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RESTITUTION OF HIGH-FREQUENCY COMPONENTS OF THE WIND BASED ON FOUR DOPPLER MEASUREMENTS - RESTITUTION OF LOCALIZATION

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RESTITUTION OF HIGH-FREQUENCY COMPONENTS OF THE WIND BASED ON FOUR DOPPLER MEASUREMENTS - RESTITUTION OF LOCALIZATION

I. INTRODUCTION

As a first phase, project "Eole" attempts to find, from the localization of a balloon on two consecutive orbits, the continuous lowfrequency component of wind. The project requires localization of the balloon at each orbital revolution as perfect as possible. Memoranda No. 10 and 11 have shown the possibility of restituting under zero wind and on the basis of two interrogations made by a Doppler measurements, the exact position of the balloon taking into account systematic errors due to terrestrial rotation and the relative speed of satellite and balloon. However, the investigation of the influence of the wind has shown that such localization resulted in considerable errors of as much as 100 to 150 km at a wind strength on the order of 200 km/hr.

At least a partial knowledge of the high-frequency components of the wind was shown to be indispensible in order to correct the error introduced. This knowledge can be furnished only by sequential data obtained by different measurments during the time of visibility. We intend in this study to propose a method on the basis of an arithmetic processing of the data which will make it possible to restitute the geographic

components of the wind assumed to be constant during the time of visibility (on the order of 8 to 10 minutes).

II. Principle of the Method

The iterative process referred to in memorandum No. 11 makes it possible to completely restitute the localization, provided the continuous components of the wind are known.

Subsequently we can consider processing of the information utilizing such a process on the basis of given wind data. Data so collected will make it possible to isolate the influence due to the wind and to restitute the two continuous components of the wind.

We consider a sequence of interrogation $t_1 t = T$, $t + 2T$, $t + 3T$ ....., $t + nT$ to which correspond the positions S_1, S_2, S_3 and S_n of the satellite and the corresponding values measured by the Doppler effect $\cos \beta_{1m}$, $\cos \beta_{2m}$, $\cos \beta_{3m}$, ... $\cos \beta_{nm}$ taking into account the terrestrial rotation, the speed of the approach satellite-balloon and the continuous components of the wind. In this first part it is assumed that the measured magnitudes are not affected by any errors. We shall see specifically in the second part the influence of the ionospheric errors on the restitution of localization. The wind is further defined by its two geographic components Vp and Vm assumed to be contant during the time of visibility (Vp = component parallel to the terrestrial equator; Vp perpendicular to Vm in the plane tangential to the sphere of the balloons at the pont considered).

The process referred to in memorandum No. 11 and consisting of two

iterative processes for compensating terrestrial rotation and an overall iterative process for compensating the influence of the speed of approach satellite-balloon, makes it possible to restitute, on the basis of the given components V_1 and V_2 and of two interrogations, the position of the balloon falsified uniquely by the erroneous components of the wind. Let us designate respectively by μ_{12} and θ_{12} the longitude and the inertial latitude of the balloon B_{21} at the instant t + T taking into account the geographic components V_1 and V_2 and the two magnitudes cos β_{1m} and cos β_{2m} corresponding to the first and second interrogation.

In similar manner, the second and third interrogation define, on the basis of the same components of the wind, the longitude μ_{23} and the latitude θ_{23} of the balloon B_{22} at the instant t + T. We thus associate with each pair of values V_1 V_2 the magnitudes $\mu_{23} - \mu_{12}$ and $\theta_{23} - \theta_{12}$ which express the variations of longitude and latitude of the balloon at the instant t + T by utilizing only the three first interrogations. It is obvious that $\mu_{23} = \mu_{12}$ and $\theta_{23} = \theta_{12}$ for $V_1 = V_p$ and $V_2 = V_m$. In order to dissociate the respective influences of the two components of the wind, we plotted (cf. fig. 1 and 1') the curves $\mu_{23} - \mu_{12} = f(V_1)$ and $\theta_{23} - \theta_{12} = g(V_1)$ for $V_2 = 0$ and the curves $\mu_{23} - \mu_{12} = h(V_2)$ and $\theta_{23} - \theta_{12} = k(V_2)$ for $V_1 = 0$ based on the three measured values cos β_{1m}, cos β_{2m} and cos β_{3m}. The linearity of these curves was verified in general and is explained by the low value of the wind in relation to the speed of rotation of the earth which results in relatively slight variations of latitude and longitude of the balloon. On the other

hand, the curves f (V_1) and g (V_1) are concurrent in a point such that $f(V_{11}) = g(V_{11}) = 0$. With one component known, the second is subsequently determined in a unique manner with the aid of three interrogations and is such that the distance d_{12} separating the two positions of the balloon B_{21} and B_{22} at the instant t + T, as calculated respectively from the first and the second interrogation as well as for the second and third interrogation, is precisely zero.

Subsequently, three interrogations and consequently two localizations make it possible to define a relation $F(V_1, V_2) = 0$ such that $d_{12} = 0$. This relation is linear with very satisfactory approximation and we again find (cf. fig.2) the magnitudes V_{11} and V_{21} such that $f(V_{11}) = g(V_{11}) = h(V_{21}) = k(V_{21}) = 0$.

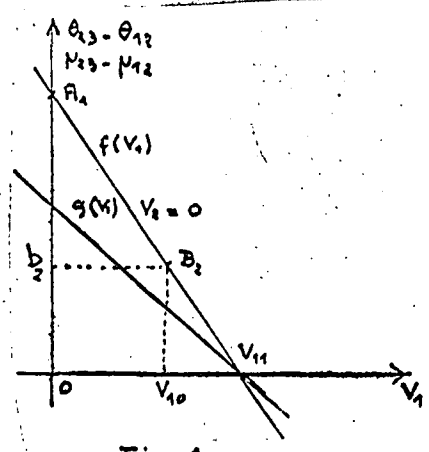

Fig 1

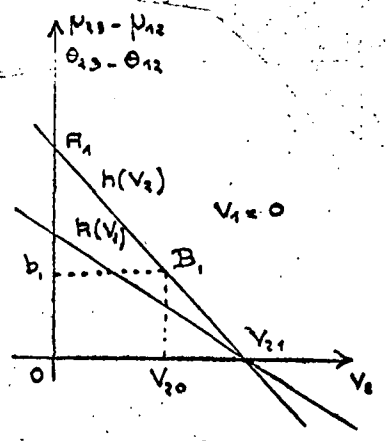

Fig 1'

Three interrogations are insufficient for restituting the two components of the wind but make it possible to determine the strenght

of the wind if its direction is known or vice versa. The foruth interrogation ($\cos \beta_{4m}$) furnishes the additional information perimtting solution of the problem. We resume the process described in the preceding by utilizing e.g. for the localization of the balloon, the second and foruth interrogation. We thus define for two given components of the wind the position B_{23} of the balloon (cf. fig.3) which we compare with the position B_{21} of the balloon obtained with the same components of the wind.

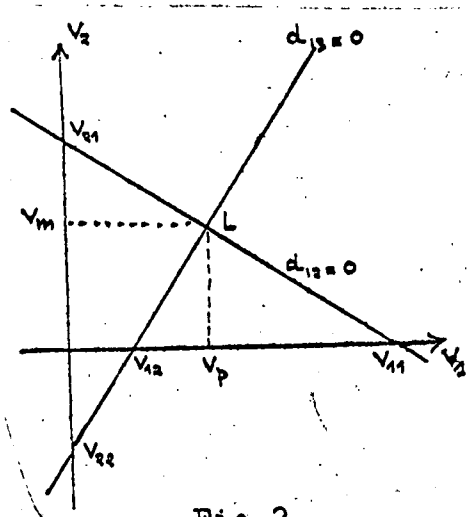

Fig.2

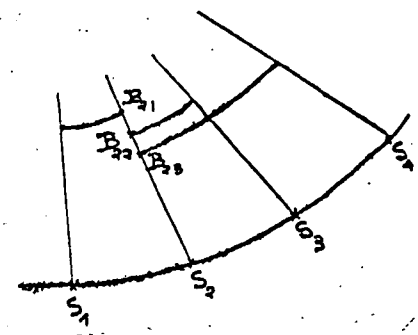

Fig.3

In similar manner, the distance $d_{13} = |\overrightarrow{B_{23}B_{21}}|$ is cancelled when the components V_1 and V_2 of the wind are subject to an approximately linear relation $G(V_1, V_2) = 0$ schematized by a straight line in the plane V_1, V_2. The point L at the intersection of the two straight lines uniquely defines

the two components Vp and Vm of the wind and these, when reitroduce into the process of localization, determine the exact position of the balloon.

This method for which we shall specify the formulation in the next paragraph, suggests some remarks. The observed linearity of the curves prevents any recurrent process for determining the magnitudes V_{11}, V_{21}, V_{12} and V_{22} and consequently considerably reduces the time of computation which is essentially only a function of the number of localizations necessary for gathering all data.

The magnitudes V_{11} and V_{21} make it necessary to know the points A_1 B_1 and B_2 (cf. fig. 1 and 1') which require the following six localitations:

$V_1 = 0$ 1) localization on base S_1 S_2

$V_2 = 0$ 2) localization on base S_2 S_3

from which $\overline{OA} = \mu_{23} - \mu_{12}$

$V_1 = V_{10}$ 3) localization on base S_1 S_2

$V_2 = 0$ 4) localization on base S_2 S_3

from which $\overline{OB_2} = \mu_{23} - \mu_{12}$

$V_2 = V_{20}$ 5) localization on base S_1 S_2

$V_1 = 0$ 6) localization on base S_2 S_3

from which $\overline{OB_1} = \mu_{13} - \mu_{12}$

In order to know the magnitudes V_{12} and V_{22} only three localizations are necessary because we can profit from the localizations 1, 3 and 5 already made on base S_1S_2.

$V_1 = 0$ 7) localization on base S_2 S_4

$V_2 = 0$

$V_1 = 0$ 8) localization on base S_2 S_4

$V_2 = V_{20}$

$V_1 = V_{10}$ 9) localization on base S_2 S_4

$V_2 = 0$

Complete restitution of the two components requires nine localizations with the aid of the general iterative process where an additional localization is necessary for the restitution of the position of the ballloon on the basis of the components of the wind. Memorandum No. 11 points out the amount of time of arithmetic calculation for a complete localization taking into account terrestrial rotation and the error of measurement. We must therefore expect a large amount of time of calculation for the restitution of the localization of a balloon. Consequently, a variant of the method will be proposed in a subsequent paragraph in order to reduce computation time.

III. Formulation

The first part of the program consists in an attempt to find from a given position of the balloon B_2 at the instant t + T, the measured values cos β_{1m}, cos β_{2m}..... cos β_{nm} at the instant t, t + T, t + 2T ....., t + nT taking into account the velocity of rotation of the earth, the speed of approach satallite-balloon, and the components

of the wind Vp and Vm.

The notations utilized are those of memorandum No. 11. Since the ballooon is defined by its longitude φ_i and its orbital latitude λ_i ($\lambda_i = (\overrightarrow{OS_i}, \overrightarrow{Ob_i})$,the different formulas for the solution of this problem are as follows:

B_2 from φ_2 and λ_2 = given position of the balloon at the instant t + T; when designating by $\alpha = (\overrightarrow{Ox}, \overrightarrow{OS_2})$ we have.

$\mu_2 = \mathrm{Arc}\sin\left(\sin i \cos\varphi_2 \sin(\lambda_2+\alpha) + \cos i \sin\varphi_2\right)$ μ_2 =geographic latitude of B_2

$\theta_2 = \mathrm{Arc\,tg}\left[\left(\cos i \cos\varphi_2 \sin(\lambda_2+\alpha) - \sin i \sin\varphi_2\right) / \cos\varphi_2 \cos(\lambda_2+\alpha)\right]$ θ_2 = inertial longitude of B_2

$\theta_1 = \theta_2 - \omega_0 T - V_p T / \cos\mu_2$ θ_1 = inertial longitude of B_1

$\mu_1 = \mu_2 - V_m T$ μ_1 = latitude of B_1 $\varphi_1 = \mathrm{Arc\,tg}\,\dfrac{\cos i \sin\theta_1 \cos\mu_1 + \sin i \sin\mu_1}{\cos\theta_1 \cos\mu_1}$

$\varphi_1 = \mathrm{Arc}\cos\dfrac{\cos\theta_1 \cos\mu_1}{\cos\varphi_1}$ φ_1 =orbital longitude of B_1 counted as starting from Ox

φ_1 =orbital latitude of B_1.

From this we deduce $\cos\beta_1 = \dfrac{\cos\varphi_1 \sin(\Omega_s T + \varphi_1 - \alpha)}{\sqrt{1+u^2-2u\cos\varphi_1\cos(\varphi_1+\Omega_s T-\alpha)}}$ and $\cos\beta_2 = \dfrac{\cos\varphi_2 \sin\lambda_2}{\sqrt{1+u^2-2u\cos\varphi_2\cos\lambda_2}}$. In order to define the measured magnitudes $\cos\beta_{1m}$ and $\cos\beta_{2m}$, we must take into account the components of the speed vector of the balloon at each instant of interrogation.

The components of the speed vector referenced to the axes OXYZ are:

$$\vec{V}_{B_i} \left| \begin{array}{l} -(\omega_0\cos\mu_i + V_p)\sin\theta_i - V_m \sin\mu_i \cos\theta_i \\ (\omega_0\cos\mu_i + V_p)\cos\theta_i - V_m \sin\mu_i \sin\theta_i \\ V_m \cos\mu_i \end{array} \right.$$

The components of vector referenced to the axis Oxyz based on the orbital plane become:

$$\vec{V}_{B_i} \left| \begin{array}{l} -(\omega_0 \cos\mu_i + V_p)\sin\theta_i - V_m \sin\mu_i \cos\theta_i \\ (\omega_0 \cos\mu_i + V_p)\cos\theta_i \cos i - V_m \sin\mu_i \sin\theta_i \cos i + V_m \cos\mu_i \sin i \\ -(\omega_0 \cos\mu_i + V_p)\cos\theta_i \sin i - V_m \sin\mu_i \sin\theta_i \sin i + V_m \cos\mu_i \cos i \end{array} \right.$$

By writing that:

$$\frac{(S_i B_i, V_{B_i})}{|S_i B_i|} = |V_{B_i}| \cos\gamma_i$$

we deduce from this

$$\cos\beta_{im} = \cos\beta_i - \left|\frac{V_{B_i}}{V_{S_i}}\right| \cos\gamma_i$$

and we have

$$\cos\beta_{im} = \cos\beta_2 - \Big(\big(-(\omega_0\cos\mu_2 + V_p)\sin\theta_2 - V_m \sin\mu_2 \cos\theta_2\big)\big(\cos\varphi_2\cos\psi_2 - u\cos\alpha\big) + \big((\omega_0\cos\mu_2 + V_p)\cos\theta_2\cos i - V_m\sin\mu_2\sin\theta_2\cos i + V_m\cos\mu_2\sin i\big)\big(\cos\varphi_2\sin\psi_2 - u\sin\alpha\big) + \big(-(\omega_0\cos\mu_2 + V_p)\cos\theta_2\sin i + V_m\sin\mu_2\sin\theta_2\sin i + V_m\cos\mu_2\cos i\big)\sin\varphi_2\Big)\Big/\Pi_2$$

$$\cos\beta_{im} = \cos\beta_1 - \Big(\big(-(\omega_0\cos\mu_1 + V_p)\sin\theta_1 - V_m\sin\mu_1\cos\theta_1\big)\big(\cos\varphi_1\cos\psi_1 - u\cos(\alpha - \Omega_S T)\big) + \big((\omega_0\cos\mu_1 + V_p)\cos\theta_1\cos i - V_m\sin\mu_1\sin\theta_1\cos i + V_m\cos\mu_1\sin i\big)\big(\cos\varphi_1\sin\psi_1 - u\sin(\alpha - \Omega_S T)\big) + \big(-(\omega_0\cos\mu_1 + V_p)\cos\theta_1\sin i + V_m\sin\mu_1\sin\theta_1\sin i + V_m\cos\mu_1\cos i\big)\sin\varphi_1\Big)\Big/\Pi_1$$

with

$$\Pi_1^2 = 1 + u^2 - 2u\cos(\alpha - \Omega_S T)\cos\varphi_1\cos\psi_1 - 2u\sin(\alpha - \Omega_S T)\cos\varphi_1\sin\psi_1$$

and

$$\Pi_2^2 = 1 + u^2 - 2u\cos\alpha\cos\varphi_2\cos\psi_2 - 2u\sin\alpha\cos\varphi_2\sin\psi_2$$

The second part of the program consists in finding, on the basis of the magnitudes cos β_{1m}, cos β_{2m}, cos β_{3m} and cos β_{4m} as calculated by this method, the exact position of the balloon and the geographic components of the wind. We utilize the arithmetic program of restitution of localization referred to in memorandum No. 11 by modifying the formulation and introducing the automatic calculation of the magnitudes V_{11}, V_{12}, V_{21} and V_{22} required for the calculation of Vp and Vm.

IV. Arithmetical Program and Experimental Results

The general arithmetical problem of restitution given in Annex 1 consists of a program of finding the measured magnitudes cos β_{nm} and of the actual restitution program. Table 1 groups the results obtained for the nine localizations considered on the basis of the four magnitudes cos β_{im} $1 \leq i \leq 4$ calculated for a balloon defined by B_2 (φ_2 = 0.261801 rad = 15°; λ_2 and α = 0.0383957 rad) and a wind with the components Vp = 0.00000500 rad/sec = 115.20 km/hr and Vm = 0.0000700= 161.2 km/hr. The curves defining the variations of latitude and longitude of the balloons were plotted (cf. diagram 1 and 2) by maintaining one of the components of the wind constant. We verify in the specific case under consideration the linearity of the curves and obtain:

$$V_{11} = 9{,}5 \times 10^{-5} \text{ rad/sec} = 218{,}88 \text{ km/h}$$
$$V_{12} = -35 \times 10^{-5} \quad " \quad = -806{,}04 \text{ km/h}$$
$$V_{21} = 13{,}5 \times 10^{-5} \text{ rad/sec} = 311{,}04 \text{ km/h}$$
$$V_{22} = 6{,}5 \times 10^{-5} \text{ rad/sec} = 149{,}76 \text{ km/h}$$

By carrying these values over to the plane V_1V_2, we determine the magnitudes Vp and Vm at the intersections of the two straight lines (diagram 3). The numerical solution (cf. Annex II which reproduces the arithmetical results) furnishes the following values:

$$Vp = 0{,}00000512 \text{ rad/sec} \qquad Vm = 0{,}00000709 \text{ rad/sec}$$

The precision obtained is very satisfactory (on the order of 2 %) and the corresponding position of the balloon calculated by the last iterative process furnishes the following results.

$$\varphi_2 = 0{,}261674 \text{ rad} \qquad \lambda_2 + \alpha = 0{,}038381 \text{ rad}$$

The error in the position of the balloon after restitution of the continuous components is on the order of 800 m where the error in orbital latitude is predominant. The method therefore permits restitution of the localization of the balloon with very satisfactory approximation by nearly completely eliminating the influence of the wind; (the error due to the wind in the particular case under consideration was 50 km), it is now reduced to 800 m. It is possible to improve the localization by a more appropriate selection of the base and of the wind components. It was shown in fact that such a choice is linked essentially to the ionospheric errors which can rather appreciably disturb the result. Very generally, it will be noted (cf. diagram2) that the determintation of V_{12} will be as much better as the component V_{10} selected a priori is very large in absolute value and clse to V_{12}. This observation becomes increasingly justified with the decreasing slope of the curves. However, such a choice requires a rather rough knowledge of the components of the wind obtaine

TABLE 1

$\cos\beta_{1m} = 0,53763181$ $\cos\beta_{2m} = 0,13054304$ $\cos\beta_{3m} = -0,35723468$ $\cos\beta_{4m} = -0,62966364$

Localization	V_1 (rad/sec)	V_2 (rad/sec)	θ_1 (rad)	μ_1 (rad)
1 with S_1 S_2	0	0	-0,16274589	0,21895238
2 with S_1 S_2	0,00000350	0	-0,16185602	0,21692967
3 with S_1 . S_2	0	0,00000350	-0,16067090	0,21770575
4 with S_2 S_3	0	0	-0,16178245	0,21779094
5 with S_2S_3	0,00000350	0	-0,16123181	0,21616990
6 with S_2S_3	0	0,00000350	-0,15994328	0,21682217
7 with S_2S_4	0	0	-0,16309782	0,21937796
8 with S_2S_4	0,00000350	0	-0,16224846	0,21740102
9 with S_2S_4	0	0,00000350	-0,16082130	0,21788610

e.g. during the preceding revolution. It will be noted, however, that even a very poor choice of V_{10} (like the one made by us) furnishes very satisfactory results.

V. Alternate Method

The necessity for carrying out nine complete localizations on the basis of the given wind requires a relatively large amount of time of arithmetical calculation. In order to limit the latter, we have considered a method consisting in a comparison at hte instants of interrogation of the magnitudes effectively measured ($\cos \beta_{1m}$, $\cos \beta_{2m}$ ..... $\cos \beta_{nm}$) and of the calculated magnitudes. Localization is made on a given base, (e.g. base S_1S_2) and furnishes a position B_2 of the balloon with given wind components V_1V_2. From this position and with the same components of the wind, we calculate, at the instants of interrogation $t + 2T$,..... $t + nT$, the magnitudes $\cos \beta_3$, $\cos \beta_4$,....., $\cos \beta_n$ which are compared to the measured magnitudes $\cos \beta_{3m}$, $\cos \beta_{4m}$,..... $\cos \beta_{nm}$. Each pair of values V_1V_2 is associated with two variations of $\cos \beta$ at any two instants of interrogation other than those having served for localization (e.g. the instants $t + 2T$ and $t + 3T$). The relations

$$\cos\beta_3 - \cos\beta_{3m} = f(V_1) \text{ for } V_2 = C_1 \qquad \cos\beta_3 - \cos\beta_{3m} = g(V_2) \text{ for } V_1 = C_2$$

$$\cos\beta_4 - \cos\beta_{4m} = h(V_1) \text{ for } V_2 = C_1 \qquad \cos\beta_4 - \cos\beta_{4m} = k(V_2) \text{ for } V_1 = C_2$$

complete the determination of the components of the wind Vp and Vm in

B

an entirely analogous manner to that considered in the preceding paragraph. We verify the very satisfactory linearity of the functions f, g, h and k which limit to only three the number of localizations made on the base S_1 and S_2 and necessary for the restitution of the wind.

Table 2 groups the experimental results obtained for the case considered in the preceding.

$\cos\beta_{1m} = 0,53763151$ $\cos\beta_{2m} = 0,15054304$ $\cos\beta_{3m} = -0,35723468$ $\cos\beta_{4m} = -0,62966364$

Localization	V_1 (rad/sec)	V_2 (rad/sec)	θ_1 (rad)	μ_1 (rad)	$\cos\beta_3$	$\cos\beta_4$
1 with S_1S_2	0	0	-0,16274589	0,21895238	-0,35524293	-0,63028085
2 with S_1S_2	0,00000350	0	-0,16185602	0,21692967	-0,35594457	-0,63034963
3 with S_1S_2	0	0,00000350	-0,16067 090	0,21770575	-0,35571911	-0,62992483

TABLE 2

We arrive at the following results:

$V_{11} = 0,00000994$ $V_{21} = 0,00001464$ $V_{12} = -0,00003141$ $V_{22} = 0,00000607$

where

$V_p = 0,00000514$ rd/s $V_m = 0,000000706$ rd/sec

The accuracy for the components of the wind is approximately the same (on the order of 2 %) and the computation time was reduced at a ratio of 1:4. The computation time for $\cos \beta_3$ and $\cos \beta_4$ is negligible in relation to the time required for complete localization. We have definitely adopted this method which makes possible rapid exploitation of the resultas with very satisfactory accuracy.

VI. Influence of Ionospheric Errors

The selection of the base of localization as well as that of the bases requiring comparison of magnitudes has so far not been considered since the accuracy obtained is very satisfactory and approximately the same for the different bases. However, since ionospheric errors appreciably disturb the measurements, the problem of selection of bases becomes essential. It might be imagined that, with the selection of the base of localization made, a statistical investigation of the results obtained for all other combinations of bases would produce valuable information. However, the restricted number of interrogations during one orbital revolution and for a given balloon considerably reduces the quantity of information required for a statistical investigation.

In accordance with the report of J.C. Thoin and by utilizing the mean values of the absolute and relative errors for cos β due to the contribution of the integrated mean index and of the horizontal gradient of η, we considered an absolute error of 10^{-4} in the measured magnitude. On the basis of the magnitudes calculated for a given position of the balloon (φ_2= 0.261801 rad = 15° and $\lambda_2 + \alpha$ = 1.383957 rad) and for two components of the wind (Vp = 0.00000700 and Vm = 0. 00000500). Localization is made from measured magnitudes falsified by excess of 10^{-4} since the compution time at the present stage does not permit generation of a Gauss noise and localization for each corresponding value. On the other hand, in order to more precisely indicate the importance of the selection of bases, we intentionally reduced the interval between interrogation in order to profit from a large number of bases (we profit from seven possible interrogations (cf. Figure 4) for T = 97.5 sec.)

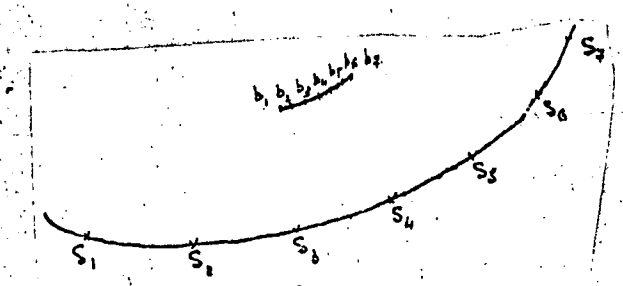

fig 4

Since localization is made on the base S_1 S_3 and restitution then requires only two interrogations, we considered all possible combinations of the other interrogations and the corresponding results are shown in Table 3.

TABLE NO. 3

Vp = 0,00000700 rad/sec Vm = 0,00000500 rad/sec

	S_2S_4	S_2S_5	S_2S_6	S_2S_7	S_4S_5
Vp	0,00000550	0,00000647	0,00000667	0,00000675	0,00000918
Vm	0,00001176	0,00000688	0,00000586	0,00000547	0,00000214
Vp	S_4S_6	S_4S_7	S_5S_6		
Vp	0,00000837	0,00000798	0,0000750		
Vm	0,00000425	0,00000528	0,00000507		

We should immediately point out that the spacing of the bases definetely has an influence on the restituted components of the wind. Diagram 4 reproduces the curves representing Vp and Vm as a function of the base utilized. The error is a direct function of the spacing of the bases to a very appreciable degree. For the base S_2 S_4, the restituted components of the wind are very erroneous whereas the base S_2 S_7 comes to within better than 10% of the real components of the wind and restitutes the position of the balloon with the ionospheric errors within better than 3km.

The increase of the interval between interrogation reduces in a very appreciable manner the errors of localization due to ionospheric errors (cf. Memorandum No. 13) so that we could expect that a large spacing of the bases would sensibly diminish the errors. On the other hand, by referring to diagram No. 4, the asymptotic trend of the curves will be noted which appears to tend toward the true positions at the rate at which the spacing at the bases increases.

The problem of the initial selection of the first base of localization does not appear in such evident fashion. However, if we desire to profit from the most distant base, we must reject a priori the interrogations at the midpoint of the zone of visibility. Although the independence of the ionospheric error as a function of the geometric trajectory appears confirmed, the method consisting in utilizing the first two and the last two interrogations appears to be the most satisfactory in view of the sensitivity of the error of localization to the interval between interrogation T. It would be possible to

utilize the first and the last interrogation as support of the base serving for the comparison of the measured and calculated magnitudes and the second and next to last interrogation as base of localization or vice versa. On the other hand, the bases thus obtained are approximately centered so that we are freed of the large amount of the error of localization consequent on eccentric bases.

VII. Conclusion

With the aid of four suitably selected interrogations, we can therefore restitute with satisfactory approximation the continuous components of the wind taking into account ionospheric errors. The selection of centered and widespaced bases appreciably reduces the ionospheric errors and brings the error of localization within acceptable limits (on the order of 2 to 3 km for a 200 - km/hr wind) where the continuous components of the wind are restituted within 8%. However, it should be carefully noted that such performances are essentially dependent on the bases utilized. In order to justify the conclusions of this note in regard to the selection of these bases, we will investigate in the subsequent memorandum the influence of the various parameters (interval between interrogation T, position of the base, distance from the trace) on the ionospheric errors, on the errors due to balloon altitude, and to the inaccuracies in the ephemerides.

In order to carry out a satisfactory restitution of the continuous components of the wind, it is necessary to dissociate the errors due to the wind from the other errors and very specifically the ionospheric errors. The increase of the interval between interrogation

for centered bases reduces the influence of the ionospheric errors and considerably increases the effect of the wind. The increase of the ratio of signal/noise = error due to wind/ionospheric error provides for a better restitution of the components of the wind and consequently definitely improves the localization.

ANNEX I - RESTITUTION OF WIND - RESTITUTION OF LOCALIZATION

PAF

$_i$ = 10 C D P V F L Z H J O G A X K N

U = S/6400

G = √(398599/S^3)

W = (1+U)/2U

Q = 1/U

Y = GT

1 Read C_1 C_2 Z_5 Z_6 S T M B

2 Calculate U G W Q Y

3 State $H_5=C_1$ $H_6=C_2$ $V_5=C_1$ $V_6=C_2$ $F_5=0$ $F_6=0$ $F_2=0$ $P_4=1$ $L_2=0$ $G_1=0$

4 State $P_3=1$ $H_3=C_1$ $H_2=C_2$ $H_1=C_1$

5 If T<0 go to 90

6 State (13440)=C_1 (13441)=C_2

7 Program 100,0

8 State D_1=(13449) D_2=(13450) E=(13454)

9 Calculate $F_3=F_2-E$

10 Calculate $F_4=L_2-D_2-B$

11 If $|F_3|<0,00000002$ SI $|F_4|<0,00000002$ go to 28

12 If $P_3>11$ go to 28

13 Make $P_3=P_3+1$

14 Calculate $L_2=D_2+B$

15 State $F_2=E$

16 Calculate O_2= ARC SIN ((COS F_2)(SIN L_2)(SIN M)+(SIN F_2)(COS M))

17 Calculate J_2= ARC TG (((COS M)(COS F_2)(SIN L_2)-(SIN M)(SIN F_2))/(COS F_2)(COS L_2))

ANNEX I (Continued)

18 Calculate $J_1=J_2-0,0000768T-Z_5T/\cos O_2$

19 Calculate $O_1=O_2-Z_6T$

20 Calculate $L_1=\text{ARC TG}(((\cos M)(\sin J_1)(\cos O_1)+(\sin M)(\sin O_1))/(\cos J_1)(\cos O_1))$

21 State $L_1=L_1$

22 Calculate $F_1=\text{ARC COS}((\cos J_1)(\cos O_1)/\cos L_1)$

23 Calculate $C_1=(\cos F_1)(\sin(L_1+Y-B))/\sqrt{(1+U^2-2U(\cos F_1)(\cos(L_1+Y-B)))}$

24 Calculate $C_3=H_3+H_1-C_1$

25 State $C_1=C_3$ $H_3=C_3$

26 Go to 5

27 Go to 5

28 If $G\geqslant 0$ go to 36

29 Print with 6 DEC RC F_2 TAB L_2 TAB G RC

30 Calculate $J_3=F_5-F_2$

31 Calculate $J_4=F_6-L_2$

32 State $F_5=F_2$ $F_6=L_2$

33 If $|J_3|\leqslant 0,000002$ SI $|J_4|\leqslant 0,000002$ go to 50

34 If $P_4\geqslant 7$ go to 50

35 Make $P_4=P_4+1$

36 State $Z_1=H_1$ $Z_2=C_2$ $I=1$

37 Calculate $C_i=Z_i+((-(0,0000768(\cos O_i)+Z_5)(\sin J_i)-Z_6(\sin O_i)(\cos J_i))((\cos F_i)(\cos L_i)-U\cos(B-Y))+((0,0000768(\cos O_i)+Z_5)((\cos J_i)(\cos M))-Z_6(\sin O_i)(\sin J_i)(\cos M)+Z_6(\cos O_i)(\sin M))((\cos F_i)(\sin L_i)-U\sin(B-Y))+(Z_6(\cos O_i)(\cos M)+Z_6(\sin J_i)(\sin O_i)(\sin M)-(0,0000768(\cos O_i)+Z_5)(\sin M)(\cos J_i))(\sin F_i))/GU\sqrt{(1+U^2-2U(\cos F_i)(\cos L_i)(\cos(B-Y))-2U(\cos F_i)(\sin L_i)(\sin(B-Y)))}$

38 State $Y = 0$

39 Make $I=I+1$

ANNEX I - (Continued)

```
40  If I ≤ 2 go to 37
41  Calculate Y=G1T
42  State G=-G
43  If G ≤ 0 go to 4
44  State V3=C1 V4=C2
45  Calculate C1=V5+H5-V3
46  Calculate C2=V6+H6-V4
47  State V5=C1 V6=C2
48  Calculate Y=G1T
49  Go to 4
50  Calculate I=X0+1
51  State A1=L2
52  Print with 8 DEC RC A1
53  State N1=J2 K1=O2 X0=I
54  Print with 8 DEC RC N1 TAB K1
55  If I ≥ 9 go to 57
56  Go to 1
57  Calculate X1=0,00000350(N4-N1)/(N4-N1-N5+N2)
58  Calculate X2=0,00000350(K4-K1)/(K4-K1-K6+K3)
59  Calculate X3=0,00000350(N7-N4)/(N7-N4-N8+N5)
60  Calculate X4=0,00000350(K7-K4)/(K7-K4-K9+K6)
61  Calculate X5=X1X3(X2-X4)/(X3X2-X4X1)
62  Calculate X6=X2X4(X3-X1)/(X3X2-X4X1)
63  Print with 8 DEC RC X5 TAB X6
64  Go to 1
```

ANNEX I (Continued)

```
90   State Y=|Y|
91   State (13440)=C2 (13441)=C1
92   Interrogation
93   Program 100,0
94   State D1=(13450) D2=(13449) E=(13454)
95   State Y=-Y
96   Go to 9
98   State X0=0
99   Go to 1
100  END
```

VERIFICATION PROGRAM

```
101  Read F2 L2 Z5 Z6 S T M B
102  Print 8 DEC RC S TAB T TAB M RC
103  Calculate U G W Q Y
104  Calculate O2= ARC SIN (( COS F2)( SIN L2)( SIN M)+( SIN F2)( COS M)
105  Calculate J2= ARC TG ((( COS M)( COS F2)( SIN L2)-( SIN M)( SIN F2)
     )/( COS F2)( COS L2))
106  Calculate J1=J2-0,00007268T-Z5T/ COS O2
107  Calculate O1=O2-Z6T
108  Calculate L1= ARC TG ((( COS M)( SIN J1)( COS O1)+( SIN M)( SIN O1))/
     ( COS J1)( COS O1))
```

ANNEX I (Continued)

SF1ER5N5	110,0M	EV15	EV51	A2EM105,1
SF1M105,12	EV24	EV46	EV15	IF1E0F8
RV46ΔP1	EV25	EV23	EV54	EV30
ARC1R6	EV44	EV33	EV53	EV35
T1EM105,14	EV37	EV24	EV25	EV23
T6EM182,91	EV15	X	EV30	EV56
RM105,30	EV34	EV37	EV35	EV24
X	EV33	EV51	EV23	EV14
X	EV44	EV52	EV55	IF2E0F8
X	EV44	EV15	EV24	RM180,1
X	EV47	EV48	EV14	X
FE	EV46	EV33	A1EM105,0	

SECOND METHOD OF LOCALIZATION

PAF

i = 10 C D P V F L Z H J O G A X K N

U = S/6400

G = √(398599/S^3)

W = $(1+U^2)/2U$

Q = 1/U

Y = GT

1 State X=0 C_3=-0,35826262 C_4=-0,63251829

2 Read J_2 O_2 Z_5 Z_6 S T M B

3 Calculate U G W Q Y

4 Calculate J_1=J_2-0,0000268T-Z_5T/ COS O_2

5 Calculate O_1=O_2-Z_6T

6 Calculate L_1= ARC TG (((COS M)(SIN J_1)(COS O_1+(SIN M)(SIN O_1)/(COS J_1)(COS O_1))

7 Calculate F_1= ARC COS ((COS J_1)(COS O_1)/ COS L_1)

SECOND METHOD OF LOCALIZATION (Continued)

8 Calculate $C_1=(\cos F_1)(\sin(L_1+Y-B))/\sqrt{(1+U^2-2U(\cos F_1)(\cos(L_1+Y-B))}$

9 State $Z_1=C_1$ $I=1$

10 Calculate $C_i=Z_i+((-(0,0000)7268(\cos O_i)+Z_5)(\sin J_i)-Z_6(\sin O_i)(\cos J_i))((\cos F_i)(\cos L_i)-U\cos(B-Y))+((0,0000)7268(\cos O_i)+Z_5)((\cos J_i)(\cos M))-Z_6(\sin O_i)(\sin J_i)(\cos M)+Z_6(\cos O_i)(\sin M))((\cos F_i)(\sin L_i)-U\sin(B-Y))+(Z_6(\cos O_i)(\cos M)+Z_6(\sin J_i)(\sin O_i)(\sin M)-/0,0000)7268(\cos O_i)+Z_5)(\sin M)(\cos J_i))(\sin F_i))/-GU\sqrt{(1+U^2-2U(\cos F_i)(\cos L_i)(\cos(B-Y))-2U(\cos F_i)(\sin L_i)(\sin(B-Y)))}$

11 Calculate $I=X_0+1$

12 State $N_i=C_1$ $X_0=I$

13 Print with 8 DEC RC N_i RC

14 If $I \geq 3$ go to 16

15 Go to 2

16 If $T \leq -200$ go to 21th

17 Calculate $X_1=0,00000350(N_1-C_3)/(N_1-N_2)$

18 Calculate $X_2=0,00000350(N_1-C_3)/(N_1-N_3)$

19 State $X_0=0$

20 Go to 2

21 Calculate $X_3=0,00000350(N_1-C_4)/(N_1-N_2)$

22 Calculate $X_4=0,00000350(N_1-C_4)/(N_1-N_3)$

23 Calculate $X_5=X_1X_3(X_2-X_4)/(X_3X_2-X_4X_1)$

24 Calculate $X_6=X_2X_4(X_3-X_1)/(X_3X_2-X_4X_1)$

25 Print with 8 DEC RC X_1 TAB X_2 TAB X_3 TAB X_4 RC

26 Print with 8 DEC X_5 TAB X_6

27 END

SECOND METHOD OF LOCALIZATION (Continued)

109 Calculate F_1= ARC COS ((COS J_1)(COS Q_1)/ COS L_1)

110 Calculate C_1=(COS F_1)(SIN (L_1+Y-B))/(1+U^2-2U(COS F_1)(COS (L_1+Y-B)))

111 Calculate C_2=(COS F_2)(SIN L_2)/(1+U^2-2U(COS F_2)(COS L_2))

1129 State Z_1=C_1 Z_2=C_2 I=1

113 Calculate C_i=Z_i+((-(0,00007268(COS O_i)+Z_5)(SIN J_i)-Z_G(SIN O_i)(COS J_i)
)((COS F_i)(COS L_i)-U COS (B-Y))+((0,00007268(COS O_i)+Z_5)((COS J_i)(COS M))
-Z_G(SIN O_i)(SIN J_i)(COS M)+Z_G(COS O_i)(SIN M))((COS F_i)(SIN L_i)-U SIN (
B-Y))+(Z_G(COS O_i)(COS M)+Z_G(SIN J_i)(SIN O_i)(SIN M)-(0,00007268(COS O_i)
+Z_5)(SIN M)(COS J_i))(SIN F_i))/-GU/(1+U^2-2U(COS F_i)(COS L_i)(COS (B-Y))-2U(COS
F_i)(SIN L_i)(SIN (B-Y)))

114 Interrogation

115 State Y=0

116 Make I=I+1

117 If I≤ 2 go to 113

118 Print with 8 DEC C_1 TAB C_2 TAB L_2 TAB F_2 RC

119 State T=-T

120 Go to 103

100,OM	A4EM182,94	SF6R5	A1EM105,22+	RV10,0ΔN7
T12M105,30	A5ER2	SF6R3	AF1R2	A5EM182,96
A1EM105,0	MF5R4	T6M105,21	QF1R6	SF5R6
A2ER1	T5M105,22	SF2ER6	SF2ER1	QF5V2
MF2R1	MF5R4	A5EV0	AF2EV1	A1ER7
A1EM105,1	A6ER5	A3ER1	RV10,0ΔN2	MF1R5
A3ER1	MF6R2	SF1R2	ARC2R1	X
MF3R1	SF6R5	A6ER3P1	A1EM105,0+	RV82ΔP1
A1ER2	SF6R2	A6ER2N1	A1ER2P1	SF1ER5
SF1R3	T6M105,20	A1ER6	SF1ER2N1	QF1V2
A1ER3N1	X	YF6R1	T1M105,9+	A5ER1
A1ER2P1	SF1ER6	A15EV0	RV67ΔY15	AF6R5
X	A5ER3	A1EM105,20+	A15EV1	A1ER7P7
A4EM182,90	MF5R4	A2ER6	RV47Δ	SF1ER7N7
A5ER4	T5M105,23	MF2R6	A7EM105,9	SF1M105,11
MF5R4	MF5R4	AF1R2	SF7M105,10	X
SF5R1	A6ER5	A1EV0N1	SF7M182,98	RV92ΔN1
RV0,0ΔN5	MF6R3	YF2R1	RV75ΔY5	A1ER5P5

ANNEX II

$C_1 = 0,53763151$ $C_2 = 0,15054304$
$C_3 = -0,35723468$ $C_4 = -0,62966364$ } $\cos \beta_{im}$

Position of Balloon

$\varphi_2 = F_2 = 0,261801$ $L_2 = 0,0383957$
$V_b = 0,00000500$ $V_m = 0,00000700$

Wind

F_2= 0,269975	L_2= 0,039175	
F_2= 0,269358	L_2= 0,038512	
F_2= 0,269358	L_2= 0,038512	Localization 1
A_1= 0,03851151		
N_1= 0,16274589	K_1= 0,21895238	
F_2= 0,267992	L_2= 0,038374	
F_2= 0,267317	L_2= 0,037652	Localization 2
F_2= 0,267318	L_2= 0,037654	
A_2= 0,03765368		
N_2= 0,16185602	K_2= 0,21692967	
F_2= 0,267904	L_2= 0,039722	
F_2= 0,267042	L_2= 0,039068	Localization 3
F_2= 0,267041	L_2= 0,039070	
A_3= 0,03906953.		
N_3= 0,16067090	K_3= 0,21770575	
F_2= 0,267977	L_2= 0,038871	
F_2= 0,267869	L_2= 0,038339	
F_2= 0,267873	L_2= 0,038341	Localization 4
F_2= 0,267873	L_2= 0,038341	
A_4= 0,03834092		
N_4= 0,16178245	K_4= 0,21779094	
F_2= 0,266466	L_2= 0,038119	
F_2= 0,266357	L_2= 0,037544	
F_2= 0,266358	L_2= 0,037546	Localization 5
A_5= 0,03754580		
N_5= 0,16123181	K_5= 0,21617990	

F_2= 0,266180	L_2= 0,039458	
F_2= 0,265914	L_2= 0,038935	
F_2= 0,265915	L_2= 0,038936	Localization 6
A_6= 0,03893649		
N_6= 0,15994328	K_6= 0,21682217	
F_2= 0,269469	L_2= 0,038923	
F_2= 0,269902	L_2= 0,038576	Localization 7
F_2= 0,269902	L_2= 0,038575	
A_7= 0,03857493		
N_7= 0,16309782	K_7= 0,21937796	
F_2= 0,267409	L_2= 0,038085	
F_2= 0,267925	L_2= 0,037723	
F_2= 0,267922	L_2= 0,037722	Localization 8
F_2= 0,267922	L_2= 0,037722	
A_8= 0,03772163		
N_8= 0,16224846	K_8= 0,21740102	
F_2= 0,267096	L_2= 0,039445	
F_2= 0,267271	L_2= 0,039095	Localization 9
F_2= 0,267273	L_2= 0,039095	
A_9= 0,03909540		
N_9= 0,16082130	K9= 0,21788610	
X_5= 0,00000512	X_6= 0,00000709	Wind restituted
F_2= 0,262907	L_2= 0,039103	
F_2= 0,261679	L_2= 0,038380	
F_2= 0,261671	L_2= 0,038380	
F_2= 0,261674	L_2= 0,038381	Balloon position restituted
F_2= 0,261674	L_2= 0,038381	

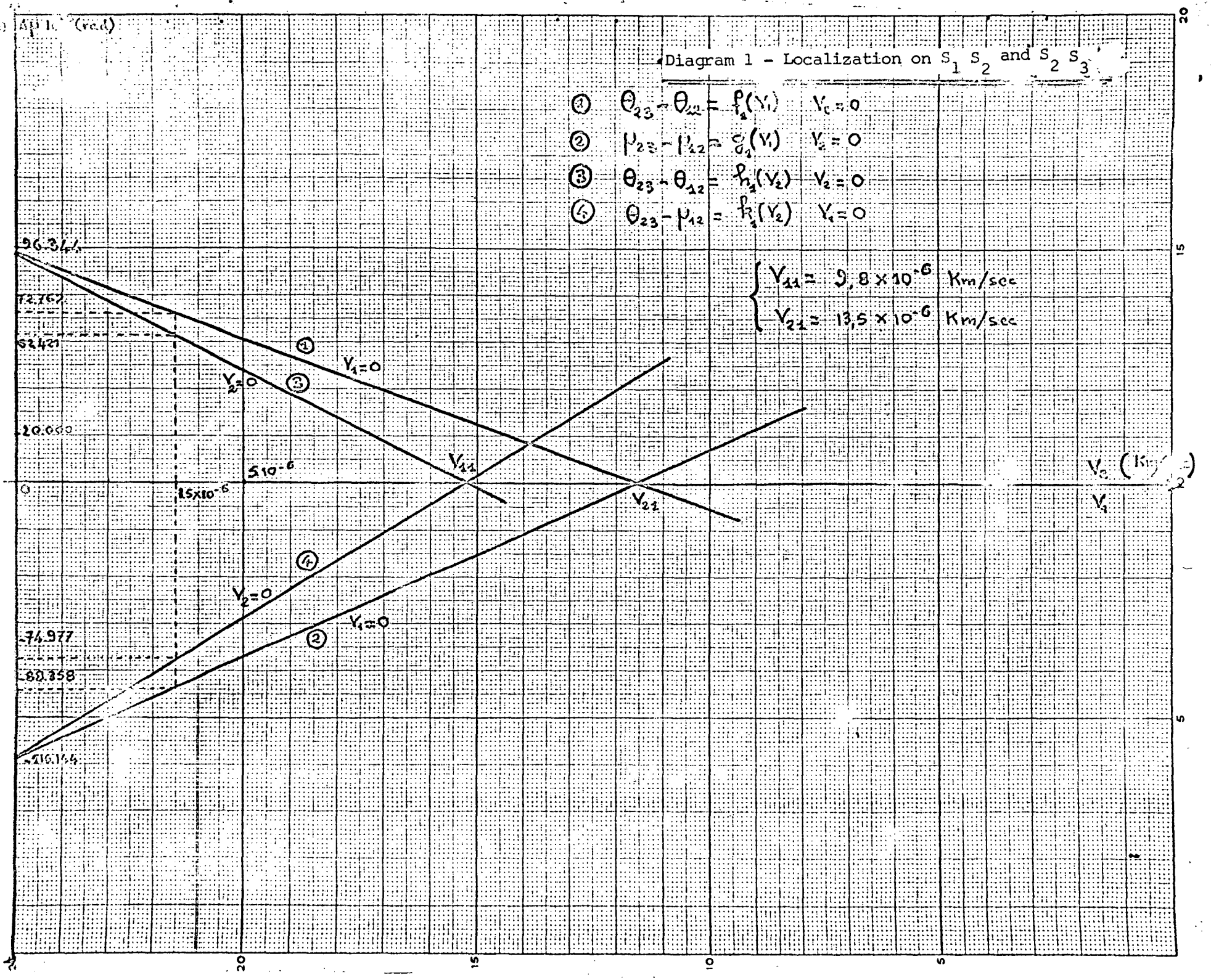

Diagram 1 - Localization on S_1 S_2 and S_2 S_3
① $\theta_{23} - \theta_{12} = f_1(V_1)$ $V_2 = 0$
② $\mu_{23} - \mu_{12} = g_1(V_1)$ $V_2 = 0$
③ $\theta_{23} - \theta_{12} = h_1(V_2)$ $V_2 = 0$
④ $\theta_{23} - \mu_{12} = k_1(V_2)$ $V_1 = 0$
$V_{11} = 9,8 \times 10^{-6}$ Km/sec
$V_{21} = 13,5 \times 10^{-6}$ Km/sec
96.344
72.767
62.421
20.060
0
-74.977
-80.358
-216.144
$5 \cdot 10^{-6}$
15×10^{-6}
$V_1 = 0$
$V_2 = 0$
V_{11}
V_{21}
V_2 (Km/sec)
V_1
20
15
10
5

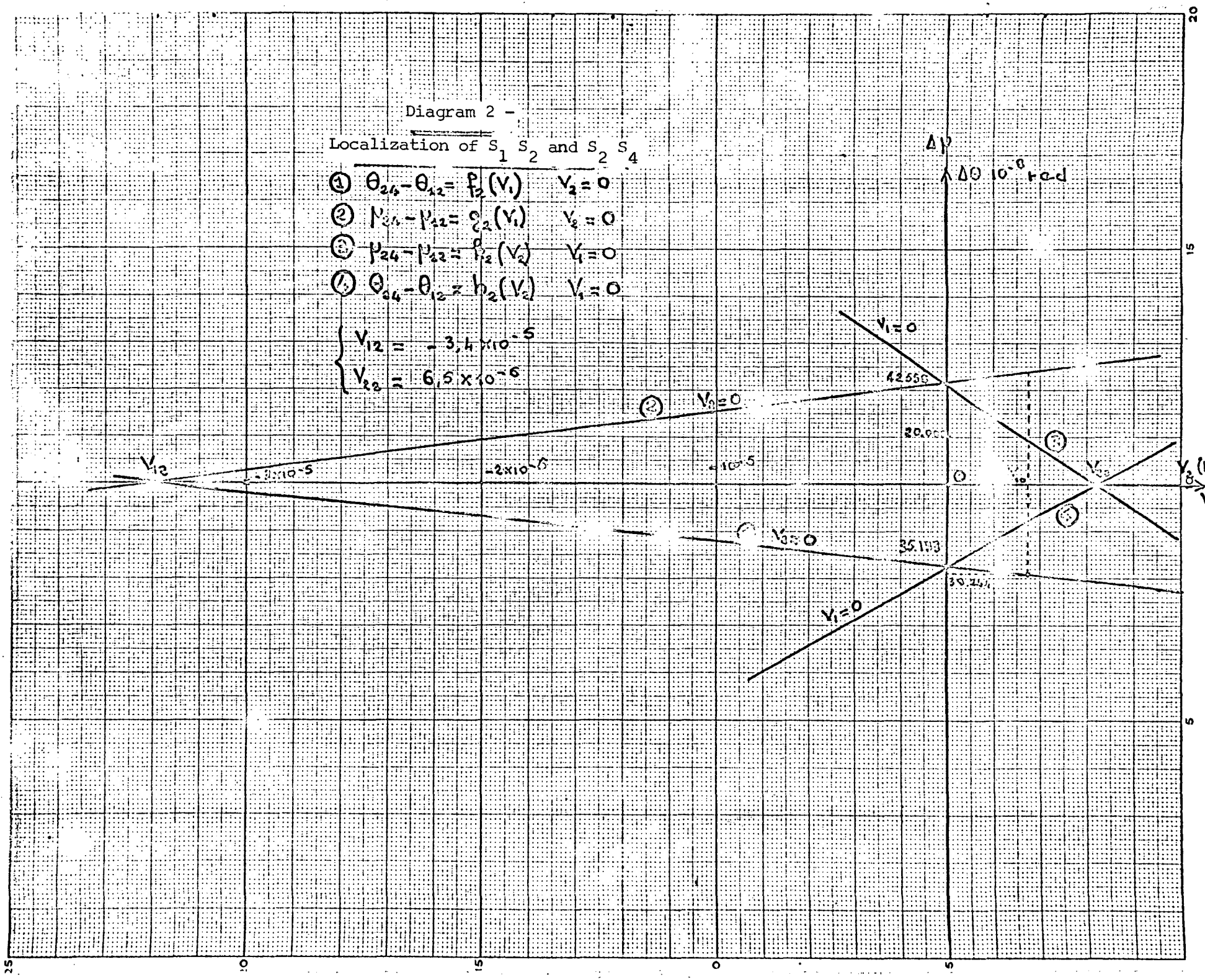

Diagram 2 -

Localization of S_1 S_2 and S_2 S_4

① $\theta_{24} - \theta_{12} = f_2(V_1)$ $V_2 = 0$

② $\mu_{24} - \mu_{12} = g_2(V_1)$ $V_2 = 0$

③ $\mu_{24} - \mu_{12} = k_2(V_2)$ $V_1 = 0$

④ $\theta_{24} - \theta_{12} = h_2(V_2)$ $V_1 = 0$

$$\begin{cases} V_{12} = -3{,}4 \times 10^{-5} \\ V_{22} = 6{,}5 \times 10^{-6} \end{cases}$$

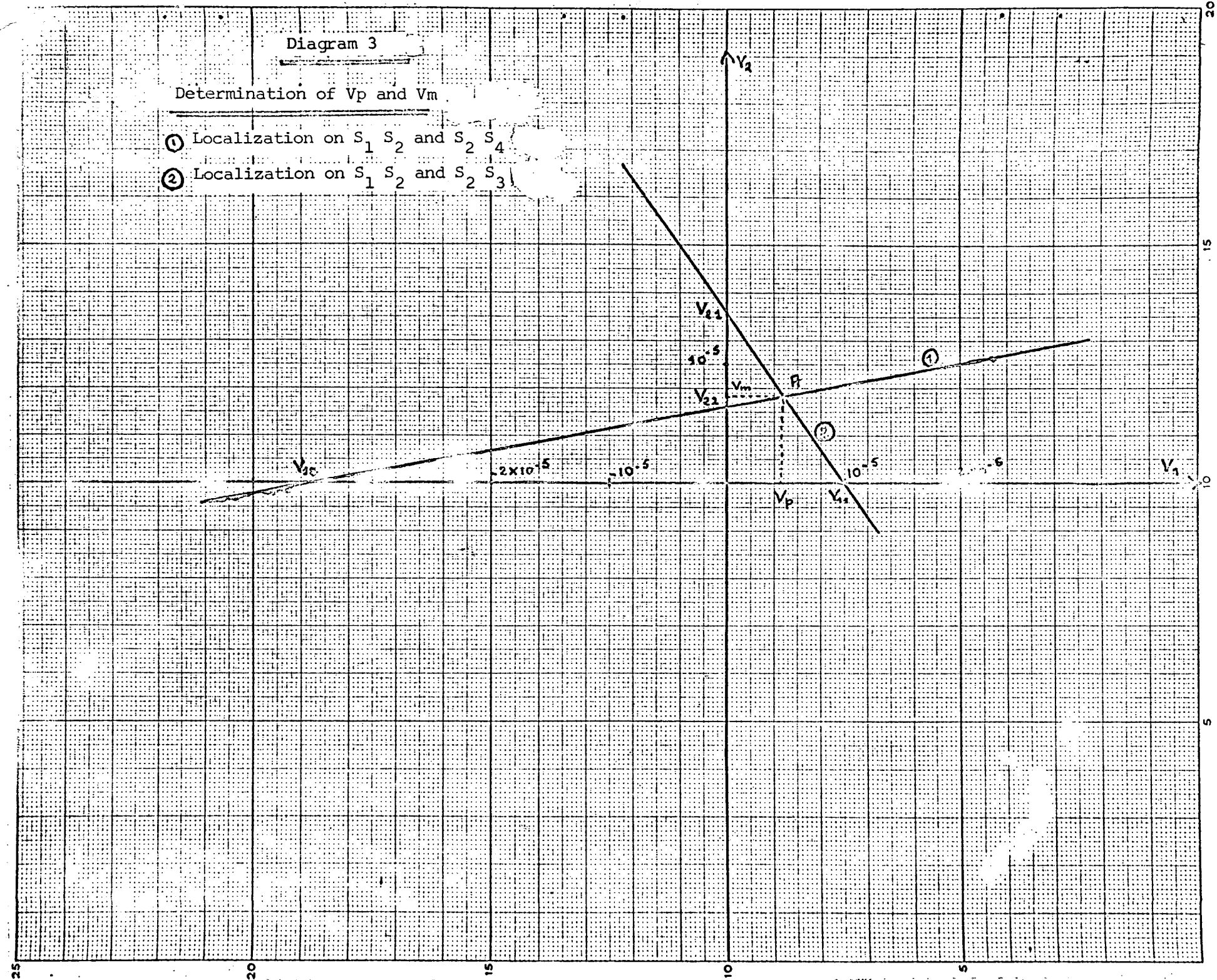
Diagram 3
Determination of Vp and Vm
① Localization on S_1 S_2 and S_2 S_4
② Localization on S_1 S_2 and S_2 S_3
V_2
V_{21}
10^{-5}
V_m
V_{22}
P
①
②
V_{12}
2×10^{-5}
10^{-5}
10^{-5}
V_1
V_p
V_{11}
25
20
15
10
5
20
15
10
5

Diagram 4
Influence of ionospheric errors
Base of localization
1
2