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A SOLUTION TO THE CONSTRAINED ALTITUDE PLANETARY FLYBY PROBLEM USING THE WEIGHTED LEAST SQUARES ITERATION TECHNIQUE

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Advanced Mission Design Branch

July 24, 1969

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By Victor R. Bond

1.0 SUMMARY

The weighted least squares iteration technique is used to solve the constrained altitude flyby problem. It was found that the iterator could be calibrated by use of data from a known solution of the problem for one synodic period of Mars. The remainder of the solutions were then found for the other synodic periods of Mars.

2.0 INTRODUCTION

Probably the most important parameters that must be known to design manned interplanetary missions are the flight times and departure dates associated with a given type of mission. These flight times must be chosen in a way that optimizes some performance criterion in the presence of mission constraints. The relations between the constraints and times are nonlinear in most cases and must be solved by iteration. The application of a weighted least squares iterative scheme (ref. 1) to the solution of this problem will be discussed in this document. As an example, the iteration technique is applied specifically to the constrained altitude Mars flyby problem.

This problem has been solved by other methods such as those presented in reference 2. The purpose here is not only to present flyby mission opportunities not given in reference 2, but also to demonstrate the technique used to solve the constrained altitude problem by a general purpose iterative scheme that can be applied just as well to other manned interplanetary missions such as the conjunction and opposition Mars missions. The iterative scheme to be applied here requires that the independent variables (the flight times) and the dependent variables (the mission constraints and performance criterion) be assigned numerical weights to assure dependable and rapid convergence. The study of a previously investigated problem permits the calibration of the iterative scheme by experimental determination of the most effective weighting factors.

3.0 SYMBOLS

$f(x)$	expresses the functional dependence of the independent and dependent variables
$h_{\pi T}$	periapsis altitude at the target planet
P	matrix of partial derivatives $\frac{\partial f}{\partial x}$
V_{AT}	hyperbolic excess velocity at target planet arrival
V_D	hyperbolic excess velocity at the departure planet
V_{DT}	hyperbolic excess velocity at target planet departure
X	vector of independent variables
Y	vector of dependent variables
λ	inhibitor parameter

Subscripts

ij	arbitrary indices of the elements of P

Operators

$\Delta(\ )$	deviations of ( ) from desired values
$\delta(\ )$	small increment in ( )
$\overline{(\ )}$	desired value

4.0 APPLICATION OF THE ITERATION TO THE CONSTRAINED ALTITUDE FLYBY PROBLEM

The constrained altitude flyby problem may be stated as follows. Find the departure date, the flight time to the target planet, and the flight time to return to another planet such that a single impulsive thrust at departure is minimized and the periapsis altitude at the target planet is specified. In the notation of the appendix, this problem may be formulated as follows.

$$\bar{Y}_1 = V_{AT} - V_{DT} = 0 \tag{1}$$

$$\bar{Y}_2 = h_{\pi T} \tag{2}$$

$$\bar{Y}_3 = V_D \tag{3}$$

Equations (1) and (2) represent the flyby constraint and periapsis altitude constraint, respectively, and equation (3) represents the variable to be minimized. The velocities V_D, V_{AT}, and V_{DT} are, respectively, the hyperbolic excess velocities at the departure planet, target planet arrival, and target planet departure. The independent variables are the following.

X_1 the time from the departure planet to the target planet

X_2 the time from the target planet to return

X_3 the time of departure measured from a fixed calendar date (the fixed calendar date is arbitrary but will be assumed to be a Mars opposition date in this paper)

The relations between the dependent variables (Y_1, Y_2, Y_3) and the independent variables (X_1, X_2, X_3) cannot be written in closed form for this problem. However, a program (ref. 3) is available that computes the solution for the dependent variables when the independent variables are given. With this program, the conic trajectory of a particle between any two planets of the solar system can be computed when values are given for the date of departure from the departure planet and the date of arrival at the arrival planet. The output of the program consists of the relative velocity vectors of the particle with respect to each of the planets. For all practical purposes, these relative velocity vectors are the same as the hyperbolic excess velocity vectors. The partial derivative matrix P may be obtained numerically from the same program if small variations are made in each of the independent variables from the initial guess values of X. If $Y = f(X)$ is the solution for the initial guess of the vector X, then the components of the P matrix are computed from

$$P_{ij} = \frac{Y_i(X_j + \delta X_j) - Y_i(X_j)}{\delta X_j} \tag{4}$$

If it is assumed that the proper values of the weighting matrices W_Y and W_X have been determined and that the inhibitor λ is properly controlled, equation (5) is solved to give the correction ΔX to the guess for the vector X.

$$\Delta X = (P^T W_Y P + \lambda W_X)^{-1} P^T W_Y (\bar{Y} - Y) \tag{5}$$

Equation (5) is then applied repeatedly until equations (1) and (2) are satisfied and equation (3) is minimized.

To control the inhibitor λ during each iteration step, its value is automatically chosen to satisfy the conditions that the error in the estimate of ΔY is minimized and that ΔX is chosen such that linearity assumptions are valid. These conditions are given by equations (6) and (7).

$$R = e^T W_Y e \tag{6}$$

$$S = \Delta X^T W_Y \Delta X < S_0 \tag{7}$$

5.0 RESULTS AND DISCUSSION

A computer program was written to solve equation (5) in its application to the constrained altitude planetary flyby problem. The flyby problem is solved in reference 2 for a few cases which were used as trial problems to calibrate the iterative scheme. During this calibration, the proper values of W_Y, W_X, and S_0 were selected to minimize the number of iterations and at the same time to maintain reasonable accuracy. These values were then used to solve the cases that were unknown.

In reference 2, it is pointed out that within any given synodic period (time between successive oppositions, varies between 764 and 811 days) of Mars there are two minimum departure velocity opportunities to depart from Earth, fly by Mars at a specified altitude, and return to Earth. These missions require just less than 2 years total flight time. There are other flyby opportunities during a synodic period, but all of these opportunities require very long total flight times or excessively high minimum departure velocities. The first minimum departure velocity after opposition is called long-short because the outbound time X_1 is greater than the return time X_2. This first minimum occurs approximately 122 days after opposition. The second minimum departure velocity is called short-long because the outbound

time X_1 is less than the return time X_2. This minimum occurs approximately 93 days prior to opposition.

As a test of this iteration scheme, it was decided to use the times found in reference 2 for the short-long missions that occur prior to the 1975 opposition and for the long-short missions that occur after the 1975 opposition as initial guesses to obtain all of the minimum short-long and long-short opportunities through the 1999 Mars opposition date for a constrained altitude of 200 n. mi. The iteration scheme exhibited remarkable stability and speed in both the short-long and long-short mission searches. All of the short-long missions, a total of twelve, were obtained in one computer run by use of the same initial guesses for the times X_1, X_2, and X_3 and required approximately 10 seconds per opportunity on the UNIVAC 1108 digital computer. The twelve long-short opportunities were also found in a single computer run that averaged 10 seconds per opportunity. The results of these runs are presented in tables I and II. Note from these results that the variations in the flight times from the initial values were quite large in several cases.

6.0 CONCLUDING REMARKS

It has been shown in this paper that the weighted least squares iteration technique can be used to obtain the solution rapidly for the constrained altitude Mars flyby problem. Initial guesses for the flight times for the departure opportunities were obtained from an independent source. These initial guesses were then used to calibrate the weights of the dependent variables in a digital program developed to obtain the solutions for the two departure opportunities (of total flight time approximately 2 years) during each synodic period of Mars from 1975 through 1999.

TABLE I.- MINIMUM DEPARTURE VELOCITY (V_D) FOR MARS SHORT-LONG FLYBY OPPORTUNITIES[a]

Date of Earth/Mars opposition		Time from Earth to Mars, X_1, days	Time from Mars to Earth X_2, days	Time of departure, X_3, days	Minimum departure velocity at Earth, V_D, fps
Calendar date, mo., day, yr.	Julian date				
12-15-75	2 442 762.1	131.77	538.37	-83.68	[b]19 138
1-22-78	2 443 530.5	145.69	534.08	-87.30	[b]18 351
2-25-80	2 444 294.7	142.52	535.35	-87.60	[b]18 011
3-31-82	2 445 059.9	128.20	538.58	-85.35	[b]18 438
5-11-84	2 445 831.9	108.84	553.81	-80.79	19 336
7-10-86	2 446 621.8	91.31	578.81	-77.08	19 971
9-28-88	2 447 432.7	90.84	566.72	-77.65	20 964
11-27-90	2 448 223.3	119.74	541.02	-81.02	19 804
1-07-93	2 448 995.4	142.68	535.02	-86.20	18 587
2-12-95	2 449 760.5	145.18	534.32	-87.56	18 090
3-17-97	2 450 524.8	134.86	537.34	-86.44	18 128
4-24-99	2 451 293.2	115.25	541.79	-82.36	19 353

[a]Periapsis altitude at Mars was 200 n. mi. Initial guesses for all cases were X_1 = 125 days, X_2 = 545 days, and X_3 = -85 days. The opportunities found in ref. 2 are indicated by b.

TABLE II.- MINIMUM DEPARTURE VELOCITY (V_D) FOR MARS LONG-SHORT FLYBY OPPORTUNITIES[a]

Date of Earth/Mars opposition		Time from Earth to Mars, X_1, days	Time from Mars to Earth, X_2, days	Time of departure, X_3, days	Minimum departure velocity at Earth, V_D, fps
Calendar date, mo., day, yr.	Julian date				
12-15-75	2 442 762.1	582.90	99.01	122.43	[b]16 746
1-22-78	2 443 530.5	568.92	108.02	122.06	17 077
2-25-80	2 444 294.7	563.34	110.47	122.11	17 112
3-31-82	2 445 059.9	570.49	103.76	122.45	16 789
5-11-84	2 445 831.9	600.63	83.25	126.43	16 559
7-10-86	2 446 621.8	624.18	68.35	145.01	16 615
9-28-88	2 447 432.7	609.01	79.79	134.53	16 330
11-27-90	2 448 223.3	589.52	94.15	124.03	16 589
1-07-93	2 448 995.4	573.32	105.17	122.30	16 973
2-12-95	2 449 760.5	564.41	110.42	122.10	17 144
3-17-97	2 450 524.8	565.53	107.74	122.32	16 947
4-24-99	2 451 293.2	586.05	93.11	123.23	16 572

[a]Periapsis altitude at Mars was 200 n. mi. Initial guesses for all cases were X_1 = 580 days, X_2 = 110 days, and X_3 = 122 days. The opportunities found in ref. 2 are indicated by b.

APPENDIX A

THE WEIGHTED LEAST SQUARES ITERATION EQUATION

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THE WEIGHTED LEAST SQUARES ITERATION EQUATION

The iterator is based on the weighted least squares method which is presented in reference 1 but will be derived here for completeness. Consider the system of equations

$$Y = f(X) \tag{A1}$$

where Y is an n vector of dependent variables and X is an m vector of independent variables. The problem is to solve equation (A1) for independent variables $\bar{X}$ when the desired values of dependent variables $\bar{Y}$ have been given. Because equation (A1) is nonlinear, it must be solved by iteration. Therefore, a correction formula must be found to predict the changes ΔX in the independent variables.

Expand equation (A1) in a Taylor series about the initial guess for the vector X.

$$\bar{Y} = Y + \frac{\partial f}{\partial X}(\bar{X} - X) + \ldots \tag{A2}$$

Use the definitions

$$P = \frac{\partial f}{\partial X} \tag{A3}$$

and

$$\bar{X} = X + \Delta X \tag{A4}$$

where ΔX is the correction to the guess X to write equation (A2).

$$\bar{Y} = Y + P\Delta X + \text{higher order terms} \tag{A5}$$

The residual error vector e may be written as

$$e = \bar{Y} - Y - P\Delta X \tag{A6}$$

This vector represents the difference between the true variation in Y and the variation that would be predicted from the linear approximation (A5).

Because m is not necessarily equal to n, the matrix P does not, in general, have an inverse; therefore, equation (A5) cannot be solved directly for ΔX. In addition, it is desirable to obtain an expression for ΔX that minimizes the length of the error vector e. Because the components of the vector Y may have different dimensions and because the nonlinear system of equations may be more sensitive to some components of Y than to others, it is necessary to obtain a correction ΔX that minimizes the sum of the weighted squares of the components of the error vector e. This condition may be stated as follows: minimize the scalar

$$R = e^T W_Y e \tag{A7}$$

with respect to the vector ΔX.

To avoid very large values of ΔX, a constraint may also be imposed on the weighted squares of the components of ΔX, which may be expressed as

$$S = \Delta X^T W_X \Delta X < S_0 \tag{A8}$$

The matrices W_Y and W_X are n by n and m by m, respectively, and are both diagonal matrices; S_0 is a constant. The constraint [eq. (A8)] may be added to the scalar [eq. (A7)] by the use of a Lagrange multiplier λ, and the minimization is done by differentiation of this sum with respect to the correction vector ΔX. This operation may be expressed as follows.

$$\frac{\partial}{\partial \Delta X}(R + \lambda S) = 0 \tag{A9}$$

When the differentiation is performed, the equation for the correction ΔX is found to be the following.

$$\Delta X = (P^T W_Y P + \lambda W_X)^{-1} P^T W_Y (\bar{Y} - Y) \tag{A10}$$

An iterative scheme that employs equation (A10) must have the means to control the multiplier λ to satisfy the constraint [eq. (A8)] and to minimize the function R, defined by equation (A7). The method used to control λ is given in reference 1. The desired values of the dependent variables $\bar{Y}$ are specified to some tolerance based upon the accuracy desired and the computation time limitations. The dependent variable that is to be optimized is assigned a desired value that is impossible to attain.

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