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1 MARCH 1965

DIGITAL COMPUTER PROGRAMS
FOR
ROCKET NOZZLE DESIGN AND ANALYSIS

VOLUME IX
MULTIPLE EXPANSION PLUG NOZZLE DESIGN

Prepared under Contract NAS9-2487
for NASA Manned Spacecraft Center

AVAILABLE TO U.S. GOVERNMENT AGENCIES ONLY

Pratt & Whitney Aircraft DIVISION OF UNITED AIRCRAFT CORPORATION

FLORIDA RESEARCH & DEVELOPMENT CENTER

[redacted]

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FOREWORD

This manual provides the necessary background for successful operation of the Multiple Expansion Plug Nozzle Design computer program. The manual was prepared under Contract NAS9-2487, Digital Computer Programs for Rocket Nozzle Design and Analysis, with the NASA Manned Spacecraft Center, Houston, Texas, and is the ninth of ten volumes specified in Parts I and II of the contract.

CONTENTS

ABSTRACT

The necessary information for successful operation of the Multiple Expansion Plug Nozzle Design computer program is presented in this manual. Design criteria for the construction of a perfect plug nozzle and secondary contour and the order of calculations of the computer program are given with a discussion and flow diagram of each subroutine. The input required by the program is described and a sample output given.

No attempt is made in this manual to derive the equations used by the program. A general derivation of the basic equations, along with applications, is given in Volume I of this report.

SECTION I
INTRODUCTION

The Multiple Expansion Plug Nozzle Design computer program mathematically constructs the supersonic nozzle contours of a perfect plug nozzle and secondary contour and calculates the corresponding performance. The method of characteristics for steady, supersonic potential flow is used in constructing the flow field. Thermodynamic properties can be either based on an ideal gas, where the gas properties of the combustion products are known, or approximated by a perfect gas.

Design criteria and a detailed description of the order of calculations are presented herein. Each of the subroutines used in the program is discussed, and flow diagrams are given for clarification. The input and output formats for the program are included with recommended procedures to use in the event of unsuccessful runs.

Construction of the contours for a perfect multiple expansion plug nozzle requires that the throat flow be expanded to a uniform flow at the exit. However, the length and weight of such a nozzle may be excessive for use in any practical space vehicle. A significant reduction in weight and length can be effected with little loss in thrust by simply truncating the plug.

SECTION II
TECHNICAL DESCRIPTION

In the design of a supersonic multiple expansion plug nozzle, the annular throat flow is expanded toward the centerline of the nozzle and then undergoes a second expansion at the end of the secondary contour or shroud, thereby turning the flow to an axial direction. The expansions occur in a manner such that uniform, shock-free flow is obtained at the exit of the nozzle. The initial starting condition, exit Mach number, and gas model are required parameters for constructing the contours of the perfect nozzle when the location of the first secondary contour point is specified. The method of characteristics for two-dimensional or axisymmetric potential flow is used to construct the necessary flow field.

The main program of the deck is used to control the order of calculations, while calculations such as determining the fluid properties at an interior point in the flow field or performance parameters along the constructed nozzle contours are made in subroutines that are "called" by the main program or other subroutines. The functions of the main program and calculations made therein are described in Paragraph B, and descriptions of the individual subroutines are given in Paragraph C.

A. DESIGN CRITERIA

A typical supersonic multiple expansion plug nozzle is shown in figure 1. The initial expansion occurs along the entire secondary contour, $S_1 S_2$, with the point of expansion located at the beginning of the primary or plug contour, C_1. The second point of expansion is located at the end of the secondary nozzle with the flow expanding

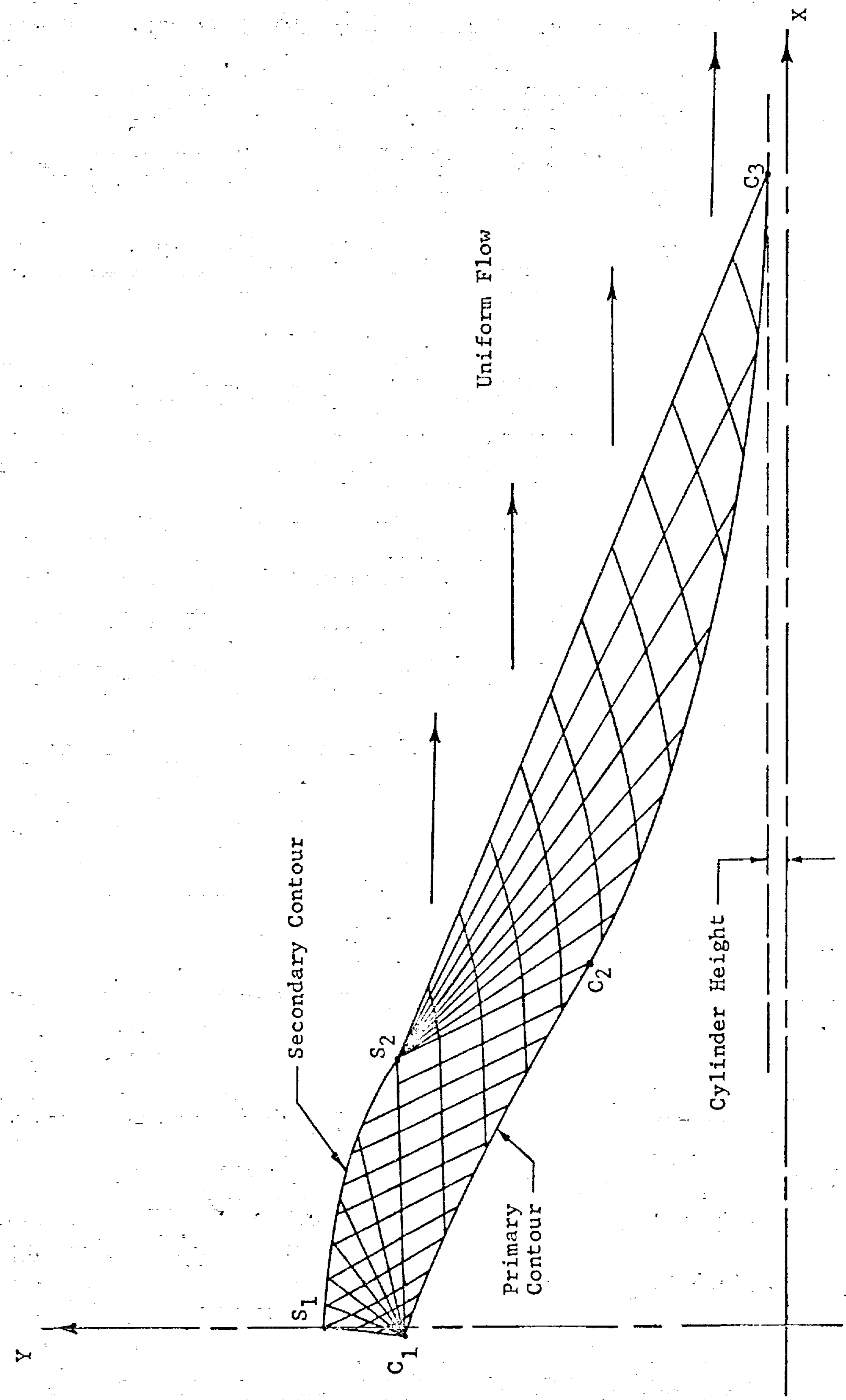


Figure 1. Typical Perfect Multiple Expansion Plug Nozzle

between C_2 and C_3 on the primary contour. The region C_1 S_2 C_2 is a transition region in which little or no variation in the flow exists. All parameters are nondimensionalized within the program, the point S_1 being located at (0, 1).

1. Gas Model

In many cases, the thermodynamic properties of the nozzle exhaust may be approximated by assuming a perfect gas; in which case only the specific heat ratio, γ, must be input and the thermodynamic properties are calculated using perfect gas relationships. For this condition, the local velocity is nondimensionalized with respect to the maximum velocity (V_{max}), and the critical velocity ratio (i.e., where M = 1) and density ratio at the throat are

$$W_{sonic} = \frac{V_{sonic}}{V_{max}} = \sqrt{\frac{\gamma - 1}{\gamma + 1}}$$

and

$$\frac{\rho_{sonic}}{\rho_o} = \left[1 - W^2_{sonic}\right]^{\frac{1}{\gamma - 1}} .$$

The flow field for an ideal gas (usually equilibrium or frozen flow) may be calculated by specifying the thermodynamic properties (in tabular form) as a function of specific impulse. These properties may be obtained from conventional one-dimensional combustion programs*, and consist of pressure, density, local frozen sound speed (optional), and specific impulse. The SONICP Subroutine "beam fits" these properties as a function of

*Zelenik, F.S., and S. Gordon, NASA TN D-1454, "A General IBM 704 or 7090 Computer Program for Computation of Chemical Equilibrium Compositions, Rocket Performance, and Chapman-Jouguet Detonations."

specific impulse and determines the sonic velocity and density at the throat. The local velocities are then nondimensionalized with respect to the sonic velocity and the thermodynamic properties are beam fit as a function of the velocity ratio.

2. Starting Conditions

The starting conditions (i.e., the flow properties along S_1 C_1 in figure 1) can be specified by any of the following methods.

1. Uniform flow. This type of flow can be obtained when the slope of the velocity vector at point S_1, figure 1, is zero.
2. A constant Mach number line. When the slope of the velocity vector at S_1 is nonzero, a line of constant Mach number or velocity ratio is developed (see TFLOW Subroutine).
3. A given Mach line, the coordinates and flow properties of which are known along the line.

For all of these methods, the initial starting line must be slightly supersonic.

3. Design Mach Number

Although parameters such as the nozzle area ratio, design pressure ratio or the mass flow rate may be used to describe the exit conditions, this program uses the Mach number at the exit of the nozzle as a design parameter. (An area ratio option can be used with a perfect gas.) The input parameter used to describe the design Mach number is EMI.

B. FLOW FIELD CONSTRUCTION

The first function of the main program is to call the INPUT Subroutine, which initializes parameters and reads in the input data. After the fluid properties at the exit of the nozzle are calculated, the TFLOW

Subroutine is used to develop the initial Mach line (if the Mach line is not input) and to calculate the total mass flow through the nozzle.

To begin the flow field construction, the flow at the throat is expanded through a small increment of velocity ratio, W, using the procedure in EXPAN Subroutine. For circular expansion, this requires that a down Mach line (I_1 B_1 in figure 2a) from the initial Mach line be constructed such that the change in velocity ratio is within the expansion tolerance of WEXPAN. From the expansion point, an up Mach line (B_1 B_n in figure 2a) is constructed by calculating the interior point intersections with down Mach lines from points on the initial Mach line (INTX Subroutine). The construction using corner expansion is similar, except that the coordinates of the point B_1 are the same as A_1 (figure 2b). Since the mass flow between B_1 B_n is less than the total mass flow for the nozzle, this line must be extended. Assuming dW/dX constant, the Mach line is temporarily extended, using the procedure in INTEG Subroutine, until the total mass flow is reached (B_i).

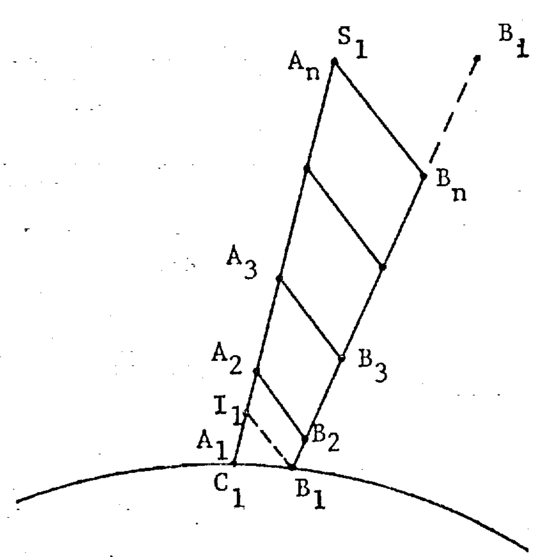


Figure 2a. Circular Expansion

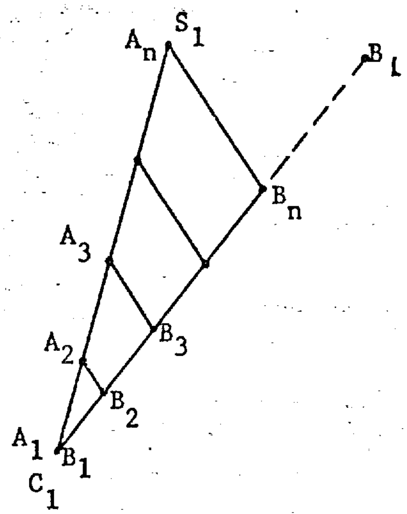


Figure 2b. Corner Expansion

If the slope of the velocity vector at B_i is positive, the procedure of expanding the flow and constructing up Mach lines is continued. If the slope is negative, a test is made to determine if there has been enough internal expansion. Assuming B_i to be the last point on the secondary contour, the velocity vector corresponding to the design Mach number is turned from its horizontal position to the slope of the velocity vector at B_i. If the compressed velocity is greater than W_{B_i}, the internal expansion is continued until the compressed velocity is equal to W_{B_i}.

Depending on the value in the variable TIFLOW, the program will either iterate to obtain a specified cylinder height by adjusting the throat size or calculate a cylinder height from the throat mass flow. In the latter instance, all calculations in the program will be discontinued if the calculated cylinder height is a negative number.

The exit Mach line is not extended to the axis of symmetry since, for axisymmetric flow, numerical difficulties arise due to an indeterminate term at Y = 0 in the compatibility equations. Therefore, a cylinder of very small radius is placed around the axis (figure 1). This cylinder permits the solution to be approximated quite accurately, while eliminating the instabilities that may occur when trying to determine the limiting value as $Y \rightarrow 0$. Although this procedure is not required for two-dimensional flow, it is usually used to eliminate problems that may arise when calculating succeeding down Mach lines.

When the option is used to converge on a specific cylinder height, the throat mass flow is compared to the exit mass flow. If the two mass flows are not within a specified tolerance, the throat size is adjusted and the development of the first section is repeated.

After satisfying the above iteration, or when the cylinder height is to be calculated for given initial conditions, the minimum cross sectional area is calculated from

$$A^* = \frac{\dot{\omega}}{\rho_{sonic} W_{sonic}},$$

and the gross thrust coefficient at the first point on both the primary and secondary contours is determined by integrating the rate of change of momentum and static pressure forces along the initial up Mach line (CTGT Subroutine).

Region II, described by $A_n B_n B_i$ in figure 3, contains the construction of the secondary contour. Up Mach lines are extended from points on $A_n B_n$ by calculating the interior point intersections with down Mach lines from $B_n B_i$. Each Mach line is extended until the accumulated mass flow exceeds the total mass flow of the nozzle and a secondary contour point is then determined by linear interpolation on the total mass flow.

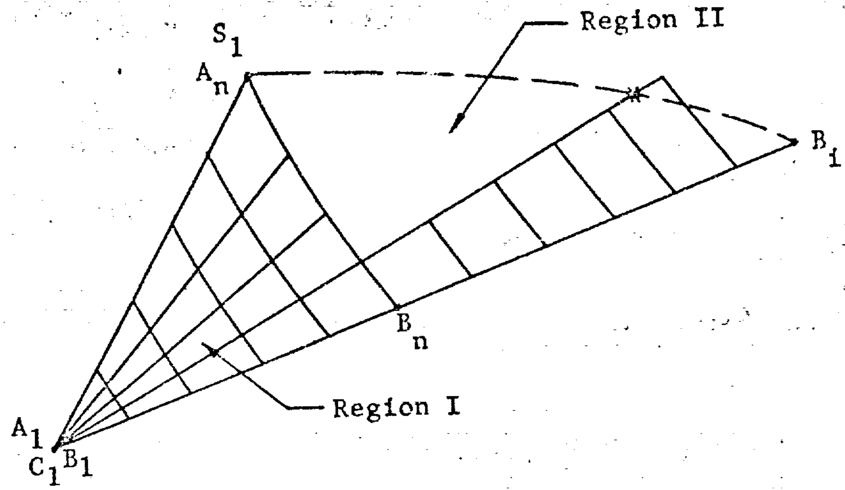


Figure 3.

The exit Mach line is developed by extending a straight Mach line from the last point on the secondary contour to the cylinder height at the design Mach angle. The number of points along this line are determined by the input mesh size. From the external expansion point at B_i (figure 4), the flow is compressed by a small decrement in the velocity ratio (EXPAND Subroutine) and a new down Mach line, $B_i\ P_{n-1}$, is developed by interior point intersections with up Mach lines from points on $B_i\ P_n$. The accumulated mass flow at each interior point is obtained by summing the mass flow across each up Mach line to the exit line with the accumulated mass flow at the corresponding point on the exit Mach line (MFLOWT Subroutine). The new down Mach line is extended in this manner until the total mass flow is exceeded; then a contour point on the primary contour is calculated by linear interpolation on the total mass flow through the nozzle.

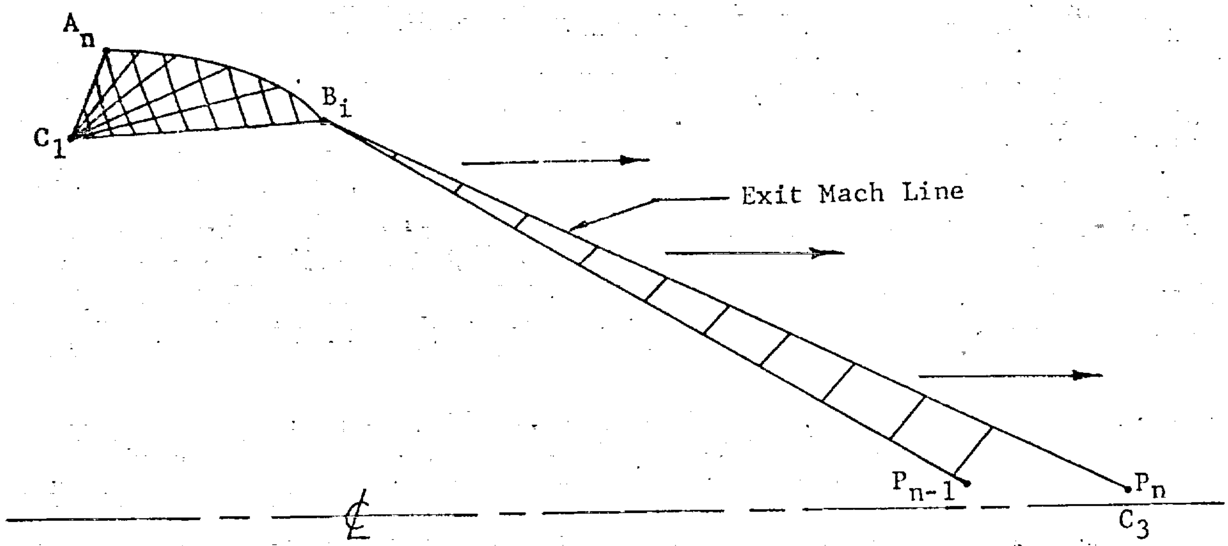


Figure 4.

The procedure of compressing the flow in the external expansion region is continued until the slope of the velocity vector at B_i is the same for both Regions II and III (figure 5).

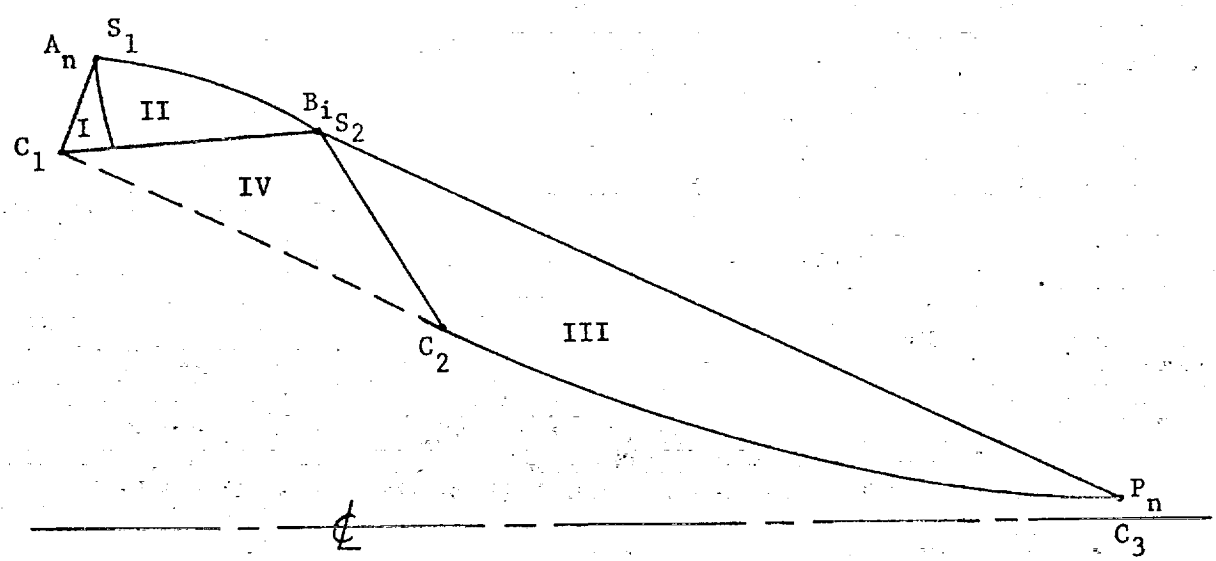


Figure 5.

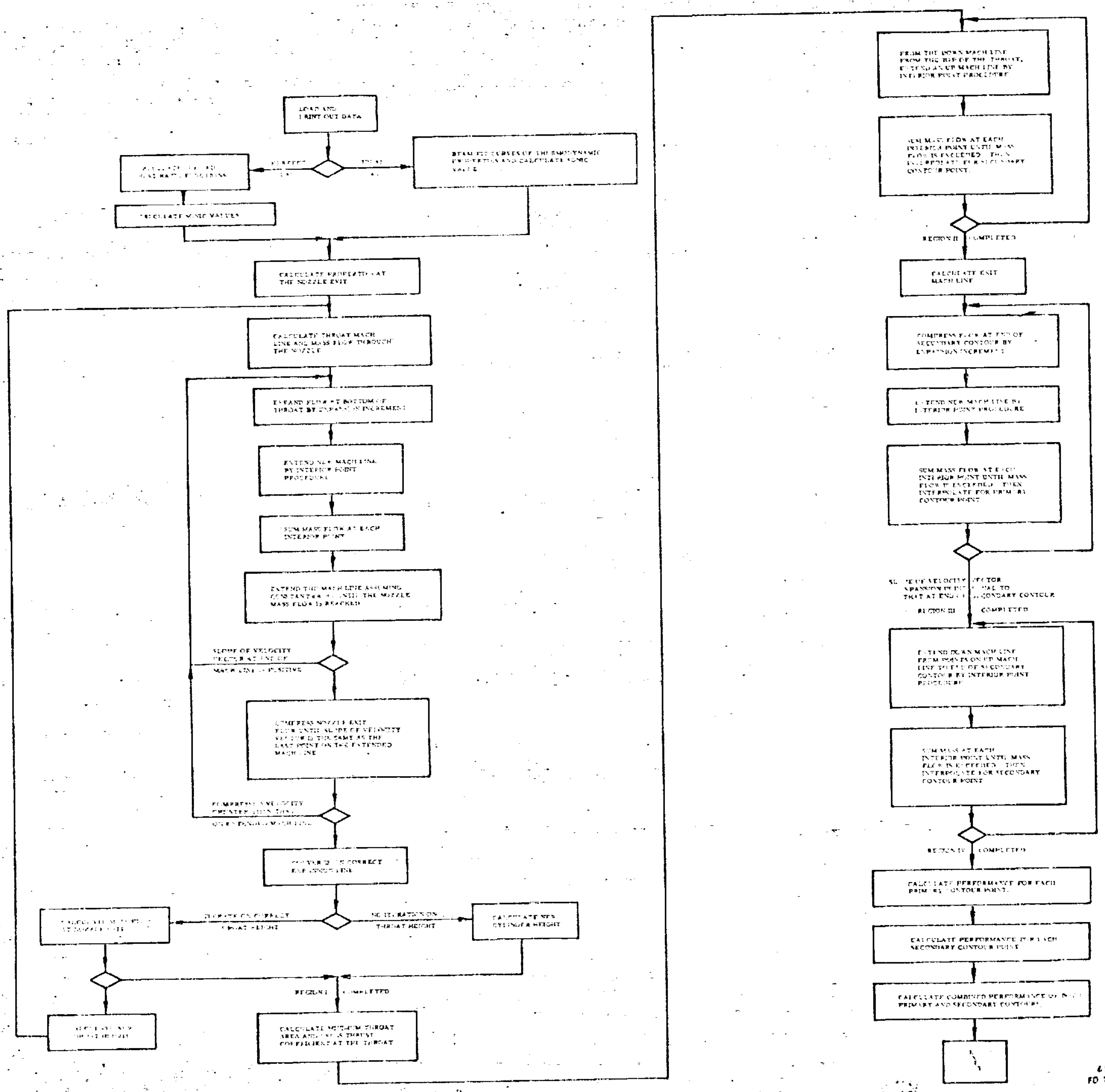
The remainder of the primary contour and Region IV is calculated by extending down Mach lines from points on $C_1 B_i$. Interior point intersections with up Mach lines from $B_i C_2$ are used in the construction of the lines with the primary contour points determined from the mass flow balance procedure.

Having constructed both the primary and secondary contours, the PERFM Subroutine is used to calculate and print the following performance parameters for each contour:

1. X/R
2. Y/R
3. TAN THETA (Contour slope)
4. MACH NO.
5. P/PC (Local static pressure/chamber pressure)
6. GAMMA (Specific heat ratio)
7. AS/A* (Local surface area/throat area)
8. CTG (Gross thrust coefficient)
9. CTN (Net thrust coefficient)

The combined effect of both contours is then calculated by the MAIN program.

Multiple Expansion Plug Nozzle Design



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C. SUBROUTINES

Since most of the subroutines are used many times in constructing a flow field or calculating performance, they are discussed individually in this section. The purpose of each subroutine, the equations used, and flow diagrams are given.

1. INTX Subroutine

Under certain conditions, the numerical solution of the characteristic system becomes difficult or impossible in determining the intersection of an up and a down Mach line. These conditions may occur if the slope of one or both of the Mach lines is extremely large or small. The problem can be eliminated by rotating the coordinate axis when solving the physical characteristics, and by modifying the axisymmetric term in the compatibility equations. The physical characteristic equations are invariant under this transformation. The INTX Subroutine performs the function of determining if rotation is needed, the form required for the axisymmetric term, and calls one of the following subroutines:

INT1 Subroutine - No rotation is used and the axisymmetric terms of the compatibility equations for both up and down Mach lines use the differential dX.

INT2 Subroutine - The coordinate system is rotated and the axisymmetric terms of the compatibility equations for both up and down Mach lines use the differential dX (for an up or down Mach line with very small slope).

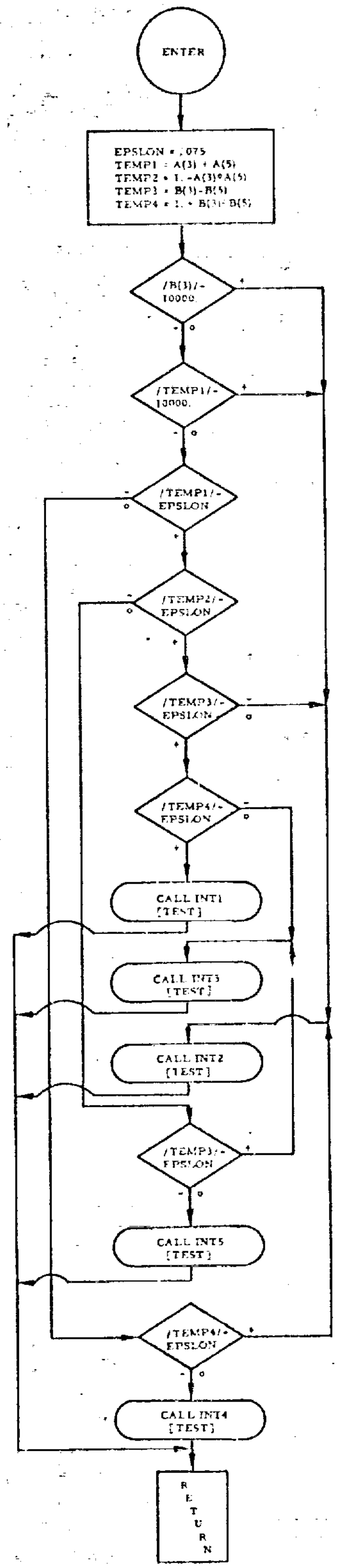
INT3 Subroutine - The coordinate system is rotated and the axisymmetric terms of the compatibility equations for both up and down Mach lines use the differential dY (for an up or down Mach line with very large slope).

INT4 Subroutine - The coordinate system is rotated and the axisymmetric term for the up Mach line uses dX and the down Mach line dY (for an up Mach line with very small slope combined with a down Mach line with very large slope).

INT5 Subroutine - The coordinate system is rotated and the axisymmetric term for the up Mach line uses dY and the down Mach line dX (for an up Mach line with very large slope combined with a down Mach line with very small slope).

The coordinates and flow conditions (W, tan θ, and tan α) at the points on an up and down Mach line must be stored into the variables A(I) and B(I), respectively.

Subroutine INTX



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2. INTI Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at two points in the flow field not on the same Mach line, the INT1 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point, and a down Mach line from the other. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the interior point, 3, (figure 6) will be stored in the variable C(I).

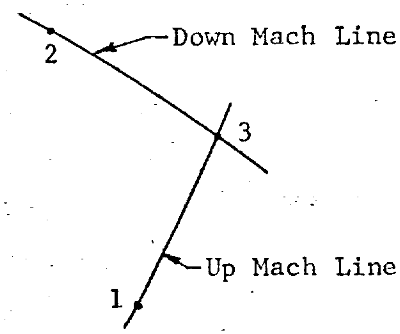


Figure 6

Written in finite difference form, the characteristic system is

$$(Y_3 - Y_1) = \left[\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha\tan\theta}\right]_1 (X_3 - X_1)$$

and (2.1)

$$(Y_3 - Y_2) = \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_2 (X_3 - X_2),$$

$$(W_3-W_1)\left[\frac{1}{W\tan\alpha}\right]_1 - (\tan\theta_3 - \tan\theta_1)\left[\frac{1}{1+\tan^2\theta}\right]_1 = \sigma\left[\frac{\tan\alpha\tan\theta}{(1-\tan\alpha\tan\theta)Y}\right]_1 (X_3-X_1)$$

and (2.2)

$$(W_3-W_2)\left[\frac{1}{W\tan\alpha}\right]_2 + (\tan\theta_3 - \tan\theta_2)\left[\frac{1}{1+\tan^2\theta}\right]_2 = \sigma\left[\frac{\tan\alpha\tan\theta}{(1+\tan\alpha\tan\theta)Y}\right]_2 (X_3-X_2).$$

The subscripts 1 and 2 indicate that the quantities in brackets are evaluated at these points.

Solving equations (2.1) simultaneously, the coordinates at point 3 are:

$$X_3 = \frac{Y_1 - \left[\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha\tan\theta}\right]_1 X_1 - Y_2 + \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_2 X_2}{\left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_2 - \left[\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha\tan\theta}\right]_1}$$

and (2.3)

$$Y_3 = Y_2 + (X_3 - X_2)\left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_2 .$$

Flow conditions W_3 and $\tan\theta_3$ are then obtained from the simultaneous solution of equations (2.2).

$$W_3 = \frac{K_1 + K_2}{\left[\frac{1}{W\tan\alpha}\right]_2 \div \left[\frac{1}{1+\tan^2\theta}\right]_2 + \left[\frac{1}{W\tan\alpha}\right]_1 \div \left[\frac{1}{1+\tan^2\theta}\right]_1}$$

and (2.4)

$$\tan\theta_3 = K_2 + W_3\left[\frac{1}{W\tan\alpha}\right]_1 \div \left[\frac{1}{1+\tan^2\theta}\right]_1 ;$$

where:

$$K_1 = \tan\theta_2 + W_2\frac{\left[\frac{1}{W\tan\alpha}\right]_2}{\left[\frac{1}{1+\tan^2\theta}\right]_2} + \sigma\,(X_3 - X_2)\frac{\left[\frac{\tan\alpha\tan\theta}{(1+\tan\alpha\tan\theta)Y}\right]_2}{\left[\frac{1}{1+\tan^2\theta}\right]_2}$$

and

$$K_2 = \tan\theta_1 - W_1\frac{\left[\frac{1}{W\tan\alpha}\right]_1}{\left[\frac{1}{1+\tan^2\theta}\right]_1} + \sigma\,(X_1 - X_3)\frac{\left[\frac{\tan\alpha\tan\theta}{(1-\tan\alpha\tan\theta)Y}\right]_1}{\left[\frac{1}{1+\tan^2\epsilon}\right]_1}$$

The tangent of the Mach angle ($\tan\alpha_3$), which is a function of W_3, is determined by the procedure described in TAGAL Subroutine for an ideal gas or by the procedure in the PRFCT Subroutine for a perfect gas.

Since the evaluation of equations (2.3) and (2.4) gives first approximations to X_3, Y_3, $\tan\theta_3$, W_3, and $\tan\alpha_3$, improved solutions are obtained by replacing the quantities in brackets with average values; that is,

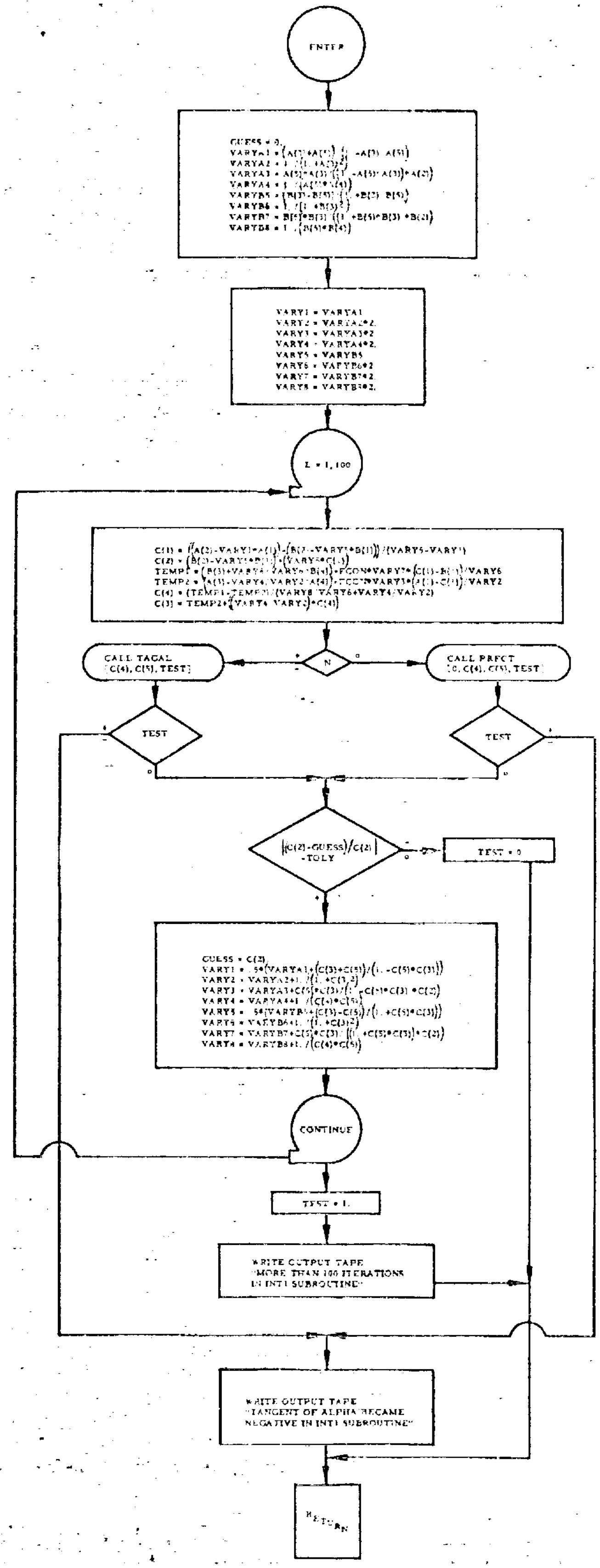
replace $\left[\dfrac{\tan\theta + \tan\alpha}{1-\tan\alpha\tan\theta}\right]_1$ by

$$\left[\frac{\tan\theta + \tan\alpha}{1-\tan\alpha\tan\theta}\right]_{1,3} = 1/2\left\{\left[\frac{\tan\theta + \tan\alpha}{1-\tan\alpha\tan\theta}\right]_1 + \left[\frac{\tan\theta + \tan\alpha}{1-\tan\alpha\tan\theta}\right]_3\right\}$$

This procedure for obtaining the improved solutions is repeated until successive values of X_3 are within a specified tolerance:

$$\left|X_3^i - X_3^{i-1}\right| \leq \text{XTOL.}$$

Subroutine INT 1



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3. INT2 Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at two points in the flow field not on the same Mach line, the INT2 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of either Mach line is very small. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 7) will be stored in the variable C(I).

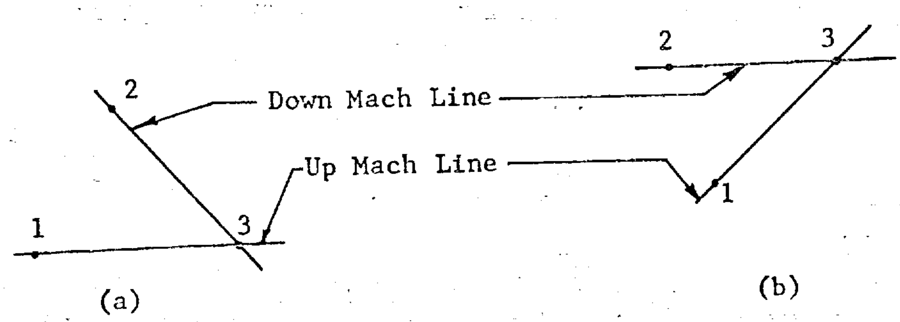


Figure 7

The characteristic system is the same as for the INT1 Subroutine (equations 2.1 and 2.2), except that the coordinate axes are rotated. The coordinate transformation is given by the following:

$$X' = X \cos \varphi + Y \sin \varphi$$

$$Y' = Y \cos \varphi - X \sin \varphi,$$

and (3.1)

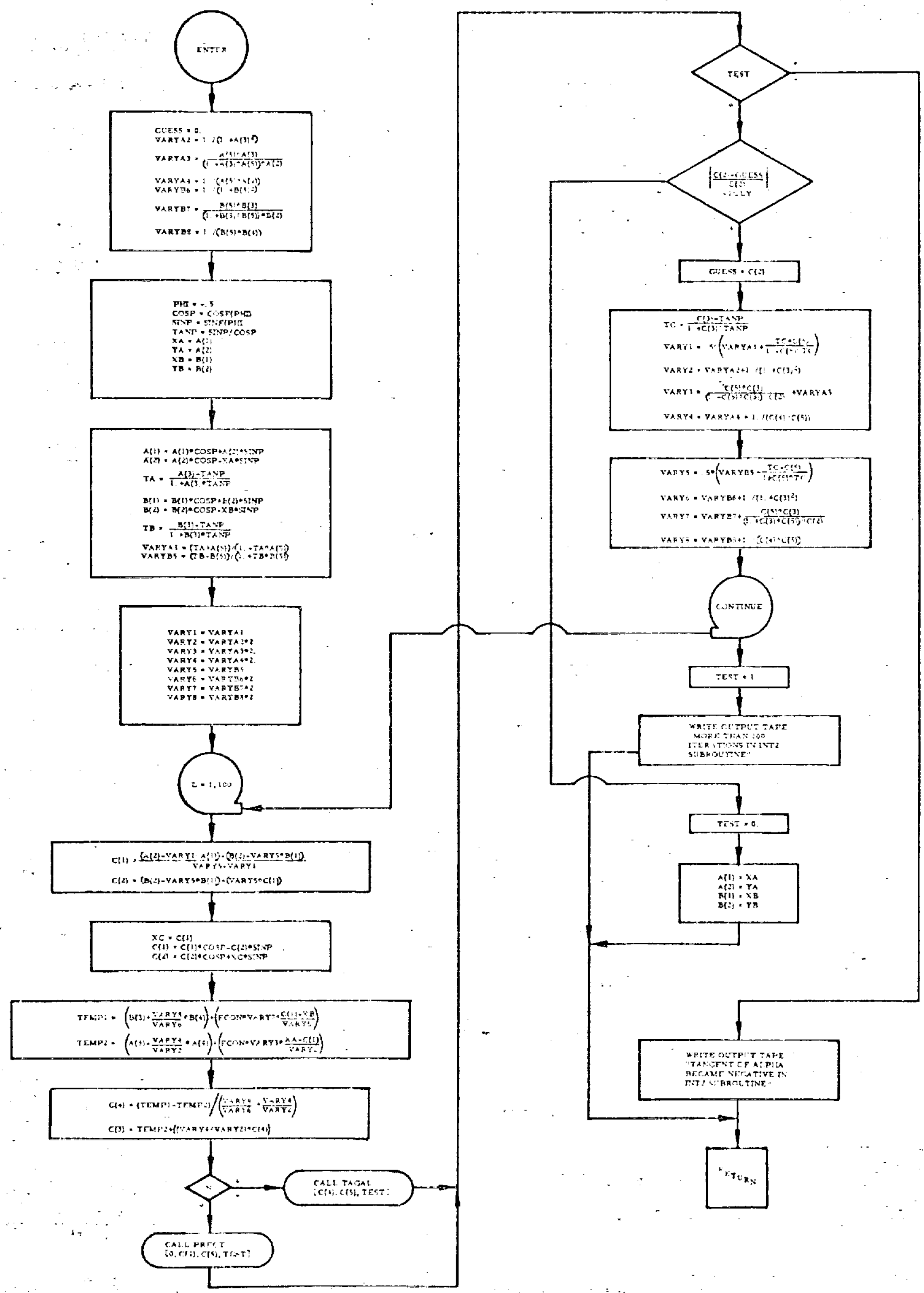
$$\tan \theta' = (\tan \theta - \tan \varphi) \div (1 + \tan \theta \tan \varphi)$$

where:

the prime indicates the rotated value and φ the angle of rotation.

Solving the physical characteristics (equation 2.1) simultaneously using the rotated values, the coordinates at point 3' are determined. The coordinate axes are then rotated back to their original position, and the flow conditions W_3 and tan θ_3 are obtained as in the INT1 Subroutine.

Subroutine INT2



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4. INT3 Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at two points in the flow field not on the same Mach line, the INT3 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of either Mach line is very large. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 8) will be stored in the variable C(I).

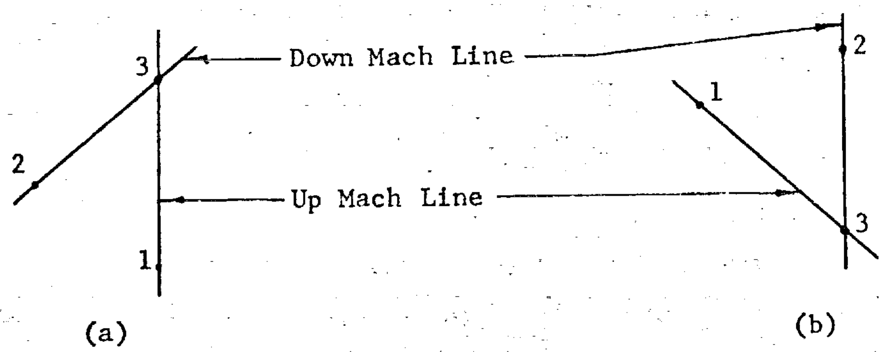


Figure 8

The characteristic system is the same as for the INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equations have the form

$$\sigma \left[\frac{\tan \alpha \ \tan \theta}{(\tan \theta + \tan \alpha)\ Y}\right]_1 (Y_3 - Y_1) \tag{4.1}$$

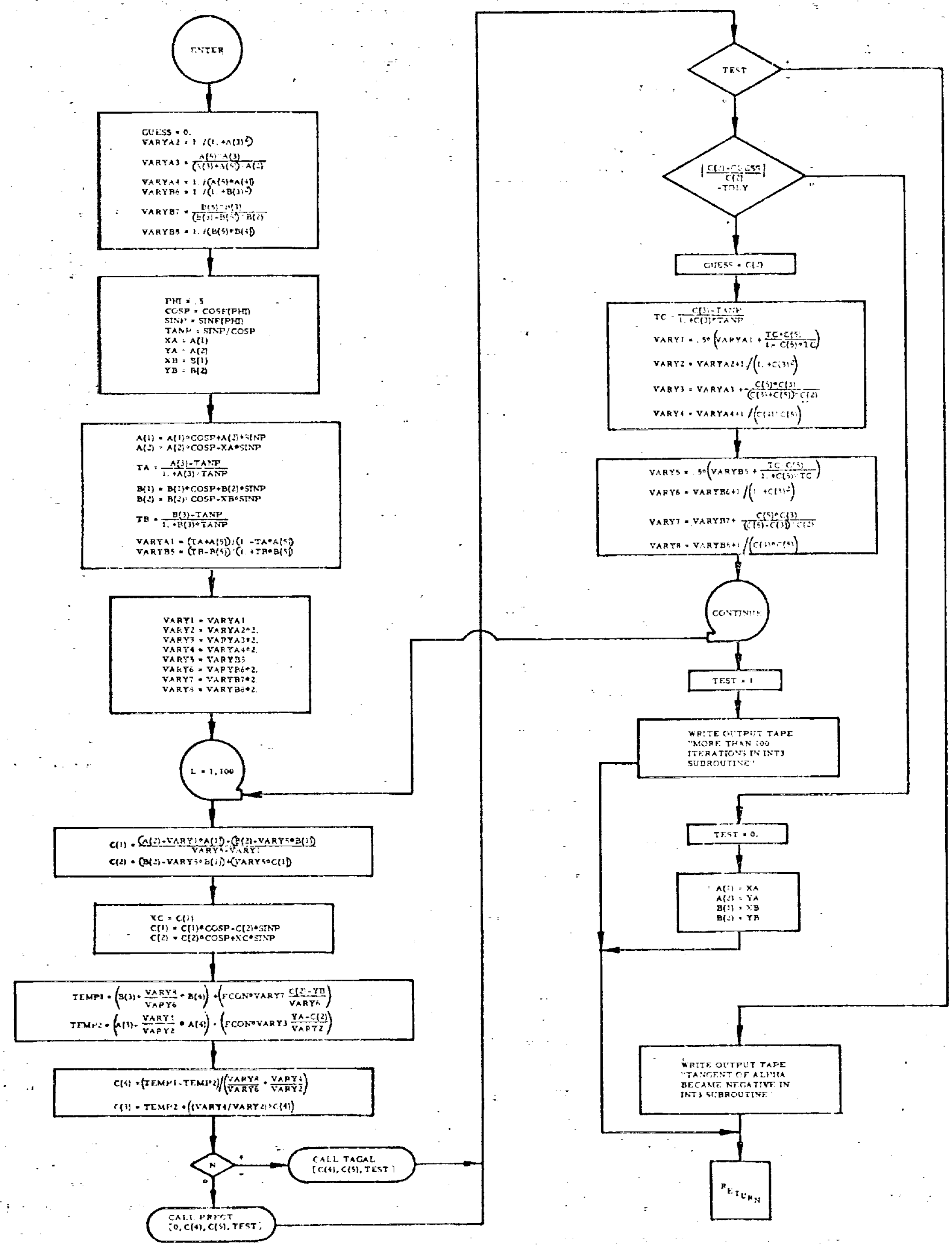
for an up Mach line, and

$$\sigma \left[\frac{\tan \alpha \ \tan \theta}{(\tan \theta - \tan \alpha)\ Y}\right]_2 (Y_3 - Y_2) \tag{4.2}$$

for a down Mach line.

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is very large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

Subroutine INT3



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5. INT4 Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at two points in the flow field not on the same Mach line, the INT4 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of the up Mach line is very small and the slope of the down Mach line very large. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 9) will be stored in the variable C(I).

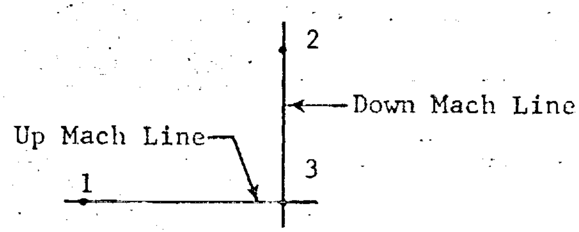


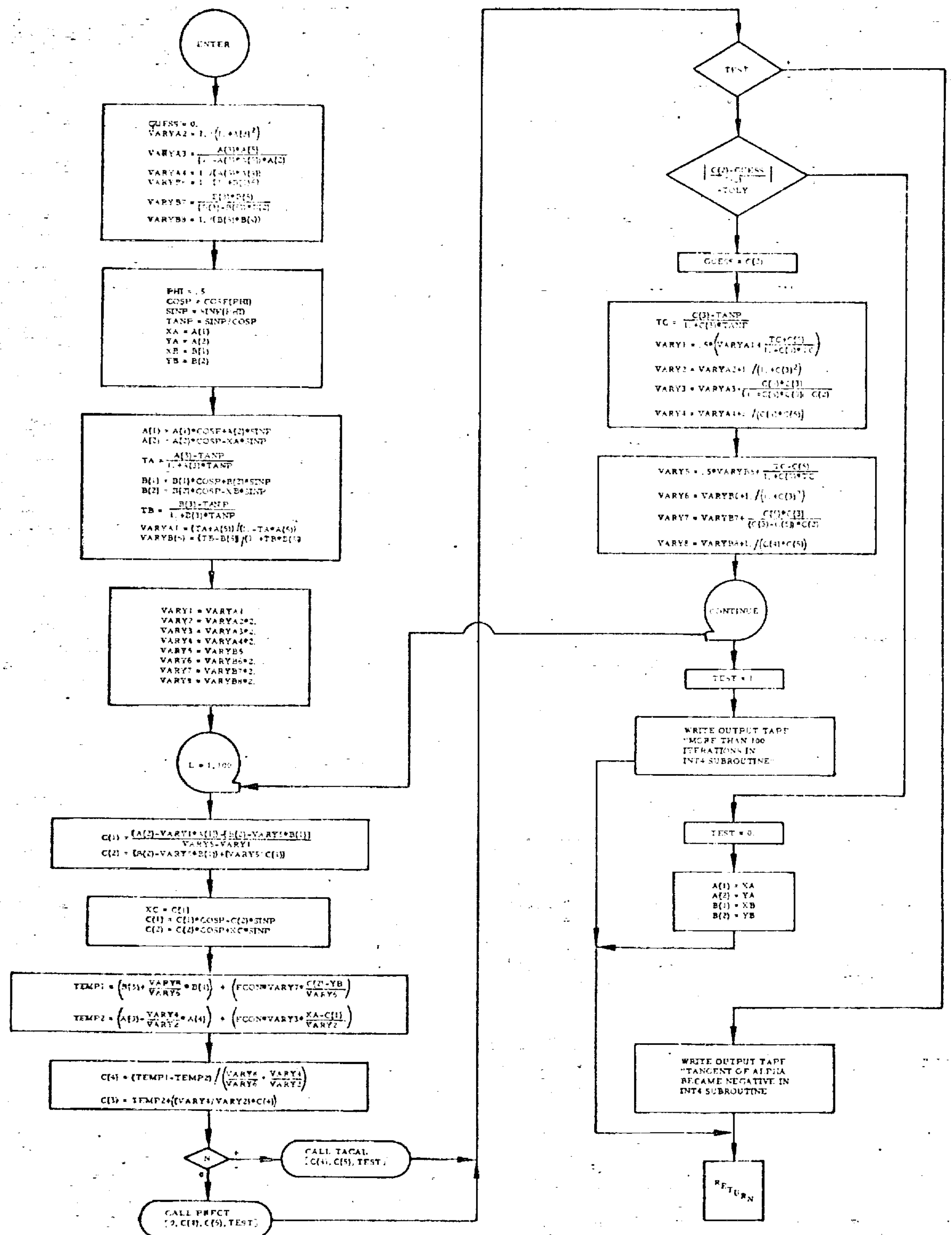
Figure 9

The characteristic system is the same as in INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equation for the down Mach line is

$$\sigma \left[\frac{\tan \alpha \quad \tan \theta}{(\tan \theta - \tan \alpha)\ Y} \right]_2 (Y_3 - Y_2). \qquad (5.1)$$

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

Subroutine INT4



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6. INT5 Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at two points in the flow field not on the same Mach line, the INT5 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of the up Mach line is very large and the slope of the down Mach line very small. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 10) will be stored in the variable C(I).

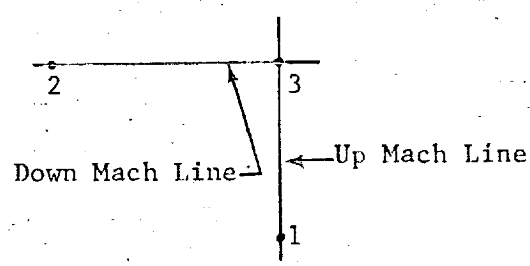


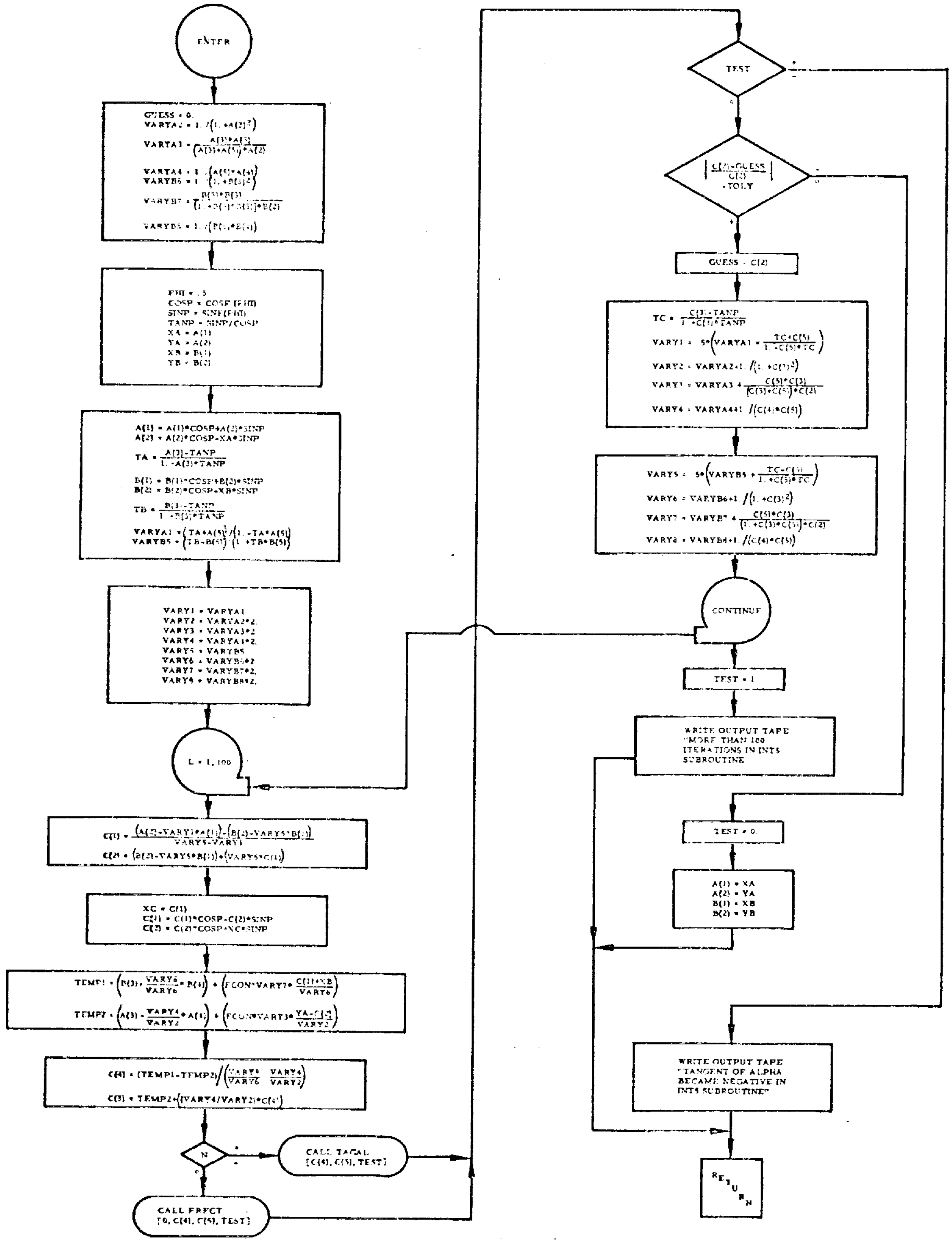
Figure 10

The characteristic system is the same for the INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equation for the up Mach line is

$$\sigma \left[\frac{\tan \alpha \tan \theta}{(\tan \theta + \tan \alpha) Y} \right]_1 (Y_3 - Y_1). \qquad (6.1)$$

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

Subroutine INT5



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7. EXPAN Subroutine

Depending on the value stored in the variable N3, the EXPAN Subroutine calculates the flow properties after expanding about a sharp corner on a circular arc. The sharp corner expansion can be accomplished by either turning the flow through a specified angle (N3 = 0) or by expanding the flow through an increment in the velocity ratio (N3 = 1).

For sharp corner expansion through a small angle δ (figure 11), the expanded velocity ratio, W_c, is found

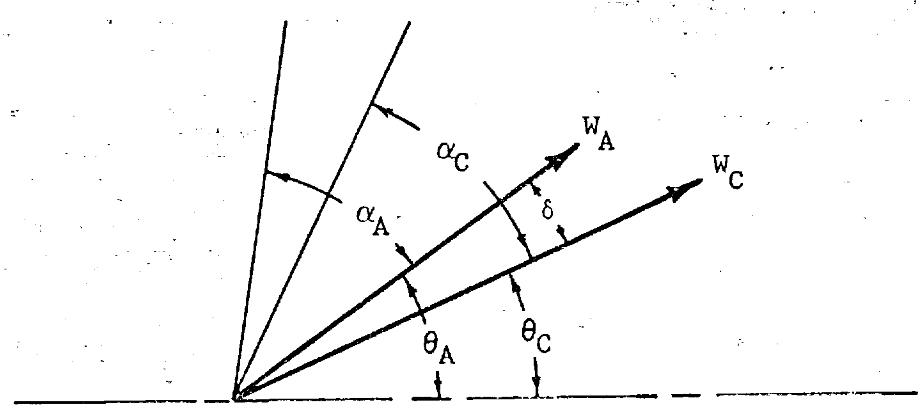


Figure 11

by integrating

$$\frac{dW}{d(\tan\theta)} = \frac{W \tan\alpha}{1 + \tan^2\theta} = f(\tan\theta, W). \tag{7.1}$$

Using the method of Runge - Kutta over ten intervals,

let

$$h = \frac{\tan\theta_C - \tan\theta_A}{10}$$

$$\tan\theta_1 = \tan\theta_A$$

and

$$W_1 = W_A.$$

Solve equations 7.2 to 7.8 as j = 1, 10.

$$K_1 = f\ (\tan\theta_j,\ W_j)\ h \qquad (7.2)$$

$$K_2 = f\ (\tan\theta_j + h/2,\ W_j + K_1/2)h \qquad (7.3)$$

$$K_3 = f\ (\tan\theta_j + h/2,\ W_j + K_2/2)h \qquad (7.4)$$

$$K_4 = f\ (\tan\theta_j + h,\ W_j + K_3)h \qquad (7.5)$$

$$\Delta W = 1/6\ (k_1 + 2K_2 + 2K_3 + K_4) \qquad (7.6)$$

$$W_j+1 = W_j + \Delta W \qquad (7.7)$$

$$\tan\theta_{j+1} = \tan\theta_j + h \qquad (7.8)$$

The properties corresponding to W_C are

$$W_C = W_{11}$$

$$\tan\alpha_C = f\ (W_C)$$

$$\tan\theta_C = \tan\theta_{11},$$

and are stored in the variable C(I). Since the expansion occurs about a sharp corner, the X and Y coordinates remain unchanged.

When expanding through an increment in the velocity ratio, the slope $\tan\theta_C$ of the expanded velocity ratio, W_C, is found by integrating

$$\frac{d(\tan\theta)}{dW} = -\frac{1+\tan^2\theta}{W\tan\alpha} = f\ (W,\ \tan\theta). \qquad (7.9)$$

By interchanging the values of W and $\tan\theta$, the previous method of Runge - Kutta can be used.

For circular expansion (N3=2), the flow is expanded by determining the intersection of the down Mach line from point 1 (figure 12) with the arc of a circle of radius R with its center at point $(X_o,\ Y_o)$.

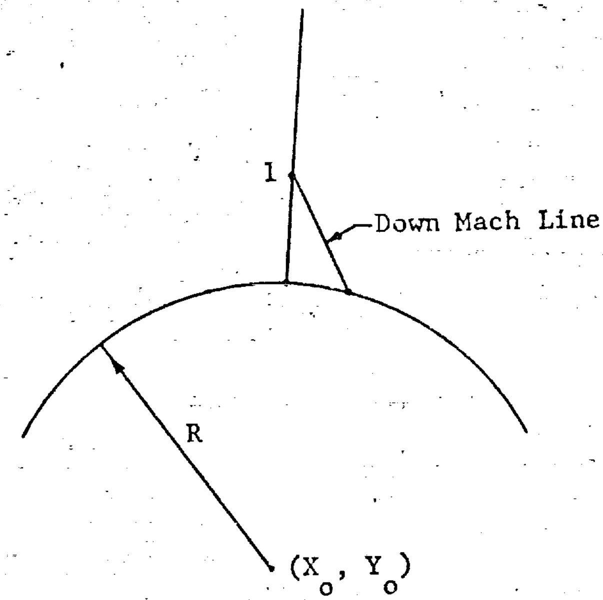


Figure 12

By solving simultaneously the equation of circle and the characteristic equation in finite difference form, the coordinates of the intersection are

$$Y_3 = K_2/K_1 + \sqrt{K_3}$$

and (7.10)

$$X_3 + X_1 + (Y_3 - Y_1) \Big/ \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_1 ;$$

where

$$K_1 = 1 + \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_1^2 ,$$

$$K_2 = \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_1 \left\{ \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_1 Y_o - (X_1 - X_o) \right\} + Y_1 ,$$

and

$$K_3 = \left(\frac{K_2}{K_1}\right)^2 - \frac{\left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_1 \left\{ \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_1 \left(Y_o^2 - R^2 + (X_1 - X_o)^2\right) - 2Y_1 (X_1 - X_o) \right\} + Y_1^2 .}{K_1}$$

The flow properties are

$$\tan\theta_3 = \frac{X_o - X_3}{Y_3 - Y_o} ,$$

$$W_3 = W_1 + \frac{(\tan\theta_1 - \tan\theta_3)\left[\frac{1}{1+\tan^2\theta}\right]_1 \div \left[\frac{\tan\alpha\tan\theta}{(1+\tan\alpha\tan\theta)Y}\right]_1 (X_3 + X_1)}{\left[\frac{1}{W\tan\alpha}\right]_1},$$

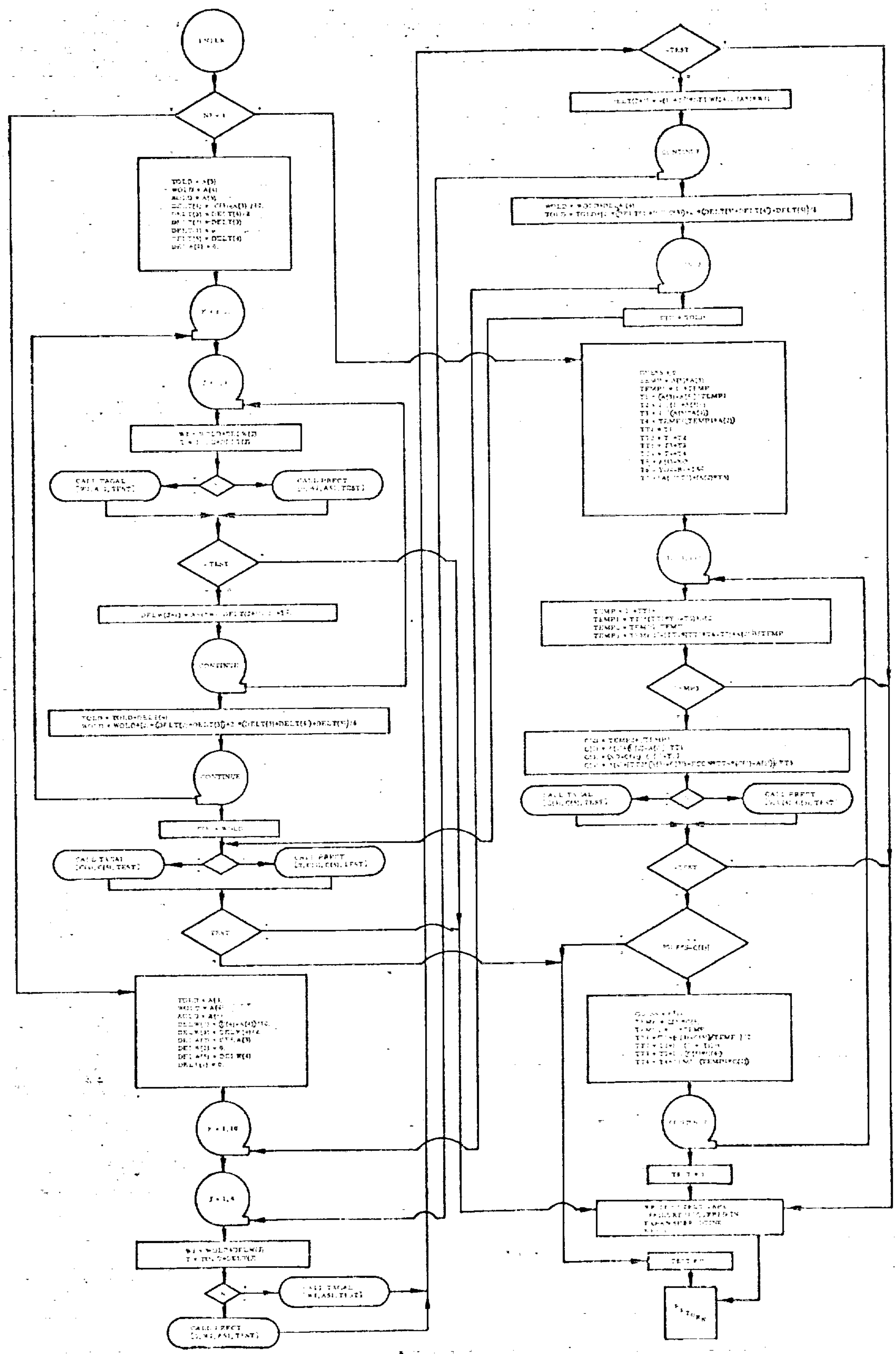
and

$$\tan\alpha_3 = f\,(W_3).$$

Improved solutions for the coordinates and flow properties are obtained by replacing the quantities in brackets with average values and repeating the calculations until

$$\left| X_3^i - X_3^{i-1} \right| \le 0.000005.$$

Subroutine EXPAN



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8. EXPAND Subroutine

The EXPAND Subroutine calculates the flow properties at a point after the flow has been compressed by a decrement in the velocity ratio (figure 13). The slope, $\tan \theta_C$, of the compressed velocity ratio, W_C, is found

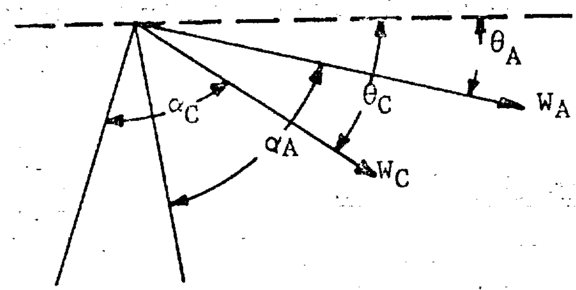


Figure 13

by integrating

$$\frac{d(\tan \theta)}{dW} = \frac{1 + \tan^2 \theta}{W \tan \alpha} = f(W, \tan \theta). \qquad (8.1)$$

Using the method of Runge-Kutta over ten intervals,

let

$$h = \frac{W_C - W_A}{10}$$

$$W_1 = W_A$$

and

$$\tan \theta_1 = \tan \theta_A .$$

Solve equations 8.2 to 8.8 as j = 1, 10.

$$K_1 = f(W_j, \tan \theta_j)h \qquad (8.2)$$

$$K_2 = f(W_j + h/2, \tan \theta_j + K_1/2)h \qquad (8.3)$$

$$K_3 = f(W_j + h/2, \tan \theta_j + K_2/2)h \qquad (8.4)$$

$$K_4 = f(W_j + h, \tan \theta_j + K_3)h \qquad (8.5)$$

$$\Delta(\tan \theta) = 1/6(K_1 + 2K_2 + 2K_3 + K_4) \qquad (8.6)$$

$$\tan \theta_{j+1} = \tan \theta_j + \Delta(\tan \theta) \qquad (8.7)$$

$$W_{j+1} = W_j + h \qquad (8.8)$$

The properties corresponding to W_C are

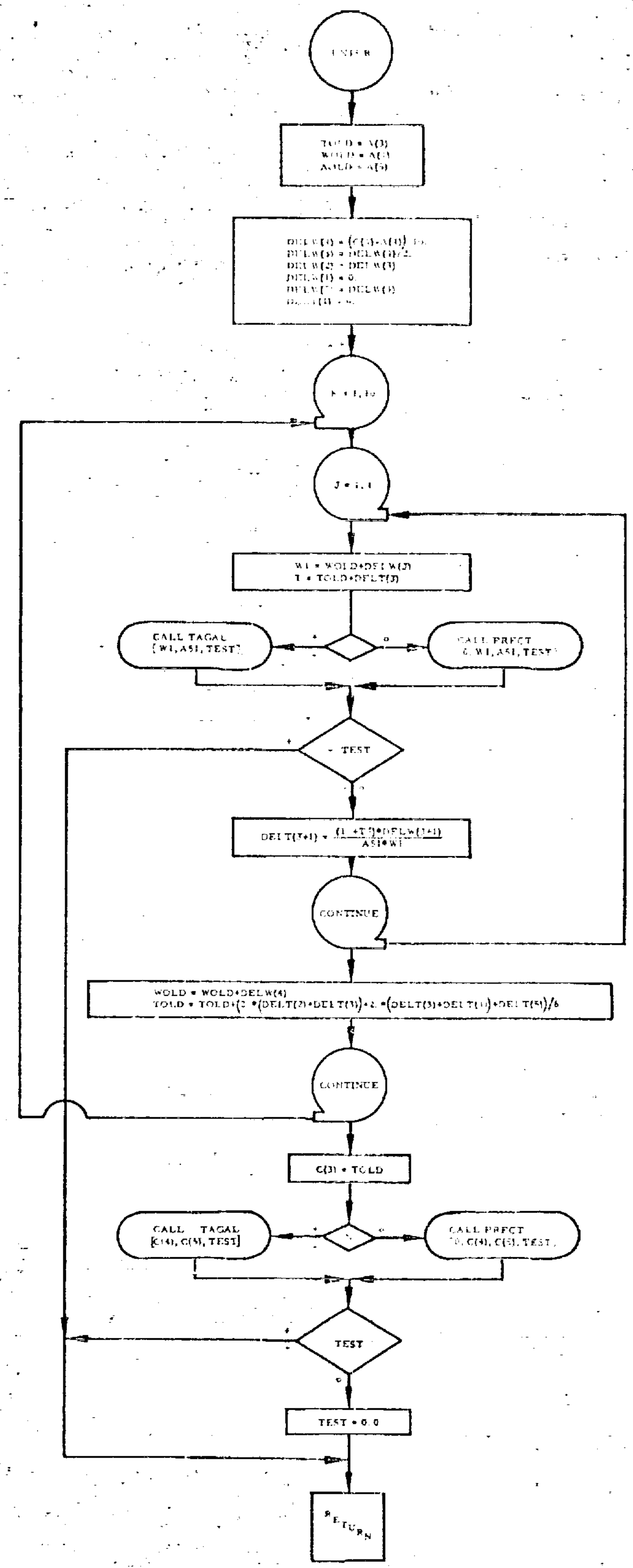
$$W_C = W_{11}$$

$$\tan \alpha_C = f(W_C)$$

$$\tan \theta_C = \tan \theta_{11},$$

and are stored in the variable C(I). Since the compression occurs about a sharp corner, the X and Y coordinates remain unchanged.

Subroutine EXPAND



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9. PLUGPT Subroutine

The PLUGPT Subroutine is used to determine points on the primary contour by calculating the intersection of a down Mach line with the streamline corresponding to the total mass flow through the nozzle.

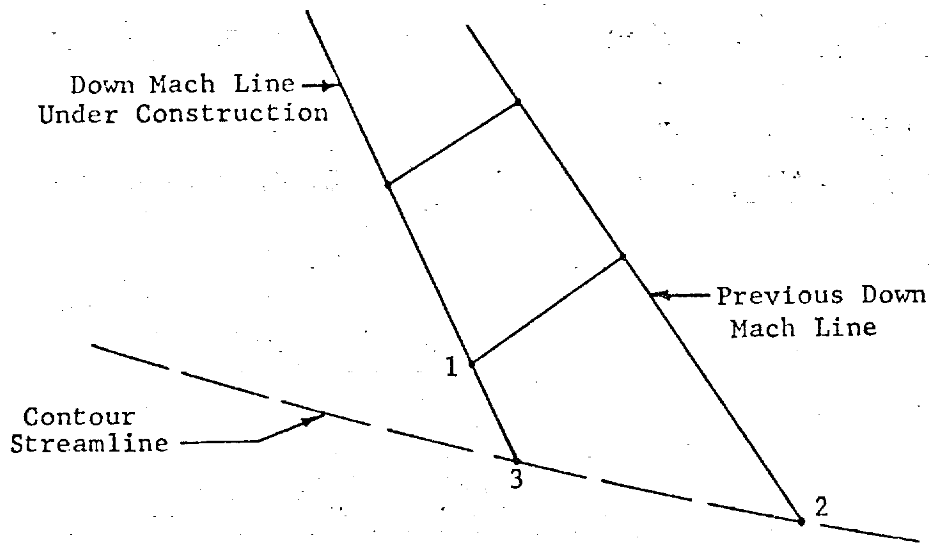


Figure 14

Before entering this subroutine the total mass flow must be stored in TMF; the mass flow between the expansion point and point 1 must be in ACM; and the flow conditions at points 1 and 2 must be in B(I) and BLINE (NEPP, I), respectively. The calculated values at point 3 are stored in C(I).

Using the slope of the previously calculated contour point as a first guess for the slope of the streamline, the equations for the characteristic curve passing through point 1 and the streamline through point 2 are solved simultaneously to obtain the coordinates at point 3.

Thus

$$X_3 = \frac{Y_1 - Y_2 + X_2[\tan\theta]_2 - X_1\left[\dfrac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_1}{[\tan\theta]_2 - \left[\dfrac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_1}, \tag{9.1}$$

and

$$Y_3 = Y_2 + (X_3 - X_2)\,[\tan\theta]_2. \tag{9.2}$$

The velocity ratio at point 3 as obtained from the compatibility equation is

$$W_3 = W_1 - \frac{(\tan\theta_3 - \tan\theta_1)\left[\dfrac{1}{1 + \tan^2\theta}\right]_1 - \sigma(Y_3 - Y_1)\left[\dfrac{\tan\alpha\tan\theta}{(\tan\theta - \tan\alpha)Y}\right]_1}{\left[\dfrac{1}{W\tan\alpha}\right]_1}, \tag{9.3}$$

and

$$\tan\alpha_3 = f(W_3). \tag{9.4}$$

Improved solutions to these equations are obtained by replacing the quantities in the brackets that are subscripted 1 with average values, and repeating the calculations until

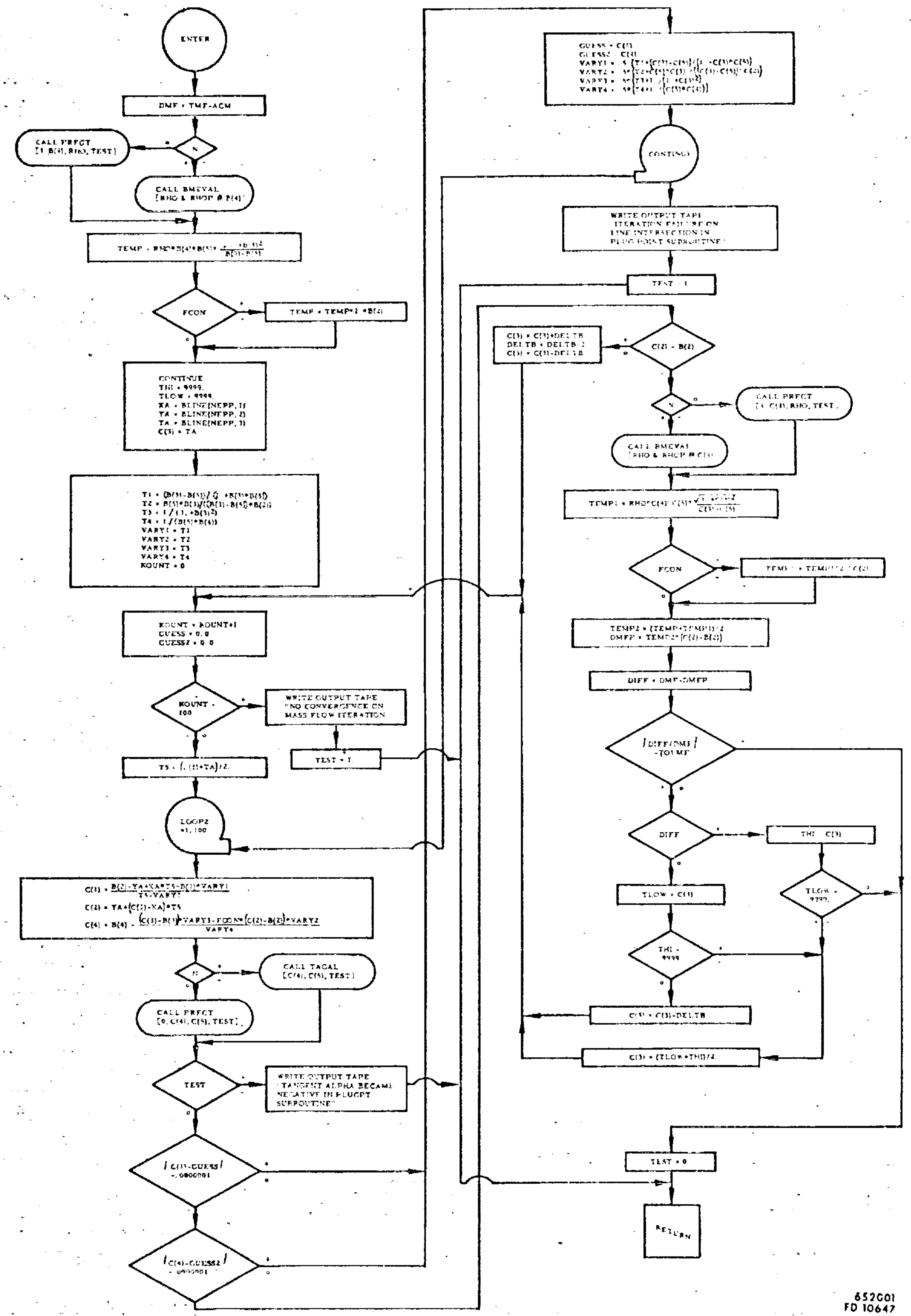
$$\left| X_3^{i} - X_3^{i-1} \right| \leq 0.0000001,$$

and

$$\left| W_3^{i} - W_3^{i-1} \right| \leq 0.0000001.$$

Obtaining a new value for $\tan\theta_3$ and replacing $[\tan\theta]_2$ with the average value, $[\tan\theta]_{2,3}$, the above procedure is repeated until the calculated mass flow between points 1 and 3 is equal to $\left| TMF - ACM \right|$ within the optional input tolerance, TOLMF.

Subroutine PLUGPT



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10. PRFCT Subroutine

The PRFCT Subroutine is made up of four perfect gas relationships, which are a function of velocity ratio and a constant specific heat ratio. Depending on the value of the parameter (L), this subroutine calculates either tan α, ratio of static to total density, ratio of static to total pressure, or Mach number.

The following equations are evaluated by the PRFCT Subroutine.

$$\tan \alpha = \sqrt{\frac{1 - W^2}{W^2 \left(\frac{\gamma + 1}{\gamma - 1}\right) - 1}} \qquad (L = 0)$$

$$\rho/\rho_o = [1 - W^2]^{\frac{1}{\gamma - 1}} \qquad (L = 1)$$

$$P/P_o = [1 - W^2]^{\frac{\gamma}{\gamma - 1}} \qquad (L = 2)$$

$$M = \sqrt{\frac{W^2 \left(\frac{2}{\gamma - 1}\right)}{1 - W^2}} \qquad (L = 3)$$

The following is an explanation of the subroutine call list.

L - Indicates parameter to be calculated

Q - The known or input value of velocity ratio V/V_{max}

T - Variable that will contain the calculated value

TEST - An error signal in case of a subsonic velocity

11. TAGAL Subroutine

For an ideal gas, the TAGAL Subroutine is used to calculate tan α as a function of a known velocity ratio ($W = V/V_{sonic}$). For the option where the local frozen sound speeds from the table of gas properties are not used,

$$\tan \alpha = \sqrt{\frac{1}{V_{sonic}^2 \quad W^2 \left(\frac{d\rho/dW}{dP/dW}\right) - 1}} .$$

A beam fit evaluation of the gas properties is necessary to determine the values of $d\rho/dW$ and dP/dW.

If the local frozen sound speed option is used, then

$$\tan \alpha = \sqrt{\frac{1}{[V_{sonic} \; W/c]^2 - 1}} ,$$

where the local frozen sound speed (c) is also determined from a beam fit evaluation of the gas properties table.

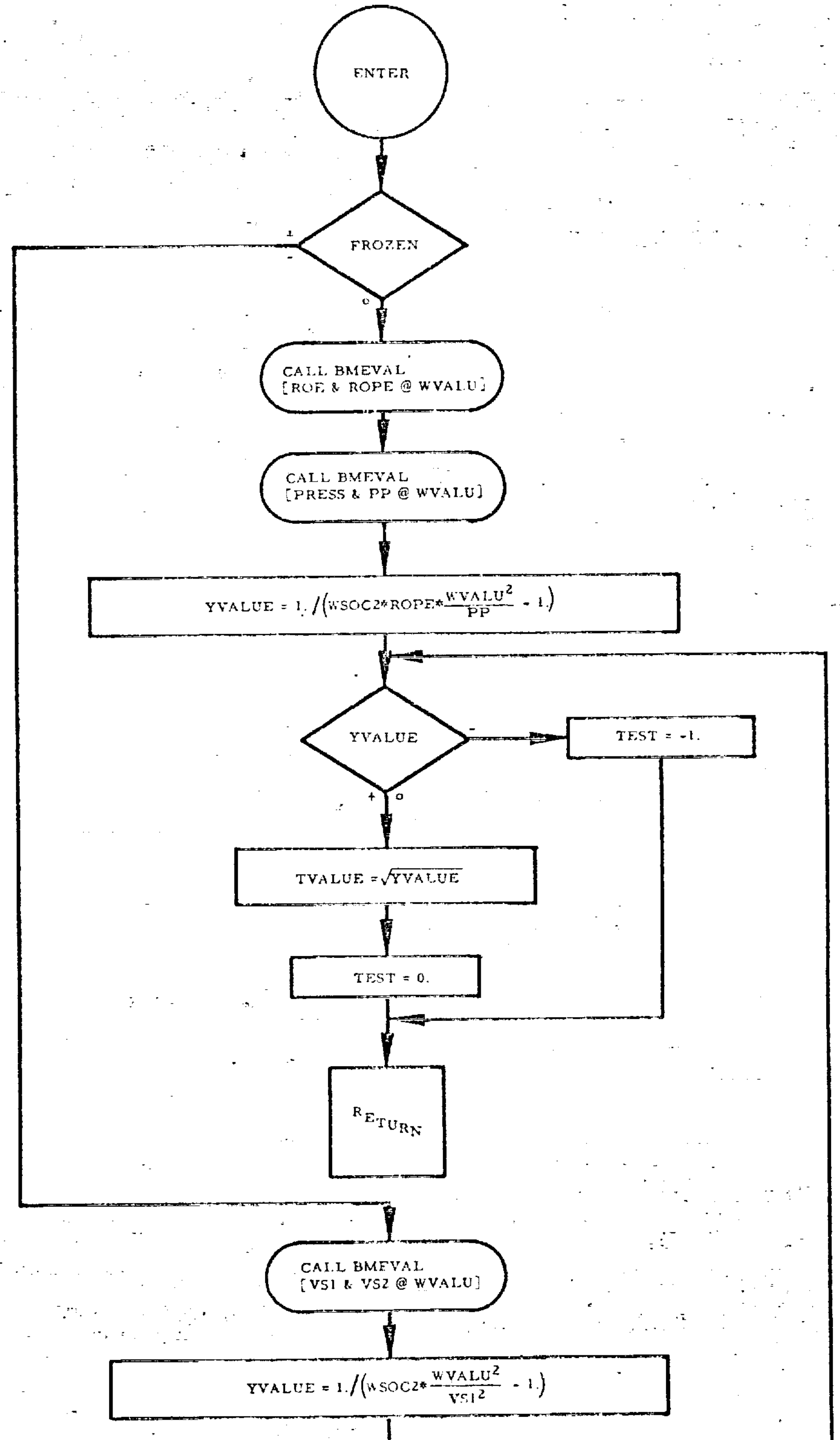
The following is an explanation of the subroutine call list.

WVALU - The known value of velocity ratio

TVALU - Value of tan α corresponding to WVALU

TEST - A signal that the input velocity ratio is subsonic. If TEST = -1, subsonic; TEST = 0, supersonic.

Subroutine TAGAL



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12. SONICP Subroutine

For an ideal gas, the SONICP Subroutine is called to adjust the units, determine the sonic values, and beam fit gas properties. Corresponding values of specific impulse, $\frac{\text{lbf sec}}{\text{lbm}}$; density, lbm/ft^3; pressure, lbf/in^2; and local frozen sound speed, ft/sec, must be stored into variables W(I), RO(I), P(I), and VS(I), respectively. The subroutine converts the units of pressure to lbf/ft^2 and density to $\frac{\text{lbf sec}^2}{\text{ft}^4}$.

If the program is to calculate local sound speeds, the pressure and density is beam fit as a function of specific impulse to calculate the sonic velocity at the throat. The sonic I_s is first bracketed by two values of specific impulse and a halving process is used until

$$\frac{dP/dI_s}{d\rho/dI_s} = V^2_{sonic} \pm 0.00001,$$

where:

V_{sonic}/g_o is the value of I_s at which dP/dI_s and $d\rho/dI_s$ are evaluated. The velocity and density at this point are stored into variables SONICV and RHOSON, respectively. All of the specific impulse values are converted to velocity ratios by dividing each one by the sonic I_s. The pressure and density is then beam fit again as a function of velocity ratio.

For the option where the local frozen sound speeds are used, the curve of c vs W is beam fit for the purpose of evaluating the local speed of sound throughout the flow field. Also, the iteration to determine the sonic velocity at the throat is eliminated because this value is given in the input.

13. REALV Subroutine

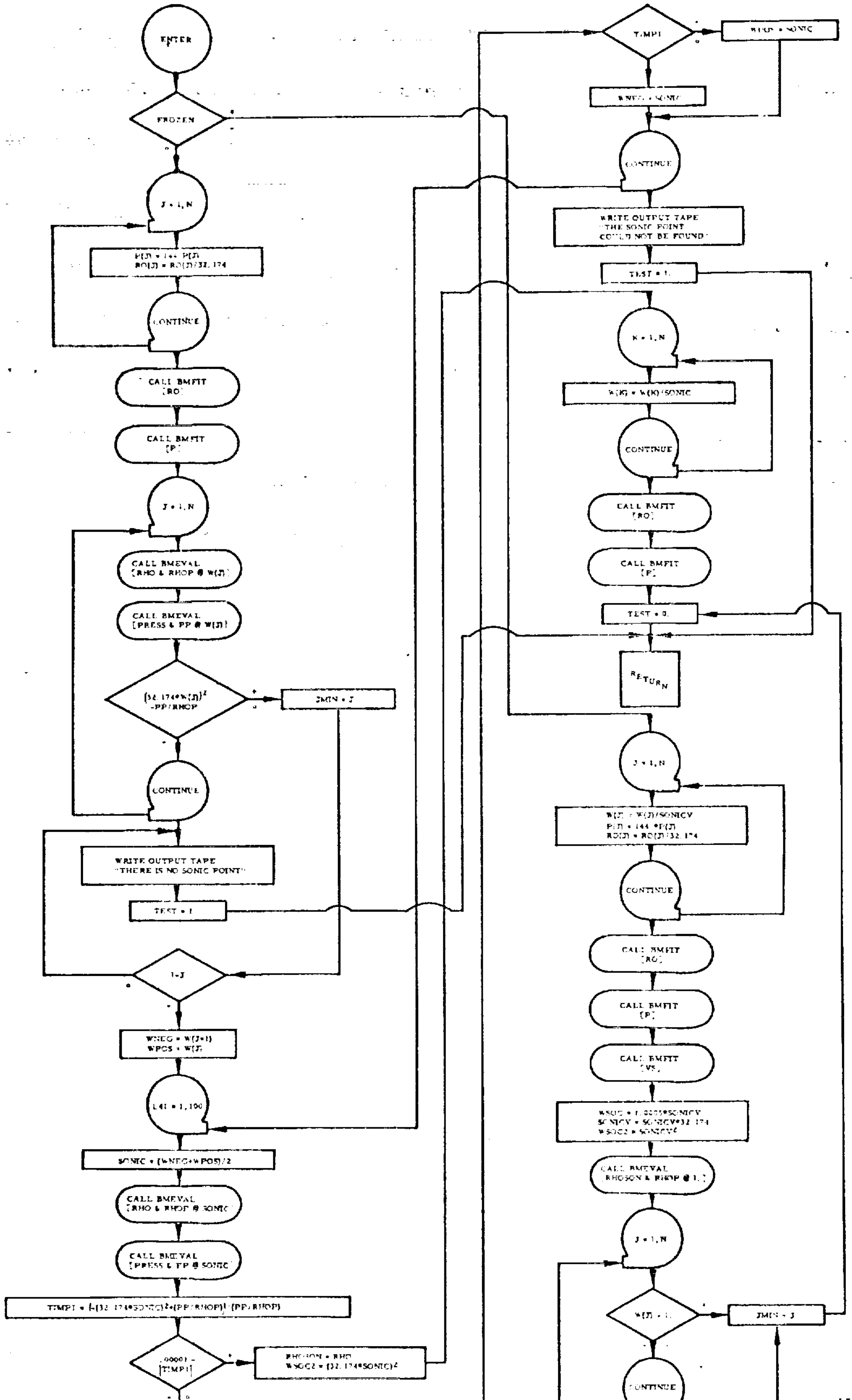
For an ideal gas, the REALV Subroutine calculates the velocity ratio (W) for the Mach number given in the call list. The corresponding value of tan α is calculated by

$$\tan \alpha = 1.0 \Big/ \sqrt{M^2 - 1} \;.$$

An iteration is necessary to determine the velocity ratio. This is accomplished by using the TAGAL Subroutine for increasing values of W to calculate the corresponding values of $(\tan \alpha)_G$ until tan α is bracketed. Knowing the bracketed values of W, a halving process is used; for each guess on W, the corresponding value of $(\tan \alpha)_G$ is calculated until

$$(\tan \alpha)_G = \tan \alpha \pm 0.00005.$$

Subroutine SONICP



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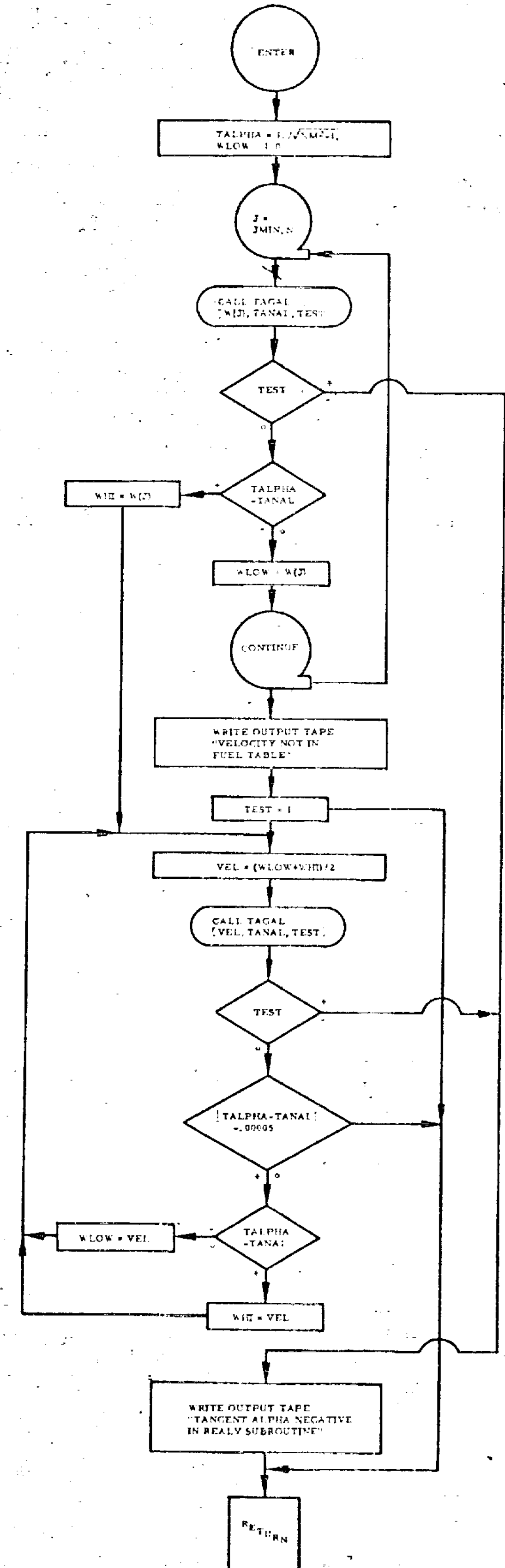
Subroutine REALV



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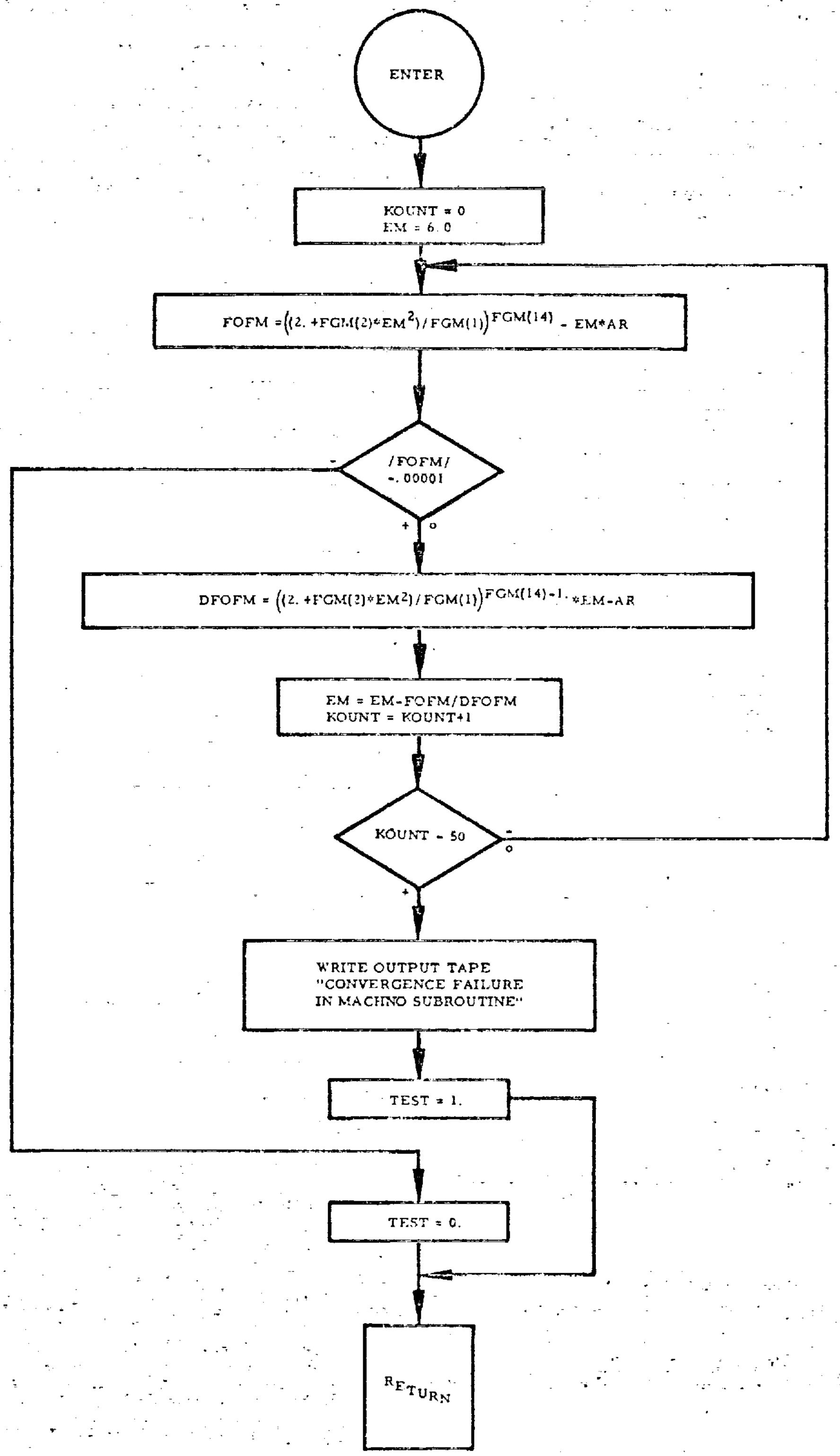
14. MACHNO Subroutine

For a perfect gas, the MACHNO Subroutine calculates Mach number as a function of area ratio and specific heat ratio from the following equation:

$$A/A^* = \frac{1}{M}\left(\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

Since the above equation cannot be solved explicitly for Mach number, this subroutine uses Newton's iteration technique for the solution within a tolerance of 0.00001.

Subroutine MACHNO



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15. INTEG Subroutine

The INTEG Subroutine is used to extend an up Mach line assuming dW/dX is constant. This Mach line is extended until the mass flow across the line is equal to the total mass flow through the nozzle.

The characteristic and compatibility equations can be written

$$\frac{dY}{dX} = \frac{\tan\theta + \tan\alpha}{1 - \tan\alpha \tan\theta}$$

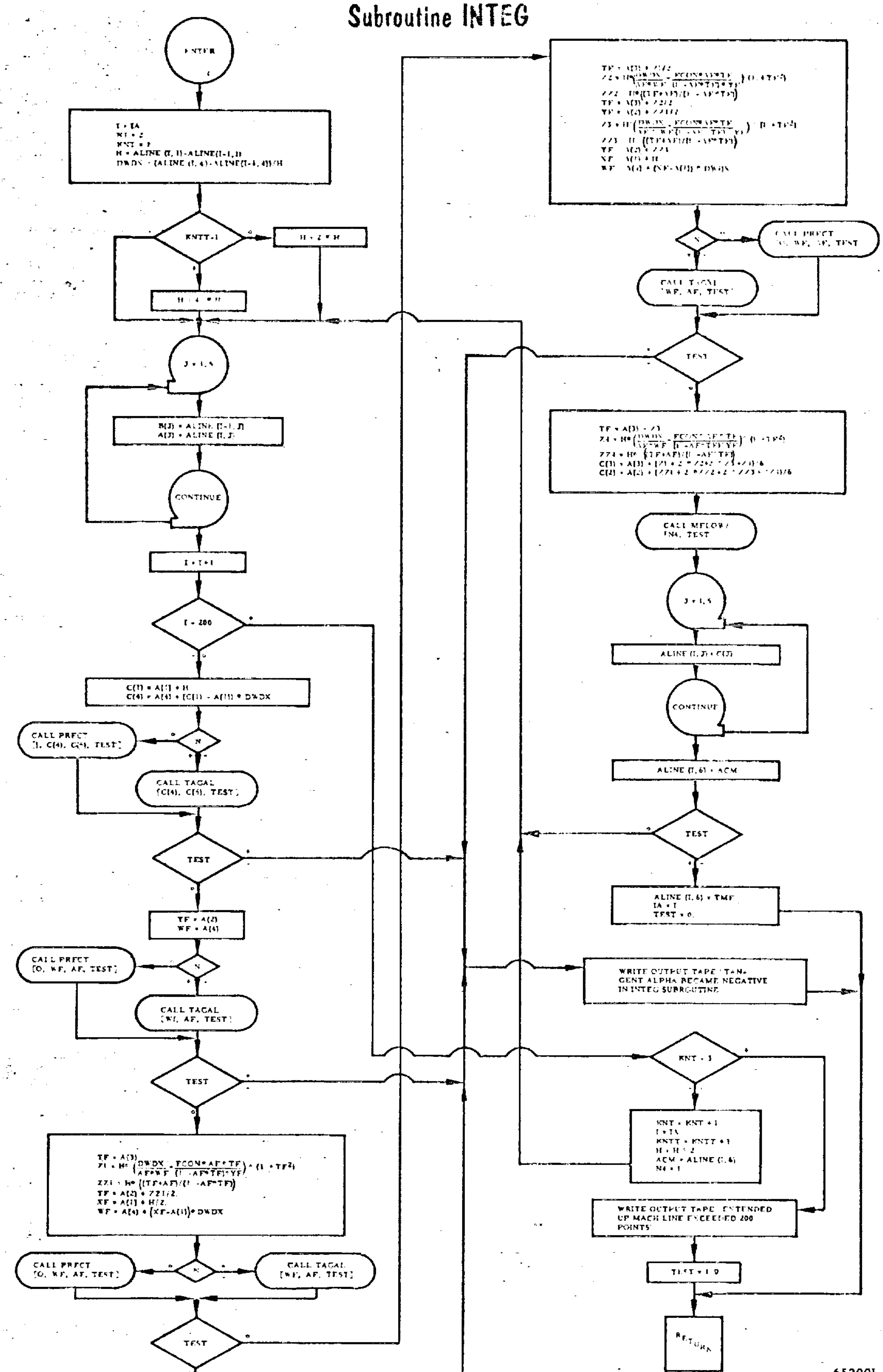
and

$$\frac{d(\tan\theta)}{dX} = \left\{ \frac{dW/dX}{W \tan\alpha} - \sigma \frac{\tan\alpha \tan\theta}{(1 - \tan\alpha \tan\theta)\, Y} \right\} (1 + \tan^2\theta).$$

Using the method of Runge-Kutta for two first-order differential equations, the above equations are integrated to obtain the Y coordinates and corresponding values of tan θ for points along the extended Mach line.

The increments of ΔX and velocity ratio, W, are initially assumed to be the same as the last two points on the Mach line before the extension. If the number of points along the Mach line exceeds 200 before the total mass flow is reached, the increment is doubled and the extension is again calculated.

Subroutine INTEG



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16. TFLOW Subroutine

When the initial Mach line is not part of the input data, the TFLOW Subroutine calculates the initial velocity ratio from the given Mach number and then develops the initial Mach line. The total mass flow through the nozzle is calculated for both options.

If the slope of the velocity vector at the first point on the secondary contour is zero, a straight Mach line having uniform properties and extending to the input throat height is developed; otherwise, a line of constant velocity ratio is developed using the following procedure.

The characteristic equation for an up Mach line can be written

$$\frac{dX}{dY} = \frac{1 - \tan\alpha \tan\theta}{\tan\theta + \tan\alpha} ,$$

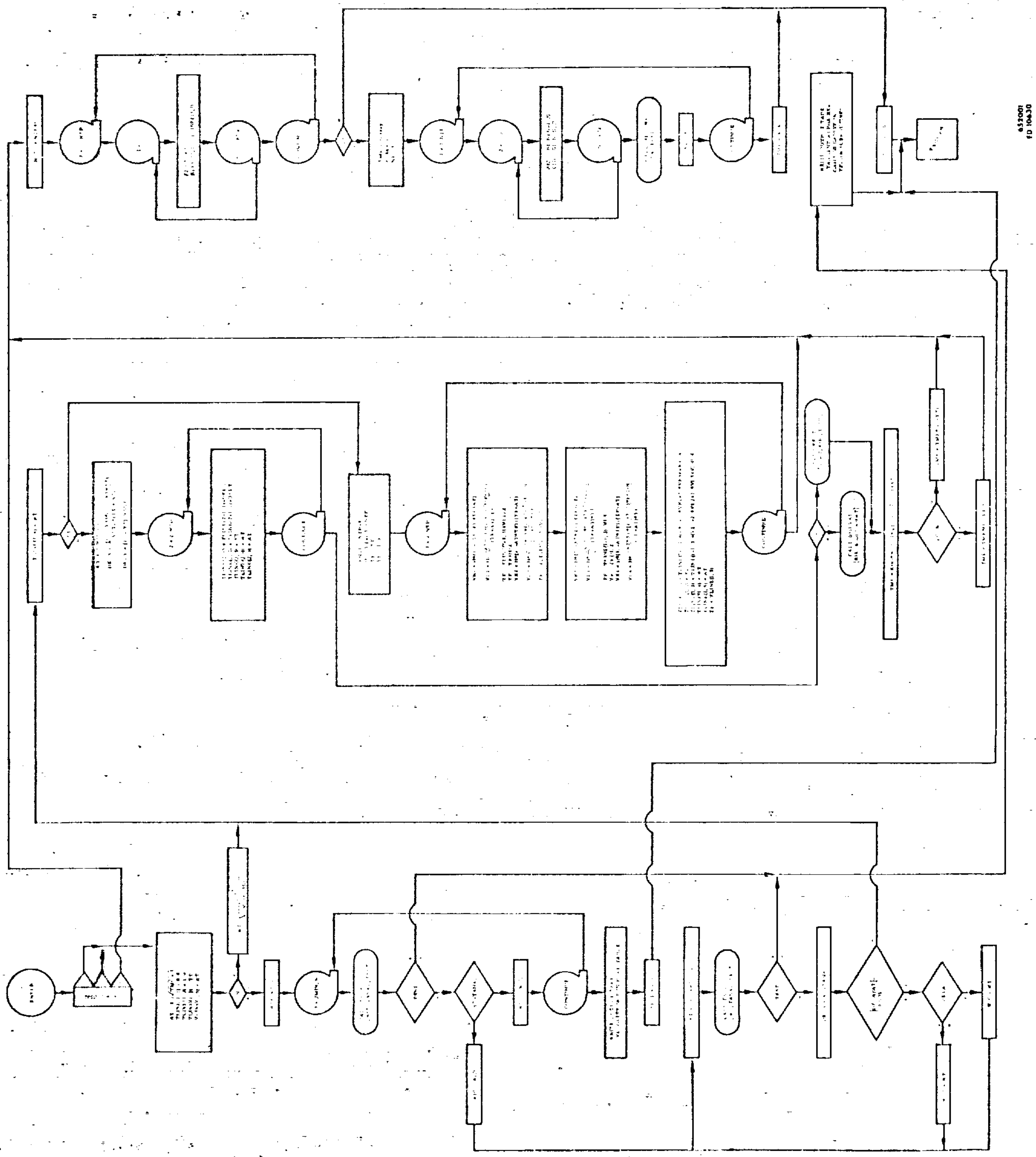
and the compatibility equation, assuming the velocity ratio constant, is

$$\frac{d(\tan\theta)}{dY} = -\sigma \frac{\tan\alpha \tan\theta \, (1 + \tan^2\theta)}{(\tan\theta + \tan\alpha)\, Y} .$$

Using the method of Runge-Kutta for two first-order differential equations, these equations are integrated to obtain the X coordinate and $\tan\theta$ for XNTP points along the initial Mach line at specified ΔY increments.

The mass flow across a uniform initial Mach line is ρAV, while the mass flow for an input Mach line or a constant velocity ratio Mach line of nonuniform values of $\tan\theta$ is determined by integrating $\rho V_n dA$ along the Mach line (MFLOW1 Subroutine).

The initial up Mach line of XNTP points is stored in BLINE (I,J), and the total mass flow through the nozzle is stored in TMF.



Subroutine TFLOW

17. MFLOW1 Subroutine

The MFLOW1 Subroutine calculates the mass flow between two points along an up Mach line. After each interior intersection in Regions I and II of the flow field is calculated, this subroutine is called to determine when the accumulated mass flow along an up Mach line has exceeded the total mass flow through the nozzle.

To calculate the mass flow for axisymmetric flow,

$$\dot{\omega} = \int \left(\rho W \tan \alpha \frac{\sqrt{1 + \tan^2 \theta}}{\tan \theta - \tan \alpha} \; 2\pi Y \right) dX \tag{17.1}$$

For two-dimensional flow, the $2\pi Y$ term is eliminated.

The known conditions at points 1 and 2 must be stored in the variables A(I) and C(I), respectively.

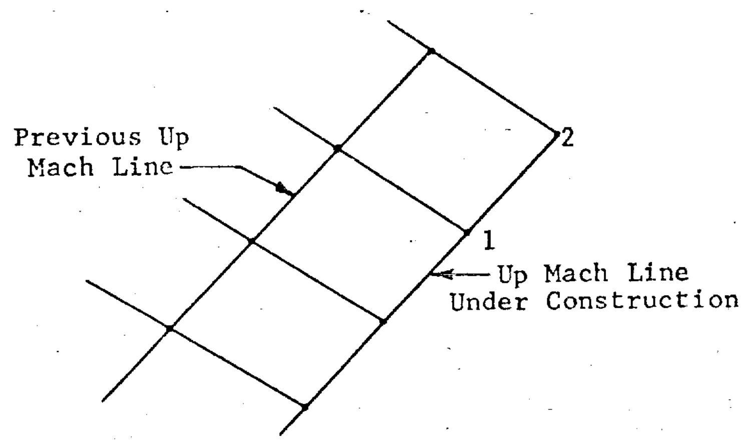


Figure 15

The increment of mass flow between the first two points on the Mach line, which is indicated by the value in N4, is calculated by trapezoidal integration. In equation (17.1), let Q represent the quantity in parentheses; then the first mass flow increment is found by

$$\dot{\omega}_2 = \left(\frac{Q_1 + Q_2}{2} \right) (X_2 - X_1). \tag{17.2}$$

The remaining increments of mass flow, as illustrated in figure 16, are determined by the parabolic integration of equation (17.1).

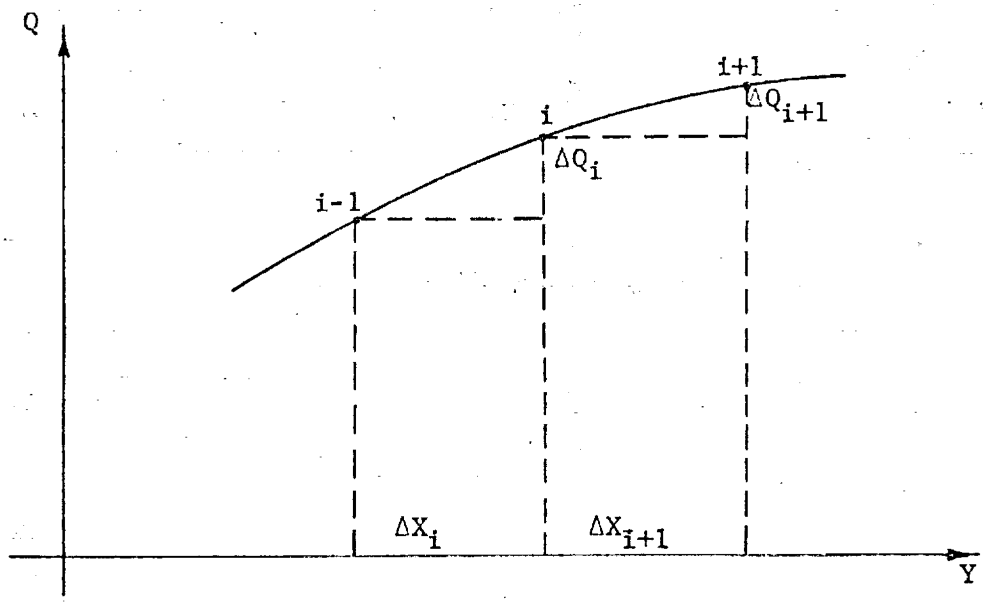
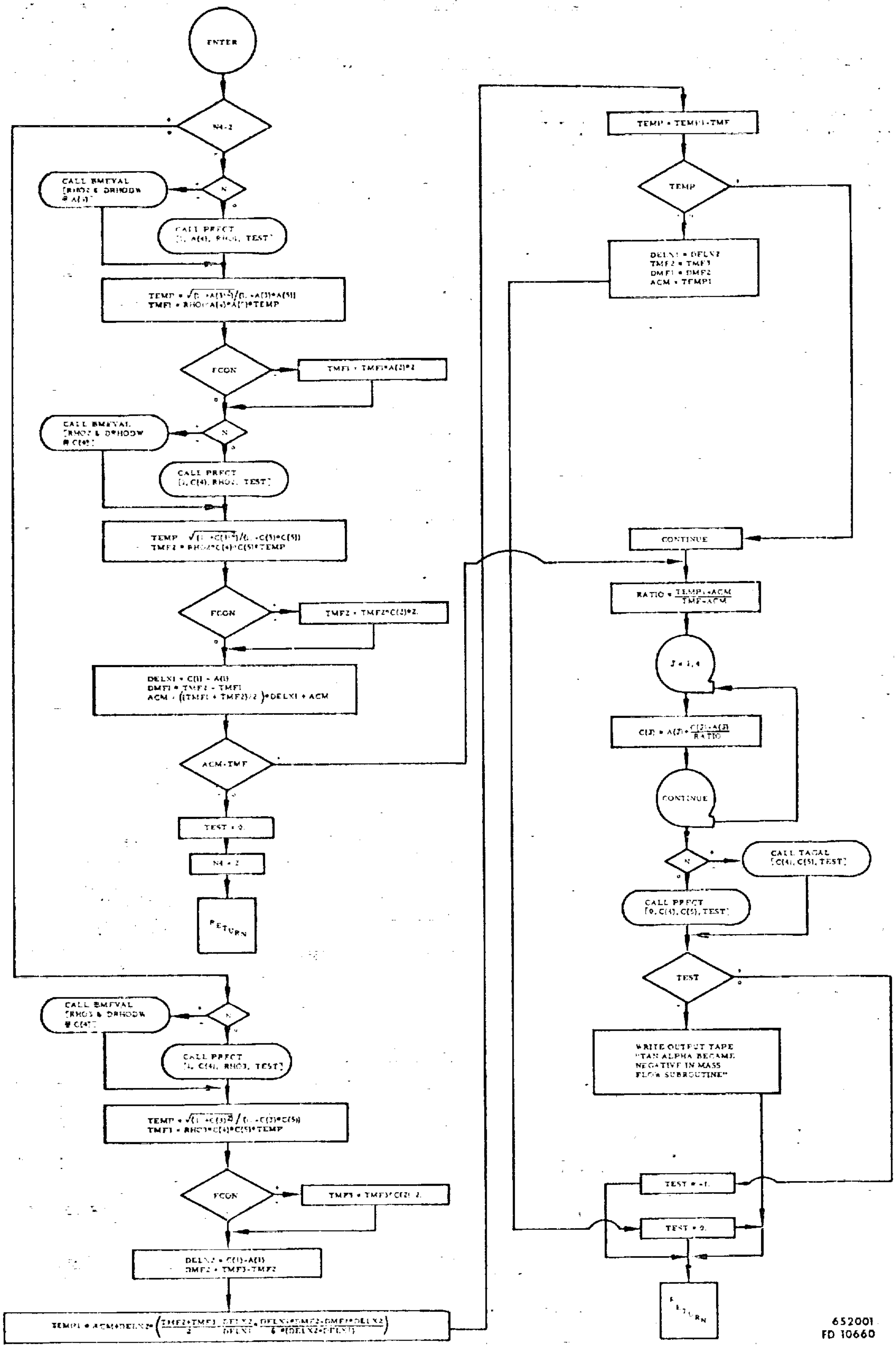


Figure 16

$$\dot{\omega}_{i+1} = \dot{\omega}_i + \Delta X_{i+1} \left[\frac{1}{2}(Q_i + Q_{i+1} - \frac{1}{6}\left(\frac{\Delta X_{i+1}}{\Delta X_i}\right)\left(\frac{\Delta X_i \ \Delta Q_{i+1} - \Delta X_{i+1} \ \Delta Q_i}{\Delta X_{i+1} + \Delta X_i}\right)\right]$$

The calculated mass flow is then added to ACM; if the resultant value is greater than the total mass flow through the nozzle, a point between points 1 and 2 corresponding to the total mass flow is calculated by linear interpolation. The coordinates and flow conditions are stored in C(I) and a signal parameter, TEST, is set equal to one (1.0). If ACM is less than the total mass flow, TEST is set equal to zero.

Subroutine MFLOW1



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18. MFLOW2 Subroutine

The MFLOW2 Subroutine calculates the mass flow between two points along a down Mach line. After the calculation of each interior intersection in Regions III and IV of the flow field, this subroutine is called to determine when the accumulated mass flow along a down Mach line has exceeded the total mass flow through the nozzle.

To calculate the mass flow for axisymmetric flow,

$$\dot{\omega} = \int \left(\rho W \tan\alpha \frac{\sqrt{1 + \tan^2\theta}}{\tan\theta - \tan\alpha} 2\pi Y \right) dY. \quad (18.1)$$

For two-dimensional flow, the $2\pi Y$ term is eliminated .

The same integration procedure as explained in the MFLOW1 Subroutine is used for solving the preceding equation; that is, trapezoidal integration is used for the first increment on the Mach line and parabolic integration is used for the remaining increments.

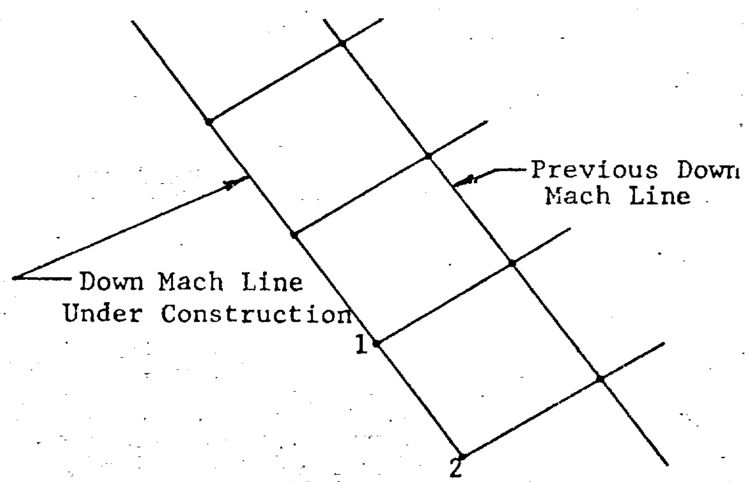
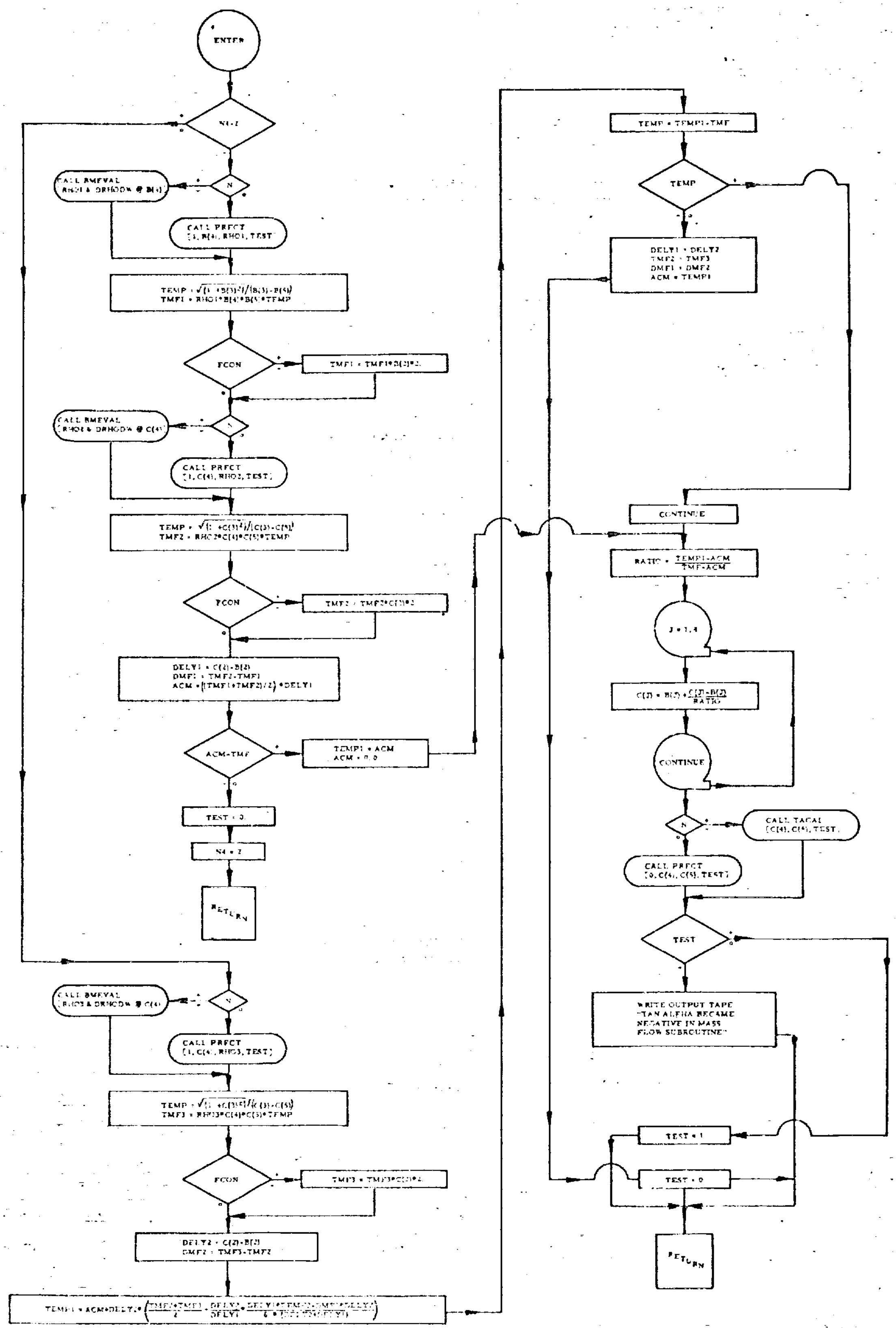


Figure 17

The calculated mass flow is then added to ACM; if the resultant value is greater than the total mass flow through the nozzle, a point between points 1 and 2 corresponding to the total mass flow is calculated by linear interpolation. The coordinates and flow conditions are stored in C(I) and a signal parameter, TEST, is set equal to one (1.0). If ACM is less than the total mass flow, TEST is set equal to zero.

Subroutine MFLOW 2



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19. MFLOWT Subroutine

The MFLOWT Subroutine calculates the mass flow between two points on an up Mach line using trapezoidal integration.

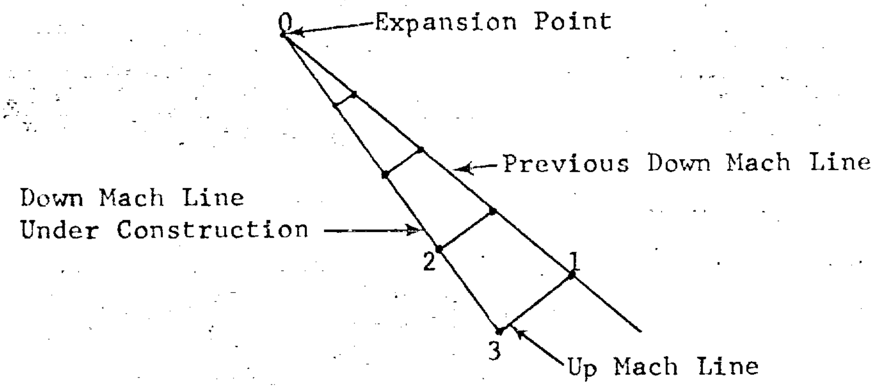


Figure 18

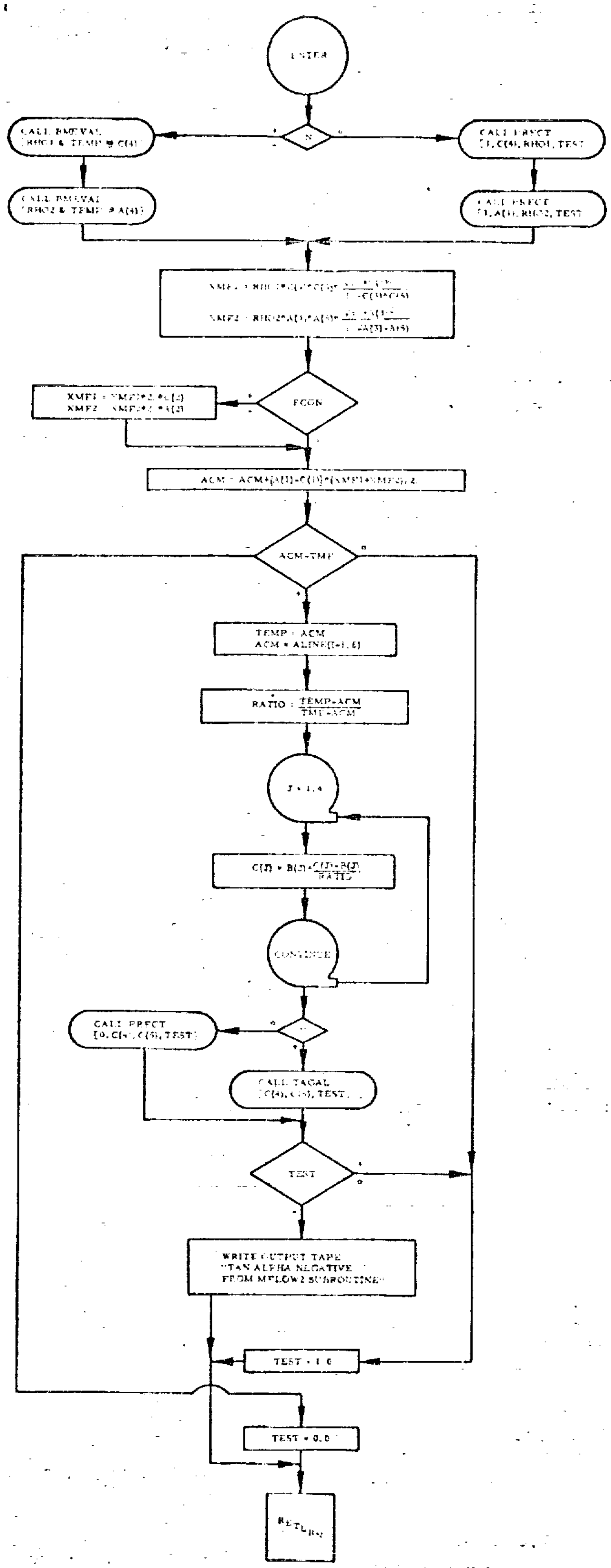
The mass flow between points 0 and 1 must be stored in the variable ACM, and the flow conditions at points 1 and 3 must be stored into A(I) and C(I), respectively. The integral for calculating mass flow along an up Mach line for axisymmetric flow is

$$\dot{\omega} = \int \rho W \tan \alpha \frac{\sqrt{1 + \tan^2 \theta}}{1 - \tan \theta \tan \alpha} 2\pi Y dX.$$

For two-dimensional flow, the $2\pi Y$ term is omitted.

The calculated mass flow is then added to ACM; if the resultant value is greater than the total mass flow through the nozzle, a point between points 2 and 3 corresponding to the total mass flow is calculated by linear interpolation. The coordinates and flow conditions are stored in C(I) and a signal parameter, TEST, is set equal to one (1.0). If ACM is less than TMF, TEST is set equal to zero.

Subroutine MFLOWT



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20. CTGT Subroutine

The gross thrust coefficient at the first point on both the primary and secondary contours is calculated by the CTGT Subroutine. In this subroutine, the starting up Mach line is expected to be stored in TLINE (I,J), and the calculated thrust coefficient will be stored in the variable CTG.

For a perfect gas and axisymmetric flow, the gross thrust coefficient is determined by integrating the following equation along the starting Mach line:

$$C_{TG_T} = \int \frac{1}{A^*} \left[\frac{\tan \alpha}{\tan \alpha - \tan \theta} \quad M^2 \gamma + 1 \right] \frac{P}{P_o} \quad 2\pi Y \; dY.$$

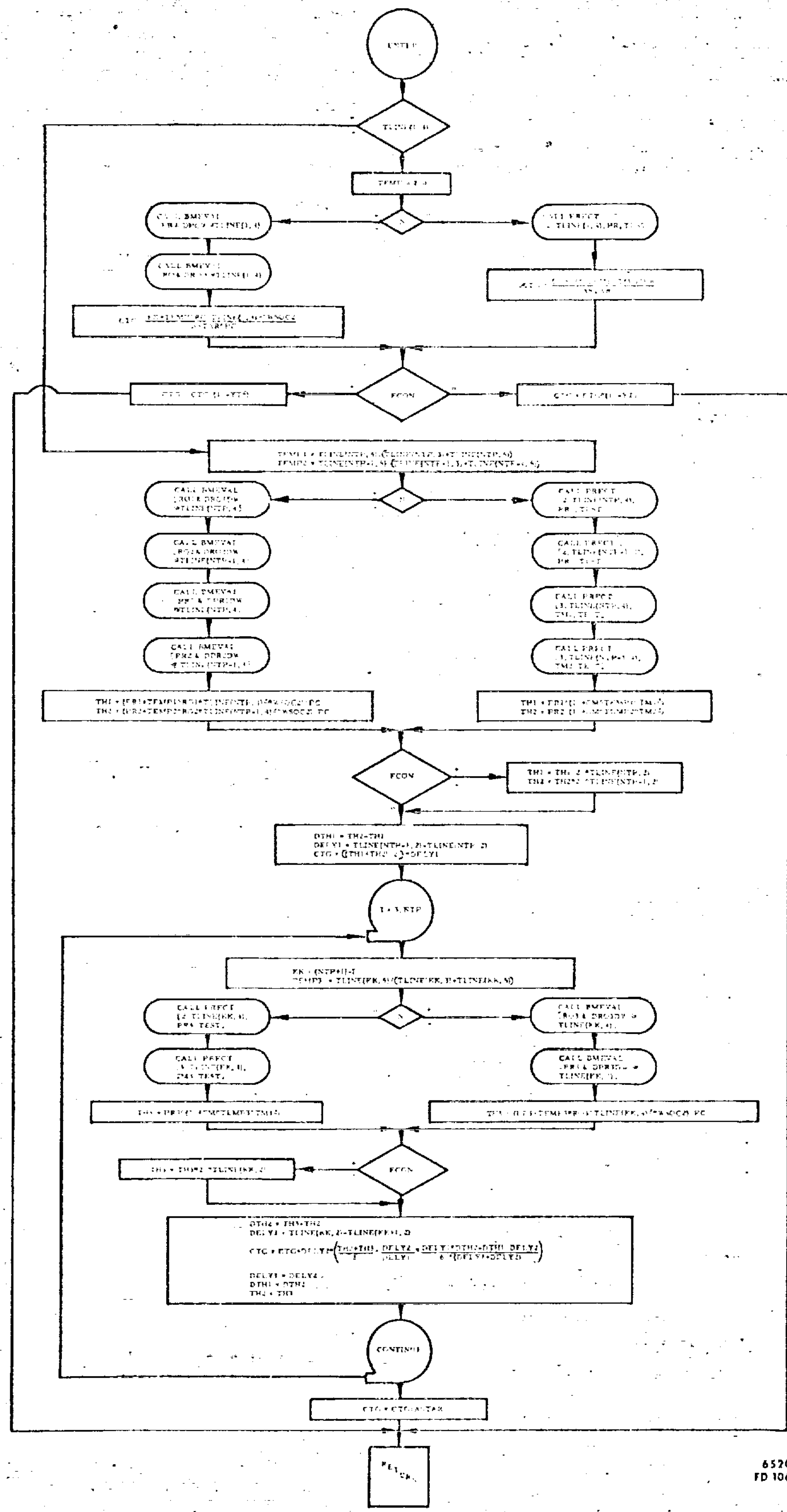
For an ideal gas and axisymmetric flow,

$$C_{TG_T} = \int \left[\frac{\frac{\tan \alpha}{\tan \alpha - \tan \theta} \quad \rho W^2 \, V^2{}_{sonic} + P}{A^* \; P_C} \right] 2\pi Y \; dY.$$

For either gas model and two-dimensional flow, the $2\pi Y$ term in the above equations is omitted.

The same integration procedure as explained in the MFLOW1 Subroutine is used for solving the preceding equations; that is, use trapezoidal integration for the first increment on the Mach line and parabolic integration for the remaining increment.

Subroutine CTGT



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21. PERFM Subroutine

After the points describing the primary and secondary nozzle contours are calculated, the PERFM Subroutine is called to calculate the performance parameters. The flow properties of each contour point are placed in the variable C(I) before entering this subroutine. For parameters that require integration, trapezoidal integration is used for the first intersection, and parabolic integration for the remaining points. The following are the nine parameters to be printed at each contour point.

PRF(1) - The ratio of the X coordinate to the radius of the initial point on the secondary contour as stored in C(1)

PRF(2) - The ratio of the Y coordinate to the radius of the initial point on the secondary contour as stored in C(2)

PRF(3) - The slope of the contour or tan θ as stored in C(3)

PRF(4) - The Mach number calculated by $M = \sqrt{1 + \frac{1}{\tan^2 \alpha}}$, where $\tan \alpha = C(5)$

PRF(5) - The ratio of static pressure to chamber or total pressure; a function of W as stored in C(4)

PRF(6) - The ratio of specific heats that is input for a perfect gas, but for an ideal gas $\gamma = c^2 \rho/P$

PRF(7) - The ratio of accumulated surface area to the throat area (A^*)

For two-dimensional flow, $A_s/A^* = 1/A^* \int ds$.

For axisymmetric flow, $A_s/A^* = 2\pi/A^* \int Yds$

where: $ds = \sqrt{(dX)^2 + (dY)^2}$.

PRF(8) - The gross thrust coefficient where the value at the first point is stored in CTG

For axisymmetric flow, $CTG = CTG + \int \left(\frac{P}{P_o \; A^*} \right) 2\pi Y dY.$

For two-dimensional flow, the $2\pi Y$ term is omitted.

PRF(9) - The net thrust coefficient, which is the CTG less (1) frictional drag along the contour, and (2) subsonic losses

For a perfect gas and axisymmetric flow, the frictional drag coefficient is

$$DRAG = 1/2 \int \left(\frac{C_f \; P \; M^2 \; \gamma}{P_o \; A^*} \right) 2\pi Y dY,$$

where the coefficient of friction (C_f) at each point is determined from

$$C_f = C_{f_i} \left[1 + 0.72 \left(\frac{\gamma - 1}{2} \right) M^2 \right]^{-0.578} .$$

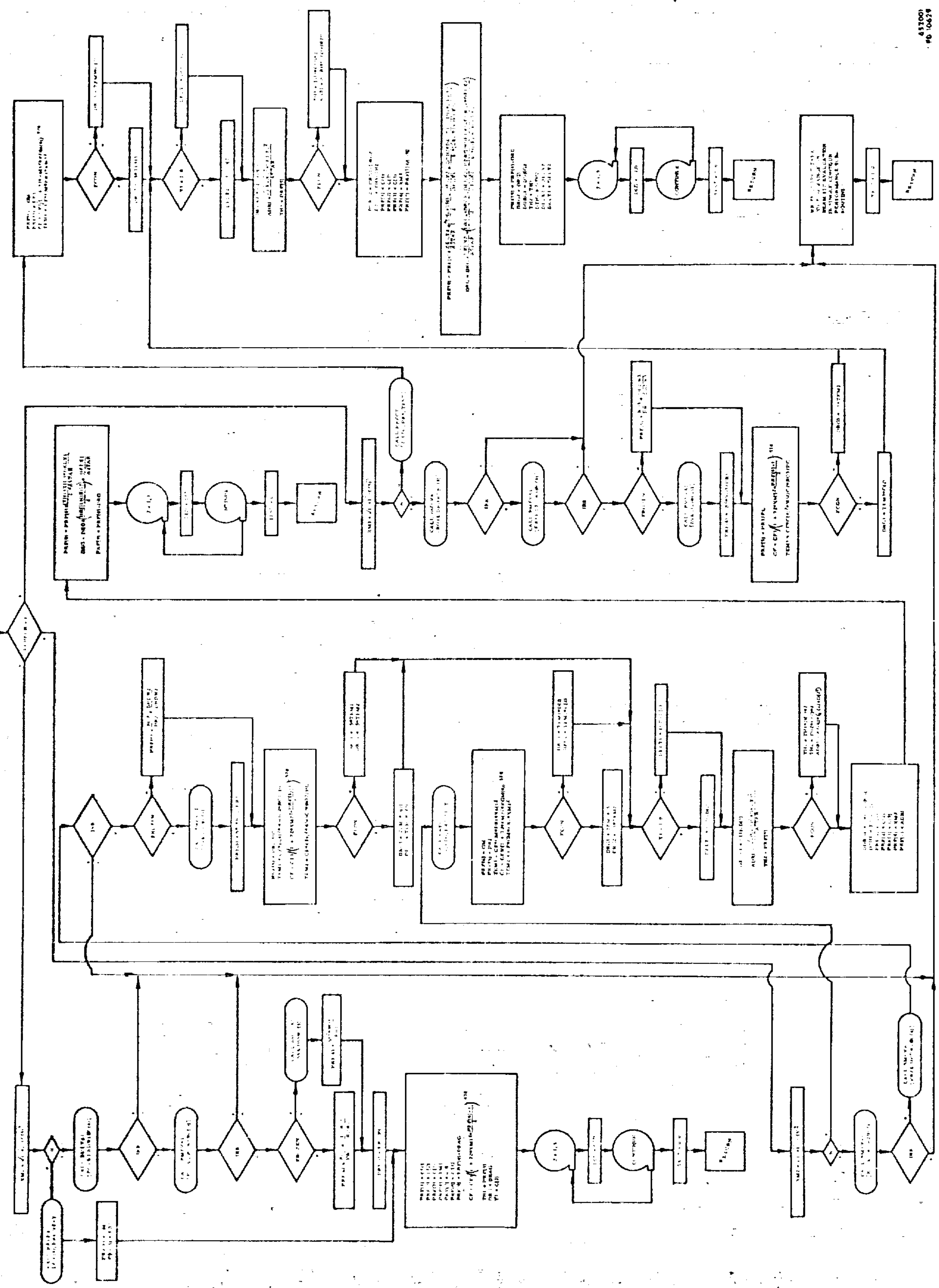
The equation for two-dimensional flow is the same except the $2\pi Y$ term is omitted. For an ideal gas

$$DRAG = 1/2 \int \left(\frac{C_f \; W^2 \; V_{sonic}^2 \; \rho}{P_o \; A^*} \right) 2\pi Y dY.$$

Again, the $2\pi Y$ term is omitted for two-dimensional flow.

The subsonic thrust coefficient loss is an input parameter.

Subroutine PERFM



22. BMFIT Subroutine

The BMFIT Subroutine is used to calculate sets of cubic coefficients for a spline curve fit through a series of input points. This method of curve fitting, commonly referred to as beam-fitting, is derived in Volume I. The calling sequence for the subroutine is:

CALL BMFIT (L,N,X,Y,EO,EN,A,B,C,D),

where:

L - A fixed point variable denoting one of the following moment or slope end-condition options

$L = 1$, $M_1 = EO$ and $M_N = EN$

$L = 2$, $M_1 = EO$ and $M_N = M_{N-1}$

$L = 3$, $M_1 = EO$ and $Y'_N = EN$

$L = 4$, $M_1 = M_2$ and $M_N = EN$

$L = 5$, $M_1 = M_2$ and $M_N = M_{N-1}$

$L = 6$, $M_1 = M_2$ and $Y'_N = EN$

$L = 7$, $Y'_1 = EO$ and $M_N = EN$

$L = 8$, $Y'_1 = EO$ and $M_N = M_{N-1}$

$L = 9$, $Y'_1 = EO$ and $Y'_N = EN$

N - A fixed point variable equal to the number of points to be fit

X - A single dimensional array containing the values of the independent variables

Y - A single dimensional array containing the values of the dependent variable

EO - The moment or slope at the leading end as required by the options L = 1,2,3,7,8, and 9 (EO is zero for L = 4,5 and 6.)

EN - The moment or slope at the trailing end as required by the options L = 1,3,4,6,7, and 9 (EN is zero for L = 2,5, and 8.)

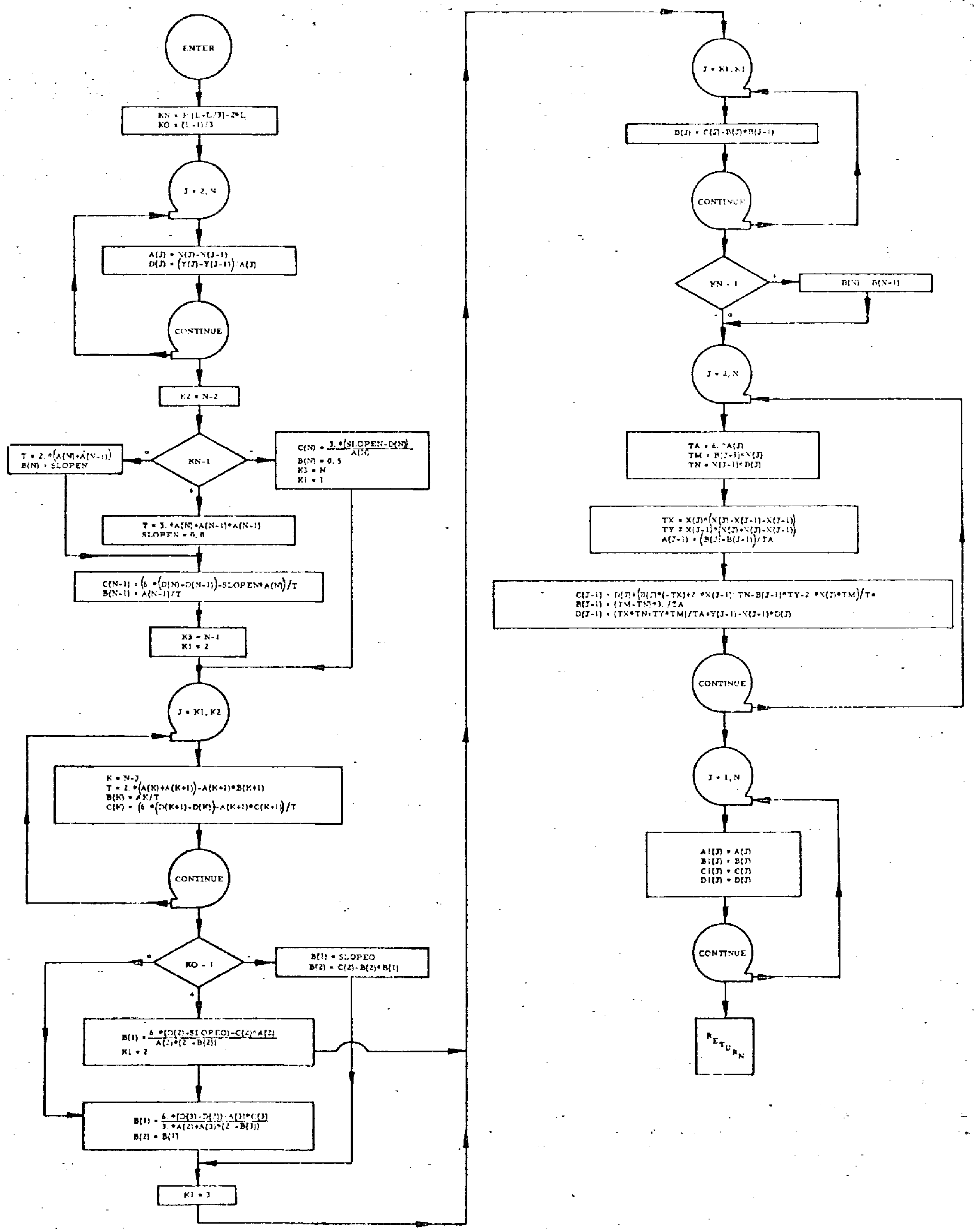
M_i - The moment at the i^{th} contour point

Y'_i - The slope at the i^{th} contour point.

A set of coefficients is calculated for the interval between each input point and then stored in the one dimensional arrays A, B, C, and D. For the i^{th} interval,

$$Y = A_i X^3 + B_i X^2 + C_i X + D_i .$$

Subroutine BMFIT



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23. BMEVAL Subroutine

After a curve that is represented by a set of input coordinates has been fit by the BMFIT Subroutine, the BMEVAL Subroutine is used to evaluate the curve and its first derivative for a given value of the independent variable. The subroutine solves the cubic equation $Y = AX^3 + BX^2 + CX + D$ after searching for the set of coefficients corresponding to the given value of the independent variable.

The eleven parameters that make up the call list of the BMEVAL Subroutine are as follows:

N - The number of coordinates describing the curve

X - Contains values of the independent variable in increasing order (maximum of 100)

Y - Corresponding values of the dependent variable (maximum of 100)

VALU - Value of the independent variable at which the curve is to be evaluated

L - Error signal

L = -1, curve is out of range to the left

L = 0, curve is in range

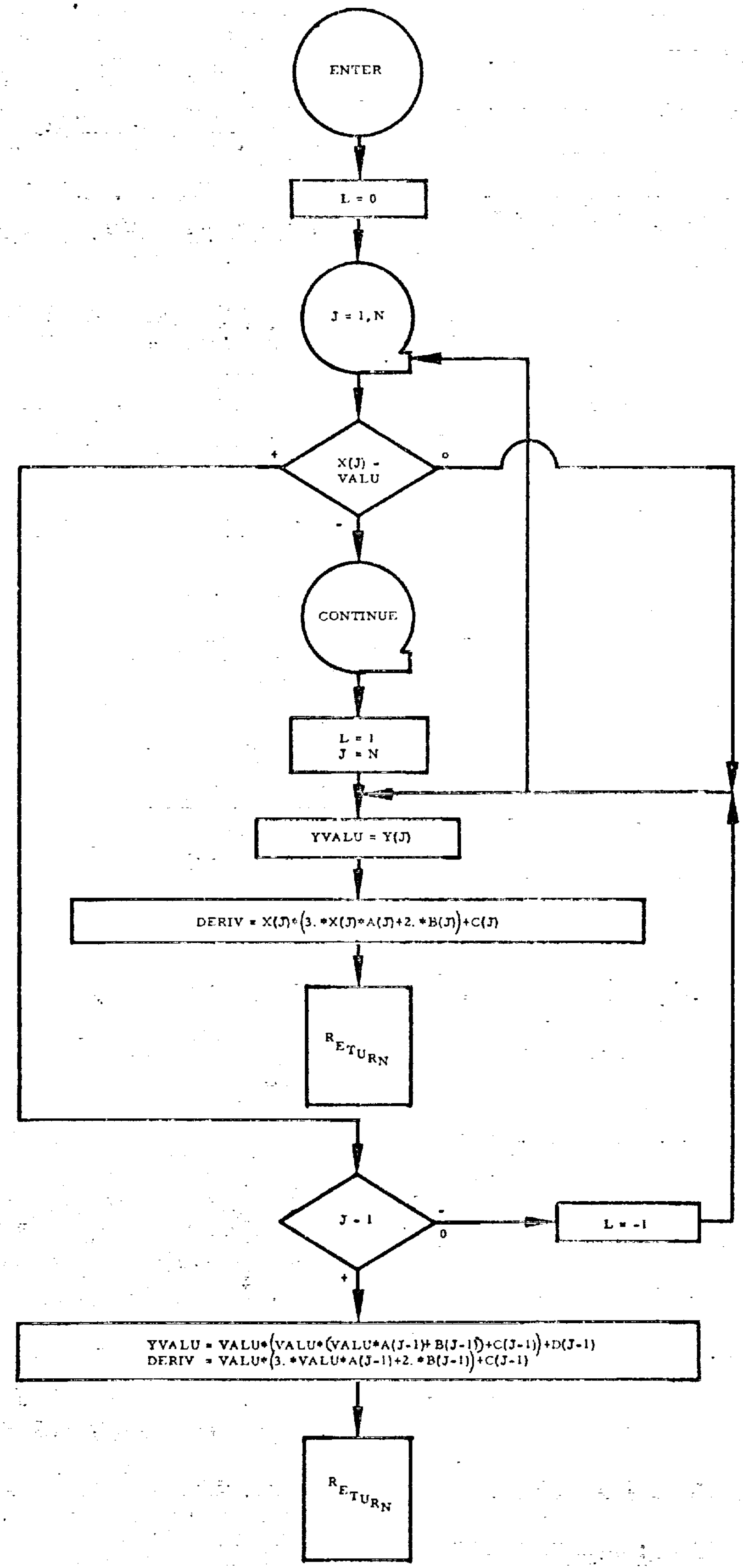
L = +1, curve is out of range to the right

A,B,C,D - Contain the coefficients of the cubic equation for intervals between each input coordinate (these coefficients are calculated in the BMFIT Subroutine)

YVALU - The calculated value of the dependent variable corresponding to VALU

DERIV - Calculated value of the first derivative corresponding to VALU

Subroutine BMEVAL



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SECTION III
INPUT - OUTPUT

The INPUT Subroutine is used to initialize constants and to load the input data. After loading the required data, this subroutine reads any of the optional input with a scatter loading feature that terminates with an END card. The following paragraphs present a detailed outline of the input procedure. A sample input sheet, which corresponds to the output results given in paragraph B, is shown in figure 19.

A. INPUT FORMAT

All data cards must contain the FORTRAN variable name in card columns 1 - 6 (adjusted to the left). For single-valued variables, the number must be in columns 7 - 16. Depending on the gas model option, the parameters under Required Input must be in the order listed. The Optional Input has no particular order and is terminated by a card with END in columns 1 - 3. Each optional input variable will equal the built-in value, unless another number is input. The built-in value is restored between multiple cases.

1. Required Input

a. General

TITLE - Any information to be printed at the beginning of the output will be placed in columns 14 - 72. A zero must be placed in column 12.

b. Gas Model Option

(1) Perfect Gas (Constant Specific Heat Ratio)

GM - Specific heat ratio

ARATIO - The design area ratio or ratio of exit area to annular throat area, (If the exit Mach number is input, this value is not necessary.)

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GENERAL INPUT FORM

SHEET ____ OF ____

ANALYST SAMPLE INPUT ENGINEER ____ EXT ____

JOB NO. ____ COST CONTROL NO. ____

```
TITLE    PERFECT GAS TEST CASE
GM       1.2
RPRINT   0.0
TPRINT   0.0
EMI      2.0
TIFLOW   2.0
YTH    0.7358811
EMSZ     0.05
TMSZ     0.02
END
```

Figure 19. Sample Input Sheet

EMI - Exit Mach number (If ARATIO is input, this value is calculated by the program.)

(2) Ideal Gas (Table of Gas Properties is required and must follow the END card.)

(a) Local Sound Speeds Calculated by Program

XNFUEL - The number of cards in the table of gas properties

PCH - Chamber pressure, psi

EMI - Exit Mach number

(b) Local Frozen Sound Speeds from Tables of Gas Properties

FROZEN - Must be nonzero

XNFUEL - The number of cards in the table of gas properties

PCH - Chamber pressure, psi

EMI - Exit Mach number

2. Optional Input

Name	Built-in Value	Description
TIFLOW	1.0	Indicates the type of starting line TIFLOW = 1.0, constant W starting line and correct value of YTH is calculated TIFLOW = 2.0, constant W starting line and input value of YTH is used TIFLOW = 3.0, the starting Mach line to be loaded into TLINE (I,J) and input value of YTH is used
YTH	0.9	Radius of plug at starting line
TT	0.0	Slope of velocity vector at intersection of starting line and secondary contour

Name	Built-in Value	Description
R	0.0	Radius of circle for circular expansion in the internal expansion region (Must be zero for corner expansion.)
TM	1.005	Mach number along a constant W starting line
ZNTP	0.0	Number of points on starting line (Must be input only when starting line is input, TIFLOW = 3.0.)
TMSZ	0.0075	Mesh size in the Y direction for a constant W starting line
EMSZ	0.025	Mesh size in the Y direction for exit Mach line
FCON	1.0	FCON = 1.0, axisymmetric flow FCON = 0.0, two-dimensional flow
FPRINT	0.0	FPRINT = 0.0, no flow field print FPRINT = 1.0, flow field print
TPRINT	0.0	If nonzero, all throat Mach line guesses will be printed
RPRINT	1.0	If nonzero, the regional dividing Mach lines will be printed
WEXPAN	0.0075	Velocity increment for internal expansion
TOLY	0.000001	Tolerance on interior point iteration
TOL1	0.0005	Tolerance on throat size iteration
TOL2	0.0005	Iteration tolerance when determining last internal expansion Mach line
DELYT	0.01	Increment of ΔY for second guess on YTH
TNO	20.0	Number of velocity decrements for external expansion

Name	Built-in Value	Description
WEXPD	0.0001	Beginning decrement of velocity for external expansion
WEXMAX	0.005	Maximum external expansion decrement
TOLMF	0.000001	Tolerance on mass flow balance in PLUGPT Subroutine
DRAG	0.011	Subsonic thrust losses
CFI	0.003	Incompressible coefficient of skin friction along nozzle contour

A card with END in columns 1 - 3 must follow the last optional input card.

For an ideal gas, a table of gas properties must be input. The first card is a title card describing the gas model, and the second card contains in columns 2 - 15 the specific impulse, $\left(\frac{\text{lbf-sec}}{\text{lbm}}\right)$, at the throat. Each of the remaining cards must contain corresponding values of specific impulse $\left(\frac{\text{lbf-sec}}{\text{lbm}}\right)$, pressure (psi), and density (lbm/ft^3) in columns 2 - 15, 16 - 29, and 30 - 43, respectively, with the local sound speed (ft/sec) in columns 58 - 71.

If a starting down Mach line is to be input, the values of X, Y, tan θ, W, and tan α at each point on the line must be on a separate card with the numbers in field widths of ten beginning in column 7. The first six columns of each card are used to identify the FORTRAN variable (TLINE).

B. OUTPUT DESCRIPTION

The first page of the output includes the title and values of important input parameters. If there is no flow field printout, the following pages of the output include the nine performance parameters at points along the contour for both the primary or plug contour and the secondary contour and the combined performance for both contours. The nine performance parameters are explained in the PERFM Subroutine. A sample of the program output is shown in figures 20a through 20f.

C. PROCEDURES FOR CORRECTING PROGRAM FAILURES

When a program failure occurs, a thorough check of the input should first be made. In most instances, a description of where the error occurred will be indicated.

If the time used in running the program is extremely long with no visible signs of error or if an iteration failure on the throat size is obtained, the initial estimate on the throat size probably is below the level of accuracy acceptable to the program. In such instances, a more accurate approximation for YTH would probably reduce the running time and allow the program to continue.

When an iteration failure in INT2 Subroutine is obtained, the rotation angle, PHI, has possibly rotated the Mach lines to a position where the numerical solution of the characteristic system is difficult or impossible to obtain. By changing the value of PHI, a solution usually can be obtained.

```
                         MULTIPLE EXPANSION PLUG NOZZLE DESIGN                          PAGE 1

                                 PERFECT GAS TEST CASE
PERFECT GAS
AXISYMMETRIC FLOW

                                    * * * I N P U T * * *

 CIRCLE EXPAN. RADIUS      (R) =   0.                 THROAT MACH NUMBER                (TM) =   1.0050
 THROAT FLOW OPTION (TIFLOW) =   2.00000              NO.SECONDARY EXPAN. POINTS (TNO) =  20.0000
 FRICTION COEFFICIENT    (CF1) =   0.00300            THROAT MESH SIZE              (TMSZ) =   0.0200
 THROAT TANGENT           (TT) =   0.                 EXIT MACH LINE MESH SIZE      (EMSZ) =   0.0500
 SUBSONIC THRUST LOSS (DRAG) =   0.01100              CYLINDER HEIGHT              (CYLHT) =   0.0250
 FLOW FIELD PRINT     (FPRINT) =   0.                 FLOW DIMENSION OPTION         (FCON) =   1.0000
 SPECIFIC HEAT RATIO      (GM) =   1.20000            AREA RATIO                  (ARATIO) =  -0.0000
 EXIT MACH NUMBER        (EMI) =   2.00000            INITIAL THROAT HEIGHT          (YTH) =0.73588110
```

Figure 20a

PRIMARY CONTOUR PERFORMANCE

X/YB	Y/YB	TAN THETA	MACH NO.	P/PC	GAMMA	AS/AT	CTG	CTN
-0.0266514	0.7338179	-0.3561399	1.6698245	0.2286232	1.2000000	0.	1.2418534	1.2308534
-0.0177130	0.7365001	-0.3551464	1.6672245	0.2295561	1.2000000	0.0297316	1.2398958	1.2288663
-0.0085006	0.7387246	-0.3546028	1.6643193	0.2306020	1.2000000	0.0600264	1.2382598	1.2271999
0.0015689	0.7396229	-0.3545985	1.6615165	0.2316146	1.2000000	0.0924105	1.2375949	1.2265016
0.0131370	0.7382066	-0.3555239	1.6588367	0.2325859	1.2000000	0.1297306	1.2386422	1.2275105
0.0253140	0.7342676	-0.3574377	1.6562472	0.2335276	1.2000000	0.1736116	1.2415739	1.2305985
0.0402849	0.7270389	-0.3598218	1.6536493	0.2344753	1.2000000	0.2209107	1.2454558	1.2342524
0.0535592	0.7248350	-0.3617295	1.6509628	0.2354585	1.2000000	0.2647760	1.2485655	1.2373005
0.0648358	0.7236892	-0.3623542	1.6481820	0.2364796	1.2000000	0.3003523	1.2494140	1.2381121
0.0741572	0.7255120	-0.3618074	1.6453165	0.2375353	1.2000000	0.3301776	1.2480642	1.2367319
0.0846852	0.7254184	-0.3620070	1.6424359	0.2385818	1.2000000	0.3632782	1.2481343	1.2367673
0.0969812	0.7224997	-0.3632891	1.6396519	0.2396331	1.2000000	0.4029279	1.2503707	1.2389533
0.1107247	0.7172259	-0.3654472	1.6367569	0.2407107	1.2000000	0.4488510	1.2543235	1.2428712
0.1245874	0.7117364	-0.3676382	1.6337722	0.2418257	1.2000000	0.4950173	1.2584244	1.2469271
0.1368667	0.7067787	-0.3687920	1.6307741	0.2429497	1.2000000	0.5338945	1.2606308	1.2490938
0.1469465	0.7093212	-0.3686217	1.6278307	0.2440570	1.2000000	0.5649124	1.2602248	1.2486552
0.1573208	0.7093732	-0.3686934	1.6249011	0.2451630	1.2000000	0.5968044	1.2601857	1.2485826
0.1696110	0.7063067	-0.3700070	1.6219097	0.2462963	1.2000000	0.6356609	1.2623911	1.2507482
0.1835204	0.7005604	-0.3723600	1.6187928	0.2474814	1.2000000	0.6815390	1.2667177	1.2550300
0.1975351	0.6946084	-0.3747357	1.6155735	0.2487101	1.2000000	0.7275694	1.2711818	1.2594493
0.2097652	0.6916069	-0.3758721	1.6123827	0.2499324	1.2000000	0.7653953	1.2734292	1.2615579
0.2199296	0.6919854	-0.3757229	1.6092979	0.2511185	1.2000000	0.7958895	1.2731449	1.2613413
0.2310855	0.6906866	-0.3762987	1.6061845	0.2523200	1.2000000	0.8295387	1.2741196	1.2622505
0.2443187	0.6858625	-0.3783115	1.6029274	0.2535815	1.2000000	0.8715512	1.2777655	1.2658844
0.2586219	0.6791770	-0.3810405	1.5995044	0.2549125	1.2000000	0.9182506	1.2827938	1.2706677
0.2719537	0.6741302	-0.3830154	1.5960542	0.2562594	1.2000000	0.9600523	1.2865759	1.2746082
0.2831524	0.6727157	-0.3834966	1.5927384	0.2575589	1.2000000	0.9929943	1.2876363	1.2756337
0.2940474	0.6717609	-0.3838831	1.5894657	0.2588460	1.2000000	1.0248560	1.2883544	1.2763179
0.3071844	0.6668650	-0.3859447	1.5860033	0.2602137	1.2000000	1.0655212	1.2920505	1.2799732
0.3215926	0.6596786	-0.3889318	1.5823235	0.2616729	1.2000000	1.1118017	1.2974417	1.2853201
0.3350459	0.6541615	-0.3911114	1.5786243	0.2631460	1.2000000	1.1531920	1.3015628	1.2894002
0.3464780	0.6521931	-0.3917868	1.5751049	0.2645532	1.2000000	1.1860341	1.3030328	1.2908554
0.3580642	0.6498609	-0.3927375	1.5715902	0.2659641	1.2000000	1.2193785	1.3047783	1.2925458
0.3719669	0.6432666	-0.3955719	1.5677800	0.2675000	1.2000000	1.2624938	1.3097124	1.2974579
0.3864846	0.6354817	-0.3988556	1.5637572	0.2691288	1.2000000	1.3081387	1.3155004	1.3031826
0.3990544	0.6312869	-0.4004174	1.5598798	0.2707056	1.2000000	1.3445118	1.3186078	1.3062527
0.4104005	0.6292344	-0.4011862	1.5561923	0.2722115	1.2000000	1.3760048	1.3201294	1.3077409
0.4236646	0.6234905	-0.4037076	1.5522270	0.2738379	1.2000000	1.4152406	1.3243912	1.3119637
0.4382848	0.6150833	-0.4074110	1.5479298	0.2756083	1.2000000	1.4605031	1.3305912	1.3181212
0.4515199	0.6093132	-0.4097316	1.5437059	0.2773568	1.2000000	1.4988086	1.3348231	1.3223149
0.4634503	0.6059373	-0.4110627	1.5397080	0.2790193	1.2000000	1.5314580	1.3372958	1.3247535
0.4768622	0.5995027	-0.4140036	1.5354365	0.2808035	1.2000000	1.5703129	1.3420028	1.3294224
0.4915917	0.5903466	-0.4182158	1.5307678	0.2827631	1.2000000	1.6150277	1.3486560	1.3360342
0.5045914	0.5846529	-0.4205186	1.5262680	0.2846613	1.2000000	1.6511609	1.3527681	1.3401103
0.5169946	0.5799466	-0.4226126	1.5219440	0.2864940	1.2000000	1.6846379	1.3561595	1.3434675
0.5311530	0.5714399	-0.4267623	1.5171581	0.2885324	1.2000000	1.7258466	1.3622643	1.3495336
0.5455002	0.5624515	-0.4310094	1.5121128	0.2906927	1.2000000	1.7674439	1.3686602	1.3558710
0.5585209	0.5561642	-0.4338320	1.5073336	0.2927496	1.2000000	1.8024908	1.3731051	1.3605013
0.5719608	0.5486304	-0.4376899	1.5023966	0.2948854	1.2000000	1.8393748	1.3784047	1.3655656
0.5867227	0.5380425	-0.4431831	1.4968968	0.2972778	1.2000000	1.8821501	1.3857877	1.3729103

Figure 20b

X/YB	Y/YB	TAN THETA	MACH NO.	P/PC	GAMMA	AS/AT	CTG	CTN
0.6003314	0.5299540	-0.4471216	1.4916143	0.2995886	1.2000000	1.9187860	1.3913730	1.3784610
0.6135217	0.5223383	-0.4512989	1.4863679	0.3018962	1.2000000	1.9535149	1.3965952	1.3836500
0.6282431	0.5109741	-0.4577005	1.4804141	0.3045301	1.2000000	1.9951549	1.4043123	1.3913308
0.6420598	0.5016246	-0.4627688	1.4746601	0.3070909	1.2000000	2.0317593	1.4105851	1.3975700
0.6557235	0.4920138	-0.4686858	1.4688109	0.3097095	1.2000000	2.0677264	1.4169666	1.4039190
0.6700289	0.4806366	-0.4755658	1.4623826	0.3126053	1.2000000	2.1062489	1.4244280	1.4113470
0.6842157	0.4691110	-0.4830766	1.4560817	0.3154619	1.2000000	2.1438651	1.4318766	1.4187633
0.6988190	0.4557331	-0.4925556	1.4491870	0.3186082	1.2000000	2.1835534	1.4403762	1.4272304
0.7132588	0.4425657	-0.5017928	1.4422619	0.3217901	1.2000000	2.2215913	1.4485826	1.4354056
0.7273602	0.4291269	-0.5126880	1.4351561	0.3250774	1.2000000	2.2583844	1.4567921	1.4435854
0.7408196	0.4175188	-0.5212142	1.4277449	0.3285302	1.2000000	2.2909911	1.4637503	1.4505162
0.7579134	0.3929337	-0.5452256	1.4199622	0.3321827	1.2000000	2.3435759	1.4780315	1.4647638
0.7579613	0.3917695	-0.5480450	1.4181349	0.3330442	1.2000000	2.3455570	1.4786898	1.4654221
0.7708597	0.3850636	-0.5382115	1.4283103	0.3282660	1.2000000	2.3700274	1.4824381	1.4691491
0.7971125	0.3707820	-0.5224595	1.4488148	0.3187787	1.2000000	2.4189747	1.4900042	1.4766674
0.8250567	0.3566638	-0.5045345	1.4691646	0.3095508	1.2000000	2.4683244	1.4969960	1.4836103
0.8547425	0.3419157	-0.4875124	1.4895946	0.3004755	1.2000000	2.5185001	1.5038047	1.4903695
0.8854480	0.3266527	-0.4707919	1.5103911	0.2914325	1.2000000	2.5694763	1.5103487	1.4968630
0.9203081	0.3114184	-0.4521347	1.5309821	0.2826730	1.2000000	2.6208113	1.5163951	1.5028383
0.9566499	0.2951198	-0.4354847	1.5520649	0.2739045	1.2000000	2.6731578	1.5223559	1.5087672
0.9957081	0.2783680	-0.4186279	1.5734224	0.2652279	1.2000000	2.7259694	1.5279676	1.5143266
1.0377378	0.2613302	-0.4007606	1.5948385	0.2567353	1.2000000	2.7790056	1.5331679	1.5194742
1.0831059	0.2438917	-0.3820226	.6163565	0.2484109	1.2000000	2.8322143	1.5379899	1.5242435
1.1322941	0.2257896	-0.3632118	1.6381692	0.2401846	1.2000000	2.8855567	1.5424908	1.5286917
1.1859186	0.2067348	-0.3453649	1.6605394	0.2319321	1.2000000	2.9388928	1.5467068	1.5328553
1.2444448	0.1871705	-0.3266763	1.6834853	0.2237705	1.2000000	2.9915638	1.5505123	1.5366093
1.3085958	0.1671308	-0.3067536	1.7067891	0.2156806	1.2000000	3.0431603	1.5538934	1.5399403
1.3796085	0.1458484	-0.2889456	1.7315081	0.2073560	1.2000000	3.0934358	1.5569473	1.5429457
1.4585173	0.1238132	-0.2707585	1.7572510	0.1989638	1.2000000	3.1413072	1.5595642	1.5455170
1.5463478	0.1019639	-0.2471253	1.7835712	0.1906711	1.2000000	3.1855855	1.5616480	1.5475589
1.6455190	0.0793317	-0.2280031	1.8149807	0.1811488	1.2000000	3.2255454	1.5633036	1.5491772
1.7578917	0.0579935	-0.1916965	1.8470457	0.1718386	1.2000000	3.2595807	1.5644260	1.5502683
1.8884839	0.0365287	-0.1608261	1.8853315	0.1612515	1.2000000	3.2866868	1.5651607	1.5509786
1.9812861	0.0274870	-0.0830104	1.9169779	0.1529232	1.2000000	3.2996205	1.5653584	1.5511650
2.0333118	0.0223613	-0.0475562	1.9461606	0.1455724	1.2000000	3.3052671	1.5654413	1.5512429
2.0557071	0.0232494	-0.0177608	1.9750348	0.1386016	1.2000000	3.3074822	1.5654288	1.5512285
2.0661109	0.0241746	-0.0069134	1.9894332	0.1352352	1.2000000	3.3085555	1.5654158	1.5512147
2.0711770	0.0247141	-0.0022050	1.9964998	0.1336093	1.2000000	3.3090952	1.5654081	1.5512065
2.0736827	0.0249999	0.	2.0000000	0.1328103	1.2000000	3.3093669	1.5654040	1.5512022

Figure 20c

SECONDARY CONTOUR PERFORMANCE

X/YB	Y/YB	TAN THETA	MACH NO.	P/PC	GAMMA	AS/AT	CTG	CTV
0.	1.0000000	0.	1.0050000	0.5613971	1.2000000	0.	1.2418534	1.2308534
0.0413500	0.9999275	-0.0042361	1.0282578	0.5471445	1.2000000	0.1791916	1.2416791	1.2305019
0.0674507	0.9997575	-0.0109271	1.0517765	0.5328428	1.2000000	0.2922884	1.2412831	1.2299921
0.0896220	0.9994246	-0.0192252	1.0755509	0.5185132	1.2000000	0.3883431	1.2405272	1.2291379
0.1099456	0.9989455	-0.0287907	1.0994585	0.5042468	1.2000000	0.4763715	1.2394676	1.2279869
0.1292901	0.9982877	-0.0394205	1.1234433	0.4900923	1.2000000	0.5601370	1.2380538	1.2264851
0.1481367	0.9974423	-0.0509741	1.1474683	0.4760855	1.2000000	0.6417198	1.2362891	1.2246335
0.1667731	0.9963754	-0.0633493	1.1715065	0.4622542	1.2000000	0.7223662	1.2341280	1.2223859
0.1854123	0.9950708	-0.0764662	1.1955327	0.4486241	1.2000000	0.8029938	1.2315658	1.2197363
0.2042145	0.9935062	-0.0902621	1.2195214	0.4352187	1.2000000	0.8842912	1.2285884	1.2166701
0.2233054	0.9916529	-0.1046868	1.2434462	0.4220607	1.2000000	0.9667973	1.2251732	1.2131644
0.2427698	0.9894645	-0.1197020	1.2672818	0.4091701	1.2000000	1.0508796	1.2212710	1.2091697
0.2627183	0.9869215	-0.1352730	1.2909951	0.3965691	1.2000000	1.1370006	1.2168859	1.2046896
0.2832341	0.9839852	-0.1513731	1.3145521	0.3842782	1.2000000	1.2255093	1.2119926	1.1996985
0.3043840	0.9806082	-0.1679820	1.3379171	0.3723165	1.2000000	1.3166840	1.2065569	1.1941622
0.3262433	0.9767470	-0.1850819	1.3610470	0.3607044	1.2000000	1.4108303	1.2005578	1.1880594
0.3488902	0.9723565	-0.2026577	1.3838928	0.3494627	1.2000000	1.5082383	1.1939766	1.1813712
0.3723935	0.9673822	-0.2206980	1.4064004	0.3386120	1.2000000	1.6092334	1.1867872	1.1740714
0.3968240	0.9617644	-0.2392038	1.4285238	0.3281662	1.2000000	1.7140219	1.1789617	1.1661320
0.4222431	0.9554454	-0.2581389	1.4501523	0.3181664	1.2000000	1.8228340	1.1704822	1.1575351
0.4487209	0.9483526	-0.2775183	1.4712390	0.3086206	1.2000000	1.9359121	1.1613167	1.1482486
0.5014715	0.9327168	-0.3150220	1.5089737	0.2920425	1.2000000	2.1601688	1.1422078	1.1289029

Figure 20d

COMBINED CONTOUR PERFORMANCE

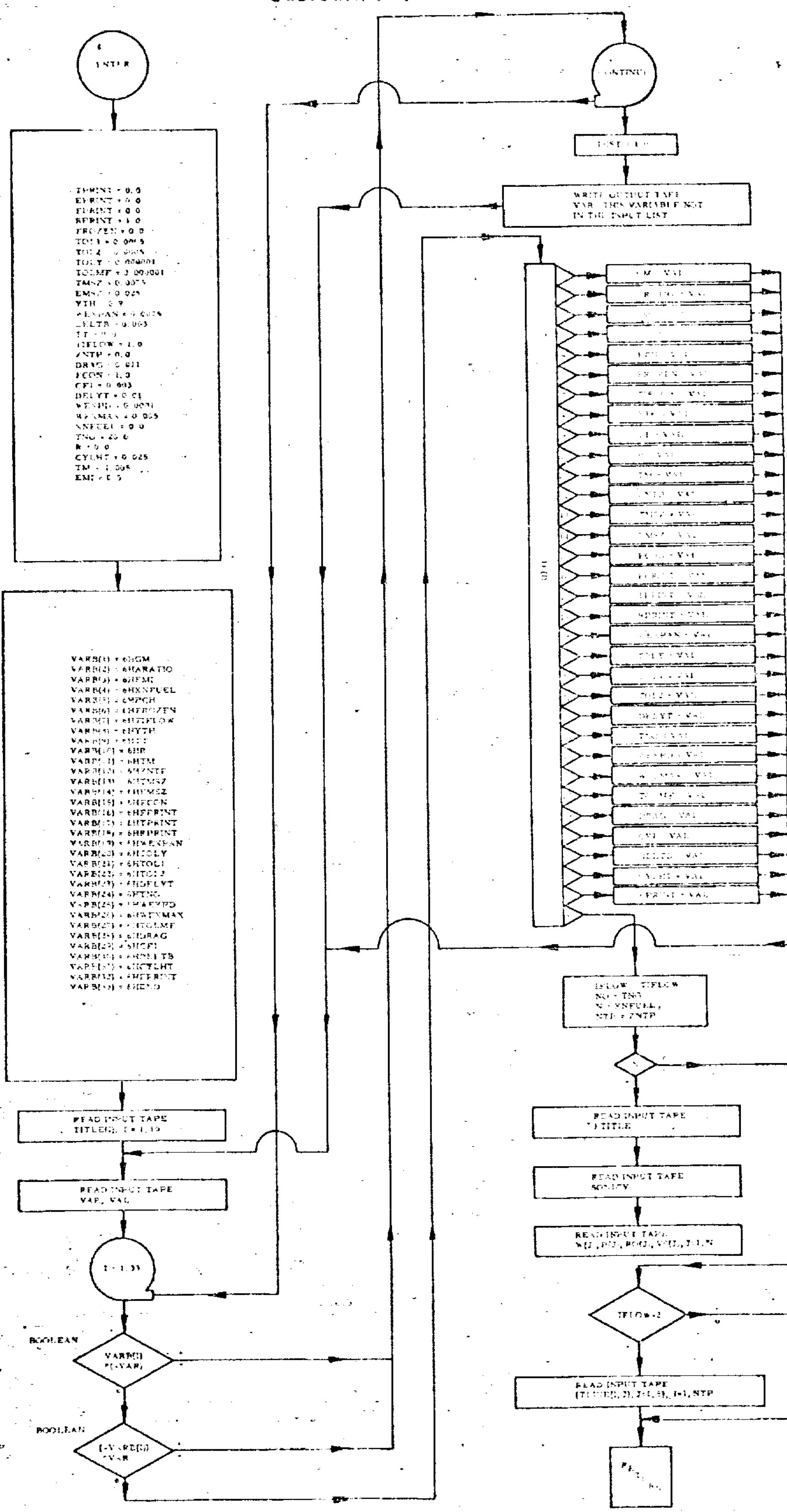
X/R	AS/A*	CTG	CTN
-0.0266514	0.	1.2418534	1.2308534
-0.0177130	0.0297316	1.2398958	1.2288663
-0.0085006	0.0600264	1.2382598	1.2271999
0.	0.0873648	1.2376985	1.2155104
0.0015689	0.0992098	1.2375955	1.2154956
0.0131370	0.1866620	1.2386302	1.2164425
0.0263140	0.2876461	1.2415093	1.2192215
0.0402849	0.3954869	1.2452891	1.2228950
0.0535592	0.4968729	1.2482551	1.2257598
0.0648358	0.5813107	1.2489043	1.2263229
0.0741572	0.6515230	1.2473099	1.2246569
0.0846852	0.7302350	1.2470117	1.2242775
0.0969812	0.8231490	1.2487052	1.2258756
0.1107247	0.9285969	1.2518887	1.2289523
0.1245874	1.0347931	1.2550054	1.2319608
0.1368667	1.1268323	1.2561698	1.2330293
0.1469465	1.2014809	1.2547842	1.2315646
0.1573208	1.2762707	1.2536082	1.2303069
0.1696110	1.3703051	1.2543011	1.2309028
0.1835204	1.4763505	1.2567082	1.2331999
0.1975351	1.5829842	1.2590184	1.2353993
0.2097652	1.6736790	1.2592128	1.2354970
0.2199296	1.7480999	1.2570986	1.2333023
0.2310855	1.8299495	1.2559284	1.2320436
0.2443187	1.9291200	1.2568560	1.2328662
0.2586219	2.0375712	1.2587571	1.2346542
0.2719537	2.1369039	1.2594531	1.2352451
0.2831524	2.2181513	1.2577957	1.2334995
0.2940474	2.2969894	1.2557623	1.2313803
0.3071844	2.3942715	1.2560062	1.2315209
0.3215926	2.5026100	1.2574528	1.2328548
0.3350439	2.6018972	1.2577521	1.2330493
0.3464780	2.6839207	1.2558737	1.2310822
0.3580642	2.7670675	1.2541372	1.2292560
0.3719669	2.8696957	1.2547797	1.2297914
0.3864846	2.9778354	1.2559616	1.2308622
0.3990544	3.0680903	1.2549854	1.2297903
0.4104005	3.1481724	1.2527498	1.2274687
0.4236646	3.2441528	1.2525362	1.2271550
0.4382848	3.3518829	1.2537041	1.2282135
0.4515199	3.4466552	1.2533001	1.2277111
0.4634503	3.5301302	1.2515339	1.2258568
0.4768622	3.6260359	1.2514067	1.2256331
0.4915917	3.7333015	1.2526730	1.2267902
0.5014715	3.8026578	1.2521357	1.2261815
0.5045914	3.8113296	1.2531226	1.2271598
0.5169946	3.8448067	1.2565140	1.2305170
0.5311530	3.8860154	1.2626187	1.2365831
0.5455002	3.9276127	1.2690147	1.2429405
0.5585209	3.9626595	1.2734596	1.2473508

Figure 20e

X/R	AS/A*	CTG	CTN
0.5719608	3.9995435	1.2787592	1.2526151
0.5867227	4.0423189	1.2861421	1.2599598
0.6003314	4.0789547	1.2917275	1.2655105
0.6135217	4.1136836	1.2969497	1.2706995
0.6282431	4.1553236	1.3046668	1.2783803
0.6420598	4.1919280	1.3109395	1.2846196
0.6557235	4.2278951	1.3173211	1.2909686
0.6700289	4.2664176	1.3247824	1.2983966
0.6842157	4.3040339	1.3322311	1.3058128
0.6988190	4.3437222	1.3407307	1.3142799
0.7132588	4.3817600	1.3489370	1.3224551
0.7273602	4.4185532	1.3571465	1.3306350
0.7408196	4.4511598	1.3641048	1.3375657
0.7579134	4.5037446	1.3783859	1.3518133
0.7579613	4.5057258	1.3790443	1.3524716
0.7708597	4.5301962	1.3827925	1.3561986
0.7971125	4.5791435	1.3903587	1.3637169
0.8250567	4.6264932	1.3973504	1.3706599
0.8547425	4.6786689	1.4041592	1.3774190
0.8864480	4.7296451	1.4107032	1.3839125
0.9203081	4.7809801	1.4167496	1.3899078
0.9566499	4.8333265	1.4227104	1.3958167
0.9957081	4.8861382	1.4283221	1.4013761
1.0377378	4.9391744	1.4335223	1.4065237
1.0831059	4.9923830	1.4383443	1.4112930
1.1322941	5.0457254	1.4428453	1.4157412
1.1859186	5.0990615	1.4470613	1.4199049
1.2444448	5.1517325	1.4508667	1.4236588
1.3085958	5.2033291	1.4542479	1.4269898
1.3796085	5.2536045	1.4573018	1.4299952
1.4585173	5.3014760	1.4599187	1.4325665
1.5463478	5.3457543	1.4620025	1.4346084
1.6455190	5.3857142	1.4636581	1.4362267
1.7578917	5.4197495	1.4647805	1.4373178
1.8884839	5.4468555	1.4655152	1.4380281
1.9812861	5.4597892	1.4657129	1.4382145
2.0333118	5.4654359	1.4657957	1.4382924
2.0557071	5.4676509	1.4657833	1.4382781
2.0661109	5.4687243	1.4657703	1.4382642
2.0711770	5.4692640	1.4657626	1.4382560
2.0736827	5.4695356	1.4657585	1.4382517

Figure 20f

Subroutine INPUT



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APPENDIX A
SYMBOL TABLE

A^*	-	Theoretical throat area
c	-	Local speed of sound
C_f	-	Coefficient of friction
C_{TG}	-	Gross thrust coefficient
C_{TN}	-	Net thrust coefficient
I_s	-	Specific impulse
M	-	Mach number
P	-	Pressure
V_{max}	-	Maximum velocity
V_{sonic}	-	Sonic velocity
W	-	Velocity ratio; either V/V_{max} or V/V_{sonic}
$\dot{w}$	-	Mass flow rate
α	-	Mach angle
γ	-	Ratio of specific heats
ρ	-	Density
σ	-	=1 for axisymmetric flow
		=0 for two-dimensional flow
θ	-	Angle between velocity vector and axis of symmetry

APPENDIX B
FORTRAN SYMBOL TABLE (COMMON)

Variable	Dimension	Description
A	5	For storing X,Y, tan θ, W, and tan α; usually represents the point on an up Mach line
A2	100	Coefficients of X^3 for ρ vs W beam fit
A3	100	Coefficients of X^3 for P vs W beam fit
A5	200	Coefficients of X^3 for AS/A* vs X beam fit
A6	200	Coefficients of X^3 for (Throat CTG-CTG) vs X beam fit
A7	200	Coefficients of X^3 for DRAG vs X beam fit
A8	100	Coefficients of X^3 for c vs W beam fit
ACM	—	Accumulated mass flow
ALINE	200, 6	Variable used to store X, Y, tan θ, W, and tan α, and $\dot{\omega}$ along a Mach line
ARATIO	—	Design area ratio or ratio of exit area to annular throat area
ASTAR	—	Theoretical or minimum throat area
B	5	For storing X, Y, tan θ, W, and tan α; usually represents the point on a down Mach line
B2	100	Coefficients of X^2 for ρ vs W beam fit
B3	100	Coefficients of X^2 for P vs W beam fit
B5	200	Coefficients of X^2 for AS/A* vs X beam fit
B6	200	Coefficients of X^2 for (Throat CTG-CTG) vs X beam fit
B7	200	Coefficients of X^2 for DRAG vs X beam fit
B8	100	Coefficients of X^2 for c vs W beam fit
BLINE	200, 6	Variable used to store X, Y, tan θ, W, tan α, and $\dot{\omega}$ along a Mach line

Variable	Dimension	Description
C	5	For storing X, Y, tan θ, W, and tan α; normally used in the calculation of a Mach line intersection
C2	100	Coefficients of X for ρ vs W beam fit
C3	100	Coefficients of X for P vs W beam fit
C5	200	Coefficients of X for AS/A* vs X beam fit
C6	200	Coefficients of X for (Throat CTG-CTG) vs X beam fit
C7	200	Coefficients of X for DRAG vs X beam fit
C8	100	Coefficients of X for c vs W beam fit
CF1	—	Incompressible coefficient of skin friction
CLINE	200, 6	Variable used to store X, Y, tan θ, W, tan α, and $\dot{\omega}$ along the secondary contour
CTG	—	Initial gross thrust coefficient
CYLHT	—	Value of Y/R at the last primary contour point (cylinder height)
D2	100	Constants for ρ vs W beam fit
D3	100	Constants for P vs W beam fit
D5	200	Constants for AS/A* vs X beam fit
D6	200	Constants for (Throat CTG-CTG) vs X beam fit
D7	200	Constants for DRAG vs X beam fit
D8	100	Constants for c vs W beam fit
DELTB	—	Increment of tan θ
DELYT	—	Increment of ΔY for second guess on YTH
DLINE	400, 5	Variable used to store X, Y, tan θ, W, and tan α along primary contour
DRAG	—	Subsonic thrust losses
EMI	—	Exit Mach number
EMSZ	—	Mesh size in the Y direction for exit Mach line

Variable	Dimension	Description
FCON	—	Indicates whether two-dimensional or axisymmetric flow is considered
FGM	15	Various functions of GM - see Gamma Function Table in program listing
FPRINT	—	Indicates whether or not to print the flow field
FROZEN	—	For an ideal gas this indicates that the input values of local frozen sound speeds are to be used
FTITLE	12	Variable used to store the title for the table of gas properties
GM	—	Specific heat ratio for a perfect gas
IFLOW	—	Fixed point variable for TIFLOW
JMIN	—	The number of the first supersonic value of W in the table of gas properties
N	—	The number of cards making up the table of gas properties
NEP	—	Number of points on the exit Mach line
NO	—	Number of velocity decrements for external expansion
NTP	—	Number of points on starting line
P	100	Contains the values of pressure for an ideal gas
PCH	—	Chamber pressure
PRF	9	Contains calculated performance parameters
R	—	Radius of circle for circular expansion in the internal expansion region
RHOSON	—	Density evaluated at the sonic velocity
RO	100	Contains the values of density for an ideal gas
RPRINT	—	Indicates whether or not to print the regional dividing Mach lines

Variable	Dimension	Description
SONICV	—	The sonic velocity
TIFLOW	—	Indicates the type of starting line
TITLE	10	To store and print the title card
TLINE	200, 6	Variable used to store X, Y, tan θ, W, tan α, and $\dot{\omega}$ along the last internal expansion line
TM	—	Mach number along a constant W starting line
TMF	—	Total mass flow through nozzle
TMSZ	—	Mesh size in the Y direction for a constant W starting line
TNO	—	Number of velocity decrements for external expansion
TOLY	—	Tolerance on interior point iteration
TOL1	—	Tolerance on throat size iteration
TOL2	—	Iteration tolerance when determining last internal expansion Mach line
TOLMF	—	Tolerance on mass flow balance in PLUGPT Subroutine
TPRINT	—	Indicates whether or not to print the throat Mach line guesses
TT	—	Slope of velocity vector at intersection of starting line and secondary contour
VS	100	Contains the values of local sound speeds for an ideal gas
W	100	Contains the values of velocity ratio for an ideal gas
WEXPAN	—	Velocity increment for internal expansion
WEXPD	—	Beginning decrement of velocity for external expansion
WEXMAX	—	Maximum external expansion decrement
WSOC2	—	The sonic velocity squared

Variable	Dimension	Description
XNFUEL	—	Number of cards in the table of gas properties
XO	—	X/R_{S_1} coordinate for center of expansion circle
YO	—	Y/R_{S_1} coordinate for center of expansion circle
YTH	—	Radius of plug at starting line
ZNEP	—	Number of points on exit Mach line
ZNTP	—	Number of points on starting line