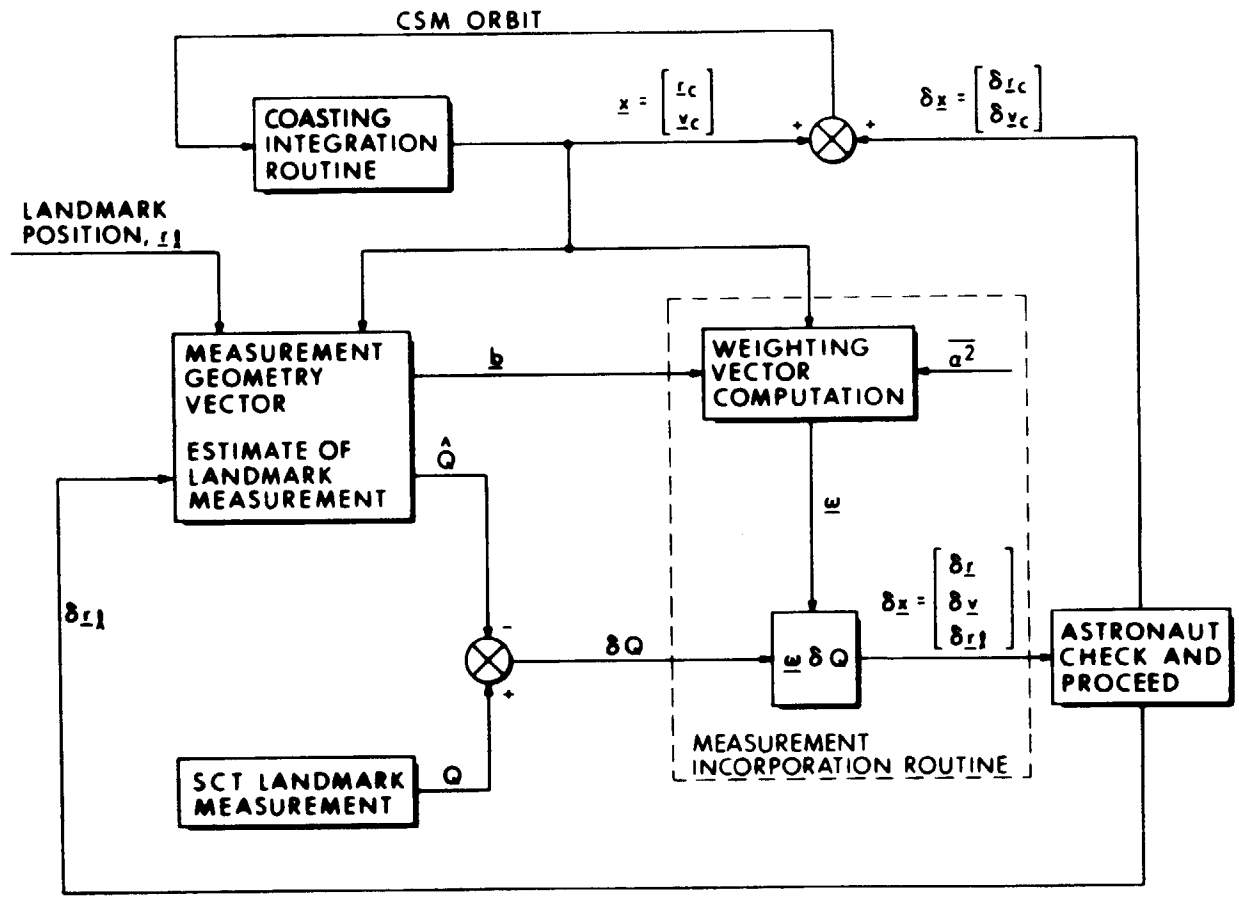


Fig. 2.1-1 Simplified Orbit Navigation Functional Diagram

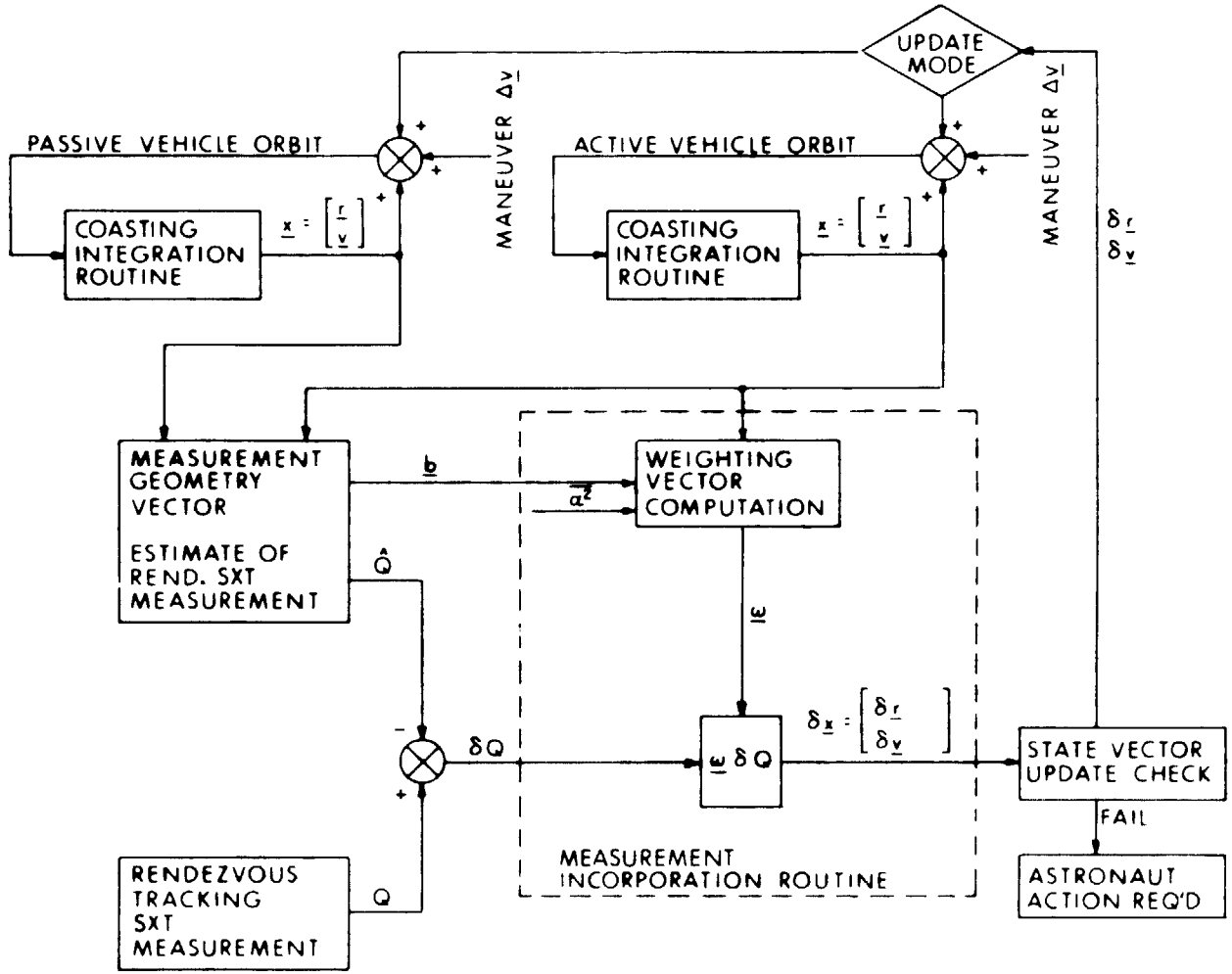


Fig. 2.1-2 Simplified CMC Rendezvous Navigation Functional Diagram

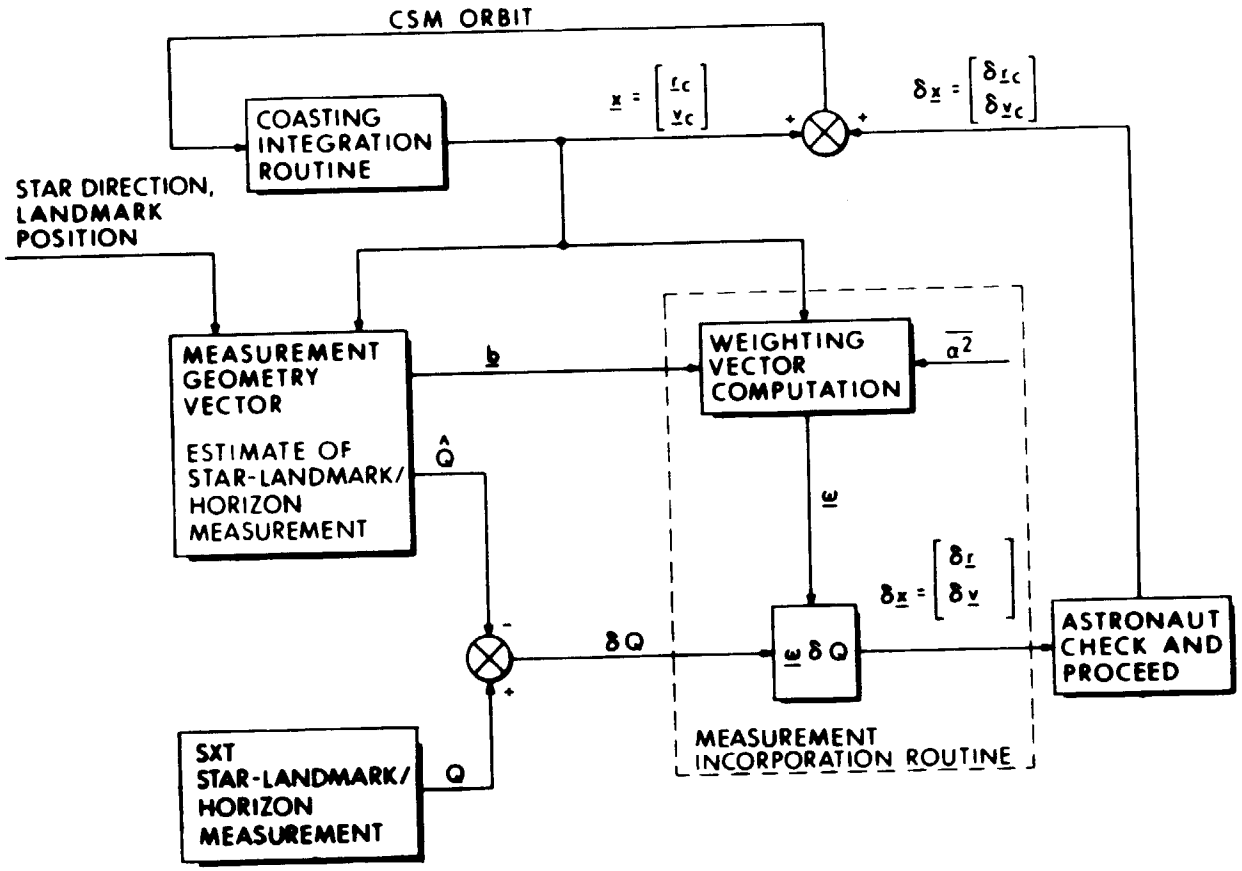


Fig. 2.1-3 Simplified Cislunar-Midcourse Navigation Functional Diagram

form in Figs. 2.1-1 to 2.1-3, involves computing an estimated tracking measurement, $\hat{Q}$, based on the current state vector estimates. This estimated measurement is then compared with the actual tracking measurement Q (optical angles in the CMC) to form a measured deviation $\delta Q$. A statistical weighting vector, $\underline{\omega}$, is computed from statistical knowledge of state vector uncertainties and tracking performance, $\overline{\sigma^2}$, plus a geometry vector, b, determined by the type of measurement being made. The weighting vector, $\underline{\omega}$, is defined such that a statistically optimum linear estimate of the deviation, $\delta \underline{x}$, from the estimated state vector is obtained when the weighting vector is multiplied by the measured deviation $\delta Q$. The vectors $\underline{\omega}$, $\underline{b}$ and $\delta \underline{x}$ are of six or nine dimensions depending upon the dimension of the state vector being estimated.

In an attempt to prevent unacceptably large incorrect state vector changes, certain validity tests have been included in the various CMC navigation routines.

In the Orbit Navigation Routine (Section 5.2.4) the astronaut tracks a landmark and acquires a number of sets of optical angle data before the state vector updating process begins. During the data processing procedure the landmark is out of sight, and it is not possible to repeat the tracking. Before the first set of data is used to update the estimated state vector, the magnitudes of the proposed changes in the estimated CSM position and velocity vectors, $\delta r$ and $\delta v$, respectively, are displayed for astronaut approval. In general, successive accepted values of $\delta r$ and $\delta v$ will decrease during the processing of the tracking data associated with one landmark. Thus, if the MARK REJECT button has been used to erase all inaccurate marks, then all state vector updates should be either accepted or rejected. If the first displayed values of $\delta r$ and $\delta v$ are judged to be valid, then all data associated with that landmark will be accepted.

The actual values of the first displayed $\delta r$ and $\delta v$ will depend upon the statistical parameters stored in the CMC and upon the following types of errors:

Type 1: Errors in the current state vector estimates

Type 2: Errors in alignment of the IMU

Type 3: Reasonable tracking performance errors, including both hardware and astronaut errors

Type 4: A CSM GNCS failure

Type 5: Gross astronaut errors, such as incorrect identification of the landmark

The existence of Type 1 errors is precisely the reason that the landmark tracking is being done. It is the function of the navigation to decrease Type 1 errors in the presence of noise in the form of errors of Types 2 and 3. Since the landmark tracking should not be performed unless the IMU is well aligned and the GNCS is functioning properly, and since bad marks should be rejected, it follows that the purpose of the state vector change validity check is to discover a Type 5 error. This validity check cannot distinguish between a Type 4 error and a Type 5 error.

Based upon the last time that the state vector was updated, when the IMU last was realigned, and an estimate of the tracking performance for the first mark, very crude reasonable values for the first $\delta r$ and $\delta v$ can be generated by the astronaut. The CMC will provide no information to assist the astronaut in his estimates of reasonable values for $\delta r$ and $\delta v$.

One further CSM GNCS item is available for help in estimating expected values for the first display of $\delta r$ and $\delta v$. If the astronaut uses the Automatic Optics Designate Routine (Section 5.6.8) during the acquisition of the landmark, he will have an approximate visual picture of the first $\delta r$. In the Automatic Optics Designate Routine the CMC will point the optics in the estimated direction of the specified landmark. The first value of $\delta r$ should be of the same order of magnitude as the displacement of the actual landmark from the center of the SCT reticle. The value can be computed by multiplying the CSM-landmark distance by the angular offset of the landmark. It should be noted that this estimate for $\delta r$ must be made while the optics mode switch is set to CMC. Again, this estimating process is an astronaut function only, with no assistance from the CMC.

In the Rendezvous Navigation Routine (Section 5.2.5) measurement data is processed periodically ~~(once~~ per minute), and it is desirable that the LM be tracked during the entire rendezvous phase up to the manual terminal maneuver. If the magnitudes of the changes in the estimated position and velocity vectors, $\delta r$ and $\delta v$, respectively, are both less than preset tracking alarm levels, then the selected vehicle's state vector is automatically updated by the computed deviation, $\delta \underline{x}$, and no special display is presented, except that the tracking measurement counter is incremented by one. If either $\delta r$ or $\delta v$ exceeds its alarm level, then the state vector is not updated, and the astronaut is alerted to this condition by a special display of $\delta r$ and $\delta v$.

In this case the astronaut should recheck the optical tracking and make sure that he is tracking the LM. Under certain conditions it is possible to mistake a star for LM reflected sunlight, and it may take a period of a few minutes to determine the LM target by watching relative motion of the target and star background. After the tracking

has been verified, and navigation data has again been acquired, the astronaut has the option of commanding a state vector update if the tracking alarm is again exceeded, or of repeating further optical checks before incorporating the measurement data. If the astronaut cannot determine the LM target due to no positive acquisition (bright background, etc.) he can terminate the ~~prog~~ram and try to achieve tracking conditions at a later time. routine

The displayed values of $\delta r$ and $\delta v$ which have not passed the tracking alarm test will depend upon the statistical parameters stored in the CMC and upon the same five types of errors discussed previously in regard to orbit navigation. The tracking alarm criterion is incorporated in the Rendezvous Navigation Routine to alert the astronaut to the fact that the state vector update is larger than normally expected, and to prevent the estimated state vector from automatically being updated in such cases. The update occurs only by specific command of the astronaut. The tracking alarm level beyond which updating is suspended is primarily chosen to avoid false acquisition and tracking conditions. As previously mentioned, this condition is possible in the CSM if a star is tracked by mistake instead of the LM reflected sun light, and it is therefore possible for the alarm level to be exceeded in such cases even though the estimated state vectors are essentially correct. It is also possible for the state vector update alarm level to be exceeded after correct initial acquisition and tracking in the case where a poor estimate of either the CSM or LM state vector exists. In this case the astronaut would have to command the initial state vector update, after which the alarm level would seldom be exceeded during the remainder of the rendezvous phase. It should be noted that this statement is true only if the estimated state vector of the active vehicle performing a powered rendezvous maneuver is updated by the Average-G Routine in the case of the CSM being the active vehicle, or by a DSKY entry of the maneuver $\Delta V$ if the LM is the active vehicle.

All of the previously discussed methods which the astronaut can use to generate crude estimates of expected $\delta r$ and $\delta v$ values in the case of the Orbit Navigation Routine can also be applied to the Rendezvous Navigation Routine. The astronaut must decide whether or not he is tracking the LM. The CMC cannot make this decision.

In the Cislunar-Midcourse Navigation Routine (Section 5.2.6) the astronaut measures the angle between a star and a planetary landmark or horizon. The data from each angle measurement is processed immediately after it is made. The values of $\delta r$ and $\delta v$ are displayed for astronaut approval before the state vector is updated by the computed deviation $\delta \underline{x}$. Thus, it is a simple matter to repeat the measurement if the astronaut is uncertain as to the validity of the proposed state vector changes.

The parameters required to initialize the navigation routines (Sections 5.2.4 - 5.2.6) are the initial estimated CSM state vector, plus the initial estimated LM state vector for the Rendezvous Navigation Routine, initial state vector estimation error covariance matrices in the form of prestored diagonal error transition matrices (as defined in Section 5.2.2.4), and a priori measurement error variances. The basic input to the navigation routines is SCT or SXT tracking angle data indicated to the CMC by the astronaut when he presses the MARK button signifying that he has centered the optical reticle on the tracking target (landmark or LM) or superimposed the two objects in the case of a star-landmark/horizon measurement. The primary output of the navigation routines is the estimated CSM state vector, plus estimated landmark coordinates in the case of the Orbit Navigation Routine or the estimated LM state vector in the case of the Rendezvous Navigation Routine. The various guidance targeting modes outlined in Section 5.4 are based on the state vector estimates which result from these navigation routines.

## 5.2.2 COASTING INTEGRATION ROUTINE

### 5.2.2.1 General Comments

During all coasting phase navigation procedures, an extrapolation of position and velocity by numerical integration of the equations of motion is required. The basic equation may be written in the form

$$\frac{d^2}{dt^2}\,\underline{r}(t) + \frac{\mu_P^*}{r^3}\,\underline{r}(t) = \underline{a}_d(t) \qquad (2.2.1)$$

where $\mu_P$ is the gravitational constant of the primary body, and $\underline{a}_d(t)$ is the vector acceleration which prevents the motion of the vehicle (CSM or LM) from being precisely a conic with focus at the center of the primary body. The Coasting Integration Routine is a precision integration routine in which all significant perturbation effects are included. The form of the disturbing acceleration $\underline{a}_d(t)$ depends on the phase of the mission.

An approximate extrapolation of a vehicle state vector in which the disturbing acceleration, $\underline{a}_d(t)$ of Eq. (2.2.1), is set to zero may be accomplished by means of the Kepler subroutine (Section 5.5.5).

### 5.2.2.2 Encke's Method

If $\underline{a}_d$ is small compared with the central force field, direct integration of Eq. (2.2.1) is inefficient. Therefore, the extrapolation will be accomplished using the technique of differential accelerations attributed to Encke.

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* In the remainder of Section 5.2 the subscripts P and Q will denote primary and secondary body, respectively. When the body is known, then the subscripts E, M, and S will be used for earth, moon, and sun, respectively. The vehicle will be indicated by the subscripts C for CSM and L for LM.

At time $t_0$ the position and velocity vectors, $\underline{r}_0$ and $\underline{v}_0$, define an osculating conic orbit. The position and velocity vectors in the conic orbit, $\underline{r}_{con}(t)$ and $\underline{v}_{con}(t)$, respectively, will deviate by a small amount from the actual position and velocity vectors.

The conic position and velocity at time t are computed as shown in Section 5.5.5. Required in this calculation is the variable x which is the root of Kepler's equation. In order to minimize the number of iterations required in solving Kepler's equation, an estimate of the correct solution for x is obtained as follows:

Let

$$\tau = t - t_0 \tag{2.2.2}$$

During the previous computation cycle the values

$$\underline{r}' = \underline{r}_{con}\left(\tau - \frac{\Delta t}{2}\right)$$

$$\underline{v}' = \underline{v}_{con}\left(\tau - \frac{\Delta t}{2}\right) \tag{2.2.3}$$

$$x' = x\left(\tau - \frac{\Delta t}{2}\right)$$

were computed. A trial value of $x(\tau)$ is obtained from

$$x_t = x' + s\left[1 - \gamma s\left(1 - 2\gamma s\right) - \frac{1}{6}\left(\frac{1}{r'} - \alpha\right) s^2\right] \tag{2.2.4}$$

where

$$s = \frac{\sqrt{\mu_P}}{r'}\left(\frac{\Delta t}{2}\right)$$

$$\gamma = \frac{\underline{r}' \cdot \underline{v}'}{2r'\sqrt{\mu_P}} \tag{2.2.5}$$

$$\alpha = \frac{2}{r_0} - \frac{v_0^2}{\mu_P}$$

After specification of $\underline{r}_0$, $\underline{v}_0$, $x_t$ and $\tau$, the Kepler subroutine (Section 5.5.5) is used to compute $\underline{r}_{con}(\tau)$, $\underline{v}_{con}(\tau)$, and $x(\tau)$. Because of the excellent initial approximation provided by Eq. (2.2.4), it has not been found necessary to iterate for x more than once (and frequently it is not necessary at all) in order to obtain a value of x for which $\tau - \tau(x)$ is less than the given tolerance level.

The true position and velocity vectors will deviate from the conic position and velocity since $\underline{a}_d$ is not zero. Let

$$\begin{aligned} \underline{r}(t) &= \underline{\delta}(t) + \underline{r}_{con}(t) \\ \underline{v}(t) &= \underline{\nu}(t) + \underline{v}_{con}(t) \end{aligned} \qquad (2.2.6)$$

where $\underline{\delta}(t)$ and $\underline{\nu}(t)$ are the position and velocity deviations from the conic. The deviation vector $\underline{\delta}(t)$ satisfies the differential equation

$$\frac{d^2}{dt^2}\,\underline{\delta}(t) = -\frac{\mu_P}{r_{con}^3(t)}\left[f(q)\,\underline{r}(t) + \underline{\delta}(t)\right] + \underline{a}_d(t) \qquad (2.2.7)$$

subject to the initial conditions

$$\underline{\delta}(t_0) = \underline{0}, \quad \underline{\nu}(t_0) = \underline{0} \qquad (2.2.8)$$

where

$$q = \frac{(\underline{\delta} - 2\underline{r}) \cdot \underline{\delta}}{r^2} \qquad (2.2.9)$$

$$f(q) = q \frac{3 + 3q + q^2}{1 + (1 + q)^{3/2}} \tag{2.2.10}$$

The first term on the right-hand side of Eq. (2.2.7) must remain small, i.e., of the same order as $\underline{a}_d(t)$, if the method is to be efficient. As the deviation vector $\underline{\delta}(t)$ grows in magnitude, this term will eventually increase in size. Therefore, in order to maintain the efficiency of the method, a new osculating conic orbit should be defined by the total position and velocity vectors $\underline{r}(t)$ and $\underline{v}(t)$. The process of selecting a new conic orbit from which to calculate deviations is called rectification. When rectification occurs, the initial conditions for the differential equation for $\underline{\delta}(t)$, as well as the variables $\tau$ and x, are again zero.

5.2.2.3 Disturbing Acceleration

The form of the disturbing acceleration $\underline{a}_d(t)$ that is used in Eq. (2.2.1) depends on the phase of the mission. In earth or lunar orbit, only the gravitational anomalies arising from the non-spherical shape of the primary body need be considered. Let $\underline{a}_{dP}$ be the acceleration due to the non-spherical gravitational anomalies of the primary body. Then, for the earth

$$\underline{a}_{dE} = \frac{\mu_E}{r^2} \sum_{i=2}^{4} J_{iE} \left(\frac{r_E}{r}\right)^i \left[P'_{i+1} (\cos \phi)\, \underline{u}_r - P'_i (\cos \phi)\, \underline{u}_z\right] \tag{2.2.11}$$

where

$$P_2'(\cos\phi) = 3\cos\phi$$

$$P_3'(\cos\phi) = \frac{1}{2}(15\cos^2\phi - 3)$$

$$P_4'(\cos\phi) = \frac{1}{3}(7\cos\phi\, P_3' - 4P_2') \qquad (2.2.12)$$

$$P_5'(\cos\phi) = \frac{1}{4}(9\cos\phi\, P_4' - 5P_3')$$

are the derivatives of Legendre polynominals,

$$\cos\phi = \underline{u}_r \cdot \underline{u}_z \qquad (2.2.13)$$

and $J_2$, $J_3$, $J_4$ are the coefficients of the second, third, and fourth harmonics of the earth's potential function. The vectors $\underline{u}_r$ and $\underline{u}_z$ are unit vectors in the direction of $\underline{r}$ and the polar axis of the earth, respectively, and $r_E$ is the equatorial radius of the earth.

In the case of the moon

$$\underline{a}_{dM} = \frac{\mu_M}{r^2}\left\{\sum_{i=2}^{4} J_{iM}\left(\frac{r_M}{r}\right)^i\left[P_{i+1}'(\cos\phi)\,\underline{u}_r - P_i'(\cos\phi)\,\underline{u}_z\right]\right.$$

$$\left. + 3J_{22M}\left(\frac{r_M}{r}\right)^2\left[4\frac{x_M y_M}{x_M^2 + y_M^2}(\underline{u}_r \times \underline{u}_z) + \frac{x_M^2 - y_M^2}{x_M^2 + y_M^2}((5\cos^2\phi - 3)\,\underline{u}_r - 2\cos\phi\,\underline{u}_z)\right]\right\}$$

(2.2.14)

where $r_M$ is the mean lunar radius, $J_{22M}$ is the coefficient of the term in the moon's gravitational potential function which describes the asymmetry of the moon about its polar axis, and $x_M$ and $y_M$ are the X and Y components of $\underline{r}$ expressed in moon-fixed coordinates. The other terms in Eq. (2. 2. 14) have definitions analogous to those in Eq. (2. 2. 11). The variables $x_M$, $y_M$, and $\underline{u}_z$ are computed by means of the Planetary Inertial Orientation Subroutine (Section 5. 5. 2).

During cislunar-midcourse flight (translunar and transearth) the gravitational attraction of the sun and the secondary body Q (earth or moon) are relevant forces. The accelerations due to the secondary body and the sun are

$$\underline{a}_{dQ} = -\frac{\mu_Q}{r_{QC}^3}\left[f(q_Q)\,\underline{r}_{PQ} + \underline{r}\right] \tag{2. 2. 15}$$

$$\underline{a}_{dS} = -\frac{\mu_S}{r_{SC}^3}\left[f(q_S)\,\underline{r}_{PS} + \underline{r}\right] \tag{2. 2. 16}$$

where $\underline{r}_{PQ}$ and $\underline{r}_{PS}$ are the position vectors of the secondary body and the sun with respect to the primary body, $r_{QC}$ and $r_{SC}$ are the distances of the CSM from the secondary body and the sun, and the arguments $q_Q$ and $q_S$ are computed from

$$q_Q = \frac{(\underline{r} - 2\,\underline{r}_{PQ}) \cdot \underline{r}}{r_{PQ}^2} \tag{2. 2. 17}$$

$$q_S = \frac{(\underline{r} - 2\,\underline{r}_{PS}) \cdot \underline{r}}{r_{PS}^2} \tag{2. 2. 18}$$

The functions $f(q_Q)$ and $f(q_S)$ are calculated from Eq. (2. 2. 10).

The position vectors of the moon relative to the earth, $\underline{r}_{EM}$, and the sun relative to the earth, $\underline{r}_{ES}$, are computed as described in Section (5. 5.4). Then,

$$\underline{r}_{PQ} = \begin{cases} \underline{r}_{EM} & \text{if } P = E \\ -\underline{r}_{EM} & \text{if } P = M \end{cases} \tag{2. 2. 19}$$

and

$$\underline{r}_{PS} = \begin{cases} \underline{r}_{ES} & \text{if } P = E \\ \underline{r}_{ES} - \underline{r}_{EM} & \text{if } P = M \end{cases} \tag{2. 2. 20}$$

Finally,

$$\begin{aligned} \underline{r}_{QC} &= \underline{r} - \underline{r}_{PQ} \\ \underline{r}_{SC} &= \underline{r} - \underline{r}_{PS} \end{aligned} \tag{2. 2. 21}$$

5. 2. 2. 4 Error Transition Matrix

The position and velocity vectors as maintained in the computer are only estimates of the true values. As part of the navigation technique it is necessary also to maintain statistical data in the computer to aid in the processing of navigation measurements.

If $\underline{\epsilon}(t)$ and $\underline{\eta}(t)$ are the errors in the estimates of the position and velocity vectors, respectively, then the six-dimensional correlation matrix E(t) is defined by

$$E_6(t) = \begin{pmatrix} \overline{\underline{\epsilon}(t)\ \underline{\epsilon}(t)^T} & \overline{\underline{\epsilon}(t)\ \underline{\eta}(t)^T} \\ \overline{\underline{\eta}(t)\ \underline{\epsilon}(t)^T} & \overline{\underline{\eta}(t)\ \underline{\eta}(t)^T} \end{pmatrix} \qquad (2.2.22)$$

In certain applications it becomes necessary to expand the state vector and the correlation matrix to more than six dimensions so as to include estimation of landmark locations in the CMC during orbit navigation, and rendezvous radar tracking biases in the LGC during the rendezvous navigation procedure. For this purpose a nine-dimensional correlation matrix is defined as follows :

$$E(t) = \begin{pmatrix} & & \overline{\underline{\epsilon}(t)\ \underline{\beta}^T} \\ & E_6(t) & \\ & & \overline{\underline{\eta}(t)\ \underline{\beta}^T} \\ \overline{\underline{\beta}\ \underline{\epsilon}(t)^T} & \overline{\underline{\beta}\ \underline{\eta}(t)^T} & \overline{\underline{\beta}\ \underline{\beta}^T} \end{pmatrix} \qquad (2.2.23)$$

where the components of the three-dimensional vector $\underline{\beta}$ are the errors in the estimates of three variables which are estimated in addition to the components of the spacecraft state vector.

In order to take full advantage of the operations provided by the interpreter in the computer, the correlation matrix will be restricted to either six or nine dimensions. If, in some navigation procedure, only one or two additional items are to be estimated, then a sufficient number of dummy variables will be added to the desired seven-or eight-dimensional state vector to make it nine-dimensional.

Rather than use the correlation matrix in the navigation procedure, it is more convenient to utilize a matrix W(t), called the error transition matrix, and defined by

$$E(t) = W(t)\, W(t)^T \tag{2.2.24}$$

Extrapolation of the nine-dimensional matrix W(t) is made by direct numerical integration of the differential equation

$$\frac{d}{dt} W(t) = \begin{pmatrix} O & I & O \\ G(t) & O & O \\ O & O & O \end{pmatrix} W(t) \tag{2.2.25}$$

where G(t) is the three-dimensional gravity gradient matrix, and I and O are the three-dimensional identity and zero matrices, respectively. If the W matrix is partitioned as

$$W = \begin{pmatrix} \underline{w}_0 & \underline{w}_1 & \cdots & \underline{w}_8 \\ \underline{w}_9 & \underline{w}_{10} & \cdots & \underline{w}_{17} \\ \underline{w}_{18} & \underline{w}_{19} & \cdots & \underline{w}_{26} \end{pmatrix} \tag{2.2.26}$$

then,

$$\left.\begin{aligned} \frac{d}{dt}\,\underline{w}_i(t) &= \underline{w}_{i+9}(t) \\ \frac{d}{dt}\,\underline{w}_{i+9}(t) &= G(t)\,\underline{w}_i(t) \\ \frac{d}{dt}\,\underline{w}_{i+18}(t) &= \underline{0} \end{aligned}\right\} \quad i = 0,\ 1, \ldots, 8 \qquad (2.2.27)$$

The extrapolation may be accomplished by successively integrating the vector differential equations

$$\frac{d^2}{dt^2}\,\underline{w}_i(t) = G(t)\,\underline{w}_i(t) \qquad i = 0, 1, \ldots, 8 \qquad (2.2.28)$$

The gravity gradient matrix G(t) for earth or lunar orbit is given by

$$G(t) = \frac{\mu_P}{r^5(t)}\left[3\,\underline{r}(t)\,\underline{r}(t)^T - r^2(t)\,I\right] \qquad (2.2.29)$$

During cislunar-midcourse flight

$$\begin{aligned} G(t) = {} & \frac{\mu_P}{r^5(t)}\left[3\,\underline{r}(t)\,\underline{r}(t)^T - r^2(t)\,I\right] \\ & + \frac{\mu_Q}{r^5_{QC}(t)}\left[3\,\underline{r}_{QC}(t)\,\underline{r}_{QC}(t)^T - r^2_{QC}(t)\,I\right] \end{aligned} \qquad (2.2.30)$$

Thus, if D is the dimension of the matrix W(t) for the given navigation procedure, the differential equations for the $\underline{w}_i(t)$ vectors are

$$\frac{d^2}{dt^2}\underline{w}_i(t) = \frac{\mu_P}{r^3(t)}\left\{3\left[\underline{u}_r(t)\cdot\underline{w}_i(t)\right]\underline{u}_r(t) - \underline{w}_i(t)\right\} + M\frac{\mu_Q}{r^3_{QC}(t)}\left\{3\left[\underline{u}_{QC}(t)\cdot\underline{w}_i(t)\right]\underline{u}_{QC}(t) - \underline{w}_i(t)\right\} \qquad (2.2.31)$$

$$i = 0, 1, \ldots, D-1$$

where $\underline{u}_r(t)$ and $\underline{u}_{QC}(t)$ are unit vectors in the directions of $\underline{r}(t)$ and $\underline{r}_{QC}(t)$, respectively, and

$$M = \begin{cases} 1 \text{ for cislunar-midcourse flight} \\ 0 \text{ for earth or lunar orbit} \end{cases} \qquad (2.2.32)$$

5.2.2.5 Numerical Integration Method

The extrapolation of navigational data requires the solution of a number of second-order vector differential equations, specifically Eqs. (2.2.7) and (2.2.31). These are all special cases of the form

$$\frac{d^2}{dt^2}\underline{y} = \underline{f}(\underline{y}, t) \qquad (2.2.33)$$

Nystrom's method is particularly well suited to this form and gives an integration method of fourth-order accuracy. The second-order system is written

$$\frac{d}{dt} y = z$$

$$\frac{d}{dt} \underline{z} = \underline{f}(\underline{y}, t) \tag{2.2.34}$$

and the formulae are summarized below.

$$\underline{y}_{n+1} = \underline{y}_n + \underline{\phi}(\underline{y}_n)\Delta t$$

$$\underline{z}_{n+1} = \underline{z}_n + \underline{\psi}(\underline{y}_n)\Delta t$$

$$\underline{\phi}(\underline{y}_n) = \underline{z}_n + \frac{1}{6}(\underline{k}_1 + 2\underline{k}_2)\Delta t$$

$$\underline{\psi}(\underline{y}_n) = \frac{1}{6}(\underline{k}_1 + 4\underline{k}_2 + \underline{k}_3) \tag{2.2.35}$$

$$\underline{k}_1 = \underline{f}(\underline{y}_n, t_n)$$

$$\underline{k}_2 = \underline{f}(\underline{y}_n + \frac{1}{2}\underline{z}_n \Delta t + \frac{1}{8}\underline{k}_1 (\Delta t)^2, \quad t_n + \frac{1}{2}\Delta t)$$

$$\underline{k}_3 = \underline{f}(\underline{y}_n + \underline{z}_n \Delta t + \frac{1}{2}\underline{k}_2 (\Delta t)^2, \; t_n + \Delta t)$$

For efficient use of computer storage as well as computing time the computations are performed in the following order:

1) Equation (2. 2. 7) is solved using the Nystrom formulae, Eq. (2. 2. 35). It is necessary to preserve the values of the vector $\underline{r}$ at times $t_n$, $t_n + \Delta t/2$, $t_n + \Delta t$ for use in the solution of Eqs. (2. 2. 31).

2) Equations (2. 2. 31) are solved one-at-a-time using Eqs. (2. 2. 35) together with the values of $\underline{r}$ which resulted from the first step.

The variable $\Delta t$ is the integration time step and should not be confused with $\tau$, the time since rectification. The maximum value for $\Delta t$ which can be used for precision integration, $\Delta t_{max}$, is given by the empirically determined formula

$$\Delta t_{max} = \text{minimum}\left(\Delta t_{\ell im}, \frac{K\, r^{3/2}}{\sqrt{\mu_P}}\right) \tag{2. 2. 36}$$

where the calculation is truncated to the nearest two centiseconds and

$$\begin{aligned} \Delta t_{\ell im} &= 4000 \text{ sec.} \\ K &= 0.3 \end{aligned} \tag{2. 2. 37}$$

Because of the form of Eq. (2. 2. 36), it is possible for the Coasting Integration Routine to use an excessive amount of time to extrapolate a state vector if the resulting trajectory passes too close to the target planet. Furthermore, the capability of extrapolating a state vector to a desired radial distance may be a useful function. For these reasons, the desired final time and the desired final radius must be specified by the calling routine.

### 5. 2. 2. 6 Coasting Integration Logic

Estimates of the state vectors of two vehicles (CSM and LM) will be maintained in the computer. In various phases of the mission it will be required to extrapolate a state vector either alone or with an associated W matrix of dimension six or nine.

To accomplish all of these possible procedures, as well as to solve the computer restart problem, three state vectors will be maintained in the computer. Let $\underline{x}_C$ and $\underline{x}_L$ be the estimated CSM and LM state vectors, respectively, and let $\underline{x}$ be a temporary state vector. The state vector $\underline{x}$ is a symbolic representation of the following set of variables:

$$
\begin{aligned}
\underline{r}_0 &= \text{rectification position vector} \\
\underline{v}_0 &= \text{rectification velocity vector} \\
\underline{r}_{con} &= \text{conic position vector} \\
\underline{v}_{con} &= \text{conic velocity vector} \\
\underline{\delta} &= \text{position deviation vector} \\
\underline{\nu} &= \text{velocity deviation vector} \\
t &= \text{time associated with } \underline{r}_{con},\ \underline{v}_{con},\ \underline{\delta} \text{ and } \underline{\nu} \\
\tau &= \text{time since rectification} \\
x &= \text{root of Kepler's equation} \\
P &= \text{primary body} = \begin{cases} \pm 1 \text{ for earth} \\ \pm 2 \text{ for moon} \end{cases}
\end{aligned}
\tag{2.2.38}
$$

where the positive sign indicates cislunar-midcourse flight and the negative sign near-earth or near-lunar orbit.

The state vectors $\underline{x}_C$ and $\underline{x}_L$ represent an analogous set of variables.

The Coasting Integration Routine is controlled by the calling program by means of the two indicators D and V. The variable D indicates the dimension of the W matrix with

$$D = 0 \tag{2.2.39}$$

denoting that the state vector only is to be extrapolated. The varible V indicates the appropriate vehicle as follows:

$$V = \begin{cases} 1 \text{ for CSM} \\ 0 \text{ for LM} \\ -1 \text{ for state vector specified by calling program} \end{cases} \tag{2.2.40}$$

In addition, the calling program must set the desired final time $t_F$; the desired final radius $r_F$; and, for V equal to -1, the desired state vector $\underline{x}$.

A simplified functional diagram of the Coasting Integration Routine is shown in Fig. 2.2-1. In the figure the indicated state vector is being integrated to time $t_F$. The value of $\Delta t$ for each time step is $\Delta t_{max}$ (Eq. (2.2.36)) or the total time-to-go whichever is smaller. The integration is terminated when the computed value of $\Delta t$ is less than $\epsilon_t$, or the magnitude of the position vector is less than $r_F$. Figures 2.2-2, 2.2-3 and 2.2-4 illustrate in more detail the logic flow of this routine. In these figures certain items which have not been discussed fully in the text are explicitly illustrated. The following is a list of these items together with the number of the figure in which each occurs.

1) Saving of $\underline{r}$ values for W matrix integration: Fig. 2.2-2.

2) Change in origin of coordinates: Fig. 2.2-3.

3) Rectification procedure: Fig. 2.2-3.

4) Selection of disturbing acceleration: Fig. 2.2-4.

The logic flow shown in these figures is controlled by the three switches or flags A, B, and F. Switch A is used in the determination

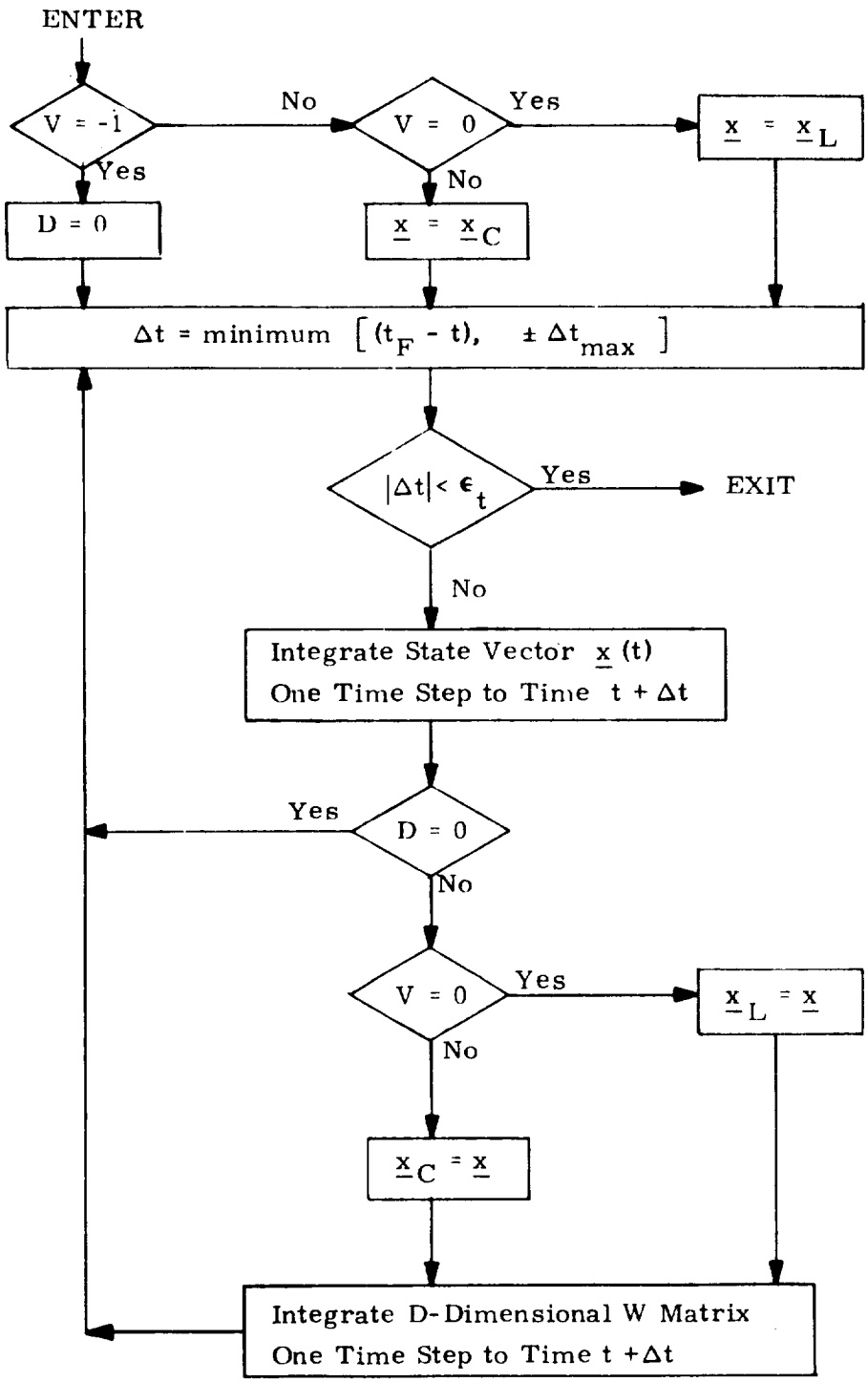



Figure 2.2-1 Simplified Coasting Integration Routine Logic Diagram

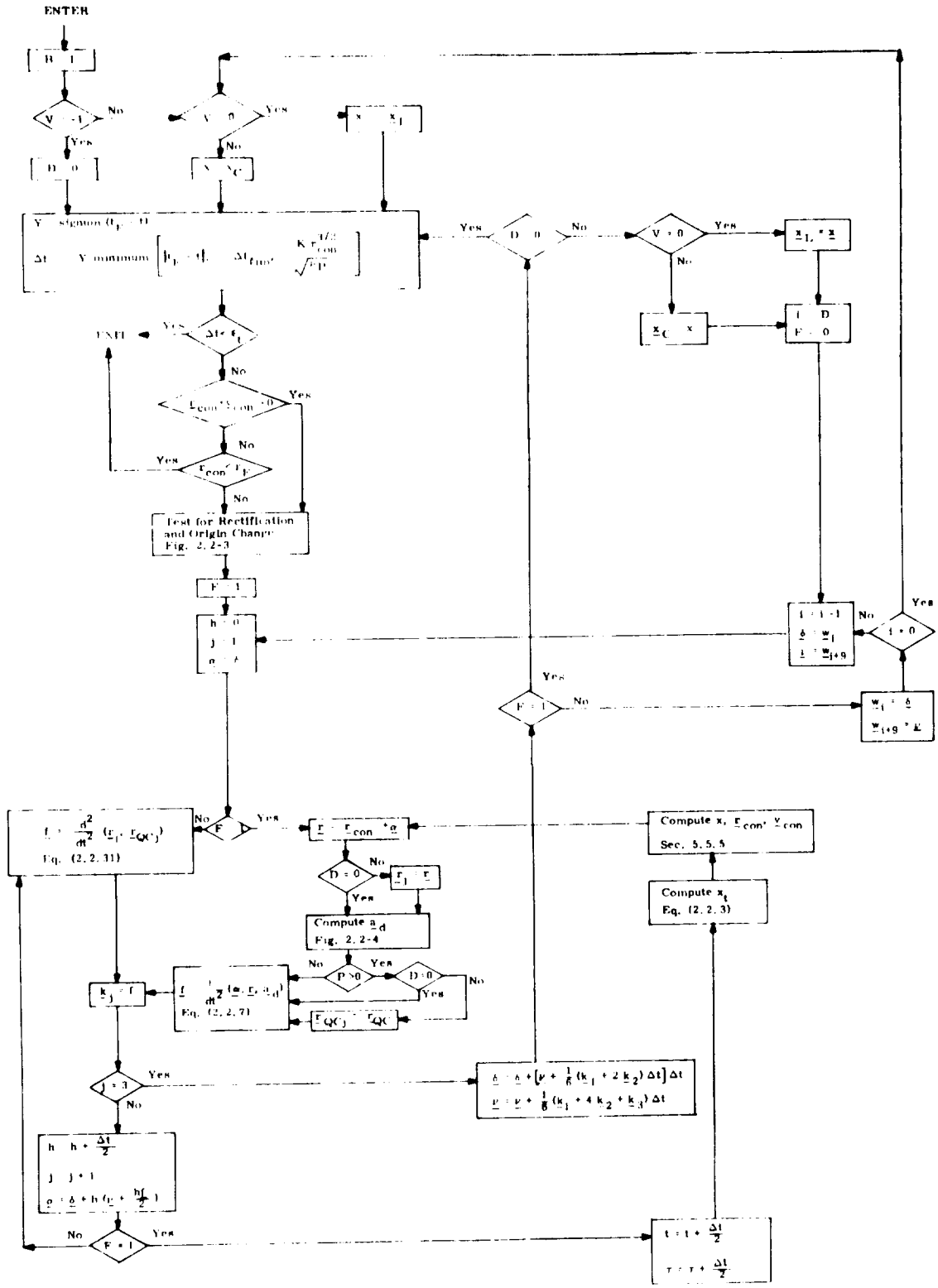



Figure 2.2-2 Coasting Integration Routine Logic Diagram

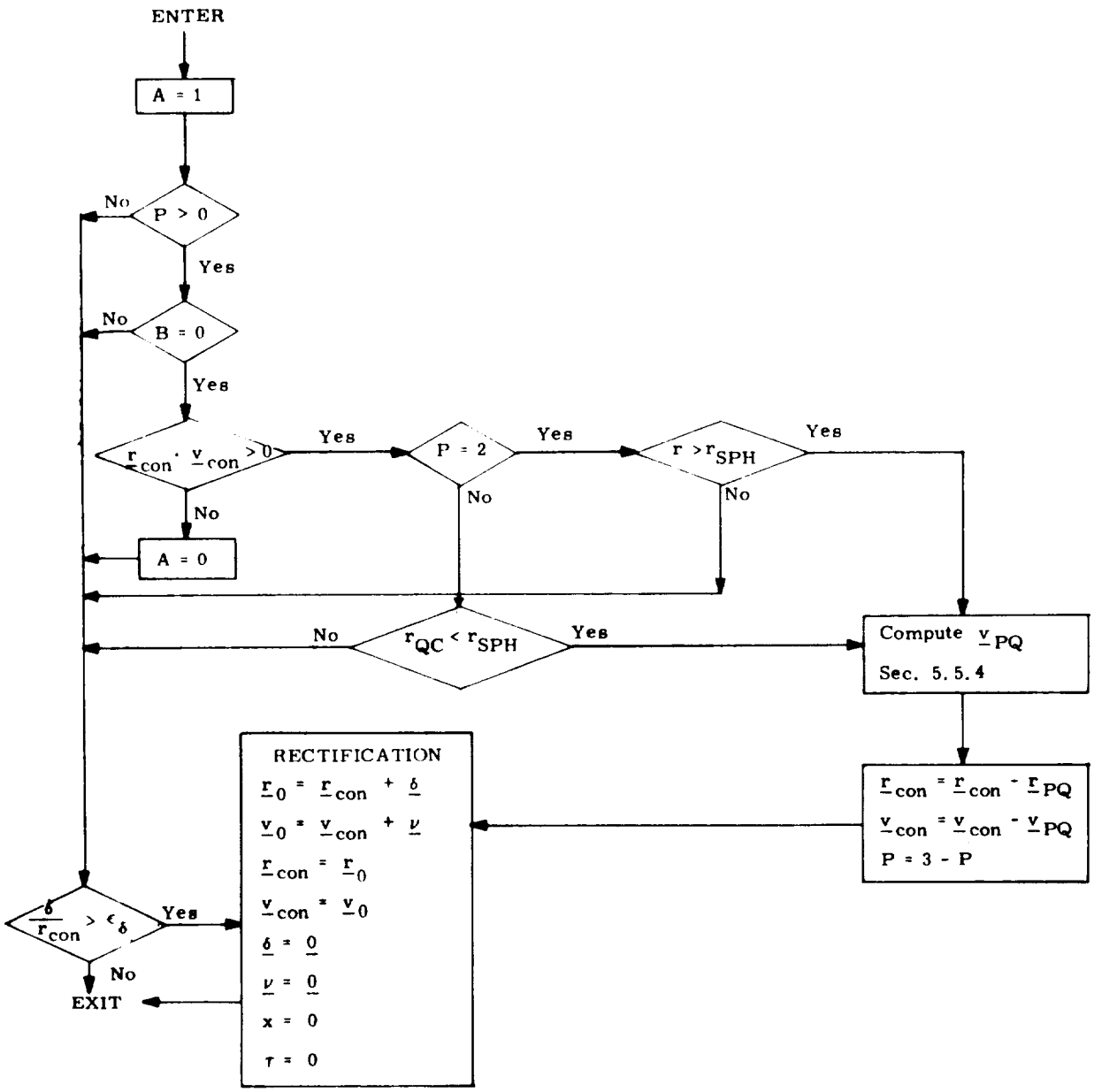



Figure 2.2-3 Rectification and Coordinate System Origin Change Logic Diagram

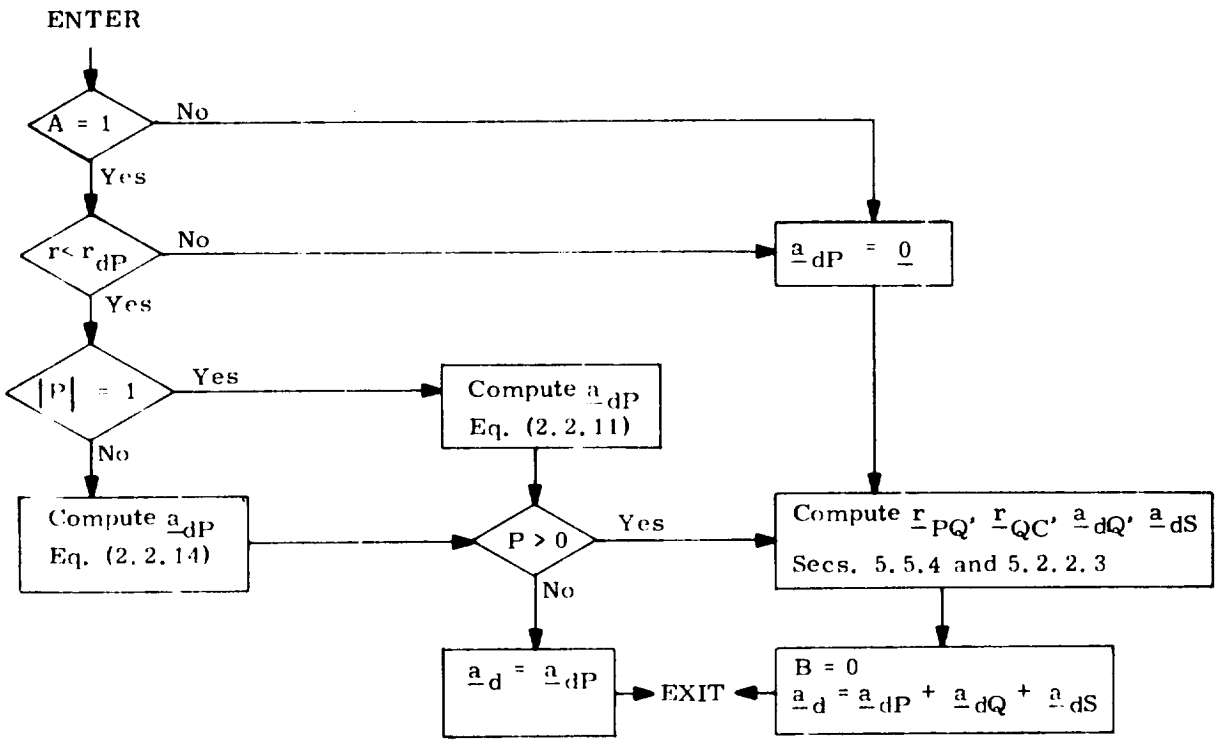



Figure 2.2-4 Disturbing Acceleration Selection Logic Diagram

of whether or not the acceleration due to the non-spherical gravitational anomalies of the primary planet, $\underline{a}_{dP}$, should be included in the disturbing acceleration. The acceleration $\underline{a}_{dP}$ is relevant during earth and lunar orbit, and during that part of cislunar-mid-course flight in which the vehicle is both departing from the primary planet and is within the prescribed radius $r_{dP}$. Switch B inhibits the test for a change in the origin of the Basic Reference Coordinate System until the vector $\underline{r}_{PQ}$ is available. Switch F is used to distinguish between state vector integration (F = 1) and W matrix integration (F = 0).

The definitions of the various control constants which appear in Figs. 2.2-1 to 2.2-4 are as follows:

$\epsilon_t$ = integration time step criterion

$\epsilon_\delta$ = rectification criterion

$r_{SPH}$ = radius of lunar sphere of influence

$r_{dE}$ = radius of relevance for earth non-spherical gravitational anomalies

$r_{dM}$ = radius of relevance for moon non-spherical gravitational anomalies

### 5.2.3 MEASUREMENT INCORPORATION ROUTINE

Periodically it is necessary to update the estimated position and velocity vectors of the vehicle (CSM or LM) by means of navigation measurements. At the time a measurement is made, the best estimate of the state vector of the spacecraft is the extrapolated estimate denoted by $\underline{x}'$. The first six components of $\underline{x}'$ are the components of the estimated position and velocity vectors. In certain situations it becomes necessary to estimate more than six quantities. Then, the state vector will be of nine dimensions. From this state vector estimate it is possible to determine an estimate of the quantity measured. When the predicted value of this measurement is compared with the actual measured quantity, the difference is used to update the indicated state vector as well as its associated error transition matrix as described in Section 5.2.1. The error transition matrix, W, is defined in Section 5.2.2.4.

This routine is used to compute deviations to be added to the components of the estimated state vector, and to update the estimated state vector by these deviations provided the deviations pass a state vector update validity test as described in Section 5.2.1.

Let D be the dimension (six or nine) of the estimated state vector. Associated with each measurement are the following parameters which are to be specified by the program calling this routine:

$\underline{b}$ = Geometry vector of D dimensions

$\overline{\alpha^2}$ = A priori measurement error variance

$\delta Q$ = Measured deviation, the difference between the quantity actually measured and the expected value based on the original value of the estimated state vector $\underline{x}'$.

The procedure for incorporating a measurement into the estimated state vector is as follows:

1. Compute a D-dimensional $\underline{z}$ vector from

$$\underline{z} = W'^{T} \underline{b} \tag{2.3.1}$$

where W' is the error transition matrix associated with $\underline{x}'$.

2. Compute the D-dimensional weighting vector, $\underline{\omega}$, from

$$\underline{\omega}^{T} = \frac{1}{z^2 + \overline{\alpha^2}} \underline{z}^{T} W'^{T} \tag{2.3.2}$$

3. Compute the state vector deviation estimates from

$$\delta \underline{x} = \underline{\omega} \, \delta Q \tag{2.3.3}$$

4. If the data pass the validity test, update the state vector and the W matrix by

$$x = \underline{x}' + \delta \underline{x} \tag{2.3.4}$$

$$W = W' - \frac{\underline{\omega} \, \underline{z}^{T}}{1 + \sqrt{\dfrac{\overline{\alpha^2}}{z^2 + \overline{\alpha^2}}}} \tag{2.3.5}$$

In order to take full advantage of the three-dimensional vector and matrix operations provided by the interpreter in the computer, the nine-dimensional W matrix will be stored sequentially in the computer as follows:

$$\underline{w}_0, \underline{w}_1, \cdots, \underline{w}_{26}$$

Refer to Section 5. 2. 2. 4 for the definition of the W matrix. Define the three-dimensional matrices

$$W_0 = \begin{pmatrix} \underline{w}_0^T \\ \underline{w}_1^T \\ \underline{w}_2^T \end{pmatrix} \qquad W_1 = \begin{pmatrix} \underline{w}_3^T \\ \underline{w}_4^T \\ \underline{w}_5^T \end{pmatrix} \qquad \cdots \qquad W_8 = \begin{pmatrix} \underline{w}_{24}^T \\ \underline{w}_{25}^T \\ \underline{w}_{26}^T \end{pmatrix} \tag{2. 3. 6}$$

so that

$$W = \begin{pmatrix} W_0^T & W_1^T & W_2^T \\ W_3^T & W_4^T & W_5^T \\ W_6^T & W_7^T & W_8^T \end{pmatrix} \tag{2. 3. 7}$$

Let the nine-dimensional vectors $\delta\underline{x}$, $\underline{b}$, $\underline{\omega}$, and $\underline{z}$ be partitioned as follows:

$$\delta\underline{x} = \begin{pmatrix} \delta\underline{x}_0 \\ \delta\underline{x}_1 \\ \delta\underline{x}_2 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} \underline{b}_0 \\ \underline{b}_1 \\ \underline{b}_2 \end{pmatrix} \qquad \underline{\omega} = \begin{pmatrix} \underline{\omega}_0 \\ \underline{\omega}_1 \\ \underline{\omega}_2 \end{pmatrix} \qquad \underline{z} = \begin{pmatrix} z_0 \\ z_1 \\ \vdots \\ z_8 \end{pmatrix} = \begin{pmatrix} \underline{z}_0 \\ \underline{z}_1 \\ \underline{z}_2 \end{pmatrix}$$

(2. 3. 8)

Then, the computations shown in Eqs. (2. 3. 1) through (2. 3. 3) are performed as follows, using three-dimensional operations:

$$\underline{z}_i = \sum_{j=0}^{\frac{D}{3}-1} W'_{i+3j}\, \underline{b}_j$$

$$a = \sum_{j=0}^{\frac{D}{3}-1} \underline{z}_j \cdot \underline{z}_j + \overline{\alpha^2}$$

$$\underline{\omega}_i^T = \frac{1}{a} \sum_{j=0}^{\frac{D}{3}-1} \underline{z}_j^T W'_{3i+j} \tag{2. 3. 9}$$

$$\delta \underline{x}_i = \delta Q\, \underline{\omega}_i \qquad \left(i = 0,\ 1,\ \ldots,\ \frac{D}{3} - 1\right)$$

Equation (2. 3. 5) is written

$$\gamma = \frac{1}{1 + \sqrt{\overline{\alpha^2}/a}} \tag{2. 3. 10}$$

$$\underline{w}_{i+9j} = \underline{w}'_{i+9j} - \gamma\, z_i\, \underline{\omega}_j \qquad \left(\begin{array}{l} i = 0,\ 1,\ \ldots,\ D - 1 \\ j = 0,\ 1,\ \ldots,\ \frac{D}{3} - 1 \end{array}\right)$$

The Measurement Incorporation Routine is divided into two subroutines, INCORP1 and INCORP2. The subroutine INCORP1 consists of Eqs. (2. 3. 9), while INCORP2 is composed of Eqs. (2. 3. 4) and (2. 3. 10). The method of using these subroutines is illustrated in Fig. 2. 3-1.

Since the estimated position and velocity vectors are maintained in two pieces, conic and deviation from the conic, Eq. (2. 3. 4) cannot be applied directly. The estimated position and velocity deviations resulting from the measurement, $\delta \underline{x}_0$ and $\delta \underline{x}_1$, are added to the vectors $\underline{\delta}$ and $\underline{\nu}$, the position and velocity deviations from the conics, respectively. Since $\underline{\delta}$ and $\underline{\nu}$ are maintained to much higher accuracy than the conic position and velocity vectors, a possible computation overflow situation exists whenever Eq. (2. 3. 4) is applied. If overflow does occur, then it is necessary to reinitialize the Coasting Integration Routine (Section 5. 2. 2) by the process of rectification as described in Section 5. 2. 2. 2. The logic flow of the subroutine INCORP2 is illustrated in detail in Fig. 2. 3-2.

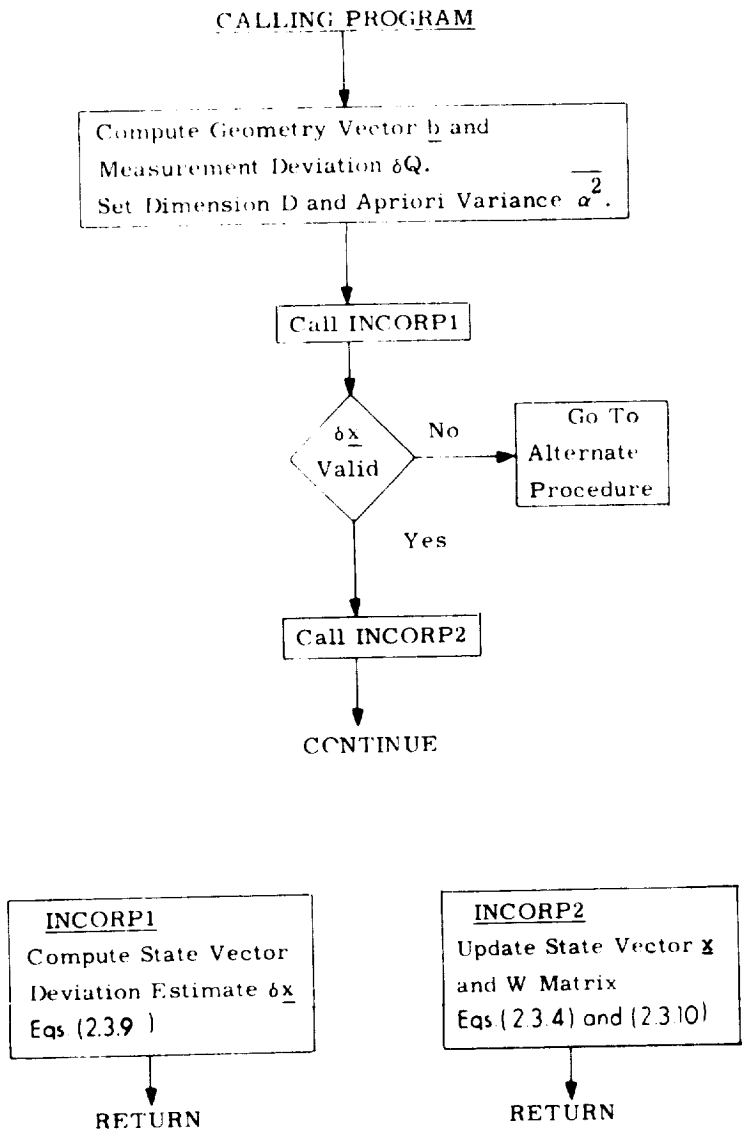



Fig. 2.3-1 Measurement Incorporation Procedure

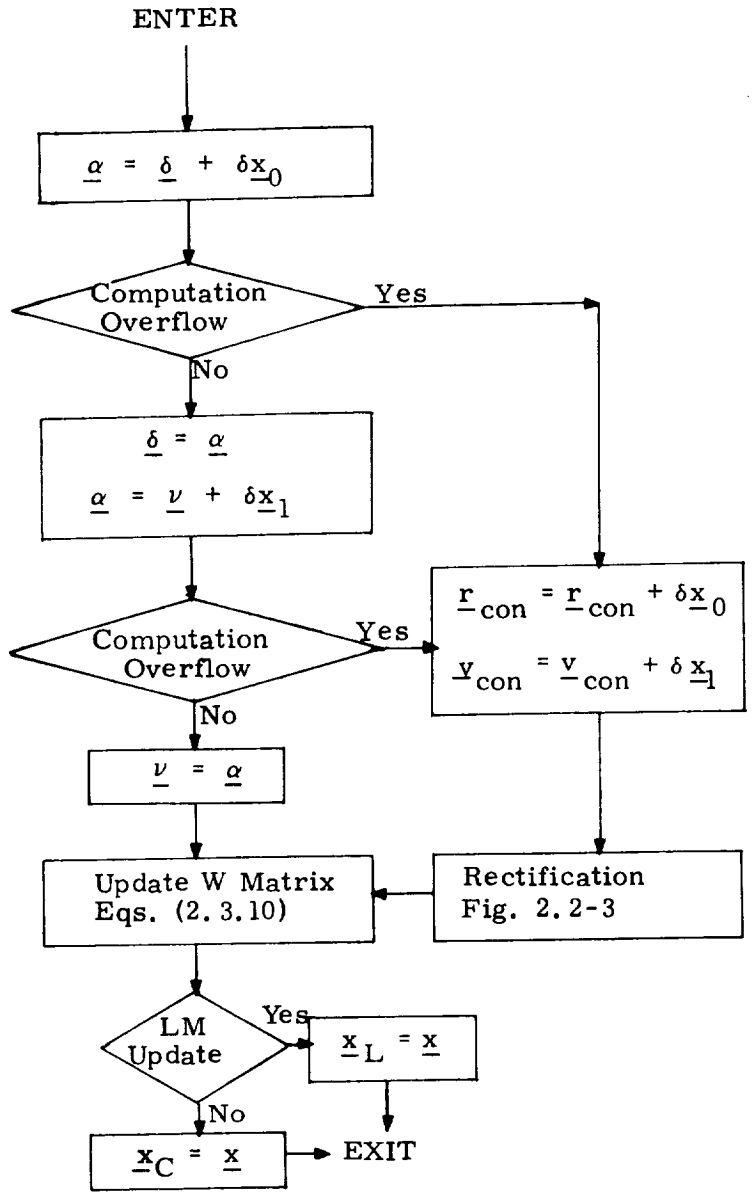



Fig. 2.3-2 INCORP2 Subroutine Logic Diagram

## 5.2.4 ORBIT NAVIGATION ROUTINE

### 5.2.4.1 Landmark Tracking Procedure

While the CSM is in lunar or earth orbit, landmark optical tracking data are used to update the estimated CSM state vector and the coordinates of the landmark that is being tracked, as described in Section 5. 2. 1. This routine is used to process the landmark-tracking measurement data, as shown in simplified form in Fig. 2. 1-1, and is used normally in lunar orbit in the lunar landing mission. The routine also can be used in earth orbit during abort situations or alternate missions.

The astronaut should use the Lunar Landmark Selection Routine (Section 5. 6. 9) or the Ground Track Routine (Section 5. 6. 5) as an aid in selecting appropriate landmarks for tracking prior to using the Orbit Navigation Routine.

In order to initially acquire and maintain optical tracking, the CSM attitude must be oriented such that the CSM-to-landmark line-of-sight falls within the SCT field of view. In the CSM GNCS there is no automatic vehicle attitude control during the landmark tracking procedure. Any desired attitude control must be accomplished manually by the astronaut.

If the astronaut wishes, he may use the Automatic Optics Designate Routine (Section 5. 6. 8) as an aid in the acquisition of the landmark. This routine has two modes which are relevant to orbit navigation. In the advanced ground track mode (which is useful in lunar orbit for surveillance, selection, and tracking of possible landing sites) the routine drives the CSM optics to the direction of the point on the ground track of the spacecraft at a time slightly more than a specified number of orbital revolutions ahead of current time.

In the landmark mode (which is useful for acquisition of a specified landmark) the routine drives the optics to the estimated direction of the specified landmark. Either the revolution number or the landmark must be specified by the astronaut. The computations and positioning commands in this routine are repeated once per second provided the optics mode switch is set to CMC. Thus, in the advanced ground track mode, the astronaut is shown continuously the ground track of the CSM for a future revolution. The reason for this mode is that it is desirable to select a landing site which is near the CSM orbital plane at the LM lunar landing time.

The Automatic Optics Designate Routine is used in other routines to align the CSM optics to the directions of the following sighting targets:

1) The LM during the rendezvous phase

2) A specified star during IMU alignment procedures

After the astronaut has acquired the desired landmark (not necessarily the one specified to the Automatic Optics Designate Routine), he switches the optics mode to MANUAL and centers the SCT or SXT reticle on the landmark. When accurate tracking is achieved, he presses the optics MARK button, upon which the time of the measurement and all optics and IMU gimbal angles are stored in the CMC. Up to five unrejected navigation sightings of the same landmark may be made during the tracking interval,and all sets of navigation data are acquired before processing of the data begins.

After the astronaut has completed the tracking of a landmark, he is asked by the CMC whether or not he wishes to identify the tracked landmark. If he does, then he enters into the CMC through

the keyboard the identification number or the coordinates of the landmark, and the data are processed as described in Section 5. 2. 4. 2.

The coordinates of twenty-five preselected lunar landmarks will be stored in the CMC fixed memory. In addition, the coordinates of the landing site will be stored in the CMC erasable memory and can be updated during the mission. In general, code numbers will be used to identify landmarks during a lunar-orbit navigation phase, but it will be necessary for the astronaut to enter the actual coordinates in the case of earth landmarks.

If the astronaut does not identify the landmark, then the Landing Site Designation procedure (Section 5.2.4.3) is used for the navigation data processing. In this process the landmark is considered to be unknown, and the first set of navigation data is used to compute an initial estimate of the landmark location. The remaining sets of data are then processed as described in Section 5. 2. 4. 2 to update the estimated nine-dimensional CSM-landmark state vector.

Whether the landmark is identified or not, one further option is available to the astronaut. He may specify that one of the navigation sightings is to be considered the designator for an offset landing site near the tracked landmark. In this case, the designated navigation data set is saved, the remaining sets of data are processed as described above, and then the estimated offset landing site location is determined from the saved data as described in Section 5. 2. 4. 4. This procedure offers the possibility of designating a landing site in a flat area of the moon near a landmark which is suitable for optical navigation tracking but not for landing.

Each set of navigation data which is used for state vector updating and not for landing site designation or offset produces two updates as described in Section 5. 2. 4. 2. For the first navigation data set the magnitudes of the first proposed changes in the estimated CSM position and velocity vectors, $\delta r$ and $\delta v$, respectively, are displayed for astronaut approval. If the astronaut accepts these proposed changes, then all state vector updates will be performed, and all the information obtained during the tracking of this landmark will be incorporated into the state vector estimates. A detailed discussion of this state vector update validity check is given in Section 5. 2. 1.

After all of the sets of navigation data have been processed, the astronaut has the option of having the updated landmark coordinates (or the coordinates of the unknown landmark) stored in the erasable memory registers allocated to the landing site coordinates. In this manner the original coordinates of the landing site can be revised, or a new landing site can be selected.

The various functions for which the sets of navigation data acquired from the line-of-sight tracking measurements are used are presented in Sections 5. 2. 4. 2 through 5. 2. 4. 4. A detailed description in the form of logic diagrams of the entire Orbit Navigation Routine with all of its options is given in Section 5. 2. 4. 5.

### 5.2.4.2 State Vector Update from Landmark Sighting

As mentioned in Section 5. 2. 1 the orbit navigation concept involves the nine-dimensional state vector

$$\underline{x} = \begin{pmatrix} \underline{r}_C \\ \underline{v}_C \\ \underline{r}_\ell \end{pmatrix} \qquad (2.4.1)$$

where $\underline{r}_C$ and $\underline{v}_C$ are the estimated CSM position and velocity vectors, respectively, and $\underline{r}_\ell$ is the estimated landmark position vector. Both the CSM state vector and the landmark position vector are estimated and updated through the processing of optical tracking data. A simplified functional diagram of the orbit navigation procedure is illustrated in Fig. 2.1-1. In this section the method of updating the estimated nine-dimensional state vector from a landmark line-of-sight navigation measurement is given.

After the preferred CSM attitude is achieved and optical tracking acquisition is established (Section 5.2.4.1), the astronaut enters tracking data into the CMC by pressing the optics MARK button when he has centered the SCT or SXT reticle on the landmark. As described in Section 5.2.4.1, each set of navigation data contains the time of the measurement and the two optics and three IMU gimbal angles. From these five angles the measured unit vector, $\underline{u}_M$, along the CSM-to-landmark line-of-sight is computed in the Basic Reference Coordinate System from

$$\underline{u}_M = [\mathrm{REFSMMAT}]^T\,[\mathrm{NBSM}]\,\underline{u}_{NB} \qquad (2.4.2)$$

where [REFSMMAT] and [NBSM] are transformation matrices and $\underline{u}_{NB}$ is the measured line-of-sight vector in navigation base coordinates. All terms of Eq. (2.4.2) are defined in Section 5.6.3.

For the purpose of navigation it is convenient to consider the measured unit vector, $\underline{u}_M$, to be the basic navigation data. This

navigation measurement of the line-of-sight vector, $\underline{u}_M$, is mathematically equivalent to the simultaneous measurement of the angles between the lines-of-sight to the landmark and two stars. The data are processed by selecting two convenient unit vectors (fictitious star directions), converting the vector $\underline{u}_M$ to an equivalent set of two artificial star-landmark measurements, and using the Measurement Incorporation Routine (Section 5.2.3) twice, once for each artificial measurement. These two unit vectors are chosen to be perpendicular to each other and to the current estimated line-of-sight vector so as to maximize the convenience and accuracy of the procedure.

Let $\underline{r}_C$ and $\underline{r}_\ell$ be the estimated CSM and landmark position vectors at the time of a given line-of-sight measurement. Then, the first state vector update for the measurement is performed as follows:

1. Calculate the estimated CSM-to-landmark line-of-sight from

$$\underline{r}_{CL} = \underline{r}_\ell - \underline{r}_C \qquad (2.4.3)$$
$$\underline{u}_{CL} = \text{UNIT}\,(\underline{r}_{CL})$$

2. Initialize the fictitious star direction to the vector

$$\underline{u}_s = \text{UNIT}\,(\underline{u}_{CL} \times \underline{u}_M) \qquad (2.4.4)$$

If the vectors $\underline{u}_{CL}$ and $\underline{u}_M$ are separated by an angle of less than $2^{-19}$ rad., then a computation overflow occurs in the execution of Eq. (2.4.4), and this set of measurement data is discarded because all of the components of the estimated state vector deviation, $\delta\,\underline{x}$, would be negligible for both state vector updates.

(3) Compute an artificial star direction from

$$\underline{u}_s = \text{UNIT}\,(\underline{u}_s \times \underline{u}_{CL}) \tag{2.4.5}$$

(4) Calculate the nine-dimensional geometry vector, $\underline{b}$, from

$$\underline{b}_0 = \frac{1}{r_{CL}}\,\underline{u}_s \tag{2.4.6}$$

$$\underline{b}_1 = \underline{0} \tag{2.4.7}$$

$$\underline{b}_2 = -\,\underline{b}_0 \tag{2.4.8}$$

(5) Determine the measured deviation, $\delta Q$, from

$$\begin{aligned}\delta Q &= \cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \cos^{-1}(\underline{u}_s \cdot \underline{u}_{CL}) \\ &= \cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2}\end{aligned} \tag{2.4.9}$$

(6) Incorporate the fictitious star-landmark measurement using the Measurement Incorporation Routine (Section 5.2.3).

Included in Step (6) is the state vector update validity check for the first proposed update.

It should be noted that the initialization of the star direction, $\underline{u}_s$, which is given by Eq. (2. 4. 4), is such that the first artificial star (computed from Eq. (2. 4. 5) ) will yield the maximum value for the measured deviation, $\delta Q$, which is obtained from Eq. (2. 4. 9). The reason for selecting the first $\underline{u}_s$ vector in this manner is that there is only one state vector update validity check even though there are two updates.

Assuming that the first state vector update was accepted by the astronaut, the second update for this measurement data set is performed by first recomputing the estimated CSM-to-landmark line-of-sight vector from Eq. (2. 4. 3) using the updated values of the estimated CSM and landmark position vectors, $\underline{r}_C$ and $\underline{r}_\ell$, respectively. Then, Steps (3) - (6) are repeated, this time with no state vector update validity check.

If the astronaut rejects the first state vector update, then all of the navigation data is discarded, and no update occurs.

The results of the processing of the measured line-of-sight vector, $\underline{u}_M$, are updated values of the estimated position and velocity vectors of the CSM, $\underline{r}_C$ and $\underline{v}_C$, respectively, and an updated value of the estimated landmark position vector, $\underline{r}_\ell$.

### 5.2.4.3 Landing Site Designation

As mentioned in Section 5. 2. 4. 1 the nine-dimensional orbit navigation procedure provides the means of mapping on the

surface of the planet a point which is designated only by a number of sets of optical tracking data. This process may be used to redesignate the landing site optically, or as an unknown landmark orbit navigation procedure.

Assume that an unmapped landmark has been tracked, and N sets of optical measurement data have been acquired as described in Section 5. 2. 4. 1. Let $\underline{u}_M$ be the measured unit CSM-to-landmark line-of-sight vector obtained from the first set of measurement data by means of Eq. (2. 4. 2). An estimate of the landmark position at the time of the first navigation sighting, $t_M$, is given by

$$\underline{r}_\ell = \underline{r}_C + r_C \left[ \cos A - \left( \frac{r_0^2}{r_C^2} - \sin^2 A \right)^{1/2} \right] \underline{u}_M \quad (2.4.10)$$

where

$$\cos A = \frac{\underline{u}_M \cdot \underline{r}_C}{r_C} \quad (2.4.11)$$

$\underline{r}_C$ is the estimated CSM position vector at time $t_M$, and $r_0$ is the estimated planetary radius. This initial estimated landmark position vector, $\underline{r}_\ell$, and the estimated CSM state vector are then updated by means of the standard Orbit Navigation Routine and the last N - 1 sets of tracking measurement data exactly as if the designated point were a mapped landmark.

The final results of this procedure are a location estimate for the designated point and an improvement in the estimated CSM state vector.

### 5.2.4.4 Landing Site Offset

During the landing site selection operation any visible landmark may be tracked that is in, or near, the desired landing area. In most cases this visible landmark will not be an acceptable touch down point, and it is desirable to offset the desired landing point away from the visible landmark used for tracking. This is accomplished by tracking the visible landmark and processing this data as previously described for either a mapped or an unknown landmark depending upon the type of landmark tracked. During this tracking operation a designated navigation data set can be taken by positioning the SXT to the desired actual landing point. This designated data set is saved, and,after the tracking data is processed for the visible landmark,the offset landing site location is computed from the saved data by means of Eq. (2.4.10). In this landing site offset calculation, the magnitude of the estimated position vector of the visible landmark is used for the estimated planetary radius $r_0$ in Eq. (2.4.10).

### 5.2.4.5 Orbit Navigation Logic

After all optical landmark tracking data have been acquired, the data processing procedure is initialized as illustrated in Fig. 2.4-1. It is assumed that the following items are stored in erasable memory at the start of the procedure shown in this figure:

$\underline{x}_C$ = Estimated CSM state vector as defined in Section 5.2.2.6.

W = Six-dimensional error transition matrix associated with $\underline{x}_C$ as defined in Section 5.2.2.4.

ORBWFLAG = $\begin{cases} \text{1 for valid W matrix} \\ \text{0 for invalid W matrix} \end{cases}$

This flag or switch is maintained by programs external to the Orbit

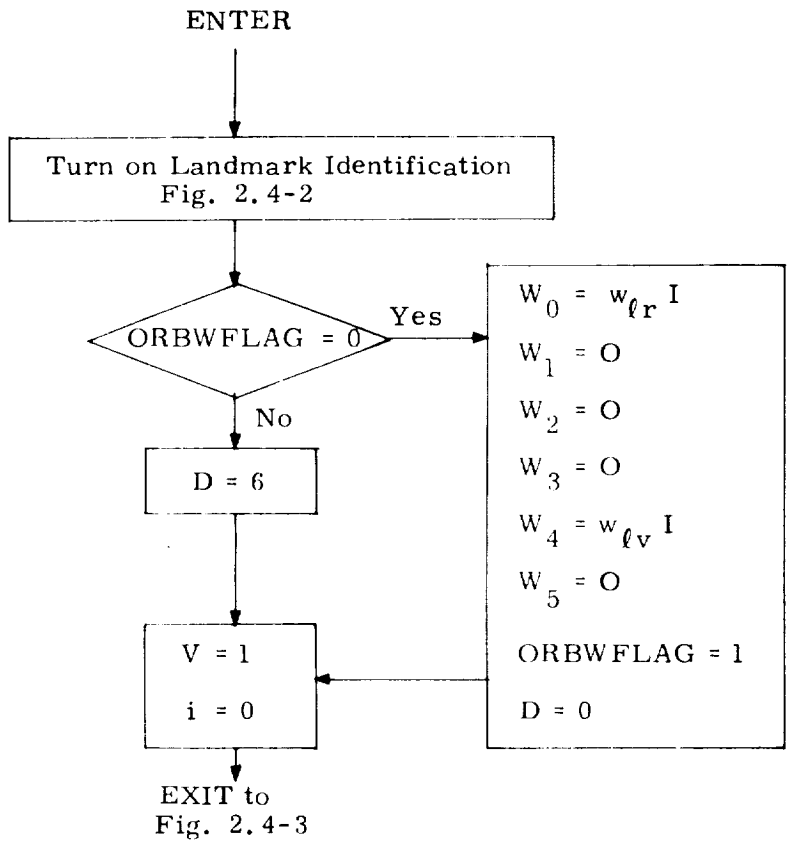



Fig. 2.4-1 Orbit Navigation Routine Initialization

Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing landmark tracking data. The flag is set to zero after each of the following procedures:

1) Powered maneuver
2) State vector update from ground
3) Rendezvous navigation
4) Astronaut Command

$[REFSMMAT]$ = Transformation Matrix: Basic Reference Coordinate System to IMU Stable Member Coordinate System

N = Number of unrejected sets of navigation data acquired during the tracking of the landmark

$t_{M1}$ to $t_{MN}$ = The N measurement times associated with the N sets of navigation data

N sets of five optics and IMU gimbal angles each

$w_{\ell r}$ and $w_{\ell v}$ preselected W matrix initial diagonal elements

OFF = 1 through 5 for index of landing site offset designator; 0 for no landing site offset designator

The variables D and V are indicators which control the Coasting Integration Routine (Section 5.2.2) as described in Section 5.2.2.6, I and O are the three-dimensional identity and zero matrices, respectively, and i is an index which is used to count the navigation data sets.

The landmark identification procedure (Fig. 2. 4-2), which is turned on by the initialization procedure, is essentially an astronaut interface routine and runs in parallel with the main computation. In the identification routine the astronaut enters into the CMC through the keyboard the following two items:

$$\text{KNOWN} = \begin{cases} 1 \text{ for mapped or known landmark} \\ 0 \text{ for unmapped landmark or landing site designation} \end{cases}$$

$$L = \begin{cases} 00 \text{ for a landmark whose coordinates are not stored in CMC memory} \\ 01 \text{ for the landing site} \\ 02 \text{ through } 26 \text{ for index number of a landmark whose coordinates are stored in CMC fixed memory} \end{cases}$$

If

$$\text{KNOWN} = 1 \quad \text{and} \quad L = 00$$

then the astronaut is further requested to enter the coordinates of the landmark; that is, latitude (Lat), longitude (Long) and altitude (Alt). Altitude is defined with respect to the mean lunar radius for lunar landmarks, and the Fischer ellipsoid for earth landmarks.

After completion of the initialization procedure the Orbit Navigation Routine begins processing the data.

For convenience of calculation in the CMC, Eqs. (2. 4. 5), (2. 4. 6), (2. 4. 7), and (2. 4. 9) are reformulated and regrouped as follows:

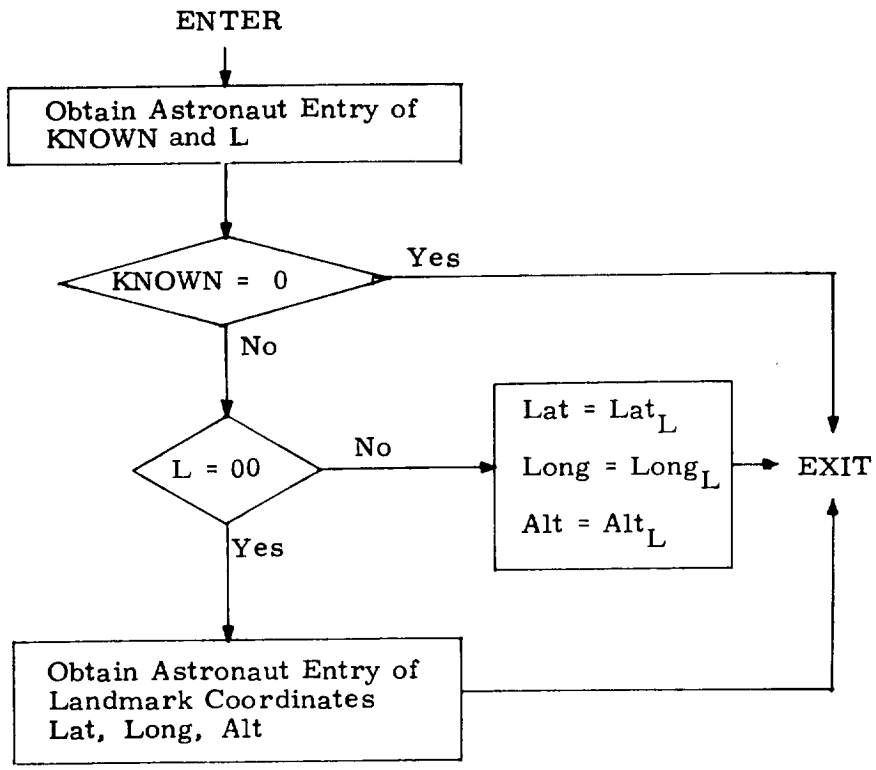



Fig. 2.4-2 Orbit Navigation Routine Landmark Identification

$$
\begin{aligned}
\underline{u}_s &= \text{UNIT}\,(\underline{u}_s \times \underline{u}_{CL}) \\
\underline{b}_0 &= \underline{u}_s \\
\underline{b}_1 &= \underline{0} \\
\delta Q &= r_{CL}\left[\cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2}\right]
\end{aligned}
\qquad (2.4.12)
$$

This set of equations is used both by the Orbit Navigation Routine and the Rendezvous Navigation Routine (Section 5.2.5) in processing optical tracking data. To validate the use of Eqs. (2.4.12) it is necessary only to let

$$
\overline{\alpha^2} = r_{CL}^{\;2}\,(\text{var}_{SCT} + \text{var}_{IMU}) \qquad (2.4.13)
$$

where $\text{var}_{SCT}$ and $\text{var}_{IMU}$ are the a priori estimates for one half the SCT and IMU total angular error variances, respectively.

The processing of the N sets of landmark-tracking navigation data is illustrated in Fig. 2.4-3. In the figure $K_0$, $K_1$ and $K_2$ are preselected constants, $w_\ell$ and $w_{un\ell}$ are preselected initial W matrix diagonal elements, and F is the altitude flag as defined in Section 5.5.3.

After the processing of the data is completed, the location of the offset landing site is computed from the saved data, if a data set was so designated, as shown in Fig. 2.4-4. Then, the final estimated landmark position vector is converted to latitude, longitude, altitude coordinates and these data are displayed. If the tracked landmark is to be the landing site, then these data are saved in erasable memory.

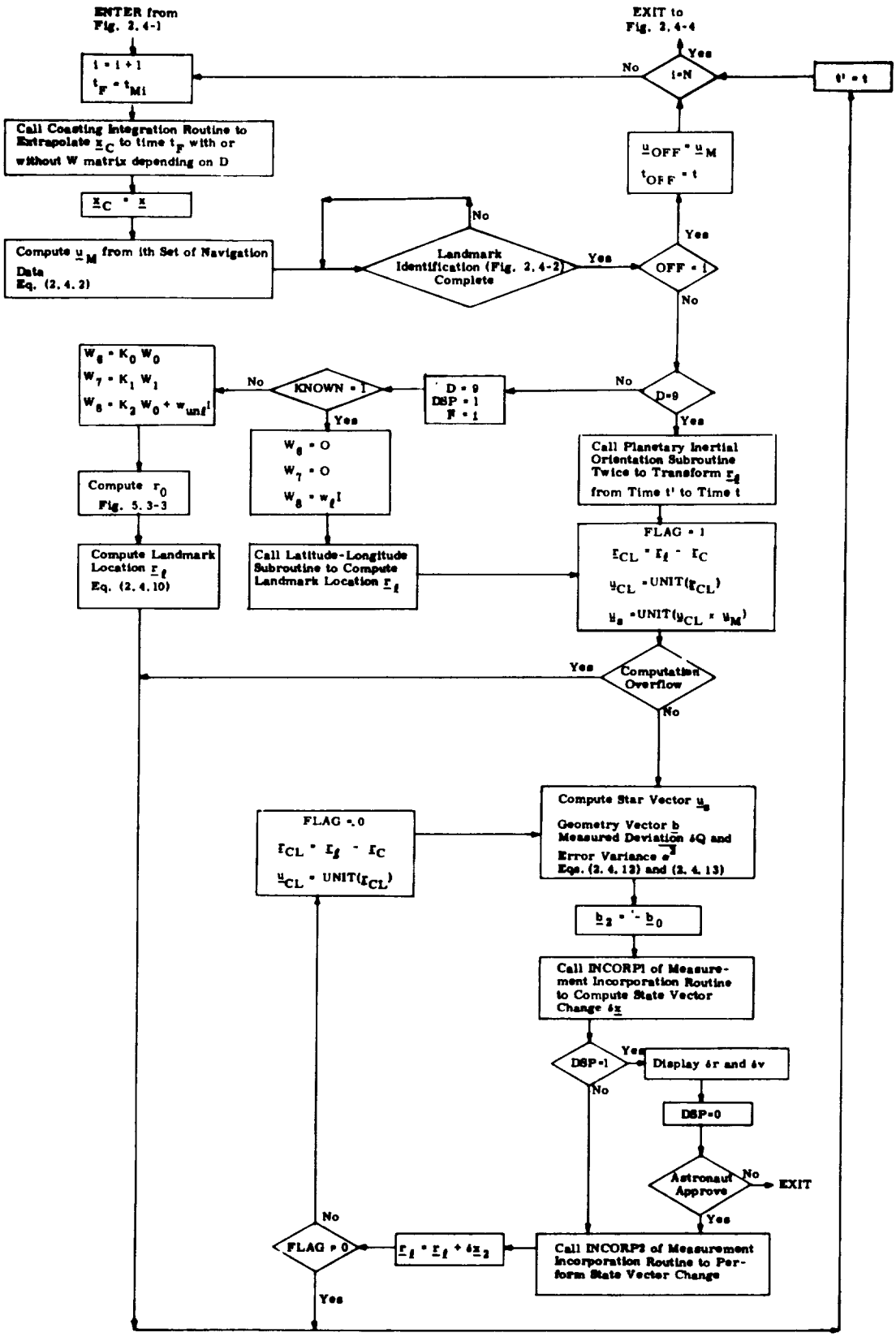



Figure 2.4-3 Orbit Navigation Routine Logic Diagram

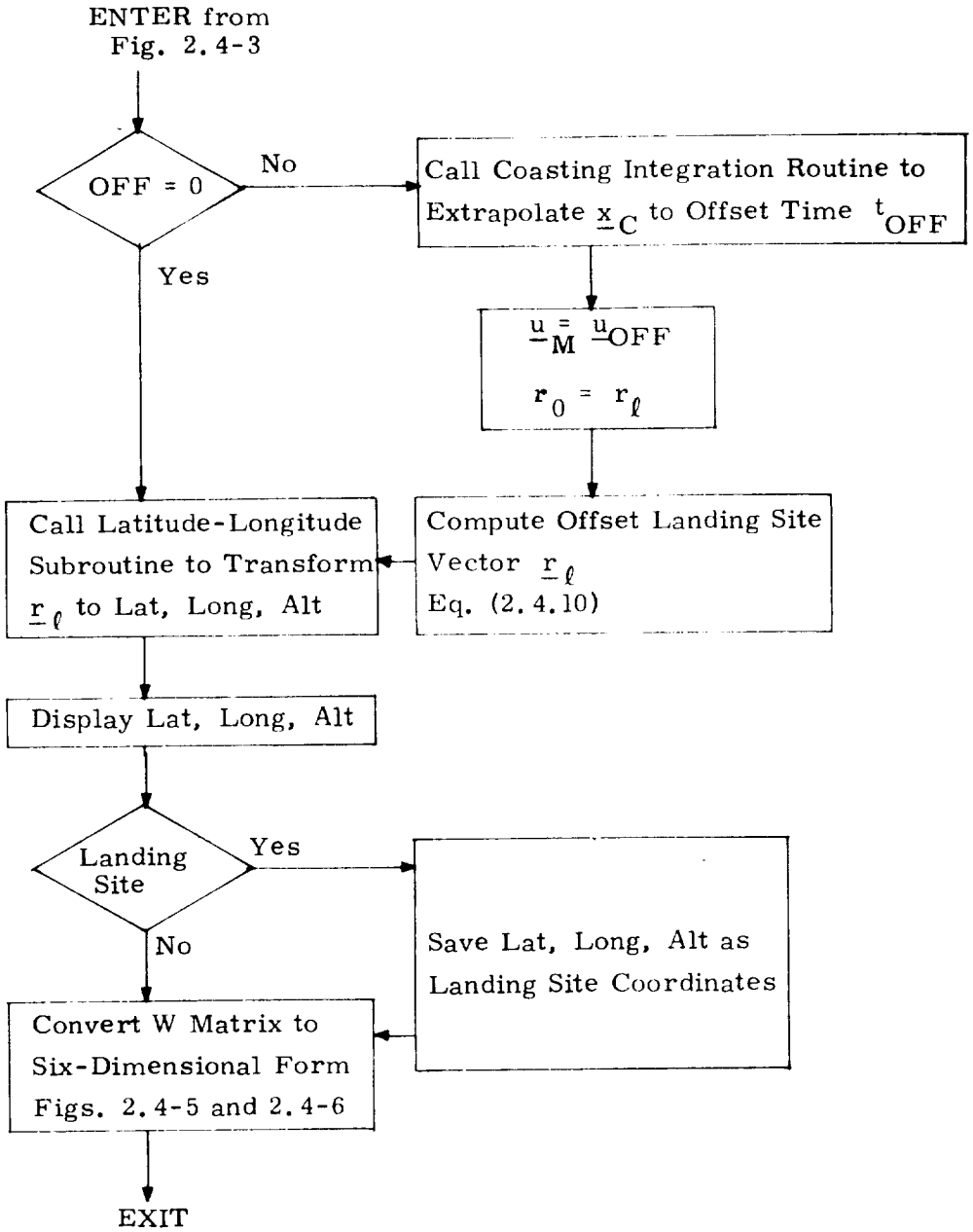



Fig. 2.4-4 Orbit Navigation Routine Termination

The final operation of the Orbit Navigation Routine is to convert the nine-dimensional error transition matrix, W, to a six-dimensional matrix with the same CSM position and velocity estimation error variances and covariances. The reason for this procedure is that the W matrix, when it is initialized for processing the data associated with the next landmark, must reflect the fact that the initial landmark location errors are not correlated with the errors in the estimated CSM position and velocity vectors. Of course, after processing measurement data, these cross correlations become non-zero, and it is for this reason that the nine dimensional procedure works, and that it is necessary to convert the final W matrix to six-dimensional form.

The solution to the conversion problem is not unique. A convenient solution is obtained in the following manner.

The error transition matrix, W, has been defined in Section 5.2.3 in terms of the nine three-dimensional submatrices $W_0, W_1, \ldots, W_8$ as follows:

$$W = \begin{pmatrix} W_0^T & W_1^T & W_2^T \\ W_3^T & W_4^T & W_5^T \\ W_6^T & W_7^T & W_8^T \end{pmatrix} \qquad (2.4.14)$$

Let the elements of each of these submatrices be defined by

$$W_i^T = \begin{pmatrix} w_{9i} & w_{9i+3} & w_{9i+6} \\ w_{9i+1} & w_{9i+4} & w_{9i+7} \\ w_{9i+2} & w_{9i+5} & w_{9i+8} \end{pmatrix} \quad (i = 0, 1, \ldots, 8) \tag{2.4.15}$$

The W matrix is then converted to the following six-dimensional form:

$$\begin{pmatrix} W_0^T & W_1^T \\ W_3^T & W_4^T \end{pmatrix} = \begin{pmatrix} w_0 & w_3 & w_6 & w_9 & w_{12} & w_{15} \\ w_1 & w_4 & w_7 & w_{10} & w_{13} & 0 \\ w_2 & w_5 & w_8 & w_{11} & 0 & 0 \\ & & & & & \\ w_{27} & w_{30} & w_{33} & 0 & 0 & 0 \\ w_{28} & w_{31} & 0 & 0 & 0 & 0 \\ w_{29} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.4.16}$$

$$W_2 = W_5 = O$$

The twenty-one non-zero elements of the converted W matrix are computed by solving the following twenty-one equations:

$$\sum_{k=0}^{5} w_{i+3k} \; w_{j+3k} = \sum_{k=0}^{8} w'_{i+3k} \; w'_{j+3k} = e_p \quad \left( \begin{array}{l} i,j = 0,1,2,27, \\ \quad 28, 29 \\ i < j \\ p = 0,1,\ldots,20 \end{array} \right) \tag{2.4.17}$$

where primes refer to quantities before conversion, and the following table gives p as a function of i and j:

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| i | 29 | 28 | 27 | 2 | 1 | 0 | 28 | 27 | 2 | 1 | 0 | 27 | 2 | 1 | 0 | 2 | 1 | 0 | 1 | 0 | 0 |
| j | 29 | 29 | 29 | 29 | 29 | 29 | 28 | 28 | 28 | 28 | 28 | 27 | 27 | 27 | 27 | 2 | 2 | 2 | 1 | 1 | 0 |
| p | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

The twenty-one elements $e_0$ to $e_{20}$ are computed as illustrated in Fig. 2.4-5. The converted W matrix is then calculated as shown in Fig. 2.4-6. Included in this procedure are negative radicand and zero divisor checks.

The Orbit Navigation Routine is now ready to process the data acquired in tracking the next landmark.

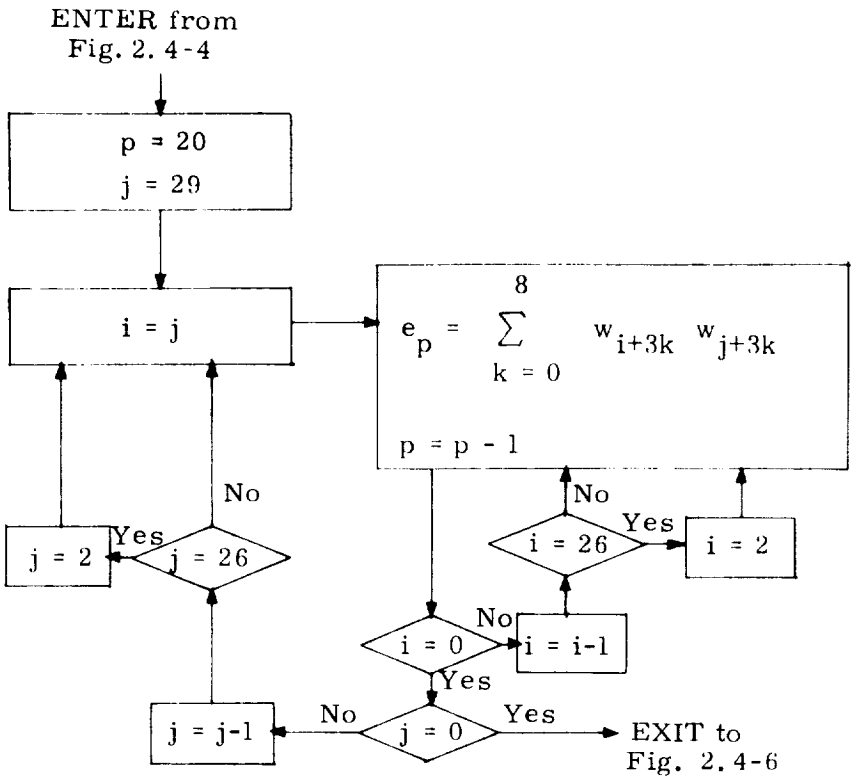



Fig. 2.4-5 Orbit Navigation Routine W Matrix Conversion I

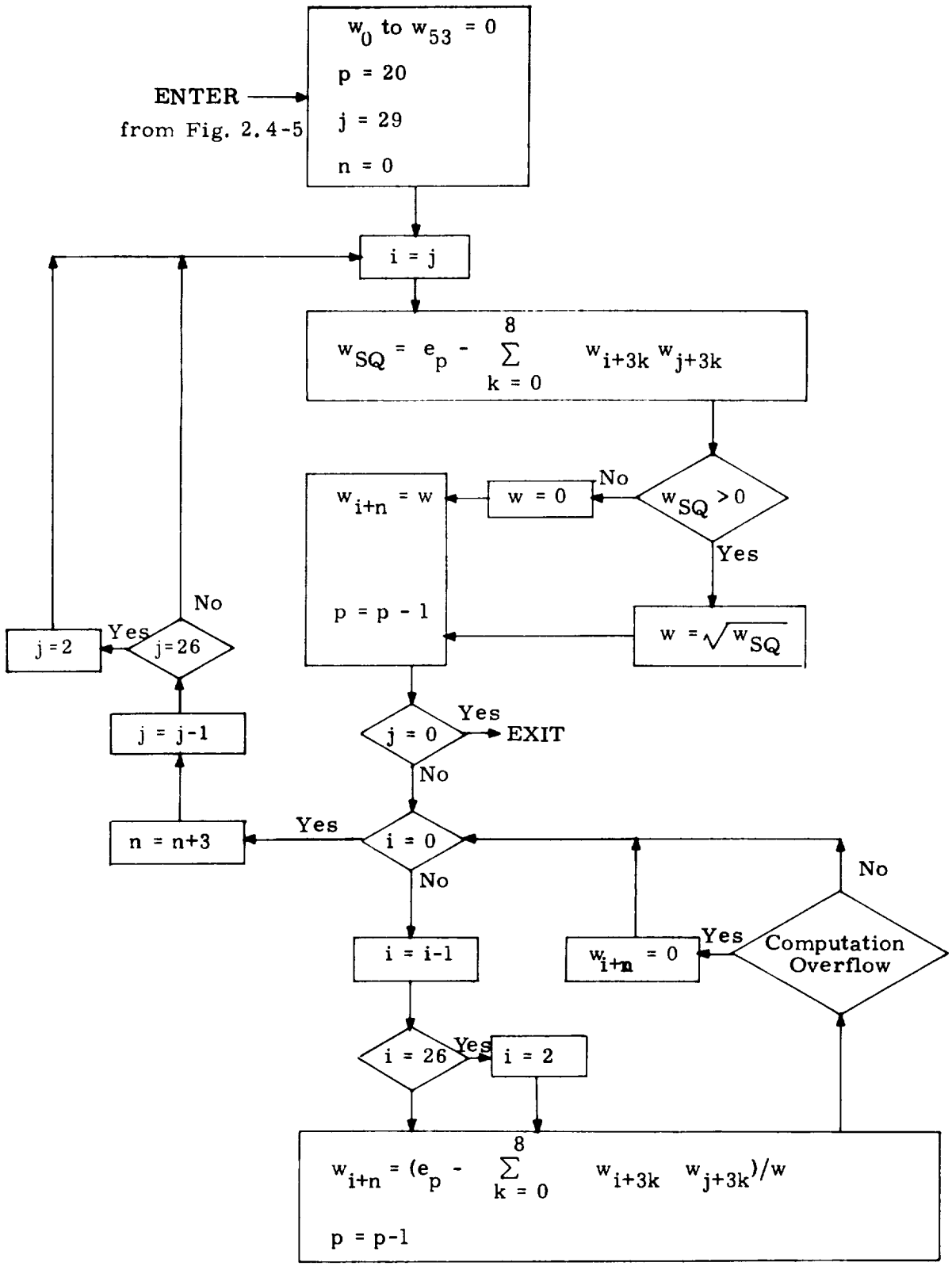



Fig. 2.4-6 Orbit Navigation Routine W Matrix Conversion II

## 5.2.5 RENDEZVOUS NAVIGATION PROGRAM

### 5.2.5.1 Target Acquisition and LM Tracking Routine

During mission phases involving rendezvous, CSM SXT tracking data of the LM are used in the Rendezvous Navigation Routine (Section 5.2.5.2) to update the estimated state vector of either the CSM or the LM, as discussed in Section 5.2.1 and outlined in Fig. 2.1-2. In order to initially acquire and maintain SXT tracking in the CSM and RR tracking in the LM, the CSM attitude must be oriented such that the tracking line-of-sight falls within the SXT and RR transponder coverage sectors.

The general routine used to achieve and maintain tracking conditions on the CSM is illustrated in Fig. 2.5-1. With reference to this figure, the estimated line-of-sight between the CSM and LM is first determined by advancing the estimated state vectors of the two vehicles to the current time using the Kepler Subroutine (Section 5.5.5). A preselected reference CSM vector (or axis) is then automatically controlled to coincide with the estimated line-of-sight. This reference CSM vector is chosen to be approximately in the center of the common coverage sector of the optics and RR transponder with allowance for the optics pole region. The reference vector currently considered for this function is illustrated in Fig. 2.5-2. The vehicle orientation about the reference axis is not controlled by the Preferred Tracking Attitude Routine and must be corrected manually by the astronaut if it is unsatisfactory. The Preferred Tracking Attitude Routine does maintain the CSM reference vector alignment to within $5^{\circ}$ of the estimated line-of-sight throughout free-fall rendezvous phases.

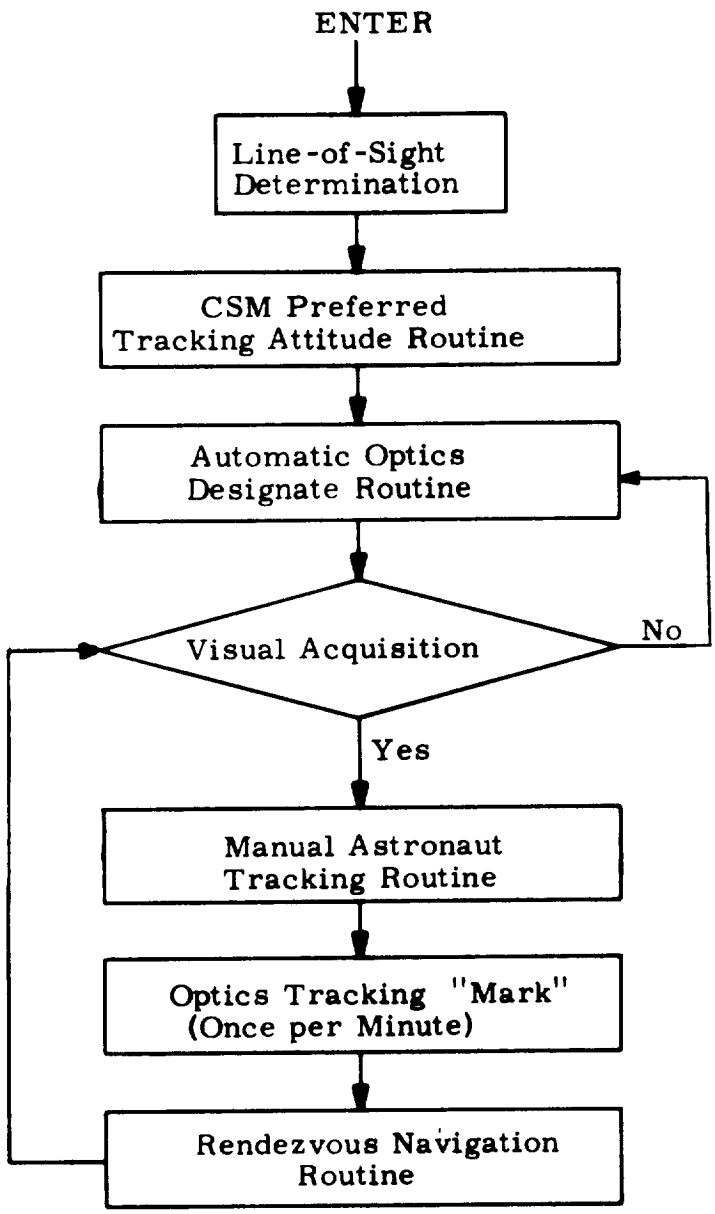



Figure 2.5-1 Target Acquisition and LM Tracking Routine

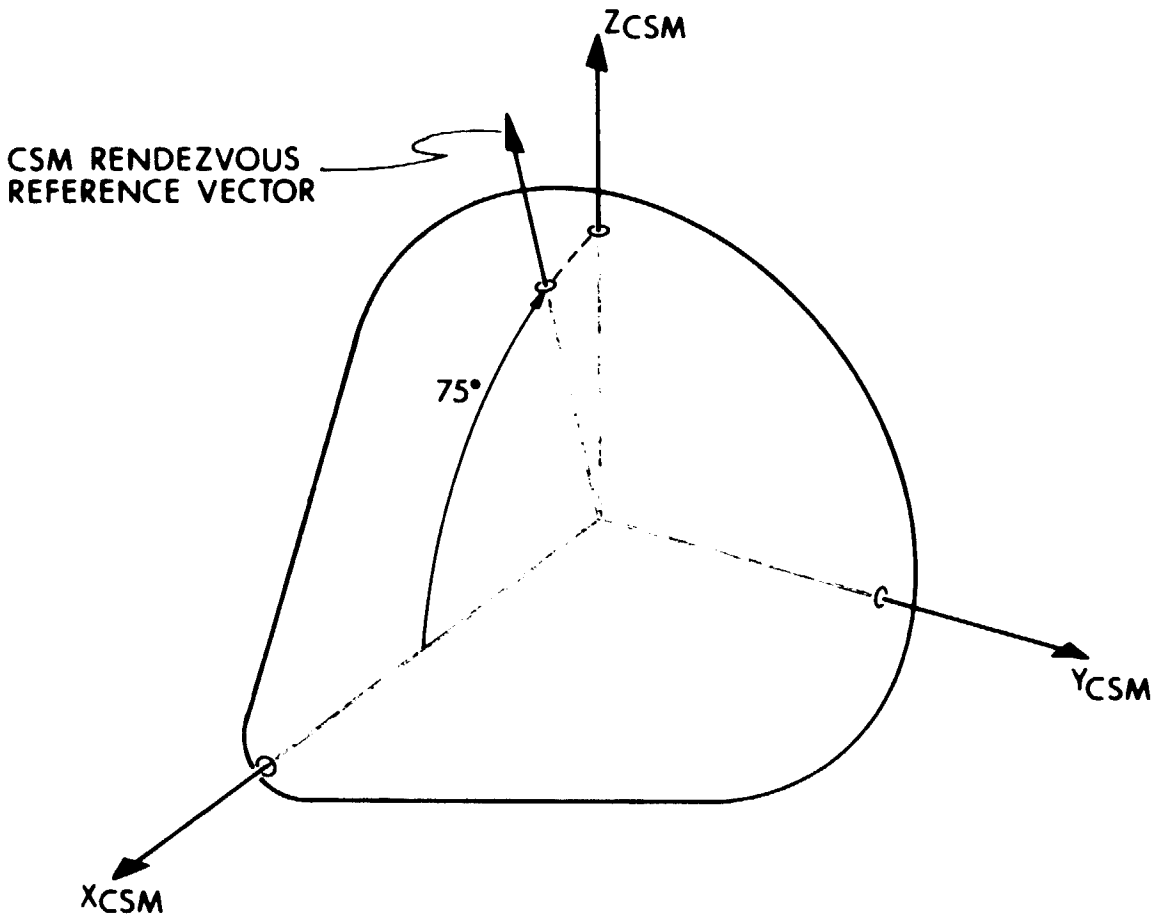



Figure 2.5-2 CSM Rendezvous Reference Vector

Referring again to Fig. 2.5-1, the Automatic Optics Designate Routine (Section 5.6.8) drives the CSM optics to the estimated line-of-sight or pointing direction. The computations and positioning commands in this routine are repeated once per second provided the optics mode switch is set to CMC.

After the astronaut has identified the target vehicle (a flashing optical beacon mounted on the LM or a steady light from LM reflected sunlight), he switches the optics mode to MANUAL and centers the SXT reticle on the target. When accurate tracking is achieved, he presses the MARK button, upon which the CMC stores the measurement time and all SXT and IMU gimbal angles. These data are used in the Rendezvous Navigation Routine (Section 5.2.5.2) to update the estimated state vector of the selected vehicle.

In order to achieve desired accuracy levels of the vehicle state vector using optical measurements, it is important that optical tracking data used in the Rendezvous Navigation Routine (Section 5.2.5.2) be processed at essentially uniform intervals over the tracking period. Uniform tracking intervals are defined as once every minute throughout the phase, as opposed to all tracking data taken over a short period. The reason for this requirement is described in Section 5.2.5.2.

Optical tracking of the LM from the CSM is suspended during rendezvous maneuvers by either vehicle. If the CSM is the passive vehicle and is tracking the LM for monitoring and possible abort retrieval, LM rendezvous maneuvers are voice-linked to the CSM as an ignition time and three velocity components in a LM local vertical coordinate system, and then entered as updates to the estimated LM state vector in the CMC. Upon receipt of these data, SXT tracking and data processing should be suspended until after the maneuver. The update is accomplished by means of the Target $\Delta V$ Routine, R-32. If the CSM is the active vehicle, then the estimated CSM state vector is updated by means of the Average G-Routine (Section 5.3.2) during the maneuver.

### 5.2.5.2 Rendezvous Navigation Routine

During rendezvous phases optical tracking data are obtained by means of SXT sightings of the LM from the CSM. These data are used to update the estimated six-dimensional state vector of either the CSM or the LM. The option of which state vector is to be updated by the SXT tracking data is controlled by the astronaut as described in Section 5.2.1 and illustrated in Fig. 2.1-2. This decision will be based upon which, if either, state vector is the most accurate and upon which vehicle is performing the rendezvous.

This routine is used to process the LM-tracking measurement data, as shown in simplified form in Fig. 2.1-2, and is used normally during lunar-orbit rendezvous in the lunar landing mission. The routine also can be used in earth orbit during alternate missions.

After the preferred CSM attitude is achieved and optical tracking acquisition is established (Section 5.2.5.1), the astronaut enters tracking data into the CMC by pressing the optics MARK button when he has centered the SXT reticle on the LM. As described in Section 5.2.5.1, each set of navigation data contains the time of the measurement and the two optics and three IMU gimbal angles. From these five angles the measured unit vector, $\underline{u}_M$, along the CSM-to-LM line-of-sight is computed in the Basic Reference Coordinate System from

$$\underline{u}_M = [\text{REFSMMAT}]^T [\text{NBSM}] \underline{u}_{NB} \qquad (2.5.1)$$

where [REFSMMAT] and [NBSM] are transformation matrices and $\underline{u}_{NB}$ is the measured line-of-sight vector in navigation base coordinates. All terms of Eq. (2.5.1) are defined in Section 5.6.3.

For the purpose of navigation it is convenient to consider the measured unit vector, $\underline{u}_M$, to be the basic navigation data. This navigation measurement of the line-of-sight vector, $\underline{u}_M$, is mathematically equivalent to the simultaneous measurement of the angles between the lines-of-sight to the LM and two stars. The data are processed by selecting two convenient unit vectors (fictitious star directions), converting the vector $\underline{u}_M$ to an equivalent set of two artificial star-LM measurements, and using the Measurement Incorporation Routine (Section 5.2.3) twice, once for each artificial measurement. These two unit vectors are chosen to be perpendicular to each other and to the current estimated line-of-sight vector so as to maximize the convenience and accuracy of the procedure.

Let $\underline{r}_C$ and $\underline{r}_L$ be the estimated CSM and LM position vectors at the time of a given line-of-sight measurement. Then, the first state vector update for the measurement is performed as follows:

(1) Calculate the estimated CSM-to-LM line-of-sight from

$$\begin{aligned} \underline{r}_{CL} &= \underline{r}_L - \underline{r}_C \\ \underline{u}_{CL} &= \text{UNIT}\,(\underline{r}_{CL}) \end{aligned} \qquad (2.5.2)$$

(2) Initialize the fictitious star direction to the vector

$$\underline{u}_s = \text{UNIT}\,(\underline{u}_{CL} \times \underline{u}_M) \qquad (2.5.3)$$

If the vectors $\underline{u}_{CL}$ and $\underline{u}_M$ are separated by an angle of less than $2^{-19}$ rad., then a computation overflow occurs in the execution of Eq. (2.5.3), and this set of measurement data is discarded because all of the components of the estimated state vector deviation, $\delta\,\underline{x}$, would be negligible for both state vector updates.

(3) Compute an artificial star direction from

$$\underline{u}_s = \text{UNIT}\,(\underline{u}_s \times \underline{u}_{CL}) \qquad (2.5.4)$$

(4) Calculate the six-dimensional geometry vector, $\underline{b}$, from

$$\underline{b}_0 = \frac{1}{r_{CL}}\,\underline{u}_s \qquad (2.5.5)$$

$$\underline{b}_1 = \underline{0} \qquad (2.5.6)$$

(5) Determine the measured deviation, $\delta Q$, from

$$\delta Q = \cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \cos^{-1}(\underline{u}_s \cdot \underline{u}_{CL})$$

$$= \cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2} \qquad (2.5.7)$$

(6) Incorporate the fictitious star-LM measurement using the Measurement Incorporation Routine (Section 5.2.3).

Included in Step (6) is the state vector update validity check for the first proposed update, as described in Section 5.2.1.

It should be noted that the initialization of the star direction, $\underline{u}_S$, which is given by Eq. (2. 5. 3), is such that the first artificial star (computed from Eq. (2. 5. 4)) will yield the maximum value for the measured deivation, $\delta Q$, which is obtained from Eq. (2. 5. 7). The reason for selecting the first $\underline{u}_S$ vector in this manner is that there is only one state vector update validity check even though there are two updates.

Assuming that the first state vector update was valid, the second update for this measurement data set is performed by first recomputing the estimated CSM-to-LM line-of-sight vector from Eq. (2. 5. 2) using the updated values of the estimated CSM and LM position vectors, $\underline{r}_C$ and $\underline{r}_L$, respectively. Then, Steps (3) - (6) are repeated, this time with no state vector update validity check.

If the first proposed state vector update does not pass the validity check, then the magnitudes of the proposed changes in the estimated position and velocity vectors, $\delta r$ and $\delta v$, respectively, are displayed. If the astronaut is sure that he is tracking the LM, then he should command the update. Otherwise he should reject the data and recheck the optical tracking. A detailed discussion of this state vector update validity check is given in Section 5. 2. 1.

The results of the processing of the measured line-of-sight vector, $\underline{u}_M$, are updated values of the estimated position and velocity vectors of the CSM or the LM. These two estimated state vectors are used to compute required rendezvous targeting parameters as described in Section 5. 4. 4.

For convenience of calculation in the CMC, Eqs. (2. 5. 4)-(2. 5. 7) are reformulated and regrouped as follows:

$$\underline{u}_s = \text{UNIT}\,(\underline{u}_s \times \underline{u}_{CL})$$

$$\underline{b}_0 = \underline{u}_s$$

$$\underline{b}_1 = \underline{0} \qquad (2.5.8)$$

$$\delta Q = r_{CL}\left[\cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2}\right]$$

This set of equations is used both by the Rendezvous Navigation Routine and the Orbit Navigation Routine (Section 5.2.4) in processing optical tracking data. To validate the use of Eqs (2.5.8) it is necessary only to let

$$\overline{\alpha^2} = r_{CL}^{\;2}\,(\text{var}_{SXT} + \text{var}_{IMU}) \qquad (2.5.9)$$

where $\text{var}_{SXT}$ and $\text{var}_{IMU}$ are the a priori estimates for one half the SXT and IMU total angular error variances, respectively.

The computation logic for the Rendezvous Navigation Routine is illustrated in Fig. 2.5-3. It is assumed that the following items are stored in erasable memory at the start of the computation shown in the figure:

| | | |
|---|---|---|
| $\underline{x}_C$ | = | Estimated CSM state vector as defined in Section 5.2.2.6. |
| $\underline{x}_L$ | = | Estimated LM state vector. |
| W | = | Six-dimensional error transition matrix associated with $\underline{x}_C$ or $\underline{x}_L$ as defined in Section 5.2.2.4. |

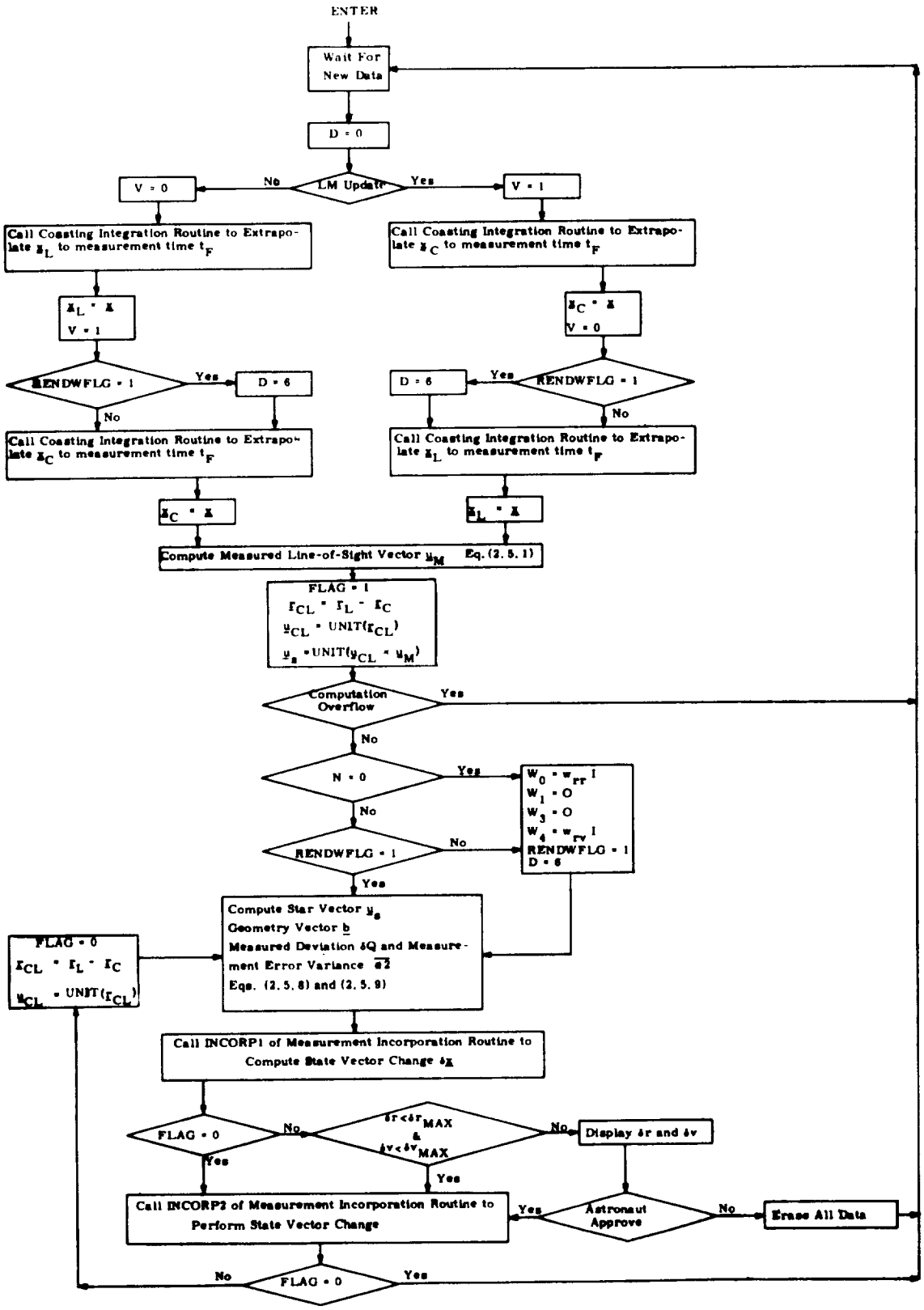



Figure 2.5-3 Rendezvous Navigation Routine Logic Diagram

$$\text{RENDWFLG} = \begin{cases} 1 \text{ for valid W matrix} \\ 0 \text{ for invalid W matrix} \end{cases}$$

This flag or switch is maintained by programs external to the Rendezvous Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing LM tracking data. The flag is set to zero after each of the following procedures:

1) Powered maneuver
2) State vector update from ground
3) Orbit navigation
4) Astronaut Command

[REFSMMAT] = Transformation Matrix: Basic Reference Coordinate System to IMU Stable Member Coordinate System.

N = Number of sets of rendezvous navigation data already processed.

$t_F$ = Measurement time.

Five optics and IMU gimbal angles

$w_{rr}$ and $w_{rv}$ = Preselected W matrix initial diagonal elements.

The variables D and V are indicators which control the Coasting Integration Routine (Section 5.2.2) as described in Section 5.2.2.6, and I and O are the three-dimensional identity and zero matrices, respectively.

The LM line-of-sight measurement incorporation procedure outlined above is repeated at one minute intervals throughout the rendezvous phase except during powered maneuvers, as described in Section 5.2.5.1. As indicated in Section 5.2.5.1, it is important that the SXT tracking data be taken over as large an angular sector in inertial space swept out by the line-of-sight as possible to achieve desired state vector accuracy levels. This essentially uniform tracking operation is required since the SXT tracking provides information only in directions normal to the line-of-sight, as indicated by Eq. (2.5.5), and the line-of-sight must be allowed to rotate in inertial space to achieve more complete update data.

### 5.2.5.3 Rendezvous Navigation Routine (Alternate Line-of-Sight)

During rendezvous phases optical tracking data are normally obtained by means of SXT sightings of the LM from the CSM and processed as described in Section 5.2.5.2. An alternate method of obtaining navigation data is by means of docking-reticle (COAS) sightings of the LM. However, since data obtained in this manner is much less accurate than SXT sighting data, the docking reticle should be used for the sightings only if the SXT has failed, or if the astronaut cannot return to the lower equipment bay to make SXT sightings.

The processing of the data from a docking-reticle sighting is identical to the procedure described in Section 5.2.5.2 with the following two exceptions:

1. The vector $\underline{u}_{NB}$, Eq. (2.5.1), is a fixed vector instead of one which is computed from the two optics angles.

2. The measurement error variance, Eq. (2.5.9), is replaced by

$$\overline{\alpha^2} = r_{CL}^2 \left( var_{DR} + var_{IMU} \right) \qquad (2.5.10)$$

where $var_{DR}$ is the a priori estimate for one-half the total angular error variance of a docking-reticle line-of-sight measurement.

## 5.2.6 CISLUNAR-MIDCOURSE NAVIGATION ROUTINE

### 5.2.6.1 General Comments

During the midcourse phase of the lunar mission, navigation data can be obtained by the measurement of the angle between the directions to a star and a planetary horizon or landmark, as described in Section 5.2.1. This routine is used to process the star-landmark/horizon measurement data, as illustrated in simplified form in Fig. 2.1-3, and is normally used only in an abort situation in conjunction with a return-to-earth targeting and maneuver procedure after the loss of ground communication. The Return-to-Earth Routine (Section 5.4.5) and this routine provide the CMC with the capability for guiding the CSM back to the earth and to safe entry conditions.

The acquisition of the star and landmark/horizon may be accomplished either automatically or manually. In the manual mode it is not necessary to have the IMU aligned or even on for this measurement since only the optics trunnion angle is used as measurement data. In the automatic acquisition mode, however, the IMU must be on and aligned prior to the initiation of this routine.

In the processing of the navigation data, it is necessary to distinguish between earth and lunar measurements, and between primary and secondary body measurements. This is accomplished by means of the variable Z, which denotes measurement planet, and which is part of the data loaded by the astronaut after the measurement. Also included in the data load are star and landmark or horizon identification.

### 5.2.6.2 Star-Landmark Measurement

Let $\underline{r}_C$ and $\underline{v}_C$ be the estimated CSM position and velocity vectors and W the error transition matrix. The SXT star-landmark angle measurement processing procedure is as follows:

1. Use the Coasting Integration Routine (Section 5. 2. 2) to extrapolate the estimated CSM state vector and the W matrix to the time of the measurement obtaining $\underline{r}'_C$, $\underline{v}'_C$, $W'$ and $\underline{r}_{QC}$. The vector $\underline{r}_{QC}$ is the estimated position of the CSM relative to the secondary body Q.

2. Let $\underline{r}_{ZC}$ be the estimated CSM position vector relative to the measurement planet Z. Then

$$\underline{r}_{ZC} = \begin{cases} \underline{r}'_C & \text{if } Z = P \\ \underline{r}_{QC} & \text{if } Z = Q \end{cases} \qquad (2.6.1)$$

3. Compute $\underline{r}_\ell$, the location of the landmark at the measurement time, by means of the Latitude-Longitude Subroutine (Section 5. 5. 3).

4. Compute the estimated pointing vector from

$$\underline{r}_{CL} = \underline{r}_\ell - \underline{r}_{ZC}$$

$$\underline{u}_{CL} = \text{UNIT}\,(\underline{r}_{CL}) \qquad (2.6.2)$$

(5) Let $\underline{u}_s$ be the star unit vector prestored in the CMC fixed memory. It is necessary to correct the star vector for aberration, i.e. the change in the observed star direction caused by velocity perpendicular to the direction. The observed star direction, $\underline{u}_s^*$, is given by

$$\underline{u}_s^* = \text{UNIT}\left(\underline{u}_s + \frac{\underline{v}'_C - \underline{v}_{ES}}{c}\right) \qquad (2.6.3)$$

where $\underline{v}_{ES}$ is the velocity of the sun relative to the earth and c is the speed of light. The velocity vector of the sun relative to the earth, $\underline{v}_{ES}$, is assumed constant for the duration of the mission, as described in Section (5.5.4). The velocity of the moon relative to the earth, for the case in which the moon is the primary body, is negligible.

(6) Correct $\underline{u}_{CL}$ for abberation as follows:

$$\underline{u}^*_{CL} = \text{UNIT}\left(\underline{u}_{CL} + \frac{\underline{v}'_C}{c}\right) \qquad (2.6.4)$$

(7) Then, if A is the measured angle, the six dimensional geometry vector, $\underline{b}$, and the measured deviation, $\delta Q$, are given by

$$\mathrm{COSQ} = \underline{u}_s^* \cdot \underline{u}_{CL}^*$$

$$\underline{b}_0 = \frac{1}{r_{CL}} \mathrm{UNIT}\,(\underline{u}_s^* - \mathrm{COSQ}\,\underline{u}_{CL}^*) \qquad (2.6.5)$$

$$\underline{b}_1 = \underline{0}$$

$$\delta Q = A - \cos^{-1}(\mathrm{COSQ})$$

(8) Incorporate the measurement into the CSM state vector estimate by means of the Measurement Incorporation Routine (Section 5.2.3) after astronaut approval.

5.2.6.3 Star-Horizon Measurement

The processing of a star-horizon measurement is the same as that of a star-landmark measurement except for Step (3) above. The estimated location of the landmark (horizon) $\underline{r}_\ell$ must be obtained from geometrical considerations.

Referring to Fig. 2.6-1, it is seen that the star unit vector, $\underline{u}_s$, and the estimated CSM position vector, $\underline{r}_{ZC}$, determine a plane. Assuming that the measurement planet is the earth and that the horizon of the earth is at a constant altitude, the intersection of this plane and the horizon ellipsoid of the earth is an ellipse, called the horizon ellipse.

To determine the orientation of the horizon ellipse, define the following three mutually orthogonal unit vectors:

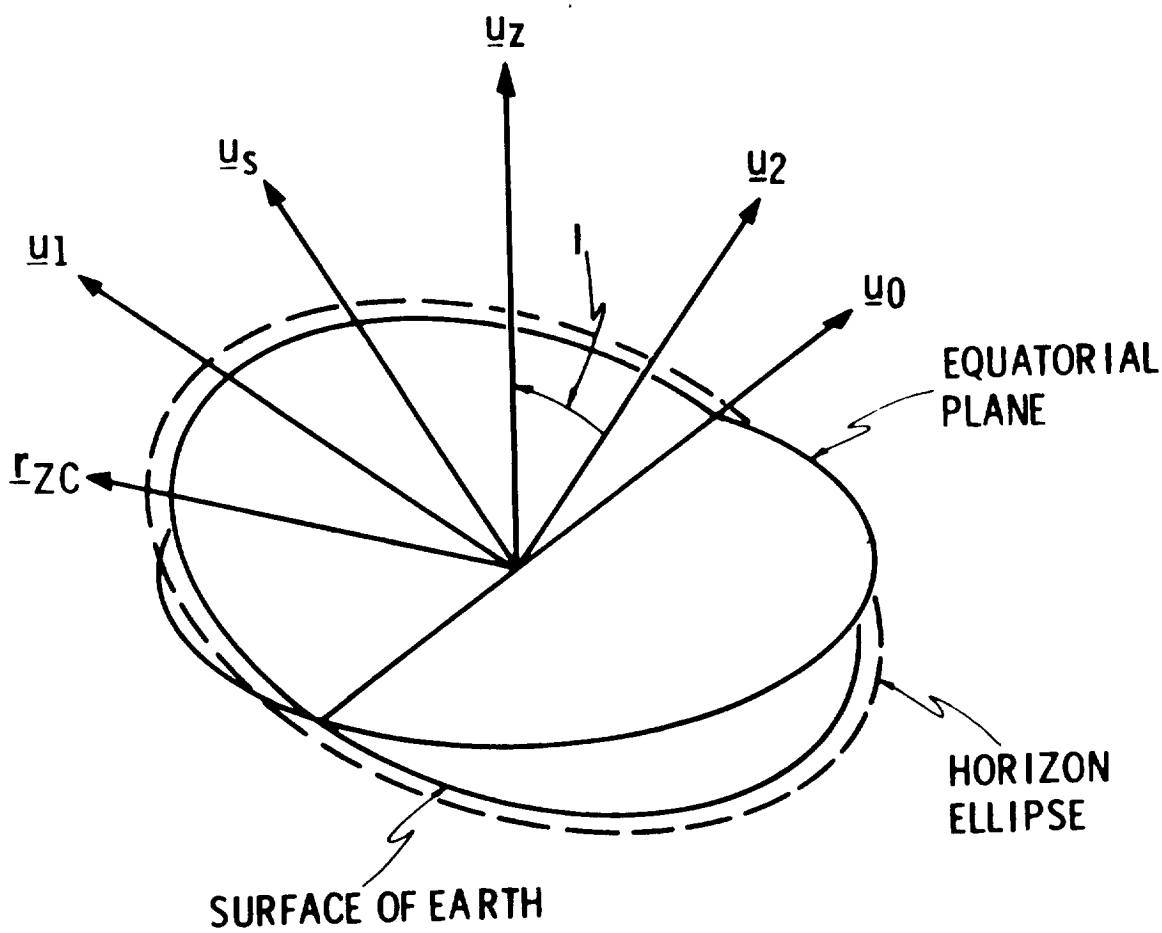



Figure 2.6-1 Definition of Horizon Coordinate System

$$\underline{u}_2 = \text{UNIT}\,(\underline{u}_s \times \underline{r}_{ZC})$$

$$\underline{u}_0 = \text{UNIT}\,(\underline{u}_Z \times \underline{u}_2) \qquad (2.6.6)$$

$$\underline{u}_1 = \underline{u}_2 \times \underline{u}_0$$

where $\underline{u}_Z$ is a unit vector along the earth's polar axis and is given by

$$\underline{u}_Z = \begin{pmatrix} -A_Y \\ A_X \\ 1 \end{pmatrix} \qquad (2.6.7)$$

The angles $A_X$ and $A_Y$ are defined in Section 5.5.2. Then, as seen in Fig. 2.6-1, the vectors $\underline{u}_0$ and $\underline{u}_1$ are along the semi-major and semi-minor axes of the horizon ellipse, respectively; and $\underline{u}_2$ is perpendicular to the ellipse. The inclination angle I of the horizon ellipse with respect to the equatorial plane of the earth is obtained from

$$\sin I = \underline{u}_1 \cdot \underline{u}_Z \qquad (2.6.8)$$

The shape of the ellipse is defined by its major and minor axes. The semi-major axis $a_H$ is given by

$$a_H = a + h \qquad (2.6.9)$$

where a is the semi-major axis of the Fischer ellipsoid and h is the horizon altitude. Let $r_F$ be the radius of the Fischer ellipsoid at the latitude equal to the inclination angle of the horizon ellipse computed from Eq. (5.3.1) of Section 5.5.3. Then, the semi-minor axis of the ellipse, $b_H$, is obtained from

$$b_H = r_F + h \tag{2.6.10}$$

The problem of determining the vector $\underline{r}_\ell$ can now be reduced to a two dimensional one. Define the Horizon Coordinate System to have its X- and Y-axes along $\underline{u}_0$ and $\underline{u}_1$, respectively, as illustrated in Fig. 2.6-2. Let

$$M = \begin{pmatrix} \underline{u}_0^T \\ \underline{u}_1^T \\ \underline{u}_2^T \end{pmatrix} \tag{2.6.11}$$

The matrix M is the transformation matrix from the Basic Reference Coordinate System to the Horizon Coordinate System. The vectors $\underline{r}_{ZC}$ and $\underline{u}_s$ are transformed to the Horizon Coordinate System as follows:

$$\begin{aligned} \underline{r}_H &= M\,\underline{r}_{ZC} \\ \underline{u}_{sH} &= M\,\underline{u}_s \end{aligned} \tag{2.6.12}$$

Let $x_H$ and $y_H$ be the two non-zero components of $\underline{r}_H$, and let the two points of tangency from $\underline{r}_H$ to the horizon be $\underline{t}_0$ and $\underline{t}_1$.

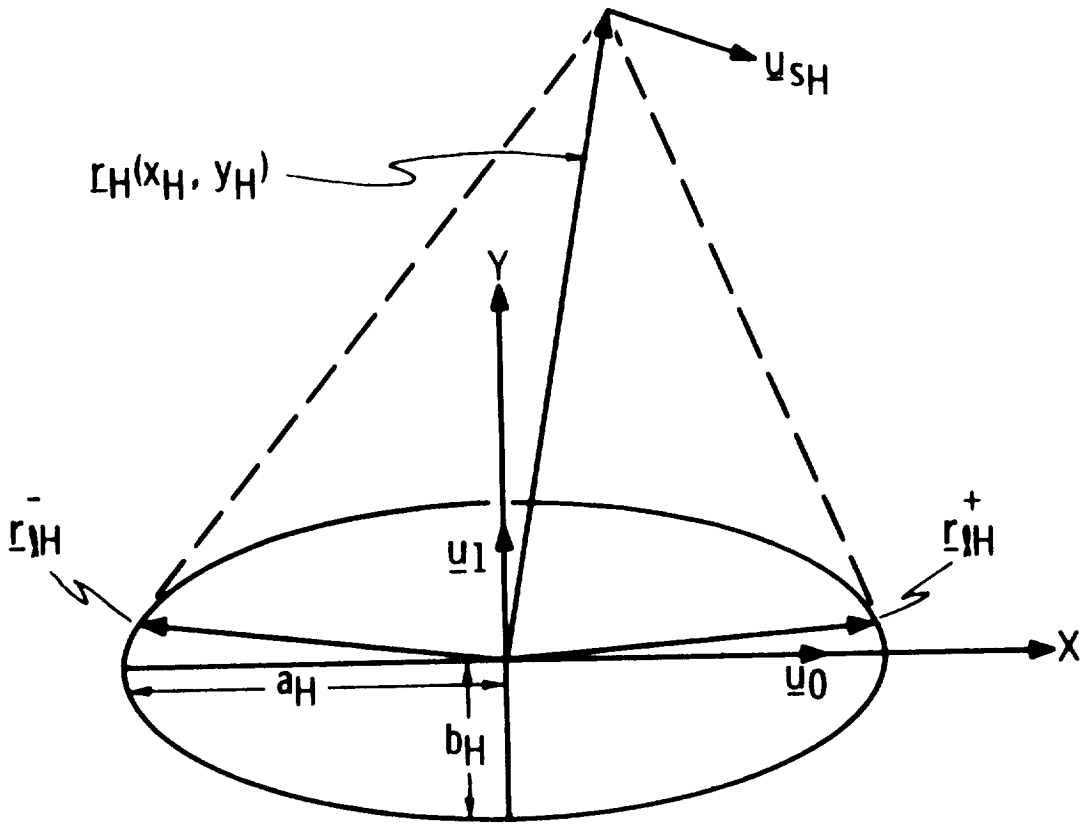



**Figure 2.6-2** Geometry of Star-Horizon Measurement

The vectors $\underline{t}_0$ and $\underline{t}_1$ are obtained by solving simultaneously the equation of the horizon ellipse

$$\frac{x^2}{a_H^2} + \frac{y^2}{b_H^2} = 1 \tag{2.6.13}$$

and the equation of the line which is tangent to the ellipse and which passes through the point $(x_H, y_H)$

$$\frac{xx_H}{a_H^2} + \frac{yy_H}{b_H^2} = 1 \tag{2.6.14}$$

The following equations result:

$$\underline{t}_0 = \frac{1}{A} \begin{pmatrix} x_H + \frac{a_H}{b_H} y_H \sqrt{A - 1} \\ y_H - \frac{b_H}{a_H} x_H \sqrt{A - 1} \\ 0 \end{pmatrix} \tag{2.6.15}$$

$$\underline{t}_1 = \frac{1}{A} \begin{pmatrix} x_H - \frac{a_H}{b_H} y_H \sqrt{A - 1} \\ y_H + \frac{b_H}{a_H} x_H \sqrt{A - 1} \\ 0 \end{pmatrix} \tag{2.6.16}$$

where

$$A = \frac{x_H^2}{a_H^2} + \frac{y_H^2}{b_H^2} \qquad (2.6.17)$$

The two points of tangency, $\underline{t}_0$ and $\underline{t}_1$, correspond to the two horizon points, $\underline{r}_{\ell H}^{+}$ (near horizon) and $\underline{r}_{\ell H}^{-}$ (far horizon). To determine which $\underline{t}_i$ corresponds to which horizon, compute the two angles

$$A_i = \cos^{-1}\left[\underline{u}_{sH} \cdot (\underline{t}_i - \underline{r}_H)\right] \quad (i = 0, 1) \qquad (2.6.18)$$

Then, the $\underline{t}_i$ which yielded the smaller angle $A_i$ is the near horizon and the other is the far horizon. Let $\underline{t}_k$ be the horizon which was used in the measurement. The horizon vector is then

$$\underline{r}_\ell = M^T \underline{t}_k \qquad (2.6.19)$$

The measurement processing is completed by following steps (4) - (8) of the star-landmark measurement procedure.

The star-horizon measurement processing procedure has been based upon the assumption that the planet involved in the measurement is the earth. If the moon is the measurement planet, and it is assumed that the moon is a sphere, then the entire procedure presented above is valid except for the computation of $a_H$ and $b_H$. For a lunar-horizon measurement Eqs. (2. 6. 8 ) - (2. 6. 10) are replaced by

$$a_H = r_M$$
$$b_H = r_M \qquad (2.6.20)$$

where $r_M$ is the mean radius of the moon.

5. 2. 6. 4 Angle Measurement Processing Logic

The computational logic for the Cislunar-Midcourse Navigation Routine is illustrated in Fig. 2. 6-3. It is assumed that the following items are stored in erasable memory at the start of the computation shown in the figure:

$\underline{x}_C$ = Estimated CSM state vector as defined in Section 5. 2. 2. 6

W = Six-dimensional error transition matrix associated with $\underline{x}_C$ as defined in Section 5. 2. 2. 4

$$\text{ORBWFLAG} = \begin{cases} 1 \text{ for valid W matrix} \\ 0 \text{ for invalid W matrix} \end{cases}$$

This flag or switch is maintained by programs external to the Cislunar-Midcourse Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing star-landmark/horizon angle measurement data. The flag is set to zero after each of the following procedures:

1) Powered maneuver
2) State vector update from ground
3) Rendezvous navigation
4) Astronaut Command

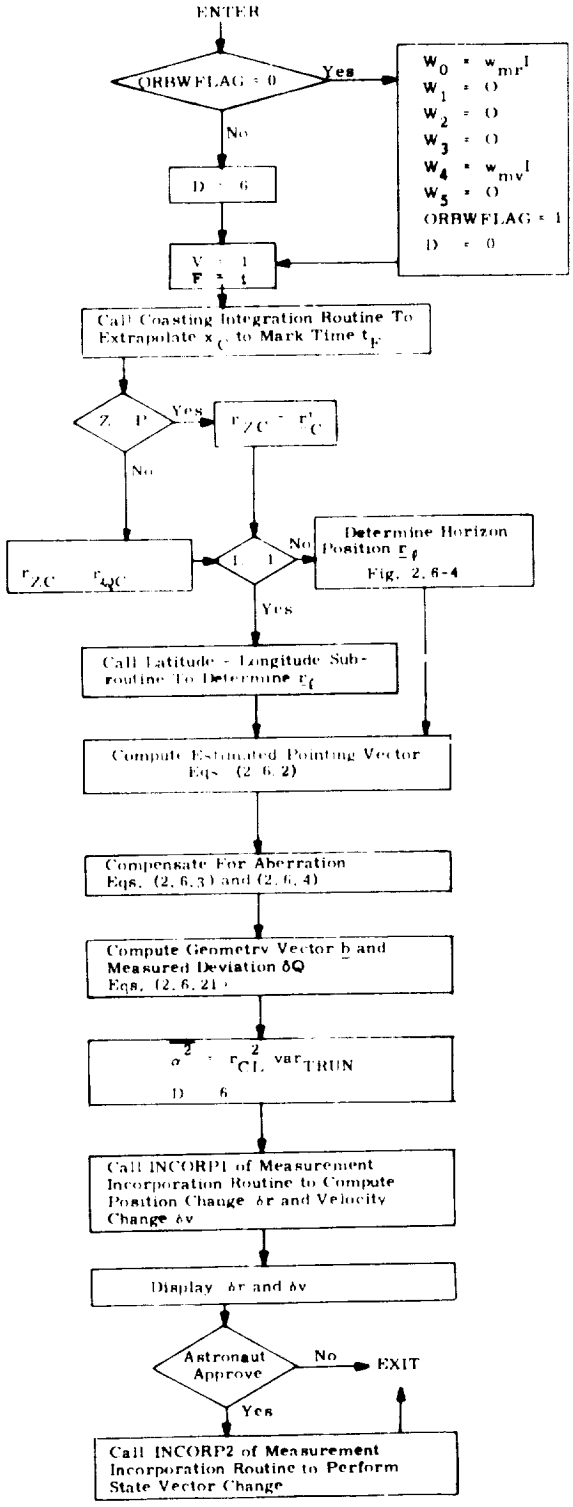



Figure 2.6-3 Cislunar-Midcourse Navigation Routine Logic Diagram

$t_F$ = Measurement time

$A$ = Measured angle

$\underline{u}_s$ = Measurement star

$Z$ = Measurement planet = $\begin{cases} 1 \text{ for earth} \\ 0 \text{ for moon} \end{cases}$

$L$ = Landmark switch or flag = $\begin{cases} 1 \text{ for landmark measurement} \\ 0 \text{ for horizon measurement} \end{cases}$

$w_{mr}$ and $w_{mv}$ = Preselected W matrix initial diagonal elements

For convenience of calculation in the CMC, Eqs. (2. 6. 5) are reformulated as follows:

$$
\begin{aligned}
COSQ &= \underline{u}_s^* \cdot \underline{u}_{CL}^* \\
\underline{b}_0 &= UNIT \left( \underline{u}_s^* - COSQ\, \underline{u}_{CL}^* \right) \\
\underline{b}_1 &= \underline{0} \\
\delta Q &= r_{CL} \left[ A - \cos^{-1} (COSQ) \right]
\end{aligned}
\tag{2. 6. 21}
$$

To make Eqs. (2. 6. 21) valid it is necessary only to let

$$\overline{\alpha^2} = r_{CL}^2 \, var_{TRUN} \tag{2. 6. 22}$$

where $var_{TRUN}$ is the a priori estimate of the SXT trunnion-angle error variance.

The variables D and V are indicators which control the Coasting Integration Routine (Section 5. 2. 2) as described in

Section 5.2.2.6, I and O are the three-dimensional identity and zero matrices, respectively, and F is the altitude flag as defined in Section 5.3.3.

Finally, there is available either the landmark coordinates or the far horizon flag H defined by

$$H = \begin{cases} 1 \text{ for far horizon} \\ 0 \text{ for near horizon} \end{cases}$$

In the case of a horizon measurement the computational logic for determining the horizon vector $\underline{r}_\ell$ is shown in Fig. 2.6-4.

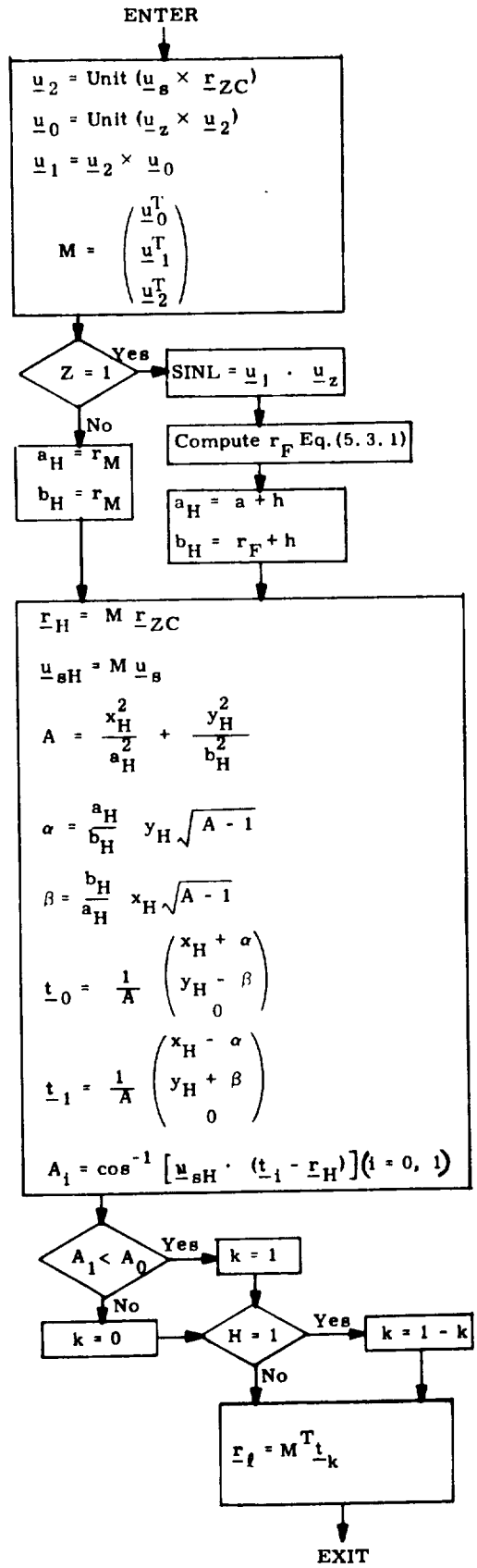



Figure 2.6-4 Horizon Vector Determination Logic Diagram

## 5.3 POWERED FLIGHT NAVIGATION AND GUIDANCE

### 5.3.1 GENERAL COMMENTS

The objective of the powered flight guidance routines is to maintain an estimate of the CSM state vector during thrusting maneuvers, and to control or steer the thrust direction such that the desired velocity cut-off conditions are achieved. The powered flight navigation program, used to maintain an estimate of the vehicle state vector during all thrusting conditions, is referred to as the Average-G Routine and is presented in Section 5.3.2. The basic powered flight guidance concept is a velocity-to-be-gained concept with cross-product steering, described in Section 5.3.3. All powered maneuvers controlled by the Command Module Computer (CMC) use this concept in one of the three following programs: External ΔV Maneuver Guidance (Sec. 5.3.4), Lunar Orbit Insertion (LOI) Maneuver Guidance (Sec. 5.3.5), or Lambert Aim Point Maneuver Guidance (Sec. 5.3.6). These three programs based on the cross product steering concept differ only in the unique generation of the required velocity vector, $\underline{v}_R$.

The primary objective of the External ΔV Maneuver Guidance program is to achieve an inertial velocity condition determined or targeted from an external source. The LOI Maneuver Guidance objective is to achieve a circular orbit at cut-off such that the orbital plane contains a specified target position vector. The objective of the Lambert Aim Point Maneuver Guidance is to establish a trajectory which will intercept a target position vector at a specified time of arrival. These three CMC basic powered flight guidance programs are described in detail in the following sections.

## 5.3.2 POWERED FLIGHT NAVIGATION--AVERAGE-G ROUTINE

During a powered flight maneuver, the state vector estimate will be maintained by numerical integration of the equations of motion using Average-G equations.

The information about the thrust acceleration comes from the IMU Pulsed Integrating Pendulous Accelerometers (PIPA) in the form of velocity increments ($\Delta \underline{v}$) over the time interval ($\Delta t$) corresponding to the time of repetitive computations (generally two seconds). The computations are, therefore, in terms of discrete increments of velocity rather than instantaneous accelerations.

If $\underline{r}_{n-1}$ and $\underline{v}_{n-1}$ are the position and velocity estimates at the beginning of the nth computational cycle, the $\underline{r}_n$ and $\underline{v}_n$ are computed from

$$\underline{r}_n = \underline{r}_{n-1} + \Delta t \left(\underline{v}_{n-1} + \underline{g}_{n-1} \Delta t / 2 + \Delta \underline{v} / 2\right) \qquad (3.2.1)$$

$$\underline{v}_n = \underline{v}_{n-1} + .5\, \Delta t \left(\underline{g}_{n-1} + \underline{g}_n\right) + \Delta \underline{v} \qquad (3.2.2)$$

where $\underline{g}_n$ is the gravitational acceleration. The gravitational acceleration outside the lunar sphere of influence is given by:

$$\underline{g}_n = -\frac{\mu_e}{r_n^2} \left\{ \underline{u}_{r_n} + \frac{3}{2} J_2 \left(\frac{r_{eq}}{r_n}\right)^2 \left[ (1 - 5 \cos^2 \phi)\, \underline{u}_{r_n} + 2 \cos \phi\, \underline{u}_z \right] \right\} \qquad (3.2.3)$$

where

$$\cos \phi = \underline{u}_{r_n} \cdot \underline{u}_z \tag{3.2.4}$$

The vectors $\underline{u}_{r_n}$ and $\underline{u}_z$ are unit vectors in the direction of $\underline{r}_n$ and the polar axis of the earth, respectively, $\mu_e$ is the gravitational constant of the earth, $r_{eq}$ is the equatorial radius of the earth, and $J_2$ is the second-harmonic coefficient of the earth's potential function.

The gravitational acceleration within the lunar sphere of influence is determined by a simplification of Eq. 3.2.2 as follows:

$$\underline{g}_n = -\frac{\mu_M}{r_n^2} \underline{u}_{r_n} \tag{3.2.5}$$

where $\mu_M$ is the gravitational constant of the moon.

The $\Delta\underline{v}$ is compensated for instrument errors as described in Section 5.6.13. Further, the $\Delta\underline{v}$ is transformed into the Basic Reference Coordinate System since the navigation calculations are made in reference coordinates.

## 5.3.3 CROSS PRODUCT STEERING

### 5.3.3.1 Velocity-to-be-Gained and Cross Product Steering Concept

The velocity-to-be-gained, $\underline{v}_G$, is defined by

$$\underline{v}_G = \underline{v}_R - \underline{v} \qquad (3.3.1)$$

where $\underline{v}_R$ is the required velocity associated with the current position and is dependent on the target parameters specified for the maneuver under consideration.

The rate of change of $\underline{v}_G$ is given by

$$\dot{\underline{v}}_G = \dot{\underline{v}}_R - \dot{\underline{v}}$$

$$\dot{\underline{v}}_G = \dot{\underline{v}}_R - \underline{g} - \underline{a}_T = \underline{b} - \underline{a}_T \qquad (3.3.2)$$

where

$$\underline{b} = \dot{\underline{v}}_R - \underline{g} \qquad (3.3.3)$$

$\underline{g}$ is the gravitational acceleration

$\underline{a}_T$ is the thrust acceleration

The desired thrust direction is defined by

$$\underline{a}_{TD} \times \underline{v}_G = c\,\underline{b} \times \underline{v}_G \qquad (3.3.4)$$

where $\underline{a}_{TD}$ is the desired thrust acceleration and c is a scalar (between 0 and 1). This equation is the cross product steering concept.

If the actual thrust acceleration $\underline{a}_T$ is all along $\underline{a}_{TD}$, the equation

$$\underline{v}_G \times (c\,\underline{b} - \underline{a}_T) = \underline{0} \qquad (3.3.5)$$

will be satisfied. However, in general there will be a directional error and the control system aligns the vehicle so that Eq. 3.3.5 is satisfied.

The command for the attitude control system is generated by

$$\underline{\omega}_c = K \frac{\underline{v}_G \times \Delta \underline{m}}{v_G\,\Delta m} \qquad (3.3.6)$$

where

| | | |
|---|---|---|
| $\underline{\omega}_c$ | = | commanded attitude rate to CSM attitude control system |
| $\Delta\underline{m}$ | = | $c\,\underline{b}\,\Delta t - \Delta\underline{v}$ |
| $\Delta\underline{v}$ | = | the accumulated velocity |
| $\Delta t$ | = | the computation interval |

The basic powered flight steering represented by the above equations is illustrated in Fig. 3.3-1. The scalar K of the figure is a guidance command gain setting required for desired dynamic response of the combined powered guidance and thrust control (autopilot) loops. The initial value of $\dot{\underline{v}}_R\Delta t$ is obtained by precomputing the $\underline{v}_R$ which corresponds to $(t_{IG} - \Delta t)$, then allowing the vehicle to free-fall to the ignition time and recomputing $\underline{v}_R$. The difference between these two values of $\underline{v}_R$ is then used as the initial $\dot{\underline{v}}_R\Delta t$.

The command-rate signal is computed every two seconds during the powered flight based on an updated $\underline{v}_G$. The computation of $\underline{v}_G$ depends on the particular maneuver and is described in the following sections.

### 5.3.3.2 Engine-Off and Short Burn Tests

The engine is turned on at a specified or computed ignition time. During the powered maneuver steering phase, a time-to-cut-off, or time-to-go ($t_{GO}$), is computed from the equation:

$$t_{GO} = K_1 \left( |\underline{v}_G \cdot \Delta\underline{m}| / \Delta\underline{m} \cdot \Delta\underline{m} \right) \Delta t + \Delta t_{tail\text{-}off} \qquad (3.3.7)$$

$$t_{GO} = K_1 \left( \frac{V_G \cdot (cb - a_T)}{|cb - a_T|^2} \right)$$

$$= K_1 \frac{|V_G|}{|cb - a_T|} \cos\angle \frac{V_G}{cb - a_T}$$

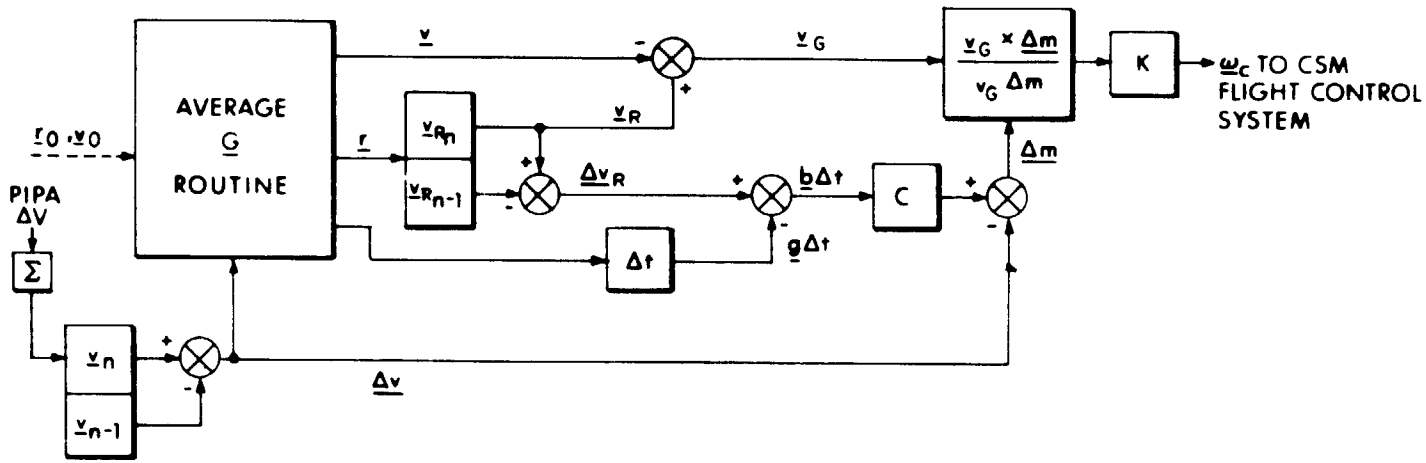



Fig. 3.3-1 CMC Powered Flight Steering Block Diagram

where

$$K_1 = 1 - .5\,|\underline{v}_G \cdot \Delta\underline{m}|/g\,I_{SP}\,|\Delta\underline{m}| \qquad (3.3.8)$$

$I_{SP}$ = specific impulse of the selected engine

$g$ = 32.17405 fps$^2$.

$\Delta t_{tail\text{-}off}$ = a time representing the duration of a burn at full or maximum thrust equivalent to the tail-off impulse after the engine-off signal is issued, and computation delays.

It should be noted that $\Delta t_{tail\text{-}off}$ is a <u>negative</u> number in erasable storage. When $t_{GO}$ falls, for the first time, below a value of 4.0 seconds, the engine-off signal is set to be issued $t_{GO}$ seconds later and the attitude control system command, $\underline{\omega}_c$, is set equal to zero.

Very short burns require special consideration since some interval of time elapses before effective steering is achieved. Not only must the autopilot react to the pointing commands, but the engine-off signal may be required before the $\Delta\underline{v}$ from the PIPA's can be measured. To this end, an estimate of the burn time, $t_{ON}$, is made on the basis of engine test data. See Fig. 3.3-2.

If $t_{ON}$ is less than 6 seconds, the engine is set to stay on only $t_{ON}$ seconds and the vehicle is held in the prealigned position. The computation and logic for short maneuver steering is shown in Fig. 3.3-3.

Maneuvers with the RCS jets also require special consideration. Since the acceleration from the RCS jets is low, the quantization error in the PIPA's can be significant, causing erroneous pointing signals when Eq. 3.3.6 is mechanized. Therefore maneuvers with the RCS jets will be manually controlled. The initial thrust alignment for such maneuvers will still be done automatically, however.

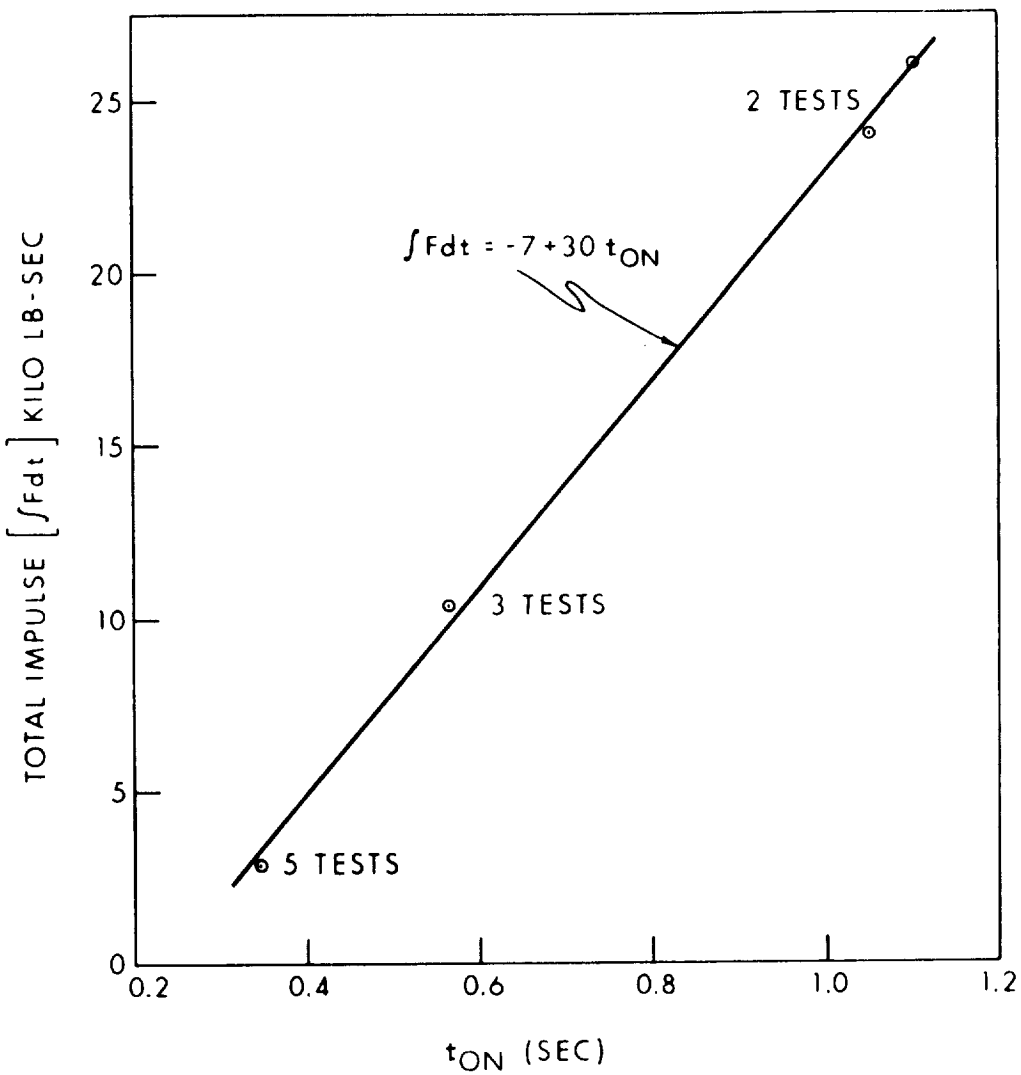
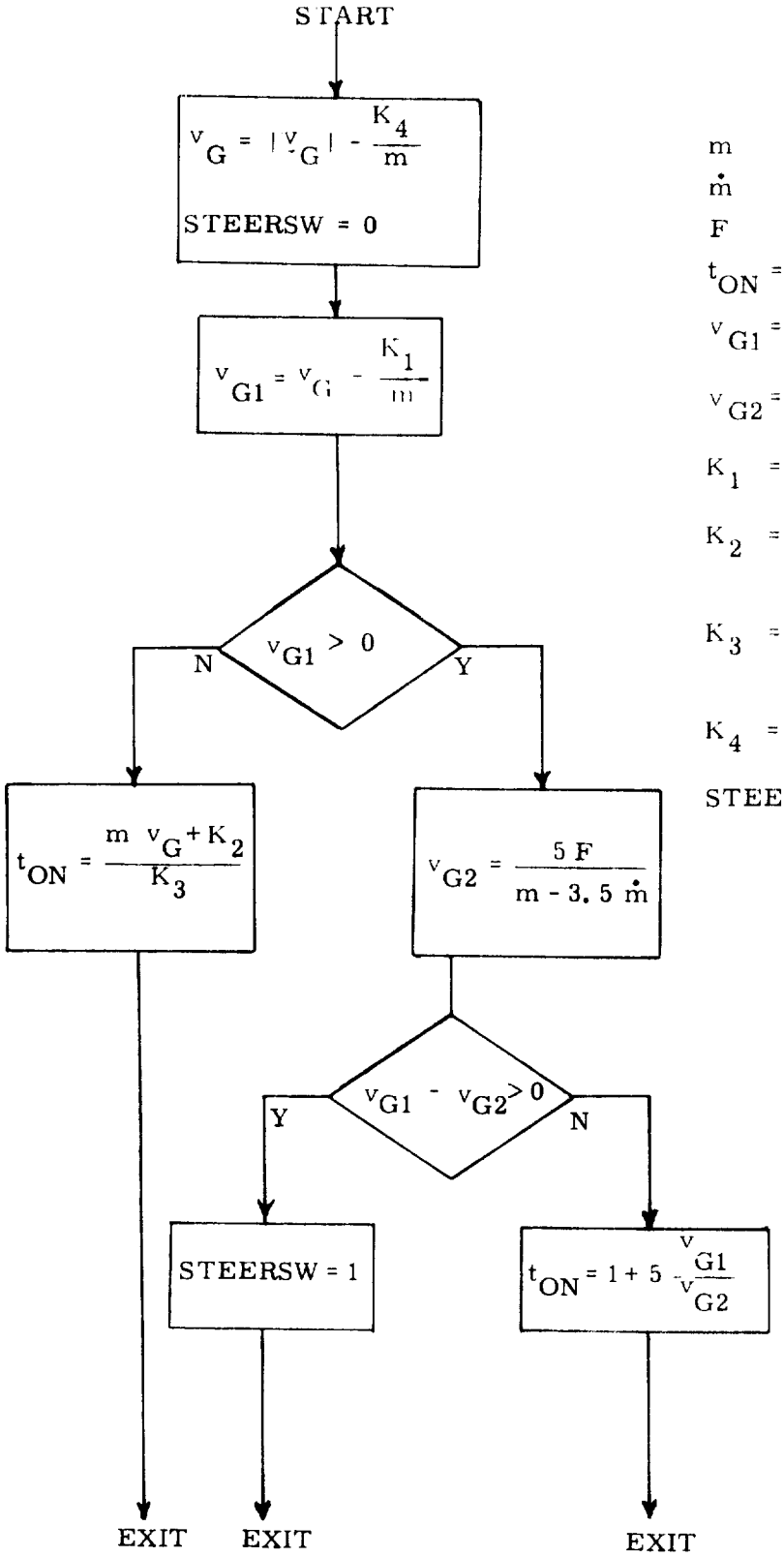



Fig. 3.3-2 SPS Minimum Impulse Test Data (NAA. 4-5-66)




LEGEND

| | |
|---|---|
| $m$ | Vehicle mass - slugs |
| $\dot{m}$ | Rate of change of mass |
| $F$ | SPS Thrust - lbs. |
| $t_{ON}$ | = SPS on time - sec. |
| $v_{G1}$ | = Velocity to be gained after 1st second of burn |
| $v_{G2}$ | = Velocity gain in burning from 1 - 6 seconds |
| $K_1$ | = Impulse in 1 sec. burn 23, 200 lb. - sec. |
| $K_2$ | = Initial value of minimum impulse curve = 7, 000 lb. -sec. |
| $K_3$ | = Slope of minimum impulse curve = 30, 000 lb. - sec. |
| $K_4$ | = Impulse due to assumed 7 sec. two jet RCS ullage |

STEERSW = 1 if $t_{ON} > 6$

Figure 3. 3 3 SPS Short Burn Time Calculation Logic

### 5.3.3.3 Initial Thrust Alignment

Before the selected engine is ignited for any particular powered maneuver, the vehicle must be oriented so that at ignition the thrust is in the desired direction. Since the time of ignition is known beforehand, the position and velocity at $\Delta t$ seconds before ignition can be computed prior to the arrival of the vehicle at that point. By integrating over $\Delta t$ seconds from that point to the ignition time, the vectors $\underline{v}_G$ and $\underline{b}\,\Delta t$ can be computed as described in Section 5.3.3.1.

The desired thrust direction can then be calculated (prior to arrival at the ignition point) as

$$\underline{u}_T = \text{UNIT}\left[\underline{q} + (a_T'^{\,2} - q^2)^{\frac{1}{2}}\,\underline{u}_G\right] \qquad (3.3.9)$$

where

$$\underline{u}_G = \text{UNIT}\,(\underline{v}_G) \qquad (3.3.10)$$

$$\underline{q} = c\,\underline{b} - (\underline{u}_G \cdot c\,\underline{b})\,\underline{u}_G \qquad (3.3.11)$$

and $a_T'$ is an estimate of the thrust acceleration.

## 5.3.4 EXTERNAL $\Delta V$ MANEUVER GUIDANCE

External $\Delta V$ Maneuver Guidance is normally used for the cislunar midcourse corrections. It may also be used to perform orbit maneuvers such as those prescribed by the concentric flight plan. The guidance program accepts input data via the DSKY or the telemetry uplink in the form of 3 components of an impulsive $\Delta \underline{V}$. An approximate compensation for the finite maneuver time is made by rotating the $\Delta \underline{V}$ vector and the guidance program issues commands to the spacecraft control system so as to apply the compensated velocity increment along an inertially fixed direction defined at the ignition time.

Input Parameters

1) Time of ignition ($t_{IG}$)

2) Specified velocity change ($\Delta \underline{V}_S$) in the local vertical coordinate system of the active vehicle at the time of ignition

3) Anticipated magnitude of the vehicle weight, wt, in pounds

4) Engine choice: SPS or RCS

Outputs

1) $\underline{\omega}_c$ = corrective angular rate command to the SPS flight control system (P-40)
2) SPS Engine-off Signal (P-40)
3) Display of velocity to be gained components in vehicle body axes for manually controlled RCS maneuvers (P-41)

Computations

The local vertical coordinate system is defined by:

$$\underline{X} = \text{UNIT} \left[ \{\underline{r}(t_{IG}) \times \underline{v}(t_{IG})\} \times \underline{r}(t_{IG}) \right]$$

$$\underline{Y} = \text{UNIT} \left[ \underline{v}(t_{IG}) \times \underline{r}(t_{IG}) \right] \qquad (3.4.1)$$

$$\underline{Z} = -\text{UNIT} \left[ \underline{r}(t_{IG}) \right]$$

where $\underline{r}(t_{IG})$ and $\underline{v}(t_{IG})$ are the position and velocity vectors at the time of ignition respectively.

The input column vector $\Delta\underline{V}_S$ is first pre-multiplied by a matrix, the columns of which are $\underline{X}$, $\underline{Y}$, and $\underline{Z}$ to yield $\Delta\underline{V}_S$ in the inertial coordinate system.

The input $\Delta\underline{V}_S$ is the required impulsive velocity correction. In order to compensate for the change in position during the maneuver, the input $\Delta\underline{V}_S$ is biased by half the estimated central angle of travel during the maneuver. The inplane velocity components of $\Delta\underline{V}_S$ are given by:

$$\Delta\underline{V}_P = \Delta\underline{V}_S - \left(\Delta\underline{V}_S \cdot \underline{u}_P\right) \underline{u}_P$$

where (3.4.2)

$$\underline{u}_P = \text{UNIT} \left[ \underline{v}(t_{IG}) \times \underline{r}(t_{IG}) \right]$$

The approximate central angle $\theta_T$ traveled during the maneuver is given by:

$$\theta_T \cong \frac{|\underline{r} \times \underline{v}|}{r^2} \quad \frac{\Delta V_S}{a'_T} \tag{3.4.3}$$

Where $a'_T$ is the estimated constant maneuver acceleration determined from

$$a'_T = \frac{F\,(32.17405)}{wt} \tag{3.4.4}$$

with F equal to a prestored nominal thrust in pounds based on the astronaut engine selection, and wt is the vehicle weight in pounds entered as a DSKY input parameter. The values of F are

$$\begin{aligned} F &= 21400 \text{ pounds} \quad \text{for the SPS} \\ F &= 100 \text{ pounds/jet} \quad \text{for the RCS} \end{aligned} \tag{3.4.5}$$

The corrected inplane velocity-to-be-gained is defined as:

$$\Delta \underline{V}_C = \Delta V_P \left\{ \text{UNIT } (\Delta \underline{V}_P) \cos \frac{\theta_T}{2} + \text{UNIT } (\Delta \underline{V}_P \times \underline{u}_P) \sin \left(\frac{\theta_T}{2}\right) \right\} \tag{3.4.6}$$

and is illustrated in Fig. 3.4-1. The corrected velocity vector-to-be-gained during the powered flight is then given by:

$$\Delta \underline{V}_T = \Delta \underline{V}_C + (\Delta \underline{V}_S \cdot \underline{u}_P)\, \underline{u}_P \tag{3.4.7}$$

The general computational sequence for the External $\Delta V$ Maneuver Guidance is illustrated in Fig. 3.4-2. The input $\underline{v}_G$ to the steering equations of Section 5.3.3 is initially set equal to $\Delta \underline{V}_T$ of Eq. (3.4.7). During the powered maneuver, $\underline{v}_G$ is updated as shown in Fig. 3.4-2 by the accumulated $\Delta \underline{v}$ measured by the accelerometers. During this program the scalar c of Eq. (3.3.6) is zero.

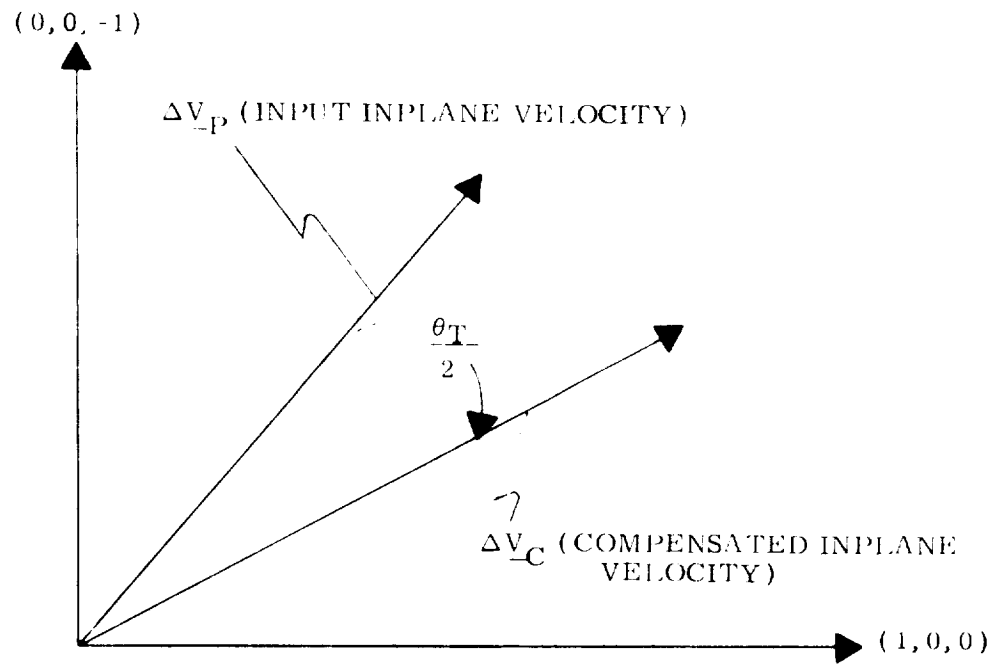



Figure 3.4-1 Inplane External $\Delta V$ Maneuver Time Compensation

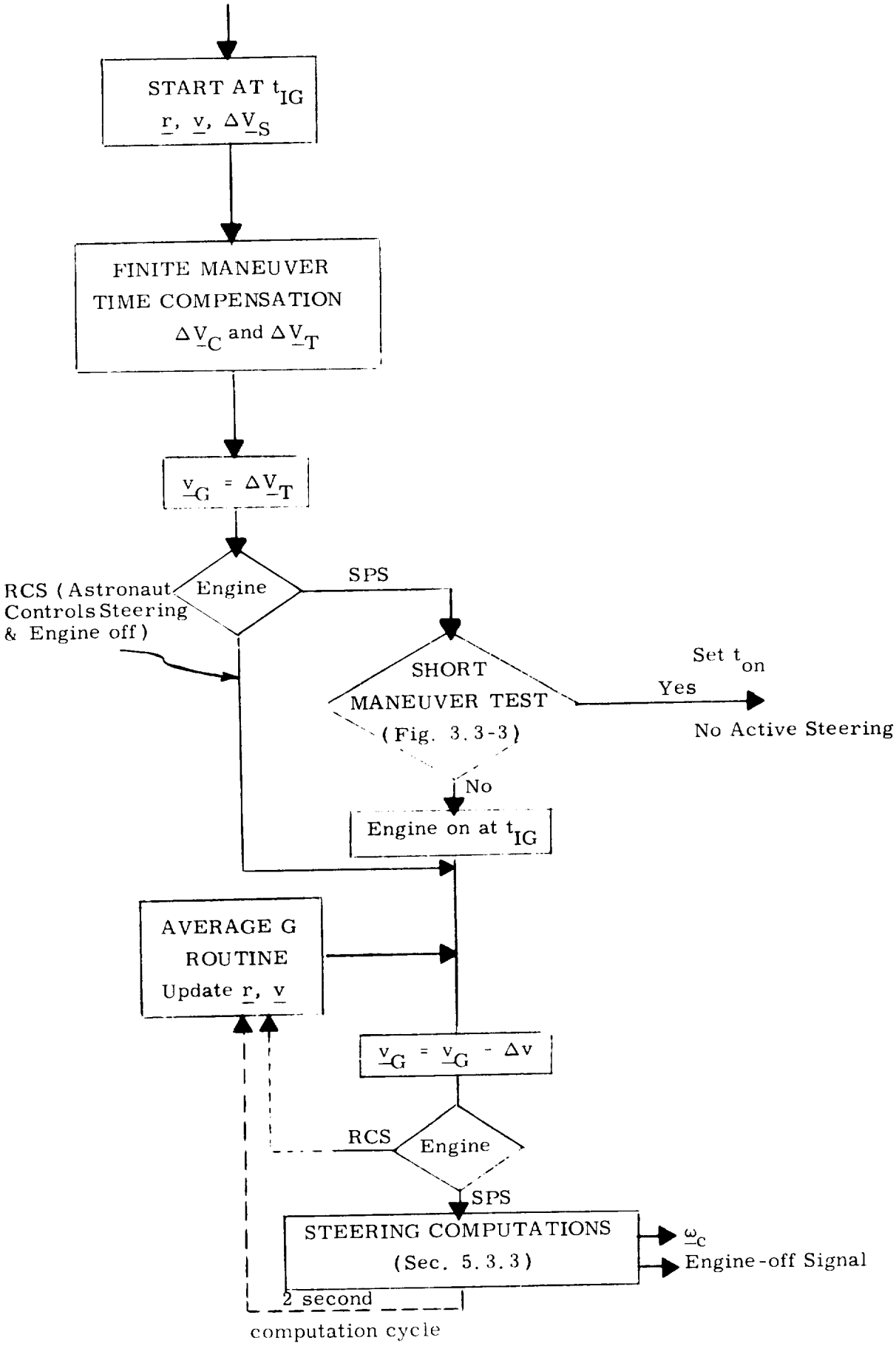



Figure 3.4-2 External ΔV Maneuver Guidance

## 5.3.5 LUNAR ORBIT INSERTION (LOI) MANEUVER GUIDANCE

The objective of the LOI Maneuver Guidance program is to establish a circular orbit with an orbital plane that contains a specified target position vector. This program is used to control the following CSM maneuvers:

1. Lunar orbit injection
2. CSM lunar orbit plane change prior to LM launch and ascent.

This program is based on the cross product steering concept of Sec. 5.3.3, and requires the following input parameters.

Input Parameters

$t_{IG}$ : time of ignition

$u_{TA}$ : selenocentric target vector to be contained by the orbital plane

c : cross product steering constant

Outputs

1. $\underline{\omega}_c$ - corrective angular rate command to the CSM flight control system
2. SPS Engine-off Signal

For the lunar orbit insertion maneuver, the above target input parameters are determined by the RTCC such that the resulting lunar orbit passes over the desired lunar landing site during the orbit in which the LM initiates the descent orbit injection maneuver.

The required target parameters are determined by the Lunar Orbit Plane Change (LOPC) Targeting Routine of Sec. 5.4.3 for the CSM plane change normally performed two orbits prior to LM ascent. The objective of this maneuver is to establish the CSM lunar orbit such that it contains the LM landing site at LM launch.

The selenocentric target vector, $\underline{u}_{TA}$, and the maneuver burn out position vector define the orbital plane for both of the above maneuvers. The target vector $\underline{u}_{TA}$ is approximately 90 degrees ahead of the expected maneuver cut-off point. The desired orbital plane will be obtained if normal propulsion system performance and maneuver ignition time are achieved.

Required Velocity for Orbital Maneuver

The instantaneous velocity to be gained is given by:

$$\underline{v}_G = \underline{v}_R - \underline{v} \tag{3.5.1}$$

where $\underline{v}$ is the present vehicle velocity and $\underline{v}_R$ is the instantaneous velocity that the vehicle should have to be in a circular orbit at the present position.

The vector $\underline{v}_R$ is computed as follows

$$\underline{v}_R = \sqrt{\frac{\mu}{r}}\, \underline{u}_z \;\; \mathrm{SGN} \left[ (\underline{r} \times \underline{u}_{TA}) \cdot \underline{z} \right] \tag{3.5.2}$$

where $\mu$ is the gravitational constant, r is the instantaneous selenocentric vehicle position vector,

$$u_z = \text{UNIT}\left[(\underline{r} \times \underline{u}_{TA}) \times \underline{r}\right] \qquad (3.5.3)$$

and $\underline{z}$ denotes the southerly direction.

The cross product steering constant, c, is set equal to approximately unity for the lunar orbit insertion program. The exact value of this constant will be determined by the RTCC to minimize errors in the resulting lunar orbit. The steering constant, c, will be set equal to unity for the LOPC maneuver when targeted by the CMC.

The LOI Maneuver Guidance Computation sequence is summarized in Fig. 3.5-1.

## Initial Thrust Attitude Calculation

In order to obtain the proper initial direction for thrust attitude, the following computations are made:

a. The vehicle state vector is extrapolated ahead to $(t_{IG} - \Delta t)$ by the Coasting Integration Routine and the corresponding $v_G$ is calculated.

b. The vehicle state vector is then extrapolated ahead to $t_{IG}$ and the corresponding $\underline{v}_G$ is obtained once again.

c. Initial pointing of the thrust vector is then accomplished as described in Section (5.3.3.1).

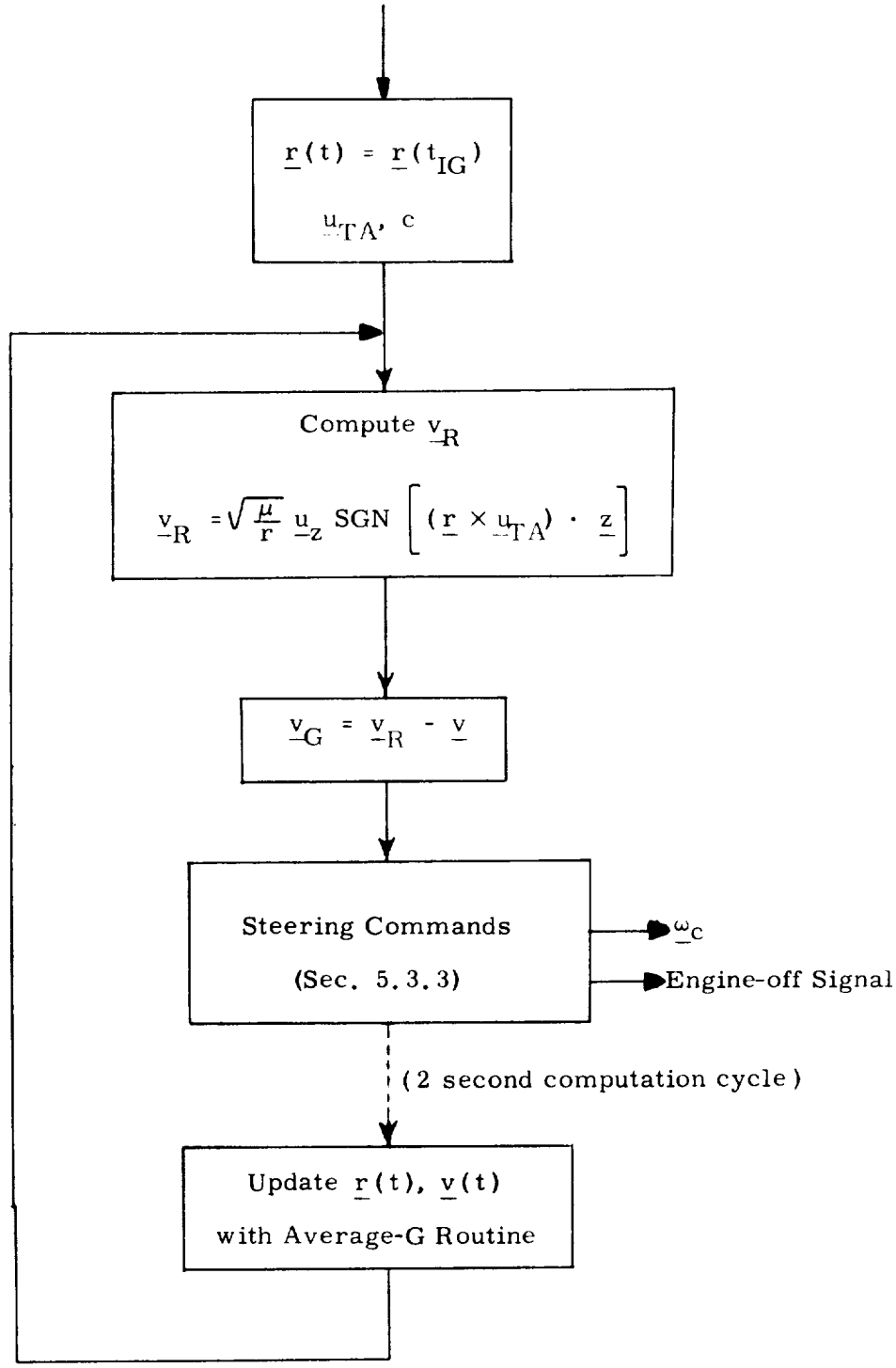



Fig. 3.5-1

LOI Maneuver Guidance

## 5.3.6 LAMBERT AIM POINT MANEUVER GUIDANCE

The objective of the Lambert Aim Point Maneuver Guidance program is to control the thrust direction to achieve a cut-off velocity vector such that the resulting trajectory intercepts a specified target position vector at a given time. This program utilizes the cross product steering concept of Sec. 5.3.3 and the Lambert Subroutine of Sec. 5.5.6.

The Lambert Aim Point Maneuver Guidance can be used to control the following maneuvers:

1.) Translunar Injection (TLI) P-15

2.) Rendezvous Intercept (P-34) and Midcourse Correction Maneuvers (P-35), Stable Orbit Rendezvous Maneuvers (P-38 and P-39).

3.) Transearth Injection (TEI) P-31

4.) Cislunar Midcourse Corrections

5.) Return-to-Earth Maneuvers

The targeting parameters for the TLI and TEI maneuvers are determined by the RTCC, and no CMC targeting routines exist for these maneuvers. Return-to-earth aborts initiated within the lunar sphere of influence must also be RTCC targeted. Either the CMC, LGC or RTCC can target Rendezvous Intercept maneuvers. Cislunar midcourse corrections and return-to-earth aborts outside the lunar sphere of influence can be targeted by either the RTCC or by the CMC Return-to-Earth Targeting Routine of Sec. 5.4.5.

### 5.3.6.1 Translunar Injection (TLI) Maneuver Guidance

This powered flight guidance routine is used to inject the vehicle out of earth parking orbit with a resulting trajectory that intercepts a given position vector in space at a fixed time.

The translunar injection maneuver routine is used in program P-15. This CMC program can actively control the TLI maneuver, or be used as a thrust monitor program for a Saturn controlled maneuver. The required inputs for program P-15 are:

Input Parameters

1. Ignition Time ($t_{IG}$)
2. Time of flight to conic target aim vector ($t_F$)
3. Conic target aim vector $\underline{r}_2$ ($t_A$)

   where $t_A = t_F + t_{IG}$

4. c which is the cross product steering constant

The target parameters $t_{IG}$, $t_F$, $\underline{r}_2$ ($t_A$) and c will be determined by the RTCC and accepted as required input parameters from the telemetry uplink.

Outputs

1. $\underline{\omega}_C$ = corrective angular rate which is transformed into an attitude error command and sent to the Saturn Flight Control System
2. SIVB Engine-off signal

## Guidance Computations

The TLI maneuver guidance computation sequence is illustrated in Fig. 3.6-1. The velocity required to intercept the target aim point $\underline{r}_2$ at $t_A$ is determined by the Lambert Subroutine. The associated velocity to be gained $\underline{v}_G$ is derived as shown in the Fig. 3.6-1. The Lambert Subroutine will normally be cycled at the same rate as the steering computations. If the computer work load prevents completion of the Lambert Subroutine at this rate, the Lambert problem is cycled at half the basic steering computation rate.

This velocity to be gained is then used with the cross product steering concept of Section 5.3.3 to determine the required angular command to the flight control system and the time for engine cut-off. The value of the cross product steering constant, c, in Eq. 3.3.6 of Section 5.3.3.1 is approximately unity and the exact value received over the telemetry uplink is a function of the general targeting calculations for determining the conic target aim vector.

## SIVB Engine-off Computation

A modified cut off scheme has been tested which would allow for thrust magnitude variation of the SIVB with time. This thrust variation has been mentioned as a possible degradation of the engine performance. The modified scheme would use the past history of the thrust magnitude measured by the PIPA's to extrapolate the thrust value near cut off. Since this extrapolation has the standard problem with derivatives of noisy signals, this scheme will not be used unless warranted by SIVB engine performance degradation.

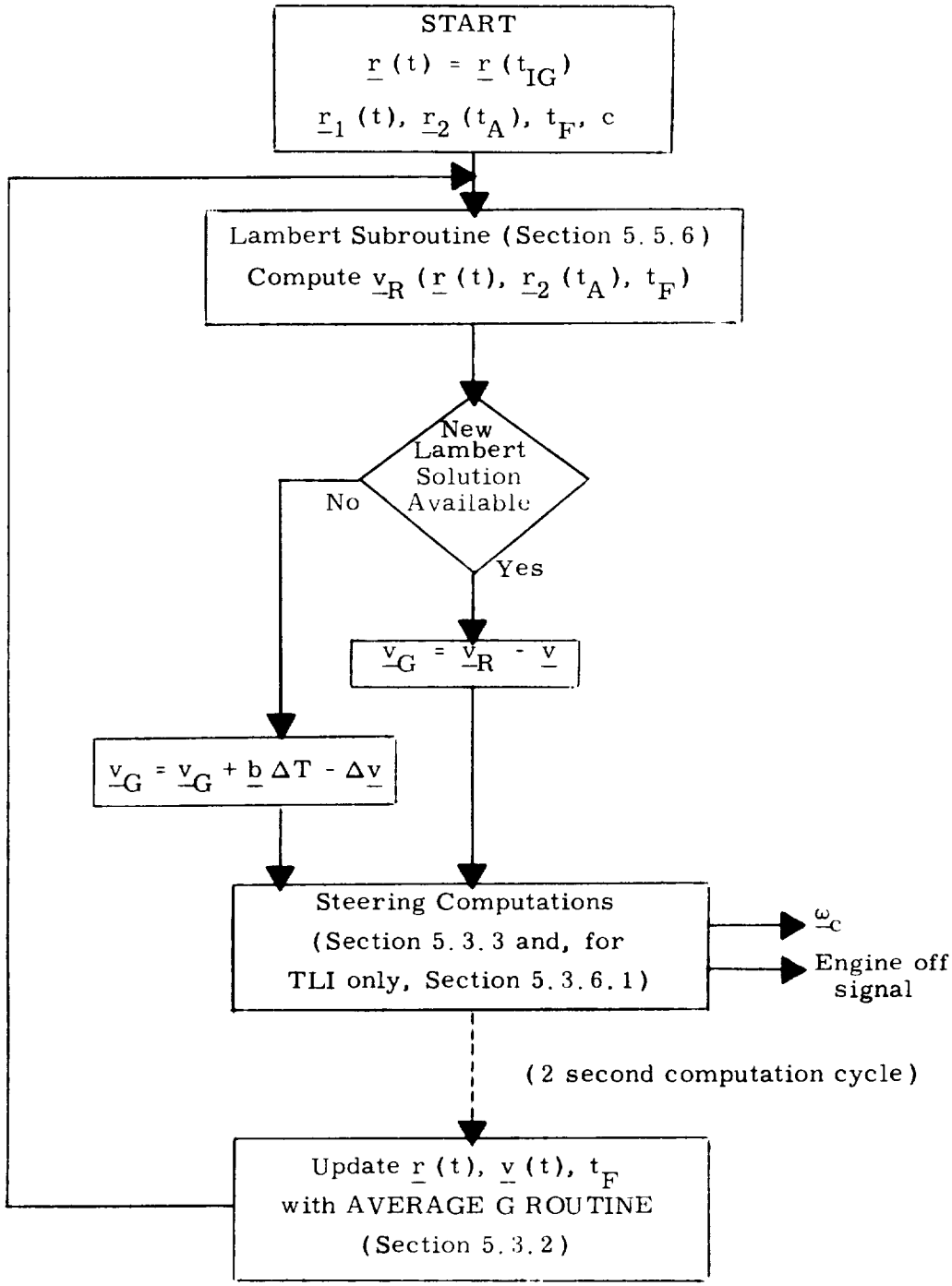



Figure 3.6-1 Translunar Injection Maneuver (TLI) Guidance
Transearth Injection Maneuver (TEI) Guidance
Return-to-Earth Abort Maneuver Guidance

### 5.3.6.2 Rendezvous Intercept Maneuver Guidance

This powered flight guidance routine is used to control the rendezvous Transfer Phase Initiation (TPI) and midcourse correction maneuvers in both the Concentric Flight Plan and the Stable Orbit Rendezvous technique. The required maneuver velocity is initially calculated before the ignition time using the Initial Velocity Subroutine (Section 5.5.11) which in turn uses the Lambert Subroutine (Section 5.5.6). During the maneuver the Lambert Subroutine is used to update the required maneuver velocity. This closed loop guidance concept, coupled with cross product steering (Section 5.3.3), controls maneuvers such that the resulting trajectory intercepts a target aim point at a designated time.

The rendezvous Intercept Maneuver Routine is used in program P-40 when the SPS is chosen for the maneuver, or program P-41 if the RCS is used to perform the maneuver. The required initial targeting inputs for this routine are determined in programs P-34, P-35, P-38 and P-39.

Input Parameters

1. Ignition time ($t_{IG}$ or $t_{TPI}$)
2. Time of flight to intercept ($t_F$) measured between TPI and intercept provided no terminal rendezvous maneuver was made.
3. Target aim point $\underline{r}_2$ ($t_A$)

   where $t_A$ is the time of intercept
4. Engine Choice: SPS or RCS

Outputs

1. $\underline{\omega}_c$ = corrective angular rate command to the SPS system (P-40).

2. SPS Engine-off Signal (P-40)

3. Display of velocity to be gained components in body axes for RCS controlled maneuvers (P-41)

Guidance Computations

The rendezvous intercept maneuver guidance computation sequence is illustrated in Fig. 3.6-2. The velocity required to intercept the target aim point $\underline{r}_2$ at $t_A$ is determined by the Lambert Subroutine. The associated velocity to be gained $\underline{v}_G$, is derived as shown in Fig. 3.6-2. The velocity to be gained is then used in the cross product steering concept of Section 5.3.3 to determine the required angular rate command to the flight control system and the time for engine cut-off. The Lambert Subroutine will ordinarily be cycled at the same rate as the steering computations. If the computer work load prevents completion of the Lambert Subroutine at this rate, the Lambert problem is cycled at half the basic steering computation rate as illustrated in Fig. 3.6-2.

The value of the cross-product steering constant c of Eqs. (3.3.5) and (3.3.6) in Section 5.3.3.1 is equal to unity in this guidance routine if the SPS was selected, or c = 0 if the RCS was chosen.

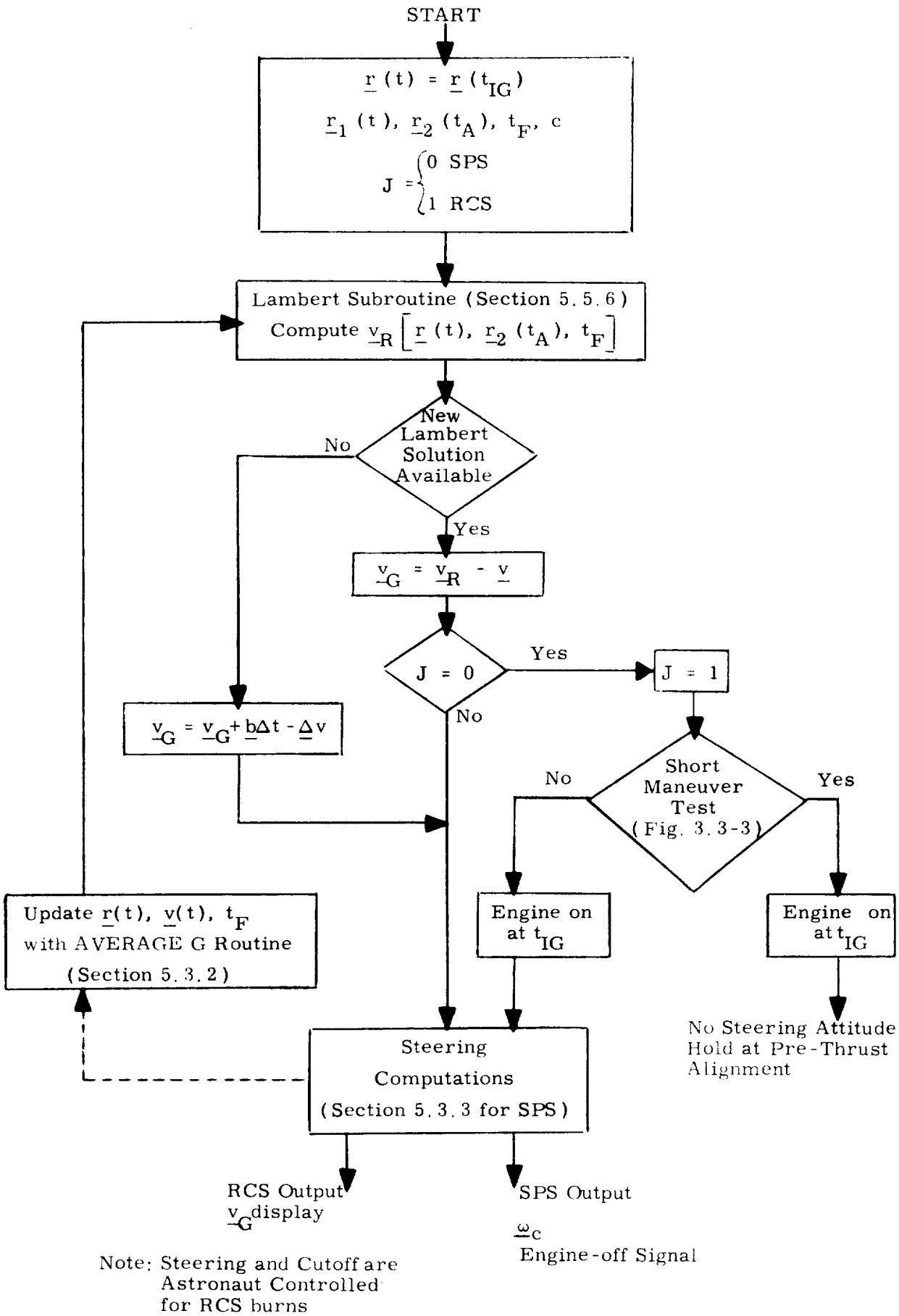



Figure 3.6-2 Rendezvous Intercept Maneuver and Cislunar Midcourse Maneuver Guidance

### 5.3.6.3 Transearth Injection (TEI) Maneuver Guidance

The objective of this maneuver is to inject the CSM out of lunar orbit on a trajectory that intercepts a given target position vector at a fixed time of flight. The basic steering concept is that presented in Section 5.3.3 with the input velocity to be gained being determined by the Lambert Subroutine of Section 5.5.6.

The initial targeting input parameters required for this guidance maneuver are determined by RTCC calculations and transmitted to the CMC over the telemetry uplink.

Input Parameters

1. Ignition Time ($t_{IG}$)
2. Time of flight to intercept ($t_F$)
3. Target aim point $\underline{r}_2$ ($t_A$)

   where $t_A = t_F + t_{IG}$
4. c = cross product steering constant
5. SPS engine

The value of c will be approximately 1/2 for a constant attitude burn. The exact value will be a function of the RTCC targeting computations.

Output

1. $\underline{\omega}_c$ = corrective angular rate command to the CSM Flight Control System
2. SPS Engine-off Signal

Guidance Computations

The TEI maneuver guidance computation sequence is illustrated in Fig. 3.6-1. The velocity required to intercept the target aim vector $\underline{r}_2$ at $t_A$ is determined by the Lambert Subroutine and then the associated velocity to be gained $\underline{v}_G$ is derived as shown in Fig. 3.6-1. The velocity to be gained is then used in the cross product steering concept of Section 5.3.3.1 to determine the required angular rate command to the flight control system, and the time for engine cut-off. As indicated in Fig. 3.6-1 the Lambert Subroutine is cycled in the computation sequence at the same rate as the steering computations. If the computer work load prevents completion of the Lambert Subroutine at this rate, the Lambert problem is cycled at half the rate of the steering computations as shown in Fig. 3.6-1. The value of the cross product steering constant c is approximately unity and the exact value is determined from RTCC targeting calculations.

### 5.3.6.4 Cislunar Midcourse Correction (MCC) Maneuver Guidance

The objective of this maneuver is to correct the current cislunar trajectory in order to arrive at a given position vector at a specified time.

This MCC maneuver guidance is essentially the same as that for the TEI maneuver of Section 5.3.6.3 with the exception that either the SPS or RCS propulsion systems can be chosen to execute the maneuver. The targeting parameters required to initialize this guidance routine are determined by the RTCC for all midcourse maneuvers made within the lunar sphere of influence, and nominal

translunar and transearth trajectories outside the lunar sphere of influence. Return to earth abort trajectory midcourse correction maneuvers outside the lunar sphere of influence can be targeted by either the RTCC or by the CMC Return-to-Earth Program P-37 or Safe Perilune Target Program P-70 of Section 5.4.5.

Input Parameters

1. Ignition Time ($t_{IG}$)
2. Time of flight to intercept ($t_F$)
3. Target aim point $\underline{r}_2$ ($t_A$)

   where $t_A = t_F + t_{IG}$
4. SPS or RCS engine

Output

1. $\underline{\omega}_c$ = corrective angular rate command to the SPS Flight Control System (P-40)
2. SPS Engine-off Signal (P-40)
3. Velocity to be gained components in vehicle axes for RCS manually controlled maneuvers (P-41)

Guidance Computation (Short Burn Test)

The cislunar MCC maneuver guidance computation sequence is illustrated in Fig. 3. 6-2. The velocity required to intercept the target aim vector $\underline{r}_2$ at $t_A$ is determined by the Lambert Subroutine and then the associated velocity to be gained $\underline{v}_G$ is derived as shown in Fig. 3. 6-2. The velocity steering concept of Section 5. 3. 3. 1 is used to determine the required angular rate command to the flight control system and the time for engine cut-off. Figure 3. 6-2 has two short maneuver tests depending on engine choice: SPS or RCS. The Lambert Subroutine is ordinarily cycled in the computation sequence at the same rate as the steering computations. If the computer work load prevents completion of the Lambert Subroutine at this rate, the Lambert problem is cycled at half the basic steering computation rate. The value of the scalar c in Eq. (3. 3. 6) is equal to zero for the MCC maneuver.

5. 3. 6. 5 Return-to-Earth Maneuver Guidance

The objective of this guidance routine is to control an abort maneuver such that the CSM is on a satisfactory return-to-earth trajectory. The Return-to-Earth Maneuver Guidance Routine is the same as that for the Transearth Injection (TEI) Guidance of Section 5. 3. 6. 3 with the exception that the routine can be targeted by the Return-to-Earth Targeting Program, P-37, of the CMC for those abort cases outside the lunar sphere of influence. The RTCC can also supply the required targeting parameters in these cases, and is required to perform this function for all return-to-earth aborts initiated within the lunar sphere of influence. For all return-to-earth maneuvers the value of the scalar c is unity.

Input Parameters

1. Ignition Time ($t_{IG}$)

2. Time of flight to intercept ($t_F$)

3. Target aim point $\underline{r}_2$ ($t_A$)

   where $t_A = t_F + t_{IG}$

4. Engine : SPS or RCS

5. c = 1 for CMC targeting, and c is approximately unity for RTCC targeting. The exact value will be determined from ground targeting computations.

Output

1. $\underline{\omega}_c$ = corrective angular rate command to CSM Flight Control System (P-40)

2. SPS Engine-off signal (P-40)

Guidance Computation

The return-to-earth abort maneuver guidance computation sequence is illustrated in Fig. 3.6-1.. The velocity required to intercept the target aim point $\underline{r}_2$ at $t_A$ is determined by the Lambert Subroutine and the corresponding velocity to be gained $\underline{v}_G$ is derived as shown in Fig. 3.6-1 . The velocity to be gained is then used in the cross product steering concept of Section 5.3.3.1 to determine the required angular rate command to the flight control system and the time for engine cut-off. As indicated in Fig. 3.6-1 , the Lambert

Subroutine is cycled in the computation sequence at the same rate as the steering computations. If the computer work load prevents completion of the Lambert Subroutine at this rate, the Lambert problem is cycled at half the basic steering computation rate. The value of the cross product steering constant c is unity if under CMC targeting, and c is approximately unity if under RTCC targeting. The exact value is a function of the ground targeting computation.

## 5.4 TARGETING ROUTINES

### 5.4.1 GENERAL COMMENTS

The objectives of the targeting routines presented in this section are to provide a CMC capability to determine the required input target parameters and set control modes for the various powered flight guidance routines of Section 5.3. It should be noted that the CMC does not have a targeting capability for every phase of the lunar landing mission, and therefore must rely on RTCC targeting for many of the nominal and abort phases of the lunar mission. The restricted targeting capability provided by the CMC is presented in the following subsections:

5.4.2 Lunar Landing Time Prediction Routine

5.4.3 Lunar Orbit Plane Change Targeting Routine

5.4.4 Rendezvous Retrieval Targeting Routines

These include TPI and Rendezvous Midcourse Correction programs, a TPI Search Program, and Stable Orbit Rendezvous routines.

5.4.5 Return-to-Earth Targeting Routines

These routines are restricted to operation outside the lunar sphere of influence for abort returns to earth.

All CMC targeting programs use the Basic Reference Coordinate System defined in Section 5.1.4.1. The Rendezvous Retrieval Targeting Routines of Section 5.4.4 can be used in either earth or lunar orbits. The basic input parameters required by the targeting routines of this section are the vehicle state vector estimates determined by the navigation programs of Section 5.2.

## 5.4.2 LUNAR LANDING TIME PREDICTION ROUTINE

### 5.4.2.1 General

The main object of this program is to compute the lunar landing time for a Hohmann descent trajectory from the CSM orbit followed by a powered flight phase to the desired landing site. The input parameters to the program are:

| | | |
|---|---|---|
| 1. | $\underline{r}_D$, $\underline{v}_D$, $t_D$ | Vehicle state vector at time $t_D$. |
| 2. | $t$, $t_E$ | Present time and earliest permissible deorbit time. |
| 3. | $\underline{r}_{LSA}$, $t_A$ | Position of the desired landing site at time $t_A$. |
| 4. | $h_{DP}$, $\theta_F$, $t_F$ | Desired ignition altitude, nominal angle and time for the powered landing maneuver (see Fig. 4.2-1). |

The program contains an iterative loop which insures that the landing position of the LM coincides with the desired landing site as specified in the LGC initialization when rotated into the CSM plane at the nominal landing time. Figure 4.2-2 illustrates the program logic and Fig. 4.2-1 defines the descent trajectory configuration and nomenclature.

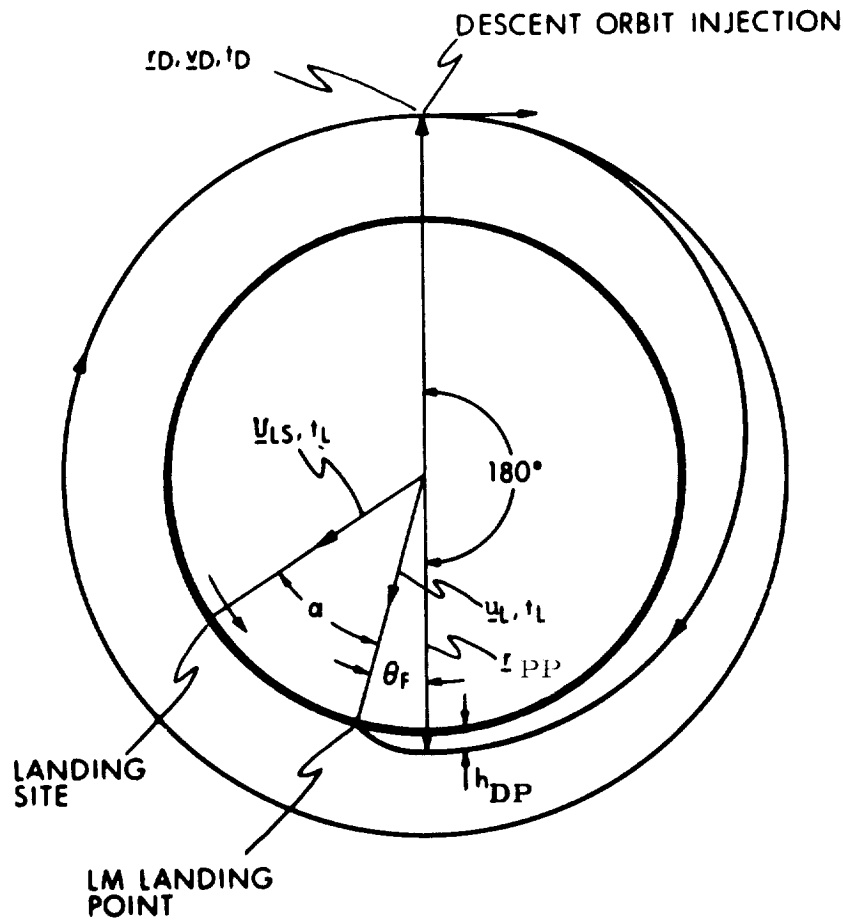



Fig. 4.2-1 Descent Trajectory Nomenclature

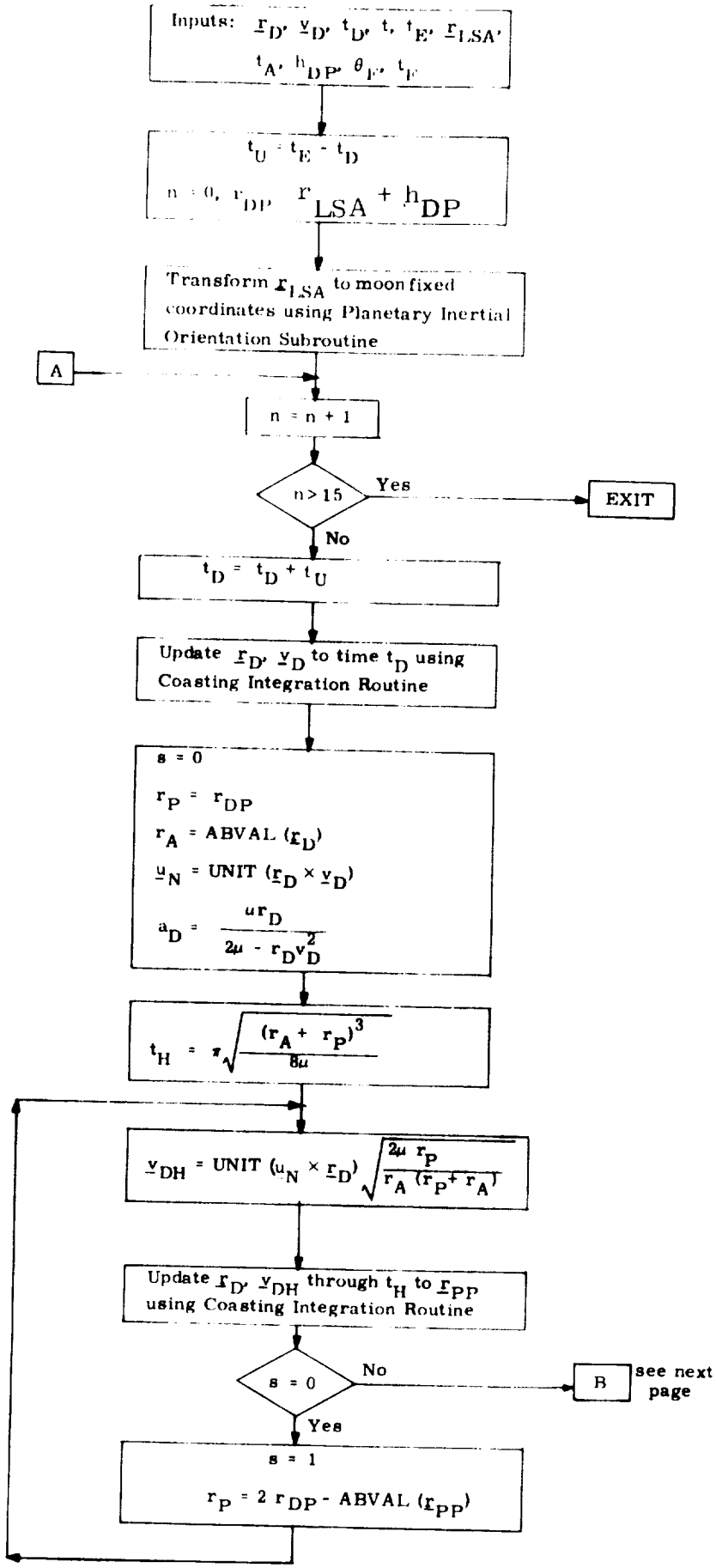



Figure 4.2-2 Lunar Landing Time Prediction Logic Diagram (page 1 of 2)

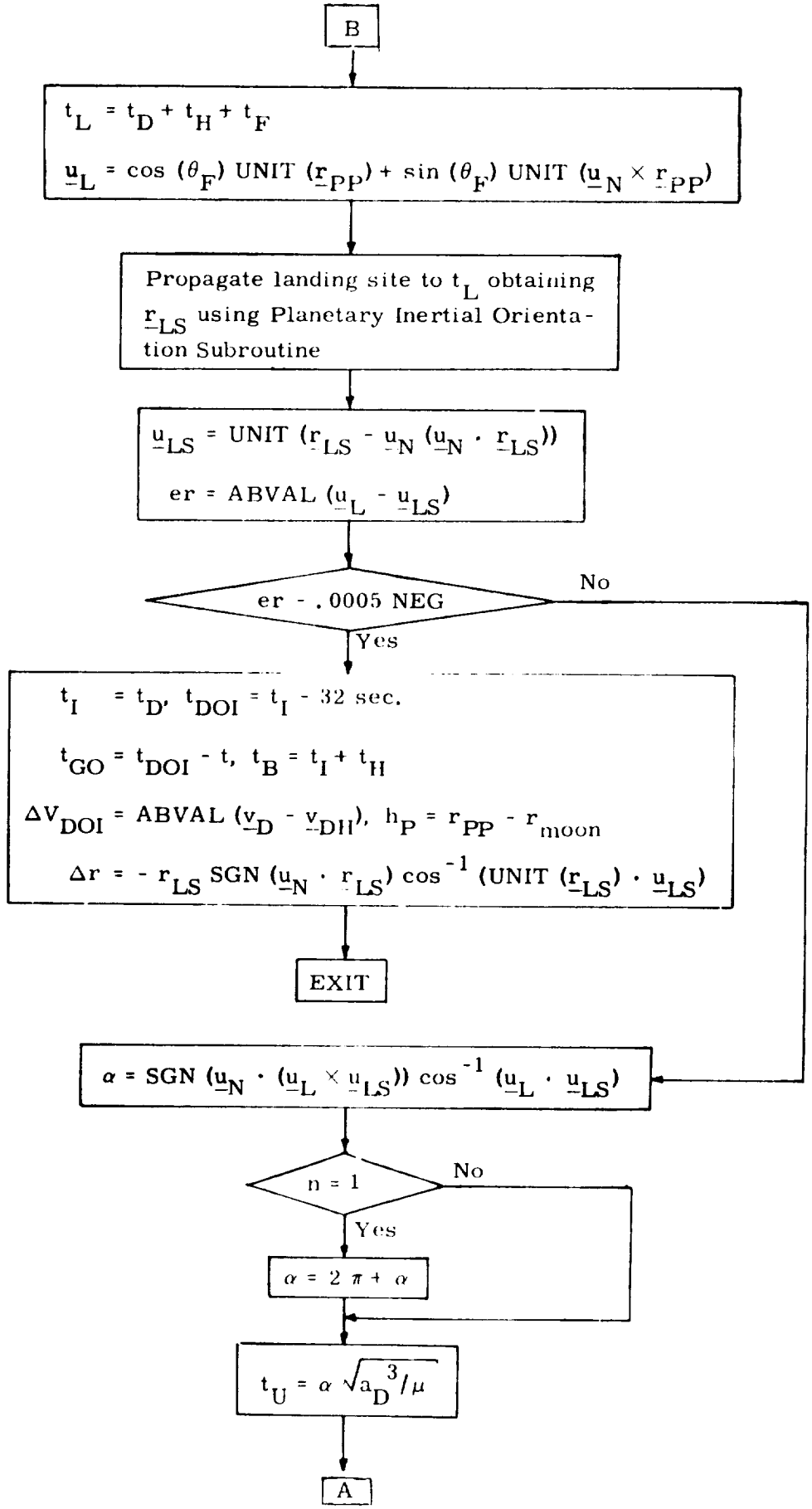



Figure 4.2-2 Lunar Landing Time Prediction Logic Diagram (page 2 of 2)

The iterative loop starts with the updating of the CSM state vector to a new time and then advancing the LM through the Hohmann transfer and the powered landing maneuver phases. The landing position of the LM is then compared with the desired landing site at the LM landing time to obtain a basis for selecting a new descent orbit injection time.

### 5.4.2.2 Determination of the LM Descent Trajectory to Perilune

After the descent orbit injection maneuver, the LM should descend on a Hohmann transfer maneuver to the desired perilune altitude. However, due to the non-spherical gravity of the moon, the actual LM descent trajectory will only approximate the conic Hohmann transfer. An offset procedure is used to partially compensate for the non-spherical effects.

According to Fig. 4.2-2, the conic Hohmann transfer time $t_H$ and velocity $\underline{v}_{DH}$ are first computed. The LM state vector ($\underline{r}_D$, $\underline{v}_{DH}$) is then advanced through $t_H$ using the Coasting Integration Routine. The perilune altitude is then offset and a new Hohmann velocity is computed. The LM state vector is again advanced through $t_H$ which will result in approximately attaining the desired perilune conditions.

### 5.4.2.3 Determination of New Descent Orbit Injection Time

Once the LM landing time is obtained, the position vector of the desired landing site is determined and rotated into the LM descent orbit plane. The angle $\alpha$ between this vector and the LM landing point is computed in the following manner. On the first iteration ( n = 1 ) $\alpha$ is measured from the LM to the desired landing site in the rotational direction of the LM. On subsequent iterations $\alpha$ is defined in the region $-\pi < \alpha < \pi$, with the positive error indicating that the landing site is ahead of the LM. When $\alpha$ is sufficiently small the iteration is terminated. Otherwise, the CSM state vector is updated through the time obtained by multiplying the mean motion of the CSM by $\alpha$.

### 5.4.2.4 Program Outputs

The outputs of the program are:

| | |
|---|---|
| $t_I$ | Nominal time for descent orbit injection (assuming impulsive burn). |
| $t_{DOI}$ | Approximate time of ignition for descent orbit injection ($t_I$ - 32 secs. ) |
| $t_{GO}$ | Time-to-go to ignition time for the descent orbit injection maneuver. |
| $t_B$ | Nominal time of ignition for the powered landing maneuver. |
| $t_L$ | Nominal time of landing. |

| | |
|---|---|
| $\Delta r$ | Cross track distance between the descent orbit and the landing site (plus if north of the descent orbital plane). |
| $h_P$ | Descent orbit insertion target altitude. |
| $\Delta V_{DOI}$ | Required velocity for the descent orbit injection (impulsive burn) maneuver. |

The landing site position vector at the predicted landing time, $t_L$, is used as the basic vector direction to which the LM IMU is aligned for the descent and landing maneuver phases. The CSM orbital plane is used to define the other IMU or stable member (SM) landing alignment parameters as follows:

$$\underline{X}_{SM} = \text{UNIT}\left[\underline{r}_{LS}(t_L)\right]$$

$$\underline{Y}_{SM} = \text{UNIT}\,(\underline{Z}_{SM} \times \underline{X}_{SM})$$

$$\underline{Z}_{SM} = \text{UNIT}\left[(\underline{r}_C \times \underline{v}_C) \times \underline{X}_{SM}\right]$$

where $\underline{r}_C$ and $\underline{v}_C$ are the CSM position and velocity vectors, respectively.

The nominal descent orbit injection time, $t_I$, and the perilune target altitude, $h_P$, are the two primary input parameters for the LM DOI pre-thrust program. The output parameters, $t_{DOI}$ and $t_{GO}$, are estimates of the DOI maneuver ignition time and time-to-go respectively which provide the astronaut with this major maneuver timing requirements before calling the

DOI pre-thrust program. The output $t_B$ is used as the nominal ignition time for the LM landing maneuver pre-ignition program. The $\Delta r$ and $\Delta V_{DOI}$ are primary display outputs to the astronaut to determine if the landing maneuver and DOI maneuver targeted by this routine are within nominal LM DPS $\Delta V$ limits.

This routine is in both the CMC and LGC. The CMC program is normally used to monitor this initial LGC targeting operation and provide a check solution to be sure that the LGC was correctly initialized.

### 5.4.3 LUNAR ORBIT PLANE CHANGE TARGETING ROUTINE

The objective of the Lunar Orbit Plane Change (LOPC) Targeting Routine is to determine the required maneuver parameters to establish a CSM orbit that will pass over the LM landing site during the orbit that the LM launch is planned. It is assumed that the approximate LM launch time is known, and that the LOPC maneuver will be initiated 2.25 revolutions before the desired passage over the LM landing site. The Lunar Orbit Insertion (LOI) Maneuver Routine of Section 5.3.5 is used to control the LOPC maneuver, and the basic output parameters of the LOPC Targeting Routine (P-33 of Section 4) are used as the required targeting parameters for the LOI pre-thrust program (P-32 of Section 4).

The required input parameters to the LOPC Targeting Routine are:

| | |
|---|---|
| $\underline{r}$, $\underline{v}$, $t$ | CSM state vector at time t |
| $\underline{r}_{LSA}$, $t_A$ | LM landing site position vector at time $t_A$ |
| $t_E$ | Earliest permissible LOPC maneuver time based upon the approximate knowledge of the LM launch time and the 2.25 revolutions desired prior to LM launch |

The primary outputs of the LOPC Targeting Routine for the LOI Maneuver Guidance Routine are:

| | |
|---|---|
| $t_{IG}$ | Maneuver ignition time |
| $\underline{u}_{TA}$ | Target unit vector that the CSM orbital plane must contain at LOPC maneuver burn-out |

In addition to the above parameters the following output parameters are required for displays and astronaut monitoring:

| | |
|---|---|
| $\Delta \underline{v}$ | LOPC maneuver impulsive velocity components displayed in the local vertical coordinate system at $t_{IG}$ |
| $t_0$ | The approximate time that the CSM will pass over the LM landing site 2.25 orbits after the LOPC maneuver |

The LOPC Targeting Routine will determine a LOPC maneuver time by an iterative procedure such that the ignition time will occur within the orbital revolution following the input time $t_E$. The general targeting parameters used in this routine are illustrated in Fig. 4.3-1, and the program logic is outlined in Fig. 4.3-2.

With reference to Fig. 4.3-2, the first step is to transform the LM landing site position vector at time $t_A$ into moon fixed coordinates. The CSM state vector is then updated to time $t_E$ using the Coasting Integration Routine of Section 5.2.2. Assuming that the plane change maneuver is made at $t_E$, the time $t_0$ corresponding to when the CSM would have completed 2.25 revolutions, is obtained by assuming a circular orbit following the maneuver. The landing site is then propagated to time $t_0$ and projected into the CSM orbital plane, obtaining the unit vector $\underline{u}_{LS}$. In general, the landing site will not be at the position obtained by advancing the CSM through 2.25 revolutions. The central angle $\theta$ between the landing site and the updated CSM position (see Fig. 4.3-1) is next computed. If the iteration counter n is one, the central angle $\theta$ is measured in the CSM rotational direction from the updated CSM position. Otherwise, the location of $\underline{u}_{LS}$ ahead or behind the updated CSM position is used to set a switch sw which controls the update direction of the CSM.

The time $t_3$ is computed for the state vector $\underline{r}$, sw $\underline{v}$ to conically traverse the angle $\theta$. The estimated plane change maneuver time $t_1$ is then increased by sw $t_3$ and the calculations, starting with an update of the CSM, are repeated until the angle $\theta$ is sufficiently small.

When the iteration of Fig. 4.3-2 is successful with respect to the central angle $\theta$, the velocity $\underline{v}_N$ following the maneuver is computed. The required $\Delta\underline{v}$ to perform the LOPC maneuver is then determined as shown. The LOPC maneuver time, $t_B$, is estimated on the basis of $\Delta\underline{v}$ and the SPS thrust acceleration magnitude, $a_T$. The LOPC maneuver ignition time, $t_{IG}$, is decreased from the impulsive ignition time, $t_1$, by one half the estimated maneuver time $t_B$ as shown.

The time $t_0$ of Fig. 4.3-2 is the approximate time that the CSM will pass over the LM landing site 2.25 orbital revolutions after the LOPC maneuver and is used as an output display parameter. The outputs $t_{IG}$ and $\underline{u}_{TA}$ are stored in special registers so that they can be used as the required input target parameters of the LOI Maneuver Guidance Routine, P-32, of Section 5.3.5.

The 2.25-orbital revolution parameter used in this targeting program is in erasable memory, and could be changed prior to the mission.

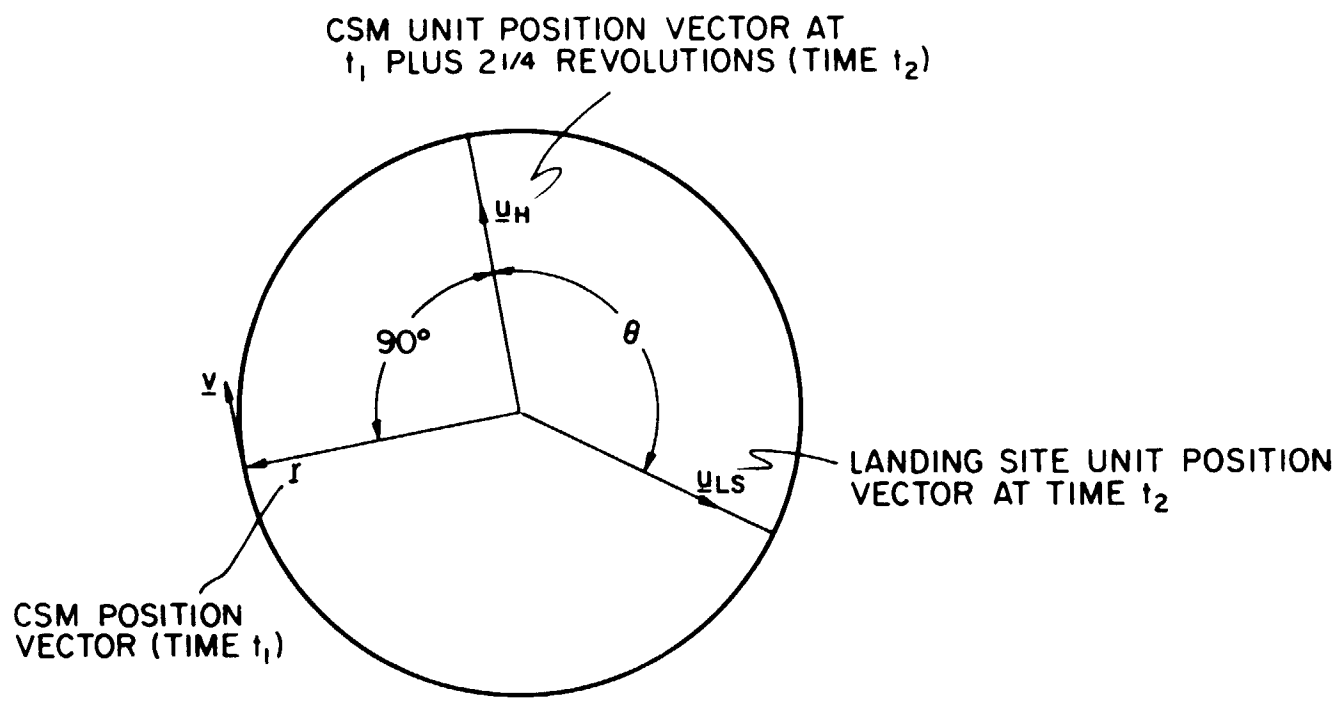



Fig. 4.3-1 Lunar Orbit Plane Change Targeting Nomenclature

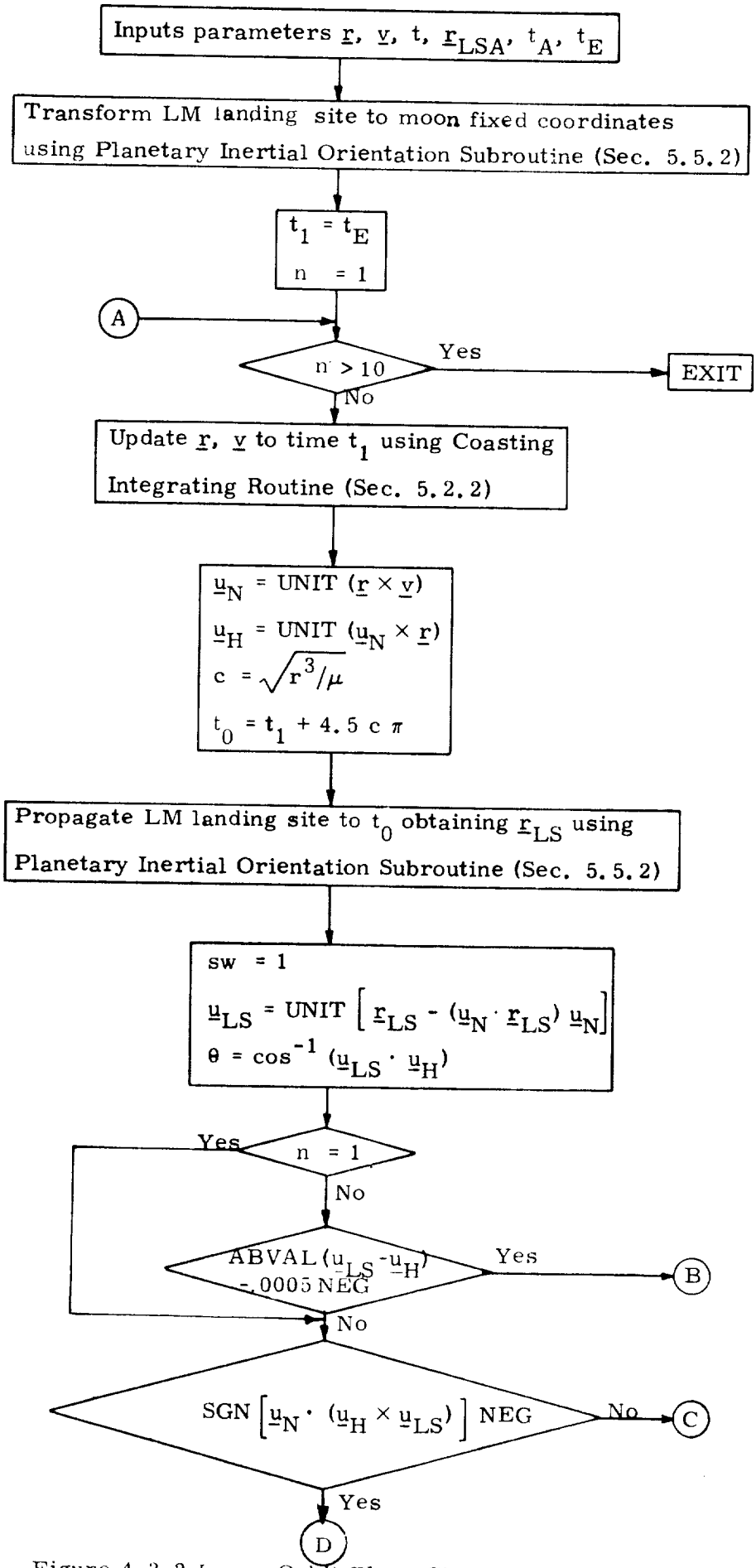



Figure 4.3-2 Lunar Orbit Plane Change (LOPC) Targeting Routine Logic Diagram (page 1 of 2)

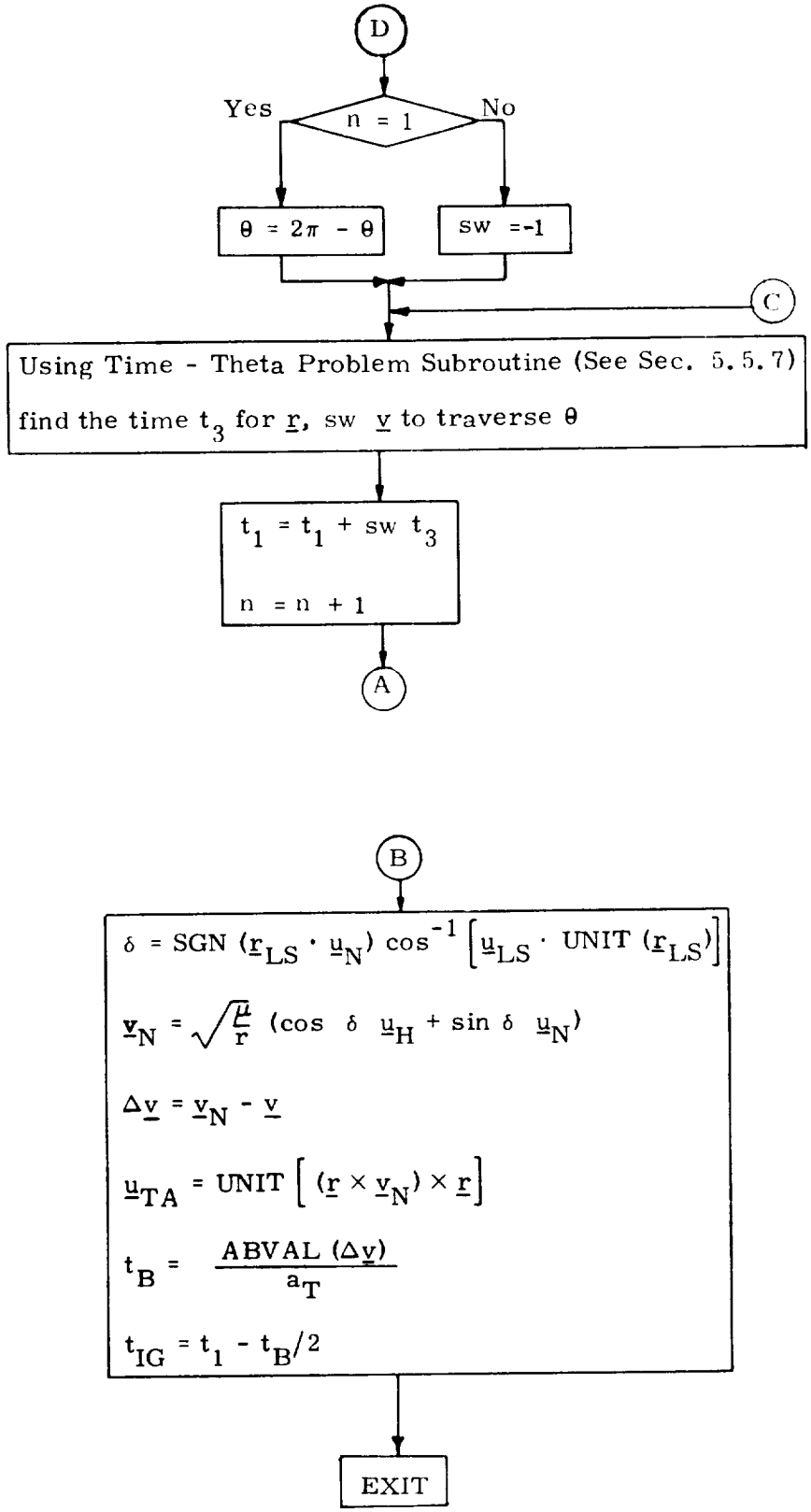



Figure 4.3-2 Lunar Orbit Plane Change (LOPC)
Targeting Routine Logic Diagram
(page 2 of 2)

## 5.4.4 RENDEZVOUS RETRIEVAL TARGETING

### 5.4.4.1 General

The Rendezvous Guidance Program in the CMC may be used in two modes which, in combination, provide the CMC with a direct intercept trajectory and the midcourse maneuvers required for rendezvous. These two modes designated pre-TPI mode (Transfer Phase Initiation) and Mid-Course mode are identical to the last two modes of the Concentric Rendezvous Program in the LM Guidance Computer (LGC).

In addition, the TPI Search Program provides a minimum fuel transfer capability. An alternate rendezvous procedure is available using the Stable Orbit Rendezvous Routines.

All computations performed in the Rendezvous Guidance Program use the Basic Reference Coordinate System defined in Section 5.1.4.

The program has a basic dependence on the state vector estimates which are obtained from the navigation routines described in Sections 5.2 and 5.3.2.

For the routines in this section there are two associated program numbers based on whether the CSM or the LM is the active vehicle. If the astronaut elects to make the LM active, the only change is the program number selected. All equations and program operations in Section 5.4.4 are identical for these two modes of operation.

### 5.4.4.2 Pre-TPI Maneuver (MODE 3)

This program mode corresponds to program P-34 (CSM active) or P-74 (LM active) of Section 4, and the computational sequence is illustrated in Fig. 4.4-1.

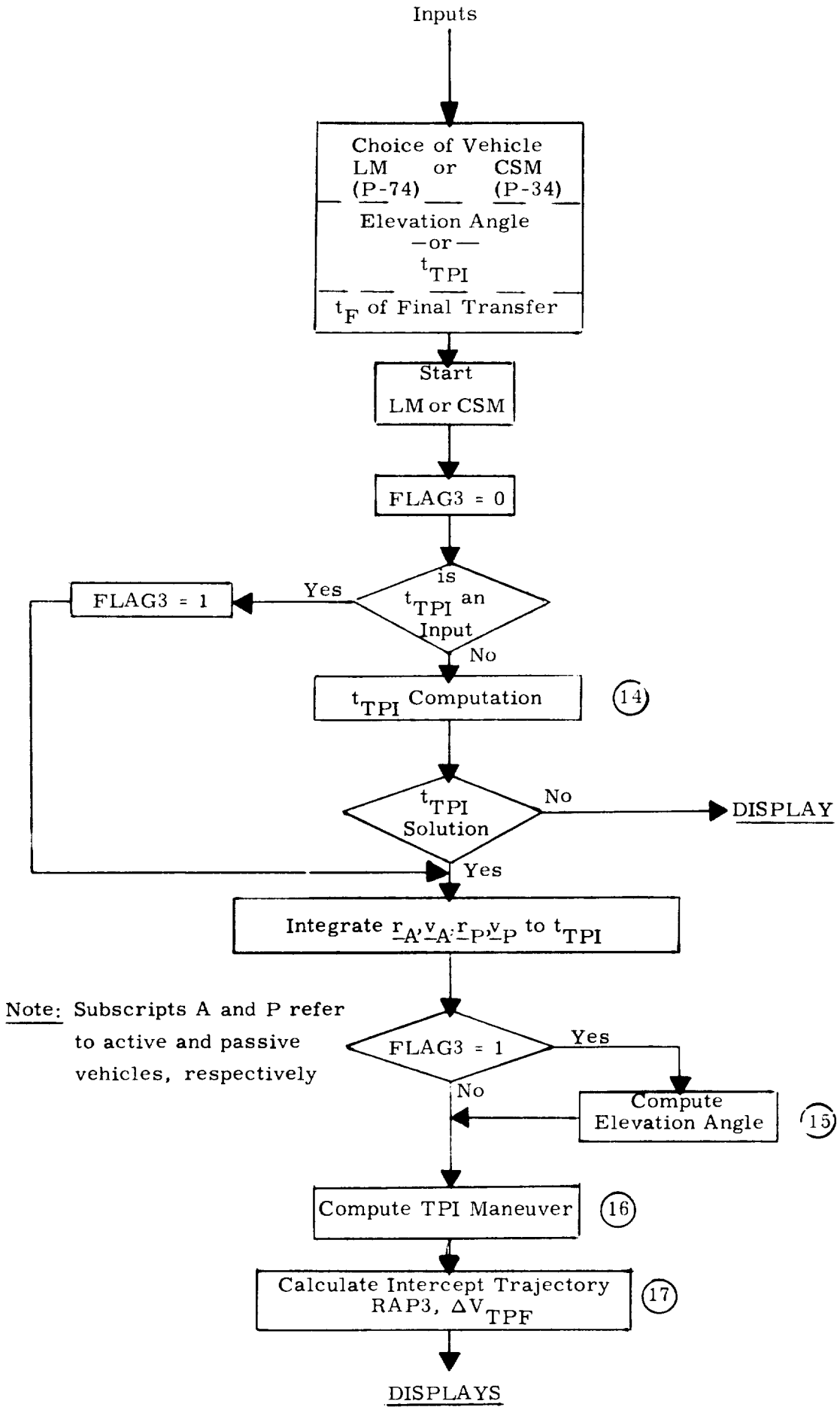



Figure 4.4-1 Pre-TPI Maneuver Mode

Inputs

1) Choice of maneuvering vehicle

2) TPI time or Elevation angle

3) $t_F$ of final transfer trajectory

(This time is the transfer trajectory time between TPI and intercept, and does not include the effects of terminal rendezvous maneuvers.)

Program Operations

If the astronaut wishes to specify an elevation angle it is necessary to input both the TPI time and E, the desired elevation angle. As discussed below a nominal $t_{TPI}$ is needed to initialize the routine used to compute $t_{TPI}$ for a given E. When the above input procedure is completed and the E option selected, the TPI time associated with the desired elevation angle will be computed with the following iterative technique.

(14)* TPI Computation

Both vehicles are advanced to the stored TPI time using the precision Coasting Integration Routine. If the difference between the true line of sight angle and the input TPI

*Circled numbers in Sections 5.4.4.2 and 5.4.4.3 refer to the logic flow diagrams of Fig. 4.4-1 and 4.4-3 and correspond to the numbering used in the analogous LGC Figures.

angle is greater than 0.1 degrees, the true TPI time is then calculated by making successive co-planar circular approximations to the current orbits (which are assumed to be no more than 5 degrees out of plane).

The time error to the true TPI for circular orbits is given by the equation:

$$TERR = \frac{ARCCOS\left[(RA/RP)\cos E\right] - E - \lambda}{WP - WA} \qquad (4.4.1)$$

where RA, RP = the magnitudes of the active and passive vehicle position vectors.

E = desired TPI angle (4.4.2)

$\lambda$ = central angle from RA to RP, positive when RP is ahead of RA.

WA, WP = the magnitudes of orbital angular velocities

It should be noted that the ARCCOS term in the above equation always gives rise to 2 solutions. If RP > RA only one such solution is meaningful; however when RA > RP, both solutions are significant. In this case the solution corresponding to the shorter line of sight between vehicles is selected.

Both vehicles are then advanced to the time given by Eq. (4.4.3) and the true line of sight angle rechecked.

$$t_{TPI} = t_{TPI} + TERR \qquad (4.4.3)$$

This process is repeated until the line of sight error becomes less than 0.1 degree. An iteration counter is used to protect against excessive iterations. When there is no TPI time associated with the elevation angle, the counter will serve as a means of initiating a program alarm which indicates that no solution can be found.

(15) Elevation Angle Computation

If a TPI time is one of the inputs, both vehicles are advanced to that time using the Coasting Integration Routine; and the elevation angle, defined in Fig. 4.4-2 which corresponds to the TPI time, is computed as follows:

$$\underline{H}3 = (\underline{r}_{A3} \times \underline{v}_{A3}) \times \underline{r}_{A3}$$

$$\underline{LOS} = \text{UNIT}\,(\underline{r}_{P3} - \underline{r}_{A3})$$

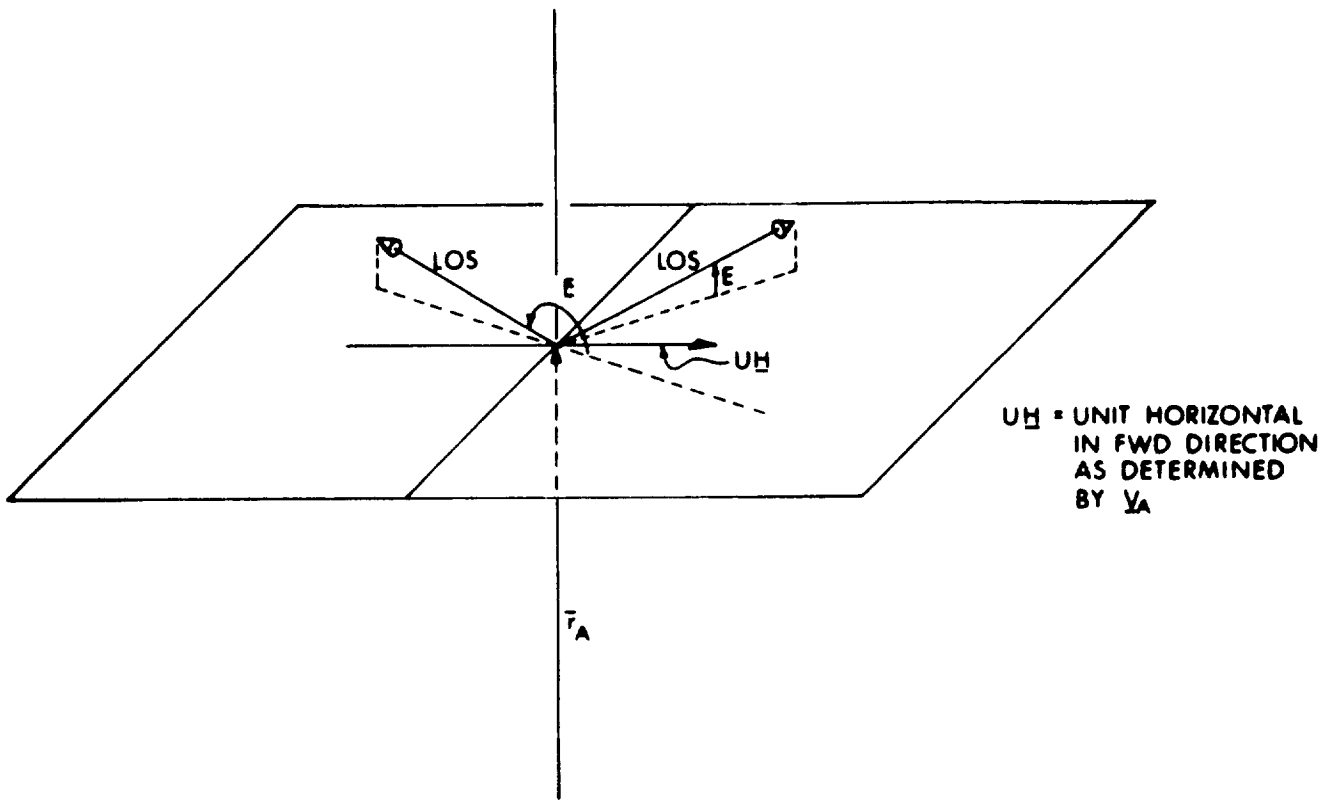
$$\underline{UA}3 = \text{UNIT}\,(\underline{r}_{A3}) \qquad (4.4.4)$$

$$E = \pi/2 - \cos^{-1}(\underline{UA}3 \cdot \underline{LOS})$$

$$\text{If } (\underline{LOS} \cdot \underline{H}3) < 0; \quad E = \pi - E$$

$$XE = |E| - \pi \qquad (4.4.5)$$

$$\text{If } XE > 0; \quad E = XE - \pi \qquad (4.4.6)$$




1) IF THE LOS PROJECTION ON UH IS POSITIVE:

   a) WHEN THE LOS IS ABOVE THE HORIZONTAL PLANE $0 < E < \pi/2$
   b) WHEN THE LOS IS BELOW THE HORIZONTAL PLANE $-\pi/2 < E < 0$

2) IF THE LOS PROJECTION ON UH IS NEGATIVE

   a) WHEN THE LOS IS ABOVE THE HORIZONTAL PLANE $\pi/2 < E < \pi$
   b) WHEN THE LOS IS BELOW THE HORIZONTAL PLANE $-\pi < E < -\pi/2$

Fig. 4.4-2 Definition of Elevation Angle, E

(16) Computation of TPI Maneuver

To compute the TPI Maneuver the Initial Velocity Subroutine (Section 5. 5. 11) is used with the proper switch set to allow for one target offset. Although the offset procedure, as well as the initialization for the Lambert Subroutine, is included in the Initial Velocity Subroutine the offset technique is repeated here for further clarity.

The estimate of both vehicle state vectors along with the $t_F$ of the final transfer (an astronaut input) are used to compute an intercept aimpoint and the maneuver required to arrive at this aimpoint at the desired time. In calculating an offset target to be used by the Lambert Routine (See Section 5. 5. 6) for computing $\Delta \underline{V}_{TPI}$, the effects of non-spherical gravitational perturbations are considered in the following way.

The position vector of the passive vehicle is advanced to the intercept time with the Coasting Integration Routine using the input transfer trajectory $t_F$. The intercept time is given by Eq. (4. 4. 7).

$$t_{TPF} = t_{TPI} + t_F \qquad (4.4.7)$$

The aimpoint for the initial $\Delta \underline{V}$ calculation is the position vector of the passive vehicle at $t_{TPF}$, that is

$$\underline{r}_{AIM_0}(t_{TPF}) = \underline{r}_P(t_{TPF}) \qquad (4.4.8)$$

The passive vehicle velocity vector at the intercept point, $\underline{v}_P(t_{TPF})$, is also obtained from the Coasting Integration Routine and stored.

The state vector of the active vehicle is advanced to $t_{TPF}$ using the Lambert solution, resulting in $\underline{r}_A(t_{TPF})$. Due to non-spherical gravitational perturbations, the active vehicle will not intercept the target. The position deviation from the desired position $\underline{r}_P(t_{TPF})$ is given by

$$\delta \underline{r}_1 (t_{TPF}) = \underline{r}_A (t_{TPF}) - \underline{r}_P (t_{TPF}) \qquad (4.4.9)$$

A new aimpoint is computed for the second iteration, by the following equation:

$$\underline{r}_{AIM_1} (t_{TPF}) = \underline{r}_{AIM_0} - \delta \underline{r}_1 (t_{TPF}) \qquad (4.4.10)$$

A new $\Delta \underline{V}$ is then computed using $\underline{r}_{AIM1}(t_{TPF})$, $t_F$ and $\underline{r}_A(t_{TPI})$. Using the new $\Delta \underline{V}$, the state vector of the active vehicle is again advanced to $t_{TPF}$ to obtain $\underline{r}_A(t_{TPF})$ and $\underline{v}_A(t_{TPF})$.

## (17) Intercept Trajectory Computation

Once a $\Delta \underline{V}_{TPI}$ has been computed the technique explained in Section 5.5.9 is used to determine the pericenter

altitude of the trajectory established by the TPI maneuver. The relative velocity at intercept, $\Delta \underline{V}_{TPF}$ is

$$\Delta \underline{V}_{TPF} = \underline{v}_P(t_{TPF}) - \underline{v}_A(t_{TPF}) \qquad (4.4.11)$$

where $\underline{v}_A(t_{TPF})$ and $\underline{v}_P(t_{TPF})$ are both available from the above computations.

Nominal Outputs (Pre-TPI Mode)

1) Elevation angle or TPI time

2) Magnitude of

   a) $\Delta \underline{V}_{TPI}$

   b) $\Delta \underline{V}_{TPF}$

3) Pericenter altitude after the TPI maneuver.

### 5.4.4.3 Rendezvous Mid-Course Maneuvers (MODE 4)

This program mode corresponds to program P-35 (CSM active) or P-75 (LM active) of Section 4. Normally midcourse maneuvers, and consequently this program, should not be used if the time-to-go to intercept is less than 10 minutes.

Inputs

1) Choice of maneuvering vehicle
   P-35 for CSM
   P-75 for LM

Program Operation

The computational sequence for this program is illustrated in Fig. 4.4-3. The objective of this mode is to maintain an intercept trajectory with midcourse correction maneuvers so that the active vehicle would intercept the passive target vehicle at the time and aimpoint position established in the previous TPI mode.

The midcourse mode of the rendezvous targeting program may be called up by the astronaut at any time after the TPI maneuver. When the program is initiated the number of navigation measurements since the last maneuver and the time to intercept are displayed. Based on this information and additional displays discussed in Section 5.6.7, the astronaut may elect to proceed with a midcourse correction maneuver at any point. When he does so the program computes the midcourse correction which would take place a specified time (A) minutes from the current time. The time delay (A) is specified by the astronaut and is the period required to prepare for a thrusting maneuver. To compute the midcourse correction, the active vehicle is advanced to a point (A) minutes from the present time and the passive vehicle is advanced to the intercept time using the Coasting Integration Routine. The correction required to intercept the passive vehicle is then computed using the techniques described in Section 5.4.4.2.

Nominal Outputs

1) Midcourse Maneuver $\Delta V_{MC}$
2) $\Delta V_{TPF}$ if midcourse maneuver is executed

MODE 4

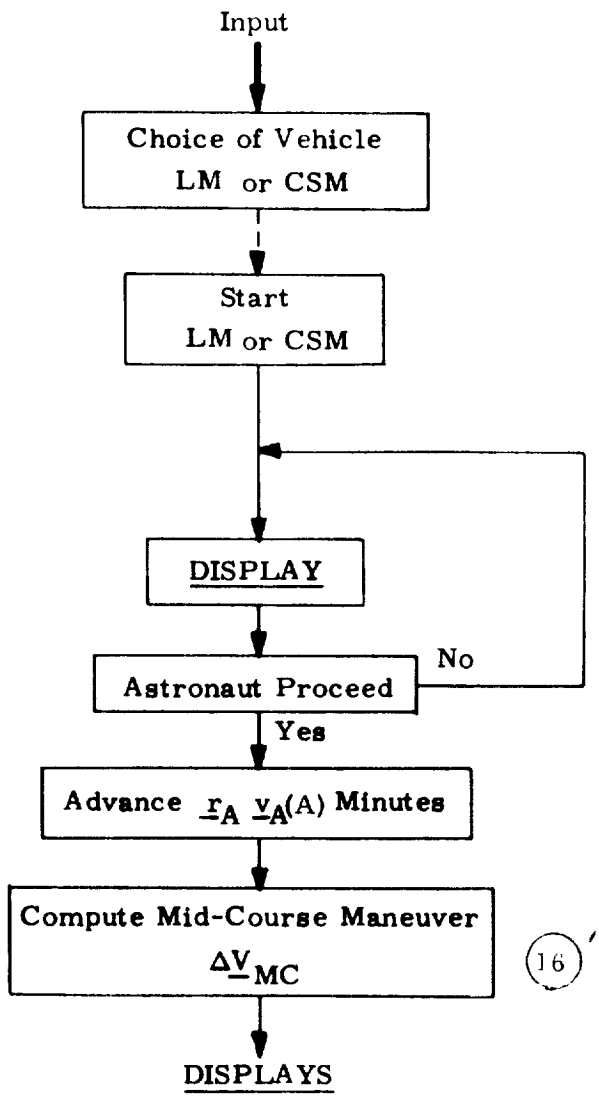



**Fig.** 4.4-3 **Mid-course Correction Mode**

### 5.4.4.4 TPI Search Program

The objective of the TPI Search Program is to determine the minimum total velocity transfer trajectory from a specified TPI maneuver time within the constraint of a safe pericenter. The minimum total velocity determined during the transfer trajectory search function of this routine is the sum of the impulsive velocities for the TPI maneuver and the TPF or ideal rendezvous maneuver. This routine is used to establish the target parameters for a direct intercept trajectory between the CSM and LM orbits controlled by a TPI maneuver. The manual terminal rendezvous maneuver is normally initiated from such an intercept trajectory. The TPI Search Program is used to provide the required TPI maneuver target parameters for either direct transfer (DT) rendezvous profiles or concentric flight plan (CFP) rendezvous TPI maneuvers if $\Delta V$ minimization is desired beyond nominal preselected astronaut input target parameters. This routine can be used in both earth and lunar orbits. The TPI Search Program is actually designated as two separate programs in Section 4. When used as program P-17, the active vehicle which establishes the intercept trajectory is the CSM. When used as program P-77, the active vehicle is the LM.

The following input parameter is required by the TPI Search Program.

1) $t_{TPI}$ desired time of transfer phase initiation (TPI)

The basic output parameters displayed to the astronaut are:

| | |
|---|---|
| 1.) $\pm \theta_0$ | Initial relative phase angle between the two vehicles at $t_{TPI}$, + indicating that the active vehicle is ahead of the passive vehicle with respect to orbital motion. |
| 2.) $\pm \Delta h$ | Altitude difference between the two vehicles at $t_{TPI}$, + indicating that the passive vehicle has a larger radial magnitude than the active vehicle. |
| 3.) K | Preferred central angle search sector. K = + 1 for central angles greater than $180^{\circ}$, and K = - 1 for central angles less than $180^{\circ}$. |
| 4.) $\Delta V_{TPI}$ | Impulsive velocity required for the TPI maneuver. |
| 5.) $\pm \Delta V_{TPF}$ | Impulsive velocity required for the terminal rendezvous maneuver, + indicating that the active vehicle is ascending at TPF, - if descending. |
| 6.) $h_P$ | Altitude of the pericenter above a reference radius vector. |
| 7.) $t_F$ | Transfer time of the intercept trajectory between the TPI and TPF maneuvers. |

In general non-coplanar transfer cases, there will normally be two minimum total $\Delta V$ intercept trajectories, one having a central angle greater than $180^{\circ}$ and the other less than $180^{\circ}$. In the TPI Search Program these two central angle sectors are searched separately. The Program initially selects the sector in which the most acceptable solution will probably be found based upon the relative position conditions existing at the selected TPI time and displays this choice to the astronaut. The astronaut may proceed with this choice or command the other search sector by changing the above output parameter K. Due to the excessive $\Delta V$ requirements for two-impulse transfers with $180^{\circ}$ degree central angles in non-coplanar conditions, the central angle search sectors are limited away from $180^{\circ}$ by an angle depending upon the magnitude of the non-coplanar angle between the active vehicle position vector at TPI and the passive vehicle orbital plane.

In general, TPI Search Program target vectors and relative angles are illustrated in Fig. 4. 4-4. The subscripts A and P refer to the active and passive vehicles, respectively.

## Program Operation

The TPI Search Program Logic diagram is shown in Fig. 4. 4-5. With reference to this figure the following major computation sequences are described:

1. Upon selection and activation of the TPI Search Program the planetary body (earth or moon) is first confirmed by the SETMU Subroutine of Section 5. 5. 10, and the appropriate gravitational constants, $\mu$, and minimum pericenter altitude $h_{PLIM}$ are set for program operation. The following minimum pericenter altitude limits for the transfer trajectory are pre-stored:

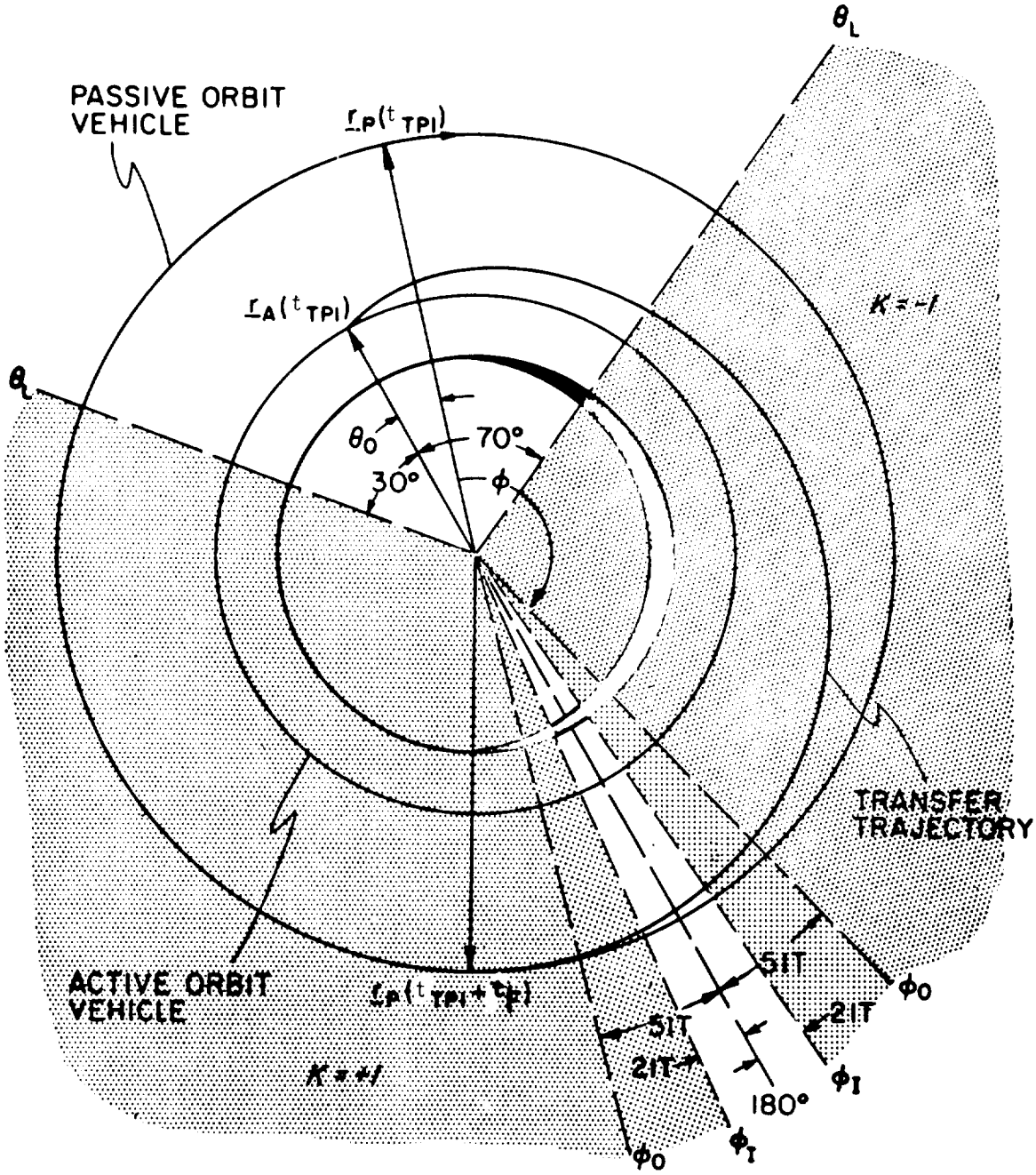



Figure 4.4-4 Central Angle Sectors for the TPI Search Program

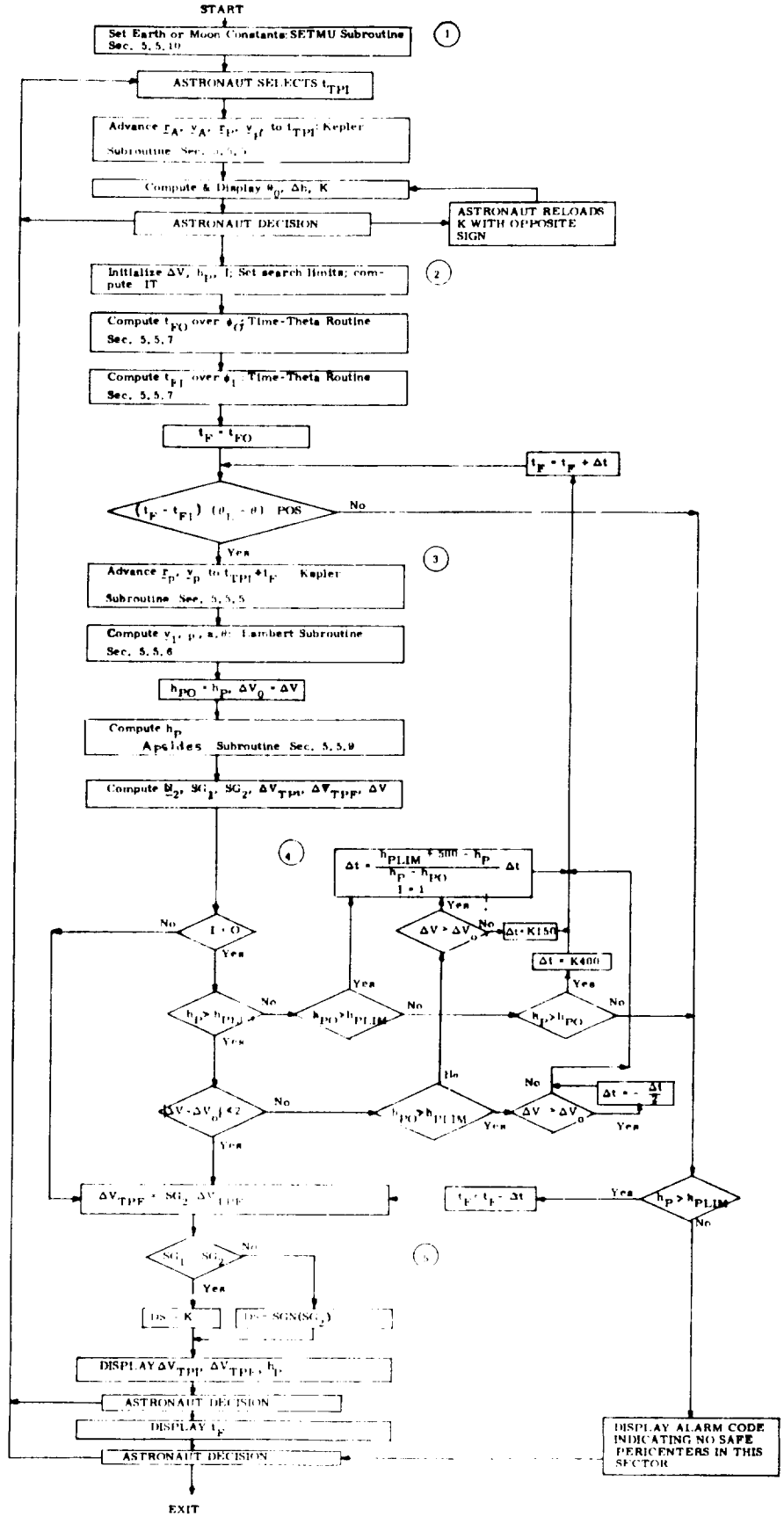





Figure 4.4-5 TPI Search Program Logic Diagram

Lunar Orbit Case : $h_{PLIM}$ = 35,000 ft above the landing site radius magnitude.

Earth Orbit Case : $h_{PLIM}$ = 85 nm above the launch pad radius.

The active and passive vehicle state vectors are next advanced to the specified TPI time by means of the Kepler Subroutine of Section 5.5.5. At this point the relative phase angle $\theta_0$, altitude difference $\Delta h$, and search sector flag K are computed as follows:

$$\underline{u}_{HP} = \text{UNIT}\,(\underline{r}_P \times \underline{v}_P) \tag{4.4.12}$$

$$\underline{r}_{PA} = \underline{r}_A - (\underline{r}_A \cdot \underline{u}_{HP})\underline{u}_{HP} \tag{4.4.13}$$

$$\theta_0 = \text{Sign}\left[(\underline{r}_P \times \underline{r}_A) \cdot \underline{u}_{HP}\right] \cos^{-1}\left[\text{UNIT}\,(\underline{r}_{PA}) \cdot \text{UNIT}\,(\underline{r}_P)\right] \tag{4.4.14}$$

$$\Delta h = \left|\underline{r}_P\right| - \left|\underline{r}_A\right| \tag{4.4.15}$$

$$K = \text{Sign}\left[\Delta h\right] \tag{4.4.16}$$

The parameters $\theta_0$, $\Delta h$ and K are then displayed to the astronaut and a response is requested. The astronaut may proceed, command the initial search to the other sector by changing K, or change the TPI time.

(2.) With reference to Fig. 4.4-5, the program next computes the parameter IT by:

$$IT = \left[ UNIT\,(\underline{r}_A) \cdot \underline{u}_{HP} \right]^{\frac{1}{2}} \qquad (4.4.17)$$

An initial guess $\phi_0$ is then made of the transfer angle of the passive vehicle corresponding to a minimum $\Delta V$ for the active vehicle:

$$\phi_0 = 180^\circ + \theta_0 + K\,5IT \qquad (4.4.18)$$

and is used to start the search iteration. An inner transfer angle limit for the passive vehicle of

$$\phi_I = 180^\circ + \theta_0 + K\,2IT \qquad (4.4.19)$$

is set to avoid excessive $\Delta V$ conditions, and the outer search limits are set as follows:

$$\begin{aligned} \theta_L &= 70^\circ \; (K = -1) \\ \theta_L &= 330^\circ \; (K = +1) \end{aligned} \qquad (4.4.20)$$

The following initial values are next set to start the iterative procedure:

$$\Delta V = 10^5 \text{ (fps)}$$

$$h_P = -10^8 \text{ (ft)} \qquad (4.4.21)$$

$$I = 0$$

The Time Theta Subroutine of Section 5.5.7 is used to determine the passive vehicle transfer time between its position at the selected TPI time and both $\phi_0$ $(t_{FO})$ and $\phi_I$ $(t_{FI})$. The initial time of flight $t_F$ is defined equal to $t_{FO}$.

3. The minimum $\Delta V$ trajectory search begins by advancing the passive vehicle to the initial aim point $t_{TPI} + t_F$ by means of the Kepler Subroutine (Section 5.5.5) and storing the resulting passive vehicle state vector at this time. The Lambert Subroutine of Section 5.5.6 is then used to determine the required velocity $\underline{v}_1$ to establish an intercept trajectory to the aim point with a transit time of $t_F$. This subroutine also provides the trajectory orbit parameters p and a that are later used with the Apsides Subroutine of Section 5.5.9 to determine the transfer trajectory pericenter altitude $h_P$. In order to determine the velocity $\underline{v}_2$ of the active vehicle at the intercept point, the Time-Theta Subroutine (Section 5.5.7) is used. The following computations are then performed:

$$SG_1 = \text{Sign}\left[\underline{r}_A \cdot \underline{v}_A\right] \tag{4.4.22}$$

$$SG_2 = \text{Sign}\left[\underline{r}_P(t_{TPI} + t_F) \cdot \underline{v}_2\right] \tag{4.4.23}$$

$$\Delta V_{TPI} = \left|\underline{v}_1 - \underline{v}_A\right| \tag{4.4.24}$$

$$\Delta V_{TPF} = \left|\underline{v}_2 - \underline{v}_P(t_{TPI} + t_F)\right| \tag{4.4.25}$$

$$\Delta V = \Delta V_{TPI} + \Delta V_{TPF} \tag{4.4.26}$$

(4) The program enters a control sequence whose effect is as follows:

1. If the height of pericenter $h_P$ and its previous value $h_{PO}$ are both less than $h_{PLIM}$ and furthermore $h_P$ is less than $h_{PO}$, an alarm is displayed indicating that there are no safe pericenters in the sector being searched.

2. If, as before, both $h_P < h_{PLIM}$ and $h_{PO} < h_{PLIM}$, but now $h_P > h_{PO}$, the program sets the time of flight increment $\Delta t$ equal to K 400 sec., increments the time of flight, and continues searching.

3. If the pericenter changes from safe to unsafe, the program interpolates to find the time of flight at which the change occurred, and accepts this value as the solution, together with $\Delta V_{TPI}$, $\Delta V_{TPF}$, $h_P$.

4. If the pericenter changes from unsafe to safe, the program also interpolates and accepts the solution provided $\Delta V$ is greater than its previous value $\Delta V_0$. If not it sets $\Delta t = K\ 150$ sec., increments the time of flight, and continues searching.

5. If the present pericenter is safe and $|\Delta V - \Delta V_0| < 2$ the present time of flight is accepted as the solution.

6. If none of the above conditions holds, the time of flight is incremented by $\Delta t$ if $\Delta V < \Delta V_0$ or diminished by $\Delta t / 2$ if $\Delta V > \Delta V_0$, and the search function continues.

7. If the time of flight exceeds the bounds set in (2) a solution is assumed if a safe pericenter exists, otherwise the alarm code is displayed indicating no safe pericenters in the sector being searched.

(5) A sign is affixed to $\Delta V_{TPF}$ and the solution set consisting of $\Delta V_{TPI}$, $\pm \Delta V_{TPF}$, $h_P$ is displayed to the astronaut. The order in which the above parameters are presented on the three display registers (R1, R2, R3) is dependent upon the polarity of the display sequence variable (DS) in Fig. 4.4-5 and indicates to the astronaut whether the pericenter of the transfer trajectory occurs between the TPI and TPF maneuvers (DS positive) or beyond the rendezvous point (DS negative) as follows:

| | |
|---|---|
| R1: $\Delta V_{TPI}$ | |
| $R_2$: $h_P$ | pericenter between TPI and TPF |
| $R_3$: $\pm \Delta V_{TPF}$ | |

| | |
|---|---|
| R1: $\Delta V_{TPI}$ | |
| R2: $\pm \Delta V_{TPF}$ | pericenter beyond rendezvous point |
| R3: $h_P$ | |

If the astronaut is satisfied with these trajectory parameters, he may request that the time of flight $t_F$ be displayed to be recorded as an input parameter for the TPI Pre-thrust program, P-34 or P-74, along with the initial $t_{TPI}$. The other central angle sector may be searched by returning to 1 and changing K, or a new trajectory solution attempted by returning to 1 and changing the input TPI time until a satisfactory transfer trajectory has been found.

### 5.4.4.5 Stable Orbit Rendezvous

The Stable Orbit Rendezvous profile is defined by the following four orbital maneuvers. The first maneuver (initial Stable Orbit TPI) defines a transfer of the active vehicle to a point on the orbit of the passive vehicle, a given distance ahead of, or behind the passive vehicle as measured along the passive vehicle orbital path. The second (Stable Orbit Rendezvous-SOR maneuver) is designed to place the active vehicle in the same orbit as the passive vehicle with a separation between the two vehicles determined by the first TPI maneuver. The position of the active vehicle relative to the passive vehicle after this second maneuver is referred to as the stable orbit point. The third and fourth maneuvers represent the final transfer from the stable orbit point (final TPI) and terminal rendezvous respectively, and are similar to the corresponding maneuvers in the final phase of the concentric rendezvous sequence.

The first two stable orbit rendezvous maneuvers will make use of program P-38 which is essentially the same as P-34 except for the following modifications. If P-38 is used for the initial Stable Orbit TPI maneuver, a special phantom target state vector is generated and used in place of the passive vehicle for targeting and Lambert steering. When P-38 is used for the stable orbit maneuver an ignition time bias is used in order to partially compensate for the finite maneuver time. Both of these modifications are discussed in detail below.

During the transfer to the stable orbit point and also during the final transfer phase a program (see Section 5.4.4.5.2) is available for computing any midcourse corrections required to intercept the target point.

All the orbital calculations required for the targeting in this program use the Coasting Integration Routine, a precision technique. The pre-thrust programs for the Stable Orbit Rendezvous (P-38, P-78, P-39, P-79) use the Lambert targeting concept with one offset aimpoint (see Section 5.4.4.2 and 5.5.6) and the cross - product steering of Section 5.3.3.

5.4.4.5.1 Stable Orbit TPI Program

The Stable Orbit TPI Program has two phases and is used to perform the first and the second maneuvers described above. This program corresponds to P-38 or P-78 of Section 4. The logic flow is shown in Fig. 4.4-6.

Input Parameters

1) Choice of maneuvering vehicle
   (P-38 CSM, P-78 LM)

2) Ignition Time of the maneuver, $t_{TPI}$

3) $t_F$, Time of flight of transfer trajectory

4) Program Phase

   PHASE = 1 TPI maneuver

   PHASE = 2 SOR maneuver

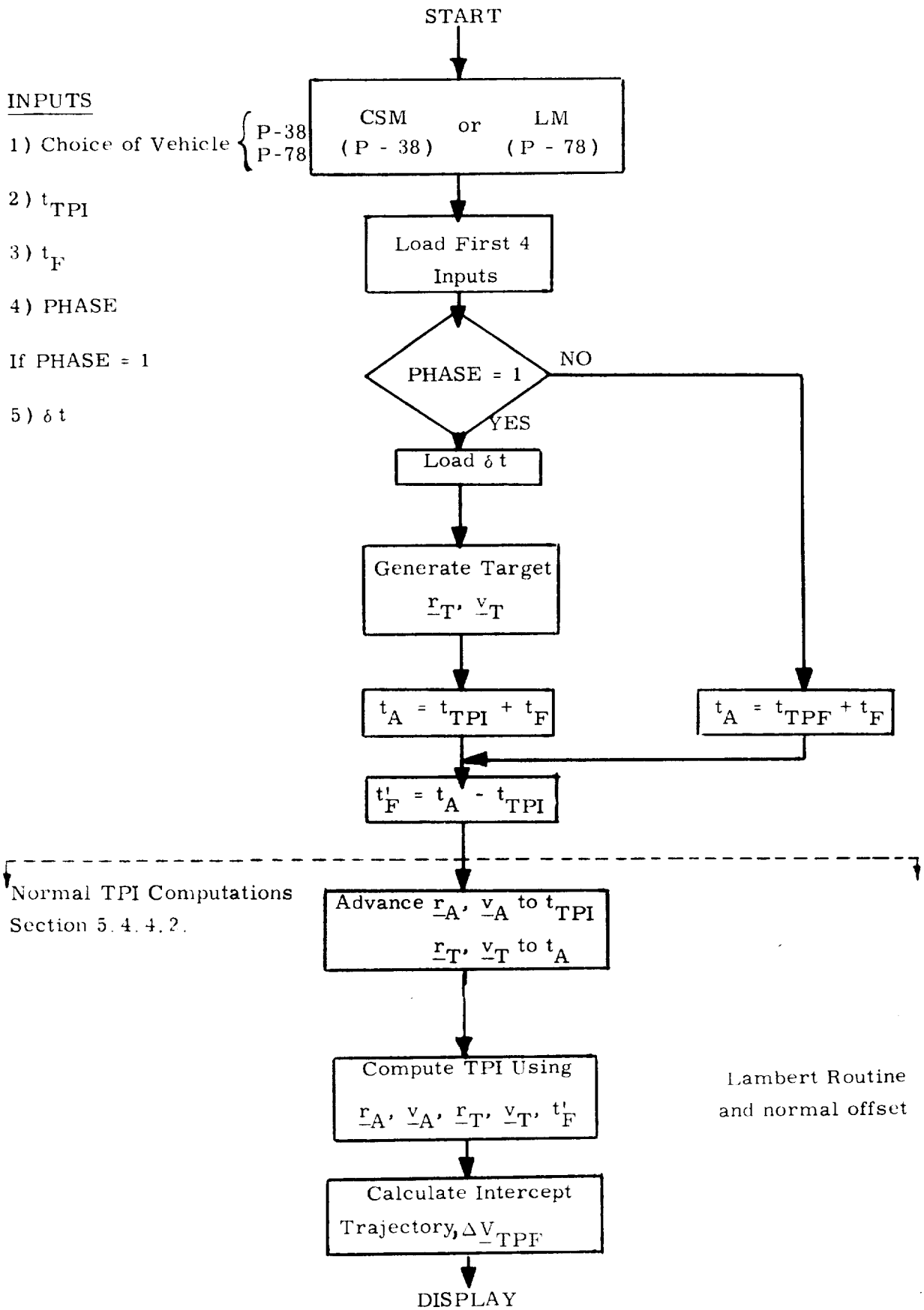



Figure 4.4-6 Stable Orbit Rendezvous Program (TPI and SOR Modes)

If PHASE = 1, the following parameter (5) is also a required input.

5) $\delta t$, The offset of the stable orbit point specified as the time required for the passive vehicle to traverse the desired offset distance. Positive $\delta t$ indicates that the stable orbit point is ahead of the passive vehicle, negative $\delta t$ behind the passive vehicle.

Program Operation

If the program is used for the first TPI transfer (PHASE = 1) a phantom or offset target is generated using precision orbit integration. This offset state vector is displaced from the passive vehicle an amount specified by the $\delta t$ input and represents a target in the same orbit as the passive vehicle. In both phases this program uses the phantom state vector to compute a transfer maneuver using the same logic as that in the pre-TPI maneuver targeting (Section 5.4.4.2). The portion of Fig. 4.4-6 below the dotted line is completely analogous to the targeting of the pre-TPI maneuver.

When this program is used for the second (SOR) maneuver in the stable orbit rendezvous sequence (PHASE = 2), the time of ignition is offset from the time corresponding to an impulsive maneuver. This offset is computed by the astronaut and the time of ignition, $t_{TPI}$ is adjusted accordingly. In order to maintain the targeting which corresponds to an impulsive maneuver and thus provide an ignition bias, the time of arrival, $t_A$, required for the definition of the intercept trajectory is based on the impulsive maneuver time, $t_{TPF}$, which was computed in the PHASE = 1 operation of this program. The logic and equations required for this ignition offset are included in Fig. 4.4-6, and the definitions of the quantities used are presented in Figure 4.4-7.

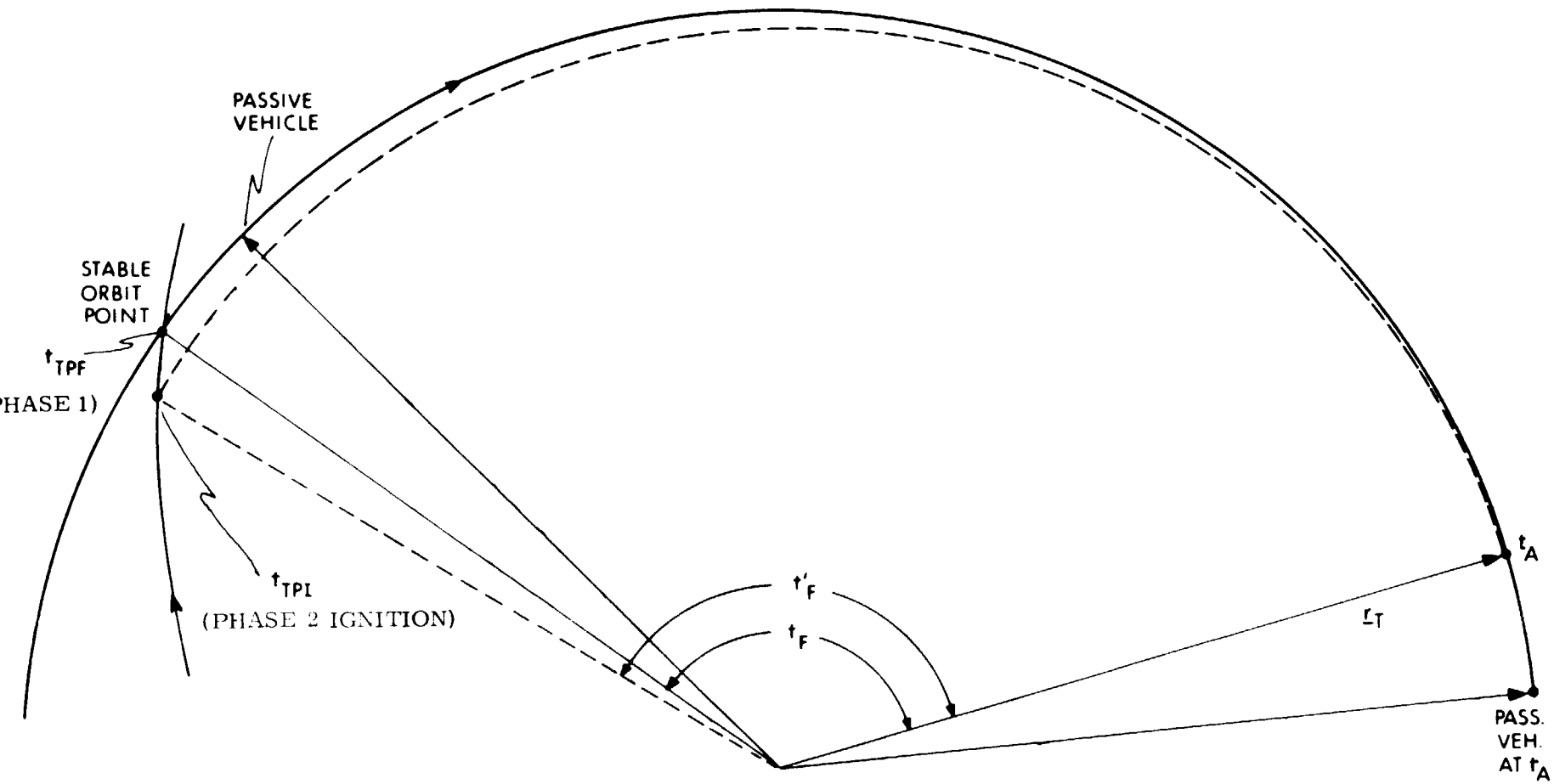



Figure 4.4-7 Stable Orbit Rendezvous TPI - Phase 2

After the state vector of the stable orbit point has been matched by the active vehicle, the PHASE = 1 mode of this program may be used again to perform the final transfer to rendezvous by setting $\delta t = 0$. Alternately, P-34 or P-74 may be used to perform the TPI maneuver included in the concentric rendezvous sequence. In either case the final braking and docking maneuvers will be accomplished manually.

Outputs

1) $\Delta V$ of maneuver $= \begin{cases} \Delta V_{TPI} & \text{PHASE} = 1 \\ \Delta V_{SOR} & \text{PHASE} = 2 \end{cases}$

2) $\Delta V_{TPF}$ = Relative velocity at target intercept (approximately zero for PHASE = 2)

3) Pericenter altitute after maneuver

5.4.4.5.2 Stable Orbit Midcourse

Input Parameters

1) Choice of maneuvering vehicle (P-39 CSM, P-79 LM)

Program Operation

This program has the same function and follows the same logic as P-35 or P-75 described in Section 5.4.4.3. except that the target used to compute the midcourse maneuver is the offset phantom state vector used in the computation of the Stable Orbit TPI or SOR maneuvers. (Section 5.4.4.5.1)

Outputs

1) Midcourse maneuver, $\Delta V_{MC}$

2) $\Delta V_{TPF}$ if midcourse maneuver is performed.

## 5.4.5 RETURN TO EARTH TARGETING

### 5.4.5.1 Introduction

The Return to Earth Targeting Program provides to the astronaut the ability to compute on board the spacecraft the targeting for a powered maneuver which returns the spacecraft to an earth reentry point with proper conditions for a safe reentry. This program is intended as a backup in the event of a communication or ground system failure and, thus, is completely independent of inputs from the ground system. The method of computation, however, does not compromise certain critical trajectory characteristics such as fuel usage and transit time for the expediency of minimum computer storage usage or computational speed.

Specifically this program provides powered maneuver targeting for

(1) Returning from near earth orbits

(2) Returning from trajectories resulting from a translunar injection powered maneuver failure.

*(3) Returning from translunar midcourse

**(4) Returning, while in the moon's sphere of influence, from trajectories resulting from lunar orbit insertion or transearth injection powered maneuver failures

**(5) Returning from nominal lunar orbits

---

* Outside moon's sphere of influence only

** This capability has been removed from this program.

*(6) Returning from transearth midcourse

*(7) Midcourse corrections during transearth midcourse

*(8) Changes in transit time during transearth midcourse

Two modes of operation are provided: time critical and fuel critical. The former establishes trajectories which return to earth in a minimum of time, the latter, with a minimum of fuel.

The return trajectory is constrained to be in the plane which the spacecraft was in prior to the return so that optimum fuel utilization will be achieved.

Although explicit control of the spacecraft landing site is not a feature of this program, it is possible to achieve landing site selection through adjustment of the astronaut input to the program. When on translunar or transearth trajectories, the adjusted parameter is the desired fuel utilization. When in orbit near the earth, the designated return time is the adjusted parameter. The astronaut need only to note the earth track of the landing site as a function of the adjusted parameter to select the proper parameter value.

The computations take place in two steps. The first is the generation of a conic two body solution to the problem. This takes a relatively short computational time and the solution characteristics are nearly the same as those of the precise solution. Once the astronaut is satisfied with the conic return trajectory he can allow the second step to proceed, the computation of a precise trajectory which takes a relatively long computational time.

The Safe Perilune Targeting Program utilizes essentially the same computational procedures as the Return to Earth Program. This program first determines the perilune of the present spacecraft trajectory. If it is not satisfactory the astronaut may select a more desirable perilune altitude. The Safe Perilune Program will then establish the targeting information for a trajectory which has the desired perilune altitude and minimizes the required velocity change.

In the following discussion the Program Input-Output is discussed first. Next, a discussion of the overall general computation procedure is presented, followed by a more detailed discussion of the program logic. Then the detailed computations are presented.

5.4.5.2 Program Input-Output

The Return to Earth Program may be called by the astronaut at any time the computer is operable. Should the designated point of return place the spacecraft at a point in space where the program is incapable of producing a successful solution, an alarm will occur. The current calling designation for this program is P-37 for all modes of Return to Earth, and P-70 for Safe Perilune.

Return-to-Earth Targeting

After calling the program, the astronaut must load the following information into erasable memory via the keyboard.

(1) objective velocity change to be utilized during the maneuver

(2) time the maneuver is to be executed

(3) reentry angle (normally set to zero and internally calculated*)

Should a minimum fuel return be desired, the objective velocity change should be set to zero. Should the objective change be less than the minimum required, the program will select the minimum. When the objective velocity change is greater than the minimum, the return will require that amount of velocity change.

The program provides the following displays when both the conic trajectory computation and the precision trajectory computation are complete.

(1) latitude of the landing site

(2) longitude of the landing site

(3) velocity change required (vector)

---

*See 5.4.5.4.2 for detailed explanation of its use

(4) velocity magnitude at 400,000 ft. entry altitude

(5) flight path angle at 400,000 ft. entry altitude

(6) transit time to 400,000 ft. entry altitude from ignition

The astronaut must select either the SPS or RCS engine after the precision trajectory calculation. The following quantities are displayed after the engine selection:

(7) middle gimbal angle at ignition

(8) time of ignition

(9) time from ignition

The precision trajectory displays will not be significantly different from the conic displays; however, the astronaut should check the new displays before accepting the precise solution.

Safe Perilune Targeting

Input is not required when the Safe Perilune Program P-70 is first called by the astronaut. The first set of displays are as follows:

(1) perilune altitude of the present trajectory

(2) minimum time of ignition*

(3) time to perilune

The astronaut must now decide to end the program or proceed. If he proceeds he must first input via the keyboard the following:

(1) desired perilune altitude

(2) desired time of ignition

The program displays the following precision target trajectory characteristics.

---

*This will be present time if the spacecraft is inside the moon's sphere of influence and sphere arrival time if the spacecraft is outside the sphere.

(1) velocity change required (vector)

(2) transit time from ignition to perilune

The astronaut must select either the RCS or SPS engine after this display. Upon completion the program will display the following quantities.

(3) middle gimbal angle at ignition

(4) time of ignition

(5) time from ignition

5.4.5.3 General Computational Procedure

This section contains a general description of the Return-to-Earth computational procedure as illustrated in Figs. 4.5-1, 4.5-2, and 4.5-3.

The Return to Earth Targeting Program, as previously mentioned, may be used in either the fuel critical or time critical mode. When used in the fuel critical mode, the program generates a trajectory which meets specified constraints on reentry flight path angle and reentry altitude, and minimizes the impulsive velocity change required to achieve this trajectory. In the time critical mode, the program generates a trajectory which meets the specified reentry constraints and returns the spacecraft in the shortest possible time. The astronaut selects either mode by entering via the keyboard the objective velocity change ($\Delta v_D$). If $\Delta v_D$ is set to zero, the routine will provide a fuel critical trajectory. If $\Delta v_D$ is not set to zero, the routine will provide a time critical trajectory if possible, or select a fuel critical trajectory if $\Delta v_D$ is less than the minimum required impulsive velocity change.

The Return to Earth Targeting Program has two major phases, an initial two body conic phase illustrated in Fig. 4.5-1 and a numerically integrated precision trajectory phase illustrated in Fig. 4.5-2. A conic routine which provides initial conditions for return trajectories for both phases is illustrated in Fig. 4.5-3.

5.4.5.3.1 Nomenclature

The following is a list of the nomenclature used in Figs. 4.5-1, 4.5-2, and 4.5-3

| | |
|---|---|
| $\underline{y}(t_0) = (\underline{r}(t_0), \underline{v}(t_0))$ | CSM state vector maintained in the CMC |
| $\underline{y}_1(t_1) = (\underline{r}(t_1), \underline{v}_1(t_1))$ | pre-return state vector |
| $\underline{y}_2(t_1) = (\underline{r}(t_1), \underline{v}_2(t_1))$ | post-return state vector |
| $\underline{y}(t_2) = (\underline{r}(t_2), \underline{v}(t_2))$ | state vector at reentry |
| $\underline{y}(t_2)_{PRE}$ | final state vector of a precision trajectory |
| $R_{CON}$ | final radius of a conic trajectory |
| $R_{PRE}$ | final radius of a precision trajectory |
| $R_D$ | desired final radius (reentry radius) |
| $\gamma(t_2)_{CON}$ | final flight path angle of a conic trajectory |
| $\gamma(t_2)_D$ | desired final flight path angle |
| $\gamma(t_2)_{PRE}$ | final flight path angle of a precision trajectory |
| $x(t_1)$ | cotangent of the post-return flight path angle |
| $\Delta v$ | impulsive velocity change |
| $\Delta v_D$ | desired velocity change |
| $\epsilon_1$ to $\epsilon_{10}$ | tolerances |

| | |
|---|---|
| $t_{21}$ | time of flight $(t_2 - t_1)$ |
| $n_1$ | counter |
| $f_1$ | flag set to $\begin{cases} 0 \text{ initial conic phase} \\ 1 \text{ precision trajectory phase} \end{cases}$ |

5.4.5.3.2 Discussion of the Conic Phase

The following discussion is intended to supplement the information contained in Fig. 4.5-1.

The pre-return CSM state vector $(\underline{y}_1(t_1))$ is obtained by numerical integration. An initial conic trajectory is then generated which meets the reentry and velocity change constraints of the problem. The reentry radius and flight path angle constraints, however, are known only as a function of the latitude and velocity magnitude at reentry. Therefore an iterative procedure is employed to generate this initial conic trajectory. Based on the pre-return radius magnitude, an estimate of the reentry constraints is made. This initial estimate is used to compute a conic trajectory which satisfies the $\Delta v_D$ requirement. The final velocity magnitude and latitude are now available for use in a more precise calculation of the reentry constraints. An iterative process has now been started which quickly coverges on a suitable initial conic trajectory.

During the above iterative process slight changes in the reentry flight path angle can produce significant changes in the minimum required velocity change. If the time critical mode is being used, it is possible that during a particular pass through the iteration loop the velocity change desired may be less than the minimum required. This does not necessarily preclude a time critical solution, and therefore the iteration is continued using the minimum required velocity change.

A description, in general terms, of the procedure CONICRETURN, used to obtain the initial state vector of the conic return trajectory, will follow later in this section.

After an initial conic trajectory is computed, an estimate is made of the landing site, and the results are displayed. Based on this display the astronaut may elect to continue with the calculation of a numerically integrated trajectory.

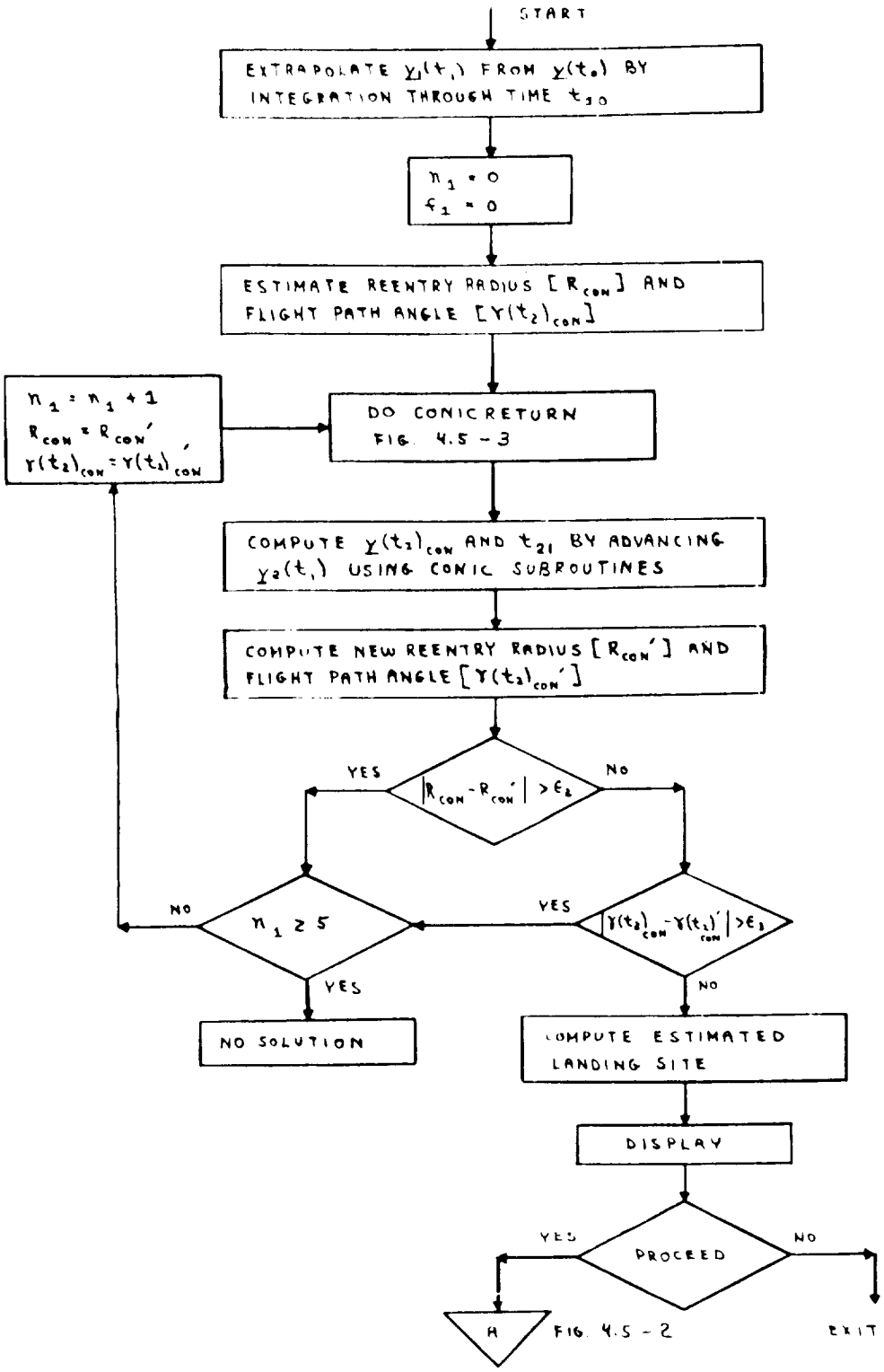



FIGURE 4.5-1 SIMPLIFIED LOGIC DIAGRAM OF RETURN TO EARTH TARGETING

5.4.5.3.3 Discussion of the Precision Phase

The following discussion is intended to supplement the information contained in Fig. 4. 5-2.

The numerically integrated trajectory must satisfy the reentry constraints achieved by the initial conic trajectory and meet the $\Delta v_D$ requirement as closely as possible. An iterative procedure is again employed to generate this trajectory. The post-return state vector of the previous conic trajectory is used to initialize the Coasting Integration Routine. The integration proceeds until the desired reentry flight path angle has been achieved. The reentry radius of the next conic trajectory is then offset to compensate for the error in the reentry radius of the numerically integrated trajectory. An iterative process has now been initiated which converges to the precise trajectory which meets the objective velocity change and the reentry constraints. Next a target is computed for the Lambert Aimpoint Powered Flight Routine, the landing site is calculated, and the results are displayed.

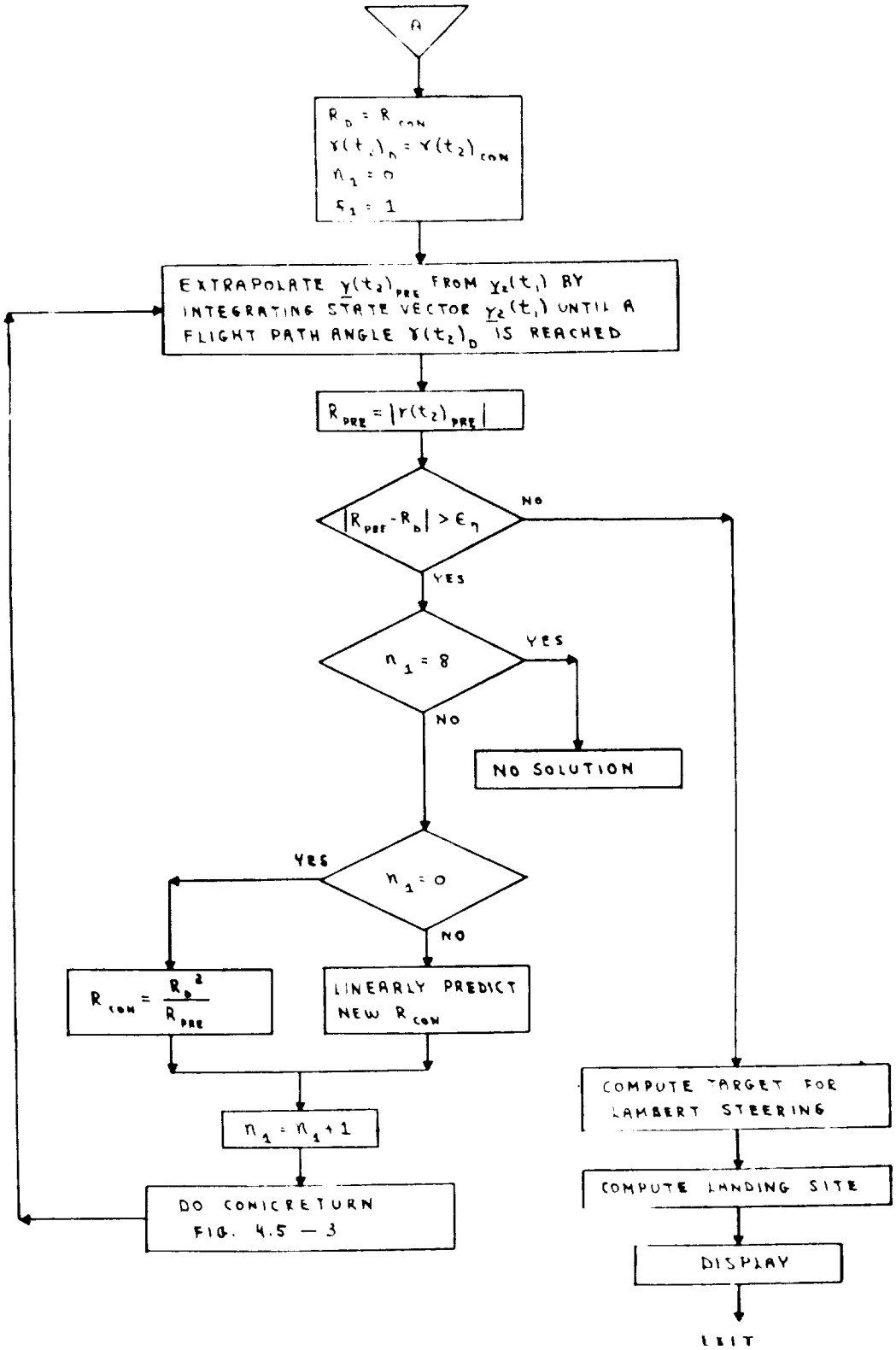



FIGURE 4.5-2 SIMPLIFIED LOGIC DIAGRAM OF RETURN TO EARTH TARGETING

5.4.5.3.4 Discussion of the Conic Routine

The following discussion is intended to supplement the information contained in Fig. 4.5-3.

The conic routine solves the following two conic problems.
(1) Given an initial position vector ($\underline{r}(t_1)$), a pre-return velocity vector ($\underline{v}_1(t_1)$), a final flight path angle ($\gamma(t_2)_{CON}$), and a final radius magnitude ($R_{CON}$), compute a post-return velocity vector ($\underline{v}_2(t_1)$) which minimizes the velocity change ($\Delta v$).
(2) Given the above quantities, compute a post-return velocity vector ($\underline{v}_2(t_1)$) which meets the objective velocity change ($\Delta v_D$).

An iterative procedure is again used. The independent variable of the procedure is the cotangent of the post-return flight path angle ($x(t_1)$), and the dependent variable is the velocity change ($\Delta v$).

Initial bounds on $x(t_1)$ are computed based on the maximum allowable semi-major axes. The upper bound constrains return trajectories which pass through apogee from going too near the moon, and the lower bound keeps the reentry velocity for direct return flights from exceeding the maximum allowable reentry velocity.

During the second phase of the Return to Earth Targeting Program, the calculation of a numerically integrated trajectory, additional bounds are imposed on the independent variable $x(t_1)$. In the case of a return from earth orbit, the resulting trajectory may be nearly circular, and will pass through the desired reentry flight path angle twice at nearly the same radius. Additional limits on $x(t_1)$ are computed to prevent conic trajectories generated during the search for a numerically integrated trajectory from oscillating between these two solutions.

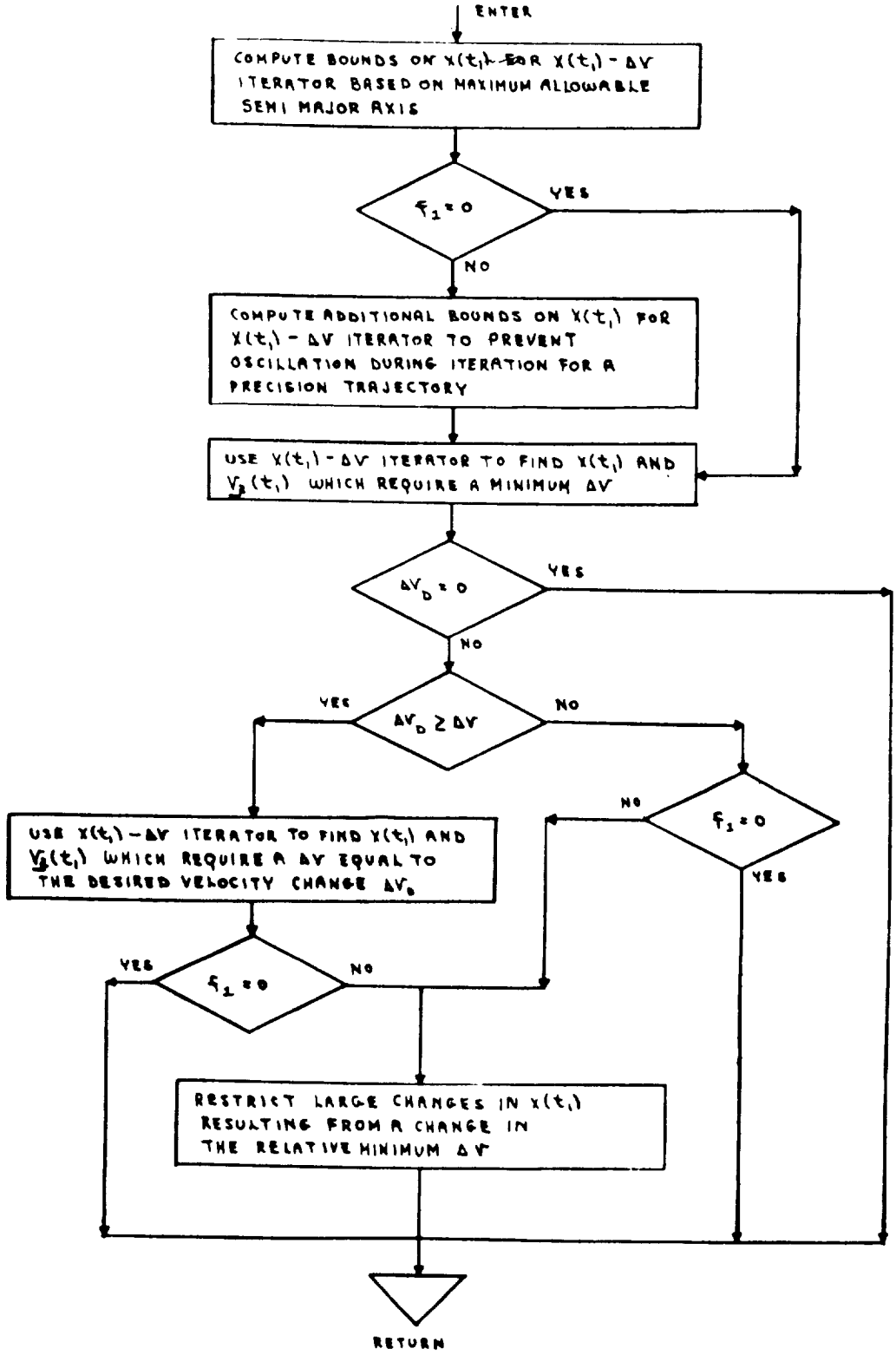



FIGURE 4.5-3 SIMPLIFIED LOGIC DIAGRAM OF RETURN TO EARTH TARGETING - CONIC RETURN ROUTINE

In all cases the $x(t_1)$ - $\Delta v$ iterator is used to compute a post-abort velocity vector which requires a minimum $\Delta v$. In the event that a time critical answer is required, another iteration using the $x(t_1)$ - $\Delta v$ iterator will be made to compute a post-return velocity vector which requires a $\Delta v$ equal to $\Delta v_D$. If the velocity change desired is less than the minimum required, the second iteration will be omitted.

During the calculation of the numerically integrated trajectory, the intermediate conics are additionally restricted to prevent large changes in the post-return flight path angle. This additional restraint is necessary in the time critical mode for certain near earth returns because both absolute and relative minimum $\Delta v$ solutions exist.

### 5. 4. 5. 4 Logical Description

This section contains logical flow diagrams of the outside the sphere portion of the Return to Earth Targeting Program. Some items not fully discussed in the text are explicitly illustrated in the diagrams. Certain self contained procedures are detailed in the next section. It should be noted that the Safe Perilune Targeting Program is integrated into the Return to Earth Targeting Program. The minor differences between the two programs are shown by branches based on the variable $\phi_1$.

5.4.5.4.1 Nomenclature

A list of the nomenclature used in this section and not previously defined follows.

| | |
|---|---|
| $t_{IG}$ | desired ignition time |
| $t_{IGMIN}$ | minimum ignition time |
| $t_{21}$ | time of flight $(t_2 - t_1)$ |
| $p(t)$ | primary body $\begin{cases} \pm 1 \text{ for earth} \\ \pm 2 \text{ for moon} \end{cases}$ |
| $x(t_1)_{LIM}$ | limit on the cotangent of the post-return flight path angle |
| $x(t_1)_{MAX}, x(t_1)_{MIN}$ | upper and lower bounds on the cotangent of the post-return flight path angle |
| $\Delta x(t_1)$ | change in $x(t_1)$ |
| $\Delta x(t_1)_{MAX}$ | maximum allowed change in $x(t_1)$ |
| $x(t_2)$ | final cotangent of the flight path angle |
| $x(t_2)_{PRE}$ | final cotangent of the flight path angle for a precision trajectory |
| $p_{CON}$ | semi-latus rectum of a conic trajectory |
| $R_{ERR}$ | precision trajectory reentry radius error |

| | |
|---|---|
| $\mu$ | gravitation constant |
| $r_P$ | radius of perilune |
| $r_F$ | minimum final radius |
| $\Lambda_{MAX1}, \Lambda_{MAX2}$ | maximum major axes |
| $\underline{n}$ | normal vector |
| $\phi_1$ | $\begin{cases} 0 \text{ Return to Earth} \\ 1 \text{ Safe Perilune} \end{cases}$ |
| $\phi_2$ | $\begin{cases} +1 \text{ near apogee solution} \\ -1 \text{ near perigee solution} \end{cases}$ |
| $\phi_3$ | switch (Safe Perilune) |
| $f_2$ | iterator mode flag |
| $n_1, n_2$ | counters |
| $\beta_1$ to $\beta_{11}$ | intermediate variables |

5.4.5.4.2 Discussion of the Logical Flow Diagram

The following discussion is intended to supplement the information contained in the referenced diagrams.

Three inputs are required in addition to the CSM state vector (Fig. 4.5-4). They are the objective velocity change desired ($\Delta v_D$), the desired time of ignition ($t_{IG}$), and the reentry angle ($\gamma(t_2)_D$). The reentry angle should nominally be set to zero, allowing the program to compute a reentry angle in the center of the reentry corridor. The astronaut may select the reentry angle if a minimum $\Delta v$ solution lying on the edge of the reentry corridor is required. This may be necessary in the event the reaction control system is used for the return. See Fig. 4.5-15 for a detailed explanation of the reentry angle calculations.

The Coasting Integration Routine is used to advance the state vector to the desired return time. A check is made to insure that the state vector is not inside the moon's sphere of influence. If the state vector outside the sphere of influence, precomputations are executed. They are explained in detail later. A conic trajectory is then generated which satisfies the estimated reentry constraints and $\Delta v_D$ requirements. The procedure CONICRETURN, which computes the initial state vector of this conic, will be discussed later. The Time Radius Subroutine, described in Section 5.5.8, is used to compute the time of flight to the desired radius and advance the state vector through that time of flight.

Based on this final state vector (Fig. 4.5-5), a new reentry radius ($R_{CON}'$) and a new value for the cotangent of the reentry flight path angle ($x(t_2)'$) can be computed. If they are sufficiently close to the previously used values, the conic trajectory is accepted. Otherwise a new conic is computed which satisfies the new reentry constraints. A counter is employed to protect against an excessive number of iterations.

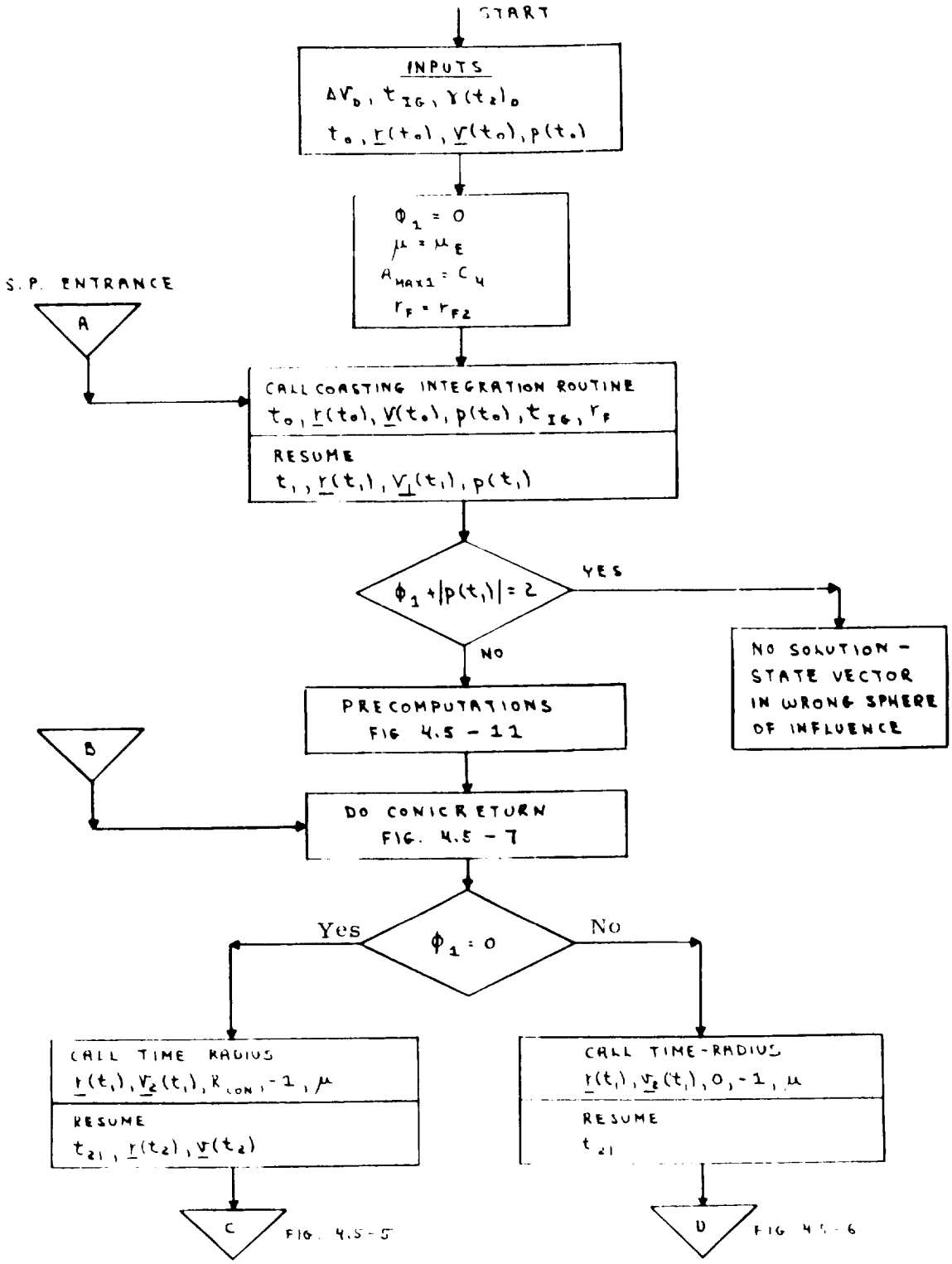



FIGURE 4.5-4 LOGICAL DIAGRAM OF RETURN TO EARTH TARGETING

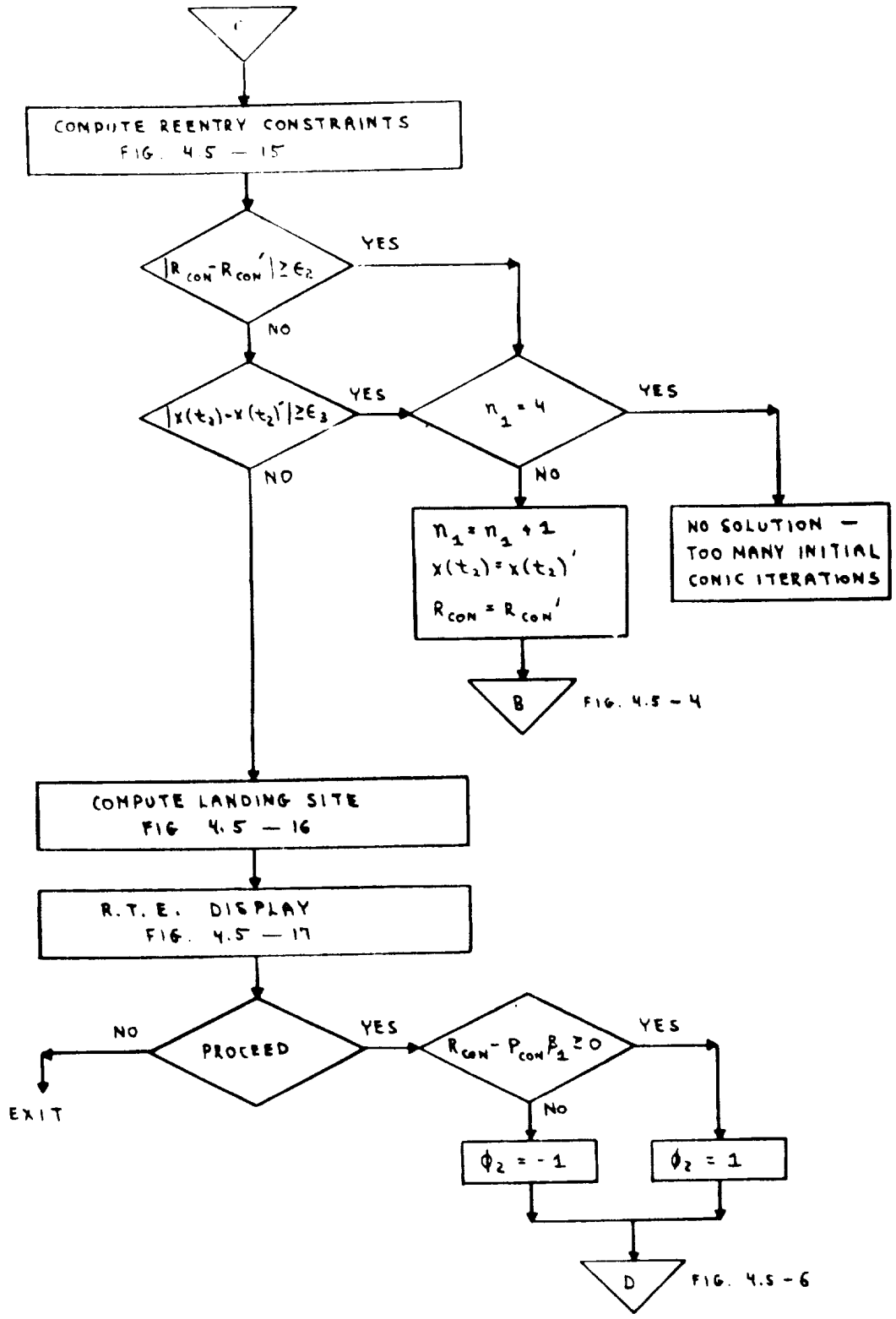



FIGURE 4.5-5 LOGICAL DIAGRAM OF RETURN TO EARTH TARGETING

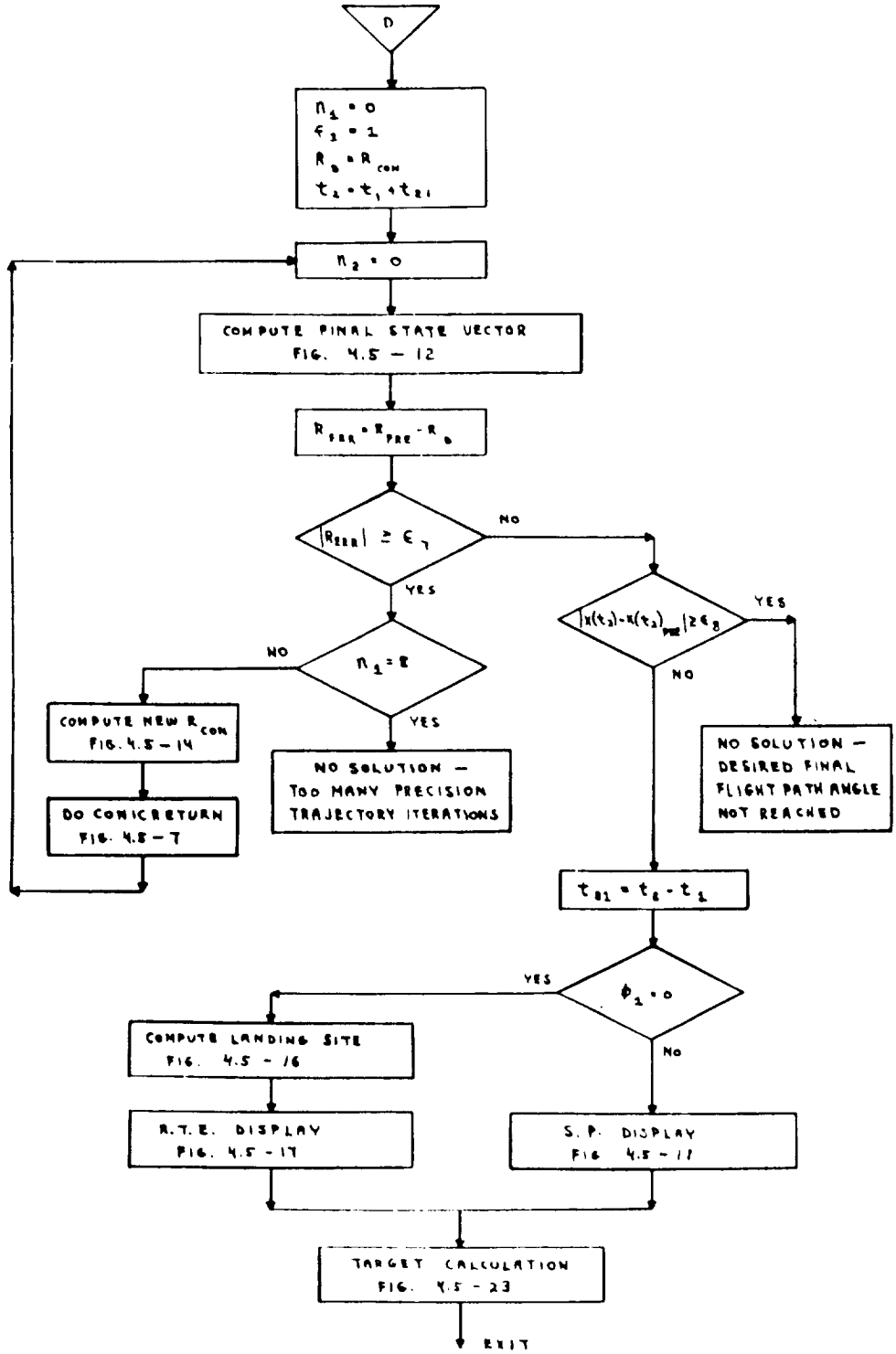



FIGURE 4.5-6 LOGICAL DIAGRAM OF RETURN TO EARTH TARGETING

If the initial conic is accepted, an estimate is made of the latitude and longitude of the landing site. For long flight times, however, differences in time of flight between the conic and numerically integrated trajectory will cause an error in the conically estimated landing site longitude. The results of the preceding calculations are displayed, and based on this information the astronaut may elect to proceed with the calculation of a numerically integrated trajectory or exit the program. As explained in Section 5. 4. 5. 3, return trajectories from near earth orbits may be nearly circular, and therefore pass through the desired flight path angle twice with approximately the same radius. The unique solution reached by the initial conic is tested to determine whether the final state vector was achieved during the first or second pass through the desired reentry flight path angle. A switch $\phi_2$ is set appropriately, and with one exception all succeeding precision and conic trajectories are required to behave in a similar manner. The exception will be explained in detail in Section 5. 4. 5. 4. 3.

The post-return state vector of the conic trajectory is now advanced until the desired flight path angle of reentry is reached (Section 5. 4. 5. 5 ). A radius error will exist between the desired final radius ($R_D$) and the final radius magnitude of the numerically integrated trajectory. If this error is small, the precision trajectory is accepted. Otherwise a new conic trajectory is generated which uses an offset final radius magnitude. If the error in the final radius is small, a final check is made to insure that the reentry flight path angle is acceptable.

5.4.5.4.3 Discussion of Logical Flow Diagram - CONICRETURN

The procedure CONICRETURN, illustrated in Figures 4.5-7 and 4.5-8, is used to generate initial state vectors for both the Return to Earth Targeting Program and the Safe Perilune Program. Equations required to initialize CONICRETURN each time it is used are described in Section 5.4.5.5. The conic problems solved by the routine were described in Section 5.4.5.3.4.

The logical structure shown in Figs. 4.5-7 and 4.5-8 is based in the following facts.

(1) There is only one minimum if the pre-return velocity vector has a negative radial component.

(2) There may be two minima if the pre-return velocity vector has a positive radial component. In this case the absolute minimum post-return velocity vector has a positive radial component and the relative minimum has a negative component.

As explained in Section 5.4.5.3.4 and 5.4.5.4.2, it is necessary to compute additional bounds on $x(t_1)$ during the search for a precision trajectory. The logic used to do this is shown on the lower portion of Fig. 4.5-7.

A check is made to determine whether the locus of possible solutions includes trajectories which achieve the radius $R_{CON}$ and the flight path angle $\gamma(t_2)$ near apogee, and if so the variable $\beta_6$ will be positive*.

*With one exception every conic solution generated by this routine achieves the desired reentry flight path angle ($\gamma(t_2)$) twice during the passage from apogee to perigee. The first time $\gamma(t_2)$ is reached during the passage from apogee to perigee will be referred to as near apogee, and the second time will be referred to as near perigee. The desired reentry angle will be achieved only once if $\gamma(t_2)$ equals the maximum flight path angle of the conic.

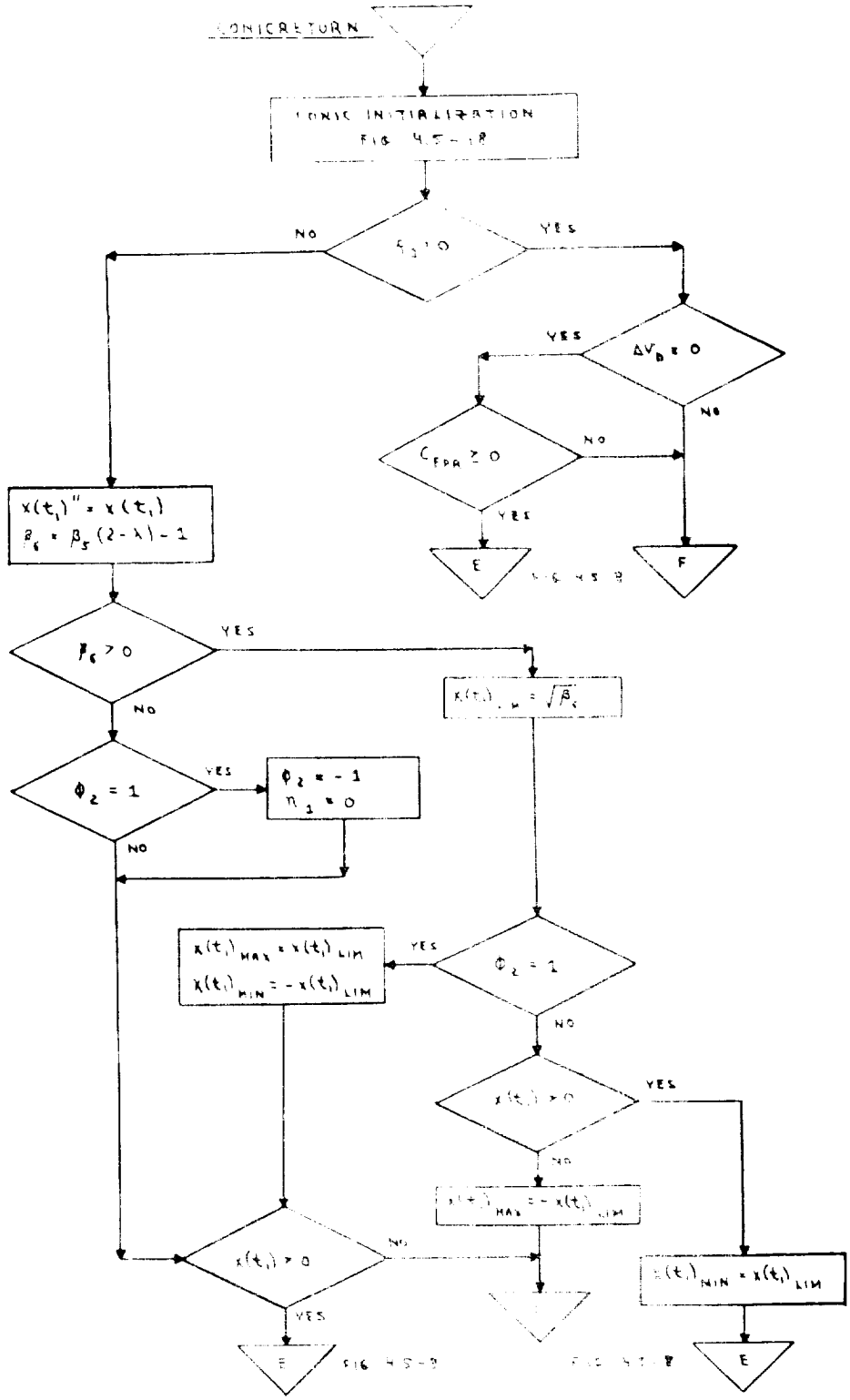



FIGURE 4.5-7 LOGICAL DIAGRAM OF [illegible] TO EARTH TARGETING - CONIC RETURN [illegible]

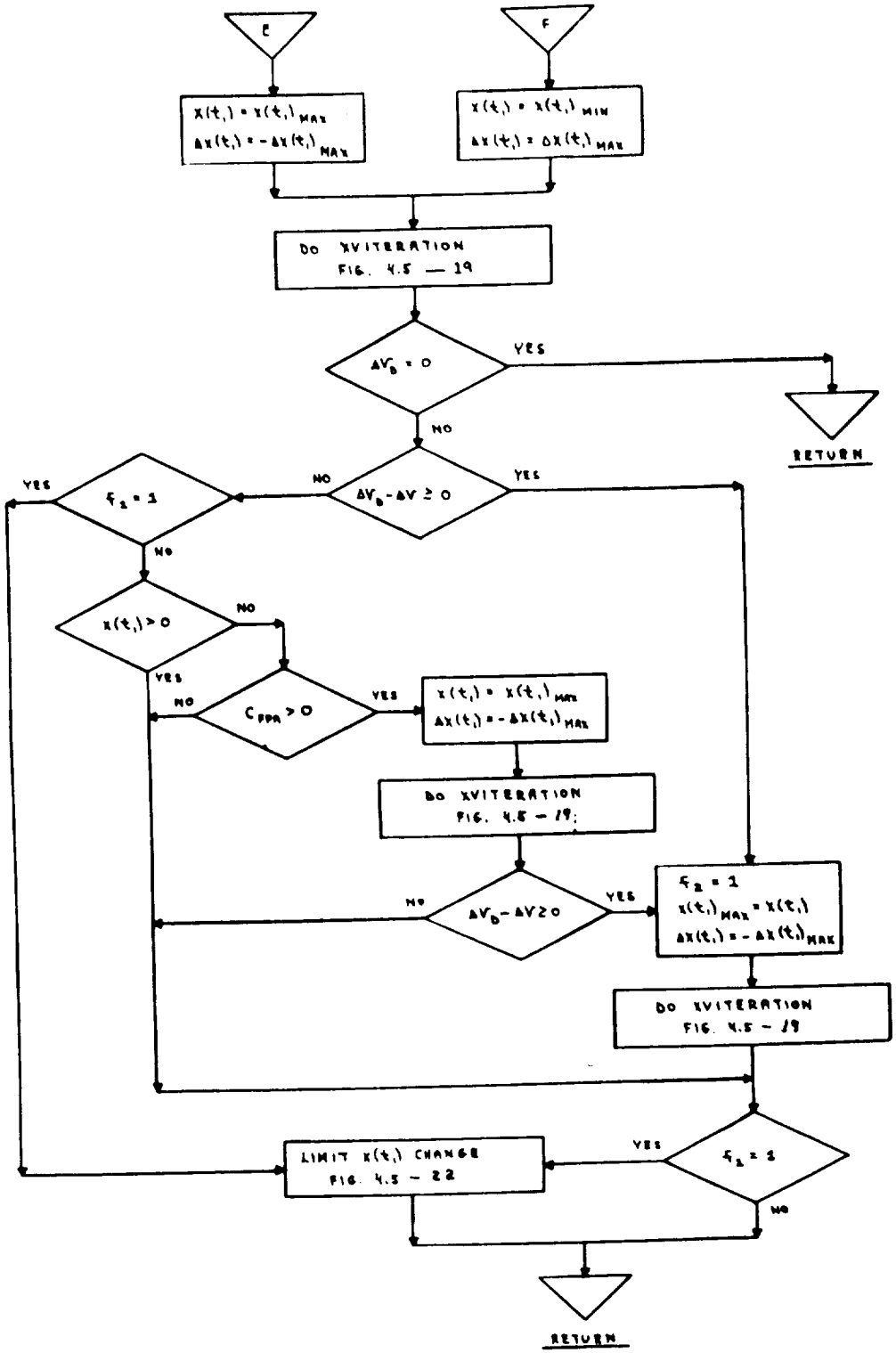



FIGURE 4.5-8 LOGICAL DIAGRAM OF RETURN TO EARTH TARGETING - CONIC RETURN ROUTINE

If $\beta_6$ is negative, only near perigee solutions are possible and hence no additional bounds on $x(t_1)$ are computed. The initial conic trajectory may have achieved the desired reentry flight path angle near apogee, however, and this type of solution is no longer possible due to a change in $R_{CON}$. This is the one exception mentioned in Section 5. 4. 5. 4. 2. The value of $\phi_2$ must be set to -1, forcing all subsequent conic solutions to reach the desired reentry flight path angle near perigee ,and then the entire precision trajectory phase is recycled.

If $\beta_6$ is positive, it is used to calculate $x(t_1)_{LIM}$. Three types of solutions may exist.

(1) $-x(t_1)_{LIM} < x(t_1) < x(t_1)_{LIM}$

If $x(t_1)$ takes a value in this region, the desired reentry flight path angle ($\gamma(t_2)$) will be achieved near apogee.

(2) $x(t_1) < -x(t_1)_{LIM}$ or $x(t_1)_{LIM} < x(t_1)$

If $x(t_1)$ takes a value in these regions, the desired reentry flight path angle ($\gamma(t_2)$) will be achieved near perigee.

(3) $x(t_1) = x(t_1)_{LIM}$ or $x(t_1) = -x(t_1)_{LIM}$

If $x(t_1)$ takes on either of these values, the desired reentry flight path angle ($\gamma(t_2)$) will equal the maximum angle achieved by the conic.

The above information is used to set new bounds on $x(t_1)$ which will force the conic to reach a near apogee or near perigee solution as prescribed by the initial conic.

If the program is being operated in the fuel critical mode, the $x(t_1)$ - $\Delta v$ iterator is used to compute an absolute minimum $\Delta v$ solution.

If the program is being operated in the time critical mode, the $x(t_1)$ - $\Delta v$ iterator will be used to compute a solution requiring a velocity change equal to the objective velocity change.

5.4.5.4.4 Discussion of Logical Diagram - Safe Perilune

The following discussion is intended to supplement the information contained in Figures 4.5-9 and 4.5-10.

No inputs other than the CSM state vector maintained in the CMC are required to start the Safe Perilune Targeting computation. The program may be called when the CSM is either inside or outside the moon's sphere of influence. As mentioned in Section 5.2.2.5, the Coasting Integration Routine has the capability to extrapolate a state vector to within a given radial distance. This capability is used in the initial portion of the Safe Perilune Targeting Program to numerically integrate the CSM state vector to a point near perilune. After a state vector near perilune is obtained, the perilune altitude of the current trajectory is computed. If the astronaut elects to adjust his perilune altitude, he must input via the keyboard a desired perilune altitude. The Safe Perilune Program then enters the flow of the Return to Earth Program.

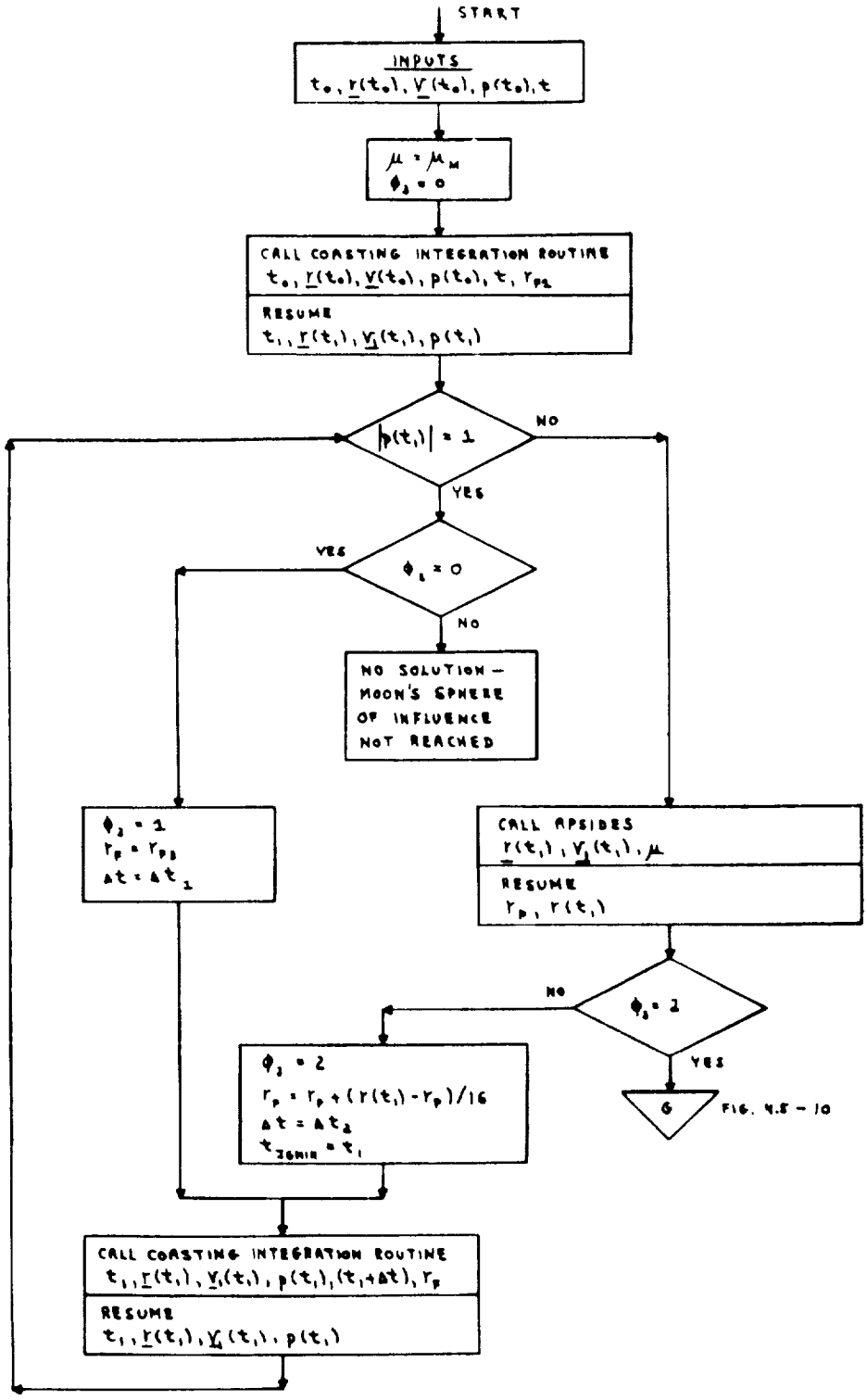



FIGURE 4.5-9 LOGICAL DIAGRAM OF SAFE PERILUNE TARGETING

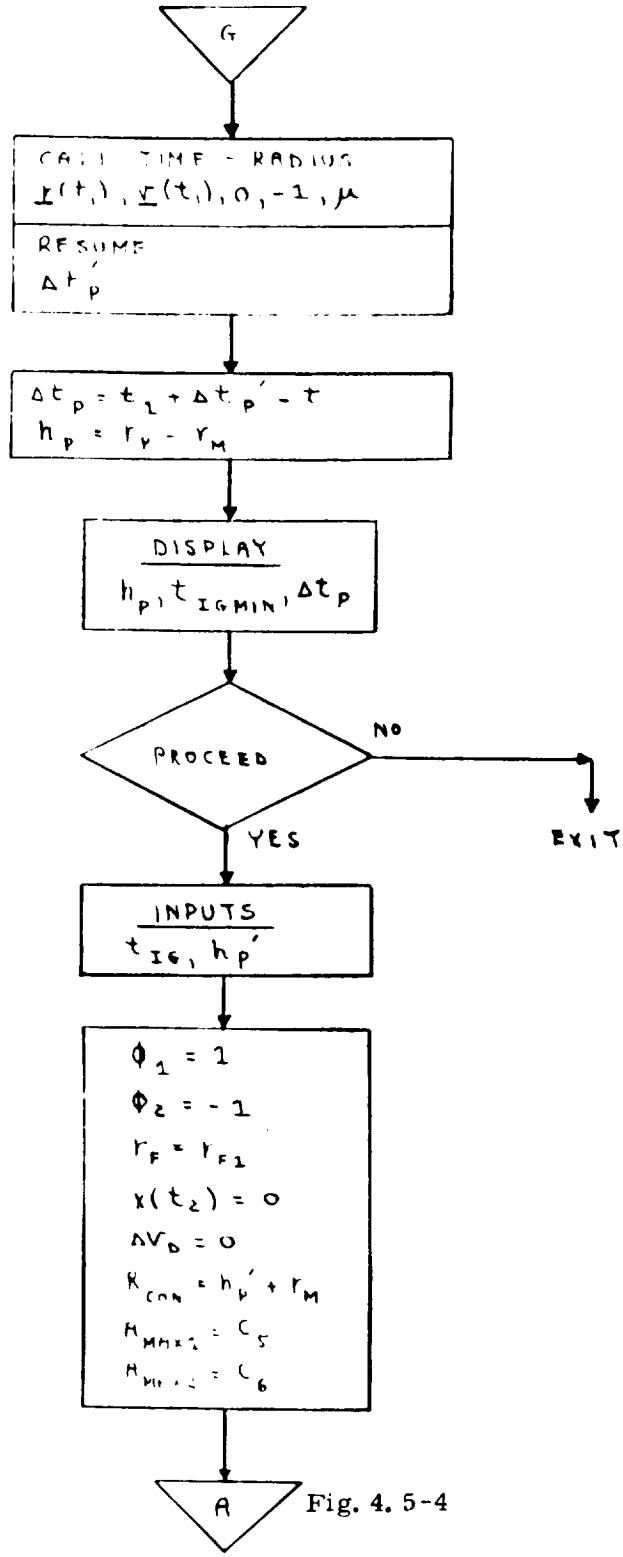



FIGURE 4.5-10 LOGICAL DIAGRAM OF SAFE PERILUNE TRAGETING

### 5.4.5.5 Detailed Flow Diagrams

Certain portions of the flow diagrams mentioned in the previous sections are illustrated in Figs. 4.5-11 through 4.5-23.

A list of nomenclature used in the following diagrams and not defined previously follows.

| | |
|---|---|
| $\lambda$ | radius ratio |
| p | semi-latus rectum |
| $\alpha$ | 1/semi-major axis |
| $\underline{r}_2(t_2)$ | target aim point |
| $\underline{r}_{LS}$ | vector through landing site |
| $\theta_{LAT}$ | landing site latitude |
| $\theta_{LONG}$ | landing site longitude |
| $\dot{m}$ | mass flow rate |
| F | thrust |
| $f_3$ | flag set to $\begin{cases} 0 & \text{SPS engine} \\ 1 & \text{RCS jets} \end{cases}$ |
| $t_{LS}$ | time of landing |
| $\Delta t_B$ | length of burn |
| $\Delta \underline{v}_{LV}$ | velocity change vector in local vertical coordinates |
| $a_{MG}$ | middle gimbal angle |
| $\theta_1, \theta_2, \theta_3$ | intermediate variables |
| $\Delta t_{21}$ | adjustment of $t_{21}$ |
| $\Delta R_{CON}$ | change in the final radius of a conic trajectory |

The diagrams are essentially self-explanatory, however, a discussion of two of the diagrams will follow.

## Precomputations

The equations illustrated in Figure 4.5-11 are used to compute basic variables required during the remaining portions of the program.

As mentioned in the introduction, the pre-return and post-return state vectors are required to lie in the same plane. If the pre-return position and velocity vectors are nearly co-linear, the plane of the return trajectory will be defined to prevent reentry near the earth's poles.

Retrograde return to earth trajectories are not allowed. The Safe Perilune Program will only allow retrograde lunar trajectories.

## Final State Vector Computation

The method illustrated in Figures 4.5-12 and 4.5-13 is used to extrapolate a post-return state vector to the desired reentry flight path angle. An iterative procedure is again used.

First the post-return state vector is advanced through the best estimate of the transit time to reentry using the Coasting Integration Routine. Since it is not necessary to explicitly compute the flight path angle of the resulting reentry state vector, the cotangent of the flight path angle is compared with the cotangent of the desired reentry flight path angle. If they are sufficiently close, the resulting reentry state vector is accepted.

If the desired cotangent of the reentry flight path angle is not achieved, an iterative procedure is started. This procedure uses Conic Subroutines to compute the time of flight ($\Delta t_{21}$) to the desired reentry flight path angle.

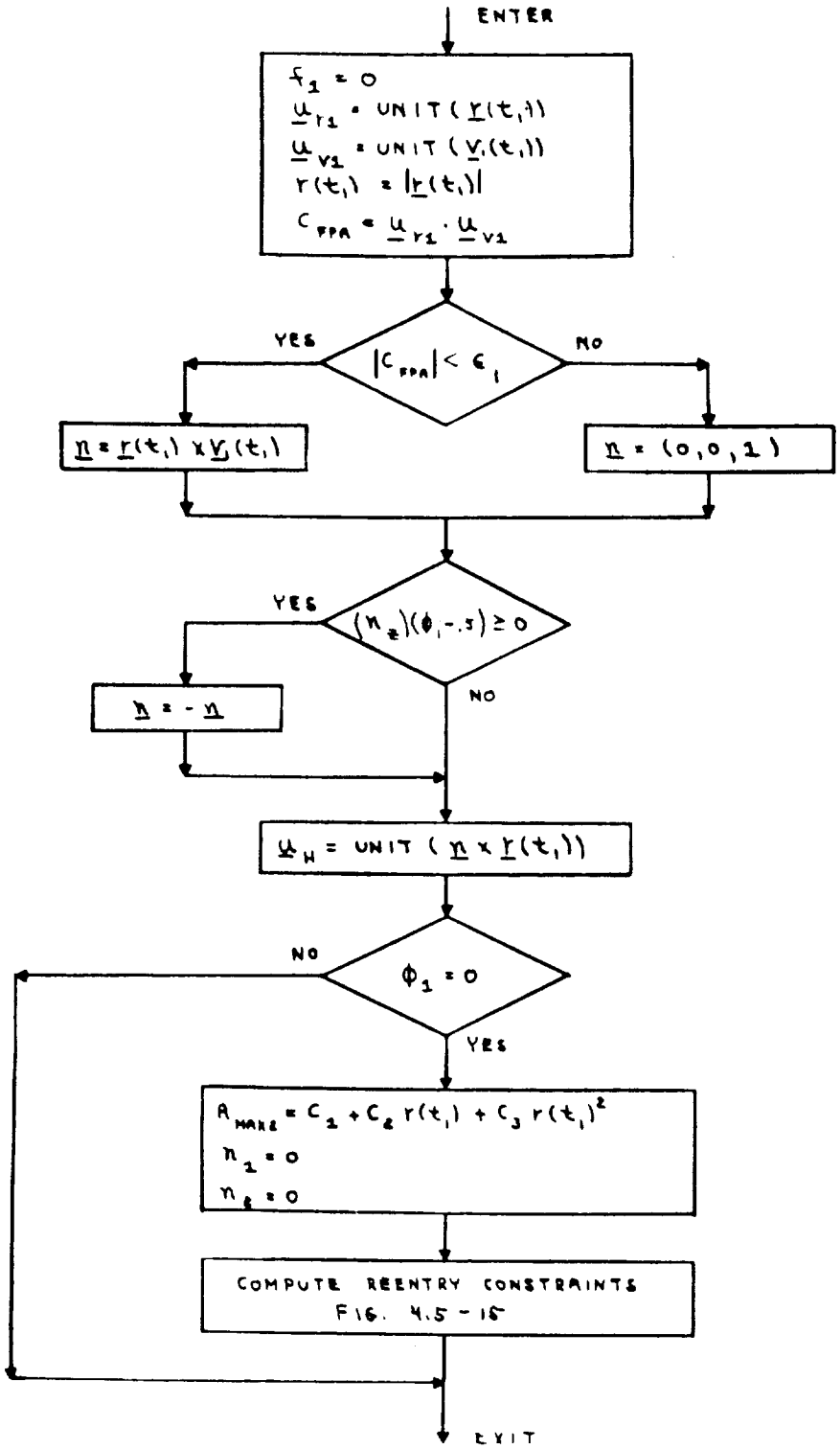



FIGURE 4.5-11 PRECOMPUTATIONS

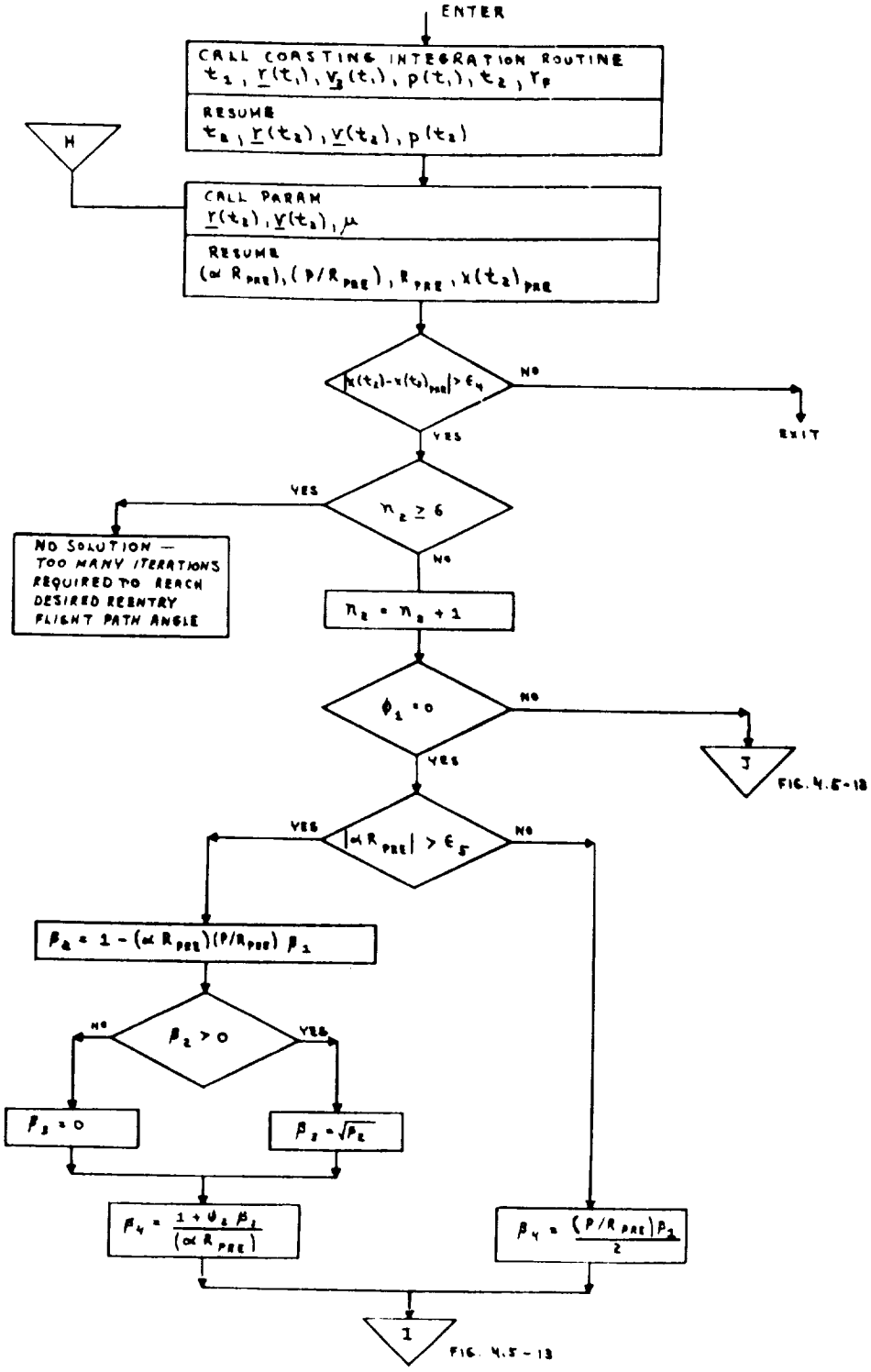



FIGURE 4.5-12 FINAL STATE VECTOR COMPUTATION

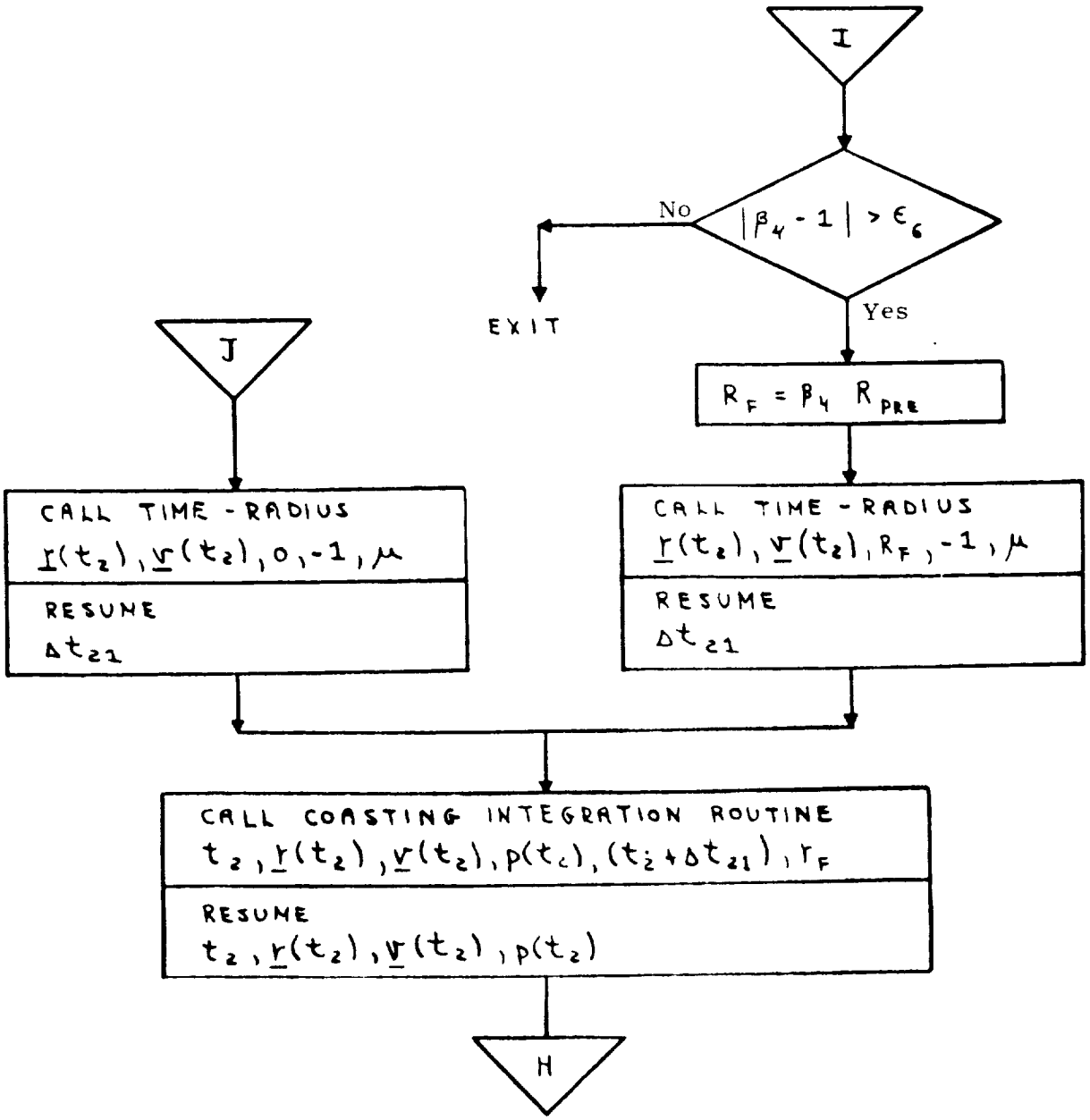



FIGURE 4.5-13 FINAL STATE VECTOR COMPUTATION

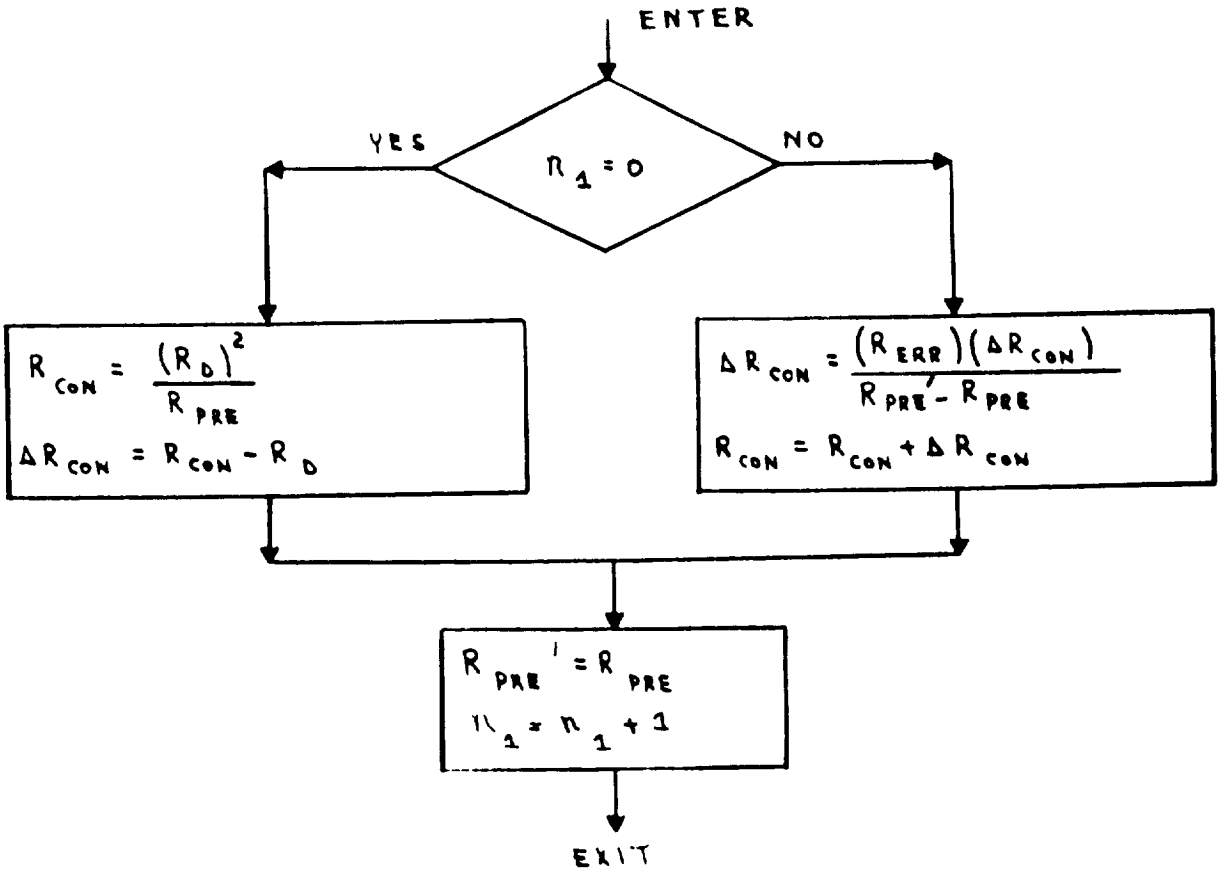



FIGURE 4.5-14 NEW $R_{CON}$ COMPUTATION

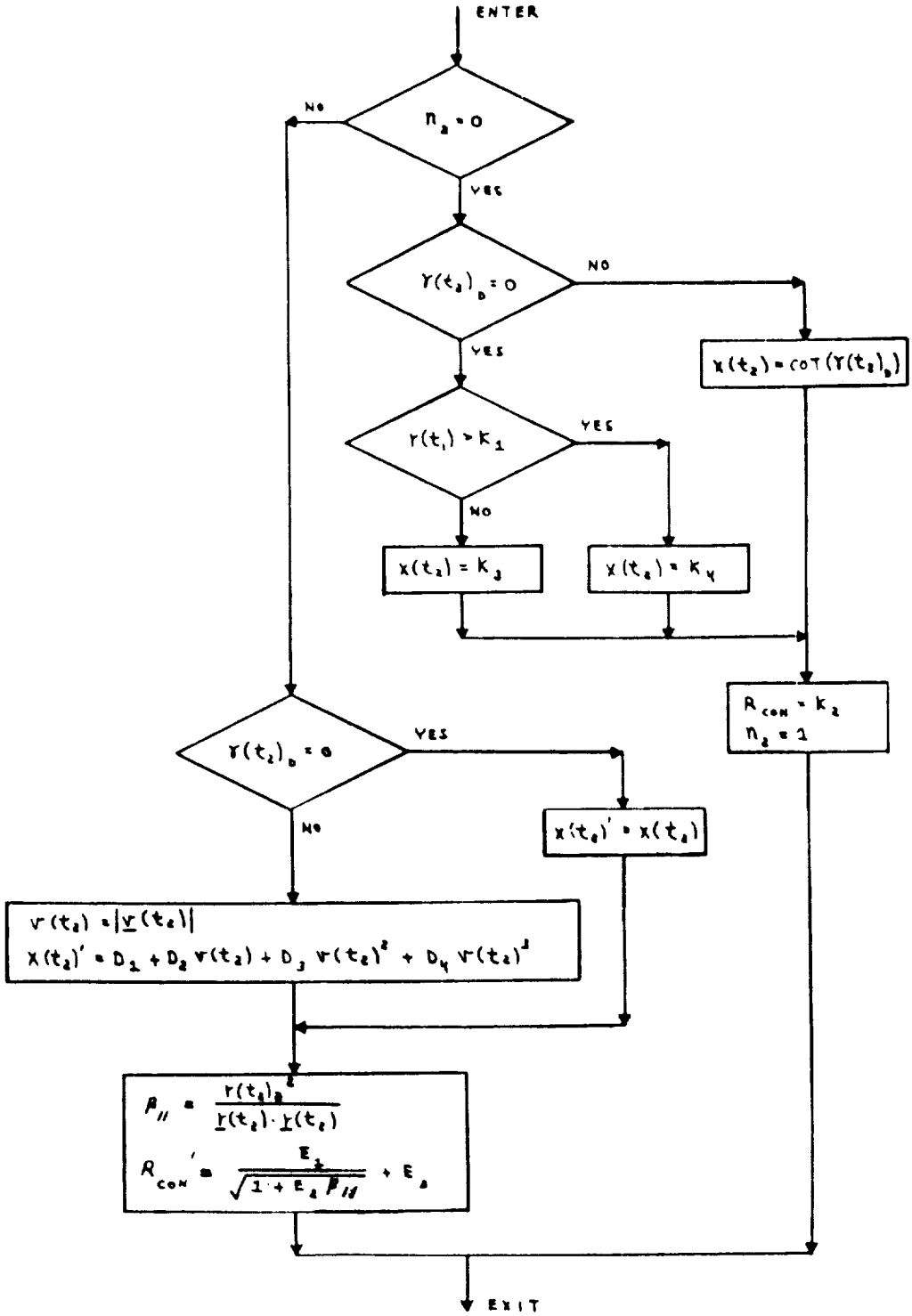



FIGURE 4.5-15 REENTRY CONSTRAINT COMPUTATION

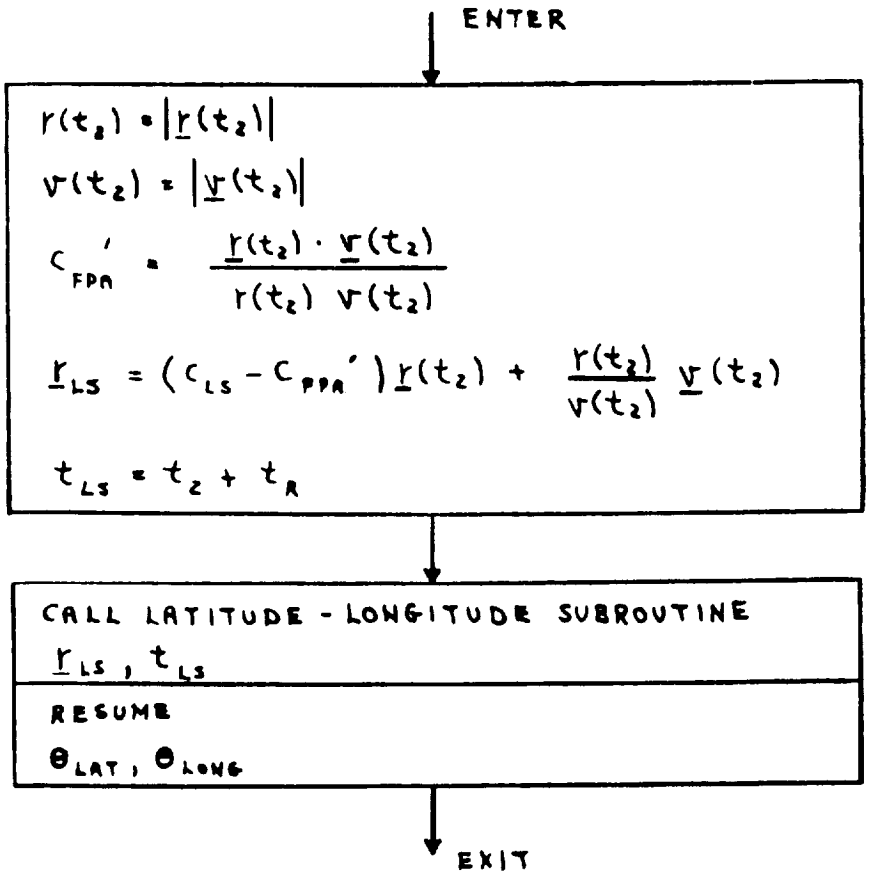



FIGURE 4.5-16 LANDING SITE COMPUTATION

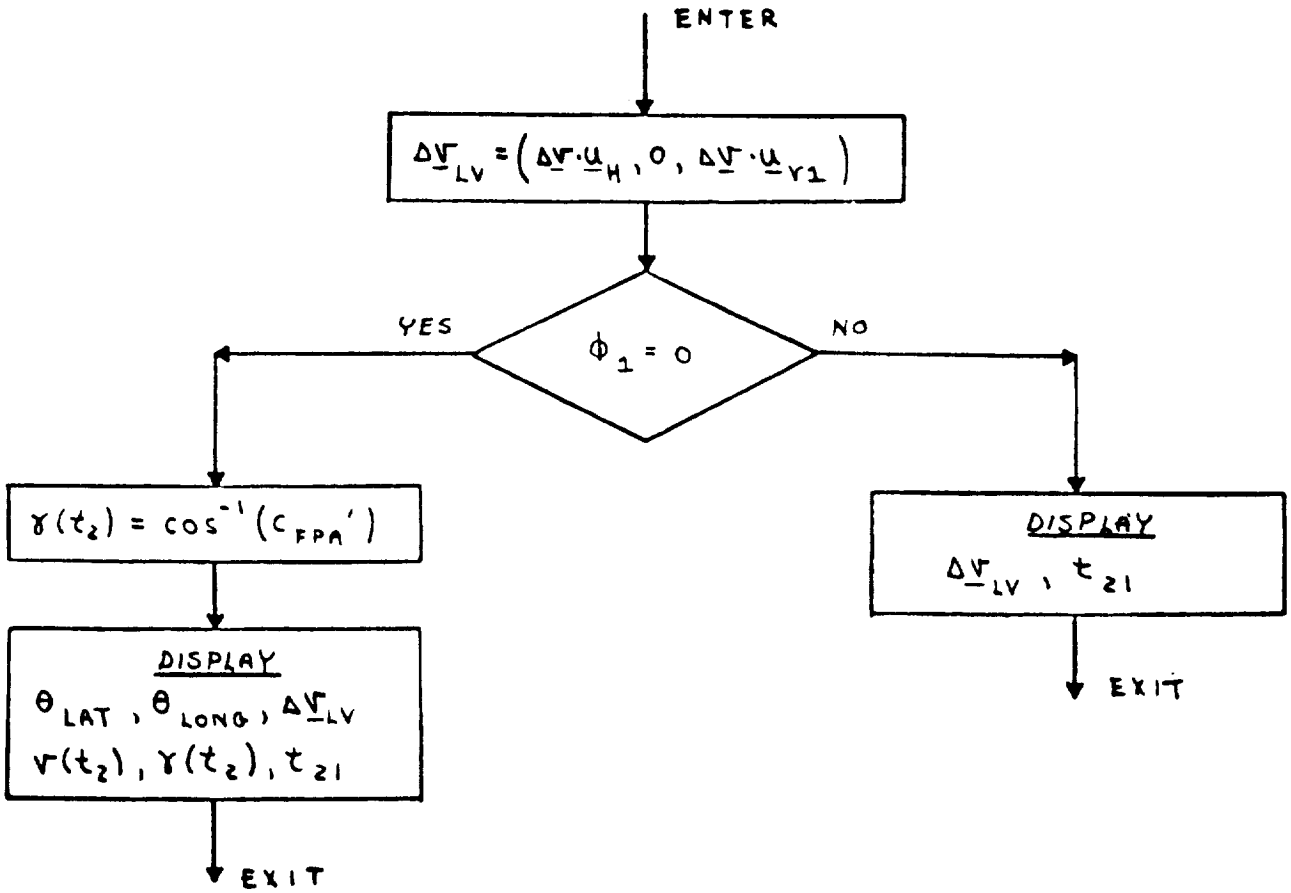



FIGURE 4.5-17 RETURN TO EARTH AND SAFE PERILUNE DISPLAY

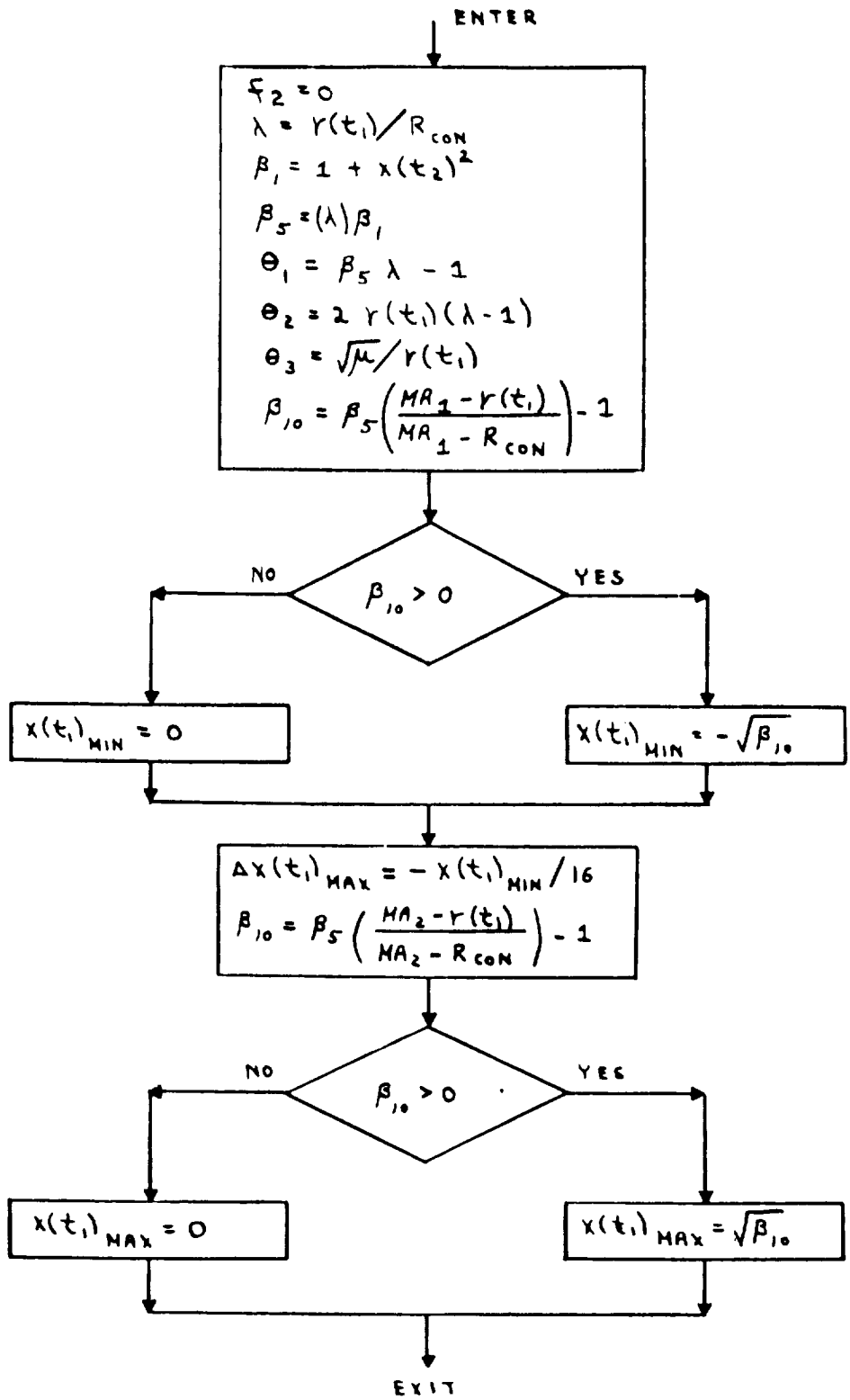



FIGURE 4.5-18 CONIC INITIALIZATION

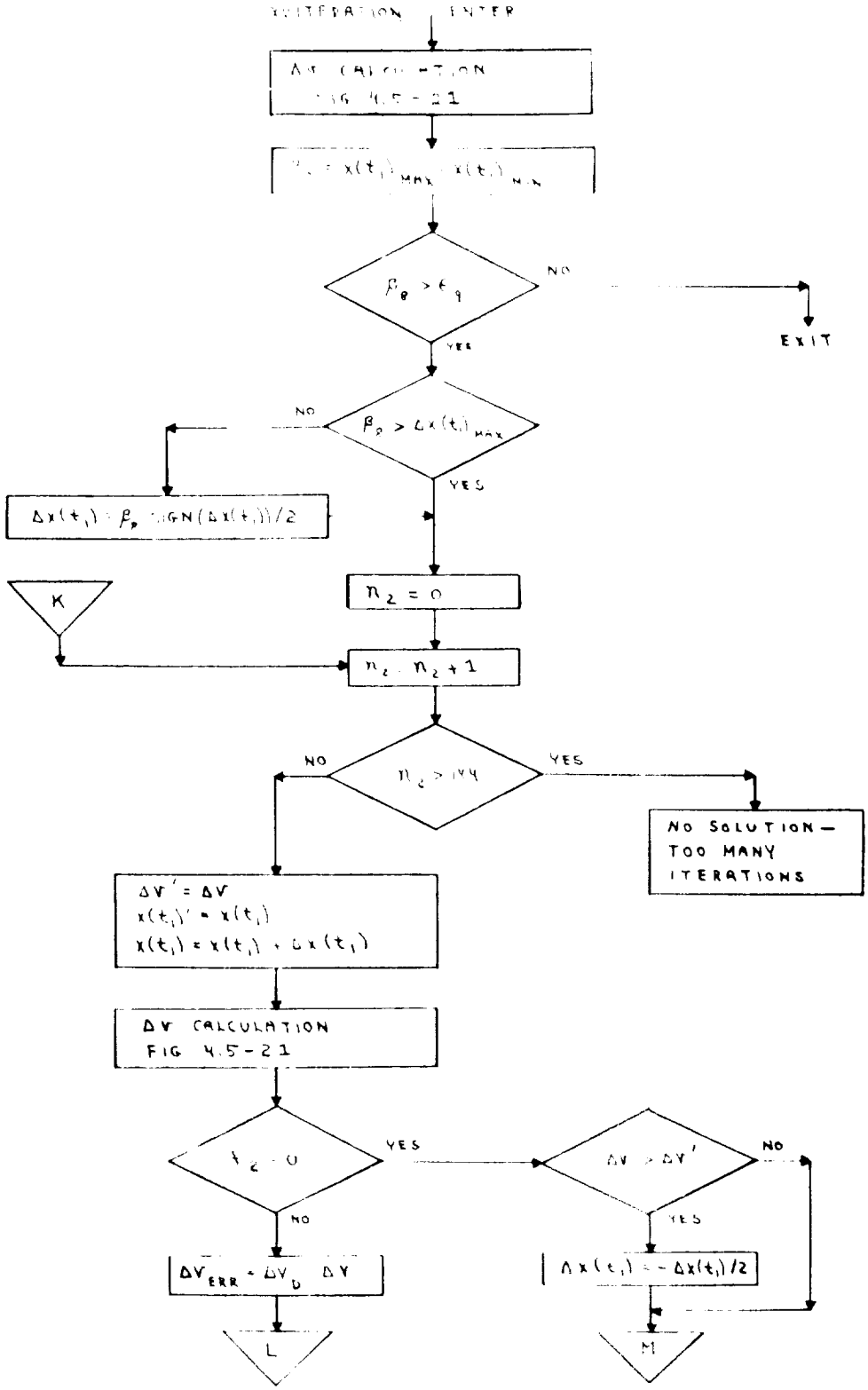



FIGURE 4.5-19 $x(t_1)$ - $\Delta V$ ITERATOR

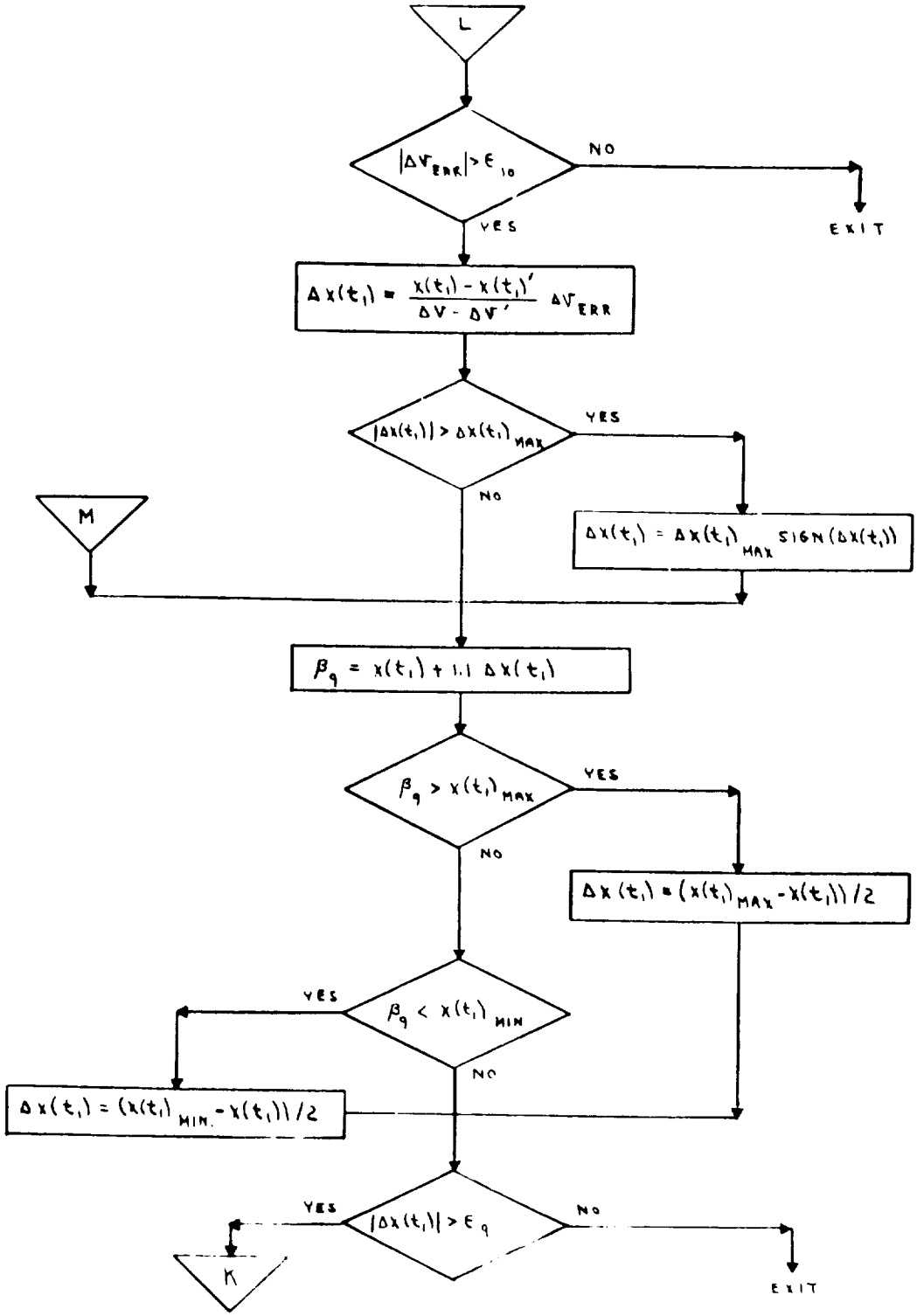



FIGURE 4.5-20 $x(t_1)$ - $\Delta V$ ITERATOR

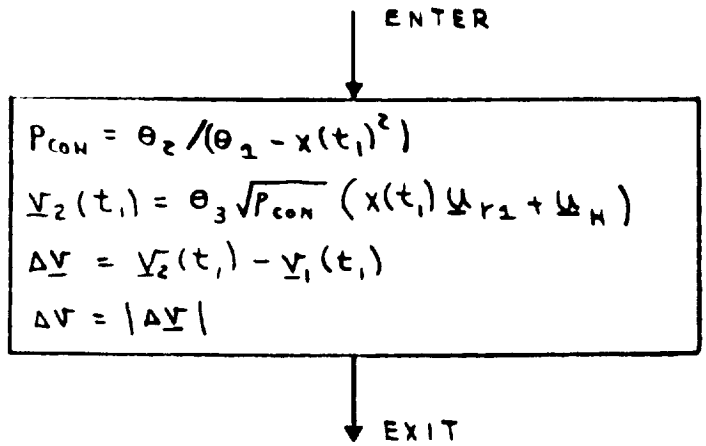



FIGURE 4.5-21 $\Delta v$ CALCULATION

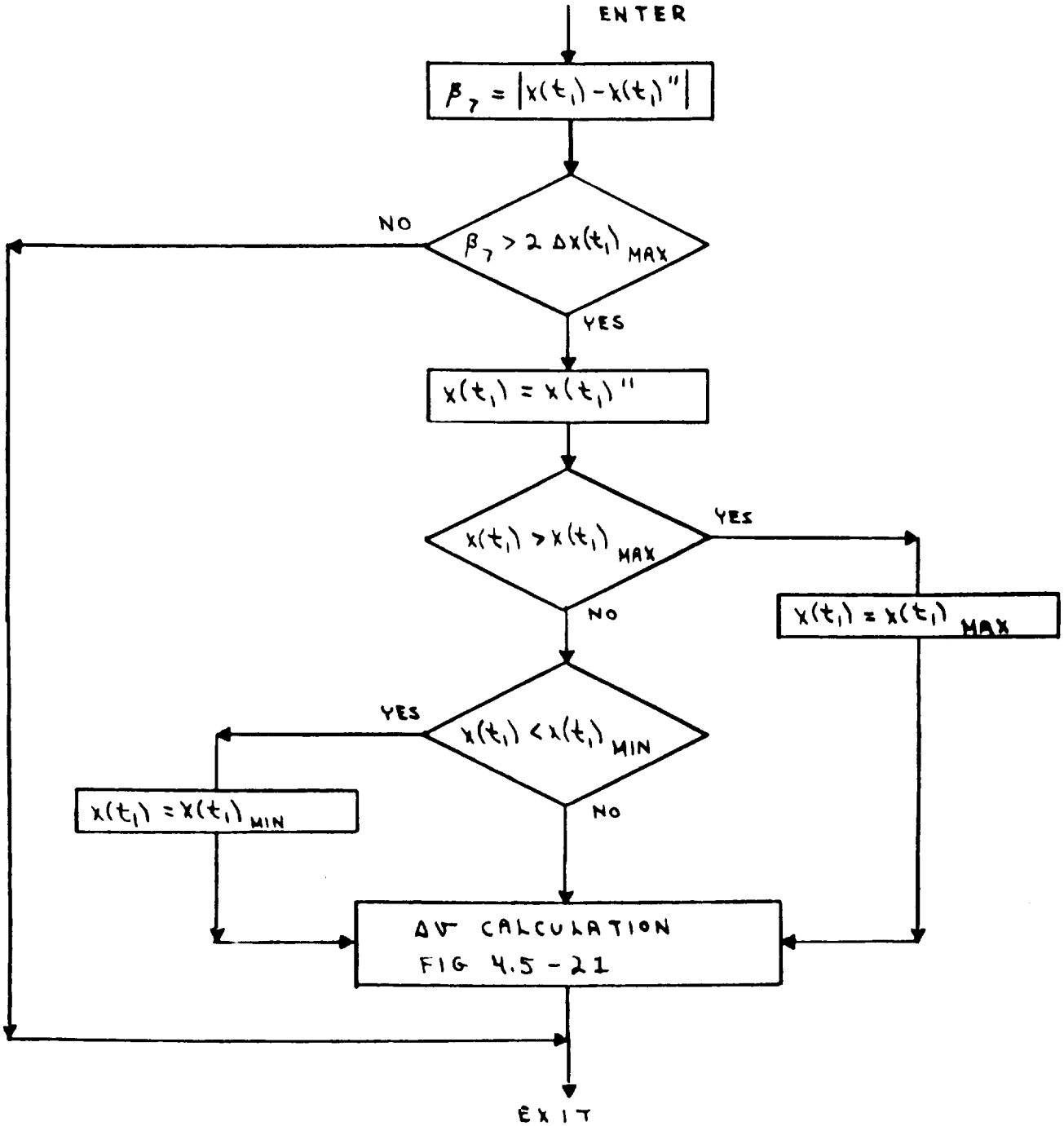



Figure 4.5-22 LIMIT x ($t_1$) CHANGE

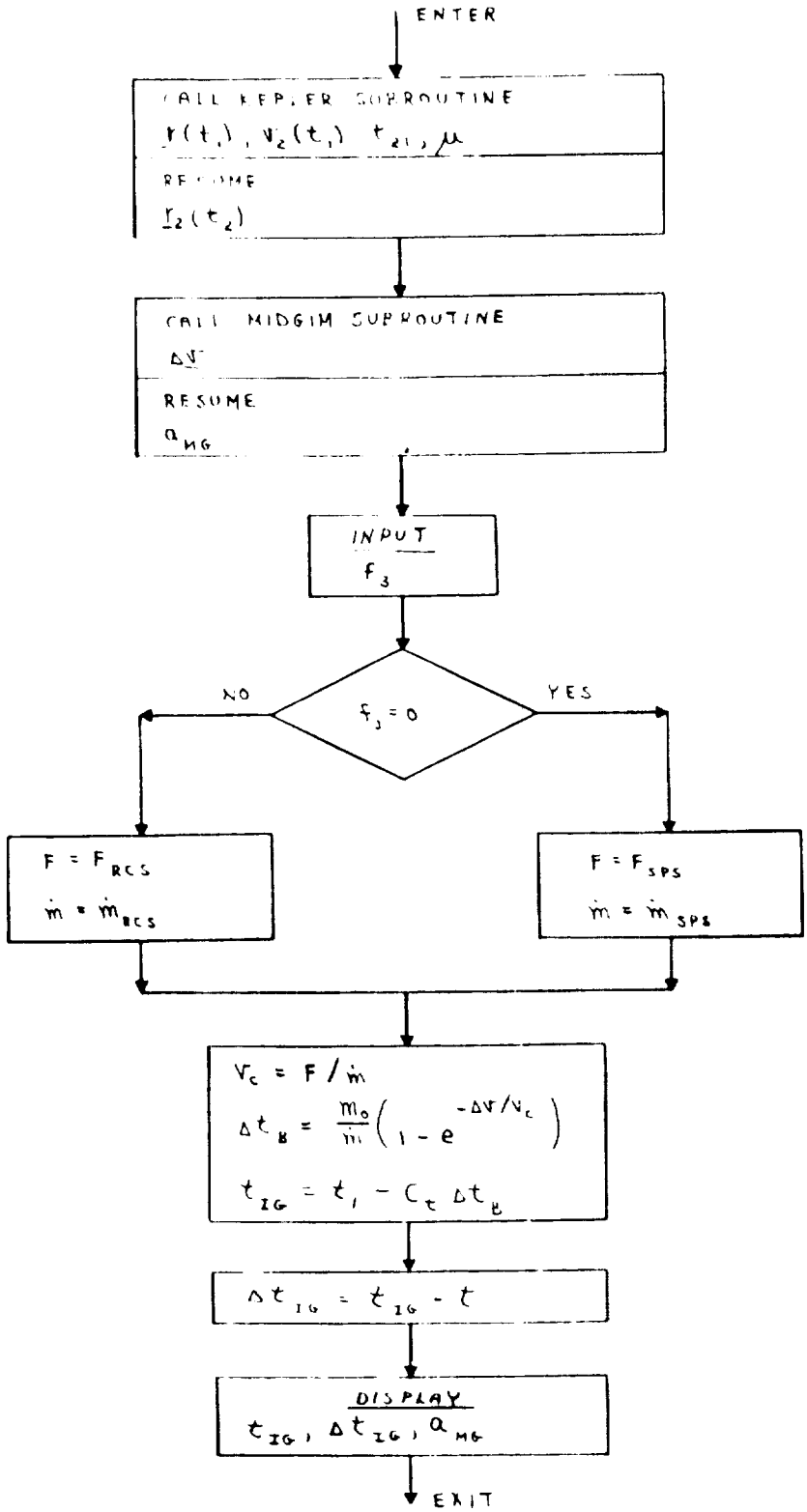



FIGURE 4.5-23 TARGET COMPUTATION

## 5.5 BASIC SUBROUTINES

### 5.5.1 GENERAL COMMENTS

The basic solar system and conic trajectory subroutines which are used by the various guidance and navigation routines are described in this section.

#### 5.5.1.1 Solar System Subroutines

The subroutines used to determine the translation and rotation of the relevant solar system bodies (earth, moon and sun) are designed specifically for a fourteen day lunar landing mission. The method of computing the position and velocity of the moon and the sun relative to the earth is given in Section 5.5.4. The transformations between the Basic Reference Coordinate System and the Earth- and Moon-fixed Coordinate Systems are described in Section 5.5.2. The procedure for transforming between vectors in the Basic Reference Coordinate System and latitude, longitude, altitude coordinates is given in Section 5.5.3. Although these subroutines are normally used in the lunar landing mission, they are valid for use in any mission of not more than fourteen days duration in earth-moon space.

## 5.5 1.2 Conic Trajectory Subroutines

This is a description of a group of conic trajectory subroutines which are frequently used by higher level routines and programs in both the Command Module and the Lunar Module computers.

These subroutines, whose block diagrams are presented in Sections 5.5.5 to 5.5.10, provide solutions to the following conic problems. (See nomenclature which follows)

(1) Given $\underline{r}(t_1)$, $\underline{v}(t_1)$, $t_D$; solve for $\underline{r}(t_2)$, $\underline{v}(t_2)$
(Kepler Subroutine)

(2) Given $\underline{r}(t_1)$, $\underline{r}(t_2)$, $t_{D21}$, $s_G$; solve for $\underline{v}(t_1)$
(Lambert Subroutine)

(3) Given $\underline{r}(t_1)$, $\underline{v}(t_1)$, $\theta$; solve for $t_{21}$, $\underline{r}(t_2)$, $\underline{v}(t_2)$
(Time-Theta Subroutine)

(4) Given $\underline{r}(t_1)$, $\underline{v}(t_1)$, $r(t_2)$, $s_r$; solve for $t_{21}$, $\underline{r}(t_2)$, $\underline{v}(t_2)$
(Time-Radius Subroutine)

(5) Given $\underline{r}(t)$, $\underline{v}(t)$; solve for $r_P$, $r_A$, $e$
(Apsides Subroutine)

In addition, the following useful subroutines are provided.

(6) Conic Parameters Subroutine (See Fig. 5.10-1).

(7) Geometric Parameters Subroutine (See Fig. 5.10-2).

(8) Iterator Subroutine (See Fig. 5.10-3).

The solutions to the above set of conic problems have stringent accuracy requirements. Programming the fixed-point Apollo computer introduces two constraints which determine accuracy limitations: the 28 bit double precision word length, and the range of variables which is several orders of magnitude for the Apollo mission.

In order to maintain numerical accuracy when these subroutines are programmed into the Apollo Computer, floating point programming techniques must be exercised. The effect is for even a simple equation to require a large number of computer instructions. The alternative to this is to separate the problem into phases, each with a different variable range. This, however, requires an even larger number of instructions. These considerations provide the incentive for efficiently organizing the conic equations as shown on the block diagrams.

In addition to the requirement for accuracy, the solution to the Kepler and Lambert Problems must be accomplished in a minimum of computation time in order that the guidance system operate satisfactorily in real time. This additional constraint dictates that a minimum of computer instructions be performed when solving the problem.

Method of Solution

To minimize the total number of computer instructions, the problems are solved in the "universal" form; i.e. only equations which are equally valid for the ellipse, parabola and hyperbola are used. Also these subroutines can be used with either the earth or the moon as the as the attracting body.

Kepler's equation, in the universal form, is utilized to relate transfer time to the conic parameters. All other necessary equations are also universal. The Kepler and Lambert problems are solved with a single iteration loop utilizing a simple first-order slope iterator. In the case of the Kepler problem a third order approximation is available to produce the initial guess for the independent variable (See Eq.(2. 2. 4) of Section 5. 2. 2. 2).

Sections 5. 5. 5 thru 5. 5. 10 provide block diagrams of the detailed computational procedures for solving the various problems. The equations are presented in block diagram form with the nomenclature defined below.

## Range of Variables

As indicated previously, the programming of the conic subroutines requires a careful balance between accuracy, computational speed and number of instructions. This balance, in the Apollo Guidance Computer, leaves very little margin in any of these areas.

Since the values of problem variables are determined by the solution of the problem being solved and since the problem may originate from the ground system, it is essential that the variable range limitations be defined. The conic routines are incapable of handing problems when the solution lies outside of the range.

The following is a list of the maximum allowable numeric values of the variables. Note that, in addition to fundamental quantities such as position and velocity, there are limitations on intermediate variables and combinations of variables.

Scaling for Conic Subroutines (Sections 5. 5. 5 to 5. 5. 10)

| Parameter | Maximum Value* Earth Primary Body | Maximum Value* Moon Primary Body |
|---|---|---|
| $r$ | $2^{29}$ | $2^{27}$ |
| $v$ | $2^{7}$ | $2^{5}$ |
| $t$ | $2^{28}$ | $2^{28}$ |
| $\alpha$** | $2^{-22}$ | $2^{-20}$ |
| $\alpha_N$** | $2^{6}$ | $2^{6}$ |
| $p_N$ | $2^{4}$ | $2^{4}$ |
| $\cot \gamma$ | $2^{5}$ | $2^{5}$ |
| $\cot \frac{\theta}{2}$ | $2^{5}$ | $2^{5}$ |
| $x$ | $2^{17}$ | $2^{16}$ |
| $\xi = \alpha x^2$*** | $-50$<br>$+4\pi^2$ | $-50$<br>$+4\pi^2$ |
| $c_1 = \underline{r} \cdot \underline{v} / \sqrt{\mu}$ | $2^{17}$ | $2^{16}$ |
| $c_2 = r v^2/\mu - 1$ | $2^{6}$ | $2^{6}$ |
| $\lambda = r(t_1)/r(t_2)$ | $2^{7}$ | $2^{7}$ |
| $\cos \theta - \lambda$ | $2^{7}$ | $2^{7}$ |

* All dimensional values are in units of meters and centiseconds.

** The maximum absolute value occurs for negative values of this parameter.

***Both the maximum and minimum values are listed since neither may be exceeded.

| Parameter | Maximum Value* Earth | Maximum Value* Moon |
|---|---|---|
| $e$ | $2^3$ | $2^3$ |
| $x^2$ | $2^{33}$ | $2^{31}$ |
| $x^2\ c\ (\xi)$ | $2^{33}$ | $2^{31}$ |
| $x^3\ s\ (\xi)/\sqrt{\mu}$ | $2^{28}$ | $2^{28}$ |
| $c_1\ x^2\ c\ (\xi)$ | $2^{49}$ | $2^{46}$ |
| $c_2\ x^2\ s\ (\xi)$ | $2^{35}$ | $2^{33}$ |
| $x\ [c_2\ x^2\ s\ (\xi) + r\ (t_1)]$ | $2^{49}$ | $2^{46}$ |
| $\xi\ s\ (\xi)$ | $2^7$ | $2^7$ |
| $x^2\ c\ (\xi)/\ r$ | $2^8$ | $2^8$ |
| $\sqrt{\mu}\ x\ (\xi\ s\ (\xi) - 1)/r\ (t_2)$ | $2^{15}$ | $2^{13}$ |

* All dimensional values are in units of meters and centiseconds.

Nomenclature for Conic Subroutines (Sections 5.5.5 to 5.5.10)

| | |
|---|---|
| $\underline{r}(t_1)$ | initial position vector |
| $\underline{v}(t_1)$ | initial velocity vector |
| $\underline{r}(t_2)$ | terminal position vector |
| $\underline{v}(t_2)$ | terminal velocity vector |
| $\underline{u}_N$ | unit normal in the direction of the angular momentum vector |
| $\alpha$ | reciprocal of semi-major axis (negative for hyperbolas) |
| $r_P$ | radius of pericenter |
| $r_A$ | radius of apocenter |
| e | eccentricity |
| $\alpha_N$ | ratio of magnitude of initial position vector to semi-major axis |
| $p_N$ | ratio to semi-latus rectum to initial position vector magnitude |
| $\gamma$ | inertial flight path angle as measured from vertical |
| $\theta$ | true anomaly difference between $\underline{r}(t_1)$ and $\underline{r}(t_2)$ |
| f | true anomaly of r $(t_2)$ |

| | |
|---|---|
| $x$ | a universal conic parameter equal to the ratio of eccentric anomaly difference to $\sqrt{+\alpha}$ for the ellipse, or the ratio of the hyperbolic analog of eccentric anomaly difference to $\sqrt{-\alpha}$ for the hyperbola |
| $x'$ | value of x from the previous Kepler solution |
| $t_{21}$ | computed transfer time from Kepler's equation $(t_2 - t_1)$ |
| $t'_{21}$ | transfer time corresponding to the previous solution of Kepler's equation |
| $t_D$ | desired transfer time through which the conic update of the state vector is to be made |
| $t_{D21}$ | desired transfer time to traverse from $\underline{r}(t_1)$ to $\underline{r}(t_2)$ |
| $t_{ERR}$ | error in transfer time |
| $\epsilon_t$ | tolerance to which transfer time must converge |
| $\Delta x$ | increment in x which will produce a smaller $t_{ERR}$ |
| $\epsilon_x$ | value of $\Delta x$ which will produce no significant change in $t_{21}$ |
| $\Delta \cot \gamma$ | increment in $\cot \gamma$ which will produce a smaller $t_{ERR}$ |
| $\epsilon_c$ | value of $\Delta \cot \gamma$ which will produce no significant change in $t_{21}$ |

| | |
|---|---|
| $\mu$ | product of universal gravitational constant and mass of the primary attracting body |
| $x_{MAX}$ | maximum value of x |
| $x_{MIN}$ | minimum value of x |
| $\cot_{MAX}$ | maximum value of cot $\gamma$ |
| $\cot_{MIN}$ | minimum value of cot $\gamma$ |
| $\ell_{MAX}$ | upper bound of general independent variable |
| $\ell_{MIN}$ | lower bound of general independent variable |
| $x_{MAX\,1}$ | absolute upper bound on x with respect to the moon |
| $x_{MAX\,0}$ | absolute upper bound on x with respect to the earth |
| k | a fraction of the full range of the independent variable which determines the increment of the independent variable on the first pass through the iterator |
| y | general dependent variable |
| y' | previous value of y |
| $y_{ERR}$ | error in y |
| z | general independent variable |

| | |
|---|---|
| $\Delta z$ | increment in z which will produce a smaller $y_{ERR}$ |
| $s_G$ | a sign which is plus or minus according to whether the true anomaly difference between $\underline{r}(t_1)$ and $\underline{r}(t_2)$ is to be less than or greater than 180 degrees |
| $s_{\dot{r}}$ | a sign which is plus or minus according to whether the desired radial velocity at $r(t_2)$ is plus or minus |
| $\underline{\eta}_1$ | general vector # 1 |
| $\underline{\eta}_2$ | general vector # 2 |
| $\phi$ | angle between $\underline{\eta}_1$ and $\underline{\eta}_2$ |
| $f_1$ | a switch set to 0 or 1 according to whether a guess of cot $\gamma$ is available or not |
| $f_2$ | a switch set to 0 or 1 according to whether Lambert should determine $\underline{u}_N$ from $\underline{r}(t_1)$ and $\underline{r}(t_2)$ or $\underline{u}_N$ is an input |
| $f_3$ | a tag set to 0 or 1 according to whether the iterator should use the "Regula Falsi" or bias method |
| $f_4$ | a flag set to 0 or 1 according to whether the iterator is to act as a first order or a second order iterator |
| $f_5$ | a flag set to 0 or 1 according to whether a feasible solution exists or not |

| | |
|---|---|
| $f_6$ | a switch set to 0 or 1 according to whether or not the new state vector is to be an additional output requirement of the Time-Theta or Time-Radius problems. |
| $f_7$ | a flag set to 1 if the inputs require that the conic trajectory must close through infinity |
| $f_8$ | a flag set to 1 if the Time-Radius problem was solved for pericenter or apocenter instead of $r(t_2)$ |

## 5.5.2 PLANETARY INERTIAL ORIENTATION SUBROUTINE

This subroutine is used to transform vectors between the Basic Reference Coordinate System and a Planetary (Earth-fixed or Moon-fixed) Coordinate System at a specified time. These three coordinate systems are defined in Section 5.1.4.

Let $\underline{r}$ be a vector in the Basic Reference Coordinate System, $\underline{r}_P$ the same vector expressed in the Planetary Coordinate System, and t the specified ground elapsed time (GET). Then,

$$\underline{r}_P = M(t)\,(\underline{r} - \underline{\ell} \times \underline{r}) \tag{5.2.1}$$

and

$$\underline{r} = M^T(t)\,(\underline{r}_P + \underline{\ell}_P \times \underline{r}_P) \tag{5.2.2}$$

where M(t) is a time dependent orthogonal transformation matrix, $\underline{\ell}$ is a small rotation vector in the Basic Reference Coordinate System, and $\underline{\ell}_P$ is the same vector $\underline{\ell}$ expressed in the Planetary Coordinate System. The vector $\underline{\ell}$ is considered constant in one coordinate system for the duration of the mission. The method of computing M(t) and $\underline{\ell}$ depends on whether the relevant planet is the earth or the moon.

### Case I - Earth

For the earth, the matrix M(t) describes a rotation about the polar axis of the earth (the Z-axis of the Earth-fixed Coordinate

System), and the vector $\underline{\ell}$ accounts for the precession and nutation of the polar axis (the deviation of the true pole from the mean pole).

Let $A_X$ and $A_Y$ be the small angles about the X- and Y-axes of the Basic Reference Coordinate System, respectively, that describe the precession and nutation of the earth's polar axis. The values of these two angles at the midpoint of the mission are included in the pre-launch erasable data load and are considered constant throughout the flight. Then,

$$\underline{\ell} = \begin{pmatrix} -A_X \\ -A_Y \\ 0 \end{pmatrix}$$

$$A_Z = A_{Z0} + \omega_E (t + t_0) \qquad (5.2.3)$$

$$M(t) = \begin{pmatrix} \cos A_Z & \sin A_Z & 0 \\ -\sin A_Z & \cos A_Z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{\ell}_P = M(t)\, \underline{\ell}$$

where $A_{Z0}$ is the angle between the X-axis of the Basic Reference Coordinate System and the X-axis of the Earth-fixed Coordinate System (the intersection of the Greenwich meridian and the equatorial plane of the earth) at July 1.0, 1968 universal time (i.e., midnight at Greenwich just prior to July 1, 1968), $t_0$ is the elapsed time between July 1.0, 1968 universal time and the time that the computer clock was zeroed, and $\omega_E$ is the angular velocity of the earth.

Case II - Moon

For the moon, the matrix M(t) accounts for the difference in orientation of the Basic Reference and Moon-fixed Coordinate Systems in exact accordance with Cassini's laws, and the rotation vector $\underline{\zeta}$ corrects for deviations from the above orientation because of physical libration.

Define the following three angles which are functions of time:

$B$ = the obliquity, the angle between the mean earth equatorial plane and the plane of the ecliptic.

$\Omega_I$ = the longitude of the node of the moon's orbit measured from the X-axis of the Basic Reference Coordinate System.

$F$ = the angle from the mean ascending node of the moon's orbit to the mean moon.

Let I be the constant angle between the mean lunar equatorial plane and the plane of the ecliptic ($1^{\circ}$ 32.1'). Then, the sequence of rotations which brings the Basic Reference Coordinate System into coincidence with the Moon-fixed Coordinate System (neglecting libration) is as follows:

| Rotation | Axis of Rotation | Angle of Rotation |
|---|---|---|
| 1 | X | B |
| 2 | Z | $\Omega_I$ |
| 3 | X | -I |
| 4 | Z | $\pi$ + F |

The transformation matrices for these rotations are, respectively,

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos B & \sin B \\ 0 & -\sin B & \cos B \end{pmatrix}$$

$$M_2 = \begin{pmatrix} \cos \Omega_I & \sin \Omega_I & 0 \\ -\sin \Omega_I & \cos \Omega_I & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos I & -\sin I \\ 0 & \sin I & \cos I \end{pmatrix} \qquad (5.2.4)$$

$$M_4 = \begin{pmatrix} -\cos F & -\sin F & 0 \\ \sin F & -\cos F & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The matrix M(t) is then given by

$$M(t) = M_4 \, M_3 \, M_2 \, M_1 \tag{5.2.5}$$

The following approximate method is used to determine the transformation between the Basic Reference and Moon-fixed Coordinate Systems.

The angles B, $\Omega_I$ and F are computed as linear functions of time. Let $\underline{\ell}_M$ be the value of the vector libration $\underline{\ell}_I$, (expressed in the Moon-fixed Coordinate System) at the midpoint of the mission. The vector $\underline{\ell}_M$ is included in the pre-launch erasable data load and is considered constant throughout the flight. Then,

$$\begin{aligned}
\underline{\ell}_I &= \underline{\ell}_M \\
t_M &= t + t_0 \\
B &= B_0 + \dot{B}\, t_M \\
\Omega_I &= \Omega_{I0} + \dot{\Omega}_I\, t_M \\
F &= F_0 + \dot{F}\, t_M \\
\underline{a} &= \begin{pmatrix} \cos\Omega_I \\ \cos B \sin\Omega_I \\ \sin B \sin\Omega_I \end{pmatrix} \\
\underline{b} &= \begin{pmatrix} -\sin\Omega_I \\ \cos B \cos\Omega_I \\ \sin B \cos\Omega_I \end{pmatrix}
\end{aligned} \tag{5.2.6}$$

$$\underline{c} = \begin{pmatrix} 0 \\ -\sin B \\ \cos B \end{pmatrix}$$

$$\underline{d} = \underline{b}\, C_I - \underline{c}\, S_I$$

$$\underline{m}_2 = \underline{b}\, S_I + \underline{c}\, C_I$$

$$\underline{m}_0 = -\underline{a} \cos F - \underline{d} \sin F \qquad (5.2.6)\ (\text{cont.})$$

$$\underline{m}_1 = \underline{a} \sin F - \underline{d} \cos F$$

$$M(t) = \begin{pmatrix} \underline{m}_0^T \\ \underline{m}_1^T \\ \underline{m}_2^T \end{pmatrix}$$

$$\underline{f} = M^T(t)\, \underline{f}_P$$

where $B_0$, $\Omega_{I0}$, and $F_0$ are the values of the angles B, $\Omega_I$ and F, respectively, at July 1.0, 1968 universal time; $\dot{B}$, $\dot{\Omega}_I$ and $\dot{F}$ are the rates of change of these angles; and $C_I$ and $S_I$ are the cosine and sine, respectively, of the angle I.

## 5.5.3 LATITUDE-LONGITUDE SUBROUTINE

For display and data load purposes, the latitude, longitude, and altitude of a point near the surface of the earth or the moon are more meaningful and more convenient to use than the components of a position vector. This subroutine is used to transform position vectors between the Basic Reference Coordinate System and Geographic or Selenographic latitude, longitude, altitude at a specified time.

In the case of the moon, the altitude is computed above either the landing site radius, $r_{LS}$, or the mean lunar radius, $r_M$. For the earth, the altitude is defined with respect to either the launch pad radius, $r_{LP}$ or the radius of the Fischer ellipsoid, $r_F$, which is computed from

$$r_F^2 = \frac{b^2}{1 - (1 - \frac{b^2}{a^2})(1 - SINL^2)} \qquad (5.3.1)$$

where a and b are the semi-major and semi-minor axes of the Fischer ellipsoid, respectively, and SINL is the sine of the geocentric latitude.

The computational procedures are illustrated in Figs. 5.3-1, 5.3-2, and 5.3-3. The calling program must specify either a vector $\underline{r}$ or latitude (Lat), longitude (Long), and altitude (Alt). In addition, the program must set the time t and the two indicators P and F where

$$P = \begin{cases} \pm 1 \text{ for earth} \\ \pm 2 \text{ for moon} \end{cases}$$

$$F = \begin{cases} 1 \text{ for Fischer ellipsoid or mean lunar radius} \\ 0 \text{ for launch pad or landing site radius} \end{cases}$$

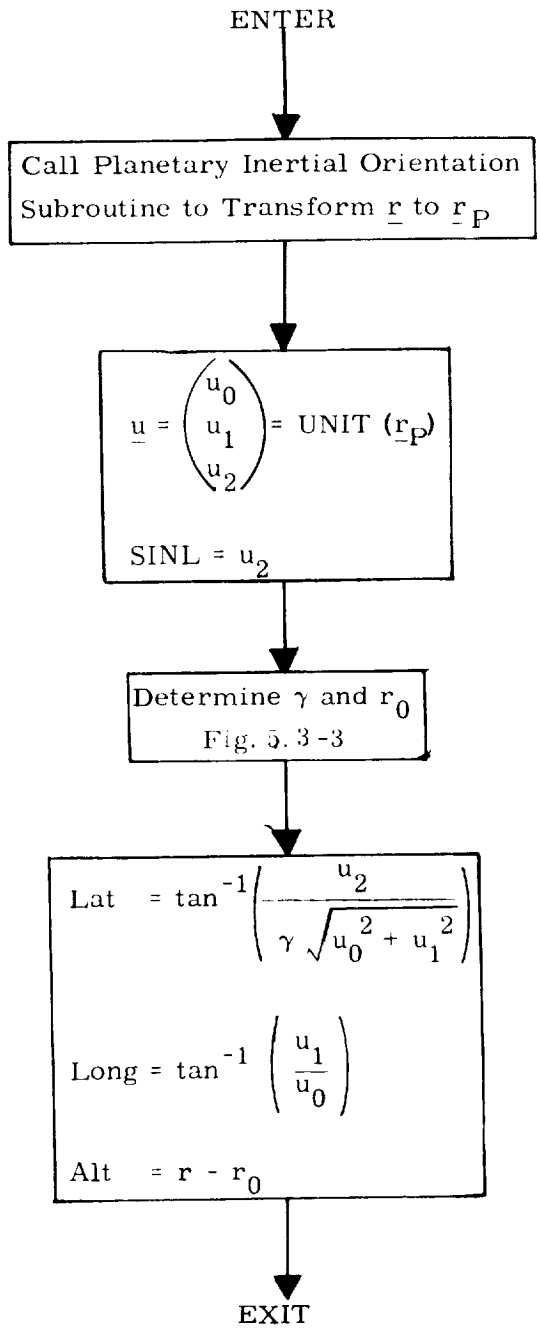



Fig. 5.3-1 Vector to Latitude, Longitude, Altitude Computation Logic Diagram

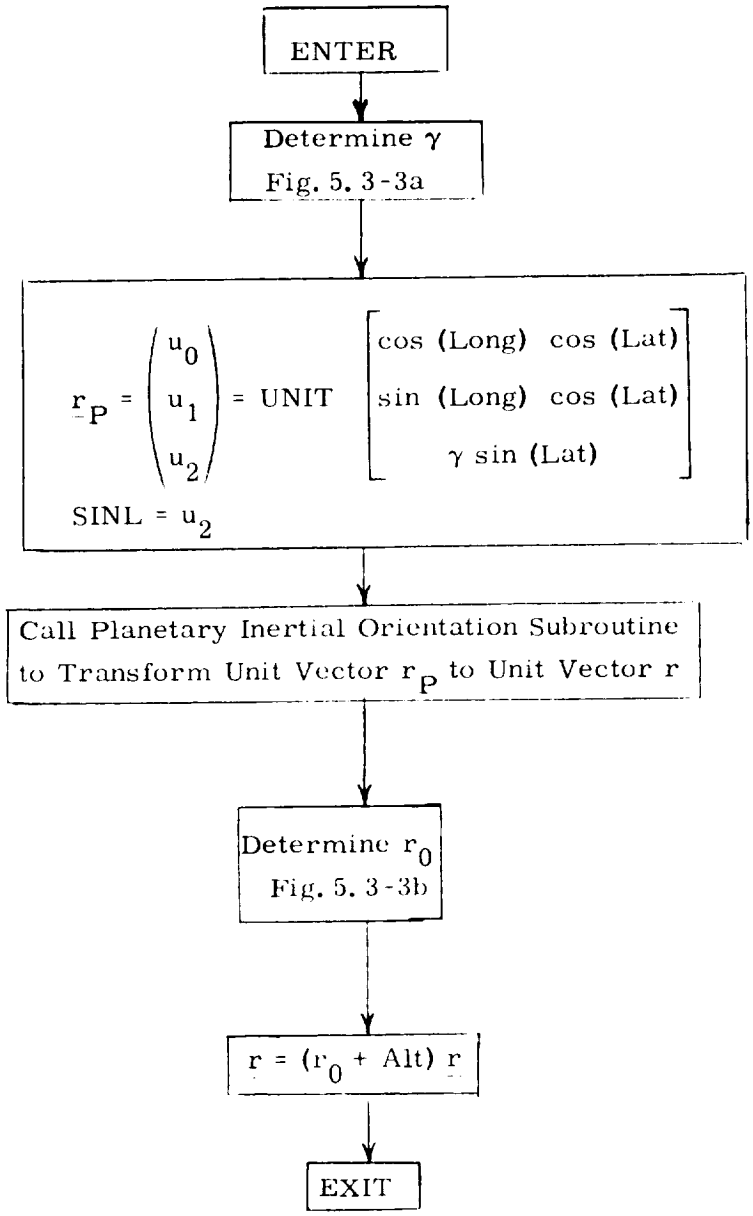



Fig. 5.3-2 Latitude, Longitude, Altitude to Vector Computation Logic Diagram

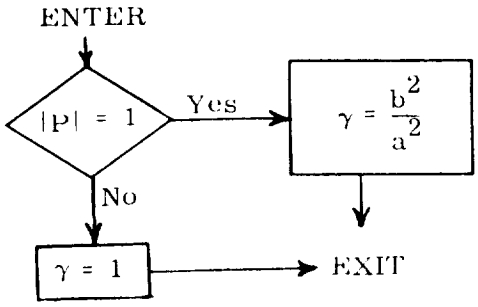



Figure 5.3-3a Determination of $\gamma$

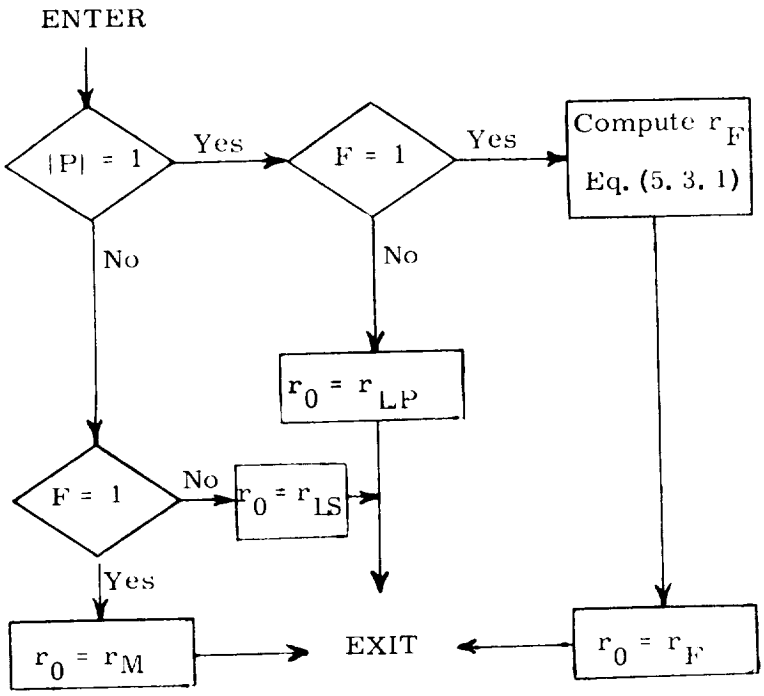



Figure 5.3-3b Determination of $r_0$

## 5.5.4 LUNAR AND SOLAR EPHEMERIDES

This subroutine is used to determine the position and velocity vectors of the sun and the moon relative to the earth. The position vectors of the moon and the sun are needed by the Coasting Integration Routine to compute gravity perturbations (Section 5.2.2.3). The velocity of the moon is used by the Coasting Integration Routine when a change in the origin of the coordinate system is performed at the sphere of influence of the moon (Fig. 2.2-3). The velocity of the sun is required, but not very accurately, to compute aberration corrections to optical sightings.

The position of the moon is stored in the computer in the form of a ninth-degree polynominal approximation which is valid over a 14.5 day interval beginning at noon ephemeris time on the day of the launch. The following parameters are included in the pre-launch erasable data load:

$t_{M0}$ = the elapsed time between July 1.0, 1968 universal time and the time at the center of the range over which the lunar-position polynominal is valid. The value of $t_{M0}$ will be an integral number of quarter days minus the difference between ephemeris time and universal time.

$\underline{c}_0$ to $\underline{c}_9$ = vector coefficients

Let t be the specified ground elapsed time (GET), and $t_0$ be the elapsed time between July 1.0, 1968 universal time and the time that the computer clock was zeroed. Then, the approximate position and velocity of the moon are computed from

$$t_M = t + t_0 - t_{M0} \tag{5.4.1}$$

$$\underline{r}_{EM} = \sum_{i=0}^{9} \underline{c}_i \, t_M^{\,i} \tag{5.4.2}$$

$$\underline{v}_{EM} = \sum_{i=1}^{9} i \, \underline{c}_i \, t_M^{\,i-1} \tag{5.4.3}$$

The approximate position and velocity of the sun are computed from the following items which are included in the pre-launch erasable data load:

$\underline{r}_{ES0}$, $\underline{v}_{ES0}$ = the position and velocity vectors of the sun relative to the earth at time $t_{M0}$.

$\omega_{ES}$ = the angular velocity of the vector $\underline{r}_{ES0}$ at time $t_{M0}$

Then,

$$\underline{r}_{ES} = \underline{r}_{ES0} \cos(\omega_{ES} t_M) + \left[\underline{r}_{ES0} \times \mathrm{UNIT}(\underline{v}_{ES0} \times \underline{r}_{ES0})\right] \sin(\omega_{ES} t_M) \tag{5.4.4}$$

$$\underline{v}_{ES} = \underline{v}_{ES0} \tag{5.4.5}$$

## 5.5.5 KEPLER SUBROUTINE

The Kepler Subroutine solves for the two body position and velocity vectors at the terminal position given the initial position and velocity vectors and a transfer time to the terminal position.

This section contains information to aid the reader in understanding the less obvious aspects of the Kepler Subroutine block diagram depicted in Figs. 5.5-1 and 5.5-2. The subroutines referred to in these figures are presented in Section 5.5.10 and the nomenclature is found in Section 5.5.1.2.

Prior to entering the Kepler Subroutine, an initial estimate of x can be generated via Eq. (2.2.4) of Section 5.2.2.2 with $x_1 = x'$, $\tau_1 = t_{21}$, and $\tau = t_D$. However, $x'$ and $t_{21}$ are non-zero only if the subroutine is being used repetitively.

Although, theoretically, there is no upper bound on x, the practical bound is set to $x_{MAX0}$ or $x_{MAX1}$ to eliminate non-feasible trajectories and increase the accuracy to which x can be computed. In addition, $\alpha x^2$ has a practical range of $-50 < x^2 < 2\pi$ which determines an independent upper bound on x. The $x_{MAX}$ used, then, corresponds to the smaller of the two values.

The transfer time convergence criterion is approximately the same as the granularity of the time input. Since, for some of the problems to be solved, the sensitivity of time to x is so large that the granularity in x, $\epsilon_x$, produces a change in time which exceeds the granularity in time, it is necessary to introduce $\epsilon_x$ as a redundant convergence criterion.

The Kepler Subroutine, provided the parameter range constraints are satisfied and $t_D$ is less than one period, will always produce a solution.

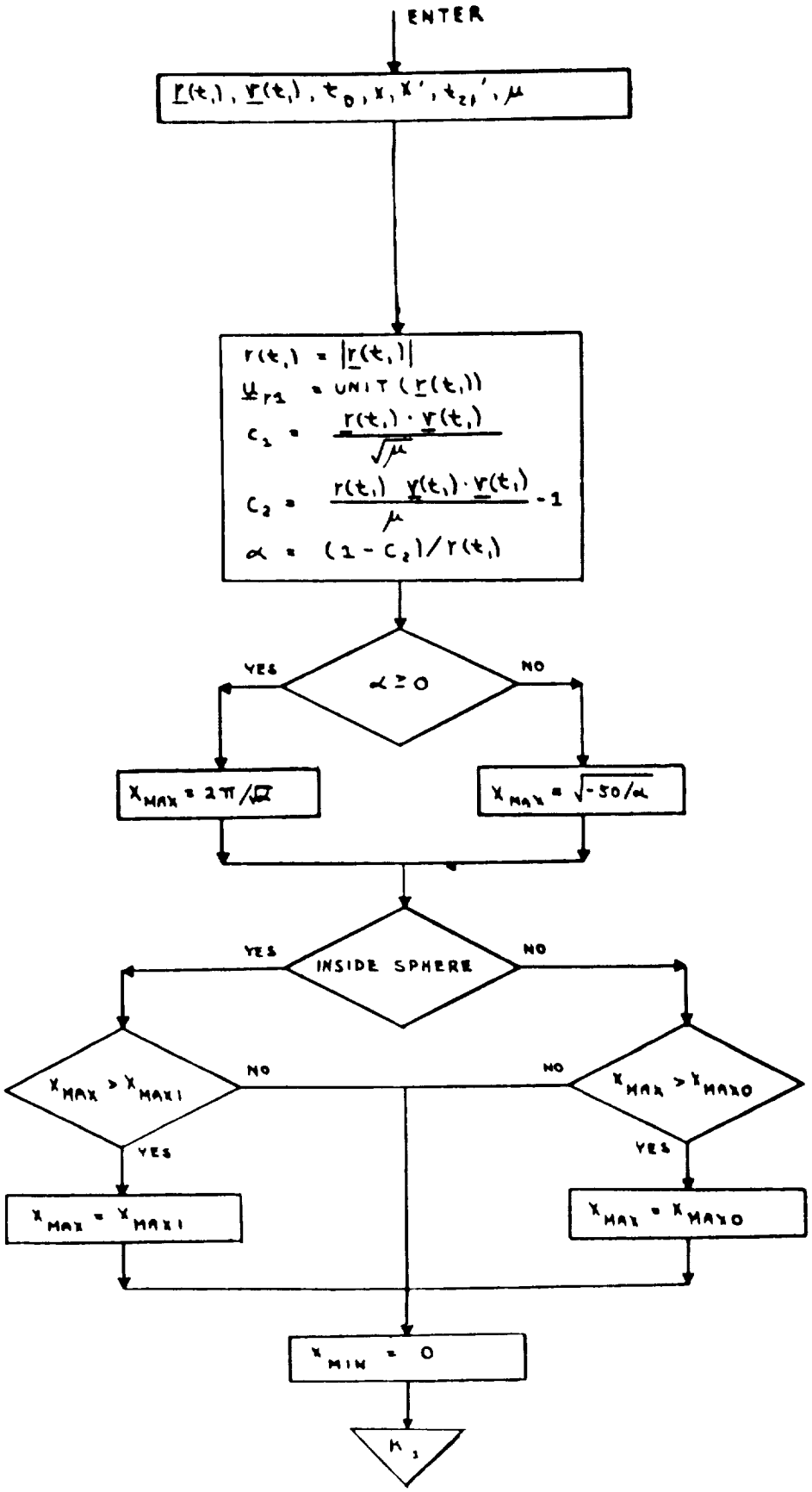



Figure 5.5-1 Kepler Subroutine

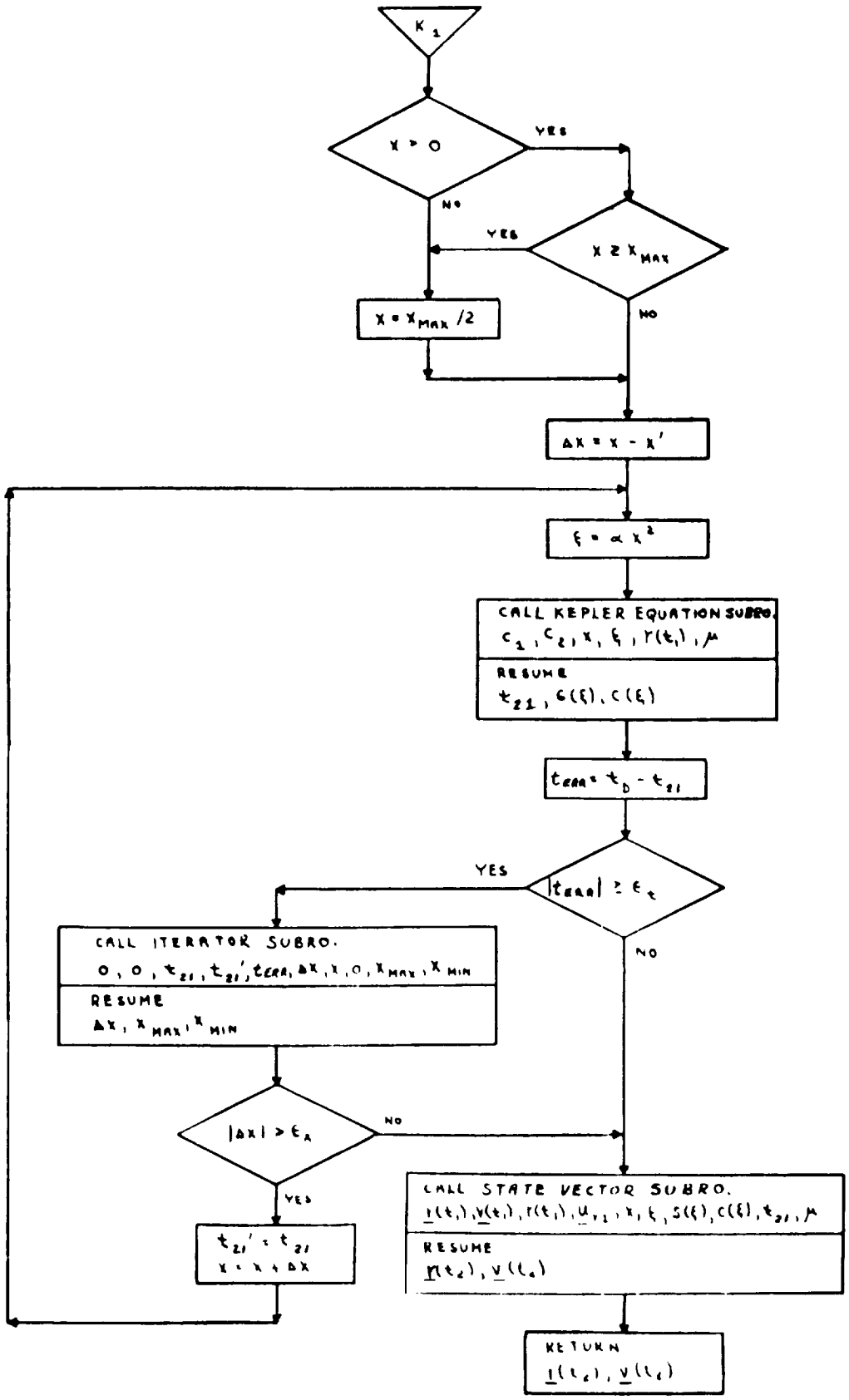



Figure 5.5-2 Kepler Subroutine

## 5.5.6 LAMBERT SUBROUTINE

The Lambert Subroutine solves for the two body initial velocity vector given the initial and terminal position vectors and a transfer time between the two.

This section contains information to aid the reader in understanding the less obvious aspects of the Lambert Subroutine block diagrams depicted in Figs. 5.6-1 and 5.6-2. The subroutines referred to in these figures are presented in Section 5.5.10 and the nomenclature is found in Section 5.5.1.2.

If the Lambert Subroutine is used repetitively and rapid computation is required, the previous value of the independent variable, $\cot \gamma$, can be used as a starting point for the new iteration along with "k", a measure of the quality this guess is expected to have. Flag $f_3$ provides this option.

The Lambert Subroutine computes the normal to the trajectory, $\underline{u}_N$, using the two input position vectors. If these vectors are nearly colinear, it is desirable to specify the normal as an input rather than rely on the ill-defined normal based on the two input position vectors. Flag $f_2$ provides this option. The presence of the inputs in parentheses, therefore, is contingent upon the setting of these flags.

The theoretical bounds on the independent variable, $\cot \gamma$, correspond to the infinite energy hyperbolic path and the parabolic path which closes through infinity. These bounds are dynamically reset by the iterator to provide a more efficient iteration scheme. In addition, if during the course of the iteration, $\cot \gamma$ causes a parameter of the problem to exceed its maximum as determined by its allowable range, the appropriate bound is reset and the iterator continues trying to find an acceptable solution. (This logic does not appear in Figs. 5.6-1 and 2

as it is pertinent only to fixed-point programming). If no acceptable solution is reached, the transfer time input was too small to produce a practical trajectory between the input position vectors. When this happens, $\Delta \cot \gamma$ approaches its granularity limit $\epsilon_x$ before time converges to $\epsilon_t$. However, this same granularity condition exists when the sensitivity problem described in the Kepler Subroutine, Section 5.5.5, occurs In this case an acceptable solution does exist. This dual situation is resolved via a third convergence criterion. If the error in transfer time is greater than $\epsilon_t$ but less than a small percentage, $k_1$, of the desired transfer time and $\Delta \cot \gamma$ is less than $\epsilon_x$, then the solution is deemed acceptable and the required velocity is computed.

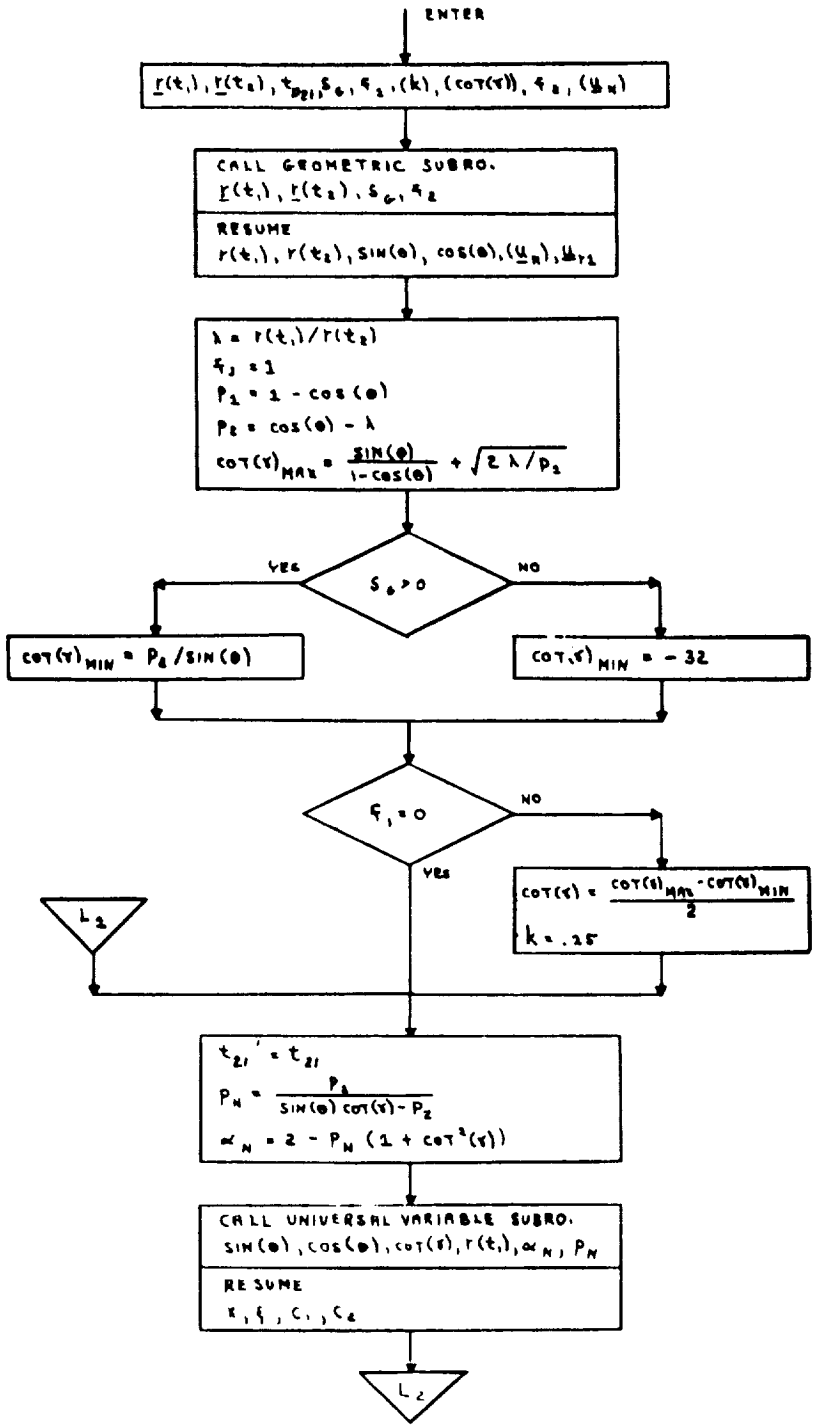



Figure 5.6-1 Lambert Subroutine

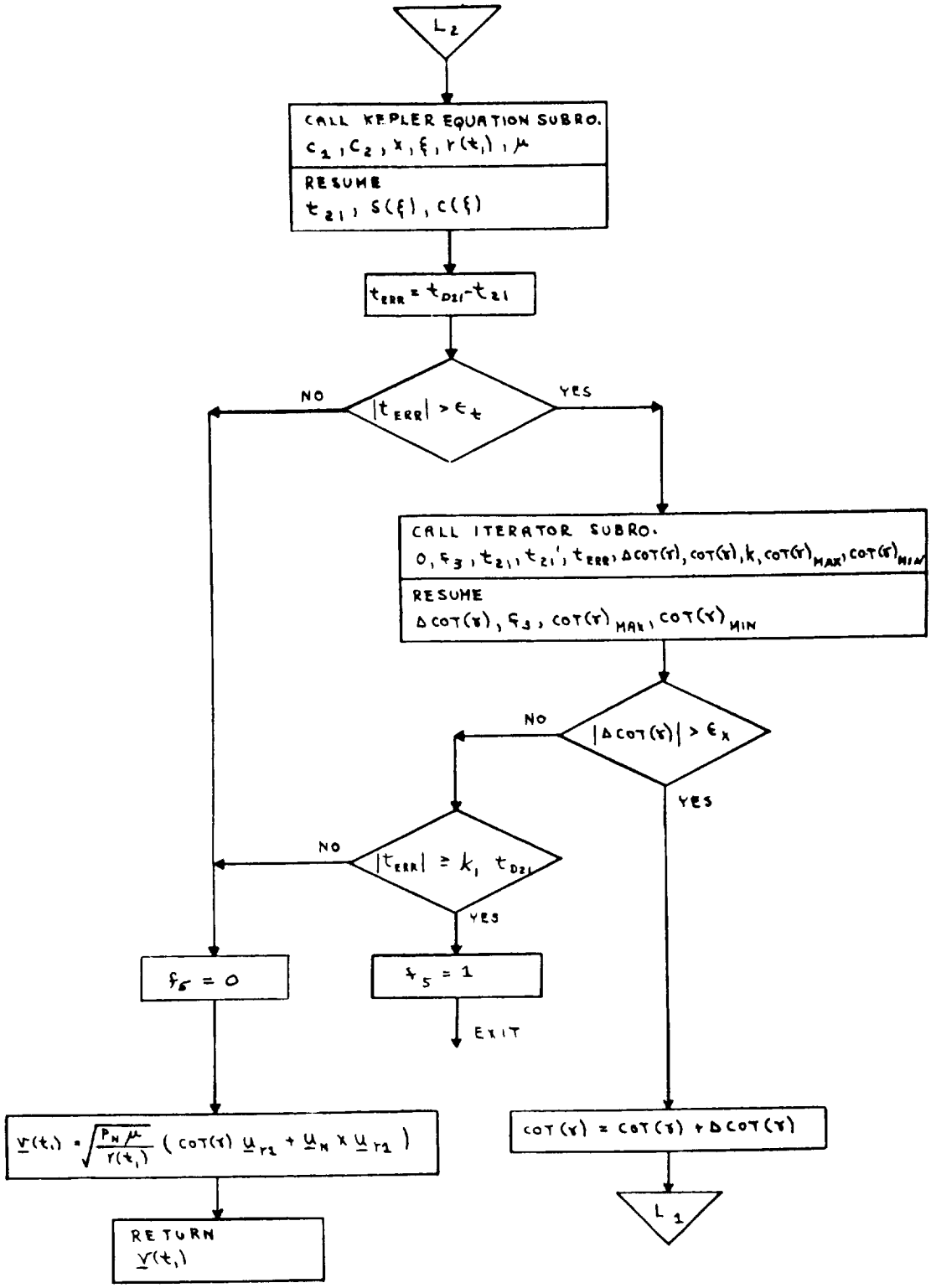



Figure 5.6-2 Lambert Subroutine

## 5.5.7 TIME-THETA SUBROUTINE

The Time-Theta Subroutine solves for the two body transfer time given the initial position and velocity vectors and the true anomaly difference (transfer angle) to the terminal position.

This section contains information to aid the reader in understanding the less obvious aspects of the Time-Theta Subroutine block diagram depicted in Fig. 5.7-1. The subroutines referred to in this figure are presented in Section 5.5.10 and the nomenclature is found in Section 5.5.1.2.

The flag $f_6$ must be zero if the user desires computation of the terminal state vector in addition the transfer time.

If the conic trajectory is a parabola or hyperbola and the desired transfer angle, $\theta$, lies beyond the asymptote of the conic, $f_7$ will be set indicating that no solution is possible.

In addition to the parameter range constraints imposed on Kepler's equation, the additional restriction on Time-Theta that the trajectory must not be near rectilinear is indicated by the range of cot $\gamma$.

The Time-Theta problem is not well defined for near rectilinear trajectories, i e. the transfer angle $\theta$ is no longer a meaningful problem parameter. This will not cause difficulties provided the input variables are within the specified ranges.

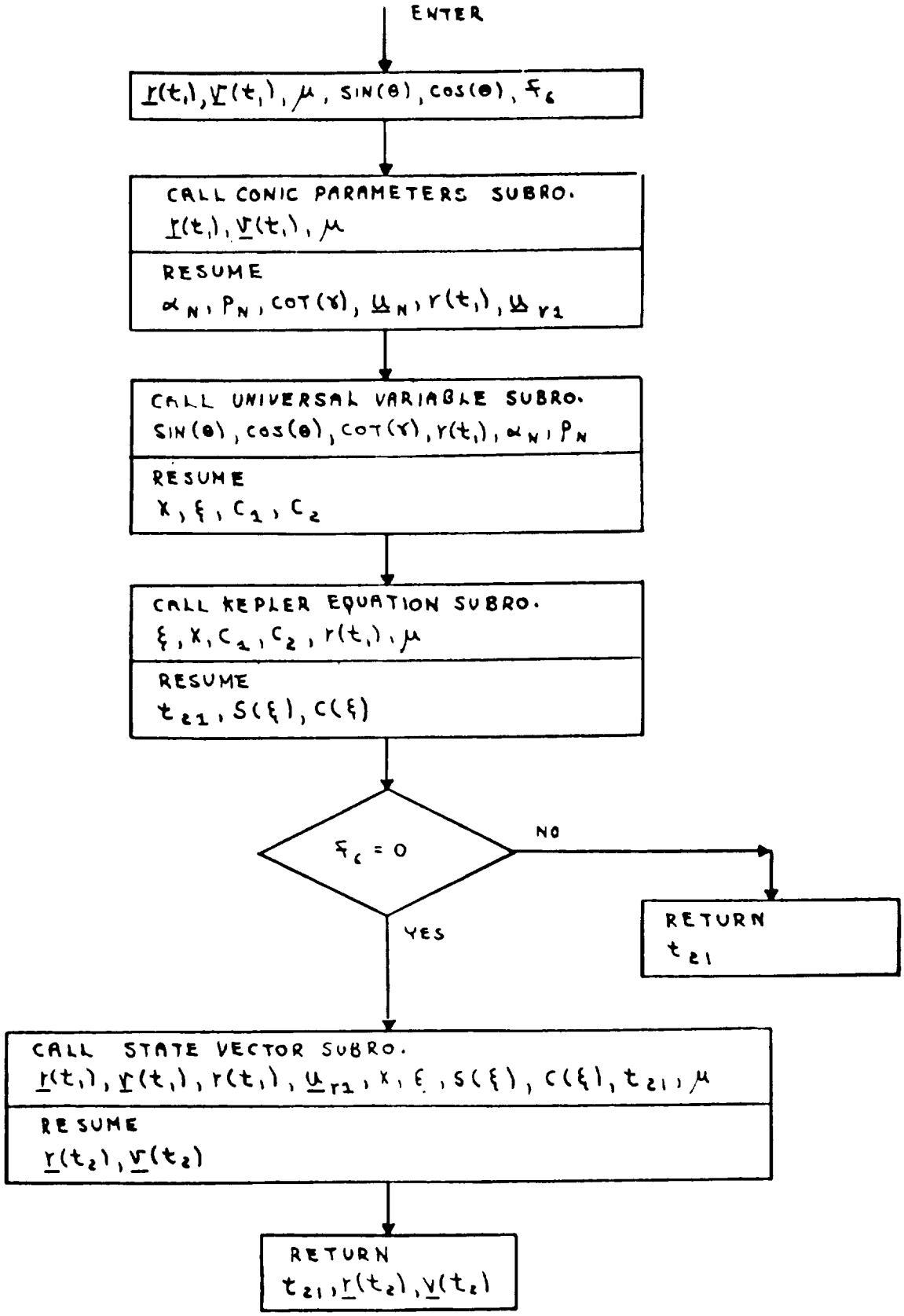



Figure 5.7-1 Time-Theta Subroutine

## 5.5.8 TIME-RADIUS SUBROUTINE

The Time-Radius Subroutine solves for the two body transfer time to a specified radius given the initial position and velocity vectors and the radius magnitude.

This section contains information to aid the reader in understanding the less obvious aspects of the Time-Radius Subroutine block diagrams depicted in Figs. 5.8-1 and 5.8-2. The subroutines referred to in this figure are presented in Section 5.5.10 and the nomenclature is found in Section 5.5.1.2.

Paragraphs 3, 4, and 5 of Section 5.5.7 apply to the Time-Radius Subroutine as well.

Since an inherent singularity is present for the circular orbit case, near-circular orbits result in a loss of accuracy in computing both the transfer time, $t_{21}$, and the final state vector. This is caused by the increasing sensitivity of $t_{21}$ to $r(t_2)$ as the circular orbit is approached

If $r(t_2)$ is less than the radius of pericenter or greater than the radius of apocenter, then $r(t_2)$ will be ignored and the pericenter or apocenter solution, respectively, will be computed. A flag, $f_8$, will be set to indicate this.

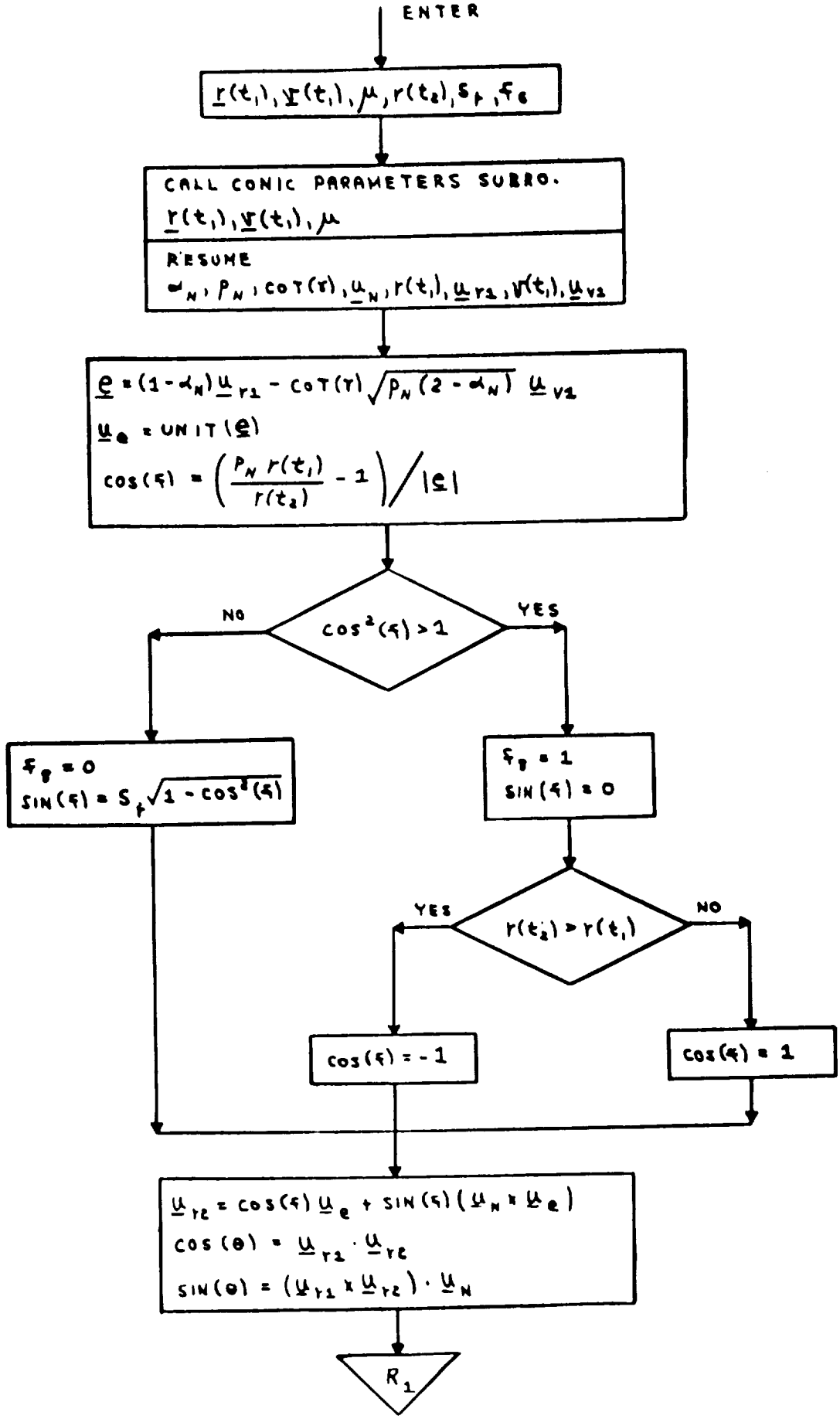



Figure 5.8-1 Time-Radius Subroutine

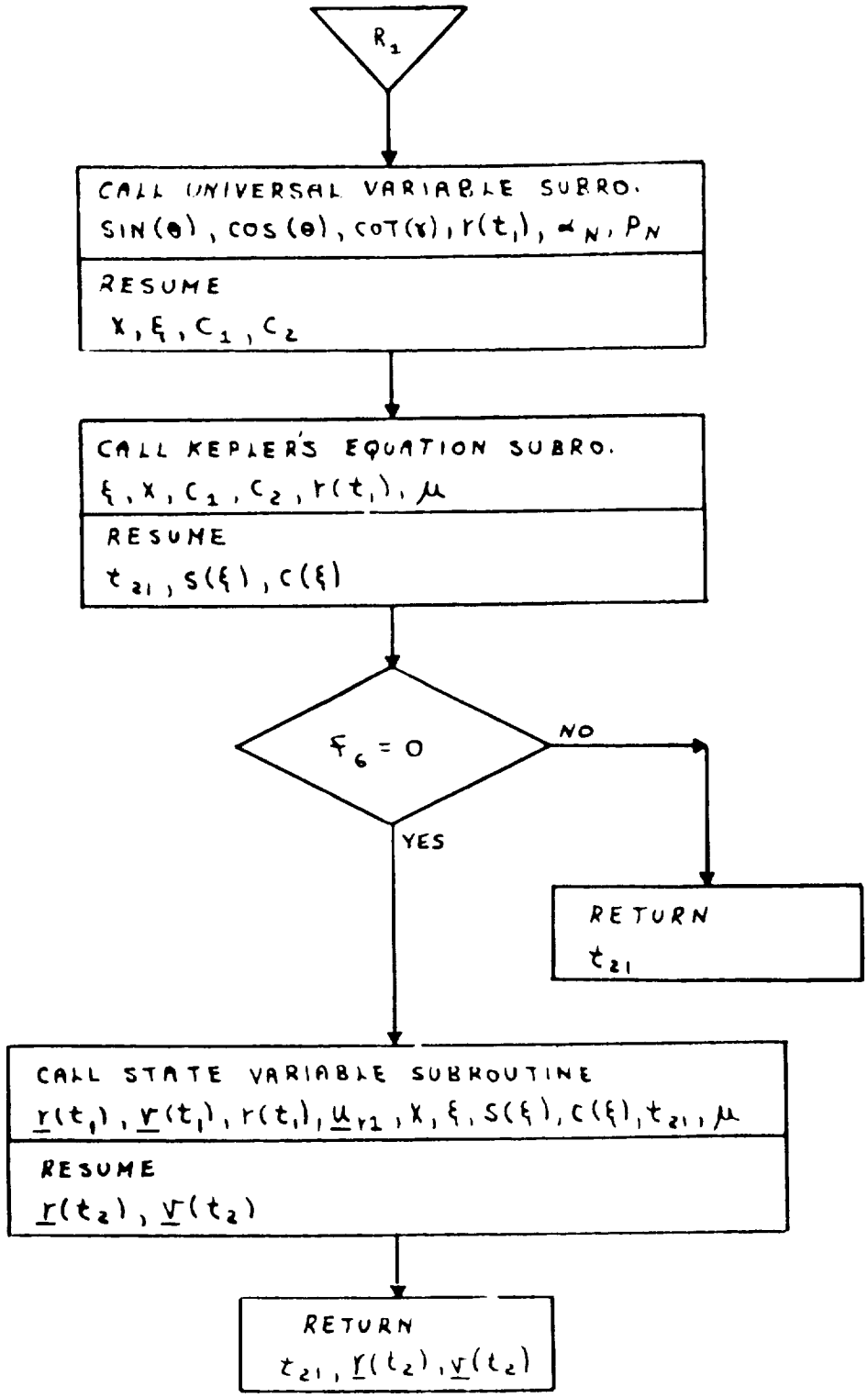



Figure 5.8-2 Time-Radius Subroutine

## 5.5.9 APSIDES SUBROUTINE

The Apsides Subroutine solves for the two body radii of apocenter and pericenter and the eccentricity of the trajectory given the position and velocity vectors for a point on the trajectory.

This subroutine is depicted in Fig. 5.9-1. The subroutines referred to in this figure are presented in Section 5.5.10 and the nomenclature is found in Section 5.5.1.2.

It is characteristic of this computation that the apsides become undefined as the conic approaches a circle. This is manifested by decreasing accuracy. When the conic is nearly parabolic, or hyperbolic, the radius of apocenter is not defined. In this event the radius of apocenter will be set to the maximum positive value allowed.

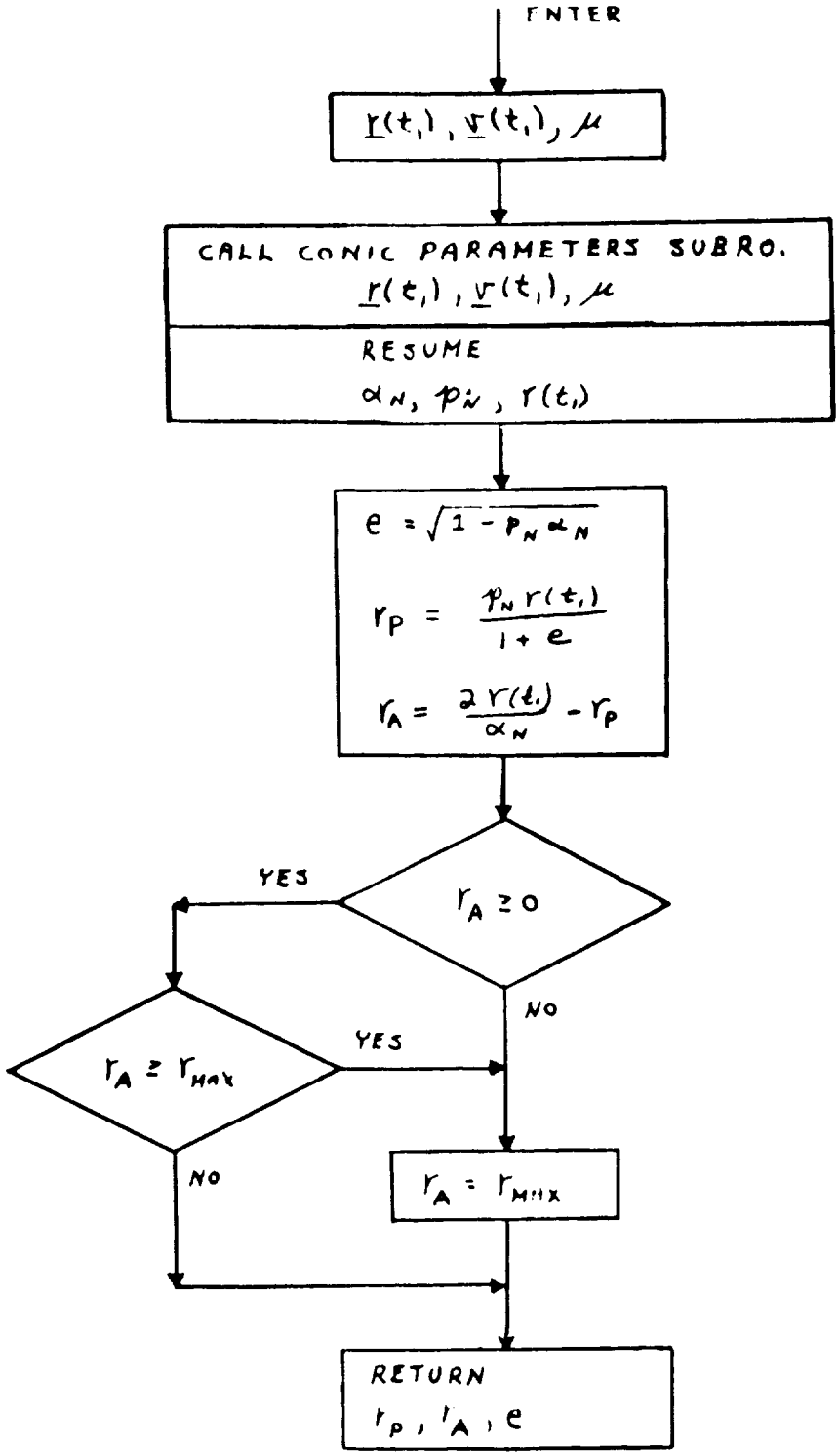



Figure 5.9-1 Apsides Subroutine

## 5.5.10 MISCELLANEOUS SUBROUTINES

There are, as part of the Conic Trajectory Subroutines, three subroutines which are useful in their own right. There are the Conic Parameters, the Geometric Parameters and the Iterator Subroutines which are depicted in Figs. 5.10-1, 5.10-2 and 5.10-3, respectively.

The Conic Parameters and Geometric Parameters Subroutines are self explanatory.

The Iterator Subroutine serves several purposes. It is used when flag $f_4$ is set to zero to solve for the value of the independent variable which drives the error in the dependent variable to zero, provided the function is monotonically increasing. To improve convergence for functions whose derivative changes rapidly, the limits are reset as shown in the block diagram.

With $f_4$ set to 1, the Iterator seeks a minimum of the function, provided the first derivative is single-valued between the limits. The inputs are redefined so that "y" is the derivative of the independent variable with respect to the dependent variable, and "x" is the value at which the derivative was computed or approximated. Since the desired value of y is zero, $y_{ERR} = -y$.

Since the Iterator uses the "Regula Falsi" technique, it requires two sets of variables to begin iteration. If only one set is available, flag $f_3$ must be set to 1, causing the iterator to generate the independent variable increment from a percentage of the full range.

In addition to the above subroutines there are three other subroutines of primary interest to the five basic conic subroutines described in Sections 5. 5. 5 to 5. 5. 9. These are the Universal Variable Subroutine, the Kepler Equation Subroutine and the State Vector Subroutine shown in Figs. 5. 10-4, 5. 10-5 and 5. 10-6 respectively.

The Universal Variable Subroutine is utilized by the Lambert, the Time-Theta and the Time-Radius Subroutines to compute the universal parameter x required for the time equation. There are two different formulations required according to the size of the parameter w.

If the input to the subroutine requires the physically impossible solution that the trajectory "close through infinity", the problem will be aborted, setting flag $f_7$.

The Kepler Equation Subroutine computes the transfer time given the variable x and the conic parameters.

The State Vector subroutine computes the position and velocity vectors at a point along the trajectory given an initial state vector, the variable x and the transfer time.

The final miscellaneous subroutine, the SETMU Subroutine, is depicted in Fig. 5. 10-7. It sets $\mu$ to the appropriate primary body gravitational constant consistent with the estimated CSM state vector as defined in Section 5. 2. 2. 6.

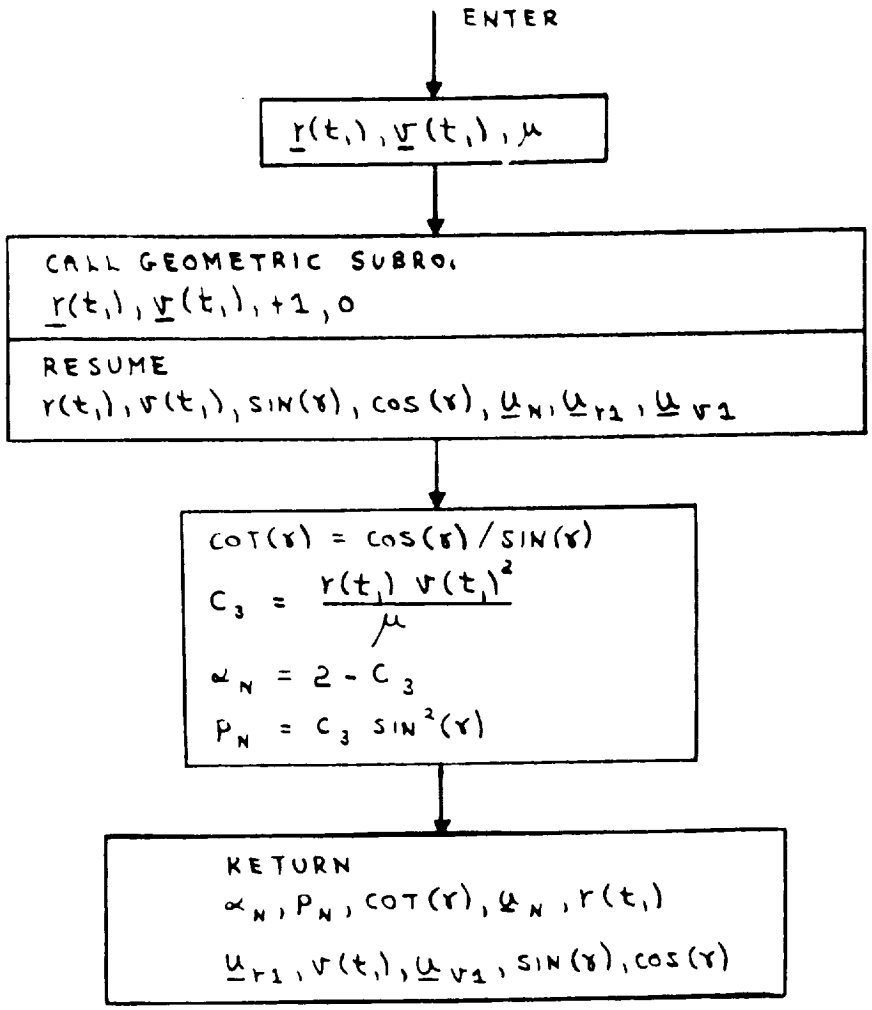



Figure 5.10-1 Conic Parameters Subroutine

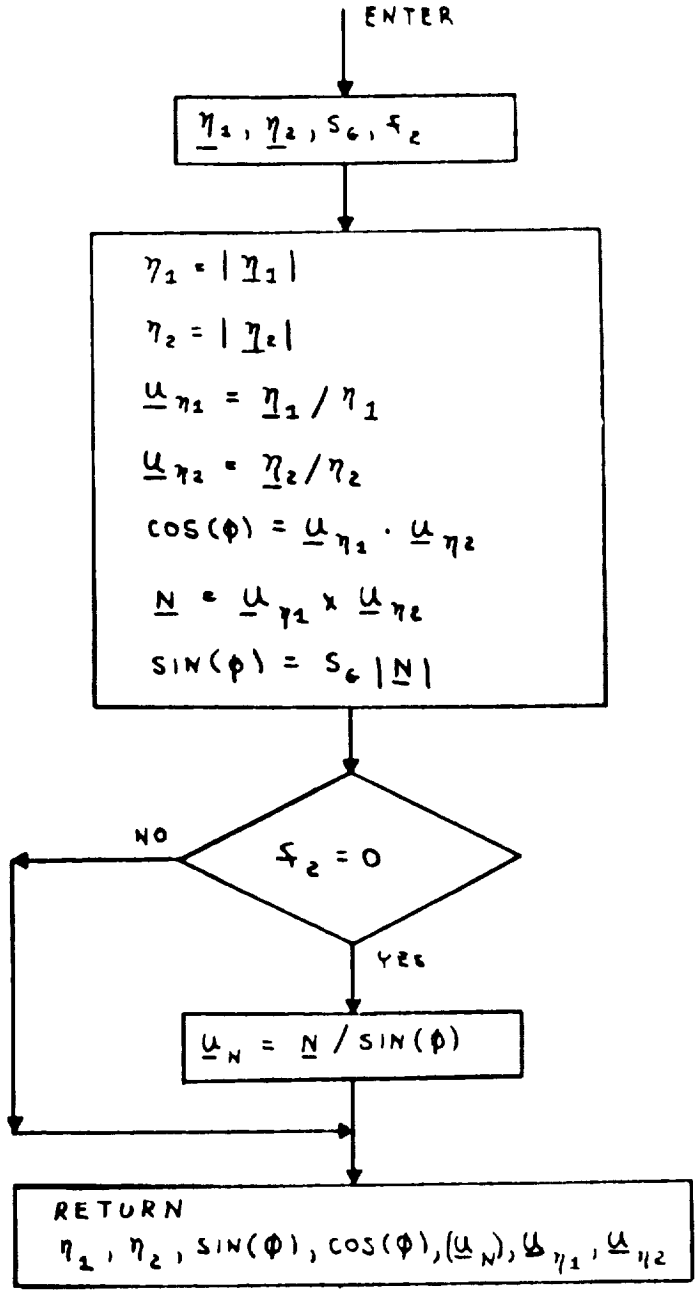
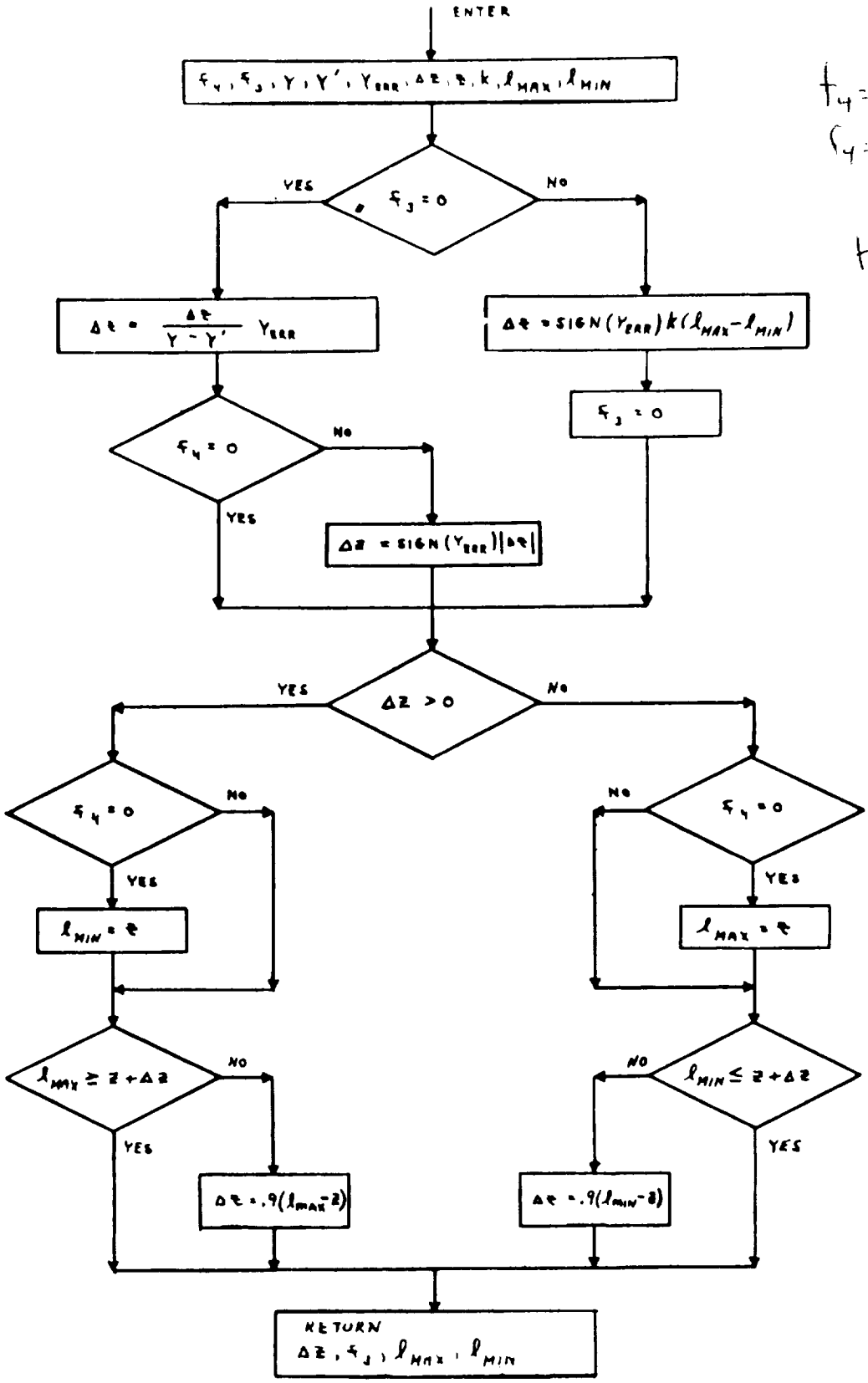



Figure 5.10-2 Geometric Parameters Subroutine




$f_4 = 0$ yes = 0

$f_4 = 1$ ~~yes~~ minimize

$f_3 = 1$ use $\Delta z$ = To $f_3$ range

Figure 5.10-3 Iterator Subroutine

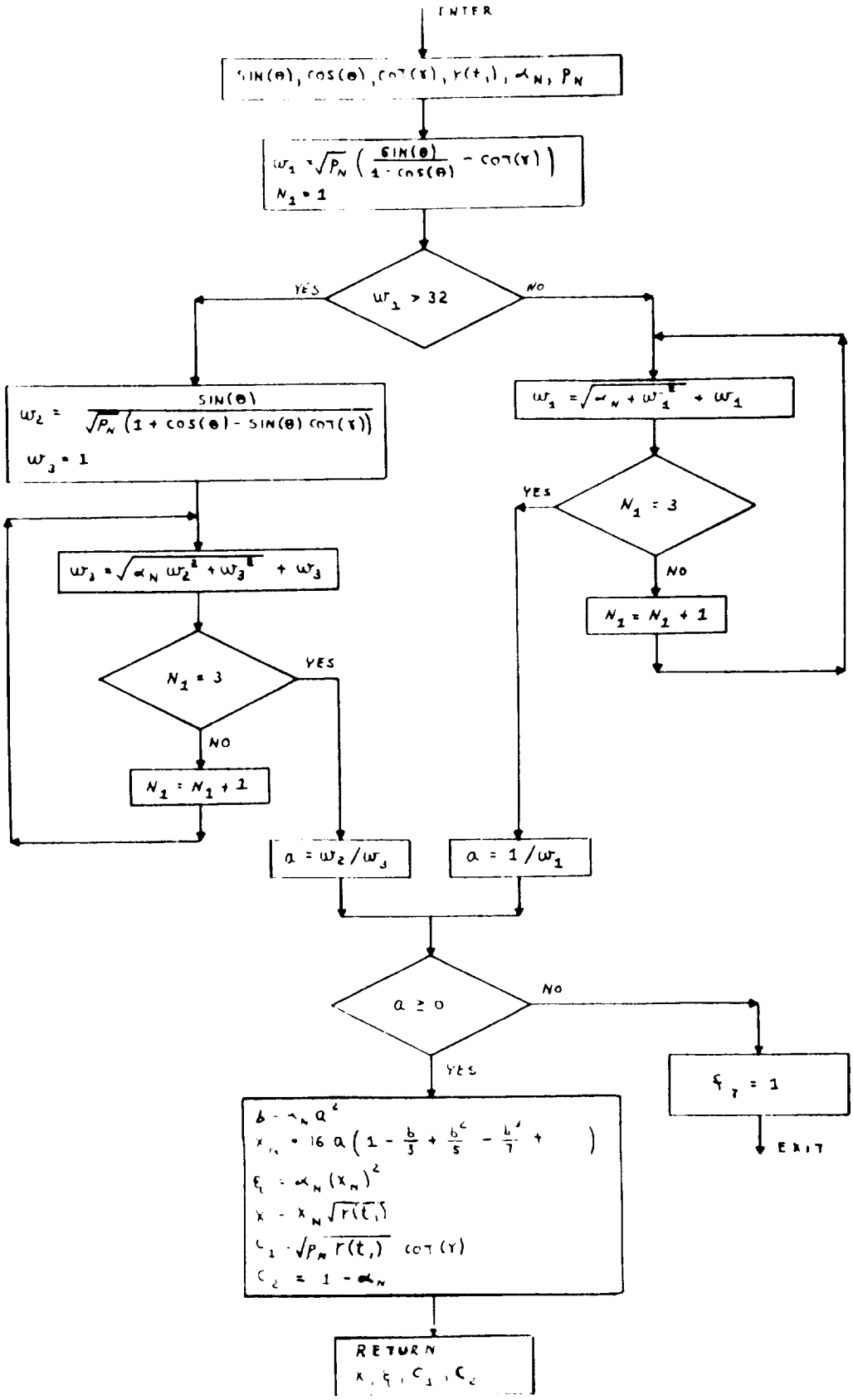



Figure 5.10-4 Universal Variable Subroutine

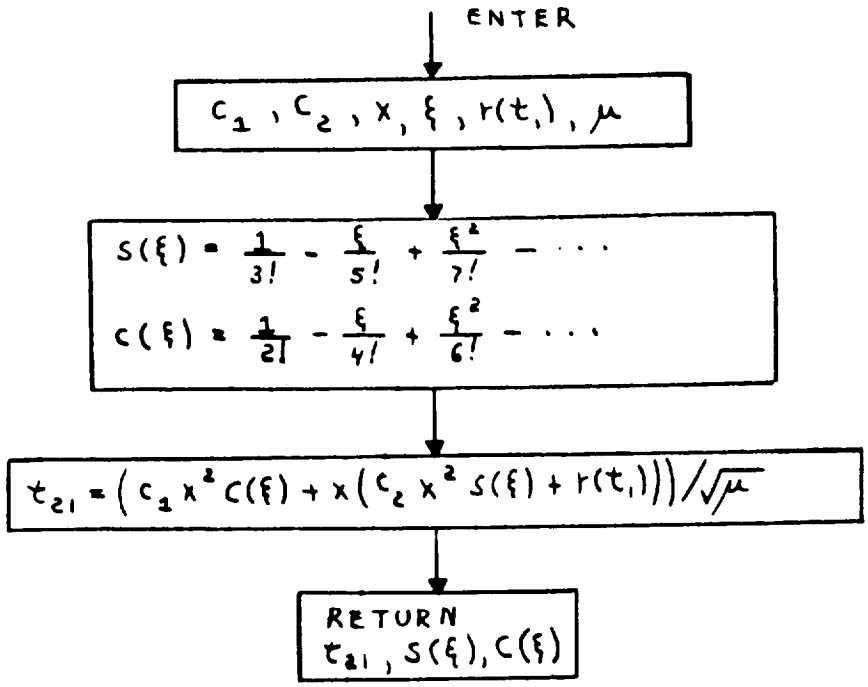



Figure 5.10-5 Kepler Equation Subroutine

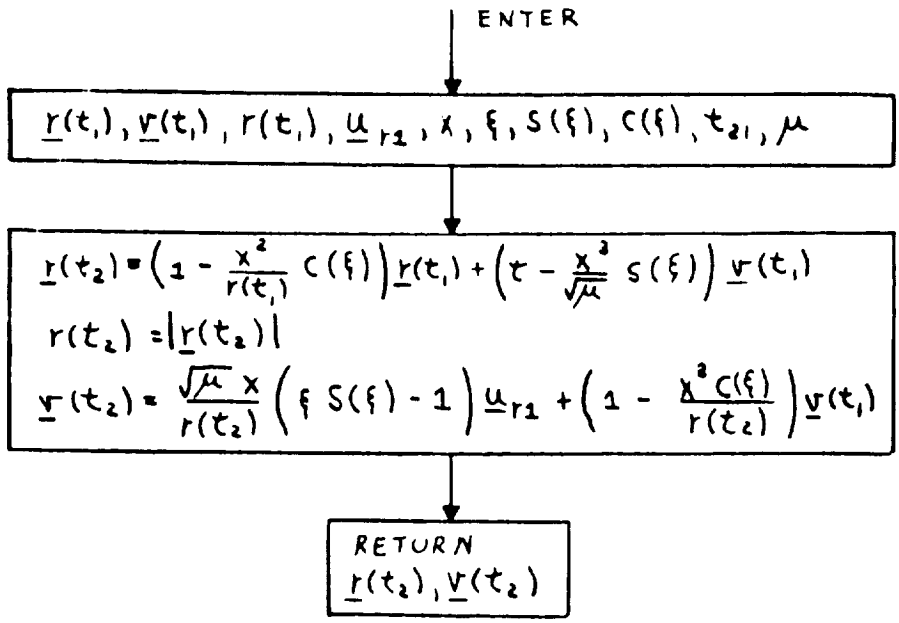



Figure 5.10-6 State Vector Subroutine

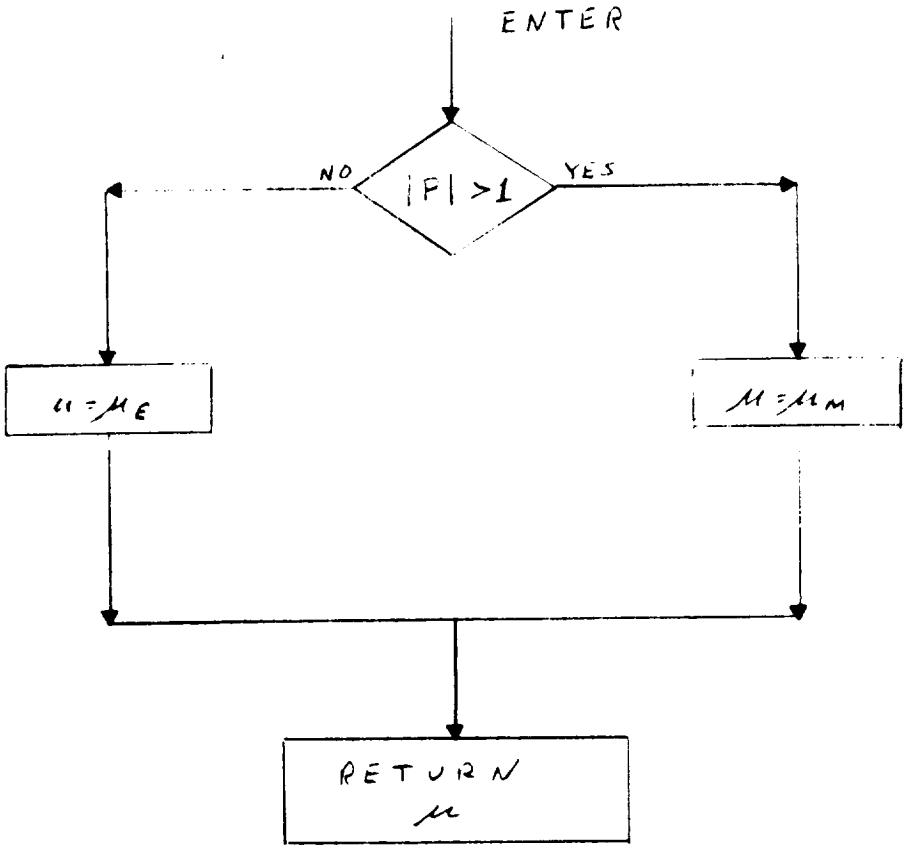



Figure 5.10-7 SETMU Subroutine

## 5.5.11 INITIAL VELOCITY SUBROUTINE

The Initial Velocity Subroutine computes the initial velocity vector for a trajectory of specified transfer time between specified initial and target position vectors. The trajectory may be either two body or precise depending on the number of iterations allowed within the subroutine.

In addition, at the users option, the target vector will be projected into the plane defined by the initial position vector and another arbitrary input vector. This option is exercised when the transfer angle between the initial and target position vectors is within a specified angle measured from 180 degrees.

This subroutine is depicted on Fig. 5.11-1. As shown on the figure the Lambert Subroutine, Section 5.5.6, is utilized to compute the two body initial velocity. The Coasting Integration Subroutine, Section 5.2.2, is utilized in computing the precise trajectory. This is an iterative process. An off-set target vector is used as input to the Lambert Subroutine which then provides the initial velocity for the Coasting Integration Subroutine.

## Nomenclature for the Initial Velocity Subroutine

| | |
|---|---|
| $\underline{r}(t_1)$ | Initial position vector |
| $\underline{v}(t_1)$ | Vector sometimes used to establish the trajectory plane - usually a velocity vector |
| $\underline{r}_T(t_2)$ | Target vector |
| $t_D$ | Transfer time to target vector |
| $N_1$ | Number of iterations allowed |
| $\epsilon$ | Angle to 180 degrees below which the target vector is rotated into an arbitrary plane |
| $f_1$ | See nomenclature in Section 5.5.1.2 |
| K | See nomenclature in Section 5.5.1.2 |
| $\underline{v}_T(t_1)$ | The initial velocity vector of the trajectory passing thru the target |
| $\underline{v}_T(t_2)$ | The velocity vector at the target position |

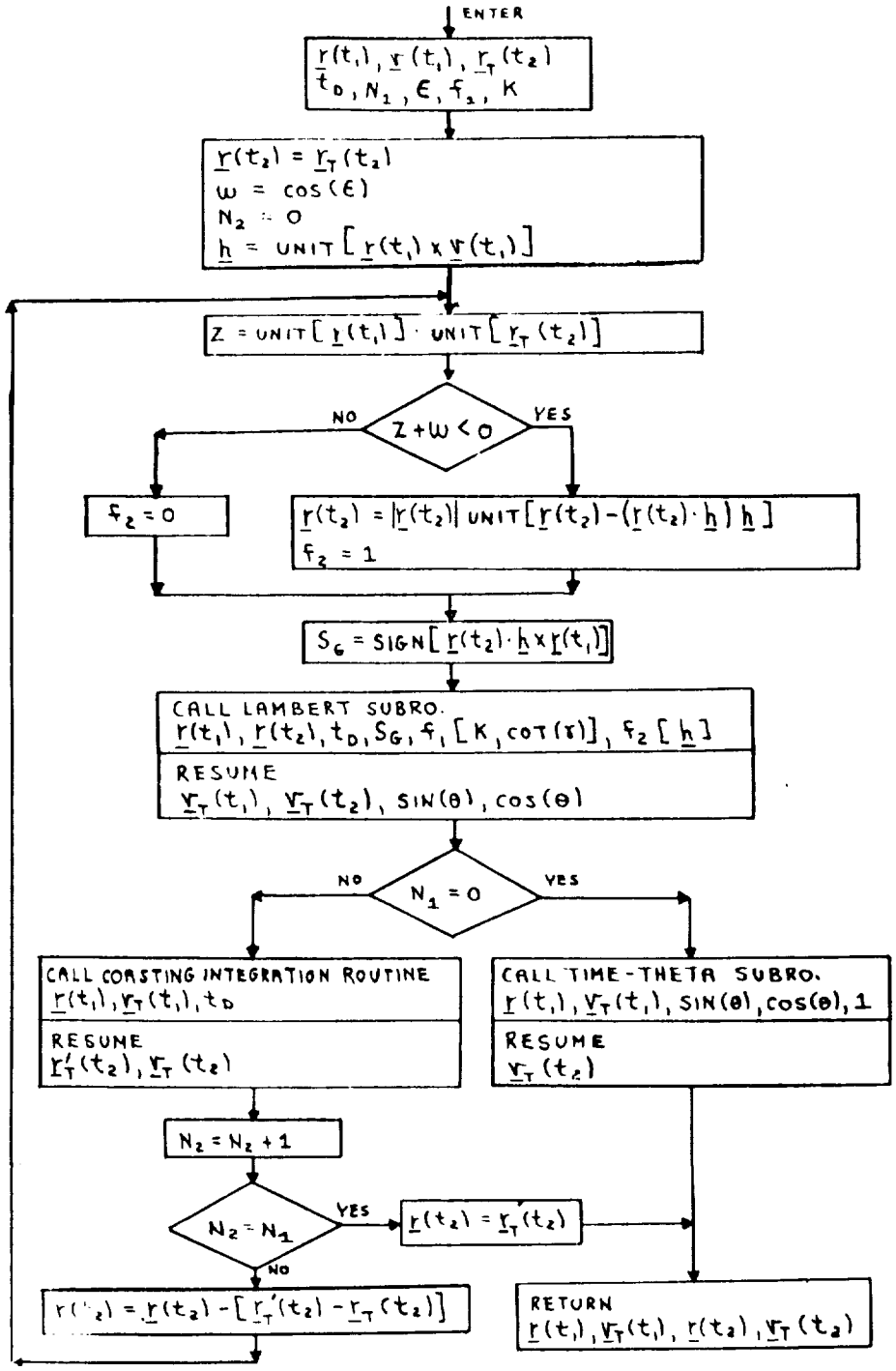



Figure 5.11-1 Initial Velocity Subroutine

## 5.6 GENERAL SERVICE ROUTINES

### 5.6.1 GENERAL COMMENTS

The routines presented in this section include the following general service functions:

1) IMU alignment modes

2) Basic Coordinate Transformations

3) Computer initialization procedures

4) Special display routines which can be called by the astronaut

5) Automatic optics designate routines

## 5.6.2 IMU ALIGNMENT MODES

### 5.6.2.1 IMU Orientation Determination Program

The IMU Orientation Determination Program (P-51) is used during free-fall to determine the present IMU stable member orientation with respect to the Basic Reference Coordinate System by sighting on two navigation stars with the sextant (SXT) or scanning telescope (SCT). These stars must be selected by the astronaut and sighted upon in the manner indicated in the Sighting Mark Routine (R-53) of Section 4. Once the line-of-sight vectors to the two stars have been obtained in stable member coordinates, a test is made by the Star Data Test Routine (R-54 of Section 4) to see if the angle between the two vectors is in close agreement with the angle between the corresponding line-of-sight vectors stored in basic reference coordinates. If the star vectors in stable member coordinates pass the test, they can be used with the same vectors stored in basic reference coordinates to determine the present stable member orientation and REFSMMAT (defined in Section 5.6.3.3) using the procedure given in Section 5.6.3.3.1.

### 5.6.2.2 IMU Realignment Program

The IMU Realignment Program (P-52) is used during free-fall to align the IMU from a known orientation to one of the desired orientations given in Section 5.6.3.3 and in P-52 of Section 4. Initially, the astronaut selects the desired stable member orientation, and the program computes and displays the IMU gimbal angles for the desired stable member orientation using the present vehicle attitude. If the computed gimbal angles are unsatisfactory, the astronaut maneuvers the vehicle to a

more suitable attitude and has the program re-compute and display the new gimbal angles. Once satisfactory angles have been obtained, the Coarse Alignment Routine (R-50 of Section 4) is used to coarse align the IMU to the desired orientation. Afterwards, the astronaut maneuvers the vehicle to a desired attitude for star acquisition and the Star Selection Routine of Section 5.6.4 is used to select two stars. If the Star Selection Routine is unable to find two satisfactory stars at the present vehicle attitude, the astronaut either repeats the above process of changing the vehicle attitude and using the Star Selection Routine, or selects his own stars.

After the stars have been selected, the Automatic Optics Positioning Routine (R-52) is used to point the CSM optics at each star, and the Sighting Mark Routine (R-53) is used with the CSM optics to perform the star sightings. Afterwards, the Star Data Test Routine (R-54) is used to check the angle between the two measured star directions, and the Gyro Torquing Routine (R-55) is used to torque the IMU stable member to the desired orientation.

#### 5.6.2.3 IMU Orientation Determination Backup Program

The IMU Orientation Determination Backup Program (P-53) provides the capability of using some other optical device other than the sextant or scanning telescope to sight on stars for purposes of IMU orientation determination. An optical device which is considered to be a backup to the normal CSM optics is the Crew Optical Alignment Sight (COAS).

This program is the same as program P-51 except that the Alternate LOS Sighting Mark Routine (R-53) is used in place of the Sighting Mark Routine (R-55) to sight on each star. When routine R-56 is called by program P-53, the equivalent shaft and trunnion angles of the sighting axis of the backup optical device are displayed to the astronaut, if they were previously stored in the appropriate erasable memory locations. If the astronaut is not satisfied with these angles, he may load the desired ones. Afterwards, the astronaut maneuvers the spacecraft until the star is along the sighting axis and keys in "ENTER", instead of depressing the MARK button as is done when using the normal CSM optics. The routine then stores the present IMU CDU angles and the two optics angles (shaft and trunnion) previously mentioned. If the astronaut is not satisfied with the star sighting, he may repeat it. Once a satisfactory star sighting has been made, the astronaut loads the appropriate star code and repeats the above process on the second star. The remaining steps in program P-53 are the same as previously given for program P-51 in Section 5.6.2.1.

For each star sighting made with the Alternate LOS Sighting Mark Routine, the unit vector defining the LOS to that star in stable member coordinates is:

$$\underline{s}_{SM} = \left[NBSM\right] \left[SBNB\right] \begin{pmatrix} \sin TA \cos SA \\ \sin TA \sin SA \\ \cos TA \end{pmatrix} \quad (6.2.1)$$

where $\left[NBSM\right]$ is the transformation matrix in Section 5.6.3.2.1 using the stored IMU CDU angles, $\left[SBNB\right]$ is the transformation matrix in Eq. 6.3.1, and SA and TA are the equivalent shaft and trunnion angles, respectively, of the backup optical device.

### 5.6.2.4 IMU Realignment Backup Program

The IMU Realignment Backup Program (P-54) provides the capability of using some other optical device other than the sextant or scanning telescope to sight on stars for purposes of IMU realignment.

This program is the same as program P-52 except that the Alternate LOS Sighting Mark Routine (R-56) is used in place of the Sighting Mark Routine (R-53) to sight on each star. In addition, the Automatic Optics Positioning Routine (R-52) is not used in this program. A description of the Alternate LOS Sighting Mark Routine is given in Section 5.6.2.3 and in routine R-56 of Section 4.

## 5.6.3 IMU ROUTINES

### 5.6.3.1 Sextant Transformations

#### 5.6.3.1.1 Sextant Base - Navigation Base

To transform a vector from sextant base to navigation base coordinates, use is made of the following transformation matrix:

$$\left[SBNB\right] = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix} \qquad (6.3.1)$$

where $\alpha = 32^{\circ}\ 31'\ 23.19''$

The above matrix and its transpose $\left[NBSB\right]$ are both present in computer memory.

The sextant base coordinate system referred to above is frequently called the Block I navigation base coordinate system in deference to the present navigation base coordinate system (Block II). Any transformation of vectors between the sextant and the present navigation base requires use of the above transformations.

5.6.3.1.2 Sextant to Navigation Base (SXTNB)

To obtain a unit vector ($\underline{u}_{NB}$), which specifies the direction of the line-of-sight of the sextant in navigation base coordinates, use is made of the following:

$$\underline{u}_{NB} = [SBNB] \begin{pmatrix} \sin TA \ \cos SA \\ \sin TA \ \sin SA \\ \cos TA \end{pmatrix} \qquad (6.3.2)$$

where $[SBNB]$ is the transformation matrix given in Section 5.6.3.1.1, and TA and SA are the trunnion (precision) and shaft angles, respectively, of the sextant.

5.6.3.1.3 Calculation of Sextant Angles (CALCSXA)

Given a unit star vector $\underline{s}_{SM}$ in stable member coordinates, this routine computes the angles SA and TA required to position the optics such that the line of sight lies along the star vector.

The star vector is first transformed to the sextant base coordinate system as follows:

$$\underline{s}_{SB} = [NBSB] \ [SMNB] \ \underline{s}_{SM} \qquad (6.3.3)$$

where $\left[NBSB\right]$ and $\left[SMNB\right]$ are given in Sections 5.6.3.1.1 and 5.6.3.2.1, respectively.

Next, a unit vector $\underline{u}_{TDA}$, defining the direction of the trunnion (precision) drive axis in sextant base coordinates, is obtained:

$$\underline{u}_{TDA} = \text{UNIT}\left[\underline{z}_{SB} \times \underline{s}_{SB}\right] \qquad (6.3.4)$$

where

$$\underline{z}_{SB} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Using $\underline{s}_{SB}$ and $\underline{u}_{TDA}$, the angles SA and TA are determined as follows:

$$\begin{aligned} \sin SA &= \underline{u}_{TDA} \cdot (-\underline{x}_{SB}) \\ \cos SA &= \underline{u}_{TDA} \cdot \underline{y}_{SB} \\ SA &= \text{ARCTRIG}(\sin SA, \cos SA) \\ TA &= \sin^{-1}\left[(\underline{u}_{TDA} \times \underline{z}_{SB}) \cdot \underline{s}_{SB}\right] \end{aligned} \qquad (6.3.5)$$

where ARCTRIG implies computing the angle, choosing either $\sin^{-1}$ or $\cos^{-1}$ so as to yield maximum accuracy, and

$$\underline{x}_{SB} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \underline{y}_{SB} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

### 5.6.3.2 IMU Transformations

#### 5.6.3.2.1 Stable Member-Navigation Base

Let IGA, MGA, OGA be the IMU inner, middle and outer gimbal angles, respectively. Define the following matrices:

$$Q_1 = \begin{pmatrix} \cos \text{IGA} & 0 & -\sin \text{IGA} \\ 0 & 1 & 0 \\ \sin \text{IGA} & 0 & \cos \text{IGA} \end{pmatrix} \qquad (6.3.6)$$

$$Q_2 = \begin{pmatrix} \cos \text{MGA} & \sin \text{MGA} & 0 \\ -\sin \text{MGA} & \cos \text{MGA} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (6.3.7)$$

$$Q_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos OGA & \sin OGA \\ 0 & -\sin OGA & \cos OGA \end{pmatrix} \qquad (6.3.8)$$

Stable Member to Navigation Base Transformation

$$\underline{U}_{NB} = Q_3 Q_2 Q_1 \underline{U}_{SM}$$

$$(6.3.9)$$

$$\left[SMNB\right] = Q_3 Q_2 Q_1$$

Navigation Base to Stable Member Transformation

$$\underline{U}_{SM} = Q_1^T Q_2^T Q_3^T \underline{U}_{NB}, \quad \left[NBSM\right] = Q_1^T Q_2^T Q_3^T$$

$$(6.3.10)$$

5.6.3.2.2 Calculation of Gimbal Angles (CALCGA)

Given a stable member orientation and a navigation base orientation both referred to the same coordinate system, the following procedure is used to compute the corresponding gimbal angles.

$$\underline{a}_{MG} = \text{UNIT}\,(\underline{x}_{NB} \times \underline{y}_{SM})$$

$$\cos \text{OGA} = \underline{a}_{MG} \cdot \underline{z}_{NB}$$

$$\sin \text{OGA} = \underline{a}_{MG} \cdot \underline{y}_{NB}$$

$$\text{OGA} = \text{ARCTRIG}\,(\sin \text{OGA},\ \cos \text{OGA})$$

$$\cos \text{MGA} = \underline{y}_{SM} \cdot (\underline{a}_{MG} \times \underline{x}_{NB})$$

$$\sin \text{MGA} = \underline{y}_{SM} \cdot \underline{x}_{NB} \qquad (6.3.11)$$

$$\text{MGA} = \text{ARCTRIG}\,(\sin \text{MGA},\ \cos \text{MGA})$$

$$\cos \text{IGA} = \underline{a}_{MG} \cdot \underline{z}_{SM}$$

$$\sin \text{IGA} = \underline{a}_{MG} \cdot \underline{x}_{SM}$$

$$\text{IGA} = \text{ARCTRIG}\,(\sin \text{IGA},\ \cos \text{IGA})$$

where the inputs are three vectors along the stable member axes and three vectors along the navigation base axes.

### 5.6.3.2.3 Calculation of Gyro Torquing Angles (CALCGTA)

In the fine align procedure, after the present platform orientation is determined, the torquing angles required to move the platform into the desired orientation must be computed. This is achieved as follows:

Let $\underline{x}_D$, $\underline{y}_D$, and $\underline{z}_D$ be the desired stable member axes referred to present stable member orientation. The rotations are performed in three steps: (1) rotating through $\theta_y$ about the y axis, yielding $\underline{x}'_D$, $\underline{y}_D$, $\underline{z}'_D$; (2) rotating through $\theta_z$ about the z' axis, yielding $\underline{x}''_D$, $\underline{y}'_D$, $\underline{z}'_D$; (3) and finally rotating through $\theta_x$ about the x'' axis, yielding $\underline{x}''_D$, $\underline{y}''_D$, $\underline{z}''_D$. The relevant equations are as follows:

$$\underline{z}'_D = \text{UNIT}\,(-x_{D,\,3},\ 0,\ x_{D,\,1})$$

$$\sin\theta_y = z'_{D,\,1}$$

$$\cos\theta_y = x_{D,\,3} \qquad (6.3.12)$$

$$\theta_y = \text{ARCTRIG}\,(\sin\theta_y,\ \cos\theta_y)$$

$$\sin\theta_z = x_{D,\,2}$$

$$\cos \theta_z = z'_{D,3}\, x_{D,1} - z'_{D,1}\, x_{D,3}$$

$$\theta_z = \text{ARCTRIG}\,(\sin \theta_z, \cos \theta_z)$$

$$\cos \theta_x = \underline{z}'_D \cdot \underline{z}_D \qquad (6.3.12 \text{ cont})$$

$$\sin \theta_x = \underline{z}'_D \cdot \underline{y}_D$$

$$\theta_x = \text{ARCTRIG}\,(\sin \theta_x, \cos \theta_x)$$

The required inputs are the three coordinate axes of the desired stable member orientation referred to the present stable member orientation.

5.6.3.2.4 Coordinate Axes Generator (AXISGEN)

Given two unit vectors (usually star vectors), $s_A$ and $s_B$, expressed in two coordinate systems, denoted by primed and unprimed characters, i.e., $s'_A$, $s'_B$, $s_A$, $s_B$, this routine computes the unit vectors x, y, z which are the primed coordinate system axes referred to the unprimed coordinate system. This is accomplished by defining two ortho-normal coordinate sets, one in each system, in the following manner:

$$
\begin{aligned}
u'_X &= s'_A \\
u'_Y &= \text{UNIT}\,(s'_A \times s'_B) \\
u'_Z &= u'_X \times u'_Y \\
u_X &= s_A \\
u_Y &= \text{UNIT}\,(s_A \times s_B) \\
u_Z &= u_X \times u_Y
\end{aligned}
\qquad (6.3\text{-}13)
$$

The primed coordinate system axes expressed in terms of the unprimed coordinate system axes are:

$$\underline{x} = u'_{X1}\,\underline{u}_X + u'_{Y1}\,\underline{u}_Y + u'_{Z1}\,\underline{u}_Z$$

$$\underline{y} = u'_{X2}\,\underline{u}_X + u'_{Y2}\,\underline{u}_Y + u'_{Z2}\,\underline{u}_Z \qquad (6.3.14)$$

$$\underline{z} = u'_{X3}\,\underline{u}_X + u'_{Y3}\,\underline{u}_Y + u'_{Z3}\,\underline{u}_Z$$

It should be noted that vectors can be transformed from the unprimed to the primed coordinate systems by using the following matrix constructed with the output (Eq. 6.3.14) of AXISGEN:

$$\begin{bmatrix} \underline{x}^T \\ \underline{y}^T \\ \underline{z}^T \end{bmatrix} \qquad (6.3.15)$$

### 5.6.3.3 REFSMMAT Transformations

The matrix required to transform a vector from the Basic Reference Coordinate System to the IMU Stable Member Coordinate System is referred to as REFSMMAT. This matrix can be constructed as follows with the unit vectors $u_{XSM}$, $u_{YSM}$, and $u_{ZSM}$ defining the orientations of the stable member axes with respect to reference coordinates.

$$\text{REFSMMAT} = \begin{bmatrix} u_{XSM}^T \\ u_{YSM}^T \\ u_{ZSM}^T \end{bmatrix} \qquad (6.3.16)$$

#### 5.6.3.3.1 Present REFSMMAT From Star Sightings

The present IMU stable member orientation with respect to the Basic Reference Coordinate System, and the associated REFSMMAT, can be determined by sighting on two navigation stars with the CSM optics. If $\underline{s}'_A$ and $\underline{s}'_B$ are the unit vectors defining the measured directions of the two stars in the present Stable Member Coordinate System, and $\underline{s}_A$ and $\underline{s}_B$ are the unit vectors to the corresponding stars as known in the Basic Reference Coordinate System, then these vectors can be used as the input to the routine AXISGEN (Section 5.6.3.2.4) to obtain the present IMU orientation and REFSMMAT (Eqs. (6.3.14) and (6.3.15)).

5.6.3.3.2 Alignment for Thrusting Maneuvers (Preferred Orientation)

During certain thrusting maneuvers the IMU will be aligned according to the following equations.

$$\underline{u}_{XSM} = \text{UNIT}\,(\underline{x}_B)$$

$$\underline{u}_{YSM} = \text{UNIT}\,(\underline{x}_B \times \underline{r}) \qquad (6.3.17)$$

$$\underline{u}_{ZSM} = \underline{u}_{XSM} \times \underline{u}_{YSM}$$

where $\underline{x}_B$ is the CSM body X-axis and $\underline{r}$ is the CSM position vector.

The associated transformation matrix (REFSMMAT) is given by Eq. (6.3.16).

5.6.3.4.3 Alignment to Local Vertical in Orbit (Nominal Orientation)

The IMU stable member may be aligned to the local vertical at a specified time. For this type of orientation the stable member axes are found from the following.

$$\underline{u}_{XSM} = (\underline{u}_{YSM} \times \underline{u}_{ZSM})$$

$$\underline{u}_{YSM} = \text{UNIT}\,(\underline{v} \times \underline{r}) \qquad (6.3.18)$$

$$\underline{u}_{ZSM} = \text{UNIT}\,(-\underline{r})$$

where $\underline{r}$ and $\underline{v}$ are the position and velocity vectors of the CSM at the specified time.

The REFSMMAT associated with this IMU orientation is found from Eq. (6.3.16).

5.6.3.3.4 Lunar Landing or Launch Orientation

The proper IMU orientation for lunar landing or launch is defined by the following equations:

$$\underline{u}_{XSM} = \text{UNIT}\left\{\underline{r}_{LS}\,[T\,(L)]\right\}$$

$$\underline{u}_{YSM} = u_{ZSM} \times \underline{u}_{XSM} \qquad (6.3.19)$$

$$\underline{u}_{ZSM} = \text{UNIT}\left\{\underline{h}_C \times \underline{r}_{LS}\,[T\,(L)]\right\}$$

where

$$\underline{h}_C = r_C \times \underline{v}_C$$

is the orbital angular momentum vector of the CSM, and $r_{LS}$ is the landing site position vector at the nominal time of lunar landing or launch, T (L), depending on the alignment mode.

The REFSMMAT associated with this IMU orientation is found from Eq. (6.3.16).

### 5.6.4 STAR SELECTION ROUTINE

The Star Selection Routine is used by the IMU Realignment Program (P - 52) to select the best pair of stars for fine alignment of the IMU. The logic diagram for this routine is shown in Fig. 6.4-1.

Each pair from the computer catalog of 37 stars is tested to see if either is occulted by the earth, moon, or sun, and that the angle of separation between them is 40 to 66 degrees. In addition, a check is made to see if both stars of a pair are within 33 degrees of the shaft axis of the sextant (SXT), which is referred to as the SXT viewing cone in Fig. 6.4-1. Although this 66 degree viewing cone is less than the total angular coverage of the SXT, it is used to enhance the probability that a selected pair will remain visible during attitude limit cycle operation of the spacecraft, and to also allow for existing errors in stable member orientation.

The pair of stars passing the above tests and having the largest angular separation are choosen by this routine. If the routine is unable to find a satisfactory pair of stars after testing all combinations, an alarm code is displayed. In this latter case, it is seen in Program P - 52 of Section 4 that the astronaut can repeat the star selection process at a different spacecraft attitude, if he so desires.

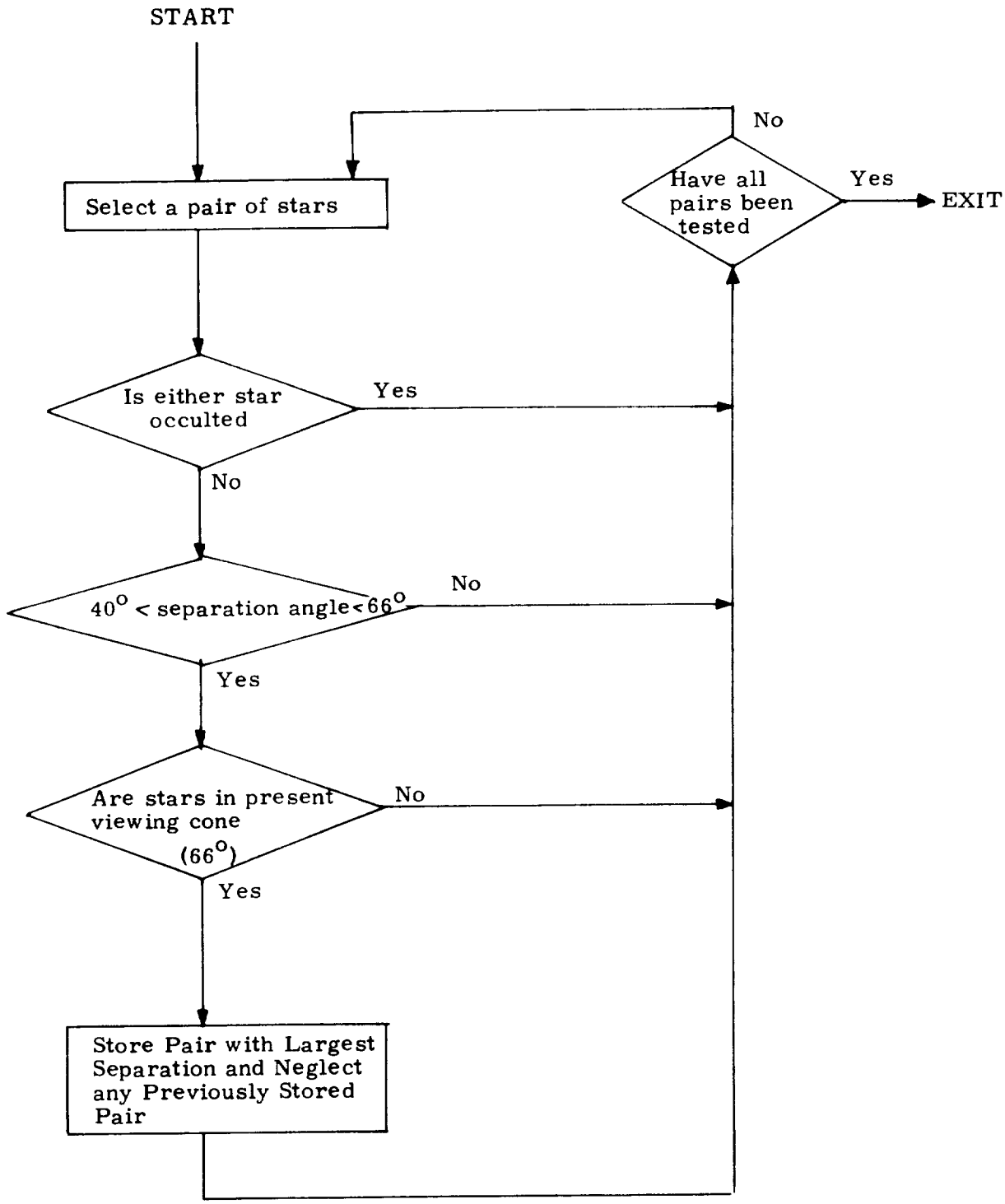



Fig. 6.4-1 Star Selection Routine

### 5.6.5 GROUND TRACK ROUTINE

This routine is used by the astronaut in near-earth or near-moon orbit to obtain CSM or LM trajectory information. The astronaut specifies a time (GET) and a vehicle (CSM or LM). The routine uses the Coasting Integration Routine (Section 5.2.2) to extrapolate the desired vehicle's state vector to the specified time. The resulting estimated position vector is converted to latitude, longitude, altitude coordinates by means of the Latitude-Longitude Subroutine (Section 5.5.3) and these data are displayed. Altitude is defined with respect to the landing site radius for lunar orbit, and the launch pad radius for earth orbit. The astronaut can request the state vector extrapolation to continue in ten minute steps, or to another specified time, and obtain additional displays of the coordinates of points in the spacecraft's orbit.

## 5.6.6 S-BAND ANTENNA ROUTINE

The S-Band Antenna Routine (R - 05) is used to compute and display 2 angles $\gamma$ and $\rho$ defined in Fig. 6.6-1. This routine can be initiated by the astronaut during coasting flight only, when the inertial system is operative and when computer activity is at a minimum. Once the program is initiated the computer will automatically update the display at a rate no less than once every three seconds. This updating will continue until S-Band Lock-on is achieved and signaled to the CMC by the astronaut through the DSKY. If the S-Band Routine is interrupted the display of $\gamma$ and $\rho$ will be stopped until the astronaut reinitiates the routine.

The computational and display accuracy of $\gamma$ and $\rho$ will be $\pm$ 1.0 degrees. This accuracy is only associated with the displayed angles and does not indicate the pointing accuracy of the associated antenna alignment, or the accuracy associated with the antenna gimbal angles.

The angles shown in Fig. 6.6-1 are computed without consideration of any constraints imposed by the actual antenna and therefore, if the present vehicle attitude results in antenna angles beyond the gimbal limit, the CMC will not indicate that the antenna line-of-sight cannot be pointed according to these angles.

The equations and logic flow used to compute $\gamma$ and $\rho$ are shown in Fig. 6.6-2. With reference to the first portion of this figure, the program will automatically determine whether the Basic Reference Coordinate System is earth or moon centered and will compute the proper values of $\gamma$ and $\rho$ for either case.

$\gamma$ is defined as a positive rotation from the plus body X axis about the CSM body Z axis. $(0 \leq \gamma < 360^{\circ})$

$\rho$ is defined as a positive rotation about the Y' axis. Y' is an axis in the CSM body XY plane rotated from the CSM Y body axis about the +Z axis direction an amount $\gamma$. $(-90^{\circ} \leq \rho \leq 90^{\circ})$

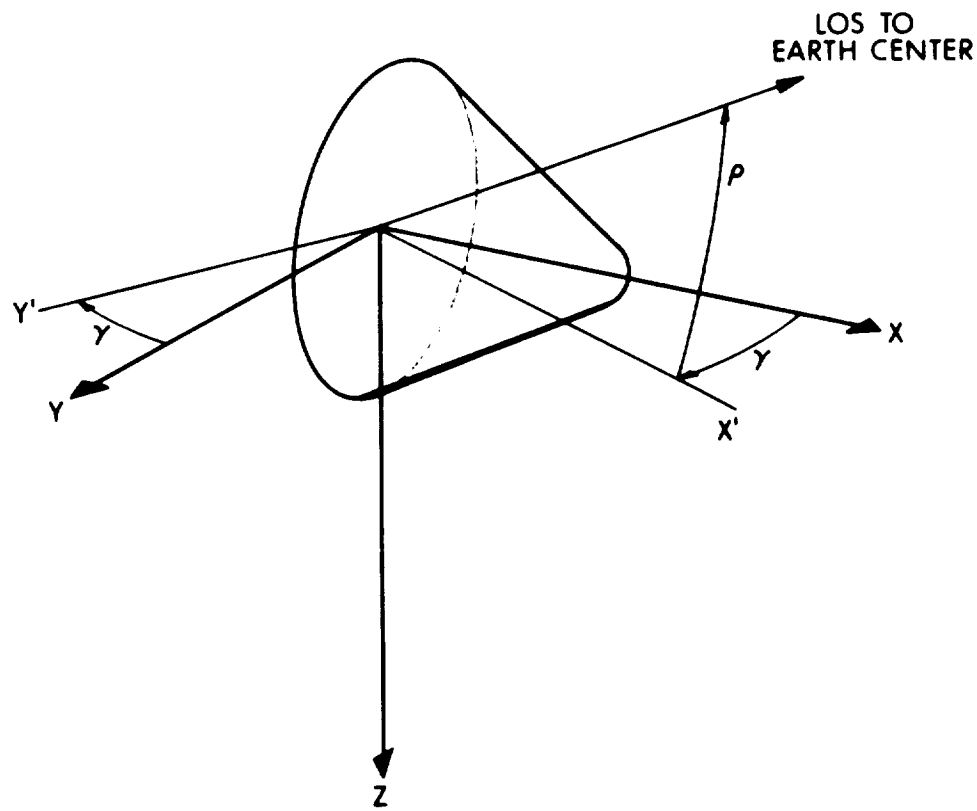



Figure 6.6-1 Definition of CSM S-Band Display Angles

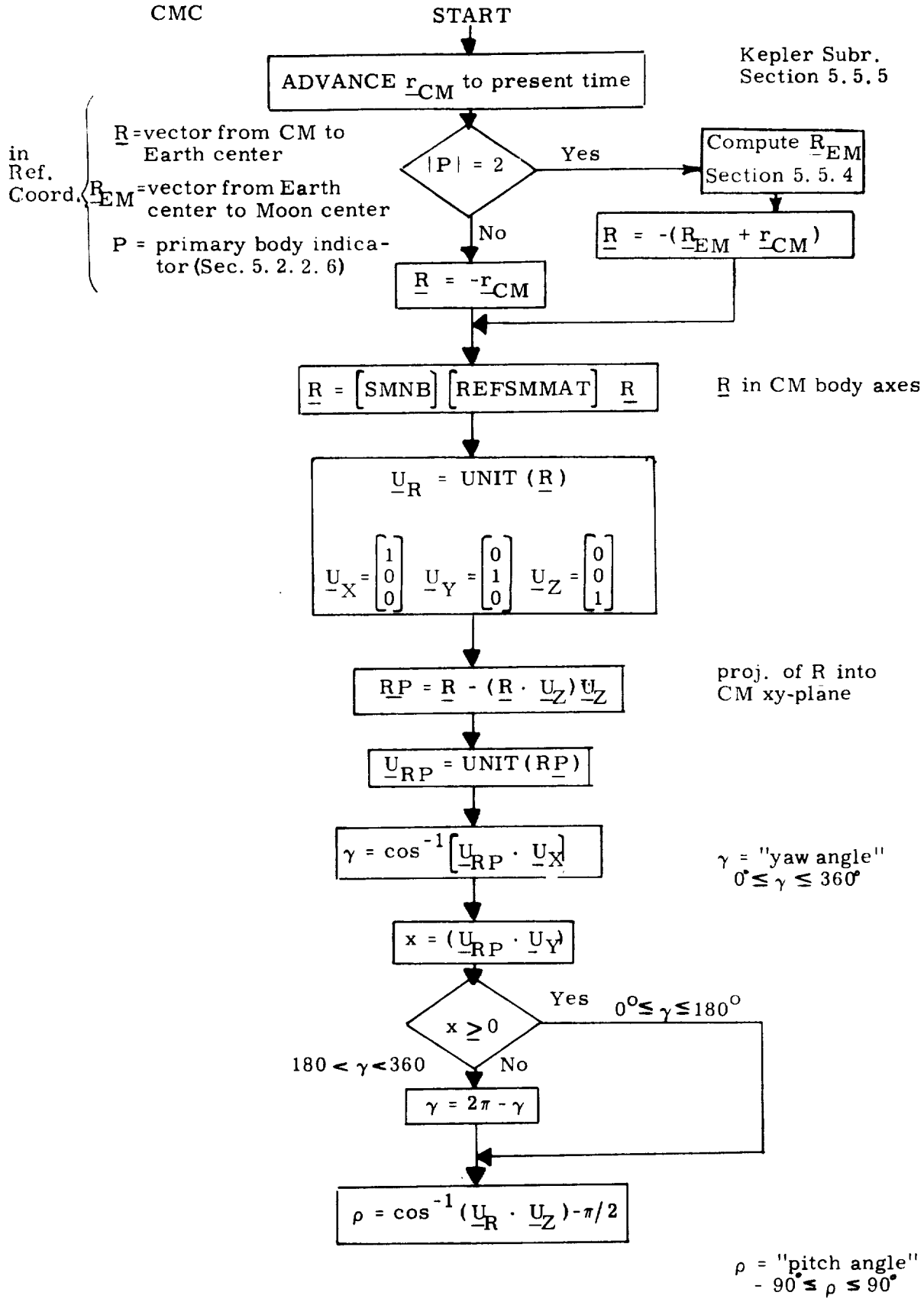



Figure 6.6-2 S-Band Angle Computations and Logic

## 5.6.7 ADDITIONAL RENDEZVOUS DISPLAYS

During the final phases of rendezvous the following three routines may be called by the astronaut for the purpose of computing and displaying special quantities related to the rendezvous geometry.

### 5.6.7.1 Range, Range Rate, Theta Display

Routine R-31 may be called upon to compute and display the range and range rate between the two vehicles and an angle $\theta$ shown in Fig. 6.7-1.

The angle $\theta$ represents the angle between the CSM X-body axis and the local horizontal plane. It, therefore, does not uniquely define the orientation of the X-axis, but only defines a cone above or below the horizontal plane on which the X axis can be found. Theta is defined between $\pm 90^{\circ}$ with the sign positive when the X axis is above the horizontal plane.

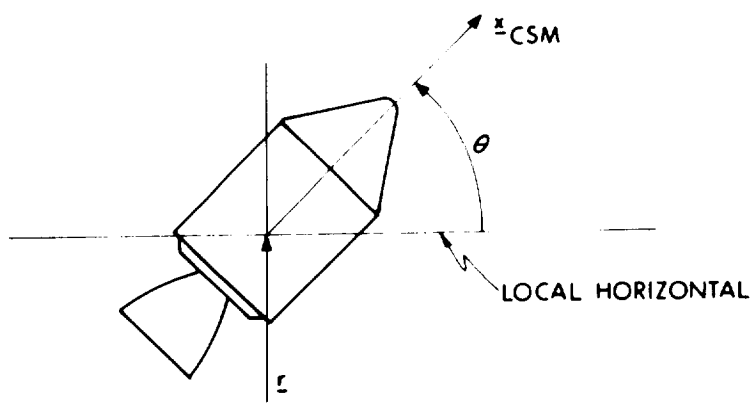



Figure 6.7-1 Definition of Theta

The three displays of R-31 are automatically updated every two seconds until R-31 is terminated by the astronaut.

5.6.7.2 Range, Range Rate, Phi Display

Routine R-34 may be initiation by the astronaut to display the computed range and range rate between the two vehicles as well as an angle $\phi$ shown in Fig. 6.7-2.

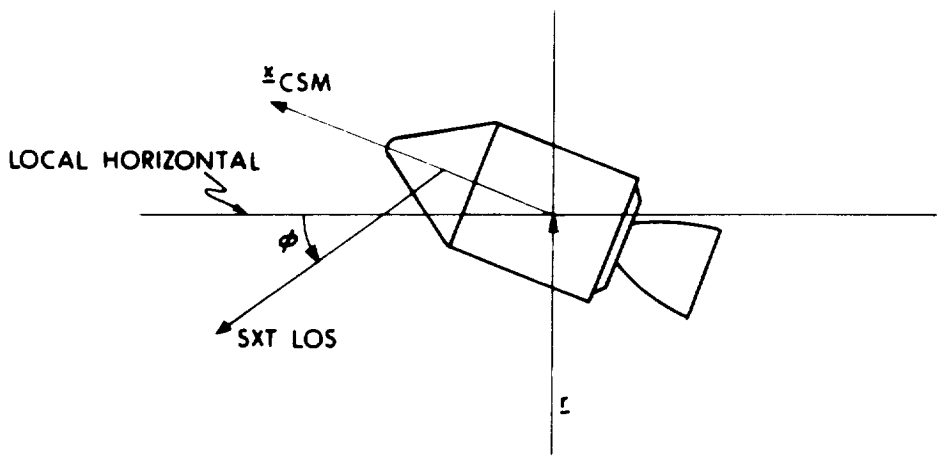



Figure 6.7-2 Definition of Phi

The angle $\phi$ represents the angle between the SXT line of sight and the local horizontal plane. It is analogous to the angle $\theta$ described in Section 5.6.7.1 and the definition and limitations of $\phi$ are the same as those for $\theta$.

The three displays of R-34 are automatically updated every two seconds until the astronaut terminates the routine.

5.6.7.3 Final Attitude Display

Routine R-63 may be used to compute and display the gimbal angles required to point the CSM X-axis at the LM. After initiation of this routine the state vectors of both vehicles are extrapolated to the present time plus one minute. Based on these new state vectors the required gimbal angles are computed and displayed.

There is no automatic display update. However, R-63 may easily be recycled to manually accomplish an update of the display.

## 5.6.8 AUTOMATIC OPTICS DESIGNATE ROUTINE

### 5.6.8.1 General

This routine is used during alignment and navigation procedures to automatically point the optics in the direction of a specified tracking target to aid the astronaut in the acquisition of the target. The four target modes available in this routine are as follows:

1) Star mode
2) LM mode
3) Landmark mode
4) Advanced ground track mode

The calculations and positioning commands in this routine are repeated once per second provided the optics mode switch is set to CMC.

The routine is divided into two timed subroutines, Line-of-Sight and Command. Let $t_n$ be the time (GET) at the beginning of a one second cycle and let

$$t_{n+1} = t_n + 1 \text{ second} \tag{6.8.1}$$

During the first part of the cycle the desired line-of-sight vector at time $t_{n+1}$, $\underline{u}_{LOS}$, is computed. The Line-of-Sight Subroutine sets two WAITLIST*calls; one for time $(t_{n+1} - \delta t)$ for the Command Subroutine, and the other for time $t_{n+1}$ for a repetition of itself. At time $(t_{n+1} - \delta t)$ the Command Subroutine computes updated values of the desired optics angles from the desired line-of-sight vector, $\underline{u}_{LOS}$,

*The WAITLIST is a basic CMC service routine which is used to control the timing of computations and procedures.

and the current vehicle attitude. The entire procedure is then repeated beginning at time $t_{n+1}$. The value of $\delta t$ is chosen to minimize the pointing error at time $t_{n+1}$.

A basic CMC routine drives the optics to the orientation indicated by the desired angles.

### 5.6.8.2 Line-of-Sight Subroutine

#### 5.6.8.2.1 Star Mode

This mode is used during IMU alignment procedures to point the optics in the direction of a specified star. The desired line-of-sight vector, $\underline{u}_{LOS}$, is the specified unit star vector stored in the CMC fixed memory.

#### 5.6.8.2.2 LM Mode

This mode is used during LM tracking phases to point the optics in the direction of the LM. The Kepler Subroutine (Section 5.5.5) is used to compute $\underline{r}_C$ and $\underline{r}_L$, the estimated conic position vectors of the CSM and the LM, respectively, at time $t_{n+1}$. Then, the desired line-of-sight vector is given by

$$\underline{u}_{LOS} = \text{UNIT}\,(\underline{r}_L - \underline{r}_C) \tag{6.8.2}$$

#### 5.6.8.2.3 Landmark Mode

This mode is used during the orbit navigation phase to point the optics in the direction of a landmark specified either by

code number for a stored landmark, or by latitude, longitude, altitude for a landmark whose coordinates are not stored in the CMC memory. The Kepler Subroutine (Section 5. 5. 5) is used to compute $\underline{r}_C$, the estimated conic position vector of the CSM at time $t_{n+1}$. Then, $\underline{r}_\ell$, the estimated position of the landmark at time $t_{n+1}$ is computed by means of the Latitude-Longitude Subroutine (Section 5. 5. 3). Finally, the desired line-of-sight is given by

$$\underline{u}_{LOS} = \text{UNIT}\ (\underline{r}_\ell - \underline{r}_C) \tag{6. 8. 3}$$

5. 6. 8. 2. 4 Advanced Ground Track Mode

This mode is used during the lunar-orbit navigation phase to aid in the surveillance, selection and tracking of possible landing sites by driving the optics to the direction of a point on the ground track of the CSM at a time slightly more than a specified number of orbital revolutions from current time. Let the specified revolution number be N, and assume that N is no greater than four.

Let $\underline{r}_C$ be the estimated CSM position vector at time $t_{n+1}$. The local vertical vector, $\underline{u}_{LV}$, at time $t_{n+1}$ is given by

$$\underline{u}_{LV} = -\ \text{UNIT}\ (\underline{r}_C) \tag{6. 8. 4}$$

The desired line-of-sight vector, $\underline{u}_{LOS}$, is determined such that the angle between the vectors $\underline{u}_{LOS}$ and $\underline{u}_{LV}$ is approximately $60^\circ$. Thus, the astronaut can survey the advanced ground track of the CSM, and select a landing site over which the CSM will pass in N orbital revolutions plus about 1/2 (5 - N) minutes. After the astronaut selects the landing site, he switches the optics mode to MANUAL in order to obtain navigation data as described in Section 5. 2. 4. 1.

The value of the angle between the vectors $\underline{u}_{LOS}$ and $\underline{u}_{LV}$ is chosen to be $60^{\circ}$ in order to ensure that the astronaut will have sufficient time to center the selected landing site in the field of view of the optical instrument before the first navigation sighting should be made. After about one minute of manual tracking, the angle between the line-of-sight to the selected landing site and the current local vertical direction (not the vector $\underline{u}_{LV}$) will be approximately $45^{\circ}$. At this time it is desirable for accurate tracking conditions to have been attained, and for the MARK button to be pressed. Further navigation sightings of the landing site should then be made at about one minute intervals. The data obtained are processed by the Orbit Navigation Routine (Section 5. 2. 4).

The desired line-of-sight vector, $\underline{u}_{LOS}$, is computed by first rotating the local vertical vector about the polar axis of the moon to account for lunar rotation, and then rotating the resulting vector about the normal to the CSM orbital plane to attain the desired $60^{\circ}$ angle. The computation sequence is as follows:

1. Call the Kepler Subroutine (Section 5. 5. 5) to determine $\underline{r}_C$ and $\underline{v}_C$, the estimated CSM conic position and velocity vectors at time $t_{n+1}$.

2. Let

$$\underline{u}_{LOS} = - \text{UNIT} (\underline{r}_C) \qquad (6.8.5)$$

3. Call the Planetary Inertial Orientation Subroutine to transform the lunar polar vector (0, 0, 1) to the Basic Reference Coordinate System, obtaining the first rotation vector $\underline{u}_R$.

(4) Let the first rotation angle A be

$$A = A_M N \tag{6.8.6}$$

where $A_M$ is the angle through which the moon rotates in one nominal lunar-orbital period.

(5) Rotate the vector $\underline{u}_{LOS}$ about $\underline{u}_R$ through the angle A by

$$\underline{u}_{LOS} = (1 - \cos A)\,(\underline{u}_R \cdot \underline{u}_{LOS})\,\underline{u}_R + \underline{u}_{LOS} \cos A + \underline{u}_R \times \underline{u}_{LOS} \sin A \tag{6.8.7}$$

(6) Compute the second rotation vector from

$$\underline{u}_R = -\,\mathrm{UNIT}\,(\underline{v}_C \times \underline{r}_C) \tag{6.8.8}$$

(7) Let the second rotation angle be

$$A = \frac{\pi}{3} - A \tag{6.8.9}$$

(8) Compute the final desired line-of-sight vector $\underline{u}_{LOS}$ by repeating Eq. (6.8.7)

5.6.8.3 Command Subroutine

After this subroutine is activated by the WAITLIST, the line-of-sight vector, $\underline{u}_{LOS}$, is transformed to stable member coordinates by means of

$$\underline{s}_{SM} = \left[\text{REFSMMAT}\right] \underline{u}_{LOS} \qquad (6.8.10)$$

The desired optics trunnion (precision) and shaft angles, TA and SA, respectively, are computed as described in Section 5.6.3.1.3.

## 5.6.9 LUNAR LANDMARK SELECTION ROUTINE

The purposes of this routine are (1) to determine the time that the CSM will pass over the landing site, and (2) to select five landmarks which are suitable for orbit navigation. The use of this routine is restricted to lunar orbit since the coordinates of only lunar landmarks are stored in the CMC fixed memory.

The landmark selection procedure is based upon the assumption that the moon is mapped by five north-south bands of landmarks at approximately the $-60^{\circ}$, $-30^{\circ}$, $0^{\circ}$, $+30^{\circ}$ and $+60^{\circ}$ meridians of selenographic longitude as shown in Fig. 6.9-1. The number of landmarks in each band (approximately five) is assumed to be sufficient to cover all lunar orbits within the prescribed maximum inclination angle. The reason for the $30^{\circ}$ separation between landmark bands is to allow about ten minutes of time for the identification, acquisition and tracking of a lunar landmark, including all DSKY operations as described in Section 5.2.4.

From each longitudinal band, the landmark which is closest to the ground track of the CSM is chosen. In this manner, five primary landmarks are selected for navigation.

Let the selenographic coordinates of the landing site ($Lat_0$, $Long_0$ and $Alt_0$) be stored in the CMC erasable memory. These coordinates are included in the prelaunch erasable data load and can be changed inflight by the methods described in Section 5.2.4. The selenographic coordinates of the landmarks are stored in the CMC fixed memory in the following order:

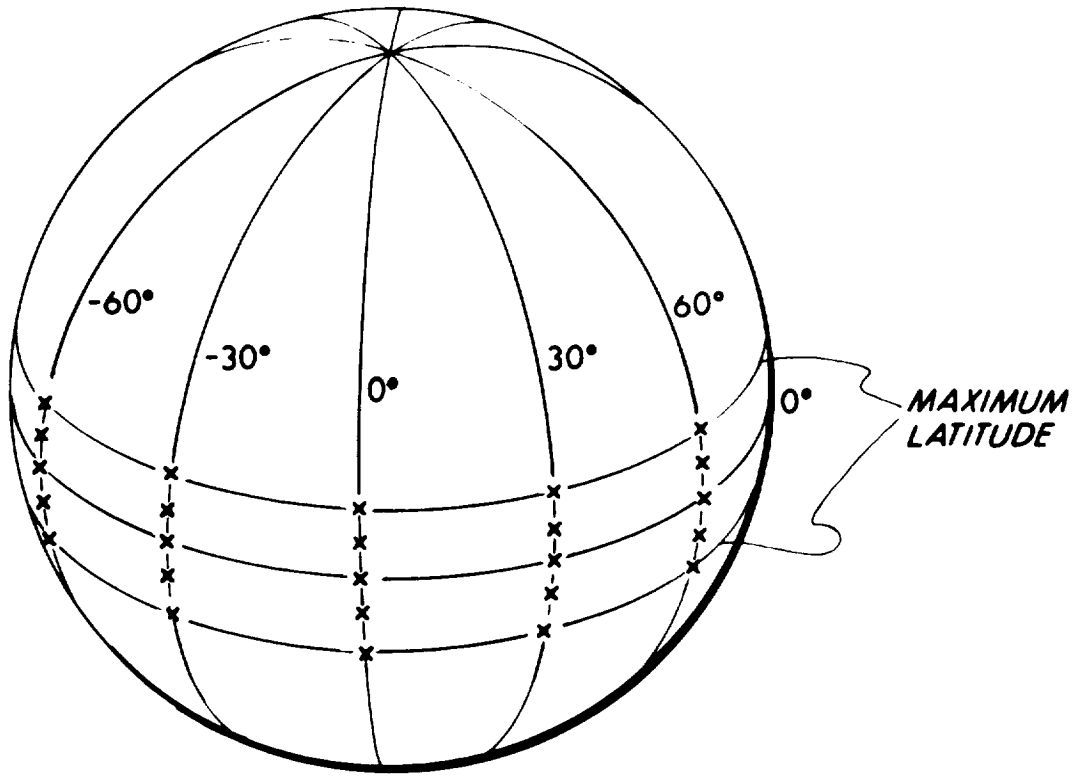



Figure 6.9-1 Mapped Lunar Landmarks

$$Lat_1, Lat_2, \ldots, Lat_{N_1}, Lat_{N_1+1}, \ldots, Lat_{N_2}, \ldots, Lat_{N_5},$$

$$Long_1, Long_2, \ldots, Long_{N_1}, Long_{N_1+1} \ldots, Long_{N_2}, \ldots, Long_{N_5},$$

$$Alt_1, Alt_2, \ldots, Alt_{N_1}, Alt_{N_1+1}, \ldots, Alt_{N_2}, \ldots, Alt_{N_5}$$

where

$$Long_1 \cong Long_2 \cong \ldots \cong Long_{N_1} \cong -60^\circ$$

$$Long_{N_1+1} \cong Long_{N_1+2} \cong \ldots \cong Long_{N_2} \cong -30^\circ$$

$$\ldots$$

$$Long_{N_4+1} \cong Long_{N_4+2} \cong \ldots \cong Long_{N_5} \cong +60^\circ$$

The numbers $N_0 = 0, N_1, N_2, \ldots, N_5$ indicate the number of landmarks contained in each longitudinal band and are assumed to be stored in the CMC fixed memory.

The astronaut specifies a future time t (GET). The estimated CSM state vector is extrapolated to the specified time by means of the Coasting Integration Routine (Section 5.2.2) obtaining $\underline{r}$ and $\underline{v}$, the estimated CSM position and velocity vectors, respectively. Let Lat and Long be the estimated selenographic latitude and longitude of the CSM at time t obtained by means of the Latitude-Longitude Subroutine (Section 5.5.3).

Next, define the following unit vectors:

$$
\begin{aligned}
\underline{u}_r &= \text{UNIT}(\underline{r}) \\
\underline{u} &= \text{UNIT}(\underline{r} \times \underline{v}) \\
\underline{u}_W &= \text{UNIT}(\underline{u}_r \times \underline{u}_Z) \\
\underline{u}_N &= \text{UNIT}(\underline{u}_W \times \underline{u}_Z)
\end{aligned}
\qquad (6.9.1)
$$

where $\underline{u}_Z$ is a unit vector along the polar axis of the moon. Then, if $Lat_j$ and $Long_j$ are the selenographic coordinates of a given landmark, the time of closest arrival to the landmark, $t_L$, is determined as shown in Fig. 6.9-2. In the figure, the factor 1.003 is used to account for the relative rates of the CSM and the landmark, and $\theta$ is the central angle from $\underline{r}$ to the point of closest arrival.

Finally, the logic for the Lunar Landmark Selection Routine is illustrated in Fig. 6.9-3.

The following data are computed and displayed in order by this routine:

1) Time of arrival at landing site
2) Time of arrival at first landmark and landmark index number
3) Time of arrival at second landmark and landmark index number
4) Time of arrival at third landmark and landmark index number
5) Time of arrival at fourth landmark and landmark index number
6) Time of arrival at fifth landmark and landmark index number

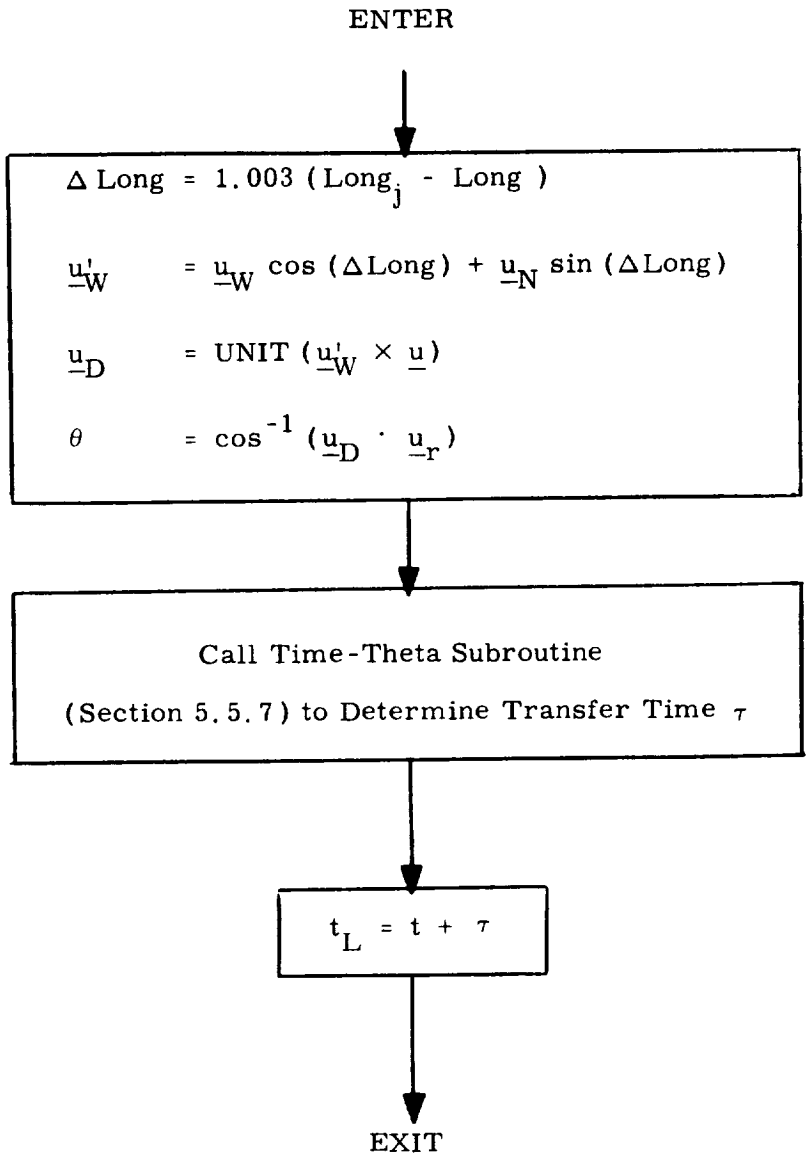



Figure 6.9-2 Determination of Time-of-Arrival

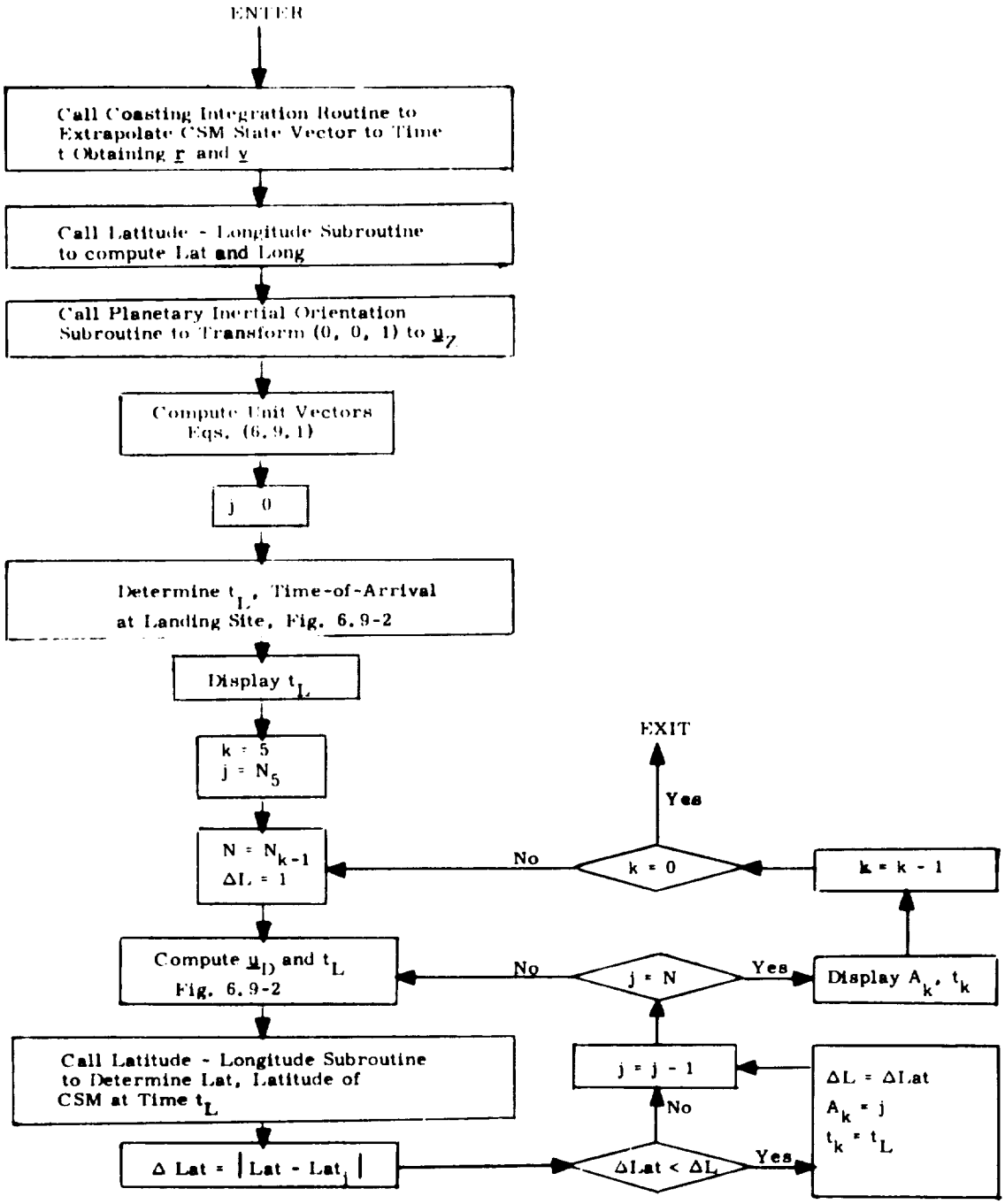



Figure 6.9-3 Lunar Landmark Selection Routine Logic Diagram

## 5.6.10 GENERAL DISPLAY COMPUTATIONS

This section presents the CMC computations required for various entry (P-61) and earth orbit injection monitoring (P-11) programs. In entry program P-61 the following three display parameters are required to initialize the Entry Monitor System, EMS:

VEMS : Predicted inertial velocity at the EMS altitude

RTOGO : Range-to-go from the EMS altitude to the predicted splash point

TTE : Time to go from the current time to the EMS altitude

The EMS altitude is measured above the Fisher Ellipsoid and is one of two fixed values stored in erasable memory. For earth orbital entries the EMS altitude is 284,643 feet. Lunar mission entries use the second EMS altitude of 297,431 feet.

The principal computation required for these display parameters is the conic calculation of time to a radius of specified length. This computation is described in Section 5.6.10.1. The intersection of a conic with an altitude above the Fisher ellipsoid is a two step iteration. An initial calculation is made using a guess of the radius length. The latitude of the conic with this assumed length is then used to calculate the radius of the Fisher ellipsoid. This is then used to calculate a second radial length, and display quantities are referred to this second radius. Additional iterations are not required to achieve the necessary display accuracy.

The computation logic for these functions is presented in Fig. 6.10-1. The earth gravitation constant used in these calculations is increased recognizing that the missions under consideration are confined to low inclination angles from the equator and oblate earth effects can be significant. The drag loss in reaching the EMS point (0.05 G) is also approximated for these display computations.

The following three additional displays computations are also made in entry program P-61:

V 400K : Predicted inertial velocity at an altitude of 400,000 feet.

$\gamma$ 400K : Predicted flight path angle at an altitude of 400,000 feet.

G MAX : Maximum predicted deceleration

The altitude of 400,000 feet is again referenced to the Fisher ellipsoid, and a two step iteration is made to calculate this radial length as in the previous EMS display parameters. The drag loss is approximated for this computation. The G MAX equation has been designed for high drag conditions (near 10 g), and may give negative results for relativly low deceleration cases of two to four g's.

During the Earth Orbit Insertion (EOI) Monitor Program, P-11, the following display parameters are computed:

Delta R : The miss distance in n. m. between the abort target landing point and the predicted splash point.

HPER : Perigee altitude above launch pad radius.

TFF : The time of free fall from the current time to an altitude of 300,000 feet above the launch pad radius.

The computation of these display parameters for P-11 is shown in Fig. 6.10-6. Again the CALCTFF routine is the primary computation. In this case the entry range and time of flight are approximated by emperical formulas. With reference to Fig. 6.10-6, RANGE from the present position to the landing site estimated for the current time is displayed if the 300,000 feet altitude will not be achieved during the trajectory.

The additional display parameters required by the callable display option in program P-11 are also computed in Fig. 6.10-6. The callable display parameters in addition to the three described above are:

HAPO : Apogee altitude above launch pad radius

TPER : Time to perigee

The pitch polynomial used to monitor the earth orbit insertion maneuver is described in Section 3.

The calculation of the IMU gimbal angles corresponding to the entry attitude at the present time is shown in Fig. 6.10-9. These entry attitude gimbal angles are displayed in program P-62 as:

OGA : Outer gimbal angle - roll

IGA : Inner gimbal angle - pitch

MGA : Middle gimbal angle - yaw

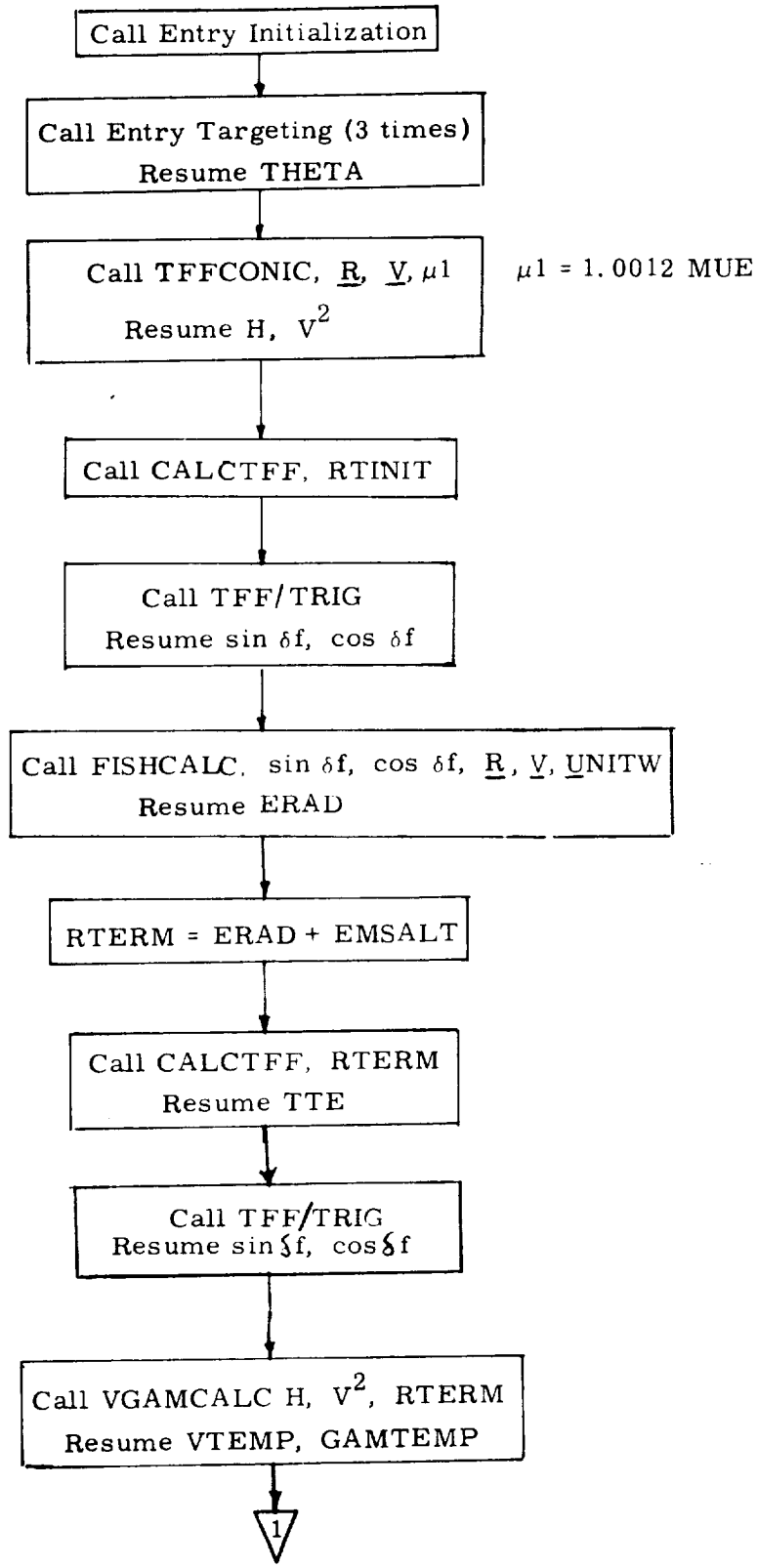



Figure 6.10-1 Entry Display Calculations

(page 1 of 2)

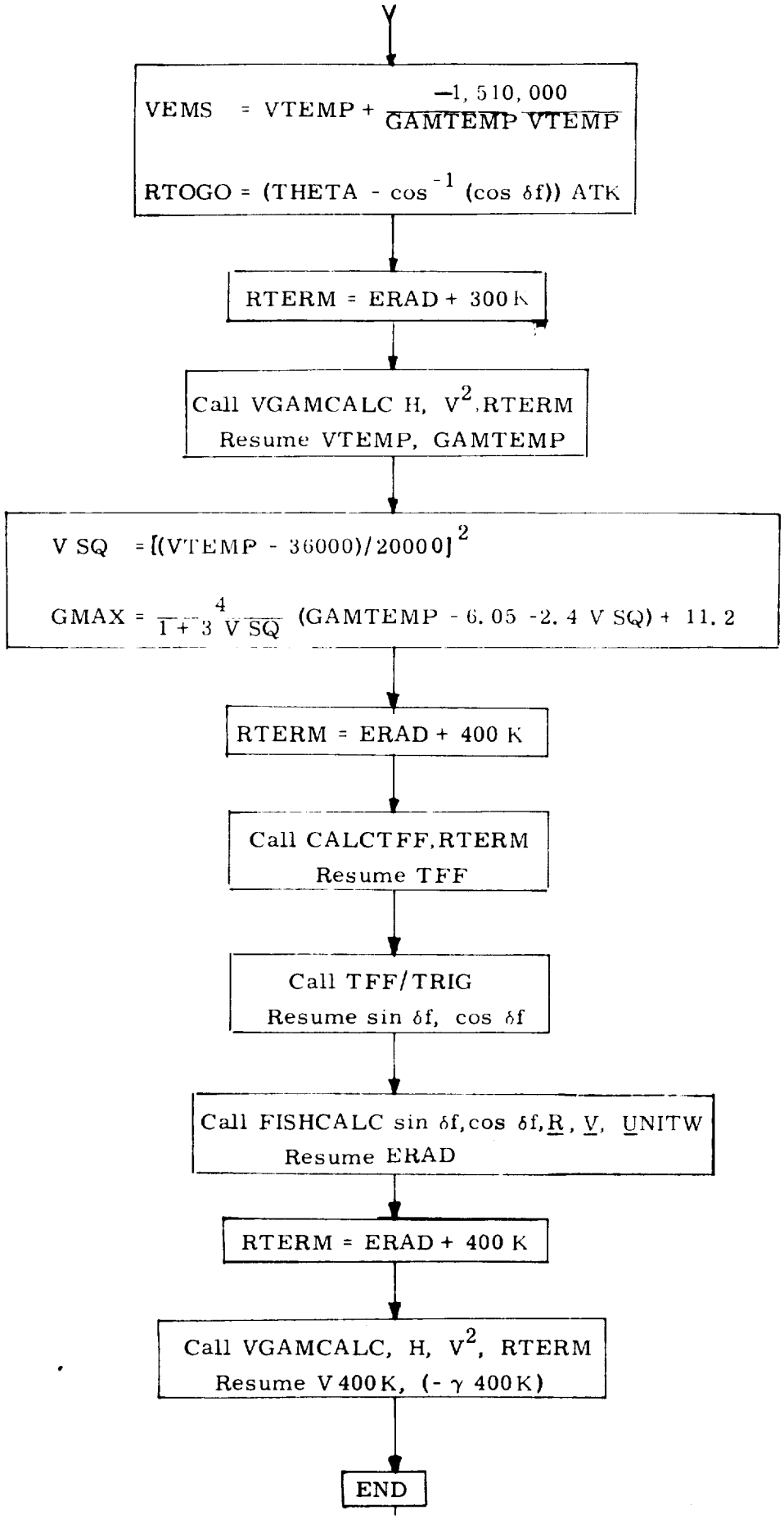



Figure 6.10-1 Entry Display Calculations (cont.)
(page 2 of 2)

### 5.6.10.1 Conic Time of Flight

The time of flight along any conic trajectory proceeding from an arbitrary radius $\underline{r}_o$ in the direction of velocity $\underline{v}_o$ to a radius of specified length $r_h$ is provided by Kepler's equation. The following formulation, while not quite universal, is explicit and direct. With a single equation, time of flight can be calculated for conic paths that are hyperbolic or parabolic and for those elliptic paths having $|\Delta E| \leq 90^o$. An alternate equation is needed for the elliptic paths having $|\Delta E| > 90^o$. Highly circular orbits are excluded by the constraint of length for the terminal radius. In the absence of a vector direction, highly circular cases are not treated.

The two conic parameters that appear in the time of flight equations are given by:

Angular momentum: $\underline{H} = \underline{r}_o \times \underline{v}_o$

Semi-latus rectum: $p = \dfrac{\underline{H} \cdot \underline{H}}{\mu}$ (6. 10. 1)

reciprocal semi-major axis: $\alpha = \dfrac{2}{r_o} - \dfrac{\underline{v}_o \cdot \underline{v}_o}{\mu}$

The equation for reciprocal semi-major axis $\alpha$ has the usual sign convention: $\alpha$ is negative for hyperbolic orbits, zero for parabolic orbits, and positive for elliptic orbits.

Two intermediate quantites, given in two useful equivalent forms, are:

$$Q_o = \mathrm{sgn}(\dot{r}_o)\sqrt{r_o(2 - \alpha r_o) - p} = \underline{r}_o \cdot \underline{v}_o / \sqrt{\mu}$$

$$Q_h = \mathrm{sgn}(\dot{r}_h)\sqrt{r_h(2 - \alpha r_h) - p} = \underline{r}_h \cdot \underline{v}_h / \sqrt{\mu} \qquad (6.10.2)$$

It is these quantities that exclude highly circular orbits, since being proportional to radial velocity they become uselessly small. For TFF application to reentry, $\underline{r}_h$ is on the returning side, so the sign of $\dot{r}_h$ is chosen as negative. Excepting ellipses for which $|\Delta E| > 90^o$, the conic time of flight is given by

$$t = (Z/\sqrt{\mu})(r_o + r_h - 2 Z^2 T(X)) \qquad (6.10.3)$$

where Z is the obtained by one of the following:

if outbound ($Q_o$ is positive):

$$Z = (Q_h - Q_o)/(2 - \alpha(r_o + r_h)) \qquad (6.10.4a)$$

if inbound ($Q_o$ is negative)

$$Z = (r_h - r_o)/(Q_h + Q_o) \qquad (6.10.4b)$$

and where if

$$\alpha Z^2 < 1.0$$

then

$$X = \alpha Z^2 \tag{6.10.5a}$$

For those elliptic cases having $|\Delta E| \geq 90^\circ$,

then

$$\alpha Z^2 \geq 1.0$$

and

$$X = 1/(\alpha Z^2). \tag{6.10.5b}$$

The conic time of flight is given by

$$t = \left(\frac{1}{\alpha\sqrt{\mu}}\right)\left(\frac{\pi}{\sqrt{\alpha}} - Q_h + Q_o + \frac{2(XT(X) - 1)}{\alpha Z}\right) \tag{6.10.6}$$

The function $T(X)$ is defined by the series

$$T(X) = \frac{1}{3} - \frac{X}{5} + \frac{X^2}{7} - \frac{X^3}{9} + \ldots$$

$$= \frac{X - \arctan(\sqrt{X})}{X\sqrt{X}} \qquad \text{if } 0 \leq X \leq 1 \qquad (6.10.7)$$

$$= -\frac{-X - \operatorname{arctanh}(\sqrt{-X})}{X\sqrt{-X}} \qquad \text{if } X \leq 0$$

The equations (6. 10. 4) are equivalent, except that each has a point of indeterminancy. The selection used in Eq. (6. 10. 4) excludes the indeterminant point of each from the region of application.

The choice of Eq. (6. 10. 3) or Eq. (6. 10. 6) depends on whether $\alpha Z^2$ is less than or greater than one. Since both Eq. (6. 10. 3) and Eq. (6. 10. 6) reduce to the same form when X = 1. 0, the division made above is arbitrary . Equation (6. 10. 3) gives both positive and negative values, indicating time to terminal point, and time since terminal point passage. On occasion, the user desires to use the negative value and a switch is provided to make this fact known. Otherwise, a positive value, time to terminal point, is always provided for the ellipse by replacing negative t by $2\pi/\alpha\sqrt{\alpha\mu} + t$. Equation (6. 10. 6) yields positive values only.

The time of flight equations are exact and involve approximations only to the extent that the function T(X) is represented by a polynominal. Futhermore Eq. (6. 10. 7) is continuous as the conic changes from hyperbolic to elliptic.

### 5.6.10.2 Application of Time of Freefall

On board the CM, the TFF routines are used to calculate time to various specified terminal radii, which may include perigee or apogee. These times when displayed on the DSKY are the negative of the calculated values, as in a countdown. In the event that the present position lies between perigee and terminal radius, a positive time representing time since terminal point is displayed. When perigee is passed, negative time to terminal radius is displayed once again. However, for outbound hyperbola or parabola, positive time since terminal point is displayed.

The following paragraphs outline several CMC Subroutines that are available.

TFFCONIC (Fig. 6.10-3) Subroutine calculates the conic parameters of Eq. (6.10.1) and stores them for short time use. It also saves other intermediate values to save duplication by following subroutines. The user must supply a value for $\mu$ appropriate to the application.

TFFRP/RA (Fig. 6.10-7) Subroutine calculates perigee and apogee radius for general conic. The equations are

$$
\begin{aligned}
e^2 &= 1 - p\alpha \\
R_p &= p/(1 + e) \\
R_a &= (1 + e)/\alpha
\end{aligned}
\qquad (6.10.8)
$$

For hyperbolic or parabolic orbits and for those elliptic where apogee radius exceeds the scaling, $R_a$ is set to double precision POSMAX. This indicates that either apogee radius is undefined, infinite or exceeds scaling.

CALCTFF / CALCTPER (Fig. 6.10-2) Subroutine uses terminal radius supplied by Caller to calculate time of free fall or time to perigee depending on entry point. The subroutine also calculates the quantity ARG required by TFF/TRIG to calculate transfer angle. In the event that the terminal radius does not lie on the present conic, t is set to POSMAX as a tag. Caller may enter CALCTFF / CALCTPER as many times as desired to obtain TFF to various radii for the same conic.

### 5.6.10.3 Other Conic Quantities

In certain applications it is desirable to calculate other conic variables at the terminal point in addition to the time of free fall. Among these are velocity magnitude, flight path angle relative to local horizontal, and sin, cos of transfer angle $\delta f$.

TFF / TRIG (Fig. 6.10-4). The sin and cos of the transfer angle $\delta f$ from $\underline{r}_o$ to $\underline{r}_h$ is given by

$$\cos(\delta f) = 1 - 2|ARG|/(r_o r_h (1 + X))$$

$$\sin(\delta f) = \mathrm{sgn}(ARG)\sqrt{1 - \cos^2(\delta f)} \qquad (6.10.9)$$

$$= \mathrm{sgn}(ARG)\sqrt{|ARG|(1 - |ARG|)}$$

$$\text{if } \alpha Z^2 < 1 \qquad ARG = p\, Z\, |Z| \tag{6.10.10}$$

$$\text{if } \alpha Z^2 \geq 1 \qquad ARG = (p/\alpha)\ \mathrm{sgn}\,(Q_o + r_o / Z)$$

The sign affixed to ARG is that of sin ($\delta f$). The computation of ARG is done during the appropriate branch in CALCTFF / CALCTPER and saved temporarily. Calculation of sin and cos occurs in subroutine TFF / TRIG and uses values available in pushlist. Caller may make only one entry for each CALCTFF/CALCTPER entry.

VGAMCALC (Fig.6.10-5). Calculation of velocity magnitude and flight path angle is performed in subroutine VGAMCALC. The terminal velocity is given by the vis-viva calculation.

$$v_h = \sqrt{(v_o^2 + 2\,\mu(r_o - r_h)/(r_o\, r_h))} \tag{6.10.11}$$

and the flight path angle by

$$\gamma_E = \arccos\,(H / r_h\, v_h) \tag{6.10.12}$$

where angular momentum $H = |\underline{H}|$. User must supply proper sign to $\gamma_E$. Program uses $r_h$ left by CALCTFF / CALCTPER. However Caller may enter at PREVGAM with other values for $r_h$ as often as desired for same conic.

FISHCALC (Fig.6.10-5). A unit vector along $\underline{r}_h$ is provided by

$$\underline{ur}_h = \underline{ur}_o \cos(\delta f) + \underline{ur}_o \times (-\underline{u}H) \sin(\delta f) \qquad (6.10.13)$$

where u represents UNIT ( ) and the vectors are as in Eq.(6.10-1). During Reentry subroutine FISHCALC generates sin (LAT) using Eq. (6.10-13) above and

$$\sin(LAT) = \underline{U}NITW \cdot \underline{ur}_h$$

where $\underline{U}$NITW is a vector along the earth's polar axis.

Finally the radius of the Fischer ellipsoid, ERAD, is calculated by calling the routine GETERAD with sin (LAT).

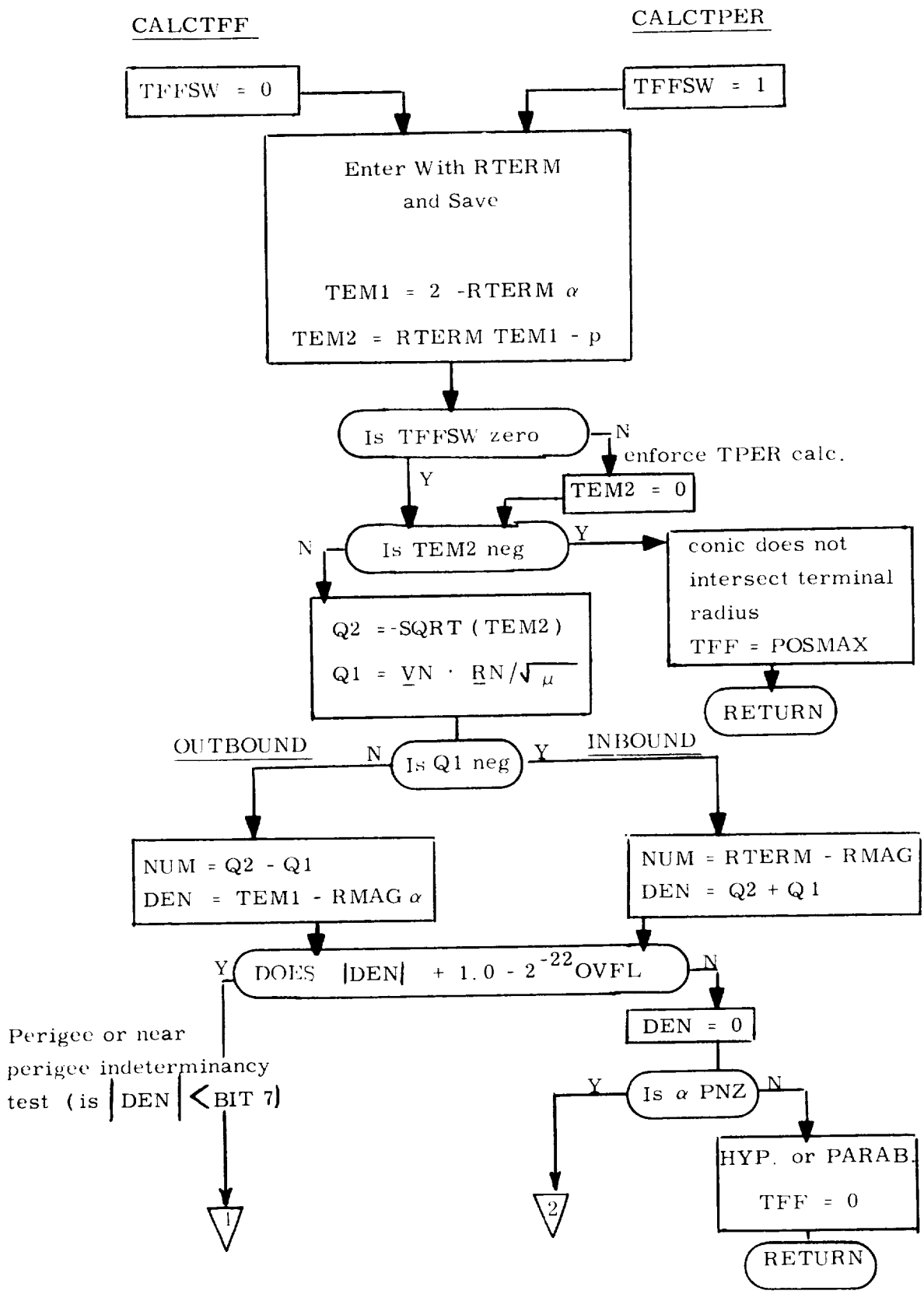



Figure 6.10-2 Time of Flight Calculation
(page 1 of 2)

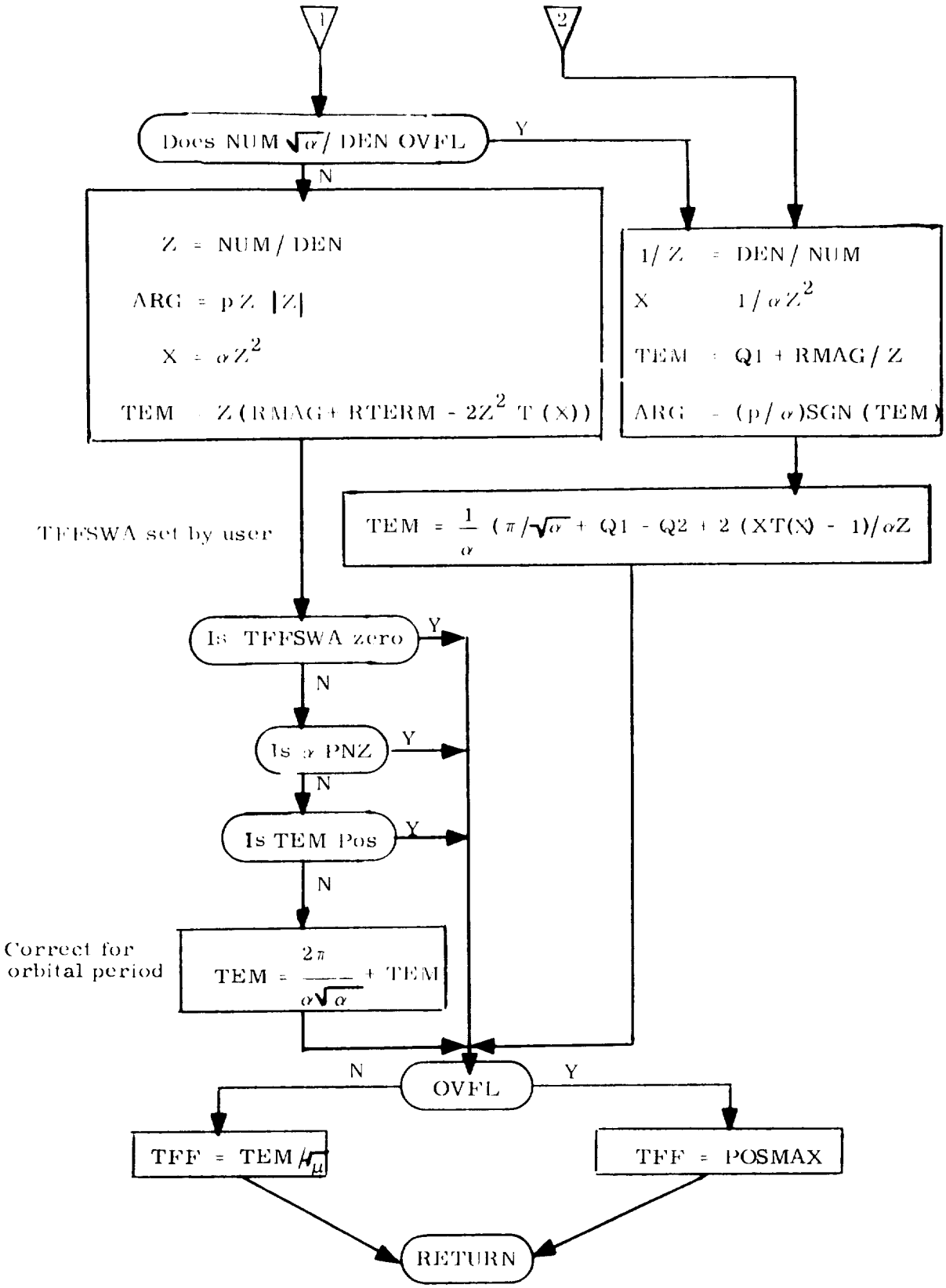



Figure 6.10-2 Time of Flight Calculation
(page 2 of 2)

TFFCONIC

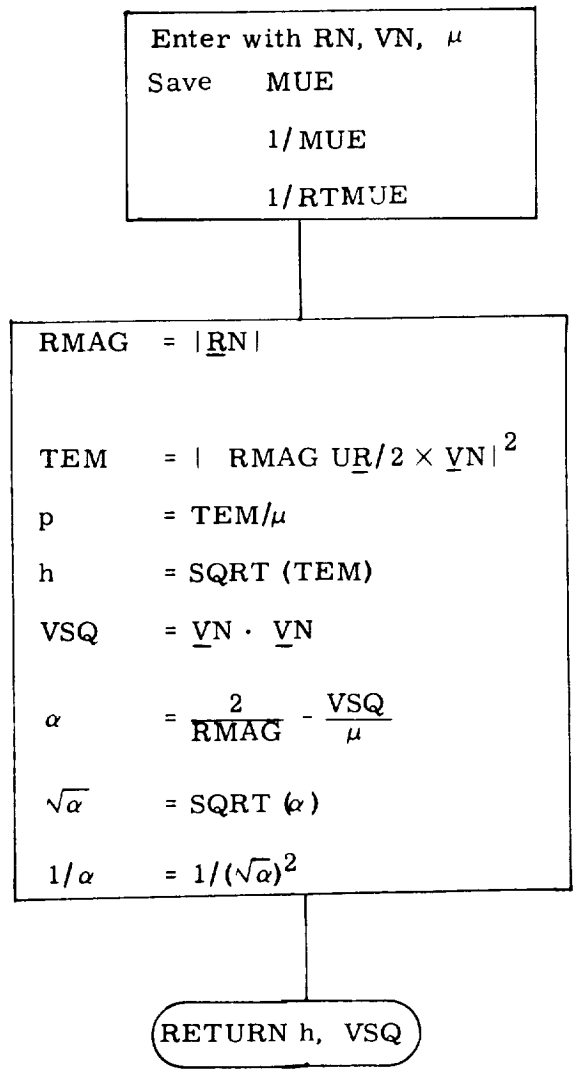



Figure 6.10-3 Conic Parameters

$$(\cos \delta f)/2 = 1/2 - \mathrm{ABS}\left[\mathrm{ARG}/(\mathrm{RMAG\ RTERM}\ (1 + X)\right]$$

$$(\sin \delta f)/2 = \mathrm{SGN}\ (\mathrm{ARG})\ \mathrm{SQRT}\left[1/4 - (\cos \delta f)/2)^2\right]$$

ARG defined in
CALCTFF/CALCTPER

RETURN

Figure 6.10-4 Sine and Cosine of Transfer Angle

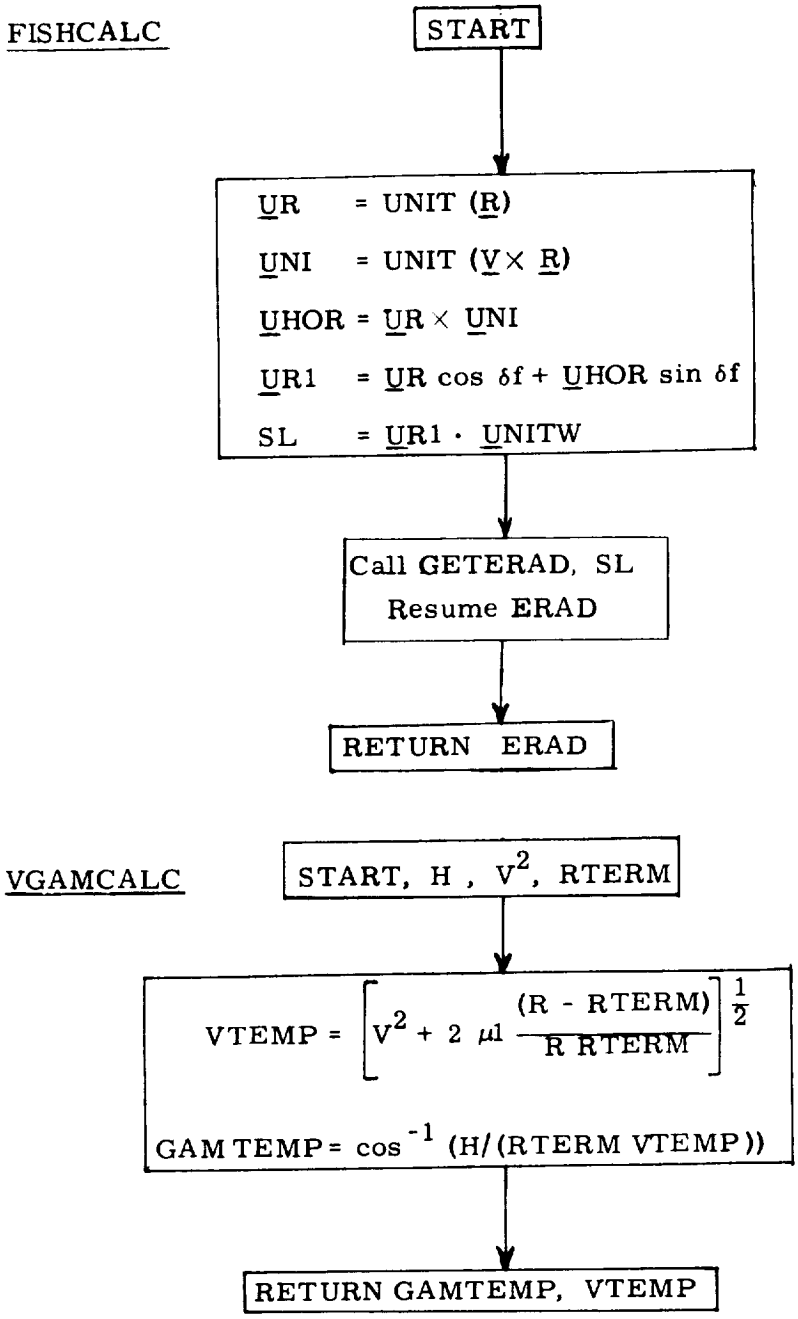



Figure 6.10-5 Subroutines for Entry Displays

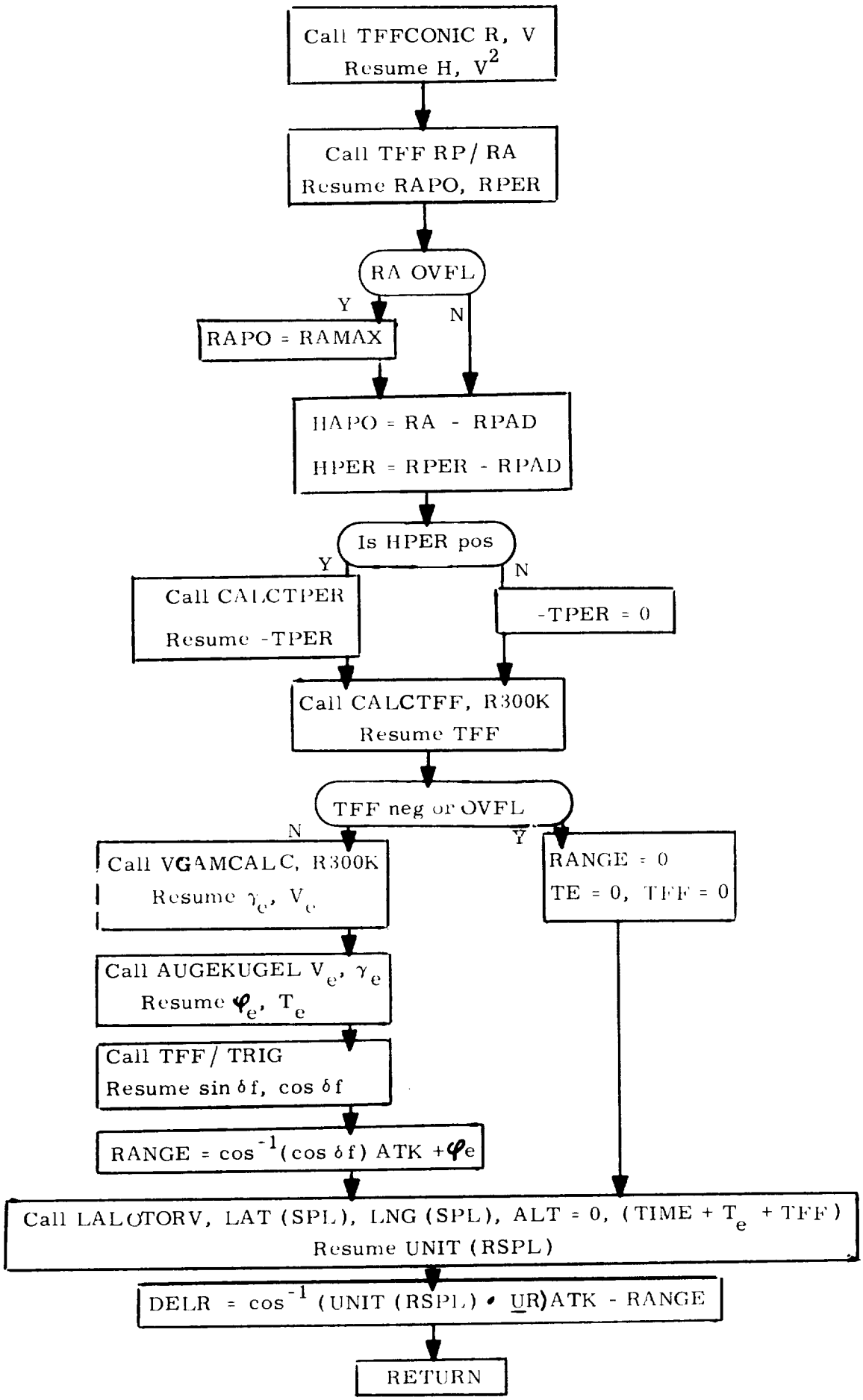



Figure 6.10-6 DeltaR Calculation

TFFRP/RA

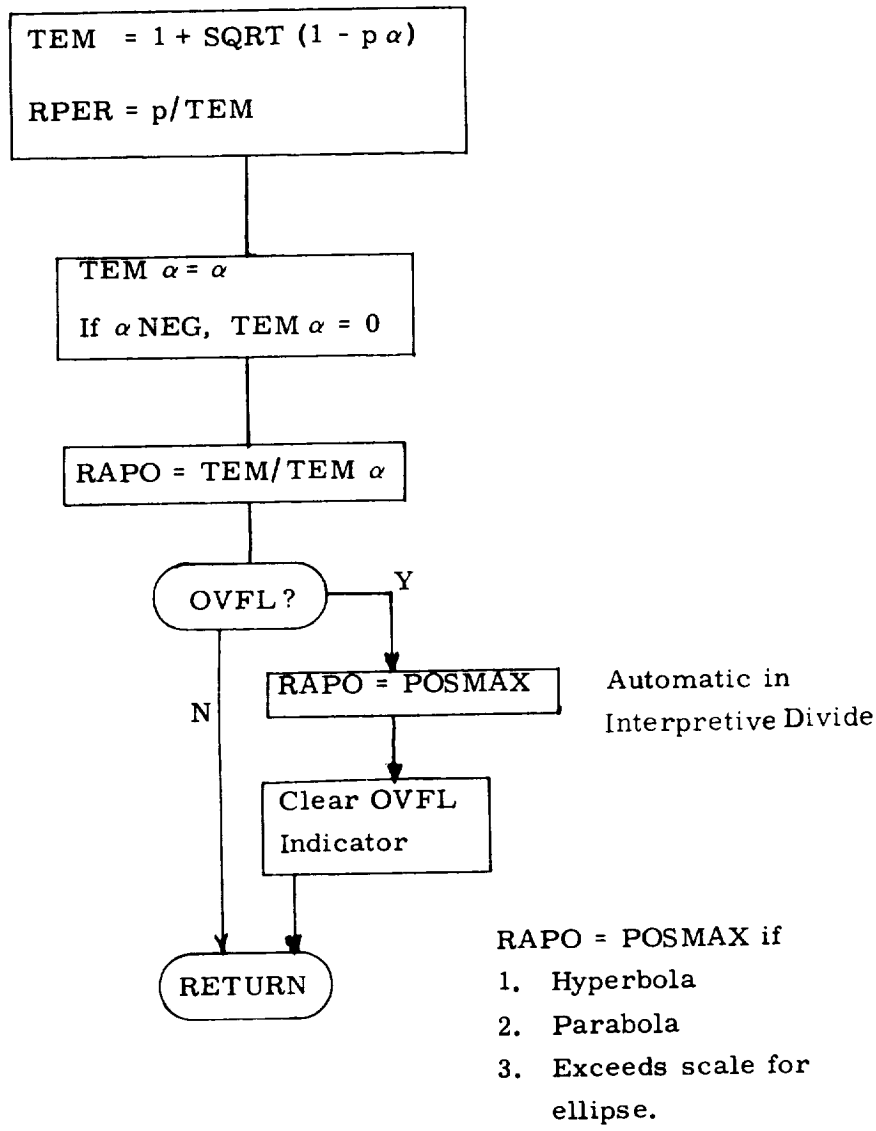



Figure 6.10-7 Apogee and Perigee Radius Calculation for General Conic

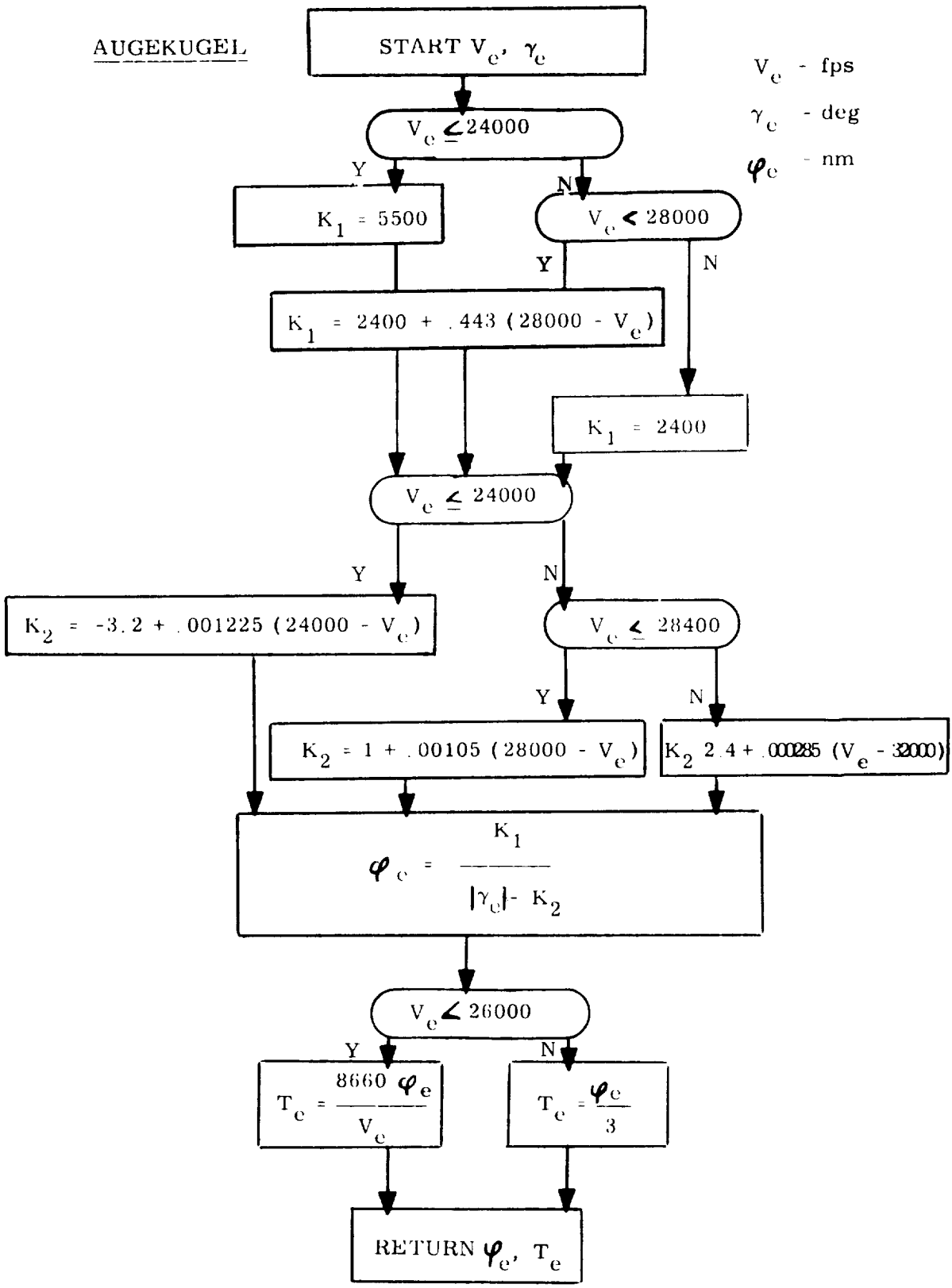



Figure 6.10-8 Empirical Relations for Entry Range & Entry Time

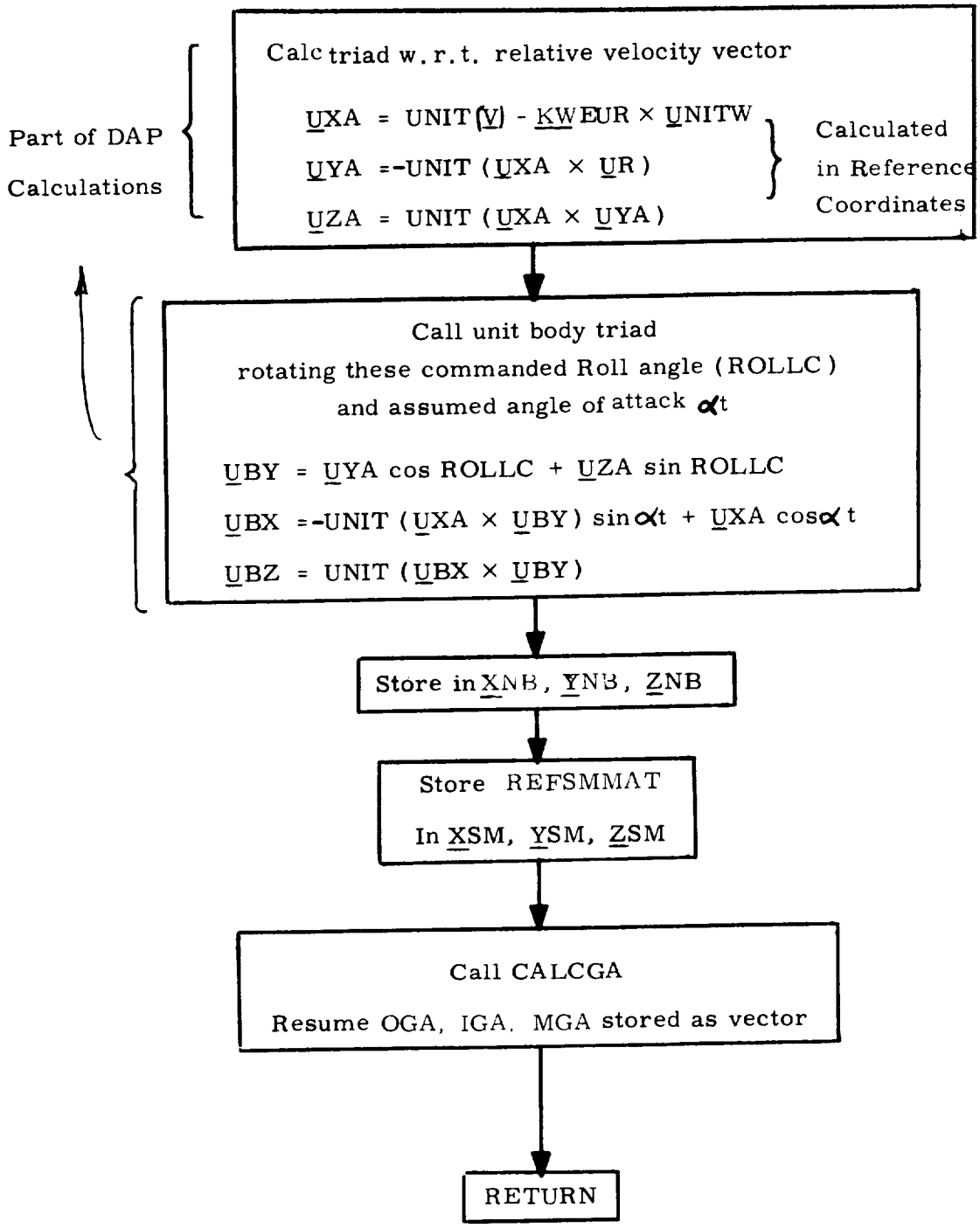



Figure 6.10-9 Calculation of Gimbal Angles Corresponding to Entry Attitude

## 5.6.11 LGC INITIALIZATION

The LGC initialization procedure prior to LM separation is a manual operation which does not involve a numbered CMC program. After the LGC is activated the first requirement is to synchronize the LGC clock with that of the CMC. This is a count-down and mark procedure described in R-30, CMC/LGC Clock Synchronization Routine of Section 4, to obtain an average clock difference which is then used to increment the LGC clock. The CMC and LGC clock synchronization can also be checked by the Mission Ground Control Center using telemetry down-link data, which can provide a more precise difference to increment the LGC clock.

In the lunar landing mission the following parameters are next voice-linked from the CSM and entered in the LGC as initialization parameters:

1) $\underline{r}_C$ : CSM position vector

2) $\underline{v}_C$ : CSM velocity vector

3) $t_C$ : CSM state vector time

4) $\underline{r}_{LS}$ : lunar landing site vector

5) $t_{LS}$ : time $\underline{r}_{LS}$ is valid

6) $t_0$ : time difference between zero GET and July 1.0, 1968 universal time

In the above procedure, all parameters are entered in octal, and position and velocity vectors are double precision. In the lunar landing mission the above items 1 through 5 are normally determined by the CMC Orbit Navigation Program P-22.

## 5.6.12 CMC IDLING PROGRAM

This program is used to maintain the CMC in a state of readiness for entry into any other program. While the idling program is in operation, the Coasting Integration Routine (Section 5.2.2) is used to advance the estimated CSM state vector (and the estimated LM state vector when the LM is active and not on the surface of the moon) to approximately current time. This procedure has the lowest priority of all programs, and is performed only when no other program is active. This periodic state vector extrapolation is not necessary from a theoretical point of view, but does have two practical purposes. First, it is advisable to maintain current (or at least nearly current) state vector estimates in case an emergency situation arises. Second, a significant amount of computation time is transferred from a period of high computer activity (navigation measurement processing, targeting, etc) to a period of low activity.

In order to use the Coasting Integration Routine in an efficient manner, the maximum value for the integration time step, $\Delta t_{max}$, is computed as described in Section 5.2.2.5. The value of $\Delta t_{max}$ is a function of radial distance and varies from step to step. Let t be the time associated with the estimated CSM state vector and $t_1$ be the current time. The estimated state vector(s) are extrapolated ahead when

$$t_1 > t + 4\,\Delta t_{max} \tag{6.12.1}$$

the integration is terminated when

$$t_1 < t + \Delta t_{max} \tag{6.12.2}$$

In this manner no extra and smaller-than-maximum integration time steps are performed, and the periodic integration is accomplished most efficiently.

The error transition matrix W (see Section 5. 2. 2. 4) is extrapolated with the estimated CSM state vector if ORBWFLAG indicates that the W matrix is valid. ORBWFLAG is defined in Sections 5. 2. 4. 5 and 5. 2. 6. 4.

The logic for the periodic state vector extrapolation is illustrated in Fig. 6. 12-1. The variables D and V are indicators which control the Coasting Integration Routine. The quantities $x_C$ and $x_L$ are the estimated CSM and LM state vectors, respectively, and x is a temporary state vector used for integration. Refer to Section 5. 2. 2. 6 for precise definitions of these items.

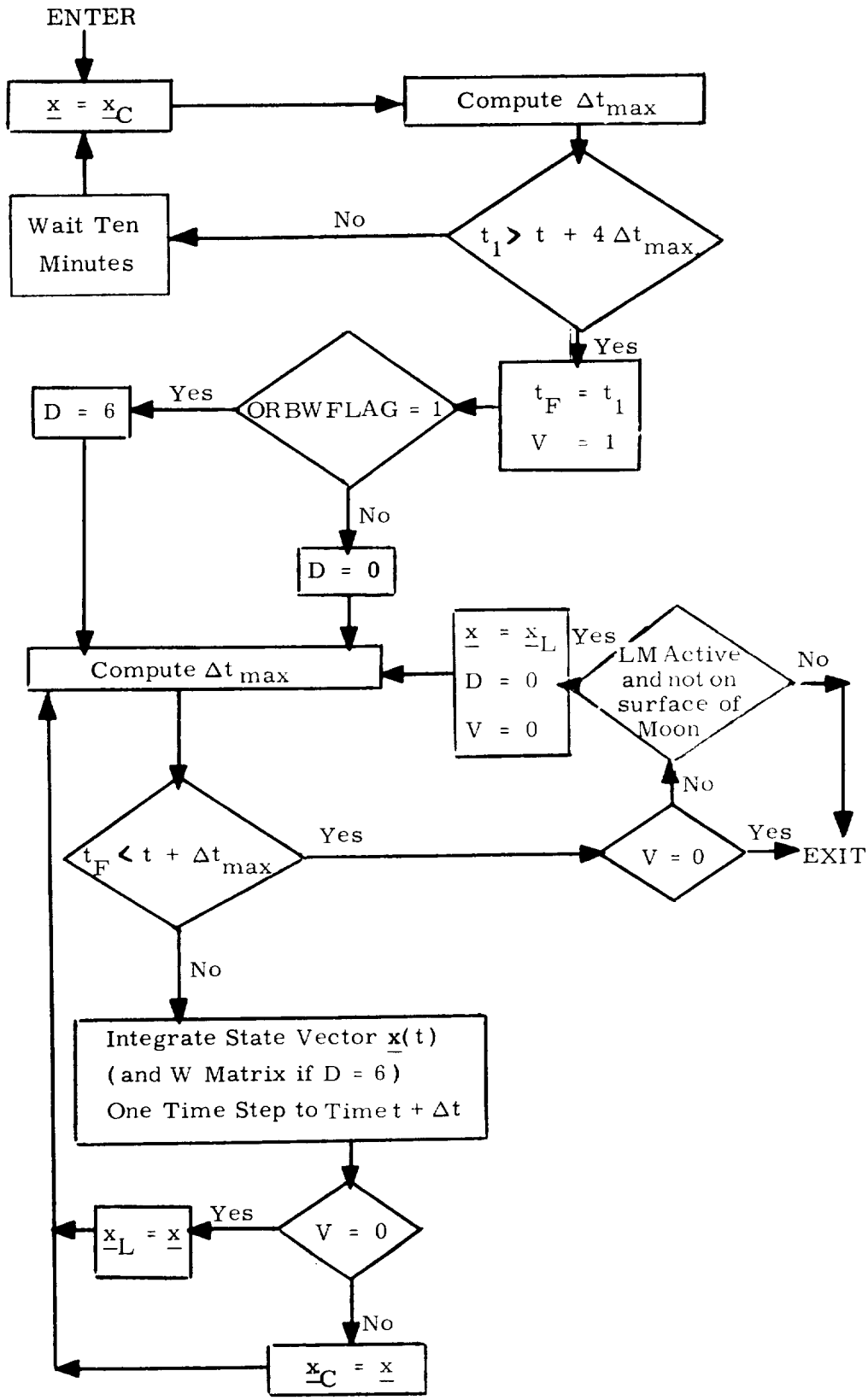



Figure 6.12-1 CMC Idling Program Logic Diagram

## 5.6.13 IMU COMPENSATION

The IMU Compensation is designed to compensate for PIPA bias and scale factor error and at the same time accumulate gyro torquing commands necessary to compensate for the associated bias and acceleration caused gyro drifts. The correction to the PIPA's is

$$PIPA_C = (1 + SFE_I)\, PIPA_I + BIAS_I\, \Delta t$$

where

$PIPA_C$ is the compensated data for the $I^{th}$ PIPA denoted $PIPAX_C$, $PIPAY_C$, $PIPAZ_C$

$$SFE = \frac{SF - SF_{nom}}{SF_{nom}} \quad \text{(erasable load)}$$

$$SF = \text{Scale-factor } \frac{CM/Sec}{Pulse}$$

$BIAS_I$ is the bias for the $I^{th}$ PIPA (an erasable load)

The compensated data is then used to compute the IRIG torquing necessary to cancel the NBD, ADIA, and ADSRA gyro coefficients. The computations are

$$XIRIG = -ADIAX\ PIPAX_C + ADSRAX\ PIPAY_C - NBDX\ \Delta t$$

$$YIRIG = -ADIAY\ PIPAY_C + ADSRAY\ PIPAZ_C - NBDY\ \Delta t$$

$$ZIRIG = -ADIAZ\ PIPAZ_C - ADSRAZ\ PIPAY_C + NBDZ\ \Delta t$$

where

XIRIG, YIRIG, ZIRIG are gyro drift compensation

NBDX, NBDY, NBDZ are gyro bias drifts (an erasable load)

ADSRAX, ADSRAY, ADSRAZ are gyro drifts due acceleration in spin reference axis (an erasable load)

ADIAX, ADIAY, ADIAZ are gyro drifts due to acceleration in the input axis (an erasable load)

When the magnitude of any IRIG command exceeds two pulses, the commands are sent to the gyros.

During free-fall only the NBDX, NBDY, NBDZ are the relevant coefficients and the routine is so ordered that only these terms are calculated for the gyro compensation.

## 5.7 ENTRY GUIDANCE

The entry guidance is used for control of the CM entry vehicle and is described by the logic flow charts in Fig. 5.7-1 thru Fig. 5.7-16. A detailed description of the guidance and steering concept is presented in MIT/IL Report R-532, "Reentry Guidance for Apollo," R. Morth, January 1966.

Figure 5.7-1 illustrates the overall picture of operations during entry. Each block in Fig. 5.7-1 is described in detail in following figures. Figure 5.7-2 defines the symbols which represent computed variables stored in erasable memory. The values and definitions of constants are given in Fig. 5.7-3.

The initialization routine shown in Fig. 5.7-4 is entered only once at the start of entry. Besides setting the appropriate variables to their initial values, this routines presets the variable SELECTOR to INITROLL.

Every pass through the entry equations (done once every 2 seconds) is begun with the section called navigation (see Fig. 5.7-5) and is the same as that presented in Section 5.3.2. This integrates to determine the vehicle's new position and velocity vectors. This navigation routine is started prior to encountering the entry interface at 400,000 feet altitude, and is continuously operated from that time to landing.

Next, the targeting section updates the desired landing site position vector and computes some quantities based on the vehicle's position and velocity and the position of the landing site. (See Fig. 5.7-6).

The mode selector chooses the next sequence of calculations depending upon the phase of entry trajectory that is currently being flown. The initial roll section maintains whatever roll angle was previously selected in the initialization routine and decides when to start the next phase, as shown in Fig. 5.7-7.

The next phase maintains a constant drag trajectory while testing to see if it is time to enter the up-control phase. The testing is presented in Figs. 5.7-8 and 5.7-9. The constant drag equations are given in Fig. 5.7-10. The other phases (up-control, ballistic, and final) are listed in Figs. 5.7-11, 5.7-12, and 5.7-13. (See Fig. 5.7-1 .) The final phase is accomplished by a stored reference trajectory with linear interpolation between the stored points. Its characteristics as well as the steering gains are stored as shown in Fig. 5.7-14. The routine that prevents excessive acceleration build-up (G-limiter) is given in Fig. 5.7-15. Finally, the section that does the lateral logic calculations and computes the commanded roll angle is shown in Fig. 5.7-16.

The parameters computed for display purposes only during the entry phase are presented in Section 5.6.10.

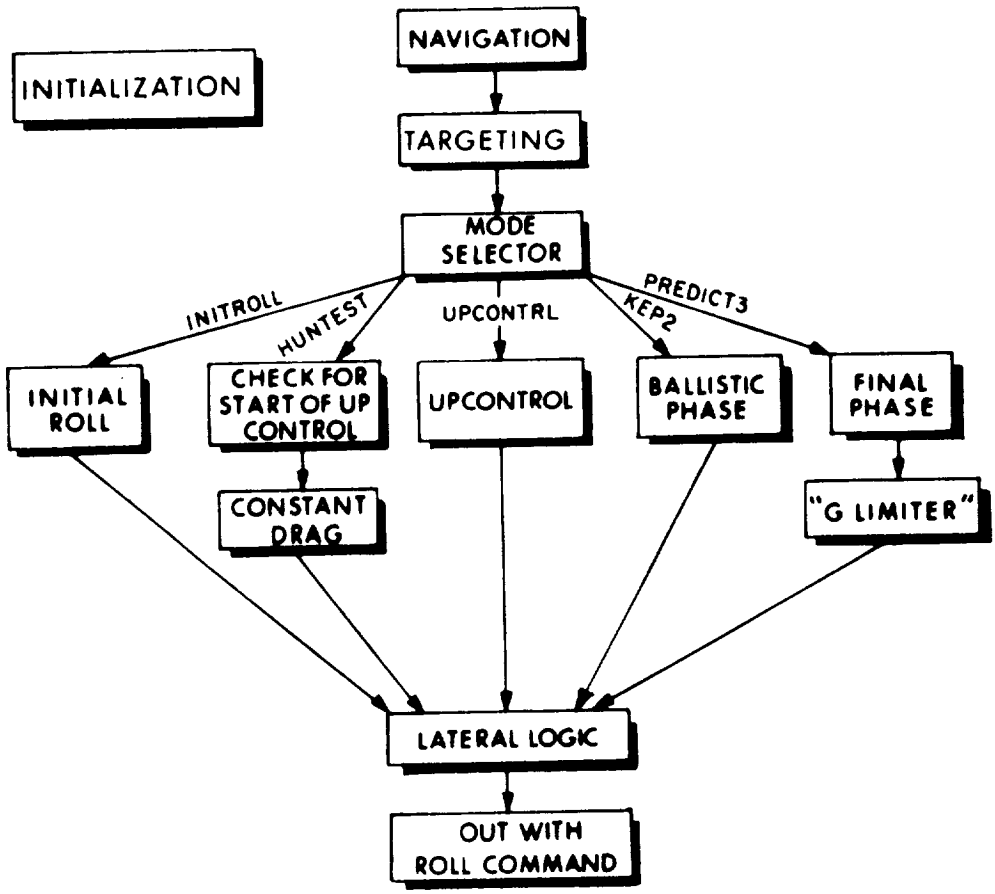



Fig. 5.7-1 Entry Computation

| VARIABLE | DESCRIPTION | MAXIMUM VALUE * | COMPUTER NAME |
|---|---|---|---|
| URTO | INITIAL TARGET VECTOR | 2 (UNIT VECTOR) | = RTINIT |
| UZ | UNIT VECTOR NORTH | 2 | = (UZ) OR UNITW |
| V | VELOCITY VECTOR | 2 VSAT | = (V) |
| R | POSITION VECTOR | 2 EXP 29 METERS | = RN |
| VI | INERTIAL VELOCITY | 128 M/CENTISEC | = VPIP |
| RTE | VECTOR EAST AT INITIAL TARGET | 2 | = RTEAST |
| UTR | NORMAL TO RTE AND UZ | 2 | = RTNORM |
| URT | TARGET VECTOR | 2 | = (RT) |
| UNI | UNIT NORMAL TO TRAJECTORY PLANE | 2 | |
| DELV | INTEGRATED ACCEL. FROM PIPAS | 5.85 16384 CM/S | |
| G | GRAVITY VECTOR | 1/32 M/CS/CS | = GRAVITY |
| A0 | INITIAL DRAG FOR UPCONTRL | 805 FPSS | FPSS=FT/SEC/SEC |
| AHOOKDV | TERM IN GAMMAL CALC. = AHOOK DVL | 805X16 VSAT FPS FPSS | |
| A1 | DRAG VALUE IN FACTOR CALCULATION | 805 FPSS | |
| ALP | CONST FOR UPCONTRL | 1 | |
| ASKEP | KEPLER RANGE | 21600 NM | NM = NAUTICAL MILE |
| ASP1 | FINAL PHASE RANGE | 21600 NM | |
| ASPUP | UP-RANGE | 21600 NM | |
| ASP3 | GAMMA CORRECTION | 21600 NM | |
| ASPDWN | RANGE DOWN TO PULL-UP | 21600 NM | |
| ASP | PREDICTED RANGE | 21600 NM | |
| COSG | COSINE(GAMMAL) | 2 | = COSG/2 |
| D | TOTAL ACCELERATION | 805 FPSS | |
| D0 | CONTROLLED CONSTANT D | 805 FPSS | |
| DHOOK | TERM IN GAMMAL COMPUTATION | 805 FPSS | |
| DIFF | THETNM-ASP (RANGE DIFFERENCE) | 21600X16 NM | |
| DIFFOLD | PREVIOUS VALUE OF DIFF | 21600X16 NM | |
| DR | REFERENCE DRAG FOR DOWNCONTROL | 805 FPSS | |
| DREFR | REFERENCE DRAG | 805 FPSS | |
| DVL | VS1-VL | 2 VSAT | |
| E | ECCENTRICITY | 4 | |
| F1 | DRANGE/D DRAG (FINAL PHASE) | 2700/805 | |
| F2 | DRANGE/D RDOT (FINAL PHASE) | 2700/2VS NM/FPS | |
| F3 | DRANGE/D (L/D) | 2700 NM | |
| FACT1 | CONST FOR UPCONTRL | 2 VSAT | |
| FACT2 | CONST FOR UPCONTRL | 1 | |
| FACTOR | USED IN UPCONTRL | 1 | |
| GAMMAL | FLIGHT PATH ANGLE AT VL | 1 RADIAN | |
| GAMMAL1 | SIMPLE FORM OF GAMMAL | 1 RADIAN | |

* MAXIMUM VALUE DENOTES UNSCALED VARIABLE VALUE WHEN SCALED VARIABLE HAS MAXIMUM VALUE OF ONE.

Fig. 5.7-2 Computer Variables

| VARIABLE | DESCRIPTION | MAXIMUM VALUE | COMPUTER NAME |
|---|---|---|---|
| HEADSUP | INDICATOR FOR INITIAL ROLL | 1 | |
| KA | DRAG TO LIFT UP IF DOWN | 805 FPSS | |
| K2ROLL | INDICATOR FOR ROLL SWITCH | | |
| LATANG | LATERAL RANGE | 4 RADIANS | |
| LEQ | EXCESS C.F. OVER GRAV=(VSQ-1)GS | 128.8 FPSS | |
| LEWD | UPCONTROL REFERENCE L/D | 1 | |
| L/D | DESIRED LIFT TO DRAG RATIO (VERTICAL PLANE) | 1 | |
| L/D1 | TEMP STORAGE FOR L/D IN LATERAL | 1 | |
| PREDANGL | PREDICTED RANGE (FINAL PHASE) | 2700 NM | |
| Q7 | MINIMUM DRAG FOR UPCONTROL | 805 FPSS | |
| RDOT | ALTITUDE RATE | 2 VSAT | |
| RDOTREF | REFERENCE RDOT FOR UPCONTROL | 2 VSAT | |
| RDTR | REFERENCE RDOT FOR DOWNCONT | 2 VSAT | |
| ROLLC | ROLL COMMAND | 1 REVOLUTION | |
| RTOGO | RANGE TO GO (FINAL PHASE) | 2700 NM | |
| SL | SINE OF LATITUDE | 1 | |
| T | TIME | 16384 SEC | = TENTRY |
| THETA | DESIRED RANGE (RADIANS) | 2 PI RADIANS | = THETAH |
| THETNM | DESIRED RANGE (NM) | 21600 NM | |
| V | VELOCITY MAGNITUDE | 2 VSAT | |
| V1 | INITIAL VELOCITY FOR UPCONTROL | 2 VSAT | |
| V1OLD | PREVIOUS VALUE OF V1 | 2 VSAT | |
| VCORR | VELOCITY CORRECTION FOR UPCONTROL | 2 VSAT | |
| VL | EXIT VELOCITY FOR UPCONTROL | 2 VSAT | |
| VREF | REFERENCE VELOCITY FOR UPCONTROL | 2 VSAT | |
| VS1 | VSAT OR V1, WHICHEVER IS SMALLER | 2 VSAT | |
| VBARS | $VL^2/VSAT^2$ | 4 | |
| VSQ | NORMALISED VEL. SQUARED = $V^2/VSAT^2$ | 4 | = VSQUARE |
| WT | EARTH RATE TIMES TIME | 1 REVOLUTION | = WIE (DTEAROT) |
| X | INTERMEDIATE VARIABLE IN G-LIMITER | 2 VSAT | |
| Y | LATERAL MISS LIMIT | 4 RADIANS | |

Fig.5.7-2a Computer Variables (cont)

EXTRA COMPUTER ERASABLE LOCATIONS NOT SHOWN ON FLOW CHARTS

| | VARIABLE | DESCRIPTION | MAXIMUM VALUE |
|---|---|---|---|
| | GOTOADDR | ADDRESS SELECTED BY SEQUENCER | |
| | XPIPBUF | BUFFER TO STORE X PIPA COUNTS | |
| | YPIPBUF | BUFFER TO STORE Y PIPA COUNTS | |
| | ZPIPBUF | BUFFER TO STORE Z PIPA COUNTS | |
| | PIPCTR | COUNTS PASSES THRU PIPA READ ROUTINE | |
| | JJ | INDEX IN FINAL PHASE TABLE LOOK-UP | |
| | M1 | INDEX IN FINAL PHASE TABLE LOOK-UP | |
| | GRAD | INTERPOLATION FACTOR IN FINAL PHASE | |
| | FX | DRANGE/D L/D = F3 | 2700 NM |
| | FX + 1 | AREF | 805 FPSS |
| | FX + 2 | RTOGO | 2700 NM |
| | FX + 3 | RDOTREF | VSAT/4 |
| | FX + 4 | DRANGE/D RDOT = F2 | 21600/2VS NM/FPS |
| | FX + 5 | DRANGE/D DRAG = F1 | 2700/805 NM/FPSS |
| | UNITV | UNIT V VECTOR | 2 |
| | UNITR | UNIT R VECTOR | 2 |
| | TEM1B | TEMPORARY LOCATION | |

| | SWITCHES | | INITIAL STATE |
|---|---|---|---|
| (6) | GONEPAST | INDICATES OVERSHOOT OF TARGET | NON-BRANCH (0) |
| (7) | RELVELSW | RELATIVE VELOCITY SWITCH | NON-BRANCH (0) |
| (8 | EGSW | FINAL PHASE SWITCH | NON-BRANCH (0) |
| (9) | HUNTIND | INITIAL PASS THRU HUNTEST | NON-BRANCH (0) |
| (10) | HIND | INDICATES ITERATION IN HUNTEST | NON-BRANCH (0) |
| (11) | INRLSW | INDICATES INIT ROLL ATTITUDE SET | NON-BRANCH (0) |
| (12) | LATSW | INHIBIT DOWNLIFT SWITCH IF NOT SET | BRANCH (1) |
| | LEWDSW | USE LEWD = .2 IF SET | NON-BRANCH (0) |

Fig. 5.7-2b Computer Variables (cont)

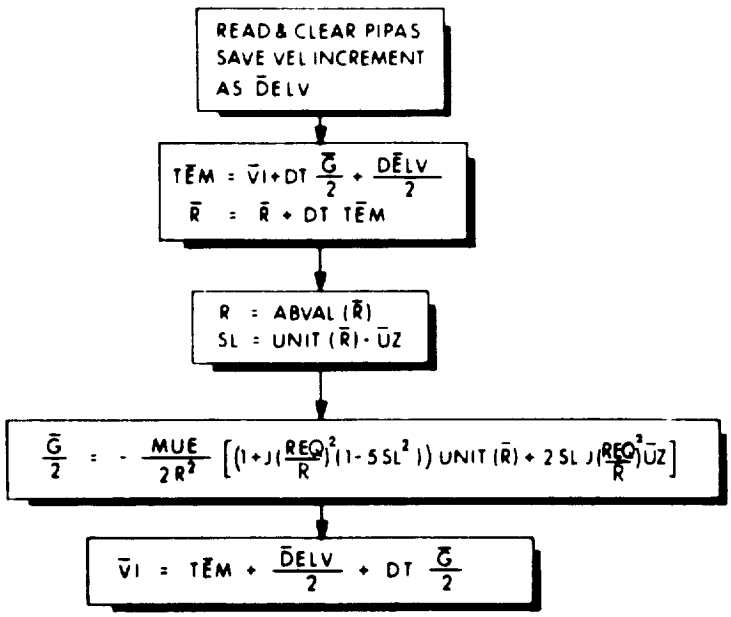
| CONSTANTS AND GAINS | | VALUE | | |
|---|---|---|---|---|
| C1 | FACTOR IN ALP COMPUTATION | 1.25 | | |
| C16 | CONSTD GAIN ON DRAG | .01 | | |
| C17 | CONSTD GAIN ON RDOT | .001 | | |
| C18 | BIAS VEL. FOR FINAL PHASE START | 500 | FPS | |
| C20 | MAX DRAG FOR DOWN-LIFT | 175 | FPSS | |
| CHOOK | FACTOR IN AHOOK COMPUTATION | .25 | | |
| CH1 | FACTOR IN GAMMAL COMPUTATION | .75 | | |
| D2 | DRAG TO CHANGE LEAD | 175 | FPSS | |
| DT | COMPUTATION CYCLE TIME INTERVAL | 2 | SEC. | |
| GMAX | MAXIMUM ACCELERATION | 322 | FPSS | (10 G-S) |
| KA1 | FACTOR IN KA CALC | 1.3 GS | | |
| KA2 | FACTOR IN KA CALC | .2 GS | | |
| KA3 | FACTOR IN DO CALC | 90 FPSS | | |
| KA4 | FACTOR IN DO CALC | 40 FPSS | | |
| KR1 | OPTIMIZED UPCONTROL GAIN | 3.4 | | |
| KR2 | OPTIMIZED UPCONTROL GAIN | .0034 | | |
| KDMIN | INCREMENT ON G7 TO DETECT END OF KEPLER PHASE | .5 | FPSS | |
| KLAT | LATERAL SWITCH GAIN | .0125 | | |
| KTETA | TIME OF FLIGHT CONSTANT | 1000 | | |
| K44 | GAIN USED IN INITIAL ROLL SECTION | 19749550 FPS | | |
| LAD | MAX L/D (MIN ACTUAL VEHICLE L/D) | .3 | | |
| LATBIAS | LATERAL SWITCH BIAS TERM | .4 | NM | |
| L/DCMINP | LAD COS(15 DEG) | .2895 | | |
| LEAD1 | NOMINAL UPCONTROL L/D | .1 | | |
| LEAD2 | UPCONTROL L/D HIGH DRAG CASES | .2 | | |
| LOD | FINAL PHASE L/D | .18 | | |
| POINT1 | FACTOR TO REDUCE UPCONTROL GAIN | .1 | | |
| Q2 | FINAL PHASE RANGE - 23500 Q3 | -1002 | NM | |
| Q3 | FINAL PHASE DRANGE/D V | .07 | NM/FPS | |
| Q5 | FINAL PHASE DRANGE/ D GAMMA | 7050 | NM/RAD | |
| Q6 | FINAL PHASE INITIAL FLIGHT PATH ANGLE | .0349 | RAD | |
| Q7F | MIN DRAG FOR UPCONTROL | 6 | FPSS | |
| Q7MIN | MIN VALUE FOR Q7 IN FACTOR CALCULATION | 50 | FPSS | |
| Q19 | FACTOR IN GAMMAL1 CALCULATION | .2 | | |
| VFINAL1 | VELOCITY TO START FINAL PHASE ON INITIAL ENTRY | 27000 | FPS | |
| VFINAL | FACTOR IN INITIAL UP-DOWN CALC | 26600 | FPS | |
| VSLOW | IF V LESS THAN VSLOW, LEAD = .2 | 35000 | FPS | |
| VLMIN | MINIMUM VL | 18000 | FPS | |
| VMIN | VELOCITY TO SWITCH TO RELATIVE VEL | VSAT/2 | | |
| VRCONTRL | RDOT TO START INTO HUNTEST | 700 | FPS | |
| VCORLIM | MAX VALUE OF VCORR | 1000 | FPS | |
| 25NM | TOLERANCE TO STOP RANGE ITERATION | 25 | NM | |
| VQUIT | VELOCITY TO STOP STEERING | 1000 | FPS | |

| CONVERSION FACTORS AND SCALING CONSTANTS | | | |
|---|---|---|---|
| ATK | ANGLE IN RAD TO NM | 3437.7468 | NM/RAD |
| GS | NOMINAL G VALUE FOR SCALING | 32.2 | FPSS |
| HS | ATMOSPHERE SCALE HEIGHT | 28500 | FT |
| J | GRAVITY HARMONIC COEFFICIENT | .00162346 | |
| KWE | EQUATORIAL EARTH RATE | 1546.70168 | FPS |
| MUE | EARTH GRAVITATIONAL CONSTANT | 3.986032233 | E 14 CUBIC M/ SEC SEC |
| RE | EARTH RADIUS | 21202900 | FT |
| VSAT | SATELLITE VELOCITY AT RE | 25766.1973 | FPS |
| WIE | EARTH RATE | .0000729211505 | RAD/SEC |
| REQ | EARTH EQUATORIAL RADIUS | 20925738.2 | FT |

Fig. 5.7-3 Constants, Gains, and Conversion Factors

```
INITIALIZATION
--------------

        -
        URTO = TARGET VECTOR AT TIME = ZERO

        -       -  -
        RTE  = UZ*URTO

        -       -  -
        UTR  = RTE*UZ

        -
        URT    = INITIAL TARGET VECTOR AT NOMINAL TIME OF ARRIVAL
                 (FOR THIS CALCULATION)

                -        -                 -
             = URTO + UTR(COS WT - 1) + RTE SIN WT
             WHERE WT = W TNOM

        -              -        -
        UNI  = UNIT (V*UNIT(R))

                -   -
        LATANG  = URT.UNI

                        -          -
        THETA = ARCCOS(URT.UNIT(R))


        K2ROLL = -SIGN(LATANG)

        Q7   =   Q7F

        SELECTOR  = INITROLL

        FACTOR = 1

        L/D = - LAD SGN(HEADSUP)

INITIALIZE SWITCHES

        GONEPAST = 0        RELVELSW = 0
        EGSW = 0            HUNTIND = 0
        MIND = 0            INRLSW = 0
        LATSW = 1           LEWDSW = 0
```

Fig. 5.7-4 Entry Program Initialization




Note:
This routine is identical to "Average-G" routine used elsewhere.

Fig. 5.7-5 "Average-G" Navigation Routine

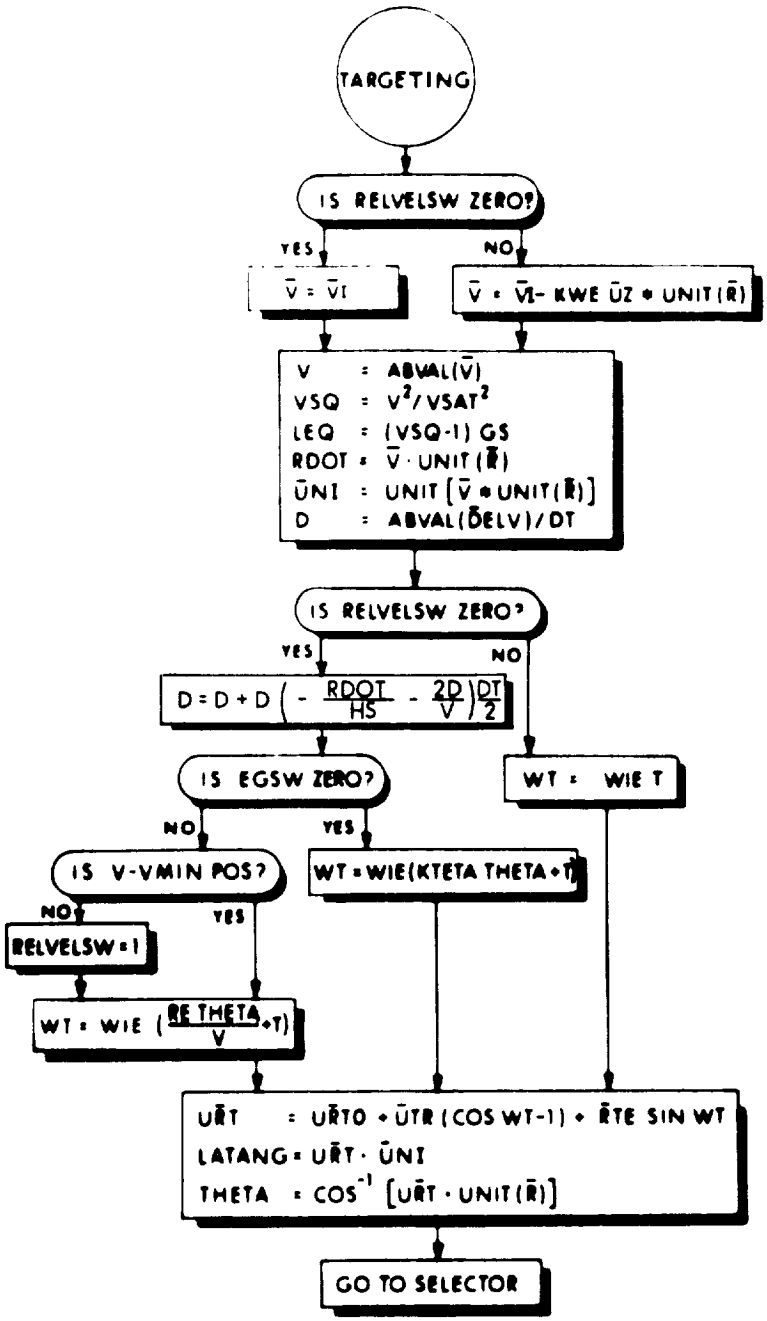



Fig. 5.7-6 Targeting

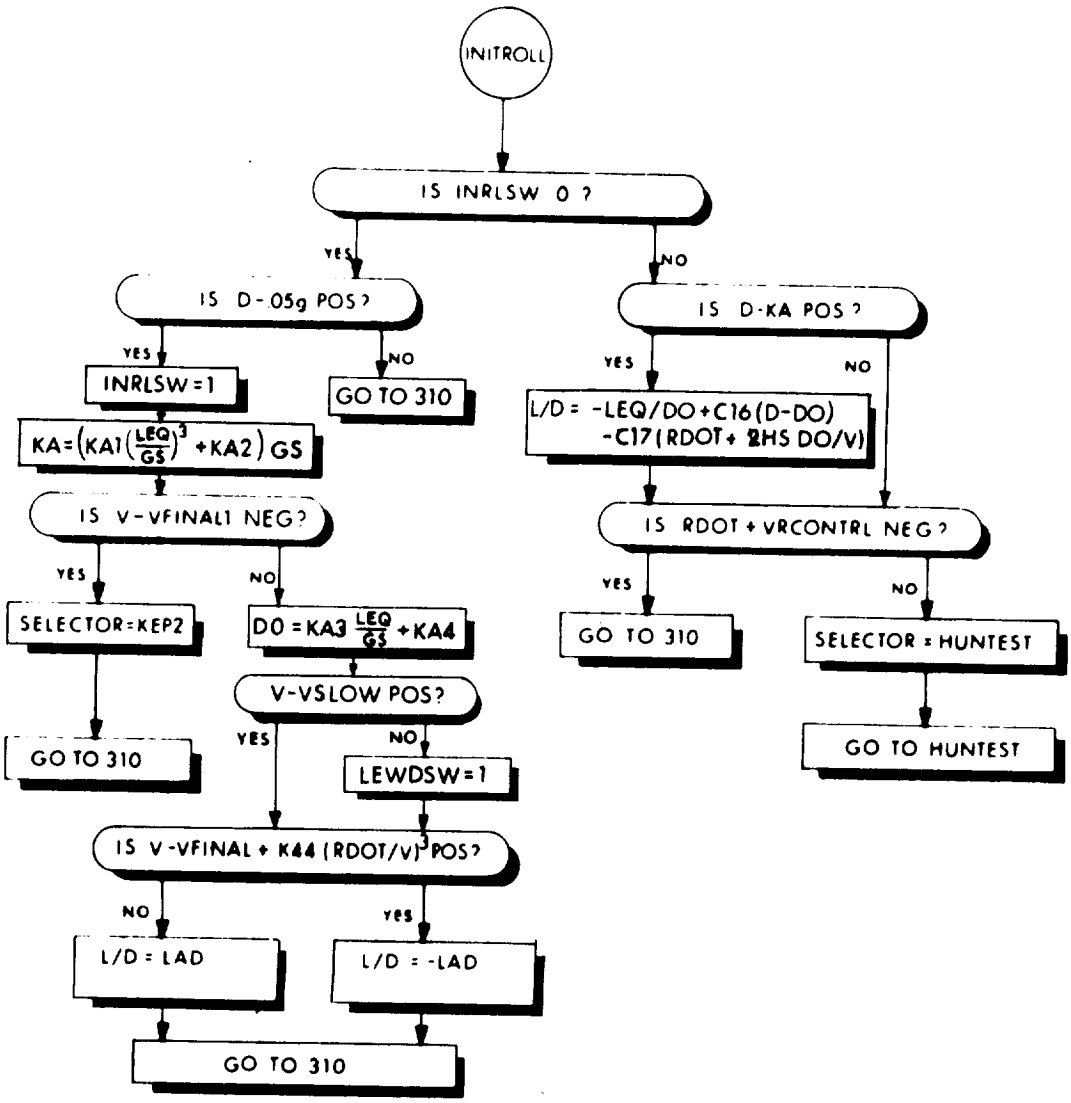



Fig. 5.7 -7 Initial Roll

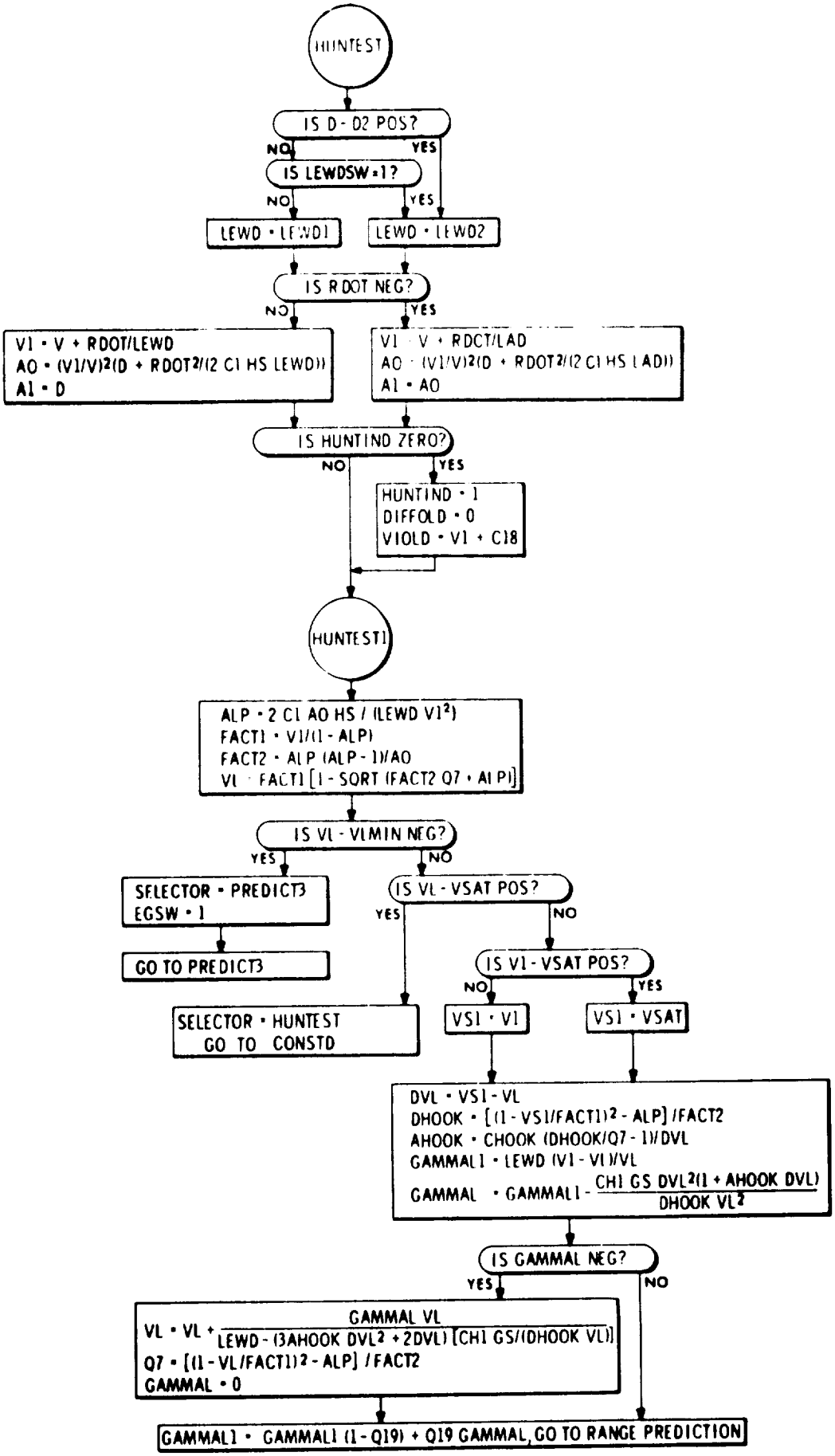



Fig. 5.7-8 Huntest

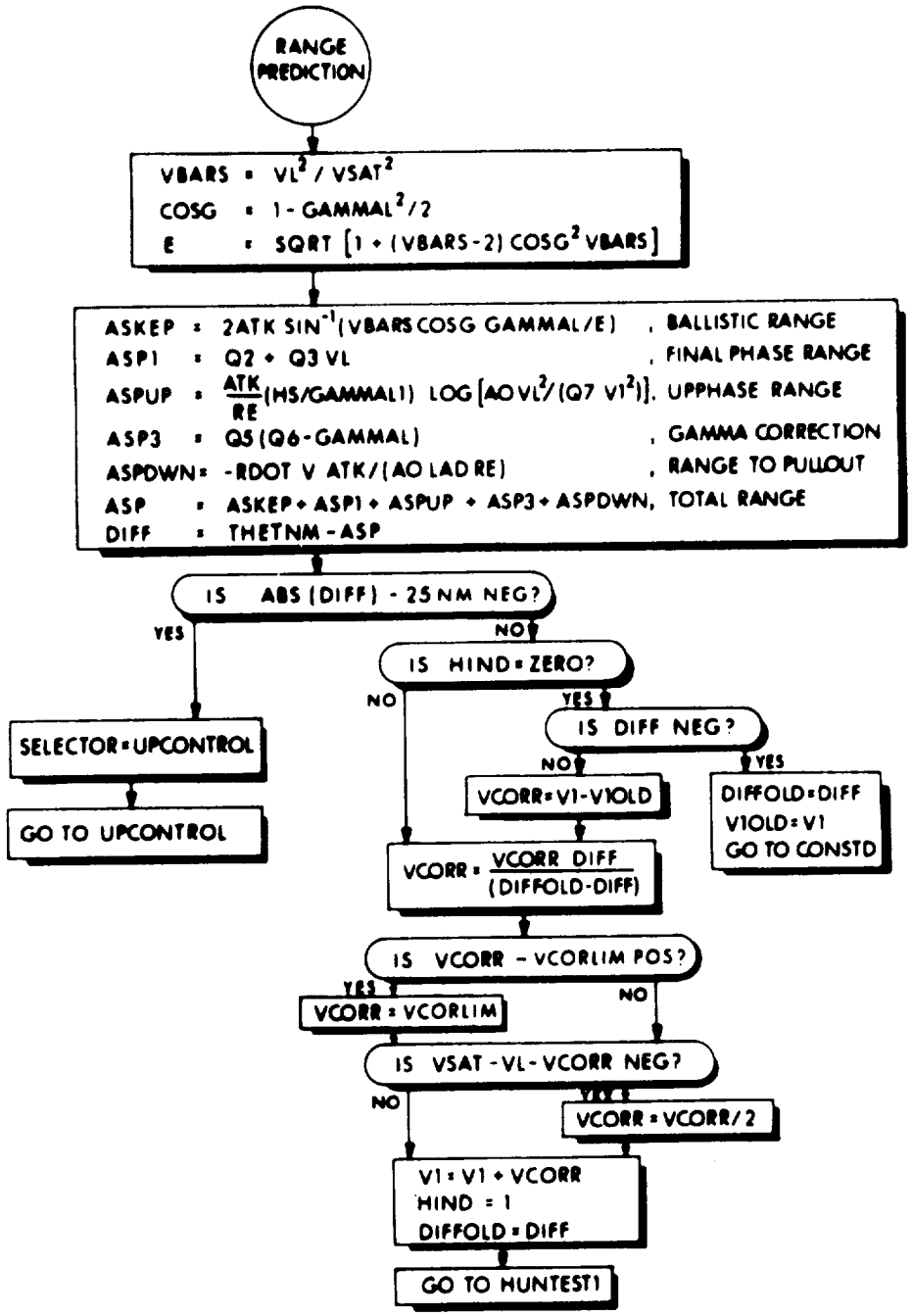



Fig. 5.7-9 Range Prediction

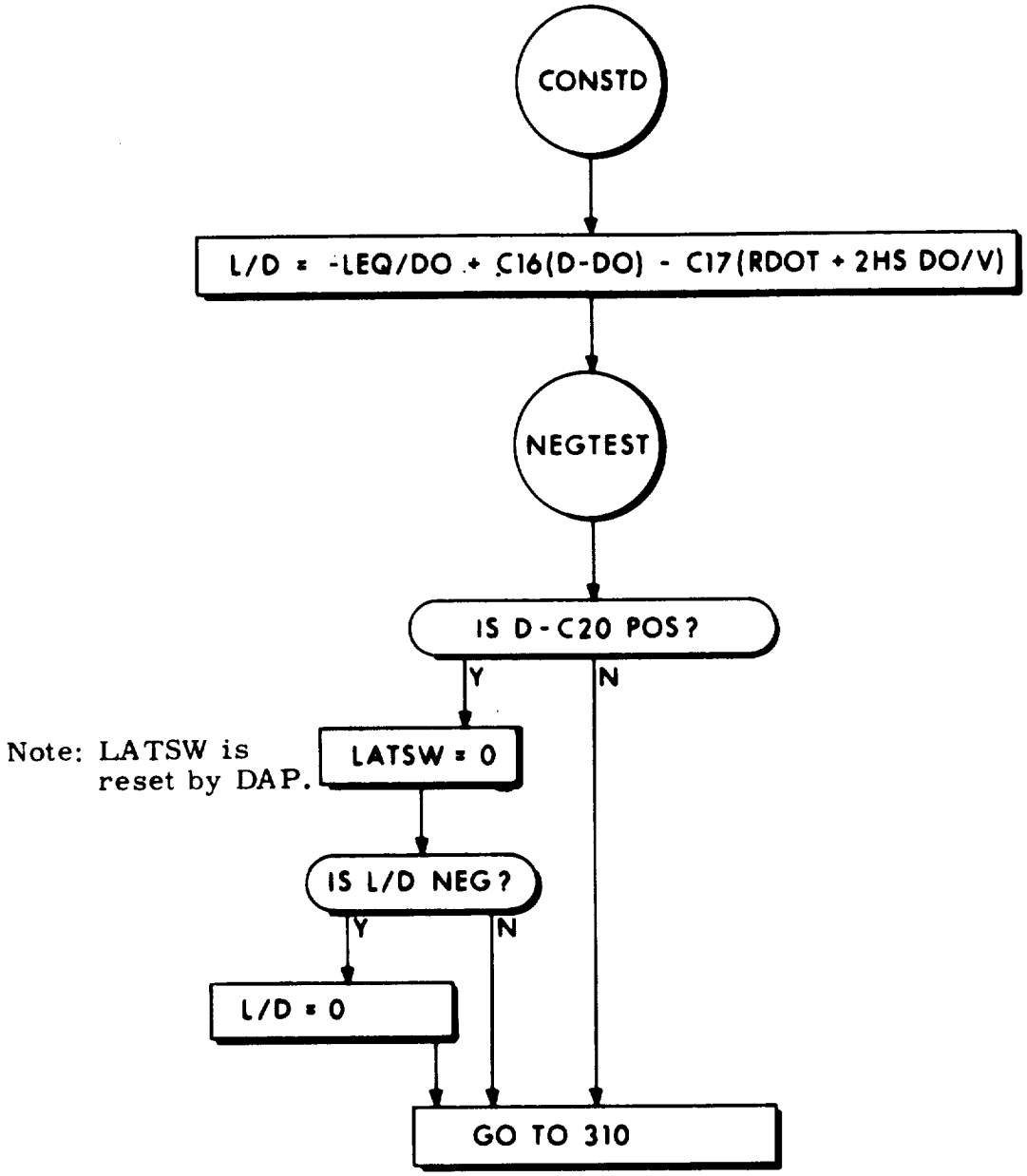



Fig. 5. 7-10 Constant Drag Control

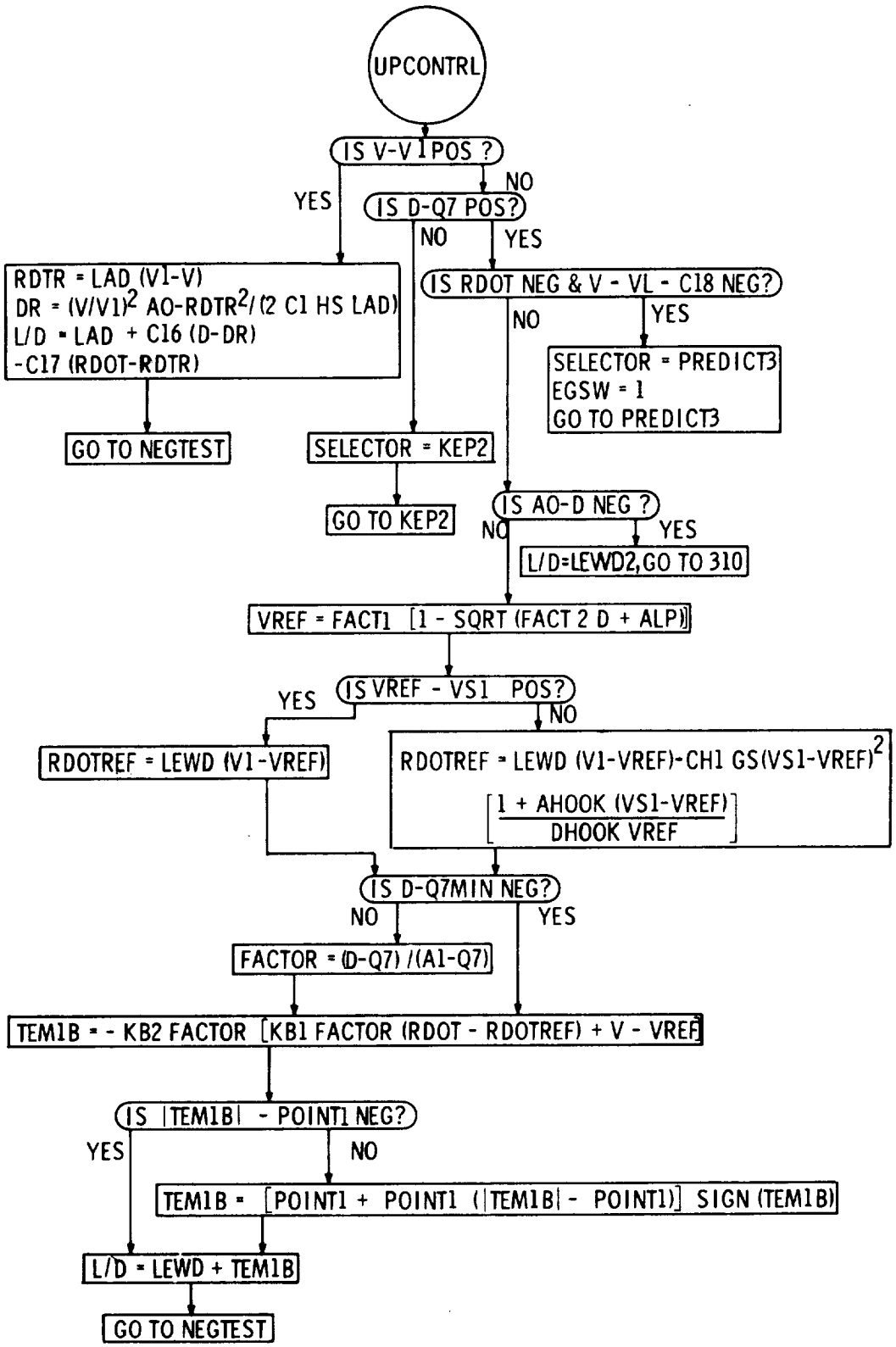



Fig. 5. 7-11 Upcontrol

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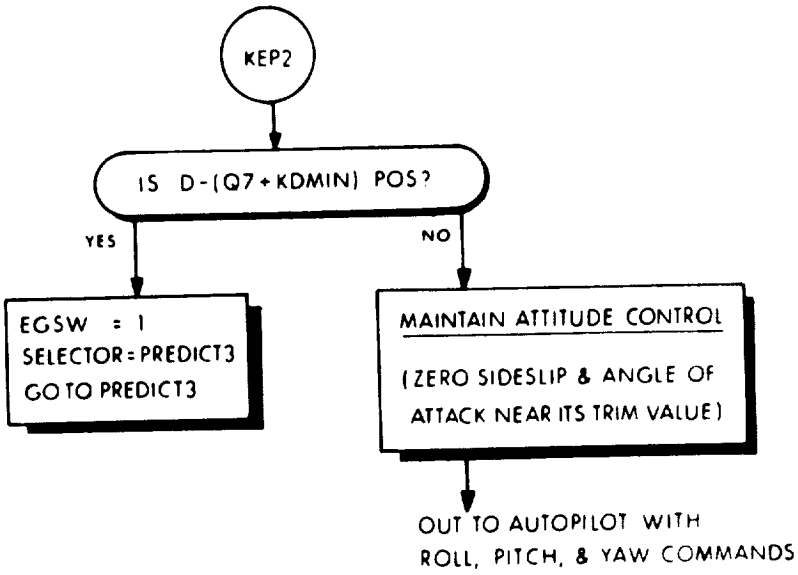


Fig. 5.7-12 Ballistic Phase

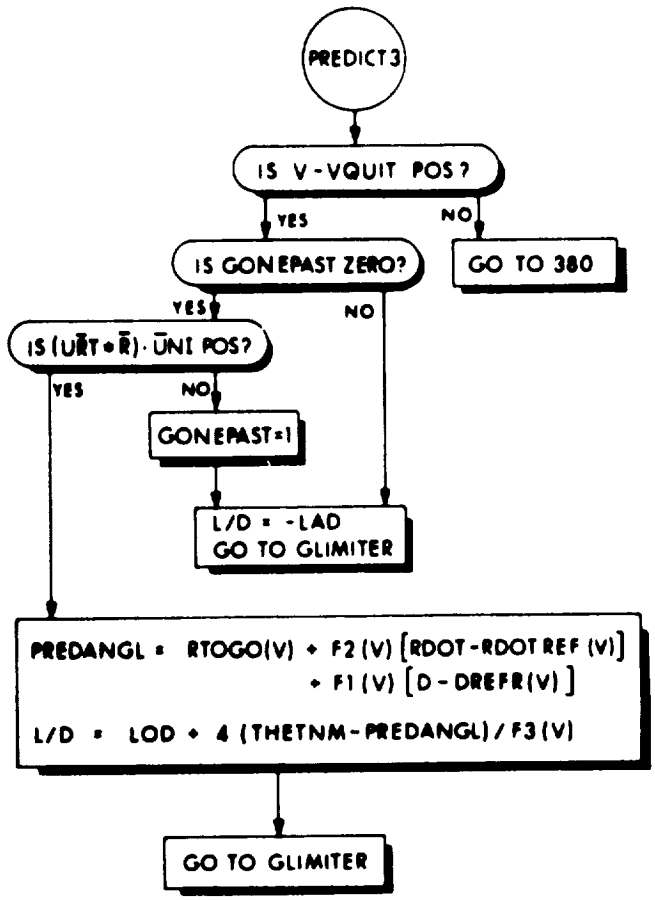


Fig. 5.7-13 Final Phase

| VREF FPS | RDOTREF FPS | DREFR FPSS | DR/DRDOT F2 NM/FPS | DR/DA F1 NM/FPSS | RTOGO NM | DR/DL/D F3 NM |
|---|---|---|---|---|---|---|
| 0 | -331 | 34.1 | 0 | -.02695 | 0 | 1 |
| 337 | -331 | 34.1 | 0 | -.02695 | 0 | 1 |
| 1080 | -693 | 42.6 | .002591 | -.03629 | 2.7 | 6.44 x 2 |
| 2103 | -719 | 60. | .003582 | -.05551 | 8.9 | 10.91 x 2 |
| 3922 | -694 | 81.5 | .007039 | -.09034 | 22.1 | 21.64 x 2 |
| 6295 | -609 | 93.9 | .01446 | -.1410 | 46.3 | 48.35 x 2 |
| 8531 | -493 | 98.5 | .02479 | -.1978 | 75.4 | 93.72 x 2 |
| 10101 | -416 | 102.3 | .03391 | -.2372 | 99.9 | 141.1 x 2 |
| 14014 | -352 | 118.7 | .06139 | -.3305 | 170.9 | 329.4 |
| 15951 | -416 | 125.2 | .07683 | -.3605 | 210.3 | 465.5 |
| 18357 | -566 | 120.4 | .09982 | -.4956 | 266.8 | 682.7 |
| 20829 | -781 | 95.4 | .1335 | -.6483 | 344.3 | 980.5 |
| 23090 | -927 | 28.1 | .2175 | -2.021 | 504.8 | 1385 |
| 23500 | -820 | 6.4 | .3046 | -3.354 | 643.0 | 1508 |
| 35000 | -820 | 6.4 | .3046 | -3.354 | 643.0 | 1508 |

Fig. 5.7-14a Final Phase Reference

Interpolation Procedure

Start with table entries. See Fig. 5.6-14

VREF(i) , i = 1,15

RDOTREF(i) , i = 1,15

etc

Vary the index i until velocity magnitude lies between two values in the table. That is:

$$VREF(i) < V < VREF(i+1).$$

Save the value i and form the linear interpolation factor

$$GRAD = \frac{V - VREF(i)}{VREF(i+1) - VREF(i)}$$

Then form interpolated values. For example

$$RDOTREF(V) = RDOTREF(i) + GRAD\left[RDOTREF(i+1) - RDOTREF(i)\right]$$

Fig. 5.7 -14b Final Phase Interpolation Procedure

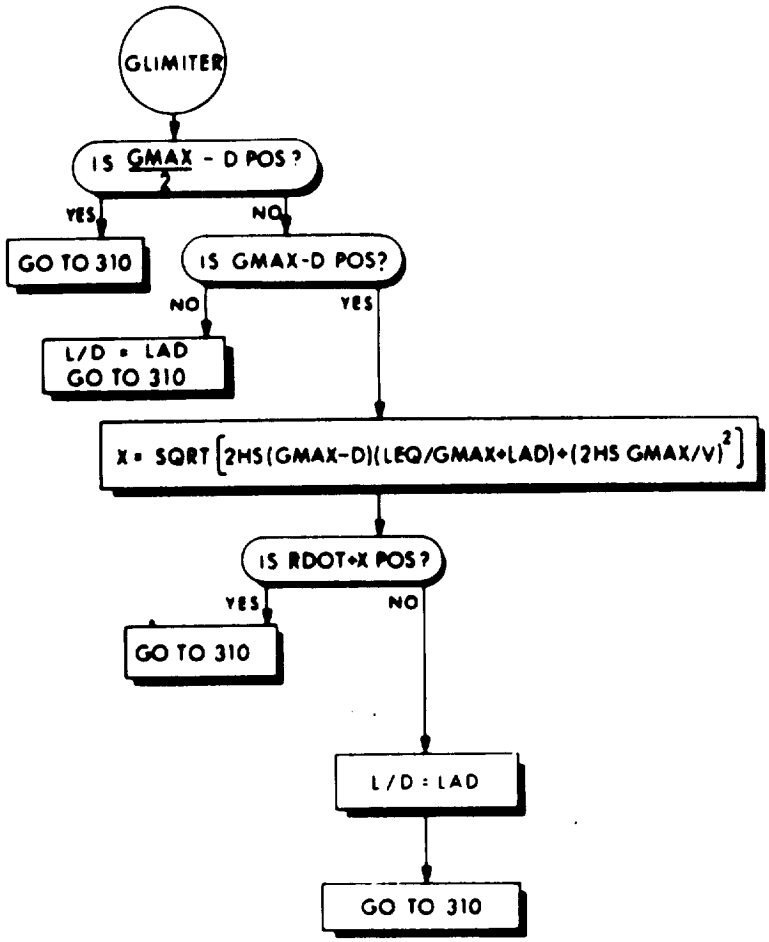


Fig. 5.7-15 G-Limiter

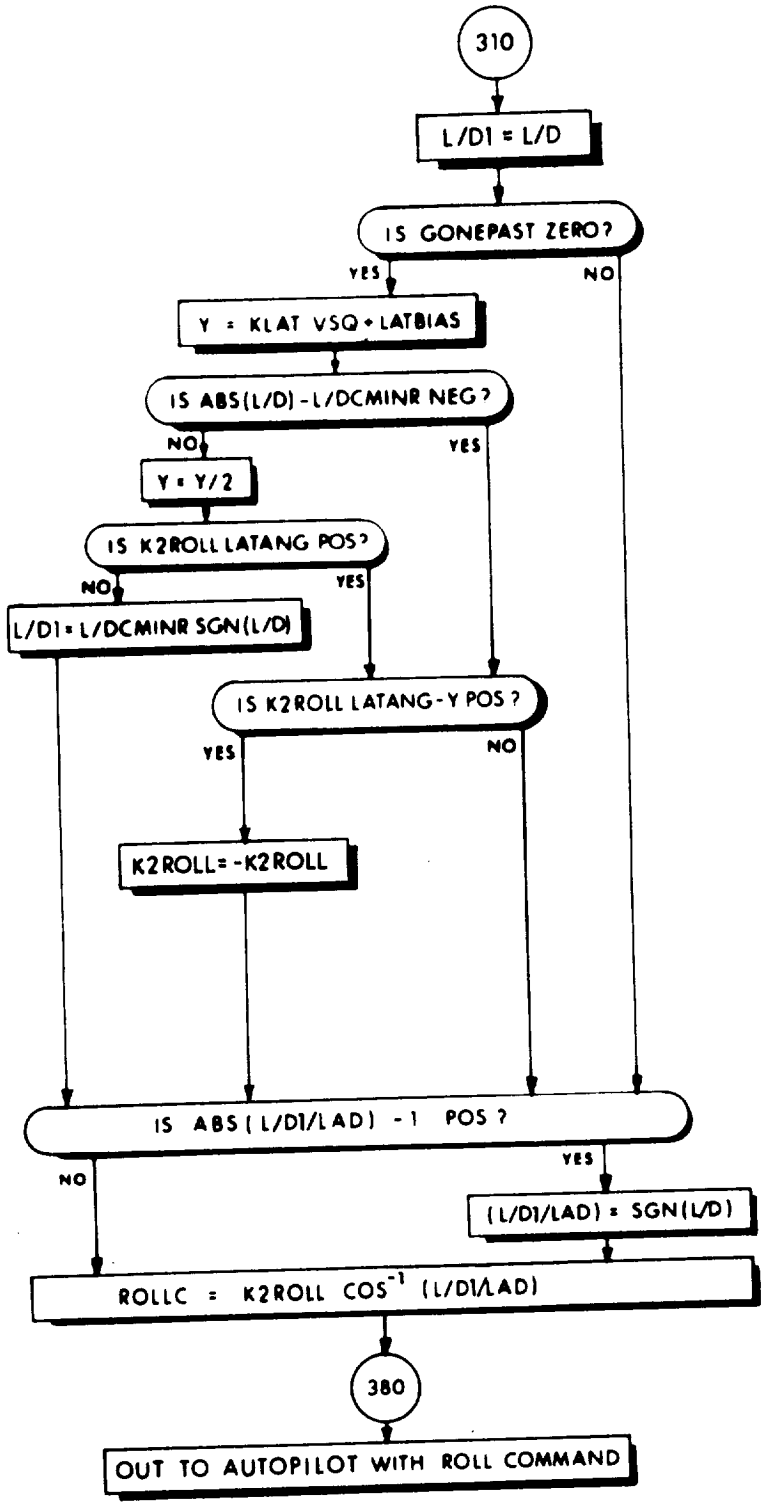


Fig. 5.7-16 Lateral Logic