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A PRELIMINARY TRAJECTORY STUDY FOR LAUNCH FROM THE MARTIAN SURFACE INTO ORBITS ABOUT THE PLANET

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
Houston, Texas
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A PRELIMINARY TRAJECTORY STUDY FOR LAUNCH FROM THE MARTIAN SURFACE INTO ORBITS ABOUT THE PLANET

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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A PRELIMINARY TRAJECTORY STUDY FOR LAUNCH FROM THE MARTIAN SURFACE INTO ORBITS ABOUT THE PLANET

SUMMARY

A preliminary trajectory study was performed to determine the characteristic velocity requirements and optimum thrust-to-weight ratios for launch of a single stage vehicle from the Martian surface into various orbits about the planet. The trajectory consisted of a launch burn phase, a long suborbital coast phase, and an initial orbit circularization phase. Subsequently, a maneuver was initiated to transfer to a higher altitude where a circularization into the final orbit was performed. Trajectory computations were made on the IBM-7094 digital computer using a two-dimensional particle trajectory program.

Results of the study are presented in a series of plots showing minimum characteristic velocities and optimum thrust-to-weight ratios. Optimum launch burn and coast times are also presented. The results indicate the presence of an optimum initial orbit altitude if transfers to higher orbits are made.

INTRODUCTION

As a part of the effort to determine the mission requirements for future manned spaceflight programs, the Advanced Spacecraft Technology Division is conducting a continuing study of manned roundtrip missions to Mars. One phase of this study is a series of trajectory analyses to determine the velocity requirements for these missions. These requirements form the basis for selection of mission profiles and the design of the spacecraft and its subsystems.

This report describes the results of a trajectory study of a single stage spacecraft launched from the Martian surface into various orbits about the planet. The primary purpose of this study was to determine the optimum thrust-to-weight ratios and minimum characteristic velocity requirements to establish these orbits. Since the Mars launch might be accomplished with any one of a variety of different propulsion systems, a secondary purpose was to investigate the effects of specific impulse. The study was based on a mission profile consisting of the eight phases illustrated in figure 1 and consisted of a detailed analysis of the following six phases:

1. Launch

2. Suborbital coast

3. Initial orbit circularization

4. Transfer thrusting

5. Transfer to high altitude

6. Final orbit circularization

The results of the study are presented in a series of plots showing the effects of launch burn time and coast time on characteristic velocity and thrust-to-weight ratio. Characteristic velocities for the transfer from the initial to final orbit are shown. A curve is presented that shows the variation of total characteristic velocity and optimum initial orbit altitude with final orbit altitude. Curves showing the effect of specific impulse on characteristic velocity, optimum launch burn time, and optimum coast time are also presented.

SYMBOLS

A	Area, ft^2
A_b	Mach number corresponding to maximum drag coefficient
a	A mach number greater than A_b
B	Drag coefficient at Mach number A_b
C_a	Drag coefficient at Mach number a
C_D	Drag coefficient
C_L	Lift coefficient
C_{L_α}	Slope of the C_L versus α curve, radian $^{-1}$
C_1	Atmospheric pressure factor
D	Drag, lbs

e	2.71828
f	Thrust, lbs
f/w	Thrust-to-weight ratio (earth)
g	Acceleration of gravity, ft/sec^2
h	Altitude, ft or n. mi.
Isp	Specific Impulse, sec
K_o	Subsonic drag coefficient
L	Lift, lbs
M	Mach number
m	Mass of vehicle, slugs
p	Pressure, lbs/ft^2
q	Dynamic pressure, lbs/ft^2
r	Radius, ft
s	Surface range, ft
t	Time, sec
v	Velocity, ft/sec
α	Angle between vehicle axis and velocity vector, radians
β	Atmosphere decay factor, ft^{-1}
θ	Angle between velocity vector and local vertical, deg
ρ	Atmosphere density, $slugs/ft^3$
ψ	Central angle between vehicle and launch point, deg

SUBSCRIPTS

e	Effective or exit
p	Planet
vac	Vacuum
0	Initial or planet surface conditions

ANALYSIS

Trajectory Program

Trajectory computations for all phases of the study were accomplished with a two-dimensional digital computer program. The program performed numerical integration of the equations of motion of a particle moving under the influence of thrust, drag, lift, and gravity over a spherical non-rotating planet. The force vectors and reference parameters are illustrated in figure 2 and the equations of motion are described in the appendix.

The vehicle was assumed to have drag characteristics typical of an Apollo type vehicle. Equations defining the drag coefficients throughout the flight regime are presented in the appendix.

Mars Atmosphere

The atmosphere used in this study was assumed to be of the exponential form described by the equations below:

$$\rho = \rho_o e^{\beta \frac{r_o h}{r_o + h}} \tag{1}$$

and

$$p = p_o \left(1 - C_1 \frac{\rho}{\rho_o}\right) \tag{2}$$

where

$$\beta = -1.82230 \times 10^{-5}\ \mathrm{ft}^{-1}$$

$$C_1 = 1.0025323$$

$$r_o = 10,930,128 \text{ ft}$$

$$\rho_o = .000258693 \text{ lbs sec}^2/\text{ft}^4$$

$$p_o = 182 \text{ lbs/ft}^2$$

The atmosphere used in this study may not agree with the current MSC mean Mars atmosphere because the MSC mean atmosphere was not available when the study began.

Trajectory Profile

The trajectory profile consisted of the eight phases described below and presented pictorally in figure 1.

Launch.- The vehicle was flown vertically for the first eight seconds at which time a small angle of attack was initiated. This angle was increased linearly to a maximum value at 12 seconds and was held constant for 24 seconds. The angle of attack was then decreased linearly to zero at 40 seconds after liftoff. During the remainder of this phase the vehicle was flown at zero angle of attack (gravity turn).

Suborbital Coast.- At the end of the launch phase the vehicle was allowed to coast with only aerodynamic and gravity forces acting on it. The time of the suborbital coast phase was optimized in conjunction with the launch burn time to establish the maximum payload delivered to various orbits about the planet.

Initial Orbit Circularization.- The third phase was the powered portion of the trajectory necessary to establish the circular orbit conditions at the desired initial orbit altitude. This circularizing portion was flown with an angle of attack. The angle was increased uniformally from zero at ignition to a maximum value 20 seconds later. This value was maintained until engine shutdown at circular orbit conditions.

Initial Orbit Coast.- No analysis was conducted for this phase.

Transfer Injection.- The angle of attack was varied to maintain a horizontal flight path angle throughout thrusting.

Transfer To High Altitude.- This phase was a coasting transfer from initial orbit to the final desired orbit altitude and included gravity and drag losses.

Final Orbit Circularization.- The velocity required to circularize into the final orbit was applied impulsively.

Final Orbit Coast.- No analysis was conducted for this phase.

Thrust-to-Weight Ratio Study

The characteristic velocity requirements and optimum thrust-to-weight ratios were determined for ascent into initial circular orbits of 75, 100, 150, and 200 nautical miles around Mars. The launch burn times, coast times, and burn times necessary to circularize the initial orbits were optimized for various f/w's to establish the maximum payload to each of the initial circular orbit altitudes. These payloads were then plotted to establish the optimum f/w for each altitude. Velocity requirements for transfers from these initial orbits to circular orbits as high as 1,000 n. mi. were obtained.

The effective specific impulse used in all characteristic velocity calculations was found to be best obtained as follows:

$$I_{sp_e} = \frac{I_{sp_o} + 4I_{sp_{vac}}}{5} \tag{3}$$

All specific impulses referred to in this report are vacuum values unless otherwise specified.

Optimization of the launch parameters was accomplished as outlined below. This optimization was conducted assuming an $I_{sp_{vac}}$ of 386.5 sec.

1. Select an initial orbit altitude
2. Select a f/w
3. Select a launch burn time
4. Obtain the burnout weight for a series of coast times
5. Repeat steps 3 and 4 for a series of launch burn times
6. Perform steps 2 through 5 for a series of thrust-to-weight ratios
7. Perform steps 1 through 6 for all desired altitudes

By selecting the coast time from step 4 that yields the maximum weight in orbit, the optimum coast time for a given launch burn time at a specified f/w and altitude was obtained.

Step 5 resulted in a launch burn time that delivered the maximum weight to orbit. From steps 4 and 5 optimum launch burn and optimum coast times were determined for a given f/w and altitude.

Results of step 6 were graphs of launch parameters (that is, characteristic velocity, optimum launch burn time, optimum coast time, and circularize burn time) versus thrust-to-weight ratio. The f/w resulting in minimum characteristic velocity was chosen from these graphs.

From step 7 optimum launch parameters were obtained for several initial orbit altitudes.

Data from the results of a study conducted simultaneously were used to determine the characteristic velocities necessary for elliptical transfers from the initial circular orbits to the final circular orbits at higher altitudes. Drag and gravity losses were included in the non-impulsive thrusting employed to initiate these transfers. These losses were also included for the transfer maneuver. A kick impulse was used to circularize the orbit at apoapsis of the transfer trajectory.

Specific Impulse Study

Specific impulse effects on characteristic velocity, optimum launch burn time, optimum coast time, and circularlize burn time were studied in a second phase of this analysis. The procedure used in this phase was similar to that used in the f/w study. The variable f/w was replaced by the variable I_{sp}, and f/w was held constant. The optimization scheme was the same as that outlined previously. No attempt was made to optimize the f/w at the various I_{sp}'s and no I_{sp} effects on the transfers were investigated. Optimum launch parameters were plotted versus I_{sp}.

RESULTS AND DISCUSSION

Thrust-to-Weight Ratio Study

The results of this portion of the study are presented in figures 3 through 8.

Figures 3a through 3d show the variation of characteristic velocity, optimum launch burn time, optimum coast time, and circularizing burn time with thrust-to-weight ratio for a vacuum specific impulse of 386.5 sec. These figures correspond to initial circular orbit altitudes of 75, 100, 150, and 200 nautical miles respectively. Tables 1a through 1d are typical trajectories for ascent to these four initial orbit altitudes.

Thrust-to-Weight Ratios.- The minimum characteristic velocity was the criteria used to determine the optimum f/w. These minimum velocities ranged between 14,695 and 15,221 ft/sec for initial orbits between 75 and 200 nautical miles. The corresponding optimum thrust-to-weight ratios varied within .918 to .871 respectively. These results were plotted versus initial orbit altitude and are shown in figure 4.

Launch Burn Times.- The launch burn time was optimized as previously described in the analysis section. The variation of optimum burn time with f/w is shown for all four initial orbit altitudes in figures 3a through 3d. The optimum values decrease with increasing f/w for all orbital altitudes. The optimum launch burn times corresponding to the optimum f/w's are 153, 184, 214, and 233 seconds for initial orbit altitudes of 75, 100, 150, and 200 n. mi. respectively.

Varying the launch burn time ±10 seconds from optimum resulted in approximately a 50 ft/sec increase in the characteristic velocity requirement to obtain the 200 n. mi. orbits. The velocity increase was less at the lower orbit altitudes.

Suborbital Coast Times.- The method used to obtain the optimum suborbital coast time was also described in the analysis section. Optimum coast time was not found to be critical. For example, a ±25 second deviation from the optimum coast time resulted in an increase of only 50 ft/sec in the characteristic velocity requirements for a 75 n. mi. initial orbit. Suborbital coast time was found to be even less sensitive for the higher initial orbital altitudes.

The optimum coast time trend with f/w showed some variation with initial orbit altitude. For a 75 n. mi. orbit, coast time increases with increasing f/w; for 100 n. mi. there is practically no variation; and for 150 and 200 n. mi. orbits, optimum coast time decreases as f/w increases.

For optimum f/w, the optimum coast time increases with increasing initial orbit altitude. Figure 5 shows these times to be 175, 245, 380, and 520 seconds for the altitudes 75, 100, 150, and 200 n. mi. respectively.

Initial Circularize Burn Times.- The initial circularize burn time was that time necessary to go from conditions at the end of coast of initial circular orbit conditions. The circularization burn times at optimum f/w were plotted against initial orbit altitude and are shown in figure 5.

Transfer to Final Orbit.- Transfers from the initial orbits to the final orbit altitudes were investigated. Figure 6 shows the characteristic velocities required to transfer from the initial orbit to the final orbit altitude and circularize.

Total Characteristic Velocity Requirements.- The total characteristic velocity requirements were obtained by combining the characteristic velocity required to establish the initial orbit with those required to transfer to the final orbit altitude and circularize. The results are presented in figure 7 where it is seen that for each final orbit there is an optimum initial orbit altitude. The optimum occurs very near 100 n. mi. for all final orbit altitudes. Figure 8 gives the total characteristic velocity requirements and optimum initial orbit altitude for various final orbits. The total characteristic velocity requirements ranged between 15,173 and 16,850 ft/sec for final orbit altitudes between 200 and 1,000 n. mi. if the initial orbit altitudes were 102.5 and 99.8 n. mi., respectively.

There exists a final orbit below which it is impractical to make the above elliptical transfer. Below this altitude the small velocity savings obtained by passing through an initial orbit do not compensate for the additional operational complexity introduced by the presence of the transfer maneuver. For example, the characteristic velocity savings for a 200 n. mi. final orbit is approximately 50 ft/sec. It is not the intent of this paper to define the orbital altitude below which the use of an initial orbit is impractical.

Specific Impulse Study

The results of the specific impulse study are presented in figures 9a through 9d showing characteristic velocity, optimum launch burn time, optimum suborbital coast time, and initial orbit circularize burn time as a function of specific impulse. These results were obtained using the procedure described in the analysis section. The analysis was based on a constant f/w of 0.828, and the specific impulses investigated were 321, 372, 386, 401, and 432 seconds. The study did not investigate the transfers from initial to final orbits.

Launch Burn Times.- Launch burn time was found to increase as I_{sp} increased for all initial orbit altitudes investigated. This increase was anticipated because increasing the I_{sp} reduces the propellant flow rate resulting in a lower acceleration.

Suborbital Coast Times.- Suborbital coast time variation with I_{sp} was found to be a function of initial orbit altitude. At 75 nautical miles orbit altitude, coast time decreased as I_{sp} was increased. For 100 nautical mile orbits there was little change. Optimum coast time increased with increasing specific impulse at 150 and 200 nautical mile orbits.

Circularize Burn Times.- Circularizing burn time showed no particular pattern except that its' variation with I_{sp} was small.

Characteristic Velocity Requirements.- At the higher I_{sp}'s, it was expected that the higher gravity losses combined with the lower drag losses would result in a larger characteristic velocity. However, characteristic velocity requirements were found to decrease with increasing I_{sp}. Table II compares I_{sp}'s 432 sec and 321 sec trajectories for launch into a 150 n. mi. initial circular orbit. Losses due to the angle of attack are not included because they are small and nearly equal for both cases.

Examination of the table reveals that the drag losses are less for the higher I_{sp}, but only by 56 ft/sec. This difference was attributed to the lower acceleration of the higher I_{sp} vehicle.

Gravity losses for the 432 sec I_{sp} case were 122 ft/sec less than for the 321 sec I_{sp} case, because the optimum trajectory for the high I_{sp} case required a more nearly horizontal flight path angle than the low I_{sp} case. The flight path angle was more nearly horizontal for the higher I_{sp} case because the longer burning allotted more time to reach the final altitude.

The result of these drag and gravity losses is a lower characteristic velocity at the higher specific impulse.

CONCLUSIONS

An investigation of the launch of a single stage vehicle from the Martian surface according to the described trajectory has been completed. The following conclusions were obtained from the results of the thrust-to-weight (fixed I_{sp}) study.

1. The optimum initial orbit altitude is approximately 100 nautical miles.

2. The optimum thrust-to-weight ratio is .895.

3. Optimum launch burn and coast times are 184 and 245 seconds, respectively.

4. Launch burn and coast times are not critical.

5. Minimum characteristic velocities are between 15,180 and 16,850 ft/sec for final orbit altitudes between 200 and 1,000 nautical miles.

The results of the specific impulse (fixed f/w) study can be summarized as follows. Increasing the specific impulse resulted in:

1. A decrease in characteristic velocity requirement

2. An increase in launch burn time

3. A decrease in coast time for initial orbits below 100 n. mi. and an increase in coast time for initial orbits above 100 n. mi.

4. Little change in circularize burn time for any given initial orbit altitude.

Multiple stage vehicle launches remain to be investigated.

REFERENCE

1. Koelle, H. H.: "Handbook of Astronautical Engineering", McGraw-Hill Book Company, Inc., New York, 1961.

TABLE I.- TYPICAL TRAJECTORY DATA FOR MARS LAUNCH

(a) Launch to 75 nautical mile initial orbit

Time (sec)	Velocity (ft/sec)	Altitude (ft)	Range (ft)	Acceleration (ft/sec^2)	Flight path angle (deg vert)	Thrust (lbs)	Weight (lbs)	Dynamic pressure (lbs/ft^2)
				Launch				
0	0	0	0	16.96	0	36,706	40,000	0
20	354	3,478	272	18.55	10.23	37,057	37,805	15
40	751	13,740	4,009	21.30	26.65	37,970	35,610	57
60	1,189	30,376	13,925	21.59	34.17	39,129	33,414	105
80	1,638	52,738	30,997	23.98	40.25	40,222	31,219	133
92	1,942	68,703	45,232	26.73	43.36	40,771	29,903	140
96	2,051	74,460	50,726	27.77	44.32	40,933	29,462	141
100	2,164	80,442	56,620	28.86	45.23	41,084	29,023	141
120	2,801	113,893	92,690	34.98	49.26	41,685	26,829	130
140	3,569	153,834	141,396	41.97	52.46	42,057	24,635	104
153	4,146	183,731	180,957	46.95	54.14	42,205	23,207	83
				Coast				
153	4,146	183,731	180,957	-11.56	54.14	0	23,207	83
160	4,071	200,404	203,978	-10.16	54.98	0	23,207	59
200	3,751	284,118	332,646	- 6.56	60.27	0	23,207	12
240	3,523	349,615	458,560	- 4.93	66.28	0	23,207	3
280	3,354	397,587	582,913	- 3.50	72.97	0	23,207	1
320	3,244	428,272	706,283	- 2.02	80.20	0	23,207	0
328	3,229	432,346	730,879	- 1.71	81.69	0	23,207	0
				Circularize				
328	3,229	432,346	730,879	57.08	81.69	42,414	23,207	0
340	3,937	437,743	771,851	61.01	83.73	42,414	21,890	1
380	6,685	451,008	973,428	77.45	88.11	42,415	17,500	2
431	11,475	455,652	1,411,836	114.28	90.00	42,415	11,883	6

TABLE I.- TYPICAL TRAJECTORY DATA FOR MARS LAUNCH - Continued

(b) Launch to 100 nautical mile initial orbit

Time (sec)	Velocity (ft/sec)	Altitude (ft)	Range (ft)	Acceleration (ft/sec^2)	Flight path angle (deg vert)	Thrust (lbs)	Weight (lbs)	Dynamic pressure (lbs/ft^2)
				Launch				
0	0	0	0	16.23	0	35,786	40,000	0
20	339	3,325	266	17.76	10.51	36,114	37,860	14
40	719	13,104	3,960	20.52	27.64	36,968	35,720	53
60	1,146	28,858	13,856	21.09	35.78	38,059	33,580	100
80	1,579	49,871	31,018	22.95	42.35	39,101	31,440	131
100	2,080	75,511	56,754	27.40	47.77	39,934	29,300	142
104	2,192	81,200	63,082	28.44	48.72	40,072	28,872	143
108	2,308	87,080	69,840	29.53	49.64	40,201	28,444	143
112	2,429	93,157	77,045	30.65	50.52	40,320	28,016	142
120	2,683	105,912	92,861	33.01	52.17	40,531	27,160	138
140	3,406	141,519	141,425	39.46	55.68	40,917	25,020	118
160	4,266	182,994	204,736	46.69	58.42	41,144	22,880	89
180	5,279	231,224	285,267	54.77	60.52	41,264	20,740	58
184	5,502	241,778	303,670	56.50	60.86	41,279	20,312	53
				Coast				
184	5,502	241,778	303,670	- 8.88	60.86	0	20,312	53
200	5,377	283,220	378,357	- 6.97	62.25	0	20,312	24
240	5,143	375,173	561,744	- 5.05	65.92	0	20,312	5
280	4,962	451,250	741,958	- 4.05	69.88	0	20,312	1
320	4,818	511,812	919,907	- 3.20	74.08	0	20,312	0
360	4,709	557,037	1,096,178	- 2.30	78.48	0	20,312	0
400	4,636	587,041	1,271,281	- 1.40	83.05	0	20,312	0
429	4,605	599,320	1,397,771	- .70	86.43	0	20,312	0
				Circularize				
429	4,605	599,320	1,397,771	64.79	86.43	41,354	20,312	0
440	5,341	602,143	1,449,516	69.04	87.57	41,354	19,135	0
480	8,479	607,256	1,708,790	89.48	89.73	41,354	14,855	0
509	11,399	607,497	1,980,926	113.23	90.00	41,354	11,745	0

TABLE I.- TYPICAL TRAJECTORY DATA FOR MARS LAUNCH - Continued

(c) Launch to 150 nautical mile initial orbit

Time (sec)	Velocity (ft/sec)	Altitude (ft)	Range (ft)	Acceleration (ft/sec^2)	Flight path angle (deg vert)	Thrust (lbs)	Weight (lbs)	Dynamic pressure (lbs/ft^2)
				Launch				
0	0	0	0	15.71	0	35,147	40,000	0
20	470	136	1,045	18.14	17.87	35,740	37,898	13
40	697	12,703	3,809	19.90	27.52	36,275	35,801	50
60	1,114	27,979	13,415	20.72	35.91	37,327	33,704	96
80	1,536	48,341	30,204	22.22	42.71	38,339	31,585	127
100	2,020	73,044	55,423	26.40	48.34	39,157	29,488	140
104	2,127	78,500	61,624	27.39	49.34	39,291	29,077	141
108	2,239	84,132	68,247	28.42	50.30	39,419	28,645	142
112	2,355	89,941	75,306	29.49	51.22	39,538	28,234	141
120	2,600	102,104	90,799	31.73	52.94	39,750	27,391	138
140	3,294	135,847	138,339	37.84	56.62	40,140	25,294	122
160	4,118	174,791	200,248	44.70	59.52	40,376	23,175	95
180	5,083	219,667	278,908	52.36	61.76	40,506	21,078	66
200	6,219	271,458	376,886	60.98	63.42	40,570	18,981	40
214	7,119	312,516	458,444	67.76	64.28	40,592	17,511	26
				Coast				
214	7,119	312,516	458,444	- 6.81	64.28	0	17,511	26
240	6,967	389,919	619,136	- 5.24	65.69	0	17,511	7
280	6,777	498,045	861,828	- 4.36	67.98	0	17,511	1
320	6,614	593,275	1,110,118	- 3.80	70.37	0	17,511	0
360	6,472	675,818	1,334,736	- 3.28	72.87	0	17,511	0
400	6,351	745,822	1,566,284	- 2.76	75.46	0	17,511	0
440	6,251	803,407	1,795,313	- 2.23	78.13	0	17,511	0
480	6,172	848,670	2,022,342	- 1.71	80.88	0	17,511	0
520	6,114	881,687	2,247,866	- 1.18	83.68	0	17,511	0

TABLE I.- TYPICAL TRAJECTORY DATA FOR MARS LAUNCH - Continued

(c) Launch to 150 nautical mile initial orbit

Time (sec)	Velocity (ft/sec)	Altitude (ft)	Range (ft)	Acceleration (ft/sec^2)	Flight path angle (deg vert)	Thrust (lbs)	Weight (lbs)	Dynamic pressure (lbs/ft^2)
				Coast				
560	6,077	902,511	2,472,364	- .64	86.52	0	17,511	0
594	6,063	910,652	2,662,729	- .19	88.95	0	17,511	0
				Circularize				
594	6,063	910,652	2,662,729	74.42	88.95	40,614	17,511	0
600	6,518	911,206	2,697,557	77.27	89.34	40,614	16,884	0
640	10,077	911,474	3,000,756	103.04	90.12	40,614	12,669	0
651	11,252	911,333	3,107,789	113.23	90.00	40,614	11,523	0

TABLE I.- TYPICAL TRAJECTORY DATA FOR MARS LAUNCH - Continued

(d) Launch to 200 nautical mile initial orbit

Time (sec)	Velocity (ft/sec)	Altitude (ft)	Range (ft)	Acceleration (ft/sec^2)	Flight path angle (deg vert)	Thrust (lbs)	Weight (lbs)	Dynamic pressure (lbs/ft^2)
				Launch				
0	0	0	0	15.46	0	34,827	40,000	0
20	322	3,167	251	16.93	10.45	35,131	37,898	13
40	686	12,486	3,782	19.63	27.82	35,929	35,844	49
60	1,098	27,463	13,359	20.59	36.45	37,957	33,747	95
80	1,515	47,370	30,147	21.90	43.43	37,951	31,672	126
100	1,919	71,386	55,365	25.92	49.22	38,758	29,596	141
104	2,097	76,669	61,563	26.88	50.25	38,892	29,164	142
108	2,206	82,114	68,180	27.88	51.23	39,018	28,753	143
112	2,320	87,722	75,231	28.91	52.18	39,135	28,342	143
116	2,438	93,495	82,732	29.98	53.09	39,245	27,932	142
120	2,560	99,436	90,699	31.07	53.93	39,346	27,499	141
140	3,239	131,748	138,110	36.99	57.76	39,739	25,424	127
160	4,044	168,736	199,751	43.64	60.76	39,984	23,348	102
180	4,990	211,013	277,959	51.08	63.08	40,120	21,251	74
200	6,094	259,437	375,267	59.48	64.82	40,191	19,176	47
220	7,378	315,185	494,437	59.15	66.05	40,226	17,100	26
233	8,323	356,089	585,055	76.39	66.60	40,235	15,739	17
				Coast				
233	8,323	356,089	585,055	- 5.85	66.60	0	15,739	17
240	8,284	379,052	636,717	- 5.39	66.85	0	15,739	11
280	8,094	503,914	927,655	- 4.32	68.33	0	15,739	1
320	7,931	618,268	1,212,381	- 3.88	69.87	0	15,739	0
360	7,783	722,307	1,491,747	- 3.51	71.47	0	15,739	0
400	7,650	816,156	1,766,402	- 3.15	73.13	0	15,739	0
440	7,531	899,920	2,036,933	- 2.80	74.84	0	15,739	0
480	7,426	973,689	2,303,885	- 2.45	76.60	0	15,739	0

TABLE I.- TYPICAL TRAJECTORY DATA FOR MARS LAUNCH - Concluded

(d) Launch to 200 nautical mile initial orbit

Time (sec)	Velocity (ft/sec)	Altitude (ft)	Range (ft)	Acceleration (ft/sec^2)	Flight path angle (deg vert)	Thrust (lbs)	Weight (lbs)	Dynamic pressure (lbs/ft^2)
				Coast				
520	7,335	1,037,544	2,567,767	- 2.10	78.41	0	15,739	0
560	7,258	1,091,553	2,829,062	- 1.75	80.26	0	15,739	0
600	7,194	1,135,774	3,088,230	- 1.40	82.14	0	15,739	0
640	7,145	1,170,254	3,345,712	- 1.06	84.05	0	15,739	0
680	7,109	1,195,028	3,601,937	- .71	85.98	0	15,739	0
720	7,088	1,210,124	3,857,320	- .36	87.92	0	15,739	0
753	7,080	1,215,303	4,067,668	- .08	89.53	0	15,739	0
				Circularize				
753	7,080	1,215,303	4,067,668	82.18	89.53	40,246	15,739	0
760	7,670	1,215,581	4,114,113	86.23	89.82	40,246	15,003	0
795	11,110	1,215,276	4,406,085	113.72	90.00	40,246	11,371	0

TABLE II.- COMPARISON OF 432 AND 321 SEC. I_{sp} TRAJECTORIES TO 150 N. MI. INITIAL CIRCULAR ORBIT ABOUT MARS

Phase	Time (sec)	Flight Path Angle (deg vert)	Altitude (ft)	Actual Velocity (ft/sec)	ΔDrag Losses (ft/sec)	ΔGravity Losses (ft/sec)	ΔCharacteristic Velocity (ft/sec)
I_{sp} = 432 seconds							
	0	0	0	0			
Launch					1002	1908	11050
	252	70.65	345421	8139			
Suborbital Coast					28	820	0
	650	88.71	908314	7291			
Initial Orbit Circularization					0	4	3976
	699	90.00	911582	11252			
I_{sp} = 321 seconds							
	0	0	0	0			
Launch					1038	1796	10008
	212	64.13	299083	7163			
Suborbital Coast					49	1039	0
	582	87.96	906737	6075			
Initial Orbit Circularization					0	19	5200
	630	90.00	911490	11252			

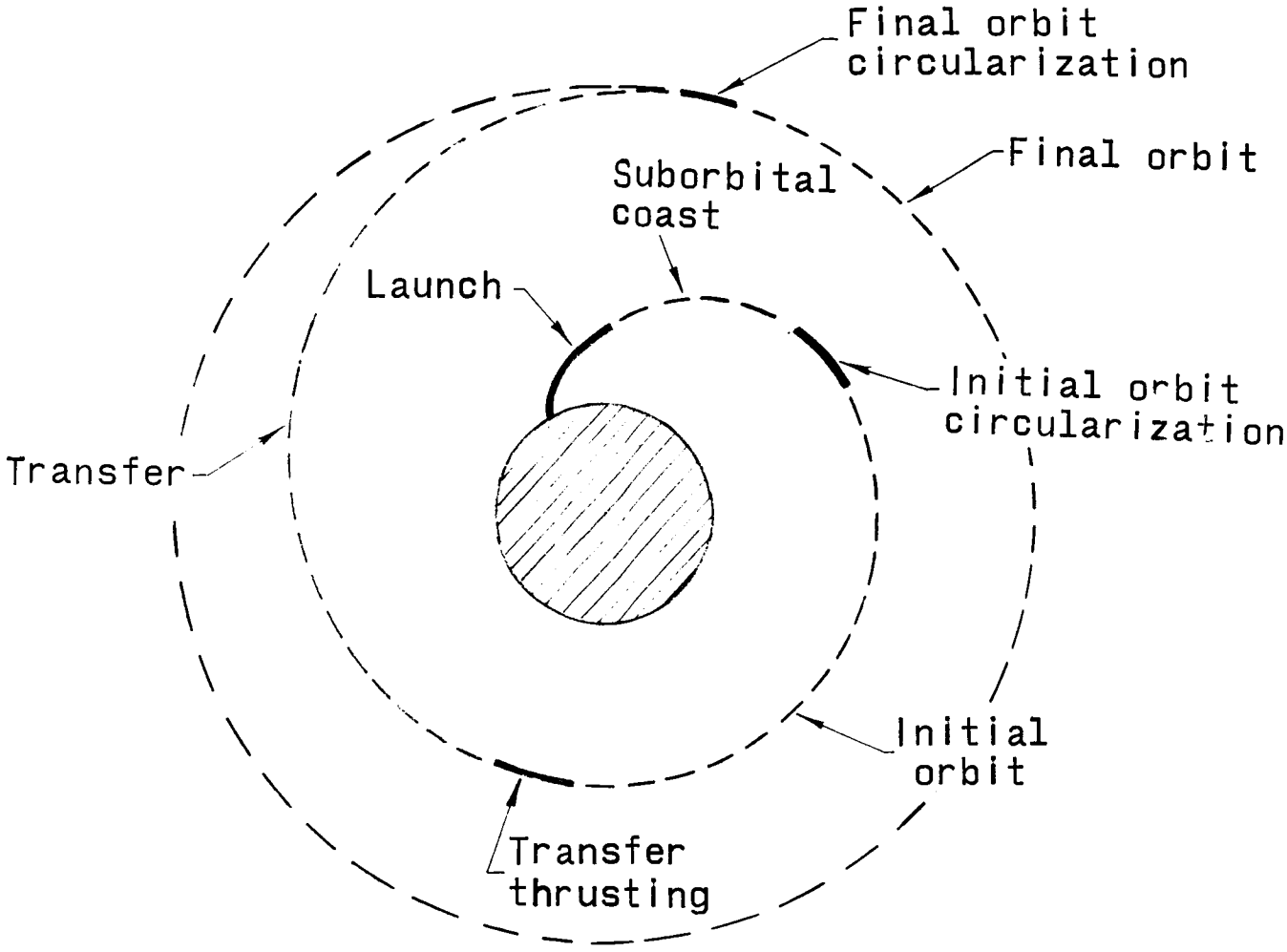


Figure 1.- Vehicle trajectory diagram.

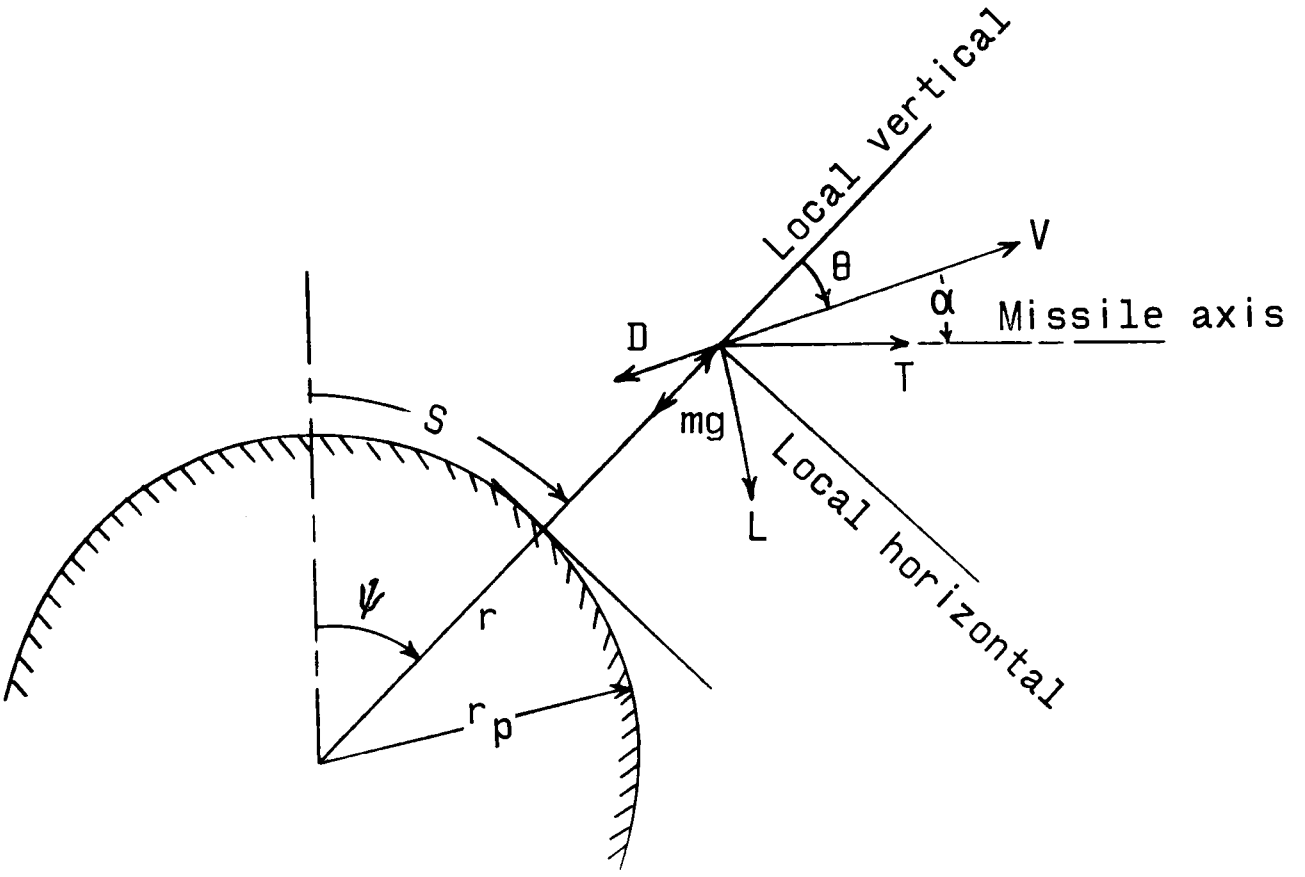
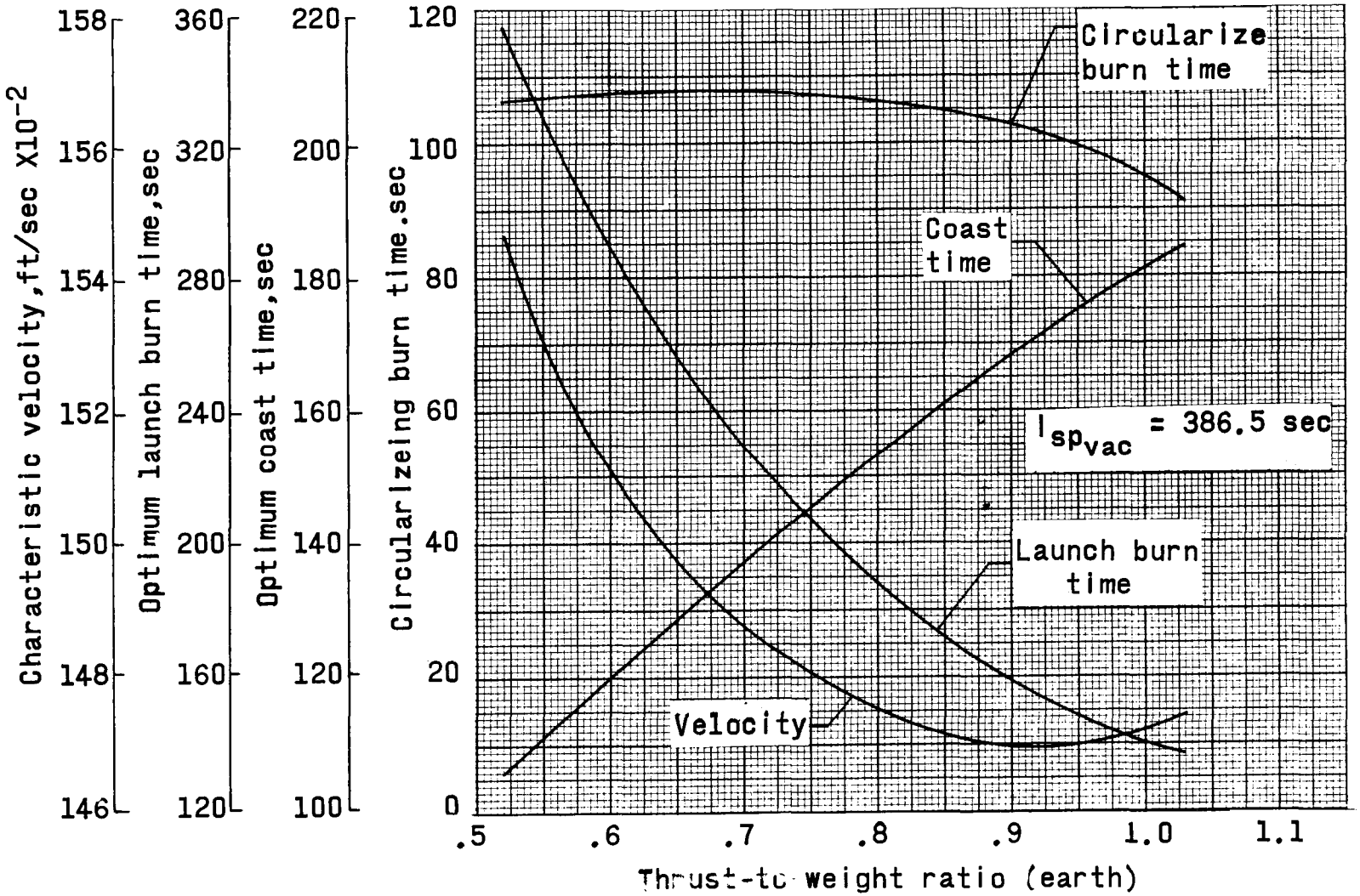
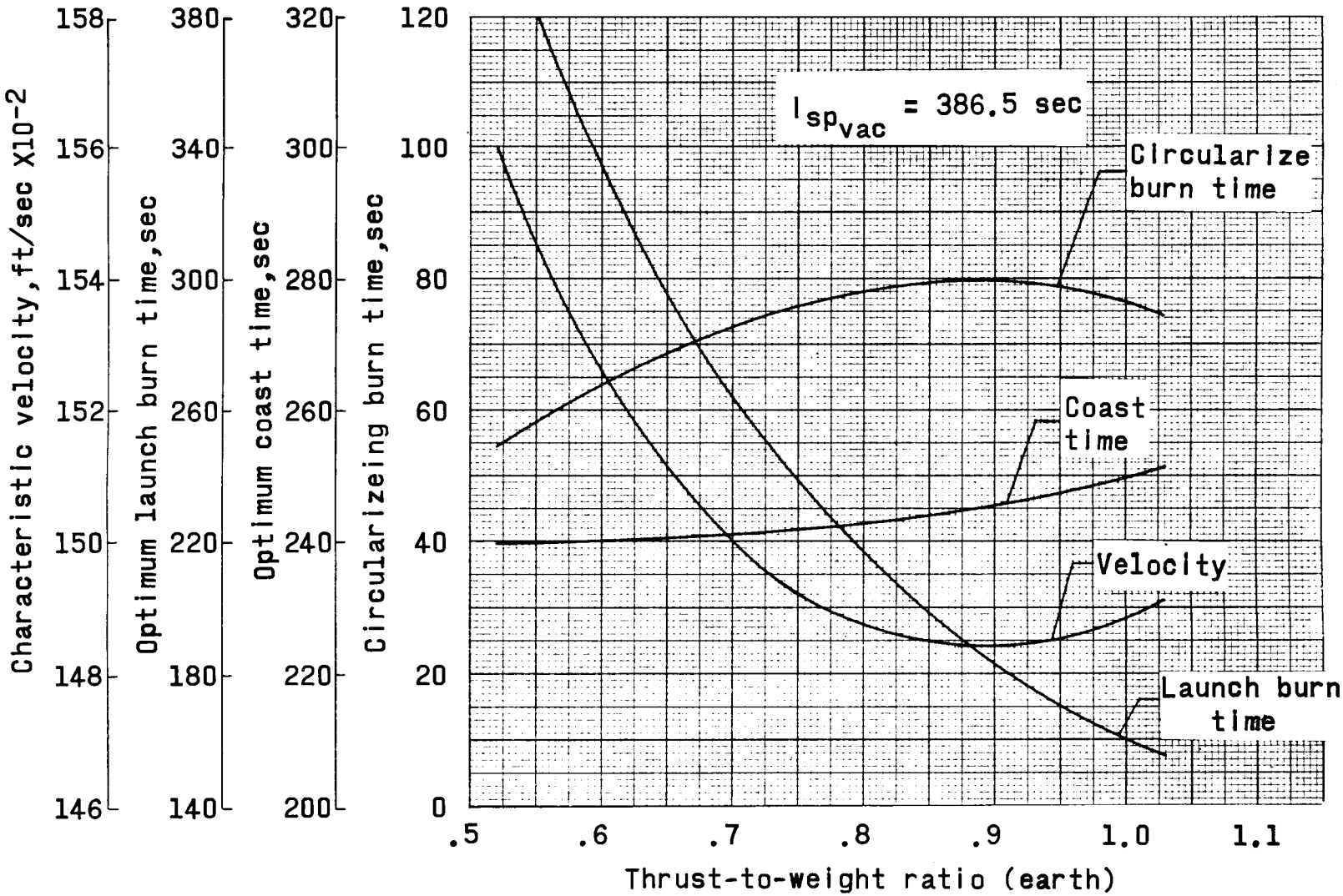


Figure 2.- Force vectors and reference parameters for an ascent trajectory.



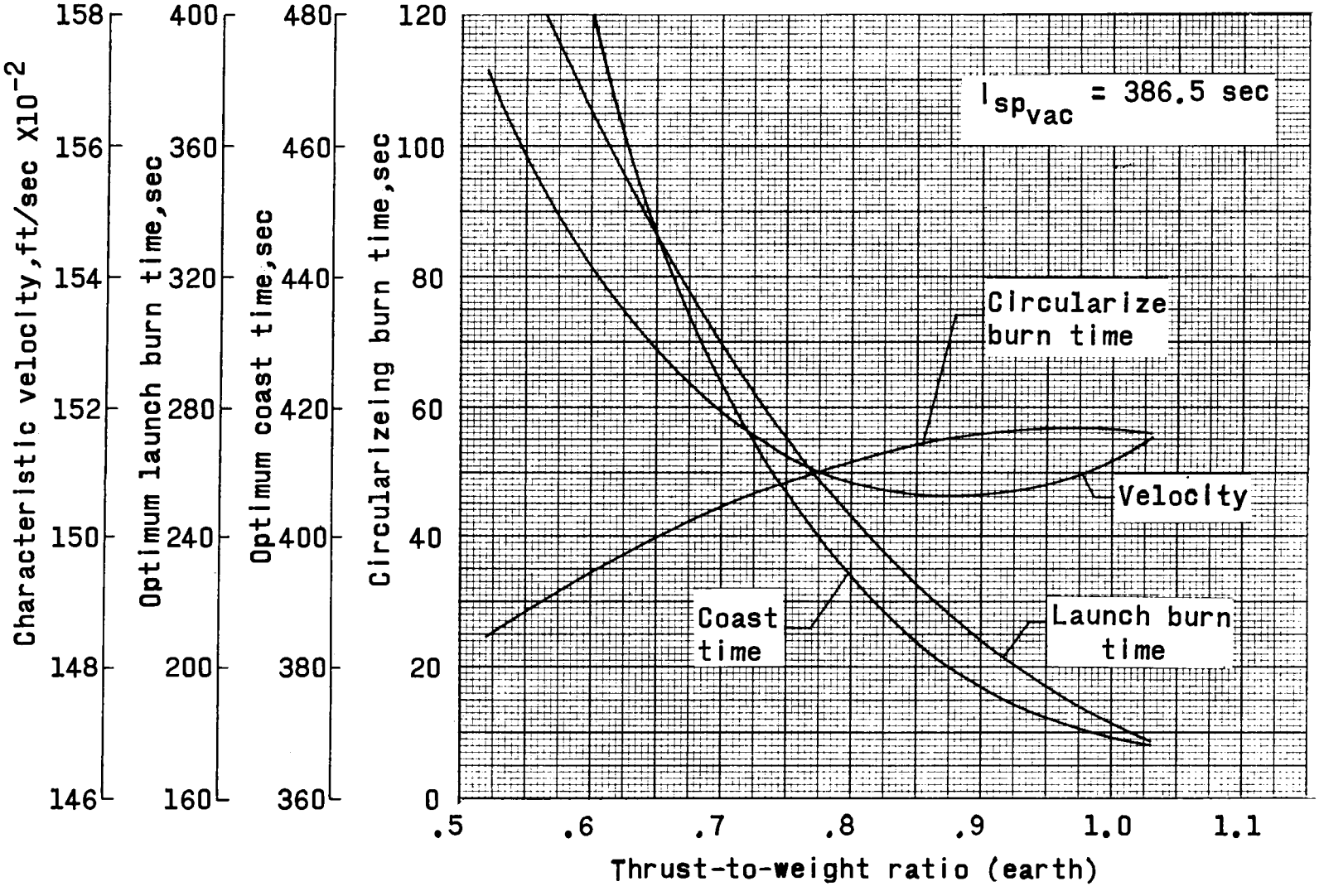
(a) Initial orbit altitude = 75 nautical miles.

Figure 3.- Thrust to weight ratio effect on Mars launch parameters.



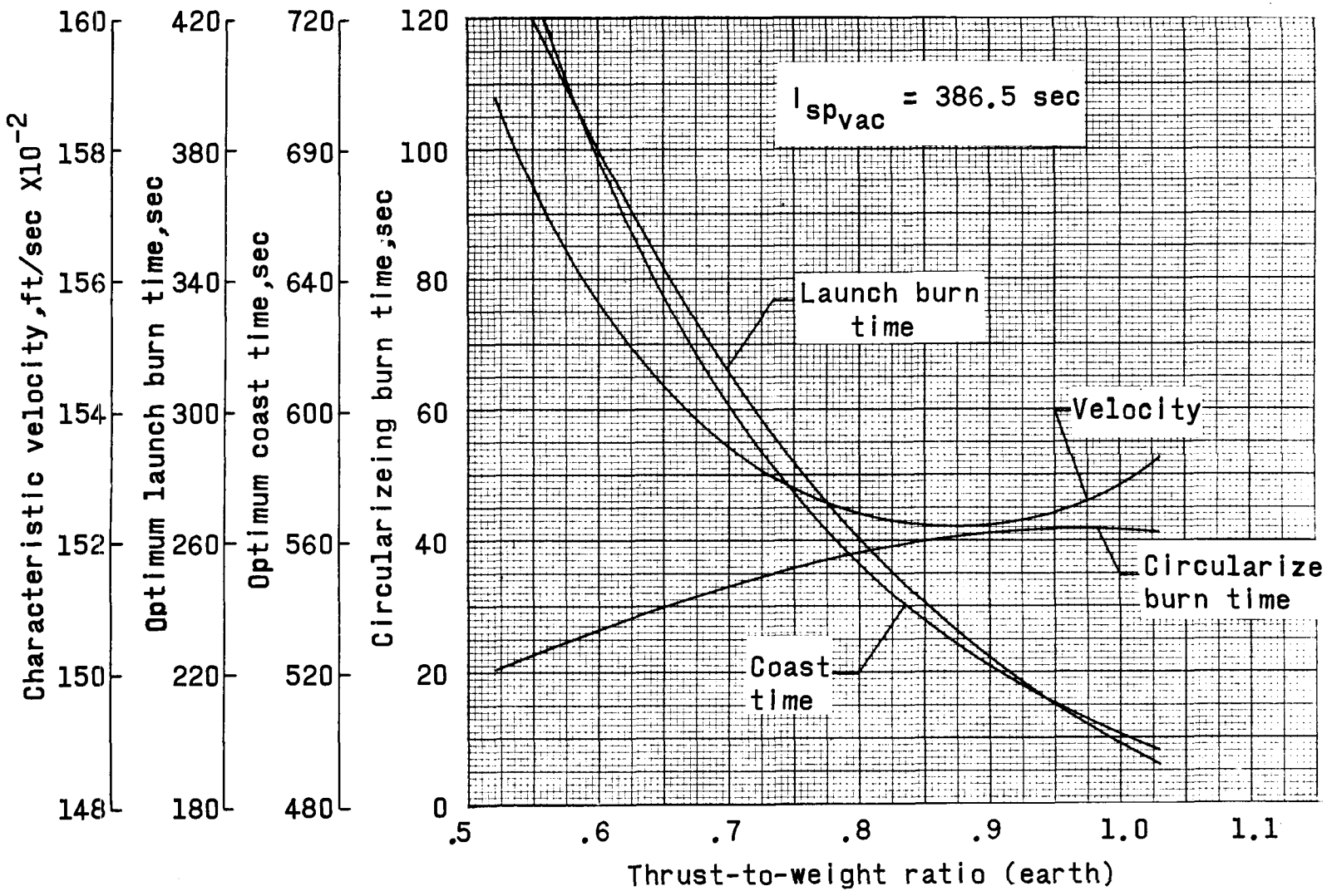
(b) Initial orbit altitude = 100 nautical miles.

Figure 3.- Continued.



(c) Initial orbit altitude = 150 nautical miles.

Figure 3.- Continued.



(d) Initial orbit altitude = 200 nautical miles.

Figure 3.- Concluded.

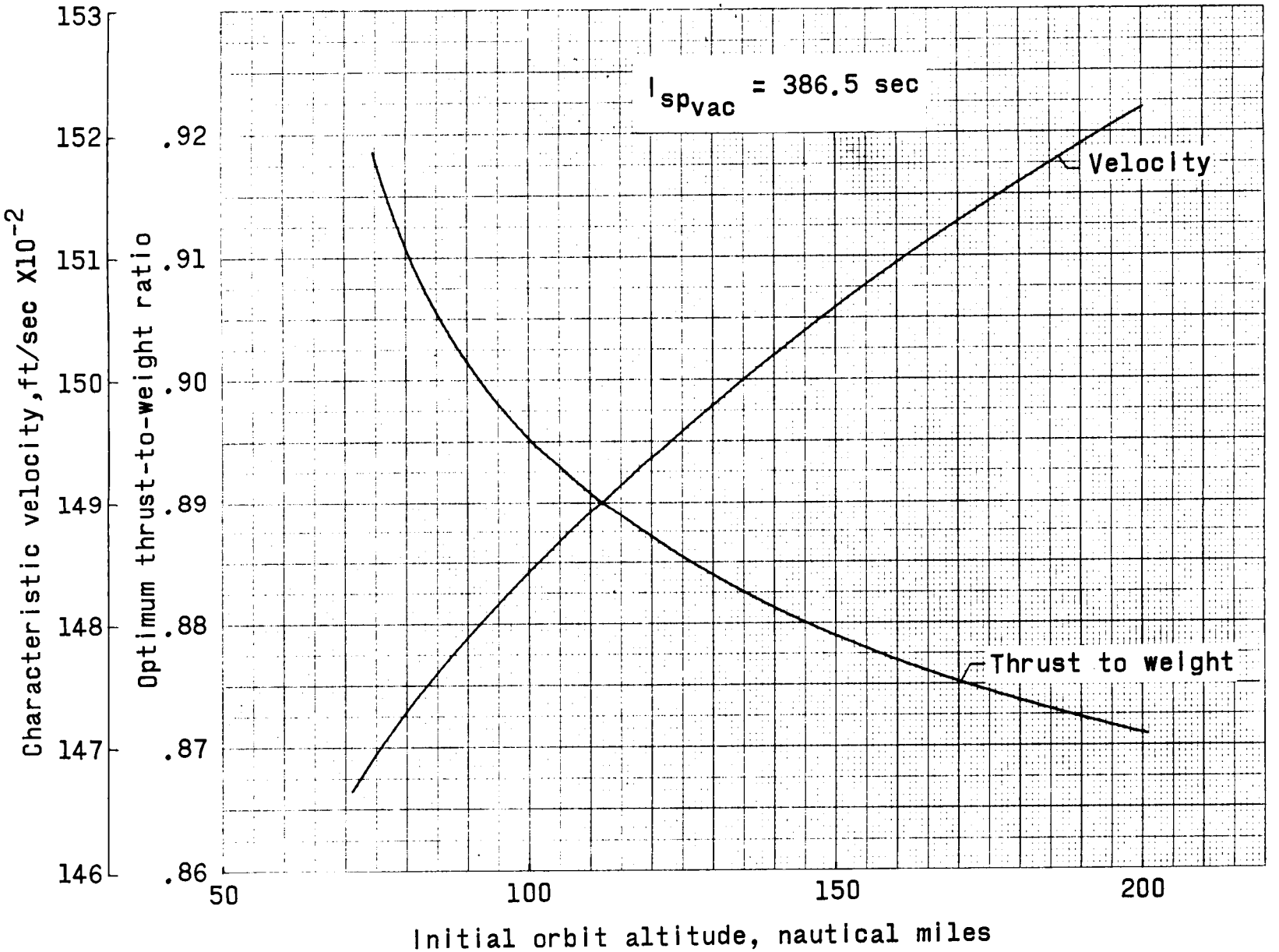


Figure 4.- Initial orbit altitude effect on characteristic velocity and thrust to weight ratio for Mars launch.

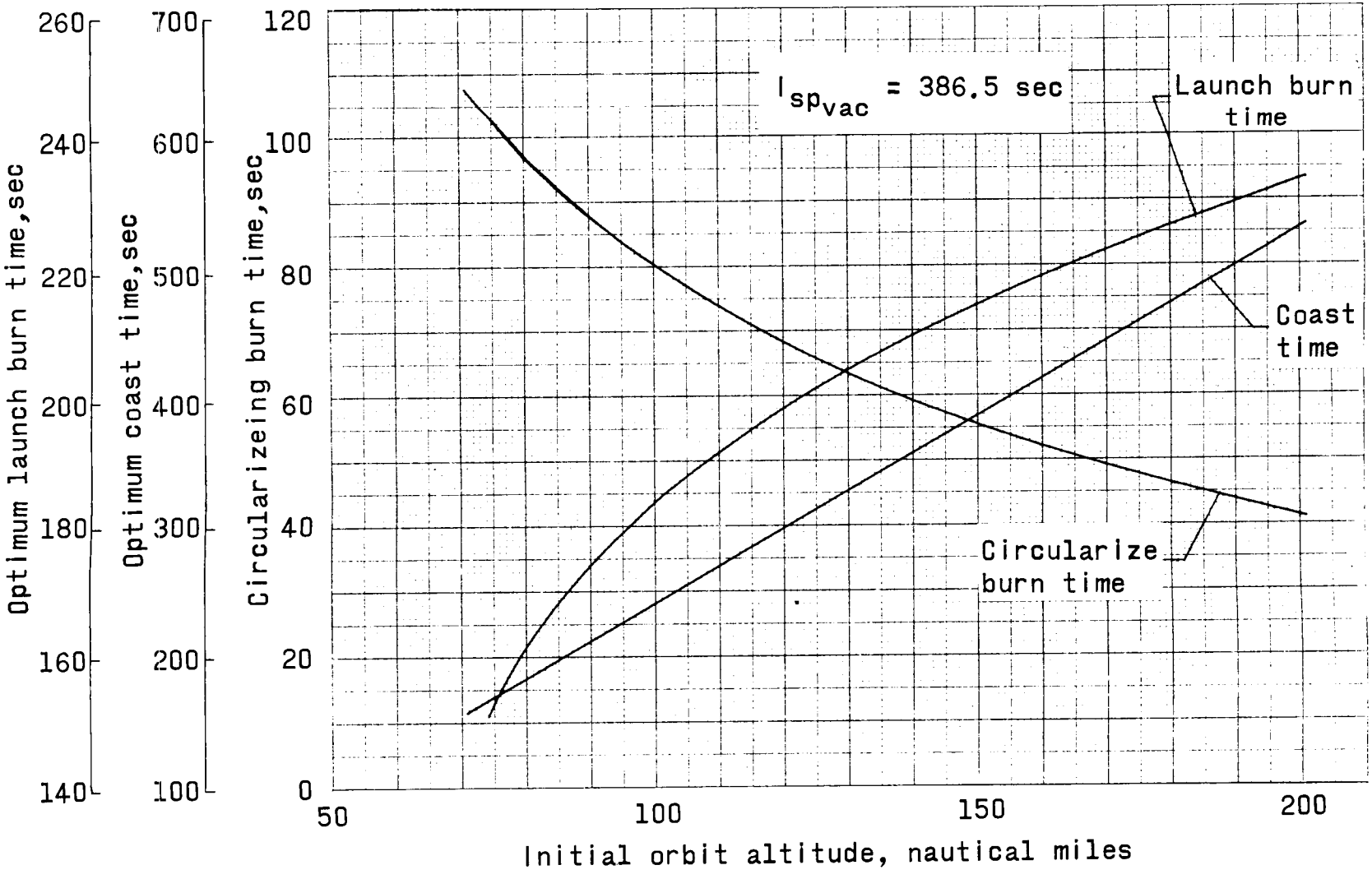


Figure 5.- Initial orbit altitude effect on burn and coast times for Mars launch.

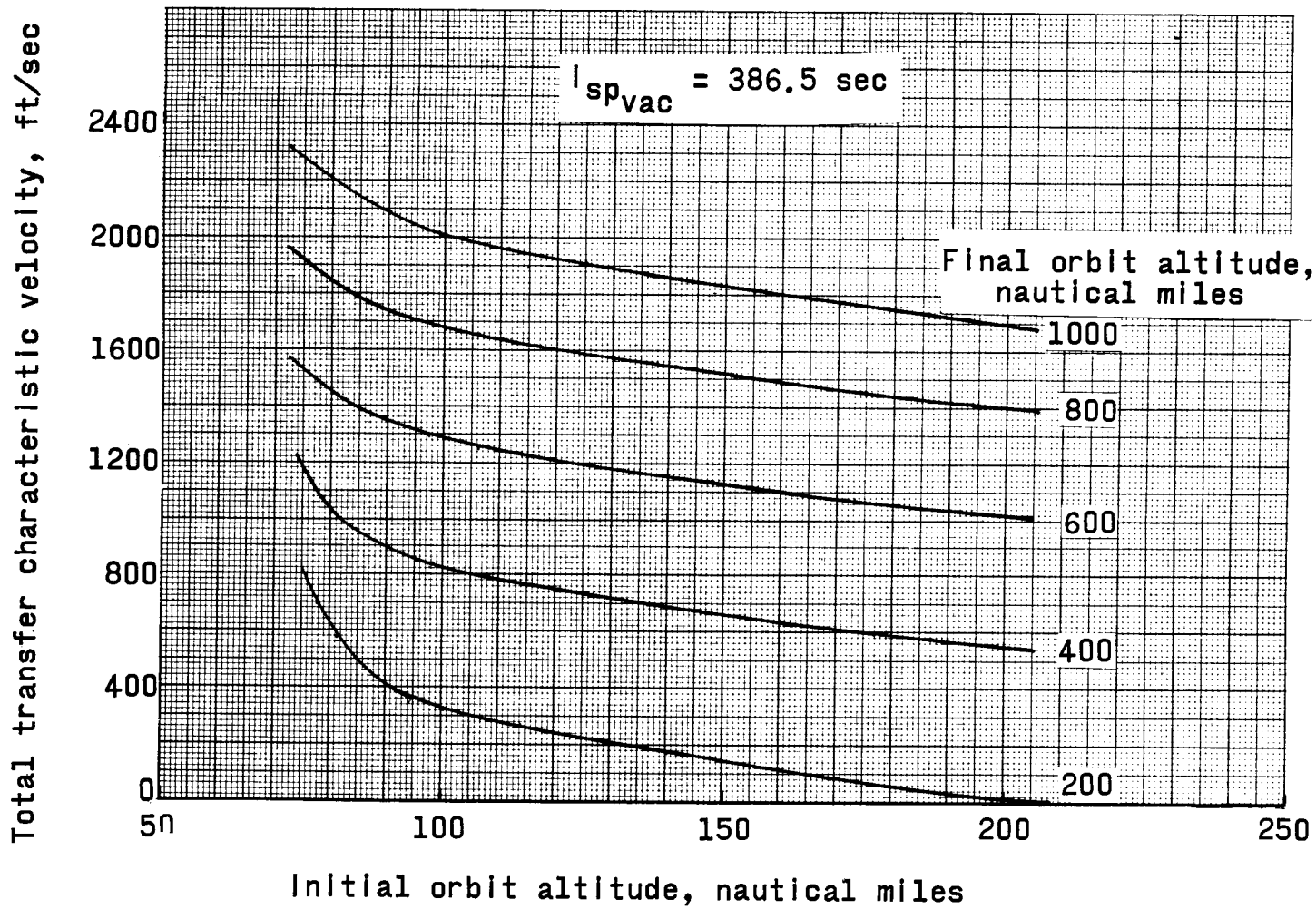


Figure 6.- Characteristic velocity requirements for Mars orbit transfer.

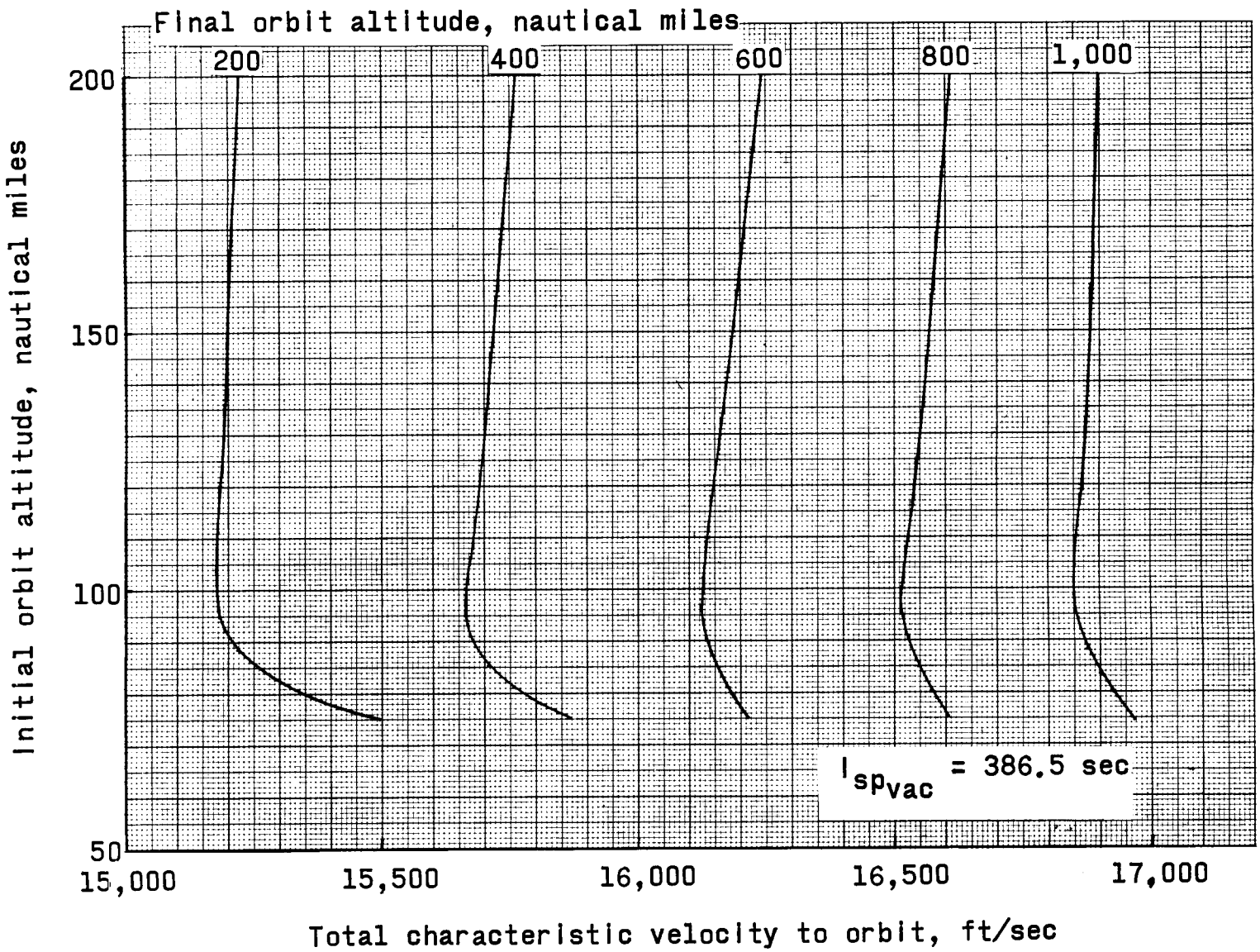


Figure 7.- Characteristic velocity variation with initial orbit altitude for launch from Martian surface to various final orbits.

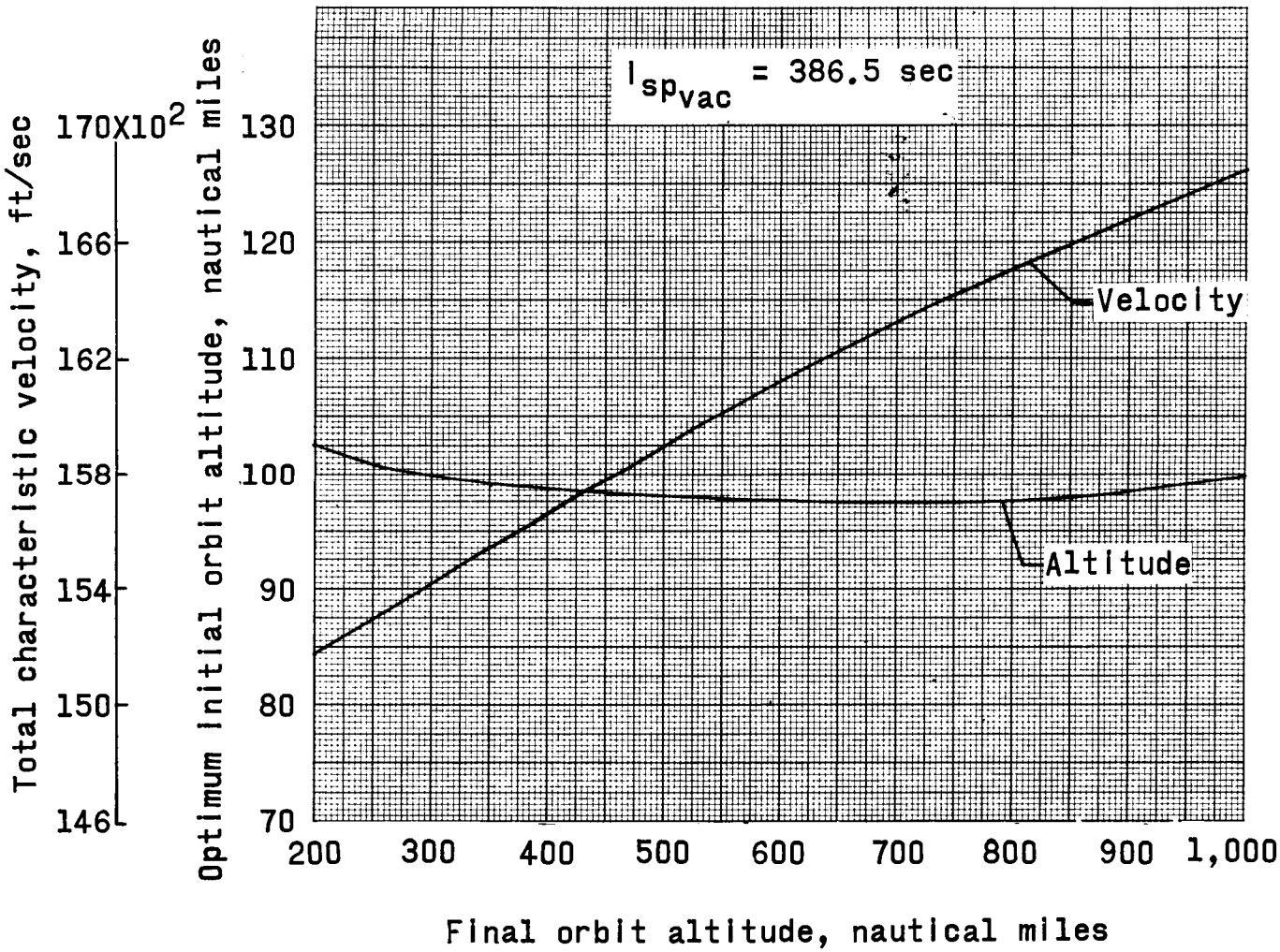
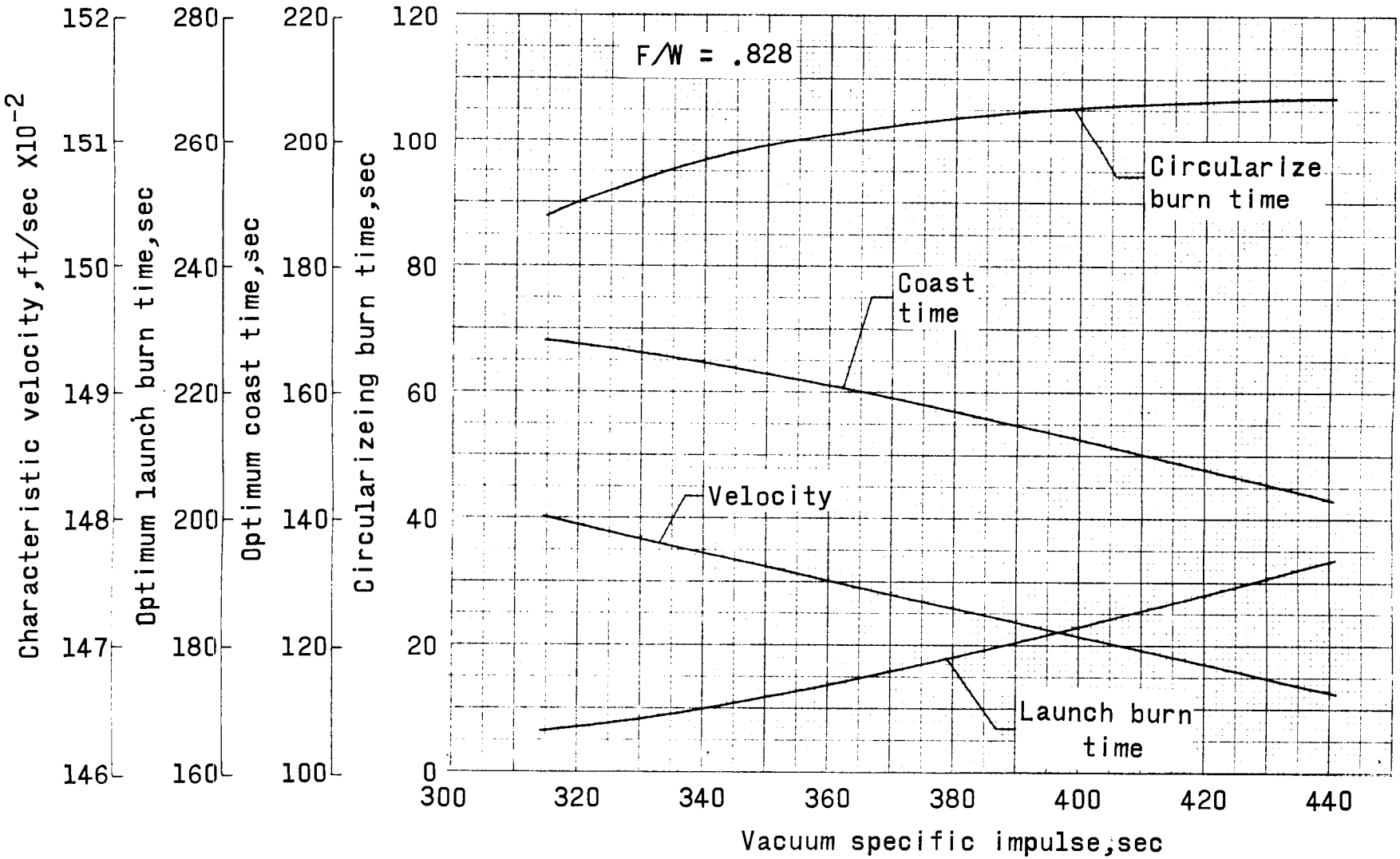
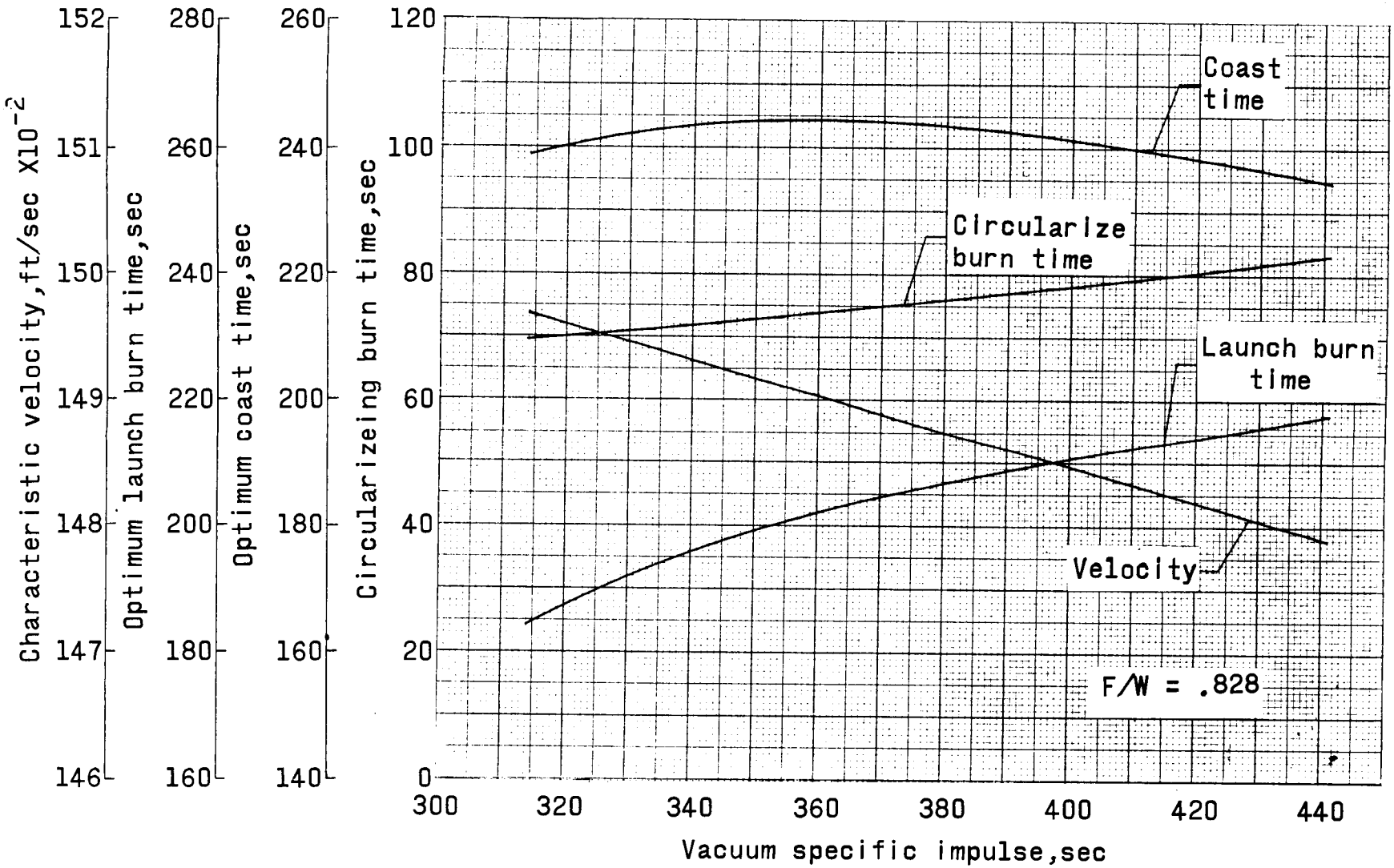


Figure 8.- Variation of characteristic velocity and optimum initial orbit altitude with final orbit altitude.



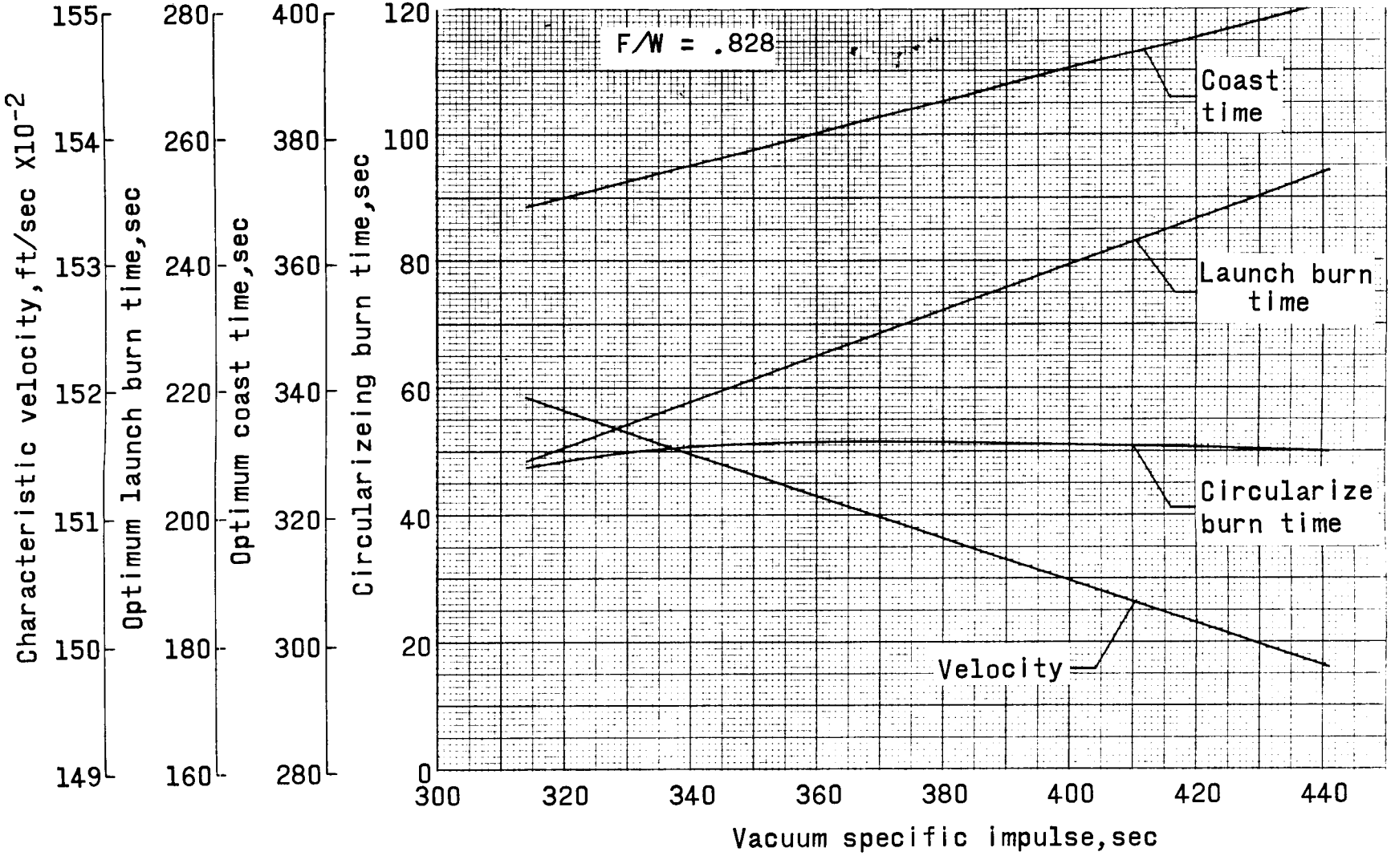
(a) Initial orbit altitude = 75 nautical miles.

Figure 9.- Specific impulse effect on Mars launch parameters.



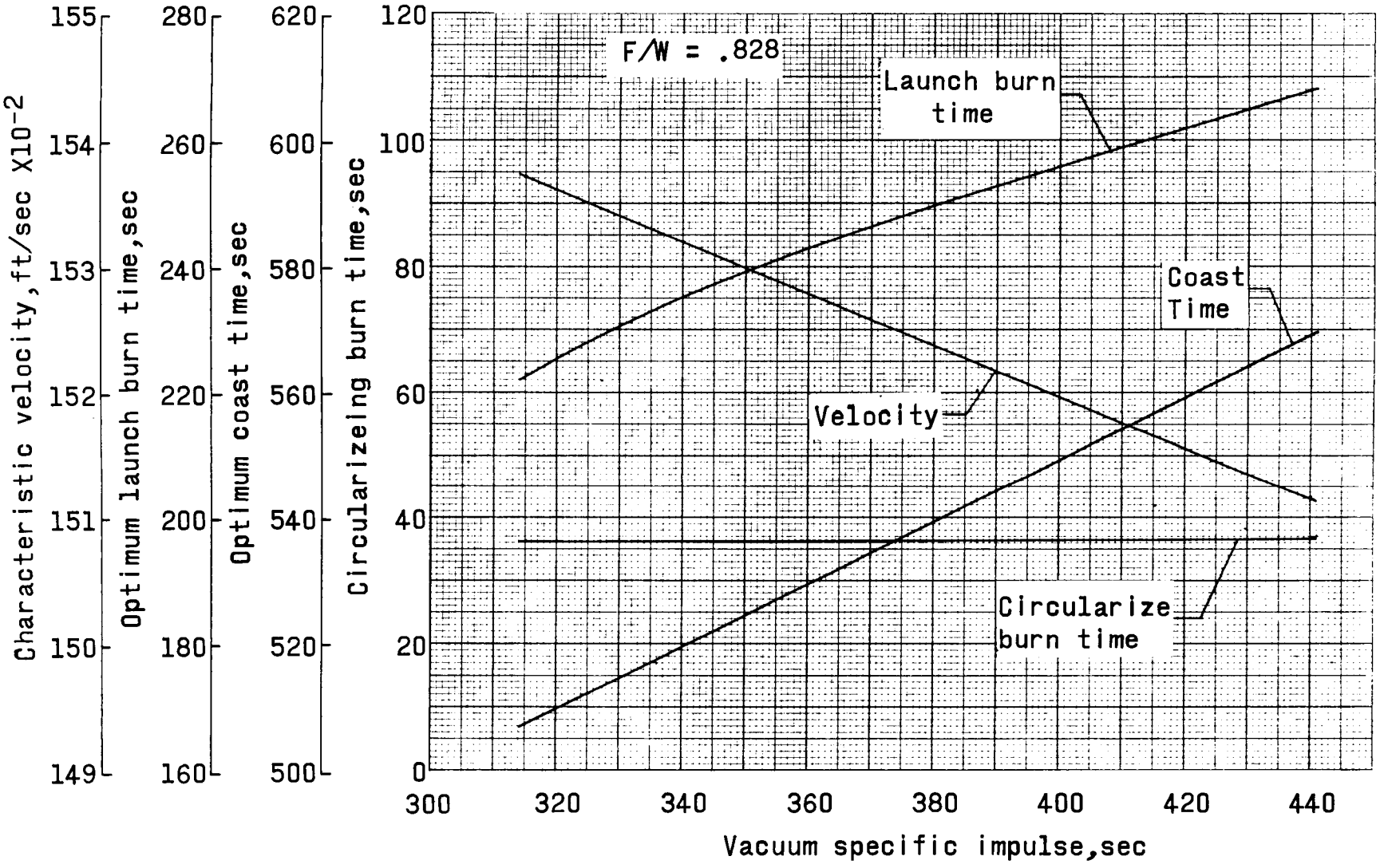
(b) Initial orbit altitude = 100 nautical miles.

Figure 9.- Continued.



(c) Initial orbit altitude = 150 nautical miles.

Figure 9.- Continued.



(d) Initial orbit altitude = 200 nautical miles.

Figure 9.- Concluded.

APPENDIX

Equations of Motion

Basic equations.- The equations of motion used in this analysis were obtained by considering a particle moving over a spherical non-rotating planet under the influence of gravity, lift, thrust, and drag. The force vectors and reference parameters are illustrated in figure 2. Following the technique of reference 1, forces were summed parallel and perpendicular to the velocity vector to obtain the equations of motion.

$$m\dot{v} = f \cos \alpha - D - mg \cos \theta \tag{1}$$

$$mv\dot{\theta} = f \sin \alpha + L + mg \sin \theta - mv\dot{\psi} \tag{2}$$

where

$$\dot{\psi} = \frac{v}{r} \sin \theta \tag{3}$$

$$f = f_o + A_e (p_o - p) \tag{4}$$

$$m = m_o - \dot{m} (t - t_o) \tag{5}$$

$$L = C_{L_\alpha} \alpha q A \tag{6}$$

$$D = C_D q A \tag{7}$$

$$q = \frac{1}{2} \rho V^2 \tag{8}$$

$$g = g_o \left(\frac{r_p}{r}\right)^2 \tag{9}$$

$$r = r_p + h \tag{10}$$

The equations of motion (1) and (2) were numerically integrated to give the velocity and flight path angle as a function of time.

$$v = \int_0^t \dot{v}\, dt \tag{11}$$

$$\theta = \int_0^t \dot{\theta}\, dt \tag{12}$$

Altitude and ground range were then obtained from the following integrations:

$$h = \int_0^t v \cos \theta\, dt \tag{13}$$

$$s = r_p \int_0^t \frac{v \sin \theta\, dt}{r} \tag{14}$$

<u>Drag Equations</u>.- Drag was varied along the trajectory as follows:

$$C_D = K_o \qquad 0 < M < Mo \tag{15}$$

$$C_D = K_1 + \frac{K_2}{M^2} - \frac{K_3}{M^4} \qquad Mo < M < \infty \tag{16}$$

where K_1, K_2, K_3, and Mo are constants given by

$$K_1 = B - \frac{a^4 (B-Ca)}{(a^2 - A_b{}^2)^2} \tag{17}$$

$$K_2 = 2 A_b{}^2 (B - K_1) \tag{18}$$

$$K_3 = \frac{1}{2} A_b{}^2 K_2 \tag{19}$$

$$M_o{}^2 = \frac{K_2 - \sqrt{K_2{}^2 - 4K_3 (K_o - K_1)}}{2(K_o - K_1)} \tag{20}$$

For the present study, the following constants were used

$Ko = 0.635$

$A_b = 1.4$

$B = 1.08$

$a = 4.0$

$C_a = 0.705$

$C_{L_\alpha} = 2.86/RAD$