NASA TECHNICAL TRANSLATION NASA TT F-14,499

CALCULATION OF THE DIRECTION OF MAGNETIC MOMENTS OF THE ESRO I SATELLITE

Contraves AG, Zurich

Translation of "Berechnung der Richtung der magnetische Momente ESRO I", Contraves AG, Zurich, Technische Mitteilung(Technical Report), TM-RES 581 39-008, May 1, 1967, 10 pages.

(NASA-TT-F-14499) CALCULATION OF THE DIRECTION OF MAGNETIC MOVEMENTS OF THE ESRO 1 SATELLITE (NASA) Sep. 1972 15 p CSCL 22B

N73-10896

Unclas

G3/31 46266

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 September 1972

Item 45

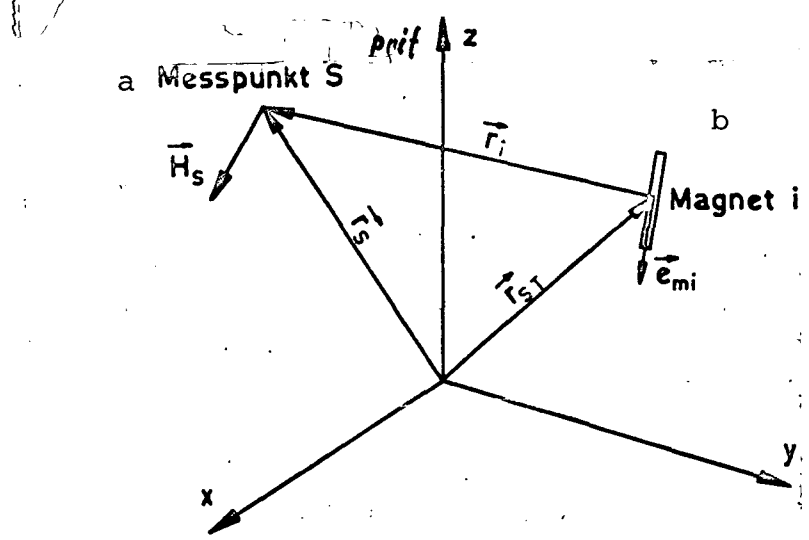
Contraves AG Zurich. Technische Mitteilung. "Berechnung der Richtung der Magnet. Momente ESRO I". TM-RES 581 39-008, dated 5.1.67. 10 p.

CALCULATION OF THE DIRECTION OF MAGNETIC MOMENTS, ESRO I

1. Formulation of the Problem

The magnetic field at various points with three components for each instance was measured for determination of the direction of the magnetic moment of the two permanent magnets in the satellite at the MSTU on the Stadlerberg. The measurement values are published in TM-RES-581.39-005, pages 14-17.

2. Derivation of Equations



a= measurement point S
b= magnet i.

Given:

Position of magnets i $\vec{r}_{STi} = (x_{STi}, y_{STi}, z_{STi})$

Position of measurement points S_i $\vec{r}_{Sl} = (x_{Sl}, y_{Sl}, z_{Sl})$

Magnetic field at points S_i $\vec{H}_{Sl} = (H_{Sxl}\ H_{Syl}\ H_{Szl})$

Magnetic moment of magnets i $M_i \quad (i=1...2)$

$\vec{r}_{il} = \vec{r}_{Sl} - \vec{r}_{STi}$ with the unit vector $\vec{e}_{ril} = \frac{\vec{r}_{il}}{r_{il}}$

(l = measurement-point index).

The magnetic field at a measurement point S is

$$\vec{H}_S = \sum_{i=1}^{2} \frac{M_i}{r_i^3} \cdot \left[3 \cdot (\vec{e}_{ri} \cdot \vec{e}_{mi}) \cdot \vec{e}_{ri} - \vec{e}_{mi} \right]$$

wanted:

Unit vector $\vec{e}_{mi} = (e_{mix}, e_{miy}, e_{miz})$

Premise: $e_{mix} < 0{,}2 \qquad e_{miy} < 0{,}2$

From this follows

$$|e_{miz}| > \sqrt{1 - 2 \cdot 0{,}2^2} = 0{,}96$$

Neglecting an error $< 4\%$

$$|e_{miz}| = 1$$

Since the two magnets have their north pole in the -z-direction $e_{miz} = -1$

and

$$\vec{e}_{mi} = (e_{mix}, e_{miy}, -1)$$

Component form of equation (1):

$$H_{Sx} = \sum_{i=1}^{2} \frac{M_i}{r_i^{\,3}} \cdot \left[e_{mix}(3e_{rix}^2 - 1) + e_{miy}(3e_{rix} \cdot e_{riy}) - 3e_{rix} \cdot e_{riz} \right] \qquad (2)$$

$$H_{Sy} = \sum_{i=1}^{2} \frac{M_i}{r_i^{\,3}} \cdot \left[e_{mix}(3e_{rix} \cdot e_{riy}) + e_{miy}(3e_{riy}^2 - 1) - 3e_{riy} \cdot e_{riz} \right] \qquad (3)$$

$$H_{Sz} = \sum_{i=1}^{2} \frac{M_i}{r_i^{\,3}} \cdot \left[e_{mix}(3e_{rix} \cdot e_{riz}) + e_{miy}(3e_{riy} \cdot e_{riz}) - 3e_{riz}^2 + 1 \right] \qquad (4)$$

Thus for a measurement point is yielded

$$a_1 e_{m1x} + a_2 e_{m1y} + a_3 e_{m2x} + a_4 e_{m2y} + a_5 = \varepsilon \qquad (5)$$

(ε = measurement error)

The coefficients for equations (2) to (4) are:

coefficient	for Equation (2)	for Equation (3)	for Equation (4)
a_1	$\frac{M_1}{r_1^{\,3}}(3e_{r1x}^2 - 1)$	$\frac{M_1}{r_1^{\,3}} 3e_{r1x} \cdot e_{r1y}$	$\frac{M_1}{r_1^{\,3}} 3e_{r1x} \cdot e_{r1z}$
a_2	$\frac{M_1}{r_1^{\,3}} 3e_{r1x} \cdot e_{r1y}$	$\frac{M_1}{r_1^{\,3}}(3e_{r1y}^2 - 1)$	$\frac{M_1}{r_1^{\,3}} 3e_{r1y} \cdot e_{r1z}$
a_3	$\frac{M_2}{r_2^{\,3}}(3e_{r2x}^2 - 1)$	$\frac{M_2}{r_2^{\,3}} 3e_{r2x} \cdot e_{r2y}$	$\frac{M_2}{r_2^{\,3}} 3e_{r2x} \cdot e_{r2z}$
a_4	$\frac{M_2}{r_2^{\,3}} 3e_{r2x} \cdot e_{r2y}$	$\frac{M_2}{r_2^{\,3}}(3e_{r2y}^2 - 1)$	$\frac{M_2}{r_2^{\,3}} 3e_{r2y} \cdot e_{r2z}$
a_5	$-\frac{M_1}{r_1^{\,3}} 3e_{r1x} \cdot e_{r1z}$ $-\frac{M_2}{r_2^{\,3}} 3e_{r2x} \cdot e_{r2z} - H_{Sx}$	$-\frac{M_1}{r_1^{\,3}} 3e_{r1y} \cdot e_{r1z}$ $-\frac{M_2}{r_2^{\,3}} 3e_{r2y} \cdot e_{r2z} - H_{Sy}$	$-\frac{M_1}{r_1^{\,3}}(3e_{r1z}^2 - 1)$ $-\frac{M_2}{r_2^{\,3}}(3e_{r2z}^2 - 1) - H_{Sz}$

The unknowns e_{mix}, e_{miy} must now be so determined that

$$S = \sum_{l} \varepsilon_l^2 = \text{Min}$$

i.e. the partial derivatives of S with respect to the unknowns must be set equal to zero. This results in a system of four linear defining equations with the following coefficient matrix:

$$C_{ik} = \begin{pmatrix} \sum_l a_{1l}^2 & \sum a_{1l} \cdot a_{2l} & \sum a_{1l} \cdot a_{3l} & \sum a_{1l} \cdot a_{4l} \\ \sum a_{2l} \cdot a_{1l} & \sum a_{2l}^2 & \sum a_{2l} \cdot a_{3l} & \sum a_{2l} \cdot a_{4l} \\ \sum a_{3l} \cdot a_{1l} & \sum a_{3l} \cdot a_{2l} & \sum a_{3l}^2 & \sum a_{3l} \cdot a_{4l} \\ \sum a_{4l} \cdot a_{1l} & \sum a_{4l} \cdot a_{2l} & \sum a_{4l} \cdot a_{3l} & \sum a_{4l}^2 \end{pmatrix} \qquad D_{ik} = -\begin{pmatrix} \sum_l a_{1l} \cdot a_{5l} \\ \sum a_{2l} \cdot a_{5l} \\ \sum a_{3l} \cdot a_{5l} \\ \sum a_{4l} \cdot a_{5l} \end{pmatrix}$$

With vector $\vec{e}_{mi}^{\,*} = (e_{m1x}, e_{m1y}, e_{m2x}, e_{m2y})$ will

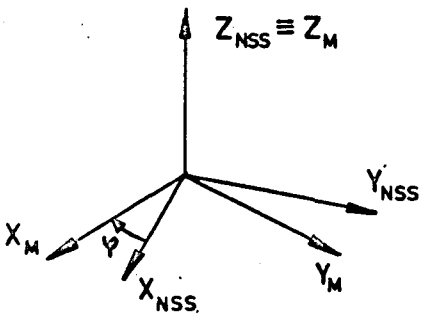
$$\underline{C_{ik} \cdot \vec{e}_{mi}^{\,*} = D_{ik}} \qquad (7)$$

A program for the SDS 920 was developed for the solution of this equation system.

3. Description of the Calculation Program

3. a Definition of the Coordinate System

The measurement arrangement was such that the satellite could be turned in the z axis to the extent of angle ϕ with respect to the space-stationary coordinate system X_{NSS}, Y_{NSS}, Z_{NSS} (compare TM-RES 581.39-003). Measurement value $\vec{r}_{Sl}$ and $\vec{H}_{Sl}$ are given in terms of the space-stationary system, the position of magnets $\vec{r}_{STi}$ and vectors $\vec{e}_{mi}$ is given in terms of the satellite-stationary system. By means of a transformation, values $\vec{r}_{STi}$ must first be converted into the space-stationary system and at the end, vectors $\vec{e}_{mi}$ must be recalculated for the satellite-stationary system.



Index NSS ≙ stationary with respect to space

Index M ≙ stationary with respect to satellite

3. b <u>Program Input</u>

Required subroutine: SYSLIN

OM (I) = magnet. Moment for Magnet i [Gauss · cm^3]

XM (I), YM (I), ZM (I) = Position of Magnet i [cm]

X = 0 (not being used)

L = I = Number of the Magnet

PHIC = Angle φ [Deg.]

XNSS, YNSS, ZNSS = Position of a Measurement Point [cm]

*H = Field Intensity (Reading Value) [γ]

HO = zero error or earth-field component subject to compensation. Give value in the direction of the positive axles ! (γ)

IND
- = ± 1, if measuring sonde lies in the ± X_{NSS} direction
- = ± 2, if measuring sonde lies in the ± Y_{NSS} direction
- = ± 3, if measuring sonde is in the ± Z_{NSS} direction

X = PHIC (control)

Reading value H corresponds to the measuring value of a measurement sonde, which however gives only half the value of the actual magnetic field (compare TM-RES-581. 39-003, page 37-39). Doubling of the measurement value is effected in the program. The measurement values given in TM-RES-581.39-005 can therefore not be used directly, since this doubling has already been carried out there.

Position of Switch SW 2

By means of this switch, the program can be so controlled that the coefficient matrix C_{ik} is calculated in terms of the space-stationary system, and vector e_m is transformed into the satellite-stationary system only at the end (SW 2 = off), or that coefficients a_k and thus also C_{ik} are calculated already in the satellite-stationary system (SW 2 = on).

In the first case, only all the measurement values with a constant ϕ can be used. The last card must then be a blank card (IND = 0).

In the second case, all measurement values can be used

independently of angle ϕ. It is necessary only to combine groups with the same ϕ and to close with a card with XNSS = 1'000'000 and IND = 0. In addition, before each such group, a card with the new (PHIG) must be entered. The very last card must again be a blank card.

3. Data Issue

In addition, the determinant of C_{ik} is calculated and printed out by means of switch SW 1 = on.

PHIG = ϕ (degree)

If SW 1 = on, i.e. if measuring values with different values of ϕ have been evaluated, the last entered value ϕ is printed out.

Magnet 1	Magnet 2	Magnet 1+2	Angle
e_{m1x}	e_{m2x}	e_{mx}	$\measuredangle e_{mx}$
e_{m1y}	e_{m2y}	e_{my}	$\measuredangle e_{my}$
e_{m1z}	e_{m2z}	e_{mz}	

Here:

$\vec{e}_{m1}$ = designates unit vector of the field of Magnet 1

$\vec{e}_{m2}$ = designates unit vector of the field of Magnet 2

$\vec{e}_{m}$ = designates unit vector of the resulting field of the 2 magnets

$\measuredangle \vec{e}_{m}$ = designates angle of $\vec{e}_{m}$ in /degree/ with respect to the -z-axis

All values are given in terms of the satellite-stationary system.

4. Calculation Program

```
C       DIRECTION OF THE MAGNETIC MOMENTS ESRO  1
C
C       SW1 OFF WITHOUT DETERMINANT
C       SW1 ON TYPE DETERMINANT
C       SW2 OFF CALCULATION IN SYSTEM SPACE STATIONARY
C       SW ON  CALCULATION IN SYSTEM STELLITE STATIONARY
C
        DIMENSION OM[2],XM[2],YM[2],ZM[2],ER[2,3],R[2],A[5],C[4,5]
        DIMENSION XX[2,2]
      1 PAUSE
        DET=1.
        PERM=2E-5
        DO 400 I=1,4
        DO 400 K=1,5
    400 C[I,K]=0.
        DO 99 I=1,2
        READ 50,OM[I],XM[I],YM[I],ZM[I],X,L
        IF[I-L] 13,99,13
     99 CONTINUE
      2 READ 50,PHIG
        PHI=PHIG/57.2957795131
        DO 100 I=1,2
        XX[I,1]=XM[I]
        XX[I,2]=YM[I]
    100 CALL TRANS[XX[I,1],XX[I,2],PHI]
      3 READ 50,XNSS,YNSS,ZNSS,H,HO,IND,X
        IF[IND] 14,4,14
     14 IF[PHIG-X] 13,5,13
      5 DO 200 I=1,2
        ER[I,1]=XNSS-XX[I,1]
        ER[I,2]=YNSS-XX[I,2]
        ER[I,3]=ZNSS-ZM[I]
        A[I]=SQRT[ER[I,1]**2&ER[I,2]**2&ER[I,3]**2]
        R[I]=OM[I]/[A[I]**3]
        DO 200 L=1,3
    200 ER[I,L]=ER[I,L]/A[I]
        H=PERM*[H*SIGN[1,IND]-HO]
        IND=IABS[IND]
        A[5]=0.
        DO 300 I=1,2
        I1=2*I-1
        I2=2*I
        L=2
        K=1
        X=0.
        IF[IND-2] 8,7,6
      6 I1=5
        L=3
        K=2
        X=A[5]
        GO TO 9
      7 I1=2*I
        I2=2*I-1
      8 A[5]=A[5]&3*R[I]*[ER[I,IND]*ER[I,3]
      9 A[I1]=X&R[I]*[3*ER[I,IND]**2-1.]
     10 A[I2]=3*R[I]*ER[I,K]*ER[I,L]
        IF[K-1] 300,300,11
     11 K=1
        I2=2*I-1
        GO TO 10
```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CZ
A 4
F 2452-T
3.66 70000

```
  300 CONTINUE
      A[5]=A[5]&H
      IF[SENSE SWITCH 2] 12,17
   12 DO 800 I=1,2
  800 CALL TRANS[A[2*I-1],A[2*I],-PHI]
   17 DO 500 I=1,4
      DO 500 K=I,5
  500 C[I,K]=C[I,K]&A[I]*A[K]
      GO TO 3
    4 IF[SENSE SWITCH 2] 18,19
   18 IF[XNSS-1E6] 19,2,2
   19 DO 600 I=1,3
      DO 600 K=I&1,4
  600 C[K,I]=C[I,K]
      CALL SYSLIN[C,4,1,4,DET]
      IF[SENSE SWITCH 2] 21,20
   20 DO 700 I=1,2
  700 CALL TRANS[C[2*I-1,5],C[2*I,5],-PHI]
   21 X=-1.
      IF[SENSE SWITCH 1] 15,16
   15 TYPE 53,DET
   16 DO 900 I=1,2
      C[I,4]=.5*[C[I,5]&C[I&2,5]]
  900 C[I&2,4]=57.2957795131*ATAN[C[I,4]]
      TYPE 52,PHI
      TYPE 51,C[I,5],C[I&2,5],C[I,4],C[I&2,4],I=1,2
      TYPE 51,X,X,X
      GO TO 1
   13 STOP
   50 FORMAT[5F10.0,I5,F10.0]
   51 FORMAT[E13.6,2E16.6,F11.3]
   52 FORMAT[$PHI=$F7.2//$    MAGNET 1          MAGNET 2          $
     ($MAGNET 1&2     WINKEL$/]
   53 FORMAT[$DET=$E13.6/]
      END
```

```
C
C
      SUBROUTINE COORDINATE TRANSFORMATION
C
      SUBROUTINE TRANS[X,Y,PHI]
      A= X*COS[PHI]&Y*SIN[PHI]
      Y=-X*SIN[PHI]&Y*COS[PHI]
      X=A
      RETURN
      END
```

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CZ
A 4
F 2452-T
3.66 10 000

CZ
A 3
F 2453-T
11.64 3000

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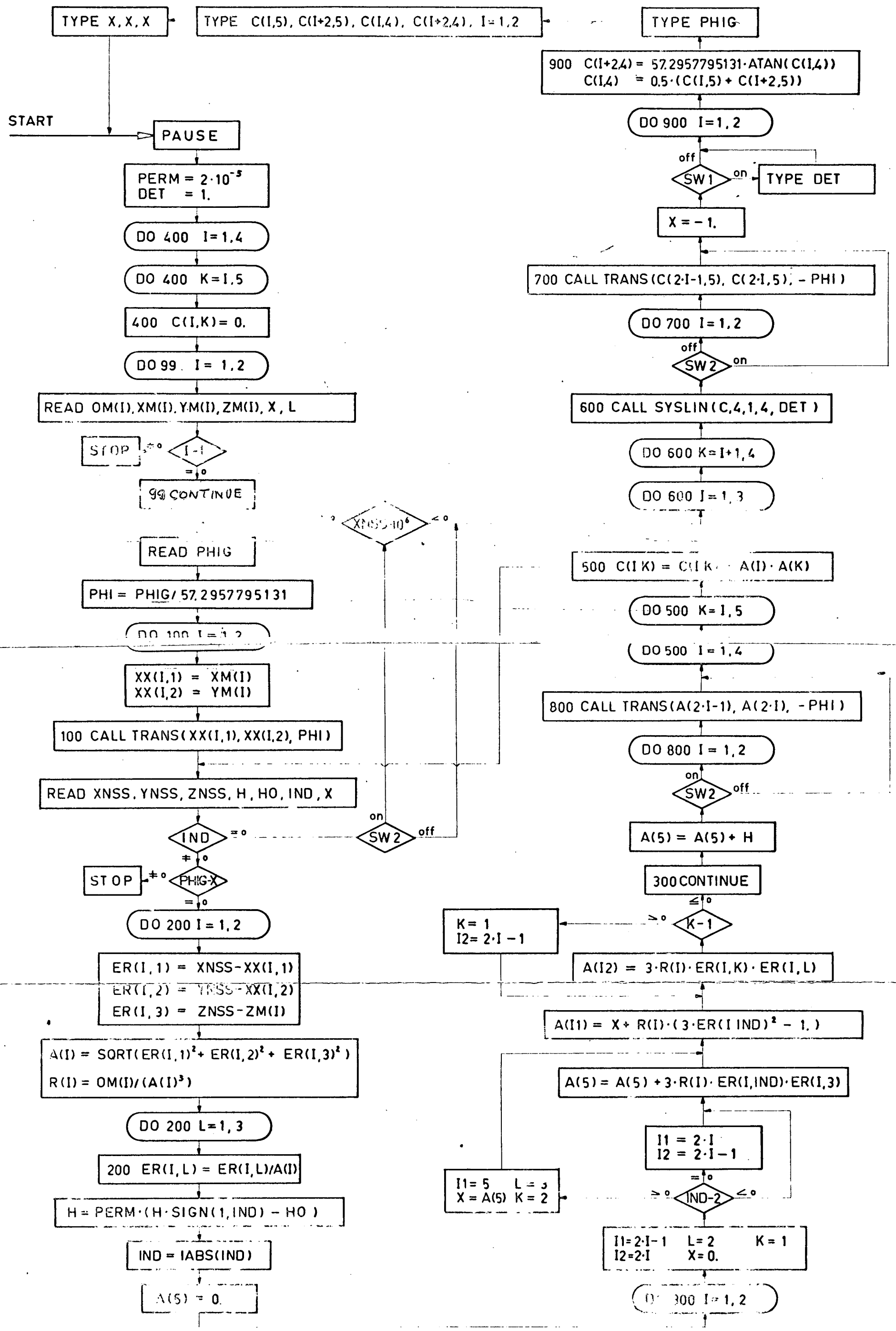

FOLDOUT FRAME 1
FOLDOUT FRAME 2
CONTINVES AG ZURICH

5. Results of the Evaluation of Measurement Values of Mov.7,66

Magnet. Moment $M_1 = M_2 = 13500$ [Gauss · cm^3]

MAGNET 1	MAGNET 2	MAGNET 1&2	ANGLE
.599733E-01	-.164566E-01	.217639E-01	1.247°
-.236286E-01	-.325362E-01	-.283824E-01	-1.639°
-.100000E 01	-.100000E 01	-.100000E 01	

Magnet. Moment $M_1 = 13000$, $M_2 = 13500$ [Gauss · cm^3]

MAGNET 1	MAGNET 2	MAGNET 1&2	ANGLE
.512733E-01	-.170081E-01	.221026E-01	1.266°
-.204508E-01	-.329740E-01	-.287024E-01	-1.644°
-.100000E 01	-.100000E 01	-.100000E 01	

Magnet. Moment $M_1 = 13500$, $M_2 = 13000$ [Gauss · cm^3]

MAGNET 1	MAGNET 2	MAGNET 1&2	ANGLE
.665161E-01	-.160196E-01	.222433E-01	1.273°
-.232077E-01	-.339122E-01	-.285599E-01	-1.636°
-.100000E 01	-.100000E 01	-.100000E 01	

Magnet. Moment $M_1 = M_2 = 1300$ [Gauss · cm^3]

MAGNET 1	MAGNET 2	MAGNET 1&2	ANGLE
.617715E-01	-.165920E-01	.225397E-01	1.290°
-.239937E-01	-.343660E-01	-.291803E-01	-1.671°
-.100000E 01	-.100000E 01	-.100000E 01	

The MSTU measurements of Mr. Ruggaber of Nov. 21/23, 1966 yielded the following values:

$\measuredangle e_{mx} = 1{,}1^\circ$

$\measuredangle e_{my} = -1{,}82^\circ$

With deviation of 0.2° the results of the two measurement methods that are independent from each other, showed good agreement.

Cancciotto

CZ
A 4
F 2452-T
3.66 70 000

Die Verwendung dieses Dokumentes wird gerichtlich verfolgt.