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THE INCLINED VISIBILITY RANGE AND THE BRILLIANCE OF DAYTIME SKY

K.S. Shifrin and N.P. Pyatovskaya

Translation of Table of Contents, Annotation and selected pages (1-19 and 126-134) of a Russian book (Translated title: The Inclinced Visibility Range and the Brilliance of the Daytime Sky), Leningrad, Hydrometeorological Publishing House, 1959.

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Main Geophysical Observatory Imeni

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Of The Inclined Visibility Range And The Brilliance of Daytime Sky

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ANNOTATION

Results of computations pertaining to the brilliance of atmospheric haze and to the daytime sky are cited in the tables. A great range of physical parameters that determine investigated values, which practically embraces all concrete atmospheric conditions and various types of terrestrial surface, has been investigated here.

The work is figured out to be of value to scientific workers and engineers employed in the fields of geophysics, aerial surveying, illumination engineering and allied sciences. Tables are presented in a form handy for use.

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PREFACE

Tables of inclined visibility range and of the brilliance of daytime cloudless sky are the result of the work that has been carried out at the Main Geophysical Observatory imeni A.I. Voyeykov for a number of years.

The work consists of two parts. In part I (tables of the inclined visibility range) is presented a theoretical computation chart of the inclined visibility range, and basic and auxiliary computation tables are cited (tables of brilliance of atmospheric haze, of spectral atmospheric transparency, etc.). An example is cited of the computation pertaining to contrasts and inclined visibility range for one of the cases of atmospheric condition. The relation of the brilliance of atmospheric haze to the albedo has been investigated.

In Part II (tables of the brilliance of daytime sky) are cited computation results of the distribution of brilliance over the cloudless sky. Tables are given of the coefficients of brilliance for various atmosphere parameters and albedo of the underlying surface for three azimuths. The possibility is indicated of the use of cited tables for computing brilliance coefficients of the cloudless sky for any azimuth and, in addition to this, a conversion formula is cited and computed auxiliary tables are given. An example of the computation of the coefficients of brilliance of cloudless sky is given for one of the cases of atmospheric condition (summer and winter conditions) for various azimuths and brilliance isophots for these cases are presented.

Part I was written by K.S. Shifrin and N.P. Pyatovskaya, part II by N.P. Pyatovskaya. L.K. Potyekhina, O.I. Yeremkina, Z.P. Koblova, K.A. Fateyeva, N.A. Chugunova and N.M. Gorb who participated in computations and compiling of tables and graphs.

Leningrad, May 1958

PART I

TABLES OF INCLINED VISIBILITY RANGE

The Chart Pertaining To The Computation Of Visibility Range And Description of Tables

The spectral theory of the inclined visibility range (IVR) developed in (1) makes it possible to compute the IVR for various conditions of real atmosphere, units, backgrounds and various geometric parameters of the problem (heights z, sight angles θ azimuths φ and zenith distance of the sun i). The aim of the present work, which is a direct extension (1), is such so that the chart developed in (1) could as much as possible simplify the practical utilization of it. For this purpose a number of auxiliary computations has been carried out here in a general way. Results of these computations are presented in the form of tables.

In conformity with (1) (see formulae (6), (42*)) the contrast K is determined according to the formula

$$K=\frac{\int [r_0(\lambda,\theta)-r_\Phi(\lambda,\theta)]\,E(\lambda)\,e^{-\tau_0^h(\lambda)\sec\theta}\,\Theta(\lambda)\,d\lambda}{\int r_0(\lambda,\theta)\,E(\lambda)\,e^{-\tau_0^h(\lambda)\sec\theta}\,\Theta(\lambda)\,d\lambda+\int D(\lambda)\,I_0(\lambda)\,\Theta(\lambda)\,d\lambda},$$

or by denoting

$$I_0(\lambda)\,\Theta(\lambda)=f(\lambda),$$

$$B(\tau_0 i)=\frac{2R_0(\tau_0 i)\cos i}{4+(3-X_1)(1-A)\tau_0},$$

we shall find

$$K=\frac{\int [r_0(\lambda,\theta)-r_\Phi(\lambda,\theta)]\,Be^{-\tau_0^h(\lambda)\sec\theta} f(\lambda)\,d\lambda}{\int r_0(\lambda,\theta)\,Be^{-\tau_0^h(\lambda)\sec\theta} f(\lambda)\,d\lambda+\int D(\lambda)\,f(\lambda)\,d\lambda}. \qquad (1)$$

For computing the contrast according to this formula, it is necessary first to determine the values of initial parameters of the problem (three spectral functions and two numerical parameters):

1) the curve of spectral sensitivity of radiation detector $-\Theta(\lambda)$.

This initial curve determines the sphere of values λ, according to which integrals are computed in K. For example, this curve is shown in (1);

2) the curve of comparative spectral brilliance of an object $r_0(\lambda,0)$ for a given sighting angle θ. For the orthotropic surface r_0 is not related to θ;

3) same for the background;

4) optical thicknwss of the entire atmosphere in the vertical direction τ_0 for $\lambda_0=0{,}550\,\mu$ during the observation time;

5) horizontal visibility range of the black body S_0 at the same moment.

Formulae in (1) make it possible now to determine remaining functions according to the values S_0 and τ_0, and also according to i, θ, φ illumination $E_{(\lambda)}$, spectral optical thickness of various air layers $\tau_0^h(\lambda)$, and the haze brilliance $D(\lambda_0)$.

The most labor consuming is the computation of haze. The brilliance of atmospheric haze is computed for $\lambda_0=0{,}550\,\mu$ for the eight atmospheric conditions $(\tau_0=0{,}2,\ S_0=50,\ 20$

and 10 км; $\tau_0 = 0{,}3$, $S_0 = 50$, 20 and 10 км; $\tau_0 = 0.5$, $S_0 = 10$ and 4 км), for the four zenith distances of the Sun i (20, 40, 60, 80°), for the five sighting angles θ (0, 20, 40, 60, 85°), for $z \leqslant 10$ км and for the four albedo values A (0, 0,5, 0,75, 1,0). For $A = 0{,}20$ the tables D are computed in (1). Computation results of azimuths $(\Delta\varphi)$ 0,90 and 180° (Table 1) are cited in Tables D.

In Table II are cited meanings of value T of the spectral transparency of the atmosphere for various levels above the Earth's surface

$$T = e^{-\tau_0^{H}(\lambda)\sec\theta}.$$

The meanings are cited in the domain $0{,}4 \leqslant \lambda \leqslant 0{,}8$ with the pace $\Delta\lambda = 0{,}05\ \mu$.

In Table III meanings of auxiliary function $B(\tau_0, i)$ for $A = 0$, 0,2, 0,5, 0,75 and 1.0 needed for the computation of illumination of the horizontal terrace $E(\lambda)$ are indicated. Spectral illumination is being easily determined now with the aid of function $B(\tau_0, i)$ according to the formula

$$E(\lambda) = I_0(\lambda)\, B(\tau_0, i),$$

where $I_0(\lambda)$ – is the density of solar radiation falling over the upper border of the atmosphere. Function $I_0(\lambda)$ is cited in Table IV, according to recent data (2, 3). We quote the table at full length for values λ from 0.22 to 7,0 μ In the first column of this table λ are given in microns, in the second one - values $I_0(\lambda)$ in watts on the square

centimeter on the interval $\Delta\lambda = 1\,\mu$, in the third one $P(\lambda) = \frac{1}{s_0}\int I_0(\lambda)\,d\lambda$ in percent.

If one considers an eye, or a receiver, whose curve of sensitivity is close to that of an eye, the function $f(\lambda)$ appears to be of the type of γ-function, it has a sharp maximum with $\lambda = \lambda_0$. The function $f(\lambda) = I_0(\lambda)\,\Theta(\lambda)$ is figured out for $\lambda = 0{,}4 - 0{,}8\,\mu$ for the eye. Values $I_0(\lambda)$ for the corresponding λ in watts on the square meter on the interval $\Delta\lambda = 0{,}01\,\mu$, are taken from Table IV, the spectral sensitivity of an eye $\Theta(\lambda)$ – from (1). Values of function $f(\lambda)$, are brought forward in table I, and the aspect of this function is given in figure 1.

TABLE 1

$\lambda\,\mu$ . . .	0,4	0,45	0,5	0,55	0,6	0,65	0,7	0,75	0,8
$f(\lambda)$ . .	0,0061	0,836	6,40	19,4	11,4	1,73	0,059	0,0015	0,00

In this case, by using such circumstance that remaining values under the integrals in formula (1) change rather slowly, one can approximately assume that

$$K = \frac{K_0(\lambda_0)}{1 + \frac{D(\lambda_0)\,e^{\tau_0^h(\lambda_0)\sec\theta}}{r_0(\lambda_0, 0)\,B(\lambda_0)}},$$

where

$$K_0(\lambda_0) = \frac{r_0(\lambda_0, 0) - r_\phi(\lambda_0, 0)}{r_0(\lambda_0, 0)}.$$

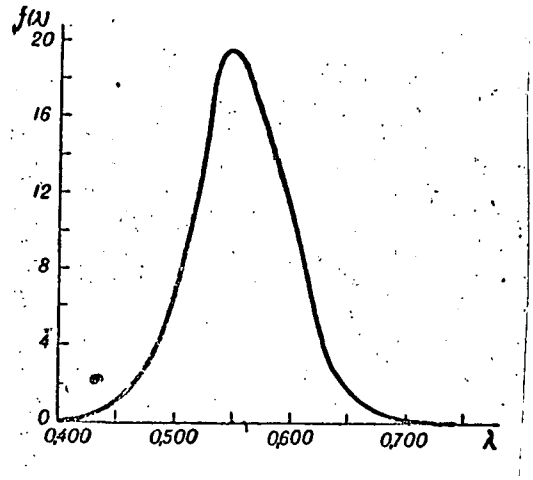

Figure 1
The Look Of Function $f(\lambda)$./

If this approximation proves to be inadequately accurate, one can recommend the chart of computation K, used previously in (1). Besides the computation of the haze is carried out in the same manner as above, i.e.

$$\int D(\lambda) f(\lambda)\, d\lambda = D(\lambda_0) \int f(\lambda)\, d\lambda,/$$

and two other integrals are computed according to λ/ numerically.

An Example Of Computation Of Inclined Visibility Range

The condition of the visibility of an object over some background is determined by the requirement $K \geqslant \varepsilon$, где ε — where is the threshold of a contrasting sensitivity of the receiver. This formula is instrumental in obtaining such distance from which the object ceases to be visible, i.e., the inclined visibility range (IVR). By using cited tables, one can figure out the inclined visibility range for any of above mentioned parameters of atmosphere according to formula (1).

In this formula we shall designate (as in (1))

$$\left.\begin{aligned} \int r_0(\lambda, 0)\, B e^{-\tau_0^h \sec\theta} f(\lambda)\, d\lambda &= F_1, \\ \int r_\phi(\lambda, 0)\, B e^{-\tau_0^h \sec\theta} f(\lambda)\, d\lambda &= F_2, \\ \int D(\lambda) f(\lambda)\, d\lambda &= F_3. \end{aligned}\right\}$$

Then

$$K = \frac{F_1 - F_2}{F_1 + F_3}.$$

Integrals F_1, F_2, F_3 — are values of the power currents, received by the transducer from the object, the background and the haze. The problem in this way leads to the computation of these three integrals.

The computation has been made for the following parameters:

$S_0 = 20$ км, $\tau_0 = 0{,}3$, $\theta = 0$, 40, 60 и 85°, $i = 40°$, $\varphi = 0°$, $z = 1$, 2 и 10 км.

The coefficients of brilliance $r_0(\lambda, \theta)$ and $r_\Phi(\lambda, \theta)$ in the expressions F_1 and F_2 should be determined in an experimental manner.

Authors have used Ye. L. Krinov's data (4) by selecting as an object a grass covered with dust (example N 161) and as a background a gray soil, podsolized (example No. 320). Not having available values $r(\lambda)$ measured with various θ the authors, with computations of IVR used $r(\lambda)$ obtained by Ye. L. Krinov with one vertical angle. This has not brought a significant error in our computations, as the object and the background can be estimated approximately orthotropic.

Data according to r_0 and r_Φ from (4) are cited in Table 2.

TABLE 2

$\lambda\varphi$	0,4	0,45	0,5	0,55	0,6	0,65	0,7	0,75
r_0	0,031	0,044	0,049	0,081	0,071	0,068	0,150	0,380
r_Φ	0,037	0,037	0,043	0,049	0,060	0,060	0,079	0,122

At first they computed the integral F_3, by using table 1. Its significance in absolute values are obtained as follows:

$$F_3 = \frac{1}{\pi} D(\lambda_0) \int f(\lambda)\, d\lambda = 63{,}4\, D(\lambda_0) \qquad \text{watt/m}^2 \text{ steradian.}$$

For more exact computations a spectral brilliance of the haze $D(\lambda)$ should be taken. For the approximate computations with such accuracy which is offered by the theory evolved in (1), one

can use the haze calculated for $\lambda_0 = 0{,}550\,\mu$. In the case under consideration the brilliance of the haze D with A = 0.2 (as the nearest for the object and background) and with corresponding significance of the above-mentioned parameters, has been used.

When integrals F_1 and F_2 have been computed the values of r_0 and r_Φ (Table 2) have been used of the auxiliary function $B(\tau_0, i)$ with A = 0.2 and with corresponding parameters (Table III), of the spectral transparency T (Table II) and functions $f(\lambda)$ (Table 1).

Obtained values of contrasts K for various heights z and sighting angles θ are cited in Table III. Distances L in kilometers, computed according to formula

$$L = \frac{z}{\cos\theta}.$$

have also been given there.

The relation of the values of contrast K to the distance L for various heights z has been presented in Figure 2 - 4. Those L, for which $K = \varepsilon = 2\%$) have been shown there by a dotted line.

Thus as far as the atmospheric condition in the case under consideration is concerned the following values of the inclined visibility range (IVR) were obtained in presence of which the observed contrast of the grass against the background of the soil is equal to the threshold one:

z км	(НДВ) км
1	8,5
2	14,9
10	57,5

With the larger L the object will not be visible.

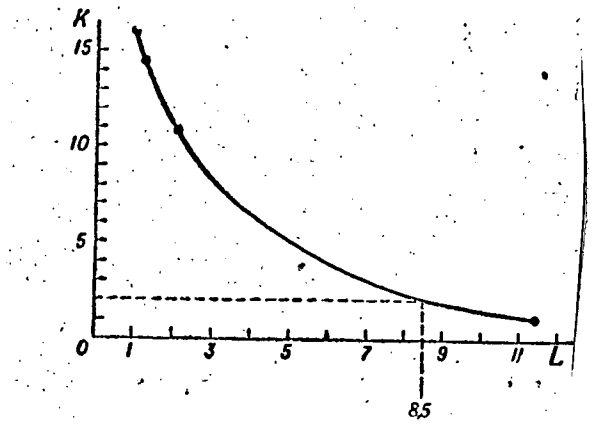

Figure 2

The Relation of Contrast K to Distance L for the Height z = 1km

TABLE 3

	z км		
	1,0	2,0	10,0
	$\theta = 0°$		
L км	1,0	2,0	10,0
K %	15,9	12,7	9,8
	$\theta = 40°$		
L км	1,31	2,61	13,1
K %	14,5	11,5	8,6
	$\theta = 60°$		
L км	2,0	4,0	20,0
K %	10,6	8,0	5,6
	$\theta = 85°$		
L км	11,5	22,9	114,5
K %	1,1	0,5	0,2

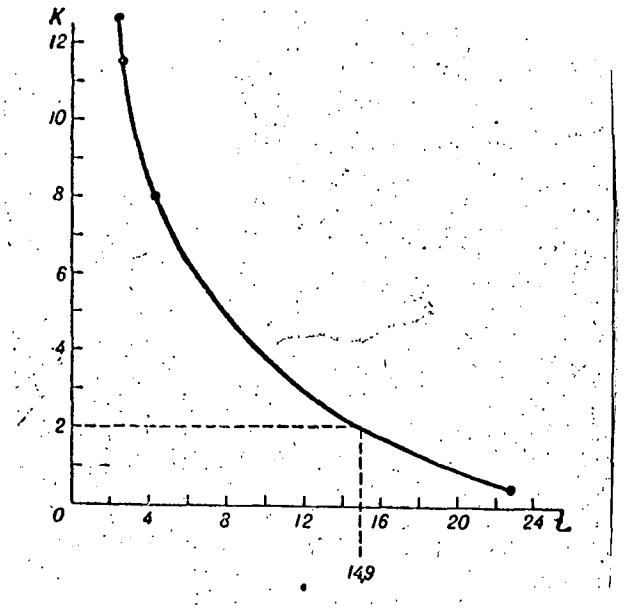

Figure 3
Relation of Contrast K to Distance L for the Height z = 2 km

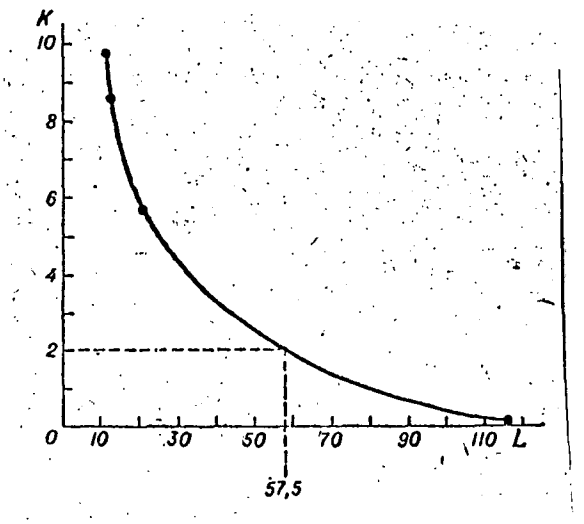

Figure 4
Relation of Contrast K to Distance L for the Height z = 10 km

The Relation of the Brilliance of Atmospheric Haze to Albedo

Tables of haze, computed here, together with the values D, published in (1), make it now possible to determine D with the error of not more than 3% for any values A - from 0 to 1. To illustrate some results in figure 5 - 6 are shown relations of D to A for one of the cases of the condition of the atmosphere (S_o = 50 km, $\tau_0 = 0{,}2$). In Figure 5 are cited curves for levels z, equal 2, 5 and 10 km, in the presence of $\Delta\varphi = 0^\circ$, $i = 60^\circ$, $\theta = 60^\circ$; in Figure 6 - curves in the presence of $\Delta\varphi = 90^\circ$, $i = 40^\circ$, $\theta = 60^\circ$ for some levels.

It is apparent from the drawings, that in all the interval A an approximate linear increase D with the increase of A takes place, especially in the interval 0 - 0.75; for $A > 0{,}75$ the increase D with the increase of value A is steeper.

In Table 4 the significance of value ΔD (with respect to the deviation from linear motion) is cited with $A = 0.5.\ 0.75$ and 1.0 in percentage. In Figure 5 and 6 the linear motion D(A) is marked by dotted lines, and the computed one by solid ones.

$$\Delta D = 100\,\frac{D(A) - D^*(A)}{D(A)}.$$

By D*(A) the linear function has been designated

$$D^*(A) = \left(\frac{dD}{dA}\right) A + D(0)$$

with slope $\left(\frac{dD}{dA}\right)$, corresponding to section $(0-0{,}20)$. Data in Table 4 are indicated for six curves (Figure 5 and 6).

TABLE 4

z км	$S_0=50$ $\tau_0=0{,}2$ $\Delta\varphi=0°$ $i=60°$ $\theta=60°$			$S_0=50$ $\tau_0=0{,}2$ $\Delta\varphi=90°$ $i=40°$ $\theta=60°$		
	$A=0{,}5$	$A=0{,}75$	$A=1{,}0$	$A=0{,}5$	$A=0{,}75$	$A=1{,}0$
2	6,3	7,7	14,4	7,1	9,8	12,2
5	4,2	7,8	13,4	9,6	12,7	15,7
10	2,7	7,2	12,5	10,0	13,1	18,7

The indicated conformity to the established rule is approximately observed also in other investigated cases of the state of atmosphere. Thus, with rough estimates the function (D(A)) for any values of A can be replaced by a straight line. Tables are indispensable for accurate computations.

The linear motion of function D(A) approximately signifies in the mentioned field A, that the derivative $\frac{dD}{dA}$ is almost permanent here. This can be also directly seen from the formula for $\frac{dD}{dA}$

$$\frac{dD}{dA}=\frac{2R\cos i}{[4+(3-X_1)(1-A)\tau_0]^2}\left[2\left(1-e^{-(\tau_0-\tau)\sec\theta}\right)+\right.$$
$$\left.+3\cos\theta\left(1-e^{-(\tau_0-\tau)\sec\theta}\right)+(3-X_1)\left(\tau-\tau_0 e^{-(\tau_0-\tau)\sec\theta}\right)\right].$$

In the investigated field of the values of parameters the right side (2) changes a little.

Some errors slipped in previously published tables:

1. V.V. Slolev. About the dispersion of light in the atmospheres of Earth and planets. Schdarly notes of the Liningrad State University, series of mathematical sciences, issue 18, 1949.

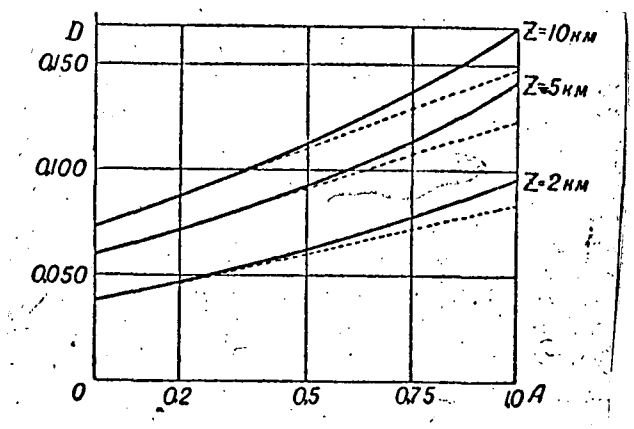

Figure 5

Relation of the Brilliance of Atmospheric Haze to Albedo for Various Heights with $S_0 = 50$ км; $\tau_0 = 0{,}2$; $\Delta\varphi = 0°$; $i = 60°$; $\theta = 60°$.

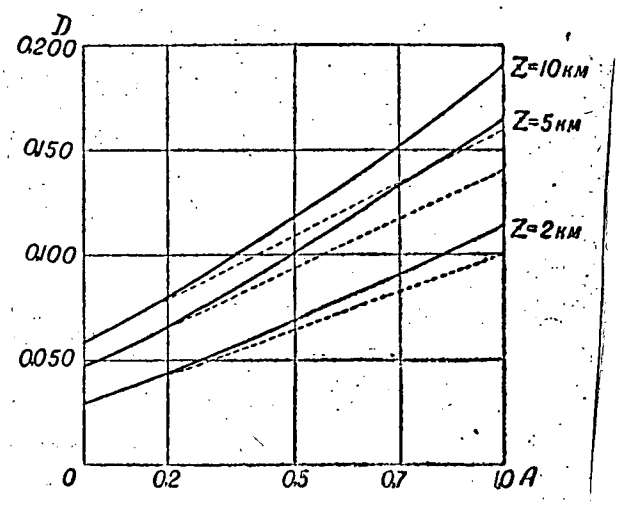

Figure 6

Relation of the Brilliance of Atmospheric Haze to Albedo for Various Heights with $S_0 = 50$ км; $\tau_0 = 0{,}2$; $\Delta\varphi = 90°$; $i = 40°$; $\theta = 60°$.

In Table 1 $(i = 60°,\ \tau_0 = 0{,}1)$ instead of 1.955 should be 1.955.

I.K.S. Shifrin, I.N. Minin. The theory of nonhorizontal visibility. Transactions of the GGO, issue 68, 1957.

In the supplement 3 (Table D_o):

a) at the tip instead of τ it should read τ_0 and instead of τ_0 it should read $q = \frac{\tau}{\tau_0}$;

b) the value D_o with $\theta = 20°$, $i = 40°$, $\tau_0 = 0{,}3$, $q = 0{,}8$ instead of 0.0199 it should be 0.0109;

c) with $\theta = 20°$, $i = 20°$, $\tau_0 = 0{,}2$, $q = 0{,}4$ instead of 0.0202 it should be 0.0263.

In the supplement 4 (Table D) on page 62 instead of $S_0 = 0{,}3$, $\tau_0 = 10$ it should read $S_0 = 10$, $\tau_0 = 0{,}3$.

Aside from this, some data in Tables D (supplement 4) should be corrected in a following manner:

Page	Case						Printed	Should be
	S_0	τ_0	i	θ	$\Delta\varphi$	z	—	
50	50	0,2	20	85	0	0,5	0,0899	0,106
						1,0	0,146	0,162
						1,5	0,182	0,196
						2,0	0,204	0,218
						2,5	0,218	0,229
						3,0	0,231	0,241
				85	90	0,5	0,0819	0,0973
						1,0	0,132	0,147
						1,5	0,165	0,179
						2,0	0,178	0,198
						2,5	0,184	0,203
						3,0	0,188	0,218
				85	180	0,5	0,0842	0,100
						1,0	0,137	0,141
						1,5	0,170	0,184
						2,0	0,190	0,204
						2,5	0,204	0,214
						3,0	0,216	0,225

Page	Case						Printed	Should be
	S_0	τ_0	i	θ	$\Delta\varphi$	z		
			40	60	0	0,5	0,0156	0,0215
					90	0,5	0,0151	0,0210
51	50	0,2	40	60	180	0,5	0,0194	0,0253
			80	85	0	1,5	0,275	0,294
52	50	0,2	80	85	90	1,5	0,0523	0,0623
					180	1,5	0,0917	0,104
52	20	0,2	20	0	0	0,5	0,0111	0,0197
					90	0,5	0,0111	0,0197
					180	0,5	0,0111	0,0197
53	20	0,2	60	40	0	0,5	0,0282	0,0171
						1,0	0,0318	0,0251
						1,5	0,0594	0,0292
						2,0	0,0522	0,0313
						2,5	0,0552	0,0331
						3,0	0,0573	0,0345
						5,0	0,0636	0,0382
						7,5	0,0692	0,0415
						10,0	0,0740	0,0428
53	20	0,2	60	40	90	1,0	0,0232	0,0242
						1,5	0,0387	0,0282
						10,0	0,0423	0,0412
54	20	0,2	60	40	180	1,0	0,0291	0,0301
						1,5	0,0456	0,0351
						2,0	0,0375	0,0387
						10,0	0,0531	0,0520
			80	85	0	1,0	0,382	0,333
					90	0,5	0,0545	0,0523
					90	1,0	0,0716	0,0668
					180	1,0	0,0976	0,0886
57	50	0,3	20	0	0	7,5	0,0744	0,0829
					90	7,5	0,0744	0,0829
					180	7,5	0,0744	0,0829
				40	0	1,0	0,0265	0,0242
					90	1,0	0,0243	0,0283
					180	1,0	0,0313	0,0290
				60	0	3,0	0,0726	0,0797
					90	3,0	0,0749	0,0857
					180	3,0	0,0814	0,0885
57	50	0,3	20	85	0	0,5	0,0844	0,112
					90	0,5	0,0730	0,102
					180	0,5	0,0791	0,105
			40	85	0	0,5	0,0888	0,122
						1,0	0,170	0,188
						1,5	0,209	0,234
						2,0	0,242	0,263
						2,5	0,270	0,285
						3,0	0,289	0,298

Page	Case						Printed	Should be
	S_0	τ_0	l	θ	$\Delta\varphi$	z		
58	50	0,3	40	85	90	0,5	0,0658	0,0951
						1,0	0,127	0,139
						1,5	0,155	0,174
						2,0	0,178	0,194
						2,5	0,198	0,210
						3,0	0,209	0,218
					180	0,5	0,0719	0,102
						1,0	0,138	0,152
						1,5	0,169	0,189
						2,0	0,195	0,212
						2,5	0,218	0,230
						3,0	0,234	0,239
59	50	0,3	80	0	0	10,0	0,0228	0,0242
					90	10,0	0,0228	0,0242
					180	10,0	0,0228	0,0242
59	50	0,3	80	85	0	3,0	0,293	0,316
					90	3,0	0,0915	0,0853
					180	3,0	0,123	0,132
61	20	0,3	80	85	0	0,5	0,214	0,251
						1,0	0,340	0,435
						1,5	0,427	0,541
						2,0	0,490	0,645
						2,5	0,546	0,721
						3,0	0,583	0,771
						5,0	0,670	0,895
						7,5	0,735	0,973
						10,0	0,777	1,029
					90	0,5	0,0549	0,0565
						1,0	0,0793	0,0864
						1,5	0,0930	0,103
						2,0	0,102	0,116
						2,5	0,110	0,126
						3,0	0,114	0,133
						5,0	0,125	0,148
						7,5	0,133	0,158
						10,0	0,138	0,165
					180	0,5	0,0716	0,0771
						1,0	0,107	0,123
						1,5	0,128	0,149
						2,0	0,143	0,172
						2,5	0,155	0,189
						3,0	0,163	0,200
						5,0	0,182	0,227
						7,5	0,196	0,244
						10,0	0,205	0,256

Page	Case						Printed	Should be
	S_0	τ_0	l	θ	$\Delta\varphi$	z		
64	10	0,5	20	85	0	0,5	0,281	0,248
						1,0	0,313	0,286
						1,5	0,314	0,292
						2,0	0,311	0,286
						2,5	0,309	0,284
						3,0	0,306	0,278
						5,0	0,299	0,273
						7,5	0,295	0,269
						10,0	0,293	0,266
					90	0,5	0,262	0,227
						1,0	0,289	0,263
						1,5	0,287	0,266
						2,0	0,284	0,260
						2,5	0,281	0,256
						3,0	0,278	0,250
						5,0	0,269	0,243
						7,5	0,264	0,238
						10,0	0,262	0,236
64	10	0,5	20	85	180	0,5	0,266	0,224
						1,0	0,286	0,260
						1,5	0,284	0,262
						2,0	0,280	0,255
						2,5	0,277	0,252
						3,0	0,273	0,245
						5,0	0,264	0,238
						7,5	0,260	0,234
						10,0	0,257	0,231
65	10	0,5	40	60	0	0,5	0,0561	0,0839
						1,0	0,0953	0,127
						1,5	0,113	0,151
						2,0	0,123	0,165
						2,5	0,130	0,175
						3,0	0,137	0,181
						5,0	0,153	0,193
						7,5	0,160	0,198
						10,0	0,165	0,202
					90	0,5	0,0523	0,0801
						1,0	0,0890	0,120
						1,5	0,106	0,143
						2,0	0,115	0,156
						2,5	0,121	0,166
						3,0	0,124	0,171
						5,0	0,142	0,182
						7,5	0,149	0,187
						10,0	0,153	0,190
					180	0,5	0,0558	0,0835
						1,0	0,0947	0,126
						1,5	0,112	0,150
						2,0	0,122	0,164
						2,5	0,130	0,174
						3,0	0,136	0,180
						5,0	0,152	0,192
						7,5	0,159	0,196
						10,0	0,164	0,200

Page	Case						Printed	Should be
	S_0	τ_0	i	θ	$\Delta\varphi$	z		
65	10	0,5	40	40	0	0,5	0,0387	0,0440
						1,0	0,0608	0,0691
						1,5	0,0742	0,0851
						2,0	0,0824	0,0950
						2,5	0,0884	0,101
						3,0	0,0917	0,106
						5,0	0,0983	0,114
						7,5	0,101	0,117
						10,0	0,103	0,120
65	10	0,5	40	40	90	0,5	0,0388	0,0441
						1,0	0,0608	0,0691
						1,5	0,0742	0,0851
						2,0	0,0825	0,0951
						2,5	0,0885	0,102
						3,0	0,0918	0,106
						5,0	0,0981	0,114
						7,5	0,101	0,117
						10,0	0,104	0,120
					180	0,5	0,0415	0,0468
						1,0	0,0654	0,0737
						1,5	0,0800	0,0909
						2,0	0,0890	0,102
						2,5	0,0956	0,109
						3,0	0,0992	0,113
						5,0	0,106	0,122
						7,5	0,110	0,126
						10,0	0,112	0,128
				20	0	0,5	0,0223	0,0263
						1,0	0,0373	0,0442
						1,5	0,0468	0,0562
						2,0	0,0541	0,0656
						2,5	0,0588	0,0717
						3,0	0,0616	0,0757
						5,0	0,0670	0,0840
						7,5	0,0694	0,0863
						10,0	0,0709	0,0883
65	10	0,5	60	20	90	0,5	0,0222	0,0262
						1,0	0,0371	0,0440
						1,5	0,0466	0,0560
						2,0	0,0538	0,0653
						2,5	0,0585	0,0714
						3,0	0,0613	0,0754
						5,0	0,0666	0,0836
						7,5	0,0690	0,0855
						10,0	0,0706	0,0874
66	10	0,5	60	20	180	0,5	0,0229	0,0269
						1,0	0,0383	0,0452
						1,5	0,0481	0,0575
						2,0	0,0557	0,0672
						2,5	0,0605	0,0734
						3,0	0,0633	0,0774
						5,0	0,0689	0,0849
						7,5	0,0715	0,0880
						10,0	0,0731	0,0899
			80	85	0	0,5	0,168	0,390
						1,0	0,377	0,431
						2,0	0,672	0,652

Page	Case						Printed	Should be
	S_0	τ_0	l	θ	$\Delta\varphi$	z		
66	10	0,5	80	85	0	2,5	0,768	0,752
						3,0	0,854	0,836
						5,0	1,052	1,008
						7,5	1,148	1,100
						10,0	1,205	1,161
					90	0,5	0,0524	0,0584
						2,0	0,0786	0,0817
						2,5	0,0804	0,0863
						3,0	0,0839	0,0900
						5,0	0,0861	0,0974
					180	0,5	0,0536	0,0618
						1,5	0,0831	0,0805
						2,0	0,0907	0,0875
						2,5	0,0960	0,0931
						3,0	0,101	0,0976
65	10	0,5	60	0	0	0,5	0,0208	0,0242
						1,0	0,0333	0,0383
						1,5	0,0439	0,0512
						2,0	0,0499	0,0585
						2,5	0,0542	0,0636
						3,0	0,0568	0,0670
						5,0	0,0623	0,0744
						7,5	0,0650	0,0779
						10,0	0,0668	0,0799
					90	0,5	0,0208	0,0242
						1,0	0,0333	0,0383
						1,5	0,0439	0,0512
						2,0	0,0499	0,0585
						2,5	0,0542	0,0636
						3,0	0,0568	0,0670
						5,0	0,0623	0,0744
						7,5	0,0650	0,0779
						10,0	0,0668	0,0799
66	10	0,5	60	0	180	0,5	0,0208	0,0242
						1,0	0,0333	0,0383
						1,5	0,0439	0,0512
						2,0	0,0499	0,0585
						2,5	0,0542	0,0636
						3,0	0,0568	0,0670
						5,0	0,0623	0,0744
						7,5	0,0650	0,0779
						10,0	0,0668	0,0799
67	4	0,5	60	60	0	0,5	0,0930	0,105
						1,0	0,119	0,135
						1,5	0,128	0,145
						2,0	0,131	0,149
						2,5	0,133	0,152

Page	Case						Printed	Should be
	S_0	τ_0	i	θ	$\Delta\varphi$	z		
68	4	0,5	60	60	0	3,0	0,135	0,153
						5,0	0,139	0,158
						7,5	0,142	0,163
						10,0	0,145	0,166
					90	0,5	0,0688	0,0807
						1,0	0,0860	0,102
						1,5	0,0915	0,109
						2,0	0,0935	0,112
						2,5	0,0947	0,113
						3,0	0,0955	0,114
						5,0	0,0979	0,118
						7,5	0,0995	0,120
						10,5	0,102	0,122
					180	0,5	0,0777	0,0896
						1,0	0,0982	0,114
						1,5	0,104	0,122
						2,0	0,107	0,125
						2,5	0,109	0,127
						3,0	0,110	0,128
						5,0	0,113	0,132
						7,5	0,115	0,136
						10,0	0,118	0,138

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4. Krinov Ye. L. Spectral reflecting power of natural formations. Reports of the Academy of Sciences of the USSR, 1947.
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TABLE III

AUXILIARY FUNCTION $B(\tau_0, i)$

A	i	S_0							
		50		20		10			4
		τ_0							
		0,2	0,3	0,2	0,3	0,2	0,3	0,5	0,5
0	20	0,880	0,864	0,884	0,874	0,886	0,888	0,878	0,896
	40	0,700	0,684	0,706	0,692	0,706	0,702	0,684	0,698
	60	0,432	0,412	0,434	0,416	0,436	0,424	0,396	0,406
	80	0,117	0,105	0,117	0,106	0,118	0,108	0,0974	0,0994
0,2	20	0,898	0,888	0,902	0,895	0,902	0,908	0,902	0,918
	40	0,715	0,702	0,718	0,708	0,720	0,718	0,702	0,712
	60	0,442	0,420	0,442	0,425	0,445	0,430	0,408	0,415
	80	0,119	0,108	0,120	0,109	0,120	0,110	0,100	0,102
0,5	20	0,924	0,924	0,928	0,932	0,928	0,936	0,944	0,956
	40	0,736	0,728	0,740	0,732	0,740	0,740	0,736	0,744
	60	0,456	0,440	0,456	0,444	0,456	0,448	0,424	0,428
	80	0,122	0,112	0,122	0,113	0,123	0,114	0,105	0,106
0,75	20	0,952	0,960	0,952	0,960	0,952	0,968	0,976	0,984
	40	0,758	0,756	0,759	0,758	0,759	0,762	0,762	0,766
	60	0,466	0,454	0,466	0,456	0,467	0,458	0,442	0,445
	80	0,126	0,116	0,126	0,117	0,126	0,117	0,108	0,109
1,0	20	0,977	0,993	0,977	0,993	0,977	0,993	1,019	1,019
	40	0,779	0,784	0,779	0,784	0,779	0,784	0,794	0,794
	60	0,480	0,472	0,480	0,472	0,480	0,472	0,460	0,460
	80	0,130	0,121	0,130	0,121	0,130	0,121	0,113	0,113

TABLE IV

SOLAR SPECTRUM ON THE ATMOSPHERE'S BOUNDARY LINE

λ	$I_0(\lambda)$	$P(\lambda)$	λ	$I_0(\lambda)$	$P(\lambda)$	λ	$I_0(\lambda)$	$P(\lambda)$
0,22	0,0030	0,02	0,455	0,219	16,7	0,85	0,1003	61,7
0,225	0,0042	0,03	0,46	0,216	17,5	0,90	0,0895	65,1
0,23	0,0052	0,05	0,465	0,215	18,2	0,95	0,0803	68,1
0,235	0,0054	0,07	0,47	0,217	19,0	1,0	0,0725	70,9
0,24	0,0058	0,09	0,475	0,220	19,8	1,1	0,0606	75,7
0,245	0,0064	0,11	0,48	0,216	20,6	1,2	0,0501	79,6
0,25	0,0064	0,13	0,485	0,203	21,3	1,3	0,0406	82,9
0,255	0,010	0,16	0,49	0,199	22,0	1,4	0,0328	85,5
0,26	0,013	0,20	0,495	0,204	22,8	1,5	0,0267	87,6
0,265	0,020	0,27	0,50	0,198	23,5	1,6	0,0220	89,4
0,27	0,025	0,34	0,505	0,197	24,2	1,7	0,0182	90,83
0,275	0,025	0,43	0,51	0,196	24,9	1,8	0,0152	92,03
0,28	0,024	0,51	0,515	0,189	25,6	1,9	0,01274	93,02
0,285	0,034	0,62	0,52	0,187	26,3	2,0	0,01079	93,87
0,29	0,052	0,77	0,525	0,192	26,9	2,1	0,00917	94,58
0,295	0,063	0,98	0,53	0,195	27,6	2,2	0,00785	95,20
0,30	0,061	1,23	0,535	0,197	28,3	2,3	0,00676	95,71
0,305	0,067	1,43	0,54	0,198	29,0	2,4	0,00585	96,18
0,31	0,076	1,69	0,545	0,198	29,8	2,5	0,00509	96,57
0,315	0,082	1,97	0,55	0,195	30,5	2,6	0,00445	96,90
0,32	0,085	2,26	0,555	0,192	31,2	2,7	0,00390	97,21
0,325	0,102	2,60	0,56	0,190	31,8	2,8	0,00343	97,47
0,33	0,115	3,02	0,565	0,189	32,5	2,9	0,00303	97,72
0,335	0,111	3,40	0,57	0,187	33,2	3,0	0,00268	97,90
0,34	0,111	3,80	0,575	0,187	33,9	3,1	0,00230	98,08
0,345	0,117	4,21	0,58	0,187	34,5	3,2	0,00214	98,24
0,35	0,118	4,63	0,585	0,185	35,2	3,3	0,00191	98,39
0,355	0,116	5,04	0,59	0,184	35,9	3,4	0,00171	98,52
0,36	0,116	5,47	0,595	0,183	36,5	3,5	0,00153	98,63
0,365	0,129	5,89	0,60	0,181	37,2	3,6	0,00139	98,74
0,37	0,133	6,36	0,61	0,177	38,4	3,7	0,00126	98,83
0,375	0,132	6,84	0,62	0,174	39,7	3,8	0,00114	98,91
0,38	0,123	7,29	0,63	0,170	40,9	3,9	0,00103	98,99
0,385	0,115	7,72	0,64	0,166	42,1	4,0	0,00095	99,05
0,39	0,112	8,13	0,65	0,162	43,3	4,1	0,00087	99,13
0,395	0,120	8,54	0,66	0,159	44,5	4,2	0,00080	99,18
0,40	0,154	9,03	0,67	0,155	45,6	4,3	0,00073	99,23
0,405	0,188	9,65	0,68	0,151	46,7	4,4	0,00067	99,29
0,41	0,194	10,3	0,69	0,148	47,8	4,5	0,00061	99,33
0,415	0,192	11,0	0,70	0,144	48,8	4,6	0,00056	99,38
0,42	0,192	11,7	0,71	0,141	49,8	4,7	0,00051	99,41
0,425	0,189	12,4	0,72	0,137	50,8	4,8	0,00048	99,45
0,43	0,178	13,0	0,73	0,134	51,8	4,9	0,00044	99,48
0,435	0,182	13,7	0,74	0,130	52,7	5,0	0,00042	99,51
0,44	0,203	14,4	0,75	0,127	53,7	6,0	0,00021	99,74
0,445	0,215	15,1	0,80	0,1127	57,9	7,0	0,00012	99,86
0,45	0,220	15,9						

PART II

TABLES OF THE BRILLIANCE OF DAYTIME SKY

1. Chart of the computation and description of tables.

The knowledge of the distribution of brilliance over the canopy of the sky is required for many practical problems in the field of meteorology, illumination engineering, biophysics (for example in computing the arrival of radiation dispersion on the slopes, for the computation of the illumination of buildings and for a number of other problems). Experimental investigation of this matter is connected with great difficulties due to the labor-consuming surveys and their duration (1 - 5). Theoretical computations of the distribution of brilliance over the canopy of sky, based on the theory of light dispersion in the atmosphere, considerably simplify the problem (5). However, a rigid theory of light dispersion in real atmosphere is complicated and awkward.

Computations can be carried out in a much simpler way for the conditions of the Rayleigh atmosphere, though their results in majority of cases are for away from reality. By not concentrating over numerous theoretical tasks pertaining to the light dispersion in the real atmosphere, we shall demonstrate that the most convenient is the method of approximation of V.V. Sobolev (6).

V.V. Sobolev obtained formulae, determining the coefficients of brilliance of the flat layer of haze enrivonment with any

of its optical thickness, with any form of dispersion index and with any reflectance of the surface adjacent to the layer.

The computation of the distribution of brilliance over the daytime, cloudless sky has been carried out according to V.V. Sobolev's method. Formula for the coefficient of brilliance, developed in (6), has the form of

$$\sigma(\theta, i, \varphi) = \frac{[(1-A)R(\tau_0, \theta) + 2A]R(\tau_0, i)}{4 + (3 - X_1)(1-A)\tau_0} - \frac{1}{2}\left(e^{-\tau_0 \sec\theta} + e^{-\tau_0 \sec i}\right) + \\ + [X(\gamma) - (3 + X_1)\cos\theta\cos i]\,\sigma_0(\theta, i). \qquad (1)$$

Here A - is albedo of the underlying surface (lambert reflection is assumed) τ_0 is the optical thickness of the atmosphere or the common attenuation factor of the atmosphere in the vertical direction, i - is the zenith distance of the sun, 0 - is the zenith distance of the investigated point of the sky, R - is the function determined by a formula

$$R(\tau_0, i) = 1 + \frac{3}{2}\cos i + \left(1 - \frac{3}{2}\cos i\right)e^{-\tau_0 \sec i},$$

X (γ) is the dispersion index, that expresses the probability of the radiation dispersion by the elimentary volume of surroundings at an angle γ to the falling rays.

The dispersion index is assumed to be standardized, i.e.,

$$\frac{1}{2}\int_0^{\pi} X(\gamma)\sin\gamma\, d\gamma = 1.$$

The angle γ can be found according to formula

$$\cos\gamma = \cos\theta\cos i + \sin\theta\sin i\cos\varphi, \qquad (2)$$

φ is azimuth, X, - is the value, conveyed in the formula

$$X_1 = \frac{3}{2}\int_0^{\pi} X(\gamma)\sin\gamma\cos\gamma\, d\gamma,$$

$\sigma_0(\theta, i)$ — is the coefficient of brightness for the spherical dispersion index stipulated by dispersion of the first order.

$$\sigma_0(\theta, i) = \frac{1}{4}\,\frac{e^{-\tau_0 \sec i} - e^{-\tau_0 \sec\theta}}{\cos i - \cos\theta}.$$

The coefficient of brightness $\sigma(\theta, i, \varphi)$ characterizes distribution of brightness over the cloudless sky, in relation to the sun's elevation over the horizon, to the transparency of the atmosphere, to the form of dispersion index and to the albedo of Earth's surface.

Computations have been conducted for $\lambda = 0{,}55\,\mu$ for the following values of initial parameters:

1) θ from 0 to 90 through 10°
2) i from 0 to 90 through 10°
3) A from 0 to 1 through 0.1
4) S_0 and τ_0 for same eight cases of atmosphere condition as the brightness of the haze (Part 1).

Function $R(\tau_0, i)$ has been tabulated in (6), and authors used its values in their tabulations. Medium dispersion indices $X(\gamma)$ for γ from 0 to 180 through 10° for the eight conditions of the atmosphere have been cited in the work (7). By proceeding from the data of table 13 (7), the authors computed $X(\gamma)$ for all γ from 0 to 180° through 1° (Table VII).

Values σ_0 corresponding to assigned valuations τ_0 and i (or 0), are cited in the work (6). Here, tables σ_0 (for τ_0 equal to 0.2, 0.3, 0.5) are reproduced indispensable (also values $X(\gamma)$ with the computation of σ for other azimuths (Table IV).

Coefficients of brilliance σ have been computed for three azimuths $\varphi: 0, 90, 180^{\circ}$.

Angles γ in these case, according to formula (2) have been computed like this:

1) if $\varphi = 0^{\circ}$ (we look at the sun) so

$$\gamma = \theta - i \text{ with } \theta > i$$
$$\gamma = i - \theta \text{ with } i > \theta$$

2) if $\varphi = 90^{\circ}$, so $\cos\gamma = \cos\theta \cos i$

3) if $\varphi = 180^{\circ}$ (we look away from the sun, the sun is behind our backs, so

$$\gamma = \theta + i.$$

Computed coefficients of brilliance for corresponding parameters in comparative units have been presented in Table V. Coefficients of brilliance of any point on the canopy of

the sky in relation to the zenith can be obtained from the Table (for this, values σ with various θ and i under consideration should be divided by σ with $\theta = 0°$ and with the same i).

One can see from the Table V that:

1) with the increase of albedo the coefficients of brilliance rise in a monotone function for all investigated cases of atmospheric condition;

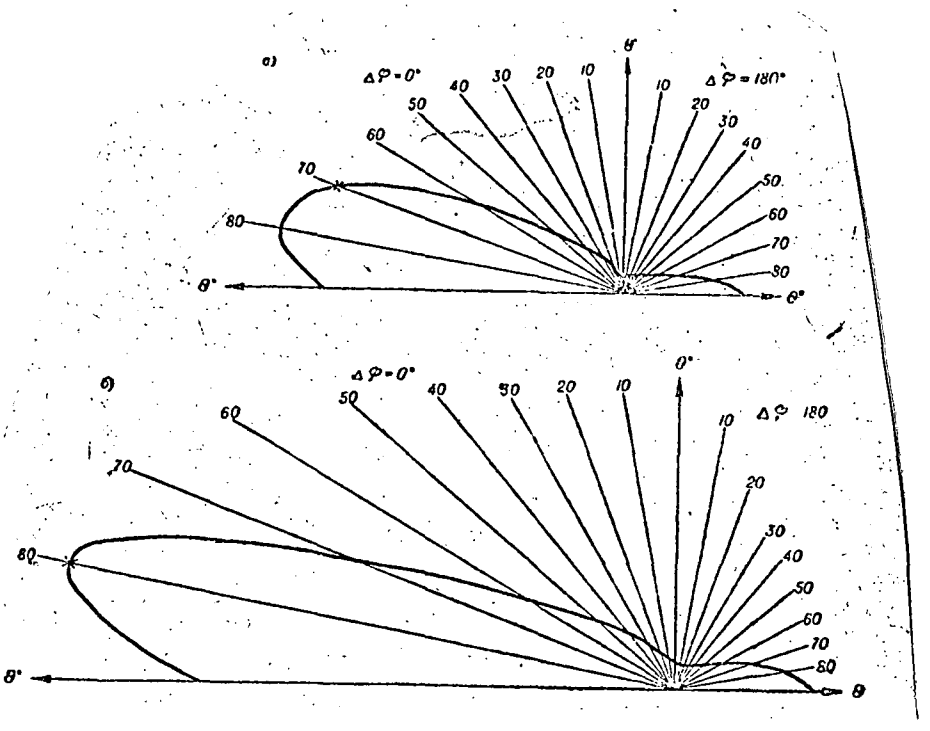

Figure 7

Distribution of Brilliance Over the Cloudless Sky in the Vertical and countervertical of the sum for winter conditions
($S_0 = 20$ км, $\tau_0 = 0{,}2$, $A = 0{,}7$).
а) $i = 70°$; б) $i = 80°$.

2) in all investigated cases two maximums of brilliance near the sun and at the horizon are distinctly observed;

3) with the identical τ_0 and the decreasing S_o and vice versa, with identical S_o, but increasing τ_0 the coefficients of brilliance increase for all φ and A;

4) with i = 0 coefficients of brilliance, with 0, A, S_o τ_0 under consideration coincide for all azimuths φ i.e., isophots in this case will serve as concentric circles, parallel to the horizon's line.

Diagrams and isophots constructed according to computed values σ may offer more descriptive presentation about the distribution of brilliance over the cloudless sky. As an example we shall cite figures 7a and b and 8a and b, where the distribution of brilliance over the cloudless sky has been given for two azimuths $\varphi = 0$ and 180° (in the vertical and countervertical of the sun) with the atmospheric condition S_o = 20km, $\tau_0 = 0{,}2$ under winter conditions (A = 0.7, i = 70 and 80°) and under summer conditions (A = 0.2, i = 40 and 60°).

In figure 8a and b, the scale has been increased twice.

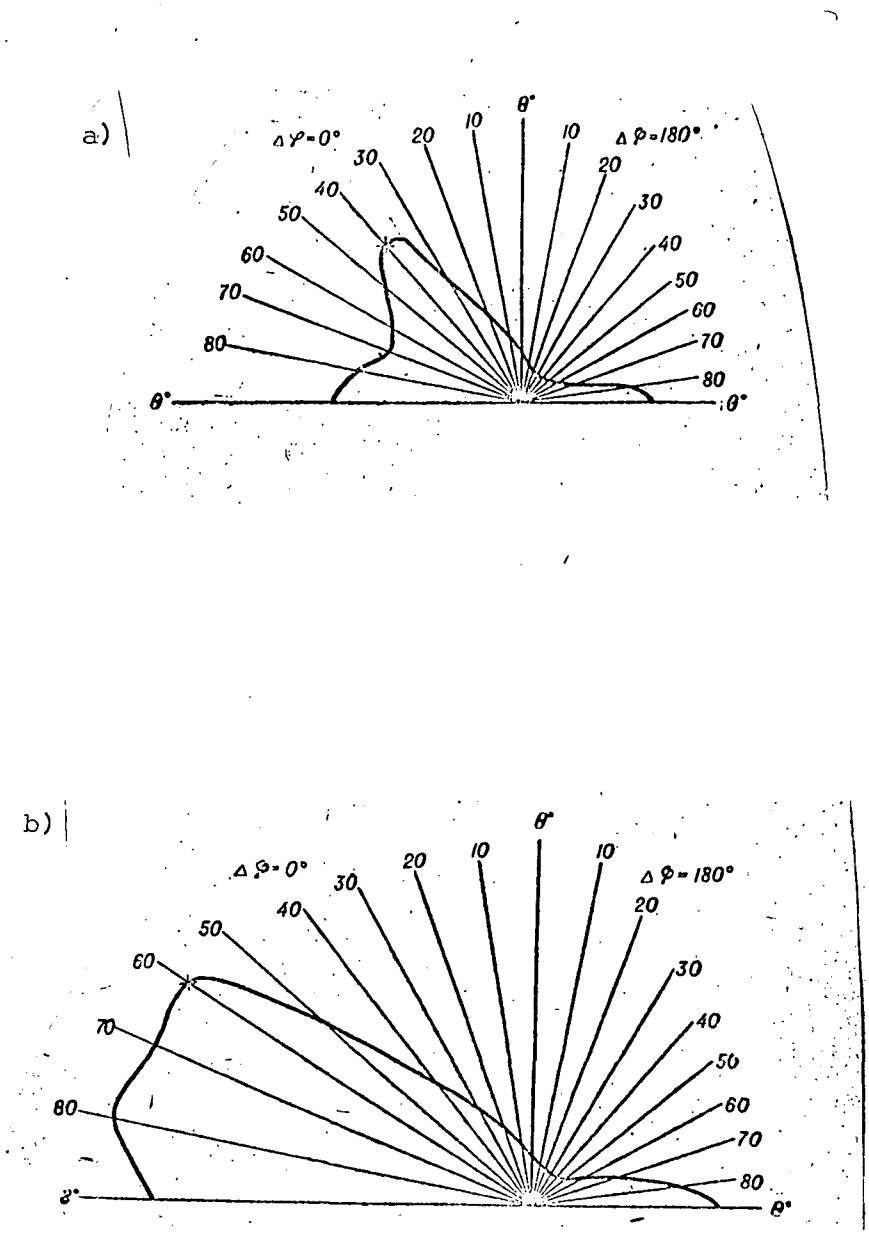

Figure 8

Distribution of brilliance over the cloudless sky in the vertical and countervertical of the sun, under summer conditions ($S_0 = 20$ км, $\tau_0 = 0{,}2$, $A = 0{,}2$). a) $i = 40°$, б) $i = 60°$.

The presented drawings distinctly confirm the deductions made about the character of the distribution of brilliance over the cloudless sky.

The Example Pertaining to the Computation of Brilliance of Daytime-Sky for Various Azimuths

In order to construct the isophots of brilliance, it is necessary to know the coefficients of brilliance for various azimuths. Presented tables can be used for computing coefficients of brilliance σ for any φ. From the formula (1) it can be seen, that only dispersion index $X(\gamma)$ depends on the azimuth φ.

Therefore for the computation of σ' for any φ the following formula can be used.

$$\sigma' = \sigma - \sigma_0 [X(\gamma) - X(\gamma)'], \qquad (3)$$

where σ' is the desired coefficient of brilliance with the parameters θ, i, S_0, τ_0. A under consideration for any φ, $X(\gamma)'$ — the dispersion index for this φ, σ_0 is the coefficient of brilliance with the spherical dispersion index σ; is the computed coefficient of brilliance with φ equal to 0.90 or 180°.

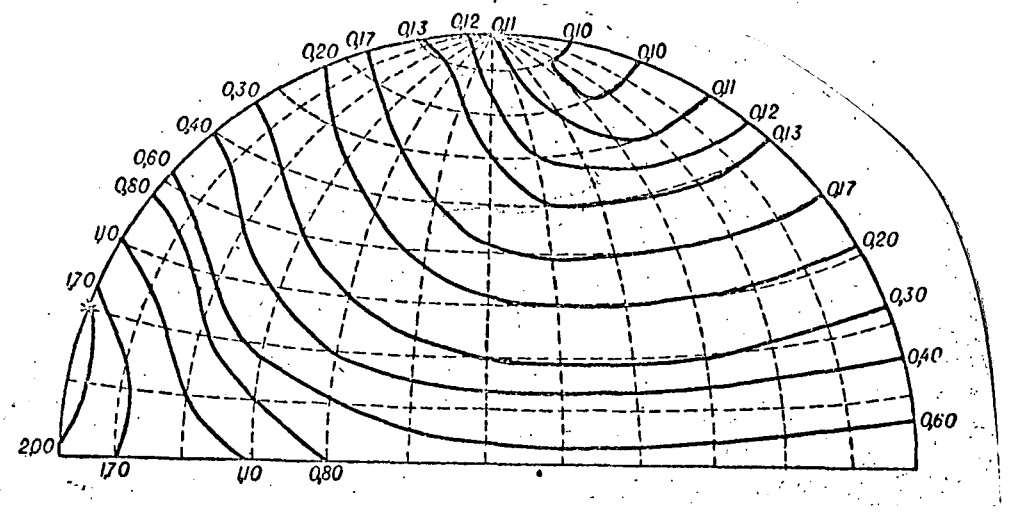

FIGURE 9

Isophots of Brilliance of Cloudless Sky $S_0 = 20$ км, $\tau_0 = 0{,}2$, $A = 0{,}7$, $i = 70°$.

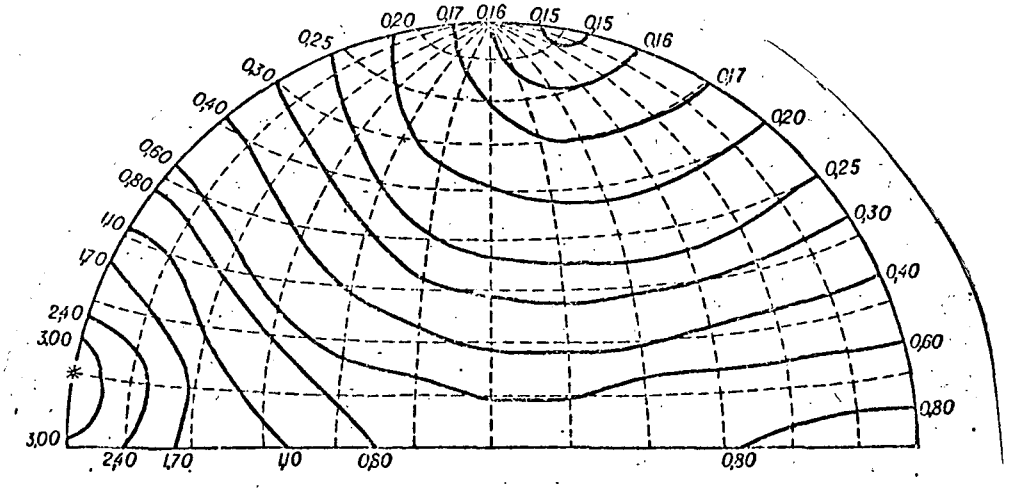

FIGURE 10

Isophots of Brilliance of Cloudless Sky $S_0 = 20$ км, $\tau_0 = 0{,}2$, $A = 0{,}7$, $i = 80°$.

Values σ_0 and $X(\gamma)$ are presented in Table VI and VII.

Significance of the angles of dispersion γ computed by us for all i, 0 (from 0 to 90° through 10°) and φ (from 0 to 180° through 15°) is presented in Table VIII.

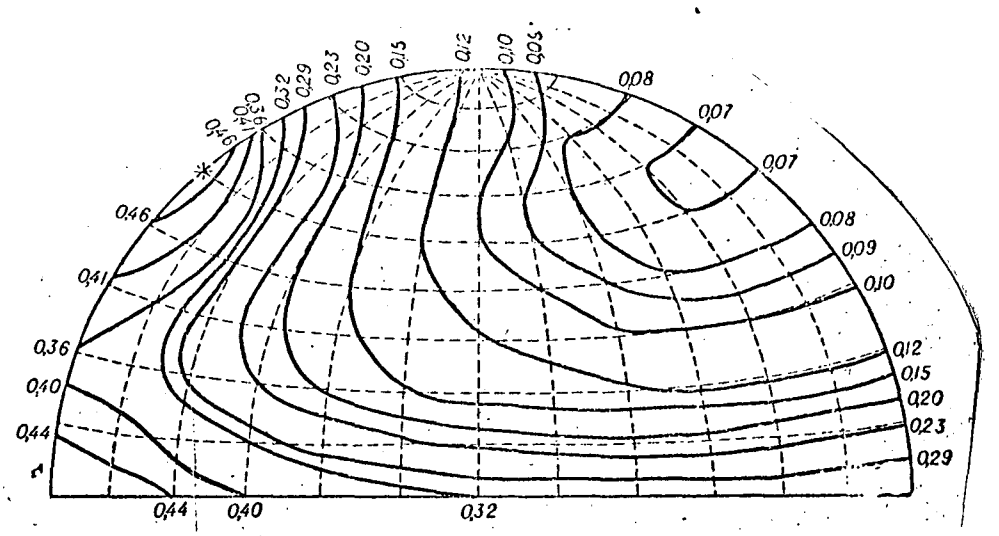

FIGURE 11

Isophots of Brilliance of Cloudless Sky $S_0 = 20$ км, $\tau_0 = 0{,}2$, $A = 0{,}2$, $i = 40°$.

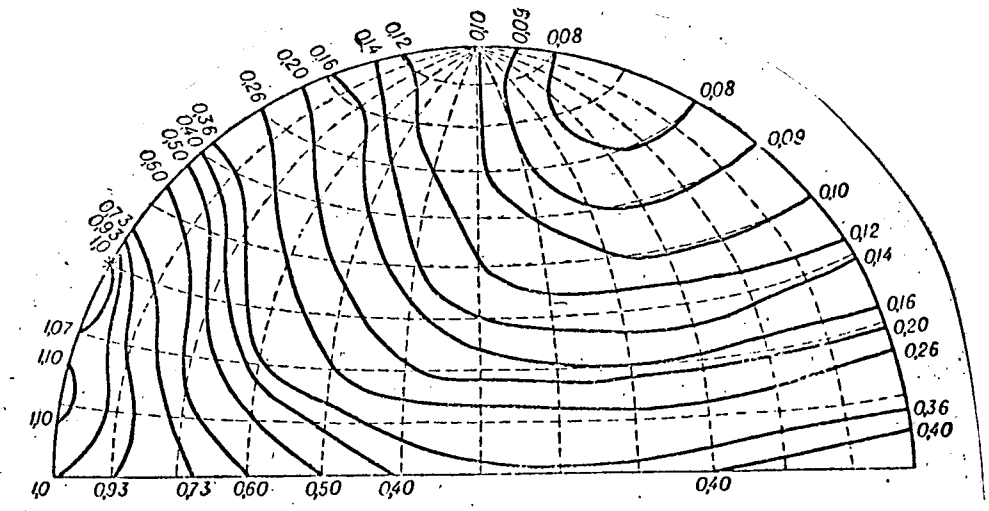

FIGURE 12

Isophots of Brilliance of Cloudless Sky $S_0 = 20$ км, $\tau_0 = 0{,}2$, $A = 0{,}2$, $i = 60°$.

for $\varphi > 180°$ the coefficients of brilliance are not computed as

$$\sigma_{180+\alpha} = \sigma_{180-\alpha}.$$

As an example for the computation of coefficients of brilliance for various azimuths σ' have been computed according to formula (3) for cases similar to these in Figure 7 and 8.

Values $X(\gamma)$ and $X(\gamma)'$ were taken from Table VII and VII, σ_0 from Table VI σ from Table V.

Values X (γ) and σ were taken with $\varphi = 0°$ (one can use these values and with φ equal to 90 or 180° accordingly). In Table IX results of these computations are cited in the same units as the data in Table V.

Isophots, presented in Figure 9 - 12, have been constructed according to computed values of the coefficients of brilliance σ' The comparison of these isophots with experimental data (3, 5) has shown that the results are complying well among themselves.

Similar isophots can be constructed under any case of atmospheric condition and underlying surface, for which basic tables of the coefficients of brilliance σ (for $\varphi = 0$, 90 and 180°) have been computed.

The following formula, resulting from (6, 7)

$$B(\lambda) = \sigma \frac{I_0(\lambda)}{\pi} \cos i \left(\frac{\text{watt}}{\text{cm}^2 \text{ steradian}}\right) \text{ on the interval}$$

$\Delta\lambda = 1\ \mu$). should be used for the purpose of computing the brilliance of daytime cloudless sky $B(\lambda)$ in absolute units.

Here I_o $(\lambda)|$ is the density of the current of solar radiation, dropping over the upper border line of the atmosphere. Values of function I_o $(\lambda)|$ have been places in Table IV.

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