VSM

Bissett-Berman Corporation

APOLLO GUIDANCE AND NAVIGATION NOTES

Volume XV

(NASA-CR-145633) APOLLO GUIDANCE AND NAVIGATION NOTES, VOLUME 15 (Bissett-Berman Corp.) 374 p

N75-77183

Unclas

00/98 39943

JAN 1972
RECEIVED
NASA STI FACILITY
INPUT BRANCH

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 470
(BBC Task 203)

H. Engel
L. Lustick
J. Gregory

ADDITIONAL CAPABILITIES OF THE RTODP-OEAP

Seven new features have been added to the RTODP-OEAP. They are:

1. Shift
2. Consolidation
3. Arbitrary weights
4. Different noise in filter and real world
5. QE
6. QR
7. QP .

Shift

At times the two spacecraft for which the program is capable of computing error covariances may come together. In such a circumstance it is desirable to ascribe the same error covariance to the estimates of both. The newly added shift operation permits us to make the covariance of the estimate of spacecraft A the same as that of spacecraft B or vice versa.

In order to illustrate what is done, let us consider a shift of A to B. In the filter world, before the shift we have

$$\begin{bmatrix} E\,\Delta\bar{x}_A\Delta\bar{x}_A^T & E\,\Delta\bar{x}_A\Delta\bar{x}_B^T & E\,\Delta\bar{x}_A\,\Delta\bar{D}^T \\ E\,\Delta\bar{x}_B\Delta\bar{x}_A^T & E\,\Delta\bar{x}_B\Delta\bar{x}_B^T & E\,\Delta\bar{x}_B\,\Delta\bar{D}^T \\ E\,\Delta\bar{D}\,\Delta\bar{x}_A^T & E\,\Delta\bar{D}\Delta\bar{x}_B^T & E\,\Delta\bar{D}\,\Delta\bar{D}^T \end{bmatrix}$$

and after the shift we have

$$\begin{bmatrix} E\,\Delta\bar{x}_A\,\Delta\bar{x}_A^T & E\,\Delta\bar{x}_A\,\Delta\bar{x}_A^T & E\,\Delta\bar{x}_A\,\Delta\bar{D}^T \\ E\,\Delta\bar{x}_A\,\Delta\bar{x}_A^T & E\,\Delta\bar{x}_A\,\Delta\bar{x}_A^T & E\,\Delta\bar{x}_A\,\Delta\bar{D}^T \\ E\,\Delta\bar{D}\,\Delta\bar{x}_A^T & E\,\Delta\bar{D}\,\Delta\bar{x}_A^T & E\,\Delta\bar{D}\,\Delta\bar{D}^T \end{bmatrix}$$

in which $\bar{D}$ represents the dynamic biases.

Similarly, for the noise, after transfer from A to B we have

$$\begin{bmatrix} E\,\Delta\hat{x}_{nA}\,\Delta\hat{x}_{nA}^T & E\,\Delta\hat{x}_{nA}\,\Delta\hat{x}_{nA}^T \\ E\,\Delta\hat{x}_{nA}\,\Delta\hat{x}_{nA}^T & E\,\Delta\hat{x}_{nA}\,\Delta\hat{x}_{nA}^T \end{bmatrix}$$

and for the $i^{\underline{th}}$ non-estimated nuisance parameter group, denoting this by β_i to avoid confusion with spacecraft B, we have

$$\begin{bmatrix} E\,\Delta\hat{x}_{\beta_i A}\,\Delta\hat{x}_{\beta_i A}^T & E\,\Delta\hat{x}_{\beta_i A}\,\Delta\hat{x}_{\beta_i A}^T & E\,\Delta\hat{x}_{\beta_i A}\,\Delta\hat{\beta}_i^T \\ E\,\Delta\hat{x}_{\beta_i A}\,\Delta\hat{x}_{\beta_i A}^T & E\,\Delta\hat{x}_{\beta_i A}\,\Delta\hat{x}_{\beta_i A}^T & E\,\Delta\hat{x}_{\beta_i A}\,\Delta\hat{\beta}_i^T \\ E\,\Delta\hat{\beta}_i\,\Delta\hat{x}_{\beta_i A}^T & E\,\Delta\hat{\beta}_i\,\Delta\hat{x}_{\beta_i A}^T & E\,\Delta\hat{\beta}_i\,\Delta\hat{\beta}_i^T \end{bmatrix}$$

Inputs that call for shifts from A to B and B to A are illustrated in Figure 1a and 1b respectively. The shift occurs before any other action. This is a regular batch card and shift may be combined with a batching operation.

If a shift from A to B (B to A) is called for and if the anchor point for spacecraft B(A) before the shift is different from that of spacecraft A (B), then the program will automatically change the anchor point and local coordinate system of spacecraft B (A) to be the same as that of A (B).

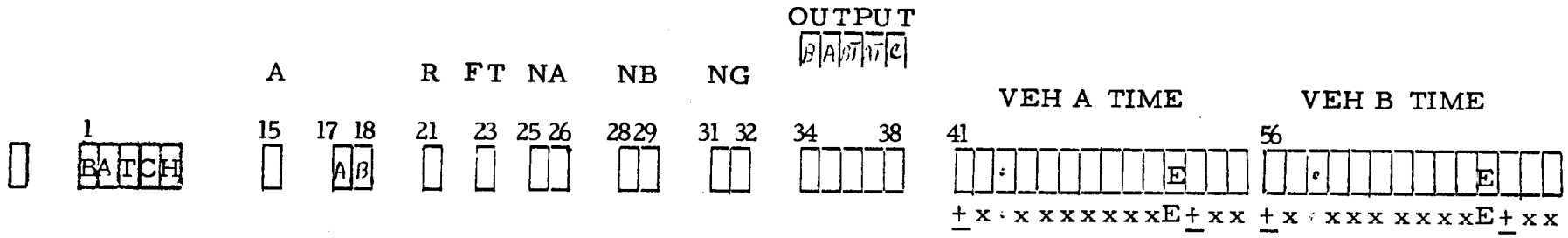


Figure 1a

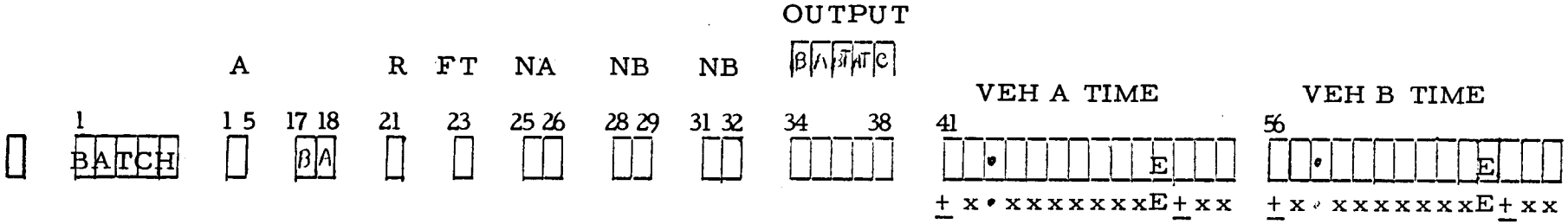


Figure 1b

Consolidation

Prior to this modification it has been necessary to maintain a non-estimated nuisance parameter group throughout all batches subsequent to that in which it was introduced. Now, if none of the parameters in a non-estimated nuisance parameter group is related to the measurables in any subsequent batch, this modification makes it unnecessary to maintain the nuisance parameter group.

From Apollo Note No. 437, pages 6 and 7

$$E \begin{bmatrix} \Delta \hat{x}_{Bi} \\ \Delta \hat{B}_i \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_{Bi} \\ \Delta \hat{B}_i \end{bmatrix}^T = \begin{bmatrix} W_a & W_{Bi} \\ O & I \end{bmatrix} E \begin{bmatrix} \Delta \hat{x}_{aBi} \\ \Delta \hat{B}_i \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_{aBi} \\ \Delta \hat{B}_i \end{bmatrix}^T \begin{bmatrix} W_a & W_{Bi} \\ O & I \end{bmatrix}^T$$

in which $\Delta \hat{x}_{Bi}$ is the error in the estimated parameters resulting from the error in the $i^{\underline{th}}$ group of non-estimated parameters B_i.

Now, W_{Bi} is zero if the measurable is independent of the nuisance parameter B_i, so in this case

$$E \begin{bmatrix} \Delta \hat{x}_{Bi} \\ \Delta \hat{B}_i \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_{Bi} \\ \Delta \hat{B}_i \end{bmatrix}^T = \begin{bmatrix} W_a & O \\ O & I \end{bmatrix} E \begin{bmatrix} \Delta \hat{x}_{aBi} \\ \Delta \hat{B}_i \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_{aBi} \\ \Delta \hat{B}_i \end{bmatrix} \begin{bmatrix} W_a & O \\ O & I \end{bmatrix}^T$$

and it follows that

$$E \Delta \hat{x}_{Bi} \Delta \hat{x}_{Bi}^T = W_a E \Delta \hat{x}_{aBi} \Delta \hat{x}_{aBi} W_a^T .$$

From page 6 of Apollo Note No. 437 we have

$$E \Delta \hat{x}_n \Delta \hat{x}_n^T = W_a \Delta \hat{x}_{an} \Delta \hat{x}_{an}^T W_a^T + C^{-1} E\, ee^T C^{-1}$$

so, adding these two equations we find

$$E(\Delta \hat{x}_n \Delta \hat{x}_n^T + \Delta \hat{x}_{Bi} \Delta \hat{x}_{Bi}^T) = W_a E\, (\Delta \hat{x}_{an} \Delta \hat{x}_{an}^T + \Delta \hat{x}_{Bi} \Delta \hat{x}_{Bi}^T)\, W_a^T + C^{-1} E\, ee^T C^{-1}.$$

In other words, if none of the parameters in a group of non-estimated nuisance parameters affect subsequent measurables, then we can add the covariance of the error in the estimated parameters resulting from that group to the covariance of the estimated parameters resulting from noise and drop that nuisance parameter group forever after.

In the program this is accomplished simply by placing the card for the covariance resulting from nuisance parameter group in the group for computing the new PN, as in Figure 2. This results in the addition of the upper left 22 x 22 of that nuisance parameter group covariance to PN.

Arbitrary Weights

Previously, the weights associated with the apriori estimate, and the estimate based on new measurements alone, depended only on the filter world covariances of the errors in those estimates, and were not under operator control; on page 4 of Apollo Note No. 437 we had

$$C = C_a + C_m$$
$$W_a = C^{-1} C_a$$

and

$$W_{bi} = -C^{-1} A_{12i}.$$

Now the program has been modified to permit downweighting of the apriori estimate. We write instead, if spacecraft A is being estimated,

$$C'_a = \begin{bmatrix} k^{-1} I_{11x11} & O \\ O & I_{(11+N_D)x(11+N_D)} \end{bmatrix} C_a \begin{bmatrix} k^{-1} I_{11x11} & O \\ O & I_{(11+N_D)x(11xN_D)} \end{bmatrix}$$

and then

$$C = C'_a + C_m$$
$$W_a = C^{-1} C'_a$$
$$W_{bi} = -C^{-1} A_{12i}.$$

This has the effect of increasing the apriori filter covariance of spacecraft A by a factor k^2. If spacecraft B is being estimated, a similar operation increases the apriori filter covariance of spacecraft B by k^2.

In using the program, if one wishes to multiply the apriori covariance by k^2 one enters k in the columns for σ on the data sheet line for the PF being brought in, as in Figure 3.

Different Noise in Filter and Real World

To weight data arbitrarily, or because he does not have an accurate estimate of the noise on the measurements, the noise variance specified for the filter by the RTODP operator may be different from the real noise variance. The RTODP-OEAP has been modified to permit simulation of this condition.

From Apollo Note No. 437, pages 6 and 9, we have

$$E\,\Delta\hat{x}_n\,\Delta\hat{x}_n^T = W_a E\,\Delta\hat{x}_{an}\Delta\hat{x}_{an}^T W_a^T + C^{-1} E\,ee^T C^{-1}$$

and

$$E\,\Delta\bar{x}_n\,\Delta\bar{x}_n^T = W_a E\,\Delta\bar{x}_{an}\Delta\bar{x}_{an}^T W_a^T + C^{-1} E\,e_f e_f^T C^{-1}$$

in which

$$E\,ee^T = \left(\frac{\partial M}{\partial x}\right)^T (E\,n_f n_f^T)^{-1}\,E\,nn^T\,(E\,n_f n_f^T)^{-1}\,\frac{\partial M}{\partial x}$$

and

$$E\,e_f e_f^T = \left(\frac{\partial M}{\partial x}\right)^T (E\,n_f n_f^T)^{-1}\,E\,n_f n_f^T\,(E\,n_f n_f^T)^{-1}\,\frac{\partial M}{\partial x} \quad .$$

If, as we assume, the noise samples are independent, then for the $i^{\underline{th}}$ kind of measurable

$$E\,e_i e_i^T = \frac{\sigma_i^2}{\sigma_{if}^2}\left(\frac{\partial M_i}{\partial x}\right)^T \frac{\partial M_i}{\partial x}$$

and

$$E\,e_{if} e_{if}^T = \frac{1}{\sigma_{if}^2}\left(\frac{\partial M_i}{\partial x}\right)^T \frac{\partial M_i}{\partial x} \quad .$$

In the modified version of the program, we use σ_{if} in computing C_m, and in the filter world still write

$$E\,\Delta\bar{x}_n\Delta\bar{x}_n^T = W_a\Delta\bar{x}_{an}\Delta\bar{x}_{an}^T W_a^T + C^{-1}C_m C^{-1}$$

PROGRAM B ALLOCATION SHEET

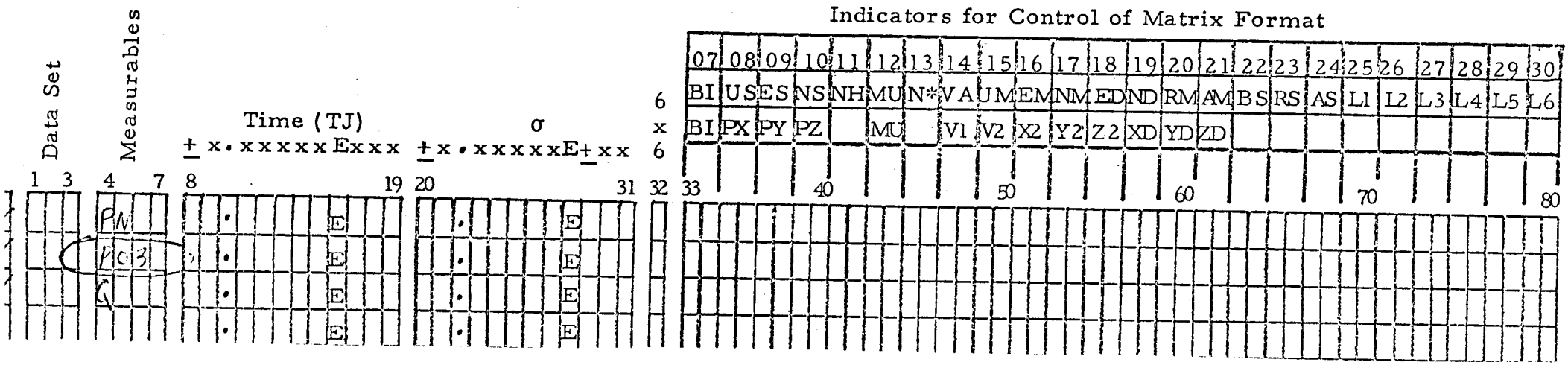


Figure 2

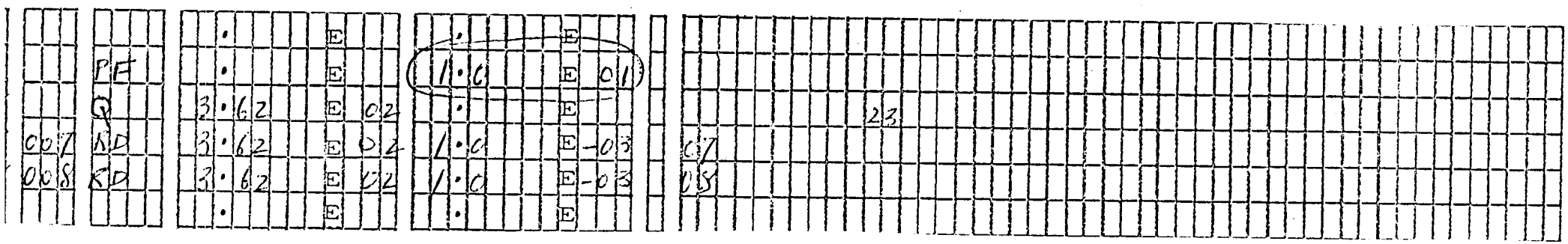


Figure 3

but in the real world we use an effective standard deviation σ_{if}^2/σ_i for the $i^{\underline{th}}$ measurable, and have

$$E\,\Delta\hat{x}_n\Delta\hat{x}_n^T = W_a\Delta\hat{x}_{an}\Delta\hat{x}_{an}^T W_a^T + C^{-1}\left(\sum_i \frac{\sigma_i^2}{\sigma_{if}^2}\left(\frac{\partial M_i}{\partial x}\right)\frac{\partial M_i}{\partial x}\right)C^{-1}$$

In the program, if we do not wish to make use of this new option, the program sheets are filled out, as before, with no information matrices used in computing the new PN. If we do wish to employ the new option, then in the PF computation portion σ_f must be used, and in the PN portion it is necessary to enter the information matrices, each with the appropriate standard deviation σ_f^2/σ, as in Figure 4.

<u>QE</u>

It has been necessary to adapt the QE computations to the RTODP-OEAP. In general we can write

$$\begin{bmatrix} dr \\ dv \\ -d\gamma_v \end{bmatrix}_T = \begin{bmatrix} x'/r & y'/r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dot{x}'/v & \dot{y}'/v & 0 \\ y'/r^2 & -x'/r^2 & 0 & -\dot{y}'/v^2 & \dot{x}'/v^2 & 0 \end{bmatrix}_T \begin{bmatrix} dx' \\ dy' \\ dz' \\ d\dot{x}' \\ d\dot{y}' \\ d\dot{z}' \end{bmatrix}_T$$

The RTODP-OEAP obtains the covariance of the errors in the estimate of the state vector in local coordinates, i.e., in a local coordinate system in which $x' = a_1$, $y' = 0$, $z' = 0$, $\dot{x}' = a_4$, $\dot{y}' = a_5$, $\dot{z}' = 0$. Thus we can write

$$\begin{bmatrix} dr \\ dv \\ -d\gamma_v \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4/v & a_5/v & 0 \\ 0 & -1/a_1 & 0 & -a_5/v^2 & a_4/v^2 & 0 \end{bmatrix} \begin{bmatrix} da_1 \\ da_2 \\ da_3 \\ da_4 \\ da_5 \\ da_6 \end{bmatrix}$$

in which

$$v = \left[a_4^2 + a_5^2\right]^{1/2} .$$

PROGRAM B ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)
	FW	. E	. E	
	103	. E	. E	
	G	. E	. E	
007	KD	3.62 E 02	5.0 E −01	
008	KD	3.62 E 02	5.0 E −01	
		. E	. E	

Indicators for Control of Matrix Format

07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									

Figure 4

The 3 x 6 matrix above is computed in Part B. The values of a_1, a_4, and a_5 used are obtained from the values of x', y', $\dot{x}'$, and $\dot{y}'$ of this point in the prime coordinate system of the anchor point by rotation to the local prime system. The 3 x 3 rotation matrix R is that normally calculated in Part B for changing anchor points. The 3 x 3 rotation matrix T is that computed from any input Euler angles. We have

$$\begin{bmatrix} a_1 \\ 0 \\ 0 \\ a_4 \\ a_5 \\ 0 \end{bmatrix} = \begin{bmatrix} T & O \\ O & T \end{bmatrix} \begin{bmatrix} R & O \\ O & R \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 0 \\ \dot{x}' \\ \dot{y}' \\ 0 \end{bmatrix} .$$

The QE output is printed each time the batch post data processing error covariance is requested.

QR

The QR computation has also been adapted to the new RTODP-OEAP. From pages A2-79 through A2-82 of "The Bissett-Berman Orbit Error Analysis Program," we have for fixed r

$$\begin{bmatrix} dv \\ -d\gamma_v \end{bmatrix} = \begin{bmatrix} \frac{\partial F}{\partial D} \end{bmatrix} \begin{bmatrix} \frac{\partial D}{\partial a} \end{bmatrix} da$$

$$= \begin{bmatrix} 0 & 1/v & 1/(rv) \\ 1/(r\dot{r}) & -H/(r\dot{r}v^2) & -H/(r^2v^2\dot{r}) \end{bmatrix} .$$

$$\begin{bmatrix} a_5 & -a_4 & 0 & 0 & a_1 & 0 & 0 \\ \mu/a_1^2 & 0 & 0 & a_4 & a_5 & 0 & -1/a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} da$$

but in our local coordinate system

$$r = a_1 \qquad v = \left[a_4^2 + a_5^2\right]^{1/2}$$
$$\dot{r} = a_4 \qquad H = a_1 a_5$$

so $\begin{bmatrix} dv \\ -d\gamma_v \end{bmatrix}$ is computable in Part B - a_1, a_4, a_5 and μ being available. a_1, a_4 and a_5 for fixed r could be computed directly from the energy and angular momentum, but that is not how they are obtained. Instead, the time T nominally corresponding to this r is determined in a preliminary program, or is otherwise available as an input and is used to determine the proper a_1, a_4, a_5.

It is necessary to have the y' and z' components of the Earth's angular rate vector. Calling these ω_{p2} and ω_{p3}, we have

$$\begin{bmatrix} \omega_{p1} \\ \omega_{p2} \\ \omega_{p3} \end{bmatrix} = \mathrm{TR}\ \mathbb{LK} \begin{bmatrix} 0 \\ 0 \\ \omega_E \end{bmatrix} .$$

The matrix ($\mathbb{LK}$) is transferred to Part B from Part A of the program, and R and T are rotation matrices similar to those used in computing QE.

Then, as on pages A2-83 and A2-84 of the reference

$$\begin{bmatrix} dp \\ \\ d\tau \end{bmatrix} = \begin{bmatrix} \dfrac{-a_1\ \omega_{p2}}{a_4} & 0 & 1 & 0 & 0 & 0 \\ \\ \dfrac{-a_5}{a_4} + \dfrac{a_1}{\omega_{p3}} & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} da_1 \\ \vdots \\ da_6 \end{bmatrix}$$

The QR output is printed each time the batch post data processing error covariance is requested, and is the result for the radius corresponding to the time of QR.

<u>QP</u>

The periapsis radius is

$$r_p = \frac{H^2}{\mu\ (1 + e)}$$

and

$$e^2 = \left(\frac{a_1\ a_4\ a_5}{\mu}\right)^2 + \left(\frac{a_1\ a_5^{\ 2}}{\mu} - 1\right)^2$$

leading to

$$dr_p = \left[\frac{2r_p}{H}\left(1 - \frac{Er_p}{\mu e}\right), \frac{-r_p^2}{\mu e}, \frac{-r_p}{\mu e}\right] \begin{bmatrix} a_5 & -a_4 & 0 & 0 & a_1 & 0 & 0 \\ \mu/a_1^2 & 0 & 0 & a_4 & a_5 & 0 & -1/a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} da_1 \\ \vdots \\ da_6 \\ d\mu \end{bmatrix}$$

in which

$$E = \frac{a_4^2 + a_5^2}{2} - \frac{\mu}{a_1} \quad ,$$

and a_1, a_4, and a_5 are the local values.

The QP output is printed each time the batch post data error covariance is requested.

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 471
(BBC Task 204)

H. Dale, Jr.
Jan. 24, 1967

THE EFFECT OF BATCHING DURING TERMINAL LUNAR RENDEZVOUS USING NEAR-OPTIMUM APRIORI PSEUDO-BIAS UNCERTAINTIES

Apollo Note No. 468 showed, that for continuous two-station tracking of the LEM during the terminal phase of Lunar Rendezvous, that the optimum filter apriori pseudo-bias was between 0.1 and 1.0 ft/sec. Note No. 468 also showed that this feasible filter design would produce state vector estimates that closely approached those of the optimum filter (all real parameters estimated).

This Note shows that batching during this interval and retaining for each batch only state parameters, does not seriously change the conclusions of the previous Note. In detail, it appears that batching does very little to total or in-plane position in fact, batching tends to degrade only the out-of-plane velocity component (and thereby 35% only on the last batch, t = 45 minutes). One can conclude that the optimum pseudo-bias apriori uncertainty is still between 0.1 and 1.0 feet per second and that intervehicle sightings will be required if out-of-plane position and velocity are to be brought below 2000 ft. and 2 ft/second.

The analyses in this Note consist of two batching runs: one with the Madrid and Ascension apriori pseudo-biases set at 0.1 ft/sec, and the other using 1.0 ft/sec. Since the previously reported continuous tracking study encompassed 45 minutes of tracking starting with the last nominal boost prior to rendezvous, this study assumes three equally spaced batches of data that terminate at 45 minutes. The orbital advancing program was used to produce Program A inputs for the vehicle at 15 and 30 minutes based upon the Program A inputs, from Note 468, at t = 0. Information matrices were then generated for $\dot{R}$ ($\sigma = 3.937 \times 10^{-2}$ ft/sec) from the two stations for three batch intervals (t initial = 1.0, TJ = 15; t initial = 16, TJ = 30; and t initial = 31, TJ = 45 min.). The input data sets for these three batches are shown in Figures 1, 2, and 3.

INPUTS - DATA SET 1

(MADRID MASTER)

1	6.3504074E 06	A1
2	-1.4772000E 01	A4
3	5.2317770E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	-6.0000000E 01	ZETA
9	4.0000000E 01	LAMBDA
10	-4.0000000E 00	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	1.5000000E 01	TIME
16	4.9917000E 01	TIME
17	-5.9386954-261	RE RAD
18	0	LT SLP
19	0	LO SLP
20	1.0000000E 00	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	1.0000000E 00	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	-5.9386954-261	LAMB2
31	-5.9386954-261	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	0	T CLK
36	0	T XYZ
37	0	T ORB
38	0	T TILD
39	0	T SHIP

INPUTS - DATA SET 2

(ASCENSION SLAVE)

1	6.3504074E 06	A1
2	-1.4772000E 01	A4
3	5.2317770E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	-6.0000000E 01	ZETA
9	-8.0000000E 00	LAMBDA
10	-1.4000000E 01	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	1.5000000E 01	TIME
16	4.9917000E 01	TIME
17	-5.9386954-261	RE RAD
18	0	LT SLP
19	0	LO SLP
20	1.0000000E 00	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	1.0000000E 00	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	4.0000000E 01	LAMB2
31	-4.0000000E 00	ALPHA2
32	0	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	0	T CLK
36	0	T XYZ
37	0	T ORB
38	0	T TILD
39	0	T SHIP

Figure 1

Data Inputs for Information Matrices for First Batch of the Lunar Rendezvous Problem

INPUTS - DATA SET 3

(MADRID MASTER)

1	6.3471317E 06	A1
2	7.8336539E 00	A4
3	5.2344771E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	-1.7472383E 01	ZETA
9	4.0000000E 01	LAMBDA
10	-4.0000000E 00	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	3.0000000E 01	TIME
16	4.9917000E 01	TIME
17	-5.9386954-261	RE RAD
18	0	LT SLP
19	0	LO SLP
20	1.0000000E 00	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	1.6000000E 01	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	4.0000000E 01	LAMB2
31	-4.0000000E 00	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	0	T CLK
36	0	T XYZ
37	1.5000000E 01	T ORB
38	0	T TILD
39	0	T SHIP

INPUTS - DATA SET 4

(ASCENSION SLAVE)

1	6.3471317E 06	A1
2	7.8336539E 00	A4
3	5.2344771E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	-1.7472383E 01	ZETA
9	-8.0000000E 00	LAMBDA
10	-1.4000000E 01	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	3.0000000E 01	TIME
16	4.9917000E 01	TIME
17	-5.9386954-261	RE RAD
18	0	LT SLP
19	0	LO SLP
20	1.0000000E 00	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	1.6000000E 01	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	4.0000000E 01	LAMB2
31	-4.0000000E 00	ALPHA2
32	0	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	0	T CLK
36	0	T XYZ
37	1.5000000E 01	T ORB
38	0	T TILD
39	0	T SHIP

Figure 2

Data Inputs for Information Matrices for Second Batch of the Lunar Rendezvous Problem

INPUTS - DATA SET 5

(MADRID MASTER)

1	6.3632377E 06	A1
2	2.6291157E 01	A4
3	5.2212281E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	2.4965263E 01	ZETA
9	4.0000000E 01	LAMBDA
10	-4.0000000E 00	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	4.5000000E 01	TIME
16	4.9917000E 01	TIME
17	-5.9386954-261	RE RAD
18	0	LT SLP
19	0	LO SLP
20	1.0000000E 00	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	3.1000000E 01	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	4.0000000E 01	LAMB2
31	-4.0000000E 00	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	0	T CLK
36	0	T XYZ
37	3.0000000E 01	T ORB
38	0	T TILD
39	0	T SHIP

INPUTS - DATA SET 6

(ASCENSION SLAVE)

1	6.3632377E 06	A1
2	2.6291157E 01	A4
3	5.2212281E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	2.4965263E 01	ZETA
9	-8.0000000E 00	LAMBDA
10	-1.4000000E 01	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	4.5000000E 01	TIME
16	4.9917000E 01	TIME
17	-5.9386954-261	RE RAD
18	0	LT SLP
19	0	LO SLP
20	1.0000000E 00	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	3.1000000E 01	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	4.0000000E 01	LAMB2
31	-4.0000000E 00	ALPHA2
32	0	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	0	T CLK
36	0	T XYZ
37	3.0000000E 01	T ORB
38	0	T TILD
39	0	T SHIP

Figure 3

Data Inputs for Information Matrices for the Third Batch of the Lunar Rendezvous Problem

Figure 4 shows the various apriori inputs for the batching run using apriori pseudo-bias uncertainties of 1.0 ft/sec. APF-7 is the apriori covariance of the state vector and pseudo-range rate biases assumed by the real time filter. A-8 is the reinitialized values of the variances in the pseudo-biases to be used at the beginning of the second and third batches by the filter. APN-9 is the actual covariance of the state at the beginning of tracking. Here no pseudo-biases exist but the state vector uncertainty matches APF assumed by the filter. AP01-10 and AP02-11 are the covariance matrices of two groups of non-estimated nuisance parameters. Figures 5 through 12 show the data inputs to the Batching Program. It should be noted that the first "Batching Result" gives answers relative to the start of the first batch. The following batch first projects these results to the end of the first batch (t = 15 min) before adding new data. These are the results desired. For this reason the last batch is followed by a "Project Covariance" routine to yield the covariance at the end of the last batch. Apollo Note No. 464 explains these batching inputs in greater detail.

Once the results for the 1.0 ft/sec pseudo-bias were obtained, the RDOT BIAS numbers in Figure 4 were changed to $(0.1)^2$ and the routine was re-run. The two sets of results are shown in Figures 13 and 14. These graphs show the continuous tracking results, from Apollo Note No. 468, and in addition, the batching results at 15, 30, and 45 minutes. These results are also tabulated in Figure 15.

The most that one can conclude from this study is that with two stations tracking during the terminal phase of lunar rendezvous, and with the assumption that both apriori pseudo-bias uncertainties will be chosen equal in the real-time filter:

1. The optimum apriori bias uncertainty appears to be between 0.1 and 1.0 ft/sec, with or without batching.

2. Batching does not significantly affect total or in-plane position nor does it affect in-plane velocity. The z component of velocity is degraded, through batching, as time progresses, reaching a maximum at 45 minutes of a 30% degradation with bias estimates equal 1.0 ft/sec, and a 45% degradation with bias estimates equal 0.1 ft/sec.

APF 7

A(1, 1)	=	3.0000000E 05	A1 LEM
A(2, 2)	=	5.0000000E 06	A2 LEM
A(3, 3)	=	7.0000000E 06	A3 LEM
A(4, 4)	=	2.5000000E 00	A4 LEM
A(5, 5)	=	1.0000000E-01	A5 LEM
A(6, 6)	=	1.5000000E 00	A6 LEM
A(7, 7)	=	1.0000000E 00	RDOTBIASMADR
A(8, 8)	=	1.0000000E 00	RDOTBIASASCN

A 8

A(7, 7)	=	1.0000000E 00	RDOTBIASMADR
A(8, 8)	=	1.0000000E 00	RDOTBIASASCN

APN 9

A(1, 1)	=	3.0000000E 05	A1 LEM
A(2, 2)	=	5.0000000E 06	A2 LEM
A(3, 3)	=	7.0000000E 06	A3 LEM
A(4, 4)	=	2.5000000E 00	A4 LEM
A(5, 5)	=	1.0000000E-01	A5 LEM
A(6, 6)	=	1.5000000E 00	A6 LEM

AP01 10

A(7, 7)	=	1.5000000E 04	MAD STA UP
A(8, 8)	=	1.5000000E 04	MAD STA EAST
A(9, 9)	=	1.5000000E 04	MAD STA NORT
A(10,10)	=	1.0000000E 05	ASN STA UP
A(11,11)	=	1.0000000E 05	ASN STA EAST
A(12,12)	=	1.0000000E 05	ASN STA NORT
A(13,13)	=	2.5000000E 19	LUNAR MU
A(14,14)	=	2.5000000E-05	MAD CL BIAS
A(15,15)	=	4.0000000E-20	MAD CL RATE
A(16,16)	=	4.0000000E-36	MAD CL ACCEL
A(17,17)	=	2.5000000E-05	ASN CL BIAS
A(18,18)	=	4.0000000E-20	ASN CL RATE
A(19,19)	=	4.0000000E-36	ASN CL ACCEL

AP02 11

A(7, 7)	=	2.5510000E 07	LUN EPH	11
A(8, 8)	=	1.5410000E 07	LUN EPH	22
A(9, 9)	=	1.0000000E 00	LUN EPH	33
A(10,10)	=	1.2520000E-04	LUN EPH	44
A(11,11)	=	9.2700000E-04	LUN EPH	55
A(12,12)	=	1.2600000E-05	LUN EPH	66
A(7,11)	=	1.3500000E 02	LUN EPH	15
A(11, 7)	=	1.3500000E 02	LUN EPH	51
A(8,10)	=	-4.3920000E 01	LUN EPH	24
A(10, 8)	=	-4.3920000E 01	LUN EPH	42

Figure 4

Apriori Inputs for: The Filter at the Start of the First Batch (APF-7), The Re-Initialized Biases for Batches 2 and 3 (A-8), The True Noise at the Start of the First Batch (APN-8), and the Two Nuisance Parameter Groups (AP01-10, AP02-11).

PROGRAM B CONTROL CARD DATA INPUT SHEET

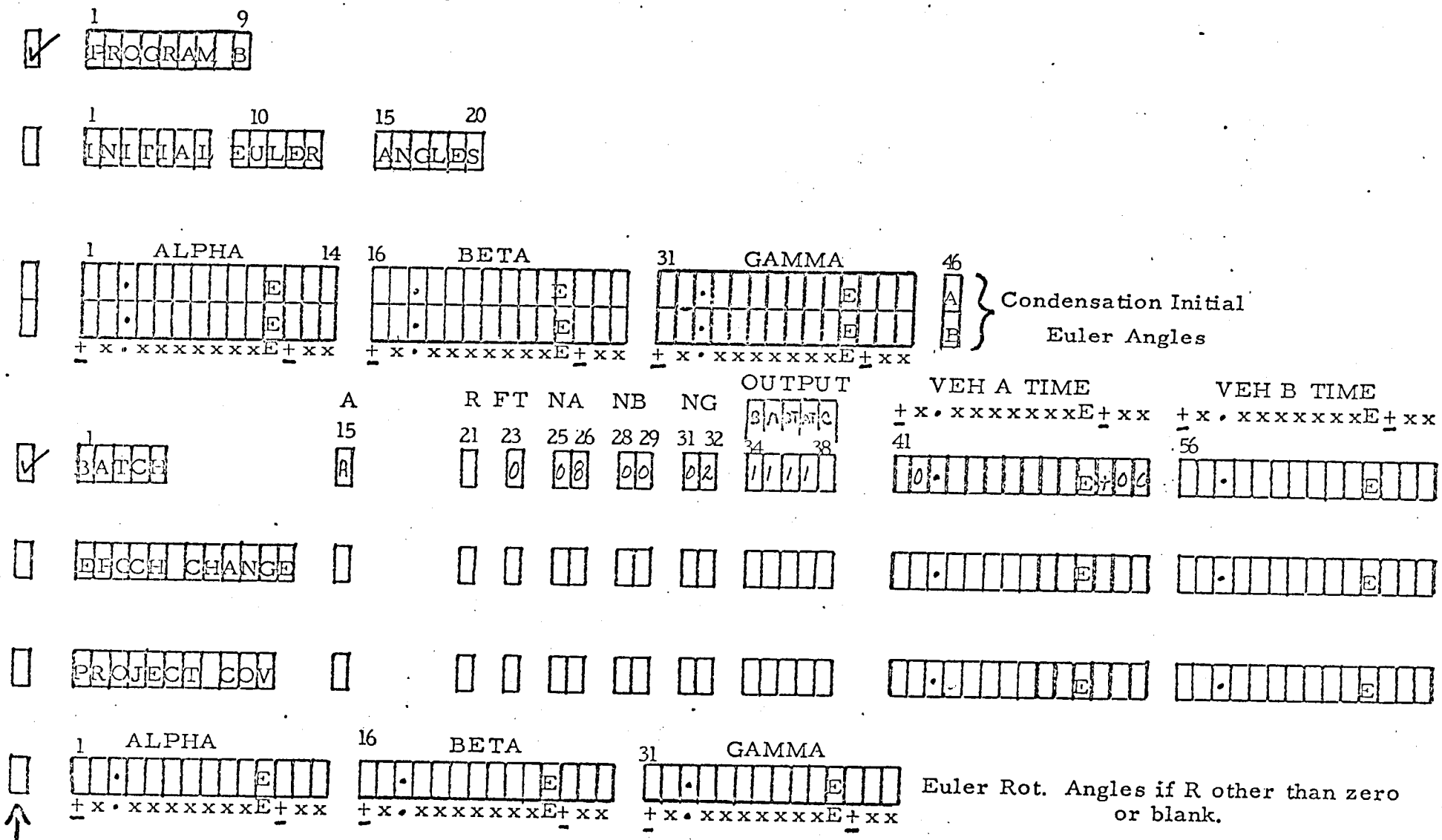


Figure 5

Batching Program Control for the First Batch (0 → 15 min.)

PROGRAM B ALLOCATION SHEET

Indicators for Control of Matrix Format

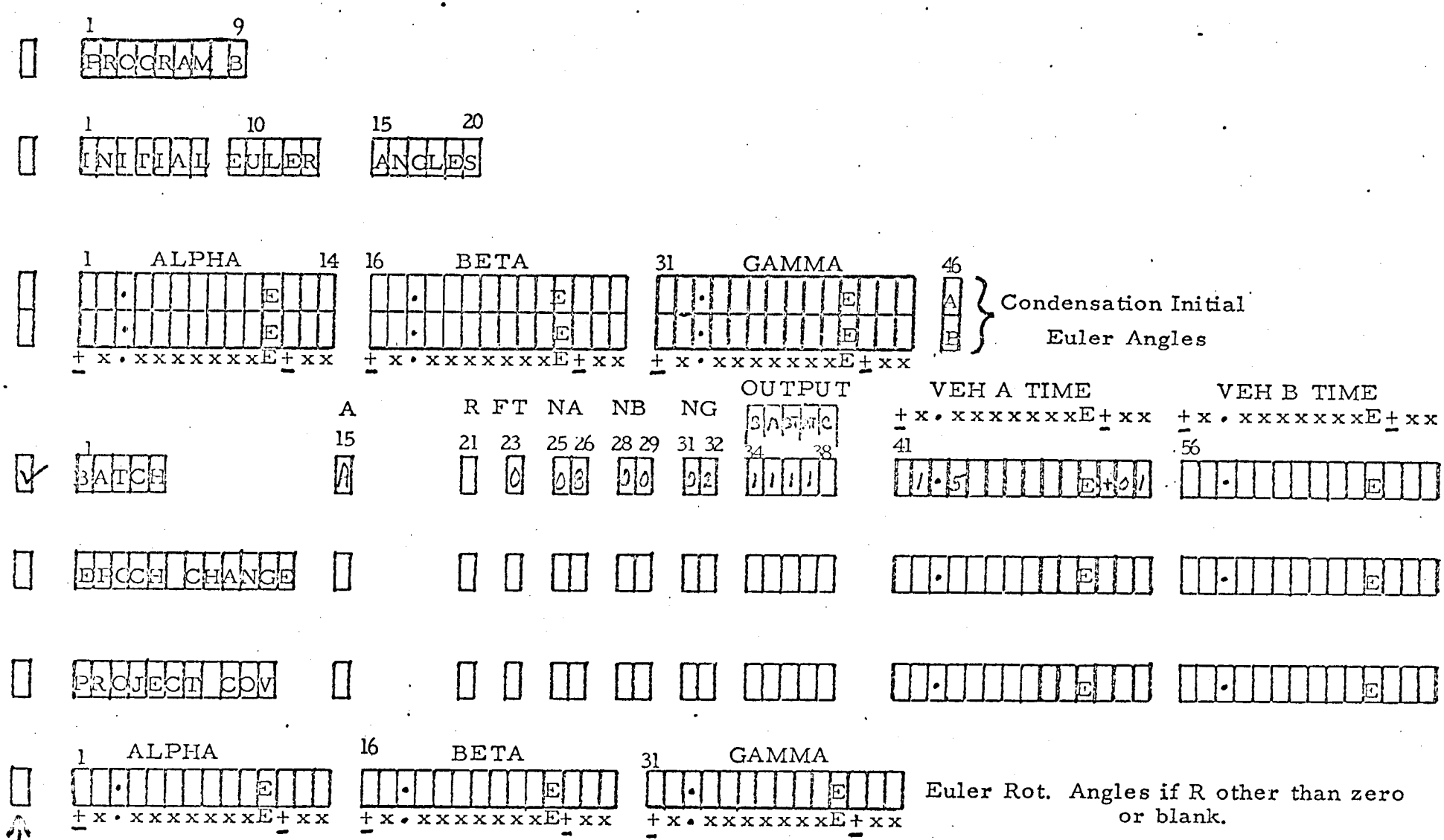
Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)	07 BI	08 US	09 ES	10 NS	11 NH	12 MU	13 N*	14 VA	15 UM	16 EM	17 NM	18 ED	19 ND	20 RM	21 AM	22 BS	23 RS	24 AS	25 L1	26 L2	27 L3	28 L4	29 L5	30 L6
					BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
✓ 007	APF	. E	. E																									
✓ 001	RD	1.5 E+01	3.9377 E-02		07																							
✓ 002	RD	1.5 E+01	3.9377 E-02		08																							
✓ BLANK CARD		. E	. E																									
✓ 009	APN	. E	. E																									
✓ BLANK CARD		. E	. E																									
✓ 010	APO1	. E	. E		23	24	25	26	27	28	29	30	31	32	33	34	35											
✓ 001	RD	1.5 E+01	3.9377 E-02		07	23	24	25		29								31	32	30								
✓ 002	RD	1.5 E+01	3.9377 E-02		08	26	27	28		27			23	24	25			31	32	33	34	35						
✓ BLANK CARD		. E	. E																									
✓ 011	APO2	. E	. E		23	24	25	26	27	28																		
✓ 001	RD	1.5 E+01	3.9377 E-02		07																		23	24	25	26	27	28
✓ 002	RD	1.5 E+01	3.9377 E-02		08																		23	24	25	26	27	28
✓ BLANK CARD		. E	. E																									

Figure 6

Batching Program Allocation for the First Batch

eypunch a card for each
ow checked (✓).

PROGRAM B CONTROL CARD DATA INPUT SHEET



Key Punch a Card for each row checked (✓).

Figure 7

Batching Program Control for the Second Batch (15→30 min.)

PROGRAM B ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)	Indicators for Control of Matrix Format (33–80)
	PF			*	
008	A				0708
001	Q	1.5 E+01		*	
003	RD	3.0 E+01	3.9337 E-02		07
004	RD	3.0 E+01	3.9337 E-02		08
BLANK CARD					
	PN			*	
BLANK CARD					
	P01				2324252627282930
003	RD	3.0 E+01	3.9337 E-02		07232425 29 3132330
004	RD	3.0 E+01	3.9337 E-02		0326272829 2324252 3132333343 5
001	Q	1.5 E+01			29
BLANK CARD					
	P02				2324252627282
003	RD	3.0 E+01	3.9337 E-02		07 2324252627282
004	RD	3.0 E+01	3.9337 E-02		08 2324252627282
001	Q	1.5 E+01	*		
BLANK CARD					

Keypunch a card for each row checked (✓).

Figure 8

Batching Program Allocation for the Second Batch

PROGRAM B CONTROL CARD DATA INPUT SHEET

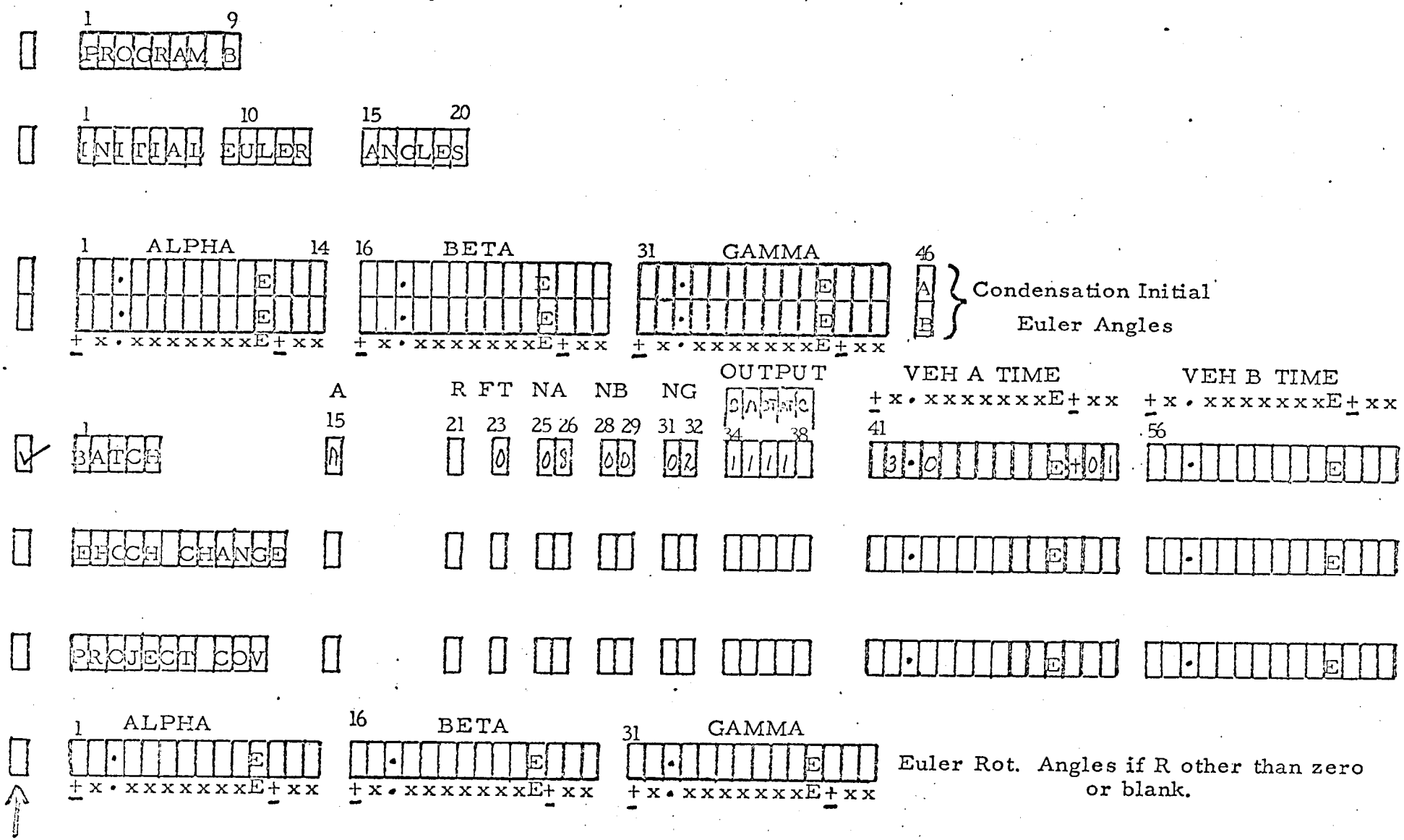


Figure 9

Batching Program Control for the Third Batch (30→45 min.)

PROGRAM B ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) ± x.xxxxxExxx (8–19)	σ ± x.xxxxxE±xx (20–31)	6 x 6 (32)	Indicators for Control of Matrix Format (33–80)
	PF			*	
008	N				0708
003	Q	3.0 E+01		*	
005	RD	4.5 E+01	3.937 E-02		07
006	RD	4.5 E+01	3.937 E-02		08
BLANK CARD					
	PN			*	
BLANK CARD					
	P01				2324252627282930
005	RD	4.5 E+01	3.937 E-02		07232425 29 3132330
006	RD	4.5 E+01	3.937 E-02		08262728 29 2324252 3132333435
003	Q	3.0 E+01		*	29
BLANK CARD					
	P02				2324252627 28
005	RD	4.5 E+01	3.937 E-02		07 23242526272 8
006	RD	4.5 E+01	3.937 E-02		08 23242526272 8
003	Q	3.0 E+01		*	
BLANK CARD					

Indicators for Control of Matrix Format (columns 07–30):

07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									

eypunch a card for each
w checked (✓).

Figure 10
Batching Program for the Third Batch

PROGRAM B CONTROL CARD DATA INPUT SHEET

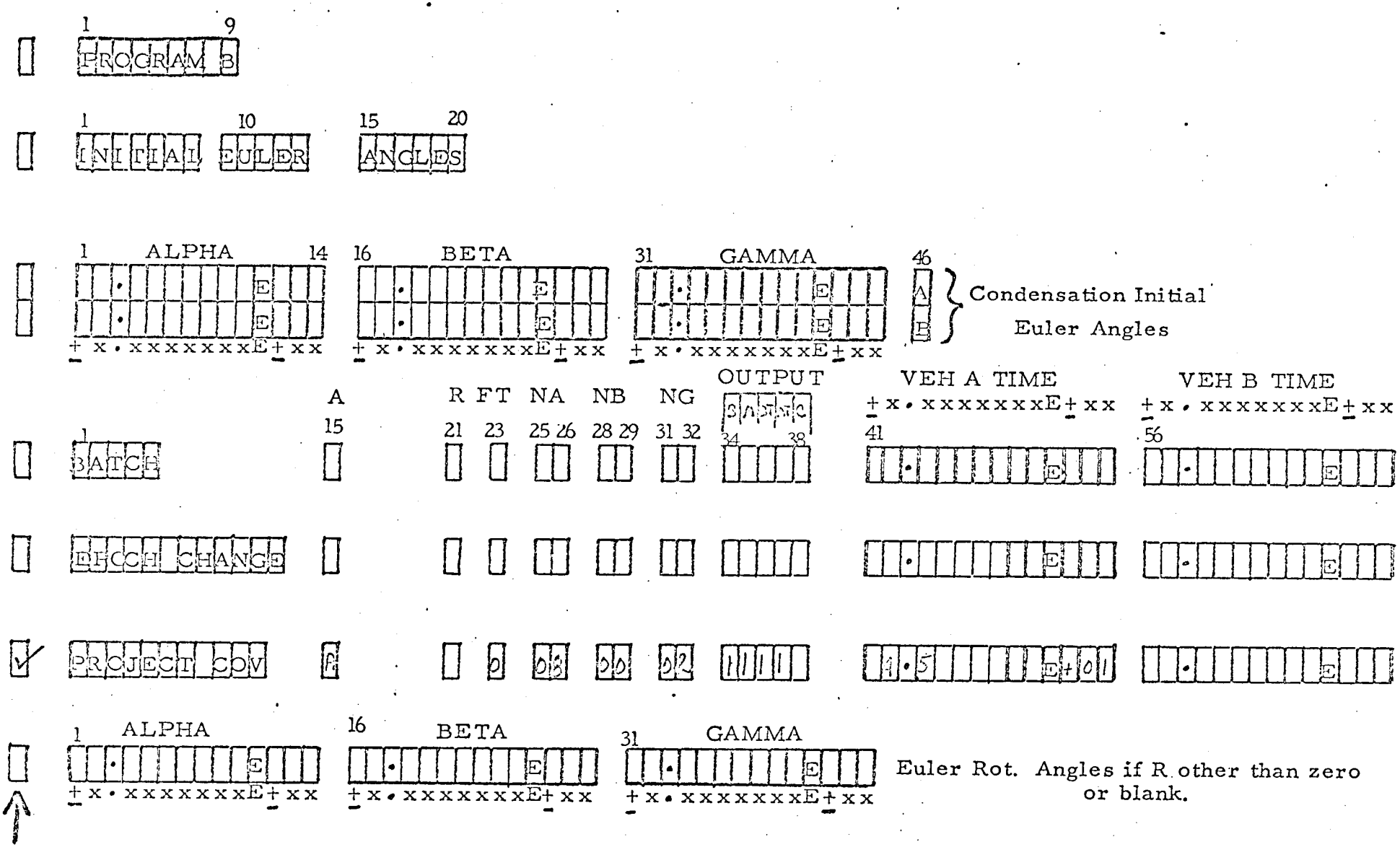


Figure 11

Projection Control to Yield Last Batch Results at 45 Min.

PROGRAM B ALLOCATION SHEET

Indicators for Control of Matrix Format

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
					BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
					BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
	PF																											
005	Q	4.5	+01																									
BLANK CARD																												
	PX																											
BLANK CARD																												
	P01																											
005	Q	4.5	+01							29																		
BLANK CARD																												
	P02																											
005	Q	4.5	+01																									
BLANK CARD																												

eypunch a card for each
w checked (✓).

Figure 12

Projection Allocation

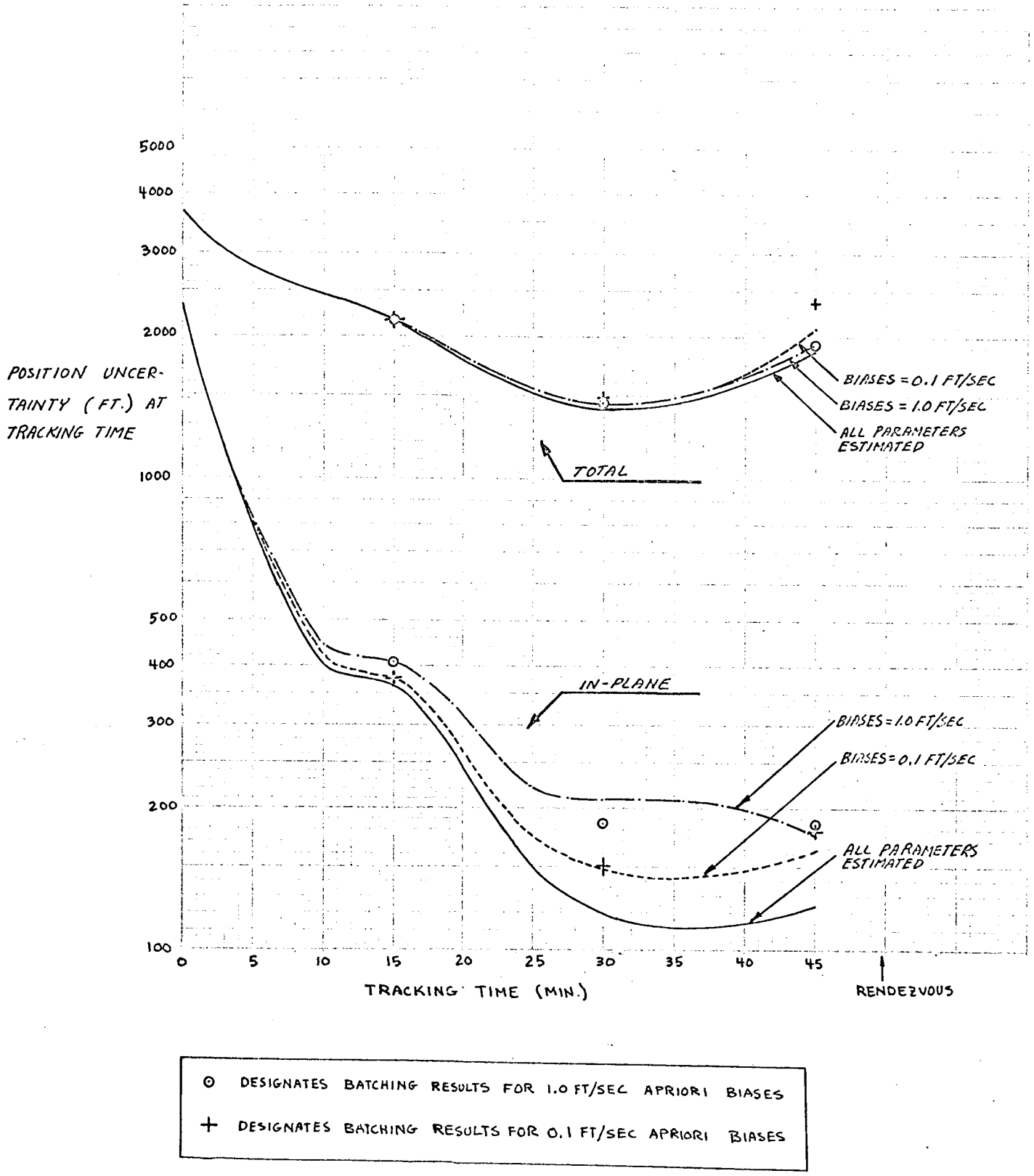


Figure 13

Position Uncertainty, A Comparison Between Batching and Continuous Tracking

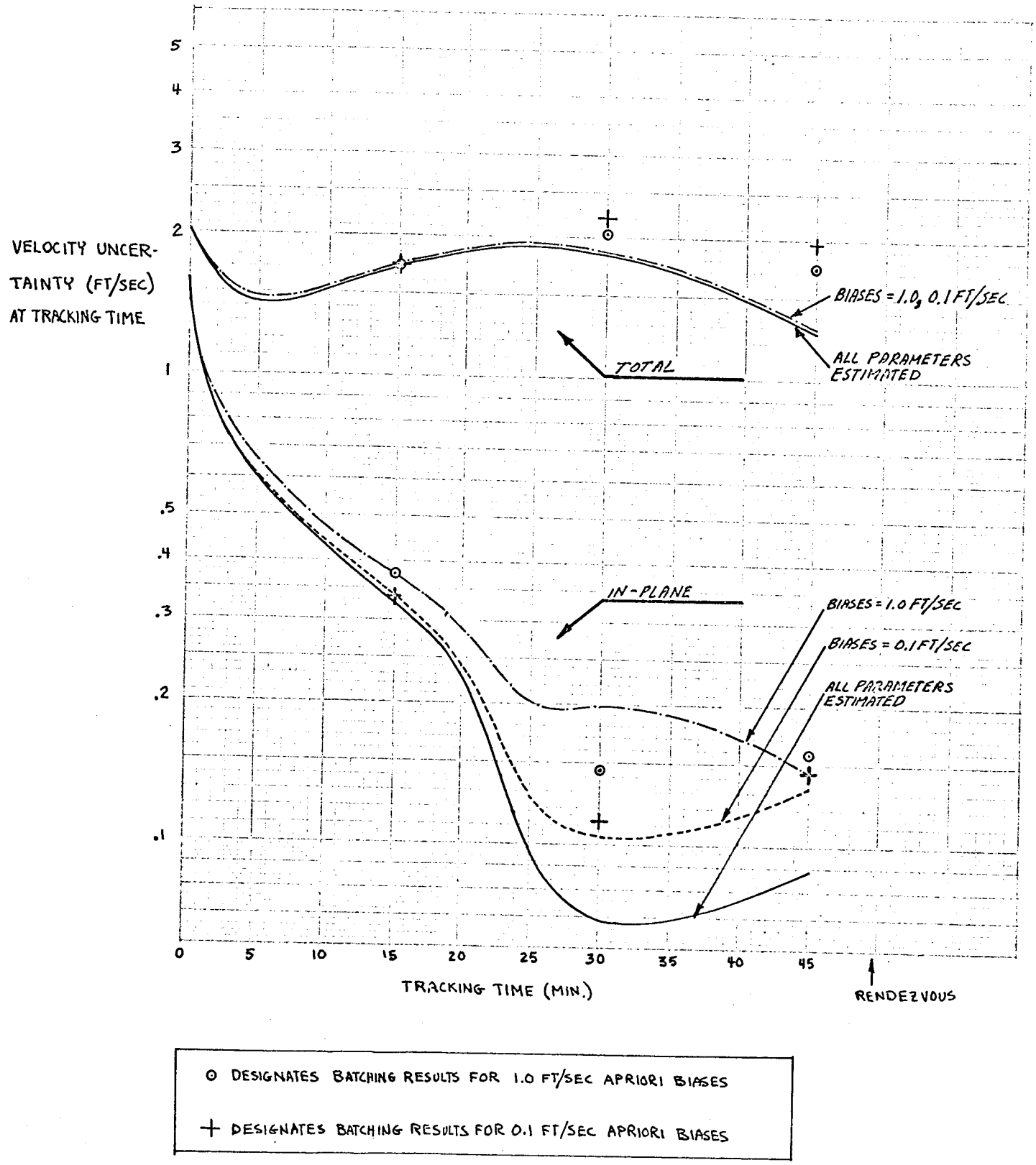


Figure 14

Velocity Uncertainty, A Comparison Between Batching and Continuous Tracking

		Apriori Bias = 1.0				Apriori Bias = 0.1		
		Apriori	t = 15	t = 30	t = 45	Apriori	t = 15	t = 30	t = 45
Actual Uncertainties	σ Pos	3.507 + 3	2.121 + 3	1.461 + 3	1.964 + 3	3.507 + 3	2.119 + 3	1.482 + 3	2.387 + 3
	σ Vel	2.025 + 0	1.720 + 0	2.037 + 0	1.741 + 0	2.025 + 0	1.741 + 0	2.228 + 0	1.989 + 0
	σ x	5.477 + 2	1.337 + 2	1.031 + 2	1.348 + 2	2.236 + 3	1.342 + 2	1.004 + 2	1.351 + 2
	σ y	2.236 + 3	3.721 + 2	1.530 + 2	1.236 + 2	2.236 + 3	3.467 + 2	1.129 + 2	1.175 + 2
	σ z	2.645 + 3	2.084 + 3	1.450 + 3	1.955 + 3	2.645 + 3	2.087 + 3	1.474 + 3	2.380 + 3
	σ $\dot{x}$	1.581 + 0	2.204 - 1	1.271 - 1	1.458 - 1	1.581 + 0	1.292 - 1	8.407 - 2	1.394 - 1
	σ $\dot{y}$	3.162 - 1	3.186 - 1	7.672 - 2	5.781 - 2	3.162 - 1	3.152 - 1	6.949 - 2	4.822 - 2
	σ $\dot{z}$	1.225 + 0	1.676 + 0	2.032 + 0	1.734 + 0	1.225 + 0	1.707 + 0	2.226 + 0	1.984 + 0
	In Plane Pos	2.26 + 3	4.08 + 2	1.84 + 2	1.83 + 2	2.26 + 3	3.73 + 2	1.51 + 2	1.79 + 2
	In Plane Vel	1.61 + 0	3.87 - 1	1.48 - 1	1.57 - 1	1.61 + 0	3.41 - 1	1.14 - 1	1.47 - 1
Filter Uncertainties	σ Pos	3.507 + 3	2.089 + 3	1.465 + 3	1.886 + 3	3.507 + 3	2.075 + 3	1.444 + 3	1.833 + 3
	σ Vel	2.025 + 0	1.727 + 0	2.009 + 0	1.707 + 0	2.025 + 0	1.681 + 0	1.964 + 0	1.662 + 0
	σ x	5.477 + 2	1.147 + 2	3.908 + 1	7.456 + 1	5.477 + 2	1.108 + 2	2.689 + 1	2.621 + 1
	σ y	2.236 + 3	4.154 + 2	2.223 + 2	1.289 + 2	2.236 + 3	3.372 + 2	8.015 + 1	3.817 + 1
	σ z	2.645 + 3	2.044 + 3	1.448 + 3	1.881 + 3	2.645 + 3	2.044 + 3	1.441 + 3	1.832 + 3
	σ $\dot{x}$	1.581 + 0	3.475 - 1	1.929 - 1	1.445 - 1	1.581 + 0	1.039 - 1	3.613 - 2	4.173 - 2
	σ $\dot{y}$	3.162 - 1	3.270 - 1	8.221 - 2	5.065 - 2	3.162 - 1	3.142 - 1	3.988 - 2	1.301 - 2
	σ $\dot{z}$	1.225 + 0	1.659 + 0	1.998 + 0	1.700 - 1	1.225 + 0	1.648 + 0	1.963 + 0	1.662 + 0
	Madrid Bias	1.0 + 0			2.104 - 1	1.0 - 1			4.649 - 2
	Ascen Bias	1.0 + 0			2.100 - 1	1.0 - 1			4.650 - 2

Figure 15

Batching Results (ft, ft/sec) at End of 15, 30, and 45 Minute Batches of Rendezvous Problem

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 472
(BBC Task 204)

C. H. Dale, Jr.
January 1967

OPTIMUM APRIORI VALUES FOR THE DOPPLER PSEUDO-BIASES FOR MSFN TRACKING DURING THE FIRST TWO HOURS AFTER TRANSLUNAR INJECTION

The object of this Note is to find the optimum apriori range-rate pseudo-biases which, if used by the real-time ODP during the first two hours after translunar injection, will minimize the state vector uncertainty. This Note parallels Apollo Note No. 468, and no batching is considered.

Burnout of the injection boost is assumed to occur at t = 0 and all three stations become visible and start tracking at t = 15 minutes. The information matrices are produced using the Program A data inputs shown in Figure 1. Figure 2 shows the apriori inputs for a run with assumed pseudo-bias uncertainties of 0.01 ft/sec. Doppler noise was assumed to be 3.937×10^{-2} ft/sec. In a like manner other runs were made using 0.1, 1.0, and 10.0 ft/sec values for the apriori pseudo-range-rate biases. The case of zero uncertainty in the biases was produced separately through the use of the old OEAP along with the results for all real parameters estimated and the results for estimating the orbit parameters alone without including the effects of nuisance parameters. This all parallels Apollo Note No. 458 which studies the Lunar Rendezvous case. The results are tabulated for tracking times of 16 (one minute of data), 25, 50, 80, and 120 minutes and appear in Figures 3 through 7. These tabulations show what the RTODP would believe the uncertainties are along with the actual state vector uncertainties for the cases of 0, 0.01, 0.1, 1.0, and 10.0 ft/sec initially assumed pseudo-bias uncertainties.

INPUTS - DATA SET 1 (MADRID MASTER)			INPUTS - DATA SET 2 (BERMUDA SLAVE)			INPUTS - DATA SET 3 (ASCENSION SLAVE)		
1	2.1952123E 07	A1	1	2.1952123E 07	A1	1	2.1952123E 07	A1
2	4.0931939E 03	A4	2	4.0931939E 03	A4	2	4.0931939E 03	A4
3	3.5376421E 04	A5	3	3.5376421E 04	A5	3	3.5376421E 04	A5
4	1.0000000E 00	GAM ID	4	1.0000000E 00	GAM ID	4	1.0000000E 00	GAM ID
5	0	BETA	5	0	BETA	5	0	BETA
6	-1.8392750E 02	XI	6	-1.8392750E 02	XI	6	-1.8392750E 02	XI
7	3.2744252E 01	ETA	7	3.2744252E 01	ETA	7	3.2744252E 01	ETA
8	8.1808099E 01	ZETA	8	8.1808099E 01	ZETA	8	8.1808099E 01	ZETA
9	4.1000000E 01	LAMBDA	9	3.2350000E 01	LAMBDA	9	-7.9700000E 00	LAMBDA
10	-4.0000000E 00	ALPHA	10	-6.4660000E 01	ALPHA	10	-1.4400000E 01	ALPHA
11	7.2921560E-05	OMEGAE	11	7.2921560E-05	OMEGAE	11	7.2921560E-05	OMEGAE
12	0	OMEGAM	12	0	OMEGAM	12	0	OMEGAM
13	2.0925738E 07	RHOE	13	2.0925738E 07	RHOE	13	2.0925738E 07	RHOE
14	0	RHOM	14	0	RHOM	14	0	RHOM
15	1.4076539E 16	MU	15	1.4076539E 16	MU	15	1.4076539E 16	MU
16	1.6000000E 01	TIME	16	1.6000000E 01	TIME	16	1.6000000E 01	TIME
16	2.5000000E 01	TIME	16	2.5000000E 01	TIME	16	2.5000000E 01	TIME
16	5.0000000E 01	TIME	16	5.0000000E 01	TIME	16	5.0000000E 01	TIME
16	8.0000000E 01	TIME	16	8.0000000E 01	TIME	16	8.0000000E 01	TIME
16	1.2000000E 02	TIME	16	1.2000000E 02	TIME	16	1.2000000E 02	TIME
17	0	RE RAD	17	0	RE RAD	17	0	RE RAD
18	0	LT SLP	18	0	LT SLP	18	0	LT SLR
19	0	LO SLP	19	0	LO SLP	19	0	LO SLP
20	1.0000000E 00	RD IND	20	1.0000000E 00	RD IND	20	1.0000000E 00	RD IND
21	0	R IND	21	0	R IND	21	0	R IND
22	1.0000000E 00	Q IND	22	1.0000000E 00	Q IND	22	1.0000000E 00	Q IND
23	0	QF IND	23	0	QF IND	23	0	QF IND
24	0	A1 IND	24	0	A1 IND	24	0	A1 IND
25	0	A2 IND	25	0	A2 IND	25	0	A2 IND
26	0	PVWIND	26	0	PVWIND	26	0	PVWIND
27	1.6000000E 01	T INIT	27	1.6000000E 01	T INIT	27	1.6000000E 01	T INIT
28	1.0000000E 00	T INCR	28	1.0000000E 00	T INCR	28	1.0000000E 00	T INCR
29	2.4000000E 01	DIMENS	29	2.4000000E 01	DIMENS	29	2.4000000E 01	DIMENS
30	0	LAMB2	30	4.1000000E 01	LAMB2	30	4.1000000E 01	LAMB2
31	0	ALPHA2	31	-4.0000000E 00	ALPHA2	31	-4.0000000E 00	ALPHA2
32	1.0000000E 00	MS IND	32	0	MS IND	32	0	MS IND
33	1.0000000E 00	FM IND	33	1.0000000E 00	FM IND	33	1.0000000E 00	FM IND
34	1.0000000E-07	VISIND	34	1.0000000E-07	VISIND	34	1.0000000E-07	VISIND
35	0	T CLK	35	0	T CLK	35	0	T CLK
36	0	T XYZ	36	0	T XYZ	36	0	T XYZ
37	0	T ORB	37	0	T ORB	37	0	T ORB
38	0	T TILD	38	0	T TILD	38	0	T TILD

Figure 1 - Program A Inputs for Three Stations, Doppler Tracking for the First Two Hours After Translunar Injection

A(1, 1)	=	1.0000000E 08	A1
A(2, 2)	=	1.0000000E 08	A2
A(3, 3)	=	1.0000000E 08	A3
A(4, 4)	=	1.0000000E 04	A4
A(5, 5)	=	1.0000000E 04	A5
A(6, 6)	=	1.0000000E 04	A6
A(7, 7)	=	1.0000000E-04	R DOT BIAS M
A(8, 8)	=	1.0000000E-04	R DOT BIAS B
A(9, 9)	=	1.0000000E-04	R DOT BIAS A

APN 14

A(1, 1)	=	1.0000000E 08	A1
A(2, 2)	=	1.0000000E 08	A2
A(3, 3)	=	1.0000000E 08	A3
A(4, 4)	=	1.0000000E 04	A4
A(5, 5)	=	1.0000000E 04	A5
A(6, 6)	=	1.0000000E 04	A6

AP01 15

A(7, 7)	=	1.0000000E 04	UP MADRID
A(8, 8)	=	1.0000000E 04	EAST MADRID
A(9, 9)	=	1.0000000E 04	NORTH MADRID
A(10,10)	=	1.0000000E 04	UP BERMUDA
A(11,11)	=	1.0000000E 04	EAST BERMUDA
A(12,12)	=	1.0000000E 04	NORTH BERMUD
A(13,13)	=	1.0000000E 04	UP ASCENSION
A(14,14)	=	1.0000000E 04	EAST ASCEN
A(15,15)	=	1.0000000E 04	NORTH ASCEN
A(16,16)	=	1.0000000E-01	NOH MADRID
A(17,17)	=	1.0000000E-01	NOH BERMUDA
A(18,18)	=	1.0000000E-01	NOH ASCEN
A(19,19)	=	1.7000000E 22	E-5 MU EARTH
A(20,20)	=	2.5000000E-05	MAD CL BIAS
A(21,21)	=	4.0000000E-20	MAD CL RATE
A(22,22)	=	4.0000000E-36	MAD CL ACCEL
A(23,23)	=	2.5000000E-05	BER CL BIAS
A(24,24)	=	4.0000000E-20	BER CL RATE

AP02 16

A(7, 7)	=	4.0000000E-36	BER CL ACCEL
A(8, 8)	=	2.5000000E-05	ASN CL BIAS
A(9, 9)	=	4.0000000E-20	ASN CL RATE
A(10,10)	=	4.0000000E-36	ASN CL ACCEL

Figure 2 - Apriori Covariance for: Filter with 0.01 ft/sec Biases (APF-10), State Vector at t = 0 (APN-14), and Two Non-Estimated Nuisance Parameter Groups (AP01, AP02-16)

Apriori Biases = 0

		Apriori	t = 16	t = 25	t = 50	t = 80	t = 120
Actual Uncertainty	σ Pos	1.732+4	3.065+4	3.088+3	7.132+3	9.720+3	8.491+3
	σ Vel	1.732+2	2.046+1	2.414+0	2.535+0	1.945+0	1.011+0
	σ x	1.000+4	6.602+3	1.887+2	6.124+2	7.499+2	6.870+2
	σ y	1.000+4	1.596+4	2.595+3	6.440+3	8.941+3	7.297+3
	σ z	1.000+4	2.532+4	1.665+3	3.003+3	3.740+3	4.113+3
	σ $\dot{x}$	1.000+2	1.414+1	1.673+0	1.313+0	8.202-1	3.607-1
	σ $\dot{y}$	1.000+2	7.076+0	1.511+0	2.058+0	1.694+0	8.847-1
	σ $\dot{z}$	1.000+2	1.298+1	8.608-1	6.818-1	4.900-1	3.309-1
	In Plane Pos	1.414+4					
	In Plane Vel	1.414+2					
Filter Uncertainties	σ Pos	1.732+4	3.063+4	3.181+2	3.281+2	4.641+2	6.154+2
	σ Vel	1.732+2	2.043+1	2.098-1	1.109-1	9.070-2	7.449-2
	σ x	1.000+4	6.599+3	6.304+1	3.339+1	3.438+1	3.068+1
	σ y	1.000+4	1.595+4	2.335+2	2.818+2	4.177+2	5.641+2
	σ z	1.000+4	2.530+4	2.066+2	1.649+2	1.994+2	2.441+2
	σ $\dot{x}$	1.000+2	1.414+1	1.354-1	5.515-2	3.661-2	2.478-2
	σ $\dot{y}$	1.000+2	7.065+0	1.303-1	8.899-2	7.863-2	6.706-2
	σ $\dot{z}$	1.000+2	1.294+1	9.137-2	3.669-2	2.652-2	2.093-2
	In Plane Pos	1.414+4					
	In Plane Vel	1.414+2					
All Parameters Estimated	σ Pos	1.732+4	3.065+4	1.710+3	1.650+3	2.455+3	3.359+3
	σ Vel	1.732+2	2.046+1	1.345+0	5.805-1	4.834-1	4.080-1
	σ x	1.000+4	6.601+3	1.515+2	1.397+2	1.722+2	1.765+2
	σ y	1.000+4	1.596+4	1.434+3	1.395+3	2.111+3	2.935+3
	σ z	1.000+4	2.532+4	9.188+2	8.706+2	1.241+3	1.623+3
	σ $\dot{x}$	1.000+2	1.414+1	8.973-1	2.684-1	1.805-1	1.256-1
	σ $\dot{y}$	1.000+2	7.076+0	8.371-1	4.474-1	3.960-1	3.471-1
	σ $\dot{z}$	1.000+2	1.298+1	5.509-1	2.546-1	2.104-1	1.736-1
	In Plane Pos	1.414+4					
	In Plane Vel	1.414+2					

Figure 3 - First Two Hours of Trans-Lunar Injection

Apriori Biases = 0.01 ft/sec

		Apriori	t = 16	t = 25	t = 50	t = 80	t = 120
Actual Uncertainties	σ Pos	1.732 + 4	3.065+4	3.067+3	4.567+3	3.831+3	4.007+3
	σ Vel	1.732 + 2	2.046+1	2.397+0	1.632+0	7.772-1	4.993-1
	σx	1.000 + 4	6.602+3	1.885+2	4.436+2	4.560+2	5.470+2
	σy	1.000 + 4	1.596+4	2.577+3	4.106+3	3.412+3	3.478+3
	σz	1.000 + 4	2.532+4	1.653+3	1.950+3	1.683+3	1.913+3
	σẋ	1.000 + 2	1.414+1	1.662+0	8.457-1	3.345-1	1.854-1
	σẏ	1.000 + 2	7.076+0	1.501+0	1.311+0	6.514-1	4.194-1
	σż	1.000 + 2	1.298+1	8.561-1	4.805-1	2.603-1	1.977-1
Filter Uncertainties	σPos	1.732 + 4	3.063+4	3.448+2	4.134+2	5.202+2	6.324+2
	σVel	1.732 + 2	2.043+1	2.324-1	1.417-1	1.017-1	7.624-2
	σx	1.000 + 4	6.599+3	6.319+1	3.804+1	3.836+1	3.538+1
	σy	1.000 + 4	1.595+4	2.589+2	3.628+2	4.702+2	5.768+2
	σz	1.000 + 4	2.530+4	2.188+2	1.945+2	2.193+2	2.570+2
	σẋ	1.000 + 2	1.414+1	1.524-1	7.101-2	4.123-2	2.553-2
	σẏ	1.000 + 2	7.065+0	1.452-1	1.145-1	8.833-2	6.852-2
	σż	1.000 + 2	1.295+1	9.854-2	4.384-2	2.895-2	2.158-2
	Bias Madrid	1.0 -02	1.00-2	9.978-3	9.098-3	7.967-3	7.115-3
	Bias Bermuda	1.0 -02	1.00-2	9.974-3	8.463-3	6.967-3	6.305-3
	Bias Ascension	1.0 -02	1.00-2	9.994-3	9.348-3	8.016-3	7.035-3

Figure 4 - First Two Hours Translunar Injection

Apriori Biases = 0.1 ft/sec

		Apriori	t = 16	t = 25	t = 50	t = 80	t = 120
Actual Uncertainties	σ Pos	1.732 + 4	3.065+5	2.083+3	1.754+3	2.583+3	3.734+3
	σ Vel	1.732 + 2	2.046+1	1.653+0	6.351-1	5.171-1	4.570-1
	σx	1.000 + 4	6.602+3	1.775+2	2.482+2	3.304+2	5.066+2
	σy	1.000 + 4	1.596+4	1.755+3	1.488+3	2.213+3	3.244+3
	σz	1.000 + 4	2.532+4	1.107+3	8.935+2	1.291+3	1.777+3
	σẋ	1.000 + 2	1.415+1	1.128+0	3.167-1	2.030-1	1.523+3
	σẏ	1.000 + 2	7.076+0	1.024+0	4.847-1	4.214-1	3.859-1
	σż	1.000 + 2	1.298+1	6.421-1	2.610-1	2.203-1	1.916-1
Filter Uncertainties	σPos	1.732 + 4	3.063+4	1.037+3	5.430+2	5.595+2	[illegible].441+2
	σVel	1.732 + 2	2.044+1	7.909-1	1.931-1	1.134-1	7.923-2
	σx	1.000 + 4	6.600+3	7.522+1	8.524+1	1.001+2	9.775+1
	σy	1.000 + 4	1.595+4	8.675+2	4.828+2	5.008+2	5.806+2
	σz	1.000 + 4	2.530+4	5.626+2	2.334+2	2.284+2	2.610+2
	σẋ	1.000 + 2	1.414+1	5.358-1	1.044-1	5.428-2	3.176-2
	σẏ	1.000 + 2	7.066+0	4.973-1	1.516-1	9.477-2	6.926-2
	σż	1.000 + 2	1.295+1	3.099-1	5.825-2	3.036-2	2.173-2
	Bias Madrid	1.0 - 1	1.0 - 1	8.704-2	5.941-2	4.714-2	3.206-2
	Bias Bermuda	1.0 - 1	1.0 - 1	8.493-2	5.407-2	4.321-2	2.974-2
	Bias Ascension	1.0 - 1	1.0 - 1	9.484-2	5.854-2	4.636-2	3.160-2

Figure 5 - First Two Hours of Translunar Injection

Apriori Biases = 1.0 ft/sec

		Apriori	t = 16	t = 25	t = 50	t = 80	t = 120
Actual Uncertainties	σ Pos	1.732 + 4	3.065+4	1.903+3	1.752+3	2.578+3	3.746+3
	σ Vel	1.732 + 2	2.046+1	1.525+0	6.283-1	5.121-1	4.595-1
	σx	1.000 + 4	6.602+3	1.772+2	3.602+2	4.591+2	6.257+2
	σy	1.000 + 4	1.596+4	1.633+3	1.456+3	2.184+3	3.241+3
	σz	1.000 + 4	2.532+4	9.606+2	9.046+2	1.291+3	1.771+3
	σẋ	1.000 + 2	1.414+1	9.962-1	3.300-1	2.112-1	1.627-1
	σẏ	1.000 + 2	7.076+0	9.244-1	4.619-1	4.102-1	3.846-1
	σż	1.000 + 2	1.298+1	6.914-1	2.693-1	2.222-1	1.918-1
Filter Uncertainties	σPos	1.732 + 4	3.098+4	2.196+3	7.457+2	5.865+2	6.474+2
	σVel	1.732 + 2	2.088+1	1.870+0	3.050-1	1.245-1	8.031-2
	σx	1.000 + 4	6.646+3	3.182+2	2.607+2	1.523+2	1.138+2
	σy	1.000 + 4	1.604+4	1.923+3	6.492+2	5.182+2	5.813+2
	σz	1.000 + 4	2.566+4	1.011+3	2.582+2	2.285+2	2.613+2
	σẋ	1.000 + 2	1.419+1	1.325+0	2.076-1	6.843-2	3.395-2
	σẏ	1.000 + 2	7.252+0	1.065+0	2.104-1	9.934-2	6.946-2
	σż	1.000 + 2	1.350+1	7.774-1	7.535-2	3.077-2	2.173-2
	Bias Madrid	1.0 + 0	9.991-1	5.839-1	2.003-1	7.437-2	3.774-2
	Bias Bermuda	1.0 + 0	9.995-1	5.881-1	1.846-1	6.839-2	3.505-2
	Bias Ascension	1.0 + 0	9.998-1	5.669-1	1.950-1	7.307-2	3.720-2

Figure 6 - First Two Hours of Translunar Injection

Apriori Biases = 10 ft/sec.

		Apriori	t = 16	t = 25	t = 50	t = 80	t = 120
Actual Uncertainties	σ Pos	1.732 + 4	3.578+4	5.590+3	1.775+3	2.579+3	3.750+3
	σ Vel	1.732 + 2	2.633+1	5.046+0	6.411-1	5.125-1	4.596-1
	σx	1.000 + 4	6.706+3	1.095+3	3.967+2	4.646+2	6.256+2
	σy	1.000 + 4	1.655+4	4.927+3	1.472+3	2.184+3	3.241+3
	σz	1.000 + 4	3.100+4	2.404+3	9.087+2	1.291+3	1.771+3
	σẋ	1.000 + 2	1.426+1	3.803+0	3.463-1	2.122-1	1.629-1
	σẏ	1.000 + 2	8.324+0	2.763+0	4.661-1	4.101-1	3.846-1
	σż	1.000 + 2	2.051+1	1.834+0	2.717-1	2.222-1	1.918-1
Filter Uncertainties	σPos	1.732 + 4	5.180+4	5.944+3	7.688+2	5.872+2	6.475+2
	σVel	1.732 + 2	4.384+1	5.376+0	3.171-1	1.248-1	8.033-2
	σx	1.000 + 4	1.021+4	1.154+3	2.760+2	1.534+2	1.140+2
	σy	1.000 + 4	2.246+4	5.249+3	6.683+2	5.186+2	5.813+2
	σz	1.000 + 4	4.548+4	2.546+3	2.612+2	2.285+2	2.613+2
	σẋ	1.000 + 2	1.871+1	4.065+0	2.178-1	6.876-2	3.398-2
	σẏ	1.000 + 2	1.716+1	2.939+0	2.171-1	9.945-2	6.946-2
	σż	1.000 + 2	3.600+1	1.935+0	7.733-2	3.078-2	2.173-2
	Bias Madrid	1.0 + 1	9.292+0	2.149+0	2.123-1	7.494-2	3.782-2
	Bias Bermuda	1.0 + 1	9.615+0	2.183+0	1.957-1	6.892-2	3.512-2
	Bias Ascension	1.0 + 1	9.821+0	2.008+0	2.067-1	7.363-2	3.727-2

Figure 7 - First Two Hours of Translunar Injection

Figures 8 and 9 show RMS position and RMS velocity uncertainties as a function of time and apriori bias assumptions. From these figures one can conclude that apriori bias assumptions between 0.1 and 1.0 ft/sec will nearly yield state vector estimates as good as if all real parameters were estimated. The results are so good as to suggest that no significant further improvement could be obtained by batching or by treating the gravitational parameter in any fancy manner in the filter for less than two hours of tracking.

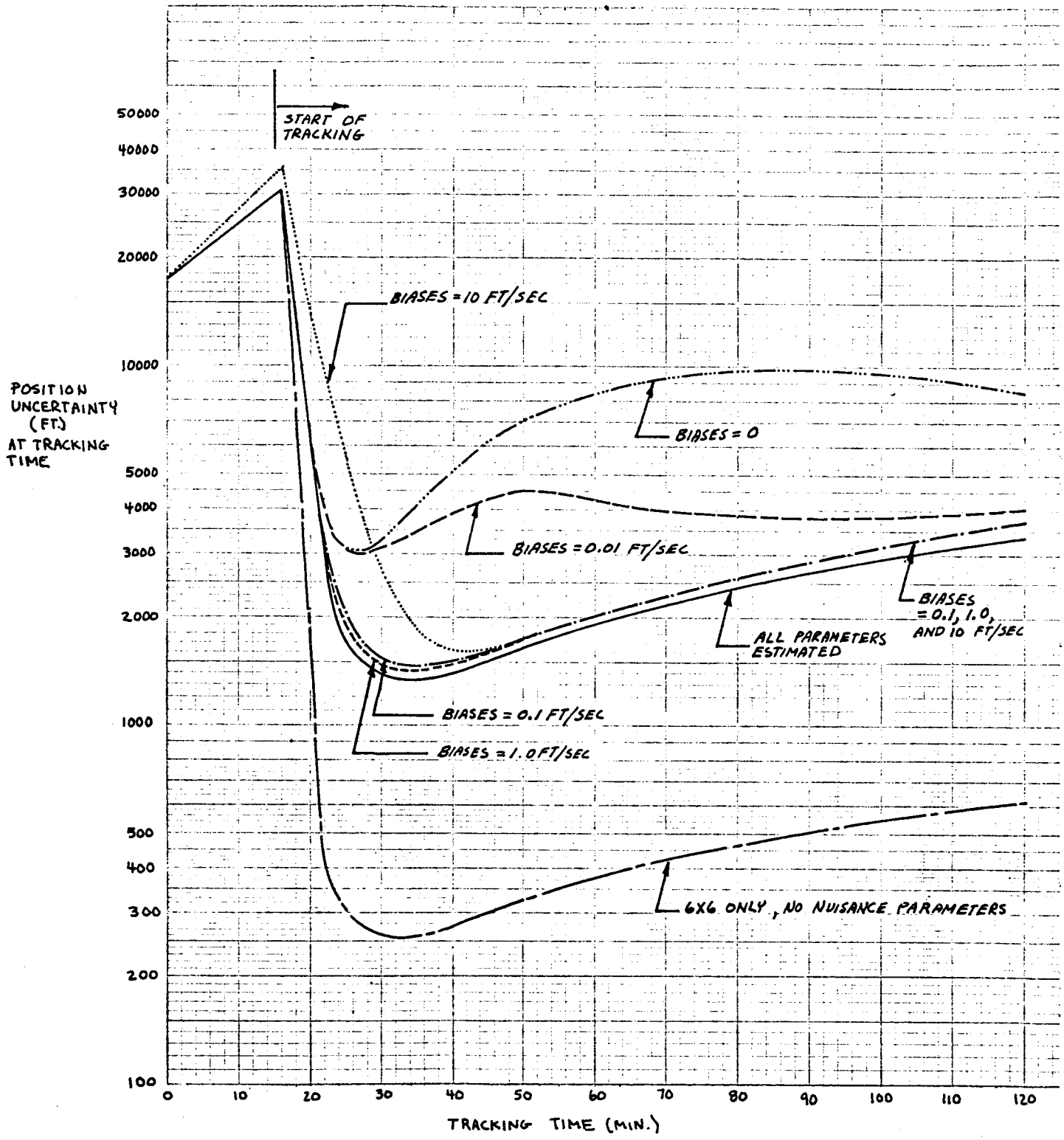


Figure 8 - Position Uncertainty with Three Stations Tracking the First Two Hours of Translunar Injection for Various Pseudo-Bias Apriori Assumptions.

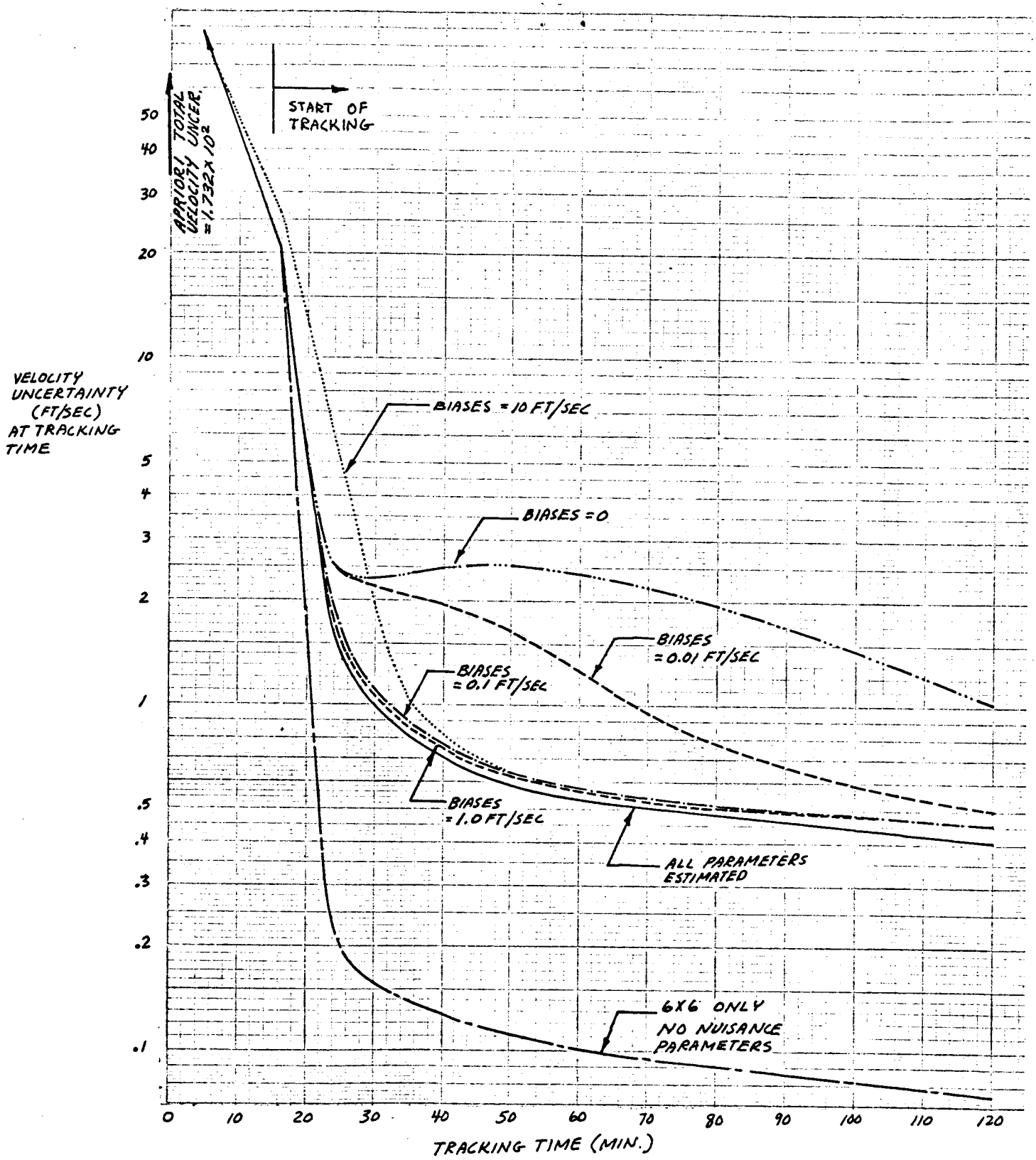


Figure 9 - Velcoity Uncertainty with Three Stations Tracking the First Two Hours of Translunar Injection for Various Pseudo-Bias Apriori Assumptions

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 473
(BBC Task 204)

L. Lustick
C. H. Dale

THE EFFECT OF ASSUMED FILTER BOOST ERRORS DURING LUNAR TRACKING

Purpose

The purpose of this note is to show the effect of filter boost assumptions on the resulting actual error.

Introduction

In the real time Apollo filter the state components and biases in the measurables are estimated. Since this filter is not in general the optimum filter the question is open as to what to assume for filter boost uncertainties in terms of the actual boost uncertainties. To illustrate the problem further let us consider the problem of estimating a constant from two batches of data separated by a "boost". From Note No. 449, the optimum filter assuming the two estimates are independent is as follows:

$$a = \frac{\text{cov } a_2}{\text{cov } a_1 + \text{cov } a_2} a_1 + \frac{\text{cov } a_1}{\text{cov } a_1 + \text{cov } a_2} a_2$$

where

a_1 = estimate from first batch of data

a_2 = estimate from second batch of data

$\text{cov } a_1$ = covariance of actual errors in estimate of a_1 from tracking

$\text{cov } a_2$ = covariance of actual errors in estimate of a_2 from tracking

let

$$\text{cov } a_3 = \text{cov } a_1 + \text{cov } \Delta b = \text{actual uncertainty in estimate subsequent to "boost"}$$

then

$$\hat{a} = \frac{\text{cov } a_2}{\text{cov } a_3 + \text{cov } a_2} a_3 + \frac{\text{cov } a_3}{\text{cov } a_3 + \text{cov } a_2} a_2$$

$$\hat{a} = \frac{\dfrac{\text{cov } a_2}{\text{cov } a_1}}{1 + \dfrac{\text{cov}\Delta b}{\text{cov } a_1} + \dfrac{\text{cov } a_2}{\text{cov } a_1}} a_3 + \frac{1 + \dfrac{\text{cov } \Delta b}{\text{cov } a_1}}{1 + \dfrac{\text{cov } \Delta b}{\text{cov } a_1} + \dfrac{\text{cov } a_2}{\text{cov } a_1}} a_2$$

From this last expression we see that the relative weighting of a_2 and a_3 depends on the ratios $(\text{cov } a_2/\text{cov } a_1)$ and $(\text{cov } \Delta b/\text{cov } a_1)$.

Since the relative weightings are determined in the filter with filter covariances a rational assumption for the filter assumed boost uncertainty, $\text{cov}_F \Delta b$ is

$$\text{cov}_F \Delta b = \left(\frac{\text{cov}_F\, a_1}{\text{cov } a_1}\right) \text{cov } \Delta b \ .$$

This study was designed to investigate this problem on a typical Moon phase Apollo trajectory.

Conditions

The trajectory in this problem is that of the LM in the terminal phase of rendezvous. The actual orbit and stations are as defined in Apollo Note No. 468.

The batching program was used to do this study and the filter estimated the state vector and pseudo biases in each of the measurables. The pseudo bias estimates were carried across the batch. The apriori uncertainty in the estimated biases was 0.1 ft/sec. No error in μ of the filter was assumed. All nuisance parameters with the exception of refraction were considered and the apriori uncertainty in these parameters are as defined in Apollo Note 468. The apriori on the state was 5,000 ft. in position and 10 ft/ sec. in velocity components.

The covariance of the actual boost errors was diagonal and the standard deviations in the boost components were as follows:

Radial velocity error	=	.3 ft/sec.
Horizontal velocity error	=	.5 ft/sec.
Out of plane velocity error	=	.3 ft/sec.

The actual errors were determined for ratios of filter boost standard deviation to actual boost standard deviations of 0, 0.1, 0.5, 1.0, 10, 100.

The tracking was with two stations, a master and one slave. The following three tracking situations were investigated:

Case I: Track for 15 minutes at which time a boost occurs. The boost is followed by 30 minutes of tracking and the results are presented for the state components at the end of tracking (45 minutes).

Case II: Track for 15 minutes at which time a boost occurs. The boost is followed by 15 minutes of tracking and the errors in the state vector at 15 minutes are presented.

Case III: Track for 30 minutes at which time a boost occurs. The boost is followed by 5 minutes of tracking and the errors in the state vector at 45 minutes are presented.

Results

Table I compares the errors in the filter world with the actual errors on the state vector at 15 minutes prior to the boost. The first thing that should be noticed is that with the exception of the z and $\dot{z}$ components the filter and real world errors are approximately the same. It can also be seen that the boost errors used are significant compared to the filter errors in the in plane velocity components and insignificant

in the out of plane component, $\dot{z}$. From the previous analysis, we might therefore expect a relative insensitivity in our results in the out of plane components and an optimum in the in plane components when the boost filter error was a little less than the actual errors.

In Table 2 and Table 3 the actual errors as a function of the ratio of assumed filter boost standard deviation to actual boost standard deviation are shown for Case I and Case II, respectively. It can be observed from these tables that the minimum in plane errors occur at a ratio of filter boost error to actual boost error between 0.5 and 1.0 and that further, this is a rather flat optimum. The out of plane components are relatively insensitive to the assumed boost error ratio.

Table 4 shows a comparison between the filter errors and the actual errors prior to a boost at 30 minutes after 30 minutes of tracking. In this case the filter and actual errors differ by more than they did in Case I at least in the in plane components and the actual boost errors in the in plane components are much larger than the filter errors.

Table 5 shows the actual error as a function of assumed filter boost error to actual boost error for Case III. Although, this case has much more tracking prior to the boost and much less after the boost than Case I the optimum filter boost error to actual boost error ratio is still approximately 1.0. Case III has much more degradation than Case I, however, for an assumed boost to actual boost, error ratios greater than 1.0.

Conclusions

For a continuous tracking interval during lunar operations in the Apollo mission of 45 minutes or less that are interrupted by an intervening boost, there is very little, if anything, to gain by assuming the filter boost errors different than the actual boost errors.

	Filter Errors at 15 Minutes	Actual Errors at 15 Minutes
Total Pos. Miss	5.712 + 3	7.240 + 3
Total Vel. Miss	5.954 + 0	8.497 + 0
Std. Dev. x	2.301 + 2	2.546 + 2
Std. Dev. y	7.47 + 2	7.65 + 2
Std. Dev. z	5.658 + 3	7.195 + 3
Std. Dev. $\dot{x}$	.2174	.2625
Std. Dev. $\dot{y}$	.6945	.7031
Std. Dev. $\dot{z}$	5.909	8.464

Table 1 - Comparison of Filter and Real Error Prior to Boost at 15 Min.

Table 2 - 15 Minute Track-Boost-30 Minute Track
(Results at 45 Minutes)

CASE I

Ratio of Filter Boost σ, to Actual Boost, σ	0	.1	.5	1.0	10	100
Total Pos. Miss	4.702 + 3	4.699 + 3	4.670 + 3	4.703 + 3	4.849 + 3	4.935 + 3
Total Vel. Miss	1.943 + 0	1.906 + 0	1.898 + 0	1.896 + 0	1.926 + 0	2.071 + 0
Std. Dev. x	2.320 + 2	1.599 + 2	1.382 + 2	1.381 + 2	1.396 + 2	1.403 + 2
Std. Dev. y	1.142 + 2	1.130 + 2	1.125 + 2	1.129 + 2	1.133 + 2	1.135 + 2
Std. Dev. z	4.695 + 3	4.695 + 3	4.696 + 3	4.699 + 3	4.845 + 3	4.932 + 3
Std. Dev. $\dot{x}$	3.157 - 1	2.260 - 1	1.596 - 1	1.589 - 1	1.623 - 1	1.632 - 1
Std. Dev. $\dot{y}$	1.751 - 1	5.830 - 2	4.530 - 2	4.445 - 2	4.452 - 2	4.461 - 2
Std. Dev. $\dot{z}$	1.909 + 0	1.892 + 0	1.891 + 0	1.888 + 0	1.919 + 0	2.064 + 0
In-Plane Pos. Miss	2.59 + 2	1.96 + 2	1.78 + 2	1.782 + 2	1.80 + 2	1.81 + 2
In-Plane Vel Miss	3.62 - 1	2.34 - 1	1.66 - 1	1.65 - 1	1.68 - 1	1.69 - 1

CASE II

Ratio of Filter Boost σ, to Actual Boost, σ	0	.1	.5	1.0	10	100
Total Pos. Miss	2.578 + 3	2.541 + 3	2.537 + 3	2.534 + 3	2.858 + 3	3.610 + 3
Total Vel. Miss	5.569 + 0	5.471 + 0	5.467 + 0	5.473 + 0	6.128 + 0	6.981 + 0
Std. Dev. x	3.899 + 2	1.147 + 2	9.712 + 1	9.680 + 1	9.751 + 1	9.879 + 1
Std. Dev. y	2.744 + 2	1.469 + 2	1.226 + 2	1.177 + 2	1.284 + 2	1.325 + 2
Std. Dev. z	2.534 + 3	2.534 + 3	2.532 + 3	2.529 + 3	2.853 + 3	3.607 + 3
Std. Dev. $\dot{x}$	1.866 - 1	1.357 - 1	1.211 - 1	1.159 - 1	1.222 - 1	1.275 - 1
Std. Dev. $\dot{y}$	1.082 + 0	3.154 - 1	1.924 - 1	1.628 - 1	1.839 - 1	1.845 - 1
Std. Dev. $\dot{z}$	5.460 + 0	5.460 + 0	5.463 + 0	5.470 + 0	6.124 + 0	6.976 + 0
In Plane Pos. Miss	477 + 0	186.5	156.5	152.0	161.0	166.0
In Plane Vel. Miss	1.1 + 0	3.44 - 1	2.28 - 1	2.00 - 1	2.21 - 1	2.25 - 1

Table 3 - 15 Minute Track-Boost-15 Minute Track
(Results at 15 Minutes)

	Filter Errors at 30 Minutes	Actual Errors at 30 Minutes
Total Pos. Miss	4.208 + 3	4.906 + 3
Total Vel. Miss	3.359 + 0	4.213 + 0
Std. Dev. x	2.762 + 1	1.006 + 2
Std. Dev. y	7.407 + 1	1.205 + 2
Std. Dev. z	4.207 + 3	4.903 + 3
Std. Dev. $\dot{x}$	4.792 - 2	9.697 - 2
Std. Dev. $\dot{y}$	3.332 - 2	6.658 - 2
Std. Dev. $\dot{z}$	3.358 + 0	4.211 + 0

Table 4 - Comparison of Filter and Real Error Prior to Boost at 30 Minutes

Table 5 - Track 30 Minutes-Boost-Track 5 Minutes
(Results at 45 Minutes)

CASE III

Ratio of Filter Boost σ, to Actual Boost, σ	0	.1	.5	1.0	10	100
Total Pos Miss	5.810 +3	5.628 + 3	5.620 + 3	5.619 + 3	6.529 + 3	8.164 + 3
Total Vel Miss	2.346 + 0	2.009 + 0	1.985 + 0	1.974 + 0	2.174 + 0	2.491 + 0
Std. Dev. x	1.435 + 3	4.453 + 2	3.480 + 2	3.179 + 2	4.848 + 2	4.921 + 2
Std. Dev. y	3.870 + 2	1.791 + 2	1.620 + 2	1.539 + 2	1.845 + 2	1.894 + 2
Std. Dev. z	5.617 + 3	5.608 + 3	5.607 + 3	5.608 + 3	6.509 + 3	8.147 + 3
Std. Dev. x	1.328 + 0	5.830 - 1	4.955 - 1	4.521 - 1	7.192 - 1	7.310 - 1
Std. Dev. y	2.251 - 1	9.669 - 2	8.276 - 2	7.544 - 2	1.099 - 1	1.123 - 1
Std. Dev. z	1.921 + 0	1.920 + 0	1.920 + 0	1.920 + 0	2.048 + 0	2.379 + 0
In Plane Pos Miss						
In Plane Vel Miss						

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 474
(BBC Task 204)

C. H. Dale
Feb. 1967

THE EFFECT ON ACTUAL STATE UNCERTAINTY OF ASSUMING AN IMPERFECTLY KNOWN GRAVITATIONAL CONSTANT IN THE RTODP FILTER

The Real Time Orbital Determination Program has, as an option, the ability of assuming that the gravitational parameter is not known perfectly. This allows the downweighting of past data. This Note shows that for tracking during the terminal phase of Lunar rendezvous, it is best to assume that μ is known perfectly. Of course, this tracking interval is only 45 minutes long and no distant projection of state covariance is studied. It might be most appropriate to assume that μ is imperfectly known when two periods of tracking are separated by an orbit or so and when the state covariance is desired near the end of the second tracking period.

The technique of this analysis is to use the batching program, set up in such a manner as to carry each entire batch estimate across. Eight parameters (six orbit parameters and two biases) are estimated during any one batch. Generally, only the orbit parameter covariance is maintained as apriori data for the next batch, while the biases are re-assumed to be zero with some apriori uncertainty. Carrying an entire batch across to the next batch means that the entire 8 x 8 is used as apriori covariance to be combined with the new batches tracking. The runs made in Apollo Note No. 471 were slightly modified for this purpose. In Note 471 three fifteen minute batches were used to study the effect of batching when the pseudo biases used in the filter were reinitialized with 0.1 ft/sec uncertainty for each batch. In this Note the estimates of the pseudo-biases are carried across each batch, and if no other changes had been made, the result would be identical to the continuous tracking case reported in Apollo Note No. 468. However, what was done was to parametrically change the assumed μ uncertainty in the filter from zero (producing the Apollo Note No. 468 result) to 1/10, 1/3 and the full actual μ uncertainty existent in the real-world ($\sigma\ \mu$ actual = 5×10^9 ft^3/sec^2).

If the above seems a bit confusing, looking at the data inputs will clear things up a bit. This is what was done:

1) The Information Matrices generated by Program A's were produced as shown in Apollo Note No. 471.

2) APF, the apriori filter covariance, was entered identically to Note No. 471 with apriori range-rate pseudo-biases of 0.1 ft/sec.

3) APN, the apriori actual state covariance, was entered identically to Note No. 471, and equalled the filter covariance on the state parameters. A "one" was entered in $APN_{12,12}$ so that the computer routine would realize that the noise covariance should be greater than a 6 x 6 (in this case an 8 x 8); this is a temporary stratagem to overcome a program deficiency.

4) AP01 and AP02, the two nuisance parameter groups, were entered identically to Note No. 471.

5) An apriori, A, input was used to enter the variance in μ assumed by the filter for all four parametric runs. This looked like:

A	8
A(7, 7) = 1.0 E-10	an insignificant μ variance
A(8, 8) = 2.5 E+17	filter uncertainty = 1/10 actual uncer.
A(9, 9) = 2.5 E+18	filter uncertainty = 1/3 actual uncer.
A(10, 10) = 2.5 E+19	filter uncertainty = actual uncer.

Now with all information matrices and apriori inputs generated we will follow through the first run, wherein an insignificant μ filter uncertainty is assumed. This is designed to produce results equal to the 0.1 apriori bias, continuous tracking, non-batching results shown

in Figures 8 and 9 of Apollo Note No. 468. This first run thus checks the analysis and computer program for our purposes before performing the three meaningful runs.

6) The first batch "Program B Control" sheet is identical to Note No. 471, however in the filter allocation the 7, 7 term of "A" data set 8 is placed in 23, i.e., the 10^{-10} μ covariance is placed in the first non-updated diagonal element. The remaining allocation is identical to Note No. 471. The resulting covariance matrices produced by this first batch (15 minutes of tracking at t = 0 epoch) are identical to those of Note No. 471 since the μ term only appears on projection.

7) The second batch Program B Control sheet is again identical to Note No. 471. In the filter group allocation PF is brought in as is, i.e., the biases are carried across the batch. The 15 minute Q matrix from data set 001 is no longer brought in as a 6 x 6; μ is allocated to column 23 to correspond to the previously allocated filter (see 6 above). In the noise group allocation PN is also brought in as is which brings in the whole previously generated 8x8 noise. Again in the nuisance parameters group allocation P01 (or P02) is brought across the batch maintaining the bias terms in 07 and 08.

8) The third batch is handled identically to the second batch.

9) Following the last batch a "Project Covariance" Program B routine is performed. Again the Control Sheet is identical to Note No. 471. For the filter group allocation, PF is brought in as is and the 45 minute Q matrix has μ allocated to column 23. In the noise group allocation PN is brought in as is. For the two nuisance parameter groups P01 and P02 are brought in as is.

This completes the first run. The second run differs from the first run in the first batch filter allocation, where A(8, 8) instead of A(7, 7), from Data Set 008, is placed in diagonal element 23. For the third run A(9, 9) is used and for the fourth run A(10, 10) is used.

The results are shown in Figure 1. What is most important is the fact that the actual results are not strongly affected by the μ filter assumption. It appears that the in-plane uncertainties get slightly larger monotonically as the filter μ uncertainty assumption grows from zero to the actual μ uncertainty. The out-of-plane position and velocity uncertainties are constant as the filter μ uncertainty varies, which is to be expected since the z and $\dot{z}$ are not effected by errors in μ for projection. These results, though appearing to suggest the simplest of μ assumptions in the filter design, should not be taken too seriously until the more interesting cases of protracted tracking and projection are studied.

		Filter μ uncer. = 0			Filt. μ = 1/10 true uncer.			Filt. μ = 1/3 true uncer.			Filt. μ = $\sigma\mu$ actual uncer.		
		t=15	t=30	t=45	t=15	t=30	t=45	t=15	t=30	t=45	t=15	t=30	t=45
Actual Uncertainty	σPos	2.119+3	1.386+3	2.101+3	2.119+3	1.387+3	2.101+3	2.119+3	1.387+3	2.101+3	2.119+3	1.387+3	2.101+3
	σVel	1.741+0	1.931+0	1.288+0	1.741+0	1.932+0	1.289+0	1.741+0	1.932+0	1.290+0	1.741+0	1.932+0	1.290+0
	σx	1.342+2	1.000+2	1.286+2	1.342+2	1.001+2	1.303+2	1.342+2	1.003+2	1.316+2	1.342+2	1.004+2	1.318+2
	σy	3.467+2	9.689+1	9.984+1	3.467+2	9.810+1	1.005+2	3.467+2	1.034+2	1.009+2	3.467+2	1.052+2	1.010+2
	σz	2.087+3	1.379+3	2.094+3	2.087+3	1.379+3	2.095+3	2.087+3	1.379+3	2.095+3	2.087+3	1.379+3	2.095+3
	$\sigma \dot{x}$	1.292-1	7.960-2	1.198-1	1.292-1	7.967-2	1.221-1	1.292-1	8.010-2	1.238-1	1.292-1	8.029-2	1.241-1
	$\sigma \dot{y}$	3.152-1	6.567-2	4.404-2	3.152-1	6.609-2	4.465-2	3.152-1	6.792-2	4.526-2	3.152-1	6.867-2	4.540-2
	$\sigma \dot{z}$	1.707+0	1.929+0	1.282+0	1.707+0	1.929+0	1.283+0	1.707+0	1.929+0	1.283+0	1.707+0	1.929-2	1.283+0
	In Plane P	3.72+2	1.39+2	1.63+2	3.72+2	1.40+2	1.65+2	3.72+2	1.43+2	1.66+2	3.72+2	1.45+2	1.66+2
	In Plane V	3.41-1	1.03-1	1.28-1	3.41-1	1.04-1	1.30-1	3.41-1	1.06-1	1.32-1	3.41-1	1.06-1	1.32-1
Filter Uncertainty	σPos	2.075+3	1.353+3	1.767+3	2.075+3	1.354+3	1.767+3	2.075+3	1.354+3	1.768+3	2.075+3	1.355+3	1.768+3
	σVel	1.681+0	1.717+0	1.225+0	1.681+0	1.718+0	1.225+0	1.682+0	1.718+0	1.226+0	1.686+0	1.723+0	1.231+0
	σx	1.108+2	2.539+1	2.653+1	1.109+2	2.629+1	2.864+1	1.118+2	3.049+1	3.295+1	1.207+2	5.483+1	5.605+1
	σy	3.322+2	6.584+1	4.707+1	3.372+2	7.030+1	4.726+1	3.373+2	7.625+1	4.789+1	3.381+2	8.100+1	5.295+1
	σz	2.044+3	1.352+3	1.767+3	2.044+3	1.352+3	1.767+3	2.044+3	1.352+3	1.767+3	2.044+3	1.352+3	1.767+3
	$\sigma \dot{x}$	1.040-1	4.152-2	5.106-2	1.047-1	4.379-2	5.327-2	1.108-1	5.749-2	6.477-2	1.599-1	1.287-1	1.313-1
	$\sigma \dot{y}$	3.142-1	3.317-2	1.555-2	3.142-1	3.554-2	1.667-2	3.144-1	4.004-2	2.071-2	3.167-1	5.529-2	4.235-2
	$\sigma \dot{z}$	1.648+0	1.717+0	1.224+0	1.648+0	1.717+0	1.224+0	1.648+0	1.717+0	1.224+0	1.648+0	1.717+0	1.224+0
	Bias Mad	7.143-2	7.119-2	6.632-2	7.143-2	7.119-2	6.738-2	7.143-2	7.119-2	6.810-2	7.143-2	7.119-2	6.823-2
	Bias Asn	7.151-2	7.125-2	6.627-2	7.152-2	7.125-2	6.735-2	7.152-2	7.125-2	6.807-2	7.152-2	7.125-2	6.820-2

Figure 1

Filter and Actual Uncertainty for Four Gravitational Uncertainty Assumptions Used in the Real Time Filter Shown at the End of Each of Three Fifteen Minute Batches

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 475
(BBC Task 204)

C. H. Dale
Feb. 1967

THE EFFECT ON ACTUAL STATE UNCERTAINTY OF ASSUMING AN IMPERFECTLY KNOWN GRAVITATIONAL CONSTANT IN THE RTODP FILTER

II

This is an addendum to Apollo Note No. 474 which studied the effect of assuming that μ is imperfectly known in the RTODP filter during 45 minutes of Lunar orbital tracking. This note shows that when two lunar tracking periods are separated by a full orbit, no significant μ downweighting effect exists, which was also the conclusion of the previous Note for a shorter projection interval. That is, the orbital estimate covariance is not affected by the uncertainty in μ used by the Real Time orbital processor.

In the analysis reported herein, a fifteen minute batch was followed by exactly one orbit measured from the beginning of this first batch, at which time a second fifteen minute batch was taken. The Program A's for these two batches correspond exactly to those used for the first fifteen minute batch reported in Note 474 except that a QF time of one orbit (128.4738 minutes) was added to the first batch and, the TI (initial), T orbit, TJ, and QF times for the second batch Program A's were appropriately changed. A "Project Covariance" routine followed the second batch giving the result at the end of the second batch. The results are shown on the following page.

		Filter $\mu = 0$		Filter $\mu = \frac{1}{10}$ Actual		Filter $\mu = \frac{1}{3}$ Actual		Filter μ = Actual	
	Estimates Projected to →	End of 2nd batch	Beg. of 2nd batch	End	Beg.	End	Beg.	End	Beg.
Actual Uncertainty	σ Pos	2.052 + 3	2.499 + 3	2.052 + 3	2.499 + 3	2.052 + 3	2.499 + 3	2.052 + 3	2.499 + 3
	σ Vel	1.700 + 0	1.233 + 0	1.700 + 0	1.233 + 0	1.700 + 0	1.233 + 0	1.700 + 0	1.233 + 0
	σ x	8.987 + 1	1.072 + 2	8.987 + 1	1.072 + 2	8.984 + 1	1.072 + 2	8.984 + 1	1.077 + 2
	σ y	6.263 + 1	9.210 + 1	6.265 + 1	9.211 + 1	6.282 + 1	9.214 + 1	6.787 + 1	9.277 + 1
	σ z	2.049 + 3	2.495 + 3	2.049 + 3	2.495 + 3	2.049 + 3	2.495 + 3	2.049 + 3	2.495 + 3
	$\sigma \dot{x}$	8.132 - 2	9.210 + 1	8.132 - 2	9.540 - 2	8.136 - 2	9.560 - 2	8.191 - 2	1.015 - 1
	$\sigma \dot{y}$	4.242 - 2	3.704 - 2	4.241 - 2	3.702 - 2	4.240 - 2	3.693 - 2	4.689 - 2	3.957 - 2
	$\sigma \dot{z}$	1.697 + 0	1.228 + 0	1.697 + 0	1.228 + 0	1.697 + 0	1.228 + 0	1.697 + 0	1.228 + 0
Filter Uncertainty	σ Pos	1.923 + 3	2.306 + 3	1.923 + 3	2.306 + 3	1.923 + 3	2.306 + 3	1.925 + 3	2.307 + 3
	σ Vel	1.600 + 0	1.210 + 0	1.600 + 0	1.210 + 0	1.601 + 0	1.211 + 0	1.607 + 0	1.216 + 0
	σ x	3.347 + 1	2.211 + 1	3.391 + 1	2.256 + 1	3.761 + 1	2.619 + 1	6.338 + 1	4.814 + 1
	σ y	4.017 + 1	6.095 + 1	4.112 + 1	6.103 + 1	4.875 + 1	6.171 + 1	9.351 + 1	6.764 + 1
	σ z	1.922 + 3	2.305 + 3	1.922 + 3	2.305 + 3	1.922 + 3	2.305 + 3	1.922 + 3	2.305 + 3
	$\sigma \dot{x}$	4.170 - 2	4.496 - 2	4.350 - 2	4.634 - 2	5.717 - 2	5.722 - 2	1.303 - 1	1.169 - 1
	$\sigma \dot{y}$	2.170 - 2	1.821 - 2	2.889 - 2	1.930 - 2	3.929 - 2	2.713 - 2	9.050 - 2	6.394 - 2
	$\sigma \dot{z}$	1.599 + 0	1.209 + 0	1.599 + 0	1.209 + 0	1.599 + 0	1.209 + 0	1.599 + 0	1.209 + 0
	Bias Mad	7.117 - 2	7.117 - 2	7.117 - 2	7.117 - 2	7.117 - 2	7.117 - 2	7.117 - 2	7.117 - 2
	Bias Asn	7.118 - 2	7.119 - 2	7.119 - 2	7.119 - 2	7.119 - 2	7.119 - 2	7.119 - 2	7.119 - 2

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 476
(BBC Task 207)

J. R. Holdsworth

AN APPROXIMATE SCALING LAW BETWEEN $\dot{R}$ BIAS ERRORS AND CLOCK PARAMETERS FOR LUNAR ORBITS

The purpose of this Note is to present some computer results which have relevance to the question as to whether it is possible to develop a simple empirical scaling law whereby the effects of $\dot{R}$ biases and clock parameter errors may be made comparable. In earlier error analyses, computer results have been obtained for various combinations of $\dot{R}$ biases. The results of the present Note should give some indication of the numerical values of the clock parameters required to give results in some sense comparable to those obtained from earlier runs using biases on the range-rate measurable.

The particular reference orbit employed was a lunar parking orbit with the following characteristics. The orbit was circular with an altitude of 80 nautical miles above the lunar surface. The inclination of the orbit plane with respect to the Earth's equatorial plane was 175 degrees, and the period is about two hours. The initial time was chosen to be such that the vehicle was directly behind the Moon at that instant. Thus it emerges from the shadow zone after 22 minutes. When it emerges it is tracked by a three-way Doppler configuration consisting of Madrid as the master station with Ascension and Grand Bahamas as the slave stations.

On the basis of measurements received, the six components of the state vector are estimated, i. e., the three components of the vehicle position vector and the three components of the vehicle velocity vector. For a fixed tracking time the six by six covariance matrix is computed at the end of the tracking time and

and its time development observed by mapping it forward around the orbit in 15 minute intervals of time. This was done for the rms. position and velocity errors due to the $\dot{R}$ measurable biases, and for the rms. position and velocity errors due to the clock parameter errors. That is, the multiple station program was employed, first to determine the rms. position and velocity errors due to an $\dot{R}$ bias and second to determine these same errors as caused by the clock errors: i.e., the clock offset and the clock rate errors.

These runs were made for tracking times of about 15, 20, 25, 30, 35, 40, 45, 60 and 75 minutes respectively. The numerical values of the clock offset and the clock rate errors were taken to be 0.5×10^{-3} secs., and 10^{-11} sec. per. sec. respectively. The following numerical values were assigned to the $\dot{R}$ measurable values and their sigmas. For the master station, Madrid, the bias on the measurable was taken to be 0.03 ft/sec with a standard deviation of 0.02 ft/sec. For each of the slave stations, Ascension and Grand Bahamas, the bias on the measurable was 0.2 ft/sec with a standard deviation of 0.038 ft/sec.

Numerical Results

Figures 1 through 9 show comparative tabulations of the ratios of r.m.s. position error due to the $\dot{R}$ bias to that due to the clock parameters; as well as the ratio of the r.m.s. velocity error due to $\dot{R}$ bias to velocity errors due to the clock parameters. For various fixed tracking lengths, these ratios are computed as functions of time along the orbit beginning with the end of tracking. The tracking times chosen were 15, 20, 25, 30, 35, 40, 45, 60 and 75 minutes.

Inspection of these figures reveals some interesting facts. First of all, from Figure 1 we observe that for a tracking time of 15 minutes that the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ are almost constant functions of time around the orbit. Moreover, these constant ratios are approximately equal. That is, even though there is a considerable variation in the sigmas $\sigma_{P\dot{R}B}(T)$, $\sigma_{PCL}(T)$, $\sigma_{V\dot{R}B}(T)$, $\sigma_{VCL}(T)$ as the vehicle makes two complete orbits, the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ are very stable and essentially constant. This suggests the possibility that the effect of the clock parameters on the uncertainties in the state vector for a lunar parking orbit might be adequately accounted for by using the $\dot{R}$ bias and multiplying by a simple scale factor.

As we tabulate the results of similar computations for different values of the tracking time an interesting, albeit somewhat disturbing pattern begins to come forth. For example, for a tracking time of 20 minutes both $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ begin to exhibit a much stronger dependence upon the epoch time than in the case of the 15 minute tracking time. The individual terms $\sigma_{P\dot{R}B}(T)$, $\sigma_{PCL}(T)$, $\sigma_{V\dot{R}B}(T)$, and $\sigma_{VCL}(T)$ are in and of themselves no more sensitive functions of epoch time T than they were for a tracking time of 15 minutes. Indeed as we have already mentioned, the four individual terms $\sigma_{PCL}(T)$, $\sigma_{V\dot{R}B}(T)$, $\sigma_{VCL}(T)$, $\sigma_{P\dot{R}B}(T)$ were quite variable functions of epoch time T for the 15 minute tracking even though the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$, $\sigma_{V\dot{R}B}/\sigma_{VCL}$ remain almost constant.

For the 20 minute tracking time, there appears to be a phase shift or misalighnment which, although it does not cause the individual sigmas to drastically alter their character, is none the less sufficient to make the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ more sensitive functions of elapsed time around the orbit.

For a tracking time of 25 minutes this tendency is even more pronounced, i. e., there is a very irregular or wavy variation of both $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ as a function of elapsed time around the orbit. This may be observed by visual inspection of Figures 2 and 3.

The situation appears to achieve its most chaotic form for a tracking time of 30 minutes. For this tracking time both of the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ vary from approximately 4 to 12. Moreover, they do not vary in a synchronous manner, i. e., $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ are no longer approximately equal for the same values of the epoch time and along the orbit. The tabulations for a tracking time of 30 minutes are exhibited in Figure 4.

As the tracking times increase, another interesting phenomenon occurs. The dependency of the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ upon the elapsed orbital time begins to die out. Figure 5 illustrates this point for a tracking time of 35 minutes. The tendency is even more pronounced for a tracking time of 40 or 45 minutes. Indeed at these larger tracking times the dependency of the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$ and $\sigma_{V\dot{R}B}/\sigma_{VCL}$ has almost disappeared so that the ratios are once again constant.

Conclusion

In this Note we have examined the dependence of the quantities $\sigma_{P\dot{R}B}/\sigma_{PCL}$, $\sigma_{V\dot{R}B}/\sigma_{VCL}$ upon the elapsed time around the orbit. This was done for a lunar parking orbit with the intention of seeing whether it would be possible to develop a simple scaling procedure to render comparable the error contributions of the clock parameters and $\dot{R}$ bias terms to the state vector position and velocity errors. It was found that for short tracking times i. e., 15 minutes that the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$, $\sigma_{V\dot{R}B}/\sigma_{VCL}$ are almost constant and equal for at least two periods of the orbit, their common constant value being approximately 11.8. For tracking times between 20 and 35 minutes the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$, $\sigma_{V\dot{R}B}/\sigma_{VCL}$ ceased to be constant along the

orbit and varied considerably. Moreover, they did not vary synchronously, i. e., the ratios were no longer approximately equal for the same tracking and epochal times.

For the larger tracking times which were examined, i. e., 40, 45, 60 and 75 minutes, the dependency of the ratios $\sigma_{P\dot{R}B}/\sigma_{PCL}$, $\sigma_{V\dot{R}B}/\sigma_{VCL}$ seems to die out again. In addition, for these greater tracking times the values of the ratios are approximately equal to the same constant which is roughly 4.

Translunar orbits and Earth parking orbits were not considered in this Note. Earth parking orbits were not mentioned since 3-way Doppler measurements are not used for these trajectories. Translunar orbits will be considered in a further note.

Root Mean Square Position and Velocity Error Ratios with Tracking Time of 15 Minutes

T	$\left(\frac{\sigma_{\dot{P}RB(T)}}{\sigma_{PCL(T)}}\right)$	$\left(\frac{\sigma_{\dot{V}RB(T)}}{\sigma_{VCL(T)}}\right)$
53	13.2	10.6
68	11.6	11.4
83	10.6	12.7
98	11.4	11.7
113	12.6	10.9
128	12.2	11.5
143	11.2	12.6
158	11.6	12.4
173	12.6	11.2
188	12.3	11.6
203	11.2	12.4
218	11.7	12.4

Figure 1

Root Mean Square Position and Velocity Error Ratios with Tracking Time of 20 Minutes

T	$\left(\frac{\sigma_{\dot{P}RB}}{\sigma_{PCL}}\right)$	$\left(\frac{\sigma_{\dot{V}RB}}{\sigma_{VCL}}\right)$
53	11.20	6.80
68	10.40	7.80
83	7.00	11.00
98	8.21	11.80
113	11.70	7.98
128	13.50	8.75
143	8.96	11.80
158	9.01	13.50
173	11.60	9.05
188	13.90	9.28
203	9.66	12.10
218	10.10	14.40

Figure 2

Root Mean Square Position and
Velocity Error Ratios With
Tracking Time of 25 Minutes

T	$\sigma_{P\dot{R}B} / \sigma_{PCL}$	$\sigma_{V\dot{R}B} / \sigma_{VCL}$
53	7.95	4.51
68	9.54	4.86
83	4.75	7.96
98	5.42	11.20
113	9.14	5.90
128	13.20	6.21
143	7.18	9.50
158	4.95	13.60
173	9.35	7.28
188	14.00	6.90
203	7.88	10.10
218	7.85	14.75

Figure 3

Root Mean Square Position and Velocity Error Ratios
With Tracking Time of 30 Minutes

T	$\frac{\text{RMS Pos } \dot{R} \text{ Bias}}{\text{RMS Pos Clock}}$	$\frac{\text{RMS Vel } \dot{R} \text{ Bias}}{\text{RMS Vel Clock}}$
53	4.02	3.77
68	7.36	3.57
83	3.98	4.18
98	3.51	8.88
113	5.24	4.46
128	11.30	4.16
143	5.30	5.86
158	4.51	12.05
173	5.80	5.34
188	12.60	4.62
203	5.64	6.38
218	5.62	13.70

Figure 4

Root Mean Square Position and Velocity Error Ratios with Tracking of 35 Minutes

T	$\sigma_{P\dot{R}B} / \sigma_{PCL}$	$\sigma_{V\dot{R}B} / \sigma_{VCL}$
68	4.58	3.32
83	3.53	3.19
98	3.34	5.25
113	3.36	3.68
128	6.92	3.44
143	3.82	3.56
158	3.66	7.74
173	3.59	3.85
188	8.42	3.55
203	3.91	3.73
218	3.63	10.30

Figure 5

Root Mean Square Position and Velocity Error Ratios with Tracking of 40 Minutes

T	$\sigma_{P\dot{R}} / \sigma_{PCL}$	$\sigma_{V\dot{R}} / \sigma_{VCL}$
68	3.48	3.29
83	3.35	3.28
98	3.30	3.64
113	3.29	3.37
128	4.05	3.33
143	3.38	3.32
158	3.33	4.36
173	3.32	3.40
188	4.71	3.34
203	3.39	3.34
218	3.66	5.76

Figure 6

Root Mean Square Position and Velocity
Error Ratios with Tracking of 45 Minutes

T	$\left(\frac{\sigma_{R_{\dot{R}}}(T)}{\sigma_{R_{CL}}(T)}\right)$	$\left(\frac{\sigma_{\dot{R}_{\dot{R}}}(T)}{\sigma_{\dot{R}_{CL}}(T)}\right)$
68	3.23	3.26
83	3.21	3.28
98	3.23	3.25
113	3.29	3.27
128	3.34	3.29
143	3.27	3.30
158	3.27	3.43
173	3.31	3.28
188	3.49	3.28
203	3.28	3.30
218	3.28	3.89

Figure 7

Root Mean Square Position and Velocity Error Ratios with Tracking of 60 Minutes

T	$\sigma_{P\dot{R}B} / \sigma_{PCL}$	$\sigma_{V\dot{R}B} / \sigma_{VCL}$
98	3.51	3.43
113	3.56	3.49
128	3.44	3.52
143	3.51	3.54
158	3.51	3.47
173	3.55	3.49
188	3.47	3.51
203	3.50	3.55
218	3.51	3.57

Figure 8

Root Mean Square Position and Velocity Error Ratios with Tracking of 75 Minutes

T	$\sigma_{P\dot{R}B} / \sigma_{PCL}$	$\sigma_{V\dot{R}B} / \sigma_{VCL}$
98	3.68	3.63
113	3.70	3.66
128	3.63	3.68
143	3.68	3.70
158	3.57	3.68
173	3.69	3.65
188	3.72	3.68
203	3.72	3.69
218	3.68	3.84

Figure 9

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 477
(BBC Task 203)

J. Gregory
G. Hempstead

REPORT OF MODIFICATIONS TO THE (DEC 10) RTODP-OEAP PROGRAM

The purpose of this Note is to report on the programming changes to the RTODP-OEAP program (Dec 10 Program). The development of these changes has been discussed in Apollo Note No. 470. The seven new features considered in Note 470 were:

1. Shift
2. Consolidation
3. Arbitrary weights
4. Different noise in the filter and real world
5. QE
6. QR
7. QP.

Appendix A gives the latest flowchart for the CALC subroutine. This subroutine is the major computational part of Program B of the RTODP-OEAP program. In particular, items 3 to 7 above will be found in CALC.

Appendix B describes the program output from CALC and the internal and external flags used by CALC. This output includes (almost) all output of Program B of the RTODP-OEAP.

Appendix C is a small test case to check items 3 to 6. In addition, the new procedure for obtaining fixed radius results is described. The results of the test case agree (almost perfectly) with the old OEA Program. The insignificant errors are probably due to the fact that the method of operation is not the same for the two programs. The corresponding run for the old OEA Program is also included in Appendix C.

Items 1 and 2 have also been checked to perform as described, but as no calculations are actually made. We will only remark that they are performing successfully as described in Apollo Note 470. Item 7 has been checked by an independent test case which is not shown.

Finally, we recall the remark in Note No. 470 pertaining to the QR section, i. e., the time T nominally corresponding to fixed r is determined in a preliminary program or is otherwise available as an input (an example of a data set which finds time T corresponding to fixed r is given below).

The following data set was used to find the time corresponding to the fixed radius. This time was then used in our new OEA test program.

1	2.1425738E 07	A1
2	-3.0000000E 02	A4
3	2.5631800E 04	A5
4	1.0000000E 00	GAM ID
5		BETA
6		XI
7	3.0000000E 01	ETA
8	2.0000000E 01	ZETA
9	1.0000000E 00	LAMBDA
10	3.8000000E 01	ALPHA
11	7.2921561E-05	OMEGAE
12		OMEGAM
13	2.0925738E 07	RHOE
14		RHOM
15	1.4076530E 16	MU
16	1.0300000E 02	TIME
17	2.1335738E 07	RE RAD
18		LT SLP
19	2.0000000E 01	LO SLP
20		RD IND
21		R IND
22		Q IND
23		QF IND
24		A1 IND
25		A2 IND
26		PVWIND
27	1.0000000E 02	T INIT
28	1.0000000E-15	T INCR
29	1.4000000E 01	DIMENS
30		LAMB2
31		ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	1.0000000E 02	T CLK
36	1.0000000E 02	T XYZ
37	1.0000000E 02	T ORB
38	1.0000000E 02	T TILD
39	1.0000000E 02	T SHIP

QR MATRIX 2 QR R = 2.1335738E 07 TIME = 1.0511455E 02

APPENDIX A

The purpose of this appendix is to provide a logical flow diagram for subroutine CALC. This subroutine is essentially described on pp. 19-24, Apollo Note 437.

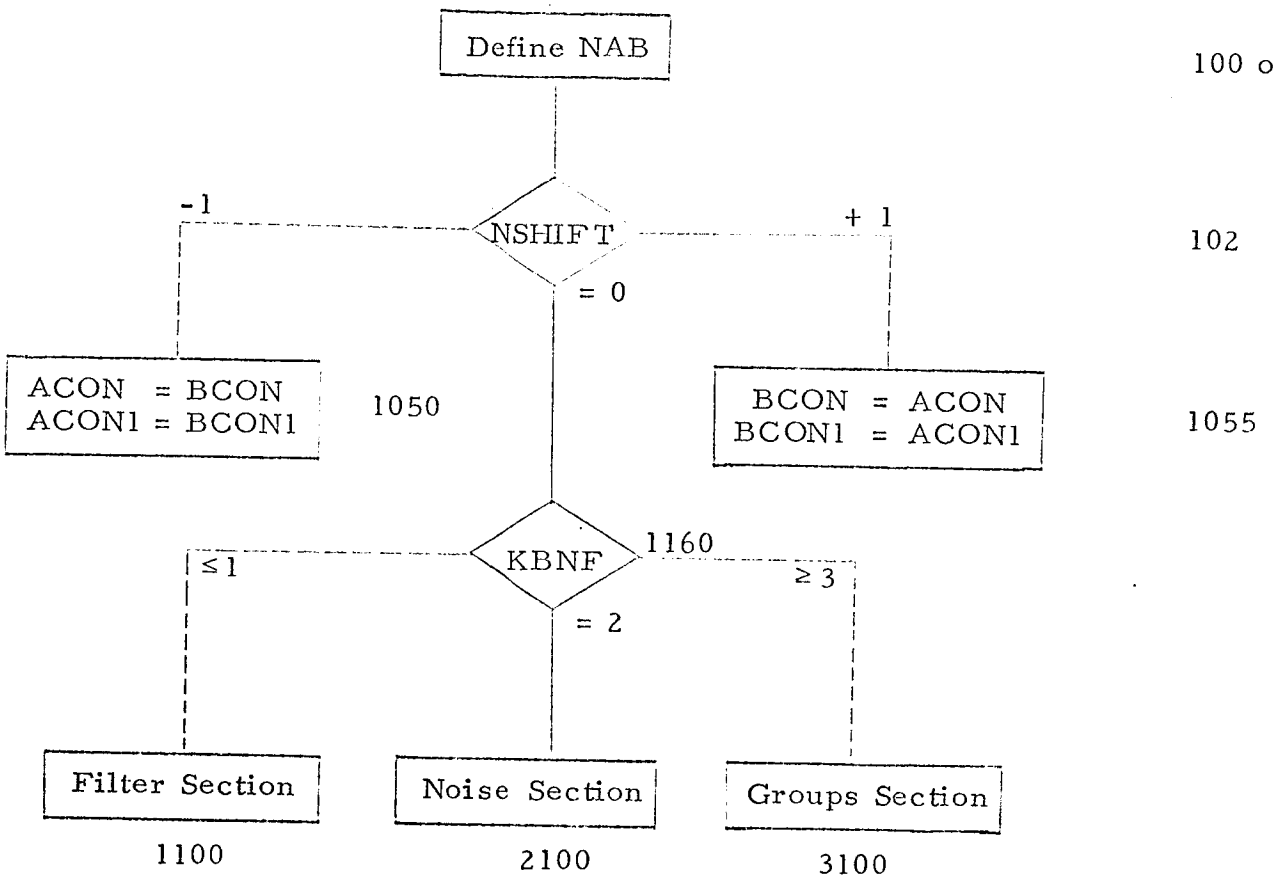
The statement numbers in the program correspond to Note 437 in that: 1, 2, 3 in the thousands position correspond, respectively, to the Filter, Noise, and Non-estimated Parameter sections of the Note; while the numbers in the hundreds position of the statement numbers in CALC are the same as those of the Note. Thus, CALC statement numbers 1200 - 1299 correspond to the subdivision number 2 of the Filter description in the note.

The flags and their meanings (which are used in CALC and in the flow diagram) are:

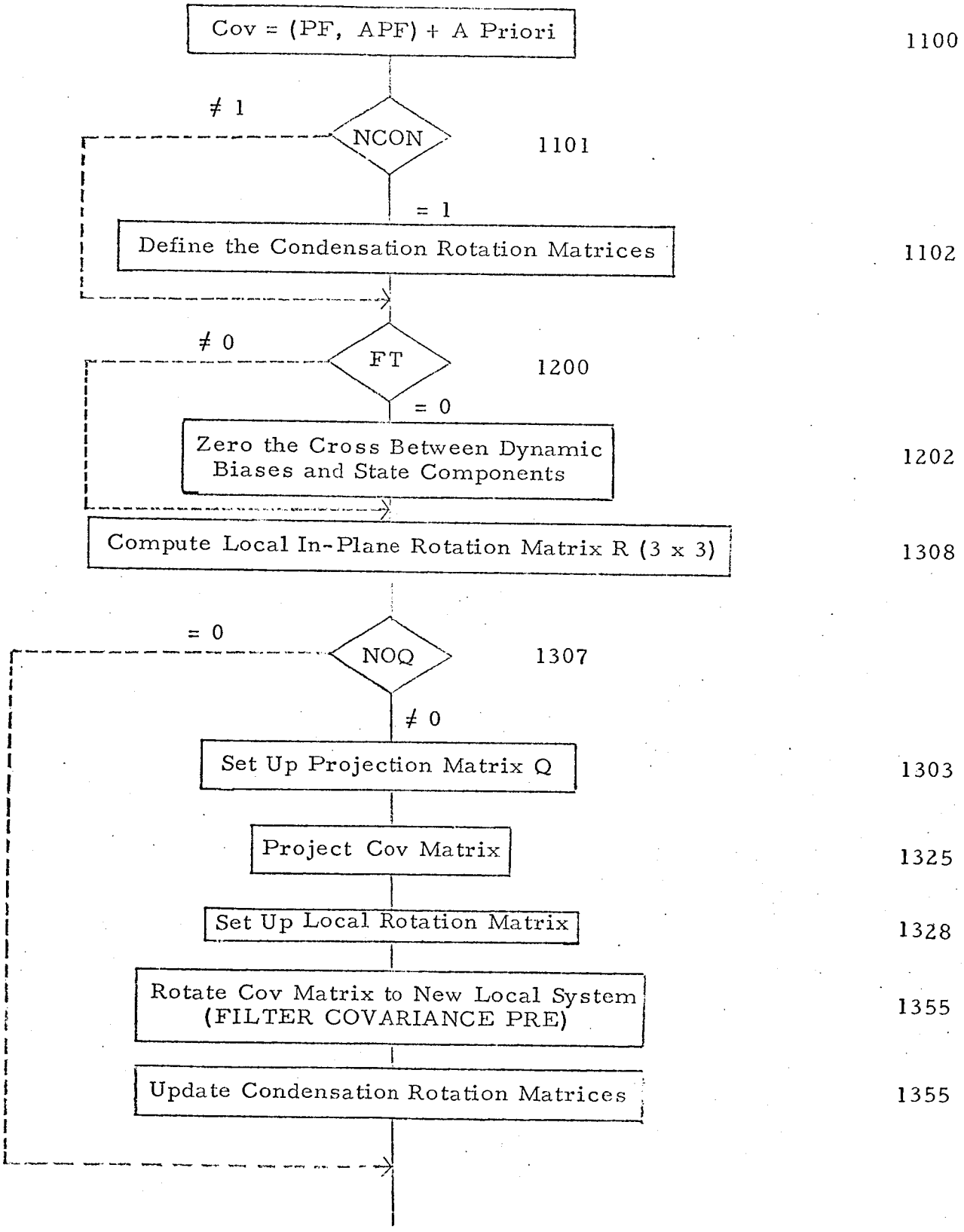
EP	=	-1 for MQ; = 0 for Epoch change; = 1 otherwise
FT	=	0 set $E(\Delta \bar{x} \Delta \bar{D}^T) = E(\Delta \bar{D} \Delta \bar{x}^T) = 0$; $\neq 0$ do nothing
KBNF	=	≤ 1 for filter; = 2 for noise; ≥ 3 for groups
KROTF	=	0 no Euler rotation; $\neq 0$ otherwise
NAB	=	NA if working on vehicle A; = NB otherwise
NBOO	=	0 if no boost; $\neq 0$ otherwise
NCON	=	≤ 0 if Condensation initialization; > 0 otherwise
NCOND	=	0 output Condensation information; = 0 otherwise
NINF	=	0 if Noise information same as Filter; $\neq 0$ otherwise
NOQ	=	0 no projection nor inplane rotation; $\neq 0$ otherwise
NPOST	$\neq$	0 Post output desired; = 0 otherwise
NPRE	$\neq$	0 Pre output desired; = 0 otherwise
NSHIFT	=	0 if no shift; -1 if B into A; + 1 if A into B
NTPOST	$\neq$	0 Post Total output desired; = 0 otherwise
NTPRE	$\neq$	0 Pre Total output desired; = 0 otherwise
VEHIND	=	0 if vehicle A; $\neq 0$ otherwise

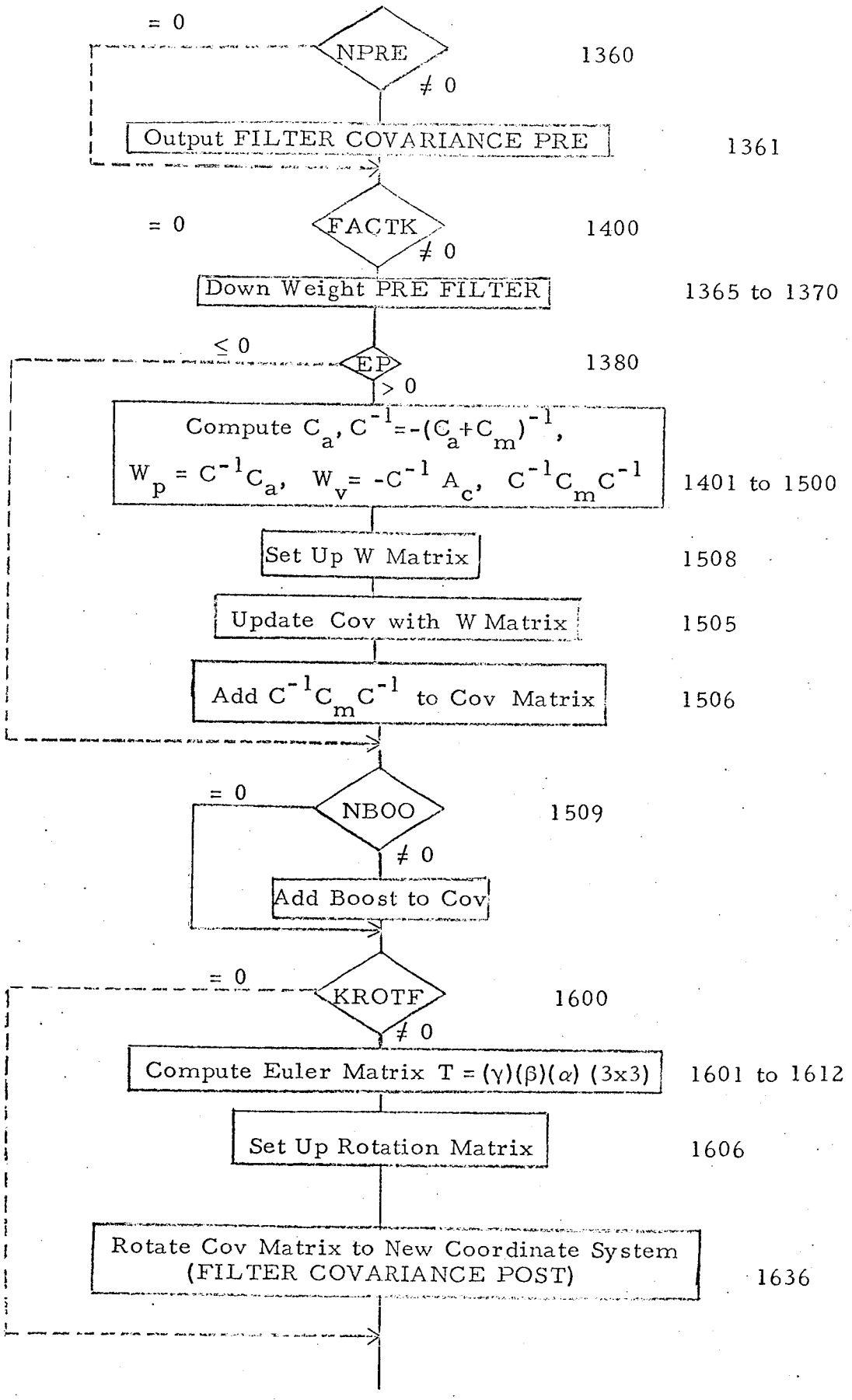
The nomenclature in the flow diagram is consistent with Apollo Note No. 437. It is usually consistent with CALC. A notable exception is that in CALC a matrix X is the "running" covariance matrix while the projection or rotation matrices are loaded in TEMP; for example, in CALC notation the projection of X is $X = (TEMP)\ X\ (TEMP)^T$.

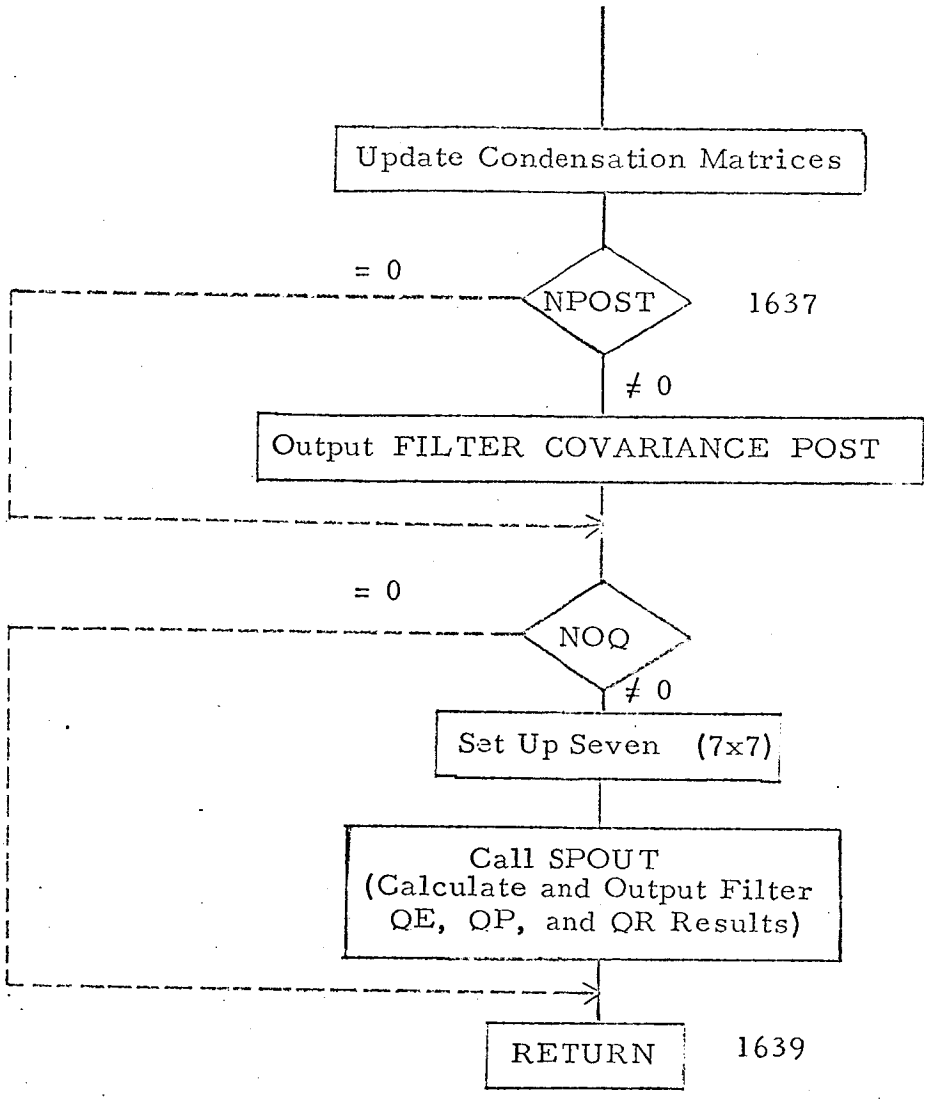
Initialization Section



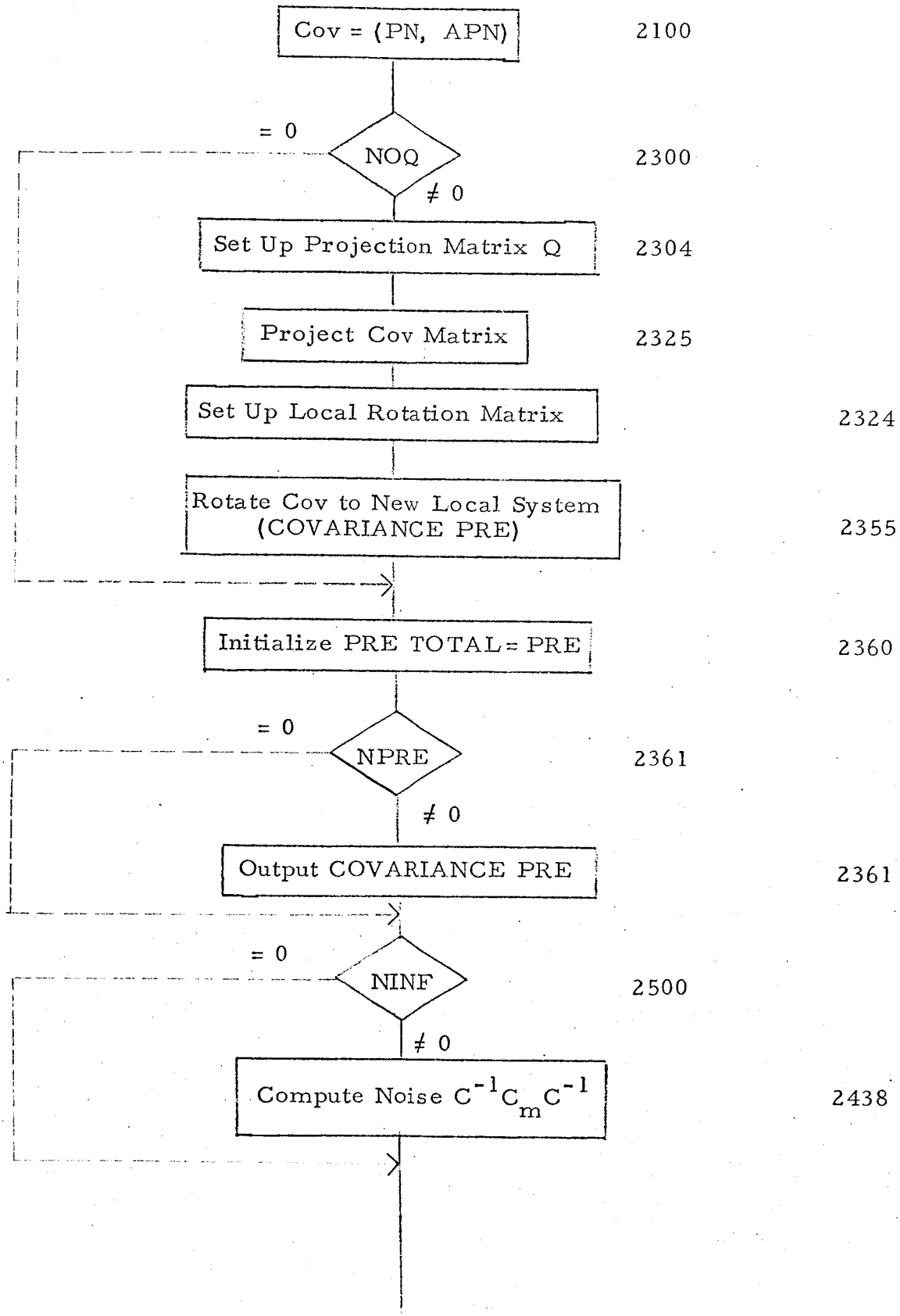
Filter Section

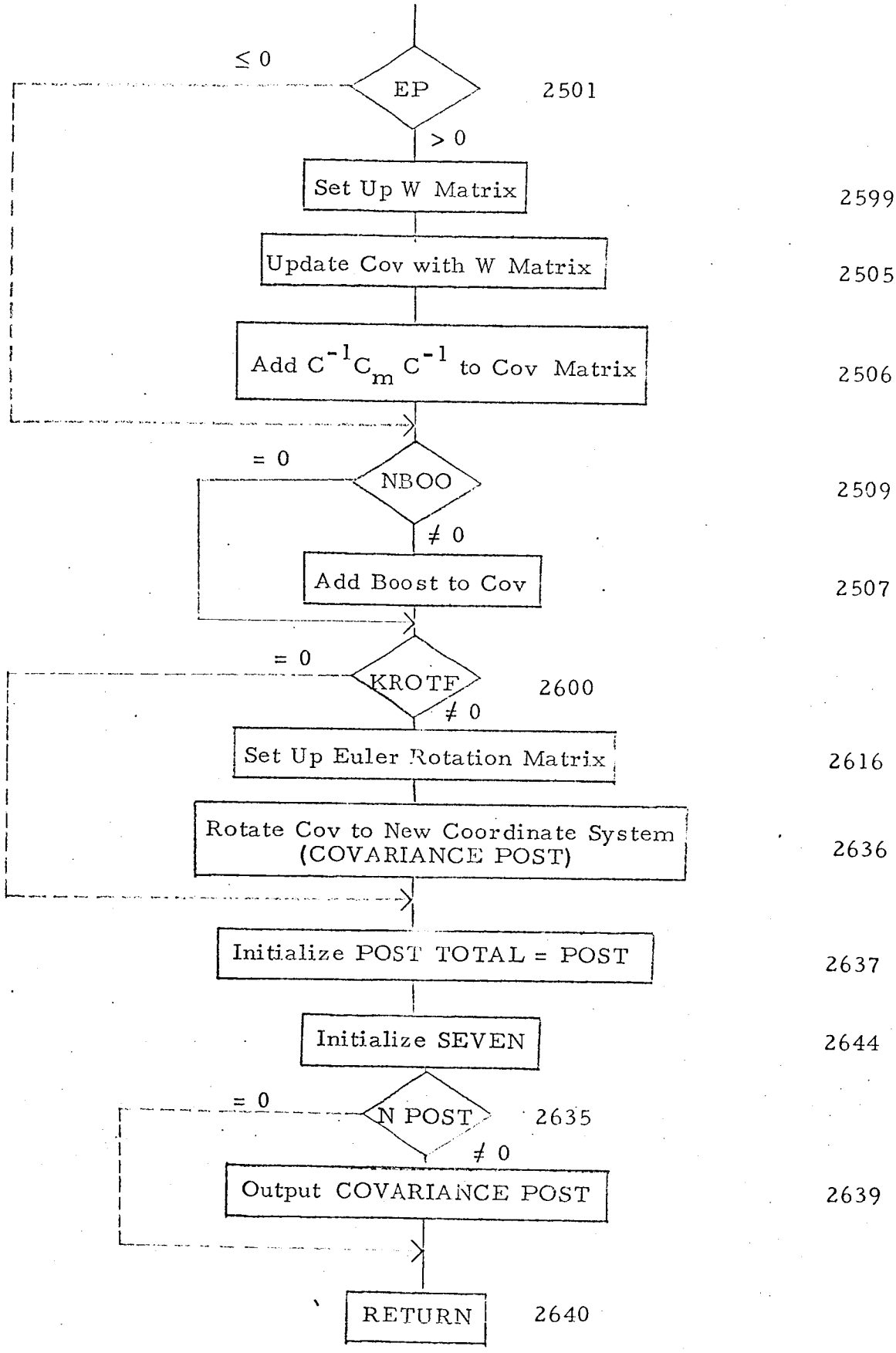



= 0
NPRE
1360
≠ 0
Output FILTER COVARIANCE PRE
1361
= 0
FACTK
1400
≠ 0
Down Weight PRE FILTER
1365 to 1370
≤ 0
EP
1380
> 0
Compute C_a, $C^{-1}=-(C_a+C_m)^{-1}$,
$W_p = C^{-1}C_a$, $W_v = -C^{-1}A_c$, $C^{-1}C_mC^{-1}$
1401 to 1500
Set Up W Matrix
1508
Update Cov with W Matrix
1505
Add $C^{-1}C_mC^{-1}$ to Cov Matrix
1506
= 0
NBOO
1509
≠ 0
Add Boost to Cov
= 0
KROTF
1600
≠ 0
Compute Euler Matrix $T = (\gamma)(\beta)(\alpha)$ (3x3)
1601 to 1612
Set Up Rotation Matrix
1606
Rotate Cov Matrix to New Coordinate System
(FILTER COVARIANCE POST)
1636

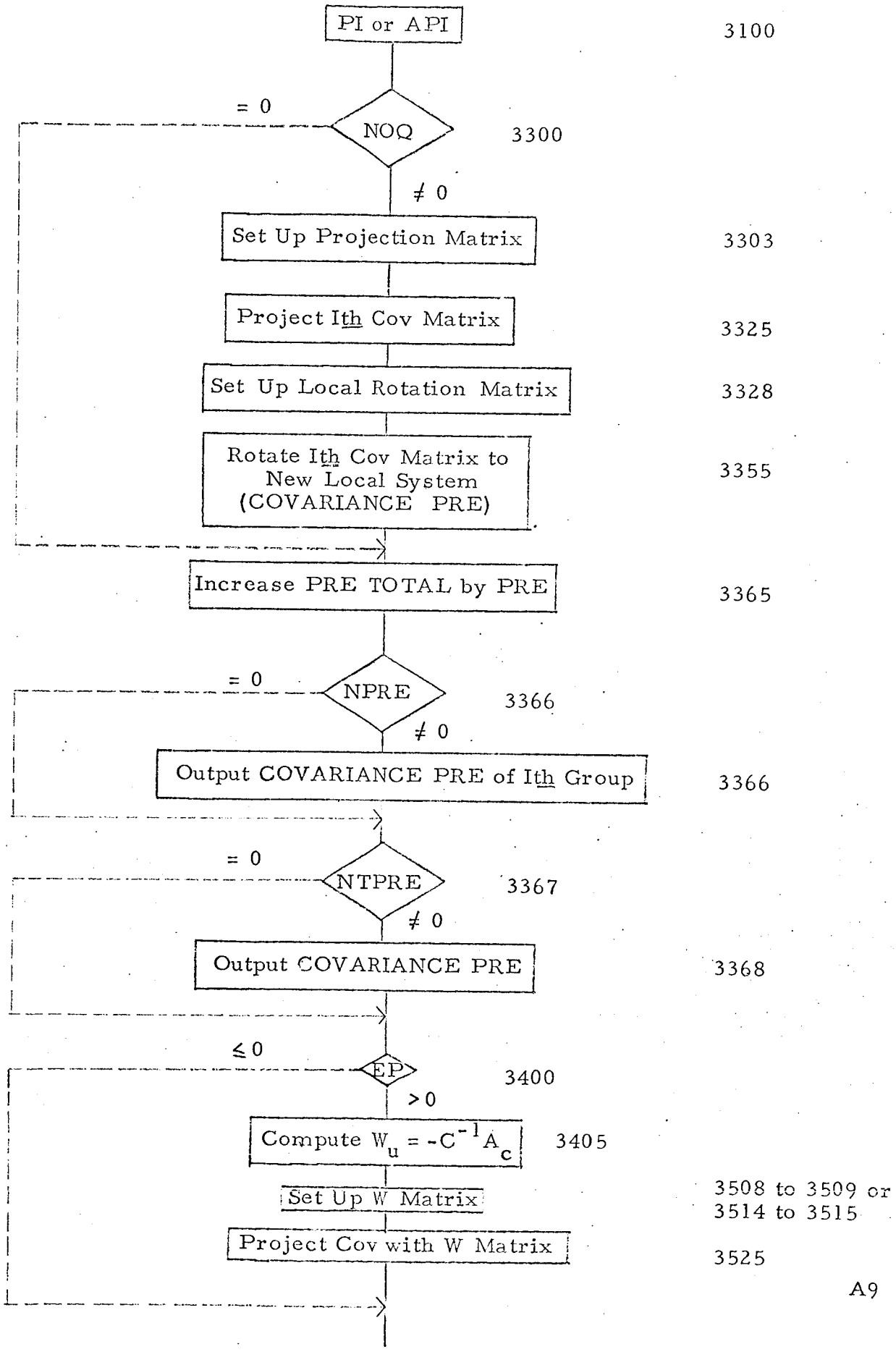

Update Condensation Matrices
1636
= 0
NPOST
1637
≠ 0
Output FILTER COVARIANCE POST
1638
= 0
NOQ
≠ 0
Set Up Seven (7x7)
Call SPOUT
(Calculate and Output Filter
QE, QP, and QR Results)
1660
RETURN
1639

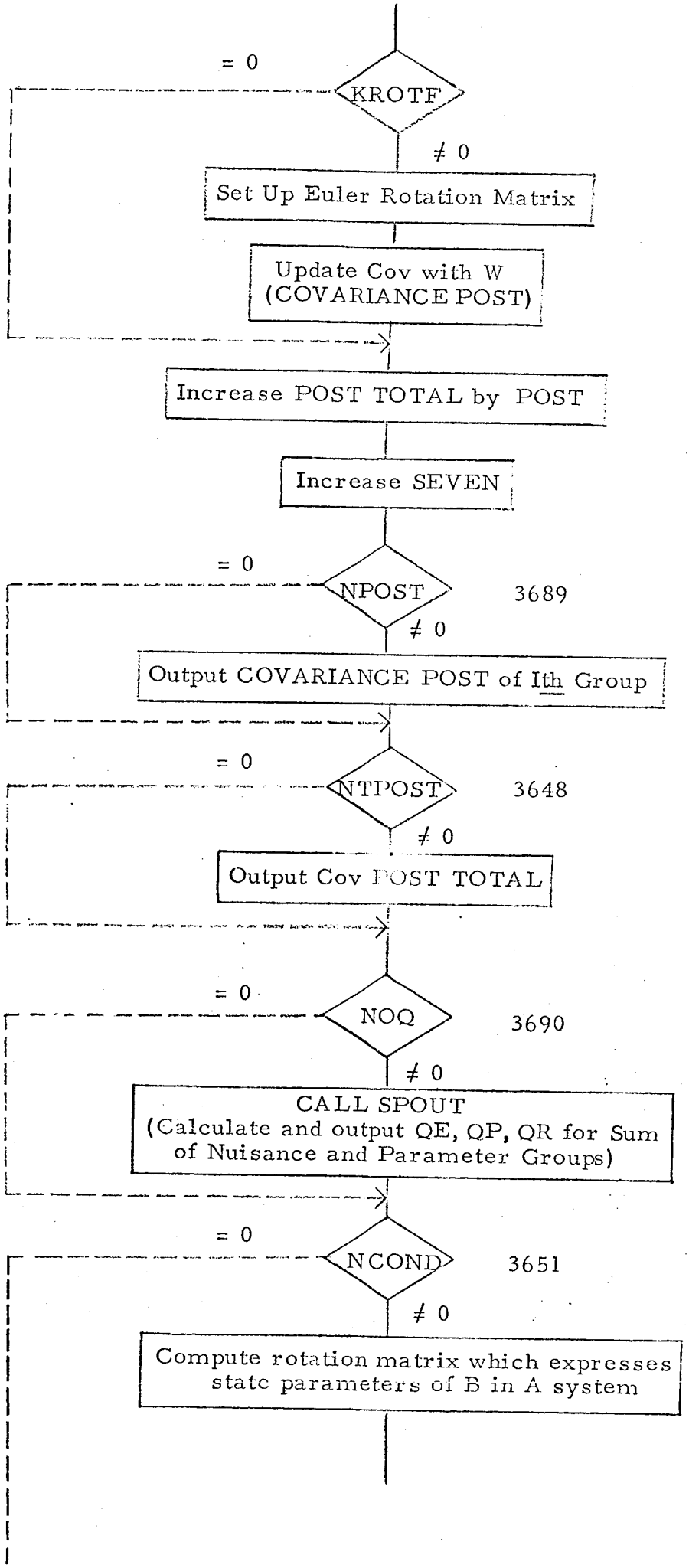
Noise Section

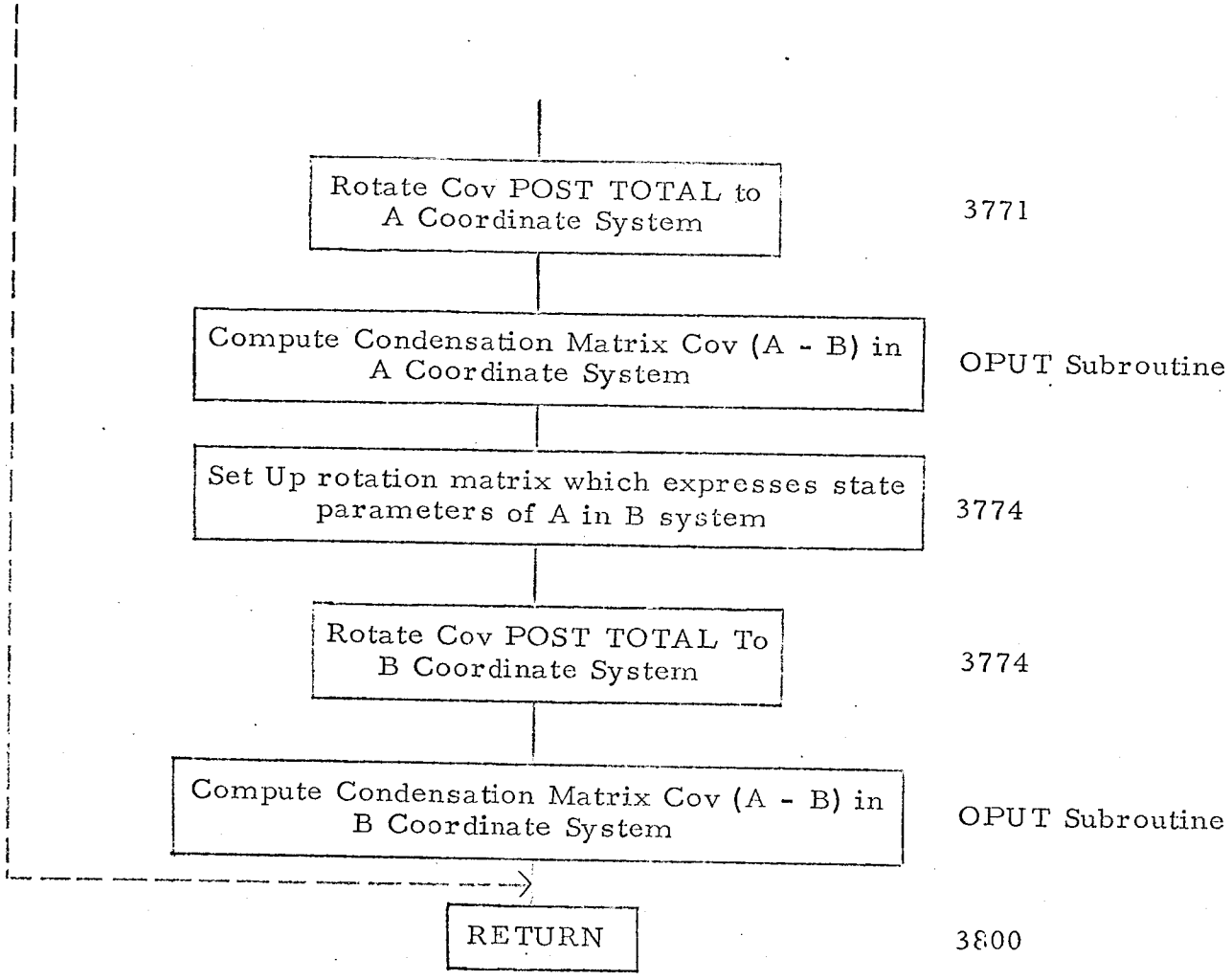



≤ 0
EP
2501
> 0
Set Up W Matrix
2599
Update Cov with W Matrix
2505
Add $C^{-1}C_m C^{-1}$ to Cov Matrix
2506
= 0
NBOO
2509
≠ 0
Add Boost to Cov
2507
= 0
KROTF
2600
≠ 0
Set Up Euler Rotation Matrix
2616
Rotate Cov to New Coordinate System (COVARIANCE POST)
2636
Initialize POST TOTAL = POST
2637
Initialize SEVEN
2644
= 0
N POST
2635
≠ 0
Output COVARIANCE POST
2639
RETURN
2640

Groups Section




= 0
KROTF
3600
≠ 0
Set Up Euler Rotation Matrix
3616
Update Cov with W
(COVARIANCE POST)
3636
Increase POST TOTAL by POST
3646
Increase SEVEN
3681
= 0
NPOST
3689
≠ 0
Output COVARIANCE POST of Ith Group
3647
= 0
NTPOST
3648
≠ 0
Output Cov POST TOTAL
3649
= 0
NOQ
3690
≠ 0
CALL SPOUT
(Calculate and output QE, QP, QR for Sum of Nuisance and Parameter Groups)
3690
= 0
NCOND
3651
≠ 0
Compute rotation matrix which expresses state parameters of B in A system
3770


Rotate Cov POST TOTAL to A Coordinate System
3771
Compute Condensation Matrix Cov (A - B) in A Coordinate System
OPUT Subroutine
Set Up rotation matrix which expresses state parameters of A in B system
3774
Rotate Cov POST TOTAL To B Coordinate System
3774
Compute Condensation Matrix Cov (A - B) in B Coordinate System
OPUT Subroutine
RETURN
3800

APPENDIX B

The purpose of this appendix is to describe the program output from CALC and the output flags for CALC. The external flags which specify the desired output are B, A, BT, AT, and C which appear respectively in columns 34-38 of the "batch" card in Program B. The output flags in CALC are NPRE, NPOST, NTPRE, NTPOST and NCOND.

By "on" we will mean a non blank alpha-numeric character has been placed in the proper column of the "batch" card. By "off" we will mean that a blank is in the proper column of the "batch" card.

If B is "on" the filter covariance matrix (before information and boost errors added), the noise covariance matrix (before information and boost errors added), and the individual parameter groups covariance matrices (before information errors added) are outputted as 22 x 22 matrices with the proper headings in outputs. In addition the corresponding standard deviation of position and velocity for vehicle A and vehicle B, and the square root of the main diagonal elements are outputted with the proper labels.

If A is "on" the situation is as B above, except that now the covariance matrices which we output have the information and/or boost errors added and are in the new Euler system.

If BT is "on" the pre filter covariance matrix and the covariance pre total matrix are outputted as 22 x 22 matrices. The latter is the sum of the noise covariance matrix (PRE) and the individual parameter groups (PRE). The standard deviation of position and velocity for vehicle A and vehicle B, and the square root of the main diagonal elements are outputted as before for each matrix.

If AT is "on" the post filter covariance matrix and the covariance post total matrix are outputted as 22 x 22 matrices. The latter is the sum of the noise covariance matrix (POST) and the individual parameter groups (POST) in the new Euler system. The standard deviation of position and velocity for vehicle A and vehicle B, and the square root of the main diagonal elements are outputted. In addition, if there has been a Q projection, the QE, QP and QR results are outputted for both these matrices. These results are not always meaningful, but are computed and outputted in any case.

The final external flag is N COND. If it is "on" the 6 x 6 matrix, COV (A - B) in the A local system is outputted with its standard deviation of position and velocity and the square root of the elements on the main diagonal. Following this is the 6 x 6 matrix, COV(A - B) in the B local system with its standard deviation of position and velocity and the square root of the elements on the main diagonal.

In CALC "off" means the flag has value zero while "on" means a non zero value. NPRE is "on" in CALC if either B or BT is "on". NPOST is "on" in CALC if either A or AT is "on". NTPRE is "on" if BT is "on" and NTPOST is "on" if AT is "on". Finally NCOND is "on" if C is "on".

APPENDIX C

This appendix lists the results of a small test case, i. e., a run with the new OEAP program and the corresponding run with the old OEAP program. We show the input data needed for these runs and the results of the runs.

New OEAP

INPUTS - DATA SET 1

1	2.1425738E 07	A1
2	-3.0000000E 02	A4
3	2.5631800E 04	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	0	XI
7	3.0000000E 01	ETA
8	2.0000000E 01	ZETA
9	1.0000000E 00	LAMBDA
10	3.8000000E 01	ALPHA
11	7.2921561E-05	OMEGAE
12	0	OMEGAM
13	2.0925738E 07	RHOE
14	0	RHOM
15	1.4076530E 16	MU
16	1.0300000E 02	TIME
17	0	RE RAD
18	0	LT SLP
19	2.0000000E 01	LO SLP
20	0	RD IND
21	1.0000000E 00	R IND
22	0	Q IND
23	-0	QF IND
24	-1.0000000E 00	A1 IND
25	-1.0000000E 00	A2 IND
26	0	PVWIND
27	1.0000000E 02	T INIT
28	1.0000000E-01	T INCR
29	1.4000000E 01	DIMENS
30	0	LAMB2
31	0	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	1.0000000E 02	T CLK
36	1.0000000E 02	T XYZ
37	1.0000000E 02	T ORB
38	1.0000000E 02	T TILD
39	1.0000000E 02	T SHIP

INPUTS - DATA SET 1

1	2.1425738E 07	A1
2	-3.0000000E 02	A4
3	2.5631800E 04	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	0	XI
7	3.0000000E 01	ETA
8	2.0000000E 01	ZETA
9	1.0000000E 00	LAMBDA
10	3.8000000E 01	ALPHA
11	7.2921561E-05	OMEGAE
12	0	OMEGAM
13	2.0925738E 07	RHOE
14	0	RHOM
15	1.4076530E 16	MU
16	1.0511455E 02	TIME
17	0	RE RAD
18	0	LT SLP
19	2.0000000E 01	LO SLP
20	0	RD IND
21	1.0000000E 00	R IND
22	0	Q IND
23	1.0511455E 02	OF IND
24	-1.0000000E 00	A1 IND
25	-1.0000000E 00	A2 IND
26	0	PVWIND
27	1.0000000E 02	T INIT
28	1.0000000E-15	T INCR
29	1.4000000E 01	DIMENS
30	0	LAMB2
31	0	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	1.0000000E 02	T CLK
36	1.0000000E 02	T XYZ
37	1.0000000E 02	T ORB
38	1.0000000E 02	T TILD
39	1.0000000E 02	T SHIP

APF 1

```
A( 1, 1)  =   2.5000000E 06
A( 2, 2)  =   2.5000000E 06
A( 3, 3)  =   2.5000000E 06
A( 4, 4)  =   1.0000000E 02
A( 5, 5)  =   1.0000000E 02
A( 6, 6)  =   1.0000000E 02
```

APN 2

```
A( 1, 1)  =   2.5000000E 06
A( 2, 2)  =   2.5000000E 06
A( 3, 3)  =   2.5000000E 06
A( 4, 4)  =   1.0000000E 02
A( 5, 5)  =   1.0000000E 02
A( 6, 6)  =   1.0000000E 02
```

AP01 3

```
A( 7, 7)  =   2.5000000E 03
A( 8, 8)  =   2.5000000E 03
A( 9, 9)  =   2.5000000E 03
A(10,10)  =   1.9600000E 22
```

```
BATCH                                   ESTIMATE SPACECRAFT A


SPACECRAFT A TIME = 1.0000000E 02 MINUTES     ****      1 HRS    40 MIN     0 SEC

SPACECRAFT B TIME =              -0 MINUTES     ****      0 HRS     0 MIN     0 SEC


                                        FILTER TYPE IS   0
NO EST PARAM IN STATE VECTOR SPACECRAFT A IS   6
NO EST PARAM IN STATE VECTOR SPACECRAFT B IS   0
                     NO NUISANCE PARAM GROUPS IS   1


                    ALPHA = BETA = GAMMA = 0


DS    1    APF    T=        -0   SG=          -0   *
DS    1    R      T=    103.00   SG=3.00000E 01   *
DS    1    A1     T=    103.00   SG=5.00000E-04   *
DS    1    A2     T=    103.00   SG=5.00000E-04   *
```

No SPOUT Results as No Q For this Batch

```
DS    2    APN    T=          -0   SG=          -0   *
```

```
DS    3    AP01   T=          -0   SG=          -0
DS    1    R      T=      103.00   SG=3.00000E 01
DS    1    A1     T=      103.00   SG=5.00000E-04
DS    1    A2     T=      103.00   SG=5.00000E-04
```

```
23 24 25 26  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0 23 24 25  0 26  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0 23 24 25  0 26  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0 23 24 25  0 26  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
```

```
EPOCH CHANGE                          ESTIMATE SPACECRAFT A


SPACECRAFT A TIME = 1.0511455E 02 MINUTES    ****      1 HRS   45 MIN    6 SEC

SPACECRAFT B TIME =            -0 MINUTES    ****      0 HRS    0 MIN    0 SEC


                                   FILTER TYPE IS  0
        NO EST PARAM IN STATE VECTOR SPACECRAFT A IS  3
        NO EST PARAM IN STATE VECTOR SPACECRAFT B IS  0
                       NO NUISANCE PARAM GROUPS IS  1


                    ALPHA = BETA = GAMMA = 0



DS  -0   PF    T=       -0  SG=        -0  *
DS   1   Q     T=   105.12  SG=        -0  *
```

```
***** FOR FIXED TIME *****

STD. DEV. IN RADIUS =  1.338960E 03
STD. DEV. IN SPEED =  1.346676E 00
STD. DEV. IN FLIGHT PATH ANGLE =  1.155627E-02 (DEGREES)

STD. DEV. IN PERIAPSIS =  4.886172E 03

***** FOR FIXED RADIUS *****

STD. DEV. IN SPEED =  8.095689E-01
STD. DEV. IN FLIGHT PATH ANGLE =  1.293930E-02 (DEGREES)

REL. POSITION ERRORS MEANINGFUL ONLY FOR EARTH ORBITS

STD. DEV. OF IN PLANE POSITION =  1.163659E 05
STD. DEV. OF OUT OF PLANE POSITION =  2.548693E 03
```

Filter Results
(BATCH # 2)

```
DS  -0   PN   T=      -0  SG=      -0  *
DS   1   Q    T=  105.12  SG=      -0  *
```

```
DS   -0    P01    T=        -0   SG=
DS    1    Q      T=    105.12   SG=

                  -0
                  -0        0  0  0  0  0 26  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
```

```
                    *****  FOR  FIXED  TIME  *****

                STD. DEV. IN RADIUS =  1.340324E 03
                 STD. DEV. IN SPEED =  1.346913E 00
        STD. DEV. IN FLIGHT PATH ANGLE =  1.155871E-02 (DEGREES)

              STD. DEV. IN PERIAPSIS =  4.891568E 03
```

Total Results
(BATCH# 2)

```
                    *****  FOR  FIXED  RADIUS  *****

                 STD. DEV. IN SPEED =  8.116088E-01
        STD. DEV. IN FLIGHT PATH ANGLE =  1.294248E-02 (DEGREES)

              REL. POSITION ERRORS MEANINGFUL ONLY FOR EARTH ORBITS

        STD. DEV. OF IN PLANE POSITION =  1.164848E 05
      STD. DEV. OF OUT OF PLANE POSITION =  2.552103E 03
```

Old OEAP

INPUTS - DATA SET 1

1	2.1425738E 07	A1
2	-3.0000000E 02	A4
3	2.5631800E 04	A5
4	1.0000000E 00	GAM ID
5	-0	BETA
6	-0	XI
7	3.0000000E 01	ETA
8	2.0000000E 01	ZETA
9	1.0000000E 00	LAMBDA
10	3.8000000E 01	ALPHA
11	7.2921561E-05	OMEGAE
12	-0	OMEGAM
13	2.0925738E 07	RHOE
14	-0	RHOM
15	1.4076530E 16	MU
16	1.0300000E 02	TIME
16	1.0511455E 02	TIME
17	-0	RE RAD
18	-0	LT SLP
19	2.0000000E 01	LO SLP
20	-0	RD IND
21	1.0000000E 00	R IND
22	-0	Q IND
23	1.0511455E 02	QF IND
24	-1.0000000E 00	A1 IND
25	-1.0000000E 00	A2 IND
26	-0	PVWIND
27	1.0000000E 02	T INIT
28	1.0000000E-01	T INCR
29	1.4000000E 01	DIMENS
30	-0	LAMB2
31	-0	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	1.0000000E 02	T CLK
36	1.0000000E 02	T XYZ
37	1.0000000E 02	T ORB
38	1.0000000E 02	T TILD
39	1.0000000E 02	T SHIP

INPUTS - DATA SET 2

1	2.1425738E 07	A1
2	-3.0000000E 02	A4
3	2.5631800E 04	A5
4	1.0000000E 00	GAM ID
5	-0	BETA
6	-0	XI
7	3.0000000E 01	ETA
8	2.0000000E 01	ZETA
9	1.0000000E 00	LAMBDA
10	3.8000000E 01	ALPHA
11	7.2921561E-05	OMEGAE
12	-0	OMEGAM
13	2.0925738E 07	RHOE
14	-0	RHOM
15	1.4076530E 16	MU
16	1.0300000E 02	TIME
16	1.0511455E 02	TIME
17	2.1335738E 07	RE RAD
18	-0	LT SLP
19	2.0000000E 01	LO SLP
20	-0	RD IND
21	1.0000000E 00	R IND
22	-1.0000000E 00	Q IND
23	1.0511455E 02	QF IND
24	-1.0000000E 00	A1 IND
25	-1.0000000E 00	A2 IND
26	-0	PVWIND
27	1.0000000E 02	T INIT
28	1.0000000E-15	T INCR
29	1.4000000E 01	DIMENS
30	-0	LAMB2
31	-0	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	1.0000000E 02	T CLK
36	1.0000000E 02	T XYZ
37	1.0000000E 02	T ORB
38	1.0000000E 02	T TILD
39	1.0000000E 02	T SHIP

A PRIORI 3

```
A( 1, 1)  =   2.5000000E 06
A( 2, 2)  =   2.5000000E 06
A( 3, 3)  =   2.5000000E 06
A( 4, 4)  =   1.0000000E 02
A( 5, 5)  =   1.0000000E 02
A( 6, 6)  =   1.0000000E 02
A( 7, 7)  =   2.5000000E 03
A( 8, 8)  =   2.5000000E 03
A( 9, 9)  =   2.5000000E 03
A(10,10)  =   1.9600000E 22
```

DATA SET 4 CASE 0 6X6

```
DS   1    R    T=    103.00  SG=3.00000E 01
DS   1    A1   T=    103.00  SG=5.00000E-04
DS   1    A2   T=    103.00  SG=5.00000E-04
DS   1    Q    T=    105.12  SG=1.00000E 00
DS   1    QE   T=    105.12  SG=1.00000E 00
DS   2    QR   R= 2.13357E 07
DS   3    A    T=        -0  SG=1.00000E 00
```

```
07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
BI US ES NS NH MU N* VA UM EM NM ED ND RM AM RS RS AS L1 L2 L3 L4 L5 L6
```

ALPHA = BETA = GAMMA = 0

```
0  7  8  9  0 10  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  7  8  9  0 10  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  7  8  9  0 10  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0 10  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
```

PARAMETERS IN COLUMNS 1- 6 UPDATED -- OTHER PARAMETERS (IF ANY) NOT UPDATED

```
1.340324E 03    ***    STD DEV IN RADIUS AT TIME =    105.1146
1.346913E 00    ***    STD DEV IN SPEED
2.017376E-04    ***    STD DEV IN FLIGHT PATH ANGLE


8.116088E-01    ***    STD DEV IN SPEED AT RAD = 2.133570E 07
2.258889E-04    ***    STD DEV IN FLIGHT PATH ANGLE
1.164847E 05    ***    STD DEV IN UNCERTAINTY IN PLANE
2.552101E 03    ***    STD DEV IN UNCERTAINTY OUT OF PLANE
1.165126E 05    ***    STD DEV IN TOTAL UNCERTAINTY
```

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 478
(BBC Task 204)

L. Lustick
March 1967

EFFECT OF RELATIVE STATE ERROR ON RENDEZVOUS MISS

Purpose

The purpose of this note is to indicate the nature of errors in the position miss at rendezvous time, t_1, even though the relative state at time t_0 is known without error.

Let,

$$\begin{pmatrix} \hat{a}_1 - r_1 \\ \vdots \\ \hat{a}_6 - r_6 \\ \hat{\mu} - r_\mu \end{pmatrix}_{t_0} = \text{the estimate of state of vehicle A with respect to reference orbit } r \text{ for vehicle A at time } t_0$$

$$\begin{pmatrix} \hat{b}_1 - s_1 \\ \vdots \\ \hat{b}_6 - s_6 \\ \hat{\mu} - r_\mu \end{pmatrix}_{t_0} = \text{the estimate of state vehicle B with respect to reference orbit } s \text{ for vehicle B at time } t_0.$$

Then, adding and subtracting the true state a_t of vehicle A,

$$\begin{pmatrix} \hat{a}_1 - r_1 \\ \vdots \\ \hat{a}_6 - r_6 \\ \hat{\mu} - r_\mu \end{pmatrix}_{t_0} \equiv \begin{pmatrix} \hat{a}_1 - a_{t_1} \\ \vdots \\ \hat{a}_6 - a_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0} + \begin{pmatrix} a_{t_1} - r_1 \\ \vdots \\ a_{t_6} - r_6 \\ \mu_t - r_\mu \end{pmatrix}_{t_0}$$

and adding and subtracting the true state b_t of vehicle B,

$$\begin{pmatrix} \hat{b}_1 - s_1 \\ \vdots \\ \hat{b}_6 - s_6 \\ \hat{\mu} - r_\mu \end{pmatrix}_{t_0} \equiv \begin{pmatrix} \hat{b}_1 - b_{t_1} \\ \vdots \\ \hat{b}_6 - b_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0} + \begin{pmatrix} b_{t_1} - s_1 \\ \vdots \\ b_{t_6} - s_6 \\ \mu_t - r_\mu \end{pmatrix}_{t_0}$$

The boost, Δv to be applied to vehicle B to put it at the same place as vehicle A at rendezvous time t_1 can be calculated from the following equation

$$({}_1\alpha, {}_1\beta, {}_1\mu) \left\{ \begin{bmatrix} a_{t_1} - r_1 \\ \vdots \\ a_{t_6} - r_6 \\ \mu_t - r_\mu \end{bmatrix}_{t_0} + \begin{bmatrix} \hat{a}_1 - a_{t_1} \\ \vdots \\ \hat{a}_6 - a_{t_6} \\ \hat{\mu} - \mu_t \end{bmatrix}_{t_0} \right\} + \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}_{t_1}$$

$$- ({}_2\alpha, {}_2\beta, {}_2\mu) \left\{ \begin{bmatrix} b_{t_1} - s_1 \\ \vdots \\ b_{t_6} - s_6 \\ \mu_t - r_\mu \end{bmatrix}_{t_0} + \begin{bmatrix} \hat{b}_1 - b_{t_1} \\ \vdots \\ \hat{b}_6 - b_{t_6} \\ \hat{\mu} - \mu_t \end{bmatrix} \right\} - \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}_{t_1}$$

$$= {}_2\beta \Delta v_p$$

where (α, β, μ) is a portion of the Q matrix,

$$(\alpha, \beta, \mu) = \begin{bmatrix} \dfrac{\partial x(t_1)}{\partial x(t_0)} & \cdots & \dfrac{\partial x(t_1)}{\partial \dot{z}(t_0)} & \dfrac{\partial x(t_1)}{\partial \mu} \\ \dfrac{\partial y(t_1)}{\partial x(t_0)} & \cdots & \dfrac{\partial y(t_1)}{\partial \dot{z}(t_0)} & \dfrac{\partial y(t_1)}{\partial \mu} \\ \dfrac{\partial z(t_1)}{\partial z(t_0)} & \cdots & \dfrac{\partial z(t_1)}{\partial \dot{z}(t_0)} & \dfrac{\partial z(t_1)}{\partial \mu} \end{bmatrix}$$

and subscript 1 preceding the symbol means for vehicle A evaluated on vehicle A reference trajectory and subscript 2 is for vehicle B evaluated on vehicle B reference trajectory.

$$\beta = \begin{bmatrix} \dfrac{\partial x(t_1)}{\partial \dot{x}(t_0)} & \dfrac{\partial x(t_1)}{\partial \dot{y}(t_0)} & \dfrac{\partial x(t_1)}{\partial \dot{z}(t_0)} \\ \dfrac{\partial y(t_1)}{\partial \dot{x}(t_0)} & \dfrac{\partial y(t_1)}{\partial \dot{y}(t_0)} & \dfrac{\partial y(t_1)}{\partial \dot{z}(t_0)} \\ \dfrac{\partial z(t_1)}{\partial \dot{x}(t_0)} & \dfrac{\partial z(t_1)}{\partial \dot{y}(t_0)} & \dfrac{\partial z(t_1)}{\partial \dot{z}(t_0)} \end{bmatrix}$$

Solving for the commanded boost,

$$\Delta v_p = \beta_2^{-1} \left\{ ({}_1\alpha, {}_1\beta, {}_1\mu) \left[\begin{pmatrix} a_{t_1} - r_1 \\ \vdots \\ a_{t_6} - r_6 \\ \mu_t - r_\mu \end{pmatrix}_{t_0} + \begin{pmatrix} \hat{a}_1 - a_{t_1} \\ \vdots \\ \hat{a}_6 - a_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0} \right] + \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}_{t_1} \right.$$

$$\left. - ({}_2\alpha, {}_2\beta, {}_2\mu) \left[\begin{pmatrix} b_{t_1} - s_1 \\ \vdots \\ b_{t_6} - s_6 \\ \mu_t - r_\mu \end{pmatrix}_{t_0} + \begin{pmatrix} \hat{b}_1 - b_{t_1} \\ \vdots \\ \hat{b}_6 - b_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0} \right] - \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}_{t_1} \right\}$$

The true positions of vehicles A and B at t_1 are

$$A_{t_1} = \begin{pmatrix} a_{t_1} \\ a_{t_2} \\ a_{t_3} \end{pmatrix}_{t_1} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}_{t_1} + ({}_1\alpha, {}_1\beta, {}_1\mu) \begin{bmatrix} a_{t_1} - r_1 \\ a_{t_6} - r_6 \\ \mu_t - r_\mu \end{bmatrix}_{t_0}$$

$$B_{t_1} = \begin{pmatrix} b_{t_1} \\ b_{t_2} \\ b_{t_3} \end{pmatrix}_{t_1} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}_{t_1} + ({}_2\alpha, {}_2\beta, {}_2\mu) \begin{bmatrix} b_{t_1} - s_1 \\ b_{t_2} - s_2 \\ \mu_t - r_\mu \end{bmatrix}_{t_0}$$

$$+ \beta_2 \Delta v_p$$

$$A_{t_1} - B_{t_1} = -\left({}_1\alpha, {}_1\beta, {}_1\mu\right) \begin{pmatrix} \hat{a}_1 - a_{t_1} \\ \hat{a}_6 - a_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0} + \left({}_2\alpha, {}_2\beta, {}_2\mu\right) \begin{pmatrix} \hat{b}_1 - b_{t_1} \\ \hat{b}_6 - b_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0}$$

Since we are assuming no error in relative state at t_0, then

$$\begin{pmatrix} \hat{a}_1 - a_{t_1} \\ \hat{a}_6 - a_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0} = \begin{pmatrix} \hat{b}_1 - b_{t_1} \\ \hat{b}_6 - b_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0}$$

then

$$\boxed{\left(A_{t_1} - B_{t_1}\right) = \left[\left({}_2\alpha, {}_2\beta, {}_2\mu\right) - \left({}_1\alpha, {}_1\beta, {}_1\mu\right)\right] \begin{pmatrix} \hat{a}_1 - a_{t_1} \\ \hat{a}_6 - a_{t_6} \\ \hat{\mu} - \mu_t \end{pmatrix}_{t_0}}$$

From this equation we see that if the partials for the different reference orbits were the same, perfect knowledge of the relative state at time t_0 would result in a rendezvous with no miss in distance. Differences in the partials on the two reference orbits will result in miss errors even though the relative state at t_0 is known perfectly.

Another way of quickly seeing the meaning of the results is to recognize that the fact that there is no error in relative state at time t_0 does not imply that there will be no error in relative state at time t_1, since the orbits are different and the effect of the same error will be different.

In order to get an idea of how much error may occur in the relative rendezvous state let us examine the following problem. Consider two vehicles around the Moon, one in a 10 mile orbit was placed in a 180° transfer orbit to rendezvous with the second vehicle in the 100 mile orbit. The portions of the Q matrices giving the partials of position components for the two vehicles are shown in the following table.

It is interesting to note that there is a considerable out-of-plane miss at time t_1 due to out-of-plane velocity errors at time t_0.

$$\frac{\partial(x, y, z)}{\partial(x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)}$$

180° Transfer Orbit

∂ \ ∂	Δx_0	Δy_0	Δz_0	$\Delta \dot{x}_0$	$\Delta \dot{y}_0$	$\Delta \dot{z}_0$	$\Delta \mu$
Δx	-3.3644	0.0	0.0	-9.1008 -03	-4.1709 +03	0.0	7.008 -08
Δy	9.8537	3.0891	0.0	3.8296 +03	9.4168 +03	0.0	-2.0665 -07
Δz	0.0	0.0	-1.0891	0.0	0.0	-3.1187 -03	0.0

100 Mile Orbit of Target Vehicle

∂ \ ∂	Δx_0	Δy_0	Δz_0	$\Delta \dot{x}_0$	$\Delta \dot{y}_0$	$\Delta \dot{z}_0$	$\Delta \mu$
Δx	-2.8918	-.7516	0.0	6.0147 +02	-4.1781 +03	0.0	6.648 -08
Δy	7.7575	4.5167	0.0	2.5073 +03	9.2533 +03	0.0	-1.853 -07
Δz	0.0	0.0	-0.9818	0.0	0.0	2.0171 + 02	0.0

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 479
(BBC Task 203)

L. Lustick
G. Hempstead

MODIFICATIONS TO RTODP ERROR ANALYSIS (DEC 10 PROGRAM)

A number of new capabilities have been added to the December 10 Program. The purpose of this Note is to give a brief description of these changes and to show how they are implemented on the data sheets.

The analyses associated with the changes are described in Apollo Note No. 470.

CHANGES

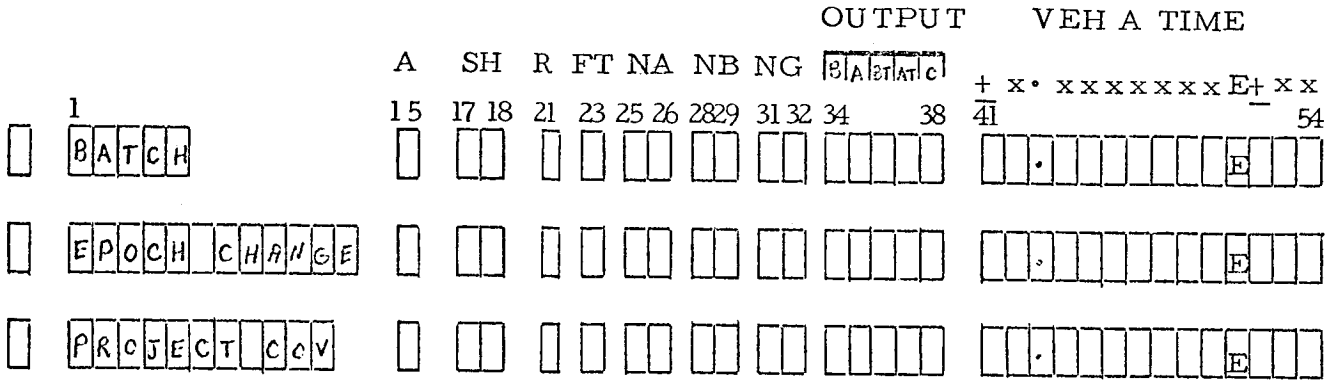
Shift

It is sometimes the case that a vehicle is tracked and subsequently the vehicle separates into two separate vehicles which are separately tracked, or two separate vehicles join together and subsequently the joined vehicle is tracked. The shift operation was added to account for this situation.

The shift can be done on a batch, epoch change, or first project in a series. It does not make sense to do the shift operation on other than the first project in a series of projects.

The indication that a shift is desired appears on the control card in columns 17 and 18 as indicated in the figure below.

17	18	
A	B	implies that the A vehicle results are shifted into the B vehicle making relative state error zero.
B	A	implies that the B vehicle results are shifted into the A vehicle making the relative state error zero.



THE SHIFT OPERATION OCCURS FIRST AND THEN THE CONTROL OPERATION (BATCH, EPOCH, PROJECT) TAKES PLACE USING THESE NEW RESULTS JUST AS IN THE DECEMBER 10 PROGRAM.

When doing a shift operation, there is no necessity to reinitialize the Euler angles that are used for "condensation" (relative state error) purposes as the shift automatically takes care of this.

Consolidation

In the Dec. 10 Program, all non-estimated parameter groups introduced had to always be carried along even if the future measurables did not depend on any of the nuisance parameters in that group. In the new version of the program if no future or current measurables depend on any parameter of a particular non-estimated parameter group, this group may be dropped, if desired, by adding it in the PN group as shown in the following figure.

PROGRAM B - ALLOCATION SHEET

Indicators for Control of Matrix Format

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6x6 (32)	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
					BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
					BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
	PN																											
	PC1																											
	PO4																											

Keypunch a card for each
row checked (✓).

The non-estimated parameter groups must be in the order previously used and may be allocated in any manner that is meaningful. The number of groups is the number remaining. At the present time time I can think of no case where it would be meaningful to allocate other than the state vector of the two vehicles (columns 1-6 and 12-17) in the non-estimated parameter groups.

Summary

The New Version of the December 10 Program has the capability of of outputting a summary of previous root mean square position and velocity uncertainty results. This option is obtained by placing a summary card at the end of a sequence of Part B runs for which a summary table is desired.

Placement of Summary Card

PROGRAM B SEQUENCE	
BLANK CARD	
1 2 3 4 5 6 7 S U M M A R Y	} Summary Card
NEXT CONTROL CARD	

The output associated with a summary is shown in the next figure.

SUMMARY

	SPCFT UPD	TIME	FILTER - PRE POS	FILTER - PRE VEL	FILTER - POST POS	FILTER - POST VEL	TOTAL COV -PRE POS	TOTAL COV -PRE VEL	TOTAL COV - POST POS	TOTAL COV - POST VEL
BATCH	A	0.100E 01	0.866E 04	0.173E 02	0.244E 03	0.396E 00	0.866E 04	0.173E 02	0.178E 04	0.102E 01
		0.200E 02	0.866E 04	0.173E 02	0.866E 04	0.173E 02	0.866E 01	0.173E-01	0.866E 01	0.173E-01
BATCH	B	0.300E 02	0.244E 03	0.396E 00	0.244E 03	0.396E 00	0.178E 04	0.102E 01	0.178E 04	0.102E 01
		0.400E 03	0.244E 03	0.396E 00	0.244E 03	0.396E 00	0.178E 04	0.102E 01	0.178E 04	0.102E 01
EPOCH	A	0.500E 04	0.201E 03	0.400E 00	0.201E 03	0.400E 00	0.121E 04	0.160E 01	0.121E 04	0.160E 01
		0.600E 03	0.244E 03	0.396E 00	0.244E 03	0.396E 00	0.178E 04	0.102E 01	0.178E 04	0.102E 01
EPOCH	B	0.500E 04	0.201E 03	0.400E 00	0.201E 03	0.400E 00	0.121E 04	0.160E 01	0.121E 04	0.160E 01
		0.800E 04	0.201E 03	0.400E 00	0.201E 03	0.400E 00	0.121E 04	0.160E 01	0.121E 04	0.160E 01

The results presented in the summary can be obtained even if none of the output options on the control card in Part B are requested.

QE, QR, QP Results

The DEC 10 Program did not give as output the results consistent with the QE, QR, or QP transformation. This option has been added to this version of the RTODP error analysis program.

This output is automatic and calculated any time there is a Q matrix in the run. The uncertainty components for fixed R (QR results) are only meaningful for Earth orbits. The fixed R results are for the R corresponding to the time of the Q matrix.

Part A has been modified so that when one obtains a QR matrix the time associated with that radius is also printed out. Hence, by running a Part A to get QR you can find the time corresponding to the fixed R. The fixed R results are obtained by obtaining results at this time with the RTODP error analysis.

Starting off New Groups

In the DEC 10 Program it was possible to start off new nuisance parameter groups whenever desired, but if the other nuisance parameter groups had a Q matrix in them the new group was required to have a Q matrix also. Physically, there is usually no need for this Q matrix as all it does is duplicate the nuisance parameter group. The program, however, required a Q matrix and if absent it used the preceeding Q matrix. This was all right as long as the proceeding Q matrix was a 6 x 6 but could lead to erroneous results if it was not.

The new version has been changed so that when adding nuisance parameter groups that have nothing in the state area and do not involve nuisance parameters such as μ or venting, it is not required to have a Q matrix when starting off this group.

K Factor

The new version allows for arbitrarily downweighting past data by a scale factor. The old filter errors are projected to a new anchor point and then the errors in the estimated quantities are multiplied by a K factor. This modified filter covariance then is assumed to be the apriori on the estimated quantities.

The K factor is placed on the PF card in the area for sigma. Nothing in this area implies K equal to one.

Real White Noise

In the DEC 10 Program whatever standard deviations were assumed for the measurables in the filter were also assumed to be the real standard deviation in the measurables. The new program allows for standard deviations in the measurables for the filter different from the real standard deviations. One may use any standard deviation desired for the measurables in the filter, but when filling out the PN group all the same information matrices as used in the filter are included in the PN group and allocated to include all estimated quantities. The area on the card or data sheet which normally contains the standard deviation for the measurable now has $(\sigma^2_{Filter}/\sigma_{Real})$ in it for each measurable. If no information matrices appear in the PN group it is assumed that filter noise and real noise are the same.

Clock Partials

The two way range clock partials have been modified from what they were in the DEC 10 version of Part A. There were terms associated with time tagging which were not treated properly in previous Part A programs.

APOLLO NOTE NO. 480
(BBC Task 203)

L. Lustick
March 1967

MODIFICATIONS TO 'THE PROBLEM OF A SMALL PERTURBATION FROM A CIRCULAR SATELLITE'

Purpose

The purpose of this Note is to present modifications to Apollo Note No. 7 (The Problem of A Small Perturbation From a Circular Satellite, by C. H. Dale, dated 7 February 1963) which allow the calculation of state partial derivatives for circular orbits consistent with the OEAP.

Introduction

Apollo Note No. 7 develops the partials of the state for a circular orbit. That note, although very useful, has caused some confusion when comparing its result with the Orbit Error Analysis Program. This confusion arises since the in-plane partials developed in Apollo Note No. 7 do not hold the same thing constant when perturbing a parameter as does the OEAP. For example, in Note No. 7 a change in the radial position also implies a change in tangential velocity. The correction to Note No. 7 to make the partials consistent with the OEAP are shown in this note. It should also be mentioned that x and y are interchanged in Note No. 7 from their meaning in the OEAP.

Partials Consistent with OEAP (except for x and y interchange)

X_I = Perturbation at time (t) in the tangential direction at time zero.

Y_I = Perturbation at time (t) in the radial direction at time zero.

x = Perturbation at time (t) in the reference tangential direction at time (t).

y = Perturbation at time (t) in the reference radial direction at time (t).

$$\frac{\partial X_I}{\partial \dot{x}_0} = \frac{\partial x}{\partial \dot{x}_0} \cos \omega t + \frac{\partial y}{\partial \dot{x}_0} \sin \omega t$$

$$\frac{\partial Y_I}{\partial \dot{x}_0} = - \frac{\partial x}{\partial \dot{x}_0} \sin \omega t + \frac{\partial y}{\partial \dot{x}_0} \cos \omega t$$

$$\frac{\partial X_I}{\partial \dot{y}_0} = \frac{\partial x}{\partial \dot{y}_0} \cos \omega t + \frac{\partial y}{\partial \dot{y}_0} \sin \omega t$$

$$\frac{\partial Y_I}{\partial \dot{y}_0} = - \frac{\partial x}{\partial \dot{y}_0} \sin \omega t + \frac{\partial y}{\partial \dot{y}_0} \cos \omega t$$

$$\frac{\partial X_I}{\partial x_0} = \frac{\partial x}{\partial x_0} \cos \omega t + \frac{\partial y}{\partial x_0} \sin \omega t + \omega \frac{\partial X_I}{\partial \dot{y}_0}$$

$$\frac{\partial X_I}{\partial y_0} = \frac{\partial x}{\partial y_0} \cos \omega t + \frac{\partial y}{\partial y_0} \sin \omega t - \omega \frac{\partial X_I}{\partial \dot{x}_0}$$

$$\frac{\partial Y_I}{\partial x_0} = - \frac{\partial x}{\partial x_0} \sin \omega t + \frac{\partial y}{\partial x_0} \cos \omega t + \omega \frac{\partial Y_I}{\partial \dot{y}_0}$$

$$\frac{\partial Y_I}{\partial y_0} = - \frac{\partial x}{\partial y_0} \sin \omega t + \frac{\partial y}{\partial y_0} \cos \omega t - \omega \frac{\partial Y_I}{\partial \dot{x}_0}$$

$$\frac{\partial \dot{X}_I}{\partial \dot{x}_0} = \frac{\partial \dot{x}}{\partial \dot{x}_0} \cos \omega t + \frac{\partial \dot{y}}{\partial \dot{x}_0} \sin \omega t + \omega \frac{\partial Y_I}{\partial \dot{x}_0}$$

$$\frac{\partial \dot{X}_I}{\partial \dot{y}_0} = \frac{\partial \dot{x}}{\partial \dot{y}_0} \cos \omega t + \frac{\partial \dot{x}}{\partial \dot{y}_0} \sin \omega t + \omega \frac{\partial Y_I}{\partial \dot{y}_0}$$

$$\frac{\partial \dot{Y}_I}{\partial \dot{x}_0} = - \frac{\partial \dot{x}}{\partial \dot{x}_0} \sin \omega t + \frac{\partial \dot{y}}{\partial \dot{x}_0} \cos \omega t - \omega \frac{\partial X_I}{\partial \dot{x}_0}$$

$$\frac{\partial \dot{Y}_I}{\partial \dot{y}_0} = - \frac{\partial \dot{x}}{\partial \dot{y}_0} \sin \omega t + \frac{\partial \dot{y}}{\partial \dot{y}_0} \cos \omega t - \omega \frac{\partial X_I}{\partial \dot{y}_0}$$

$$\frac{\partial \dot{X}_I}{\partial x_0} = \frac{\partial \dot{x}}{\partial x_0} \cos \omega t + \frac{\partial \dot{y}}{\partial x_0} \sin \omega t - \omega \left[\frac{\partial x}{\partial x_0} \sin \omega t - \frac{\partial y}{\partial x_0} \cos \omega t \right] + \omega \frac{\partial \dot{X}_I}{\partial \dot{y}_0}.$$

$$\frac{\partial \dot{X}_I}{\partial y_0} = \frac{\partial \dot{x}}{\partial y_0} \cos \omega t + \frac{\partial \dot{y}}{\partial y_0} \sin \omega t - \omega \left[\frac{\partial x}{\partial y_0} \sin \omega t - \frac{\partial y}{\partial y_0} \cos \omega t \right] - \omega \frac{\partial \dot{X}_I}{\partial \dot{x}_0}$$

$$\frac{\partial \dot{Y}_I}{\partial x_0} = -\frac{\partial \dot{x}}{\partial x_0} \sin \omega t + \frac{\partial \dot{y}}{\partial x_0} \cos \omega t - \omega \left[\frac{\partial x}{\partial x_0} \cos \omega t + \frac{\partial y}{\partial x_0} \sin \omega t \right]$$

$$+ \omega \frac{\partial \dot{Y}_I}{\partial \dot{y}_0}$$

$$\frac{\partial \dot{Y}_I}{\partial y_0} = -\frac{\partial \dot{x}}{\partial y_0} \sin \omega t + \frac{\partial \dot{y}}{\partial y_0} \cos \omega t - \omega \left[\frac{\partial x}{\partial y_0} \cos \omega t + \frac{\partial y}{\partial y_0} \sin \omega t \right]$$

$$- \omega \frac{\partial \dot{Y}_I}{\partial \dot{x}_0}$$

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 481
(BBC Task 203)

H. Engel
March 1967

REVISION FOR CLOCK PARAMETERS

It has been discovered that the clock partials in the OEAP do not properly include the time tagging error. This note indicates the required corrections.

Indicated clock time is denoted as $\bar{t}$. The corresponding real time is t, and the estimate of this real time is $\hat{t}$.

We start by considering a fictitious measurable, the 3-way range. For this measurable, the master station impresses its indicated clock time $\bar{t}_m$ on the signal it transmits, and the measured range is S_m

$$S_m = c(\bar{t}_s - \bar{t}_m) + n(t_s)$$

in which $\bar{t}_s$ is the time indicated on the slave station clock when this signal is received, having been relayed by the spacecraft.

The corresponding computed range can be expressed as S_c,

$$S_c = c(\hat{t}_s - \hat{t}_m)$$

in which $\bar{t}_s$, t_s and $\hat{t}_s$ are related by

$$\bar{t}_s = t_s + \bar{b}_s + \bar{\beta}_s t_s + \bar{\alpha}_s t_s^2 /2$$

$$\hat{t}_s = t_s + \hat{b}_s + \hat{\beta}_s t_s + \hat{\alpha}_s t_s^2 /2 \ .$$

The indicated and estimated times at the master station are related by

$$\bar{t}_m = t_m + \bar{b}_m + \bar{\beta}_m t_m + \bar{\alpha}_m t_m^2/2$$

and

$$\hat{t}_m = t_m + \hat{b}_m + \hat{\beta}_m t_m + \hat{\alpha}_m t_m^2/2$$

By substitution, the expression for S_c can be written as

$$S_c = c\left[(t_s - t_m) + (\hat{b}_s - \hat{b}_m) + (\hat{\beta}_s t_s - \hat{\beta}_m t_m) + (\hat{\alpha}_s t_s^2 - \hat{\alpha}_m t_m^2)/2\right]$$

$$= c\,(t_s - t_m) + c\left[(\hat{b}_s - \hat{b}_m) + (\hat{\beta}_s t_s - \hat{\beta}_m t_m) + (\hat{\alpha}_s t_s^2 - \hat{\alpha}_m t_m^2)/2\right]$$

The actual equivalent free space distance traveled by the signal received at the indicated time $\bar{t}_s$, however, is $c(t_s - t_m)$, which we shall denote by S. Thus,

$$S_c = S + c\left[(\hat{b}_s - \hat{b}_m) + (\hat{\beta}_s t_s - \hat{\beta}_m t_m) + (\hat{\alpha}_s t_s^2 - \hat{\alpha}_m t_m^2)/2\right]_1$$

It should be noted that S is a function only of t_s, the orbit parameters, μ, the station locations and the troposphere parameters; i.e., t_m is not independent of t_s. Then, for any parameter φ,

$$\frac{\partial S_c}{\partial \varphi} = \frac{\partial S}{\partial \varphi} + \frac{\partial S}{\partial t_s}\frac{\partial t_s}{\partial \varphi} + c\,\frac{\partial [\;]_1}{\partial \varphi}$$

Now $\partial S/\partial t_s$ is $\dot{S}$, and $\partial S/\partial \varphi = 0$ for any clock parameter, so

$$\frac{\partial S_c}{\partial \varphi} = \dot{S}\,\frac{\partial t_s}{\partial \varphi} + c\,\frac{\partial [\;]_1}{\partial \varphi}$$

The expression relating t_s and $\hat{t}_s$ may be rewritten as

$$t_s = \hat{t}_s - \hat{b}_s - \hat{\beta}_s t_s - \hat{\alpha}_s t_s^2/2$$

$$= \hat{t}_s - \hat{b}_s - \hat{\beta}_s \hat{t}_s - \hat{\alpha}_s \hat{t}_s^2/2 + \hat{\beta}_s (\hat{t}_s - t_s) + \hat{\alpha}_s (\hat{t}_s - t_s) \frac{\hat{t}_s + t_s}{2} .$$

Now t_s is of zero order, $\hat{t}_s - t_s$, $\hat{\alpha}_s$ and $\hat{\beta}_s$ are of first order, so the last two terms above are of second order. Hence we may write

$$t_s \doteq \hat{t}_s - \hat{b}_s - \hat{\beta}_s t_s - \hat{\alpha}_s \hat{t}_s^2 /2 \quad .$$

It follows that, for 3-way range,

$$\frac{\partial S_c}{\partial b_s} = - \dot{S} + c \qquad \frac{\partial S_c}{\partial b_m} = - c$$

$$\frac{\partial S_c}{\partial \beta_s} = (- \dot{S} + c)t \qquad \frac{\partial S_c}{\partial \beta_m} = - ct$$

$$\frac{\partial S_c}{\partial \alpha_s} = (- \dot{S} + c)t^2/2 \qquad \frac{\partial S_c}{\partial \alpha_m} = - ct^2/2$$

in which t_s has been replaced by t, no distinction being made between t_s and t_m in the OEAP, and no such distinction being required.

A real measurable is ordinary range, in which the master and slave station are one and the same. In this case, the expression for S_c reduces to

$$S_c = S + c\left[\hat{\beta}_m(t_s - t_m) + \hat{\alpha}_m(t_s - t_m)\frac{t_s + t_m}{2}\right]$$

$$= S\left[1 + \hat{\beta}_m + \hat{\alpha}_m\left(\frac{t_s + t_m}{2}\right)\right]$$

$$\cong S\left[1 + \hat{\beta}_m + \hat{\alpha}_m t\right] .$$

Now, in ordinary range the measurable is usually considered to be the one-way distance $S/2$, so we shall consider $S_c/2$. The clock partials are

$$\frac{\partial(S_c/2)}{\partial b_m} = -\dot{S}/2$$

$$\frac{\partial(S_c/2)}{\partial \beta_m} = (-\dot{S}t + S)/2$$

$$\frac{\partial(S_c/2)}{\partial \alpha_m} = (-\dot{S}t^2/2 + St)/2$$

The next real measurable to be considered is the 3-way range rate, $\dot{S}$. We have

$$\dot{S}_c = \dot{S} + c\left[(\hat{\beta}_s - \hat{\beta}_m) + (\hat{\alpha}_s t_s - \hat{\alpha}_m t_m)\right]$$

with the partials

$$\frac{\partial \dot{S}_c}{\partial b_s} = -\ddot{S} \qquad \frac{\partial \dot{S}_c}{\partial b_m} = 0$$

$$\frac{\partial \dot{S}_c}{\partial \beta_s} = -\ddot{S}\,t + c \qquad \frac{\partial \dot{S}_c}{\partial \beta_m} = -c$$

$$\frac{\partial \dot{S}_c}{\partial \alpha_s} = -\ddot{S}\,t^2/2 + c\,t \qquad \frac{\partial \dot{S}_c}{\partial \alpha_m} = -c\,t$$

Finally, for 2-way range rate, we have

$$\begin{aligned} \dot{S}_c &= \dot{S} + c\,\hat{\alpha}_m(t_s - t_m) \\ &= \dot{S} + S\,\hat{\alpha}_m \end{aligned}$$

with the partials

$$\frac{\partial \dot{S}_c}{\partial b_m} = -\ddot{S}$$

$$\frac{\partial \dot{S}_c}{\partial \beta_m} = -\ddot{S}\,t$$

$$\frac{\partial \dot{S}_c}{\partial \alpha_m} = -\ddot{S}\,t^2 + S\ .$$

More properly we should have written

$$\hat{t}_s = t_s + \hat{b}_s + \hat{\beta}_s (t_s - t_{s_0}) + \hat{\alpha}_s (t_s - t_{s_0})^2/2$$

$$\bar{t}_s = t_s + \bar{b}_s + \bar{\beta}_s (t_s - t_{s_0}) + \bar{\alpha}_s (t_s - t_{s_0})^2/2$$

$$\hat{t}_m = t_m + \hat{b}_m + \hat{\beta}_m (t_m - t_{m_0}) + \hat{\alpha}_m (t_m - t_{m_0})^2/2$$

$$\bar{t}_m = t_m + \bar{b}_m + \bar{\beta}_m (t_m - t_{m_0}) + \bar{\alpha}_m (t_m - t_{m_0})^2/2$$

in which t_{s_0} and t_{m_0} are the times at which the clocks are set. The only result of this change is to replace t by $t - t_{s_0}$ in the slave clock parameter partials, and t by $t - t_{m_0}$ in the master clock parameter partials.

The first term in each expression for slave clock partials for 3-way measurements, and the first term in each expression for master clock partials in 2-way measurements or ordinary range are those corresponding to the time tag errors. It is implicitly assumed that the time tags are derived from the same clock as the radar signals. If this is not so, then the above analysis can be repeated, separating these two sources of errors.

VSM

The Bissett-Berman Corporation, 4941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 482
(BBC Task 203)

L. Lustick
H. Dale
20 March 1967

ILLUSTRATIVE EXAMPLE INPUTS FOR THE RTODP ORBIT ERROR ANALYSIS PROGRAM

This note shows, by example, how data input cards are written for the Real Time Orbit Determination Program OEAP. A preliminary description of the program and how to use it is given in Apollo Note No. 464. It is intended that this note will answer detailed questions after the analyst has familiarized himself with Apollo Note No. 464.

The examples shown in this note are as follows:

1. <u>One Batch with a Projected Covariance</u> Page 2

This is essentially what is done with the old OEAP with the exception that the filter may have parameters which are estimated that do not exist in the real world.

2. <u>Multiple Batches with Re-Initialized Biases</u> Page 12

Here data over a tracking interval (batch) is used to estimate the state vector and pseudo-biases. Tracking over the following interval utilizes the previously obtained state covariance but re-estimates the biases using zero as apriori estimates and some set value for the variances in these bias estimates. The effect of using this type of filter, in the presence of real nuisance parameter noise, on the true state uncertainty is shown.

3. <u>Multiple Batches with Pseudo-Biases Carried Across but with an Uncertainty in μ Assumed By the Filter</u> Page 24

The purpose of this run is to see what happens when the RTODP filter assumes that some nuisance parameter is imperfectly known. Here both the state vector and bias estimates from one batch are used as apriori data for the next batch.

4. Multiple Batches with an Uncertain Boost Occuring Between Batches — Page 32

This shows how boost uncertainties may be included in the batching program.

5. Multiple Batches with a Change of Tracking Stations — Page 39

This typifies the Earth orbital tracking situation where many stations and thus many nuisance parameter groups may exist.

6. Tracking Two Vehicles and Estimating the Relative State Vector Using Multiple Batches — Page 46

This is highly useful when intervehicle tracking exists, and this example shows how each vehicle is separately handled.

1. One Batch With a Projected Covariance

This example is the simplest and thus a good first example. What is desired here is to study a continuous tracking interval (one batch) and to find the actual state covariance at a time later than epoch, the time at which the orbit is defined. The only way in which this differs from the old OEAP is that parameters can exist in the RTODP filter which do not exist in the real world. Thus, the filter may assume a measurement bias exists, and optimally estimate this parameter along with state parameters, while in the real world no bias exists. For this example assume the following:

a) Data Set 001 is a Program A set which builds an $\dot{R}$ information matrix and Q matrix for a single station tracking an Earth orbit. All times, entries 35 through 39, are zero. The end of tracking, TJ, occurs at 20 minutes and the Q matrix is generated through a OF input of 60 minutes.

b) The RTODP filter is to assume a bias in the $\dot{R}$ measurable whose uncertainty is 1.0 ft/sec. No $\dot{R}$ bias truly exists.

c) The actual nuisance parameters are station location, μ_{Earth}, and the station clock.

Now Data Set 001 already is assumed to exist. It is to be followed by the necessary apriori matrices, which are in turn followed by the Program B batching control cards and allocation instructions. The three apriori matrices needed for this example describe: 1) the filters apriori covariance; 2) the actual state vector apriori covariance, and finally, 3) the apriori covariance of the non-estimated parameters. These non-estimated parameters are grouped into "Non-estimated Parameter Groups" of no greater than 18 parameters per group. This is because they must be allocated into columns 23 through no greater than 40.

Let Data Set 002 be the apriori the RTODP filter assumes. It will look like:

PROGRAM A
DATA INPUT SHEET

xx: if 1st set on new tape; blank otherwise.

1–9	11–12	15–18	31–33
PROGRAM A	XX	APF	002

1–2	4–5	7–20
01	01	1. E+08
02	02	1. E+08
03	03	1. E+08
04	04	1. E+02
05	05	1. E+02
06	06	1. E+02
07	07	1. E+00
		. E
		. E
		. E
		. E
		. E
		. E
		. E
		. E
		. E
		. E
		. E

25–36 Input Description
VEH X
VEH Y
VEH Z
VEH X DOT
VEH Y DOT
VEH Z DOT
RDOT BIAS

First note that since this is the first apriori matrix to be used in the RTODP-OEAP the two "X"s appear on the first card. Now this APF apriori matrix will be used to tell the RTODP filter how well it knows each parameter it is going to estimate, and although it is not necessary, the state components are generally entered with variances equal to their uncertainty in the real world. The last card of APF has been used to enter the filter apriori uncertainty in a pseudo-bias in the $\dot{R}$ measurable. For this example it will be assumed that no true uncertainty exists in this parameter.

The next apriori matrix needed is APN, which gives the true uncertainty in the real estimated parameters. For our example APN would not have the double X, would be numbered 003, and would have identical diagonal elements 01, 01 through 06, 06. Since the measurable $\dot{R}$ bias in actuality does not exist, it is not included in APN. It is not a real parameter even though it is estimated in the filter. Thus for our example, APN contains only the six orbit parameter apriori variances. Two aspects of the real estimated parameter set should be noted here. The RTODP-OEAP has room for 22 estimated parameters. The six orbit parameters of the first vehicle must be placed in columns 1 - 6, the second vehicle's orbit parameters must, if they exist, be placed in columns 12 - 17. This leaves columns 7 - 11 for estimated parameters which involve only the first vehicle, and columns 18 - 22 for estimated parameters which involve only the second vehicle. When a parameter such as vehicle-one pseudo $\dot{R}$ measurable bias is to be in columns 7 - 11, it is only allocated in the APF input; the lack of a corresponding APN entry places zero-variance in the appropriate column. If some parameter such as μ is to be estimated, only one vehicle may be included since μ would affect both vehicles if they existed. A second point is that this program allows estimating only eleven parameters at a time. Thus, if only one vehicle is estimated, still no more than eleven columns (or 11 - 6 = 5 non-orbit parameters) may be used; one cannot utilize the unused 12 - 22 columns. Now back to the description of this example.

The final apriori inputs give the real apriori covariance of the non-estimated nuisance parameters. Since 40 - 22 = 18 parameters remain free for allocation in Part B of the program, the non-estimated parameters are broken up into groups of no greater than 18 each. The apriori covariance of each of these groups are referred to as AP01, AP02, etc. There can be no correlation between parameters in differing groups. The smallest numbered diagonal element (or row or column) that may be used for any nuisance parameter group is 07, since in later allocation instructions nothing in the first 6 x 6 may be re-allocated (this will be seen in the Allocation Instruction Sheet when we come to it). For the case at hand one Non-estimated Parameter Group is sufficient and is shown below:

PROGRAM A
DATA INPUT SHEET

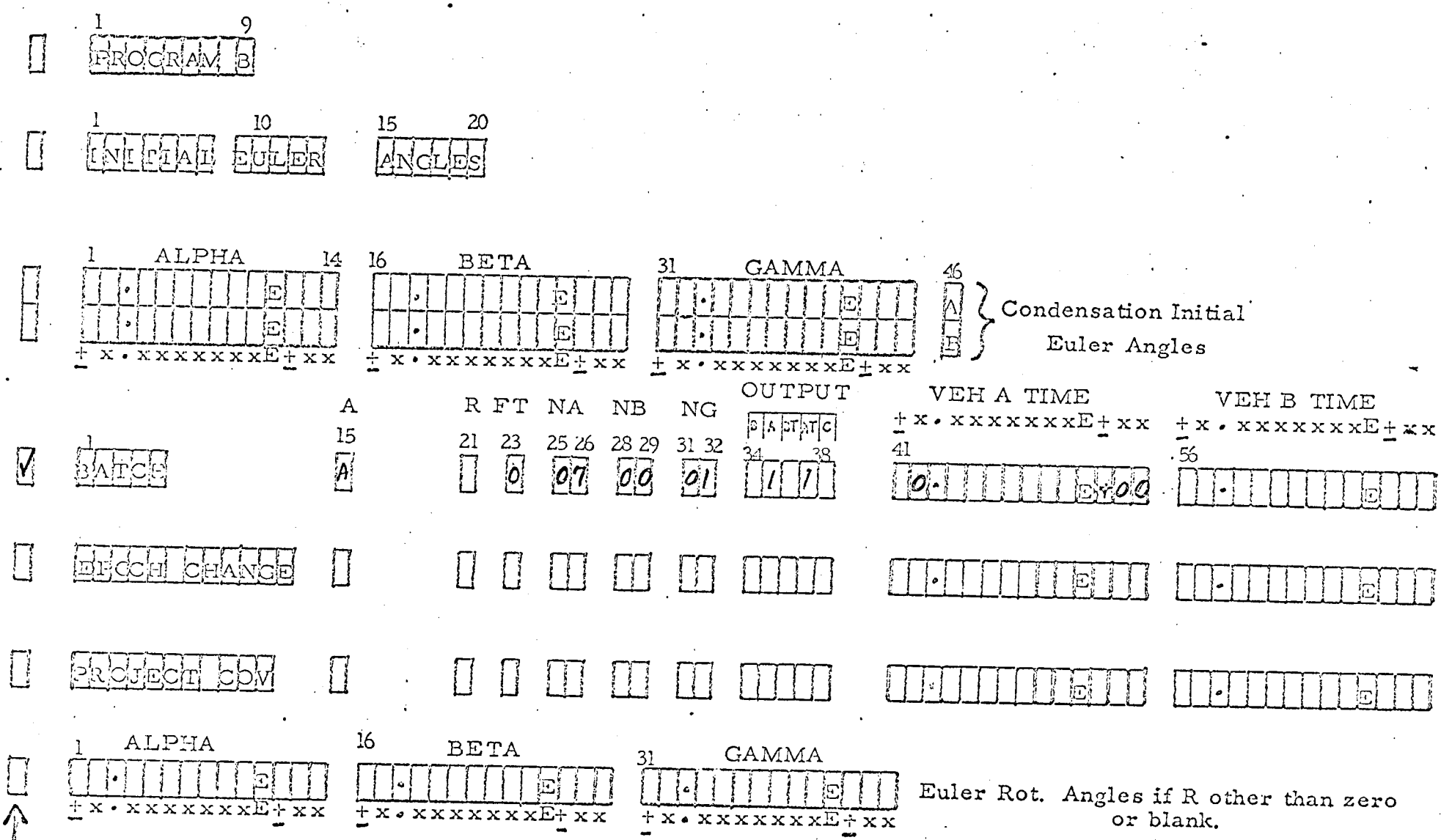
xx: if 1st set on new tape; blank otherwise.

1	9	11 12	Meas. 15 18	Data Set 31 33
PROGRAM	A		AP01	004

1 2	4 5	7	20	25 Input Description 36
07	07	1.7	E+22	MU EARTH
08	03	1.	E+04	STATION UP
09	09	1.	E+04	STATION EAST
10	10	1.	E 04	STATION NORTH
11	11	2.5	E-05	CLOCK BIAS
12	12	4.	E-20	CLOCK RATE
13	13	4.	E-36	CLOCK ACCEL
		.	E	
		.	E	
		.	E	
		.	E	
		.	E	
		.	E	
		.	E	

The next inputs involve Program B of the RTODP-OEAP. The object of this example is to take one batch of data, calculate the covariance of one vehicle's state vector at epoch, and to project that covariance to a future time. The first sheet (see Figure 1) is used to tell the OEAP that a "batch" is to be computed. Since this is the first

PROGRAM B CONTROL CARD DATA INPUT SHEET



Key Punch a Card for each row checked (✓).

Figure 1 - Program B Control of the Batch for Example 1

of a series of Program B runs, the first card states "PROGRAM B" in columns 1 through 9. The second, third and fourth cards shown in Figure 1 are not punched because only one vehicle is being considered, and no condensation is desired. The fourth card is punched indicating that a batch of data is being processed. On this card an "A" is punched in column 15 indicating that vehicle A is being batched. A zero or blank in column 21 states that no rotation (see card 8 of Figure 1) of the results is to be made and the results will be in locally defined coordinates. Columns 25 and 26 contain 07 which states that there are seven estimated parameters involving vehicle A. 01 in columns 31 through 32 states that one nuisance parameter group is being considered. The output and labeling columns (34 through 69) are best explained on pages six and seven of Apollo Note No. 464.

Figure 2 shows the next sheet of input cards which control the allocation for the single batch of data for this first example. The first card brings in the apriori data on the RTODP filter as is: no allocation is necessary. The second card brings in the measurement data used in generating the filter. Here the measurement bias is allocated. This ends the allocation of the filter and thus the third card is a blank.

The next card introduces APN, the true apriori uncertainties to the estimated parameters. This is a six by six and no further allocation is necessary. No measurement data need be allocated here since the machine knows to use the same data as in the filter group. This ends the noise group and the following card is blank.

The next card introduces the first (and in this example only) non-estimated nuisance parameter group. Here each non-estimated parameter within the group must be allocated from the measurement data. Note also that the measurement data regarding the estimated parameters must be allocated if those estimated parameters are past the first six. Thus the estimated bias (07) must be properly allocated. Again the group is terminated with a blank card.

The results of the above batch will produce, on tape, covariance matrices at epoch. Since the desired outputs of this example are at a later time a "Project Covariance" routine is required. Figure 3 shows the Program B Control Sheet for projecting the results of a batch. In this

Indicators for Control of Matrix Format

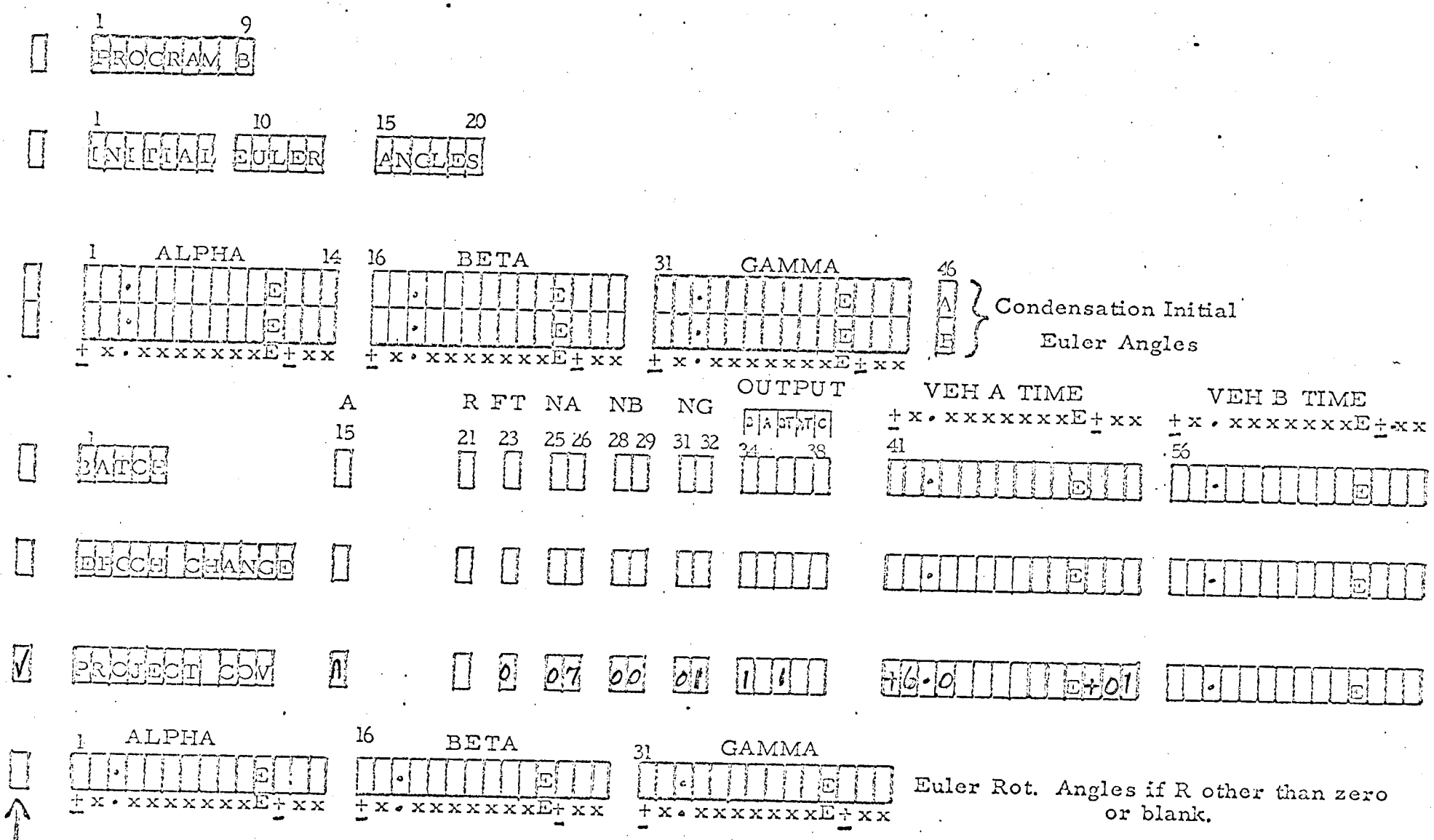
Data Set (1–3)	Measurables (4–7)	Time (TJ) (8–19) ± x.xxxxxExxx	σ (20–31) ± x.xxxxxE±xx	6 x 6 (32)
002	APF	. E	. E	
001	RD	+2.0 E +01	+4. E −02	
	BLANK	. E	. E	
003	APW	. E	. E	
	BLANK	. E	. E	
007	APO1	. E	. E	
001	RD	+2.0 E +01	+4. E −02	
	BLANK	. E	. E	

07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
07																							
23	24	25	26	27	28	29																	
07	24	25	26		23								28	29	27								

Figure 2 - Allocating the Single Batch for Example 1

ypunch a card for each
v checked (✓).

PROGRAM B CONTROL CARD DATA INPUT SHEET



Key Punch a Card for each row checked (✓).

Figure 3 - Program B Control for Projecting from Epoch to a Later Time for the Case of the First Example

case only one card is necessary since we are not starting a new run of "Program B"s. Figure 4 shows the necessary allocation for this projection. Note that P01 is allocated term by term; it could have been brought in as is. P01 could contain up to 40 terms and allocation space for only 7 through 30 is shown. If no entry is made in column 32 (truncating to a 6 x 6) the computer will automatically allocate any existing parameters past 30.

A problem which exists in the December 10 version of the program and has since been corrected involves the size of the noise group. If the situation arises, in the older version, in which only six parameters are entered into the noise group through APN, and further parameters are estimated past the first 6 x 6, then the computer will only store the first 6 x 6. Terms past the first 6 x 6 will be lost and unable to be recalled for future use. This problem may be remedied, in the older version of the program, by placing a dummy number in any diagonal term of APN past term eleven.

This concludes the first example.

PROGRAM B - ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)
	PF			
001	Q	+6.	+01	*
	BLANK			
	PN			
	BLANK			
	P01			
001	Q	+6.	+01	
	BLANK			

Indicators for Control of Matrix Format (33–80)

07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
07					23											23	24	25	26	27	28	29	

punch a card for each checked (✓).

Figure 4 - Allocation for the Project Covariance Routine for the First Example

2. Multiple Batches with Re-Initialized Biases

This example typifies the RTODP when tracking exists over an extended period. The data is broken up in time into batches. A bias in the measurable is assumed to exist. This bias is assumed to be zero with an assumed apriori variance. The data taken during each batch, in conjunction with an apriori state vector estimate, is used to estimate the state of the vehicle at the time corresponding to the beginning of the batch. The pseudo-biases in the measurables are also estimated. The state estimates are projected to the beginning of the following batch of data and used as apriori. The biases are again assumed to be zero with the old assumed apriori variance, and the estimating process repeated.

The example used for illustrating the inputs is that of Apollo Note No. 471, "The Effects of Batching During Terminal Lunar Rendezvous Using Near Optimum Pseudo-Bias Uncertainties." This example considers three consecutive fifteen minute batches of LEM tracking prior to LEM/CSM rendezvous. Two stations, Madrid and Ascension, are used, and for this example a pseudo-bias apriori variance of $(1\ \text{ft/sec})^2$ is used in the RTODP filter. Many of the required data inputs are shown in Note 471 in the form of computer output. The required inputs, however, are often simpler than the computer printed inputs. First consider the Program A inputs as shown in Figures 1, 2, and 3 of Note 471. As printed by the computer output, all input parameters, 1 through 39, are shown. Consider now how the input cards might be written. Basically only the changes need be entered. Figure 5 shows the Program A input cards.

Figure 6 shows the apriori inputs as printed in the computer output. Here again the first apriori input card should contain the XX indicating it is the first apriori on tape. For this case two nuisance parameter groups were required, thus AP01 and AP02 are shown. Data Set 008 is entered as a simple PROGRAM A A, with no designation past the second A. This is used for the apriori pseudo-bias variances at the beginning of each batch after the first batch (APF-007 has these two apriori variances built in it for the first batch).

```
1        6        11       16       21       26       31       36
PROGRAM A XX   MAD MST BCH1        001
01        6.3504074E+06
CARDS 02 THROUGH 38 NOT SHOWN
39        0.        E+00
```

Program A inputs for the Master Station for the First Batch of Tracking Data.

40 Cards

Blank Card

```
PROGRAM A      ASCEN BTCH 1      002
09       -8        E+00
10       -1.4      E+01
30        4.0      E+01
31       -4.       E+00
32        0.       E+00
```

Slave Station Batch 1 must change station Location, Master-Slave Ind. and give Master Station Location

Blank Card

```
PROGRAM A      MAD MST BCH2      003
01        6.3471317E+06
02        7.8336539E+00
03        5.2344771E+03
08       -1.7472383E+01
09        4.0       E+01
10       -4.        E+00
16        3.        E+01
23        4.9917    E+01
27        1.6       E+01
32        1.        E+00
37        1.5       E+01
```

Master Station Batch 2 Includes new a_1, a_4, a_5, new zeta, new station locations, TJ and QF times, new TI (init.), Master Station Indicator, and T ORBIT.

This Q is not used in this example.

Blank Card

```
PROGRAM A      ASCEN BTCH 2      004
09
10
30
31
32
```

Slave Station Batch 2

Blank Card

```
CONTINUE ON WITH MAST. & SLAVE BATCH 3 NOT SHOWN
```

Figure 5 - Program A Input Cards for the Second Example

APF 7

A(1, 1)	=	3.0000000E 05	A1 LEM
A(2, 2)	=	5.0000000E 06	A2 LEM
A(3, 3)	=	7.0000000E 06	A3 LEM
A(4, 4)	=	2.5000000E 00	A4 LEM
A(5, 5)	=	1.0000000E-01	A5 LEM
A(6, 6)	=	1.5000000E 00	A6 LEM
A(7, 7)	=	1.0000000E 00	RDOTBIASMADR
A(8, 8)	=	1.0000000E 00	RDOTBIASASCN

A 8

A(7, 7)	=	1.0000000E 00	RDOTBIASMADR
A(8, 8)	=	1.0000000E 00	RDOTBIASASCN

APN 9

A(1, 1)	=	3.0000000E 05	A1 LEM
A(2, 2)	=	5.0000000E 06	A2 LEM
A(3, 3)	=	7.0000000E 06	A3 LEM
A(4, 4)	=	2.5000000E 00	A4 LEM
A(5, 5)	=	1.0000000E-01	A5 LEM
A(6, 6)	=	1.5000000E 00	A6 LEM

AP01 10

A(7, 7)	=	1.5000000E 04	MAD STA UP
A(8, 8)	=	1.5000000E 04	MAD STA EAST
A(9, 9)	=	1.5000000E 04	MAD STA NORT
A(10,10)	=	1.0000000E 05	ASN STA UP
A(11,11)	=	1.0000000E 05	ASN STA EAST
A(12,12)	=	1.0000000E 05	ASN STA NORT
A(13,13)	=	2.5000000E 19	LUNAR MU
A(14,14)	=	2.5000000E-05	MAD CL BIAS
A(15,15)	=	4.0000000E-20	MAD CL RATE
A(16,16)	=	4.0000000E-36	MAD CL ACCEL
A(17,17)	=	2.5000000E-05	ASN CL BIAS
A(18,18)	=	4.0000000E-20	ASN CL RATE
A(19,19)	=	4.0000000E-36	ASN CL ACCEL

AP02 11

A(7, 7)	=	2.5510000E 07	LUN EPH	11
A(8, 8)	=	1.5410000E 07	LUN EPH	22
A(9, 9)	=	1.0000000E 00	LUN EPH	33
A(10,10)	=	1.2520000E-04	LUN EPH	44
A(11,11)	=	9.2700000E-04	LUN EPH	55
A(12,12)	=	1.2600000E-05	LUN EPH	66
A(7,11)	=	1.3500000E 02	LUN EPH	15
A(11, 7)	=	1.3500000E 02	LUN EPH	51
A(8,10)	=	-4.3920000E 01	LUN EPH	24
A(10, 8)	=	-4.3920000E 01	LUN EPH	42

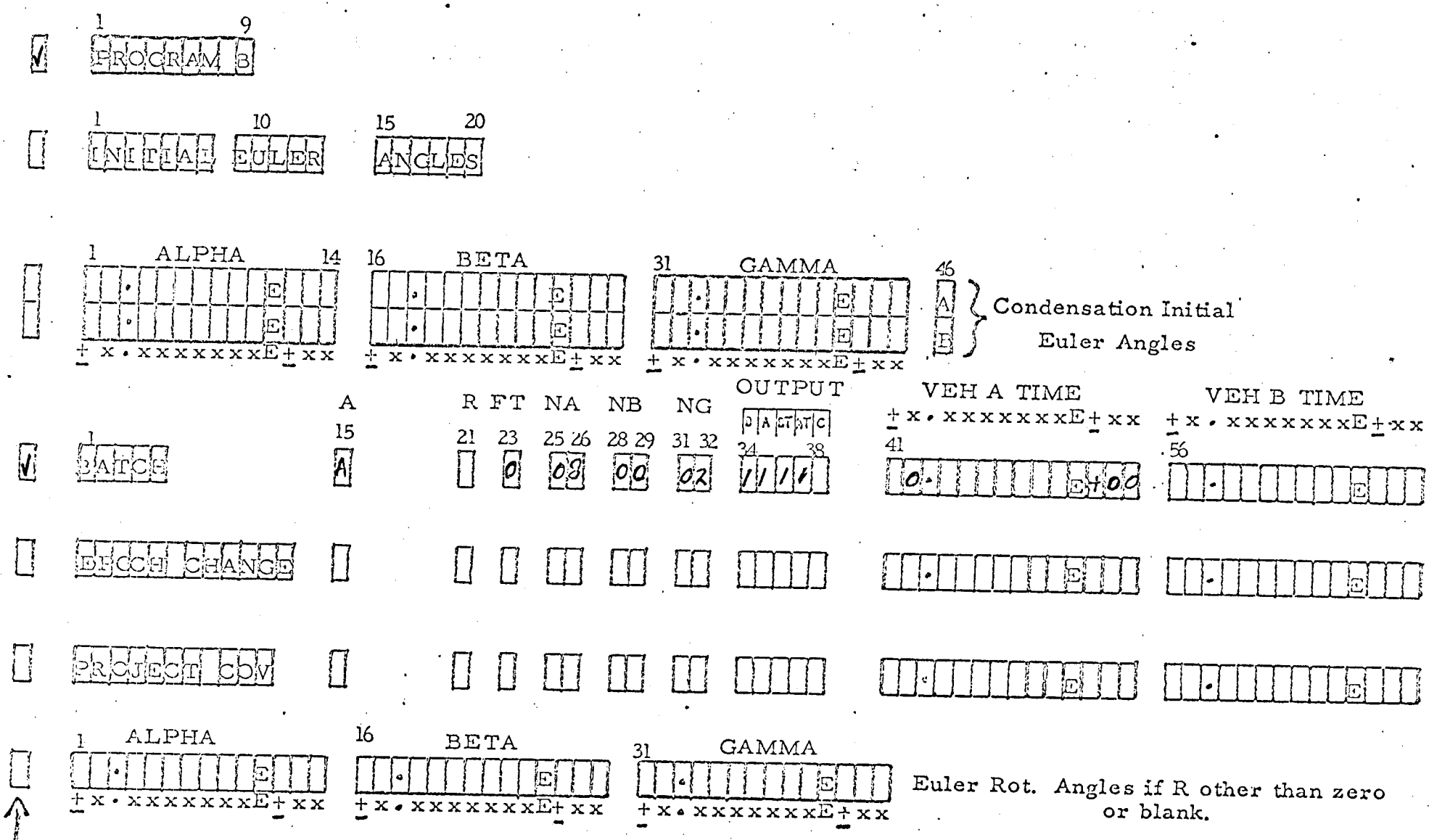
Figure 6

Apriori Inputs for: The Filter at the Start of the First Batch (APF-7), The Re-Initialized Biases for Batches 2 and 3 (A-8), The True Noise at the Start of the First Batch (APN-8), and the Two Nuisance Parameter Groups (AP01-10, AP02-11).

Figures 7 through 14 show the control and allocation cards for this second example. Note how, on the second batch, the pseudo-biases are "re-initialized". For the RTODP filter (see Figure 10) PF is called in, but truncated to a six-by-six. Data Set 008 is used to give the filter the re-initialized bias variances of $(1\ ft/sec)^2$. Since the filter assumes no μ error, the Q matrix from Data Set 001, which is used to project the previous batches results to the new epoch, is also truncated to a six-by-six. The previous batch's estimated-parameter noise, PN, is also truncated to a six-by-six to remove the previous batch's rows and columns regarding the $\dot{R}$ pseudo-biases. The third batch's control cards (Figure 11) and allocation cards (Figure 12) are essentially similar. Note that, while in one batch allocation the RD data sets come from within the batch, the Q matrices come from Program A's developed for the previous batch. The last two sets of cards (Figures 13 and 14) are used to project the result of the last batch to the end of the last batch.

This concludes the second example.

PROGRAM B CONTROL CARD DATA INPUT SHEET



Key Punch a Card for each row checked (✓).

Figure 7 - Batching Program Control for the First Batch (0 → 15 min)

PROGRAM B - ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6x6 (32)	Indicators for Control of Matrix Format (33–80)
007	APF				
001	RD	1.5E+01	3.937E-02		07
002	RD	1.5E+01	3.937E-02		08
	BLANK				
009	APN				
	BLANK				
010	AP01				23 24 25 26 27 28 29 30 31 32 33 34 35
001	RD	1.5E+01	3.937E-02		07 23 24 25 29 31 32 30
002	RD	1.5E+01	3.937E-02		08 26 27 28 29 23 24 25 31 32 33 34 35
	BLANK				
011	AP02				23 24 25 26 27 28
001	RD	1.5E+01	3.937E-02		07 23 24 25 26 27 2
002	RD	1.5E+01	3.937E-02		08 23 24 25 26 27 2
	BLANK				

07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	3
BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									

Keypunch a card for each row checked (✓).

Figure 8 - Batching Program Allocation for First Batch

PROGRAM B CONTROL CARD DATA INPUT SHEET

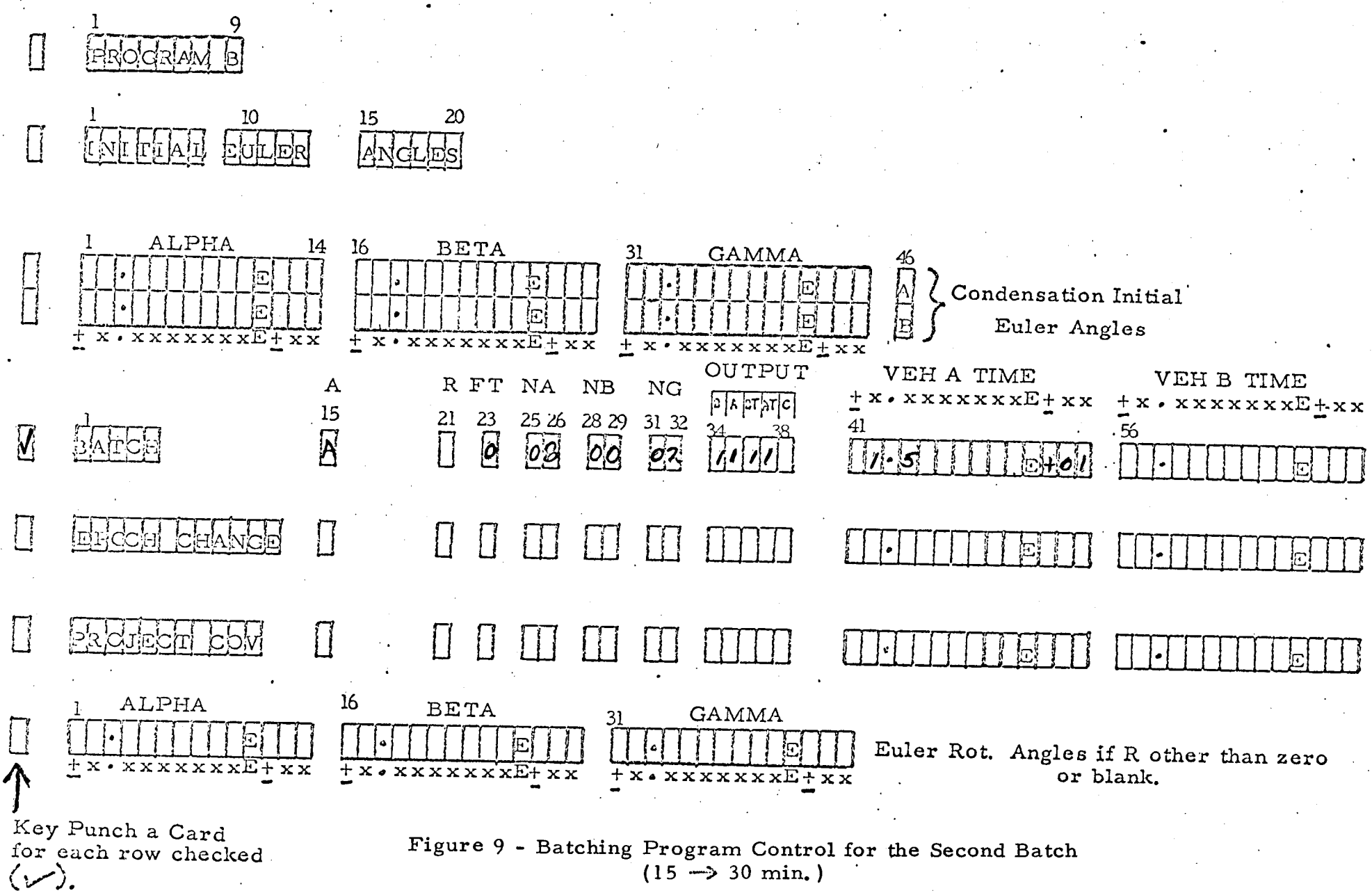


Figure 9 - Batching Program Control for the Second Batch (15 → 30 min.)

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)	Indicators for Control of Matrix Format (33–80)
					07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
					BI US ES NS NH MU N* VA JM EM NM ED ND RM AM BS RS AS L1 L2 L3 L4 L5 L6
					BI PX PY PZ MU V1 V2 X2 Y2 Z2 XD YD ZD
					33 40 50 60 70 80
	PF	.	.	*	
008	A	.	.		07 08
001	Q	1.5 E+01	.	*	
003	RD	3.0 E+01	3.937 E-02		07
004	RD	3.0 E+01	3.937 E-02		08
BLANK		.	.		
	PN	.	.	*	
BLANK		.	.		
	P01	.	.		23 24 25 26 27 28 29 30
003	RD	3.0 E+01	3.937 E-02		07 23 24 25 29 31 32 30
004	RD	3.0 E+01	3.937 E-02		08 26 27 28 29 23 24 25 31 32 33 34 35
001	Q	1.5 E+01	.		29
BLANK		.	.		
	P02	.	.		23 24 25 26 27 28
003	RD	3.0 E+01	3.937 E-02		07 23 24 25 26 27 28
004	RD	3.0 E+01	3.937 E-02		08 23 24 25 26 27 28
001	Q	1.5 E+01	.	*	
BLANK		.	.		

33 35 40 45 50 55 60 65 70 75 80

Figure 10 - Batching Program Allocation for the Second Batch

ypunch a card for each
v checked (✓).

PROGRAM B CONTROL CARD DATA INPUT SHEET

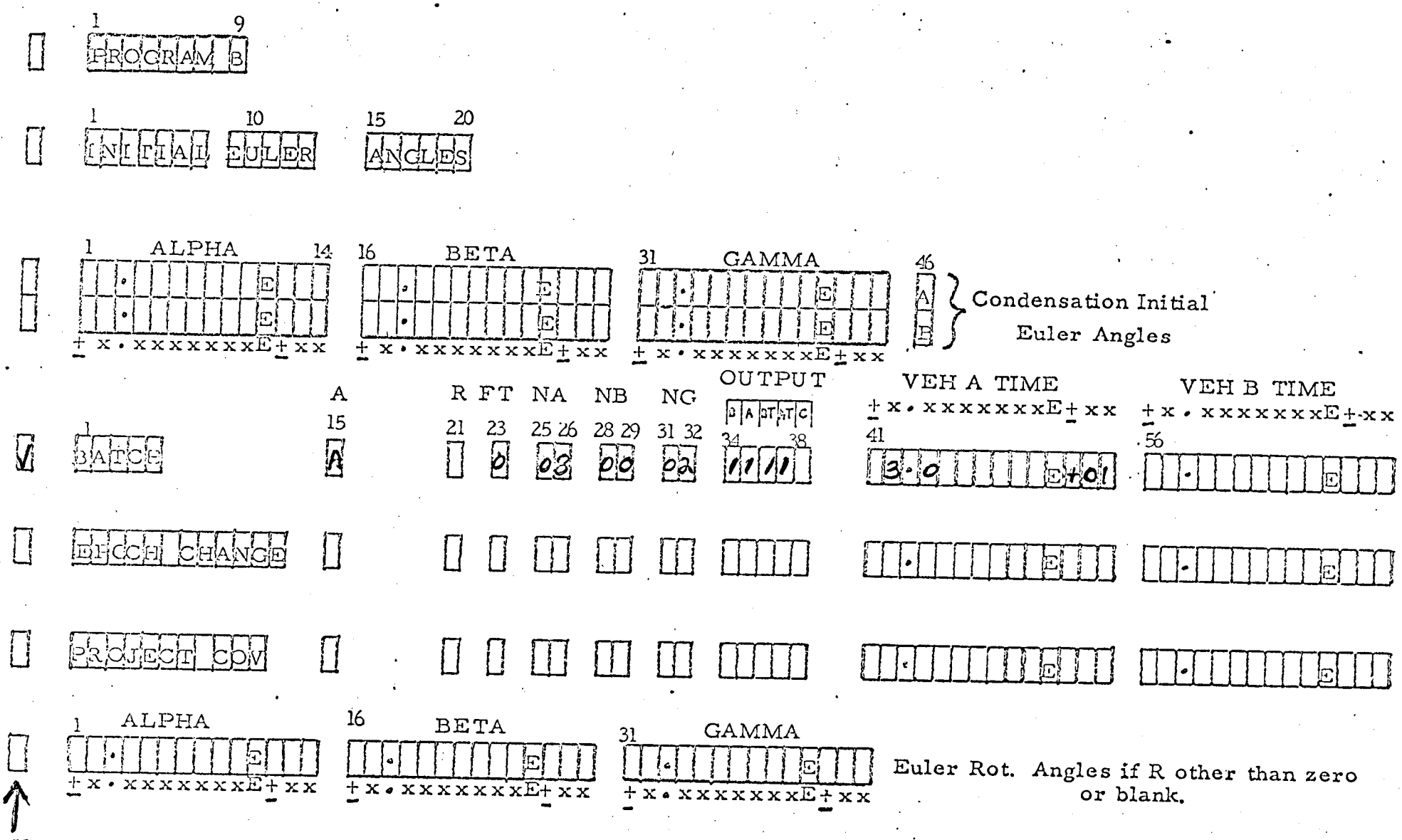


Figure 11 - Batching Program Control for the Third Batch (30 --> 45 min.)

PROGRAM B - ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) (8–19) ± x.xxxxxExxx	σ (20–31) ±x.xxxxxE±xx	6 x 6 (32)	Indicators for Control of Matrix Format (33–80)
	PF	. E	. E	*	
008	A	. E	. E		07 08
003	Q	3.0 E+01	. E	*	
005	RD	4.5 E+01	3.937 E-02		07
006	RD	4.5 E+01	3.937 E-02		08
BLANK		. E	. E		
	PN	. E	. E	*	
BLANK		. E	. E		
	P01	. E	. E		23 24 25 26 27 28 29 30
005	RD	4.5 E+01	3.937 E-02		07 23 24 25 29 31 32 30
006	RD	4.5 E+01	3.937 E-02		08 26 27 28 29 23 24 25 31 32 33 34 35
003	Q	3.0 E+01	. E		29
BLANK		. E	. E		
	P02	. E	. E		23 24 25 26 27 28
005	RD	4.5 E+01	3.937 E-02		07 23 24 25 26 27 28
006	RD	4.5 E+01	3.937 E-02		08 23 24 25 26 27 28
003	Q	3.0 E+01	. E	*	
BLANK		. E	. E		

Indicators for Control of Matrix Format

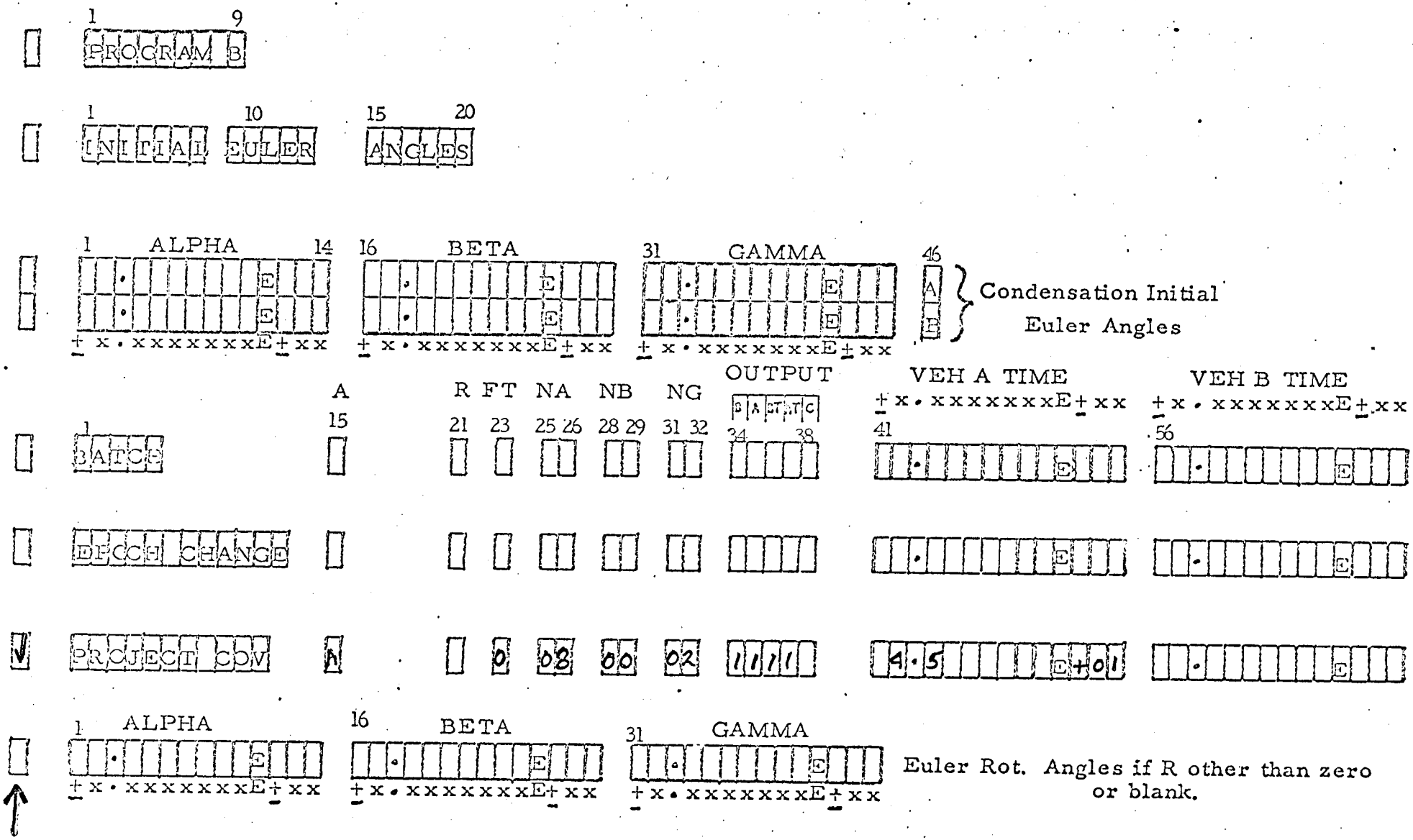
07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									

33 35 40 45 50 55 60 65 70 75 80

Figure 12 - Batching Program for the Third Batch

punch a card for each checked (✓).

PROGRAM B CONTROL CARD DATA INPUT SHEET



Key Punch a Card for each row checked (✓).

Figure 13 - Projection Control to Yield Last Batch Results at 45 Minutes

PROGRAM B - ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) (8–19) ±x.xxxxxExxx	σ (20–31) ±x.xxxxxE±xx	6 x 6 (32)
	PF			
005	Q	4.5 +01		*
	BLANK			
	PN			*
	BLANK			
	P01			
005	Q	4.5 +01		
	BLANK			
	P02			
005	Q	4.5 +01		*
	BLANK			

Indicators for Control of Matrix Format

07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
					29																		

Figure 14 - Projection Allocation

punch a card for each
checked (✓).

3. Multiple Batches with Pseudo-Biases Carried Across but with an Uncertainty in μ Assumed by the Filter

This example, if no μ uncertainty were assumed by the filter, would produce results as though no batching were done, i. e., since both the state estimates and pseudo-bias estimates from the first batch are maintained as apriori data for the second batch, it is as though no batching were done. When all three batches have been processed the results will be the same as though one long batch were processed. This is proved by the results reported in Apollo Note No. 474. Now when the RTODP filter is allowed to assume that the gravitational constant is not known perfectly, but rather has some uncertainty, $\sigma\mu$ filter, not necessarily equal to the true uncertainty, the state estimates from an early batch may be downweighted when used as apriori data for a later batch. This example can be studied, from the standpoint of required inputs, using most of inputs of the last example.

Thus, assume that Data Sets 001 through 006 exist and are given by Figure 5 (or Figures 1, 2, and 3 in Note 471). The apriori inputs look essentially the same as Figure 4 with one exception. Data Set 008, which in Figure 4 is used to re-initialize the pseudo-biases is used to parametrically supply the RTODP filter's assumed variance in μ. This is all explained further on page 2 of Note 474. The object is to run three batches first with an insignificant filter μ uncertainty (this will duplicate and check the single-batch-no-μ-error result), and then to repeat the three batches with three successively larger μ uncertainties until the filter μ uncertainty equals the actual μ uncertainty. Data Set 008 looks like

PROGRAM	A	A		008
07	07	1.0	E-10	
08	08	2.5	E+17	
09	09	2.5	E+18	
10	10	2.5	E+19	

The first filter $\sigma\ \mu$ is insignificant, the second, third and fourth are 1/10, 1/3, and equal to the actual μ uncertainty in the real world. The analysis of Apollo Note No. 474 uses these figures, one at a time, to parametrically study the effect of downweighting past data through a filter which treats μ as imperfectly known.

Figures 15 and 16 show the control and allocation for the first batch, Figures 17 and 18 show the same for the second batch. The third batch is not shown since it is quite similar to the second batch. The third batch is followed by a "Project Covariance routine which is shown in Figures 19 and 20.

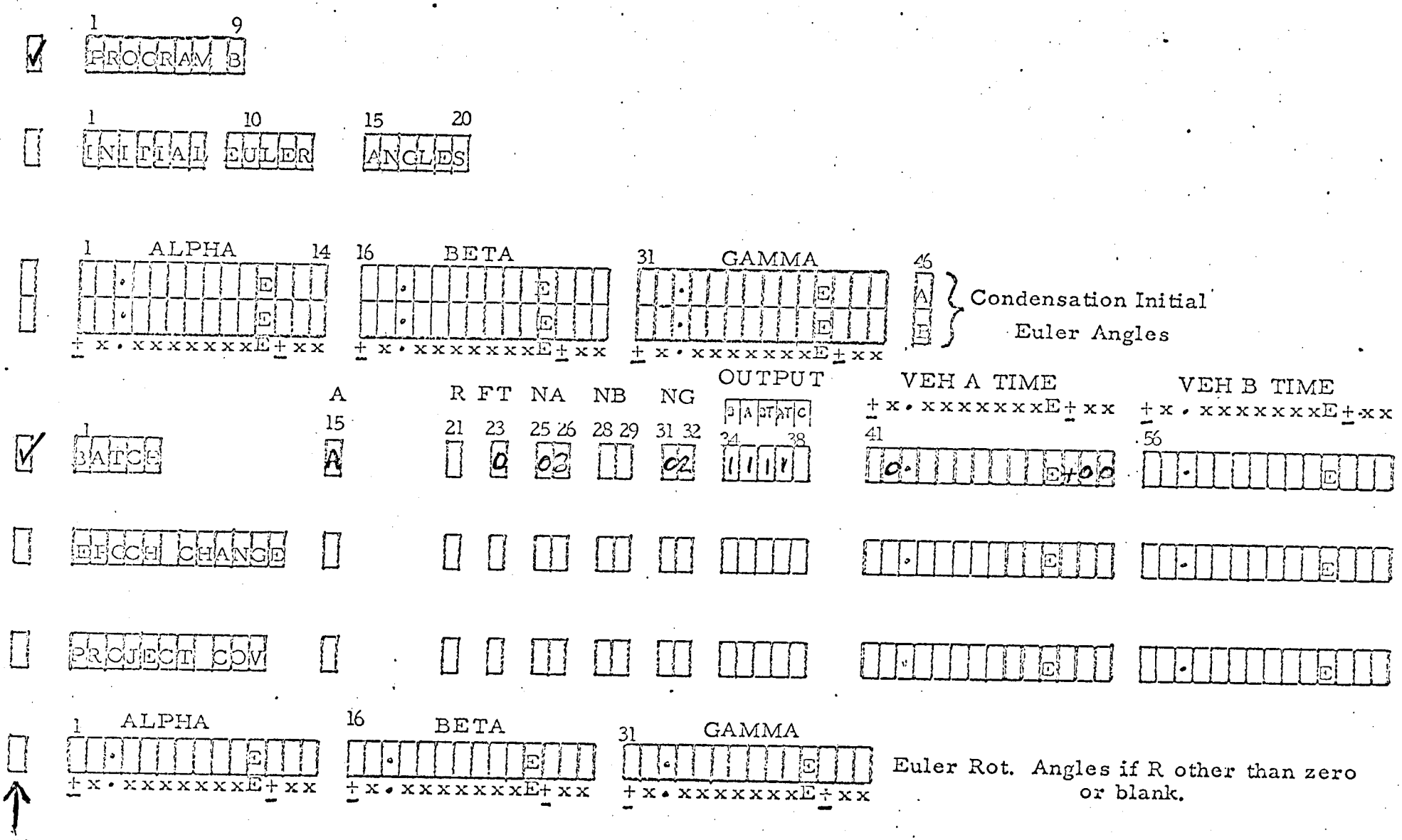
Notice in Figure 16 how the fact that μ is imperfectly known in the RTODP filter is considered by placing $\sigma^2 \mu$ from the apriori data set (008) into the first non-estimated diagonal element (23) of the filter covariance. Note that for the case shown, the second parametric value of $\sigma^2 \mu$, has been used. Now the covariance matrices produced by the first batching run will be:

PF, an 8 x 8 with $\sigma^2 \mu$ in 23, 23
PN, an 8 x 8
P01, a 35 x 35
P02, a 30 x 30

These are recalled as shown in the allocation of the second batch (Figure 18). PF is allocated as is, and is projected to the end of the first batch, which is the beginning and epoch of the second batch, with a Q matrix generated along with the information matrices of the first batch. μ terms of this Q are allocated to project the gravitational uncertainty of the filter. The estimated parameter noise, PN, is also brought in as is and the Q'ing and adding of new data is automatic. P01 and P02 are both brought in as is. (In P01 this is done automatically with no allocation shown; in P02 every term is allocated. Both examples do the same thing). Note that the biases (07, and 08) are brought in as apriori for the new batch, where they would be deleted were these pseudo-biases to be re-initialized and totally re-estimated. Figure 20, which shows the allocation for projecting the results of the third and last batch (remember that batch 3 is not shown) is simple and fairly self-explanatory. Were the effects of μ uncertainty in the filter not to be considered, PF could have been projected with a 6 x 6 Q matrix.

This concludes the third example.

PROGRAM B CONTROL CARD DATA INPUT SHEET



Key Punch a Card for each row checked (✓).

Figure 15 - Control for First Batch of Third Example

PROGRAM B - ALLOCATION SHEET

Indicators for Control of Matrix Format

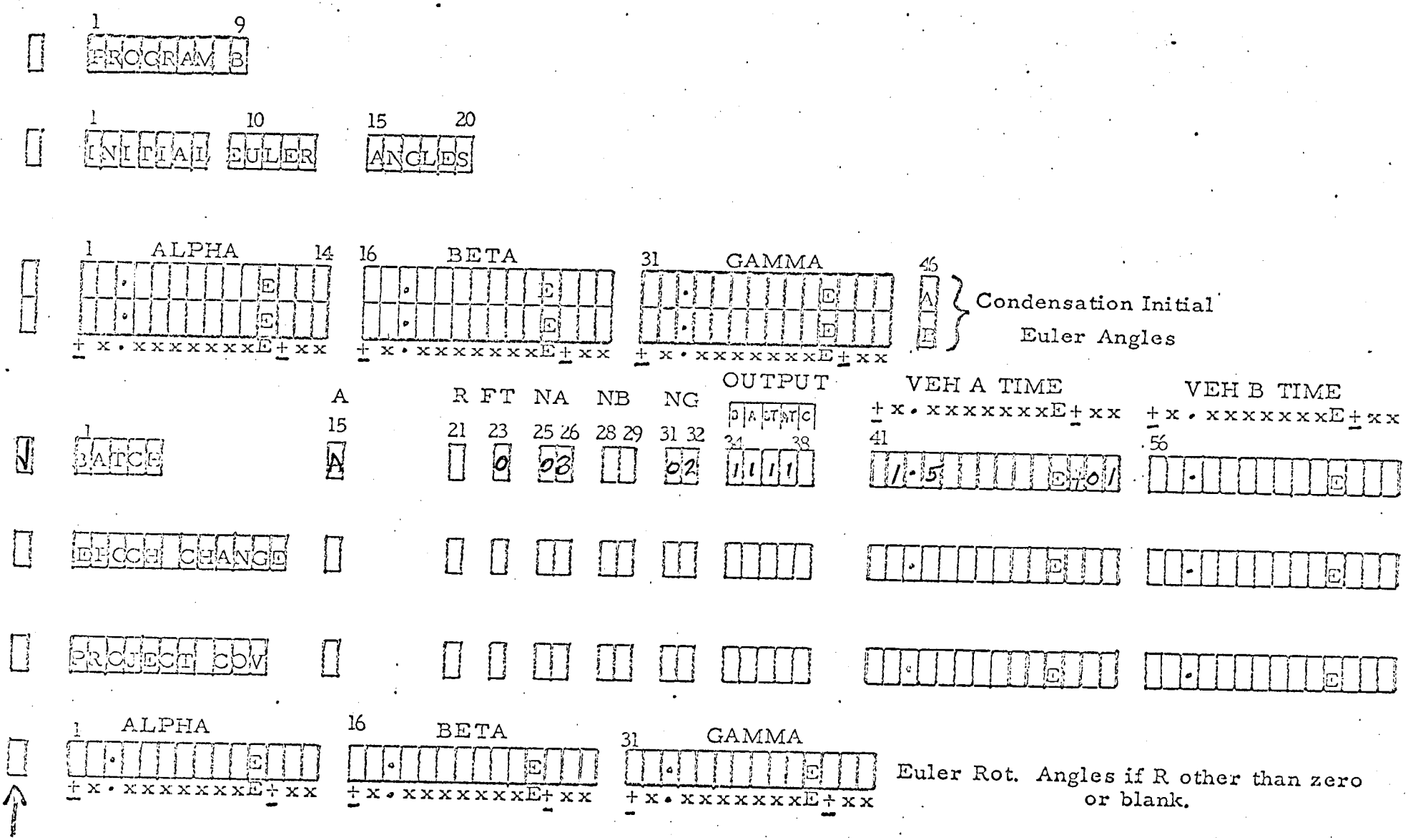
Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)	07 BI BI	08 US PX	09 ES PY	10 NS PZ	11 NH	12 MU MU	13 N*	14 VA V1	15 JM V2	16 EM X2	17 NM Y2	18 ED Z2	19 ND XD	20 RM YD	21 AM ZD	22 BS	23 RS	24 AS	25 L1	26 L2	27 L3	28 L4	29 L5	30 L6
007	APF	. E	. E																									
001	RD	1.5 E+01	3.937 E-02		07																							
002	RD	1.5 E+01	3.937 E-02		08																							
008	A	. E	. E			23																						
BLANK		. E	. E																									
009	APN	. E	. E																									
BLANK		. E	. E																									
010	AP01	. E	. E		23	24	25	26	27	28	29	30	31	32	33	34	35											
001	RD	1.5 E+01	3.937 E-02		07	23	24	25		29								31	32	30								
002	RD	1.5 E+01	3.937 E-02		08	26	27	28		29			23	24	25			31	32	33	34	35						
BLANK		. E	. E																									
011	AP02	. E	. E		23	24	25	26	27	28																		
001	RD	1.5 E+01	3.937 E-02		07																		23	24	25	26	27	28
002	RD	1.5 E+01	3.937 E-02		08																		23	24	25	26	27	28
BLANK		. E	. E																									
		. E	. E																									
		. E	. E																									
		. E	. E																									

See Page 10 regarding Dec 10 version of the program

punch a card for each checked (✓).

Figure 16 - Allocation For First Batch of Third Example

PROGRAM B CONTROL CARD DATA INPUT SHEET



Key Punch a Card for each row checked (✓).

Figure 17 - Control for Second Batch of Third Example

PROGRAM B - ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)	Indicators for Control of Matrix Format (33–80)
	PF	.	.		
001	Q	1.5	.+01		23
003	RD	3.0	3.937 -02		07
004	RD	3.0	3.937 -02		08
BLANK		.	.		
	PN	.	.		
BLANK		.	.		
	P01	.	.		
003	RD	3.0	3.937 -02		07 23 24 25 29 31 32 30
004	RD	3.0	3.937 -02		08 26 27 28 29 23 24 25 31 32 33 34 35
001	Q	1.5	. +01		29
BLANK		.	.		
	P02	.	.		07 08 23 24 25 26 27 28
003	RD	3.0	3.937 -02		07 23 24 25 26 27 28
004	RD	3.0	3.937 -02		08 23 24 25 26 27 28
001	Q	1.5	. +01	X	
BLANK		.	.		
		.	.		

Figure 18 - Allocation for Second Batch of Third Example

eypunch a card for each
ow checked (✓).

PROGRAM B CONTROL CARD DATA INPUT SHEET

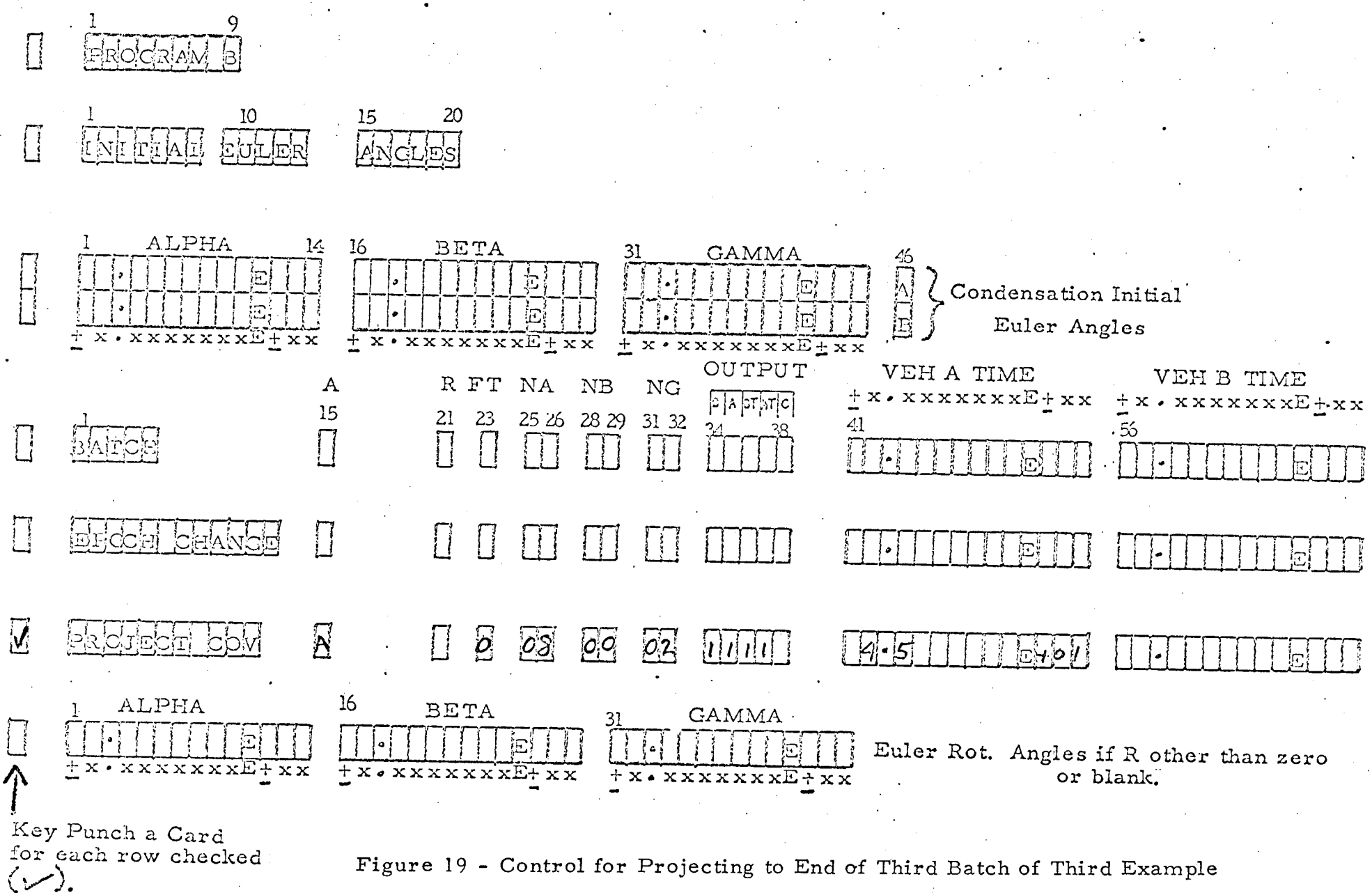


Figure 19 - Control for Projecting to End of Third Batch of Third Example

PROGRAM B - ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)
	PF			
005	Q	4.5 +01		
	BLANK			
	PN			
	BLANK			
	P01			
005	Q	4.5 +01		
	BLANK			
	P02			
005	Q	4.5 +01		*
	BLANK			

Indicators for Control of Matrix Format

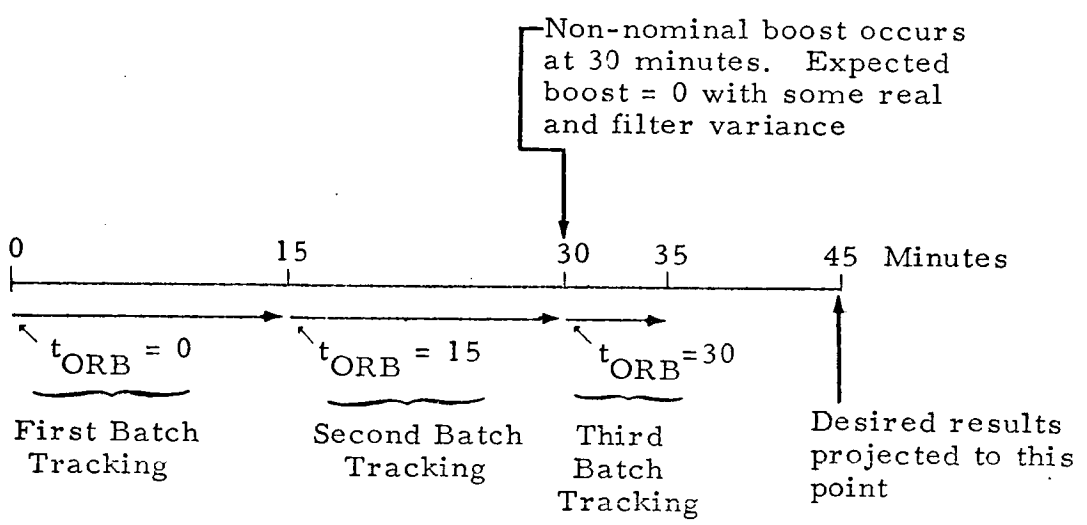
07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
					23																		
					29																		

punch a card for each
checked (✓).

Figure 20 - Allocation for Projecting to End of Third Batch of Third Example

4. Multiple Batches with an Uncertain Boost Occuring Between Batches

When a boost occurs the technique used to compute state covariance is as follows. First the covariance using all of tracking preceeding the boost is computed with respect to the state vector immediately prior to the boost. The covariance uncertainty of the boost is then added. The summed covariance is then rotated (if a rotation is required) to correspond to the boosted state vector. And, finally, new data is treated as in other batching cases (see Examples 2 and 3). There are two ways of handling the batches, one of which involves an additional epoch change operation. If the batch prior to the boost has as its epoch, the time of the boost, then no epoch change is required. If, however, the batch prior to the boost has the beginning of the batch as its epoch, then an epoch change is required to bring the covariance matrices forward and to rotate them into local pre-boost coordinates before the boost uncertainties may be added. The example shown below uses the more complicated situation in which information matrices are generated with the beginning of the batch as epoch (t_{ORB}). The following sketch shows the sequence of events.



Note that in the RTODP, the orbit parameters must be defined at t_{ORB}.

Now the above situation is interesting in that the nominal value of the boost is zero, i. e., the trajectory afterwards is identical, nominally, to the trajectory before the boost. Assume the following to exist:

BATCH 1

- PROGRAM A XX STA. 1 FST BTCH DATA SET 001 {master station}
  - T CLK = 0
  - T XYZ = 0
  - T ORB = 0 (time at which orbit for Batch 1 is defined)
  - T TILD = 0
  - TIME = 15 min. (time of end of first Batch tracking)
- PROGRAM A STA. 2 FST BTCH DATA SET 002 {slave station}
  - (Same times as above)

BATCH 2

- PROGRAM A STA. 1 SECND BTCH DATA SET 003 {master station}
  - T ORB = 15
  - TIME = 30 min.
  - New orbit parameters for 15 min. point
- PROGRAM A STA. 2 SECND BTCH DATA SET 004 {slave station}
  - (Same times as above)

BATCH 3

- PROGRAM A STA. 1 THRD BTCH DATA SET 005 {master station}
  - T ORB = 30
  - TIME = 35, 45 (need Inf. at 35 and Q at 45)
- PROGRAM A STA. 2 THRD BTCH DATA SET 006 {slave station}
  - (Same times as above)

Now the apriori matrices required are:

APF 7	
A(1, 1)	apriori orbital element variances on first (and only) vehicle
A(2, 2)	
A(3, 3)	
A(4, 4)	
A(5, 5)	
A(6, 6)	
A(7, 7)	apriori variances of measurable biases
A(8, 8)	

APN 8
A(1, 1)
A(2, 2)
A(3, 3)
A(4, 4)
A(5, 5)
A(6, 6)

A 9	
A(7, 7)	apriori variances of measurable biases to be used when re-initializing a new batch
A(8, 8)	

AP01 10		Two nuisance parameter groups
A(7, 7)		
A(8, 8)		
⋮		
A(19, 19)		
AP02 11		
A(7, 7)		
A(8, 8)		
⋮		
A(12, 12)		
ABR 12		Boost Variances*
A(4, 4)	real variances	
A(5, 5)		
A(6, 6)		
ABF 13		
A(4, 4)	filter assumed variances	
A(5, 5)		
A(6, 6)		

*Note that only velocity terms are shown; position terms are permissible if desired.

Figures 21 through 23 show the required Program B cards for the RTODP-OEAP for this example of two batches, followed by a boost, followed by a short batch, and finally, followed by a projection of the results to a particular point of interest. Each batch is processed with re-initialized pseudo range-rate biases and no μ uncertainty is assumed in the filter. Note how the epoch change and boost covariance addition is performed in a single separate epoch change run. The order of the runs is:

- Batch (Epoch at zero, tracking from 0 to 15)
- Batch (Epoch at 15, tracking from 15 to 30)
- Epoch Change and Boost (move epoch to 30 and then add boost covariance)
- Batch (Epoch at 30, tracking from 30 to 35)
- Project Covariance (Project results from 30 to 45)

Now this series of runs were chosen because the epoch of all Program A's happened to be at the beginning of the batch-tracking interval. Notice the way in which the Program B runs might be arranged if the epoch of each Program A were at the end of each batch of data:

- Batch (Epoch at 15, tracking from zero to 15)
- Batch and Boost (Epoch at 30, tracking from 15 to 30, covariance computed at 30 using this batch of data, and previous batch truncated and projected, boost covariance then added at 30)
- Batch (Epoch at 35, tracking from 30 to 35)
- Project Covariance (Project results from 35 to 45)

With this second approach one would need a Q matrix from a Program A with 35 minutes as epoch (or T ORB) which is what we have in the third batch.

This concludes the fourth example.

```
1    5    10   15   25   35   45   60   80
PROGRAM B
BATCH          A        0 08 00 02 1111    +0.        E+00
001RD   +1.5     E+01  3.937  E-02  07
002RD   +1.5     E+01  3.937  E-02  08
007APF

008APN

010AP01                          2324252627282930313233343 5
001RD   +1.5     E+01  3.937  E-02  07232425   29                  313230
002RD   +1.5     E+01  3.937  E-02  08262728   29      232425      3132333435

011AP02                          2324252627 28
001RD   +1.5     E+01  3.937  E-02  07                                  2324252627 28
002RD   +1.5     E+01  3.937  E-02  08                                  2324252627 28

BATCH          A        0 08 00 02 1111    +1.5       E+01
   PF                          *
003RD   +3.0     E+01  3.937  E-02  07
004RD   +3.0     E+01  3.937  E-02  08
001Q    +1.5     E+01          *
009A                             0708

   PN                          *

   P01                                                  2324252627282930
003RD   +3.0     E+01  3.937  E-02  07232425   29                  313230
```

FIRST BATCH

SECOND BATCH

Figure 21 - Batching with a Boost Between the Second and Third Batches
(Continued on next figure)

```
1    5    10   15   25   35   45.   60   80
004RD  +3.     E+01 3.937  E-02 08262728  29   232425   3132333435
001Q   +1.5    E+01                       29

P02                                                  232425262728
003RD  +3.0    E+01 3.937  E-02 07                          232425262728
004RD  +3.0    E+01 3.937  E-02 08                          232425262728
001Q   +1.5    E+01                *

EPOCH CHANGE   A      0 08 00 02 1111   +3.0     E+01
   PF
003Q   +3.0    E+01                *
013ABF

   PN
012ABR

   P01
003Q   +3.0    E+01                       29

   P02
003Q   +3.0    E+01                *

BATCH          A      0 08 00 02 1111   +3.0     E+01
   PF                                *
005RD  +3.5    E+01                 07
006RD  +3.5    E+01                 08
009A                                0708
```

SECOND BATCH

EPOCH CHANGE AND BOOST

THIRD BATCH

Figure 22 - Batching with a Boost Between the Second and Third Batches
(Continued on next figure)

```
THIRD BATCH
    PN                                  *

    P01                                                                 2324252627282930
005RD   +3.5    E+01  3.937   E-02  07232425   29                  313230
006RD   +3.5    E+01  3.937   E-02  08262728   29        232425    3132333435

    P02                                                                 2324252627 28
005RD   +3.5    E+01  3.937   E-02  07                                        2324252627 28
006RD   +3.5    E+01  3.937   E-02  08                                        2324252627 28

PROJECT COMMANCE TO 45 MIN.
PROJECT COV     A         0 08 00 02 1111     +4.5        E+01
    PF
005Q    +4.5    E+01                  *

    PN

    P01
005Q    +4.5    E+01                          29

    P02
005Q    +4.5    E+01                  *
```

Figure 23 - Batching with a Boost Between the Second and Third Batches (Conclusion)

5. Multiple Batches with a Change of Tracking Stations

This typifies the near-Earth orbital situation in which short tracking intervals and many stations require many nuisance parameters. These nuisance parameters are of two types: those which are common to all stations (μ and venting acceleration) and those which pertain to only one station (measurable biases, station location, station clock, atmospheric refraction parameter). With many stations the total number of nuisance parameters can be quite large and thus the RTODP-OEAP puts these nuisance parameters into groups. Generally there is enough room in one group to handle parameters regarding two stations. The parameters common to all stations can generally go in the first group (AP01). A group need not be called in until a batch is reached wherein the parameters of this group affect the measurable. Thus one might start out in the first batch with two groups and increase to three groups on some succeeding batch. Alternatively, one might put in all the groups at the beginning and "Q" them ahead each batch. However, once a group is brought in, it may not ever be deleted. (This will be changed in a later version of the program to permit deletion of groups whose parameters will never again affect the measurable.)

The example chosen is extremely simple, for purpose of illustration, and consists of but three stations tracking a vehicle for a bit over an orbit such that the first station gets a second look at the vehicle. The stations track in two-way Doppler and Az-Eℓ angle with a real uncertainty in the angle biases but no true uncertainty in R DOT bias. The filter assumes an R DOT bias exists and estimates it. No R DOT pseudo-bias is carried across a batch in the estimate (see Example 2, starting on page 12, for a full explanation of this situation). The following Figure 24 shows the tracking situation:

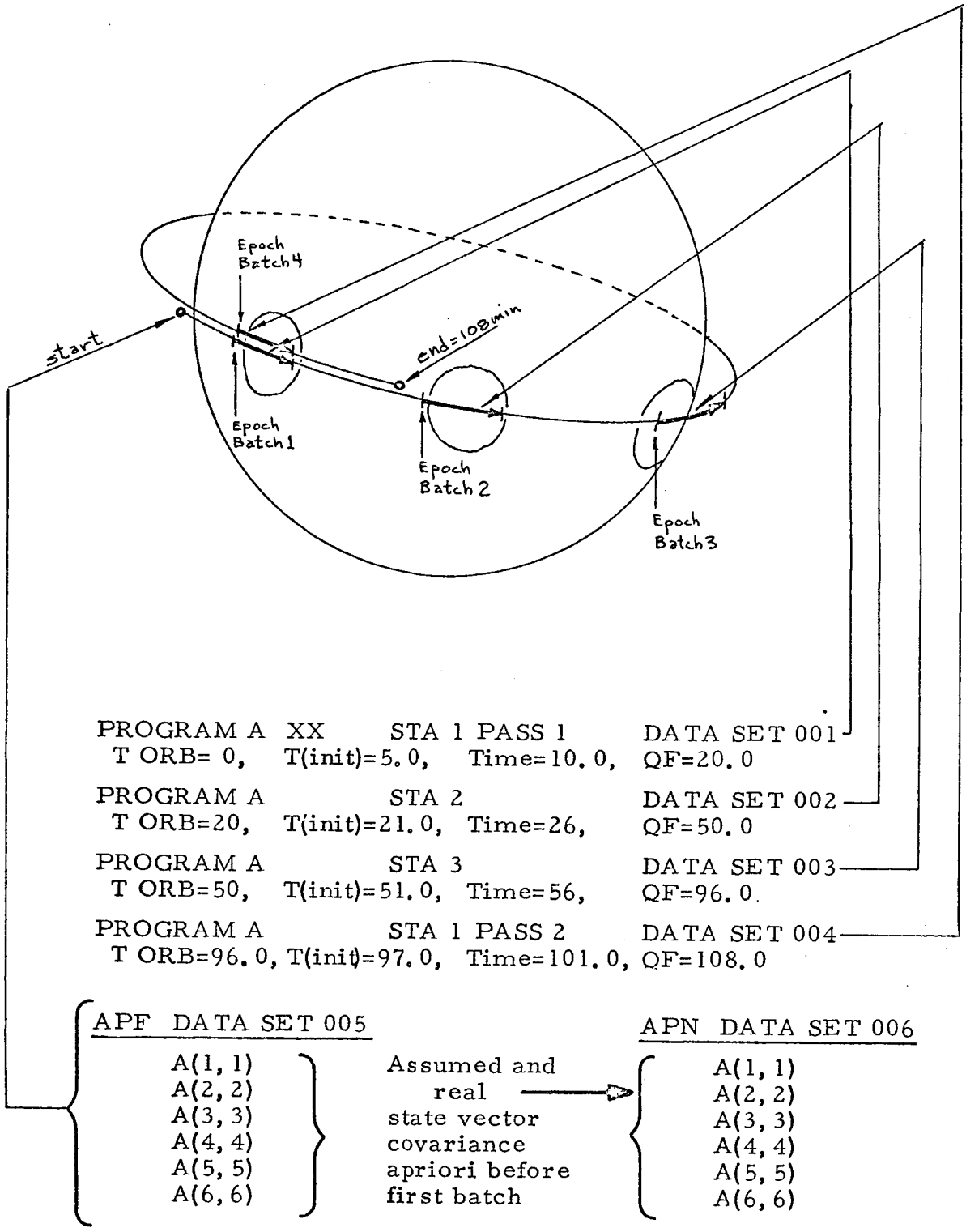


Figure 24 - Multiple Batches with a Change of Stations Showing Coverage

Now the required additional apriori matrices are shown below in Figure 25.

```
PROGRAM A     A                          DATA SET 007
     A(7, 7)  =  Pseudo - R DOT BIAS

PROGRAM A     AP01                       DATA SET 008
     A(7, 7)      =   μ          } These two parameters affect
     A(8, 8)      =   venting    } all station measurables
     A(9, 9)      =   Sta. 1 Az. Angle Bias  ┐
     A(10, 10)    =   Sta. 1 Eℓ Angle Bias   │
     A(11, 11)    =   Sta. 1 Up              │
     A(12, 12)    =   Sta. 1 East            } Station 1
     A(13, 13)    =   Sta. 1 North           │
     A(14, 14)    =   Sta. 1 Clock Bias      │
     A(15, 15)    =   Sta. 1 Clock Rate      ┘
     A(16, 16)    =   Sta. 2 Az. Bias        ┐
     A(17, 17)    =   Sta. 2 Eℓ Bias         │
     A(18, 18)    =   Sta. 2 Up              │
     A(19, 19)    =   Sta. 2 East            } Station 2
     A(20, 20)    =   Sta. 2 North           │
     A(21, 21)    =   Sta. 2 Clock Bias      │
     A(22, 22)    =   Sta. 2 Clock Rate      ┘

PROGRAM A     AP02                       DATA SET 009
     A(7, 7)      =   Sta. 3 Az Bias         ┐
     A(8, 8)      =   Sta. 3 Eℓ Angle Bias   │
     A(9, 9)      =   Sta. 3 Up              │
     A(10, 10)    =   Sta. 3 East            } Station 3
     A(11, 11)    =   Sta. 3 North           │
     A(12, 12)    =   Sta. 3 Clock Bias      │
     A(13, 13)    =   Sta. 3 Clock Rate      ┘
```

Figure 25 - Nuisance Parameter Apriori Data for Multiple Stations

One can see that the number of APXX matrices may get quite large when multiple orbits require many stations. However, three stations are sufficient for our illustrative example. The Program B Control and Allocation are shown for this example in Figures 26 through 28.

The first batch (Figure 26) shows how the $\dot{R}$ bias is allocated for estimation (term 07) while A1 and A2 biases (placed in columns 23 and 24) are not estimated. The second batch, like the first batch, has only one nuisance parameter group since the common systematic errors (μ and venting) and the station parameters for the first and second station can all fit within one nuisance parameter group. The third batch (Figure 27) uses nothing out of the first nuisance parameter group except μ (23) and venting (24); note that the estimated $\dot{R}$ bias is allocated. The second nuisance parameter group is brought in to allocate the third station's parameters.

Finally, the fourth batch (Figures 27 and 28) shows how P01 handles the first station's parameters for the second time around. Here P02 contains no parameters affecting the measurable and is brought in, dropping out the $\dot{R}$ bias estimate in 07, and carried along with a Q matrix.

This procedure can, of course, be expanded to include many orbits utilizing many stations, and this example, concluded at this point, was simplified for illustration.

```
1    5    10   15        25        35        45        60        80
PROGRAM B
BATCH          A        0 07 00 01 1111    +0.0        E+00
005APF
007A
001RD    1.      E+01 4.      E-02 07
001A1    1.      E+01 8.      E-04*
001A2    1.      E+01 8.      E-04*

006APN

008AP01                          2324252627282930313233343536373 8
001RD    1.0     E+01 4.      E-02 07272829  23  24          31  30
001A1    1.0     E+01 8.      E-04 25272829  23  24
001A2    1.0     E+01 8.      E-04 26272829  23  24

BATCH          A        0 07 00 01 1111    +2.         E+01
   PF                            *
007A
002RD    2.6     E+01 4.      E-02 07
002A1    2.6     E+01 8.      E-04*
002A2    2.6     E+01 8.      E-04*
001Q     2.0     E+01            *

   PN                            *

   P01                                                          2324252627282930
002RD    2.6     E+01 4.      E-02 07343536  23  24          38  37
```

FIRST BATCH - STATION 1

SECOND BATCH - STATION 2

Figure 26 - Batching with Many Stations in an Earth Parking Orbit
(Continued on next figure)

```
SECOND BATCH STATION 2
002A1    2.6    E+01  8.    E-04  32343536  23  24
002A2    2.6    E+01  8.    E-04  33343536  23  24
001Q                                        23  24

BATCH          A        0 07 00 02 1111   +5.0      E+01
   PF                                  *
007A
003RD    5.6    E+01  4.    E-02  07
003A1    5.6    E+01  8.    E-04 *
003A2    5.6    E+01  8.    E-04 *
002Q     5.0    E+01                *

   PN                                  *

   P01                                                  2324252627282930
003RD    5.6    E+01  4.    E-02  07        23  24
003A1    5.6    E+01  8.    E-04            23  24
003A2    5.6    E+01  8.    E-04            23  24
002Q     5.0    E+01                        23  24

009AP02                          2324252627282 9
003RD    5.6    E+01  4.    E-02  07252627                29  28
003A1    5.6    E+01  8.    E-04  23252627
003A2    5.6    E+01  8.    E-04  24252627

BATCH          A        0 07 00 02 1111   +9.6      E+01
   PF                                  *
```

The Dec. 10 version of the program would require that this group contain a Q matrix:

002Q 5.0 E + 01 *

following the rule that if a Q appears in one group it must appear in all

THIRD BATCH - STATION 3

FOURTH BATCH STATION 1 (AGAIN)

Figure 27 - Batching with Many Stations in an Earth Parking Orbit
(Continued on next figure)

FOURTH BATCH - STATION 1 (AGAIN)

```
007A
004RD    1.01    E+02  4.     E-02  07
004A1    1.01    E+02  8.     E-04 *
004A2    1.01    E+02  8.     E-04 *
003Q     9.6     E+01         *

   PN                         *

   P01                                                                  2324252627282930
004RD    1.01    E+02  4.     E-02  07272829   23   24          31   30
004A1    1.01    E+02  8.     E-04  25272829   23   24
004A2    1.01    E+02  8.     E-04  26272829   23   24
003Q     9.6     E+01                          23   24

   P02                                                                  2324252627282 9
003Q     9.6     E+01         *
```

This could be followed by a "Project Cov", otherwise the results are at 96 min.

Figure 28 - Batching with Many Stations in an Earth Parking Orbit
(Concluded)

6. Tracking Two Vehicles and Estimating the Relative State Vector Using Multiple Batches.

The RTODP-OEAP has the ability of keeping track of two vehicles at the same time. Thus, one can first process one batch of tracking data for one vehicle; follow this by processing a batch of data in which a second vehicle is tracked; and then, if desired, compute the relative-state covariance. Program C information matrices, which involve intervehicle tracking, may also be used (along with Program A's) to estimate either, but not both vehicles and to estimate the relative state uncertainty

The example chosen for illustration considers a LEM and CSM in near-proximity orbits about the Moon. We will assume the existance of two tracking stations, and will assume that these stations track the LEM (Vehicle A) while only apriori estimates are used for the CSM (Vehicle B). During the time that the LEM is being tracked we will assume that relative measurements (azimuth and elevation angles) are taken between the LEM and CSM. This relative data will be used to further estimate the LEM's state vector. It is important to note that Program C relative data may be used to update vehicle A or vehicle B but not both, in accordance with the planned RTODP procedure.

The two Program A's (for the master station at Madrid and the slave station at Ascension) and the Program C (for intervehicle angle measurements taken from vehicle A, the LEM, each 5 minutes) are shown in Figure 29. Figures 31, 32, and 33 show the control and allocation for the batching of the radar and intervehicle data.

The control inputs (Figure 31) show the three cards describing the initial Euler angles between the $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ system and each of the two vehicles at their T ORB. These correspond to XI, ETA, and ZETA from the Program A data input sheets. These angles must be entered each time a new Program B card is used, which for our example is only once. The batch shown estimates the six orbit parameters of vehicle A plus two $\dot{R}$ pseudo biases. Vehicle B is carried along to supply a base from which intervehicle measurements are used to estimate Vehicle A. Note in the allocation (Figure 32) that the second

INPUTS - DATA SET 1

(MADRID, MASTER)

1	6.3504074E 06	A1
2	-1.4772000E 01	A4
3	5.2317770E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	-6.0000000E 01	ZETA
9	4.0000000E 01	LAMBDA
10	-4.0000000E 00	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	4.5000000E 01	TIME
16	4.9917000E 01	TIME
17	-5.9386954-261	RE RAD
18	0	LT SLP
19	0	LO SLP
20	1.0000000E 00	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	1.0000000E 00	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	-5.9386954-261	LAMB2
31	-5.9386954-261	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	-0	T CLK
36	-0	T XYZ
37	-0	T ORB
38	-0	T TILD
39	-0	T SHIP

INPUTS - DATA SET 2

(ASCENSION, SLAVE)

1	6.3504074E 06	A1
2	-1.4772000E 01	A4
3	5.2317770E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	-6.0000000E 01	ZETA
9	-8.0000000E 00	LAMBDA
10	-1.4000000E 01	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	4.5000000E 01	TIME
16	4.9917000E 01	TIME
17	-5.9386954-261	RE RAD
18	0	LT SLP
19	0	LO SLP
20	1.0000000E 00	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	1.0000000E 00	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	4.0000000E 01	LAMB2
31	-4.0000000E 00	ALPHA2
32	0	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	-0	T CLK
36	-0	T XYZ
37	-0	T ORB
38	-0	T TILD
39	-0	T SHIP

INPUTS - DATA SET 3

(INTER VEHICLE ANGLES MEASURED FROM LEM)

1	6.3504074E 06	A1-1
2	-1.4772000E 01	A4-1
3	5.2317770E 03	A5-1
6	1.8000000E 02	XI-1
7	1.8000000E 02	ETA-1
8	-6.0000000E 01	ZETA-1
15	1.7290000E 14	MU
16	5.0000000E 00	TIME
16	1.0000000E 01	TIME
16	1.5000000E 01	TIME
16	2.5000000E 01	TIME
16	3.5000000E 01	TIME
16	4.5000000E 01	TIME
16	4.9917000E 01	TIME
20	0	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	1.0000000E 00	A1 IND
25	1.0000000E 00	A2 IND
27	5.0000000E 00	T INIT
28	5.0000000E 00	T INCR
33	1.0000000E 00	FM IND
41	6.3826209E 06	A1-2
42	3.8100000E-01	A4-2
43	5.2084750E 03	A5-2
44	1.8000000E 02	XI-2
45	1.8000000E 02	ETA-2
46	-5.8890750E 01	ZETA-2
47	0	T VEH1
48	-0	T VEH2

Figure 29 - Two Program A's and One Program C for Example 6.

APF 4

A(1, 1)	=	3.0000000E 05	A1LEM
A(2, 2)	=	5.0000000E 06	A2LEM
A(3, 3)	=	7.0000000E 06	A3LEM
A(4, 4)	=	2.5000000E 00	A4LEM
A(5, 5)	=	1.0000000E-01	A5LEM
A(6, 6)	=	1.5000000E 00	A6LEM
A(7, 7)	=	1.0000000E 00	RDOTBIASMADR
A(8, 8)	=	1.0000000E 00	RDOTBIASASCN
A(9, 9)	=	1.0000000E 04	A1CSM
A(10,10)	=	5.0000000E 05	A2CSM
A(11,11)	=	2.5000000E 07	A3CSM
A(12,12)	=	3.0000000E-01	A4CSM
A(13,13)	=	1.0000000E-02	A5CSM
A(14,14)	=	1.6000000E 01	A6CSM

APN 5

A(1, 1)	=	3.0000000E 05	A1LEM
A(2, 2)	=	5.0000000E 06	A2LEM
A(3, 3)	=	7.0000000E 06	A3LEM
A(4, 4)	=	2.5000000E 00	A4LEM
A(5, 5)	=	1.0000000E-01	A5LEM
A(6, 6)	=	1.5000000E 00	A6LEM
A(9, 9)	=	1.0000000E 04	A1CSM
A(10,10)	=	5.0000000E 05	A2CSM
A(11,11)	=	2.5000000E 07	A3CSM
A(12,12)	=	3.0000000E-01	A4CSM
A(13,13)	=	1.0000000E-02	A5CSM
A(14,14)	=	1.6000000E 01	A6CSM

AP01 6

A(7, 7)	=	1.5000000E 04	MAD STA UP
A(8, 8)	=	1.5000000E 04	MAD STA EAST
A(9, 9)	=	1.5000000E 04	MAD STA NORT
A(10,10)	=	1.0000000E 05	ASN STA UP
A(11,11)	=	1.0000000E 05	ASN STA EAST
A(12,12)	=	1.0000000E 05	ASN STA NORT
A(13,13)	=	1.2000000E 16	MU
A(14,14)	=	2.5000000E-05	MAD CL BIAS
A(15,15)	=	4.0000000E-20	MAD CL RATE
A(16,16)	=	4.0000000E-36	MAD CL ACC
A(17,17)	=	2.5000000E-05	ASN CL BIAS
A(18,18)	=	4.0000000E-20	ASN CL RATE
A(19,19)	=	4.0000000E-36	ASN CL ACC

AP02 7

A(7, 7)	=	2.0000000E 07	LUNAR EPH L1
A(8, 8)	=	2.0000000E 07	L2
A(9, 9)	=	1.0000000E 00	L3
A(10,10)	=	5.0000000E-04	L4
A(11,11)	=	5.0000000E-04	L5
A(12,12)	=	1.2000000E-05	L6
A(13,13)	=	1.0000000E-06	PLAT X
A(14,14)	=	1.0000000E-06	PLAT Y
A(15,15)	=	1.0000000E-06	PLAT Z

Figure 30 - Required Apriori Inputs for Example 6

PROGRAM B CONTROL CARD DATA INPUT SHEET

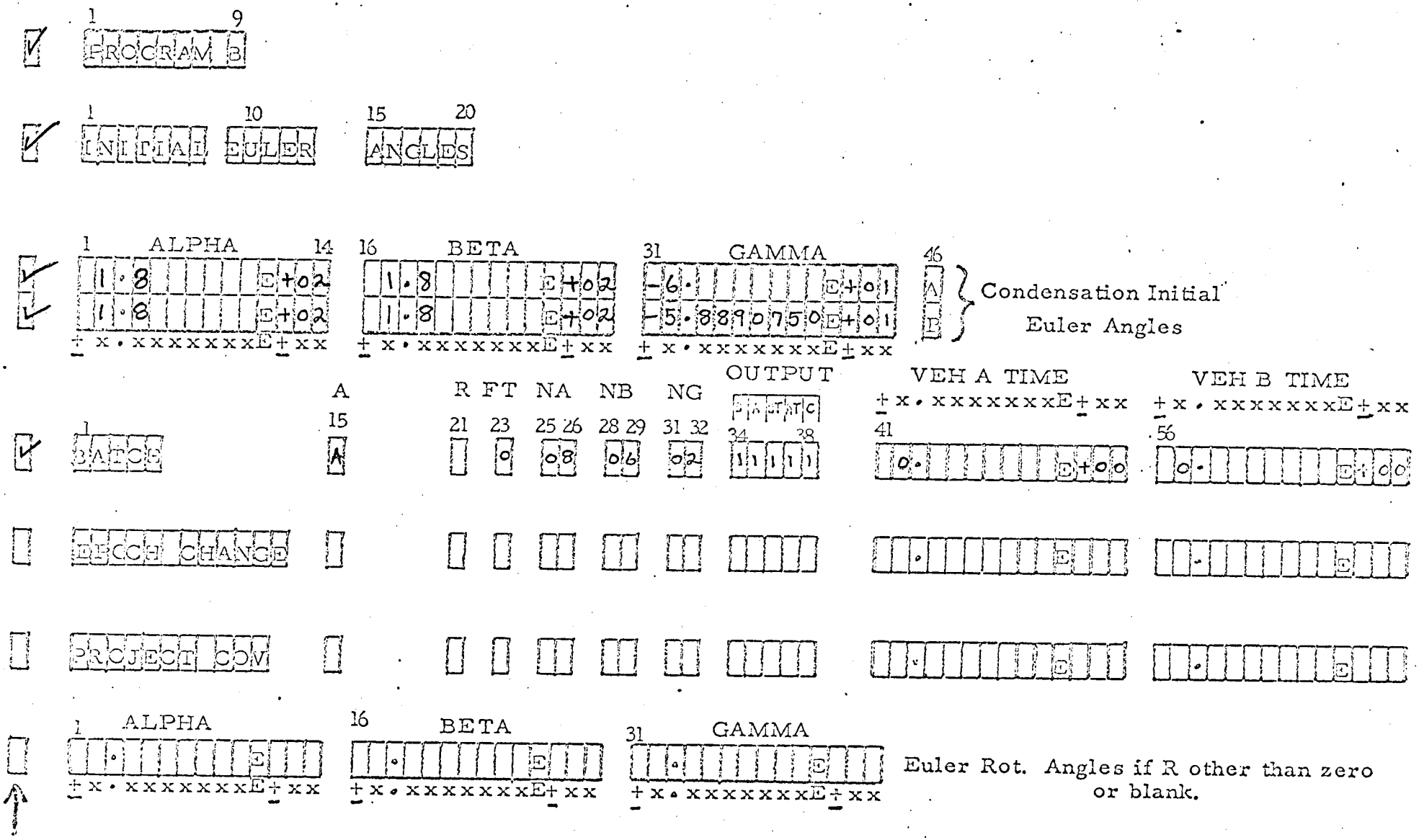


Figure 31 - Control For Single Batch of Example 6

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)	Indicators for Control of Matrix Format (33–80)
					07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
					BI US ES NS NH MU N* VA UM EM NM ED ND RM AM BS RS AS L1 L2 L3 L4 L5 L6
					BI PX PY PZ MU V1 V2 X2 Y2 Z2 XD YD ZD
004	APF	. E	. E		07 08 12 13 14 15 16 17
001	RD	4.5 E+01	4. E-02		07
002	RD	4.5 E+01	4. E-02		08
003	IA1	4.5 E+01	1. E-03		12 13 14 15 16 17
003	IA2	4.5 E+01	1. E-03		12 13 14 15 16 17
		. E	. E		
005	APN	. E	. E		12 13 14 15 16 17
		. E	. E		
006	APO1	. E	. E		23 24 25 26 27 28 29 30 31 32 33 34 35
001	RD	4.5 E+01	4. E-02		07 23 24 25 29 31 32 30
002	RD	4.5 E+01	4. E-02		08 26 27 28 29 23 24 25 31 32 33 34 35
003	IA1	4.5 E+01	1. E-03		29
003	IA2	4.5 E+01	1. E-03		29
		. E	. E		
SECOND NUISANCE		. E	. E		
PARAMETER GROUP		. E	. E		
ON NEXT FIGURE		. E	. E		
		. E	. E		

Figure 32 - Allocation for Single Batch of Example 6
(continued on next figure)

eypunch a card for each
ow checked (✓).

Indicators for Control of Matrix Format

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
					BI	US	ES	NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
					BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
1 3	4 7	8 19	20 31	32	33			40					50					60					70					80
007	APO2	.E	.E		23	24	25	26	27	28	29	30	31															
001	RD	4.5E+01	4.E-02		07																		23	24	25	26	27	28
002	RD	4.5E+01	4.E-02		08																		23	24	25	26	27	28
003	IA1	4.5E+01	1.E-03			29	30	31																				
003	IA2	4.5E+01	1.E-03			29	30	31																				

33 35 40 45 50 55 60 65 70 75 80

Figure 33 - Allocation for Single Batch of Example 6
(Concluded)

ypunch a card for each
v checked (✓).

vehicle's parameters are allocated for the intervehicle measurement part of the filter group only. The program would have performed this allocation even had these inputs been truncated to a 6 x 6. This allocation is necessary since, when estimating the parameters of vehicle A, the vehicle B parameters are treated as nuisance parameters and the appropriate weighting functions must be generated.

In order to project the results of the first batch to a later time, both vehicle A and vehicle B covariances must be projected if relative results are requested. In order to project the second vehicle, a Program A must exist for the second vehicle which, at least, generates an appropriate Q matrix. This is shown as Figure 34 and is assumed to exist on tape prior to starting the Program B. Note that the a_1, a_4, a_5, XI, ETA, and ZETA inputs are for the CSM or vehicle B.

The next series of cards are used for projecting the results from epoch (which occurred at t = 0) to t = 49.917 minutes. These are shown in Figures 35 through 38. Notice that in Figure 35, the condensation output (column 38) is not asked for since the vehicles are at different times.

This concluded the sixth and last example.

INPUTS - DATA SET 4

1	6.3626209E 06	A1
2	3.8100000E-01	A4
3	5.2084750E 03	A5
4	1.0000000E 00	GAM ID
5	0	BETA
6	1.8000000E 02	XI
7	1.8000000E 02	ETA
8	-5.8890750E 01	ZETA
9	4.0000000E 01	LAMBDA
10	-4.0000000E 00	ALPHA
11	7.2920000E-05	OMEGAE
12	2.6600000E-06	OMEGAM
13	2.0900000E 07	RHOE
14	1.2400000E 09	RHOM
15	1.7290000E 14	MU
16	4.5000000E 01	TIME
16	4.9917000E 01	TIME
17.	0	RE RAD
18	0	LT SLP
19	0	LO SLP
20	0	RD IND
21	0	R IND
22	1.0000000E 00	Q IND
23	4.9917000E 01	QF IND
24	0	A1 IND
25	0	A2 IND
26	0	PVWIND
27	1.0000000E 00	T INIT
28	1.0000000E 00	T INCR
29	3.0000000E 01	DIMENS
30	0	LAMB2
31	0	ALPHA2
32	1.0000000E 00	MS IND
33	1.0000000E 00	FM IND
34	1.0000001E 00	VISIND
35	-0	T CLK
36	-0	T XYZ
37	-0	T ORB
38	-0	T TILD
39	-0	T SHIP

Figure 34 - Program A Required to Develop a Q Matrix to Enable Projection of Vehicle B

PROGRAM B CONTROL CARD DATA INPUT SHEET

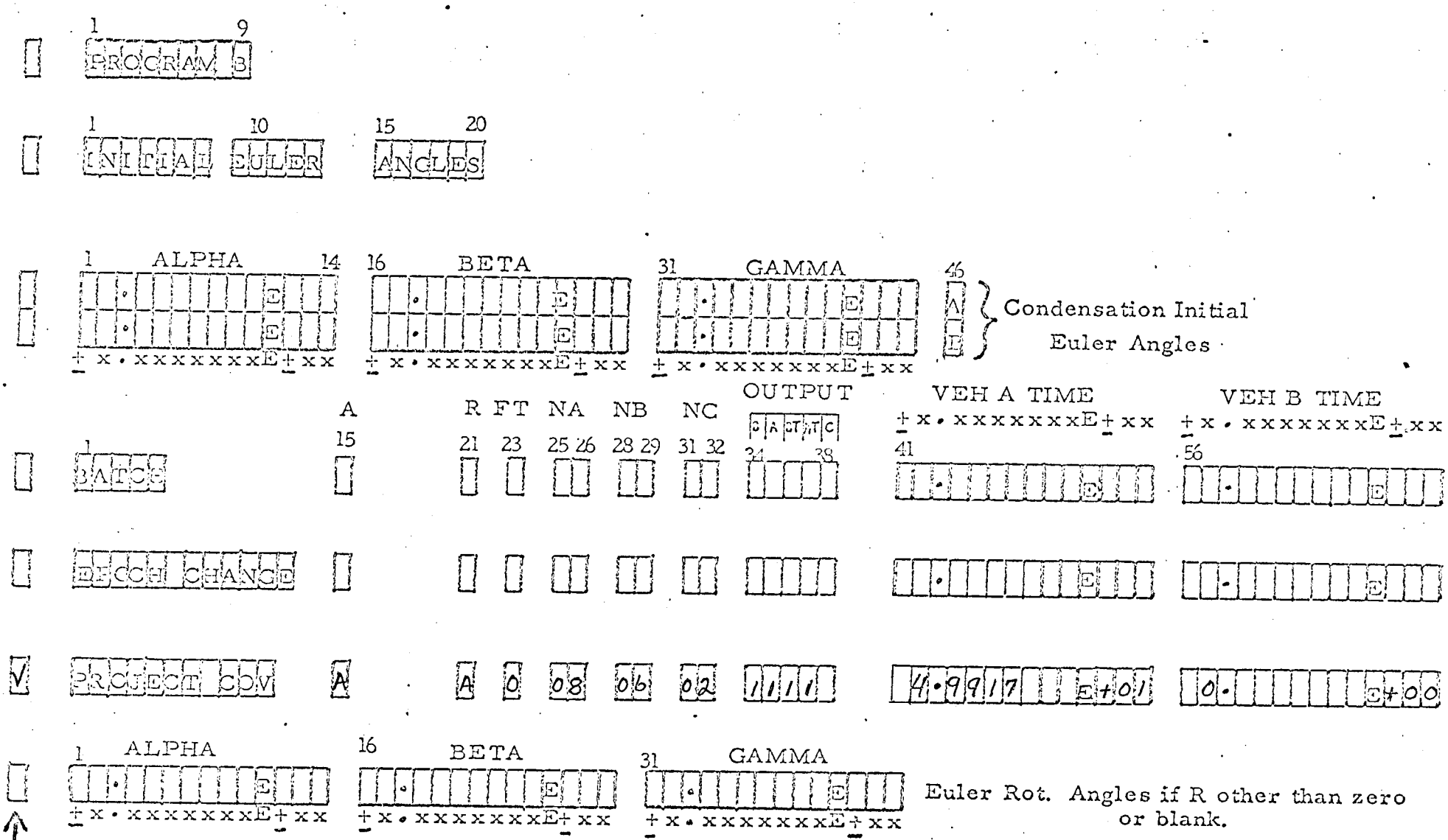


Figure 35 - Control for Projecting Vehicle A of Example 6

Indicators for Control of Matrix Format

07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	USES		NS	NH	MU	N*	VA	UM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									

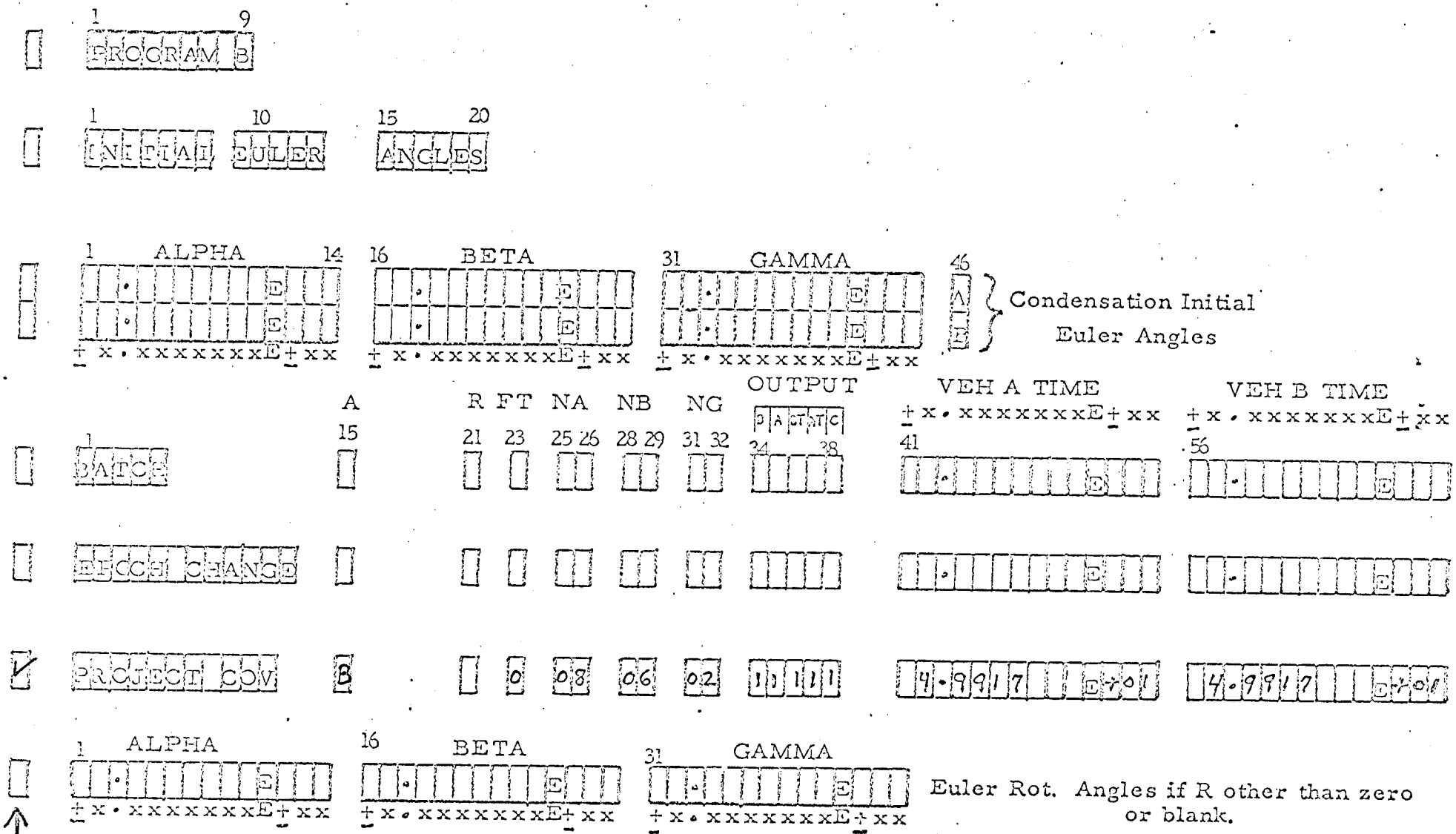
Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)
	FP			
001	Q	4.9917 E+01		
	PN			
	P01			
001	Q	4.9917 E+01		
	P02	4.9917 E+01		
001	Q			

33	35	40	45	50	55	60	65	70	75	80
			29							

Figure 36 - Allocation for Projecting Vehicle A of Example 6

ypunch a card for each
w checked (✓).

PROGRAM B CONTROL CARD DATA INPUT SHEET



Key Punch a Card for each row checked (✓).

Figure 37 - Control for Projecting Vehicle B of Example 6

PROGRAM B - ALLOCATION SHEET

Data Set (1–3)	Measurables (4–7)	Time (TJ) ±x.xxxxxExxx (8–19)	σ ±x.xxxxxE±xx (20–31)	6 x 6 (32)
	PF			
004	Q	4.9917 E+01		*
	PN			
	P01			
004	Q	4.9917 E+01		
	P02			
004	Q	4.9917 E+01		*

Indicators for Control of Matrix Format

07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BI	US	ES	NS	NH	MU	N*	VA	JM	EM	NM	ED	ND	RM	AM	BS	RS	AS	L1	L2	L3	L4	L5	L6
BI	PX	PY	PZ		MU		V1	V2	X2	Y2	Z2	XD	YD	ZD									
					29																		

Figure 38 - Allocation for Projecting Vehicle B of Example 6

ypunch a card for each
w checked (✓).

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 487
(BBC Task 105)

S. Schloss
28 March 1967

LANGLEY LUNAR ORBITER 2 - DOPPLER DATA RESIDUALS

INTRODUCTION

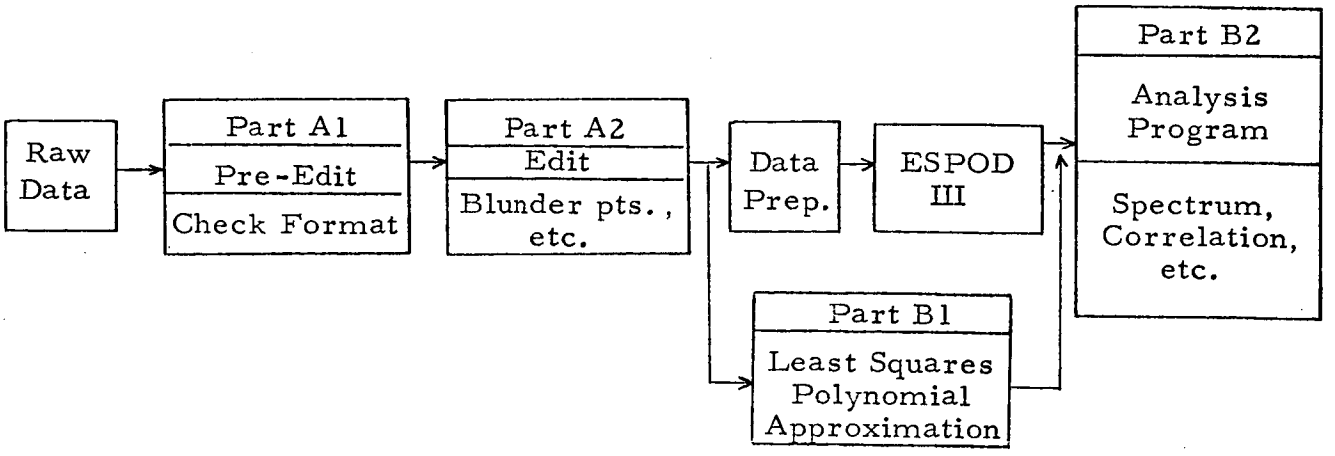
Tracking data from Langley Lunar Orbiter (LLO) Missions are being analyzed in support of the Apollo GOSS Navigation Qualification Program. The objectives of the analysis are to

(a) Check the USBS stations for proper equipment functioning and operational procedures while tracking at lunar distances;

(b) Improve the knowledge of USBS characteristics to aid in the establishment of data weights and selection criteria for the RTCC; and,

(c) Improve the USBS error model used in MSFN error analysis, dispersion studies, and RTCC simulations.

The following is an outline of the data analysis procedure:



The raw data is first checked in the Part A Program for errors in format. For details of the edit and analysis procedures see Apollo Note 465. If a data message does not conform to the appropriate format, then that entire data message is rejected. No explicit statistical rejection of data occurs at this point. However, there is a test in Part A of whether or not the data appearing at the appropriate position in the message is numerically within limits for the respective data type. If not, then it is rejected. Therefore, an implicit form of statistical rejection of data is present in Part A.

The Lunar Orbiter data presently undergoing analysis is N_1 count destruct doppler data. The quantization error is reduced by a factor of 100 in this mode and becomes a negligible error source. The sample interval for this data is 0.4 seconds. The doppler counting interval is somewhat less than 0.1 seconds each sample interval. Under these circumstances, high frequency error sources associated with random phase errors, quantization error, and high frequency clock noise will all lead to a correlation function with an expected value of zero at all lag values other than the zero lag value (the zero lag value corresponds to the combined variance of all these error sources).

We are concerned with the analysis of tracking data errors and, therefore, it would be of particular value for us to determine first a dynamic state vector or, equivalently, trajectory. This will be done in subsequent studies.

The initial LLO doppler data supplied by MSC are time measurements for $N_1 = 77,824$ zero crossings. The time is in units of 0.01 μ-seconds. To obtain approximate estimates of range-rate we can employ

$$\dot{\rho}_1 + \dot{\rho}_2 = \lambda(f_{DB} - f_B) = \frac{c}{f}(f_{DB} - f_B)$$

where

$\dot{\rho}_1 + \dot{\rho}_2$ = three-way range-rate

f = RF receiver reference frequency ~2.3 x 10^9 Hz

f_B = receiver bias frequency superimposed on the doppler frequency = 10^6 Hz

f_{DB} = bias frequency plus average doppler frequency during the counting interval.

Now

$$f_{DB} = \frac{\Delta n}{\Delta t}$$

where $\Delta n = N_1$ (for the present data) = 77,824; Δt = time for Δn zero crossings.

Variance of Approximating Polynomial

Solving the normal equations for the coefficients of the least squares approximating polynomial will result, in general, in the inversion of an ill-conditioned matrix. This matrix can be approximated by the principal minor of the Hilbert matrix.[1] However, if the degree of the polynomial is small, the normal equations can be solved accurately in a direct manner. Furthermore, the resulting polynomial will be almost identical with that obtained through the use of the orthogonal Gram polynomials. This result allows us to employ variance estimates which were derived under the assumption that the approximating polynomial was determined in this manner.

[1] Ralston, A., A First Course in Numerical Analysis, McGraw-Hill Book Co., New York, 1965.

Specifically, let σ^2 be the variance of the doppler observable $\tau(t_i)$ and $V(t_i)$, $i = 0, 1, \ldots, n-1$, the variance of the approximating polynomial, $x(t)$. Let k be the largest integer such that $2k \leq m$, where m is the degree of $x(t)$. We define $R(t)$ as

$$R(t) = \frac{V(t)}{\sigma^2} . \tag{1}$$

Then

$$R_{2k}\left(\frac{n-1}{2}\right) = \frac{1}{n}\left[1 + 5\left(\frac{1}{2}\right)^2 \frac{n^2-1}{n^2-4} + 9\left(\frac{1}{2}\cdot\frac{3}{4}\right)^2 \frac{n^2-1}{n^2-4}\cdot\frac{n^2-9}{n^2-16}\right.$$

$$+ \cdots (4t+1)\left(\frac{1}{2}\cdot\frac{3}{4}\cdots\frac{2t-1}{2t}\right)^2 \frac{n^2-1}{n^2-4}\cdot\frac{n^2-9}{n^2-16}$$

$$\left.\cdots \frac{n^2-(2t-1)^2}{n^2-(2t)^2}\right] . \tag{2}$$

For the initial studies $n = 1500$, therefore the variance reduction factor evaluated at the midpoint of the time span would be:

Degree	1	2	3	4	5	6	7
$R_{2k}\left(\frac{n-1}{2}\right)$	.00067	.00150	.00150	.00234	.00234	.00319	.00319

Here we employ the following approximation to Equation (2)

$$R_{2k}\left(\frac{n-1}{2}\right) = \frac{1}{n}\left[1 + 5\left(\frac{1}{2}\right)^2 + \cdots (4t+1)\left(\frac{1}{2}\cdot\frac{3}{4}\cdots\frac{2t-1}{2t}\right)^2\right] \tag{3}$$

Since

$$1 \leq \frac{n^2 - a}{n^2 - b} \leq 1 + 10^{-5}$$

for n = 1500 and (a, b) = (1, 4), (9, 16), (25, 36).

The previous studies of doppler residuals (e. g., Apollo Note 265) have resulted in selection of m = 4 for the degree of the approximating polynomials for translunar flight phases. For lunar orbital data, it is expected that

1) The appropriate degree will, in general, be greater than or equal to 4, and

2) The degree will not be the same for short tracking data segments from different portions of an orbit.

Initial experience with the data supports these conjectures. This will be discussed in detail in a subsequent note.

Data Summary

The LLO2 data initially considered were taken Dec. 23, 1966. Since we were employing polynomial approximation to the data, only relatively short tracking segments were considered. We considered six intervals (see Table 1) and found that in five of these, a fourth degree polynomial removed the trend adequately. Case No. 3 which begins at 4:53:35.2 GMT Dec. 23, 1966, is being studied further to determine if a higher degree polynomial is necessary for this interval.

Conservative Estimate of the Noise Error Component

A preliminary estimate of the standard deviation of the noise error was made. The blunder points were identified and removed prior to this analysis.

LL02 Hawaii December 23, 1966							
Case	Analyzed Data Point Interval	GMT Interval	Number of Raw Data Time Points	First Edit Bad Points / Format, Label Sync, Time, Etc.	Number of First Post-Edit Points	Second Edit Bad Points / 2d Diff. Sigma	Number of Second Post-Edit Points
1	51 - 1550	4:25:58.0 - 4:39:56.0	2095	595	1500	266	1234
2	1501 - 3000	4:39:27.2 - 4:53:34.8	2119	619	1500	247	1253
3	3001 - 4500	4:53:34.8 - 5:06:06.4	1879	379	1500	31	1469
4	4501 - 6000	5:06:06.8 - 5:17:23.6	1693	193	1500	20	1480
5	6001 - 7500	5:17:24.0 - 5:28:11.6	1620	120	1500	23	1477
6	7501 - 9000	5:28:12.0 - 5:38:22.4	1527	27	1500	0	1500

Table 1 - Data Interval Summary

Since

$$f = \frac{N}{t}$$

and

$$df = \frac{-N}{t^2}\, dt \quad .$$

To obtain a conservative estimate, we employ the minimum time count value.

$$t_{min} = 7.95 \times 10^{-2} \text{ sec.}$$

$$N = 77,824 = N_1$$

$$dt = 10^{-8}\, dc \quad ,$$

where C is the number of counts.

Hence,

$$df = .123 \; dc$$

and

$$\sigma_f = .123 \; \sigma_c$$

$$\sigma_{\dot{\rho}_1 + \dot{\rho}_2} = \lambda\, \sigma_f = \frac{10^9}{2.3 \times 10^9} (.123)(15.8)$$

$$\sigma_{\dot{\rho}_1 + \dot{\rho}_2} = .845 \text{ ft/sec for } .0795 \text{ second sample.}$$

From the ANWG Technical Report No. AN-1.1 the value of the error sigma for doppler destructive n count is given by

$$\sigma_{\dot{\rho}_1 + \dot{\rho}_2} = 1.6 \text{ ft/sec for } .08 \text{ second}$$

Therefore, forming the ratio

$$\frac{\sigma\,(\text{LL02 data})}{\sigma\,(\text{ANWG})} = \frac{.845}{1.6} \doteq .53 \quad ,$$

indicates that the LL02 data noise error component lies within ANWG navigation system accuracy values.

Data Interval	σ_c Count	$\sigma_{\rho_1+\rho_2}$ (ft/sec)	$\frac{\sigma(\text{LLO2 Data})}{\sigma\ (\text{ANWG})}$
51 - 1050	15.8	.845	.53
51 - 1550	13.8	.738	.46
1501 - 3000	11.1	.594	.37
3001 - 4500	11.4	.610	.38
4501 - 6000	10.4	.556	.35
6001 - 7500	10.1	.540	.34
7501 - 9000	10.4	.556	.35

Table 2 - Data Residual Sigma Values

Correlation and Spectral Analysis of Residuals

As indicated from the data summary in Table 1 and the graphs of polynomial approximation residuals, the high frequency noise of the doppler signal is composed of components from at least two different distributions. Initially, we identify and remove the "blunder" points by employing a 4-6 sigma threshold on the second differences of the data. This deletes from zero to 12.7 per cent of the format valid Hawaii, Dec. 23, 1966 data. The Nyquist frequency for this data is

$$\omega_o = (1/2\Delta t) = 1.25 \text{ cps}$$

since the time between samples is 0.4 seconds.

The correlation plots provide values of the autocorrelation coefficient, ρ_k, for 1-40 lags in initial plots and 1-100 lags in subsequent plots. One lag corresponds to approximately 0.4 seconds. Neglecting the aliasing effects, the auto correlation coefficient and spectrum plots indicate several noise error characteristics:

1. Raising the degree of the polynomial from 4 to 6 substantially reduces the one lag autocorrelation coefficient for the data point interval, 3001-4500 but not for the other intervals of the Hawaii, Dec. 23, 1966 data.

2. There is: (a) a component with a short correlation time, and (b) a noise factor in the Hawaii data with an approximate period of 11.2 to 12.4 seconds indicated in the correlation function graphs. The spectrum indicates that (b) has a period of 12.8 seconds. This is not present in the Carnarvon, Dec. 22, 1966 data.

The standard deviation in the computed value of ρ_k is $(N - k - 1)^{-1/2}$.

	Lag				
	1	2	5	20	40
σ_{ρ_k}	.02584	.02585	.02587	.02600	.02618

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 488
(BBC Task 204)

C. H. Dale
L. Lustick

TRACKING THE LEM AND CSM FROM LEM TAKEOFF TO RENDEZVOUS

This Apollo Note describes the first use of the RTODP-OEAP for the whole Lunar takeoff to rendezvous mission. In the first computer run all RTODP variables were chosen to be optimum - at least as far as is known. In further computer studies, variation in the RTODP make-up should be made. This computer study includes results with and without intervehicle radar tracking during the terminal phase of the rendezvous mission; however the assumed characteristics of the intervehicle radar are just a guess, and faith in the results should await refined assumptions.

In any case, this example is a good start at Lunar rendezvous navigation analysis and also acts as a good example for showing the capability of the RTODP-OEAP. The order of presentation within this Note will be:

		Page No.
1.0	Definition of Tracking Situation	2
2.0	Graphical Summary of Results	4
3.0	A Computer Listing of Input Card Deck	8
4.0	Computer Results	18

1.0 Definition of Tracking Situation

Tracking consists of Madrid (master), Ascension and Bermuda using one-minute, $\sigma = 0.04$ ft/sec, Doppler measurables. The tracking encompasses the Lunar take-off to rendezvous portion of the Apollo mission. The Moon is in the Earth's equatorial plane with an initial sublunar point of zero latitude and longitude. The CSM is tracked, when visible, from its first appearance from behind the Moon before LEM take-off to the last LEM boost prior to rendezvous. Three LEM boosts occur between take-off and the nominal time of rendezvous wherein the RTODP filter boost assumption equal the true component uncertainties of 0.5 ft/sec in each axis. The actual apriori state components of uncertainty for the CSM and LEM, both at the time of LEM ascent burnout, are individually equal (5,000 ft. and 10 ft/sec in each axis). Because the CSM has been tracked previously while the LEM has just undergone a large boost, it was decided to downweight the LEM apriori state more. Thus, the RTODP filter apriori state of the CSM was assumed equal to the actual, while the LEM filter assumptions were doubled (10,000 ft/ and 20 ft/sec in each axis). Batching occurs between boosts with estimated pseudo-biases in each measurable re-initialized with an apriori uncertainty of 0.1 ft/sec. each. The nuisance parameter assumptions are shown in the data input listing. Results are computed with and without 30 minutes of intervehicle radar (on the LEM) data starting after the last boost prior to rendezvous. All tracking stops 30 minutes after this boost, and results at nominal rendezvous are shown for: The CSM, the LEM with and without intervehicle data, and the LEM/CSM relative state with and without intervehicle data. It should be noted that the assumed one-minute sample noise figures for the intervehicle radar ($\sigma r = 100$ ft, $\sigma \dot{r} = 0.3$ ft/sec, $\sigma Az = \sigma E = 0.001$ radian) are not firmly known.

The LEM and CSM orbits are in the plane of the Moon's orbit which, as stated before, is in the plane of the Earth's equator. The following sketch shows the geometry. Actually it was possible to fit one ellipse to the entire LEM orbit, making rotations at boost times unnecessary. The computer orbits match perfectly at rendezvous and TPI, while the LEM orbit is not quite right during the earlier portion of the trajectory. This in no way affects the results, which are general for any near-orbit situation.

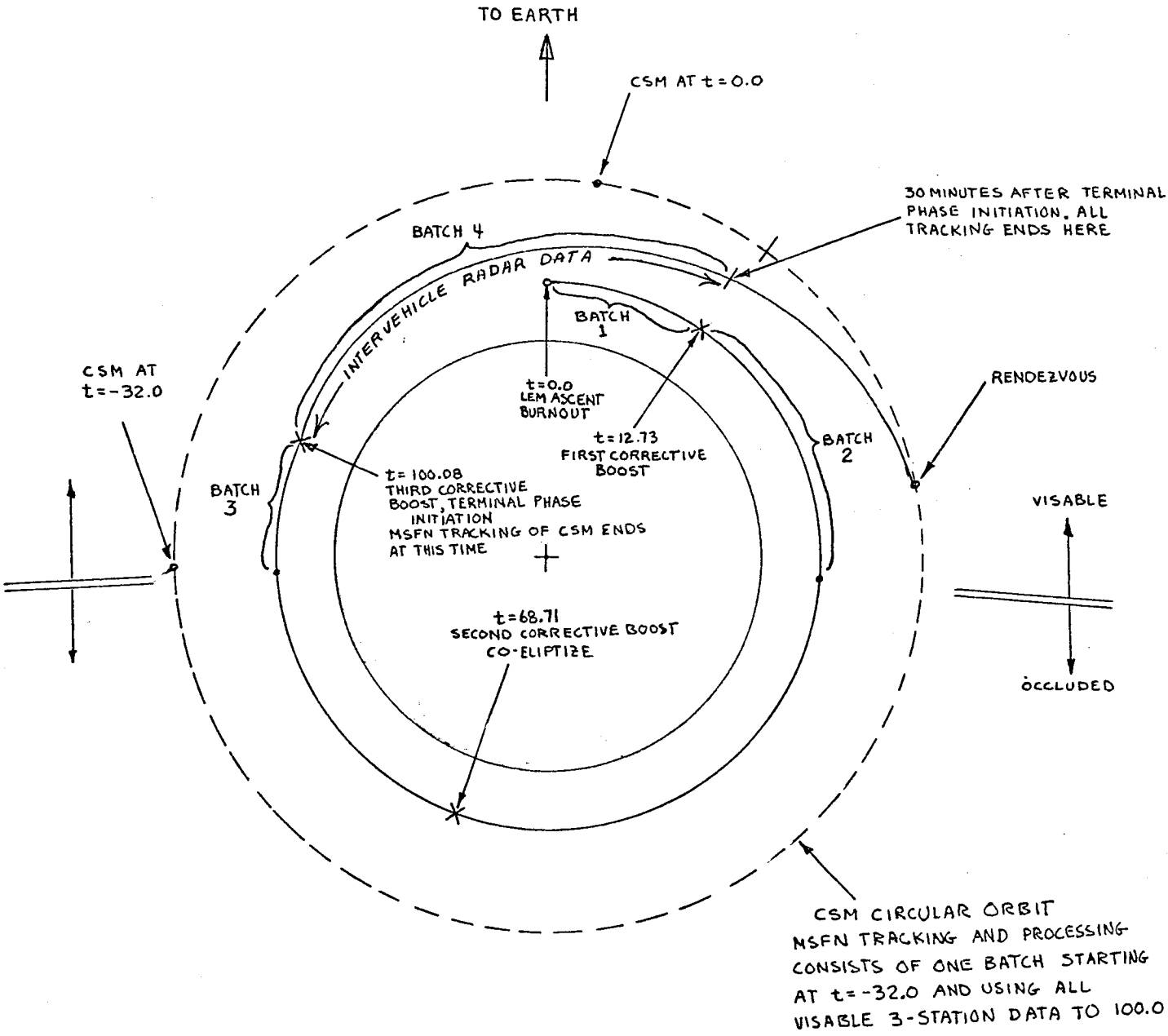


Figure 1 - Approximate Tracking Geometry

2.0 Graphical Summary of Results

Figure 2 shows the total and in-plane position uncertainty of the LEM, CSM, and LEM/CSM relative position at various events along the mission. Figure 3 shows the total velocity uncertainty in the same manner. Figure 4 shows each component of position and velocity uncertainty along the way. The coordinate system is always locally defined with x through the vehicle and away from the center of attraction, y in-plane, and z out-of-plane. Though a plot of in-plane velocity uncertainty is not shown, a glance at Figure 4 shows that the z component of velocity uncertainty swamps the in-plane terms. Since this is the first time that the entire LEM ascent and rendezvous has been analyzed by the RTODP-OEAP it is not felt that hard and fast conclusions should be offered. However, a few obvious trends should be noted:

1. The MSFN North-South separation of this example is large. The out-of-plane, z, direction parallels this separation. The absolute LEM and CSM position and velocity uncertainties are predominently in this direction, and can thus be expected to increase proportionately to any decrease in North-South station separation.

2. The LEM/CSM relative state uncertainty components, at the time of rendezvous, are not greatly different from the absolute LEM components, under the assumption of MSFN tracking only.

3. Intervehicle tracking (with the assumed measurement statistics of this example) greatly decreases the out-of-plane LEM/CSM relative position and velocity uncertainties at rendezvous, and thus greatly decreases the total relative state uncertainty at rendezvous. This effect would be even more noticable if the MSFN station North-South separation were smaller.

4. The absolute LEM position uncertainty at rendezvous is slightly improved by intervehicle data, while the velocity uncertainty is degraded.

5. Further CSM batching may improve its state uncertainty at rendezvous.

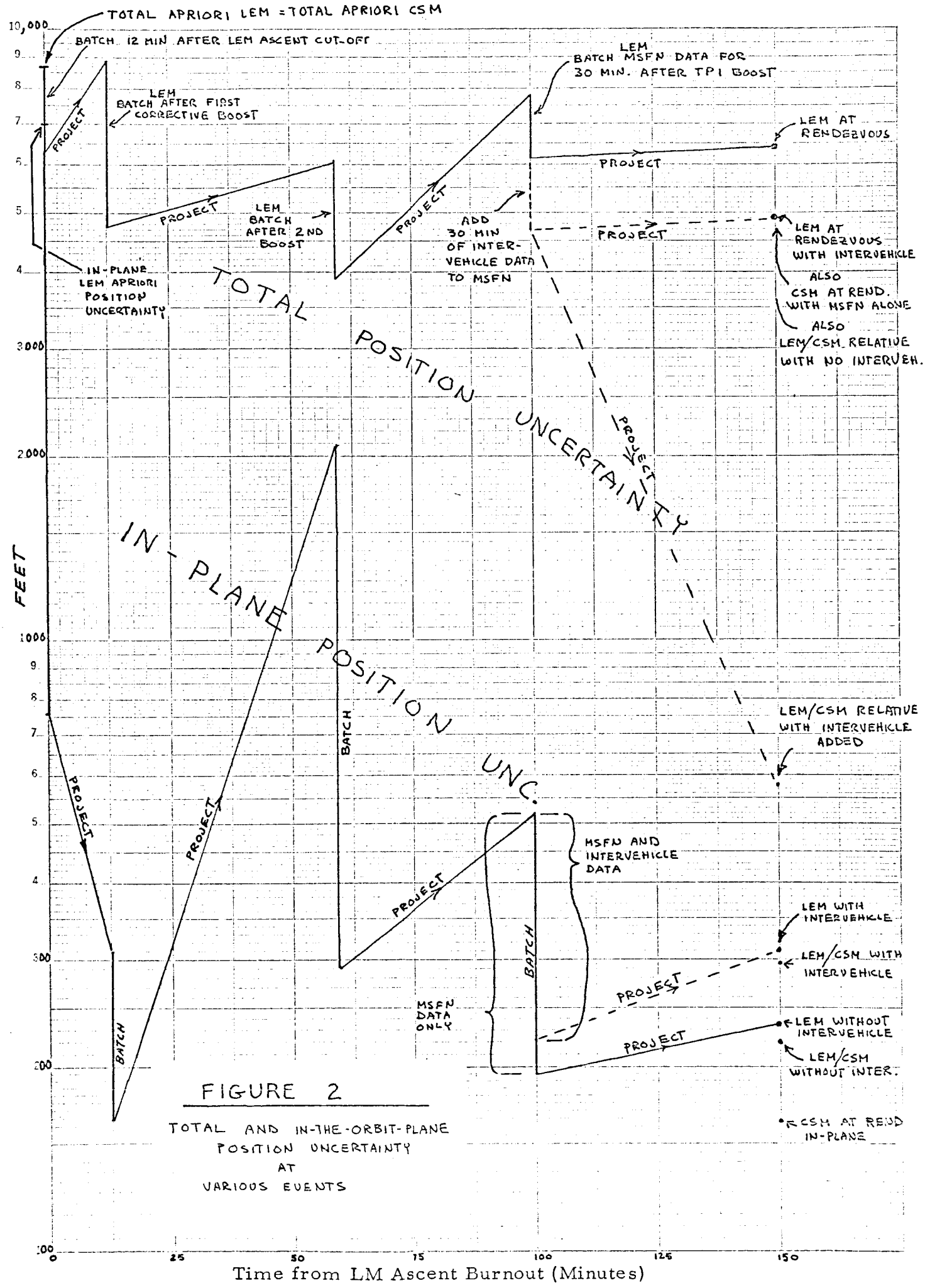
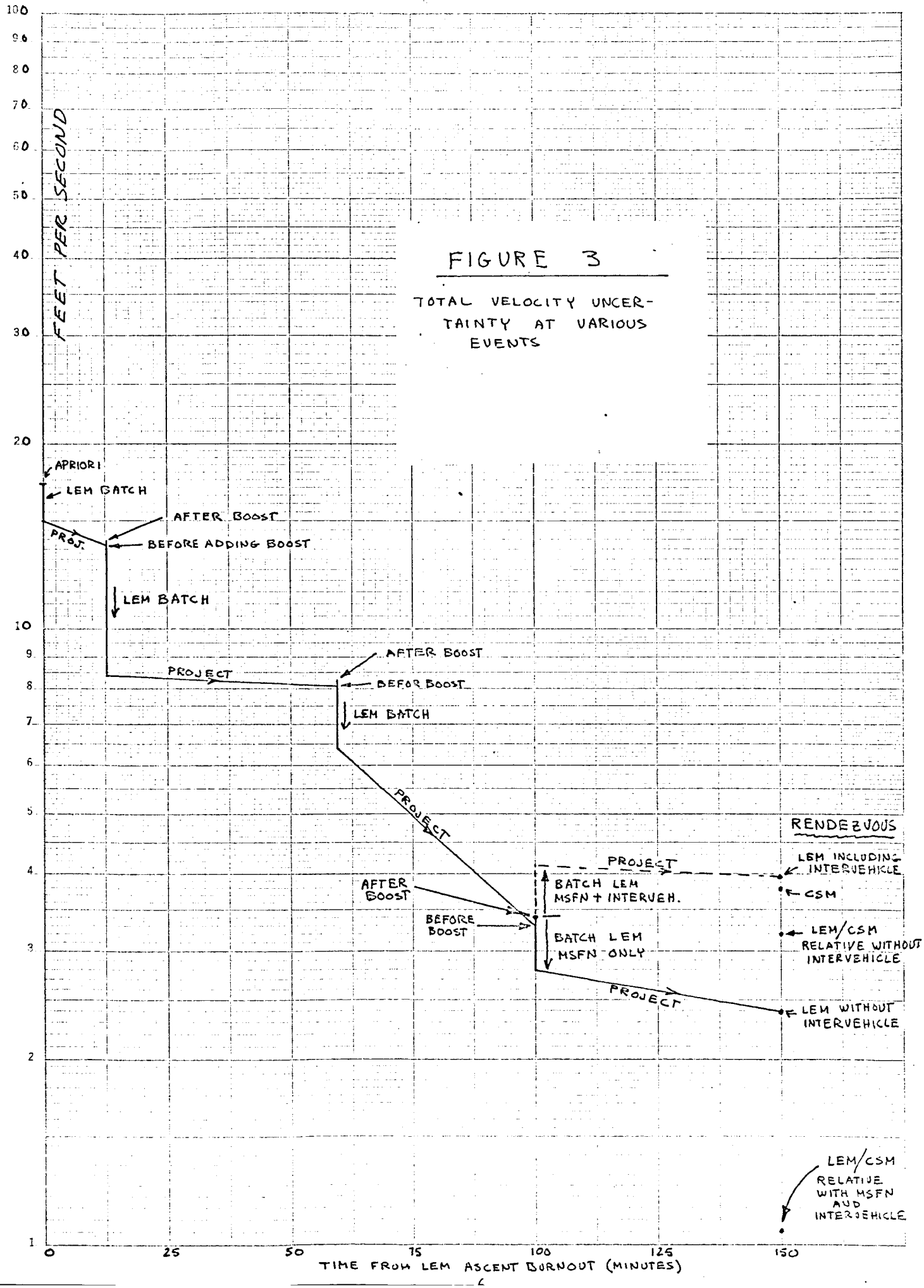


FIGURE 2

TOTAL AND IN-THE-ORBIT-PLANE POSITION UNCERTAINTY AT VARIOUS EVENTS

SEMI-LOGARITHMIC 359-61 KEUFFEL & ESSER CO. MADE IN U.S.A. 2 CYCLES X 70 DIVISIONS



SEMI-LOGARITHMIC 358-61
KEUFFEL & ESSER CO. MADE IN U.S.A.
2 CYCLES X 70 DIVISIONS

Epoch	Event	FEET					FEET PER SECOND			
		x	y	z	total position	in-plane position	$\dot{x}$	$\dot{y}$	$\dot{z}$	total velocity
0	Apriori CSM	5000	5000	5000	8660	7070	10	10	10	17.32
0	CSM After All Tracking	74.05	49.15	4823	4823	89	.0419	.0458	3.881	3.881
0	Apriori LEM	5000	5000	5000	8660	7070	10	10	10	17.32
0	LEM at t = 0 after first Batch	188.0	729.3	6302	6347	760	.1880	.8742	14.97	15.00
12.73	LEM projected to first boost	195.9	241.1	9350	9355	310	.6803	.8671	13.85	13.89
12.73	Boost Cov added to LEM	195.9	241.1	9350	9355	310	.8443	1.001	13.86	13.92
12.73	Add second batch to LEM	86.99	139.6	4764	4767	164	.2208	.1601	8.434	8.438
68.71	LEM projected to second boost	795.5	1919	5708	6074	2075	1.601	.4709	8.027	8.198
68.71	Boost cov added to LEM	795.5	1919	5708	6074	2075	1.677	.6868	8.042	8.244
68.71	Add third batch to LEM	59.17	287.1	3886	3897	293	.1908	.2910	6.425	6.434
100.1	LEM projected to third boost	501.6	105.7	7798	7815	512	.5324	.1382	3.193	3.240
100.1	Boost cov added to LEM	501.6	105.7	7798	7815	512	.7303	.5188	3.232	3.354
100.1	Add fourth MSFN batch to LEM	149.6	122.1	6195	6198	193	.2023	.0481	2.785	2.793
150.0	Project LEM to rendezvous	194.6	130.9	6447	6451	234	.1628	.0656	2.366	2.373
150.0	Project CSM to rendezvous	59.76	152.4	4916	4919	163	.1180	.0577	3.802	3.804
150.0	LEM/CSM relative (MSFN only)	166.2	144.4	4846	4851	220	.1331	.0640	3.224	3.227
	Second Case Starts with Fourth MSFN Batch Being Completed. This result is Repeated Below and Following Lines show Intervehicle Results.									
100.1	Add fourth MSFN batch to LEM	149.6	122.1	6195	6198	193	.2023	.0481	2.785	2.793
100.1	Add Intervehicle data to LEM	155.5	157.7	4702	4707	221	.3266	.5415	4.114	4.127
150.0	Project LEM to rendezvous	291.1	106.9	4894	4904	310	.2548	.1566	3.949	3.970
150.0	LEM/CSM relative (Intervehicle included)	278.4	106.9	497	580	297	.2941	.2482	.9294	1.006

Figure 4 - State Component Uncertainties at Various Events

3.0 A Computer Listing of the Input Card Deck

It is hoped that the results and inputs of this example may both be of use to analysts at MSC. Thus a 1004 computer listing of the inputs was printed and is included in this Apollo Note on the following nine pages. This listing shows each input card, including blank cards; although blank cards are not seen clearly when occurring at the beginning or ending of a page of the listing. The column of each entry on a card is not indicated, however Apollo Note No. 482 may be used to answer any such question.

```
PROGRAM A XX   MAD BTCH 1        001
01      6.3391523E+06
02      4.3299405E-01
03      5.2260479E+03
04      1.         E+00
05      0.         E+00
06      0.         E+00
07      1.80       E+02
08      1.80       E+02
09      4.0417     E+01
10     -3.667      E+00
11      7.292      E-05
12      2.66       E-06
13      2.09       E+07
14      1.24       E+09
15      1.7313945E+14
16      1.2        E+01
17      0.         E+00
18      0.         E+00
19      0.         E+00
20      1.         E+00
21      0.         E+00
22      0.         E+00
23      1.273783   E+01
24      0.         E+00
25      0.         E+00
26      0.         E+00
27      1.         E+00
28      1.         E+00
29      3.0        E+01
30      0.         E+00
31      0.         E+00
32      1.         E+00
33      1.         E+00
34      0.         E+00
35      0.         E+00
36      0.         E+00
37      0.         E+00
38      0.         E+00
39      0.         E+00

PROGRAM A       ASCN BTCH 1      002
09     -7.97       E+00
10     -1.44       E+01
30      4.0417     E+01
31     -3.667      E+00
32      0.         E+00

PROGRAM A       BERM BTCH 1      003
09      3.235      E+01
10     -3.466      E+01

PROGRAM A       MAD BTCH 2       004
01      6.3394111E+06
02      2.2171715E-01
```

```
03      5.2258546E+03
08     -1.4390127E+02
09      4.0417    E+01
10     -3.667     E+00
16      3.1       E+01
23      6.8706750E+01
27      1.3       E+01
30      0.        E+00
31      0.        E+00
32      1.        E+00
37      1.273783  E+01

PROGRAM A       ASCN BTCH 2       005
09     -7.97      E+00
10     -1.44      E+01
30      4.0417    E+01
31     -3.667     E+00
32      0.        E+00

PROGRAM A       BERM BTCH 2       006
09      3.235     E+01
10     -3.466     E+01

PROGRAM A       MAD BTCH  3       007
01      6.3384999E+06
02     -3.6351300E-01
03      5.2265653E+03
08      1.4720950E+01
09      4.0417    E+01
10     -3.667     E+00
16      1.00      E+02
23      1.000839  E+02
27      9.5       E+01
30      0.        E+00
31      0.        E+00
32      1.        E+00
37      6.8706750E+01

PROGRAM A       ASCN BTCH 3       008
09     -7.97      E+00
10     -1.44      E+01
30      4.0417    E+01
31     -3.667     E+00
32      0.        E+00

PROGRAM A       BER BTCH  3       009
09      3.235     E+01
10     -3.466     E+01

PROGRAM A       MAD BTCH  4       010
01      6.3384405E+06
02      3.1365863E-01
03      5.2266347E+03
08      1.0366934E+02
09      4.0417    E+01
```

```
10      -3.667      E+00
16       1.20       E+02
16       1.30       E+02
16       1.49       E+02
23       1.4999680E+02
27       1.01       E+02
30       0.         E+00
31       0.         E+00
32       1.         E+00
37       1.000839 E+02

PROGRAM A       ASCN BTCH 4       011
09      -7.97       E+00
10      -1.44       E+01
30       4.0417     E+01
31      -3.667      E+00
32       0.         E+00

PROGRAM A       BERM BTCH 4       012
09       3.235      E+01
10      -3.466      E+01

PROGRAM A       MAD CSM  T1       013
01       6.3835625E+06
02       0.         E+00
03       5.2079459E+03
08      -1.784579 E+02
09       4.0417     E+02
10      -3.667      E+00
16       0.         E+00
16       3.1        E+01
16       1.00       E+02
16       1.20       E+02
16       1.30       E+02
16       1.49       E+02
20       1.         E+00
21       0.         E+00
22       1.         E+00
23       1.4999680E+02
27      -3.2        E+01
32       1.         E+00
37       0.         E+00

PROGRAM A       ASN CSM  T1       014
09      -7.97       E+00
10      -1.44       E+01
30       4.0417     E+01
31      -3.667      E+00
32       0.         E+00

PROGRAM A       BER CSM  T1       015
09       3.235      E+01
10      -3.466      E+01

PROGRAM C       LEM TO CSM 4      016
```

```
01      5.5384405E+06
02      5.1365833E-01
03      5.2266347E+03
06      0.        E+00
07      1.8       E+02
08      1.0366934E+02
47      1.000839 E+02
41      6.3635625E+06
42      0.        E+00
43      5.2079459E+03
44      0.        E+00
45      1.80      E+02
46     -1.784579 E+02
48      0.        E+00
27      1.01      E+02
28      1.        E+00
16      1.20      E+02
16      1.30      E+02
16      1.49      E+02
21      1.        E+00
20      1.        E+00
24      1.        E+00
25      1.        E+00
22      0.        E+00
23      0.        E+00
33      1.        E+00
15      1.7313945E+14

PROGRAM A XX   APF                  001
01 01   1.        E+08    LEM BRNOUT X
02 02   1.        E+08    LEM BRNOUT Y
03 03   1.        E+08    LEM BRNOUT Z
04 04   4.        E+02    LEM BRNX DOT
05 05   4.        E+02    LEM BRNY DOT
06 06   4.        E+02    LEM BRNZ DOT
07 07   1.        E-02    MADLEM PBIAS
08 08   1.        E-02    ASNLEM PBIAS
09 09   1.        E-02    BERLEM PBIAS
10 10   1.        E+00    DUMMY
11 11   1.        E+00    DUMMY
12 12   2.5       E+07    CSM AT T1  X
13 13   2.5       E+07    CSM AT T1  Y
14 14   2.5       E+07    CSM AT T1  Z
15 15   1.        E+02    CSM AT X DOT
16 16   1.        E+02    CSM AT Y DOT
17 17   1.        E+02    CSM AT Z DOT
18 18   1.        E-02    MADCSM PBIAS
19 19   1.        E-02    ASNCSM PBIAS
20 20   1.        E-02    BERCSM PBIAS

PROGRAM A      APN                  002
01 01   2.5       E+07    LEM BRNOUT X
02 02   2.5       E+07    LEM BRNOUT Y
03 03   2.5       E+07    LEM BRNOUT Z
04 04   1.        E+02    LEM BRNX DOT
```

```
05 05   1.        E+02    LEM BRNY DOT
06 06   1.        E+02    LEM BRNZ DOT
12 12   2.5       E+07    CSM AT T1   X
13 13   2.5       E+07    CSM AT T1   Y
14 14   2.5       E+07    CSM AT T1   Z
15 15   1.        E+02    CSM AT X DOT
16 16   1.        E+02    CSM AT Y DOT
17 17   1.        E+02    CSM AT Z DOT

PROGRAM A      A                 003
07 07   1.        E-02    MADLEM PBIAS
08 08   1.        E-02    ASNLEM PBIAS
09 09   1.        E-02    BERLEM PBIAS
18 18   1.        E-02    MADCSM PBIAS
19 19   1.        E-02    ASNCSM PBIAS
20 20   1.        E-02    BERCSM PBIAS

PROGRAM A      AP01              004
07 07   2.5       E+19    LUNAR MU
08 08   1.5       E+04    MAD STA   UP
09 09   1.5       E+04    MAD STA EAST
10 10   1.5       E+04    MAD STA NORT
11 11   1.0       E+05    ASN STA   UP
12 12   1.0       E+05    ASN STA EAST
13 13   1.0       E+05    ASN STA NORT
14 14   1.0       E+05    BER STA   UP
15 15   1.0       E+05    BER STA EAST
16 16   1.0       E+05    BER STA NORT
17 17   2.5       E-05    MAD CLK BIAS
18 18   4.        E-20    MAD CLK RATE
19 19   2.5       E-05    ASN CLK BIAS
20 20   4.        E-20    ASN CLK RATE
21 21   2.5       E-05    BER CLK BIAS
22 22   4.        E-20    BER CLK RATE

PROGRAM A      AP02              005
07 07   2.551     E+07    LUN EPH   L1
08 08   1.541     E+07    LUN EPH   L2
09 09   1.        E+00    LUN EPH   L3
10 10   1.252     E-04    LUN EPH   L4
11 11   9.27      E-04    LUN EPH   L5
12 12   1.26      E-05    LUN EPH   L6
07 11   1.35      E+02    LUN EPH1 X 5
11 07   1.35      E+02    LUN EPH5 X 1
08 10  -4.392     E+01    LUN EPH2 X 4
10 08  -4.392     E+01    LUN EPH4 X 2
13 13   1.        E-06    LEM PLAT  PX
14 14   1.        E-06    LEM PLAT  PY
15 15   1.        E-06    LEM PLAT  PZ
16 16   9.        E+02    LMRAD R BIAS
17 17   1.        E-01    LMRAD RDBIAS
18 18   2.5       E-05    LMRAD A1BIAS
19 19   2.5       E-05    LMRAD A2BIAS

PROGRAM A      ABF               006
```

```
04 04   2.5        E-01     LEM FILTER
05 05   2.5        E-01     BOOST IN XYZ
06 06   2.5        E-01     AT T2,T3,T5

PROGRAM A       ABR                 007
04 04   2.5        E-01     LEM REAL
05 05   2.5        E-01     BOOST UNC AT
06 06   2.5        E-01     T2,T3,AND T5

PROGRAM B
INITIAL EULER ANGLES
 0.        E+00  1.80      E+02  1.80      E+02 A
 0.        E+00  1.80      E+02 -1.734579 E+02 B
BATCH          B      0 0 09 09 02 1111     0.        E+00  0.        E+00
001APF
013RD   1.      E+02 4.      E-02 07
014RD   1.      E+02 4.      E-02 08
015RD   1.      E+02 4.      E-02 09

002APN

004AP01                                 2324252627282930313233343536373 8
013RD   1.      E+02 4.      E-02 07242526  23                34   33
014RD   1.      E+02 4.      E-02 06272829  23     242526     34   3536
015RD   1.      E+02 4.      E-02 09303132  23     242526     34   3738

005AP02                                 232425262728293031323334 35
013RD   1.      E+02 4.      E-02 07                                        232425262728
014RD   1.      E+02 4.      E-02 08                                        232425262728
015RD   1.      E+02 4.      E-02 09                                        232425262728

BATCH          A      0 0 09 09 02 11111    0.        E+00  0.        E+00
   PF
001RD   1.2     E+01 4.      E-02 07
002RD   1.2     E+01 4.      E-02 08
003RD   1.2     E+01 4.      E-02 09

   PN

   P01
001RD   1.2     E+01 4.      E-02 07242526  23                34   33
002RD   1.2     E+01 4.      E-02 06272829  23     242526     34   3536
003RD   1.2     E+01 4.      E-02 09303132  23     242526     34   3738

   P02
001RD   1.2     E+01 4.      E-02 07                                        232425262728
002RD   1.2     E+01 4.      E-02 08                                        232425262728
003RD   1.2     E+01 4.      E-02 09                                        232425262728

EPOCH CHANGE   A      0 0 09 09 02 1111     1.273783 E+01  0.        E+00
   PF                                       121314151617
006ABF
001G    1.273783E+01            *

   PN                                       121314151617
```

```
007ABR

   P01
001Q     1.2737830E+01                          23

   P02
001Q     1.2737830E+01              *

BATCH          A      0 0 09 09 02 1111     1.2737830E+01  0.        E+00
   PF                                        121314151617
003A
004RD    3.1      E+01 4.      E-02 07
005RD    3.1      E+01 4.      E-02 08
006RD    3.1      E+01 4.      E-02 09

   PN                                        121314151617

   P01                                       121314151617          2324252627282930
004RD    3.1      E+01 4.      E-02 07242526  23              34  33
005RD    3.1      E+01 4.      E-02 08272829  23    242526    34  3536
006RD    3.1      E+01 4.      E-02 09303132  23    242526    34  3738

   P02                                       121314151617          2324252627282930
004RD    3.1      E+01 4.      E-02 07                                232425262728
005RD    3.1      E+01 4.      E-02 08                                232425262728
006RD    3.1      E+01 4.      E-02 09                                232425262728

EPOCH CHANGE   A      0 0 09 09 02 1111     6.8706750E+01  0.        E+00
   PF                                        121314151617
004Q     6.87067E+01                *
006ABF

   PN                                        121314151617
007ABR

   P01
004Q     6.87067E+01                          23

   P02
004Q     6.87067E+01                *

BATCH          A      0 0 09 09 02 1111     6.8706750E+01  0.        E+00
   PF                                        121314151617
003A
007RD    1.       E+02 4.      E-02 07
008RD    1.       E+02 4.      E-02 08
009RD    1.       E+02 4.      E-02 09

   PN                                        121314151617

   P01                                       121314151617          2324252627282930
007RD    1.       E+02 4.      E-02 07242526  23              34  33
008RD    1.       E+02 4.      E-02 08272829  23    242526    34  3536
009RD    1.       E+02 4.      E-02 09303132  23    242526    34  3738
```

```
   P02                                          121314151617          2324252627282930
007RD    1.      E+02 4.      E-02 07                                      232425262728
008RD    1.      E+02 4.      E-02 08                                      232425262728
009RD    1.      E+02 4.      E-02 09                                      232425262728

EPOCH CHANGE   A      0 0 09 09 02 1111     1.000839 E+02  0.        E+00
   PF                                           121314151617
007Q     1.00084E+02              *
006ABF

   PN
007ABR

   P01
007Q     1.00084E+02                           23

   P02
007Q     1.00084E+02              *

BATCH          A      0 0 09 09 02 1111     1.000839 E+02  0.        E+00
   PF                                           121314151617
003A
010RD    1.3     E+02 4.      E-02 07
011RD    1.3     E+02 4.      E-02 08
012RD    1.3     E+02 4.      E-02 09

   PN                                           121314151617

   P01                                          121314151617          2324252627282930
010RD    1.3     E+02 4.      E-02 07242526  23                 34  33
011RD    1.3     E+02 4.      E-02 08272829  23     242526      34  3536
012RD    1.3     E+02 4.      E-02 09303132  23     242526      34  3738

   P02                                          121314151617          2324252627282930
010RD    1.3     E+02 4.      E-02 07                                      232425262728
011RD    1.3     E+02 4.      E-02 08                                      232425262728
012RD    1.3     E+02 4.      E-02 09                                      232425262728

PROJECT COV    B      0 0 09 09 02          1.000839 E+02  1.4999680E+02
   PF
013Q     1.49996E+02              *

   PN

   P01
013Q     1.49996E+02                           23

   P02
013Q     1.49996E+02              *

PROJECT COV    A      0 0 09 09 02 11111    1.4999680E+02  1.4999680E+02
   PF
010Q     1.49996E+02              *

   PN
```

```
   P01
010Q     1.49996E+02                                  23

   P02
010Q     1.49996E+02                  *

BATCH          A      0 0 06 09 02 1111     1.000839 E+02  0.        E+00
   PF                                                121314151617
016R     1.30    E+02 1.      E+02                          121314151617
016RD    1.30    E+02 3.      E-01                          121314151617
016IA1   1.30    E+02 1.      E-03                          121314151617
016IA2   1.30    E+02 1.      E-03                          121314151617

   PN                                                121314151617

   P01                                               121314151617          232425262728293
016R     1.30    E+02 1.      E+02                  23
016RD    1.30    E+02 3.      E-01                  23
016IA1   1.30    E+02 1.      E-03                  23
016IA2   1.30    E+02 1.      E-03                  23

   P02                                               121314151617          232425262728293
016R     1.30    E+02 1.      E+02 32
016RD    1.30    E+02 3.      E-01 33
016IA1   1.30    E+02 1.      E-03 34293031
016IA2   1.30    E+02 1.      E-03 35293031

PROJECT COV    B      0 0 06 09 02 1111     1.000839 E+02  1.4999680E+02
   PF
013Q     1.49996E+02                  *

   PN

   P01
013Q     1.49996E+02                                  23

   P02
013Q     1.49996E+02                  *

PROJECT COV    A      0 0 06 09 02 11111    1.4999680E+02  1.4999680E+02
   PF
010Q     1.49996E+02                  *

   PN

   P01
010Q     1.49996E+02                                  23

   P02
010Q     1.49996E+02                  *

SUMMARY
```

4.0 Computer Results

Computer results for this example run are included herein. For each event (batch, epoch change or projection) the uncertainty in the state parameters is shown for the RTODP filter and for the actual real-world. These filter uncertainties are the square roots of the diagonal elements of the state covariance matrix as computed by the RTODP. The real-world uncertainties, correspondingly show the actual uncertainties that exist. The first six elements refer to the position and velocity of the LEM. Elements seven, eight, and nine are the station pseudo-biases when the LEM is estimated. The remaining elements, twelve through twenty are correspondingly for the CSM.

Batch All CSM Tracking from - 32.0 to 100.0; Epoch at 0.0 Minutes

Filter Uncertainty		Total Actual Uncertainty	
SQRT(1, 1) =	1.000000E 04	SQRT(1, 1) =	5.000000E 03
SQRT(2, 2) =	1.000000E 04	SQRT(2, 2) =	5.000000E 03
SQRT(3, 3) =	1.000000E 04	SQRT(3, 3) =	5.000000E 03
SQRT(4, 4) =	2.000000E 01	SQRT(4, 4) =	1.000000E 01
SQRT(5, 5) =	2.000000E 01	SQRT(5, 5) =	1.000000E 01
SQRT(6, 6) =	2.000000E 01	SQRT(6, 6) =	1.000000E 01
SQRT(7, 7) =	1.000000E-01	SQRT(7, 7) =	0
SQRT(8, 8) =	1.000000E-01	SQRT(8, 8) =	0
SQRT(9, 9) =	1.000000E-01	SQRT(9, 9) =	0
SQRT(10,10) =	1.000000E 00	SQRT(10,10) =	0
SQRT(11,11) =	1.000000E 00	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	1.083615E-02	SQRT(18,18) =	4.356862E-02
SQRT(19,19) =	1.129198E-02	SQRT(19,19) =	2.765555E-01
SQRT(20,20) =	1.067912E-02	SQRT(20,20) =	2.775832E-01
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

First Batch of MSFN LEM Data plus apriroi; Epoch at 0.0 min.

Filter Uncertainty		Total Actual Uncertainty	
SQRT(1, 1) =	1.625992E 02	SQRT(1, 1) =	1.880270E 02
SQRT(2, 2) =	7.081446E 02	SQRT(2, 2) =	7.292509E 02
SQRT(3, 3) =	6.441165E 03	SQRT(3, 3) =	6.302155E 03
SQRT(4, 4) =	8.060523E-02	SQRT(4, 4) =	1.879775E-01
SQRT(5, 5) =	8.671203E-01	SQRT(5, 5) =	8.741734E-01
SQRT(6, 6) =	8.541775E 00	SQRT(6, 6) =	1.497369E 01
SQRT(7, 7) =	7.276877E-02	SQRT(7, 7) =	1.308470E-01
SQRT(8, 8) =	9.073437E-02	SQRT(8, 8) =	5.048614E-02
SQRT(9, 9) =	6.448113E-02	SQRT(9, 9) =	1.509568E-01
SQRT(10,10) =	1.000000E 00	SQRT(10,10) =	0
SQRT(11,11) =	1.000000E 00	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	1.083615E-02	SQRT(18,18) =	4.356862E-02
SQRT(19,19) =	1.129198E-02	SQRT(19,19) =	2.765555E-01
SQRT(20,20) =	1.067912E-02	SQRT(20,20) =	2.775832E-01
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Project Results of First LEM Batch to First Corrective Boost (t=12.74)

Without Adding Boost Uncertainties

Filter		Total	
SQRT(1, 1) =	1.669837E 02	SQRT(1, 1) =	1.959281E 02
SQRT(2, 2) =	1.474271E 02	SQRT(2, 2) =	2.411292E 02
SQRT(3, 3) =	7.321954E 03	SQRT(3, 3) =	9.350220E 03
SQRT(4, 4) =	6.557293E-01	SQRT(4, 4) =	6.803408E-01
SQRT(5, 5) =	8.580649E-01	SQRT(5, 5) =	8.670684E-01
SQRT(6, 6) =	8.045093E 00	SQRT(6, 6) =	1.384876E 01
SQRT(7, 7) =	0	SQRT(7, 7) =	1.299274E-01
SQRT(8, 8) =	0	SQRT(8, 8) =	4.621485E-02
SQRT(9, 9) =	0	SQRT(9, 9) =	1.503684E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	0	SQRT(18,18) =	4.223623E-02
SQRT(19,19) =	0	SQRT(19,19) =	2.763308E-01
SQRT(20,20) =	0	SQRT(20,20) =	2.773833E-01
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Total Uncertainties at Time of Completing First Corrective Boost

Filter		Total	
SQRT(1, 1) =	1.669837E 02	SQRT(1, 1) =	1.959281E 02
SQRT(2, 2) =	1.474271E 02	SQRT(2, 2) =	2.411292E 02
SQRT(3, 3) =	7.321954E 03	SQRT(3, 3) =	9.350220E 03
SQRT(4, 4) =	8.246095E-01	SQRT(4, 4) =	8.443125E-01
SQRT(5, 5) =	9.931140E-01	SQRT(5, 5) =	1.000903E 00
SQRT(6, 6) =	8.060615E 00	SQRT(6, 6) =	1.385778E 01
SQRT(7, 7) =	0	SQRT(7, 7) =	1.299274E-01
SQRT(8, 8) =	0	SQRT(8, 8) =	4.621485E-02
SQRT(9, 9) =	0	SQRT(9, 9) =	1.503684E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	0	SQRT(18,18) =	4.223623E-02
SQRT(19,19) =	0	SQRT(19,19) =	2.763308E-01
SQRT(20,20) =	0	SQRT(20,20) =	2.773833E-01
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Add Second MSFN Batch to LEM; Epoch at First Corrective Boost (t = 12.73)

Filter		Total	
SQRT(1, 1) =	4.182384E 01	SQRT(1, 1) =	8.699160E 01
SQRT(2, 2) =	1.063454E 02	SQRT(2, 2) =	1.396002E 02
SQRT(3, 3) =	3.705165E 03	SQRT(3, 3) =	4.764136E 03
SQRT(4, 4) =	1.193397E-01	SQRT(4, 4) =	2.208280E-01
SQRT(5, 5) =	1.320352E-01	SQRT(5, 5) =	1.600600E-01
SQRT(6, 6) =	4.652510E 00	SQRT(6, 6) =	8.433956E 00
SQRT(7, 7) =	6.308669E-02	SQRT(7, 7) =	1.360267E-01
SQRT(8, 8) =	6.429868E-02	SQRT(8, 8) =	1.288582E-01
SQRT(9, 9) =	5.978050E-02	SQRT(9, 9) =	1.569234E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	1.000000E-01	SQRT(18,18) =	0
SQRT(19,19) =	1.000000E-01	SQRT(19,19) =	0
SQRT(20,20) =	1.000000E-01	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Project Above Results to Second Corrective Boost (t = 68.71 Min) Without Adding Second Boost Uncertainties

Filter		Total	
SQRT(1, 1) =	6.988799E 02	SQRT(1, 1) =	7.954962E 02
SQRT(2, 2) =	1.496738E 03	SQRT(2, 2) =	1.918734E 03
SQRT(3, 3) =	4.389824E 03	SQRT(3, 3) =	5.708332E 03
SQRT(4, 4) =	1.312044E 00	SQRT(4, 4) =	1.600989E 00
SQRT(5, 5) =	4.239186E-01	SQRT(5, 5) =	4.709102E-01
SQRT(6, 6) =	4.228734E 00	SQRT(6, 6) =	8.026636E 00
SQRT(7, 7) =	0	SQRT(7, 7) =	1.348318E-01
SQRT(8, 8) =	0	SQRT(8, 8) =	1.247673E-01
SQRT(9, 9) =	0	SQRT(9, 9) =	1.561681E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	0	SQRT(18,18) =	0
SQRT(19,19) =	0	SQRT(19,19) =	0
SQRT(20,20) =	0	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

LEM Total Uncertainties at Time of Completing Second Corrective Boost
(t = 68.71)

Filter		Total	
SQRT(1, 1) =	6.988799E 02	SQRT(1, 1) =	7.954962E 02
SQRT(2, 2) =	1.496738E 03	SQRT(2, 2) =	1.918734E 03
SQRT(3, 3) =	4.389824E 03	SQRT(3, 3) =	5.708332E 03
SQRT(4, 4) =	1.404086E 00	SQRT(4, 4) =	1.677250E 00
SQRT(5, 5) =	6.555204E-01	SQRT(5, 5) =	6.868452E-01
SQRT(6, 6) =	4.258191E 00	SQRT(6, 6) =	8.042194E 00
SQRT(7, 7) =	0	SQRT(7, 7) =	1.348318E-01
SQRT(8, 8) =	0	SQRT(8, 8) =	1.247673E-01
SQRT(9, 9) =	0	SQRT(9, 9) =	1.561681E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	0	SQRT(18,18) =	0
SQRT(19,19) =	0	SQRT(19,19) =	0
SQRT(20,20) =	0	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Add Third MSFN Batch to LEM; Epoch at Second Corrective Boost (t = 68.71)

Filter		Total	
SQRT(1, 1) =	6.864751E 01	SQRT(1, 1) =	5.916777E 01
SQRT(2, 2) =	9.378174E 01	SQRT(2, 2) =	2.871122E 02
SQRT(3, 3) =	3.908551E 03	SQRT(3, 3) =	3.885576E 03
SQRT(4, 4) =	1.403577E-01	SQRT(4, 4) =	1.908045E-01
SQRT(5, 5) =	2.662850E-01	SQRT(5, 5) =	2.910151E-01
SQRT(6, 6) =	4.009737E 00	SQRT(6, 6) =	6.424724E 00
SQRT(7, 7) =	6.170204E-02	SQRT(7, 7) =	1.543703E-01
SQRT(8, 8) =	6.463272E-02	SQRT(8, 8) =	1.651269E-01
SQRT(9, 9) =	6.034553E-02	SQRT(9, 9) =	1.624443E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	1.000000E-01	SQRT(18,18) =	0
SQRT(19,19) =	1.000000E-01	SQRT(19,19) =	0
SQRT(20,20) =	1.000000E-01	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Project LEM Results to TPI (t=100.08 min) Without Adding Fourth Boost Uncertainties

Filter		Total	
SQRT(1, 1) =	4.904442E 02	SQRT(1, 1) =	5.015933E 02
SQRT(2, 2) =	8.822022E 01	SQRT(2, 2) =	1.057290E 02
SQRT(3, 3) =	4.837583E 03	SQRT(3, 3) =	7.797981E 03
SQRT(4, 4) =	5.121612E-01	SQRT(4, 4) =	5.323630E-01
SQRT(5, 5) =	1.323442E-01	SQRT(5, 5) =	1.382492E-01
SQRT(6, 6) =	3.248554E 00	SQRT(6, 6) =	3.193199E 00
SQRT(7, 7) =	0	SQRT(7, 7) =	1.543703E-01
SQRT(8, 8) =	0	SQRT(8, 8) =	1.651269E-01
SQRT(9, 9) =	0	SQRT(9, 9) =	1.624443E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	0	SQRT(18,18) =	0
SQRT(19,19) =	0	SQRT(19,19) =	0
SQRT(20,20) =	0	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Total LEM Uncertainties at Time of Completing TPI Boost (Epoch = 100.08 min)

Filter		Total	
SQRT(1, 1) =	4.904442E 02	SQRT(1, 1) =	5.015933E 02
SQRT(2, 2) =	8.822022E 01	SQRT(2, 2) =	1.057290E 02
SQRT(3, 3) =	4.837583E 03	SQRT(3, 3) =	7.797981E 03
SQRT(4, 4) =	7.157577E-01	SQRT(4, 4) =	7.303495E-01
SQRT(5, 5) =	5.172185E-01	SQRT(5, 5) =	5.187609E-01
SQRT(6, 6) =	3.286807E 00	SQRT(6, 6) =	3.232108E 00
SQRT(7, 7) =	0	SQRT(7, 7) =	1.543703E-01
SQRT(8, 8) =	0	SQRT(8, 8) =	1.651269E-01
SQRT(9, 9) =	0	SQRT(9, 9) =	1.624443E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	0	SQRT(18,18) =	0
SQRT(19,19) =	0	SQRT(19,19) =	0
SQRT(20,20) =	0	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Add MSFN Batch to LEM for First 30 Min. after TPI (Epoch = TPI = 100.08 min)

Filter		Total	
SQRT(1, 1) =	5.737282E 01	SQRT(1, 1) =	1.495817E 02
SQRT(2, 2) =	4.768579E 01	SQRT(2, 2) =	1.221333E 02
SQRT(3, 3) =	2.703166E 03	SQRT(3, 3) =	6.195330E 03
SQRT(4, 4) =	1.071227E-01	SQRT(4, 4) =	2.022741E-01
SQRT(5, 5) =	2.060757E-02	SQRT(5, 5) =	4.812555E-02
SQRT(6, 6) =	2.089322E 00	SQRT(6, 6) =	2.785086E 00
SQRT(7, 7) =	5.359589E-02	SQRT(7, 7) =	1.186927E-01
SQRT(8, 8) =	5.252258E-02	SQRT(8, 8) =	1.732298E-01
SQRT(9, 9) =	5.255826E-02	SQRT(9, 9) =	1.850091E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	1.000000E-01	SQRT(18,18) =	0
SQRT(19,19) =	1.000000E-01	SQRT(19,19) =	0
SQRT(20,20) =	1.000000E-01	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Project CSM from t = 0 to Rendezvous (t=150.0 min)
(no printout requested)

Project LEM from TPI (with MSFN Data for next 30 Min) to Rendezvous

Filter		Total	
SQRT(1, 1) =	1.377675E 02	SQRT(1, 1) =	1.945842E 02
SQRT(2, 2) =	5.334535E 01	SQRT(2, 2) =	1.309018E 02
SQRT(3, 3) =	3.182490E 03	SQRT(3, 3) =	6.447428E 03
SQRT(4, 4) =	1.287494E-01	SQRT(4, 4) =	1.627930E-01
SQRT(5, 5) =	5.893253E-02	SQRT(5, 5) =	6.555219E-02
SQRT(6, 6) =	1.570710E 00	SQRT(6, 6) =	2.366019E 00
SQRT(7, 7) =	5.359589E-02	SQRT(7, 7) =	1.186927E-01
SQRT(8, 8) =	5.252258E-02	SQRT(8, 8) =	1.732298E-01
SQRT(9, 9) =	5.255826E-02	SQRT(9, 9) =	1.850091E-01
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	4.344123E 00	SQRT(12,12) =	5.976235E 01
SQRT(13,13) =	2.882309E 01	SQRT(13,13) =	1.523851E 02
SQRT(14,14) =	9.499627E 02	SQRT(14,14) =	4.916451E 03
SQRT(15,15) =	2.325922E-02	SQRT(15,15) =	1.189993E-01
SQRT(16,16) =	3.930225E-03	SQRT(16,16) =	5.771080E-02
SQRT(17,17) =	4.559898E-01	SQRT(17,17) =	3.801725E 00
SQRT(18,18) =	1.000000E-01	SQRT(18,18) =	0
SQRT(19,19) =	1.000000E-01	SQRT(19,19) =	0
SQRT(20,20) =	1.000000E-01	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Now go back to last batch where 30 minutes of MSFN data was added with Epoch at TPI (100.08 min) (See top of pg. 24)

Batch LEM Further with Intervehicle Data; Epoch still at 100.08 Min. For LEM and at 0.0 Min. for CSM

SQRT(1, 1) =	2.306098E 01	SQRT(1, 1) =	1.554856E 02
SQRT(2, 2) =	2.264478E 01	SQRT(2, 2) =	1.576795E 02
SQRT(3, 3) =	4.345651E 01	SQRT(3, 3) =	4.701757E 03
SQRT(4, 4) =	3.924797E-02	SQRT(4, 4) =	3.266276E-01
SQRT(5, 5) =	1.383098E-02	SQRT(5, 5) =	5.415269E-02
SQRT(6, 6) =	5.924855E-02	SQRT(6, 6) =	4.113997E 00
SQRT(7, 7) =	0	SQRT(7, 7) =	0
SQRT(8, 8) =	0	SQRT(8, 8) =	0
SQRT(9, 9) =	0	SQRT(9, 9) =	0
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	3.012862E 00	SQRT(12,12) =	7.405058E 01
SQRT(13,13) =	1.011745E 01	SQRT(13,13) =	4.914773E 01
SQRT(14,14) =	3.388483E 02	SQRT(14,14) =	4.822602E 03
SQRT(15,15) =	8.740513E-03	SQRT(15,15) =	4.191678E-02
SQRT(16,16) =	2.136194E-03	SQRT(16,16) =	4.586972E-02
SQRT(17,17) =	8.556591E-01	SQRT(17,17) =	3.880909E 00
SQRT(18,18) =	0	SQRT(18,18) =	0
SQRT(19,19) =	0	SQRT(19,19) =	0
SQRT(20,20) =	0	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Project CSM, again, from t = 0 to Rendezvous with LEM Left at TPI As Above

Filter		Total	
SQRT(1, 1) =	2.306098E 01	SQRT(1, 1) =	1.554856E 02
SQRT(2, 2) =	2.264478E 01	SQRT(2, 2) =	1.576795E 02
SQRT(3, 3) =	4.345651E 01	SQRT(3, 3) =	4.701757E 03
SQRT(4, 4) =	3.924797E-02	SQRT(4, 4) =	3.266276E-01
SQRT(5, 5) =	1.383098E-02	SQRT(5, 5) =	5.415269E-02
SQRT(6, 6) =	5.924855E-02	SQRT(6, 6) =	4.113997E 00
SQRT(7, 7) =	0	SQRT(7, 7) =	0
SQRT(8, 8) =	0	SQRT(8, 8) =	0
SQRT(9, 9) =	0	SQRT(9, 9) =	0
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	4.344123E 00	SQRT(12,12) =	5.976235E 01
SQRT(13,13) =	2.882309E 01	SQRT(13,13) =	1.523851E 02
SQRT(14,14) =	9.499627E 02	SQRT(14,14) =	4.916451E 03
SQRT(15,15) =	2.325922E-02	SQRT(15,15) =	1.189993E-01
SQRT(16,16) =	3.930225E-03	SQRT(16,16) =	5.771080E-02
SQRT(17,17) =	4.559898E-01	SQRT(17,17) =	3.801725E 00
SQRT(18,18) =	0	SQRT(18,18) =	0
SQRT(19,19) =	0	SQRT(19,19) =	0
SQRT(20,20) =	0	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

Project LEM from TPI to Rendezvous (Including Intervehicle Data)

Filter		Total	
SQRT(1, 1) =	5.649828E 01	SQRT(1, 1) =	2.910566E 02
SQRT(2, 2) =	3.937807E 01	SQRT(2, 2) =	1.068758E 02
SQRT(3, 3) =	7.100476E 01	SQRT(3, 3) =	4.894111E 03
SQRT(4, 4) =	6.019670E-02	SQRT(4, 4) =	2.597811E-01
SQRT(5, 5) =	2.752250E-02	SQRT(5, 5) =	1.565998E-01
SQRT(6, 6) =	3.697557E-02	SQRT(6, 6) =	3.958558E 00
SQRT(7, 7) =	0	SQRT(7, 7) =	0
SQRT(8, 8) =	0	SQRT(8, 8) =	0
SQRT(9, 9) =	0	SQRT(9, 9) =	0
SQRT(10,10) =	0	SQRT(10,10) =	0
SQRT(11,11) =	0	SQRT(11,11) =	0
SQRT(12,12) =	4.344123E 00	SQRT(12,12) =	5.976235E 01
SQRT(13,13) =	2.882309E 01	SQRT(13,13) =	1.523851E 02
SQRT(14,14) =	9.499627E 02	SQRT(14,14) =	4.916451E 03
SQRT(15,15) =	2.325922E-02	SQRT(15,15) =	1.189993E-01
SQRT(16,16) =	3.930225E-03	SQRT(16,16) =	5.771080E-02
SQRT(17,17) =	4.559898E-01	SQRT(17,17) =	3.801725E 00
SQRT(18,18) =	0	SQRT(18,18) =	0
SQRT(19,19) =	0	SQRT(19,19) =	0
SQRT(20,20) =	0	SQRT(20,20) =	0
SQRT(21,21) =	0	SQRT(21,21) =	0
SQRT(22,22) =	0	SQRT(22,22) =	0

The following page shows a summary printout. In each pair of lines, the first line refers to the LEM while the second line refers to the CSM.

SUMMARY

SPCFT UPD		TIME	FILTER -- PRE POS	FILTER -- PRE VEL	FILTER - POST POS	FILTER - POST VEL	TOTAL COV --- PRE POS	TOTAL COV --- PRE VEL	TOTAL COV -- POST POS	TOTAL COV -- POST VEL
BATCH	B	0	1.732E 04	3.464E 01	1.732E 04	3.464E 01	8.660E 03	1.732E 01	8.660E 03	1.732E 01
		0	8.660E 03	1.732E 01	3.390E 02	8.557E-01	8.660E 03	1.732E 01	4.823E 03	3.881E 00
BATCH	A	0	1.732E 04	3.464E 01	6.482E 03	8.586E 00	8.660E 03	1.732E 01	6.347E 03	1.500E 01
		0	3.390E 02	8.557E-01	3.390E 02	8.557E-01	4.823E 03	3.881E 00	4.823E 03	3.881E 00
EPOCH	A	1.274E 01	7.325E 03	8.117E 00	7.325E 03	8.163E 00	9.355E 03	1.389E 01	9.355E 03	1.392E 01
		0	3.390E 02	8.557E-01	3.390E 02	8.557E-01	4.823E 03	3.881E 00	4.823E 03	3.881E 00
BATCH	A	1.274E 01	7.325E 03	8.163E 00	3.707E 03	4.656E 00	9.355E 03	1.392E 01	4.767E 03	8.438E 00
		0	3.390E 02	8.557E-01	3.390E 02	8.557E-01	4.823E 03	3.881E 00	4.823E 03	3.881E 00
EPOCH	A	6.871E 01	4.690E 03	4.448E 00	4.690E 03	4.531E 00	6.074E 03	8.198E 00	6.074E 03	8.244E 00
		0	3.390E 02	8.557E-01	3.390E 02	8.557E-01	4.823E 03	3.881E 00	4.823E 03	3.881E 00
BATCH	A	6.871E 01	4.690E 03	4.531E 00	3.910E 03	4.021E 00	6.074E 03	8.244E 00	3.897E 03	6.434E 00
		0	3.390E 02	8.557E-01	3.390E 02	8.557E-01	4.823E 03	3.881E 00	4.823E 03	3.881E 00
EPOCH	A	1.001E 02	4.863E 03	3.291E 00	4.863E 03	3.403E 00	7.815E 03	3.240E 00	7.815E 03	3.354E 00
		0	3.390E 02	8.557E-01	3.390E 02	8.557E-01	4.823E 03	3.881E 00	4.823E 03	3.881E 00
BATCH	A	1.001E 02	4.863E 03	3.403E 00	2.709E 03	2.092E 00	7.815E 03	3.354E 00	6.198E 03	2.793E 00
		0	3.390E 02	8.557E-01	3.390E 02	8.557E-01	4.823E 03	3.881E 00	4.823E 03	3.881E 00
PROJ	B	1.001E 02	2.709E 03	2.092E 00	2.709E 03	2.092E 00	6.198E 03	2.793E 00	6.198E 03	2.793E 00
		1.500E 02	9.504E 02	4.566E-01	9.504E 02	4.566E-01	4.919E 03	3.804E 00	4.919E 03	3.804E 00
PROJ	A	1.500E 02	3.186E 03	1.577E 00	3.186E 03	1.577E 00	6.452E 03	2.373E 00	6.452E 03	2.373E 00
		1.500E 02	9.504E 02	4.566E-01	9.504E 02	4.566E-01	4.919E 03	3.804E 00	4.919E 03	3.804E 00
BATCH	A	1.001E 02	2.709E 03	2.092E 00	5.416E 01	7.240E-02	6.198E 03	2.793E 00	4.707E 03	4.127E 00
		0	3.390E 02	8.557E-01	3.390E 02	8.557E-01	4.823E 03	3.881E 00	4.823E 03	3.881E 00
PROJ	B	1.001E 02	5.416E 01	7.240E-02	5.416E 01	7.240E-02	4.707E 03	4.127E 00	4.707E 03	4.127E 00
		1.500E 02	9.504E 02	4.566E-01	9.504E 02	4.566E-01	4.919E 03	3.804E 00	4.919E 03	3.804E 00
PROJ	A	1.500E 02	9.892E 01	7.582E-02	9.892E 01	7.582E-02	4.904E 03	3.970E 00	4.904E 03	3.970E 00
		1.500E 02	9.504E 02	4.566E-01	9.504E 02	4.566E-01	4.919E 03	3.804E 00	4.919E 03	3.804E 00

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 489
(BBC Task 203)

L. Lustick
G. Hempstead

CHECKOUT OF LANDMARKS AND STAR SIGHTINGS PROGRAM

Purpose

The purpose of this note is to show the manner in which the landmark, and star sighting programs were checked out and to indicate how to generate the landmark and star sightings information matrices.

Introduction

The analysis for the star sighting and landmark measurables is shown in Note 484. Program C has been modified such that is generates information matrices for the following conditions:

1. Intervehicle (as before)
   a. Range
   b. Range-Rate
   c. Az, El. angles.
2. Landmark sightings on central body
   a. Az, El. angles
3. Angle between horizon of central body and a star (minimum angle).

A part C data sheet will run one of the three tracking situations mentioned above. The tracking condition is controlled by the Card 26 input as described on the modified Part C data sheet shown in Figure 1.

Platform rate parameters (described in Note 486) are available for both intervehicle and landmark measurables. As shown on Figure 1 these rate parameters are in columns 22, 23, and 24. The star sighting measurable has no platform angles partials or platform rate partials.

Columns 8, 9, and 10 are the platform orientation parameters for both intervehicle and landmark sightings.

Columns 16, 17 and 18 are partials of the measurables with respect to Up, East and North errors in the landmark and latitude, longitude and horizon altitude for the star sightings.

Figure 1 is an input data sheet for either intervehicle measurements, landmarks, or star sightings. The table below indicates the inputs that are not required for each of the measurements.

INTERVEH	LANDMARKS	STAR SIGHTINGS
✓ 04	✓ 04	✓ 11
✓ 05	✓ 05	20
✓ 11	13	21
13	✓ 18	25
✓ 18	✓ 19	33
✓ 19	20	34
33	21	41
34	33	42
50	34	43
51	41	44
52	42	45
53	43	46
54	44	48
	45	49
	46	50
	48	51
	53	
	54	

If Q matrices are to be generated in this program and used in the December 10th program or modifications of it and it is desired to have the in-plane and out-of-plane miss for the "fixed radius" results meaningful, the following rules regarding the checked inputs should be followed. If one is not interested in the fixed radius results then the checked inputs as well as the others indicated do not have to be filled in.

	EARTH ORBIT	LUNAR ORBIT
Intervehicle	1. Orient Veh 1 with respect to x, y, z axes with Euler angles. 2. Cards 04, 05, 18, 19 are all equal to zero. 3. Card 11 should be set equal to Earth rate	1. Orient Veh 1 with respect to $\tilde{x}, \tilde{y}, \tilde{z}$ with Euler angles. 2. Set 04, 05, 18, 19 = 0. 3. Set 11 equal to Moon angular rate.
Landmark	1. Orient Veh with respect to x, y, z axes with Euler angles. 2. Cards 04, 05, 18, 19 are all equal to zero.	1. Orient Veh 1 with respect to $\tilde{x}, \tilde{y}, \tilde{z}$ with Euler angles. 2. Set 04, 05, 18, 19 = 0. 3. Set 11 equal to Moon angular rate.
Star Sighting	1. Orient Veh with respect to x, y, z axes with Euler angles 2. Cards 04, 05, 18, 19 all equal to zero. 3. Card 11 should be Earth rate.	The results of in-plane and out-of-plane miss for fixed radius will always be meaningless.

Check Out of Landmarks and Star Sightings

The first thing that was checked was that the alterations made to the Part C program did not alter the intervehicle type results. This check is not shown in this report.

The landmark sightings were checked out by checking the intervehicle sightings against the landmark sighting for a problem in which they both should give the same answer. The input data sheet for the intervehicle run is shown in Figure 2. The landmark input data sheet is the same except for the type run indicator (Card 26).

The partial derivatives with respect to all parameters are shown for both the landmarks and the intervehicle in Figure 3.

The only difference between the landmarks and the intervehicles results are in the sign of parameters 17 and 18 and this is as it should be since the perturbations in these parameters are in the opposite direction in the intervehicle than in the landmark.

The star sightings were checked out by setting up a run in which the star was in the plane of the vehicle orbit and checking the partial derivatives against the simple analytical results for this case.

The input data sheet for the star sighting is shown in Figure 4 and the partial derivatives of the measurable with respect to all parameters is shown in Figure 5.

✓ PROGRAM C (1–9) | 11 12 | 15 Station 26 | 31 33 ← Data Set

XX: if first set on new tape

1 2	7 … 20			
01	. E	A1 (1)	m	Init. orbit radius, radius rate, and horizontal velocity for Veh. (1) at its t orbit.
02	. E	A4 (1)	m/sec	
03	. E	A5 (1)	m/sec	
04	. E	GAMID		+1 if $\lvert\gamma\rvert<\pi/2$; -1 otherwise
05	. E	BETA	deg	Inclination of E-M orbit to Eq.
06	. E	XI (1)	deg	Euler angles relating x', y', z' to $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ for Veh. (1) at its t orbit.
07	. E	ETA (1)	deg	
08	. E	ZETA (1)	deg	
11	. E	OMEGA	rad/sec	Angular rate of orbited body.
13	. E	RHO	m	Radius of orbited body.
15	. E	MU	m^3/sec^2	Gravitational constant
16	. E	TJ(s)	min	Maximum time(s) of observation.
16	. E			
16	. E			
16	. E			
16	. E			
16	. E			
18	. E	BIG L	deg	Lat. sublunar pt at T = 0
19	. E	SMA L	deg	Long. sublunar pt at T = 0
20	. E	RR IND		1: calc $\dot{R}$; 0 otherwise
21	. E	R IND		1: calc R; 0 otherwise
22	. E	Q IND		1: calc Q; 0 otherwise
23	. E	QF	1	
24	. E	PS1 IND	min	Time for additional Q
25	. E	PS2 IND		1: calc <'s ; 0 otherwise
26	. E	TYPE RUN		1: calc <'s ; 0 otherwise
27	. E	TI (init)		*
28	. E	TI (inter)	min	Time of 1st observation
33	. E	F-M IND	min	Interval between observations
34	. E	VIS IND		1: Feet; 0: Meters
41	. E	A1 (2)		0: Vis in; 1: Vis Out
42	. E	A4 (2)	m	Initial orbit radius, radius rate, and horizontal velocity for Veh. (2) at its t orbit.
43	. E	A5 (2)	m/sec	
44	. E	XI (2)	m/sec	
45	. E	ETA (2)	deg	Euler angles relating x', y', z' to $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ for Veh. (2) at its t orbit.
46	. E	ZETA (2)	deg	
47	. E	T ORB (1)	deg	
48	. E	T ORB (2)	min	Time orbit specified Veh. (1)
49	. E	PSET	min	Time orbit specified Veh. (2)
50	. E	LMRK LAT	min	Time platform set
51	. E	LMRK LONG	deg	Landmark latitude
52	. E	LMRK RAD	deg	Landmark longitude
53	. E	STAR LAT	m	Landmark radius
54	. E	STAR LONG	deg	Star latitude
			deg	Star longitude

Figure 1

PROGRAM C
DATA INPUT SHEET

VEHICLE (1) TO VEHICLE (2) LANDMARK & STAR SIGHTING MEASUREMENTS

a_1	→ x	Vehicle (1)
a_2	→ y	
a_3	→ z	
a_4	→ $\dot{x}$	
a_5	→ $\dot{y}$	
a_6	→ $\dot{z}$	
a_7	→ Bias	
a_8	→ $\delta x'$	Platform rotations about x', y', z' axes; nom = 0
a_9	→ $\delta y'$	
a_{10}	→ $\delta z'$	
a_{12}	→ Gravitational constant	
a_{14}	→ Venting Acc. Veh. (1)	
a_{15}	→ Venting Acc. Veh. (2)	
a_{16}	→ x	Vehicle (2) (If Intervehicle run requested.)
a_{17}	→ y	
a_{18}	→ z	
a_{19}	→ $\dot{x}$	
a_{20}	→ $\dot{y}$	
a_{21}	→ $\dot{z}$	
a_{16}	→ Up	Landmark Uncertainties (If Landmark run requested
a_{17}	→ East	
a_{18}	→ North	
a_{16}	→ Lat.	Star Uncertainties (If star sighting run requested.)
a_{17}	→ Long.	
a_{18}	→ Horiz. Alt.	
a_{22}	→ $\delta\dot{x}'$	Platform rotation rates about x', y', z' axes (If either Intervehicle or Landmark run requested
a_{23}	→ $\delta\dot{y}'$	
a_{24}	→ $\delta\dot{z}'$	

*Indicator specifying type of run desired: -1: star sighting; 0: intervehicle; or +1: landmark.

INPUTS - DATA SET 1

1	6.3384405E 06	A1-1
2	3.1365883E-01	A4-1
3	5.2266347E 03	A5-1
4	1.0000000E 00	GAMID
5	1.8000000E 01	BETA
6	-0	XI-1
7	1.7800000E 02	ETA-1
8	2.0000000E 01	ZETA-1
11	-0	OMEGA
13	5.7030000E 06	RHO
15	1.7313945E 14	MU
16	1.0000000E 01	TIME
16	1.0100000E 01	TIME
18	1.5000000E 01	LT SLP
19	-0	LO SLP
20	1.0000000E 00	RR IND
21	1.0000000E 00	R IND
22	1.0000000E 00	Q IND
23	-0	QF IND
24	1.0000000E 00	A1 IND
25	1.0000000E 00	A2 IND
26	-0	RUNTYP
27	1.0000000E 01	T INIT
28	1.0000000E-01	T INCR
33	1.0000000E 00	FM IND
34	1.0000000E 00	VISIND
41	5.7030000E 06	A1-2
42	-0	A4-2
43	5.5100000E 03	A5-2
44	-0	XI-2
45	1.8000000E 02	ETA-2
46	2.1000000E 01	ZETA-2
47	1.0000000E 01	TORB-1
48	1.0000000E 01	TORB-2
49	-0	T PSET
50	-0	LD LAT
51	-2.1000000E 01	LD LON
52	5.7030000E 06	LD RAD
53	0	ST LAT
54	0	ST LON

Figure 2

AZIMUTH PARTIAL		ELEVATION PARTIAL	
Intervehicle	Landmark	Intervehicle	Landmark
1.172881E-06	1.172881E-06	1.245637E-07	1.245637E-07
3.222463E-06	3.222463E-06	1.345750E-06	1.345750E-06
6.026486E-06	6.026486E-06	-7.438380E-07	-7.438380E-07
-6.027263E-14	-6.027263E-14	-3.339512E-14	-3.339512E-14
9.322527E-13	9.322527E-13	2.804703E-13	2.804703E-13
0	0	0	0
1.000000E 00	1.000000E 00	1.000000E 00	1.000000E 00
-1.000000E 00	-1.000000E 00	0	0
-3.730643E 00	-3.730643E 00	5.246003E-01	5.246003E-01
2.298819E 00	2.298819E 00	8.513487E-01	8.513487E-01
0	0	0	0
-1.302348E-23	-1.302348E-23	-4.171592E-24	-4.171592E-24
0	0	0	0
0	0	0	0
0	0	0	0
-1.303566E-06	-1.303566E-06	-1.384429E-07	-1.384429E-07
-3.395905E-06	3.395905E-06	-1.318393E-06	1.318392E-06
-5.903135E-06	5.903135E-06	7.890053E-07	-7.890053E-07
0	0	0	0
0	0	0	0
0	0	0	0
-6.000000E 02	-6.000000E 02	0	0
-2.238386E 03	-2.238386E 03	3.147602E 02	3.147602E 02
1.379291E 03	1.379291E 03	5.108092E 02	5.108092E 02

Figure 3

INPUTS - DATA SET 3

1	6.3384405E 06	A1-1
2	3.1365883E-01	A4-1
3	5.2266347E 03	A5-1
4	1.0000000E 00	GAMID
5	1.8000000E 01	BETA
6	-0	XI-1
7	1.8000000E 02	ETA-1
8	2.0000000E 01	ZETA-1
11	-0	OMEGA
13	5.7030000E 06	RHO
15	1.7313945E 14	MU
16	1.0000000E 01	TIME
16	1.0100000E 01	TIME
18	-0	LT SLP
19	-0	LO SLP
20	1.0000000E 00	RR IND
21	1.0000000E 00	R IND
22	1.0000000E 00	Q IND
23	-0	QF IND
24	1.0000000E 00	A1 IND
25	1.0000000E 00	A2 IND
26	-1.0000000E 00	RUNTYP
27	1.0000000E 01	T INIT
28	1.0000000E-01	T INCR
33	1.0000000E 00	FM IND
34	1.0000000E 00	VISIND
41	5.7030000E 06	A1-2
42	-0	A4-2
43	5.5100000E 03	A5-2
44	-0	XI-2
45	1.8000000E 02	ETA-2
46	2.1000000E 01	ZETA-2
47	1.0000000E 01	TORB-1
48	1.0000000E 01	TORB-2
49	-0	T PSET
50	-0	LD LAT
51	-2.1000000E 01	LD LON
52	5.7030000E 06	LD RAD
53	-1.8000000E 01	ST LAT
54	9.0000000E 01	ST LON

Figure 4

Star Sighting Partials (Computer)

```
 3.252702E-07
-1.577675E-07
-1.063337E-17
 1.156799E-14
 6.840951E-14
            0
 1.000000E 00
            0
            0
            0
            0
-6.901204E-25
            0
            0
            0
 3.283751E-11
-9.510565E-01
-3.615125E-07
```

The analytical partials that are different than zero for this run are:

$$\frac{\partial \varphi}{\partial a_1} = \frac{r_0}{A_1 \sqrt{A_1^2 - r_0^2}} , \qquad \frac{\partial \varphi}{\partial a_2} = - \frac{1}{A_1} , \qquad \frac{\partial \varphi}{\partial a_7} = 1.0 ,$$

$$\frac{\partial \varphi}{\partial a_{17}} = - \cos (\text{ST LAT}) , \qquad \frac{\partial \varphi}{\partial a_{18}} = - \frac{1}{\sqrt{A_1^2 - r_0^2}} .$$

Figure 5

APOLLO NOTE NO. 490
(BBC Task 105)

H. Epstein
31 May 1967

A PRIORI EDITING THRESHOLD FOR DOPPLER MEASUREMENT DATA

It is desirable that simple editing procedures be employed for the separation of "good" doppler data from "bad" doppler data. In addition, these procedures should facilitate error classification. MSC has made LLO measurement data available to BBC for analysis. The LLO data will be mostly comprised of data while in lunar orbit, supplemented by some translunar data. Data presently on hand consists of high speed destruct and non-destruct data taken at 0.4 second sample intervals and low speed non-destruct data taken at 6 second sample intervals.

The present editing procedure tests the second difference of raw measurement data. Two threshold levels are employed. The first threshold level tests for doppler counts which are inconsistent with crude trajectory information. This crude trajectory threshold level is set according to whether the spacecraft data is lunar or translunar. The second threshold is set to reject unlikely high frequency error sources.

The true value of the distance from the spacecraft to the USB station may be closely approximated by a low degree polynomial for short periods of time. A second difference editor is being used for either destruct or non-destruct data. The time interval involved for the polynomial must then correspond approximately to twice the measurement interval. A satisfactory representation for present purposes is as follows:

$$R_m \simeq R_p = R_o + \dot{R}_o t + \frac{\ddot{R}_o t^2}{2} + \frac{\dddot{R}_o t^3}{6} \qquad (1)$$

where R_m = true distance between the spacecraft and the USB

$R_p = R_p(t)$ = polynomial approximation

and R_o, $\dot{R}_o$, $\ddot{R}_o$, and $\dddot{R}_o$ are true derivatives of the measurements.

Considering first the editing of non-destruct data, one can show that:

$$R_p(t+T) - 2R_p(t) + R_p(t-T) = \ddot{R}_o T^2 + \dddot{R}_o T^2 t \tag{2}$$

where T = time interval between successive measurements.

Then,

$$E_N = R_p(T) - 2R_p(0) + R_p(-T) = \ddot{R}_o T^2 \tag{3}$$

where E_N = non-destruct data editing threshold.

Destruct data represents a range change in each measurement interval. Letting

$$\Delta R_p(t) = R_p\left(t + \frac{T_c}{2}\right) - R_p\left(t - \frac{T_c}{2}\right) \tag{4}$$

where T_c = destruct count interval.

Substituting Eq. (1) into Eq. (4), one can obtain

$$\Delta R_p(t) = R_o T_c + R_o T_c t + \frac{\dddot{R}_o}{2}\left(T_c t^2 + \frac{T_c^3}{24}\right) \tag{5}$$

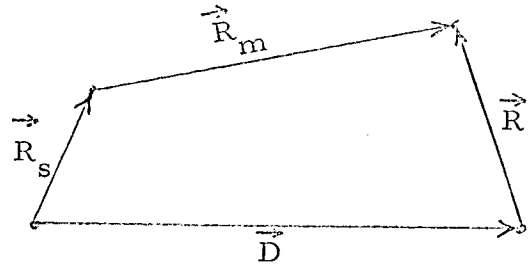
then

$$E_D = \Delta R_p(t+T) - 2\Delta R_p(t) + \Delta R_p(t-T) = \dddot{R}_o T_c T^2 \tag{6}$$

where E_D = destruct data editing threshold.

It is now only necessary to find upper bounds to $\ddot{R}_o$ and $\dddot{R}_o$ to determine this crude editing threshold level.

The geometry involved is indicated in the figure below.



where $\vec{R}_s$ = vector from the center of the earth to the USB station

$\vec{R}_m$ = vector from the USB station to the spacecraft.

In the ESOI,

$\vec{D} = 0$

$\vec{R}$ = vector from the center of the earth to the spacecraft.

In the LSOI,

$\vec{D}$ = vector from the center of the earth to the center of the moon.

$\vec{R}$ = vector from the center of the moon to the spacecraft.

From the figure,

$$\vec{R}_m = \vec{D} + \vec{R} - \vec{R}_s \tag{7}$$

then

$$R_m^2 = \vec{R}_m \cdot \vec{R}_m \tag{8}$$

Successive differentiation of Eq. (8) yields,

$$R_m \dot{R}_m = \vec{R}_m \cdot \dot{\vec{R}}_m \tag{9}$$

$$R_m \ddot{R}_m + \dot{R}_m^2 = \vec{R}_m \cdot \ddot{\vec{R}}_m + \dot{\vec{R}}_m \cdot \dot{\vec{R}}_m \tag{10}$$

$$R_m \dddot{R}_m + 3 \dot{R}_m \ddot{R}_m = \vec{R}_m \cdot \dddot{\vec{R}}_m + 3 \dot{\vec{R}}_m \cdot \ddot{\vec{R}}_m \tag{11}$$

Letting

$$\vec{R}_m = R_m \vec{1}_m \tag{12}$$

$$\dot{\vec{R}}_m = \vec{V}_m \tag{13}$$

$$\ddot{\vec{R}}_m = \vec{A}_m \tag{14}$$

$$\dddot{\vec{R}}_m = \dot{\vec{A}}_m \tag{15}$$

$$\vec{V}_m \cdot \vec{V}_m = V_m^2 \tag{16}$$

then equations (9), (10), and (11) may be written as follows:

$$\dot{R}_m = \vec{1}_m \cdot \vec{V}_m \tag{17}$$

$$\ddot{R}_m = \vec{1}_m \cdot \vec{A}_m + \frac{\left[V_m^2 - \dot{R}_m^2\right]}{R_m} \tag{18}$$

$$\dddot{R}_m = \vec{1}_m \cdot \dot{\vec{A}}_m + \frac{3}{R_m}\left[\vec{V}_m \cdot \vec{A}_m - \dot{R}_m \ddot{R}_m\right] \tag{19a}$$

$$= \vec{1}_m \cdot \dot{\vec{A}}_m + \frac{3}{R_m}\left[\vec{V}_m \cdot \vec{A}_m - \dot{R}_m (\vec{1}_m \cdot \vec{A}_m)\right]$$

$$- \frac{3 \dot{R}_m}{R_m^2}\left[V_m^2 - \dot{R}_m^2\right] \tag{19b}$$

Equations (18) and (19b) may be simplified if one recognized that bracketed quantities represent differences between total vectors and vectors along $\vec{1}_m$.

$$\vec{V}_m = \left(\vec{1}_m \cdot \vec{V}_m\right) \vec{1}_m + \vec{1}_m X \left(\vec{V}_m X \vec{1}_m\right) \tag{20}$$

$$\vec{A}_m = \left(\vec{1}_m \cdot \vec{A}_m\right) \vec{1}_m + \vec{1}_m X \vec{A}_m X \vec{1}_m \tag{21}$$

then one can write

$$\ddot{R}_m = \vec{1}_m \cdot \vec{A}_m + \frac{\left(\vec{1}_m X \vec{V}_m\right) \cdot \left(\vec{1}_m X \vec{V}_m\right)}{R_m} \tag{22}$$

and

$$\dddot{R}_m = \vec{1}_m \cdot \dot{\vec{A}}_m + \frac{3}{R_m} \left[\vec{1}_m X \vec{V}_m \cdot \vec{1}_m X \vec{A}_m\right] - \frac{3 \dot{R}_m}{R_m^2} \left[\vec{1}_m X \vec{V}_m \cdot \vec{1}_m X \vec{V}_m\right] \tag{23}$$

The equations obtained to this point will now be approximated to obtain simple upper bounds for the crude editing level.

Planning assumptions are as follows:

1. Restricted two-body solutions.
2. Cruise flight condition.
3. Lunar ephemeris is circular motion about earth center.

From Eqs. (22) and (23),

$$\ddot{R}_m \leq \left|\vec{A}_m\right| + \frac{V_m^2}{R_m} \tag{24}$$

$$\dddot{R}_m \leq \left|\dot{\vec{A}}_m\right| + \frac{3 V_m}{R_m} \left|\vec{A}_m\right| + \frac{3 V_m^2}{R_m^2} \tag{25}$$

I. Lunar Phases

The bulk of the LLO doppler tracking data will be taken while the spacecraft is in lunar orbit. The initial or crude editing threshold will accommodate lunar cruise data prior to de-boost as well as the cruise orbital data. Upper bounds will be given for station range rate, range acceleration, and range acceleration rate for the station measurements in the absence of measurement errors.

Necessary analytical expressions associated with Eqs (24) and (25) are as follows:

$$\vec{V}_m = \dot{\vec{D}} + \dot{\vec{R}} - \dot{\vec{R}}_s \tag{26}$$

$$|\vec{R}| < (V_e)\ \text{moon} \tag{27}$$

where (V_e) moon = surface escape velocity for the moon.

$$\vec{R}_s = \vec{\omega}_E X \vec{R}_s \tag{28}$$

$$|\vec{R}_s| \leqslant \omega_E R_s \tag{29}$$

$$\vec{D} = \vec{\omega}_M X \vec{D} \tag{30}$$

$$|\dot{\vec{D}}| \leqslant \omega_M D \tag{31}$$

$$\dot{\vec{V}}_m < (V_e)\text{moon} + \omega_E R_s + \omega_M D \tag{32}$$

$$\vec{A}_m = \ddot{\vec{D}} + \ddot{\vec{R}} - \ddot{\vec{R}}_s \tag{33}$$

$$\ddot{\vec{R}} \leqslant 1 \text{ lunar surface g} \tag{34}$$

$$\ddot{\vec{R}}_s = \vec{\omega}_E X \left(\vec{\omega}_E X \vec{R}_s\right) = \left(\vec{\omega}_E \cdot \vec{R}_s\right) \vec{\omega}_E - \omega_E^2 \vec{R}_s \tag{35}$$

$$\left|\ddot{\vec{R}}_s\right| \leq \omega_E^2 R_s \tag{36}$$

$$\ddot{\vec{D}} = \left(\vec{\omega}_M \cdot \vec{R}\right) \vec{\omega}_M - \omega_M^2 \vec{D} \tag{37}$$

$$\left|\ddot{\vec{D}}\right| \leq \omega_M^2 D \tag{38}$$

$$\ddot{\vec{A}}_m = \ddot{\vec{D}} + \ddot{\vec{R}} - \ddot{\vec{R}}_s \tag{39}$$

For the restricted two-body problem,

$$\dot{\vec{R}} = -\frac{\mu}{R^3} \vec{R} \tag{40}$$

$$\ddot{\vec{R}} = \frac{3\mu \dot{R}}{R^4} R - \frac{\mu}{R^3} \dot{\vec{R}} \tag{41}$$

Letting

$$g_R = \frac{\mu}{R^2} \tag{42}$$

$$\vec{R} = R \vec{1}_R \tag{43}$$

and

$$\dot{\vec{R}} = \vec{V} = V \vec{1}_V = \dot{R} \vec{1}_R + V_\perp \vec{1}_\perp \tag{44}$$

then Eq. (41) may be written as

$$\ddot{\vec{R}} = \frac{g_R}{R} \left[2 \dot{R} \vec{1}_R - V_\perp \vec{1}_\perp\right] \tag{45}$$

$$\left|\ddot{\vec{R}}\right| < \frac{2 g_R}{R} \left(V_e\right) \text{ moon} \tag{46}$$

$$\dddot{\vec{R}}_s = -\omega_E^2 \left(\vec{\omega}_E X \vec{R}_s\right) \tag{47}$$

$$\left|\dddot{\vec{R}}_s\right| < -\omega_E^3 R_s \tag{48}$$

$$\left|\dddot{\vec{R}}_s\right| \quad -\omega_M^2 D \tag{49}$$

Numerical results are indicated in the table below for the upper bounds.

Table I. Lunar Limitations

Quantity	Numerical Value
$\left\|\dot{\vec{R}}\right\|$	7750 ft/sec
$\left\|\dot{\vec{R}}_s\right\|$	1530 ft/sec
$\left\|\dot{\vec{D}}\right\|$	3360 ft/sec
$\left\|\dot{\vec{V}}_m\right\|$	12,640 ft/sec
$\left\|\ddot{\vec{R}}\right\|$	5.31 ft/sec^2
$\left\|\ddot{\vec{R}}_s\right\|$	.111 ft/sec^2
$\left\|\ddot{\vec{D}}\right\|$	.004 ft/sec^2
$\left\|\vec{A}_m\right\|$	5.43 ft/sec^2
$\frac{V_m^2}{R_m}$	.13 ft/sec^2
$\ddot{R}_m$	5.6 ft/sec^2
$\dddot{\vec{R}}$	.0144 ft/sec^3
$\left\|\dddot{\vec{R}}_s\right\|$	8.1×10^{-6} ft/sec^3
$\left\|\dddot{\vec{D}}\right\|$	10^{-8} ft/sec^3
$\frac{3 V_m \left\|\vec{A}_m\right\|}{R_m}$	1.6×10^{-4} ft/sec^3
$\frac{3 V_m^3}{R_m^2}$	4×10^{-6} ft/sec^3
$\dddot{R}_m$	.015 ft/sec^3

The primary limitation in this case is associated with the spacecraft motion about the moon.

II. Translunar Phases

Midcourse corrections were performed between about 20 and 40 hours after translunar injection on Orbiters I, II, III, and IV. Corresponding earth-to-spacecraft distances vary from about 25 to 40 e.s. Numerical results will be given for earth-to-spacecraft distances greater than 25 e.r. For this case, D and its derivates are neglected. The analytical expressions indicated for the lunar phase may be used to arrive at the desired results. The numerical results are indicated below.

Table II. Translunar Limitations

Quantity	Numerical Value
$\lvert \dot{\vec{R}} \rvert$	7340 ft/sec
$\lvert \dot{\vec{R}}_s \rvert$	1530 ft/sec
$\lvert \vec{V}_m \rvert$	8870 ft/sec
$\lvert \ddot{\vec{R}} \rvert$	.0515 ft/sec^2
$\lvert \ddot{\vec{R}}_s \rvert$	.111 ft/sec^2
$\lvert \vec{A}_m \rvert$	.163 ft/sec^2
$\frac{V_m^2}{R_m}$	.15 ft/sec^2
$\ddot{R}_m$	.32 ft/sec^2
$\lvert \dddot{\vec{R}} \rvert$	1.45×10^{-6} ft/sec^3
$\lvert \dddot{\vec{R}}_s \rvert$	8.1×10^{-6} ft/sec^3
$\frac{3 V_m \lvert \vec{A}_m \rvert}{R_m}$	10^{-5} ft/sec^3
$\frac{3 V_m^3}{R_m^2}$	10^{-5} ft/sec^3
$\dddot{R}_m$	3×10^{-5} ft/sec^3

It may be noted that the values of $\ddot{R}_m$ and $\dddot{R}_m$ associated with the translunar phase are very small as compared to the corresponding values for the lunar phase.

III. A Priori Threshold Count for Non-destruct Doppler Data Editing

This a priori threshold level is set in terms of the maximum count level associated with the second difference of continuous count doppler data. This counter indicates one count for each wavelength change in the two-way (or three-way) range between spacecraft and the USB station. The associated number of counts in the non-destruct mode (N_N) is then given using Eq. (3) as follows:

$$N_N = \frac{2}{\lambda} E_N = \frac{2\ddot{R}_m}{\lambda} T^2 \sim 4.6\, \ddot{R}_m T^2 \tag{50}$$

Using the numerical values indicated in the previous section,

$$N_N \sim 25\, T^2 \text{ (lunar phases)} \tag{51}$$

$$N_N \sim 1.5\, T^2 \text{ (translunar phases)} \tag{52}$$

Recalling that T is the time interval between successive samples the following tables may be obtained using Eqs. (51) and (52):

Table III. A Priori Threshold Levels (non-destruct doppler data)

T Sec	N_N (counts)	
	Translunar	Lunar
.1	---	.25
.2	---	1
.4	.4	4
1.0	1.5	25
6	54	900
10	150	2,500
30	1350	22,500
60	5400	90,000

Quantization errors would constitute second difference readings bounded by ± 2 counts. The use of rubidium atomic standards at USB stations insure that the standard deviation of the frequency standard high frequency noise will not exceed the quantization noise for sample intervals indicated in Table III. An a priori threshold level of 100 counts is being used for data presently under going analysis. This data is 0.4 second lunar data and six (6) second translunar data. The a priori threshold is greater than required to insure that the presence of crystal oscillators rather than the desired atomic oscillators can be discerned in the data. The majority of the USB data is expected to be taken at sample intervals no less frequent than one (1) every six seconds. A 100 or 1,000 count threshold could then be used for most non-destruct doppler data taken on the LLO spacecraft.

IV. A Priori Threshold Count for Destruct Doppler Data Editing

Some destruct count data has been submitted by MSC to BBC for analysis. This data was taken at a sample rate of 2.5 samples per second during lunar phases. Therefore, it is desirable that appropriate threshold levels be indicated for destruct data. The counter readings in the destruct doppler mode represent the time interval for a fixed number of cycles of doppler plus bias frequency. This time interval is indicated in units of 0.01 μseconds. Either the time for 77,854 cycles (N_1 mode) or 778,540 cycles (N_2 mode) may be indicated. Count time intervals (T_c) of approximately 0.08 and 0.8 seconds are associated with the N_1 and N_2 modes, respectively. The noise free threshold levels will be indicated first. The expressions will be written as though two-way doppler was the measurable. A priori threshold levels for both three-way doppler and two-way doppler are both considered the same at this time. For this case, one can write

$$\frac{N}{T_c} - f_B = \frac{2(\delta R)}{\lambda T_c} \tag{53a}$$

or

$$N - f_B T_c = \frac{2}{\lambda} \delta R \tag{53b}$$

where

N = either N_1 or N_2

f_B = bias frequency = 10^6 Hz

$\frac{2}{\lambda}$ = 4.6 cycles/ft

δR = change in range (ft)

The second difference of Eq. (53b) yields:

$$-\Delta^2 T_c = \frac{2}{f_B \lambda} \Delta^2 (\delta R) \tag{54}$$

Since T_c is given in units of 10^{-8} seconds, the a priori editing level for the destruct mode (N_D) using Eq. (6) is given by

$$N_D = \frac{2 \times 10^8}{f_B \lambda} (\dddot{R}_m T_c T^2) = 460\ T_c \dddot{R}_m T^2 \tag{55a}$$

$$N_D \sim 36.8\ \dddot{R}_m T^2 \qquad (N_1 \text{ mode}) \tag{55b}$$

$$N_D \sim 368\ \dddot{R}_m T^2 \qquad (N_2 \text{ mode}) \tag{55c}$$

The following table may be made using the upper bounds for $\dddot{R}_m$ indicated earlier.

N_D(counts)

T Sec	Lunar		Translunar	
	N_1 Mode	N_2 Mode	N_1 Mode	
.1	---	---	---	---
.2	---	---	---	---
.4	---	.88	---	---
1.0	.55	5.5	---	---
6	20	200	---	.4
10	55	550	.11	1.1
30	500	5,000	1.0	10
60	2,000	20,000	4	40

Table IV. A Priori Threshold Levels (Destruct Mode)

Present destruct count data under going analysis here was taken in the N_1 mode with T = 0.4 seconds. As a consequence, the data is detrended by editing and a priori threshold levels need to only accommodate the high frequency noise. Using Eq. (53b), one can obtain

$$\Delta T_c \times 10^8 = \frac{2 \times 10^8}{\lambda f_B} \delta R = 460 (\delta R) \text{ft} = 36.8 \tag{56}$$

with

$$\Delta T_c \times 10^8 = \text{number of counts of 100 mcps frequency}$$

and

$$(\delta R)_{ft} = .08 \text{ ft } (1 \sigma) \text{ from ANWG-1.2.}$$

The numerical value of Eq. (56) is a 1 σ uncertainty with atomic frequency standards employed. The use of crystal standards could increase this value considerably. The a priori threshold level has been selected to be of at least a 10 σ value of the high frequency noise. The actual high frequency noise levels indicated by data already analyzed

is about a factor of three or four lower than ANWG specifications. As a consequence, a priori thresholds of about 100 or 200 counts should be appropriate for high speed destruct data.

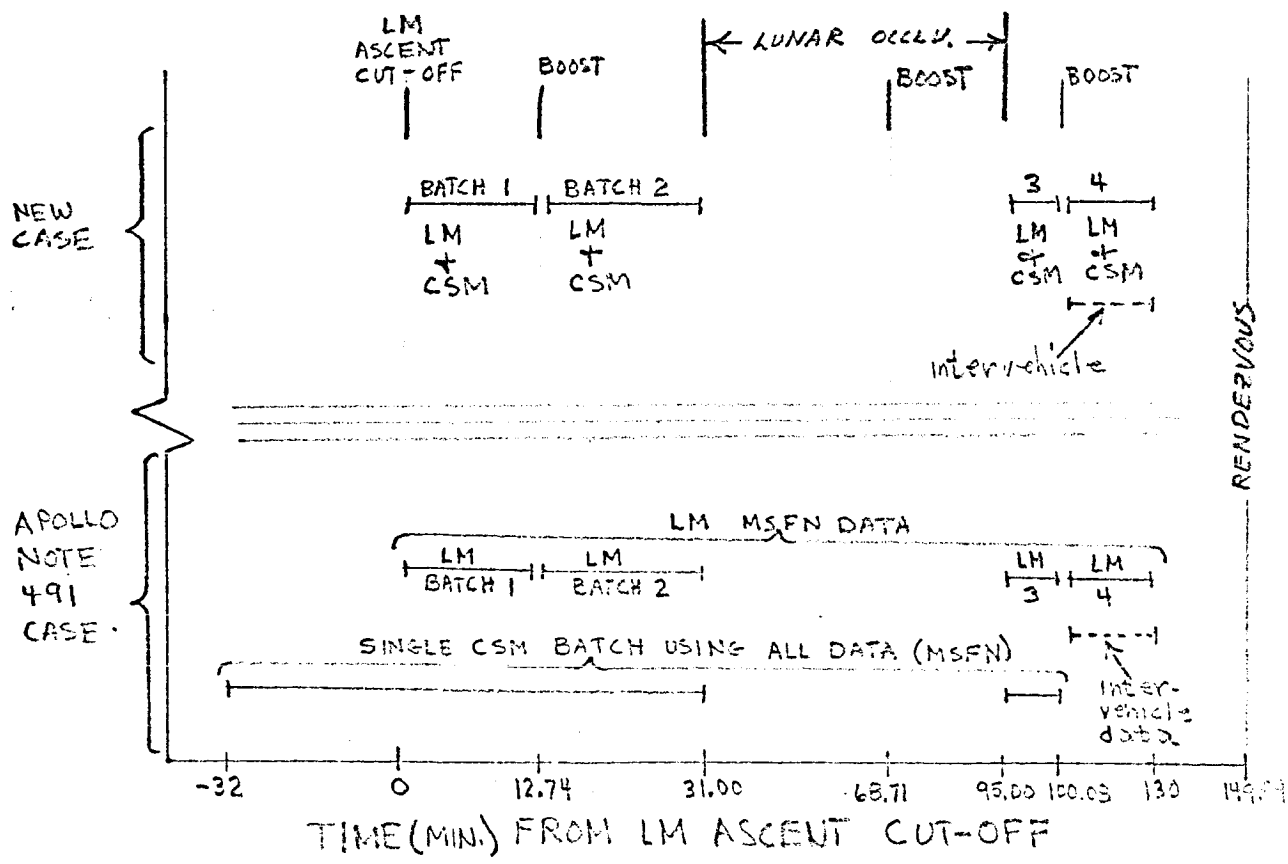
The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 492
(BBC Task 204)

C. H. Dale, Jr.
22 June 1967

TRACKING THE LM AND CSM FROM TAKE-OFF TO RENDEZVOUS WITH SIMILAR BATCHING

The purpose of this note is to report a modified running of the case described in Apollo Note No. 488, wherein the single CSM MSFN batch included all data between the first CSM appearance prior to LM take-off and the point of the terminal phase initiation. In this new case both LM and CSM are batched with MSFN data during the old LM batch intervals up to and including the last 30 minutes after the LM TPI burn. The filter assumptions regarding CSM boosts (at the times of the LM boosts) are the same as the LM. Of course there are no real CSM boost uncertainties since no boosts occur. The object of all this is to significantly reduce the total relative state uncertainty by making the LM and CSM covariance contributions from the non-estimated parameters sufficiently alike. The following sketch shows the two runs diagramatically.



Thus the Note 491 case has some early CSM tracking and lacks some later tracking as opposed to the present case. Both cases present results with and without intervehicle tracking during the first 30 minutes after TPI.

In both cases the filter-assumed and true vehicle ephemeris uncertainties are described at LM ascent cut-off. However, they are different:

NOTE 491 CASE

LM Filter State Variances:

1×10^8 ft^2 each axis

4×10^2 ft^2/sec^2 each axis

CSM Filter State Variances:

2.5×10^7 ft^2 each axis

1×10^2 ft^2/sec^2 each axis

LM True State Variances:

2.5×10^7 ft^2 each axis

1×10^2 ft^2/sec^2 each axis

CSM True State Variances:

2.5×10^7 ft^2 each axis

1×10^2 ft^2/sec^2 each axis

NEW CASE

LM Filter, LM True, CSM Filter, CSM True State Variances are all equal:

2.5×10^7 ft^2 each axis

1×10^2 ft^2/sec^2 each axis

Now the logic for changing these assumptions is based upon the desire, first to make the CSM and LM filters as identical as possible, and second to make the filter apriori assumptions equal the real-world assumptions since this has proved to be fairly optimum in past investigations.

Though the results of this case should not be considered as any sort of final proof, they do not indicate any great improvement due to implementing equal filters for the CSM and LM. Part of the improvement shown by the results can be attributed to the fact that the LM apriori filter assumptions equal the true (and smaller) state uncertainties. It also seems apparent that the single batch encompassing the entire orbit and a half of CSM data used in Note 491 is non-optimum,

as shown by the relatively large velocity error near rendezvous. A future run will be made in which the tracking is the same as this case, but no pseudo-biases are estimated. This will be done to keep the nuisance parameter errors in their place rather than to absorb them in bias estimates which are thrown away for each batch. It is hoped that the relative covariance will then reflect the cancelling of nuisance parameter effects.

This still may not prove worthwhile due to a large dependence upon apriori estimates which are assumed to be uncorrelated between the LM and CSM. It is nevertheless worth a try.

The following figures show:

Figure 1 - A comparison between this case and the case reported in Apollo Note 491.

Figure 2 - Total and in-plane position uncertainties for the LM and CSM for equal batching.

Figure 3 - Total and in-plane velocity uncertainties for the LM and CSM.

Time	Event	This Case				Note 491 Case			
		Position (ft)		Velocity (ft/sec)		Position (ft)		Velocity (ft/sec)	
		Total	In-Plane	Total	In-Plane	Total	In-Plane	Total	In-Plane
0	Apriori CSM (true)	8660	7070	17.32	14.14	8660	7070	17.32	14.14
	Apriori LM (true)	8660	7070	17.32	14.14	8600	7070	17.32	14.14
	Batch CSM	4570	861.8	10.59	.9453	5386	88	4.193	.0624
	Batch LM	4531	735.9	10.56	.8800	6347	760	15.00	.8947
	Relative (after batch)	6093	1100	7.37	1.256	----------not computed--------			
12.74	CSM	7541	340.8	9.40	1.217	----------not computed--------			
	LM Pre Boost	7467	275.1	9.37	1.085	9355	310	13.85	1.1020
	LM Post Boost	7467	275.1	9.41	1.296	9355	310	13.86	1.3095
	Batch CSM	4775	191.4	5.82	.2032	----------not computed--------			
	Batch LM	4296	158.2	6.52	.2570	4767	164	8.44	.2725
	Relative (after batch)	4637	172.5	5.07	.3069	----------not computed--------			
68.71	CSM	4659	1792	6.23	1.488	----------not computed--------			
	LM Pre Boost	4560	1987	6.23	1.602	6074	2075	8.20	1.6688
	LM Post Boost	4560	1987	6.84	1.751	6074	2075	8.24	1.8121
	Batch CSM	4789	264.5	5.88	.1532	----------not computed--------			
	Batch LM	3503	290.7	5.86	.3491	3897	293	6.43	.3478
	Relative (after batch)	4625	121.3	4.78	.3141	----------not computed--------			
100.08	CSM	7201	157.6	3.91	.1191	----------not computed--------			
	LM Pre Boost	7128	513.2	2.91	.5501	7815	512	3.24	.5499
	LM Post Boost	7128	513.2	3.04	.8958	7815	512	3.35	.8957
MSFN Only	Batch CSM	5974	147.7	3.63	.1166	-----no CSM data after TPI-----			
	Batch LM	5966	192.1	2.58	.2121	6198	193	2.79	.2078
	Relative (after batch)	3587	102.2	3.16	.1424	----------not computed--------			
MSFN and Inter-veh.	Batch CSM	5974	147.7	3.63	.1166	-----no CSM data after TPI-----			
	Batch LM	5962	222.5	3.63	.2520	5094	100	4.19	.1603
	Relative (after batch)	Not Computed				Not Computed			
149.99	CSM	6589	141.7	2.84	.1183	5311	135	4.26	.1090
	LM (MSFN Only)	6123	234.7	2.32	.1752	6451	234	2.37	.1754
	Relative (MSFN Only)	4407	183.4	2.37	.1737	4821	205	3.69	.1466
	LM (MSFN and Interveh.)	6555	324.1	2.88	.3600	5270	236	4.33	.2513
	Relative (MSFN and Interveh.)	389	378.3	0.38	.2320	394	242	0.45	.3154

Figure 1 - Results for Two Cases

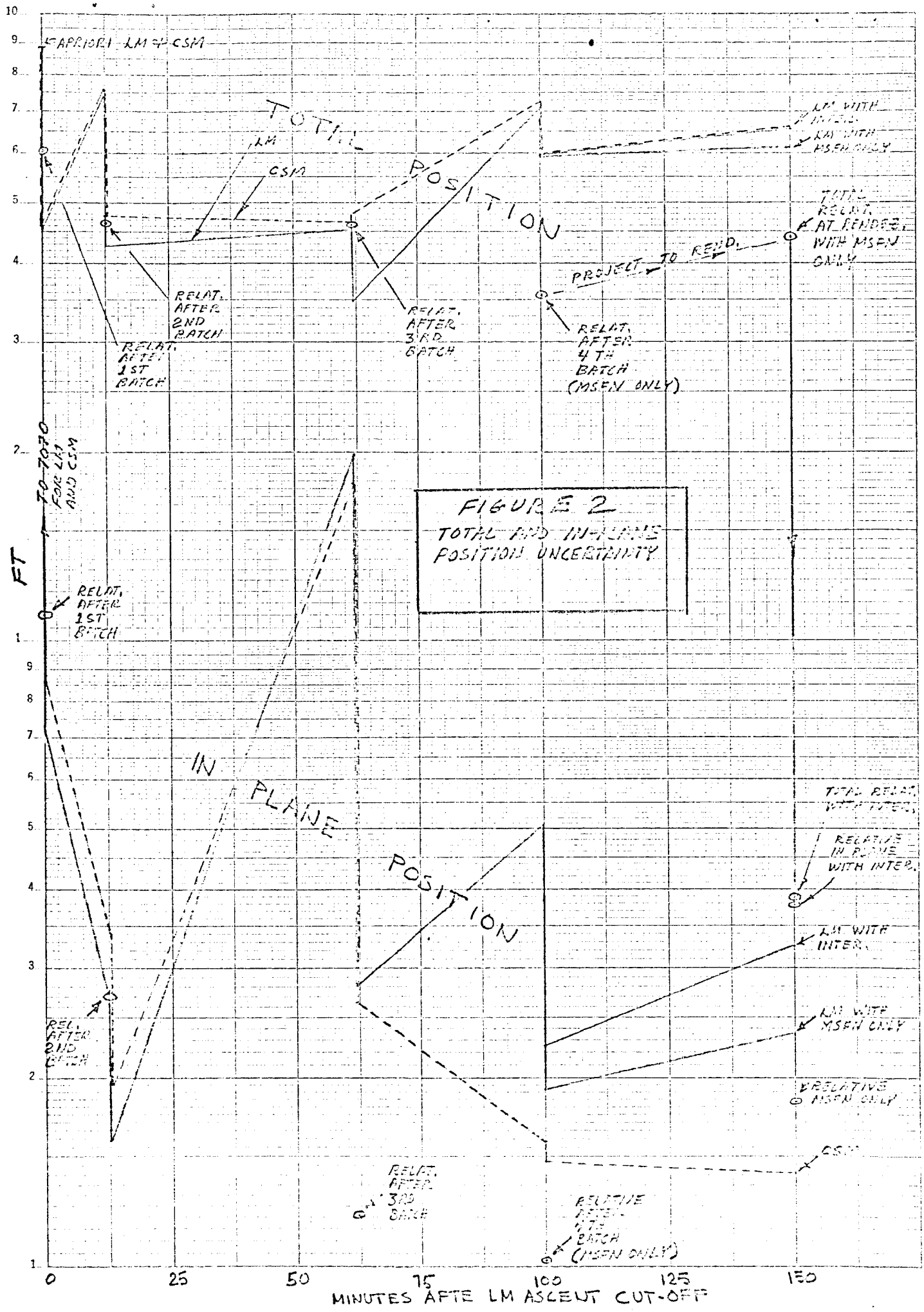
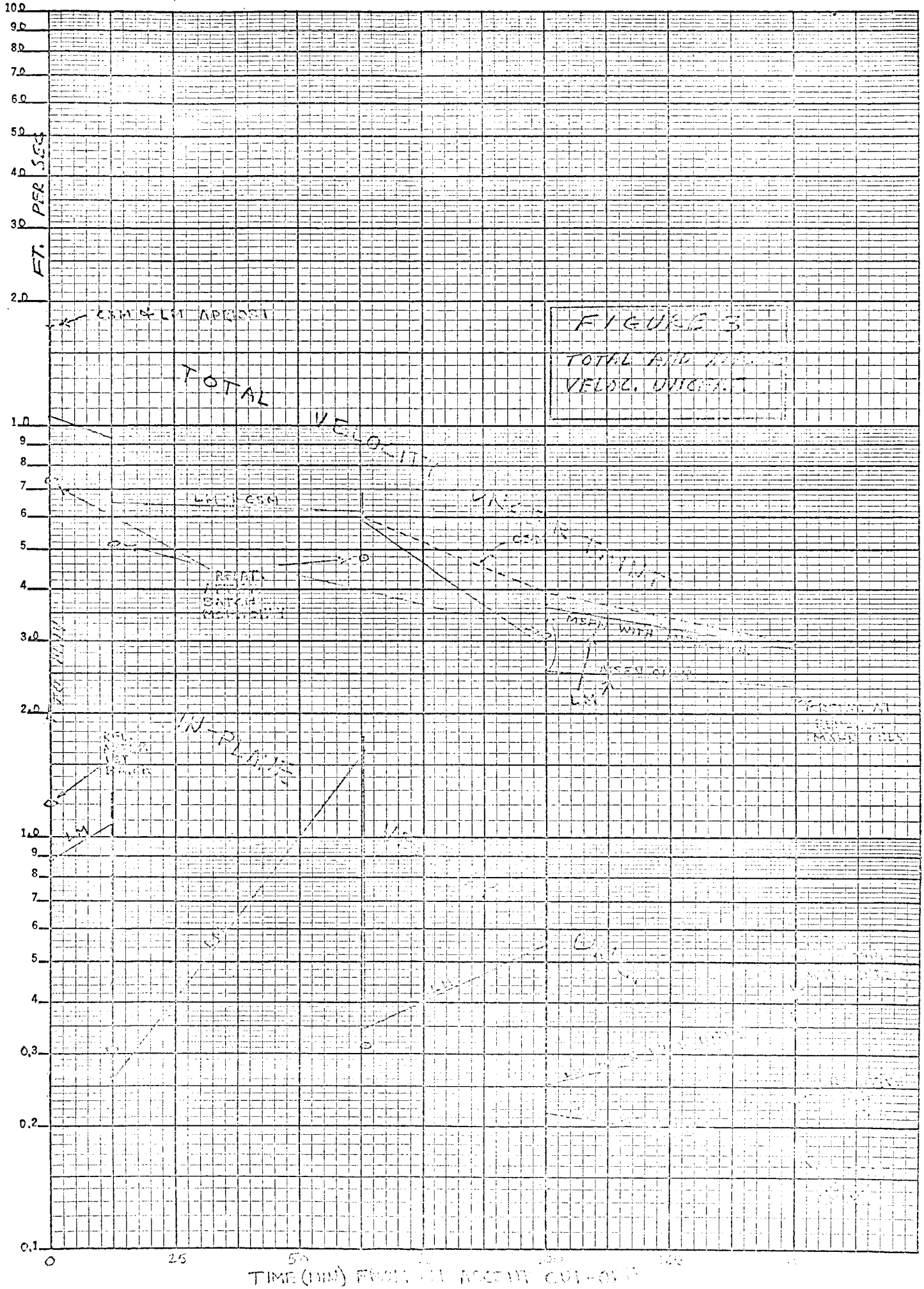


FIGURE 2
TOTAL AND IN-PLANE POSITION UNCERTAINTY

358-61
SEMI-LOGARITHMIC
KEUFFEL & ESSER CO.
2 CYCLES X 70 DIVISIONS
MADE IN U.S.A.



SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 493
(BBC Task 106)

D. Matta
28 June 1967

FAST FOURIER TRANSFORM

We require the computation of the discrete Fourier transform of the N points A_k, $k = 0, 1, 2, 3, \cdots, N-1$. This transform B_j is defined by:

$$B_j = \sum_{k=0}^{N-1} A_k \omega^{jk} \tag{1}$$

where

$$\omega = e^{\frac{2\pi i}{N}} .$$

Here j assumes values from 0 to N-1. We see that for each value of j , the computation of B_j requires the evaluation of an $(N-1)^{th}$ order polynomial in ω^j and hence requires (N-1) comples multiply-adds in addition to a comples exponentiation to obtain ω^j. The entire B_j array then requires N (N-1) complex multiply-adds plus N complex exponentiations.

For N of the order of 10^3 these computations would require over a minute of machine time. For N of the order of 10^4 the time required makes the computation infeasible.

Let us suppose that N is highly composite and has the form: $N = N_1 \times N_2 \times N_3 \times \cdots \times N_n$. Express k and j in a mixed radix system with radices $N_1, N_2, \cdots, N_n$ such that k has the form:

$$k = k_n L_n + k_{n-1} L_{n-1} + \cdots + k_2 L_2 + k_1 L_1 \tag{2}$$

where $L_1 = 1$, $L_\alpha = L_{(\alpha-1)} \cdot N_{(\alpha-1)}$; $\alpha = 2, \cdots, n$ and k_α assumes values $0, 1, 2, \cdots, (N_{\alpha-1})$. And j has the form:

$$j = j_n M_n + j_{n-1} M_{n-1} + \cdots + j_2 M_2 + j_1 M_1 \qquad (3)$$

where $M_1 = 1$, $M_\alpha = M_{(\alpha-1)} \cdot N_{(n-\alpha+2)}$ $\alpha = 2, \cdots, n$ and j_α assumes values $0, 1, 2, \cdots, (N_{(n-\alpha+1)} - 1)$. We will denote the right hand side of (2) by the n-tuple $(k_n k_{n-1} \cdots k_2 k_1)_L$ and any n-tuple enclosed in brackets subscripted by L will have this meaning i. e., indicate a number in the mixed radix system associated with k. A subscript M will indicate a number in the mixed radix system associated with j, (right half of equation (3)). We now re-write equation (1) utilizing the expressions for k and j.

$$B_j = \sum_k A_k \omega^{jk} = \sum_{k_1} \sum_{k_2} \cdots \sum_{k_n} A_{(k_n k_{n-1} \cdots k_2 k_1)_L} \omega^{j(k_n \cdots k_1)_L} \qquad (4)$$

Now

$$\omega^{jk} = \omega^{j(k_n \cdots k_1)_L} = \omega^{j\left[k_n L_n + \cdots + k_2 L_2 + k_1 L_1\right]}$$

$$= \omega^{jk_n L_n} \cdot \omega^{jk_{n-1} L_{n-1}} \cdots \omega^{jk_2 L_2} \cdot \omega^{jk_1 L_1}$$

Let us consider the 1st term in the above product and utilize the representation for j:

$$\omega^{jk_n L_n} = \omega^{\left[j_n M_n + j_{n-1} M_{n-1} + \cdots + j_2 M_2 + j_1 M_1\right] L_n k_2}$$

$$= \omega^{M_n L_n j_n k_n} \cdot \omega^{M_{n-1} L_n j_{n-1} k_n} \cdots \omega^{M_1 L_n j_1 k_n}$$

Noting the definitions for M_α and L_n we see that each of $M_n L_n$ $M_{n-1} L_{n-1}, \cdots, M_2 L_2$ is divisible by N and hence

$$\omega^{jk_n L_n} = \omega^{j_1 k_n L_1}$$

since

$$\omega^{\lambda N} = 1$$

In general $M_\alpha L_\beta$ will be divisible by N if $\alpha + \beta > n + 1$ and therefore

$$\omega^{jk_\beta L_\beta} = \omega^{j_n M_n + j_{n-1} M_{n-1} + \cdots + j_2 M_2 + j_1 M_1 \; L_\beta k_\beta}$$

$$= \omega^{M_{n-\beta+1} L_\beta \, j_{n-\rho+1} k_\beta} \cdot \omega^{M_{n-\beta} L_\beta j_{n-\beta} k_\beta} \cdots \omega^{M_1 L_\beta j_1 k_\rho}$$

Using this information equation(4) becomes:

$$B_{(j_n n_{n-1} \cdots j_2 j_1)_M} = \sum_{k_1} \omega^{(j_n M_n + \cdots j_1 M_1) L_1 k_1} \sum_{k_2} \omega^{(j_{n-1} M_{n-1} + \cdots + j_1 M_1) L_2 k_2}$$

$$\cdots \sum_{k_n} A_{(k_n k_{n-1} \cdots k_2 k_1)_L} \; \omega^{j_1 K_n k_n}$$

There are N values in the B array to be computed each uniquely identified by an n-tuple $(j_n \cdots j_1)$. Each summation must be recomputed not for every B but only for each new set of j_α which are involved in that summation.

For computational purposes we will think of the set of values A_k, $k = 1, \cdots, N$ as an n-dimensional array with dimension

$$N_n \times N_{n-1} \times N_{n-2} \times \cdots \times N_2 \times N_1$$

(which are the factors of N). Each value A_k is then located in the $(k_n, k_{n-1}, \cdots, k_2, k_1)$ entry in this array where $k = (k_n k_{n-1} \cdots k_2 k_1)_L$. We now define intermediate result arrays

$$A_1 (j_1, k_{n-1}, k_{n-2}, \cdots, k_1) = \sum_{k_n} A(k_n, k_{n-1}, \cdots, k_1)\, \omega^{j_1 L_n k_n}$$

and in general:

$$A_\ell(j_1, j_2, \cdots, j_\ell, k_{n-\ell}, \cdots, k_1) = \sum_{k_{n-\ell+1}} A_{\ell-1}(j_1, \cdots, j_{\ell-1}, k_{n-\ell+1}, \cdots, k_1)$$

$$\omega^{\left[j_\ell M_\ell + \cdots + j_1 M_1\right] L_{n-\ell+1} k_{n-\ell+1}}$$

We note that all of these arrays have the same dimensions as the original A array. And, therefore, each array requires the computation of N entries. The final array $A_k(j_1, j_2, \cdots, j_n)$ will contain the value for $B_{(j_n j_{n-1} \cdots j_2 j_1)_M}$. That is,

$B_0 = B_{(0 \cdots 0)_M}$ will be in the cell $A_{(0, 0, \cdots, 0)}$,

$B_1 = B_{(0 \cdots 1)_M}$ will be in the cell $A_{(1, 0, \cdots, 0)}$ etc.

For economy of storage area the intermediate result arrays may occupy the same area. Let us consider how the A_1 array is obtained from the A array: To obtain the value for the $A_1(0, k_{n-1}, \cdots, k_1)$ entry, we require the elements:

$$A(0, k_{n-1}, \cdots, k_1),\ A(1, k_{n-1}, \cdots, k_1),\ \cdots,\ A(N_n-1, k_{n-1}, \cdots, k_1)$$

from the A array. To compute the values for the entries $A_1(\alpha, k_{n-1}, \cdots, k_1)$ $\alpha = 1, 2, \cdots, N_n-1$ these same elements from the A array are required. We can first extract these N_n elements from the original A array, place them in a temporary location, and compute the N_n new values for the A_1 array and place these in the vacated cells in the original array. Compute the new elements in sets of N_n until all N are computed. This same procedure is used to obtain each successive A_ℓ array in the space occupied by the original.

To obtain the $A_{\ell+1}$ array from the A_ℓ array N values must be computed. Recalling the definition of $A_{\ell+1}$ and letting

$$\rho = \omega^{\left[j_1 M_1 + \cdots + j_\ell M_\ell + j_{\ell+1} M_{\ell+1}\right] L_{n-\ell}}$$

which can be represented as $\rho = \rho_0\ \rho_1^{j_{\ell+1}}$ where

$$\rho_0 = \omega^{\left[j_1 M_1 + \cdots + j_\ell M_\ell\right] L_{n-\ell}}$$

and

$$\rho_1 = \omega^{M_{\ell+1} L_{n-\ell}}$$

We obtain:

$$A_{\ell+1}(j_1, j_2, \cdots, j_{\ell+1}, k_{n-\ell-1}, \cdots, k_1) = \sum_{k_{n-\ell}=1}^{N_{n-\ell}} A_\ell(j_1, \cdots, j_\ell, k_{n-\ell}, k_{n-\ell-1}, \cdots k_1)\rho^{k_{n-\ell}}$$

Now each of the entries in $A_{\ell+1}$ requires the evaluation of an $N_{n-\ell}^{th}$ order polynomial in ρ. This requires $(N_{n-\ell}-1)$ complex multiply-adds. Let us define a cycle as a fixed set of $j_1, \cdots, j_\ell$ (there are then $M_{\ell+1}$ cycles each consisting of a block of $L_{n-\ell+1}$ elements). The parameter ρ_1 depends only on the index $(n-\ell)$ of the summation; ρ_0 however, must be computed for each new cycle. The cost of obtaining ρ is then one complex multiply-add for each entry in the array, $\rho = \rho_1\rho_0$, one complex exponentiation to get ρ_0 for each new cycle excluding the zeroth cycle, and one complex exponentation to get ρ_1 for each new summation index. The cost of obtaining $A_{\ell+1}$ from A_ℓ is then

$$N \cdot \left\{(N_{n-\ell}-1) + 1\right\} = N \cdot N_{n-\ell} \text{ complex multi-adds}$$

plus $M_{\ell+1}$ complex exponentiations. The total cost of obtaining the final array is therefore

$$N\left[N_n + N_{n-1} + \cdots + N_1\right] \text{ complex multiply adds plus}$$

$$\left[M_1 + M_2 + \cdots + M_{n-1}\right] = \left[1 + N_n + N_n \cdot N_{n-1} + \cdots + N_n \cdot N_{n-1} \cdots N_3 \cdot N_2\right]$$

complex exponentiations.

A FORTRAN subroutine to perform this fast Fourier transform has been written. The subroutine name is FASTF and the calling statement is CALL FASTF (A, NUM, L, NFAC) where:

(1) A is a complex single dimensional array containing the data points to be transformed. (The original data is real valued and so each element in A will have its imaginary part equal to zero.) A will contain the transform upon exit.

(2) NUM is the number of data points.

(3) L is an integer array containing the factors of NUM (decreasing order is optimum). Each $L_i \leq 100$.

(4) NFAC is the number of factors, i.e.

$$NUM = L_{(1)}, L_{(2)}, \cdots, L_{(NFAC)},$$

and

$$NFAC \leq 15.$$

After computation is complete, the A_n array is unscrambled utilizing a scratch tape (FORTRAN Tape No. 1) leaving the transform array, B, in the space occupied by the original data.

```
      SUBROUTINE FASTF(A,NUMI,L,NFACI)
C A IS THE ARRAY OF DATA POINTS
C AT EXIT THE TRANSFORM IS IN THE A ARRAY
C NUMI IS THE NUMBER OF POINTS
C L IS THE ARRAY OF FACTORS
C NFAC IS THE NUMBER OF FACTORS
      DIMENSION A(1),L(1)
      TYPE COMPLEX W,A,R,R0,R1,B,F,ONE
      DIMENSION RW(2),N(15),B(100)
      EQUIVALENCE (W,RW)
      DATA (PI2=6.2831853072),(ONE=(1.,0.))
      DIMENSION JI(15)
      EQUIVALENCE (JI( 1),W     ),(JI( 3),R     )
      EQUIVALENCE (JI( 5),R0    ),(JI( 7),R1    )
      EQUIVALENCE (JI( 9),F     ),(JI(11),FN    )
      EQUIVALENCE (JI(12),THETA),(JI(13),INDEX)
      EQUIVALENCE (JI(14),IND   ),(JI(15),IS    )
      NUM=NUMI
      NFAC=NFACI
      FN=NUM
      THETA=PI2/FN
      RW(1)=COS(THETA)
      RW(2)=SIN(THETA)
      N(1)=1
      DO 10 I=2,NFAC
   10 N(I)=N(I-1)*L(I-1)
      INDEX=NFAC
   15 ISTART=1
      LL=L(INDEX)
      NN=N(INDEX)
      IBLOCK=NN*LL
      LLNN=IBLOCK-NN
      NCYC=NUM/IBLOCK
C FIND R1 FOR THIS INDEX
      J2=NCYC*NN
      R1=W**J2
      DO 100 ICYC=1,NCYC
C FIND R0 FOR THIS INDEX AND ICYC
      IF(INDEX-NFAC) 17,16,16
   16 R0=ONE
      GO TO 25
   17 K1=ICYC-1
      KR0=0
      J1=INDEX+1
      DO 20 IND=J1,NFAC
      LIND=L(IND)
      K2=K1/LIND
      JJ=K1-K2*LIND
      K1=K2
   20 KR0=KR0*LIND+JJ
      KR0=KR0*NN
      R0=W**KR0
   25 IS=ISTART
      DO 60 K=1,NN
C FETCH LL ELEMENTS STARTING AT A(IS) SEPARATED BY NN
```

```
C AND PLACE IN B-ARRAY IN REVERSE ORDER
      K2=IS+LLNN
      J1=LL
      DO 30 I=IS,K2,NN
      B(J1)=A(I)
   30 J1=J1-1
      R=R0
C COMPUTE THE LL VALUES TO REPLACE THESE (BACK INTO A ARRAY)
      DO 50 I=IS,K2,NN
      F=B(1)
      DO 40 J=2,LL
   40 F=F*R+B(J)
      A(I)=F
   50 R=R*R1
   60 IS=IS+1
  100 ISTART=ISTART+IBLOCK
      INDEX=INDEX-1
      IF(INDEX) 180,180,15
C FOURIER TRANSFORM NOW IN A-ARRAY (INDICIES REVERSER ORDER)
C TRANSPOSE ARRAY BY WRITING ON TAPE IN BLOCKS OF 100
C AND THEN READING
  180 JJ=0
      DO 185 I=1,NFAC
  185 JI(I)=0
      JI(NFAC)=-1
      REWIND 1
      NCYC=NUM/100
      IF(NCYC) 250,250,190
  190 IBLOCK=100
  200 DO 240 I=1,NCYC
      DO 230 J=1,IBLOCK
      JJ=JJ+1
C FIND CELL NUMBER J1 CORRESPONDING TO THE JJ ELEMENT
      K1=NFAC
  210 JI(K1)=JI(K1)+1
      IF(JI(K1)-L(K1)) 220,215,215
  215 JI(K1)=0
      K1=K1-1
      GO TO 210
  220 J1=1
      DO 225 K=1,NFAC
  225 J1=J1+JI(K)*N(K)
  230 B(J)=A(J1)
      WRITE (1) B
  240 CONTINUE
      IF(IBLOCK-100) 260,250,250
C ARE THERE REMAINING POINTS TO WRITE
  250 IBLOCK=NUM-NCYC*100
      NCYC=1
      IF(IBLOCK) 260,260,200
  260 REWIND 1
      J1=1
      J2=100
      IF(NUM-100) 320,320,300
  300 DO 310 I=1,NUM,100
```

```
      READ (1)(A(J),J=J1,J2)
      J1=J1+100
  310 J2=J2+100
      IF(J1-NUM) 320,330,330
  320 READ (1)(A(J),J=J1,NUM)
  330 RETURN
      END
```

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 494
(BBC Task 204)

C. H. Dale
5 July 1967

TRACKING THE LM AND CSM FROM LM ASCENT CUT-OFF TO RENDEZVOUS USING SIMILAR FILTERS AND NO PSEUDO-BIASES IN MEASURABLES

Previous notes (No's. 488, 491 and especially 492) describe the basic tracking situation and lead up to the idea that relative (CSM-LM) estimates, using MSFN data only, might be optimized if:

1. The RTODP filters used for the CSM and LM were made to be as similar as possible; and

2. The batches of data were used to estimate the state vectors alone without estimating pseudo-biases.

In Note No. 492 the above point 1 was implemented but pseudo-range rate biases for each station were estimated for each batch of data for each vehicle. These pseudo-bias estimates were thrown away at the beginning of each new batch, thus following the intended design of the RTODP filters. However, the throwing away of such estimates changes the correlation between vehicle estimates and the non-estimated nuisance parameters. And it would seem that these systematic uncertainties should cause almost identical errors in two vehicle estimates when these two vehicles have trajectories as near as the CSM and LM during the rendezvous mission. Another way of looking at this point is to say that the MSFN measurable error due to North station location error, for example, should be almost identical for the LM and CSM. Thus, in LM/CSM relative estimates, North station location errors should cancel assuming that the effect of station location error has not been swallowed up in a pseudo-bias estimate which has been thrown away.

The computer study reported herein is identical to that reported in Apollo Note No. 492 except that all apriori pseudo-bias variances have been set to a very small number (10^{-20} ft^2/sec^2) so that, in effect, there are no pseudo-biases. It would be expected that the individual state estimates (CSM and LM) would suffer, and this is clearly shown in Figure 1. Toward the end of the Rendezvous Mission, when uncorrelated apriori estimates are less important, it can be seen in Figure 1 that an improvement in relative position (due to the out-of-plane component) has accrued due to not estimating pseudo-biases. Unfortunately the velocity at rendezvous has deteriorated.

It would thus seem that using the MSFN alone will result in about a mile in rendezvous position error with around 3 ft/sec of velocity error. These errors will be essentially out-of-plane, dependent upon apriori estimates, and dependent also on the North-South station separation. It can be concluded further that the position and velocity uncertainty at rendezvous can be reduced by an order of magnitude with good terminal relative radar tracking. Landmark tracking should be investigated also.

Time	Event	$\dot{R}$ pseudo-biases = 0				$\dot{R}$ pseudo-biases = 0.1 ft/sec.			
		Position		Velocity		Position		Velocity	
		Total	In-Plane	Total	In-Plane	Total	In-Plane	Total	In-Plane
0	Apriori CSM = LM	8660	7070	17.32	14.14	8660	7070	17.32	14.14
	CSM (after Batch)	8853	7152	21.36	7.97	4570	861.8	10.59	.9453
	LM (after Batch)	7917	5935	21.12	7.27	4531	735.9	10.56	.8800
	Relative (after Batch)	6105	1607	2.58	1.207	6093	1100.	7.37	1.256
12.74	CSM (After Batch)	3818	500	20.65	.922	4775	191.4	5.82	.2032
	LM (after Batch)	4042	701	20.91	1.015	4296	158.2	6.52	.2570
	Relative (after Batch)	3149	261	1.25	.275	4637	172.5	5.07	.3069
68.71	CSM (after Batch)	11309	472	24.86	.322	4789	264.5	5.88	.1532
	LM (after Batch)	5735	520	24.10	.999	3503	290.7	5.86	.3491
	Relative (after Batch)	7519	186	2.40	.937	4625	121.3	4.78	.3141
100.08	CSM (after Batch)	28884	353	7.60	.296	5974	147.7	3.63	.1166
	LM (after Batch)	27796	409	7.02	.478	5966	192.1	2.58	.2121
	Relative (after Batch)	2063	154	3.90	.268	3587	102.2	3.16	.1424
149.99	CSM (projected)	27307	348	10.80	.296	6589	141.7	2.84	.1183
	LM (projected)	27001	491	8.89	.369	6123	234.7	2.32	.1752
	Relative (projected)	2357	411	3.92	.409	4407	183.4	2.37	.1737
	MSFN Only.								
	With Intervehicle Data From 101. to 130.								
149.99	CSM (projected)	27307	348	10.80	.296	6589	141.7	2.32	.1737
	LM (projected)	27301	578	10.60	.518	6555	324.1	2.88	.3600
	Relative (projected)	469	453	0.52	.429	389	378.3	0.38	.2320

*From Note No. 492

Figure 1. The Effect of Estimating State Parameters Alone During Each Batch

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 495
(BBC Task 101)

H. Engel
10 July 1967

MORE ON THE SPEED OF LIGHT, AND ATOMIC TIME VERSUS EPHEMERIS TIME

The problem of whether an error in the speed of light is an error source in trajectory estimation has been a source of argument in the ANWG. This note presents the present Bissett-Berman view of this problem. This note draws heavily from JPL Technical Report 32-816, "Determination of the Masses of the Moon and Venus and the Astronomical Unit from Radio Tracking Data of the Mariner II Spacecraft," by John D. Anderson. Thanks are due John Anderson for his exposition on fundamental constants and planetary theory.

We first define the units of mass, length and time used in astronomy.

1. The fundamental unit of mass is the mass of the Sun.

2. The tropical year is defined as the interval between successive crossings of the equator by the Sun from South to North. The tropical year is not constant because the period of the Earth's orbit is perturbed by the other planets and because the crossing point, the vernal equinox, is not fixed with respect to the stars because the Earth precesses and nutates.

   The unit of time is, by definition, the instaneous value of the tropical year at the beginning of 1900. The theories of celestial mechanics are so accurate that measurements made today can be used to establish the length of the tropical year 1900. The ephemeris second is, by definition, 1/31556925.9747 of the tropical year 1900. The ephemeris day is 86400 ephemeris seconds.

3. The period of a body in motion about the Sun may be written as

$$T = 2\pi \sqrt{\frac{a^3}{k^2(1+m)}}$$

in which a is the semi-major axis of the ellipse, k the Gaussian constant, and m the mass of the body. The Gaussian constant k is, by definition, 0.017202098950000.

The <u>unit of length,</u> the astronomical unit (a. u.), is the semi-major axis of a fictitious planet on an undisturbed orbit having the mass and sidereal period that Gauss adopted for the Earth ($m_\oplus = 1/354,710$ and $P_\oplus = 365.2563835$ days).

The equations of motion for the nine planets of the solar system are

$$\ddot{\vec{r}}_i = -k^2 (1 + m_i) \frac{\vec{r}_i}{\vec{r}_i^{\,3}} + k^2 \sum_{j=1}^{9} (1 - \delta_{ij})\, m_j \left(\frac{\vec{r}_{ij}}{\vec{r}_{ij}^{\,3}} - \frac{\vec{r}_j}{\vec{r}_j^{\,3}} \right),$$

$$i = 1, \cdots, 9$$

except for small relativistic corrections and a perturbation due to motion of the Earth-Moon system about its barycenter. The only constants in these equations are k and m_i. Now, k is fixed by definition. If the estimated of the masses of the planets are improved, then the m_i will change. Otherwise, these equations of motion are invariant with respect to the values adopted for the speed of light, the length of the standard meter, and so forth.

A similar argument can be made for the lunar ephemeris.

The speed of light is important with respect to these equations only in that measurements involving the speed of light are used to check the theory and to establish the values of the constants in the solution of these equations. Even here, the speed of light need be known only in units of a. u. 's per ephemeris second. According to Anderson, and I accept it on faith, the speed of light in terms of a. u. 's per ephemeris second is known to about 1 part in 10^8. As a result, if range measurements from the Earth to a spacecraft near the Moon are made, the error in the measurement in a. u. 's due to this uncertainty will be equivalent to about 3 meters. The error in the measurement of radial velocity in a. u. 's per second, for a radial velocity of 12,000 meters/sec, due to this same source, will be equivalent to just 1.2×10^{-4} meters/sec.

Note that for ephemeris construction or for determination of the orbit of a spacecraft it is not necessary to employ the meter as a unit of distance. If results are to be expressed in meters (or feet) for ease of comprehension, an arbitrary constant may be used for the speed of light to perform the conversion of units. This constant may be the ANWG and International Astronomical Union adopted value of 2.997925×10^8 meters/sec. The uncertainty of 100 meters/sec associated with this value in ANWG Technical Report No. AN-1.2 is the uncertainty of the speed of light in meters per second, and does not affect the orbit estimates.

Atomic time standards are now being built with accuracies of one part in 10^{11} or one part in 10^{12}. Since the year has 3.16×10^7 seconds, this apparently corresponds to an accuracy of 3×10^{-4} or 3×10^{-5} ephemeris seconds per year, but this is not so. John Anderson, in a discussion at JPL pointed out that atomic clocks do not keep ephemeris time. That is, the rate of an atomic clock on the Earth is not constant. This occurs because the distance of the Earth from the Sun varies and consequently the gravitational field in which the clock operates is changing. According to the general theory of relativity

the rate of a clock - any clock - depends upon the gravitational field in which it exists. A simple explanation in terms of relativity theory can be found on pages 42-9, 10 and 11 of Volume 2 of "The Feynman Lectures on Physics." The equation that results is that the clock rate is proportional to

$$1 + \frac{gH}{c^2}$$

in which g is the gravitational field and H is the height in the field.

The value of g at the Earth due to the gravitational field of the Sun can be computed from

$$T = 2\pi \sqrt{\frac{a^3}{u}} = 2\pi \sqrt{\frac{a^2}{u} a} = 2\pi \sqrt{\frac{a}{g}}$$

so that

$$g = \left(\frac{2\pi}{T}\right)^2 a \ .$$

The distance H we take as half the difference between the aphelion and perihelion radii

$$H = ae$$

so that

$$\frac{gH}{c^2} = \left(\frac{2\pi a}{cT}\right)^2 e$$

$$= \left(\frac{2\pi \ \ 2 \times 10^{11}}{3 \times 10^8 \times 3.16 \times 10^7}\right)^2 \ 0.0167$$

$$= 2.97 \times 10^{-10} \ .$$

If this rate error were to persist for one quarter year it would result in a clock error of 2.4 sec.

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 496
(BBC Task 105)

H. Epstein
12 July 1967

ANALYSIS OF SOME LLO TRANSLUNAR PHASE USB TRACKING DATA

MSC has made available to BBC some USB three-way doppler tracking data taken during the translunar phases of LLO 3. This data was taken at a sample rate of six seconds with the equipment in the non-destruct doppler cycle counting mode. This was card data using the low speed format. This card data was processed in the fashion outlined in Apollo Note 497. The Copy Program for this card data removed illegible symbols and provided a low speed tape to the Pre-edit Program. The card data format indicated that all data had been taken with a crystal standard for reference. The analysis performed indicated that the data contained both crystal standards and rubidium standards as references. (It was not necessarily expected that this early data would conform to the LLO format as indicated in AS-501.) With no ODP, emphasis was placed on the high frequency error components indicated by the analysis program.

A sample input data sheet for the Pre-edit Program is included in Figure 1. The Edit Program requires two threshold constants. The F count threshold was set at 100. This means that if the magnitude of a second difference of the doppler count exceeds 100 that the doppler counter reading is rejected (see Note 490). The second threshold (K) was set to eliminate data points which resulted in second difference being more than $K\sigma$ from the median of the distribution of data point (K was set at a value of 6). The stations and their time intervals involved are indicated on Table 1 below.

Punch: 1 for yes; 2 for no; 9 for test to be ignored

		Yes	No	Test to be Ignored
1	Is this high speed 240-Bit Data		✓	/////
2	Is this non-destruct data	✓		/////
3	Is this high data (Bit 15)			✓
4	Is range-rate in standard position	✓		/////
5	Is range-rate N_1 mode			✓
	Only one of four applies:			
6	Is this one way doppler mode			/////
7	Is this two way doppler mode			/////
8	Is this multiple non-coherent mode	✓		/////
9	Is this multiple coherent mode			/////
10	Vehicle ID is [0]	/////	/////	
11	Is frequency standard rubidium		✓	
12	Is manual R-R test to be made	✓		/////
13	Is VCO lock test to be made		✓	/////
14	Is automatic range-rate test to be made	✓		/////
15	Is test to be made on real/test Bit	✓		/////
16	Is station ID test to be made	✓		/////
17	Is doppler mode test to be made	✓		/////
18	Is test to be made on R-R field Indicator	✓		/////

STATION ID

20		Bermuda
21		Merritt Island
22		Grand Bahama Island
23		Antigua
24	x	Carnarvon
25		Hawaii
26		Guaymas
27		Texas
28		Guam
29		Goldstone
30		Ascension
31		Canberra
32		GSFC
33		Madrid
34		Grand Canary Islands

EXPECTED START TIME

35-37	0 3 7	Day (If greater than 31 month will be ignored)
38-39		Month
40-41	6 7	Year
42-43	0 0	Hour
44-45	1 5	Minute
46-48	6	Expected Delta Time
49-51	3 6 0	Maximum Time Interval in Minutes
52-55	5 0	Print Rejected Data
56-59	1 0 0	Print Raw Data

Figure 1. Input Data Sheet for Pre-edit Program

Station & Station ID	Start Time		End	
	Day	GMT	Day	GMT
Carnarvon (08)	037	00:15:30	037	03:11:06
Guam (24)	036	20:22:12	036	23:12:36
Goldstone (28)	036	22:53:24	036	23:43:00
Texas (16)	036	22:01:42	036	22:50:30

Table 1. Time Interval for Translunar Data

During the analysis it was found that the Guam data contained both crystal and rubidium standards as references. This data set was then split into two groups. The results further indicated that some data labeled crystal was actually rubidium. It was also noted that the least significant bit in the Goldstone doppler data was always 0, 2, 5, or 7 (1, 3, 4, and 6 never occurred). This either indicates a malfunction of equipment or a deviation from the normal practice in the use of the equipment. The number of measurement values, acceptable second difference, and type frequency standards are indicated in Table 2 below.

Station	Number of Measurement Values		Accepted Second Differences	Type Frequency Standard
	Output of Pre-edit	Output of Edit Program		
Carnarvon	1159	1157	1143	Rubidium
Guam-1	1055	1037	999	Crystal
Guam-2	556	551	529	Rubidium
Goldstone	441	422	418	Crystal
Texas	488	488	486	Rubidium

Table 2. Type Frequency Standard and Number of Points

A simple analysis on a data point rejected by the editing program indicated a frequency shift present in the Guam 1 data. This frequency shift took place between a GMT of 21:30:00 and 21:30:06. This frequency shift could probably be associated with a tuning operation at either the master DSN station or the slave USB station. The frequency shift was about 3190 cps. This would correspond to an average $\dot{R}$ change of about 700 ft/sec during this six second period.

The detrending operation performed on this data consisted of taking the second difference of the doppler counter readings and generating residuals from a fourth degree polynomial fit to this data. These residuals may be considered as the higher frequency components of the second difference of doppler count data.

The Analysis Program takes the correlation function and makes spectral estimates on this residual data. The rubidium residuals will be treated first. The dominant error source in the six second data for these residuals should be expected to be the counter quantization noise. The expected values for the variance and correlation function are indicated in Apollo Note 351. A reference noise model can then be taken for this second difference data. Computed values and the reference values are indicated in Table 3. The results obtained are clearly consistant with the noise model. The number of points and the data span involved do not allow the clock white frequency noise component or other random phase noise components to be reliably estimated.

Station	$\hat{\sigma}$ (counts)	$\hat{\rho}(1)$	$\hat{\rho}(2)$	$\hat{\rho}(3)$
Carnarvon	.735	-.649	.135	-.001
Guam-2	.713	-.662	.151	-.015
Texas	.791	-.647	.116	.062
Noise Model	.707	-.667	.167	0

Table 3. Rubidium Residuals

Residuals based on crystal standards may be dominated by crystal white frequency noise in place of the quantization noise. Assuming that the USB station employs a crystal standard and the DSN station a rubidium standard, estimates can be made of the six second sample rate crystal white frequency noise component. The results obtained are consistent with such a model as indicated in Table 4 below. The parameter f and T indicated in the Table have numerical values of 2.3×10^9 cps and 6 seconds respectively for this data. The $\sqrt{2}$ factor arises from the difference operation on this noise. The crystal white frequency noise component is about 1 part in 10^9 for this data.

Station	$\hat{\rho}(1)$	$\hat{\rho}(2)$	$\hat{\sigma}$ (counts)	$\hat{\sigma}$ (cps)	$\Delta\hat{f}/f$
Guam 1	-.523	.064	21.31	2.51	1.1×10^{-9}
Goldstone	-.456	.096	7.94	.94	$.4 \times 10^{-9}$
Noise Model	-.5	0	σ_c	$\sigma_c/T\sqrt{2}$	$\sigma_c/fT\sqrt{2}$

Table 4. Crystal Residuals

The plots of the residuals are extremely helpful to analysis. The raw residual data has been plotted as count error as a function of time. This is particularly appropriate to indicate when quantization error is pre-dominant. The second difference of quantization error would allow value varying between -2 counts and +2 counts. A plot of the Guam data prior to separation according to frequency standards is indicated in Figure 2. Clearly this data should be divided into two regions for analysis.

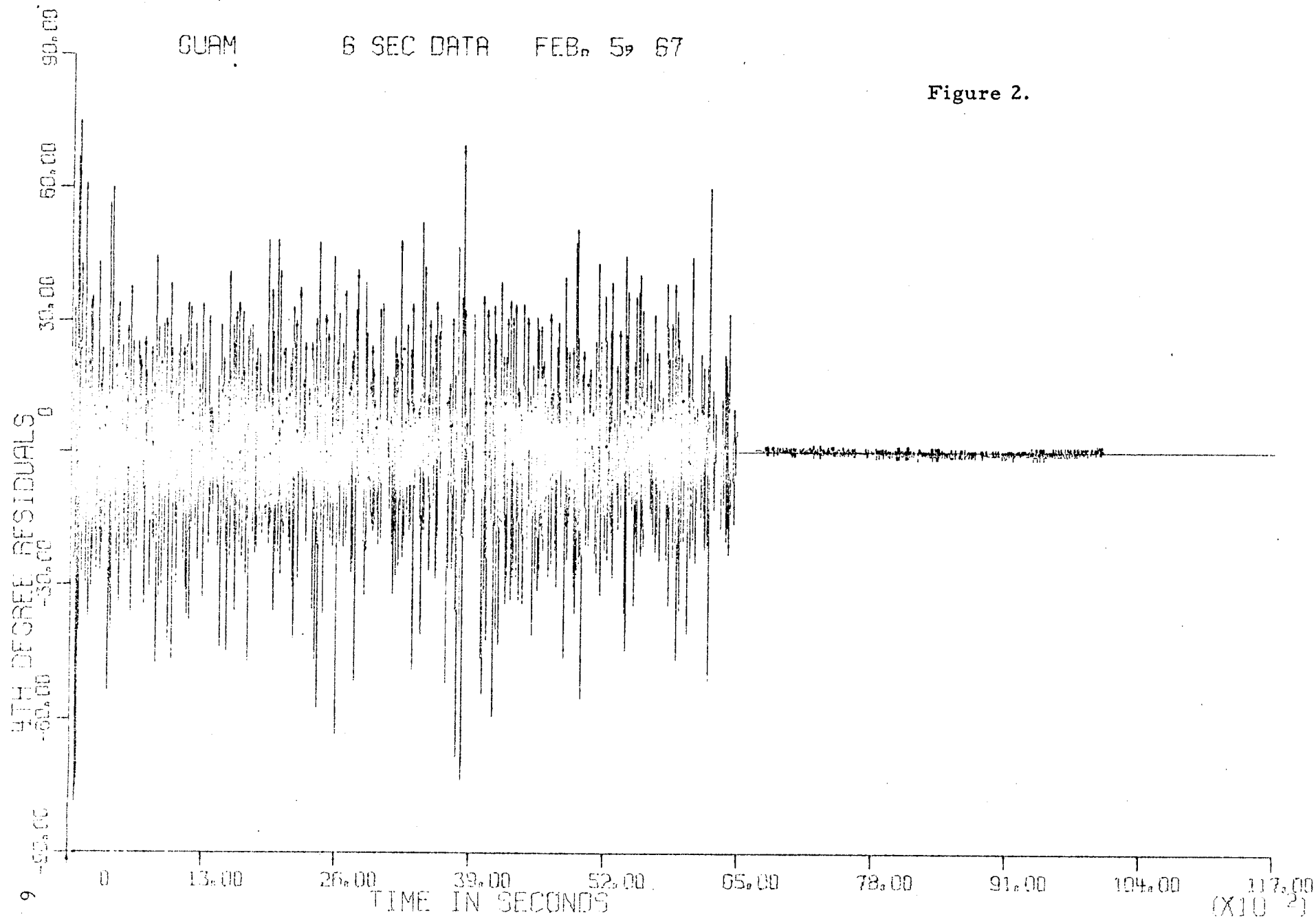

GUAM 6 SEC DATA FEB. 5, 67
90.00
60.00
30.00
0
-30.00
-60.00
-90.00
4TH DEGREE RESIDUALS
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
TIME IN SECONDS
(X10 2)

Figure 2.

Plots obtained with atomic frequency standards used throughout are indicated in Figures 3-17 for Carnarvon, Guam 2, and Texas stations. A set of station data consists of one plot of raw data, one correlation function, and three spectral analysis plots. The plot of the raw residuals from Carnarvon is indicated in Figure 3. The patterns obtained clearly indicate that quantization error is the dominant error source. The corresponding normalized autocorrelation function is indicated in Figure 4 and the spectral estimates in Figures 5, 6, and 7. These plots do not provide significant additional information regarding the error sources present. The raw residual plots for Guam 2 (Figure 8) and Texas (Figure 13) again clearly indicate a dominant quantization pattern to the error source present.

On the other hand, the raw residual plots for the data associated with the crystal frequency standard do not show a significant quantization error. The plots for the crystal data are indicated in Figures 18-27 for Guam 1 and Goldstone stations. The plots for the raw residuals for Guam 1 and Goldstone are indicated in Figures 18 and 23, respectively. The plots clearly indicated a different noise characteristic from the quantization noise.

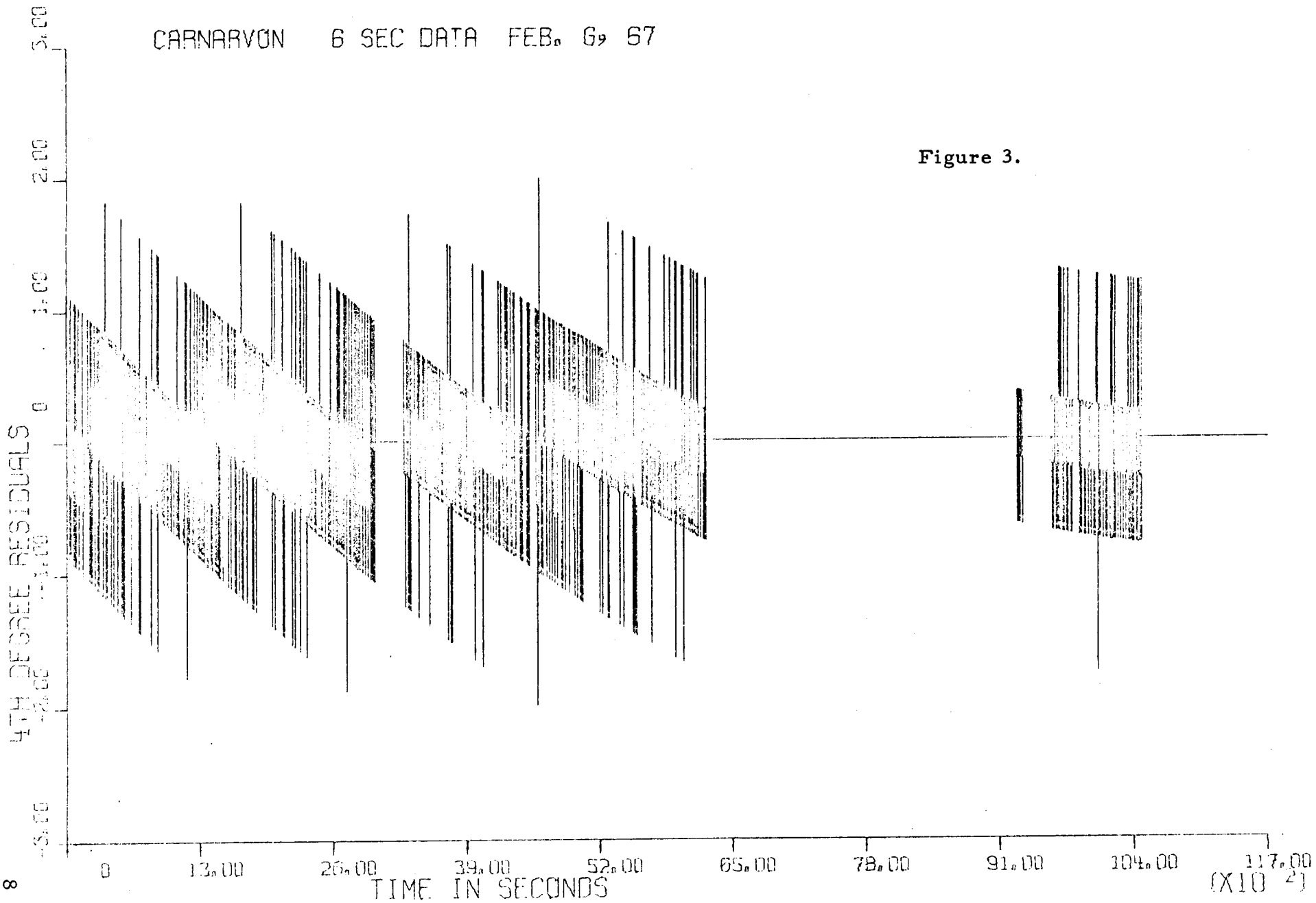

CARNARVON 6 SEC DATA FEB. 6, 67
4TH DEGREE RESIDUALS
3.00
2.00
1.00
0
-1.00
-2.00
-3.00
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
(X10 2)
TIME IN SECONDS

Figure 3.

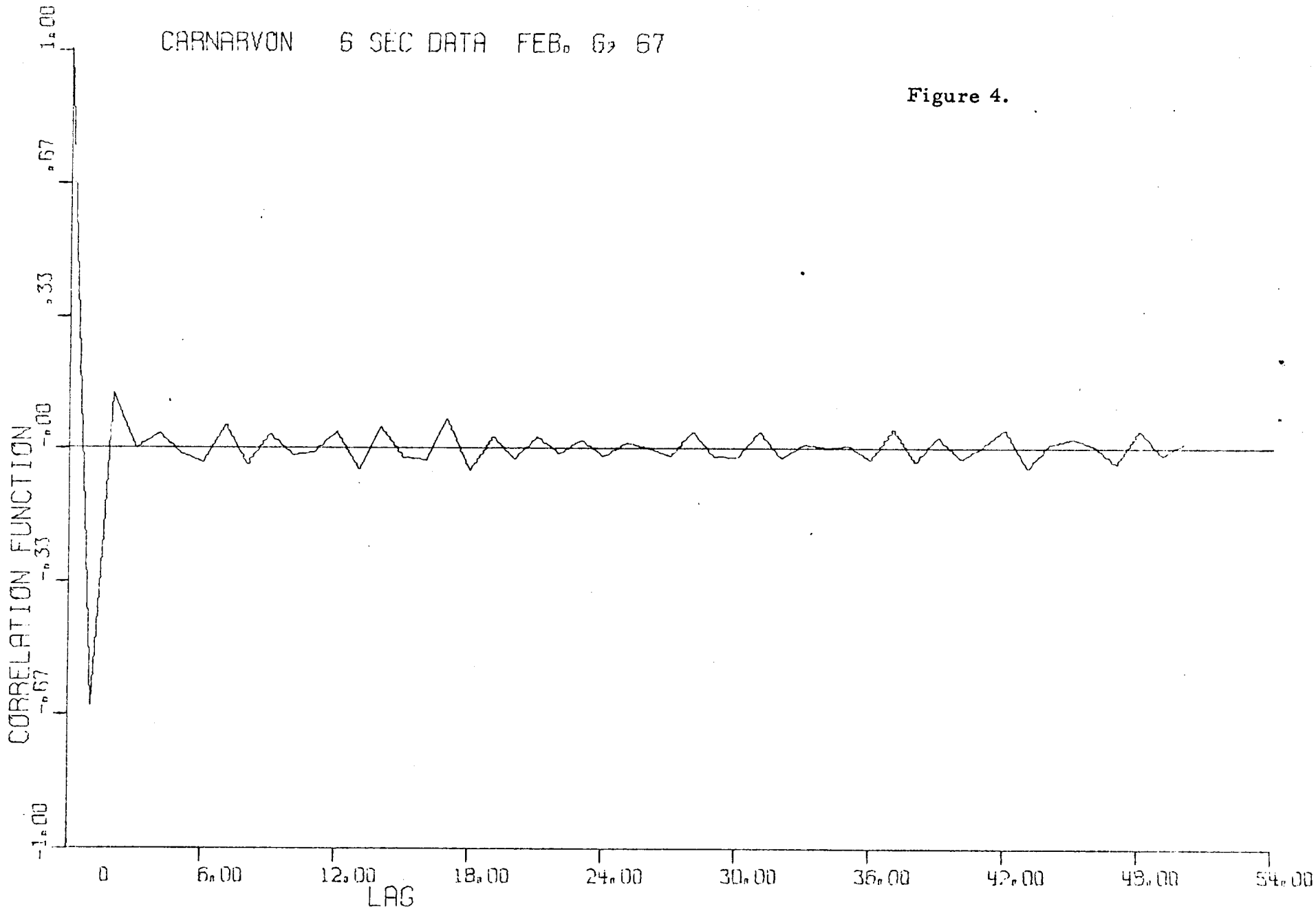

CARNARVON 6 SEC DATA FEB. 6, 67
CORRELATION FUNCTION
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 4.

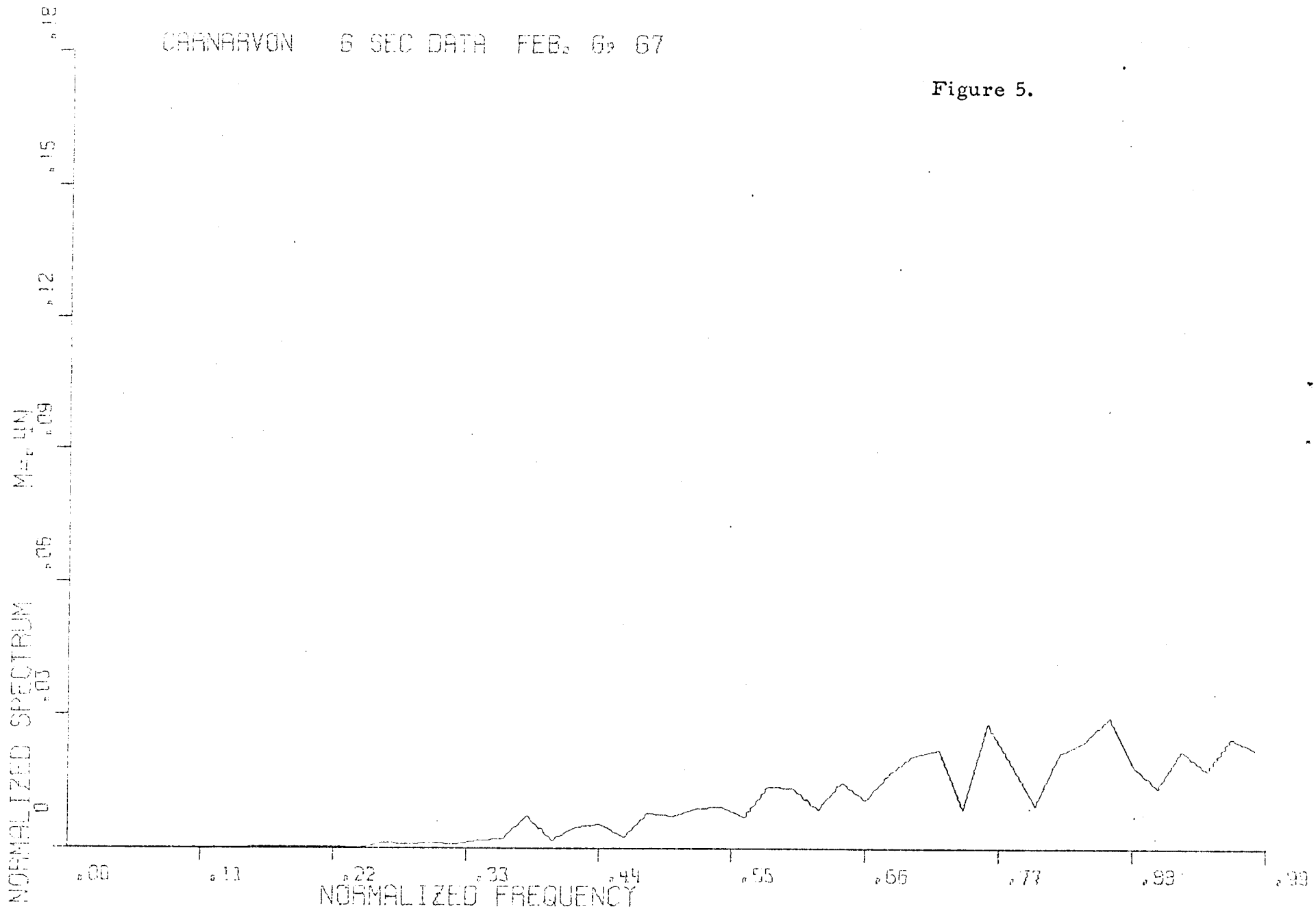

CARNARVON 6 SEC DATA FEB. 6, 67
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 5.

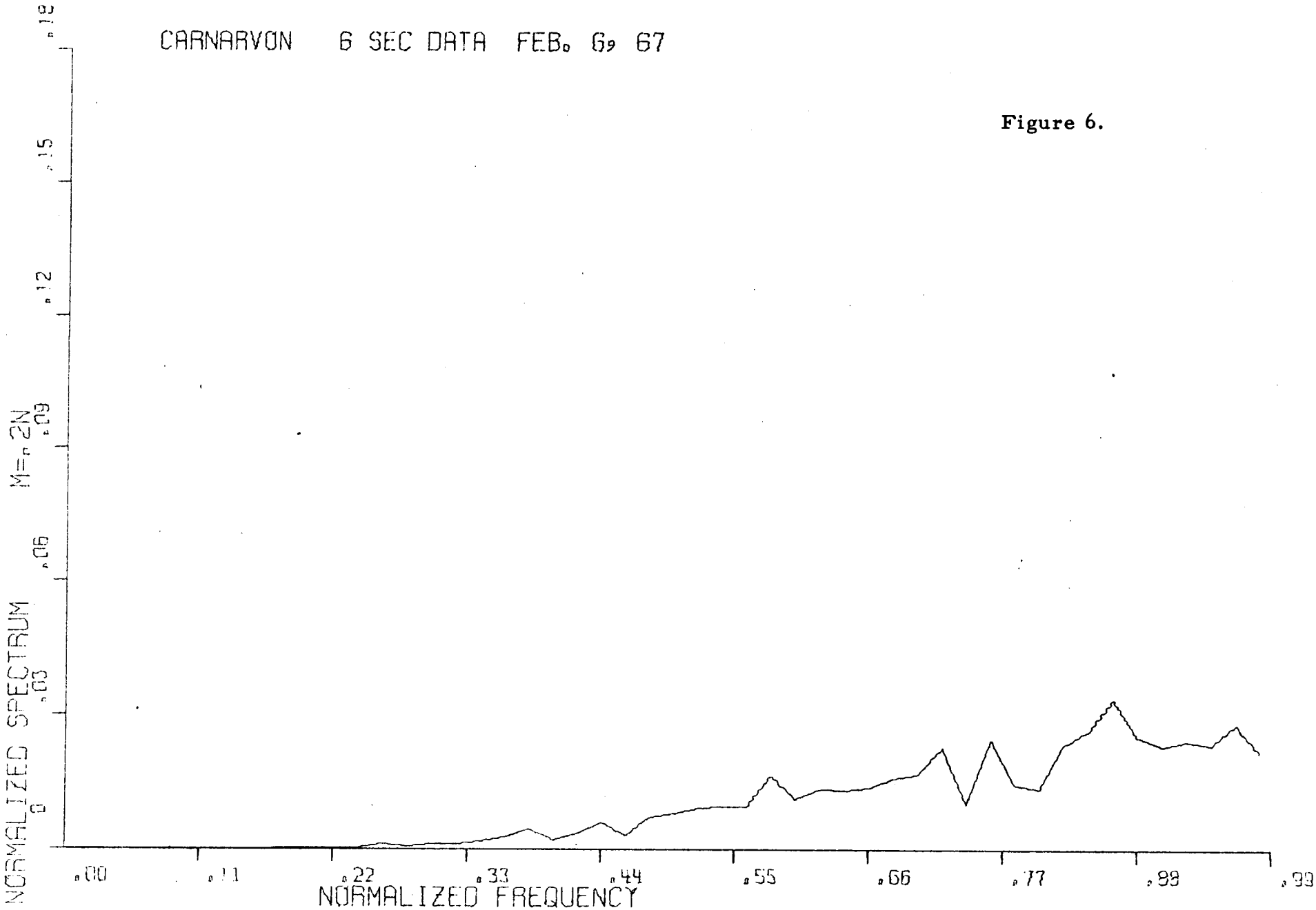

CARNARVON 6 SEC DATA FEB. 6, 67
NORMALIZED SPECTRUM
M=.2N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 6.

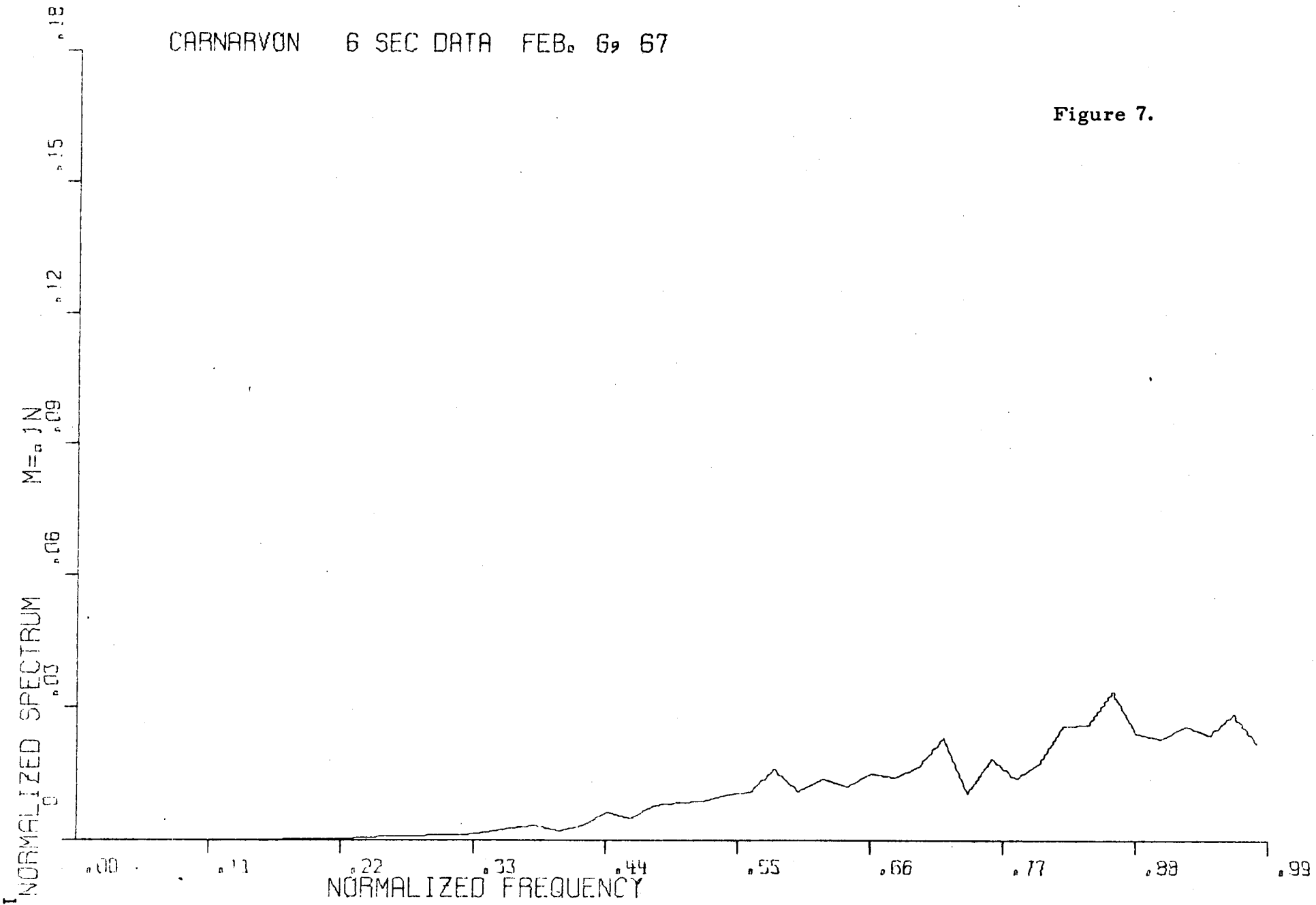

CARNARVON 6 SEC DATA FEB. 6, 67
M=. 1N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 7.

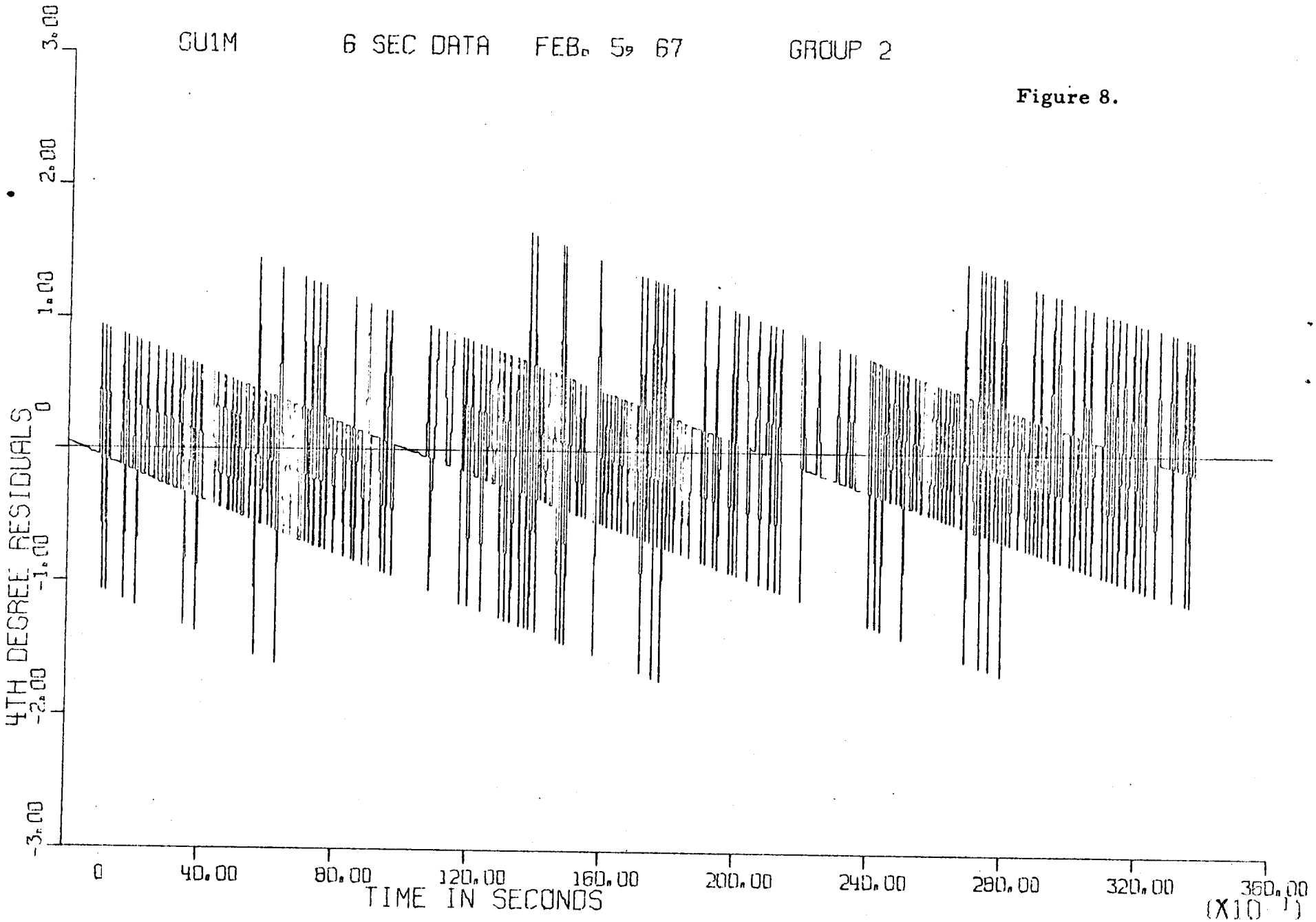

GU1M
6 SEC DATA
FEB. 5, 67
GROUP 2
4TH DEGREE RESIDUALS
3.00
2.00
1.00
0
-1.00
-2.00
-3.00
0
40.00
80.00
120.00
160.00
200.00
240.00
280.00
320.00
360.00
TIME IN SECONDS
(X10)

Figure 8.

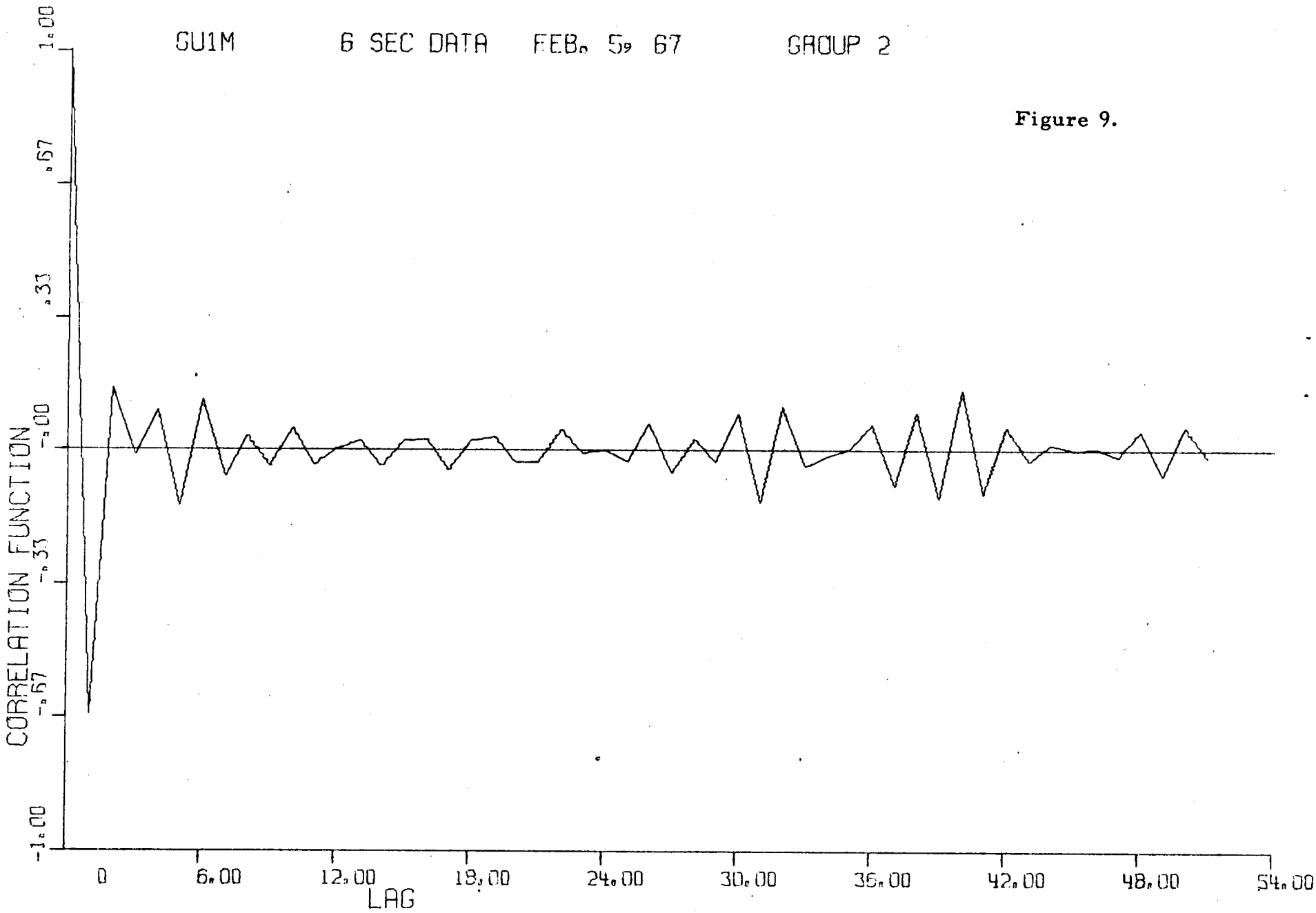

SU1M 6 SEC DATA FEB. 5, 67 GROUP 2
CORRELATION FUNCTION
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 9.

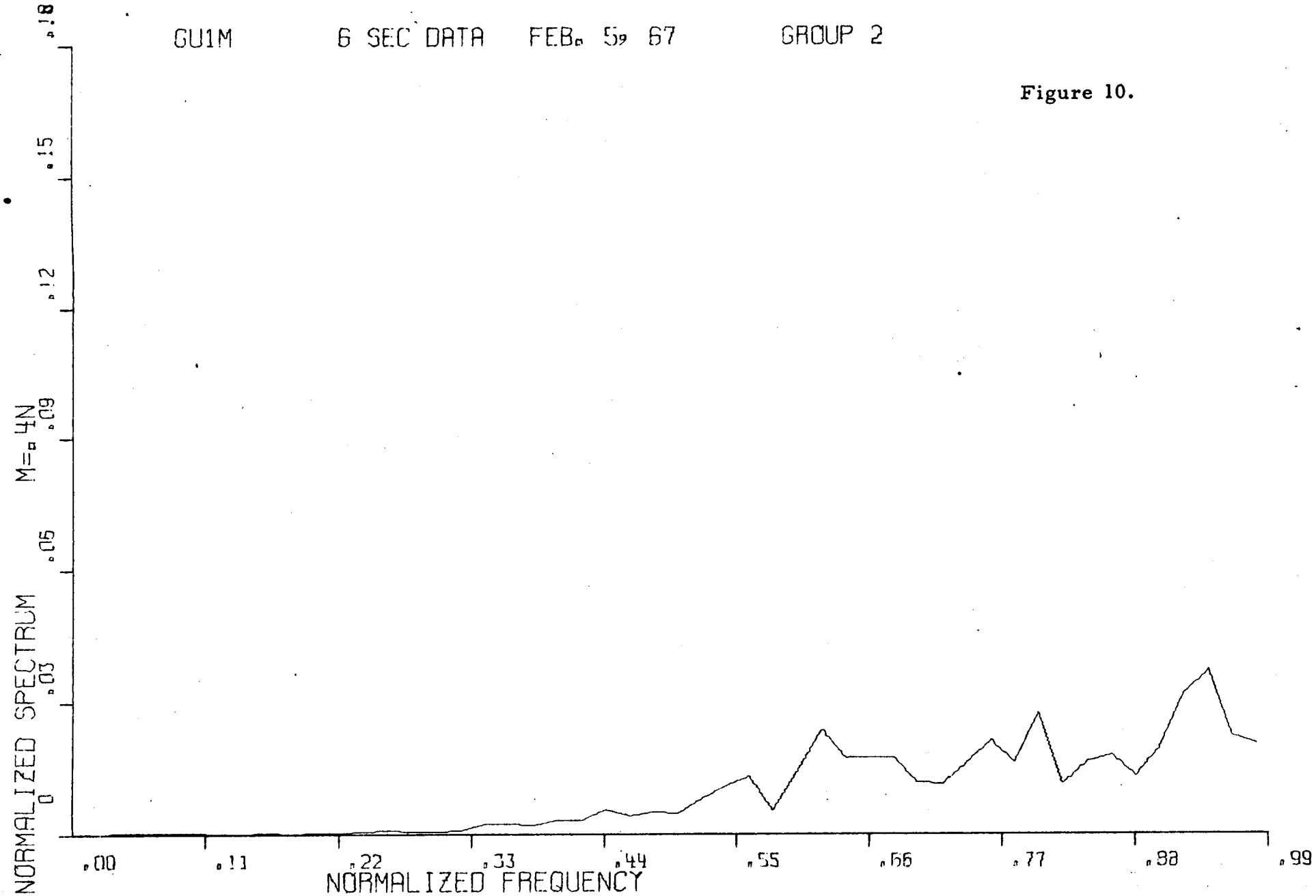

GU1M 6 SEC DATA FEB. 5, 67 GROUP 2
NORMALIZED SPECTRUM
M=.4N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 10.

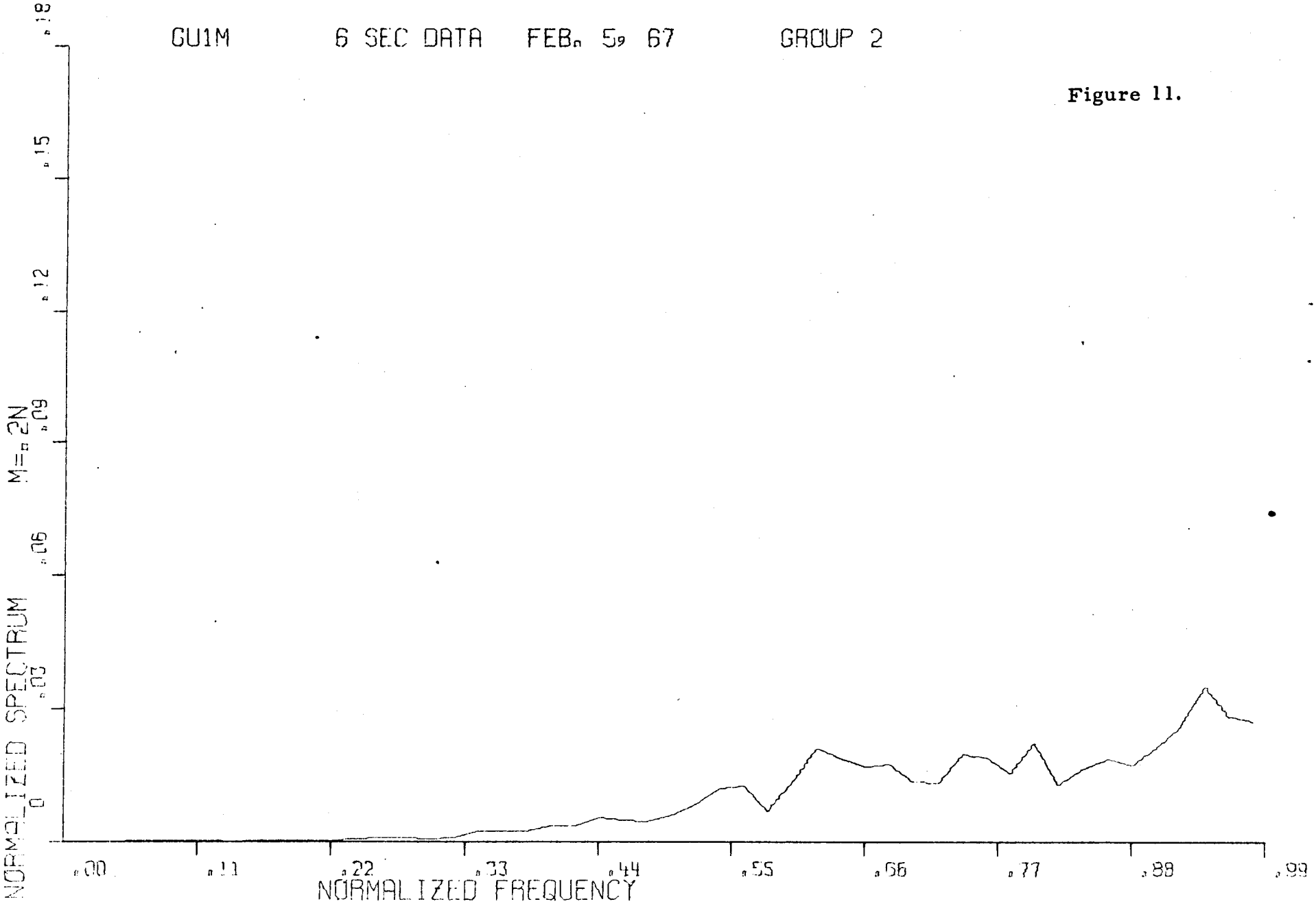

GU1M 6 SEC DATA FEB. 5, 67 GROUP 2
NORMALIZED SPECTRUM M=.2N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 11.

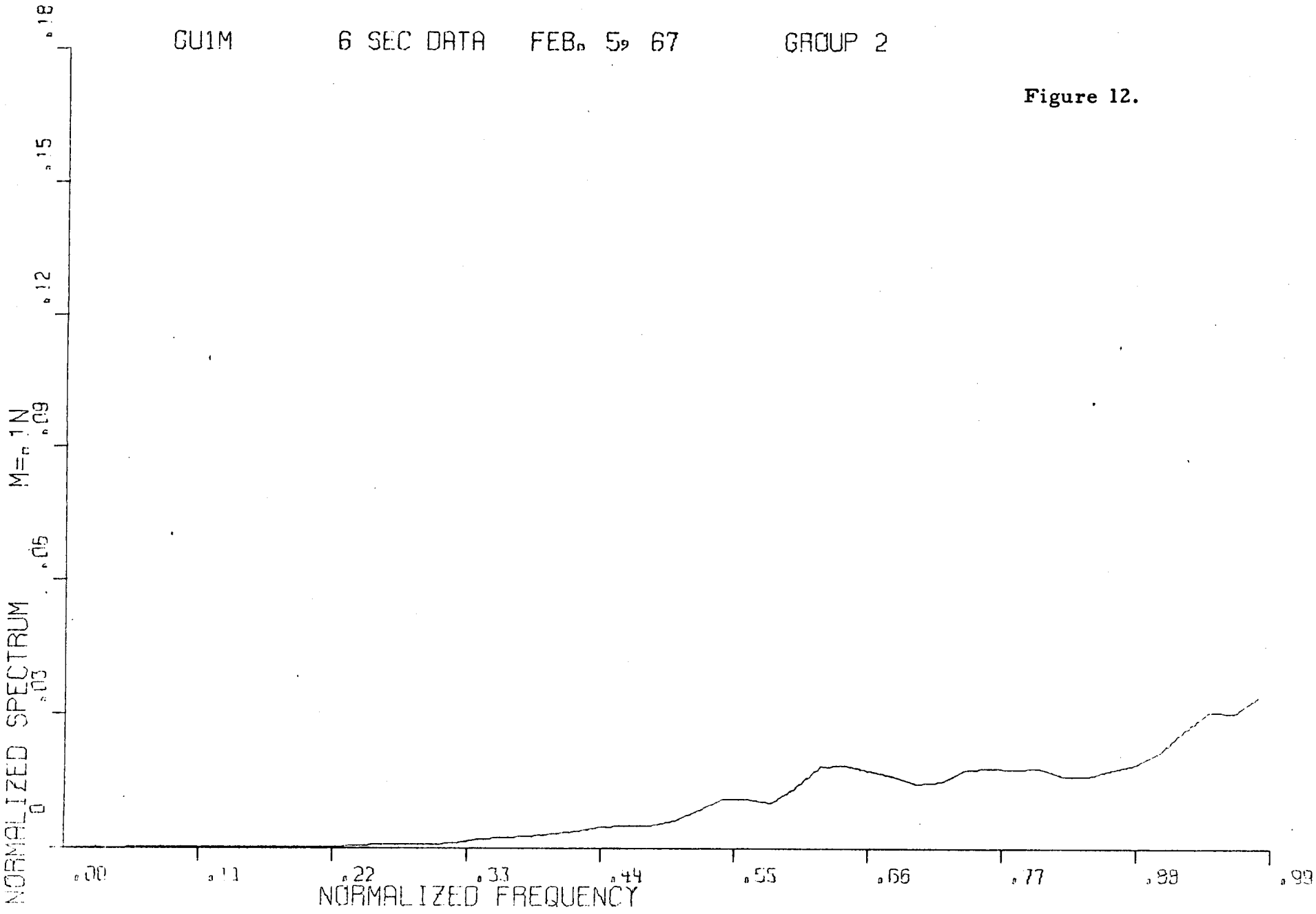

GU1M 6 SEC DATA FEB. 5, 67 GROUP 2
M=.1N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 12.

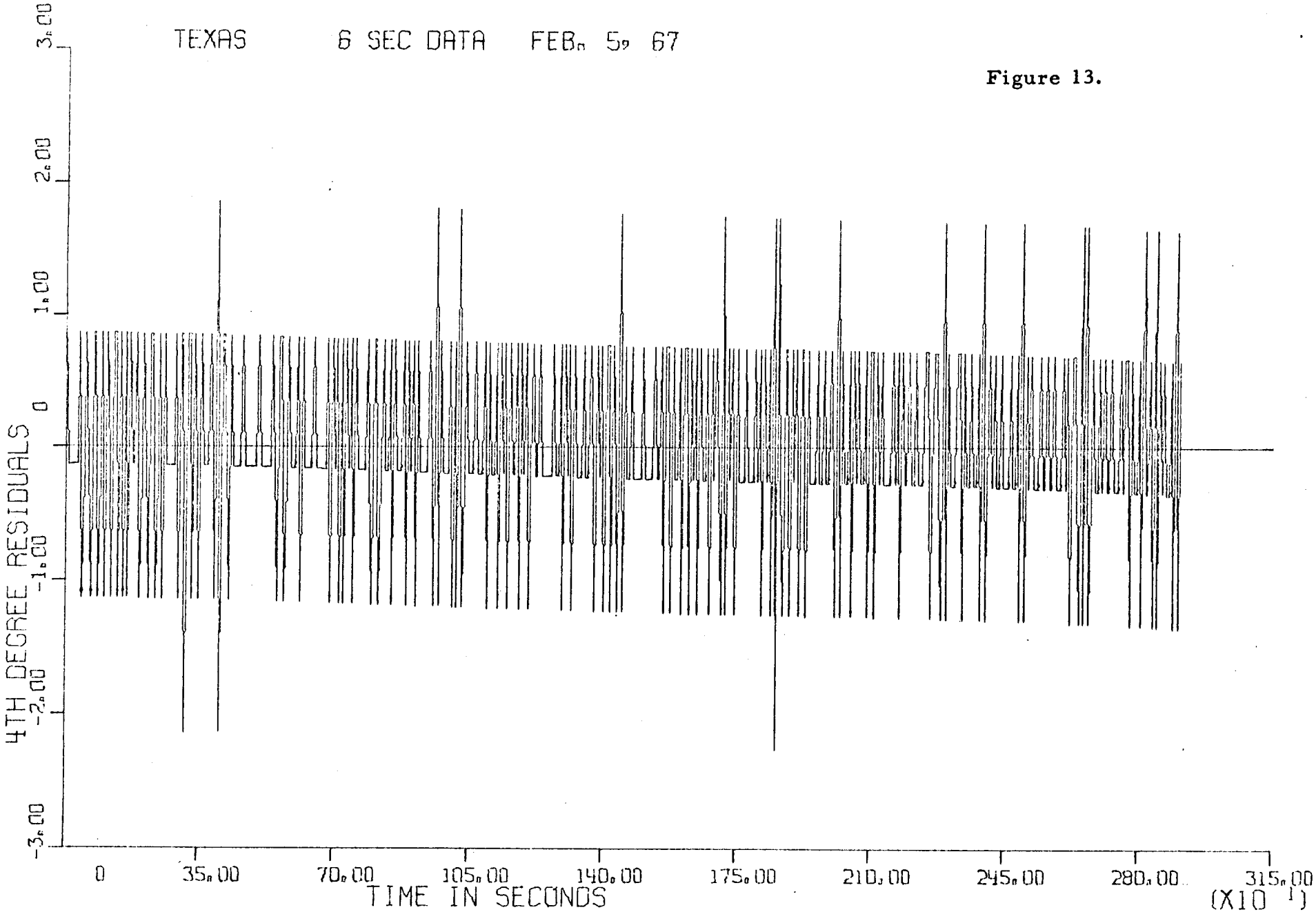

TEXAS 6 SEC DATA FEB. 5, 67
4TH DEGREE RESIDUALS
3.00
2.00
1.00
0
-1.00
-2.00
-3.00
0
35.00
70.00
105.00
140.00
175.00
210.00
245.00
280.00
315.00
TIME IN SECONDS
(X10 1)

Figure 13.

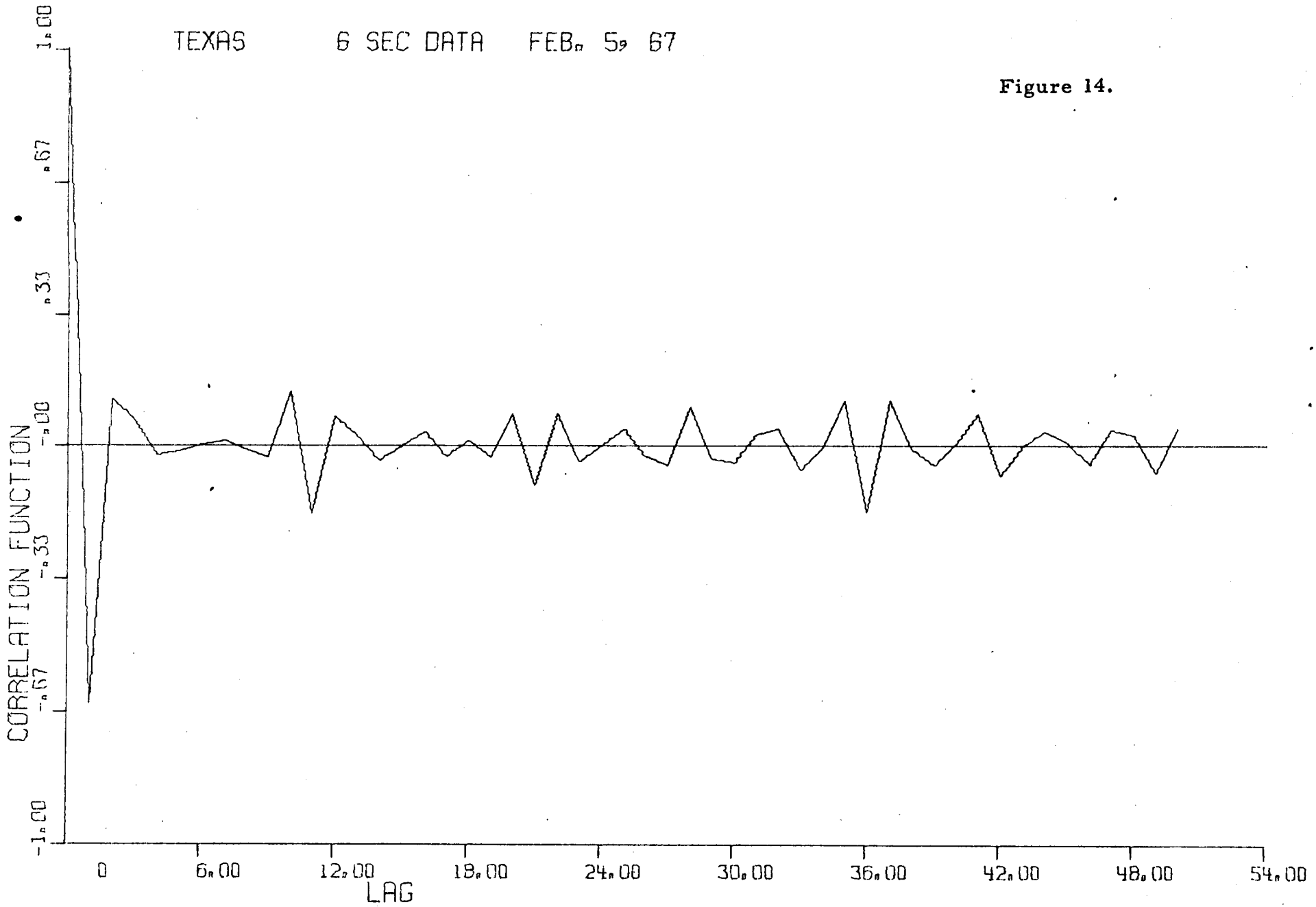

TEXAS 6 SEC DATA FEB. 5, 67
CORRELATION FUNCTION
LAG
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00

Figure 14.

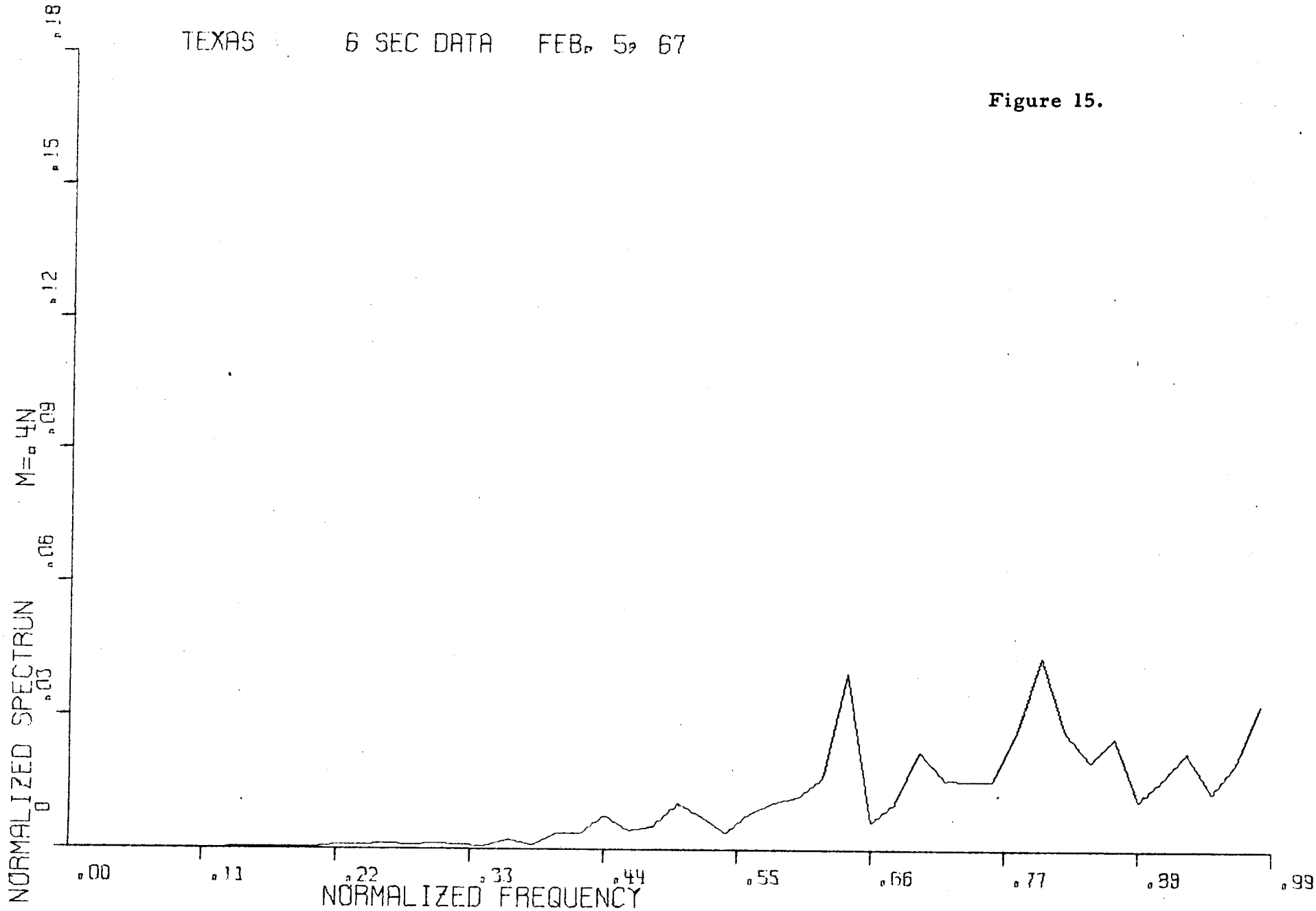

TEXAS 6 SEC DATA FEB. 5, 67
M=.4N
NORMALIZED SPECTRUN
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 15.

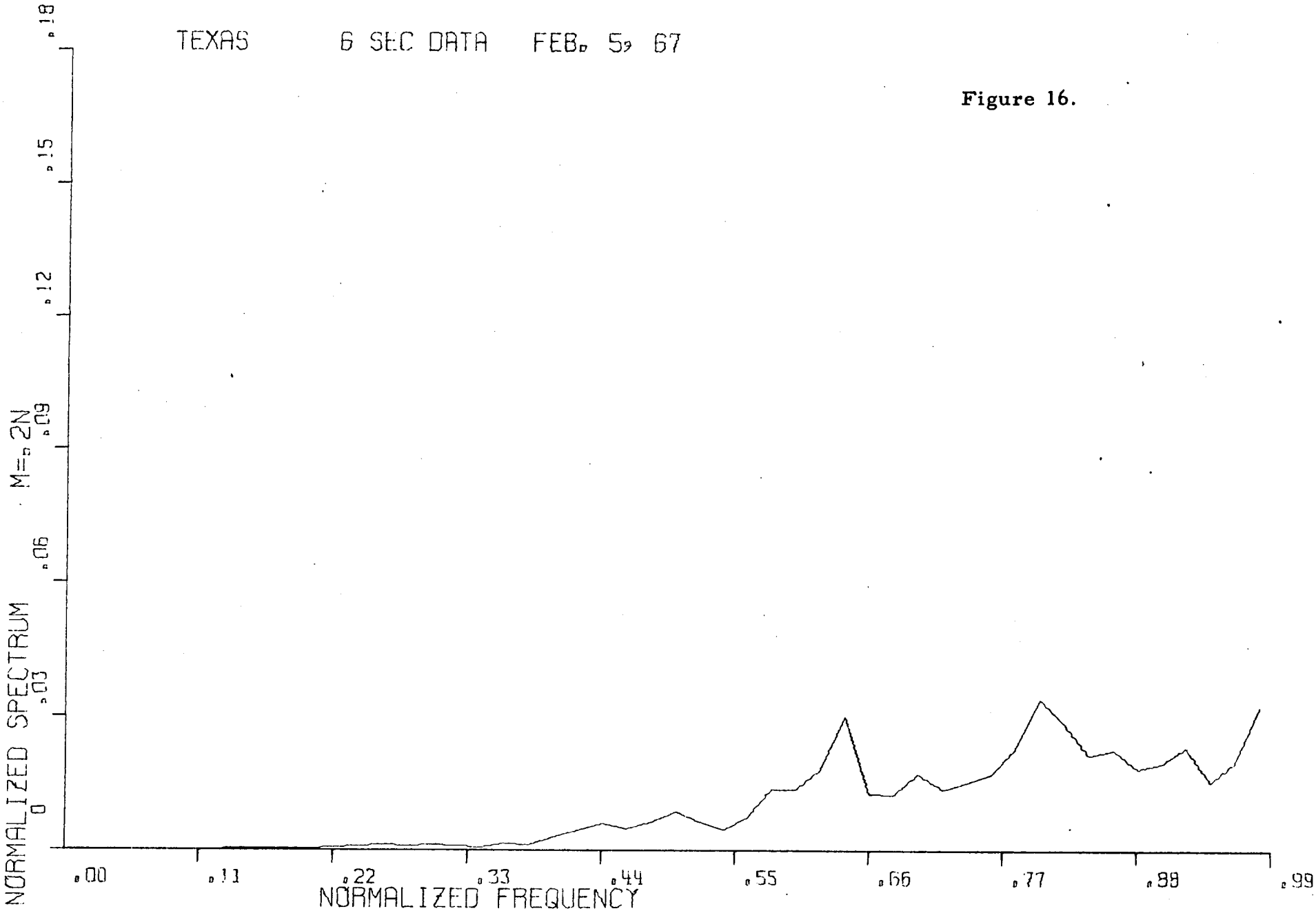

TEXAS 6 SEC DATA FEB. 5, 67
M=. 2N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 16.

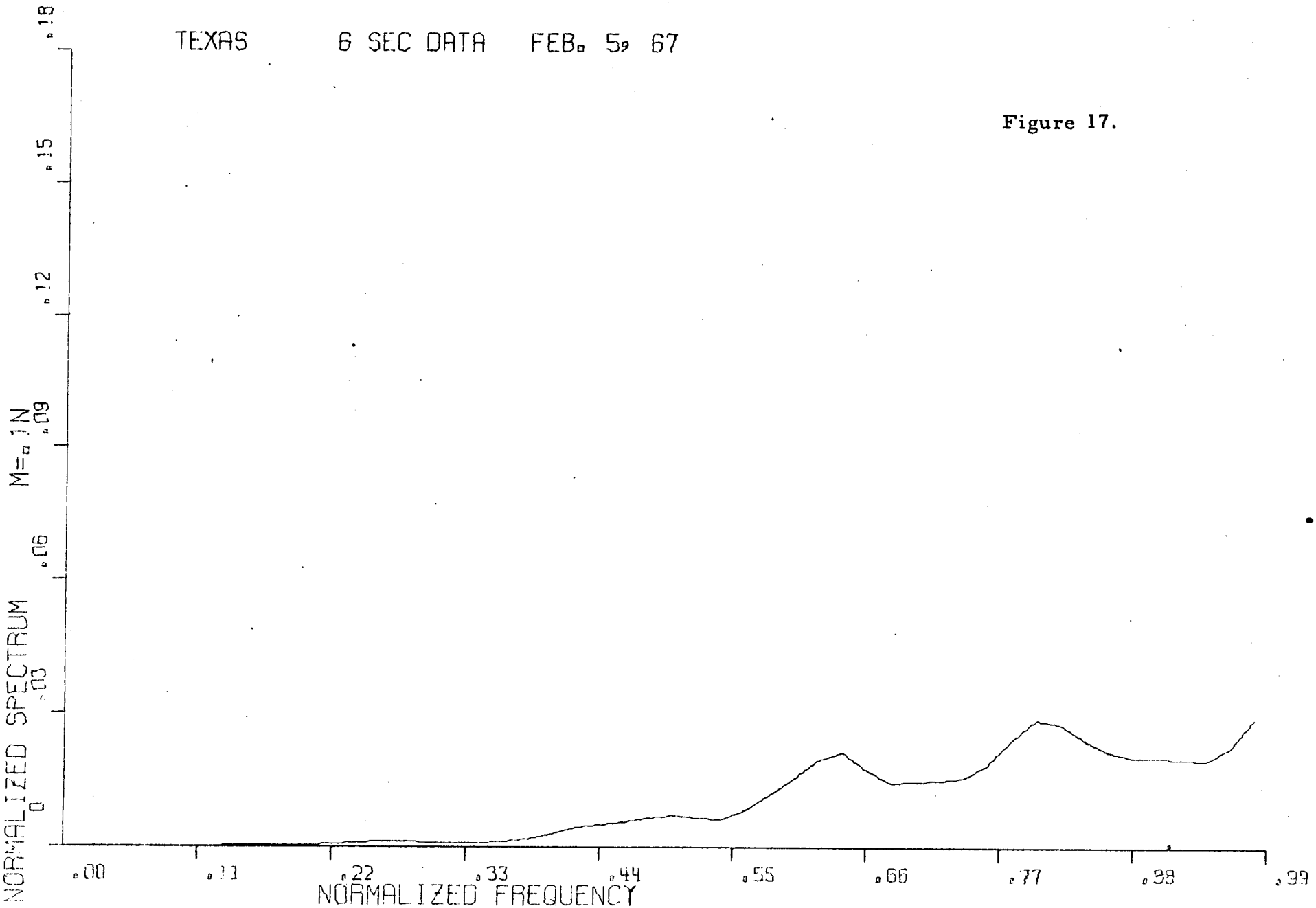

TEXAS 6 SEC DATA FEB. 5, 67
NORMALIZED SPECTRUM
M=.1N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 17.

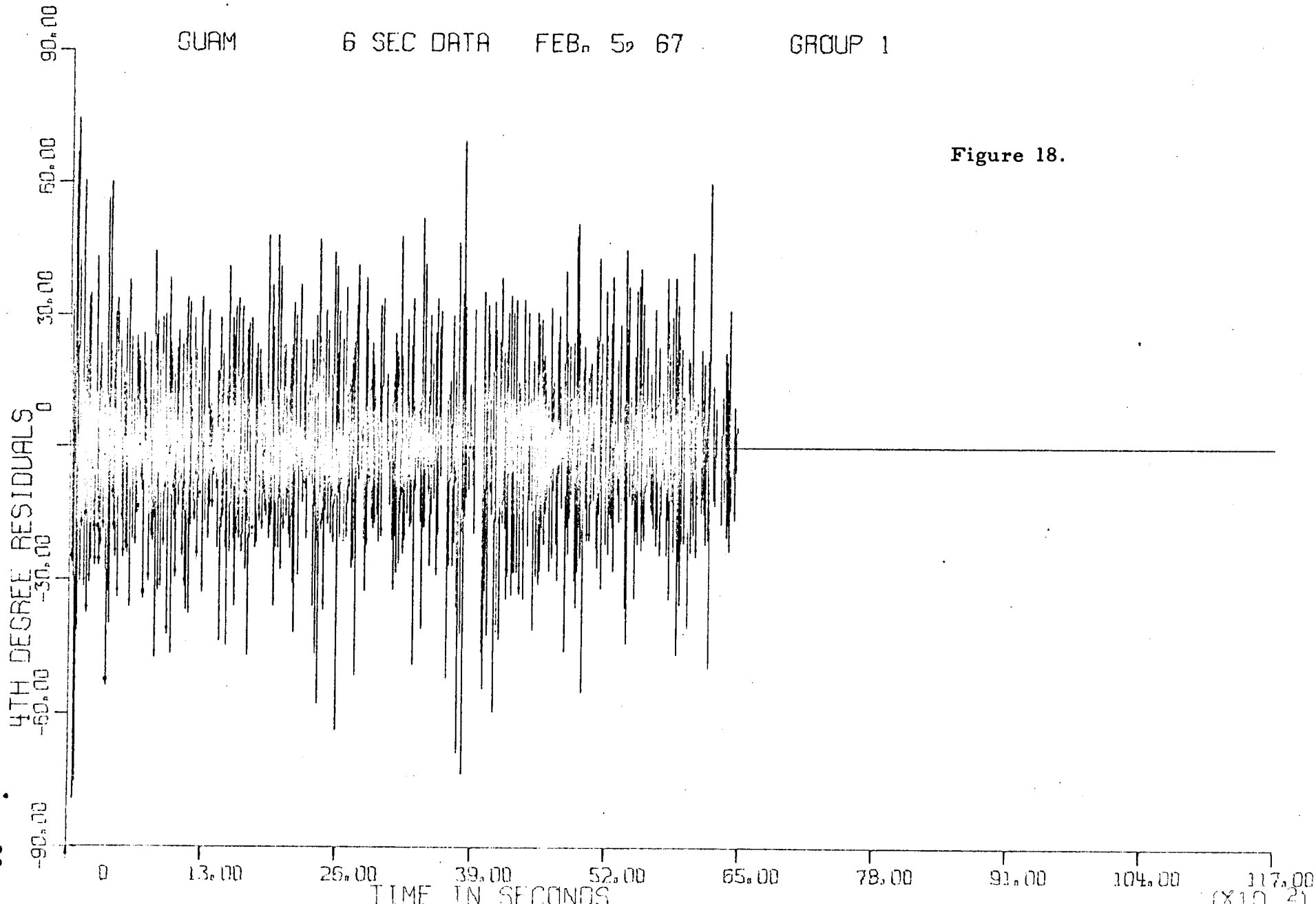

GUAM
6 SEC DATA
FEB. 5, 67
GROUP 1
4TH DEGREE RESIDUALS
90.00
60.00
30.00
0
-30.00
-60.00
-90.00
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
TIME IN SECONDS

Figure 18.

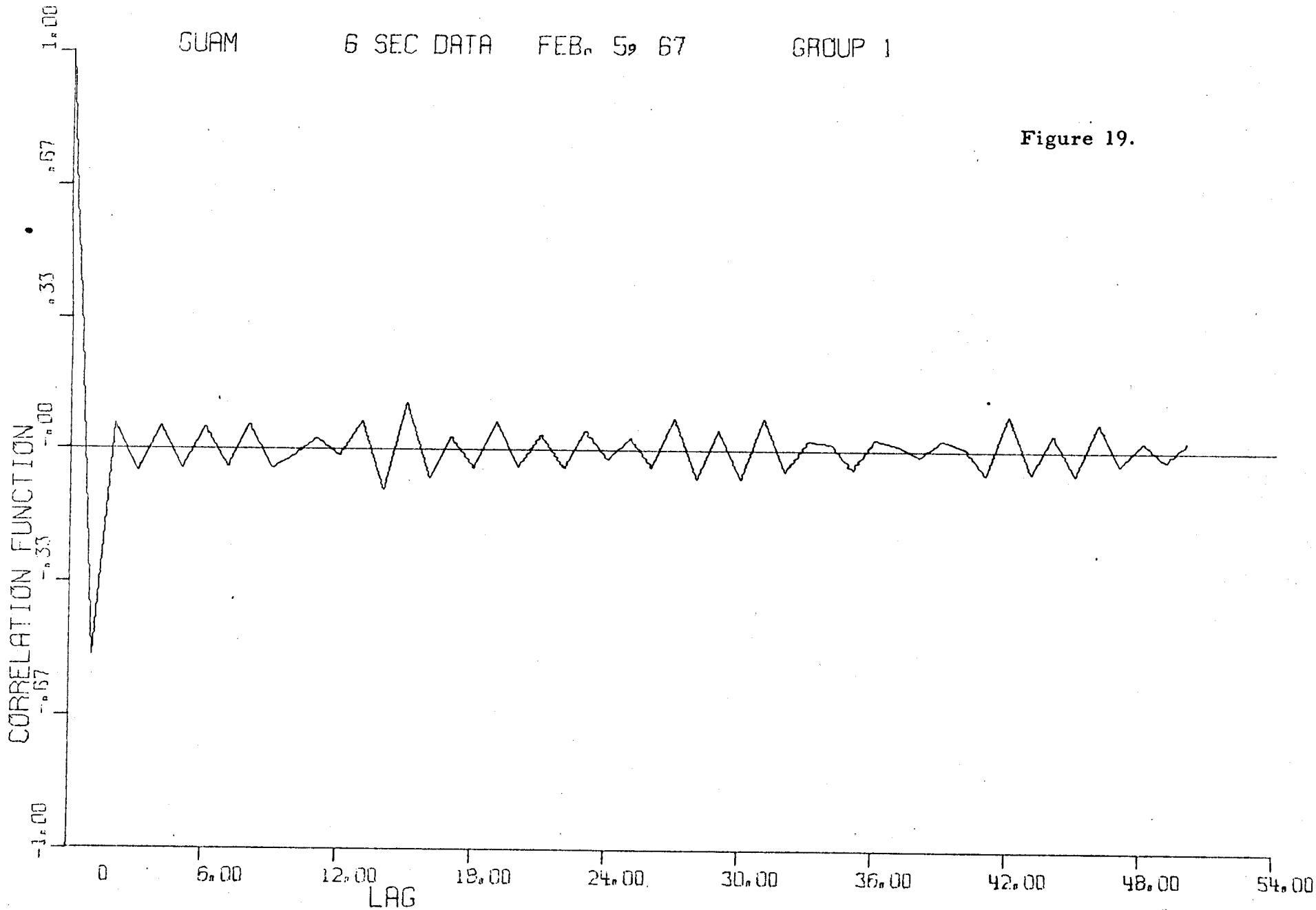

GUAM
6 SEC DATA
FEB. 5, 67
GROUP 1
CORRELATION FUNCTION
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 19.

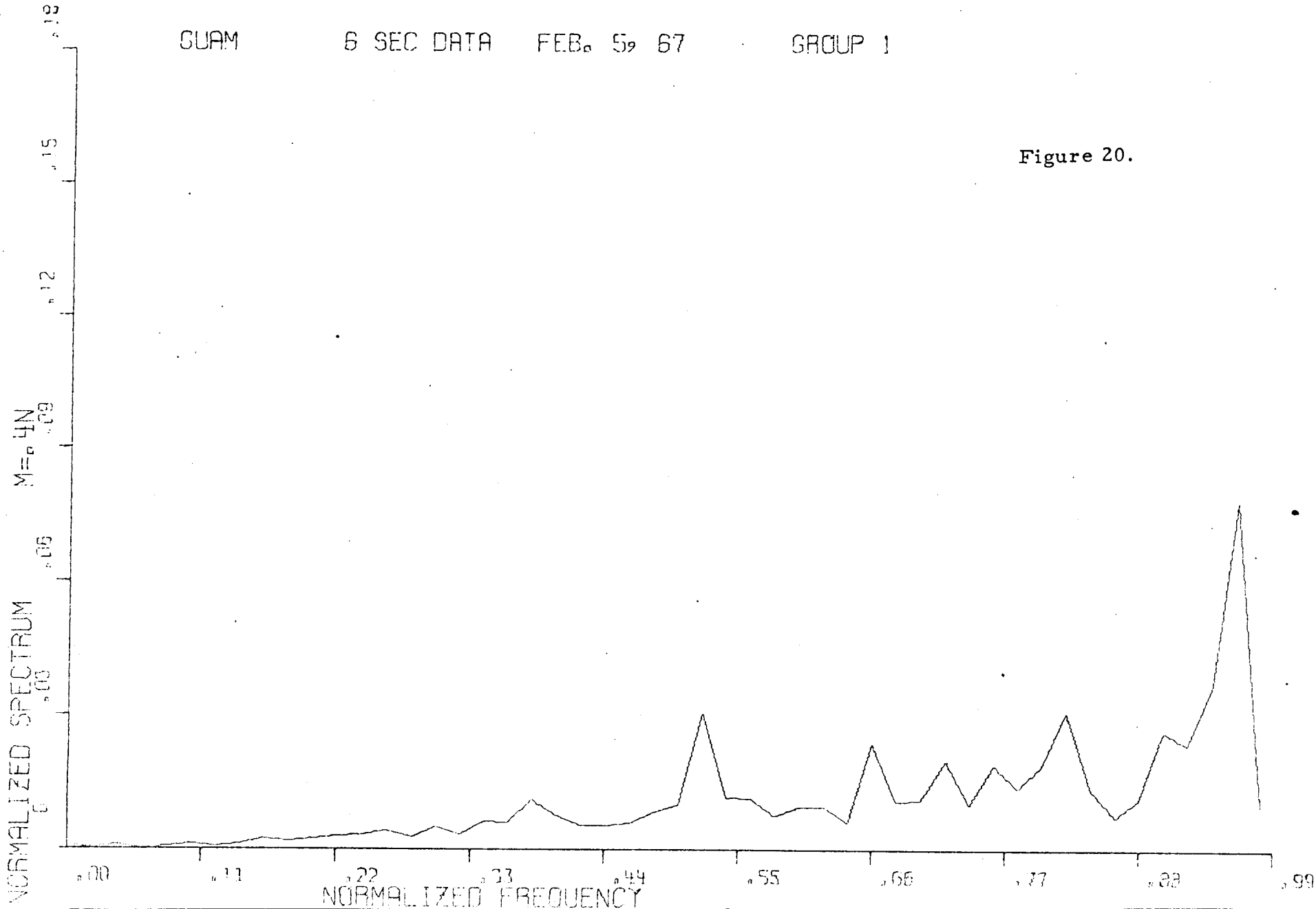

GUAM 6 SEC DATA FEB. 5, 67 GROUP 1
M=.4N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 20.

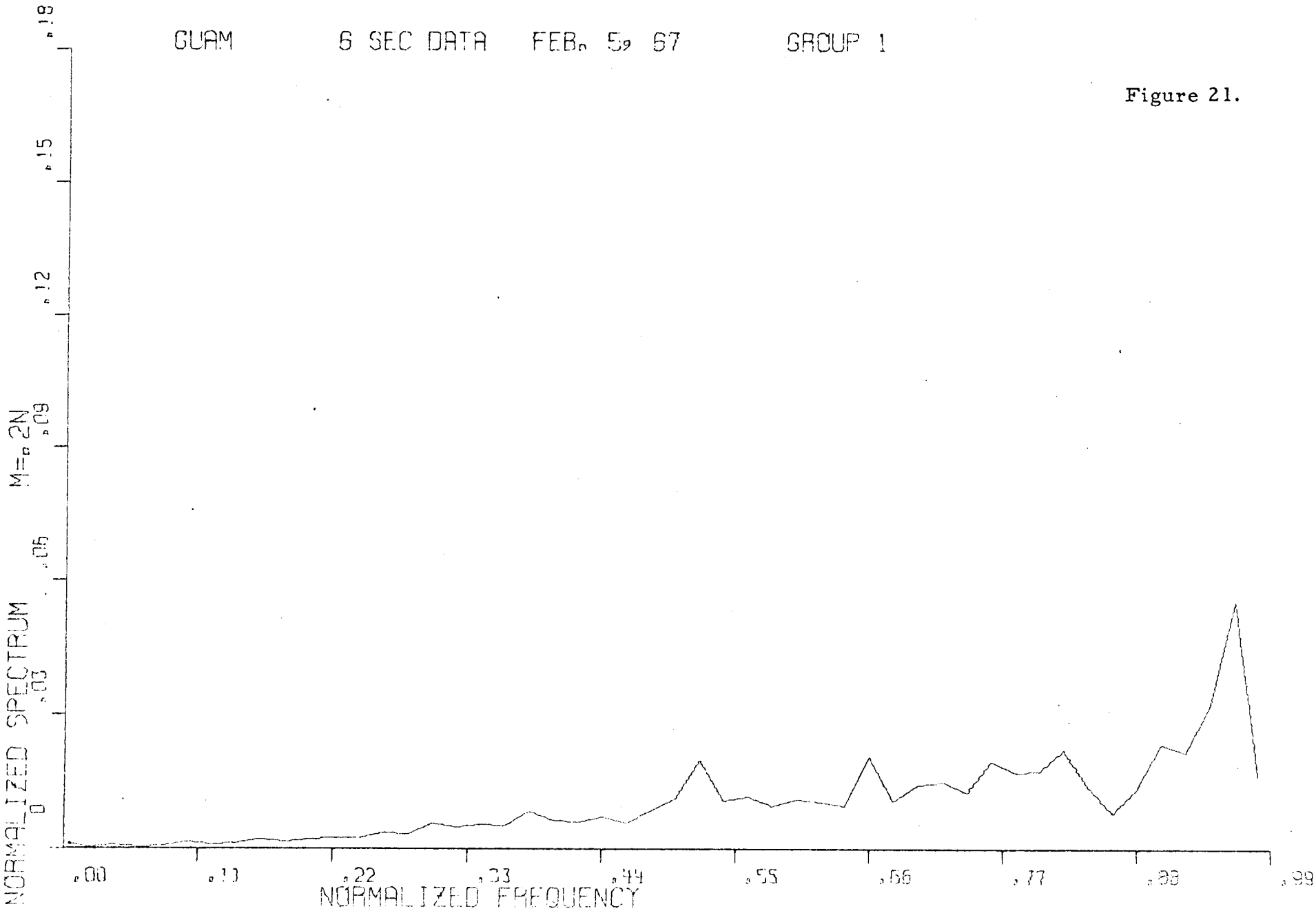

GUAM 6 SEC DATA FEB. 5, 67 GROUP 1
NORMALIZED SPECTRUM M=.2N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 21.

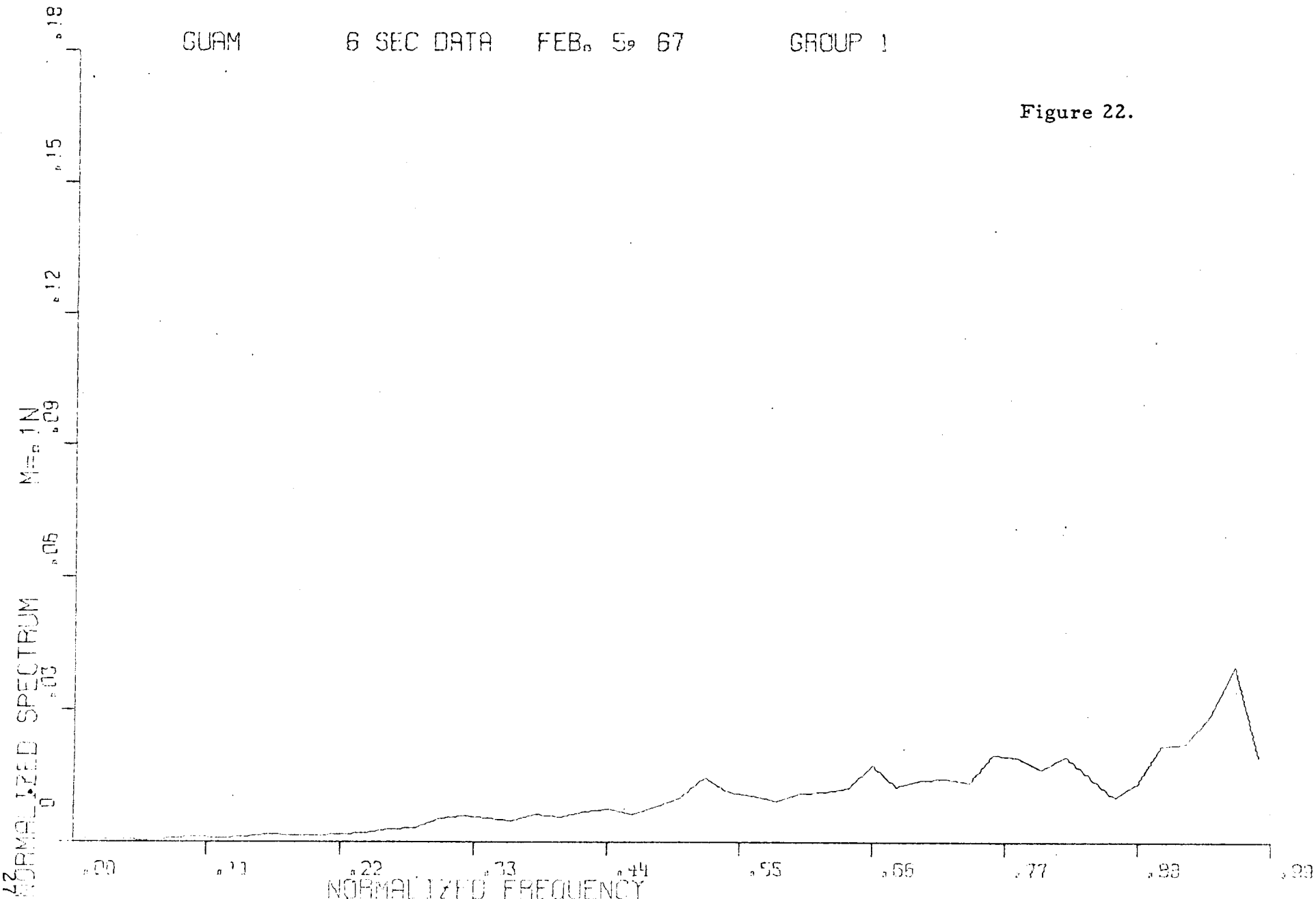

GUAM 6 SEC DATA FEB. 5, 67 GROUP 1
M=. 1N
NORMALIZED SPECTRUM
NORMALIZED FREQUENCY
.18
.15
.12
.09
.06
.03
0
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99

Figure 22.

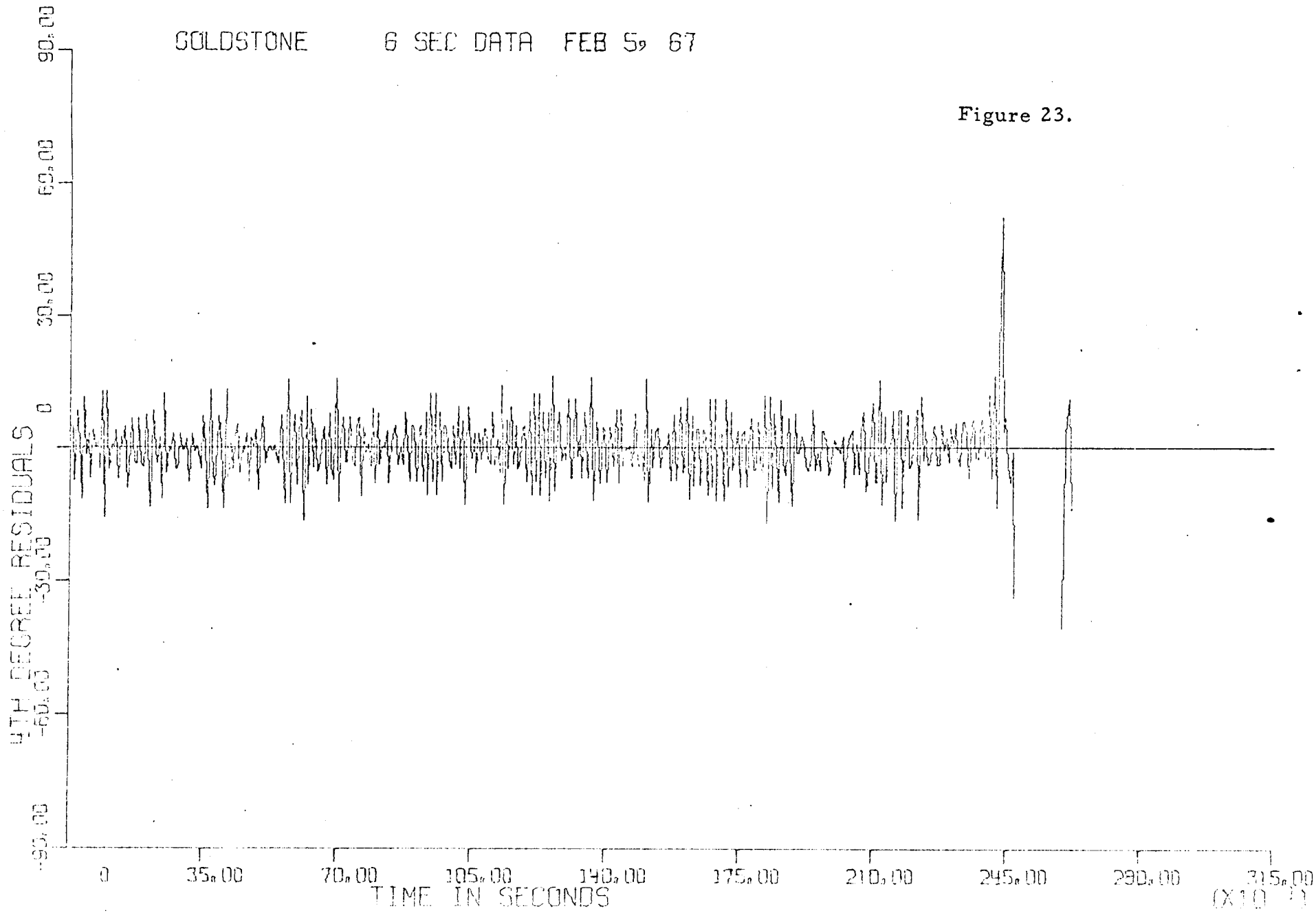

GOLDSTONE 6 SEC DATA FEB 5, 67
90.00
60.00
30.00
0
-30.00
-60.00
-90.00
4TH DEGREE RESIDUALS
0
35.00
70.00
105.00
140.00
175.00
210.00
245.00
280.00
315.00
TIME IN SECONDS

Figure 23.

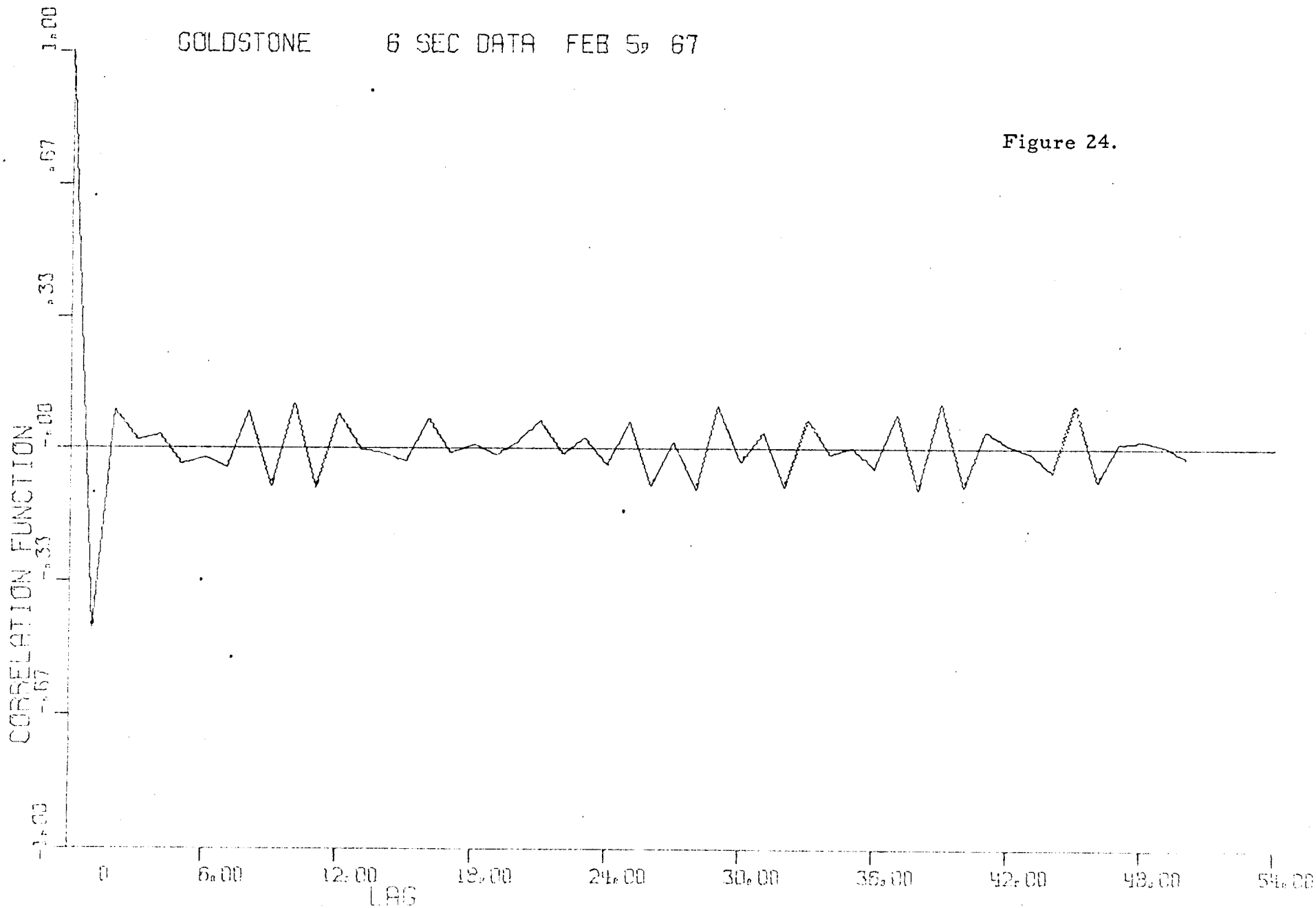

GOLDSTONE 6 SEC DATA FEB 5, 67
CORRELATION FUNCTION
1.00
.67
.33
-.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 24.

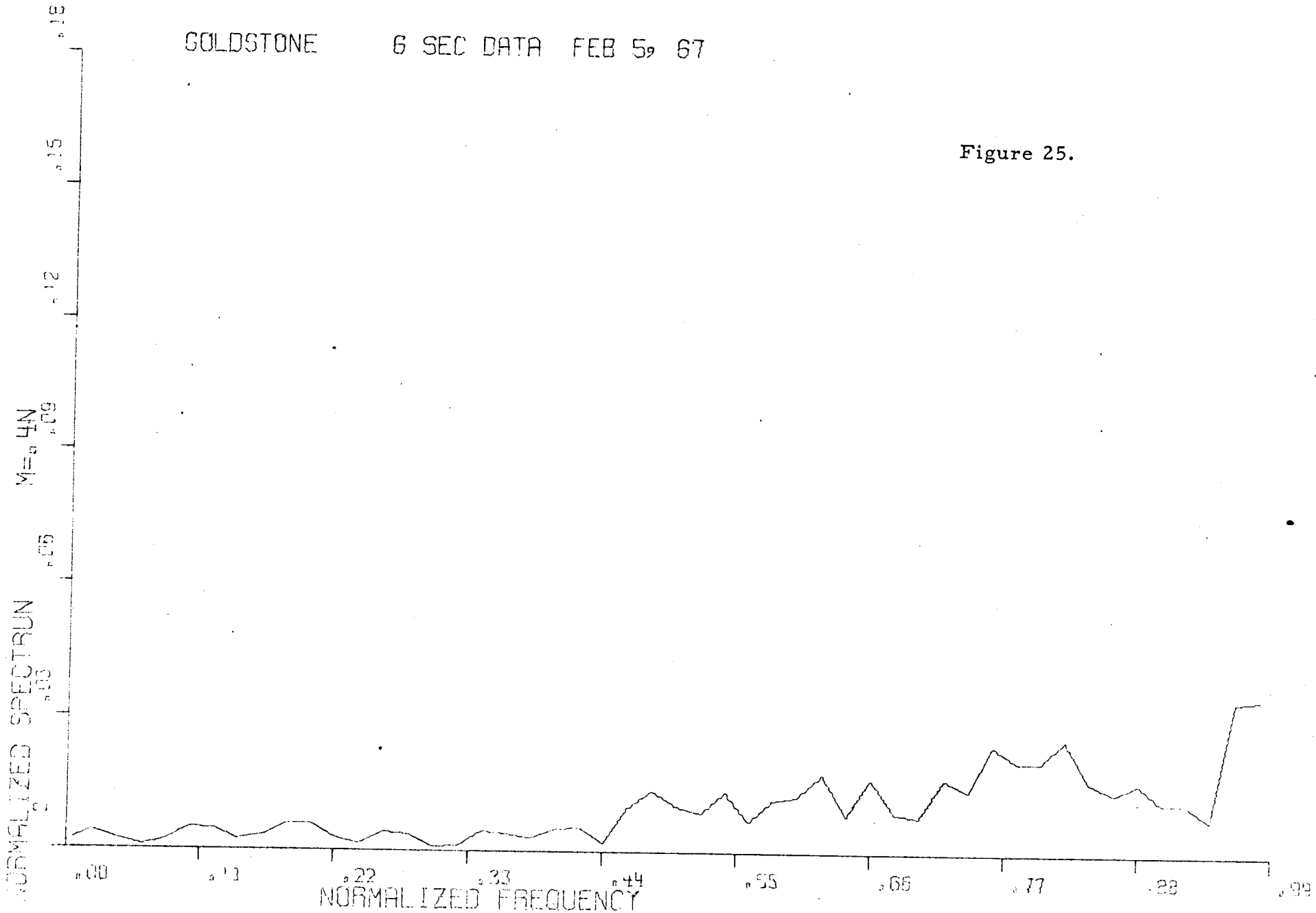

GOLDSTONE 6 SEC DATA FEB 5, 67
NORMALIZED SPECTRUM
NORMALIZED FREQUENCY
.00
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99

Figure 25.

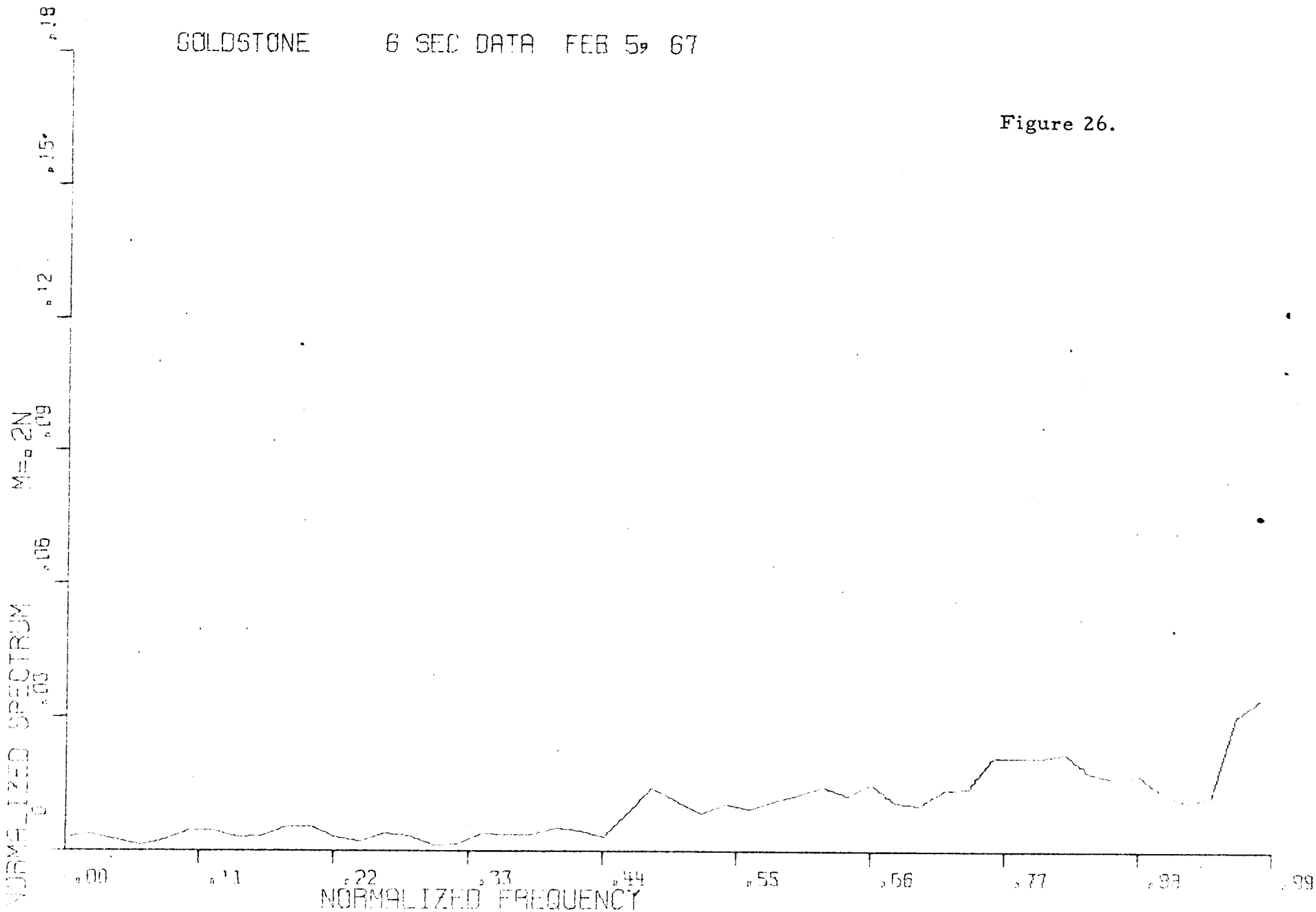

GOLDSTONE 6 SEC DATA FEB 5, 67
M=. 2N
NORMALIZED SPECTRUM
NORMALIZED FREQUENCY
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
.18
.15
.12
.09
.06
.03
0

Figure 26.

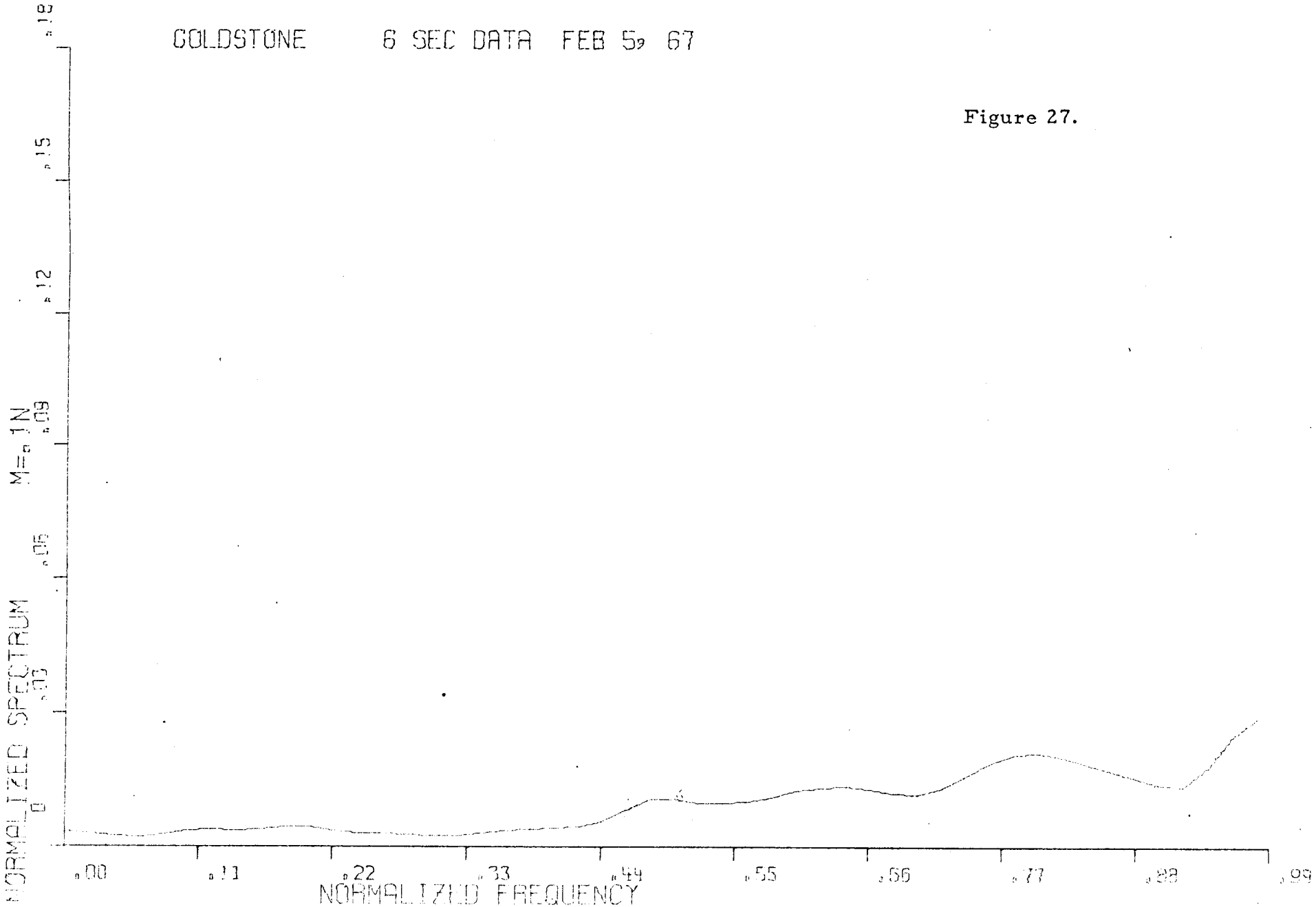

GOLDSTONE 6 SEC DATA FEB 5, 67
NORMALIZED SPECTRUM M=.1N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 27.

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 497
(BBC Task 105)

H. Epstein
12 July 1967

HIGH FREQUENCY ANALYSIS OF LLO USB TRACKING DATA

The purpose of this note is to indicate the data processing techniques involved in the analysis of LLO USB tracking data. Three-way doppler data is the only type measurable involved at this point in time. Relatively minor modifications are required to extend the capability to other doppler types, range, and angle measurables. This note pertains specifically to the high frequency error components present in the data. The restriction to analysis of high frequency error components has been necessitated by the absence of a suitable working ODP. High frequency analyses would predominantly be made with the program to be indicated shortly even if a suitable ODP was on hand (this minimizes computing costs and time involved).

Doppler data requirements as set forth in the GOSS should consist only of non-destruct count doppler with atomic frequency standards employed by all DSN and MSFN stations involved in the test program. Data on hand at BBC includes destruct count data and measurement data with crystal standards. This data has been and is being processed.

The stages involved in doppler processing for analysis are illustrated in Figures 1, 2, and 3. Raw data from MSC has been made available in the following formats: special tape formats, cards, and log tapes In addition, TTY formats are expected shortly. A special Copy Program is written for each type format. The Copy Program converts an input format to a format similar to the USB format. In addition, a compaction of the USB data is made.

The Pre-edit Program accepts the output of the Copy Program and processes data on a station basis. At least one input data sheet (see Figure 4) is required for each station. The primary function of the Pre-edit Program is to insure that a proper data format is present. The 240 bit

Figure 1. Block Diagram for Data Analysis

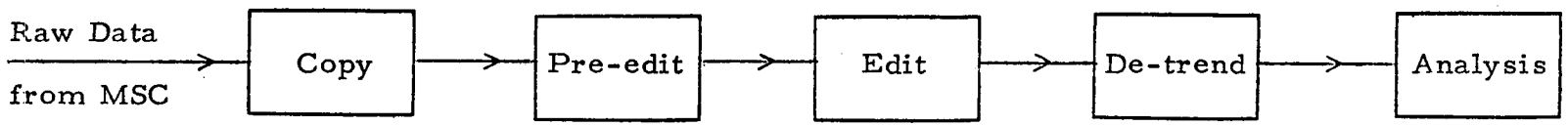


Figure 2. De-trend Program

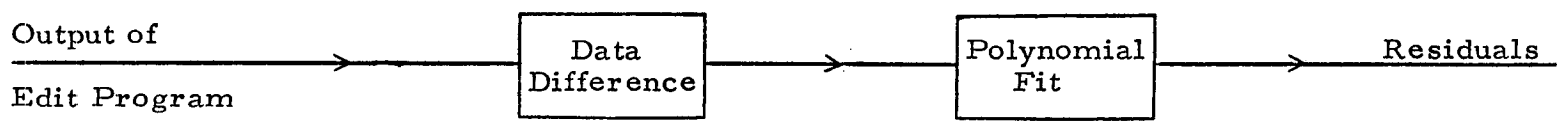
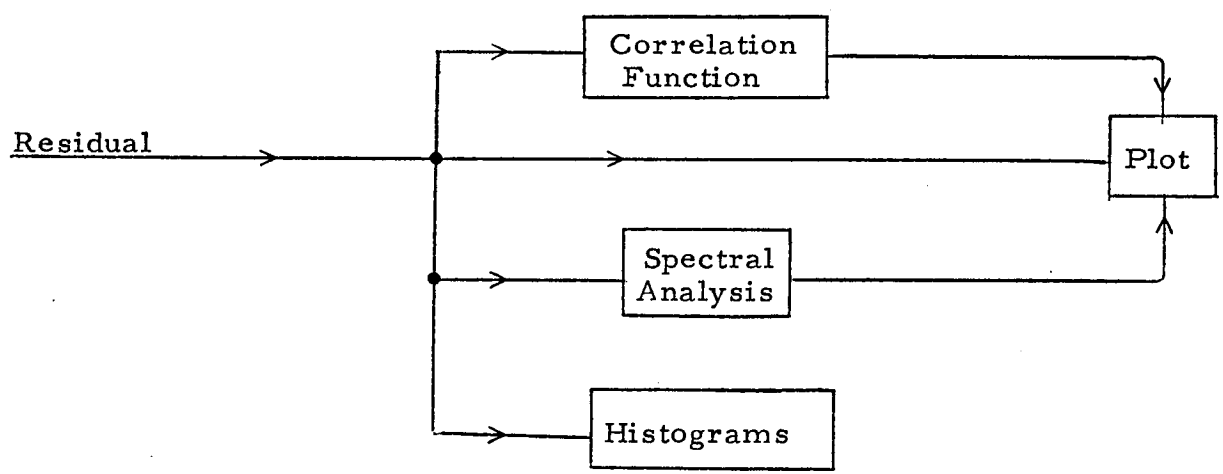


Figure 3. Analysis Program



INPUT DATA SHEET FOR PRE-EDIT PROGRAM

Punch: 1 for YES; 2 for NO; 9 for TEST TO BE IGNORED

		Yes	No	Test to be Ignored
1	Is this high speed 240-Bit data			/////
2	Is this non-destruct data			/////
3	Is this high data (Bit 15)			
4	Is range-rate in standard position			/////
5	Is range-rate N_1 mode			
	Only one of four applies:			
6	Is this one way doppler mode			/////
7	Is this two way doppler mode			/////
8	Is this multiple non-coherent mode			/////
9	Is this multiple coherent mode			/////
10	Vehicle ID is [ ]	///	///	
11	Is frequency standard rubidium			
12	Is manual R-R test to be made			/////
13	Is VCO lock test to be made			/////
14	Is automatic range-rate test to be made			/////
15	Is test to be made on real/test Bit			/////
16	Is station ID test to be made			/////
17	Is doppler mode test to be made			/////
18	Is test to be made on R-R field indicator			/////

STATION ID

20	[ ]	Bermuda
21	[ ]	Merritt Island
22	[ ]	Grand Bahama Island
23	[ ]	Antigua
24	[ ]	Carnarvon
25	[ ]	Hawaii
26	[ ]	Guaymas
27	[ ]	Texas
28	[ ]	Guam
29	[ ]	Goldstone
30	[ ]	Ascension
31	[ ]	Canberra
32	[ ]	GSFC
33	[ ]	Madrid
34	[ ]	Grand Canary Islands

EXPECTED START TIME

35-37	[ ][ ][ ]	Day (if greater than 31 month will be ignored)
38-39	[ ][ ]	Month
40-41	[ ][ ]	Year
42-43	[ ][ ]	Hour
44-45	[ ][ ]	Minute
46-48	[ ][ ][ ]	Expected Delta Time
49-51	[ ][ ][ ]	Maximum Time Interval (in minutes)
52-55	[ ][ ][ ][ ]	Print Rejected Data
56-59	[ ][ ][ ][ ]	Print Raw Data
60-68	[ ][ ][ ][ ][ ][ ][ ][ ][ ]	Start Time

Figure 4 - Pre-edit Input Data Sheet

high speed data format for early LLO USB data had not been established. As a consequence, it was necessary to remove or modify a number of format tests. A number of tests have been established which cannot be ignored. Presently, these consist of items 1, 2, and 4 of Figure 4 and the data time interval. Item 1 pertains to whether the high speed or low speed USB format should be used; item 2 indicates whether non-destruct or destruct data is to be selected, and item 4 indicates the field of the doppler cycle counter reading. The start time for the data to be examined may be indicated in three ways:

1. Calendar data and GMT,
2. Day of year and GMT, and
3. Time in seconds or .1 seconds from the beginning of the calendar year.

The third option is exercised by the number 999 for the day (35-37). An additional test was made for uniform data time intervals. Also, a gross doppler counter reading test is made on destruct count data.

The data which passes the pre-edit tests is then used as an input to the edit program. The edit program takes second differences of the doppler counter readings. Input data will be rejected for any of three reasons:

1. The apriori second difference threshold is exceeded (see Apollo Note 490);
2. No second differences can be formed on a given data for the specified time interval between data points; or,
3. The second difference lies outside of a limit placed by estimates of the standard deviation (see Apollo Note 465).

The trend or low frequency characteristic of the "good" data output of the editing program needs to be removed from the data to form high frequency residuals or error components for analysis. The nature of

the detrending program is indicated in Figure 2. Basically, a given order difference (usually, first or second difference) is performed on the doppler counter reading. The output of the difference program is then fit to a polynomial (typically of the fourth degree) and residuals are formed by the subtraction of the corresponding value of the polynomial from the output of the difference program.

The residuals are then processed by the analysis program (see Figure 3). A histogram, correlation function, and spectral density are then estimated from these residuals. Plots may then be made of the raw residuals, the correlation function, and the spectral estimates.

A few comments are in order regarding the expected nature of the high frequency doppler error components. Doppler sample intervals of .1, .2, .4, 1.0 and 6 seconds are expected for most of the measurement data to be analyzed. Past experience with DSN doppler data has indicated the predominant high frequency error source for these data rates. The crystal oscillator frequency standard has not been extensively analyzed here to date since Apollo USB stations are expected to always employ atomic frequency standards. Expected predominant error sources are indicated in Table I below for the indicated sample intervals.

Table I - Predominant High Frequency Error Components

Doppler Counter Mode	Frequency Standard	
	Rubidium	
Non-Destruct	Quantization	Crystal and Quantization
Destruct	Random Phase	Crystal and Random Phase

APOLLO NOTE NO. 498
(BBC Task 204)

L. Lustick
J. R. Holdsworth
13 July 1967

ORBIT DETERMINATION WITH STATISTICALLY CONSTRAINED DATA

The purpose of this note is to describe a computationally efficient method of handling what we have called "statistically constrained data." The term statistically constrained data is a neologism of our own invention and hence requires some explanation. We have employed this term because it is suggestively descriptive of what we have in mind, and because none of the standard terminology expresses what we mean in a sufficiently succinct manner.

The meaning which we intend to convey by this expression may be explained as follows. We assume that we have some a priori estimates for a set of orbit parameters, and that the covariance matrix of the errors of these a priori estimates is singular. As we shall shortly show, when the a priori errors are singular, the optimum combination of the old and new data is such that the final errors also have a singular distribution. It is the purpose of this note to show how to capitalize upon this singularity and to efficiently reduce the dimensions of the matrices with which we have to deal.

First we note that if $A(1)$ and $A(2)$ are covariance matrices of independent estimates of some set of orbit parameters, and if the matrix $A(1) + A(2)$ is non singular, then the optimal linear weighting of the estimators will yield an error covariance matrix A given by:

$$A = \left(A(1) + A(2)\right)^{-1} A(1)\, A(2) \tag{1}$$

Now, from (1) we note that if $A(1) + A(2)$ is non singular, but $A(1)$ say is singular we obtain:

$$\det A = \frac{\det A(1) \det A(2)}{\det (A(1) + A(2))} = 0 \tag{2}$$

since $\det A(1) = 0$, hence singularity of the a priori error distribution implies singularity of the final error distribution, if it is optimally computed.

We now turn our attention to the fundamental question. How, when one is given a singular covariance matrix, can one determine the existing linear dependencies amongst the various random variables in a computationally efficient manner. Let us now consider the Choleski decomposition of a covariance matrix V, where we factorize V into the product of two lower and upper triangular matrices as shown below:

$$V = LL^T \tag{3}$$

The matrix L in (3) has the form indicated below in equation (4)

$$L = \begin{bmatrix} \ell_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \ell_{n1} & \cdots & \ell_{nn} \end{bmatrix} \tag{4}$$

with all entries above the main diagonal being identically equal to zero.

If the matrix V is given, then by formally computing the product LL^T, from (4) and equating the resulting elements to those of the matrix V, yields the following system of equations for sequentially computing the elements ℓ_{ij} of L.

$$\ell_{11}^2 = v_{11} \qquad \text{or} \qquad \ell = \sqrt{v_{11}} \tag{5}$$

$$\ell_{11}\ell_{21} = v_{12}, \qquad \ell_{21} = \frac{v_{12}}{\ell_{11}} = \frac{v_{12}}{\sqrt{v_{11}}} \tag{6}$$

and in general:

$$\ell_{ij} = \frac{\left(v_{ij} - \sum_{k=1}^{j-1} \ell_{ik}\,\ell_{jk}\right)}{\ell_{jj}} \qquad j < i \tag{7}$$

$$\ell_{ii} = \sqrt{v_{ii} - \sum_{k=1}^{i-1} \ell_{ik}^2} \tag{8}$$

$$\ell_{ij} = 0 \quad \text{for } j > i \tag{9}$$

Of particular interest to us are the diagonal elements ℓ_{ii} of the matrix L. From (3) it follows that:

$$\begin{aligned} \det V &= \det(LL^T) \\ &= \det(L)\det(L^T) \\ &= \det^2(L) \end{aligned} \tag{10}$$

and from (8) we see that:

$$\det V = \prod_{i=1}^{n} \ell_{ii}^2 \tag{11}$$

so that the covariance matrix V is singular if and only if one or more of the diagonal elements ℓ_{ii} in the matrix L is zero.

Now assume that for some $i > 1$ we have:

$$\ell_{ii} = 0$$

and (12)

$$\ell_{jj} > 0 \quad \text{for } 1 \leq j < i.$$

In other words ℓ_{ii} is the first zero that appears as a diagonal element in the Choleski decomposition. This means that the ith error component $\Delta\theta_i$ may be exactly written as some linear combination of the first $i - 1$ components as in equation (13) below:

$$\Delta\theta_i = \sum_{k=1}^{i-1} C_k \Delta\theta_k \tag{13}$$

where the C_k coefficients are constant and may be determined as follows. If we multiply both sides of equation (13) by ϵ_j for $j = 1, 2, \cdots, i - 1$, and take the expected value we get:

$$E(\Delta\theta_i \, \Delta\theta_j) = v_{ij} = \sum_{k=1}^{j-1} C_k \, E(\Delta\theta_k \, \Delta\theta_j) \tag{14}$$

or:

$$\sum_{k=1}^{i-1} v_{jk} \, C_k = v_{ij} \qquad \text{for } 1 \leq j \leq i - 1. \tag{15}$$

Equation (15) is a system of $i - 1$ linear equations in the $i - 1$ unknowns $C_1, C_2, \cdots, C_{i-1}$, hence the coefficients may be determined by solving this system. The system is non singular since i was assumed to be the <u>first</u> integer for which we obtained $\ell_{ii} = 0$ in the Choleski

decomposition, hence the coefficients $C_{k(i)}$ are uniquely defined by the system given by equation (15). If we let $V(i-1)$ denote the upper left hand $(i - 1)$ by $(i - 1)$ submatrix of V, $C(i-1)$ be the $i - 1$ dimensional column vector with components $C_1, C_2, \cdots, C_{i-1}$, and $v^{(i)}$ denote the column vector consisting of the first $i - 1$ components of the ith row of the covariance matrix V, then (15) may be written as:

$$V(i-1)\, C(i-1) = v^{(i)} \tag{16}$$

so that the constant coefficients may be formally solved for as:

$$C(i-1) = V^{-1}(i-1)\, v^{(i)} \tag{17}$$

We now demonstrate briefly how the foregoing remarks are applied to the problem of revising the estimates of the orbit parameters based upon a combination of a priori estimates and new data. It has been shown in many earlier Apollo Notes that the modified estimate $\hat{\theta}$ may be written as the following function of the a priori estimate $\hat{\theta}_0$ and the measurement vector m.

$$\hat{\theta} = \theta_0 + \left[\mathrm{cov}^{-1}\theta_0 + \frac{\partial m_c^T}{\partial \theta} \mathrm{cov}^{-1}\epsilon \frac{\partial m}{\partial \theta} \right]^{-1} \mathrm{cov}^{-1}\epsilon \frac{\partial m^T}{\partial \theta} \left(m - m_c(\theta_0) \right) \tag{18}$$

where as usual we assume that the measurement vector m may be expressed as:

$$m = m_c(\theta) + \epsilon \tag{19}$$

where the functional form $m_c(0)$ is known and where ϵ is some zero mean additive noise with covariance cov ϵ. Notice in Eq. (18) that the term

$$m - m_c(\theta_0) \tag{20}$$

is the difference between the actual observed measurements and their predicted values, where the predicted values are computed on the basis of the a priori estimates θ_0.

We shall assume for convenience that the noise vector ϵ comes from a zero mean white noise process so that

$$\text{cov}\,\epsilon = E(\epsilon\,\epsilon^T) = \sigma_\epsilon^2\, I \tag{21}$$

where I is an nxn identity matrix. In this most usual case, a typical element, say the k, ℓ element of the matrix $(\partial m_c^T/\partial\theta)$ $\text{Cov}^{-1}\,\epsilon\,(\partial m_c/\partial\theta)$, may be written:

$$\left(\frac{\partial m_c^T}{\partial\theta}\,\text{cov}^{-1}\,\epsilon\,\frac{\partial m_c}{\partial\theta}\right)_{k,\ell} = \frac{1}{\sigma_\epsilon^2}\sum_{r=1}^{n}\frac{\partial m_c}{\partial\theta_k}(t_r)\,\frac{\partial m_c}{\partial\theta_\ell}(t_r) \tag{22}$$

where: $1 \le k, \ell \le p$, where p denotes the dimension of the parameter vector and where $m_c(t_r)$ denotes the noise free computed value of the component of the measurement vector at the time of the rth measurement.

Now, if the a priori covariance matrix cov θ_0 is singular then it no longer makes sense to talk about the matrix

$$\left[\text{cov}^{-1}\,\theta_0 + \frac{\partial m_c^T}{\partial\theta}\,\text{cov}^{-1}\,\epsilon\,\frac{\partial m_c}{\partial\theta}\right] \tag{23}$$

The dimensionality of the problem may be reduced in the following way, if we assume that in the Choleski decomposition of the matrix cov θ_0, that there is exactly one zero element say the ith, where $i > 1$, on the main diagonal of the lower-triangular matrix L. Instead of computing the pxp matrix as shown in equation (23), we make the following modifications. First, we replace cov θ_0 in (23) by the (p-1) x (p-1) matrix obtained by deleting the ith row and column from the pxp matrix cov θ_0. We will call this reduced matrix cov*θ_0.

Secondly, in the most common case of stationary white noise, the pxp matrix $\{(\partial m_c{}^T/\partial\theta)\ \mathrm{cov}^{-1}\ \epsilon\ (\partial m_c/\partial\theta)\}$ in Eq. (23) is replaced by the (p-1) x (p-1) matrix, M whose elements are defined below.

$$M_{k\ell} = \frac{1}{\sigma_\epsilon^2} \sum_{r=1}^{n} \left(\frac{\partial m_c}{\partial \theta_k}(t_r) + C_k \frac{\partial m_c}{\partial \theta_i}(t_r)\right)\left(\frac{\partial m_c}{\partial \theta_\ell}(t_r) + C_\ell \frac{\partial m_c}{\partial \theta_i}(t_r)\right) \tag{24}$$

for $1 \le k, \ell \le i-1$. In Eq. (24), the terms C_k and C_ℓ are the appropriate components of the vector which is the solution of Eq. (17) where the reduced matrix $V_{(i-1)}$ is the upper left hand (i-1) x (i-1) sub matrix of cov θ_0 and, where $v^{(i)}$ in Eq. (17) is the column vector consisting of the first i-1 elements of the ith row of the covariance matrix cov θ_0. If $i < p$, and $i \le k, \ell \le p-1$, then Eq. (24) assumes the term:

$$M_{k\ell} = \frac{1}{\sigma_\epsilon^2} \sum_{r=1}^{n} \frac{\partial m_c}{\partial \theta_{k+1}}(t_r) \frac{\partial m_c}{\partial \theta_{\ell+1}}(t_r) \tag{25}$$

Finally, if one of the indices, say k is less than i and the other, ℓ, is greater than or equal to i, we obtain:

$$M_{k\ell} = \frac{1}{\sigma_\epsilon^2} \sum_{r=1}^{n} \left[\frac{\partial m_c}{\partial \theta_k}(t_r) + C_k \frac{\partial m_c}{\partial \theta_i}(t_r) \right] \frac{\partial m_c}{\partial \theta_{\ell+1}}(t_r) \tag{26}$$

It is important to remember that the index i, in equations (24) through (30) inclusively, has a particular, fixed significance. Namely, it is the index of that component of the parameter vector θ, which is being replaced by a linear combination of components with indices strictly less than i. Equivalently, it is the row and column index of the first zero element appearing as a diagonal entry of one of the factor matrices in the Choleski decomposition of the a priori covariance matrix cov θ_0. For brevity, let F denote the information matrix whose typical element is given by equation (22), and let F_{rs} denote the r, s^{th} element of this information matrix. Expanding the right hand side of equation (24) we obtain:

$$\begin{aligned} M_{k\ell} = {} & \frac{1}{\sigma_\epsilon^2} \sum_{r=1}^{n} \frac{\partial m_c}{\partial \theta_k}(t_r) \frac{\partial m_c}{\partial \theta_\ell}(t_r) \\ & + \frac{1}{\sigma_\epsilon^2} C_\ell \sum_{r=1}^{n} \frac{\partial m_c}{\partial \theta_k}(t_r) \frac{\partial m_c}{\partial \theta_i}(t_r) \\ & + \frac{1}{\sigma_\epsilon^2} C_k \sum_{r=1}^{n} \frac{\partial m_c}{\partial \theta_i}(t_r) \frac{\partial m_c}{\partial \theta_\ell}(t_r) \\ & + \frac{1}{\sigma_\epsilon^2} C_k C_\ell \sum_{r=1}^{n} \left(\frac{\partial m_c}{\partial \theta_i}(t_r) \right)^2 \end{aligned} \tag{27}$$

Referring again to equation (22) we see that for $1 \le k, \ell \le i-1$, the element in the modified information matrix may be written as:

$$M_{k\ell} = F_{k\ell} + C_\ell F_{ki} + C_k F_{i\ell} + C_k C_\ell F_{ii} \tag{28}$$

where $F_{k\ell}$, F_{ki}, $\cdots$, etc. are elements of the original information matrix and the quantities C_k, C_ℓ are those quantities obtained from the solution of equation (17). Similarly, from (25), and for $i < p$, and $i \leq k$, $\ell \leq p - 1$, we obtain:

$$M_{k\ell} = F_{k+1,\ \ell+1} \tag{29}$$

and from (26) if $k < i$ and $\ell \geq i$, then:

$$M_{k\ell} = F_{k,\ \ell+1} + C_k F_{i,\ \ell+1} \tag{30}$$

Thus, the reduction in dimension of the information matrix is obtained by working with the (p-1) x (p-1) modified information matrix M whose elements are computed as functions of the original information elements F_{ij}, and the constants C_k, as shown in equations (28), (29), (30).

The reduction in dimension is now affected by replacing $\text{cov}^{-1}\ \theta_0$ in (23) by $(\text{cov}^*\theta_0)^{-1}$, and $\left\{(\partial m_c{}^T/\partial m_c)\ \text{cov}^{-1}\epsilon\ (\partial m_c/\partial\theta)\right\}$ by M, so that we are now working with the (p-1) x (p-1) matrix:

$$\left[(\text{cov}^*\theta_0)^{-1} + M\right]^{-1} \tag{31}$$

If the rank of the a priori covariance matrix cov θ_0 is less than p-1, the dimensionality may be further reduced by a completely analogous method. In this case there will be more than one diagonal zero in the Choleski factorization and we proceed exactly as described.

VOLUME XV NAS 9-4435

TABLE OF CONTENTS

Apollo Guidance and Navigation Notes

Note No.	Title	Author(s)
467	OEAP Reference Orbits for 17 September 1969 . . .	A. Burkow/C. Siska
468	Optimum A priori Values for the Doppler Pseudo-Biases for MSFN Tracking During the Terminal Phase of Lunar Rendezvous	C. H. Dale/ G. F. Floyd
469	Powered-Flight Position Monitoring	H. Epstein
470	Additional Capabilities of the RTODP-OEAP	H. Engel/L. Lustick/ J. Gregory
471	The Effect of Batching During Terminal Lunar Rendezvous Using Near-Optimum A Priori Pseudo-Bias Uncertainties	C. H. Dale
472	Optimum A Priori Values for the Doppler Pseudo-Biases for MSFN Tracking During the First Two Hours After Translunar Injection	C. H. Dale
473	The Effect of Assumed Filter Boost Errors During Lunar Tracking	L. Lustick/C.H. Dale
474	The Effect on Actual State Uncertainty of Assuming an Imperfectly Known Gravitational Constant in the RTODP Filter	C. H. Dale
475	The Effect on Actual State Uncertainty of Assuming an Imperfectly Known Gravitational Constant in the RTODP Filter (II)	C. H. Dale
476	An Approximate Scaling Law Between $\dot{R}$ Bias Errors and Clock Parameters for Lunar Orbits. . .	J. R. Holdsworth
477	Report of Modifications to the (DEC 10) RTODP-OEAP Program	J. Gregory/ G. Hempstead
478	Effect of Relative State Error on Rendexvous Miss	L. Lustick
479	Modifications to RTODP Error Analysis (DEC 10 Program).	L. Lustick/ G. Hempstead
480	Modifications to 'The Problem of a Small Perturbation From a Circular Satellite'	L. Lustick
481	Revision for Clock Parameters.	H. Engel
482	Illustrative Example Inputs for the RTODP Orbit Error Analysis Program	L. Lustick/C.H. Dale

Note No.	Title	Author(s)
483	Approximate Scaling for $\dot{R}$ Biases and Clock Parameter Errors for Translunar Trajectories	J. Holdsworth
484	Information Matrices, Landmark and Star Sighting Measurements	E. Cortina
485	Definition of Measurables	H. Engel
486	Addition of Platform Rate Uncertainty Parameters to Program C and the Resulting New Data Input Sheets for Programs: C, B(OEAP), and B (RTODP-OEAP)	C. H. Dale/ G. Hempstead/ D. Matta
487	Langley Lunar Orbiter 2 - Doppler Data Residuals	S. Schloss
488	Tracking the LEM and CSM from LEM Takeoff to Rendezvous	C. H. Dale/ L. Lustick
489	Checkout of Landmarks and Star Sightings Program	L. Lustick/ G. Hempstead
490	A Priori Editing Threshold for Doppler Measurement Data	H. Epstein
491	Tracking the LEM and CSM from LEM Take-Off to Rendezvous	C. H. Dale
492	Tracking the LM and CSM from Take-Off to Rendezvous with Similar Batching	C. H. Dale
493	Fast Fourier Transform	D. Matta
494	Tracking the LM and CSM from LM Ascent Cut-Off to Rendezvous Using Similar Filters and No Pseudo-Biases in Measurables	C. H. Dale
495	More on the Speed of Light, and Atomic Time Versus Ephemeris Time	H. Engel
496	Analysis of Some LLO Translunar Phase USB Tracking Data	H. Epstein
497	High Frequency Analysis of LLO USB Tracking Data	H. Epstein
498	Orbit Determination with Statistically Constrained Data	J. Holdsworth/ L. Lustick
499	Analyses of NASA Tapes 1146 and 1135	H. Engel
500	Analyses of NASA Tape 1028	H. Engel

vs 11,

The Bissett-Berman Corporation 2541 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 499
(BBC Task 105)

H. Engel
19 July 1967

ANALYSES OF NASA TAPES 1146 and 1135

These tapes are MCC-H communications processor log tapes of high speed data from Merritt Island, Carnarvon, Guaymas, Texas and Goldstone. The data were recorded in real time with data intermixed from the various stations.

These tapes contained 45,762 messages, of which the copy program found 25,043 to be of the proper length (27 words of 30 bits each) so that they could be reformatted to standard 240 bit messages. The copy program produced a tape of 240 bit messages, BBC tape number 03.

BBC tape number 03 was processed by the pre-edit program, separating the data by stations and performing the pre-edit operations. Data from Merritt Island, Carnarvon, Texas and Goldstone were placed on BBC tape number 189. Data from Guaymas was placed on BBC tape number 226.

The input data sheets for the pre-edit program appear as Figures 1 through 5. Note that for all these stations the vehicle I. D. is 6, although the AS501 Tracking Data Format Control Book, Revision 2 states that it should be 7. Note also that bit 24 of the messages indicates a crystal clock, but residuals show a rubidium clock.

The results of the pre-edit program are summarized in Figures 6 through 10. For Merritt Island the only points rejected were those for which the Doppler mode indicator in the data did not indicate multiple non-coherent mode. In addition, inspection of the data indicated that this was artificial data taken at 0.1 sec. intervals with a constant change in Doppler count indicative of the bias frequency only. Carnarvon records were rejected because of the control bits for automatic range rate quality and manual range rate quality. Guaymas data were rejected because data rate bits and automatic range rate bits were incorrect, and because

the indicated times on the data and the changes in time between successive data points were incorrect. Texas records were rejected because of data rate bits and automatic range rate bits were incorrect, and because the indicated times on the data were incorrect, and because of the auto range rate quality bit. Goldstone records were rejected for a wide variety of reasons.

The results of the edit program are indicated in Figures 11 through 15. The data from each station were broken into groups of less than 2500 points and each group analyzed separately. The F count threshold is 100. The points rejected for F count threshold are those for which the second difference of the counter readings exceeds 100. The $K\sigma$ threshold is 6. The points rejected for $K\sigma$ threshold are those for which the second difference differs from the median second difference by more than 6 times the standard deviation of these residuals.

The results of the pre-edit and edit programs are summarized in Figure 16. With regard to this figure, the number of points into the edit program should be equal to the number of points accepted by the pre-edit program, so that the following relation holds:

$$\frac{\text{Points accepted}}{\text{Points input}} = \left(\frac{\text{Points accepted by-pre-edit}}{\text{Points input to pre-edit}}\right)\left(\frac{\text{Points accepted by edit}}{\text{Points input to edit}}\right)$$

In actual fact, because of bookkeeping difficulties in the data reduction programs, the points accepted by pre-edit is greater than the points into edit; still, the ratio on the left in the above equation is calculated using that equation.

The remaining figures are plots of residuals, correlation functions and power density spectra for the residuals resulting from fitting the first differences of the input data with fourth degree polynomials.

Figures 17 through 21 are for Merritt Island. The residuals are clearly due almost entirely to quantization noise. The normalized correlation function is approximately - 0.5 at one lag, and zero at larger lags, as would be expected if quantization were the

principal source of error. The power spectral density is shown for scanning windows encompassing 0.1, 0.2 and 0.4 of the data points; no unusual peaks occur in these spectra.

Figure 22 is a plot of the residuals for group 1 of the Carnarvon data. Note the large value of the residuals. This occurs because either the transmitter or receiver frequency was changed by about 30,000 hertz during this interval, making it impossible to de-trend the data. The resultant correlation functions and power density spectra are meaningless, and so are not shown. This group could have been rerun as two separate groups in order to get meaningful results, but it was felt that enough other data was available to make this not worthwhile.

Figures 23 through 27 are for Carnarvon group 2 data. The results are similar to those for Merritt Island, indicating that quantization is the principal error source.

Figure 28 shows the residuals for Carnarvon group 3 data. The residuals are large for the same reason as in Carnarvon group 1. The frequency change here is about 40,000 hertz.

Additional correlation functions or power density spectra are shown only for Guaymas because the residuals are typical of quanitzation noise.

Figure 29 shows the residuals for Carnarvon group 4 data. This is the typical pattern of quantization errors.

Figures 30 through 34 correspond to Guaymas group 1 data. Again the results are those that would be expected with quantization being the principal error source.

Figures 35 through 39 are for Guaymas group 2 data, and are typical quantization error residuals.

Figures 40 and 41 are residuals for Texas groups 1 and 2, respectively. The patterns here, also, are typical of quantization error, except that in group 1 there are a few residuals that are too large to be accounted for by quantization. Since no data from other stations are available for this same time it is not possible to trace the source of these errors.

Figures 42 and 43 show similar results for Goldstone groups 1 and 2 data.

INPUT DATA SHEET FOR PRE-EDIT PROGRAM

Punch: 1 for YES; 2 for NO; 9 for TEST TO BE IGNORED

		Yes	No	Test to be Ignored
1	Is this high speed 240-Bit data	✓		/////
2	Is this non-destruct data	✓		/////
3	Is this high data (Bit 15)	✓		
4	Is range-rate in standard position	✓		/////
5	Is range-rate N_1 mode			✓
	Only one of four applies:			
6	Is this one way doppler mode			/////
7	Is this two way doppler mode			/////
8	Is this multiple non-coherent mode	✓		/////
9	Is this multiple coherent mode			/////
10	Vehicle ID is [6]	///	///	
11	Is frequency standard rubidium		✓	
12	Is manual R-R test to be made	✓		/////
13	Is VCO lock test to be made		✓	/////
14	Is automatic range-rate test to be made	✓		/////
15	Is test to be made on real/test Bit	✓		/////
16	Is station ID test to be made	✓		/////
17	Is doppler mode test to be made	✓		/////
18	Is test to be made on R-R field indicator	✓		/////

MSC Tape No.

1146
1135

BBC Tape No.

03 In
189 Out

STATION ID

20		Bermuda
21	✓	Merritt Island
22		Grand Bahama Island
23		Antigua
24		Carnarvon
25		Hawaii
26		Guaymas
27		Texas
28		Guam
29		Goldstone
30		Ascension
31		Canberra
32		GSFC
33		Madrid
34		Grand Canary Islands

EXPECTED START TIME

35-37	125	Day (if greater than 31 month will be ignored)
38-39		Month
40-41	67	Year
42-43	21	Hour
44-45	00	Minute
46-48	.4	Expected Delta Time
49-51	180	Maximum Time Interval (in minutes)
52-55	50	Print Rejected Data
56-59	100	Print Raw Data
60-68		Start Time

Figure 1

INPUT DATA SHEET FOR PRE-EDIT PROGRAM

Punch: 1 for YES; 2 for NO; 9 for TEST TO BE IGNORED

		Yes	No	Test to be Ignored
1	Is this high speed 240-Bit data	✓		/////
2	Is this non-destruct data	✓		/////
3	Is this high data (Bit 15)	✓		
4	Is range-rate in standard position	✓		/////
5	Is range-rate N_1 mode			✓
	Only one of four applies:			
6	Is this one way doppler mode			/////
7	Is this two way doppler mode			/////
8	Is this multiple non-coherent mode	✓		/////
9	Is this multiple coherent mode			/////
10	Vehicle ID is [6]	///	///	
11	Is frequency standard rubidium		✓	
12	Is manual R-R test to be made	✓		/////
13	Is VCO lock test to be made		✓	/////
14	Is automatic range-rate test to be made	✓		/////
15	Is test to be made on real/test Bit	✓		/////
16	Is station ID test to be made	✓		/////
17	Is doppler mode test to be made	✓		/////
18	Is test to be made on R-R field indicator	✓		/////

MSC Tape No.

1146
1135

BBC Tape No.

03 In
189 Out

STATION ID

20		Bermuda
21		Merritt Island
22		Grand Bahama Island
23		Antigua
24	✓	Carnarvon
25		Hawaii
26		Guaymas
27		Texas
28		Guam
29		Goldstone
30		Ascension
31		Canberra
32		GSFC
33		Madrid
34		Grand Canary Islands

EXPECTED START TIME

35-37	125	Day (if greater than 31 month will be ignored)
38-39		Month
40-41	67	Year
42-43	21	Hour
44-45	00	Minute
46-48	4	Expected Delta Time
49-51	180	Maximum Time Interval (in minutes)
52-55	50	Print Rejected Data
56-59	100	Print Raw Data
60-68		Start Time

Figure 2

INPUT DATA SHEET FOR PRE-EDIT PROGRAM

Punch: 1 for YES; 2 for NO; 9 for TEST TO BE IGNORED

		Yes	No	Test to be Ignored
1	Is this high speed 240-Bit data	✓		/////
2	Is this non-destruct data	✓		/////
3	Is this high data (Bit 15)	✓		
4	Is range-rate in standard position	✓		/////
5	Is range-rate N_1 mode			✓
	Only one of four applies:			
6	Is this one way doppler mode			/////
7	Is this two way doppler mode			/////
8	Is this multiple non-coherent mode	✓		/////
9	Is this multiple coherent mode			/////
10	Vehicle ID is [6]	///	///	
11	Is frequency standard rubidium		✓	
12	Is manual R-R test to be made	✓		/////
13	Is VCO lock test to be made		✓	/////
14	Is automatic range-rate test to be made	✓		/////
15	Is test to be made on real/test Bit	✓		/////
16	Is station ID test to be made	✓		/////
17	Is doppler mode test to be made	✓		/////
18	Is test to be made on R-R field indicator	✓		/////

MSC Tape No.

1 1 4 6
1 1 3 5

BBC Tape No.

0 3 In
2 2 6 Out

STATION ID

20		Bermuda
21		Merritt Island
22		Grand Bahama Island
23		Antigua
24		Carnarvon
25		Hawaii
26	✓	Guaymas
27		Texas
28		Guam
29		Goldstone
30		Ascension
31		Canberra
32		GSFC
33		Madrid
34		Grand Canary Islands

EXPECTED START TIME

35-37	1 2 5	Day (if greater than 31 month will be ignored)
38-39		Month
40-41	6 7	Year
42-43	2 3	Hour
44-45	0 0	Minute
46-48	4	Expected Delta Time
49-51	1 8 0	Maximum Time Interval (in minutes)
52-55	5 0	Print Rejected Data
56-59	1 0 0	Print Raw Data
60-68		Start Time

Figure 3

INPUT DATA SHEET FOR PRE-EDIT PROGRAM

Punch: 1 for YES; 2 for NO; 9 for TEST TO BE IGNORED

		Yes	No	Test to be Ignored
1	Is this high speed 240-Bit data	✓		/////
2	Is this non-destruct data	✓		/////
3	Is this high data (Bit 15)	✓		
4	Is range-rate in standard position	✓		/////
5	Is range-rate N_1 mode			✓
	Only one of four applies:			
6	Is this one way doppler mode			/////
7	Is this two way doppler mode			/////
8	Is this multiple non-coherent mode	✓		/////
9	Is this multiple coherent mode			/////
10	Vehicle ID is [✓]	///	///	
11	Is frequency standard rubidium		✓	
12	Is manual R-R test to be made	✓		/////
13	Is VCO lock test to be made		✓	/////
14	Is automatic range-rate test to be made	✓		/////
15	Is test to be made on real/test Bit	✓		/////
16	Is station ID test to be made	✓		/////
17	Is doppler mode test to be made	✓		/////
18	Is test to be made on R-R field indicator	✓		/////

MSC Tape No.

1146
1135

BBC Tape No.

03 In
189 Out

STATION ID

20		Bermuda
21		Merritt Island
22		Grand Bahama Island
23		Antigua
24		Carnarvon
25		Hawaii
26		Guaymas
27	✓	Texas
28		Guam
29		Goldstone
30		Ascension
31		Canberra
32		GSFC
33		Madrid
34		Grand Canary Islands

EXPECTED START TIME

35-37	1 2 5	Day (if greater than 31 month will be ignored)
38-39		Month
40-41	6 7	Year
42-43	2 3	Hour
44-45	0 0	Minute
46-48	4	Expected Delta Time
49-51	1 8 0	Maximum Time Interval (in minutes)
52-55	5 0	Print Rejected Data
56-59	1 0 0	Print Raw Data
60-68		Start Time

Figure 4

INPUT DATA SHEET FOR PRE-EDIT PROGRAM

Punch: 1 for YES; 2 for NO; 9 for TEST TO BE IGNORED

		Yes	No	Test to be Ignored
1	Is this high speed 240-Bit data	✓		/////
2	Is this non-destruct data	✓		/////
3	Is this high data (Bit 15)	✓		
4	Is range-rate in standard position	✓		/////
5	Is range-rate N_1 mode			✓
	Only one of four applies:			
6	Is this one way doppler mode			/////
7	Is this two way doppler mode			/////
8	Is this multiple non-coherent mode	✓		/////
9	Is this multiple coherent mode			/////
10	Vehicle ID is [6]	///	///	
11	Is frequency standard rubidium		✓	
12	Is manual R-R test to be made	✓		/////
13	Is VCO lock test to be made		✓	/////
14	Is automatic range-rate test to be made	✓		/////
15	Is test to be made on real/test Bit	✓		/////
16	Is station ID test to be made	✓		/////
17	Is doppler mode test to be made	✓		/////
18	Is test to be made on R-R field indicator	✓		/////

MSC Tape No.

1146
1135

BBC Tape No.

STATION ID

20		Bermuda
21		Merritt Island
22		Grand Bahama Island
23		Antigua
24		Carnarvon
25		Hawaii
26		Guaymas
27		Texas
28		Guam
29	✓	Goldstone
30		Ascension
31		Canberra
32		GSFC
33		Madrid
34		Grand Canary Islands

EXPECTED START TIME

35-37	1 2 5	Day (if greater than 31 month will be ignored)
38-39		Month
40-41	6 7	Year
42-43	2 3	Hour
44-45	0 0	Minute
46-48	4	Expected Delta Time
49-51	1 8 0	Maximum Time Interval (in minutes)
52-55	5 0	Print Rejected Data
56-59	1 0 0	Print Raw Data
60-68		Start Time

Figure 5

PRE-EDIT PROGRAM SUMMARY

1. Station Merritt Island
2. Station ID (decimal 1

MSC Tape No. 1146
1135

BBC Tape No. 03 Input
189 Output

Number	Rejects[1]	
	1	Station ID
	2	Destruct/Non Destruct
	3	Data Rate
	4	Real Test Data
	5	Vehicle ID
1582	6	Doppler Mode
	7	Frequency Standard
	8	Range Rate Field Indicator
	9	Gross Time
	10	Time Interval
*	11	VCO Lock
	12	Auto Range Rate Quality
	13	Manual Range Rate Quality
*	14	Gross Range Rate Test[2]
*	15	Destruct Count N_1/N_2 Indicator[2]
1582	Total Rejects	
538	Total Good Data	
2120	Total Input Point	

[1] *Indicates that test was not made

[2] These tests not performed with non-destruct count doppler data.

Figure 6

PRE-EDIT PROGRAM SUMMARY

1. Station Carnarvon
2. Station ID (decimal 8

MSC Tape No. 1146
1135

BBC Tape No. 03 Input
189 Output

Number	Rejects[1]	
	1	Station ID
	2	Destruct/Non Destruct
	3	Data Rate
	4	Real Test Data
	5	Vehicle ID
	6	Doppler Mode
	7	Frequency Standard
	8	Range Rate Field Indicator
	9	Gross Time
	10	Time Interval
*	11	VCO Lock
670	12	Auto Range Rate Quality
2660	13	Manual Range Rate Quality
*	14	Gross Range Rate Test[2]
*	15	Destruct Count N_1/N_2 Indicator[2]
3330	Total Rejects	
6699	Total Good Data	
10029	Total Input Point	

[1] *Indicates that test was not made

[2] These tests not performed with non-destruct count doppler data.

Figure 7

PRE-EDIT PROGRAM SUMMARY

1. Station Guaymas
2. Station ID (decimal) 10

MSC Tape No. 1146
1135

BBC Tape No. 03 Input
Output

Number	Rejects[1]	
	1	Station ID
	2	Destruct/Non Destruct
146	3	Data Rate
	4	Real Test Data
	5	Vehicle ID
	6	Doppler Mode
	7	Frequency Standard
	8	Range Rate Field Indicator
39	9	Gross Time
3	10	Time Interval
*	11	VCO Lock
14	12	Auto Range Rate Quality
	13	Manual Range Rate Quality
*	14	Gross Range Rate Test[2]
*	15	Destruct Count N_1/N_2 Indicator[2]
202	Total Rejects	
4789	Total Good Data	
4991	Total Input Point	

[1] *Indicates that test was not made

[2] These tests not performed with non-destruct count doppler data.

Figure 8

PRE-EDIT PROGRAM SUMMARY

1. Station Texas
2. Station ID (decimal 11

MSC Tape No. 1146
1135

BBC Tape No. 03 Input
189 Output

Number	Rejects[1]	
	1	Station ID
	2	Destruct/Non Destruct
42	3	Data Rate
	4	Real Test Data
	5	Vehicle ID
	6	Doppler Mode
	7	Frequency Standard
	8	Range Rate Field Indicator
13	9	Gross Time
	10	Time Interval
*	11	VCO Lock
186	12	Auto Range Rate Quality
	13	Manual Range Rate Quality
*	14	Gross Range Rate Test[2]
*	15	Destruct Count N_1/N_2 Indicator[2]
241	Total Rejects	
2560	Total Good Data	
2801	Total Input Point	

[1] *Indicates that test was not made

[2] These tests not performed with non-destruct count doppler data.

Figure 9

PRE-EDIT PROGRAM SUMMARY

1. Station Goldstone
2. Station ID (decimal 13

MSC Tape No. 1146
1135

BBC Tape No. 03 Input
189 Output

Number	Rejects[1]	
	1	Station ID
6	2	Destruct/Non Destruct
	3	Data Rate
	4	Real Test Data
	5	Vehicle ID
56	6	Doppler Mode
	7	Frequency Standard
2	8	Range Rate Field Indicator
174	9	Gross Time
	10	Time Interval
*	11	VCO Lock
27	12	Auto Range Rate Quality
138	13	Manual Range Rate Quality
*	14	Gross Range Rate Test[2]
*	15	Destruct Count N_1/N_2 Indicator[2]
403	Total Rejects	
4313	Total Good Data	
4716	Total Input Point	

[1] *Indicates that test was not made

[2] These tests not performed with non-destruct count doppler data.

Figure 10

EDIT PROGRAM SUMMARY

1.	Station	Merritt Island	MSC Tape No.	1146
2.	Station ID (decimal)	1		1135
3.	Sample Interval (seconds)	0.4		
4.	Count Threshold	100		
5.	K σ threshold	6		
6.	Start Time (seconds)	10,877,418.4		
7.	Group Interval (seconds)	1,000	BBC Tape No.	189 In
				148 Out

Group Number	Time (seconds from beginning of Calendar Year)		
	Input Start	First Good Data Point	Last Good Data Point
1	10,877,418.4	10,877,418.4	10,877,633.6
2			
3			
4			
5			
6			
7			
8			
9			
10			

Group Number	Number of Rejects				Good Data Points	Total Input Points
	Fcount Threshold	Kσ Threshold	No formed 2nd Diff. *	Total Rejects		
1	0	0		0	538	538
2						
3						
4						
5						
6						
7						
8						
9						
10						
Total	0	0		0	538	538

*When no number is indicated in this column the points rejected for Kσ and those which no second difference could be formed have been combined in the Kσ column.

Figure 11

EDIT PROGRAM SUMMARY

1.	Station	Carnarvon	MSC Tape No.	1146
2.	Station ID (decimal)	8		1135
3.	Sample Interval (seconds)	0.4		
4.	Count Threshold	100		
5.	K σ threshold	6		
6.	Start Time (seconds)	10,878,577.6		
7.	Group Interval (seconds)	1,000	BBC Tape No.	189 In
				148 Out

Group Number	Time (seconds from beginning of Calendar Year)		
	Input Start	First Good Data Point	Last Good Data Point
1	10,878,377.6	10,878,377.6	10,879,377.2
2	10,879,377.6	10,879,377.6	10,880,105.2
3	10,880,377.6	10,880,715.2	10,881,377.2
4	10,881,377.6	10,881,377.6	10,882,220.4
5			
6			
7			
8			
9			
10			

Group Number	Number of Rejects				Good Data Points	Total Input Points
	Fcount Threshold	Kσ Threshold	No formed 2nd Diff. *	Total Rejects		
1	3	4		7	2255	2262
2	0	0		0	1820	1820
3	0	0		0	412	412
4	0	0		0	2108	2108
5						
6						
7						
8						
9						
10						
Total	3	4		7	6595	6602

*When no number is indicated in this column the points rejected for Kσ and those which no second difference could be formed have been combined in the Kσ column.

Figure 12

EDIT PROGRAM SUMMARY

1.	Station	Guaymas	MSC Tape No.	1146
2.	Station ID (decimal)	10		1135
3.	Sample Interval (seconds)	0.4 *		
4.	Count Threshold	100		
5.	Kσ threshold	6		
6.	Start Time (seconds)	10,884,000.4		
7.	Group Interval (seconds)	1,000	BBC Tape No.	226 In
				136 Out

Group Number	Time (seconds from beginning of Calendar Year)		
	Input Start	First Good Data Point	Last Good Data Point
1	10,884,000.4	10,884,000.4	10,885,000.0
2	10,885,000.4	10,885,002.8	10,886,000.0
3			
4			
5			
6			
7			
8			
9			
10			

Group Number	Number of Rejects				Good Data Points	Total Input Points
	Fcount Threshold	Kσ Threshold	No formed 2nd Diff. *	Total Rejects		
1	0	110		110	2131	2241
2	3	114		117	2181	2298
3						
4						
5						
6						
7						
8						
9						
10						
Total	3	224		227	4312	4539

*When no number is indicated in this column the points rejected for Kσ and those which no second difference could be formed have been combined in the Kσ column.

Figure 13

EDIT PROGRAM SUMMARY

1.	Station	Texas	MSC Tape No.	1146
2.	Station ID (decimal)	11		1135
3.	Sample Interval (seconds)	0.4		
4.	Count Threshold	100		
5.	K σ threshold	6		
6.	Start Time (seconds)	10,883,004.4		
7.	Group Interval (seconds)	1,000	BBC Tape No.	189 In
				148 Out

Group Number	Time (seconds from beginning of Calendar Year)		
	Input Start	First Good Data Point	Last Good Data Point
1	10,883,004.4	10,883,004.4	10,884,004.0
2	10,884,004.4	10,883,004.4	10,884,071.2
3			
4			
5			
6			
7			
8			
9			
10			

Group Number	Number of Rejects				Good Data Points	Total Input Points
	Fcount Threshold	Kσ Threshold	No formed 2nd Diff. *	Total Rejects		
1	1	0		1	2411	2412
2	0	4		4	145	149
3						
4						
5						
6						
7						
8						
9						
10						
Total	1	4		5	2556	2561

*When no number is indicated in this column the points rejected for Kσ and those which no second difference could be formed have been combined in the Kσ column.

Figure 14

EDIT PROGRAM SUMMARY

1.	Station	Goldstone	MSC Tape No.	1146
2.	Station ID (decimal)	13		1135
3.	Sample Interval (seconds)	0.4		
4.	Count Threshold	100		
5.	K σ threshold	6		
6.	Start Time (seconds)	10,884,098.4		
7.	Group Interval (seconds)	1,000	BBC Tape No.	189 In
				148 Out

Group Number	Time (seconds from beginning of Calendar Year)		
	Input Start	First Good Data Point	Last Good Data Point
1	10,884,098.4	10,884,102.4	10,885,098.0
2			
3			
4			
5			
6			
7			
8			
9			
10			

Group Number	Number of Rejects				Good Data Points	Total Input Points
	Fcount Threshold	Kσ Threshold	No formed 2nd Diff. *	Total Rejects		
1	66	286		352	1805	2157
2	41	122		163	1990	2153
3						
4						
5						
6						
7						
8						
9						
10						
Total	107	408		515	3795	4310

*When no number is indicated in this column the points rejected for Kσ and those which no second difference could be formed have been combined in the Kσ column.

Figure 15

STATION	Points Accepted by Pre-Edit / Points Input to Pre-Edit	Points Accepted by Edit / Points Input by Edit	Points Accepted / Points Input
Merritt Island	.254	1.000	.254
Carnarvon	.669	.997	.666
Guaymas	.959	.952	.912
Texas	.914	.997	.910
Goldstone	.914	.897	.802

Figure 16

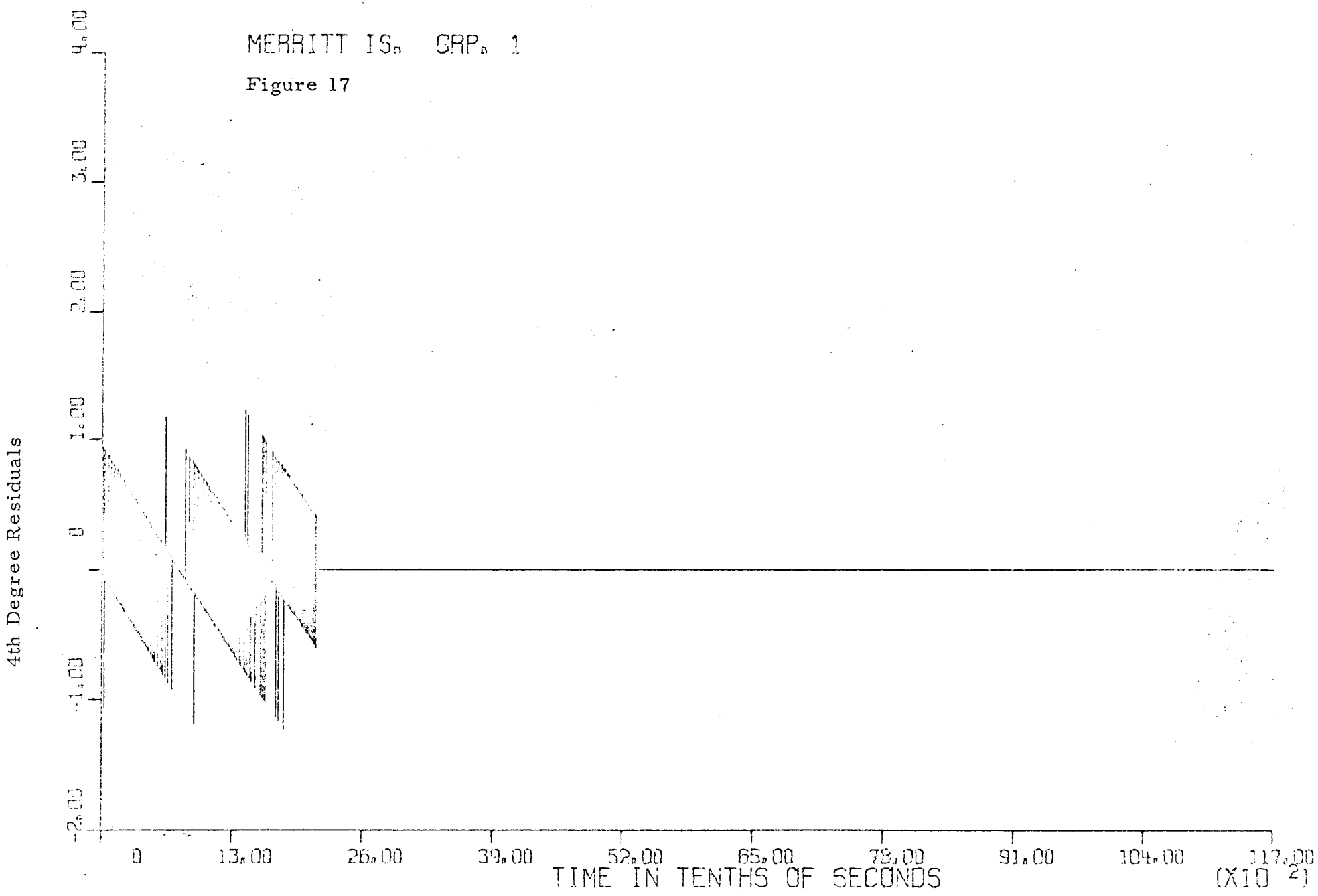

MERRITT IS. GRP. 1
4th Degree Residuals
4.00
3.00
2.00
1.00
0
-1.00
-2.00
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
TIME IN TENTHS OF SECONDS
(X10 2)

Figure 17

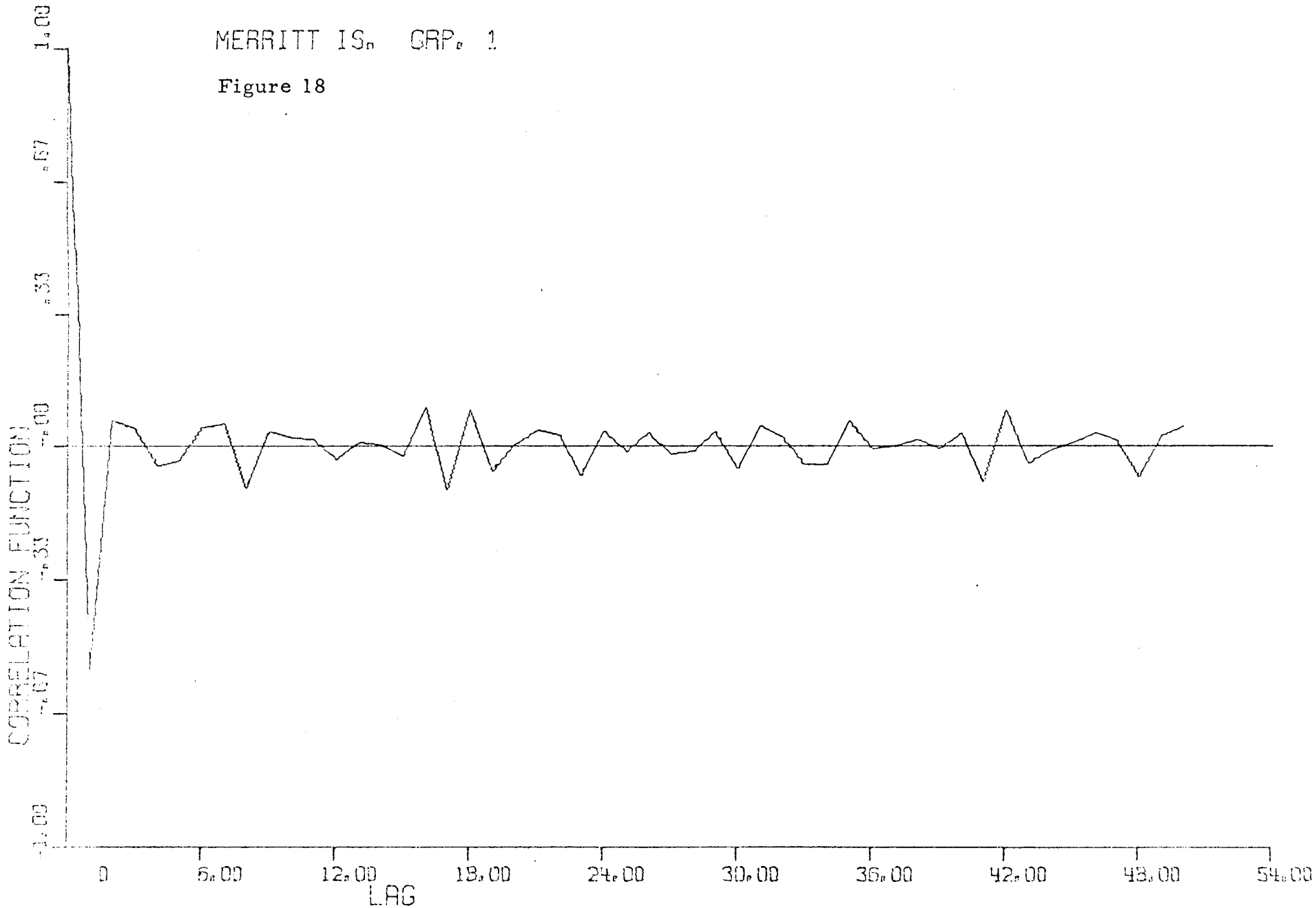
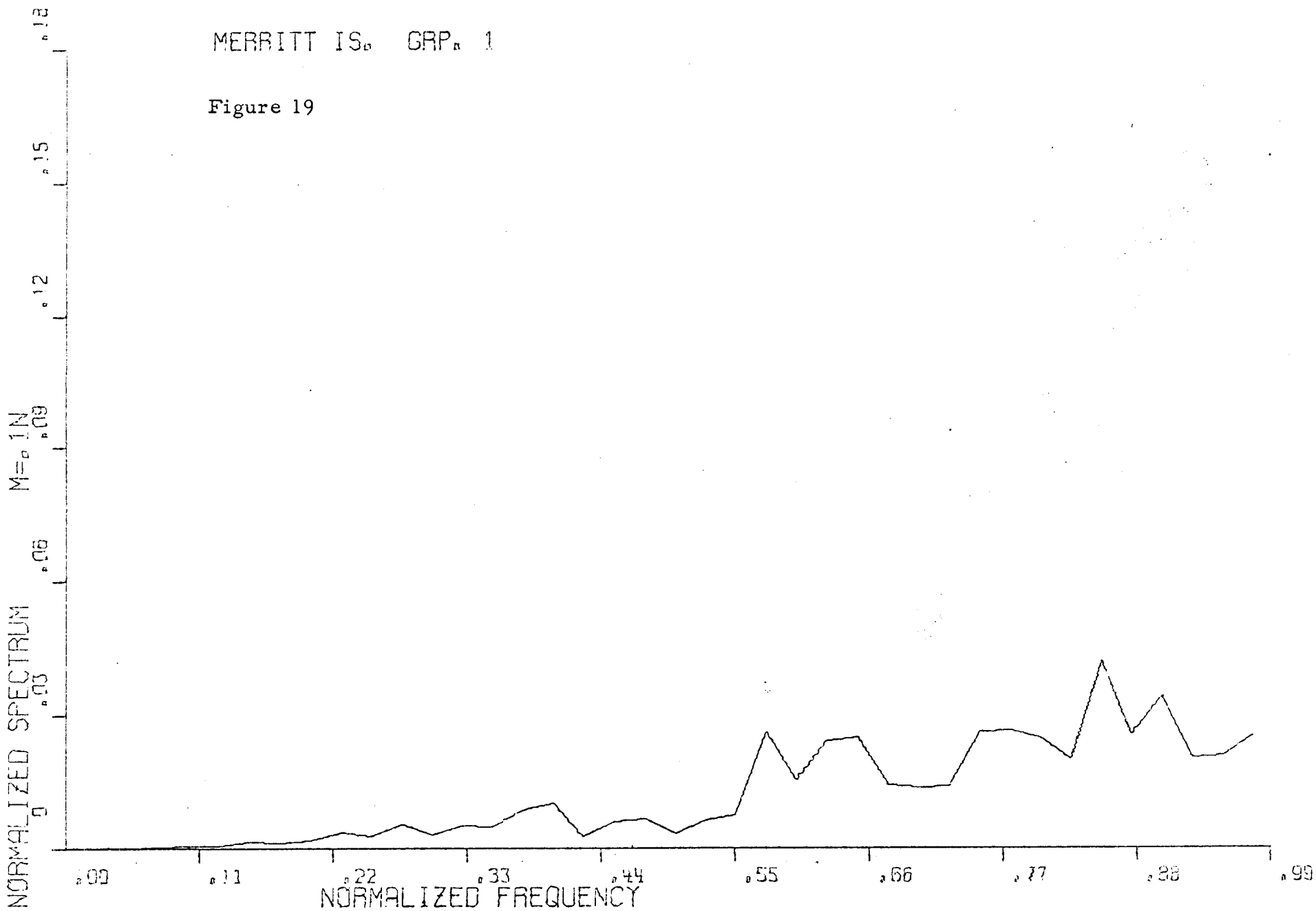

MERRITT IS. GRP. 1
CORRELATION FUNCTION
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 18

MERRITT IS. GRP. 1

Figure 19



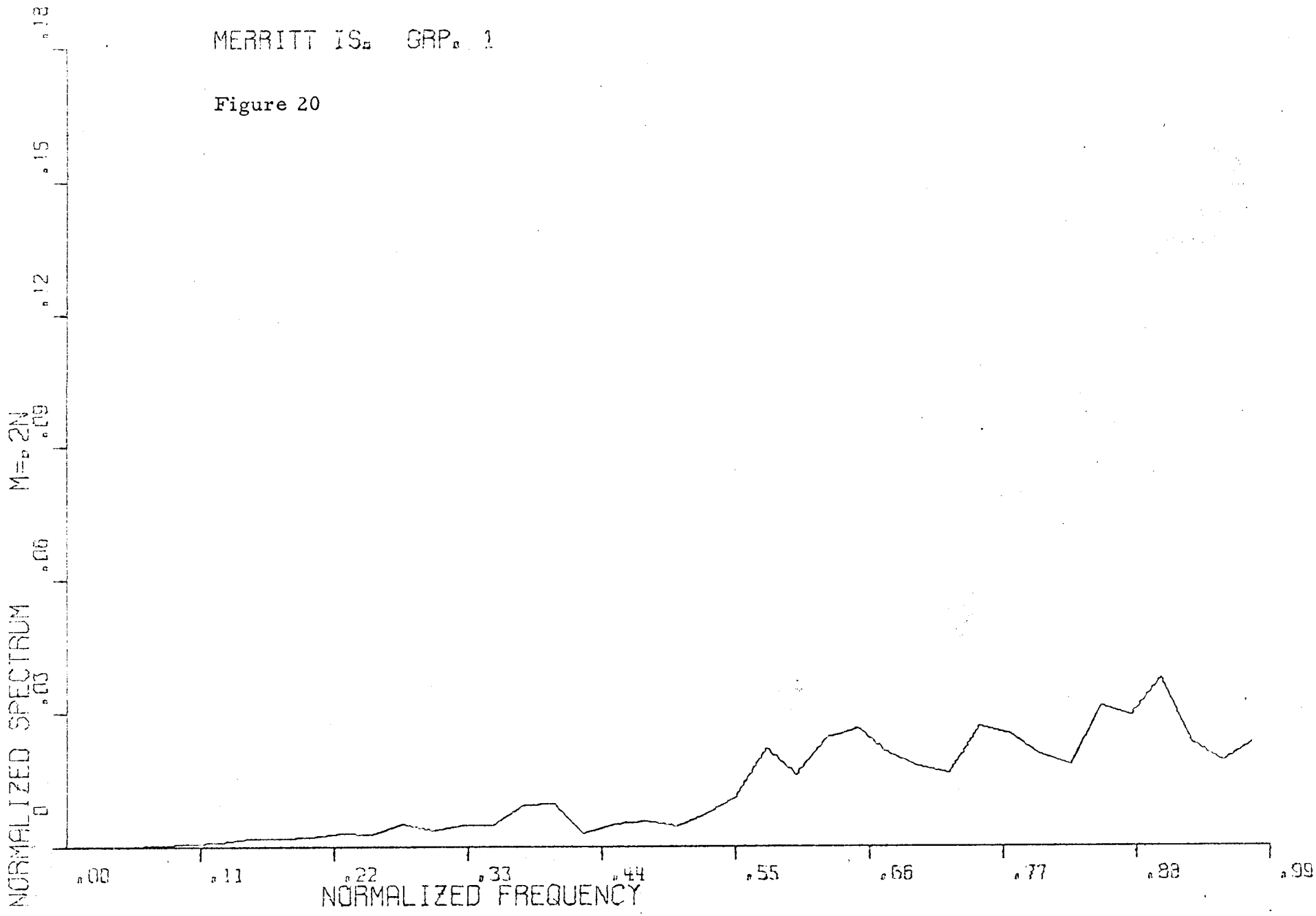

MERRITT IS. GRP. 1
M=.2N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 20

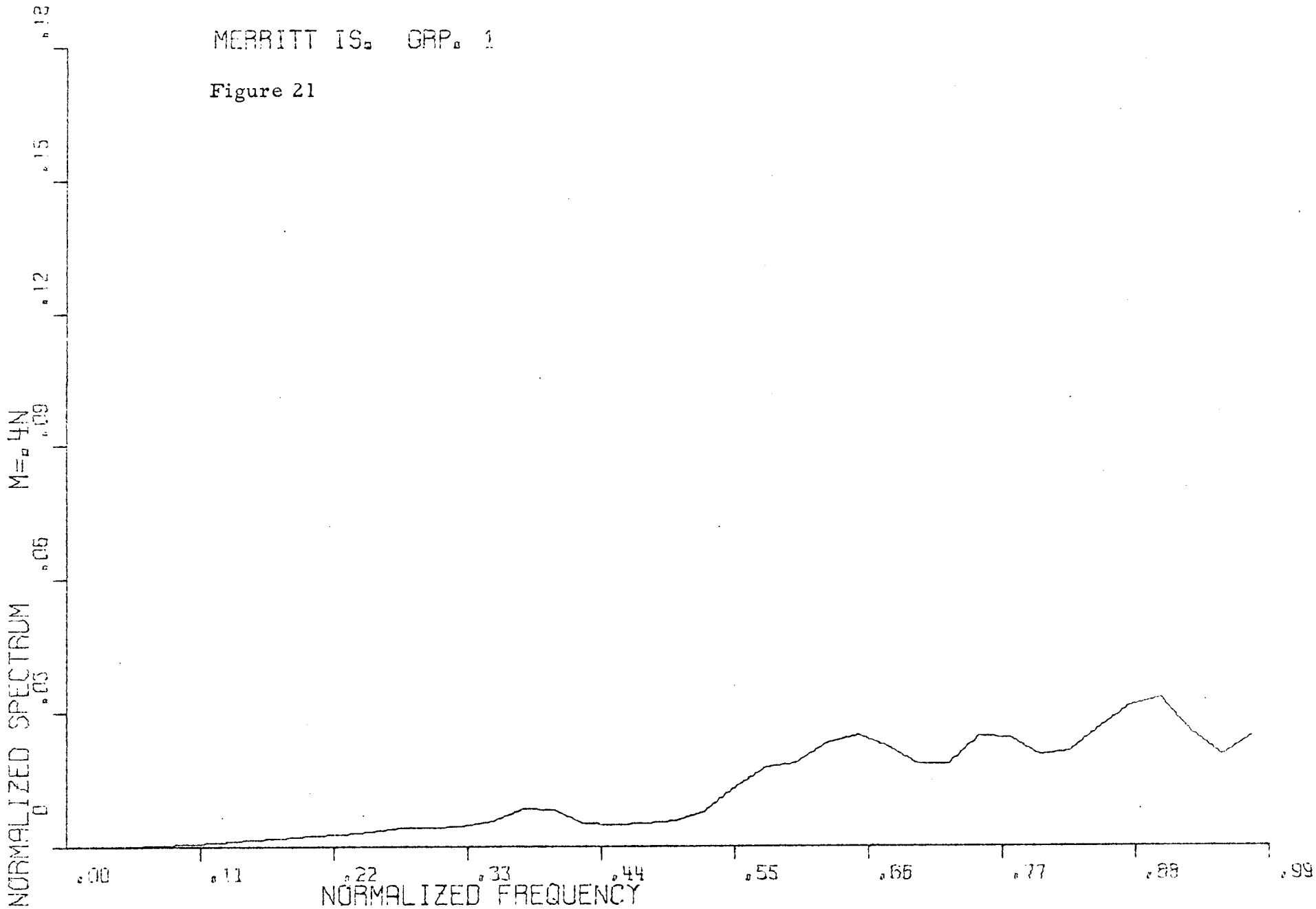

MERRITT IS. GRP. 1
M=.4N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 21

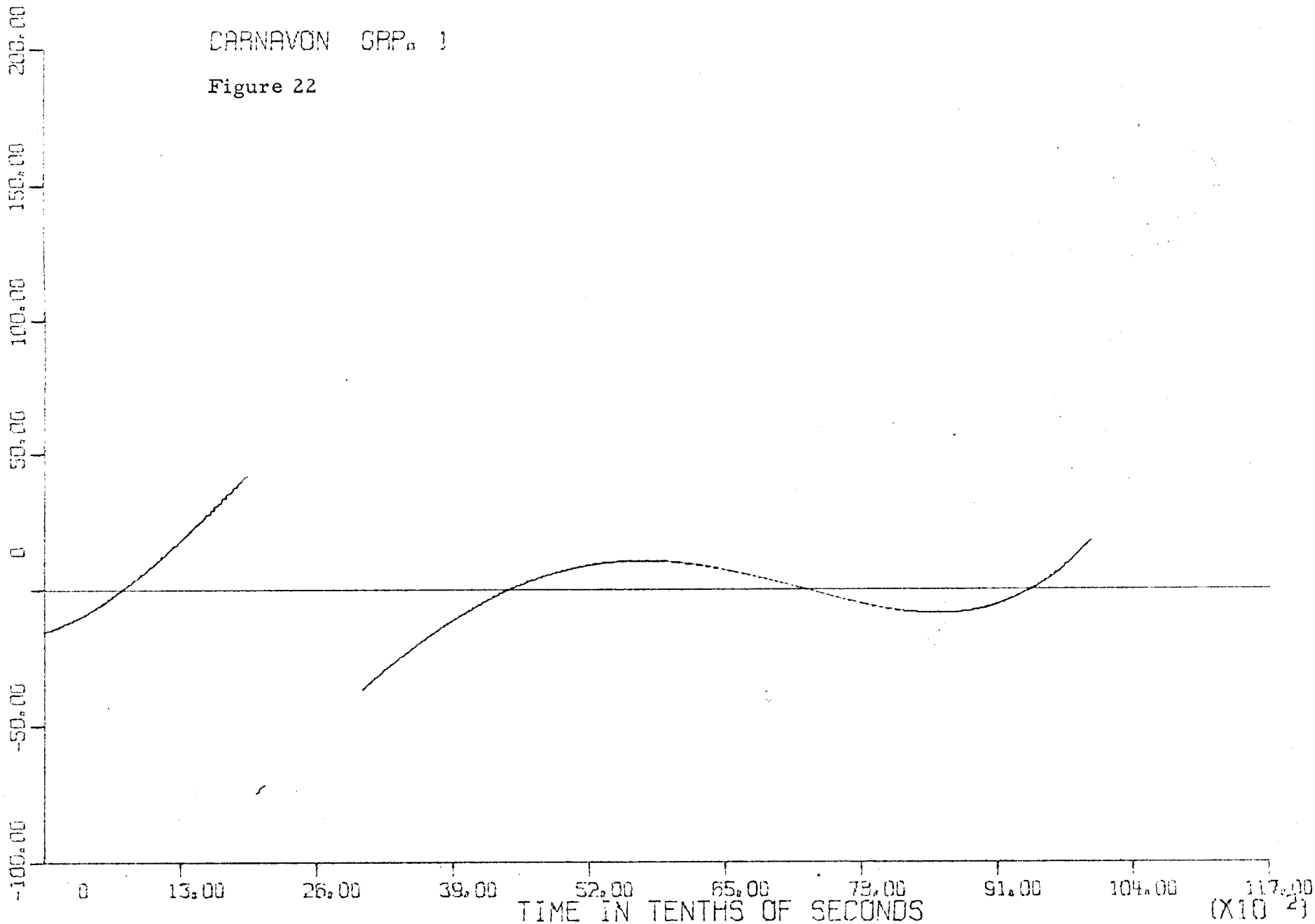
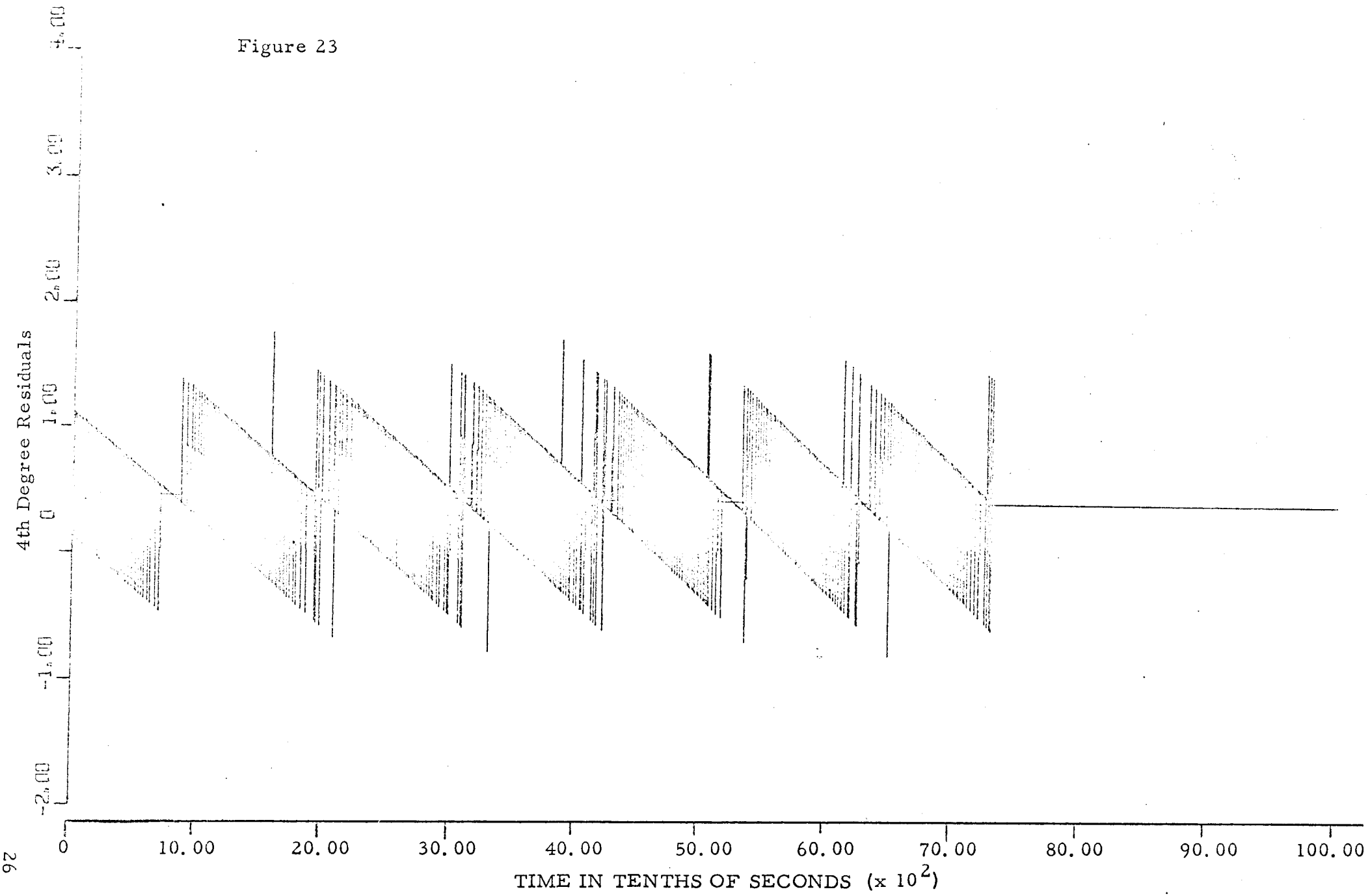


Figure 22

4th Degree Residuals

CARNAVON GRP. 2

Figure 23



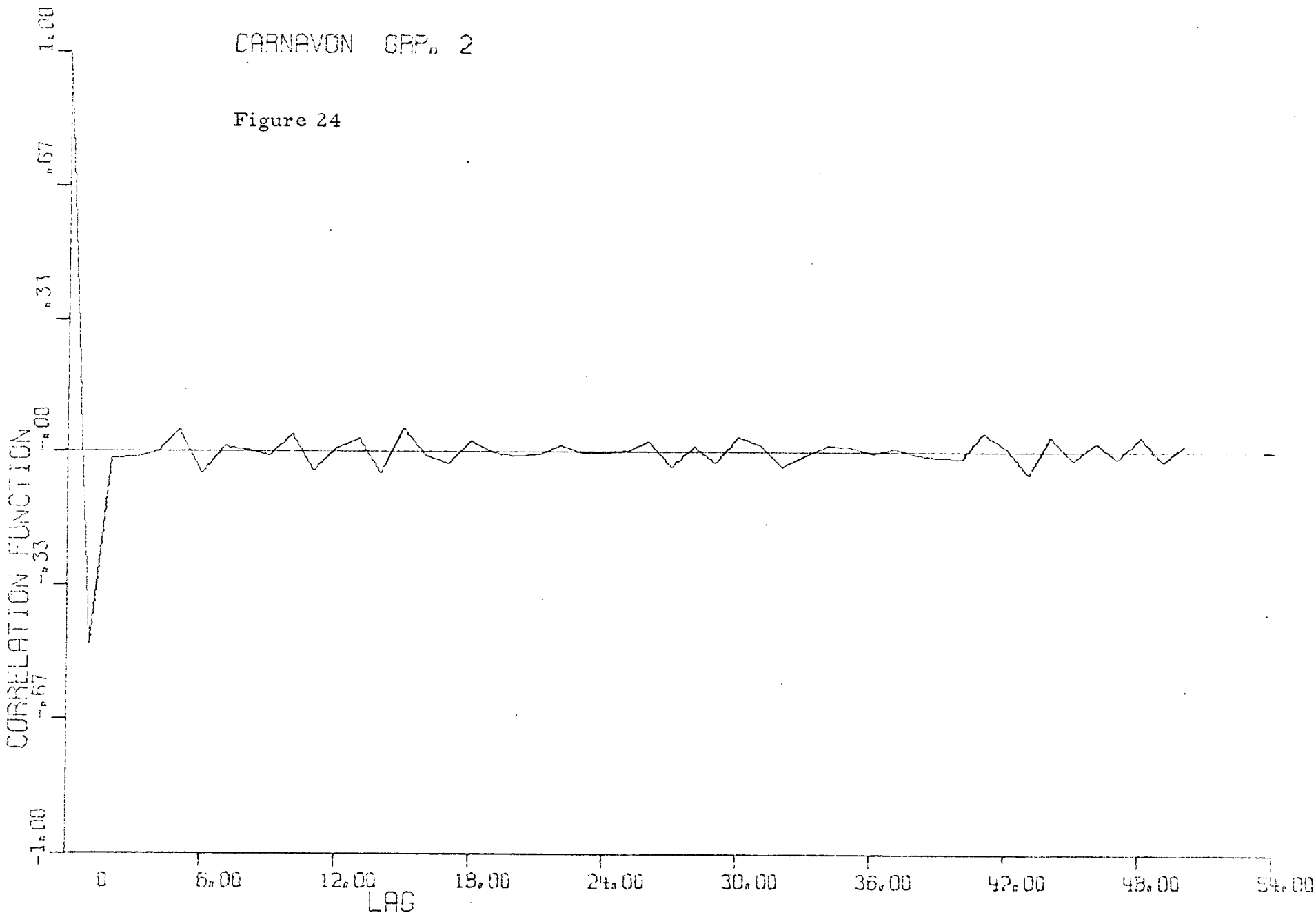
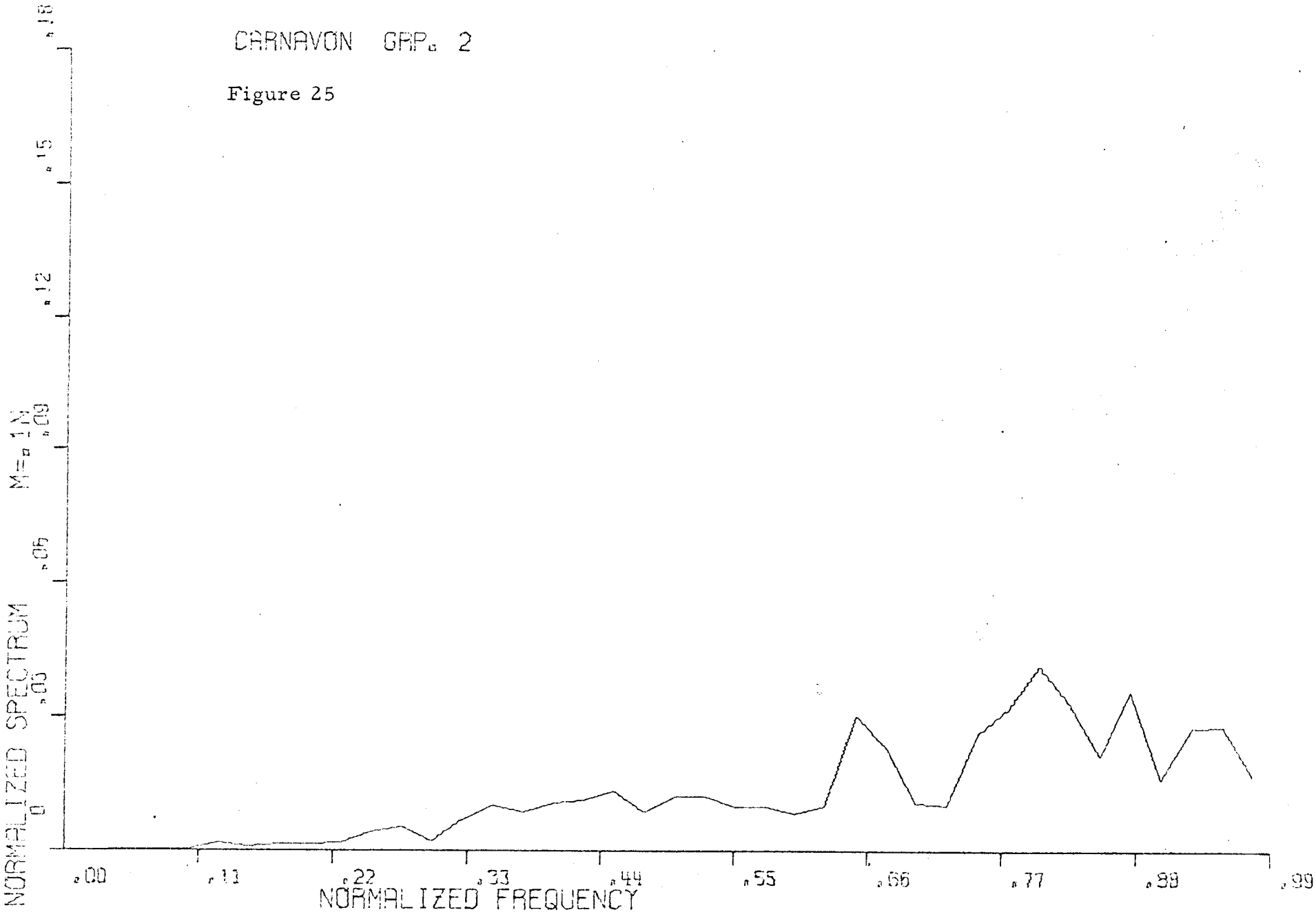

CARNAVON GRP. 2
CORRELATION FUNCTION
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 24

CARNAVON GRP. 2

Figure 25



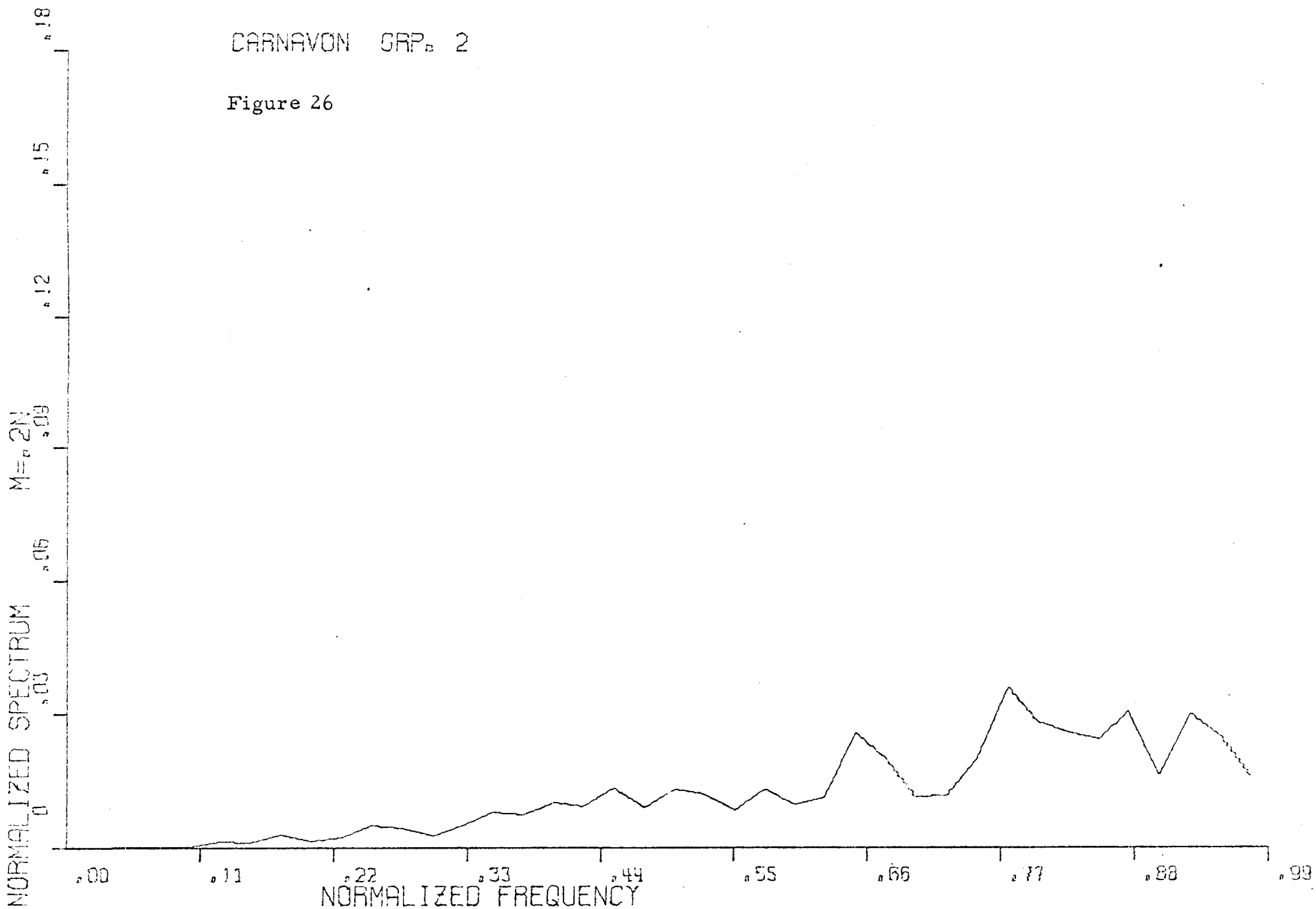
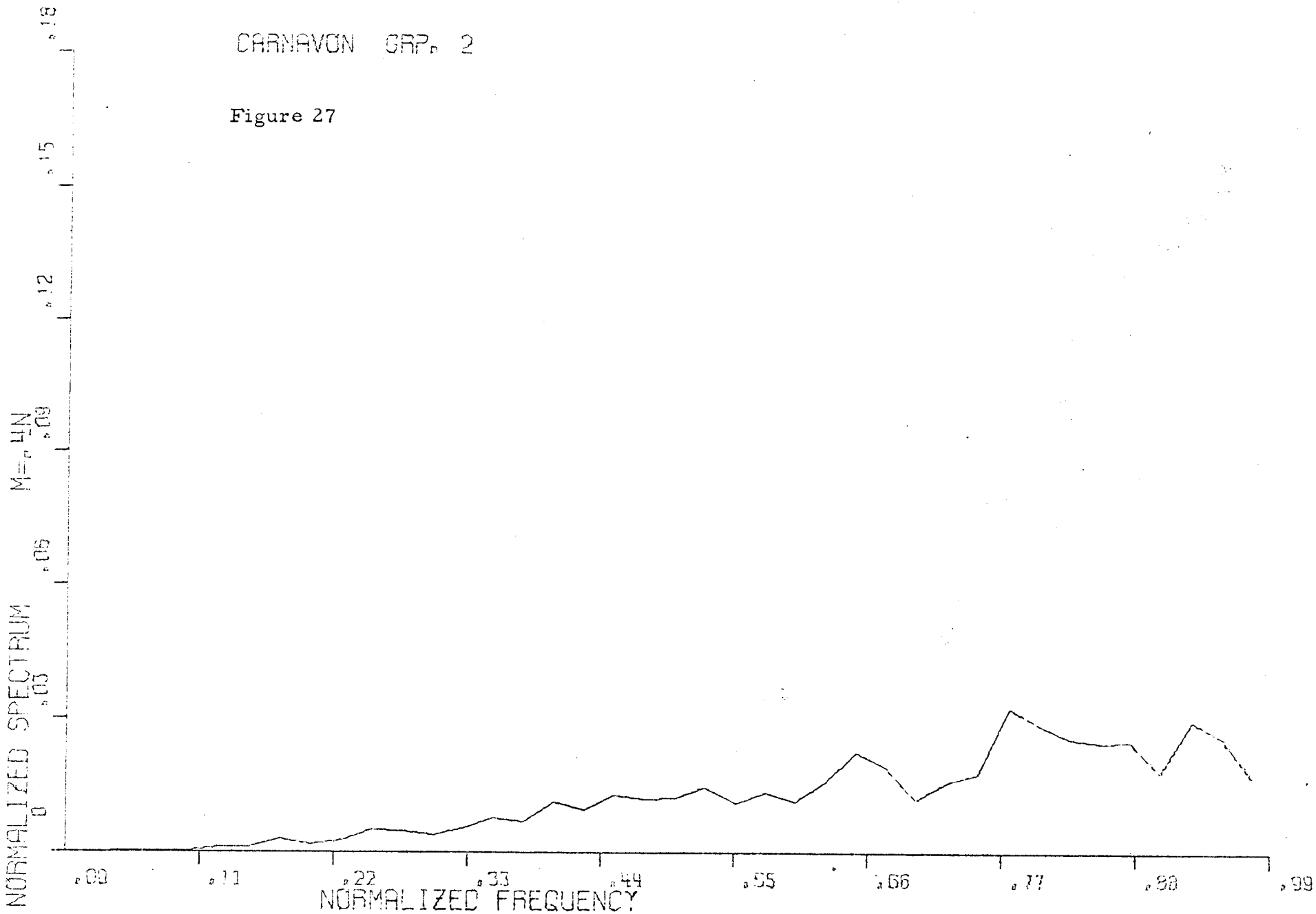

CARNAVON GRP. 2
M=.2N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 26

CARNAVON GRP. 2

Figure 27



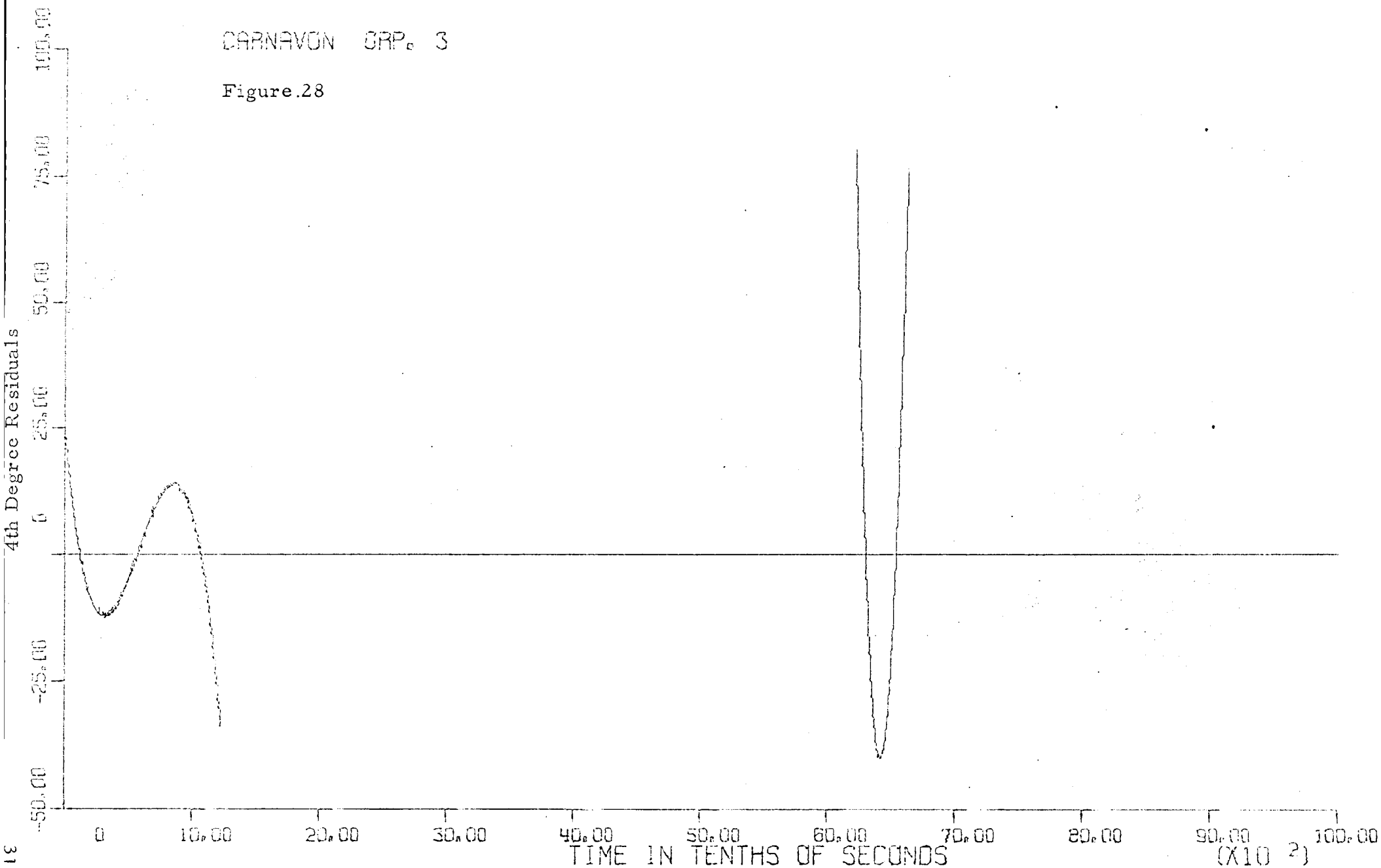

CARNAVON GRP. 3
4th Degree Residuals
100.00
75.00
50.00
25.00
0
-25.00
-50.00
0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
TIME IN TENTHS OF SECONDS
(X10 2)

Figure.28

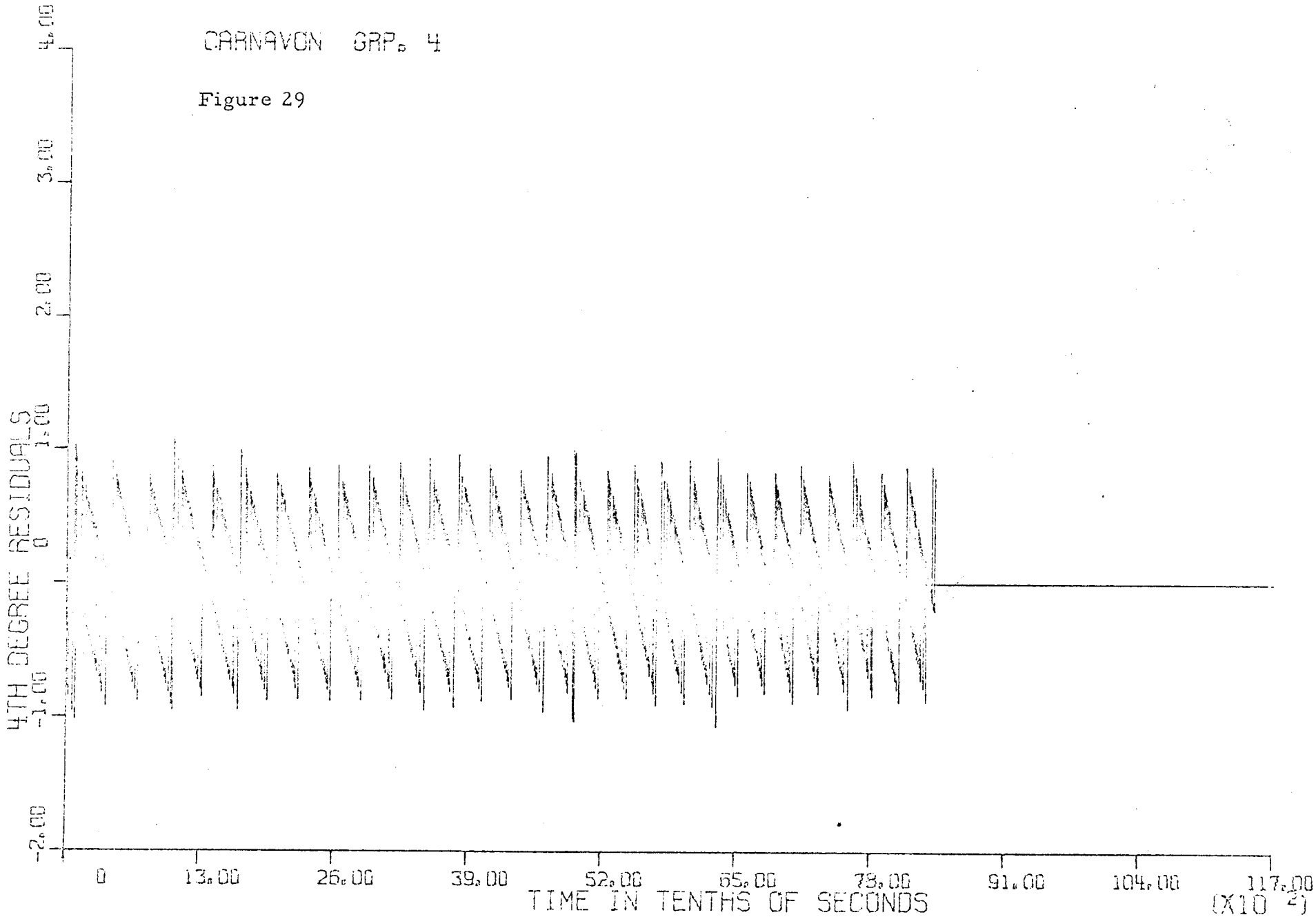
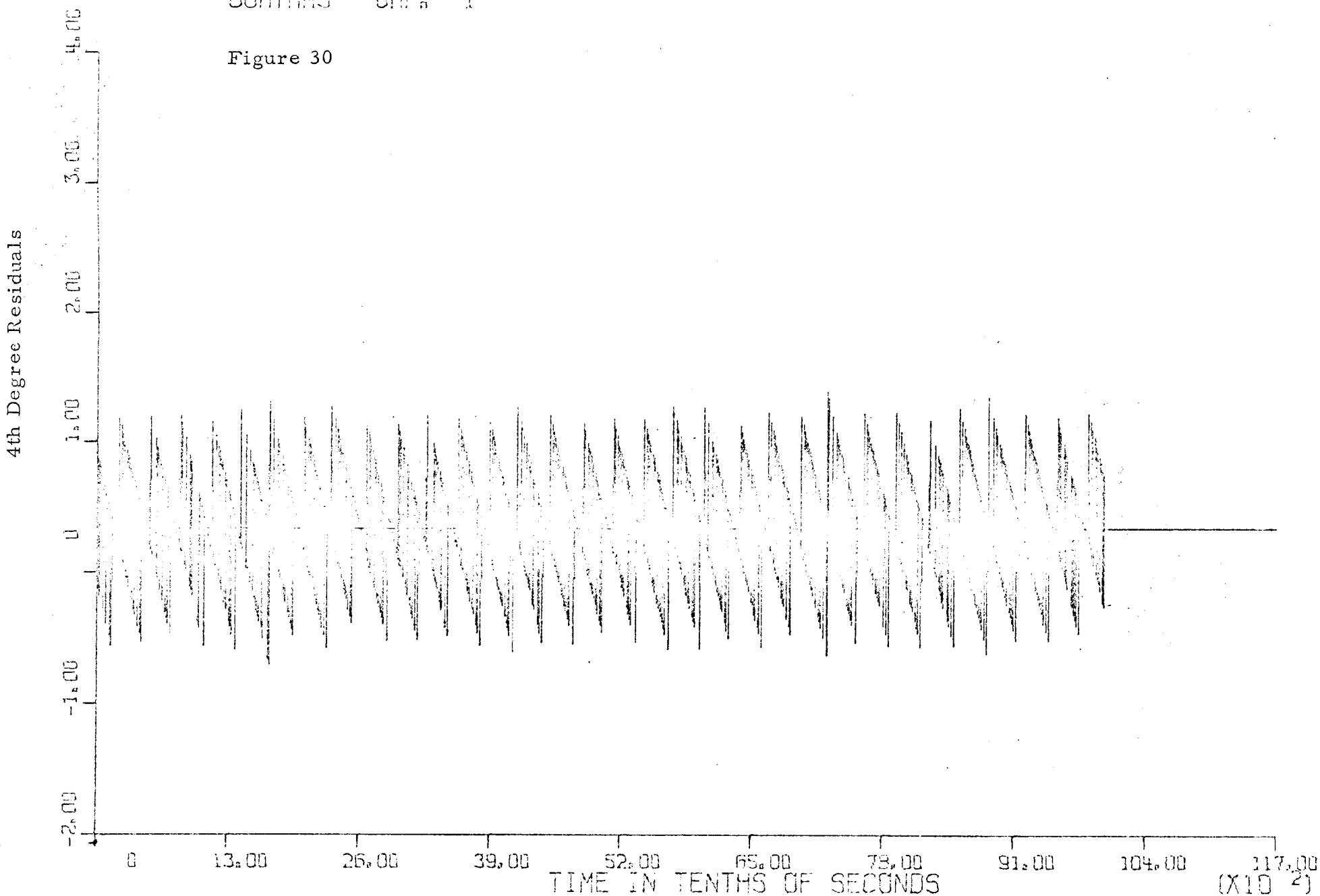

CARNAVON GRP. 4
4TH DEGREE RESIDUALS
4.00
3.00
2.00
1.00
0
-1.00
-2.00
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
TIME IN TENTHS OF SECONDS
(X10 2)

Figure 29

GUAYMAS GRP. 1

Figure 30



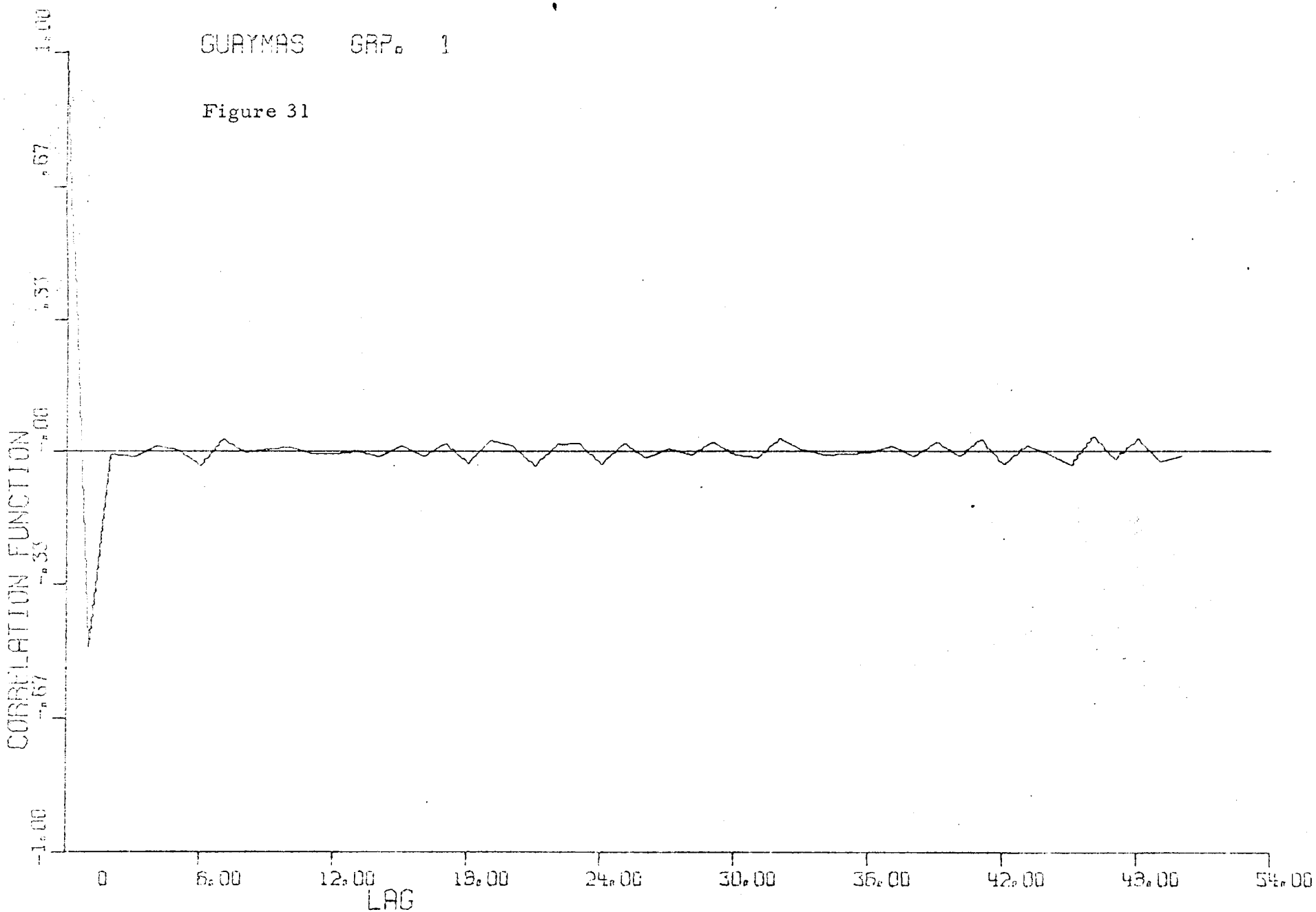

GUAYMAS GRP. 1
CORRELATION FUNCTION
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 31

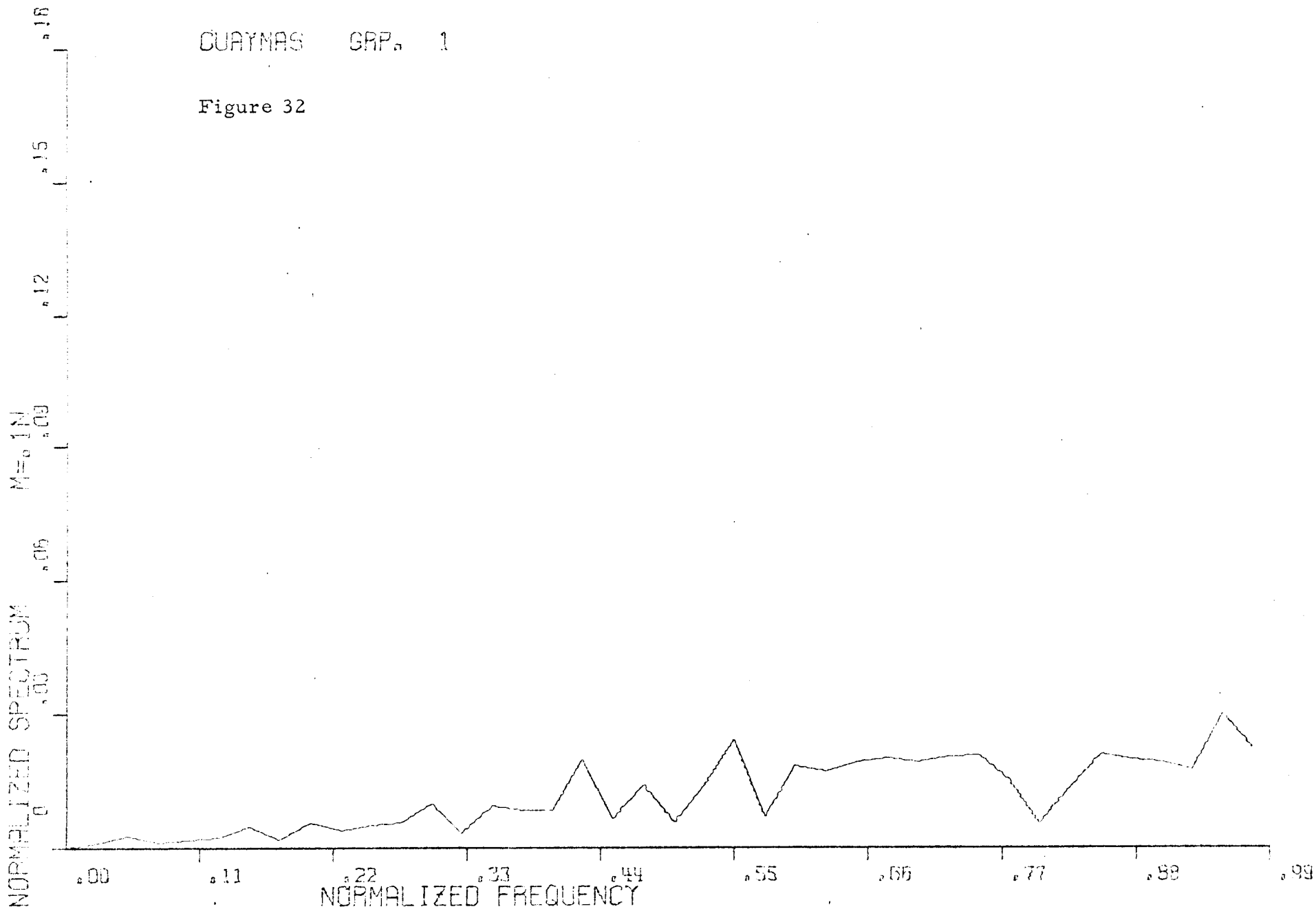

GUAYMAS GRP. 1
NORMALIZED SPECTRUM
M=.1N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 32

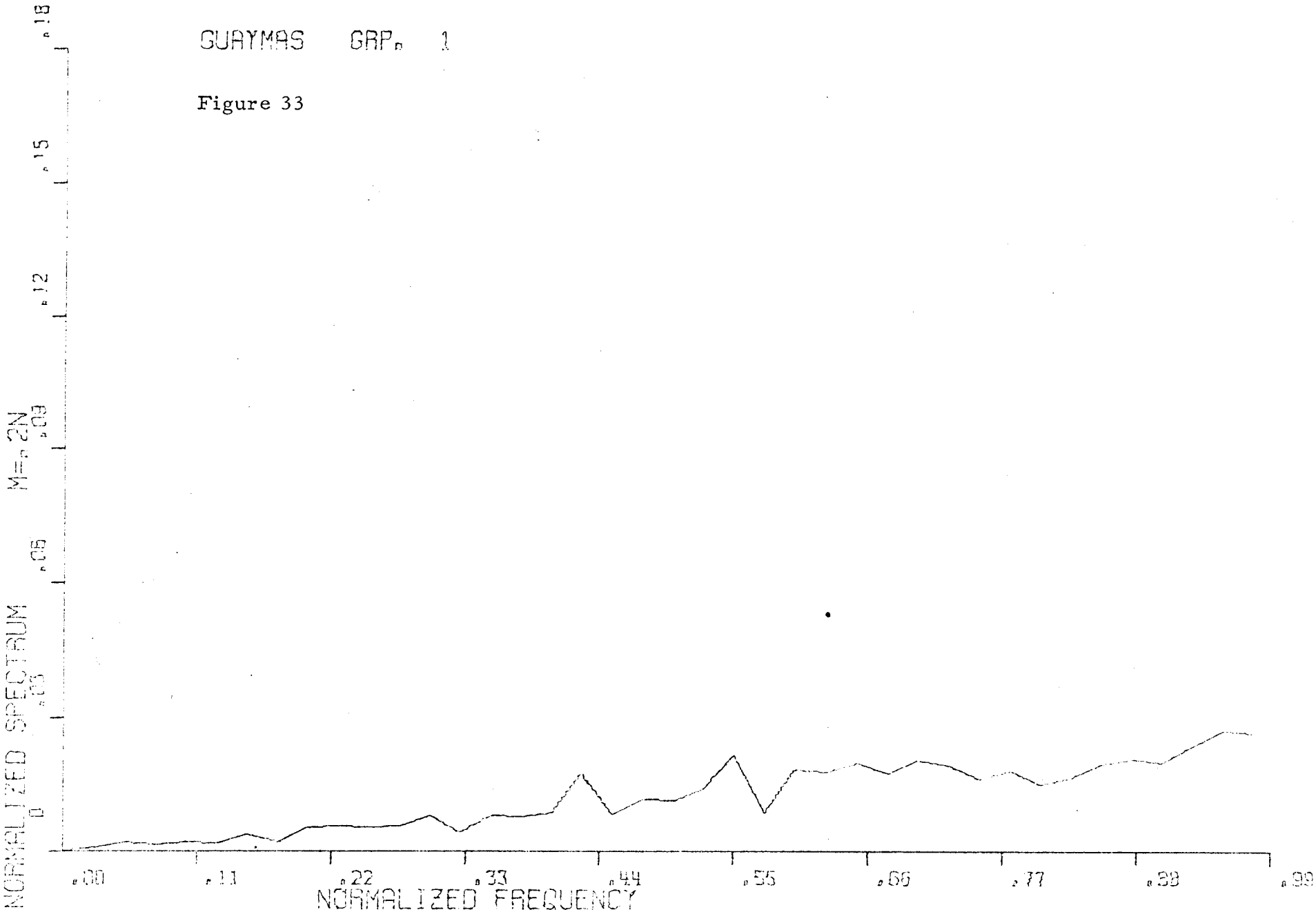
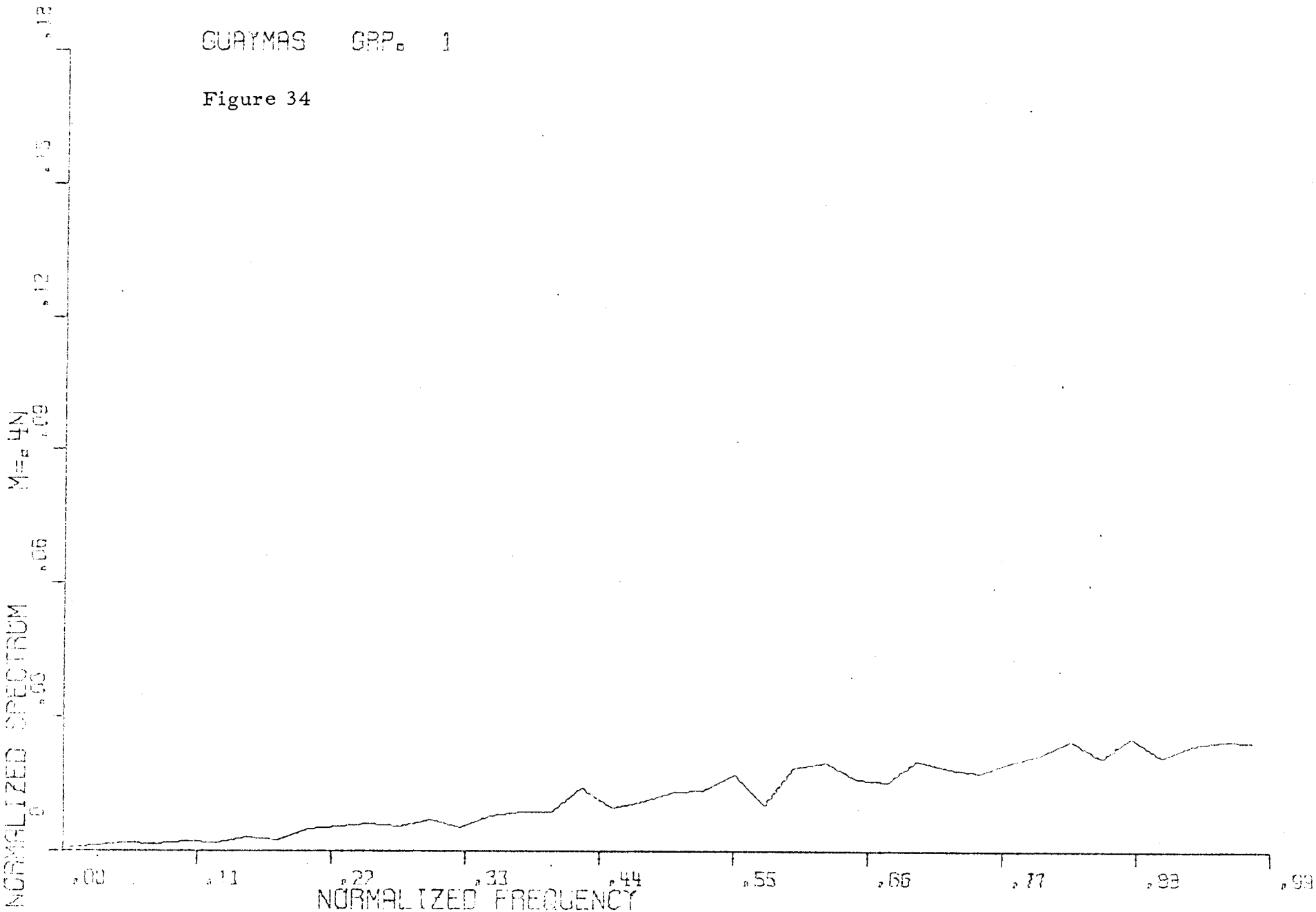

GUAYMAS GRP. 1
NORMALIZED SPECTRUM
M=.2N
.00
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 33

GUAYMAS GRP. 1

Figure 34



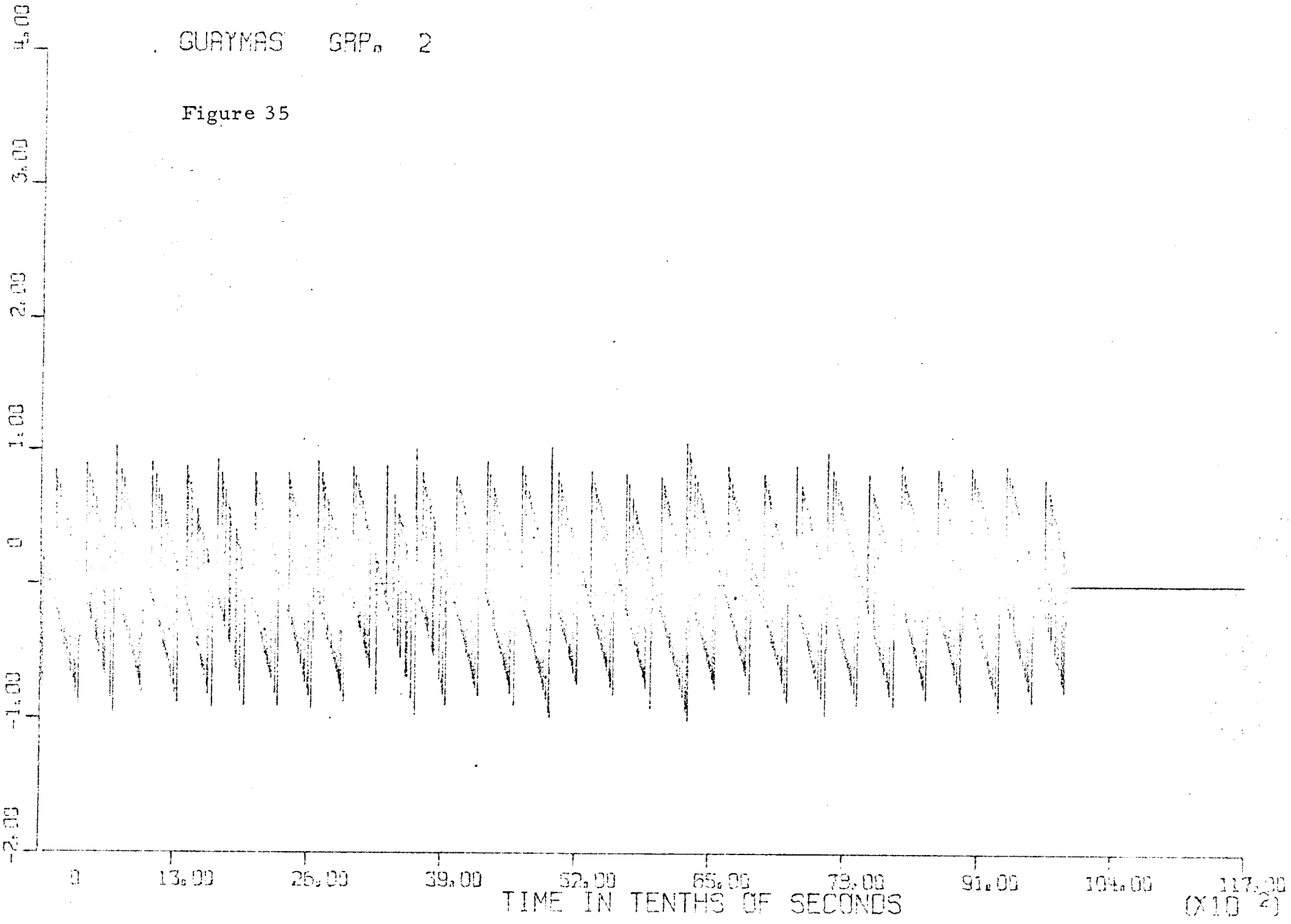

GUAYMAS GRP. 2
4th Degree Residuals
4.00
3.00
2.00
1.00
0
-1.00
-2.00
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
TIME IN TENTHS OF SECONDS
(X10 2)

Figure 35

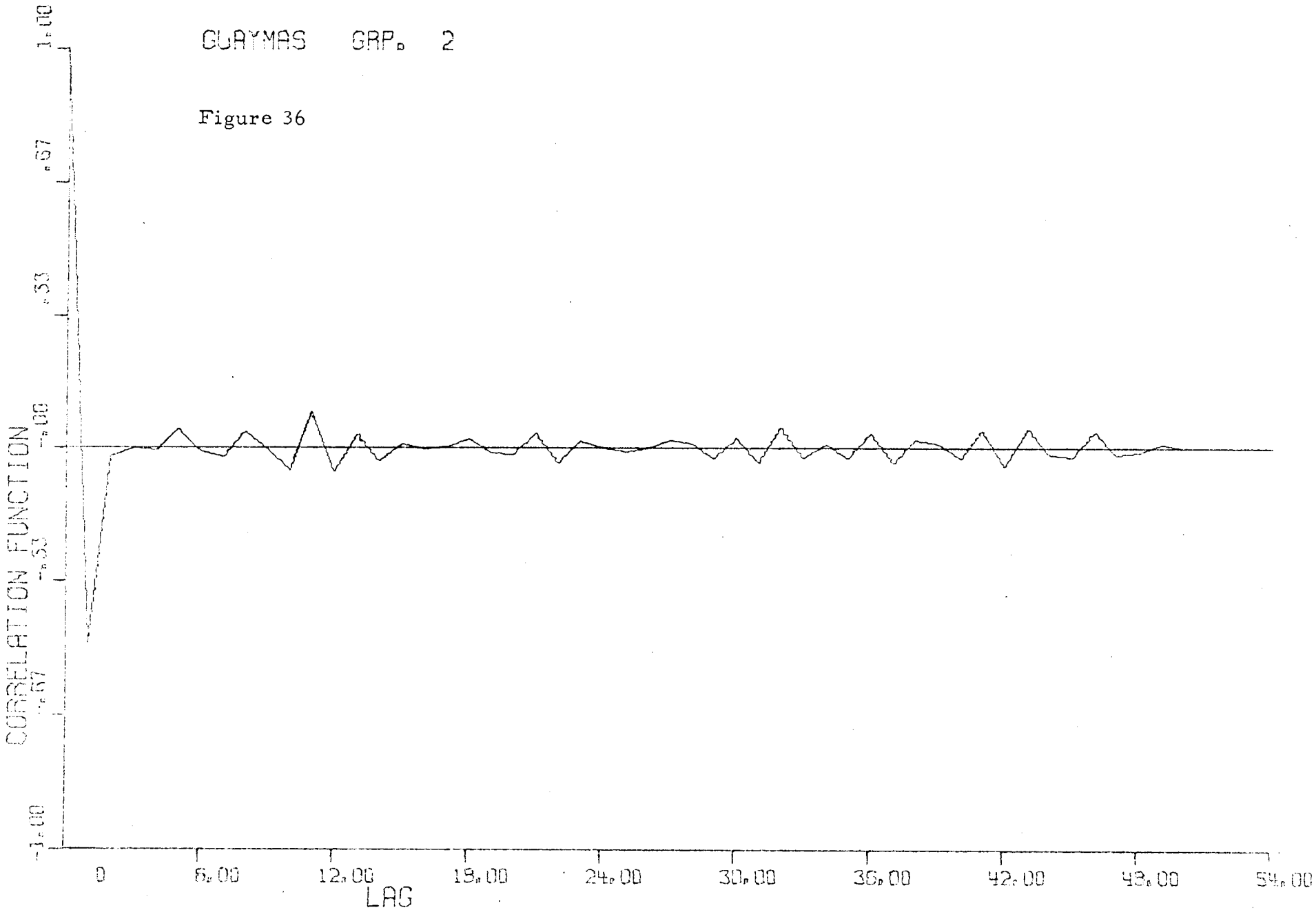
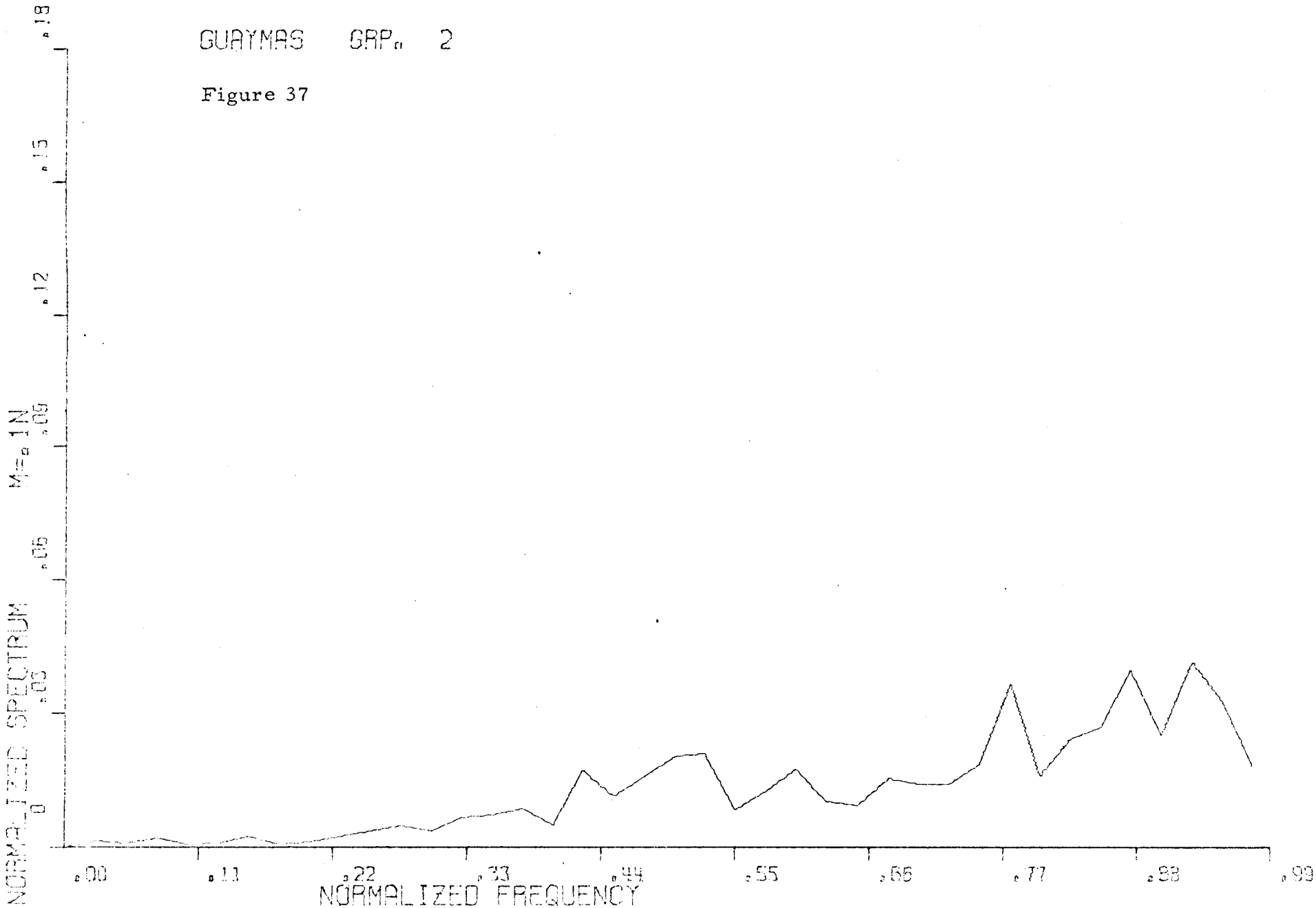

GUAYMAS GRP. 2
CORRELATION FUNCTION
1.00
.67
.33
0.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 36

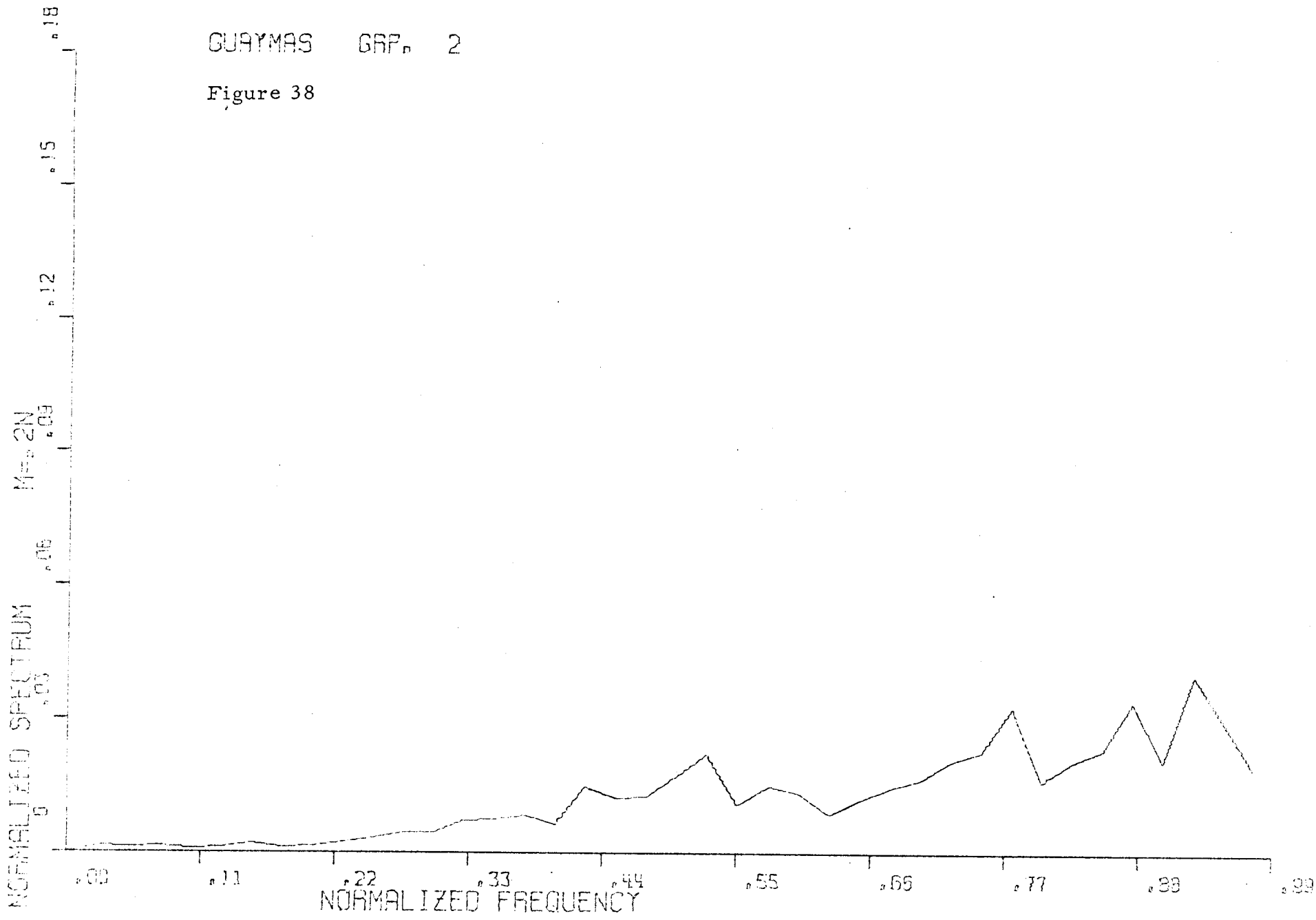
GUAYMAS GRP. 2

Figure 37



GUAYMAS GRP. 2

Figure 38



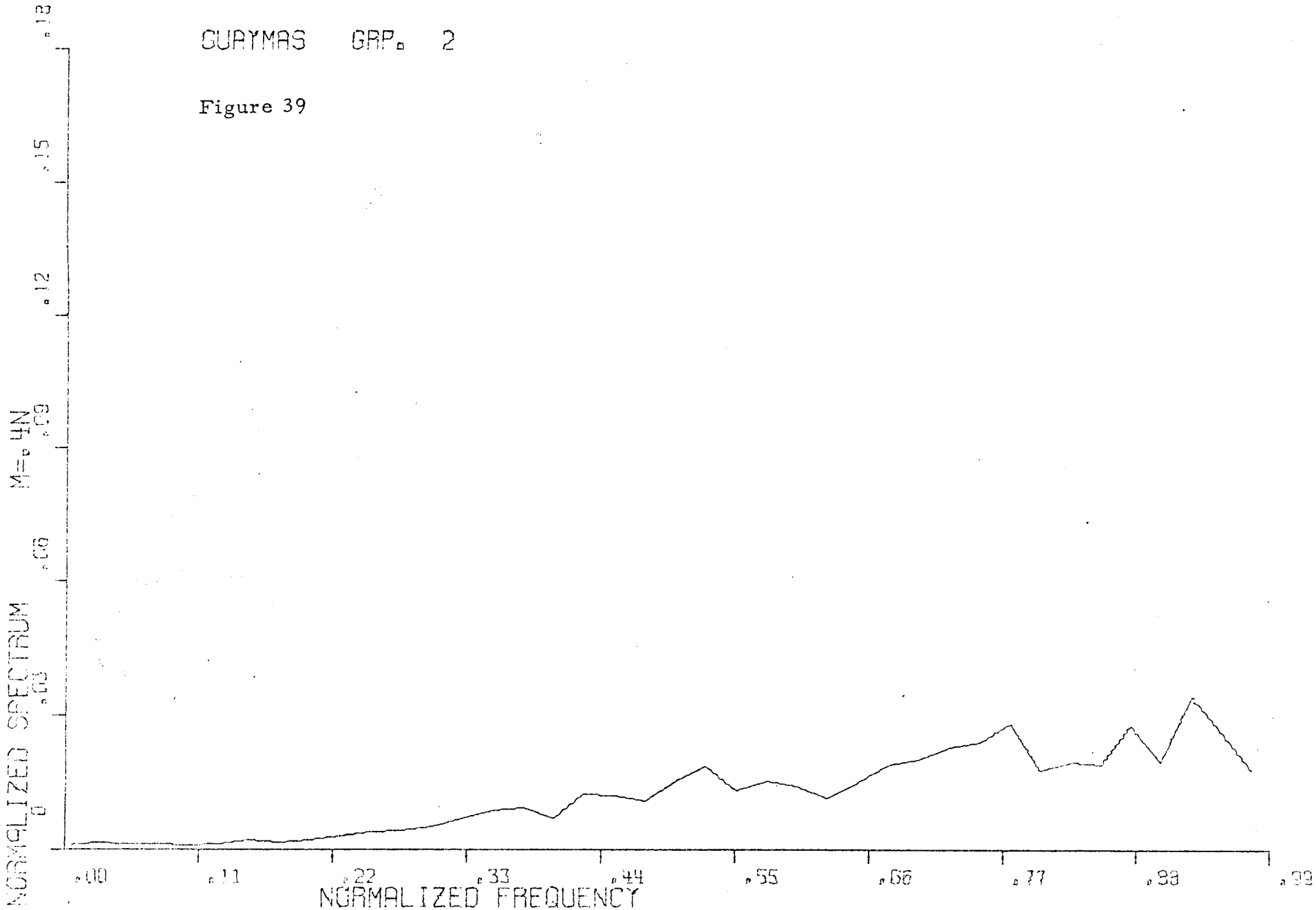
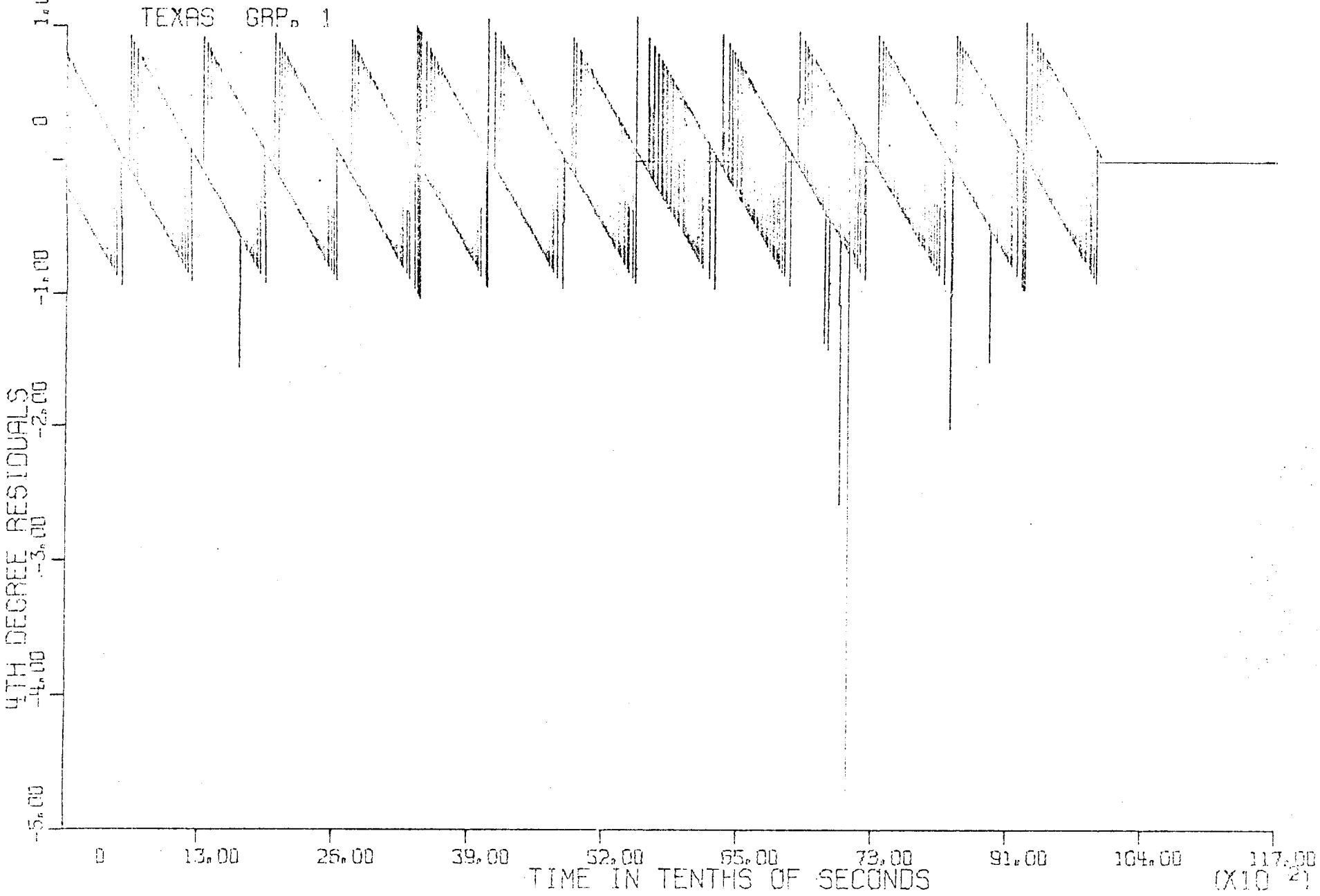

GUAYMAS GRP. 2
M= 4N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 39

Figure 40


TEXAS GRP. 1
1.00
0
-1.00
-2.00
-3.00
-4.00
-5.00
4TH DEGREE RESIDUALS
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
TIME IN TENTHS OF SECONDS
(X10 2)

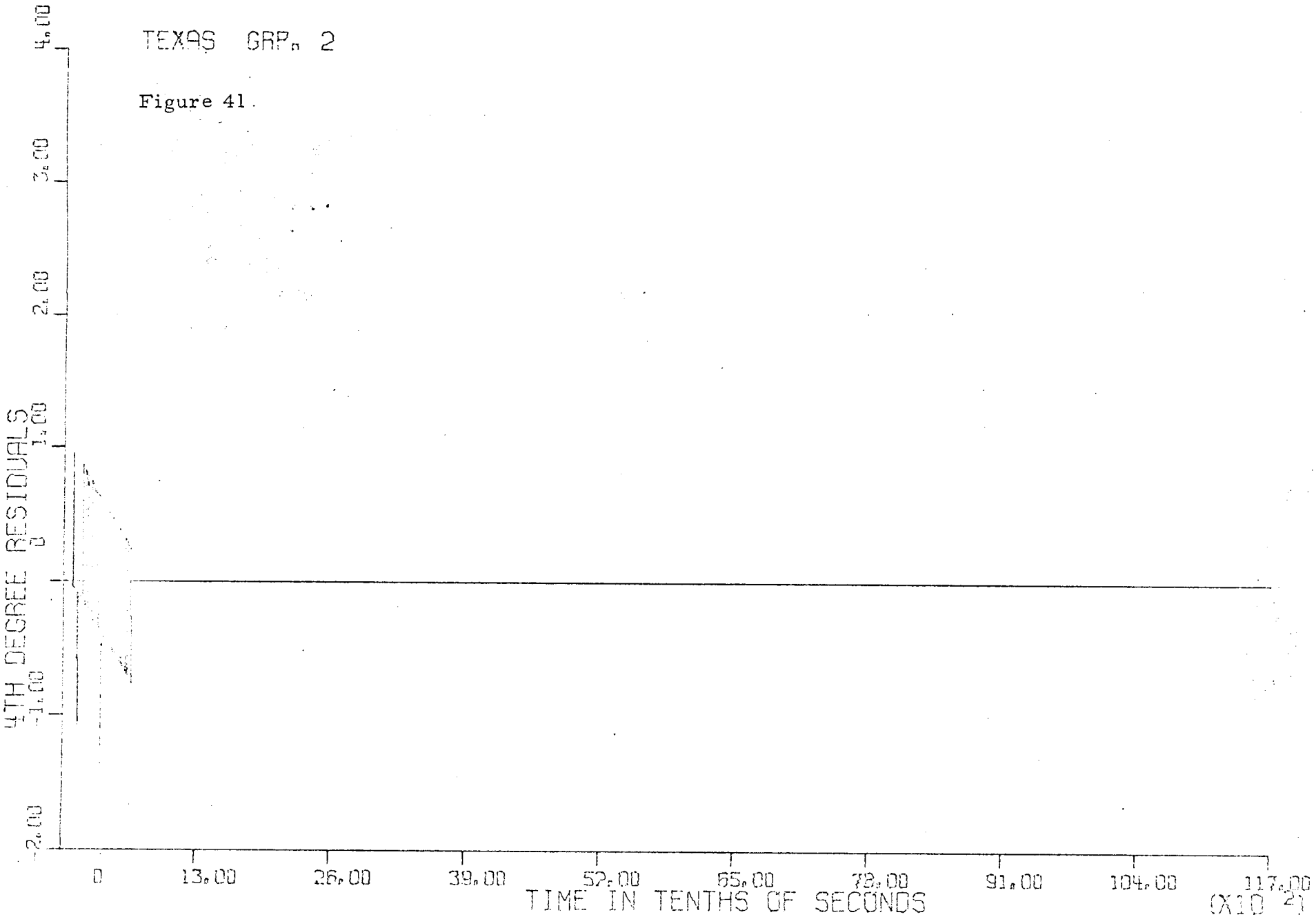

TEXAS GRP. 2
4TH DEGREE RESIDUALS
4.00
3.00
2.00
1.00
0
-1.00
-2.00
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
TIME IN TENTHS OF SECONDS
(X10 2)

Figure 41.

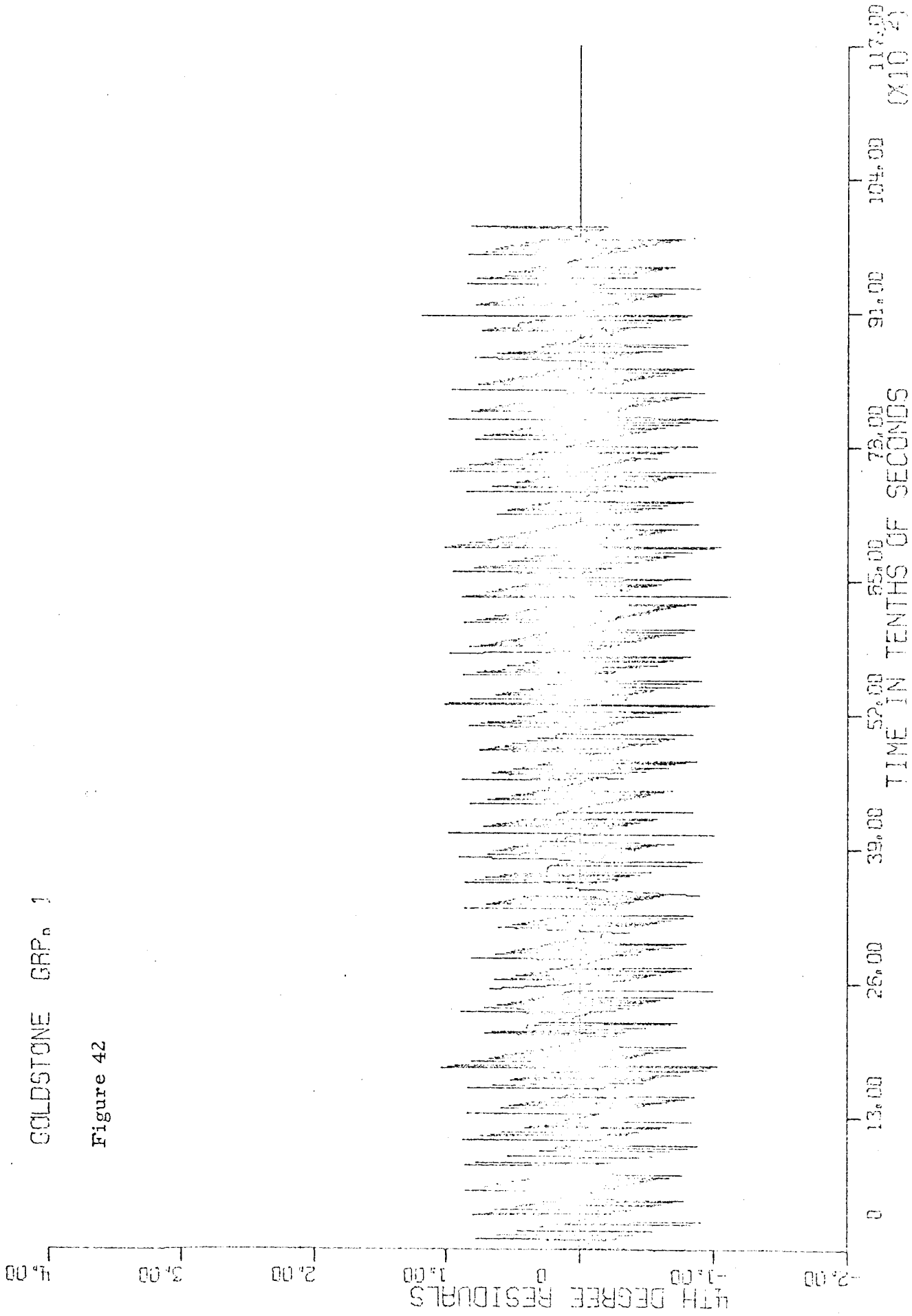
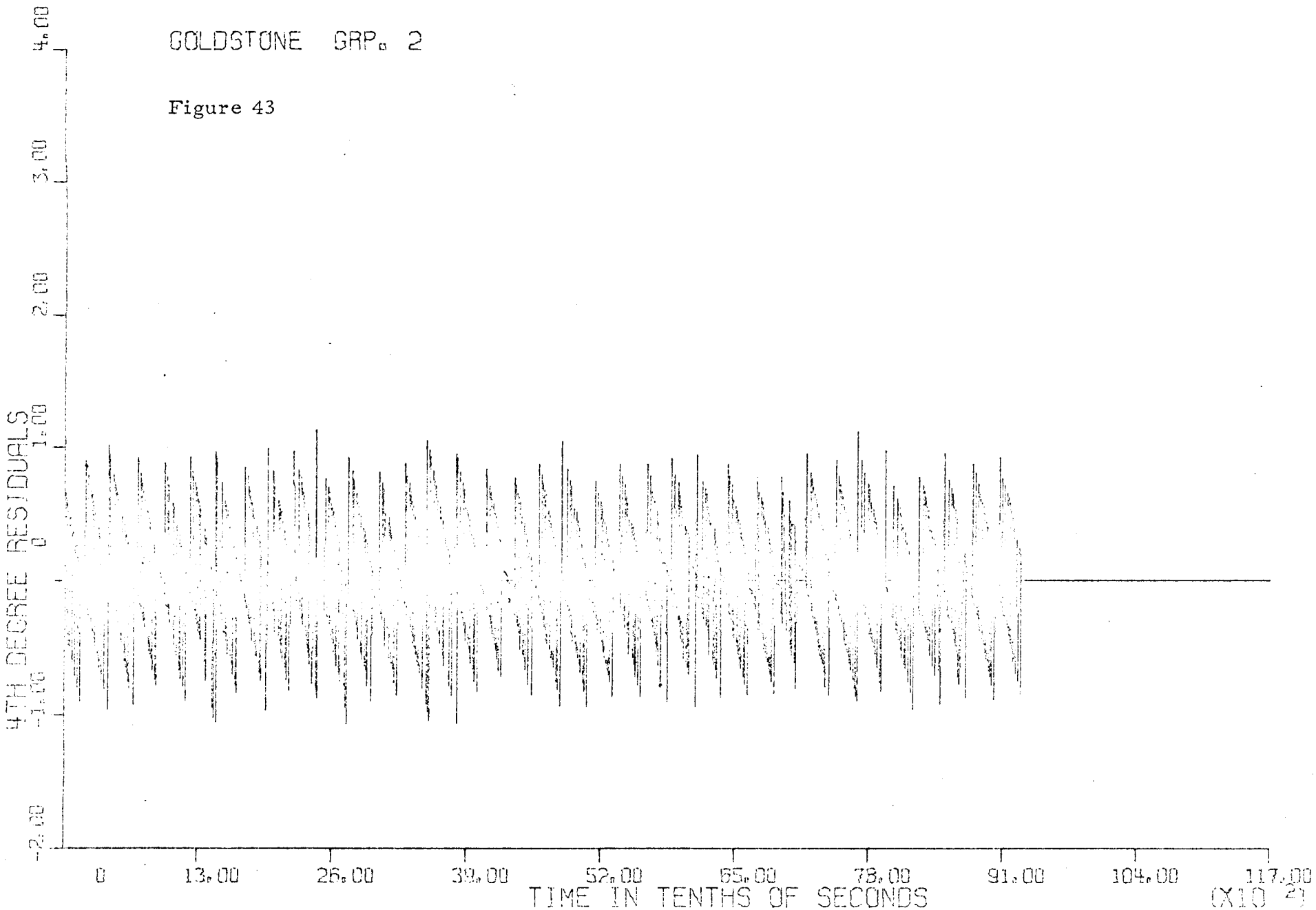

GOLDSTONE GRP. 1
4TH DEGREE RESIDUALS
-2.00
-1.00
0
1.00
2.00
3.00
4.00
TIME IN TENTHS OF SECONDS
0
13.00
26.00
39.00
52.00
65.00
78.00
91.00
104.00
117.00
(X10 2)

Figure 42

GOLDSTONE GRP. 2

Figure 43



The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. 500
(BBC Task 105)

H. Engel
19 July 1967

ANALYSES OF NASA TAPE 1028

This tape is a MCC-H communications processor log tape of high speed data from Hawaii, recorded in real time.

The tape contains 8543 messages, of which the last 8539 are of the correct length to be high speed data messages.

The input data sheet for the pre-edit program is shown as Figure 1. Note that the vehicle ID is 6, although for LLO it should be 7, according to the AS501 Tracking Data Format Control Book, Revision 2. According to that document, also, bit 24 of the message should be 1 to indicate rubidium and 0 to indicate crystal; it turns out this in this entire tape the frequency standard is rubidium but bit 24 is 0.

Figure 2 summarizes the results of the pre-edit program, and Figure 3 summarizes the edit program. The ratio of points accepted by the pre-edit program to points input to the pre-edit program is 0.996. The ratio of points accepted by the edit program to points input to the edit program is 0.9993.

The residuals of the first differences of the input data, after fitting by fourth degree polynomials, the correlation functions of these residuals, and the power spectral densities of these residuals are shown in Figures 4 through 23. The results are typical of quantization error. The group 4 residuals, however, display some anomalies about 360 seconds after the start of the group. Since there is no data available from other stations at this time, the source of the error can not be located.

INPUT DATA SHEET FOR PRE-EDIT PROGRAM

Punch: 1 for YES; 2 for NO; 9 for TEST TO BE IGNORED

		Yes	No	Test to be Ignored
1	Is this high speed 240-Bit data	✓		/////
2	Is this non-destruct data	✓		/////
3	Is this high data (Bit 15)	✓		
4	Is range-rate in standard position	✓		/////
5	Is range-rate N_1 mode			✓
	Only one of four applies:			
6	Is this one way doppler mode			/////
7	Is this two way doppler mode			/////
8	Is this multiple non-coherent mode	✓		/////
9	Is this multiple coherent mode			/////
10	Vehicle ID is [6]	///	///	
11	Is frequency standard rubidium		✓	
12	Is manual R-R test to be made	✓		/////
13	Is VCO lock test to be made		✓	/////
14	Is automatic range-rate test to be made	✓		/////
15	Is test to be made on real/test Bit	✓		/////
16	Is station ID test to be made	✓		/////
17	Is doppler mode test to be made	✓		/////
18	Is test to be made on R-R field indicator	✓		/////

MSC Tape No.

1 0 2 8

BBC Tape No.

1 9 8 In

2 5 Out

STATION ID

20		Bermuda
21		Merritt Island
22		Grand Bahama Island
23		Antigua
24		Carnarvon
25	✓	Hawaii
26		Guaymas
27		Texas
28		Guam
29		Goldstone
30		Ascension
31		Canberra
32		GSFC
33		Madrid
34		Grand Canary Islands

EXPECTED START TIME

35-37	1 2 6	Day (if greater than 31 month will be ignored)
38-39		Month
40-41	6 7	Year
42-43	0 2	Hour
44-45	0 0	Minute
46-48	0 4	Expected Delta Time
49-51	1 8 0	Maximum Time Interval (in minutes)
52-55	5 0	Print Rejected Data
56-59	5 0	Print Raw Data
60-68		Start Time

Figure 1

PRE-EDIT PROGRAM SUMMARY

1. Station Hawaii
2. Station ID (decimal 9

MSC Tape No. 1028

BBC Tape No. 195 In
25 Out

Number	Rejects[1]	
	1	Station ID
	2	Destruct/Non Destruct
	3	Data Rate
	4	Real Test Data
	5	Vehicle ID
	6	Doppler Mode
	7	Frequency Standard
	8	Range Rate Field Indicator
	9	Gross Time
	10	Time Interval
*	11	VCO Lock
6	12	Auto Range Rate Quality
34	13	Manual Range Rate Quality
	14	Gross Range Rate Test[2]
*	15	Destruct Count N_1/N_2 Indicator[2]
40	Total Rejects	
8499	Total Good Data	
8539	Total Input Point	

[1] *Indicates that test was not made

[2] These tests not performed with non-destruct count doppler data.

Figure 2

EDIT PROGRAM SUMMARY

1.	Station	Hawaii	MSC Tape No.	1028
2.	Station ID (decimal)	9		
3.	Sample Interval (seconds)	0.4		
4.	Count Threshold			
5.	K σ threshold	6		
6.	Start Time (seconds)	10,896,090.4		
7.	Group Interval (seconds)	1,000	BBC Tape No.	25 In 81 Out

Group Number	Time (seconds from beginning of Calendar Year)		
	Input Start	First Good Data Point	Last Good Data Point
1	10,896,090.4	10,896,090.4	10,897,090.0
2	10,897,090.4	10,897,090.4	10,898,090.0
3	10,898,090.4	10,898,120.8	10,899,090.0
4	10,899,090.4	10,899,090.4	10,899,521.6
5			
6			
7			
8			
9			
10			

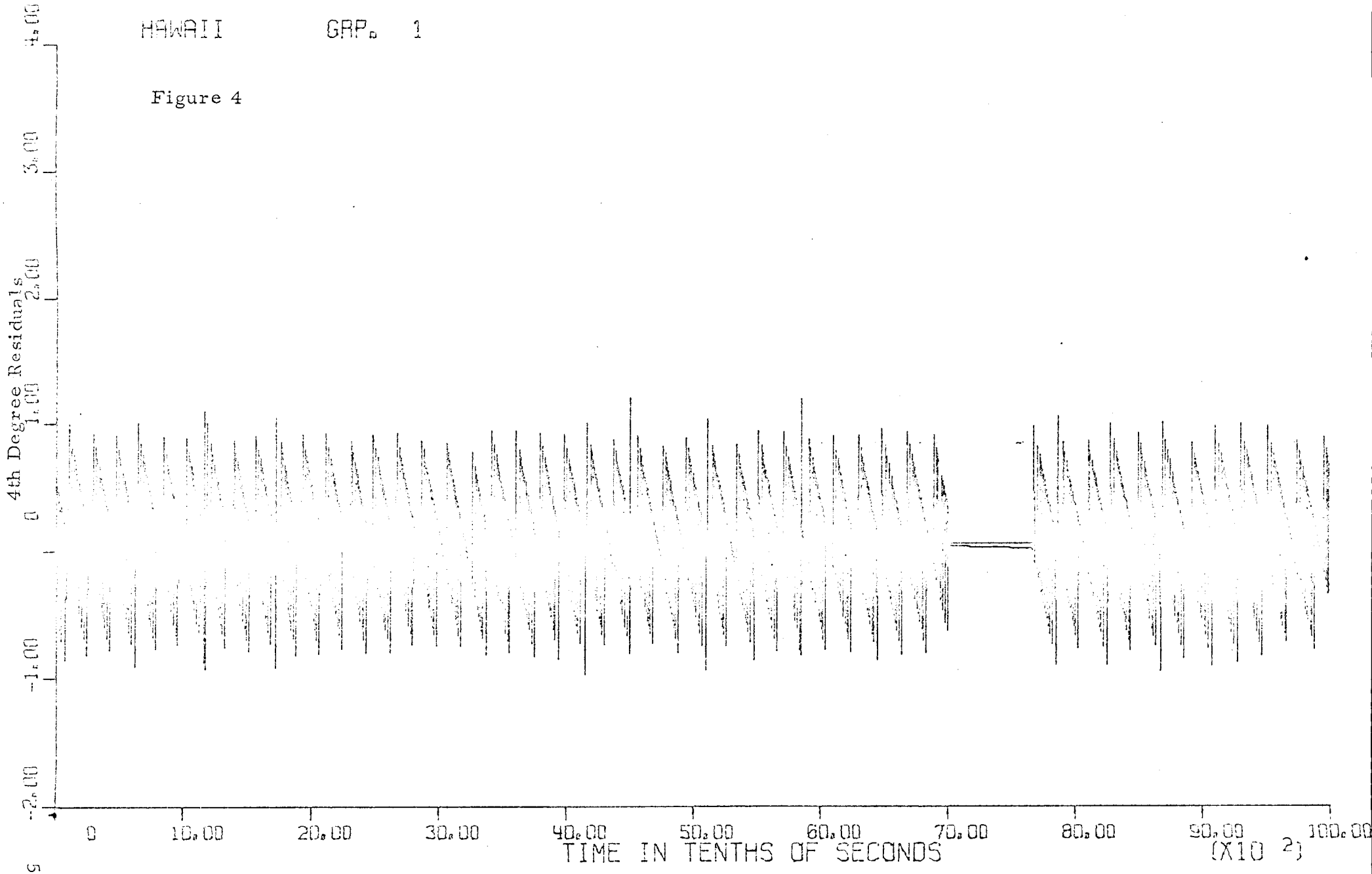
Group Number	Number of Rejects				Good Data Points	Total Input Points
	Fcount Threshold	Kσ Threshold	No formed 2nd Diff. *	Total Rejects		
1	1	4		5	2320	2325
2	0	1		1	2499	2500
3	0	0		0	2422	2422
4	0	0		0	1079	1079
5						
6						
7						
8						
9						
10						
Total	1	5		6	8320	8326

*When no number is indicated in this column the points rejected for Kσ and those which no second difference could be formed have been combined in the Kσ column.

Figure 3

HAWAII GRP. 1

Figure 4



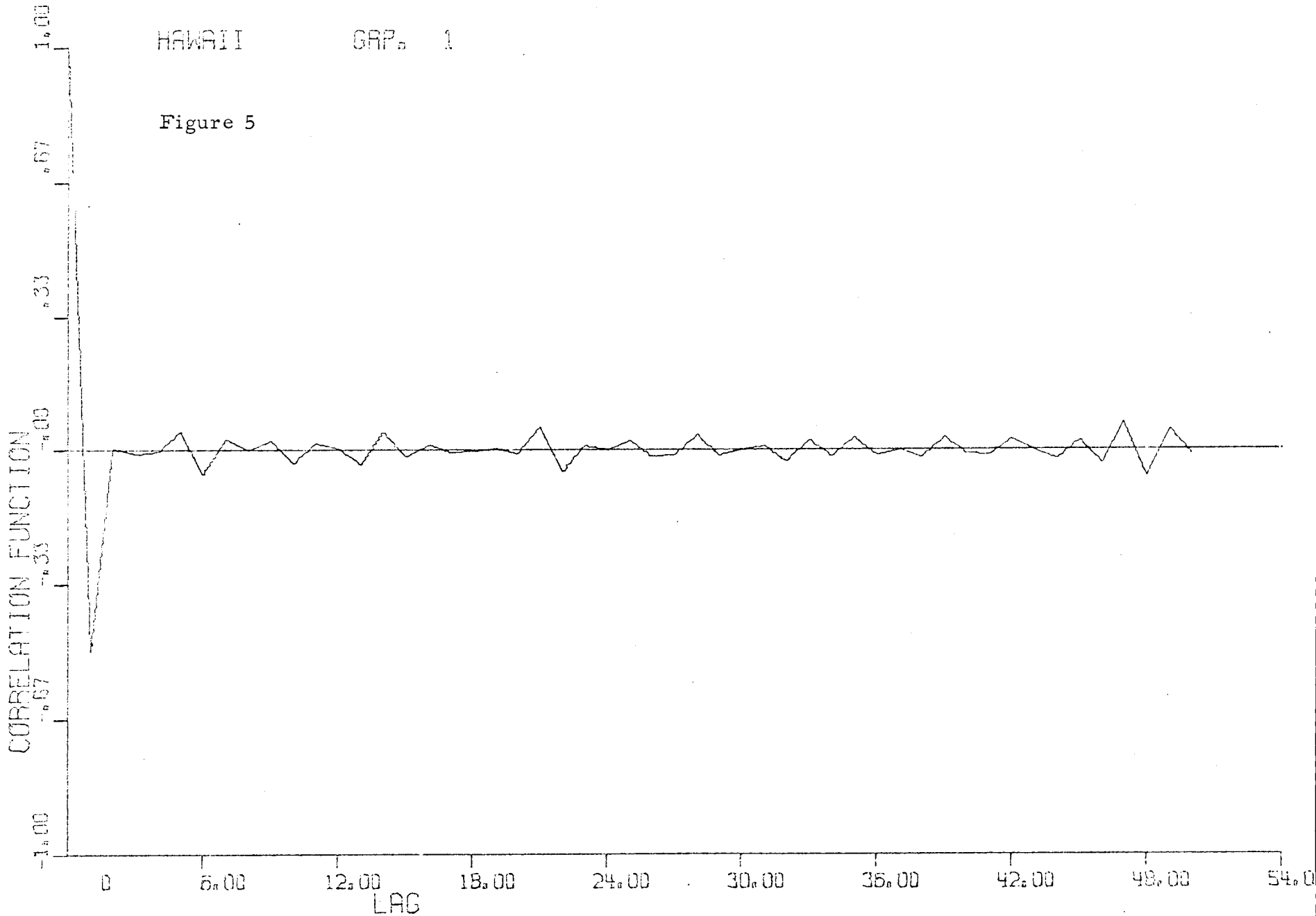
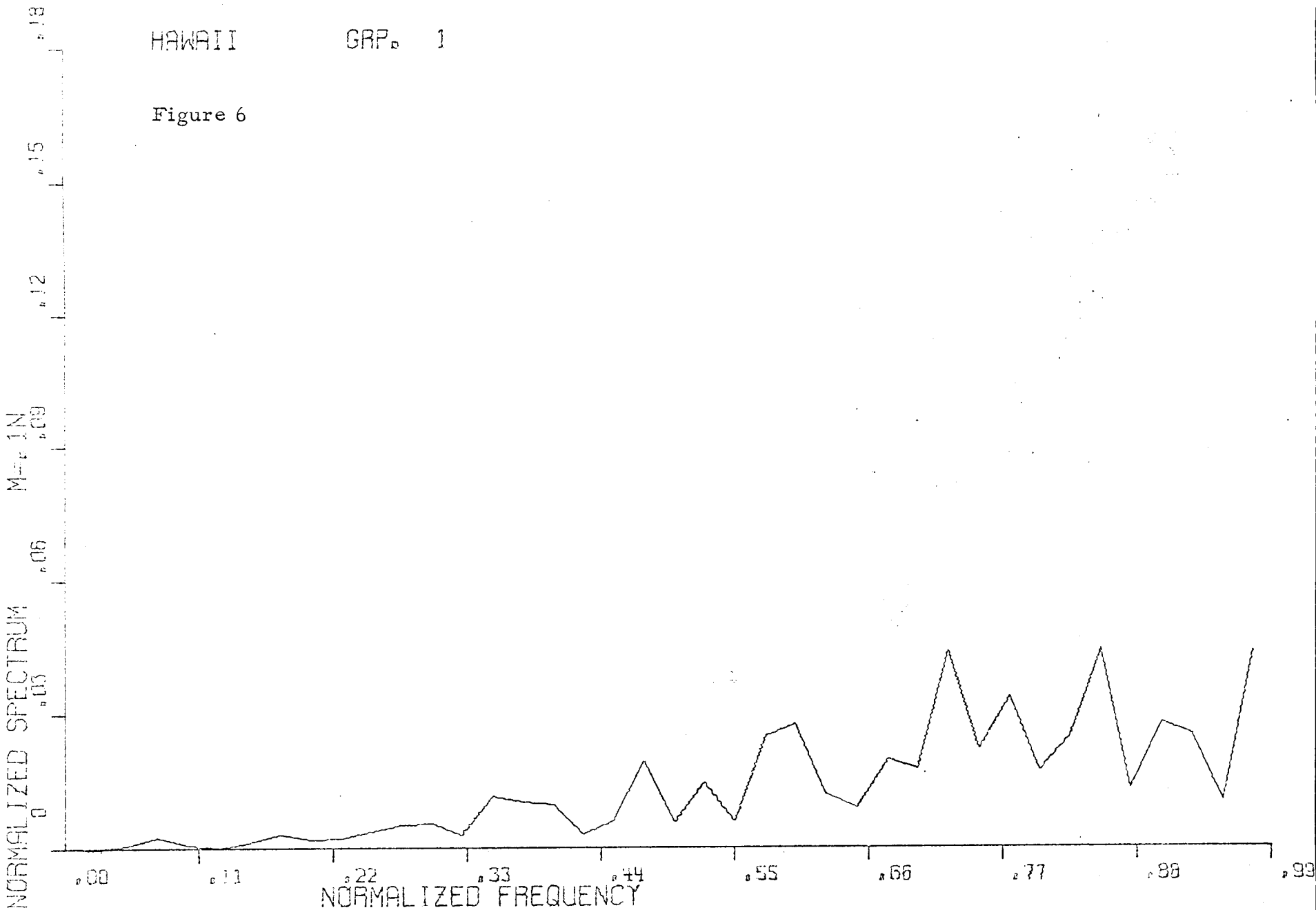

HAWAII GRP. 1
CORRELATION FUNCTION
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 5

HAWAII GRP. 1

Figure 6



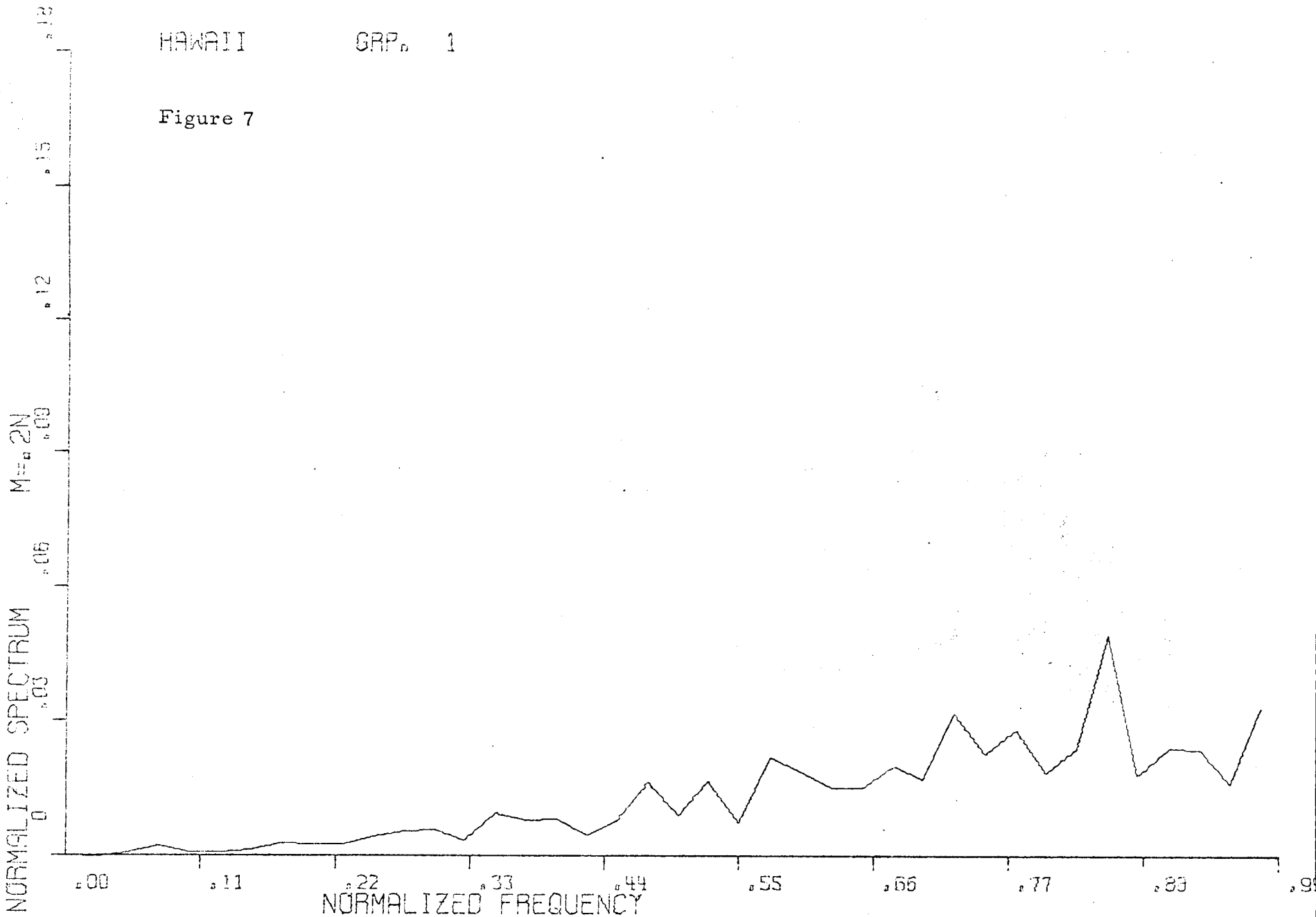
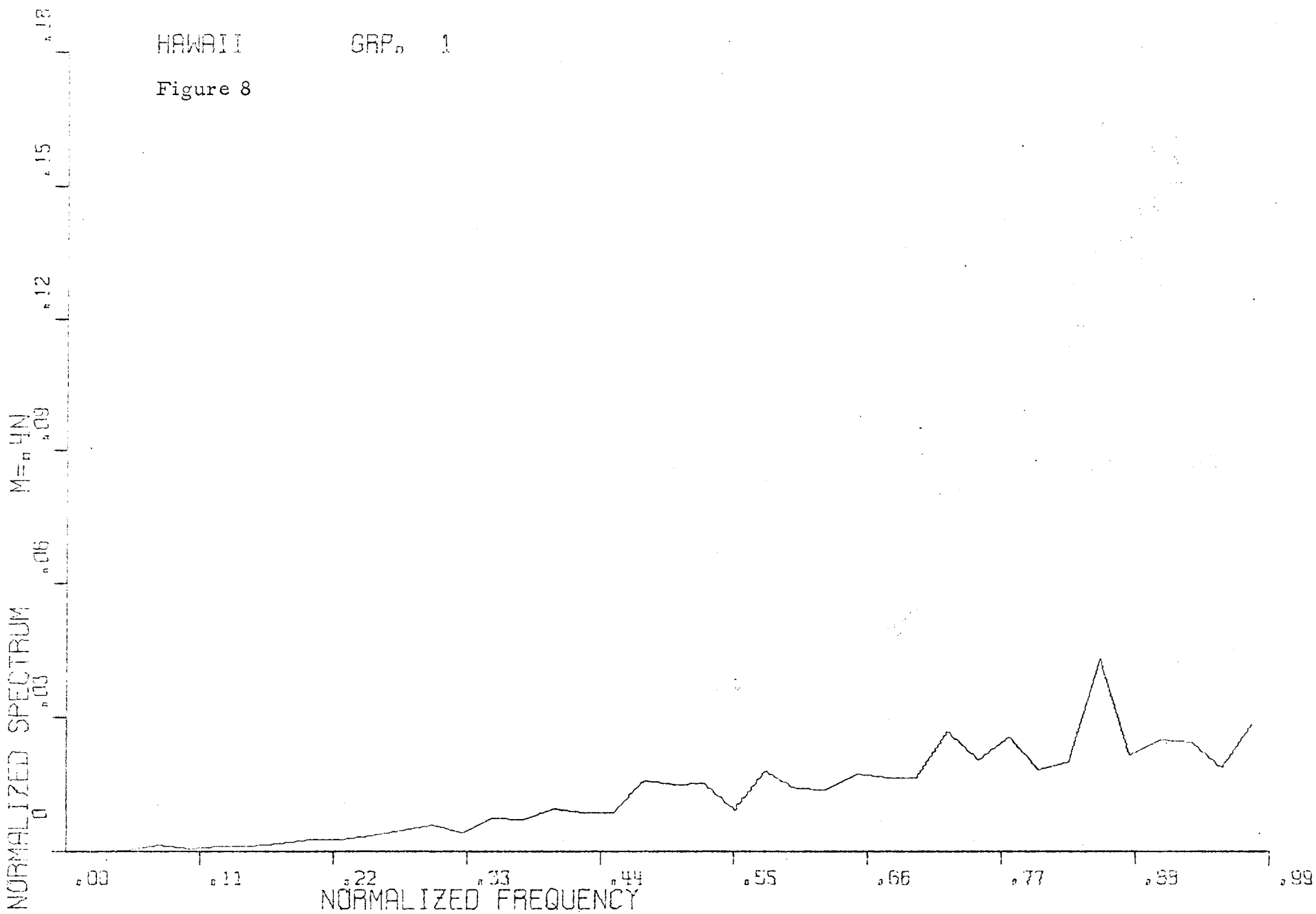

HAWAII GRP. 1
NORMALIZED SPECTRUM
M=. 2N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 7

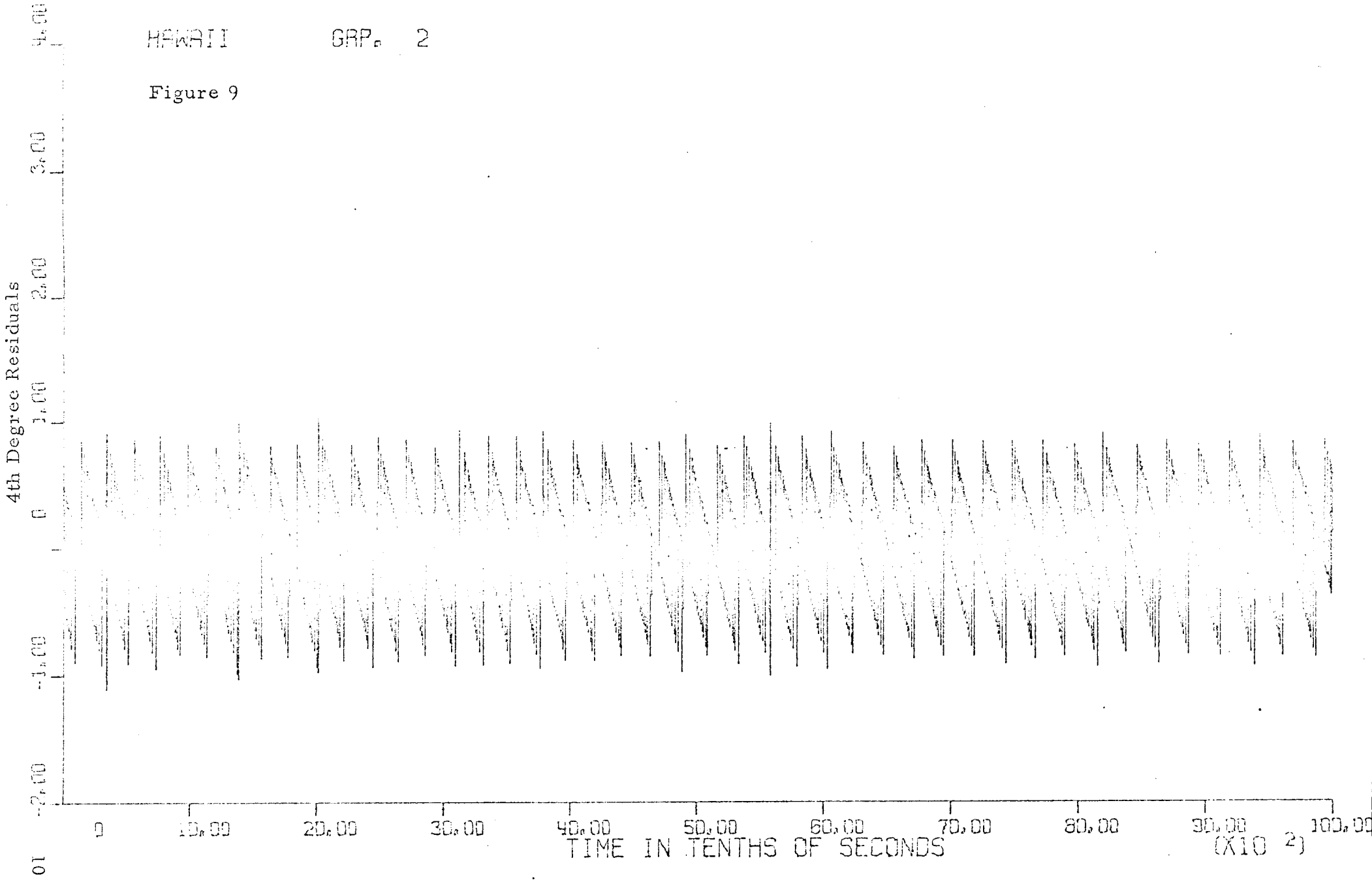
HAWAII GRP. 1

Figure 8



HAWAII GRP. 2

Figure 9



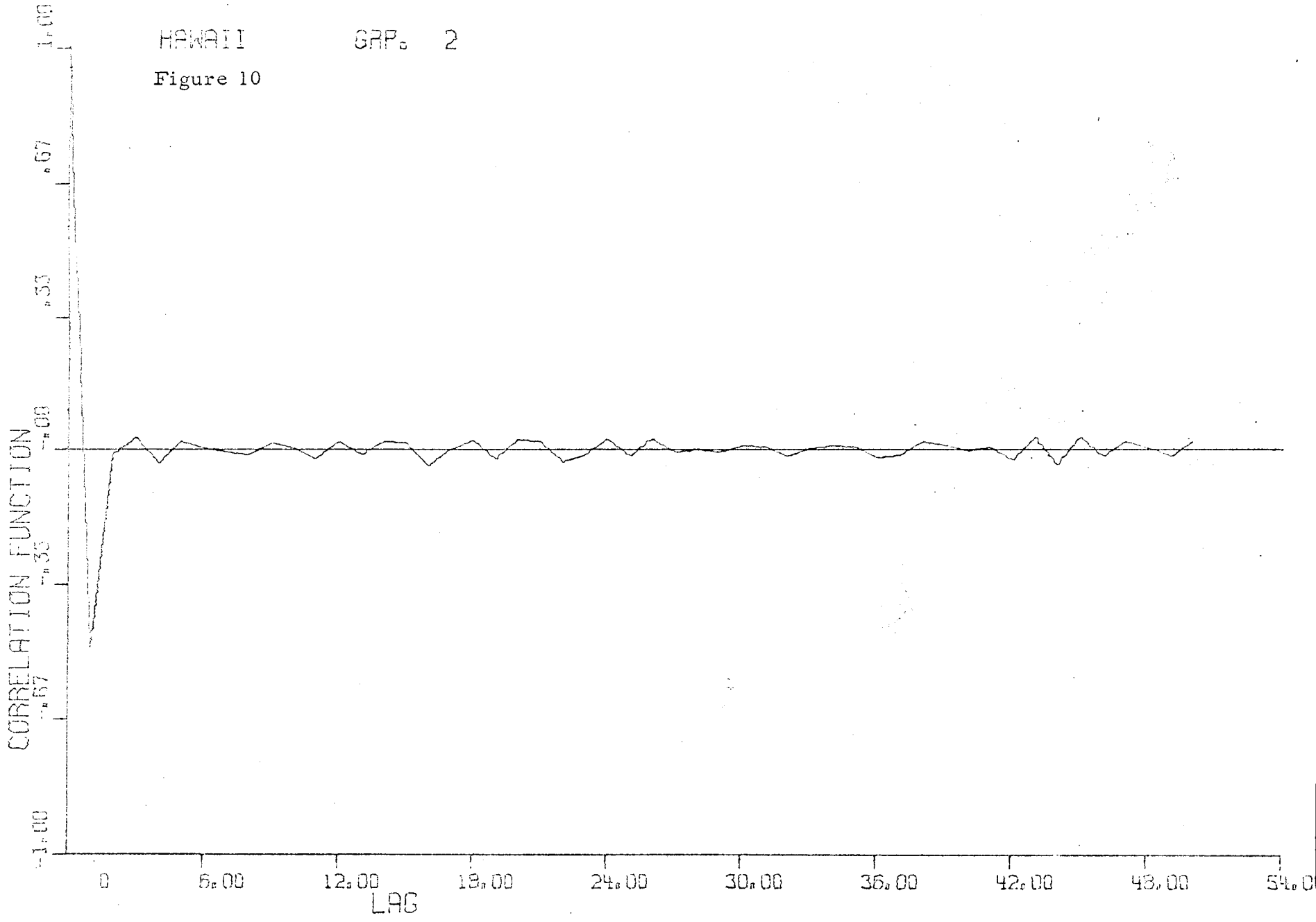

HAWAII GRP. 2
CORRELATION FUNCTION
1.00
.67
.33
.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 10

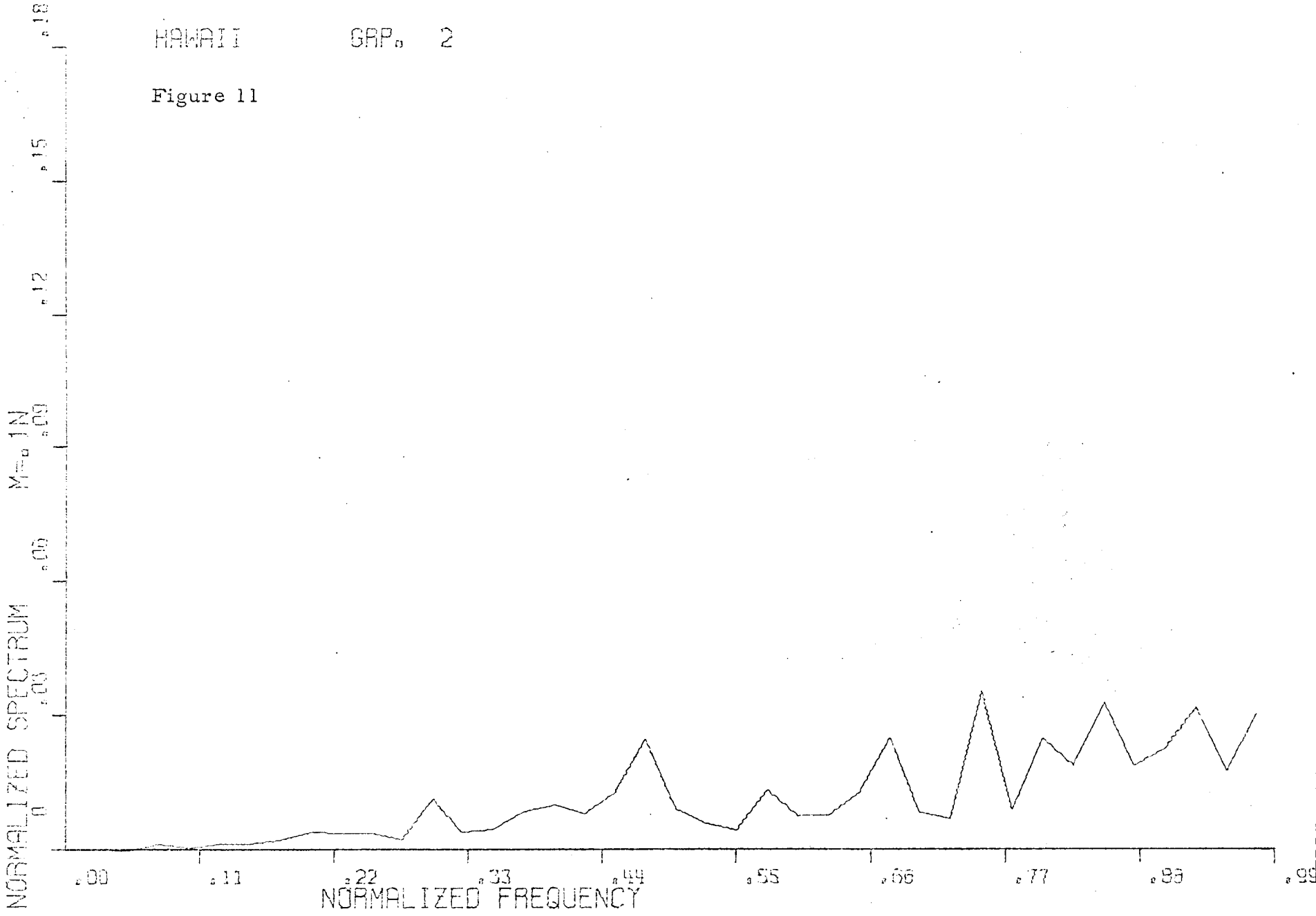

HAWAII GAP. 2
NORMALIZED SPECTRUM
M=. 1N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 11

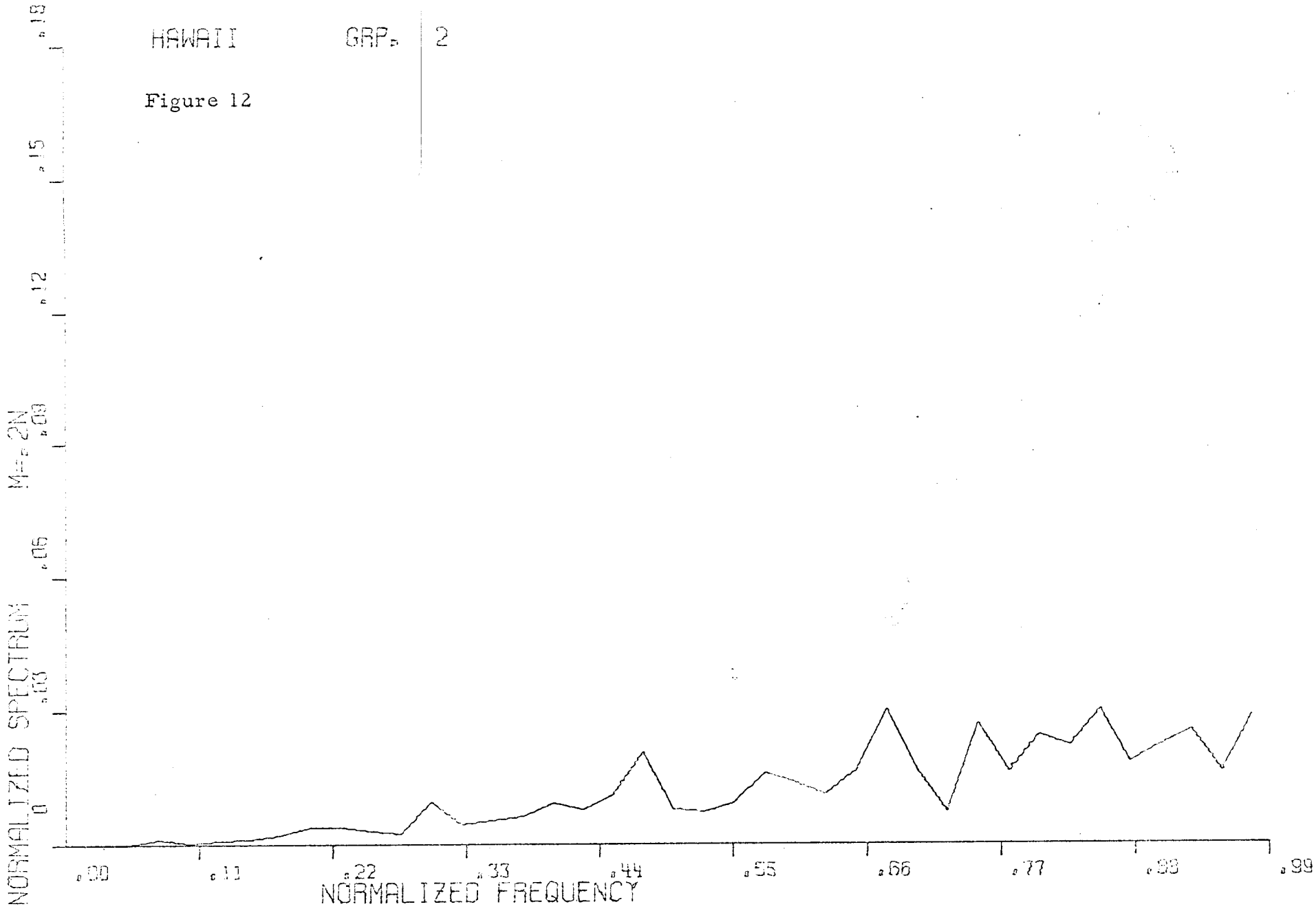
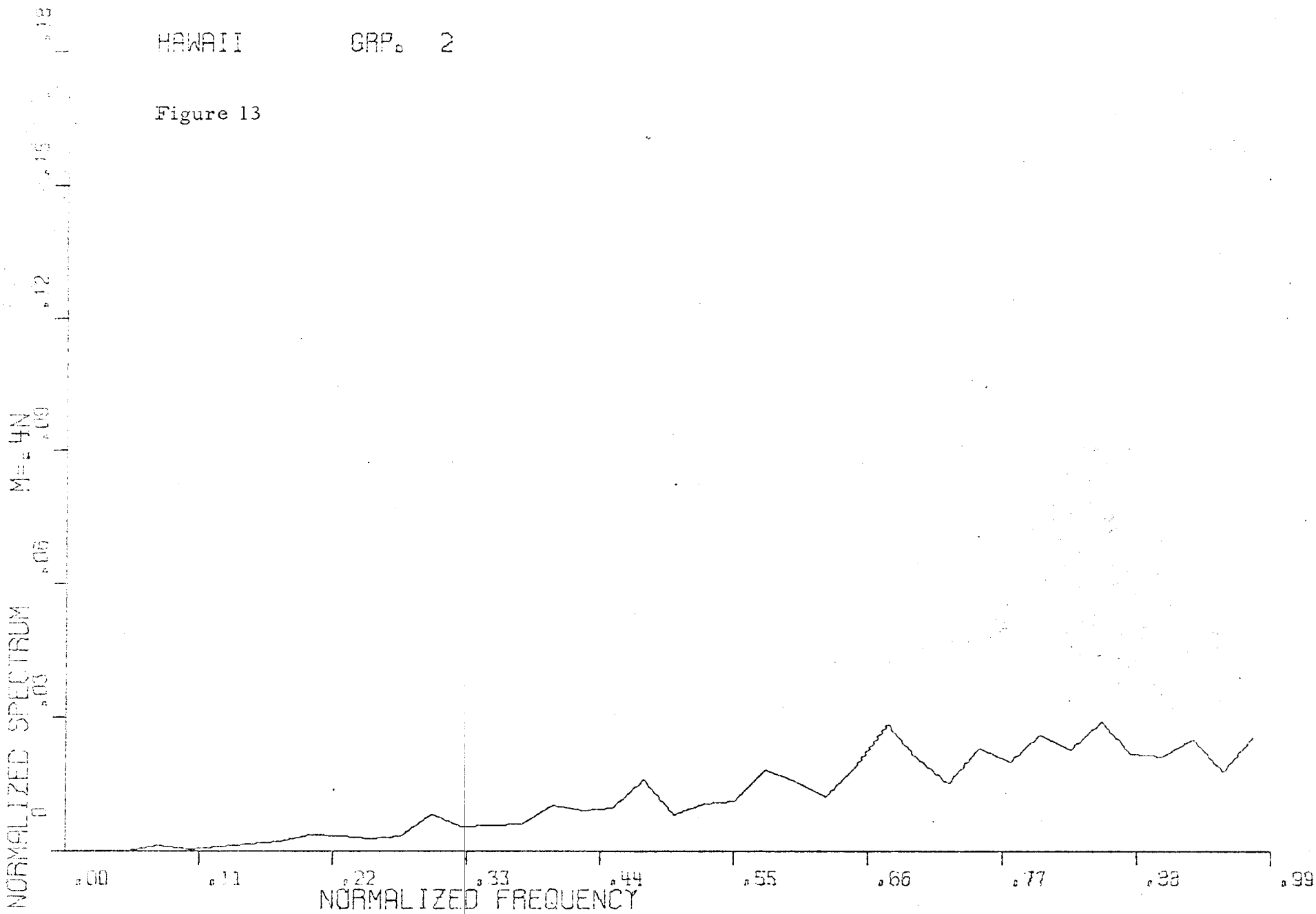

HAWAII
GRP. 2
NORMALIZED SPECTRUM
M=.2N
.18
.15
.12
.09
.06
.03
0
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 12

HAWAII GRP. 2

Figure 13



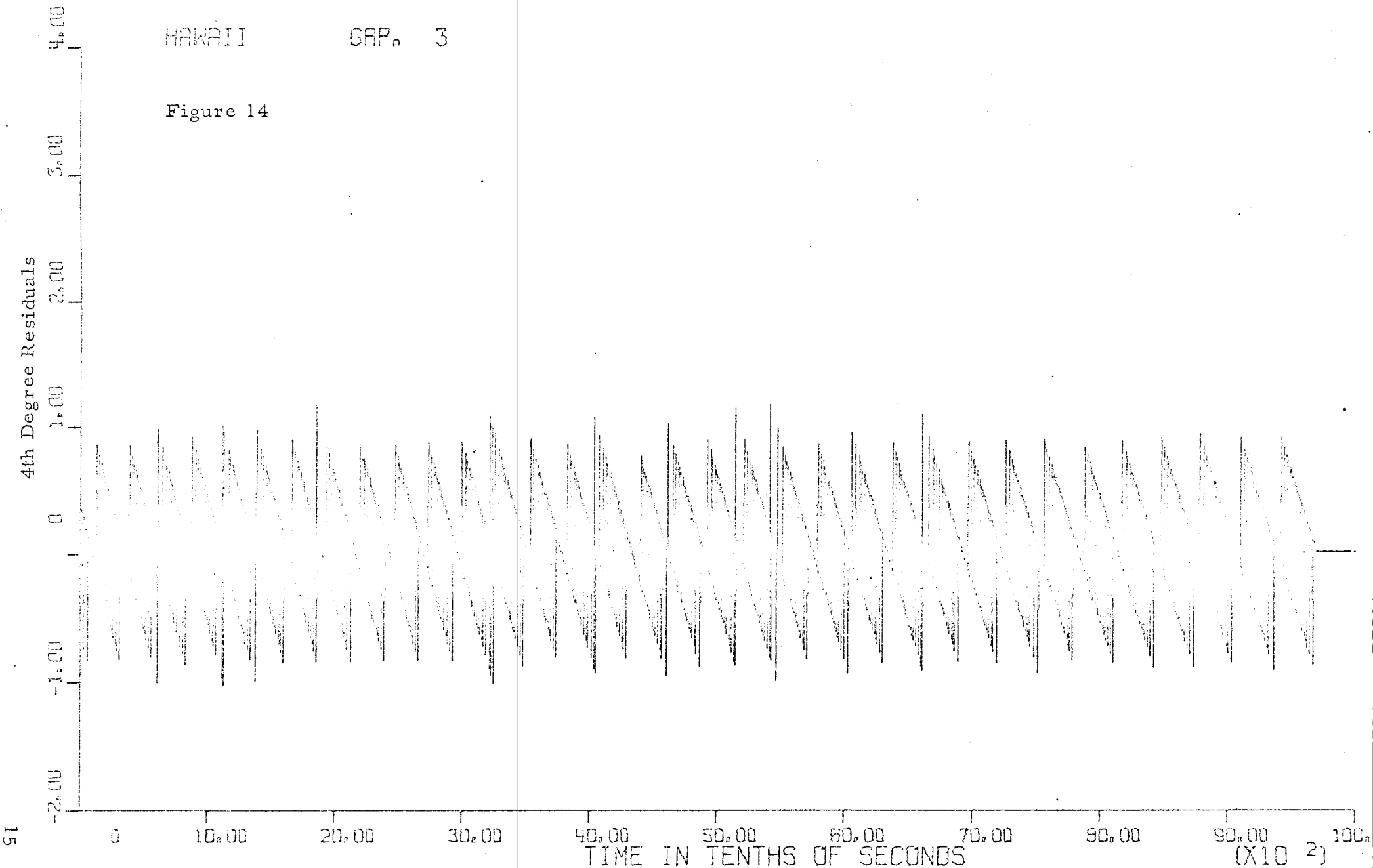

HAWAII
GRP. 3
4th Degree Residuals
4.00
3.00
2.00
1.00
0
-1.00
-2.00
0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.
TIME IN TENTHS OF SECONDS
(X10 2)

Figure 14

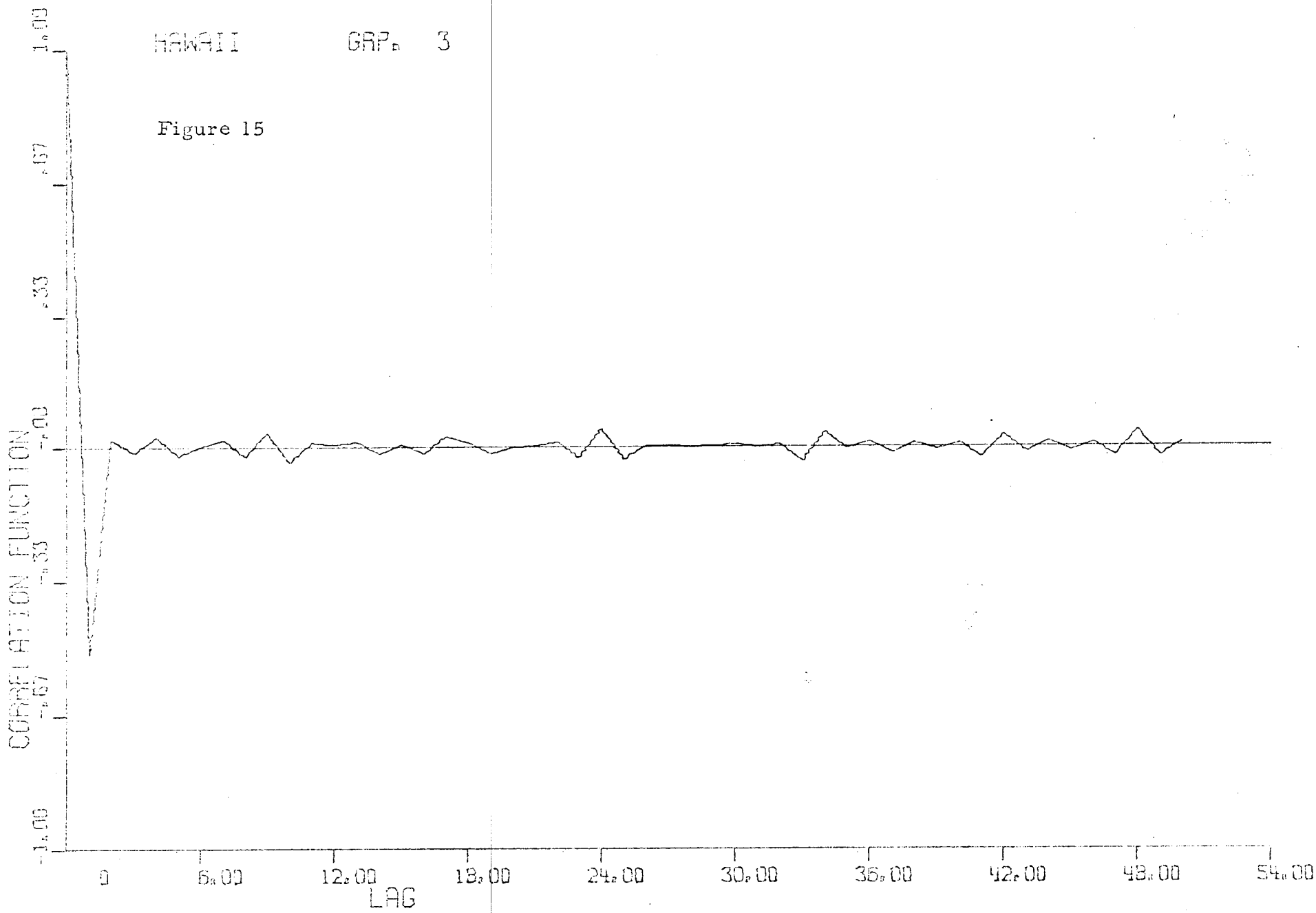

HAWAII GRP. 3
CORRELATION FUNCTION
1.00
.67
.33
0.00
-.33
-.67
-1.00
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 15

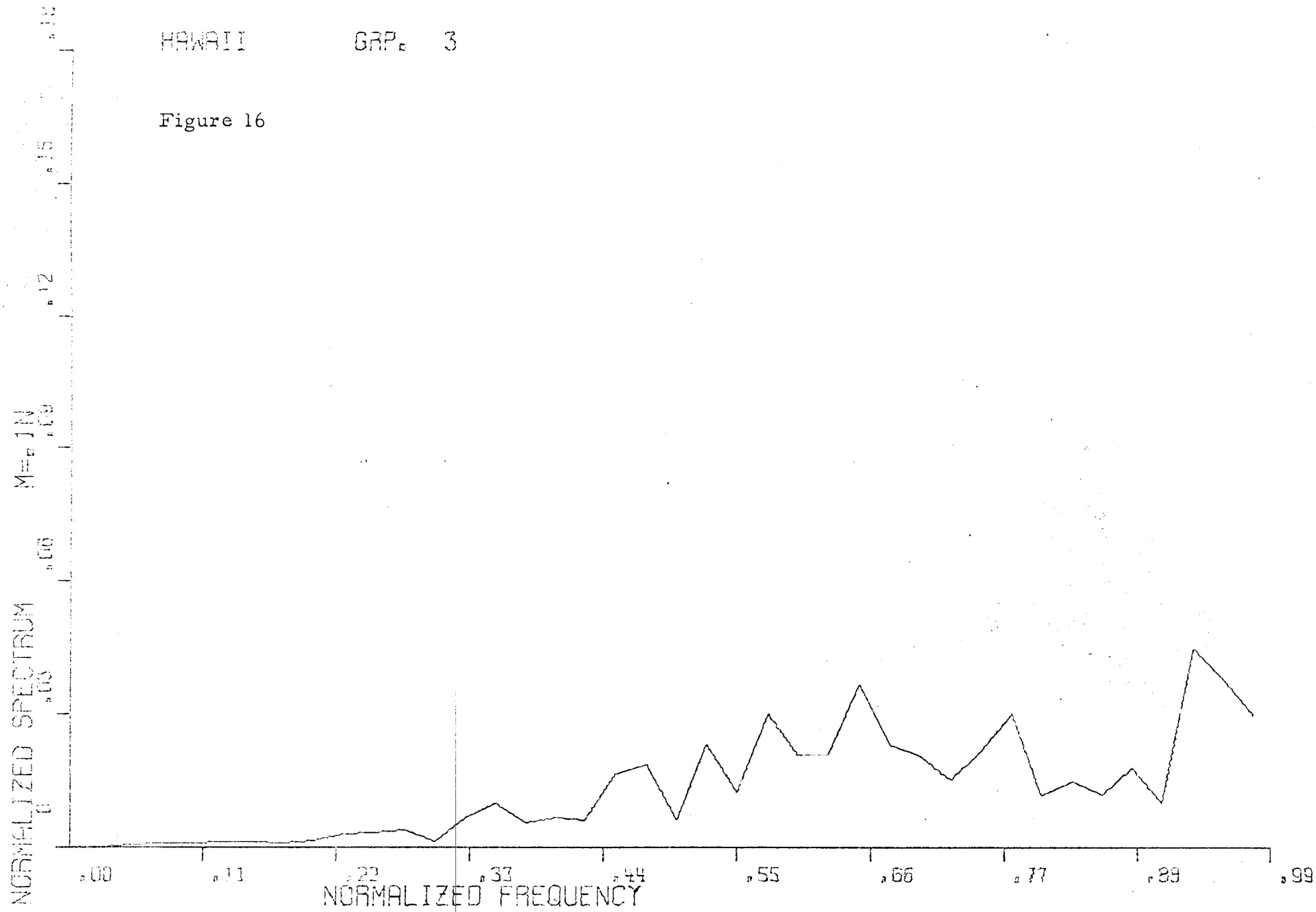

HAWAII GRP. 3
NORMALIZED SPECTRUM
M=. 1N
.00
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 16

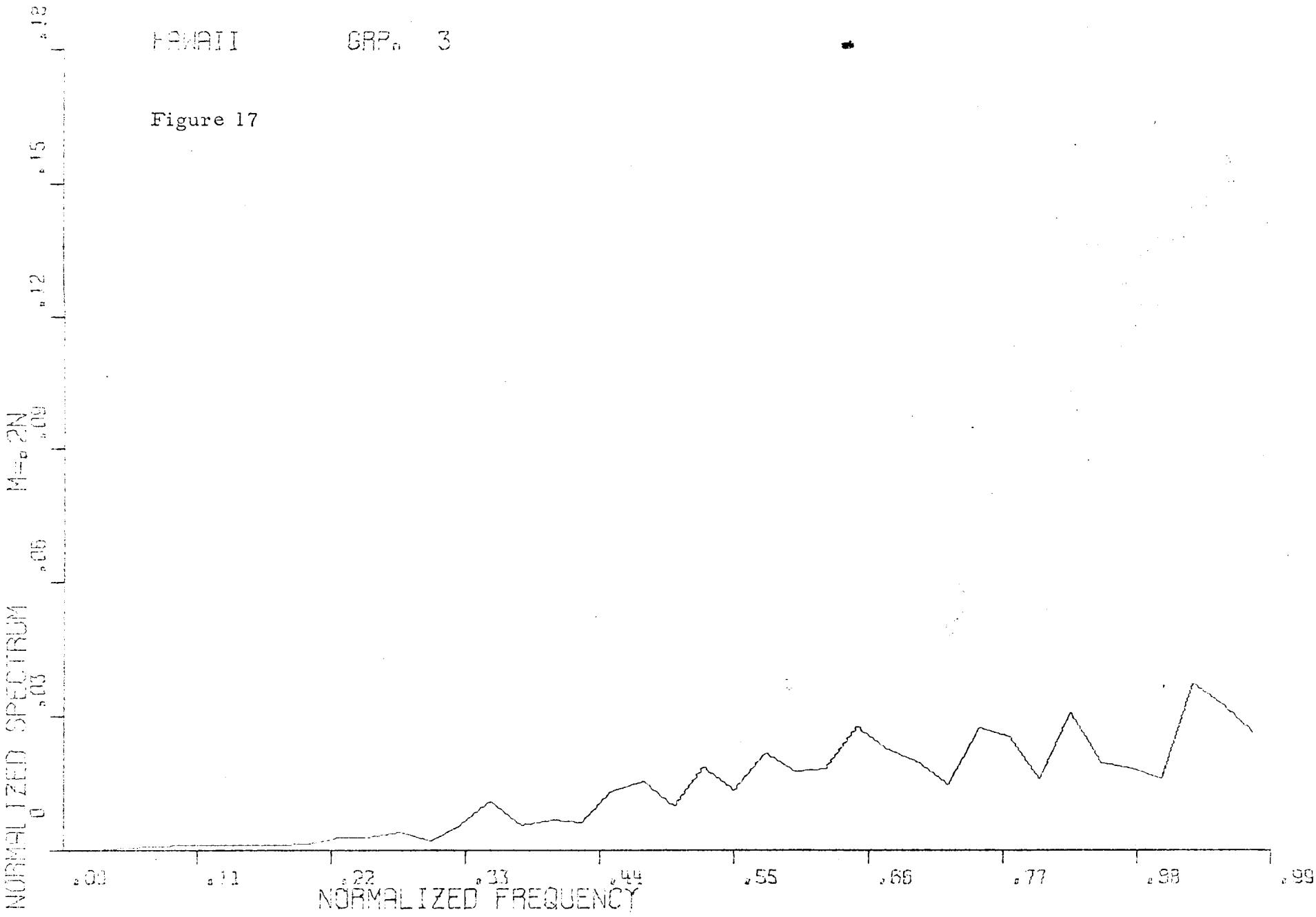

HAWAII GRP. 3
NORMALIZED SPECTRUM M-.2N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 17

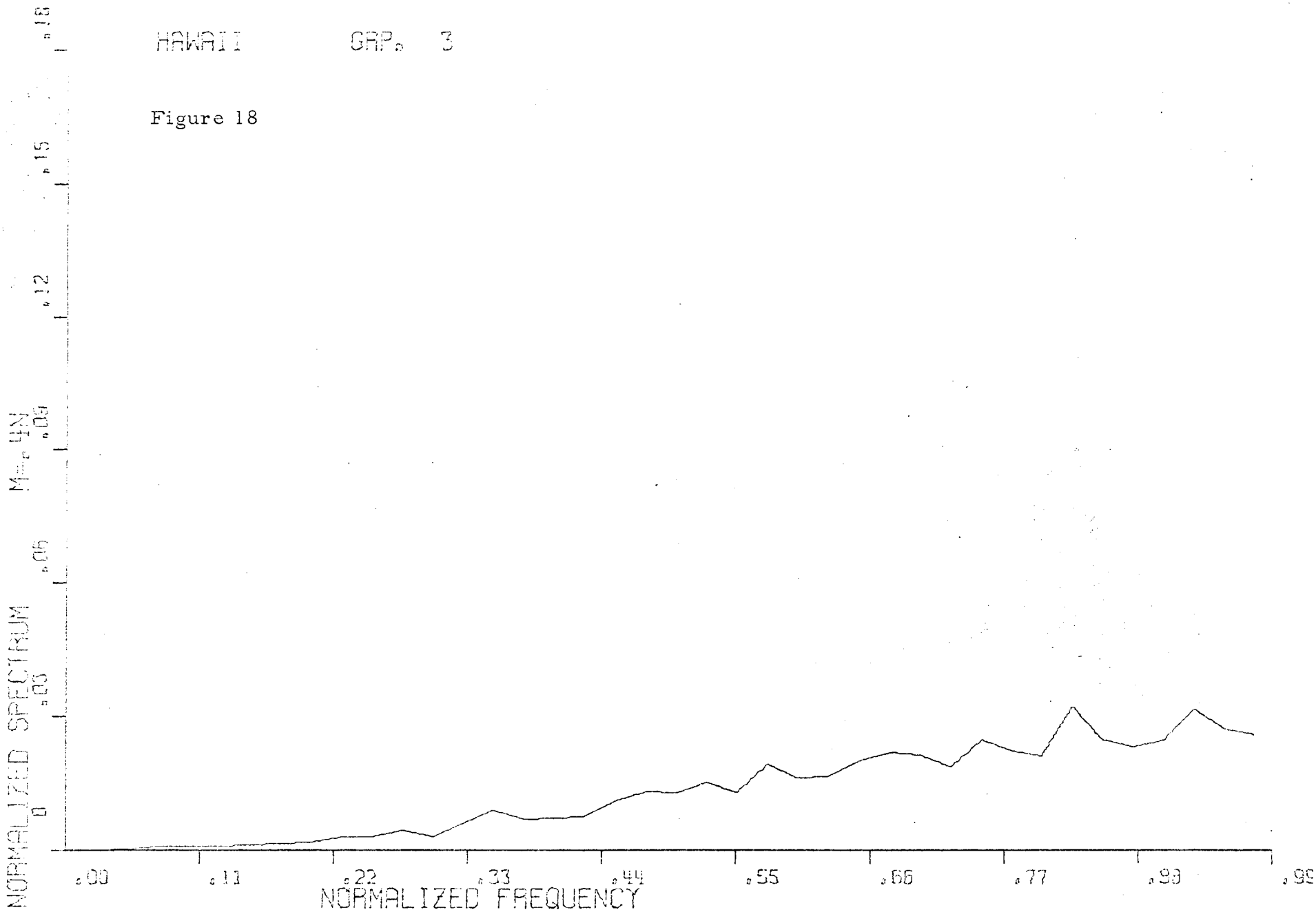
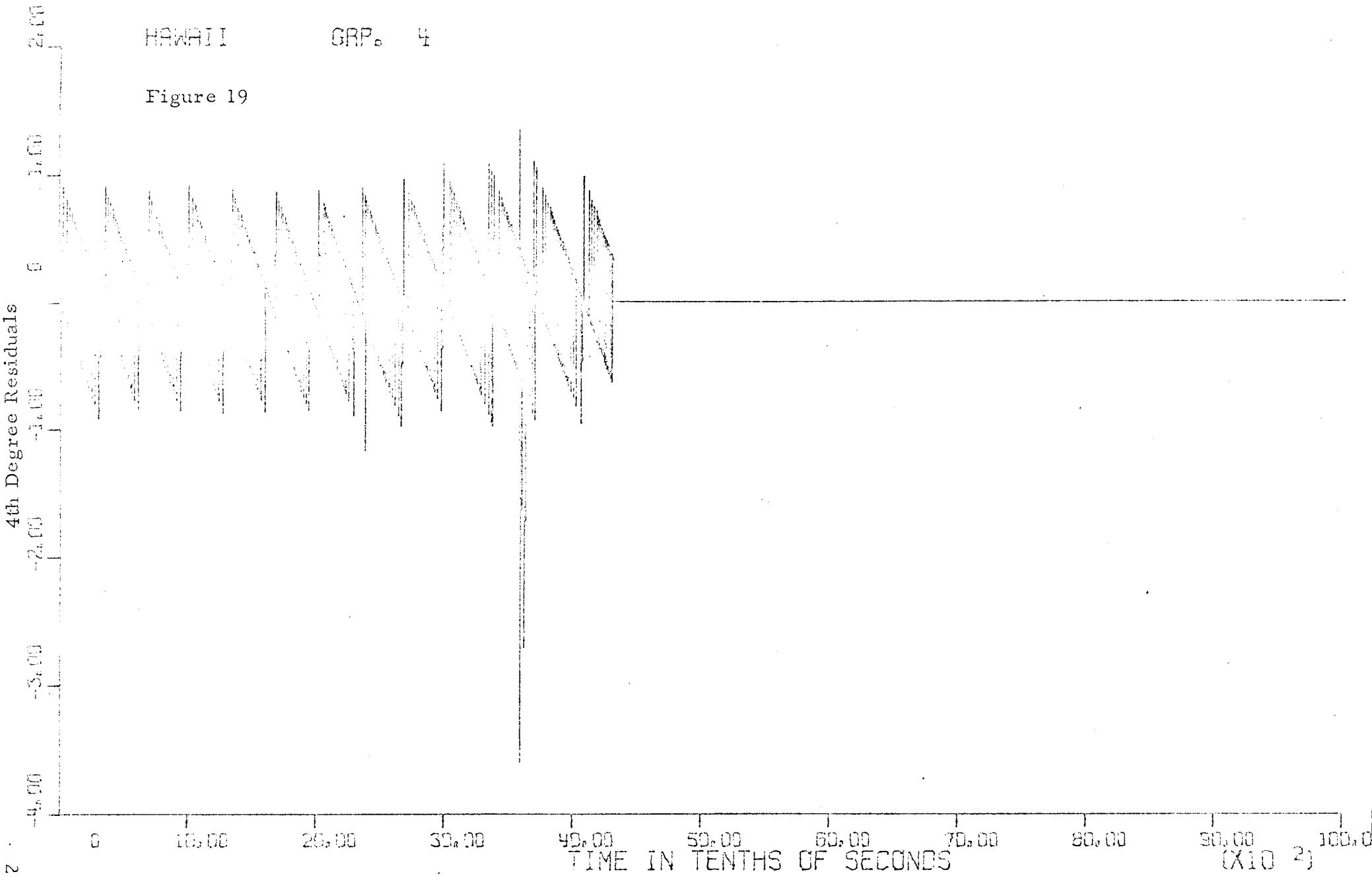

HAWAII GAP. 3
NORMALIZED SPECTRUM M=.4N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 18

HAWAII GRP. 4

Figure 19



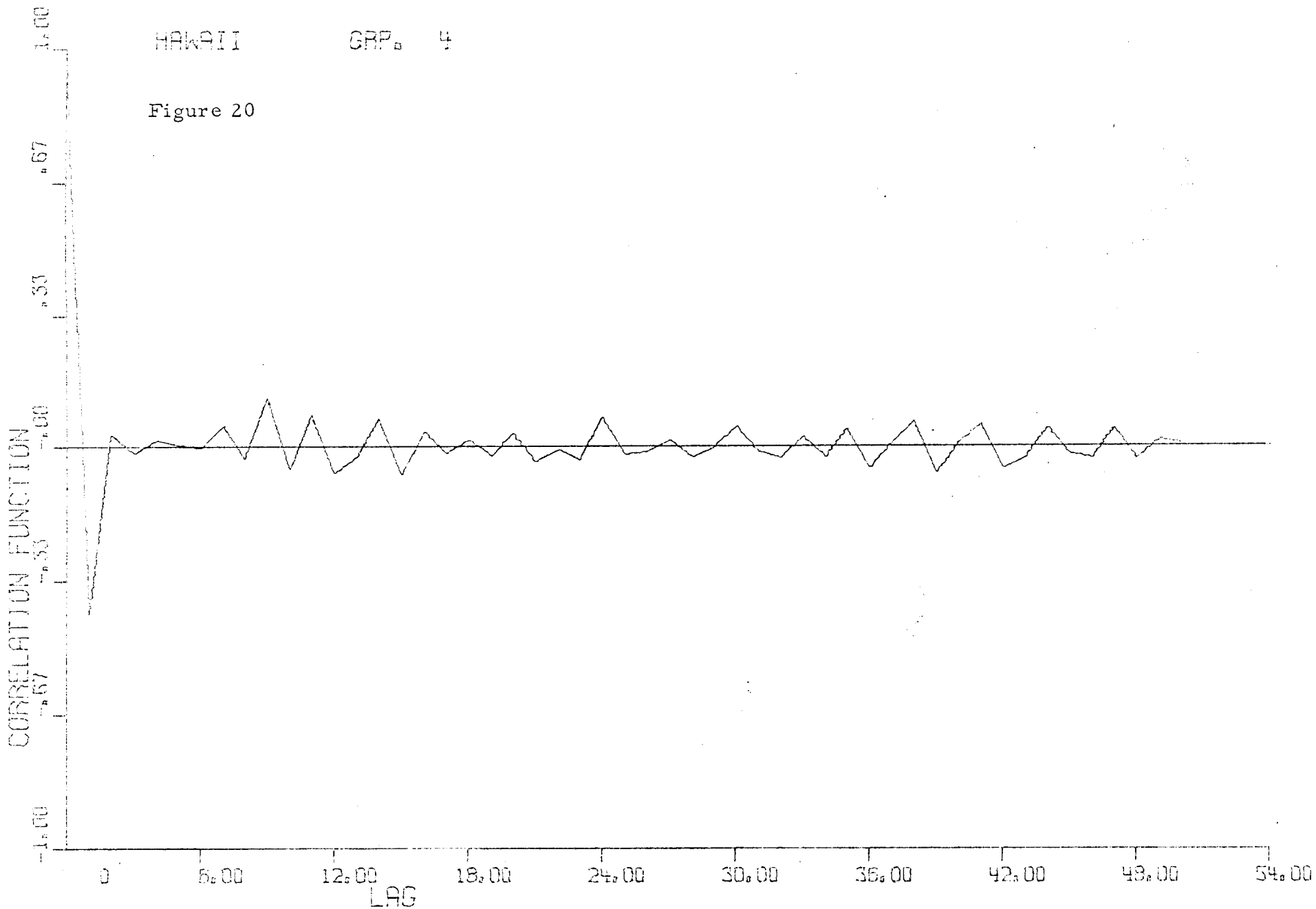

HAWAII GRP. 4
1.00
.67
.33
.00
-.33
-.67
-1.00
CORRELATION FUNCTION
0
6.00
12.00
18.00
24.00
30.00
36.00
42.00
48.00
54.00
LAG

Figure 20

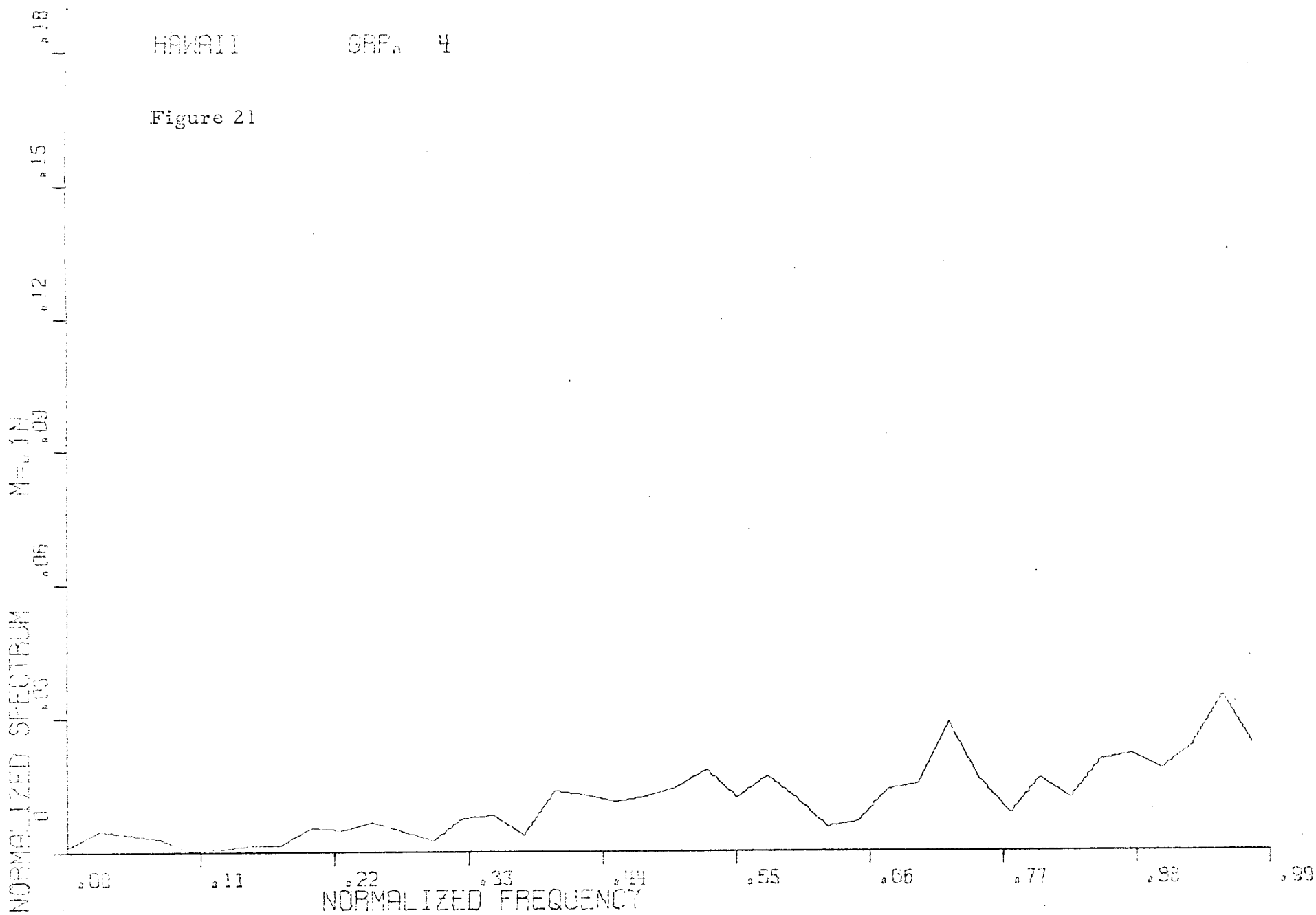

HAWAII GRP. 4
NORMALIZED SPECTRUM M=.1N
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 21

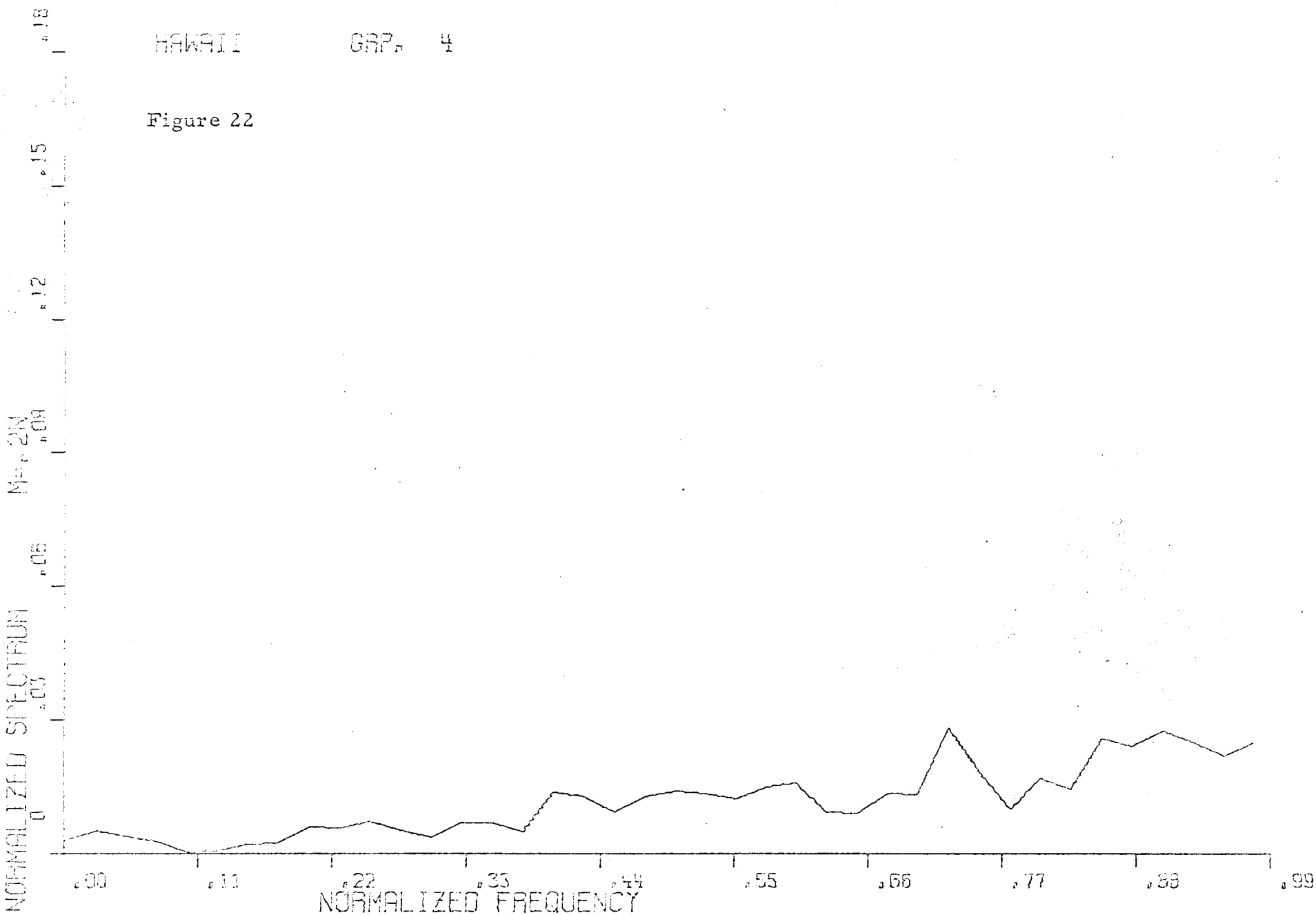

HAWAII GRP. 4
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 22

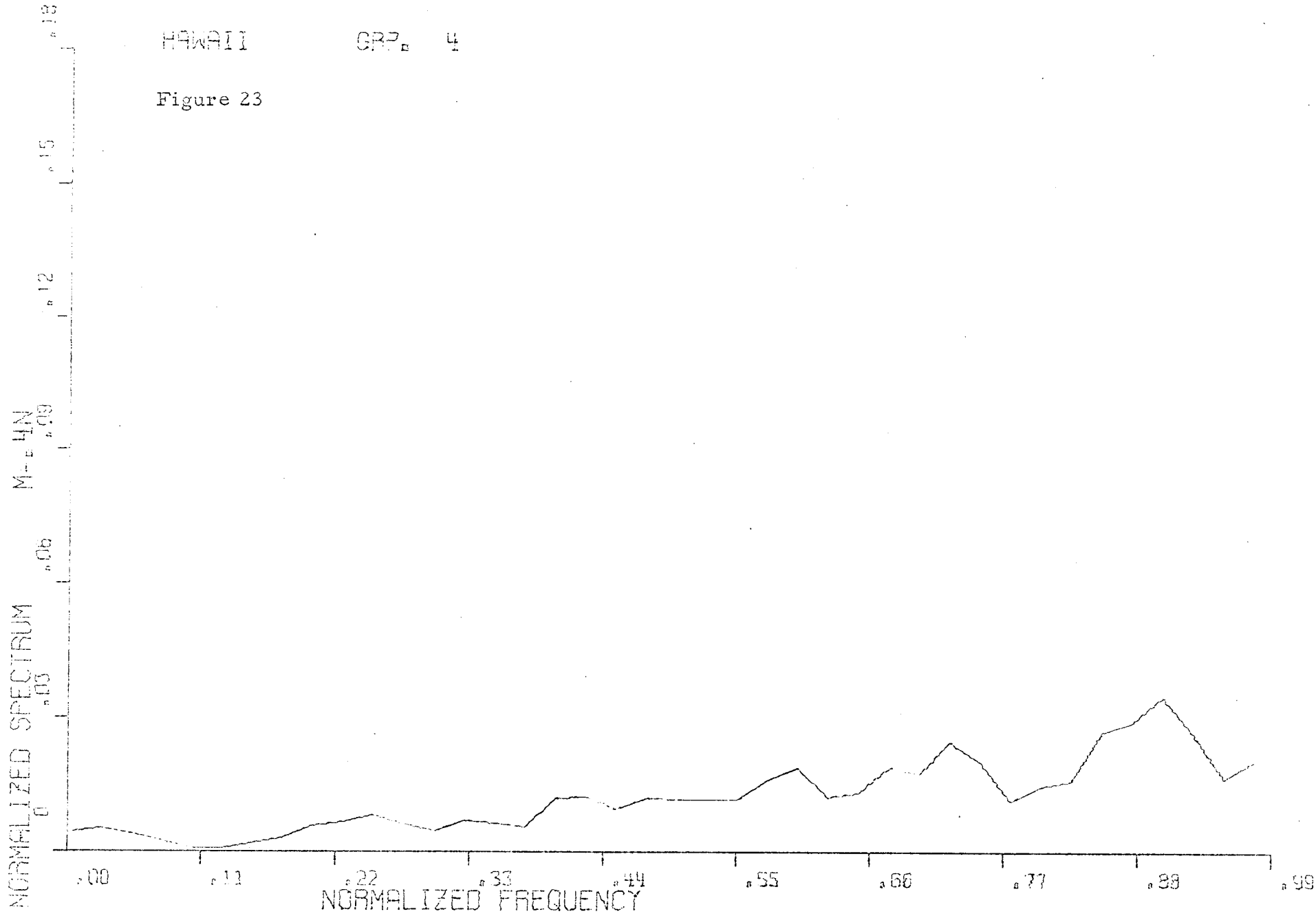

HAWAII GRP. 4
M-.4N
NORMALIZED SPECTRUM
0
.03
.06
.09
.12
.15
.18
.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
NORMALIZED FREQUENCY

Figure 23