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ABSTRACT

The geometrical theory governing direct ascent trajectories (no earth parking orbit) is developed and analytic computations are made relating launch azimuth, launch time and date, and the declination of the moon. The results of integrated direct ascent trajectories are then described. These provide more accurate results than the analytical computations, and in addition, give quantitative information about the relation between payload, launch date, and launch window.

The direct injection payload capability is shown to depend strongly on the required lengths of the launch window and upon the lunar declination (launch date). Using the earth parking orbit mode as a reference, the use of a direct injection profile results in payload increases up to 7200 pounds. A requirement for a one-hour launch window for 7 days per month reduces the advantage to 5100 pounds.

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BELLCOMM, INC.

SUBJECT: Direct-Ascent Trajectories for Lunar Missions Case 310

DATE: February 21, 1967

FROM: L. P. Gieseler

TM-67-2013-2

TECHNICAL MEMORANDUM

I. INTRODUCTION AND SUMMARY

Present plans for the Apollo trajectory call for launching the spacecraft into an earth parking orbit having an altitude of approximately 100 n. mi. After two or three revolutions in orbit, the S-IVB stage is to be reignited, and the vehicle then develops enough energy to carry it to the moon.

There are some advantages in eliminating the parking orbit and allowing the S-IVB stage to burn continuously until the required energy is attained. Among the primary advantages are that payload is increased during certain portions of the lunar cycle and that no S-IVB restart is required. (See References 1 and 2).

The geometrical factors linking direct injection trajectories with launch date and launch window are quite different from those of the earth parking orbit mode. For each launch date, an optimum launch azimuth exists which is not normally equal to 90 degrees. Deviations from this optimum are required to develop a launch window, and a payload penalty results which increases as the launch window is widened. In addition, there are a limited number of days during the month when a direct launch is advantageous, and when used, direct injection provides only one daily launch window instead of two.

In this memorandum, the characteristics of these direct ascent trajectories are first examined using simple geometrical concepts. It is shown that the optimum launch azimuth has a minimum value when the moon is at southern lunstice, and increases as the moon's declination becomes more positive. The upper range-safety limit of 108 degrees for the launch azimuth considerably restricts the number of launch windows per month.

To supplement the approximate analysis, integrated trajectories were generated using the Bellcomm Apollo Simulation

Program (BCMASP) using the weight and propulsion characteristics of the AS-503 vehicle.* Both the standard Apollo trajectory having an earth parking orbit and the direct ascent trajectory were simulated through translunar injection. The results show that with a one-hour launch window the direct ascent trajectory will inject a payload into a translunar orbit approximately 5100 pounds greater than the standard earth parking orbit trajectory for seven days per lunar month.** If the duration of the launch window is reduced to a small value (5 minutes or less), the increase will approach 7200 pounds for two days per month. For comparison, note that the standard earth parking orbit trajectory has two launch opportunities per day, each approximately four hours long.

The BCMASP targeting mode was then used to generate direct ascent integrated free return trajectories for ten launch dates, September 5 through September 14, 1967. The optimum launch time during each direct injection launch window was used for these simulations. For seven of the ten dates, the injected payload fell between 101,350 and 101,650 pounds. The smallest payload for the three remaining dates was 99,700 pounds. Over a similar period, the payload capability using the parking orbit mode is about 94,000 pounds.

The launch window for the September 7 trajectory was then investigated by making additional computer runs at non-optimum launch times. The results show that an 1,800 pound penalty must be paid for a one-hour launch window.

II. SIMPLIFIED ANALYTICAL ANALYSIS

Figure 1 illustrates a direct ascent trajectory with the powered portion shown in heavy lines. Thrust is applied continuously from launch to S-IVB cutoff except for brief periods during staging. Angle η_1 is the true anomaly of the S-IVB cutoff point, and η_2 is the true anomaly of RE_8, the position vector of the moon when the spacecraft arrives there. (See Appendix I for the list and definition of symbols). The plane of the orbit is determined by RE_8 and RE_1, the position vector of the launch site. The total sweep angle (SW) between launch and arrival is given by

$$SW = \eta_2 - \eta_1 + PF_L \qquad (1)$$

where PF_L is the powered flight central arc.

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*A payload comparison between the direct injection mode and earth parking orbit mode is relatively independent of the Apollo/Saturn V configuration used in the simulation.

**For this study, a launch opportunity is determined only by direct injection geometry. Constraints such as the lighting conditions at lunar landing will further reduce the number of available launch dates.

Some assumptions about the magnitudes of the quantities in eq. (1) are required for a simplified analysis, and the following values are chosen as typical:

$$\eta_1 \simeq 12°$$

$$\eta_2 \simeq 170°$$

$$PF_L \simeq 52°$$

$$\text{then } SW \simeq 210°$$

If the above quantities may be considered optimum, deviations from these values will result in a reduction in payload. Deviations are made necessary by the launch window requirement and are accomplished by changes to the powered flight profile as discussed in Section IV. This restriction on the value of SW contrasts with the earth parking orbit trajectory, where SW is a free variable.

Referring to Figure 1, RE_1 revolves about the earth's spin axis, making one revolution per day.* $\vec{RE}_8$ moves in the lunar orbital plane, making one revolution in approximately 27.32 days. A launch opportunity of the optimum type will occur when the angle between these two vectors equals approximately 210°.

Figure 2 shows a spherical triangle whose vertices are the south pole, and the intersection of $\vec{RE}_8$ and $(-\vec{RE}_1)$ with the surface of the earth. The latter vector is the reversed launch position vector, as illustrated in Figure 1. Three positions of the moon are shown, corresponding to three times during the lunar month. It can be seen that AL, the launch azimuth, will be minimum when the moon is at southern lunstice (maximum negative declination).

The relation between AL, ϕ_L (the latitude of the launch site) and δ_8 (the lunar declination) is given by

$$\cos AL = \frac{\cos (90° + \delta_8) - \cos (90° - \phi_L) \cos SW1}{\sin (90° - \phi_L) \sin SW1} \qquad (2)$$

$$\text{where } SW1 = SW - 180 \simeq 30° \text{ for the optimum case.} \qquad (3)$$

*Relative to an inertial frame, the earth makes one revolution in 23 hours 56.068 minutes. However, the assumption of one revolution in 24 hours or 1/4 degree per minute is accurate enough for an approximate analysis.

Figure 3 is a plot of δ_8 and AL against the day of the month for a maximum lunar declination, δ_m, of 28° (typical for 1967-1969) and 18.3° (1976-1978). The instant of southern lunstice is used as a reference for counting the days of the month. The curves for lunar declination were calculated from equation (4), which gives the approximate relationship between the declination of the moon, the time in days from southern lunstice, and the maximum declination.

$$\sin \delta_8 = \sin \delta_m \sin \frac{2\pi}{P_L} \left(T_d - \frac{P_L}{4}\right) \tag{4}$$

where T_d = time from southern lunstice, days

P_L = period of orbital motion of moon, ≈ 27.32 days

After a value of δ_8 was obtained from eq. (4), it was used in eq. (2) to calculate the corresponding value of AL, and the two upper curves of Figure 3 were then plotted. Launch windows occur at intervals somewhat greater than one day because of the moon's motion in its orbit. The calculation is complicated by the fact that the lunar orbit is not in the equatorial plane, but, for an approximate analysis, it is sufficiently accurate to use

$$\Delta T_d = 1 + \frac{1}{P_L} = 1.04 \text{ days} \tag{5}$$

From Figure 3, it can be seen that considering the range safety restriction and using the assumed optimum value of SW, there will be eight or nine launch windows per lunar month for the 1967-1969 period, and four or five launch windows per lunar month for the 1976-1978 period. Additional launch windows are available with optimum SW by disregarding the range safety limit, but the payload loss becomes excessive for launch azimuths greater than about 140 degrees. The number of launch windows can also be increased, at a sacrifice in payload, by the use of non-optimum values of SW.

A launch window having appreciable duration is obtained by allowing the sweep angle SW (and therefore SW1) to deviate from the optimum value. This is illustrated in Figure 4, which is similar to Figure 2, except that the change in time is small enough that the declination of the moon can be assumed constant. During the launch window, the rotation of the earth

moves the launch site along a line of constant latitude. The figure shows four positions of the reversed position vector which correspond to four times during the launch window.

If SW1, δ_8, and ϕ_L are given, AL can be calculated by eq. (2). Then C is given by

$$\sin C = \sin (SW1) \frac{\sin AL}{\sin (90 + \delta_8)} \qquad (6)$$

Since the earth rotates one-fourth degree per minute, C is related to the launch time by the formula

$$C = \frac{1}{4} T_L \qquad (7)$$

where T_L is referenced to the time that C equals zero.

A tabulation of corresponding values of SW1, AL, T_L and δ_8 was obtained by specifying various values of SW1 and δ_8, and calculating AL by equation (2) and T_L by equations (6) and (7). The results are given in Figure 5. The assumed optimum value of 30° for SW1 has been indicated on the lower set of curves. The slope of the linear portion of these curves is such that a one-hour launch window requires a change in SW1 of approximately thirteen degrees. The slope becomes nearly zero for small values of T_L, a favorable circumstance since an appreciable launch window will require little change in SW1. However, the launch azimuth as shown by the upper set of curves approaches 180°, and this will result in an excessive payload penalty. (See Section IV). The launch azimuth which maximizes the payload is 90°, and by combining this with the optimum value of SW1 mentioned earlier, one can conclude that the value of the lunar declination which maximizes the payload is approximately minus twenty-five degrees. This is borne out by the results of Section IV where a method of implementing the change in SW1 will be discussed.

III. POWERED FLIGHT, PARKING ORBIT MODE

The Bellcomm Apollo Simulation Program was used to compute the trajectory of the first part of a mission to the moon using the vehicle parameters for AS-503 as defined in Reference (3). The events list contained in the program was used and this resulted in the standard Apollo trajectory having

a 100 n. mi. earth parking orbit. The purpose of this simulation was to provide a reference against which to compare the direct ascent trajectories described in Section IV.

A launch azimuth of 72 degrees was used and the trajectory shaping parameters were chosen so that a maximum payload is obtained in the parking orbit. After a variable number of revolutions in the parking orbit, the thrust of the S-IVB stage is turned on a second time, and continues until an energy of -8.05×10^6 ft^2/sec^2 is obtained. This energy corresponds to a 72-hour flight time to a mean earth-moon distance; it is a standard cutoff energy for launch vehicle parametric studies. Venting of the fuel tanks is simulated during the parking orbit by subtracting 1,060 pounds per revolution from the weight of the vehicle. This venting rate was obtained from Reference (4). The reference trajectory was assumed to be one making three revolutions in the parking orbit, with fuel provided to make a plane change at injection, thus making possible two injection opportunities from the parking orbit. The calculated payload for this strategy was 93,900 pounds. If the number of revolutions is reduced to one, with no plane change capability, the payload rises to 96,900 pounds. Details of the computation of these payloads are given in Appendix II.

IV. POWERED FLIGHT, DIRECT ASCENT MODE

The events list of the Bellcomm Apollo Simulation Program was modified so that the S-IVB burned continuously until the standard energy used in Section III was attained. The weight of S-IVB cutoff was maximized by varying the trajectory shaping parameters.* A typical trajectory is shown in Figure 6. Note that an altitude of approximately 400,000 feet is obtained 365 seconds from launch and this diminishes to a minimum of 268,000 feet 755 seconds from launch. Thereafter the altitude increases rapidly, attaining a value of over 600,000 feet at S-IVB cutoff. For a ninety-degree launch azimuth, the payload equalled 101,100 pounds.

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*For simulations using the earth parking orbit mode or using direction injection, the first stage is constrained to fly at a zero angle of attack; the upper stages fly to an optimum angle of attack. The aerodynamic heating is not constrained.

The flight path angle at cutoff, β_7, is related to the true anomaly η_1 by the equation

$$\tan \beta_7 = \frac{e \sin \eta_1}{1 + e \cos \eta_1} \tag{8}$$

When e, the eccentricity of the translunar ellipse, is nearly equal to 1, and for small values of β_7, equation (8) can be replaced by

$$\beta_7 \simeq \frac{\eta_1}{2} \tag{9}$$

Thus the 13° spread in η_1 required for a one-hour launch window (Figure 5) can be achieved by a spread in β_7 of approximately 6-1/2°. Figure 7 shows the results of a series of open loop computer runs made by imposing the constraint that β_7 take on a spread of values from two to twelve degrees. From the curve, it can be seen that the spread of 6-1/2° in β_7 results in a weight penalty of 2,000 pounds. Smaller launch windows than one-hour will, of course, result in smaller weight penalties.

The effect of launch azimuth was studied using the same open loop simulations used for generating the data of Figure 7. The launch azimuth was varied from 72 to 140 degrees, and the results are plotted in Figure 8. Maximum payload occurs when AL = 90°. A 900-pound penalty results when AL is allowed to vary from 72 to 108°. The penalty when AL is increased to 140 degrees equals approximately 6,000 pounds.

Figures 7 and 8 can be used in conjunction with Figures 3 and 5 to determine the launch window duration for various dates and payloads. A date and the corresponding lunar declination is first selected from Figure 3. The lunar declination is assumed to remain constant during the launch window, and this restricts the consideration of the curves of Figure 5 to a single curve from each of the two sets shown in the figure. (Interpolated curves will be required if the value of the lunar declination does not correspond to one of the given set.) A segment of the lower curve is selected and values of SW1 and T_L are determined for each end of the segment. The upper curve will give the two corresponding values of AL. The launch window duration is the difference between the two T_L's. It is assumed

that total payload loss will be the sum of the losses from non-optimum AL and SW1, as determined from Figures 7 and 8. The maximum loss during the launch window should be assigned to the entire launch window. The best launch window will be one which has the same payload loss at the beginning and at the end of the launch window.

Figure 9 is a summary of a number of these determinations. Launch window duration is plotted against launch date for payloads equal to 95,000, 97,000 and 99,000 pounds. The curve for the latter payload shows a launch window of nearly one hour for seven consecutive days. Two more days are available with reduced launch window duration. With a payload of 95,000 pounds, a launch window duration of nearly two hours can be had for seven consecutive days, with four additional days available at a reduced launch window duration. For these results the launch azimuth was not constrained by the range safety limits.

V. INTEGRATED FREE-RETURN SIMULATIONS

The theoretical considerations discussed in Section II were incorporated in the Bellcomm Apollo Simulation Program, making possible the computation of direct ascent trajectories for an entire free-return lunar mission. The AS-503 vehicle parameters were used as before. The results can be expected to differ somewhat from the results of Section IV because the position of the moon relative to the earth is determined accurately, and the required translunar energy can be computed instead of being assumed, as before. In addition, no flight geometry reserves are needed. (See Appendix II, E-3)

In the first series of computer runs, fully optimized trajectories were flown. Ten simulations were made for ten successive days centered around September 9, 1967, and the results are summarized in Figure 10. (The payload figures shown are the maximum for the indicated dates, and correspond to a vanishingly small launch window.) The variation in payload with date is caused by the variation in launch azimuth, which ranged from 81 to 122 degrees. There are two maxima in Figure 9 because there are two dates (September 7 and September 11) when the launch azimuth is nearly 90 degrees. This will maximize the payload, as shown in Figure 8. The slight difference in payload for the two peaks is caused by the fact that the program makes use of actual lunar data which would not be quite the same for the two dates. The launch azimuth for seven of the ten simulations fell within the range of 81-101 degrees, and the payload has a spread of only 300 pounds. For the remaining three dates, the payload drops sharply, as shown in the figure. The maximum payload was approximately 550 pounds greater than that obtained in Section IV.

A number of free-return simulations were also made for the purpose of studying the launch window for September 7, 1967. Non-optimum trajectories, having a spread of values of β_7, were used and the results are given in Figure 11. As indicated in the figure, a one-hour launch window requires a 1,800 pound payload penalty. This agrees fairly well with the 2,000 pounds for the same length launch window computed with the approximate analysis (see Figure 7). A fifteen-minute launch window has a 150-pound penalty as shown. The penalties of Figure 11 are exact only for September 7, 1967. However, Figures 10 and 11 can be combined with good accuracy to illustrate payload as a function of launch date and launch window duration during the time period covered by Figure 10.

VI. CONCLUSIONS

Simple geometrical considerations show that there will be eight or nine launch windows per lunar month for the 1967-69 period if the range safety restrictions on the launch azimuth are satisfied, and if the powered-flight profile is such that the total sweep angle (SW) is optimum. There will be four or five launch windows per lunar month for the 1976-78 period subject to the same restrictions. Additional launch windows are available at reduced payloads by allowing the launch azimuth to become larger than the upper range safety limit of 108°, or by using powered-flight profiles which result in non-optimum sweep angles.

The results of integrated trajectories show a payload capability for various strategies as given in the table below. For comparison, note that the earth parking orbit strategy has two launch windows per day, each having a duration of approximately four hours.

		Payload	Increase Over 1
1.	Earth parking orbit, 72° launch azimuth, 3 revolutions in parking orbit, plane change at injection.	93,900 lbs.	----
2.	Earth parking orbit, 72° launch azimuth, 1 revolution in parking orbit, no plane change at injection.	96,900	3,000 lbs.

		Payload	Increase Over 1
3.	Direct ascent mode, 2 hr. launch window duration for 7 days per month, reduced launch window duration for 4 additional days. (Launch azimuth may exceed 108°.)	95,000	1,100
4.	Direct ascent mode, 1 hr. launch window duration for 7 days per month, reduced launch window for 2 additional days. (Launch azimuth may exceed 108°.)	99,000	5,100
5.	Direct ascent mode, 15-minute launch window duration for 7 days per month.	100,700	6,800
6.	Direct ascent mode, 5-minute launch window duration for 2 days per month.	101,100	7,200

L. P. Gieseler

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Attachments
References
Appendixes I,II & III
Figures 1-11

REFERENCES

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2. "Feasibility of Direct Ascent Mode for Saturn V" George C. Marshall Space Flight Center. Memorandum dated April 13, 1966 addressed to Dr. Rees, DEP-T.

3. "AS-503 Preliminary Launch Vehicle Reference Trajectory" Vol. I, Boeing Aircraft Co. No. D5-15480, June 7, 1966. (CONFIDENTIAL)

4. "AS-504 Preliminary Launch Vehicle Reference Trajectory" Vol. I, Boeing Aircraft Co. No. D-15481, July 13, 1966. (CONFIDENTIAL)

APPENDIX I

LIST OF SYMBOLS

AL	Launch azimuth	
e	Eccentricity of translunar orbit	
PF_L	Sweep angle of powered flight	
P_L	Period of orbital motion of moon, 27.32 days	
RE_1	Position vector of launch site	
RE_8	Position vector of moon at arrival of spacecraft	
SW	Total sweep angle from launch to arrival at the moon	
SW1	SW - 180°	
T_d	Time from southern lunstice, days	
β_7	Flight path angle at S-IVB cutoff, deg.	
δ_m	Maximum declination of moon during the month	
δ_8	Declination of the moon at arrival	
T_L	Launch time in minutes, measured from the instant that Angle C of Figures 2 and 4 equals zero	
η_1	True anomaly of S-IVB cutoff point	
η_2	True anomaly of the position vector of the moon	
ϕ_L	Latitude of launch site (28.65°)	
ΔT_d	Time between launch opportunities, days	
θ_1	First stage kick angle (deg.)	Trajectory Shaping Parameters
θ_2	Second stage kick angle (deg.)	
$\dot{\theta}_2$	Second and third stage turning rates	

APPENDIX II

CALCULATION OF PAYLOADS

A. DEFINITION OF SYMBOLS

W_{co} = Weight of vehicle at S-IVB cutoff. This quantity is determined by the computer simulations described in Sections III, IV, and V.

P_n = Payload for the nominal mission, assumed to equal 95,000 lbs.

W_e = Weight change when the S-IVB is jettisoned, that is, the weight of empty S-IVB plus the instrument unit plus the fuel reserves.

B. CALCULATION OF PAYLOAD P

Let ΔF be the weight of the usuable fuel remaining in the tanks of the S-IVB. Then

$$\Delta F = W_{co} - \left(W_e + P_n\right)$$

For a first approximation the payload could be increased by ΔF and the fuel decreases by ΔF. The vehicle will then have the same weight history as before during the flight and the same trajectory will be flown. There will be no usable fuel remaining at S-IVB cutoff. However, the fuel tanks of the S-IVB will not be completely full at lift-off. To take full advantage of the vehicle capabilities, the fuel tanks should be filled. This results in a lift-off weight which is greater than before by ΔF pounds. Computer simulations have shown that the lift-off weight (and hence the payload) should be increased by approximately 1.1 ΔF to reduce the usable fuel at cutoff to zero. Then

$$P = P_n + 1.1\ \Delta F$$

C. EARTH PARKING ORBIT MODE, THREE REVOLUTIONS

1. Calculation of W_e (See Reference 4, Page 28)

Empty S-IVB stage	26,400 pounds
Instrument Unit	4,100
Flight Performance Reserves	2,838
Flight Geometry Reserves	2,453
W_e =	35,791 pounds

2. Calculation of P

The simulation described in Section III gave W_{co} = 129,800 pounds. Then ΔF = 129,800 - 130,791 = -991 pounds. Finally P = 95,000 + 1.1 ΔF ≈ 93,900 pounds.

D. EARTH PARKING ORBIT MODE, ONE REVOLUTION

1. Calculation of W_e

Empty S-IVB stage	26,400
Instrument Unit	4,100
Flight Performance Reserves	2,838
Flight Geometry Reserves	754*
W_e =	34,092

*The nominal assumed figure of 2,453 lbs. corresponds to a ΔV of 180 fps for plane change, and 80 fps for variations in the required translunar energy. Since the plane change is not required, the flight geometry reserves become

$$2,453 \times \frac{80}{260} = 754 \text{ pounds}$$

2. Calculation of ΔF and P.

The simulation for one parking orbit described in Section III gave W_{co} = 130,800 pounds. Then $\Delta F = W_{co}$ - 34,092 - 95,000 = 1,708 pounds. Finally P = 95,000 + 1,708 x 1.1 ≅ 96,900 pounds.

E. DIRECT ASCENT MODE

1. Calculation of W_e

Empty S-IVB stage	26,400
Instrument Unit	4,100
Flight Performance Reserves	2,838
Flight Geometry Reserves	754*
Fuel Stratification (Reference 2, Page 61)	1,350

W_e = 35,442 pounds

2. Calculation of ΔF and P for the results of Section IV.

The simulation gave W_{co} = 135,972 pounds. Then $\Delta F = W_{co}$ - 35,442 - 95,000 = 5,530 pounds. Finally P = 95,000 + 5,530 x 1.1 ≅ 101,100

3. Relation between P and W_{co} for the results of Section V.

For these simulations the exact launch date is assumed to be known. The required translunar energy can be calculated in advance and no flight geometry reserves are required. Then W_e = 34,688 pounds and the formula for converting W_{co} to P becomes

$$P = 95{,}000 + 1.1\,(W_{co} - 34{,}688 - 95{,}000)$$

*The nominal assumed figure of 2,453 lbs. corresponds to a ΔV of 180 fps for plane change, and 80 fps for variations in the required translunar energy. Since the plane change is not required, the flight geometry reserves become

$$2{,}453 \times \frac{80}{260} = 754 \text{ pounds.}$$

APPENDIX III

MATHEMATICAL FORMULAS AND TECHNIQUES

A. CHANGES TO SUBROUTINE HITMSI

Inputs

GME - gravitational constant of the earth.

ERG7P - injection energy.

XLMMSP, PHIMSP - longitude and latitude of piercing point on MSI.

REO - spacecraft distance from center of earth at S-IVB cutoff.

β_7 - flight path angle at S-IVB cutoff.

λ_L, ϕ_L - longitude and latitude of launch pad.

t_f - earliest time for beginning of mission.

Ω - rotational rate of the earth.

t_{PF} - time required for powered flight.

PF_L - powered flight arc.

Outputs

ALP - launch azimuth for patched conic.

t_L - launch time for patched conic (DATLP, TIMELP).

VM8PX - spacecraft selenocentric velocity at MSI.

Formulae

$$VE7P = \sqrt{2\left(ERG7P + \frac{GME}{RE0}\right)} = \text{velocity at S-IVB cutoff}$$

Iteration

Statement 2000

RE8PX, the earth-moon aim vector is computed as before from XLMMSP, PHIMSP, and the position of the moon at an estimated time of arrival.

$$DCLPP = \tan^{-1}\left(\frac{RE8PX(3)}{\sqrt{RE8PX(1)^2 + RE8PX(2)^2}}\right) = \text{declination of } \overrightarrow{RE8PX}$$

$$RA8 = \tan^{-1}\left(\frac{RE8PX(2)}{RE8PX(1)}\right) = \text{right ascension of } \overrightarrow{RE8PX}$$

$$a = -\frac{GME}{2\ ERG7P}$$

$$p = 2\cos^2(\beta_7) \cdot RE0 \cdot \left(\frac{RE0 \cdot ERG7P}{GME} + 1\right)$$

$$e = \sqrt{1 - \frac{p}{a}}$$

$$\eta_1 = \sin^{-1}\left(\frac{p \tan(BETA7)}{e \cdot RE0}\right)$$

$$\eta_2 = \cos^{-1}\frac{1}{e}\left(\frac{p}{RE8P} - 1\right)$$

$$SW1 = \eta_2 - \eta_1 - 180 + PF_L$$

$$ALP = \cos^{-1}\left(\frac{-\sin(DCLPP) - \sin(\phi_L)\cos(SW1)}{\cos(\phi_L)\sin(SW1)}\right)$$

$$C = \sin^{-1}\left(\frac{\sin(ALP)\sin(SW1)}{\cos(DCLPP)}\right)$$ = angle shown in Figure 2 and 4.

$RAL = RA8 - C - 180°$ = right ascension of launch vector.

$$t_L = t_F + \left(\frac{RAL - \lambda_L - GHA(t_F)}{\Omega}\right)$$ = launch time

$$DT8 = \frac{a^{1.5}}{GME^{.5}}\left(E_2 - E_1 - e(\sin E_2 - \sin E_1)\right)$$

= time in elliptic orbit.

where E_2 and E_1 are the eccentric anomalies corresponding to η_2 and η_1.

$TIM8P = t_L + DT8 = t_{PF}$ = arrival date and time.

If TIM8P does not agree with the estimated time of arrival within the specified tolerance, the above calculations are repeated, beginning with statement 2000.

$\overrightarrow{R1PX}$, the position vector of the launch site is computed from RAL and ϕ_L.

$$\overrightarrow{H7PX} = \frac{\overrightarrow{RE8PX} \times \overrightarrow{RE1PX}}{|RE8PX|\ |RE1PX|} = \text{unit normal to orbit plane}$$

$$\beta_8 = \tan^{-1}\left(\frac{e \sin \eta_2}{1 + e \cos \eta_2}\right) = \text{flight path angle at MSI.}$$

$$VE8PX = \sqrt{2\left(ERG7P + \frac{GME}{RE8PX}\right)} = \text{scalar velocity at MSI.}$$

$$\overrightarrow{VE8PX} = \frac{|VE8P|}{|RE8PX|} \cdot \left[\overrightarrow{RE8PX} \text{ rotated about } \overrightarrow{H7PX} \text{ by } (90-\beta_8) \text{ degrees}\right]$$

= vector velocity at MSI.

$\overrightarrow{VM8PX} = \overrightarrow{VE8PX} - \overrightarrow{VEMX}$ = selenocentric vector velocity at MSI. (VEMX is the inertial velocity of the moon).

B. CHANGES TO SUBROUTINE TRJSEL

a. Imaging variables. In the original formulation of BCMASP one of the imaging variables was DF6, the time that the spacecraft spends in the earth parking orbit. For the direct ascent case there is no parking orbit and the launch azimuth is used for an imaging variable. The following changes were made:

DF6 is replaced by AL = launch azimuth
DF6PR is replaced by ALPR = reference launch azimuth
DFBIAS is replaced by ALBIAS = launch azimuth bias

b. Patched Conic Variables. As indicated in section 1 of Appendix III, β_7, REO, t_{PF} and PF_L are inputs to HITMSI. These quantities remain constant during the

imaging process. However they should be reasonably close to the same quantities computed by the powered-flight simulator, otherwise the patched-conic trajectory will differ too much from the integrated trajectory. This problem is solved in TRJSEL by the following sequence:

1. Read in approximate values of β_7, REO, t_{PF} and PF_L.

2. Call ANLSIS. This calls HITMSI and CNSTRN iteratively, and values of ALP, DATEL, and TIMEL are computed which yield a patched-conic free return trajectory.

3. Using these quantities the powered flight simulator is used to compute improved values of β_7, REO, t_{PF} and PF_L.

4. Begin imaging.

c. Trajectory Shaping. When BCMASP is used to generate trajectories of different launch dates a considerable spread of values of AL results. To each value of AL optimum values of θ_1, θ_2, and $\dot{\theta}_2$ exist. Subrouting LOKUP5 contains a table of these optimized values for various values of AL. (The entries in the table had been previously determined by making open loop runs). A table lookup is used to determine the required quantities for values of AL between the tabular entries.

For generating trajectories during the launch window on a specific date it is assumed that θ_1, θ_2, and $\dot{\theta}_2$ can be held constant during the computer search for a free return trajectory. The table lookup instruction is bypassed by setting ILOKUP equal to 1, and inputting appropriate values of θ_1, θ_2, and $\dot{\theta}_2$ by means of the data deck.

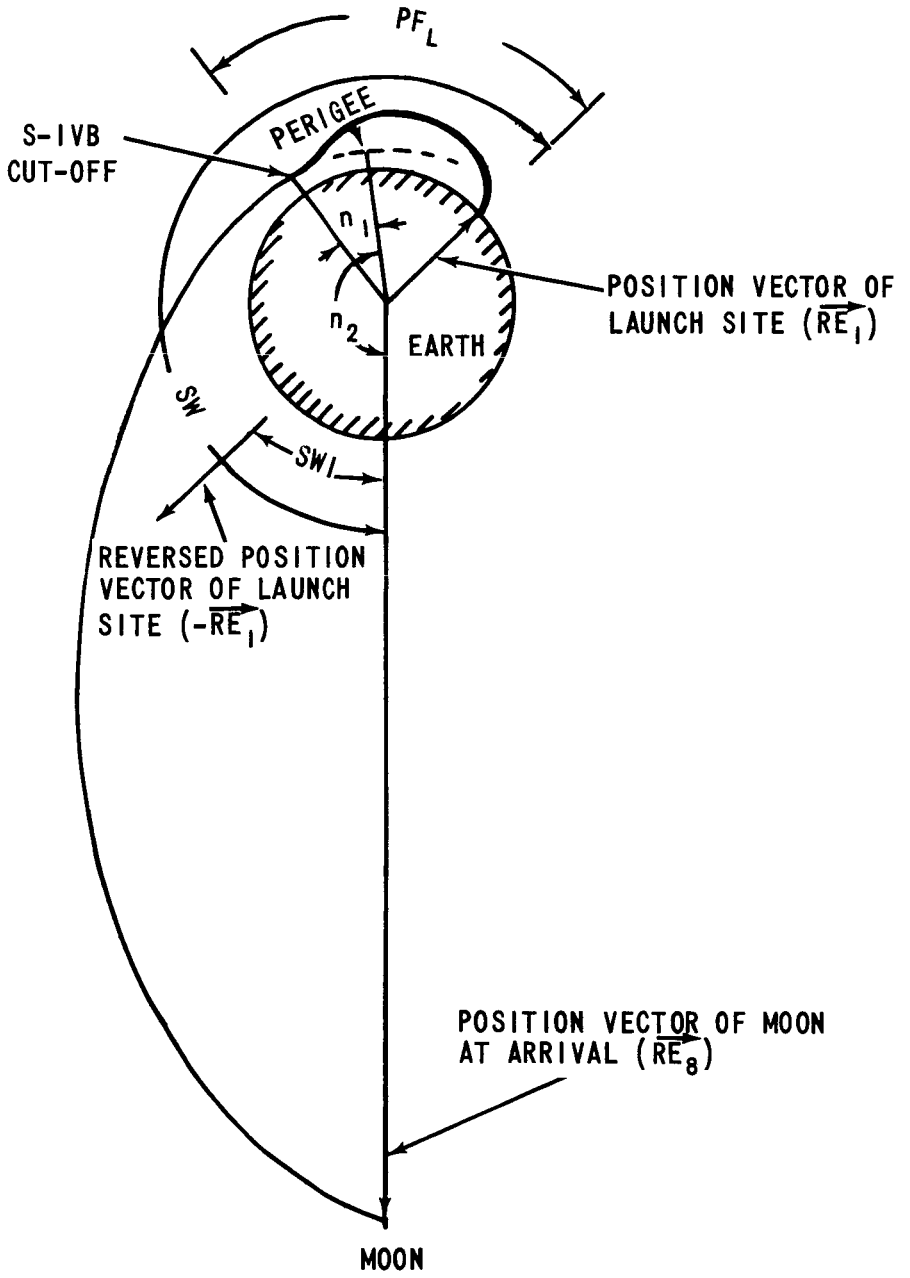

FIGURE I DIRECT ASCENT TRAJECTORY PROFILE

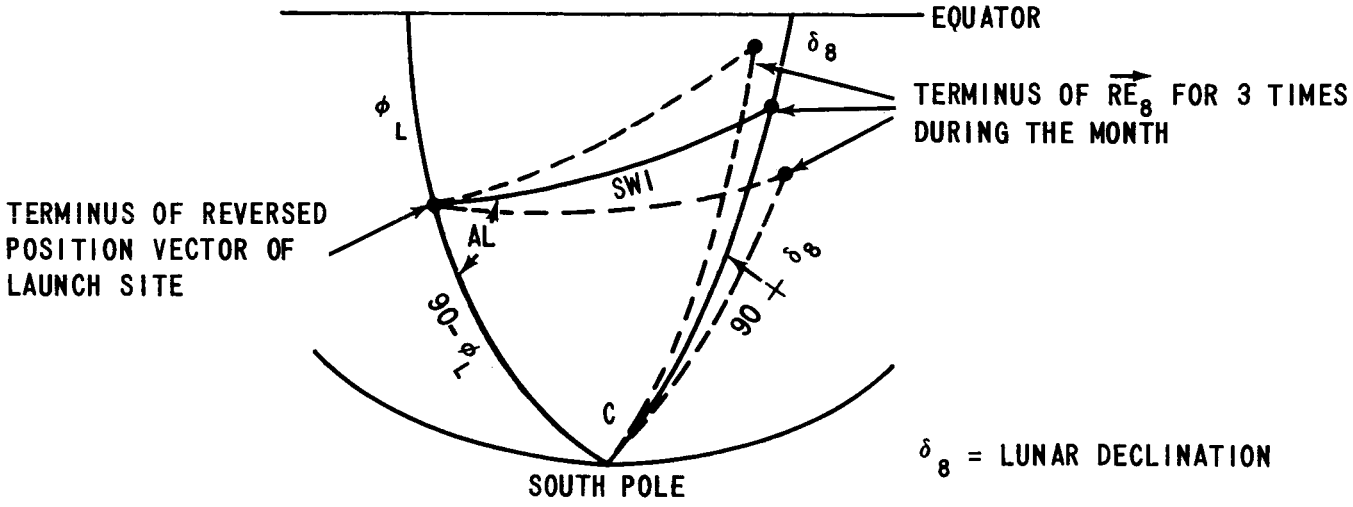

FIGURE 2 - LAUNCH OPPORTUNITIES DURING THE MONTH

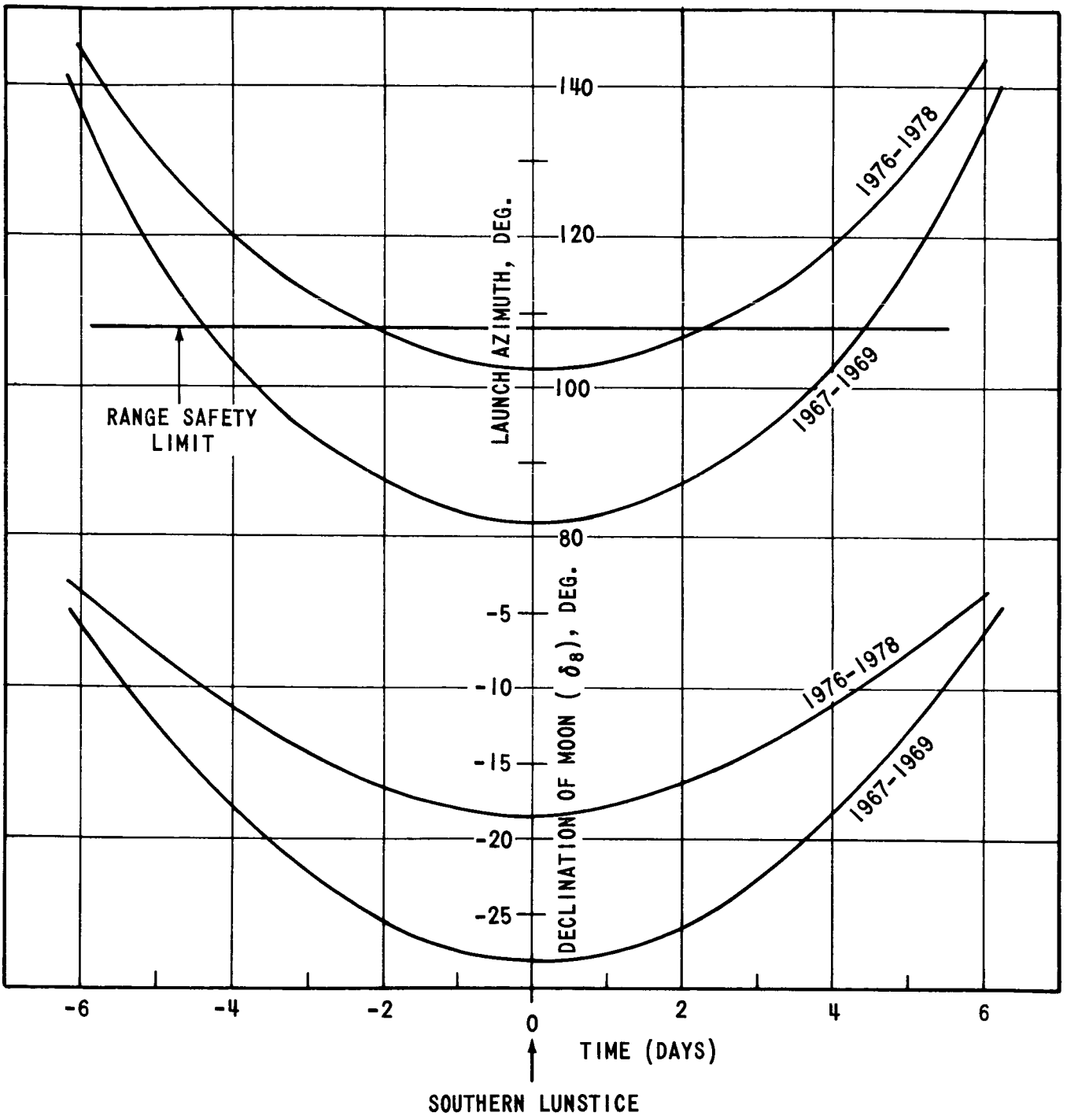

FIGURE 3 DECLINATION OF MOON AND LAUNCH AZIMUTH AS A FUNCTION OF TIME

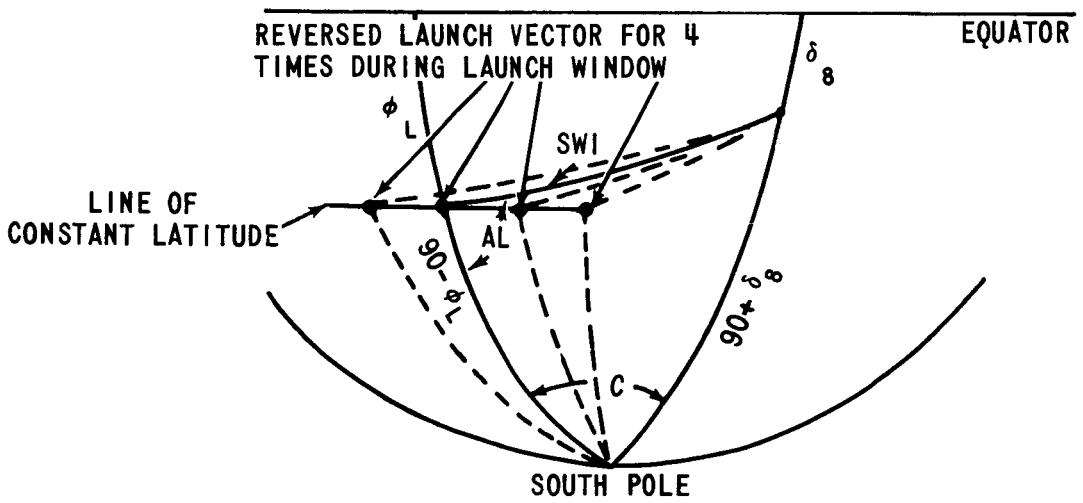

FIGURE 4 - SPHERICAL TRIANGLES DURING LAUNCH WINDOW

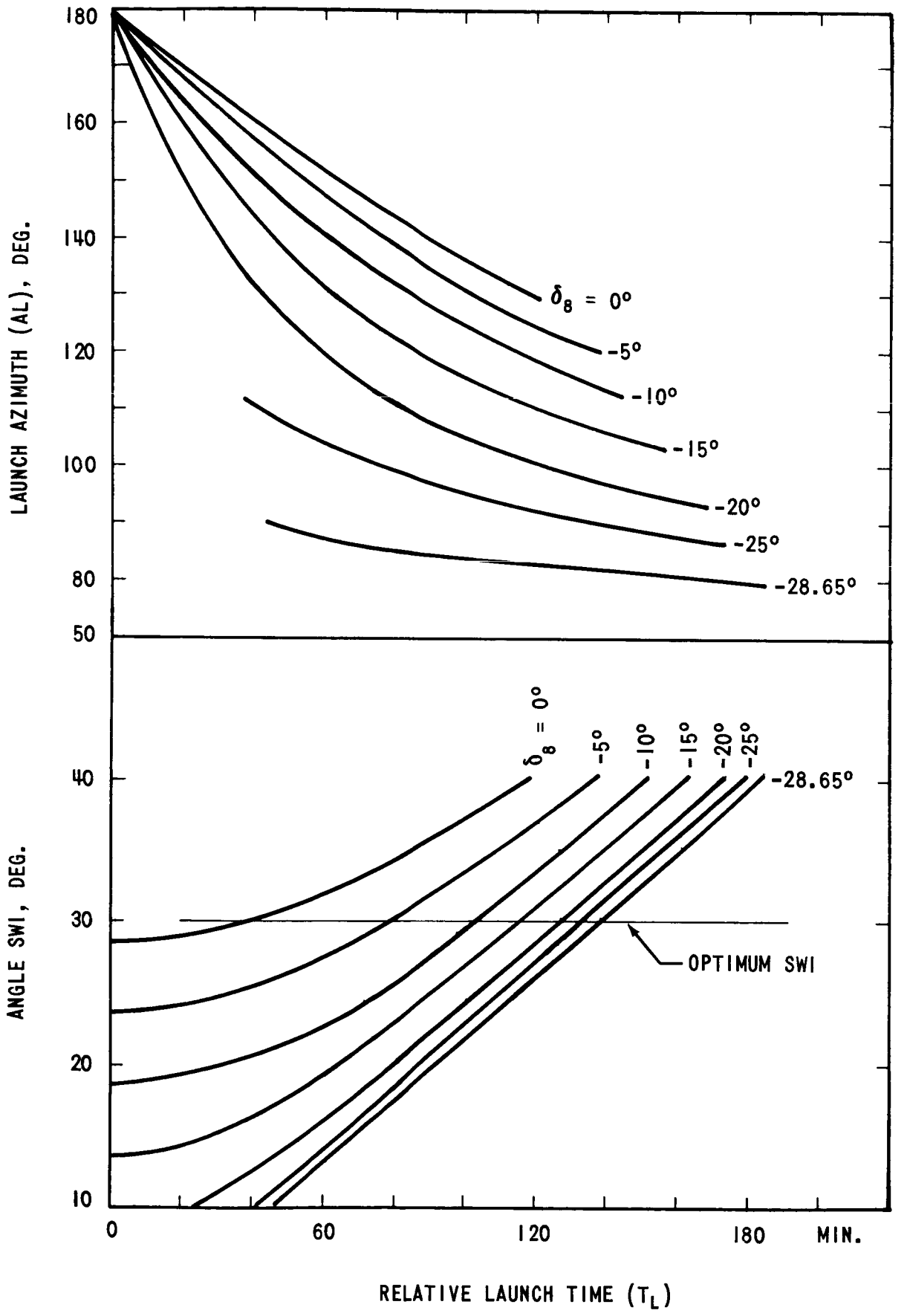

FIGURE 5 RELATION BETWEEN SWI, AL, T_L , AND δ_8

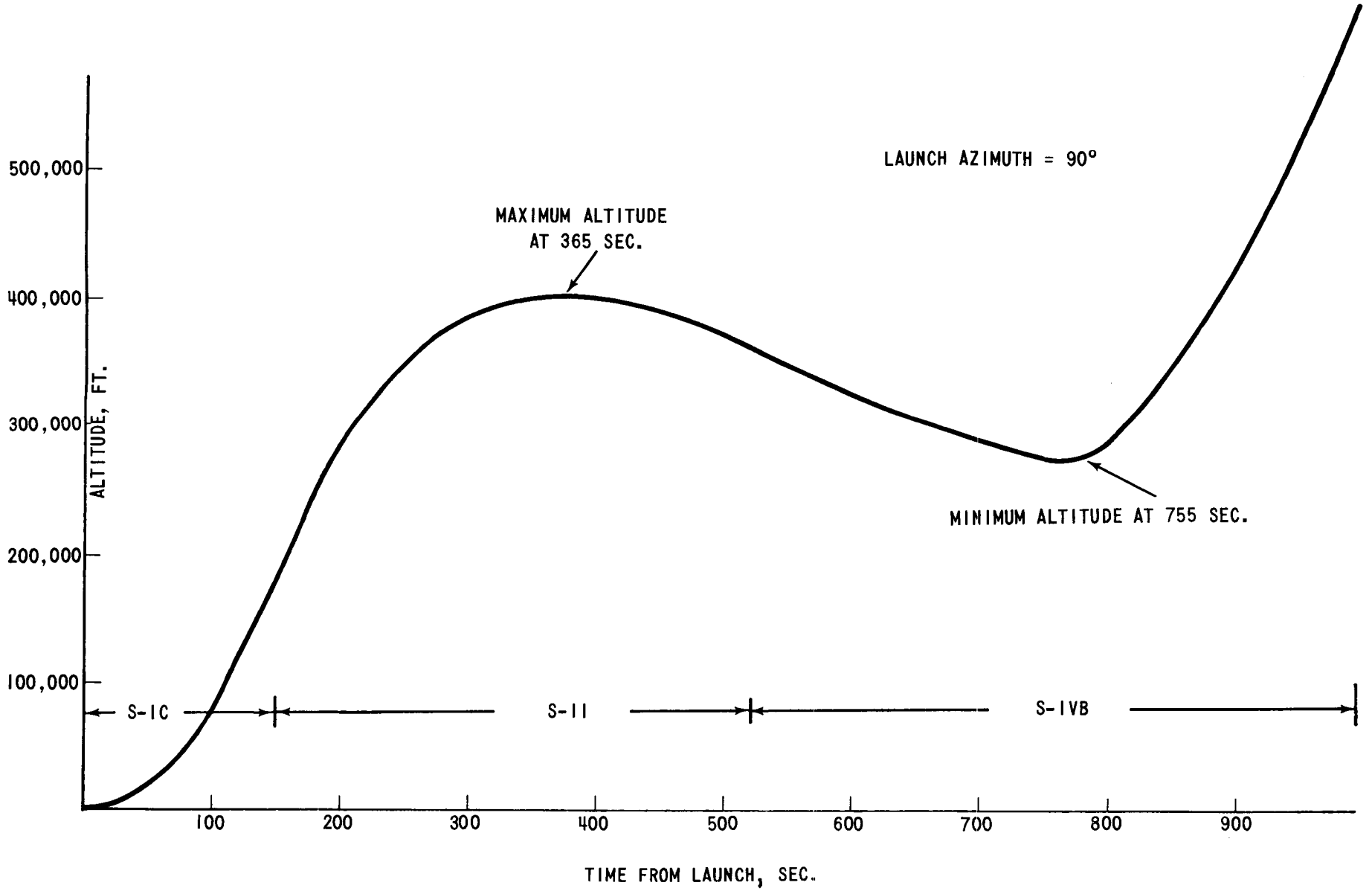

FIGURE 6 - TYPICAL DIRECT ASCENT TRAJECTORY

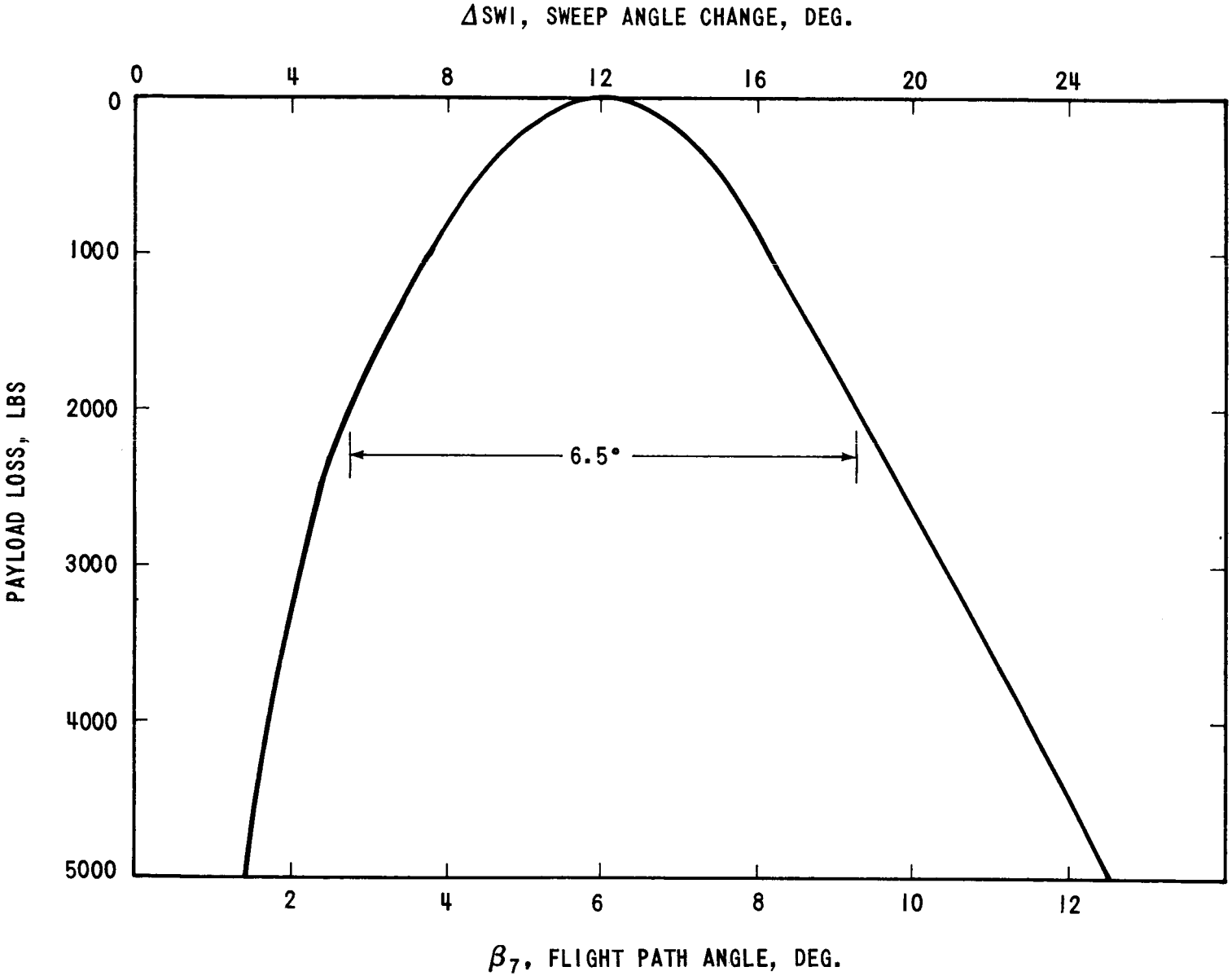

FIGURE 7 EFFECT OF CUT-OFF FLIGHT PATH ANGLE AND SWEEP ANGLE CHANGE ON PAYLOAD LOSS

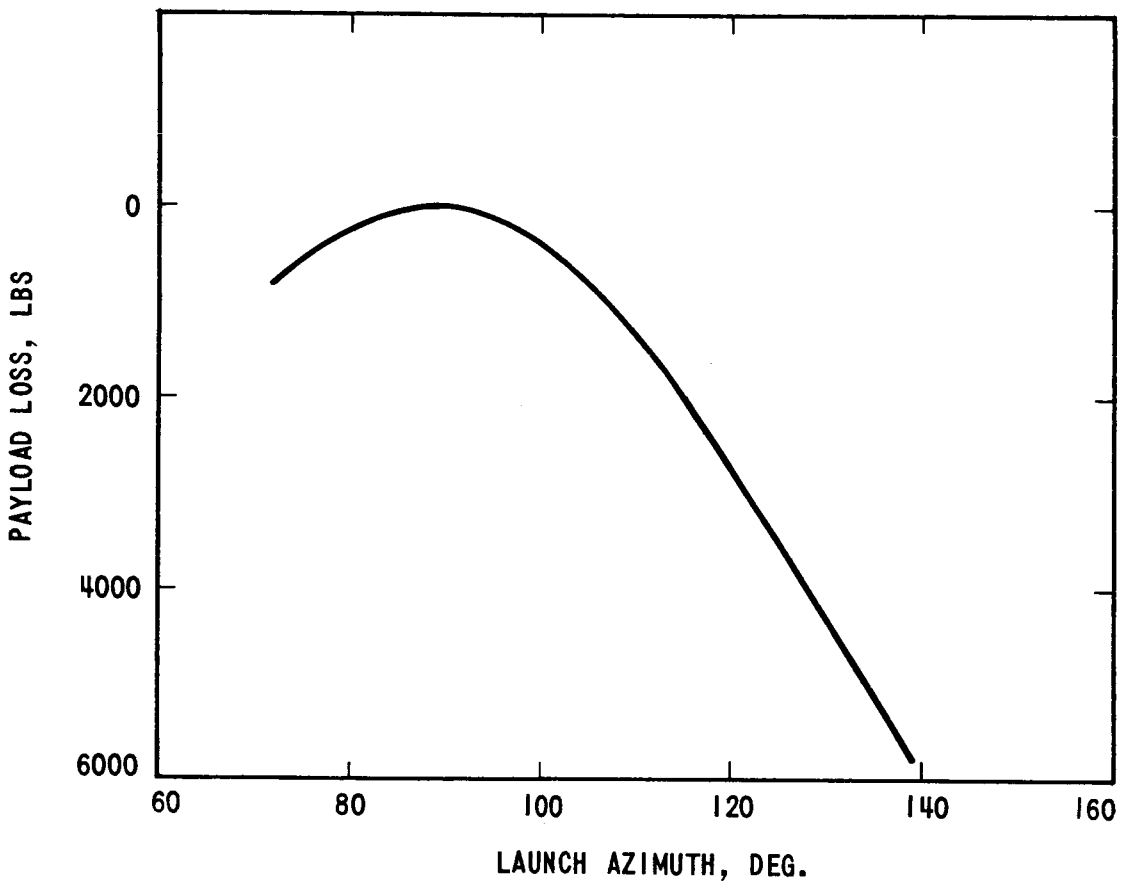

FIGURE 8 EFFECT OF LAUNCH AZIMUTH ON PAYLOAD LOSS

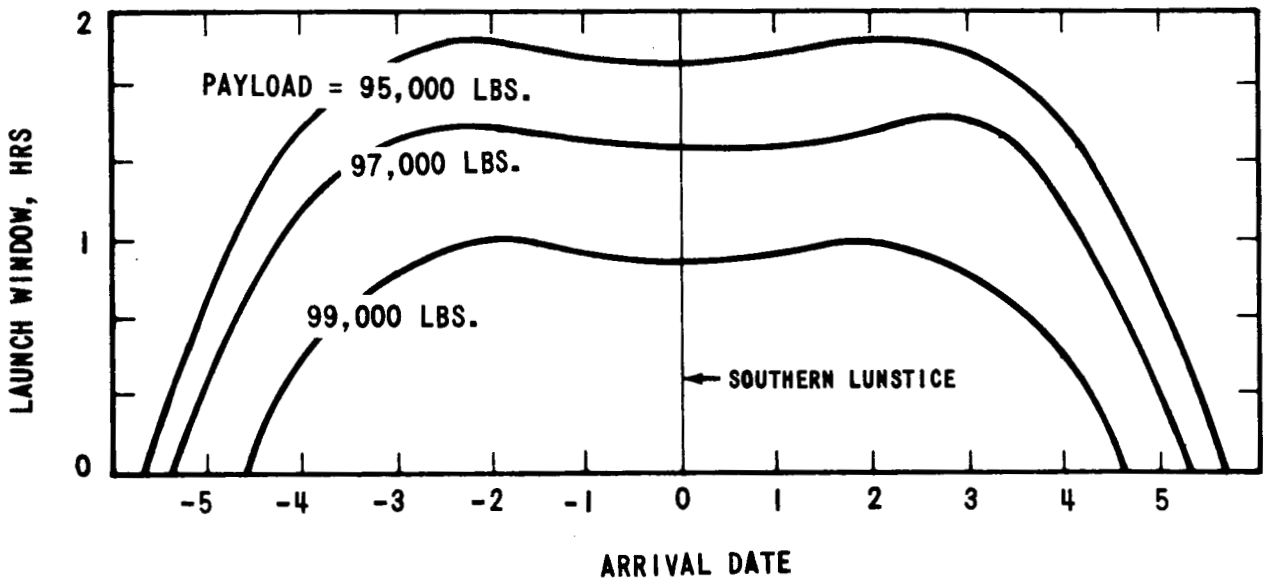

FIGURE 9 RELATIONSHIP BETWEEN LAUNCH WINDOW AND ARRIVAL DATE

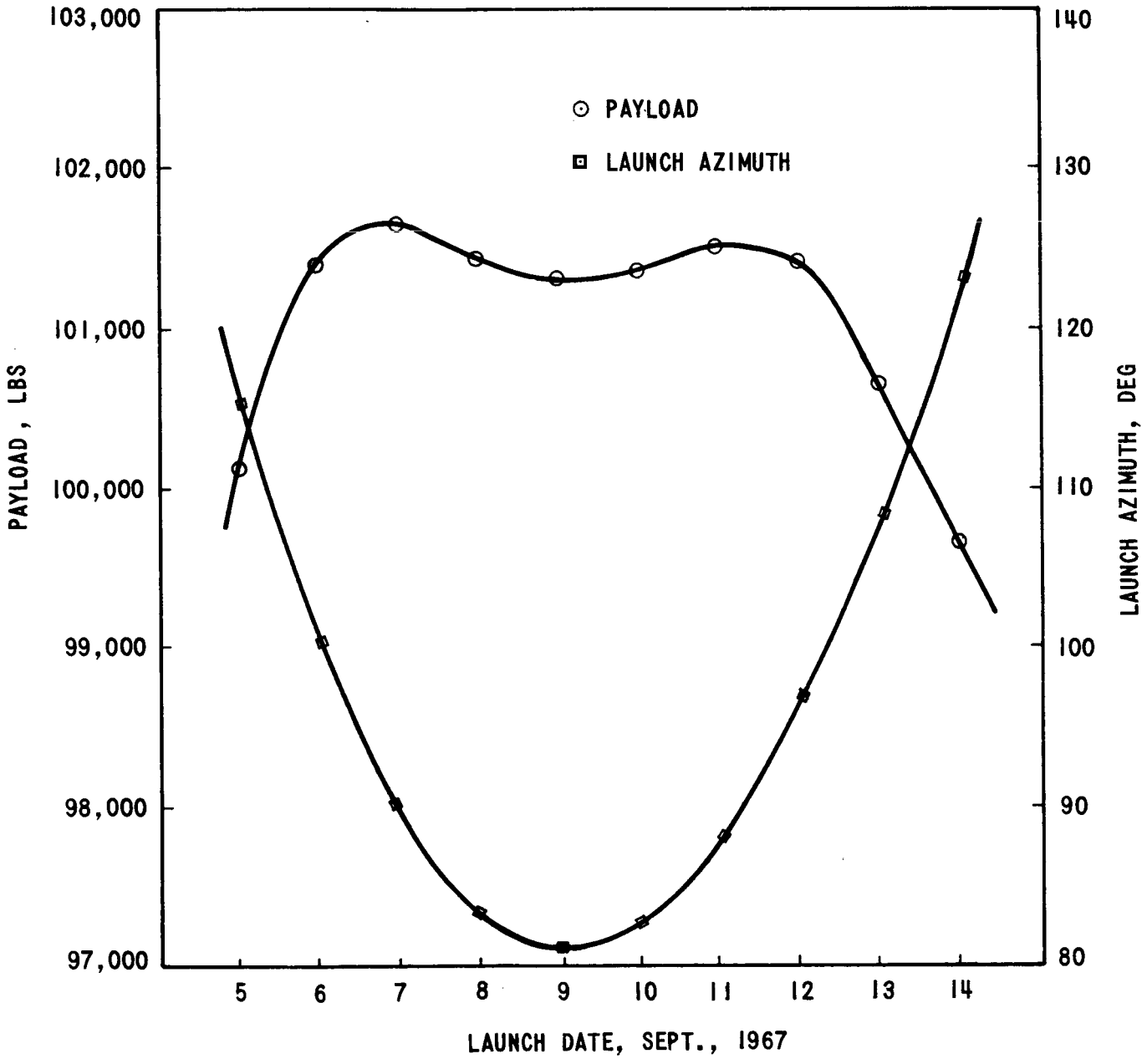

FIGURE 10 LAUNCH OPPORTUNITIES FOR SEPT., 1967

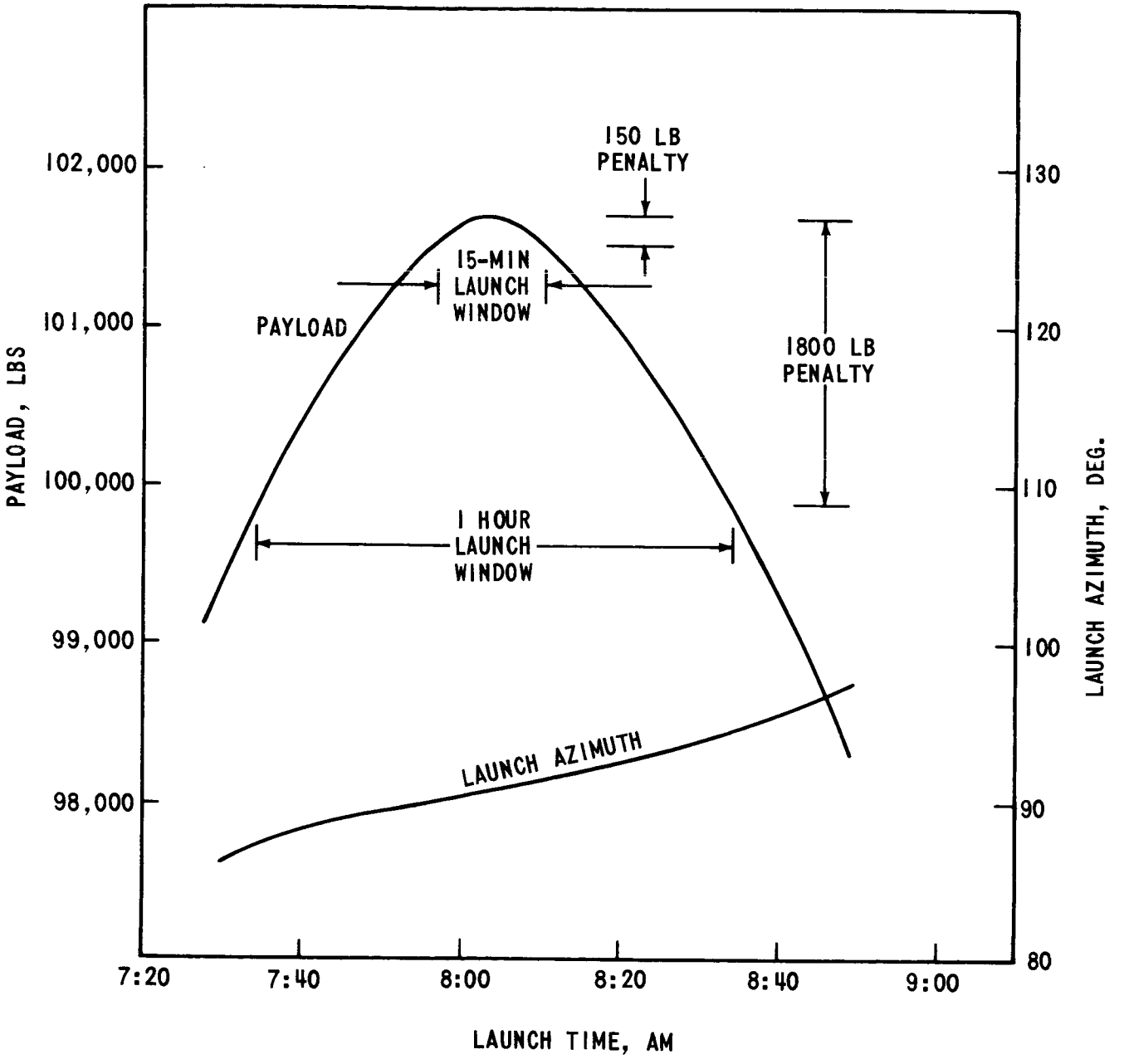

FIGURE 11 LAUNCH WINDOW FOR SEPT. 7, 1967