NASA TECHNICAL TRANSLATION NASA TT F- 14,703

ERRORS DUE TO TERRESTIAL ROTATION
ERRORS DUE TO INHERENT BALLOON SPEED
COMPLETE RESTITUTION OF LOCALIZATION (DOPPLER)

Centre National d'Etudes Spatiales

Translation of Memorandum No. 11/65, Centre National d'Etudes Spatiales, 368, March 1966, 32 pages.
Toulouse, France

(NASA-TT-F-14703) ERRORS DUE TO TERRESTRIAL ROTATION. ERRORS DUE TO INHERENT BALLOON SPEED. COMPLETE RESTITUTION OF LOCALIZATION (DOPPLER) (NASA) Sep. 1972 33 p CSCL 17G G3/21
N73-12679
Unclas
47144

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 SEPTEMBER 1972

#14

ADVANCED SCHOOL OF AERONAUTICS

Automation Research Center

CNES 368 - March 1966

Memorandum No. 11/65

ERRORS DUE TO TERRESTRIAL ROTATION

ERRORS DUE TO INHERENT BALLOON SPEED

COMPLETE RESTITUTION OF LOCALIZATION (DOPPLER)

TABLE OF CONTENTS

		Page Number
I.	ERRORS DUE TO TERRESTRIAL ROTATION AND TO INHERENT BALLOON SPEED	3
II.	CORRECTION OF THE ERROR DUE TO TERRESTRIAL ROTATION	5
	2.1 Notation	5
	2.2 Principle of the Method	6
	2.3 Formulation	9
	2.4 Convergence of the iterative procedure and results	13
III.	COMPLETE RESTITUTION OF LOCALIZATION	14
	3.1 Notations	14
	3.2 Principle of the Method	15
	3.3 Formulation	19
	3.4 Convergence of the Process and Results	20
	3.5 Data Restitution program (Doppler effect)	23
IV.	CONCLUSION	29

I. ERRORS DUE TO TERRESTRIAL ROTATION AND TO INHERENT BALLOON SPEED

The principle of localization described in the preceding memorandum consisted in attempting to find the intersection of the two cones with the respective vertex, and the position of the satellite at the instants t and t + T where the velocity vectors of the satellite at these two instants constitute the axes of the two cones, and where β_1 and β_2 define the angles at the vertex of the two cones of revolution.

Such a localization defines the position of a stationary balloon during the interval between interrogation and implicitly assumes as known through Doppler measurement the angles β_1 and β_2 of the two cones. Such a succinct localization is affected by very large errors and it becomes necessary, with the aid of iterative procedures, to correct the systematic errors (1) in the displacement of the balloon during the interval between interrogation T, and (2) in the angles of β_1 and β_2 of the two cones consecutive to the speed of the balloon in its motion toward or away from the balloon.

During the time T, the balloon is carried along by the rotation of the earch and describes an arc $B_1 B_2$ whose length depends on the latitude (cf. Figure 1) so that the preliminary localization is affected by systematic error and effectively furnishes the position of the balloon at B_c.

On the basis of this first estimate, an iterative procedure will be outlined in the next paragraph which makes it possible to rigorously determine the effective positions of the balloon at the two instants.

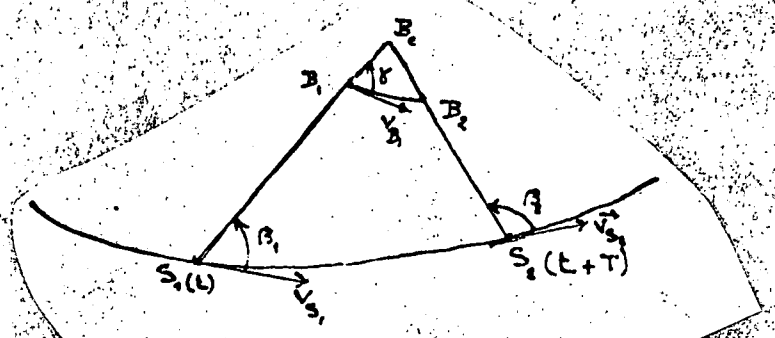


Figure 1

On the other hand, since the Doppler measurement in fact defines the radial velocity of the satellite in relation to the balloon, it is necessary to take into account the instantaneous speed of the balloon consecutive to its rotational motion. If we design as γ the angle between the velocity vector of the balloon and $\overrightarrow{S_1 B_1}$ as V_{B1} the inherent speed of the balloon, we have the relation

$$V_{S_1} \cos \beta_m = V_{S_1} \cos \beta_1 - V_{B_1} \cos \gamma$$

which defines, from the known position of the balloon, the angle of the cone as a function of the angle β_m defined by the Doppler measurement. We therefore find necessary a second process of information processing and the two iterative procedures together constitute the general problem of the restitution of localization.

II. CORRECTION OF THE ERROR DUE TO TERRESTRIAL ROTATION

2.1 Notation

We designate as $B_{ij} \begin{cases} i = 1,2 \\ j = 1,2 \ldots\ldots m \end{cases}$ the successive positions of the balloons localized during the entire iterative procedure.

The first index indicates the position of the balloon on which iteration is carried out:

$i = 1$: the positions B_{1j} converge at the end of the procedure toward B_1 which is the true position of the balloon at the instant t;

$i = 2$: the positions B_{2j} converge at the end of the procedure toward B_2 which is the true position of the balloon at the instant t + T.

The second index indicates the order of iteration:

$j = 1$: first iteration;

$j = 2$: second iteration.

We designate as β_{ij} the angle defined by the relation

$$\beta_{ij} = \left(\overrightarrow{S_1 B_{ij}}, \overrightarrow{V_{S_1}}\right)$$

Where S_1 is the position of the satellite at the instant $\underline{t}$ and $\vec{V}_{S_1}$ is the velocity vector of the satellite at the same instant. All angles β_{ij} must be counted starting with the position S_1 of the satellite. Only the angle $\beta_{2m} = (\vec{S_2 B_2}, \vec{V}_{S_2})$ is counted starting from S_2. We designate as b_{ij} the projections of the point B_{ij} on the orbital plane.

2.2 Principle of the Method

We attempt to find as a first stage an iterative method similar to that exposed in memorandum no. 5 exclusively for compensating the influence of terrestrial rotation. Let us designate as B_{21} the balloon as localized with the aid of the two measured values

$$\cos\beta_{1m} = \cos\beta_{21} \quad \text{et} \quad \cos\beta_{2m}$$

If we do not take into account the speed of the balloon, these values correspond to the angles of the cones. Let us designate as B_1 and B_2 the true positions of the balloon at the two instants of interrogation (t and t + T). The point B_{21} is projected by b_{21} at the intersection of the two corresponding parabolas, b_1 and b_2 are the projection of B_1 and B_2 and are at the intersection of the two parabolas with the ellipse which is the projection of the parallel of the balloon in the orbital plane (cf. Figure 2). The point b_{21} furnishes a first estimate of the position b_2. On the basis of b_{21}, we define the position b_{11} of the balloon at the instant $\underline{t}$

by taking into account terrestrial rotation. A parabola with vertex S_1 passes through this point and is characterized by $\cos\beta_{11} = (\overrightarrow{S_1B_{11}}, \overrightarrow{V_{S_1}})$. The point b_{11} then furnishes a first estimate of the true position b_1 and the magnitude $\cos\beta_{1m} - \cos\beta_{11} = \cos\beta_{21} - \cos\beta_{11}$ furnishes a first estimate of the error made in the cosine of the angle.

We start again with the localization of the balloon on the basis of the magnitudes

$$\cos\beta_{2m} \quad \text{et} \quad \cos\beta_{22} = \cos\beta_{21} + (\cos\beta_{21} - \cos\beta_{11})$$

This second localization defines a balloon B_{21}. On the basis of this position, we define the position B_{12} of this balloon at the instant t and obtain from this a second estimate of the error made $(\cos\beta_{21} - \cos\beta_1)$ and the third localization is made on the basis of the magnitudes

$$\cos\beta_{2m} \quad \text{and} \quad \cos\beta_{23} = \cos\beta_{22} + (\cos\beta_{21} - \cos\beta_{12})$$

In a general manner, the iterative procedure is defined by the recurrent relation

$$\cos\beta_{2n} = \cos\beta_{2,n-1} + (\cos\beta_{21} - \cos\beta_{1,n-1}) \quad \text{with} \quad n \geq 2$$

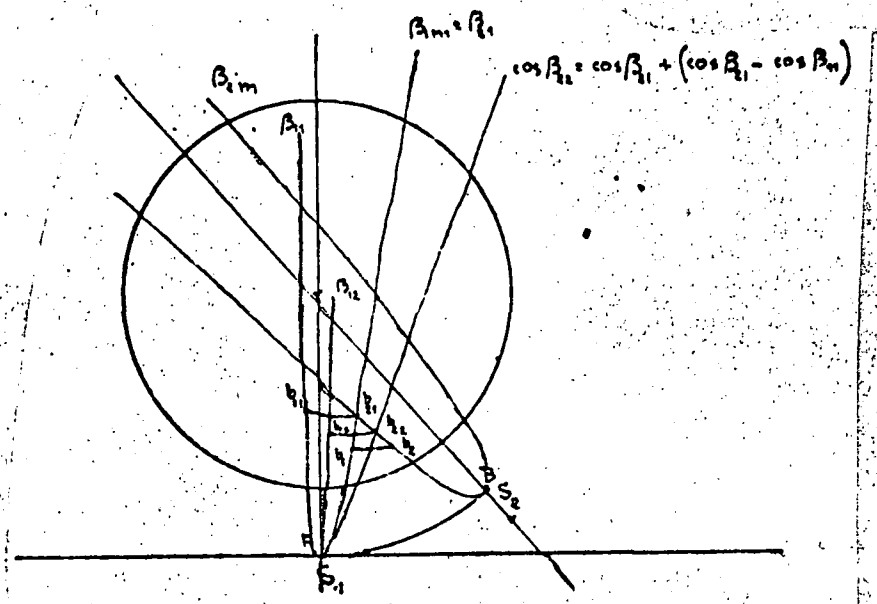


Figure 2.

Such a procedure shows that, when the term of error $\cos\beta_{21} - \cos\beta_{1,n-1}$ tends toward zero, $\beta_{1,n-1} = \beta_{21} = \beta_{1,n}$ and then $\beta_{2n} = \beta_{2m}$

the points b_{1j} tend toward the true position of the balloon b_1; the points b_{2j} tend toward the true position of the balloon b_2.

The iterative procedure can be schematized in the block diagram shown in Figure 3 where the operations indicated are made in sequence

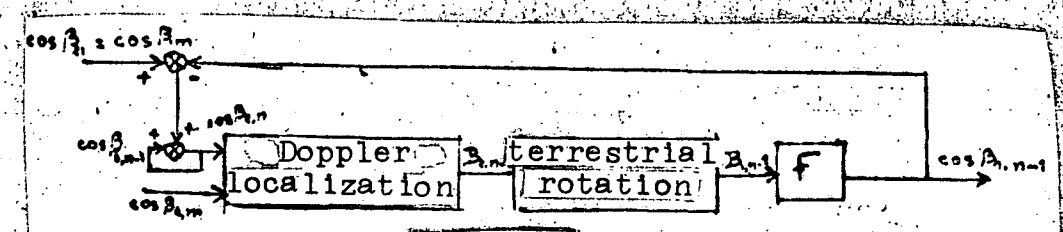


Figure 3

The magnitudes indicated in the Figure are those obtained prior to the Doppler localization with the aid of $\cos \beta_{2,n}$ and $\cos \beta_{2,m}$

2.3 Formulation

Since the Doppler localization and completely defined in the preceding memorandum, the complete solution of the problem requires the localization of the balloon $B_{1,n-1}$ from the known position of the balloon $B_{2,n-1}$

In the solution of the problem we utilize the following trihedra:

- the inertial trihedron XYZ where OZ passes through the axis of the poles and OX is an inertial axis of reference passing through the vernal point, and the trihedron x, y, z based on the orbital plane where

  Ox is in the direction of the ascending node,

  Oy is perpendicular to Ox in the orbital plane, and

  Oz is in the direction of the vector $\vec{\Omega}_s$ and

where the orbital plane is identified by its inclination $\underline{i}$ and the inertial longitude Ω of the ascending node.

Without detracting from the general character of the problem, we can neglect, in regard to localization, the advance of the perigee and the orbital precision. We can then take, as inertial axis OX, the axis ox which amounts to starting $\Omega = 0$

In general, we designate as φ_i the latitude of the balloon B_i in relation to the orgibal plane, and as λ_i the angle $(\vec{OS_i}, \vec{Ob_i})$ by designating as S_1 and S_2 the positions of the satellite at the instants t and t + T (cf. Figure 4). Let us designate as α the angle $\alpha = (\vec{Ox}, \vec{OS_1})$ which locates the satellite on its orbit. We deduce from this

$$(\vec{Ox}, \vec{Ob_2}) = \alpha + \lambda_2 = \varphi_2$$

$$(\vec{Ox}, \vec{Ob_1}) = \alpha + \lambda_1 - \Omega_3 T = \varphi_1$$

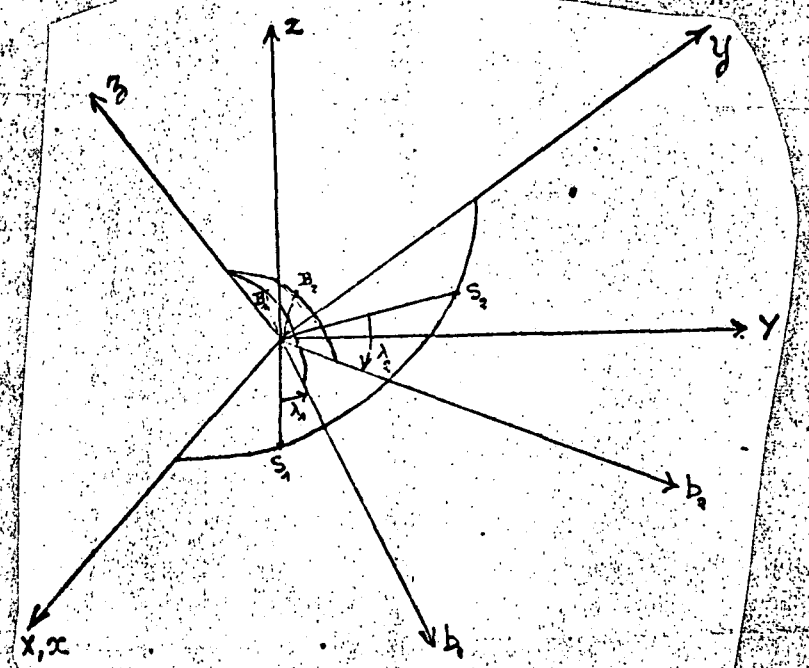


figure 4

The coordinates of the balloon B_2 in relation to the axis OXYZ are

$$\text{I} \quad \begin{cases} X_2 = R \cos \varphi_2 \cos(\lambda_2 + \alpha) \\ Y_2 = R\left(\cos i \cos \varphi_2 \sin(\lambda_2+\alpha) - \sin i \sin \varphi_2\right) \\ Z_2 = R\left(\sin i \cos \varphi_2 \sin(\lambda_2+\alpha) + \cos i \sin \varphi_2\right) \end{cases}$$

We can state these expressions by designating as the inertial longitude of B_1, and as μ_i the latitude of the balloon in the form

$$\text{II} \quad \begin{cases} X_2 = R \cos \mu_2 \cos \theta_2 \\ Y_2 = R \cos \mu_2 \sin \theta_2 \\ Z_2 = R \sin \mu_2 \end{cases}$$

By identification, we deduce from this

$$\sin \mu_1 = \sin \mu_2 = \sin i \cos \varphi_2 \sin(\lambda_2+\alpha) + \cos i \sin \varphi_2 \qquad (1)$$

$$\operatorname{tg} \theta_2 = \frac{\cos i \cos \varphi_2 \sin(\lambda_2+\alpha) - \sin i \sin \varphi_2}{\cos \varphi_2 \cos(\lambda_2+\alpha)} \qquad (2)$$

From this we deduce $\theta_1 = \theta_2 - \omega_0 T$ (3) where ω_0 is the angular velocity of terrestrial rotation

$$\text{III} \quad \begin{cases} X_1 = R \cos \mu_2 \cos \theta_1 \\ Y_1 = R \cos \mu_2 \sin \theta_1 \\ Z_1 = R \sin \mu_2 \end{cases}$$

Let us now calculate the positions of B_1 in relation to the trihedron $O\,x\,y\,z$ i.e.,

$$\varphi_1 = (\overrightarrow{Ob_1}, \overrightarrow{OB_1})$$

and $\varphi_1 = (\overrightarrow{Ox}, \overrightarrow{Ob_1}) = \alpha + \lambda_1 - \Omega_s T$

With the aid of relations II and III, we deduce from this

$$\operatorname{tg} \varphi_1 = \frac{\cos i \sin \theta_1 \cos \mu_2 + \sin i \sin \mu_2}{\cos \theta_1 \cos \mu_2} \qquad (4)$$

$$\cos \varphi_1 = \frac{\cos \theta_1 \cos \mu_2}{\cos \varphi_1} \qquad (5)$$

From this we deduced

$$\cos \beta_1 = \frac{\cos \varphi_1 \sin (\Omega_s T + \varphi_1 - \alpha)}{\sqrt{(1 + u^2 - 2u \cos \varphi_1 \cos (\varphi_1 + \Omega_s T - \alpha)}} \qquad (6) \qquad u = \frac{r_0 h}{R}$$

and obtained

$$\cos \beta_2 = \frac{\cos \varphi_2 \sin \lambda_2}{\sqrt{(1 + u^2 - 2u \cos \varphi_2 \cos \lambda_2)}} \qquad (7)$$

Accordingly, the expressions 1 and 2 define the latitude and the inertial longitude of the balloon B_2 whereas the relations 4 and 5 define, with the aid of expression 3, the latitude in relation to the orbital plane and the orbital longitude of the balloon B_1 and the formulas 6 and 7 define the cosine of the angles of the cones with the respective vertices S_1 and S_2.

These six formulas therefore make it possible to calculate $\cos \beta_{1,n-1}$ at the $(n-1)^{th}$ iteration when we know the position of the balloon B_2 at the $(n-1)^{th}$ iteration $(B_{2,n-1})$.

2.4 Convergence of the iterative procedure and results

In order to verify the satisfactory convergences of the iterative process, we investigated, with the aid of the arithmetical program (Annex 1), the error of localization in accordance with the position of the base. For this purpose and for a given distance from the trace, the localization was calculated for all possible bases beginning with the base at the limit of visibility. Table 1 groups the successive results for an interval between interrogation of 195 sec and a balloon located at 15° from the trace for the case where the first interrogation takes place at the limit of visibility.

	base 1	symmetrical base	base 3	base 4	base 5
position of the base $\lambda_2 = (\vec{OS_1}, \vec{OS_2})$	0,038580	-0,0986935	-0,15881	-0,356378	-0,42000
Localization 1	176,448	143,539	131,156	77,774	54,263
localization 2	5,210	7,664	8,821	5,997	3,760
localization 3	0,163	0,372	0,589	0,481	0,220
localization 4	0,029	0,032	0,085	0,044	0,186
localization 5	0,020	0,042	0,156	0,011	0,135
localization 6	0,015	0,022	0,033	0,035	0,223
localization 7	0,004	0,022	0,033		0,064
localization 8					0,007

T = 195 sec h = 900 km φ_1 = 0,261801 rad = 15°

The convergence of the process is snown to be very satisfactory and remains on the average less than 50 m after 6 iterations which is very satisfactory. However, the limits of accuracy of the arithmetical machine, particularly in the iterative calculation of the localization of the balloon, do not make it possible to anticipate greater accuracy.

On the other hand, it will be noted that the error due to terrestrial rotation varies greatly with the base utilized and in a continuous manner. Very high (on the order of 180 km) when the satellite is within sight of the balloon, it decreases rather sharply and is on the order of 50 km when the balloon leaves the zone of visibility of the satellite. However, in spite of a very large first error, the iterative process utilized makes it possible to compensate the influence of terrestrial rotation very satisfactorily.

III. COMPLETE RESTITUTION OF LOCALIZATION

3.1 Notations

From now on B_{ij}^{*} designates the position of the balloon obtained after the iterative process of compensation for terrestrial rotation $i = 1, 2$; $j = 1, 2 \ldots n$ where first index characterizes the balloon at the instant t $(i = 1)$ or at the instant $t + T$ $(i = 2)$, and the second index defines the order of complete iteration.

$j = 1$ indicates that the first iterative process of terrestrial rotation has taken place and that the process itself consisted of n iterations; the sign * indicates that the influence of the balloon speed is not taken into account in the localization and the compensation of terrestrial rotation.

By analogy, B_{ij} designates a balloon performing under the same conditions as the foregoing and only the elimination of the sign * indicates that the speed of approach of the balloon was taken into account.

With the points B_{ij} and B_{ij}^* are associated the magnitudes $\cos \beta_{ij}$ and $\cos \beta_{ij}^*$ indicates the angle γ_{ij}^*

$\gamma_{ij}^* = (\vec{V}_{B_{ij}}, \vec{S_i B_{ij}^*})$ velocity vector of the balloon B_{ij}^*.

3.2 Principle of the Method

On the basis of the magnitudes $\cos \beta_{1m} = \cos \beta_{1m}^*$ et $\cos \beta_{1m} = \cos \beta_{2}^*$ measured by the Doppler effect at the two instants of interrogation t and t + T (magnitudes not corresponding to the cone angles), we make a first elementary localization (memorandum No. 10) followed by the iterative process of compensation of the earth defined in the preceding paragraph. At the end of this operation, we obtain the position B_{11}^* and B_{21}^* of a balloon at the instants t and t + T. When we know B_{11}^* and B_{21}^*, we can calculate the cone angles

in consideration of the speed of the balloon at these two instants or

$$\cos\beta_{11} = \cos\beta_{11}^{*} + \frac{V_{B_1}^{*}}{V_{S_2}} \cos\gamma_{11}^{*}$$

$$\cos\beta_{21} = \cos\beta_{21}^{*} + \frac{V_{B_2}^{*}}{V_{S_2}} \cos\gamma_{21}^{*}$$

We thus obtain a first estimate of the cone angles. On the basis of $\cos\beta_{11}$ and $\cos\beta_{21}$, we start on a second elementary localization and a second process of compensation of terrestrial rotation and obtain at the end of the operation the positions B_{12} and B_{22} of the balloon which furnish a first estimate of the true positions of the balloon at the two instants.

On the basis of these two positions obtained from a frist estimate of the angles of the two cones, we calculate the corresponding values measured by the Doppler effect in consideration of the speed of approach of the balloon with the aid of the two expressions

$$\cos\beta_{12}^{*} = \cos\beta_{11} - \frac{V_{B_{12}} \cos\gamma_{12}}{V_{S_1}}$$

$$\cos\beta_{22}^{*} = \cos\beta_{21} - \frac{V_{B_{22}} \cos\gamma_{22}}{V_{S_2}}$$

Reproduced from best available copy.

We thus obtain a first estimate of the errors committed which are

$$\cos\beta_{11}^{*} - \cos\beta_{12}^{*} \quad \text{and} \quad \cos\beta_{21}^{*} - \cos\beta_{22}^{*}$$

and start on the next process with the aid of

$$\cos \beta_{12}^{*} = \cos \beta_{11}^{*} + \left(\cos \beta_{21}^{*} - \cos \beta_{12}^{*}\right)$$
$$\cos \beta_{21}^{*} = \cos \beta_{21}^{*} + \left(\cos \beta_{21}^{*} - \cos \beta_{22}^{*}\right)$$

or more generally

$$\cos \beta_{1n}^{*} = \cos \beta_{1,n-2}^{*} + \left(\cos \beta_{11}^{*} - \cos \beta_{1,n-1}^{*}\right)$$
$$\cos \beta_{2n}^{*} = \cos \beta_{2,n-2}^{*} + \left(\cos \beta_{21}^{*} - \cos \beta_{2,n-1}^{*}\right)$$

Figure 5 schematizes the different operations used in the general program of restitution of the data. The process converges in order to cancel the errors $\cos \beta_{11}^{*} - \cos \beta_{1,n-1}^{*}$ and $\cos \beta_{21}^{*} - \cos \beta_{2,n-1}^{*}$ when the two magnitudes are zero. The positions B_{1n} and B_{2n} represent the true positions of the balloon B at the two instants of interrogation and $\cos \beta_{1,n-1}$ · $\cos \beta_{2,n-1}$ represent the cosines corresponding to the angles of two cones.

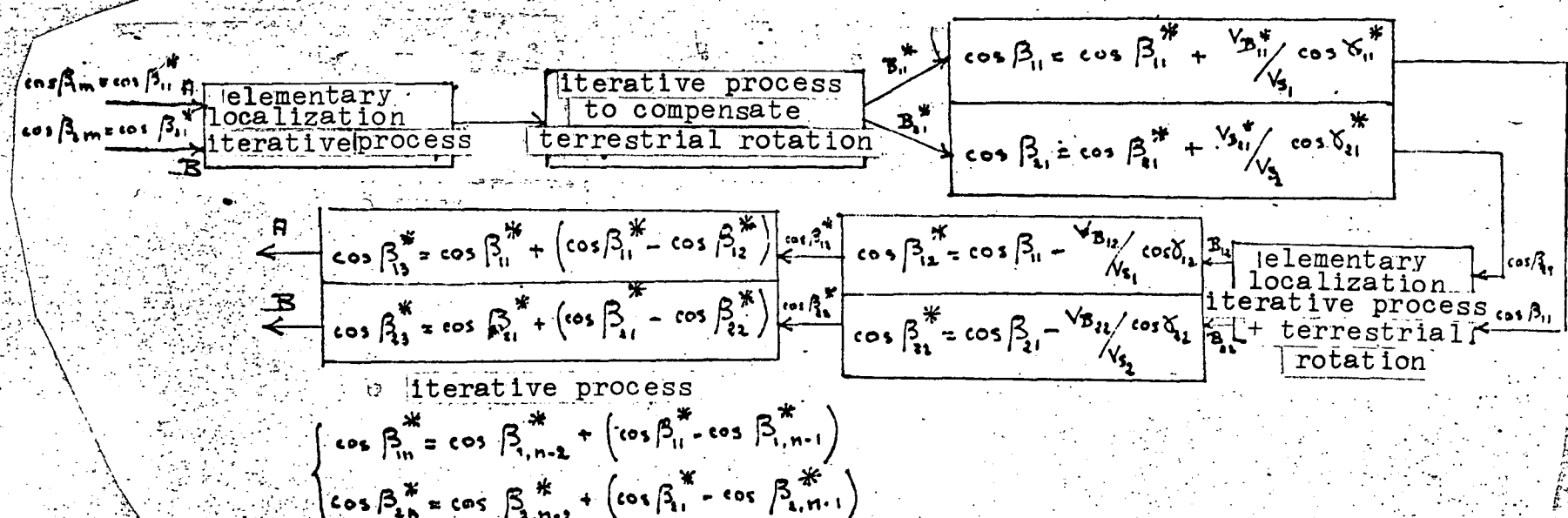

$\cos\beta_{1m} = \cos\beta_{11}^*$ A
$\cos\beta_{2m} = \cos\beta_{21}^*$ B
elementary localization iterative process
iterative process to compensate terrestrial rotation
B_{11}^*
B_{21}^*
$\cos\beta_{11} = \cos\beta_{11}^* + V_{B_{11}}^*/V_{S_1} \cos\gamma_{11}^*$
$\cos\beta_{21} = \cos\beta_{21}^* + V_{S_{21}}^*/V_{S_2} \cos\gamma_{21}^*$
$\cos\beta_{13}^* = \cos\beta_{11}^* + (\cos\beta_{11}^* - \cos\beta_{12}^*)$
$\cos\beta_{23}^* = \cos\beta_{21}^* + (\cos\beta_{21}^* - \cos\beta_{22}^*)$
$\cos\beta_{12}^* = \cos\beta_{11} - V_{B_{12}}/V_{S_1} \cos\gamma_{12}$
$\cos\beta_{22}^* = \cos\beta_{21} - V_{B_{22}}/V_{S_2} \cos\gamma_{22}$
B_{12}
B_{22}
elementary localization iterative process + terrestrial rotation
$\cos\beta_{11}$
$\cos\beta_{21}$
A
B
iterative process
$\cos\beta_{1n}^* = \cos\beta_{1,n-2}^* + (\cos\beta_{11}^* - \cos\beta_{1,n-1}^*)$
$\cos\beta_{2n}^* = \cos\beta_{2,n-2}^* + (\cos\beta_{21}^* - \cos\beta_{2,n-1}^*)$

Figure 5

3.3 Formulation

Referenced to the trihedra OXYZ, the coordinates of the points S_1, S_2, B_1, B_2 (cf. Figure 4) are as follows:

$$S_2 \left| \begin{array}{l} (R+h)\cos\alpha \\ (R+h)\sin\alpha \\ 0 \end{array} \right. \qquad S_1 \left| \begin{array}{l} (R+h)\cos(\alpha-\Omega_s T) \\ (R+h)\sin(\alpha-\Omega_s T) \\ 0 \end{array} \right.$$

$$B_2 \left| \begin{array}{l} R\cos\varphi_2\cos(\lambda_2+\alpha) \\ R\cos\varphi_2\sin(\lambda_2+\alpha) \\ R\sin\varphi_2 \end{array} \right. \qquad B_1 \left| \begin{array}{l} R\cos\varphi_1\cos(\lambda_1+\alpha-\Omega_s T) \\ R\cos\varphi_1\sin(\lambda_1+\alpha-\Omega_s T) \\ R\sin\varphi_1 \end{array} \right.$$

and we have

$$|V_{B_1}| = |V_{B_2}| = R\,\omega_0\cos\mu_2 \quad \text{avec} \quad \sin\mu_2 = \sin i\cos\varphi_2\sin(\lambda_2+\alpha) + \sin\varphi_2\cos i$$

$$\overrightarrow{S_2B_2} \left| \begin{array}{l} R\cos\varphi_2\cos(\lambda_2+\alpha) - (R+h)\cos\alpha \\ R\cos\varphi_2\sin(\lambda_2+\alpha) - (R+h)\sin\alpha \\ R\sin\varphi_2 \end{array} \right. \qquad \overrightarrow{S_1B_1} \left| \begin{array}{l} R\cos\varphi_1\cos\varphi_1 - (R+h)\cos(\alpha-\Omega_s T) \\ R\cos\varphi_1\sin\varphi_1 - (R+h)\sin(\alpha-\Omega_s T) \\ R\sin\varphi_1 \end{array} \right.$$

On the other hand, the coordinates of the vectors $\overrightarrow{V_{B_1}}$ and $\overrightarrow{V_{B_2}}$ referenced to the trihedron OXYZ are as follows:

$$\overrightarrow{V_{B_1}} \left| \begin{array}{l} -|V_B|\sin\theta_1 \\ |V_B|\cos\theta_1\cos i \\ -|V_B|\cos\theta_1\sin i \end{array} \right. \qquad \overrightarrow{V_{B_2}} \left| \begin{array}{l} -|V_B|\sin\theta_2 \\ |V_B|\cos\theta_2\cos i \\ -|V_B|\cos\theta_2\sin i \end{array} \right.$$

On the other hand, we have $\dfrac{(\overrightarrow{S_iB_i}\cdot\overrightarrow{V_{B_i}})}{|S_iB_i|} = |V_{B_i}|\cos\gamma_i$

Reproduced from best available copy.

and consequently

$$|V_{B_1}|\cos\gamma_1 = |V_B|\Big(-\sin\theta_1\big(\cos\varphi_1\cos\psi_1 - u\cos(\alpha-\Omega_3 T)\big) + \cos\theta_1\cos i\,\big(\cos\varphi_1\sin\psi_1 - u\sin(\alpha-\Omega_3 T)\big) - \cos\theta_1\sin i\sin\varphi_1\Big)\Big/ D_1$$

$$|V_{B_2}|\cos\gamma_2 = |V_B|\Big(-\sin\theta_2\big(\cos\varphi_2\cos\psi_2 - u\cos\alpha\big) + \cos\theta_2\cos i\,\big(\cos\varphi_2\sin\psi_2 - u\sin\alpha\big) - \cos\theta_2\sin i\sin\varphi_2\Big)\Big/ D_2$$

with

$$D_1^2 = 1 + u^2 - 2u\cos(\alpha-\Omega_3 T)\cos\varphi_1\cos\psi_1 - 2u\sin(\alpha-\Omega_3 T)\cos\varphi_1\sin\psi_1$$

$$D_2^2 = 1 + u^2 - 2u\cos\alpha\cos\varphi_2\cos\psi_2 - 2u\sin\alpha\cos\varphi_2\sin\psi_2$$

and we have

$$\cos\beta_2^* = \cos\beta_2 - \left|\frac{V_{B_2}}{V_{S_2}}\right|\cos\gamma_2 \quad \text{avec} \quad \left|\frac{V_{B_2}}{V_{S_2}}\right| = \frac{\omega_0\cos\mu_2}{\Omega_3 u} \qquad u = \frac{R+h}{R}$$

On the basis of the positions B_2 and B_1 localized by the magnitudes $\cos\beta_1$ and $\cos\beta_2$, these formulas accordingly make it possible to define the measured magnitudes $\cos\beta_2^*$ and $\cos\beta_1^*$ inversely.

3.4 Convergence of the Process and Results

The iterative process of complete restitution of the localization was simulated on the arithmetical computer. In the following two paragraphs, the computation program and the PAF program are reproduced. Table 2 reproduces the intermediate results of the different iterations and illustrates the rapid convergence of the process.

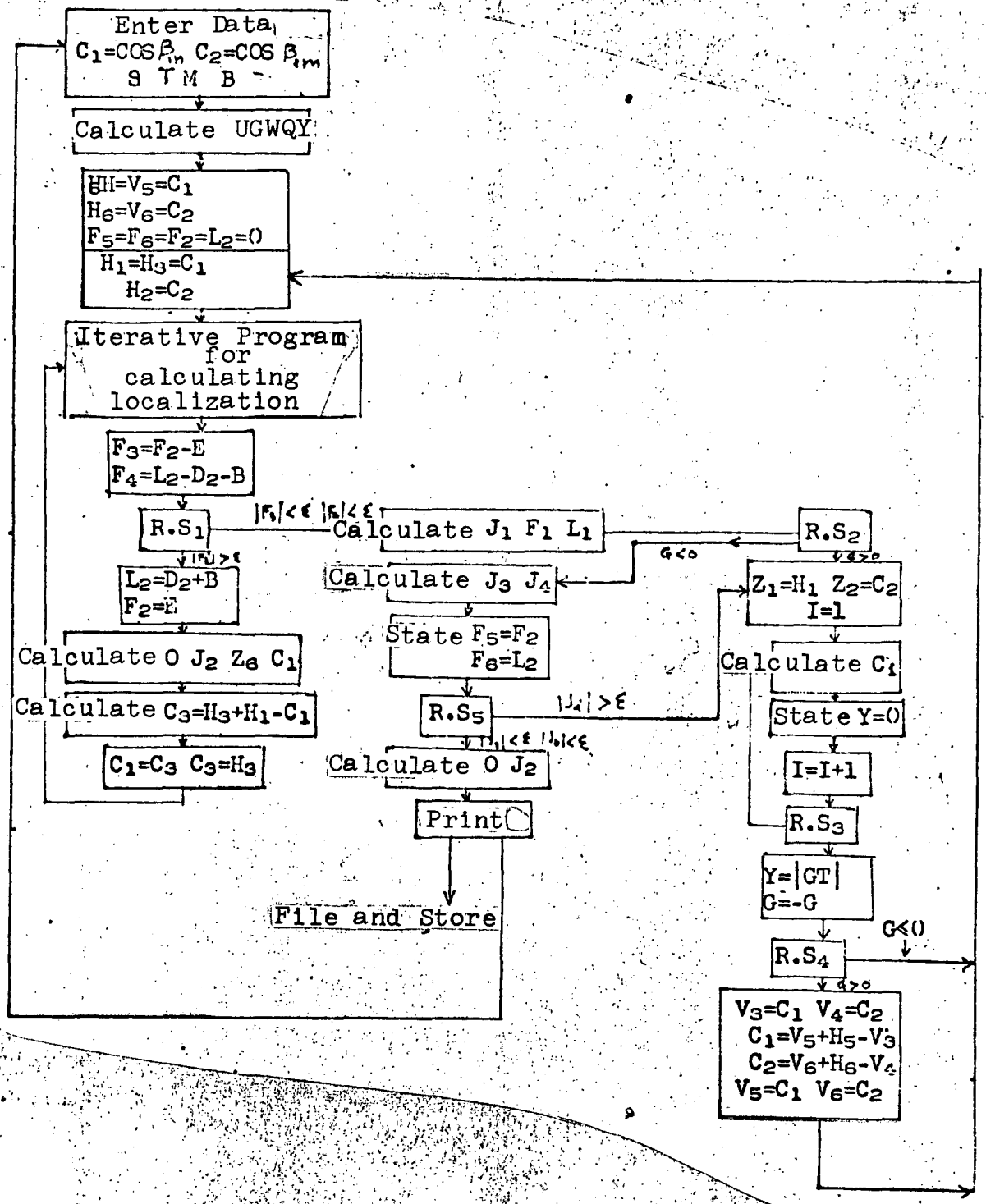
An intermediate computation program defines, for a given position of a balloon ($F_2 = 0.261801 = 15^{\circ}$; $L_2 = 0.03839570$), the two magnitudes $\cos \beta_1^*$ and $\cos \beta_2^*$ given by a Doppler measurement where wind is assumed as zero. By reintroducing these two magnitudes in the iterative program, we intend to restitute the exact position of the balloon.

The first iterative process including terrestrial rotation requires, in the example selected, 9 iterations and the second process, including both terrestrial rotation and the error of measurement, requires 6 iterations. On the other hand, the overall iterative process requires 3 iterations and furnishes the effective position of the balloon with an accuracy on the order of 10^{-6}. The convergence of the process is very satisfactory and relatively rapid in consideration of the elementary time of computation for the preliminary localization. The process has been tested for different positions of the balloon and furnished entirely similar results.

$C_1 = 0,54532814$	$C_2 = 0,15268245$	true position of balloon $F_2=0,26180100$	$L_2=0,03839570$
$F_2 = 0,303510$	$L_2 = 0,057184$		
$F_2 = 0,276243$	$L_2 = 0,052488$	$F_2=0,274672$	$L_2=0,051660$
$F_2 = 0,275371$	$L_2 = 0,052341$	$F_2=0,289155$	$L_2=0,041931$
$F_2 = 0,275340$	$L_2 = 0,052335$	$F_2=0,262610$	$L_2=0,038498$
$F_2 = 0,275343$	$L_2 = 0,052335$	$F_2=0,261793$	$L_2=0,038397$
$F_2 = 0,275338$	$L_2 = 0,05233$	$F_2=0,261832$	$L_2=0,038400$
$F_2 = 0,275342$	$L_2 = 0,052335$	$F_2=0,261801$	$L_2=0,038395$
$F_2 = 0,275335$	$L_2 = 0,052335$	$F_2=0,261801$	$L_2=0,038395$
$F_2 = 0,275349$	$L_2 = 0,052336$	position of the balloon at the end of localization	
$F_2 = 0,275334$	$L_2 = 0,052335$		
$F_2 = 0,289850$	$L_2 = 0,042631$		
$F_2 = 0,263268$	$L_2 = 0,039141$		
$F_2 = 0,262477$	$L_2 = 0,039039$		
$F_2 = 0,262458$	$L_2 = 0,039036$		
$F_2 = 0,262455$	$L_2 = 0,039036$		
$F_2 = 0,262455$	$L_2 = 0,039036$		
$F_2 = 0,302808$	$L_2 = 0,056446$		
$F_2 = 0,275570$	$L_2 = 0,051810$		
$F_2 = 0,274698$	$L_2 = 0,051664$		
$F_2 = 0,274677$	$L_2 = 0,051661$		
$F_2 = 0,274674$	$L_2 = 0,051661$		
$F_2 = 0,289155$	$L_2 = 0,041930$		
$F_2 = 0,262612$	$L_2 = 0,038498$		
$F_2 = 0,261795$	$L_2 = 0,038397$		
$F_2 = 0,261837$	$L_2 = 0,038401$		
$F_2 = 0,261801$	$L_2 = 0,038395$		
$F_2 = 0,261801$	$L_2 = 0,038395$		
$F_2 = 0,302803$	$L_2 = 0,056445$		
$F_2 = 0,275568$	$L_2 = 0,051808$		
$F_2 = 0,274697$	$L_2 = 0,051663$		

Table 2

3.5 Data Restitution program (Doppler effect)



PROGRAM FOR RESTITUTION OF LOCALIZATION

PRF

$_1$ = 6 C D P V F L Z H J

U = S/6400

G 2 $\sqrt{(398599/S^3)}$

W = $(1 + U^2)/2U$

Q = 1/U

Y = GT

1 Read C_1 C_2 S T M B

2 State $H_5=C_1$ $H_3=C_2$ $V_5=C_1$ $V_6=C_2$ $F_5=0$ $F_6=0$ $F_2=0$ $P_4=1$ $L_2=0$

3 Calculate U G W Q Y

4 State $P_3=1$ $H_3=C_1$ $H_2=C_2$ $H_1=C_1$

5 State $(13440)=C_1$ $(13441)=C_2$

6 Interrogation

7 Program 100,0

8 State $D_1=(13449)$ $D_2=(13450)$ $E=(13454)$

9 Calculate $F_3=F_2-E$

10 Calculate $F_4=L_2-D_2-B$

11 If $|F_3|<0,000002$ If $|F_4|<0,000002$ Go to 24

12 If $P_3>11$ Go to 24

13 Make $P_3=P_3+1$

14 Calculate $L_2=D_2+B$

15 State $F_2=E$

16 Calculate $O=(\cos F_2)(\sin L_2)(\sin M)+(\sin F_2)(\cos M)$

17 Calculate $J_2=$ ARC TG $(((\cos M)(\cos F_2)(\sin L_2)-(\sin M)(\sin F_2))/(\cos F_2)(\cos L_2))$

PROGRAM FOR RESTITUTION OF LOCALIZATION (Continued)

18 Calculate Z_6= ARC TG (((COS M)(SIN (J_2-0,00007268T) (√(1-O^2))+O SIN M)/(COS (J_2-0,00007268T))(√(1-O^2)))

19 Calculate C_1=(COS (J_2-0,00007268T))(√(1-O^2))(SIN (Z_6+Y-B))/(COS Z_6)√(1+U^2-2U(COS (Z_6+Y-B))(COS (J_2-0,00007268T))(√(1-O^2))/(COS Z_6))

20 Calculate C_3=H_3+H_1-C_1

21 State C_1=C_3 H_3=C_3

22 Print with 6 DEC RC F_2 TAB L_2

23 Go to 5

24 State L_1=Z_6

25 Calculate J_1=J_2-0,00007268T

26 Calculate F_1= ARC COS ((COS J_1)(√(1-O^2))/ COS L_1)

27 If G≥0 go to 35

28 Print with 6 DEC RC F_2 TAB L_2 TAB G RC

29 Calculate J_3=F_5-F_2

30 Calcula J_4=F_6-L_2

31 State FB=F_2 F_6=L_2

32 If |J_3|≤0,00002 SI |J_4|≤0,00002 go to 50

33 If P_4≥11 go to 50

34 Make P_4=P_4+1

35 State Z_1=H_1 Z_2=C_2 I=1

36 Calculate C_i=Z_i+0,00007268(√(1-O^2))(-(SIN J_i)((COS F_i)(COS L_i) -U COS (B-Y))+(COS J_i)(COS M)((COS F_i)(SIN L_i)-U SIN (B-Y))-(COS J_i)(SIN M)(SIN F_i))/+GU√(1+U^2-2U(COS F_i)(COS L_i)(COS (B-Y))-2U(COS F_i)(SIN L_i)(SIN (B-Y)))

37 State Y = 0

38 Make Y=0

39 If I<2 go to 36

40 Calculate Y=GT

PROGRAM FOR RESTITUTION OF LOCALIZATION (Continued)

41 State $Y=|Y|$

42 State $G=-G$

43 If $G \leq 0$ go to 4

44 State $V_3=C_1$ $V_4=C_2$

45 Calculate $C_1=V_5+H_5-V_3$

46 Calculate $C_2=V_6+H_6-V_4$

47 State $V_5=C_1$ $V_6=C_2$

48 Interrogation

49 Go to 4

50 Calculate $O=180(\text{ ARC SIN } O)/\pi$

51 Calculate $J_2=180J_2/\pi$

52 Print with 6 DEC O TAB J_2 RC RC

53 Go to 1

END

VERIFICATION PROGRAM

PM

101 Read F_2 S T M B

102 Print with 6 DEC RC S TAB T TAB M

103 Calculate U G W Q Y

104 Calculate $A=\text{ ARC COS } (1/U \text{ COS } F_2)$

105 State $L_2=A$

106 Calculate $O=(\text{ COS } F_2)(\text{ SIN } L_2)(\text{ SIN } M)+(\text{ SIN } F_2)(\text{ COS } M)$

107 Calculate $J_2=\text{ ARC TG } (((\text{ COS } M)(\text{ COS } F_2)(\text{ SIN } L_2)-(\text{ SIN } M)(\text{ SIN } F_2))/(\text{ COS } F_2)(\text{ COS } L_2))$

108 Calculate $Z_6=\text{ ARC TG } (((\text{ COS } M)(\text{ SIN } (J_2-0,0000\,7268T))(\sqrt{(1-O^2)})+O \text{ SIN } M)/(\text{ COS } (J_2-0,0000\,7268T))(\sqrt{(1-O^2)}))$

VERIFICATION PROGRAM (Continued)

109 Calculate $C_1=(\cos(J_2-0,00007268T))(\sqrt{(1-0^2)})(\sin(Z_6+Y))/(\cos Z_6)\sqrt{(1+U^2-2U(\cos(Z_6+Y))(\cos(J_2-0,00007268T))(\sqrt{(1-0^2)})/(\cos Z_6))}$

110 Calculate $C_2=(\cos F_2)(\sin L_2)/\sqrt{(1+U^2-2U(\cos F_2)(\cos L_2))}$

111 State $L_1=Z_6$

112 Calculate $J_1=J_2-0,00007268T$

113 Calculate $F_1=\text{ARC}\cos((\cos J_1)(\sqrt{(1-0^2)})/\cos L_1)$

114 State $Z_1=C_1$ $Z_2=C_2$ $I=1$

115 Calculate $C_1=Z_1+0,00007268(\sqrt{(1-0^2)})(-(\sin J_1)((\cos F_1)(\cos L_1)-U\cos Y)+(\cos J_1)(\cos M)((\cos F_1)(\sin L_1)+U\sin Y)-(\cos J_1)(\sin M)(\sin F_1))/-GU\sqrt{(1+U^2-2U(\cos F_1)(\cos L_1)(\cos Y)+2U(\cos F_1)(\sin L_1)(\sin Y))}$

116 Interrogation

117 State $Y=0$

118 Make $I=I+1$

119 If $I \leqslant 2$ go to 115

120 Print with 8 DEC C_1 TAB C_2 RC

121 Calculate Y

122 State $P_1=F_2$ $P_2=L_2$

123 Go to 2

124 State $F_2=P_1$ $L_2=P_2$

125 Calculate G Y

126 If L_2 -A go to 101

127 Make $L_2=P_2-Y$

128 Go to 106

1 Go to 124

END

LOCALIZATION PROGRAM

```
100,OM
T12M105,30
A1EM105,0
A2ER1
MF2R1
A1EM105,1
A3ER1
MF3R1
A1ER2
SF1R3
A1ER3N1
A1ER2P1
X
A4EM182,90
A5ER4
MF5R4
SF5R1
RV10,0ΔN5
A4EM182,94
A5ER2
MF5R4
T5M105,22
MF5R4
A6ER5
MF6R2
SF6R5
SF6R2
T6M105,20
X
SF1ER6
A5ER3
MF5R4
T5M105,23
MF5R4
A6ER5
MF6R3
SF6R5
SF6R3
T6M105,21
SF2ER6
A5EV0
A3ER1
SF1R2
A6ER3P1
A6ER2N1
A1ER6
YF6R1
A15EV0
A1EM105,20+
A2ER6
MF2R6
AF1R2
A1EV0N1
YF2R1
A1EM105,22+
AF1R2
QF1R6
SF2ER1
AF2EV1
RV10,0ΔN2
ARC2R1
A1EM105,0+
A1ER2P1
SF1ER2N1
T1M105,9+
RV67ΔY15
A15EV1
RV47Δ
A7EM105,9
SF7M105,10
SF7M182,98
RV75ΔY5
RV10,0ΔN7
A5EM182,96
SF5R6
QF5V2
A1ER7
MF1R5
X
RV82ΔP1
SF1ER5
QF1V2
A5ER1
AF6R5
A1ER7P7
SF1ER7N7
SF1M105,11
X
RV92ΔN1
A1ER5P5
SF1ER5N5
SF1M105,12
RV46ΔP1
ARC1R6
T1EM105,14
T6EM182,91
RM105,30
X
X
X
X
FE
110,OM
EV24
EV25
EV44
EV37
EV15
EV34
EV33
EV44
EV44
EV47
EV46
EV15
EV46
EV23
EV33
EV24
X
EV37
EV51
EV52
EV15
EV48
EV33
EV51
EV15
EV54
EV53
EV25
EV30
EV35
EV23
EV55EV24
EV14
A1EM105,0
A2EM105,1
IF1EOF8
EV30
EV35
EV23
EV56
EV24
EV14
IF2EOF8
RM180,1
X
X
X
X
X
FE
```

IV. CONCLUSION

On the assumption of zero wind, this study shows the possibility of restituting, with the aid of an iterative process, the exact position of the balloon by taking into account the systematic errors due to terrestrial rotation and due to the Doppler effect itself. The accuracy obtained on the order of 10^{-6} is very satisfactory but requires a relatively prolonged time of calculation since the elementary localization of the balloon by the Doppler measurement requires iteration. Subsequent memoranda will investigate the influence of the wind on the localization of the balloon on the basis of this process of restitution.

ANNEX 1 - TERRESTRIAL ROTATION RESTITUTION PROGRAM (DOPPLER EFFECT)

PAF

$_1$ = 3 C D P V F L

U = S/6400

G = $\sqrt{(398599/S^3)}$

W = $(1+U^2)/2U$

Q = $1/U$

1 Read F_1 S T M

2 Calculate U G W Q

3 Go to 50

50 Calculate L_1= ARC COS $(1/U \cos F_1)$

51 Go to 54

52 Calculate $L_3=L_1+GT$

53 Make $L_1=L_3$

54 State I=1

55 Go to 4

4 Calculate $O=(\cos F_1)(\sin L_1)(\sin M)+(\sin F_1)(\cos M)$

5 Calculate $J=$ ARC TG $(((\cos M)(\cos F_1)(\sin L_1)-(\sin M)(\sin F_1))/(\cos F_1)(\cos L_1))$

6 Calculate $Z=$ ARC TG $(((\cos M)(\sin (J-0,0000 7268T))(\sqrt{(1-O^2)})+O \sin M)/(\cos (J-0,0000 7268T))(\sqrt{(1-O^2)}))$

7 Calculate $C_1=(\cos (J-0,0000 7268T))(\sqrt{(1-O^2)})(\sin (Z+GT))/(\cos Z)\sqrt{(1+U^2-2U(\cos (Z+GT))(\cos (J-0,0000 7268T))(\sqrt{(1-O^2)})/(\cos Z))}$

8 If I = 1 go to 11

9 Calculate $C_2=(\cos F_1)(\sin L_1)/\sqrt{(1+U^2-2U(\cos F_1)(\cos L_1))}$

10 State $H=C_1$ $P_3=C_1$

11 Calculate $C_3=P_3+(H-C_1)$

ANNEX 1 - (Continued)

12 State $I=1$ $C_1=C_3$ $P_3=C_3$

13 State $V_i=|C_i|$

14 Calculate $P_i=\sqrt{(1/U^2-2C_i^2+(1+U^2)C_i^2)}$

15 If $V_1 > Q$ go to 52

16 If $I > 1,5$ go to 19

17 Make $I=I+1$

18 Go to 13

19 If $P_1 > P_2$ State $R=P_1$ $I=1$

20 If $P_1 \leqslant P_2$ State $R=P_2$ $I=1$

21 Make $K=(W-R)/2$

22 Calculate $N=U^2C_i^4-(1+U^2)C_i^2+R^2$

23 If $N<0$ go to 1

24 Calculate $D_i=(C_i/V_i)$ ARC COS $((UC_i^2+\sqrt{N})/R)$

25 If $I > 1,5$ go to 28

26 Make $I=I+1$

27 Go to 22

28 Interrogation

29 Calculate $A=D_1-D_2-GT$

30 If $R \neq P_1$ If $R = P_2$ go to 32

31 If $A<0$ go to 52

32 Calculate $B=AK$

33 If $B > 0$ go to 35

34 If $B \leqslant 0$ make $K = -K/2$

35 Make $R=R+K$

36 State $I=1$

ANNEX 1 - (Continued)

37 If $|A| \geqslant 0,000005$ go to 22

38 Calculate $E = \text{ARC COS } R$

39 State $F_2 = E$ $L_2 = D_2$

40 Calculate $Y = 6400 / (R^2 (L_2 - L_1)^2 + (F_2 - F_1)^2)$

41 Print with 6 DEC RC L_2 TAB L_1 TAB F_2 TAB F_1 TAB Y

42 Make $I = I + 1$

43 If $Y \leqslant 0,030$ go to 52

44 Go to 4

45 END

NOTATIONS

S: Radius of satellite orbit

G: Angular velocity of satellite

W: Distance from vertex of parabola to center of circle

F_1: Orbital latitude of balloon

T: Interval between interrogation

M: Inclination of orbit

O: Geographic latitude of balloon

J: Geographic longitude of balloon

C_1: $\cos \beta_1$

C_2: $\cos \beta_2$

L_1: Orbital longitude of balloon

Y: Error in distance