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BELLCOMM, INC.

Reference No. 65-2023

SUBJECT: Sensitivity Matrix Data for LEM Ascent to 50,000 Foot Orbit (U) Case 209

DATE: March 3, 1965

FROM: I. Bogner

ABSTRACT

[redacted]

A set of perturbation equations is presented for a nominal LEM ascent in which deviations in state at a fixed time after cutoff are related to initial deviations. Numerical values are given which relate estimated and actual position and velocity, and weight to 34 independent variables.

(NASA-CR-78589) SENSITIVITY MATRIX DATA FOR LEM ASCENT TO 50,000 FOOT ORBIT (Bellcomm, Inc.) 16 p

Author

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BELLCOMM, INC. Reference No. 65-2023

SUBJECT: Sensitivity Matrix Data for LEM Ascent to 50,000 Foot Orbit (U) Case 209

DATE: March 3, 1965

FROM: I. Bogner

MEMORANDUM FOR FILE

GENERAL DISCUSSION

This memo presents the matrix of partials for LEM ascent relating deviations in dependent variables at a fixed time shortly after engine cutoff to deviations in independent variables at the start of the ascent. The variables are as follows:

Dependent (at time T_2)		Independent (at time T_1)	
X_{A2}	Actual position Actual velocity	X_{A1}	Actual position Actual velocity
X_{E2}	Estimated position Estimated Velocity	X_{E1}	Estimated position Estimated velocity
W_2	Weight	P_{PF}	Vehicle performance parameters
		P_S	Sensor performance parameters

To carry out this study a nominal closed-loop ascent was simulated in which all (independent) deviations were set to zero. The nominal simulation consisted of an equatorial launch at an azimuth of 270° to a 50,000 foot orbit.

Lunar constants and missile parameters used in the simulation were the same as those listed in Reference 1. The guidance equations are the explicit type described in References 2 and 3. These equations attempt to simultaneously null radius, radial rate and horizontal velocity errors. Actual cutoff takes place when the horizontal velocity error goes to zero.

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The matrix of partials are obtained by considering deviations from the reference at time T_2 in response to perturbations at time T_1. Thus one can write a set of perturbation equations, essentially a Taylor series expansion, about the reference trajectory. In matrix notation with the δ's indicating deviations from the nominal, the equations take the form

$$\delta X_{A2} = \phi_{AA}\delta X_{A1} + \phi_{AE}\delta X_{E1} + \phi_{AP}\delta P \tag{1}$$

$$\delta X_{E2} = \phi_{EA}\delta X_{A1} + \phi_{EE}\delta X_{E1} + \phi_{EP}\delta P \tag{2}$$

$$\delta W_2 = W_A\delta X_{A1} + W_E\delta X_{E1} + W_P\delta P \tag{3}$$

$$\delta P = \delta P_{PF} + \delta P_S \tag{4}$$

δX_A and δX_E are 6 x 1 column matrices, the deviation state vectors. δP is a N x 1 column matrix where N is the total number of vehicle performance and sensor parameters of interest. Note that δP is the sum of performance and sensor deviations.

The ϕ matrices, the matrices of partials, are determined experimentally by making a series of closed-loop simulations, one simulation for each variable to be perturbed. From the equations one notes that:

$$\phi_{AA} = \left.\frac{\delta X_{A2}}{\delta X_{A1}}\right|_{\delta X_{E1} = \delta P = 0} \tag{5}$$

$$\phi_{EA} = \left.\frac{\delta X_{E2}}{\delta X_{A1}}\right|_{\delta X_{E1} = \delta P = 0} \tag{6}$$

$$W_A = \left.\frac{\delta W_2}{\delta X_{A1}}\right|_{\delta X_{E1} = \delta P = 0} \tag{7}$$

Similar equations can be written for the other terms. Equations 1, 2 and 3 may be combined into one matrix equation.

$$\underbrace{\begin{bmatrix} \delta X_{A2} \\ \delta X_{E2} \\ \delta W_2 \end{bmatrix}}_{13\times 1} = \underbrace{\begin{bmatrix} \phi_{AA} & \phi_{AE} & \phi_{AP} \\ \phi_{EA} & \phi_{EE} & \phi_{EP} \\ W_A & W_E & W_P \end{bmatrix}}_{13\times[6+6+N]} \underbrace{\begin{bmatrix} \delta X_{A1} \\ \delta X_{E1} \\ \delta P \end{bmatrix}}_{[6+6+N]\times 1} \quad (8)$$

This ϕ matrix of 13 rows by (12 + N) columns in the matrix of interest, to be used to propagate errors through the powered flight portion of the LEM ascent.

In this analysis δX_{A1} and δX_{E1} were considered to be 3 x 1 column matrices* in which only initial position deviations were assumed. Implicit in this approach is the assumption that the affect of initial velocity uncertainties is negligibly small. The ϕ matrix is therefore reduced to a 13 x (3+3+N) matrix with N in this case being 28. Thus the generation of the matrix required 34 separate perturbation simulations.

Error Sources

The 34 error sources are listed in Table 1A along with their assumed 1σ values. To improve computational accuracy 2σ perturbation were used in the ϕ computations. The units given are the units used in computing the individual terms in the ϕ matrix.

Coordinates

Computation of the terms was carried out in the orbit plane coordinates of the nominal simulation at the reference times. These are defined as follows:

---

*Instead of 6 x 1 matrix.

$\bar{V}$ = vehicle velocity vector

$\bar{R}$ = vehicle position vector

$1_N = \dfrac{\bar{R}}{|\bar{R}|}$ unit vector in the N or nominal direction.

$1_T = \dfrac{\bar{V} \times \bar{R}}{|\bar{V} \times \bar{R}|}$ unit vector in the T or track direction.

$1_R = 1_N \times 1_T$ unit vector in the R (down) range direction.

The dependent variables, listed in Table 1B are N,T,R and $\dot{N},\dot{T},\dot{R}$ corresponding to vehicle orbit plane position and velocity components along 1_N, 1_T and 1_R. A subscript A or E indicates whether the term is an actual or estimated variable.

Description of Presented Data

In order to facilitate printing out the data in a simple and compact form the dependent variables are arranged by column with the independent variables as the rows. Table 2 is the data for the ϕ_A and W matrices and Table 3 is the data for the ϕ_E matrices both corresponding to $+2\sigma$ perturbations. Tables 4 and 5 are the corresponding data for -2σ perturbations. A key to the information in the tables may be found in Figures 1 and 2.

I. Bogner

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I. Bogner

Attachments
1. References
2. Tables 1 thru 5
3. Figure 1 and 2

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REFERENCES

1. STL Report, "Lunar Orbit Rendezvous Reference Trajectory Data Package - Issue 4", dated August 26, 1964

2. "A Class of Unified Explicit Methods for Steering Throttleable and Fixed Thrust Rockets, G. W. Cherry (Preprint 63-335, AIAA G&C Conference 8/12-14/63)

3. "A Unified Explicit Technique for Performing Orbital Insertion, Soft Landing, and Rendezvous with a Throttleable Rocket - Propelled Space Vehicle", G. W. Cherry (Paper presented to AIAA G&C Conference, August 12-14, 1963)

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TABLE I

A Independent Variables

Perturbation No.	Perturbation	Value
1	Misalignment about X-axis	.0286 degrees
2	Misalignment about Z-axis	.0286 degrees
3	Misalignment about Y-axis	.0286 degrees
4	Bias drift about X-axis*	.15 deg/hr
5	Bias drift about Z-axis*	.15 deg/hr
6	Bias drift about Y-axis*	.15 deg/hr
7	Acceleration sensitive drift about the X-axis due to accelerations along the X-axis	.15 deg/hr/G
8	Acceleration sensitive drift about the X-axis due to accelerations along the Y-axis	.15 deg/hr/G
9	Acceleration sensitive drift about the Z-axis due to accelerations along the X-axis	.15 deg/hr/G
10	Acceleration sensitive drift about the Z-axis due to accelerations along the Y-axis	.15 deg/hr/G
11	Acceleration sensitive drift about the Y-axis due to accelerations along the X-axis	.15 deg/hr/G
12	Acceleration sensitive drift about the Y-axis due to accelerations along the Y-axis	.15 deg/hr/G
13	X- accelerometer scale factor	110 PPM
14	Y- accelerometer scale factor	110 PPM
15	X- accelerometer bias	.0002 G
16	Z- accelerometer bias	.0002 G
17	Y- accelerometer bias	.0002 G
18	X- accelerometer linearity	10 PPM/G
19	Y- accelerometer linearity	10 PPM/G

*Alignment time = 1800 seconds prior to launch

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Table I (Cont'd)

Perturbation No.	Perturbation	Value	
20	Nonorthogonality of Z-accelerometer input axis to Z-axis, in Z X-plane	.1	MR
21	Nonorthogonality of Z-accelerometer input axis to Z-axis, in Z Y-plane	.1	MR
22	Nonorthogonality of Y-accelerometer input axis to X-axis, in X Y-plane	.1	MR
23	Nonorthogonality of Y-accelerometer input axis to Y-axis, in Y X-plane	.1	MR
24	Launch azimuth error	.25	deg.
25	Actual altitude deviation	1000.	ft.
26	Actual cross range deviation	2000.	ft.
27	Actual down range deviation	2000.	ft.
28	Thrust increase ($\dot{W}$ increase)	2.	percent
29	Thrust increase (Isp increase)	2.	percent
30	$\dot{W}$ increase (Thrust unchanged)	2.	percent
31	Initial weight	20	pounds
32	Estimated altitude deviation	1000	ft.
33	Estimated cross range deviation	2000	ft.
34	Estimated down range deviation	2000	ft.

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TABLE I (cont'd)

B Dependent Variables

1	Actual normal position component	N_A	ft.
2	Actual track position component	T_A	ft.
3	Actual range position component	R_A	ft.
4	Actual normal velocity component	$\dot{N}_A$	ft/sec
5	Actual track velocity component	$\dot{T}_A$	ft/sec
6	Actual range velocity component	$\dot{R}_A$	ft/sec
7	Estimated normal position component	N_E	ft.
8	Estimated track position component	T_E	ft.
9	Estimated range position component	R_E	ft.
10	Estimated normal velocity component	$\dot{N}_E$	ft/sec
11	Estimated track velocity component	$\dot{T}_E$	ft/sec
12	Estimated range velocity component	$\dot{R}_E$	ft/sec
13	Weight of vehicle	W	lbs.

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THE MATRIX OF PARTIALS

Table 2 (+ 2 σ perturbations) [redacted]TIAL

	N_A	T_A	R_A	$\dot{N}_A$	$\dot{T}_A$	$\dot{R}_A$	W
1	7.2115384E 01	7.8708426E 03	4.0974650E 00	1.5428183E 00	2.1197185E 01	-2.7743253E-02	-1.1844235E-01
2	1.7590690E 04	0.	-1.0238199E 04	8.5107896E 01	-0.	-3.8849090E 01	9.6034336E-01
3	-7.2115384E 01	1.8930245E 04	-4.0974650E 00	-3.7752165E 00	9.4423321E 01	-1.0670482E-02	-3.3932132E-01
4	-4.7708333E 01	4.5135633E 03	-1.0937500E 00	-2.0408122E 00	1.3096883E 01	4.5979817E-02	-1.6235351E-01
5	1.0405000E 04	0.	-5.7654166E 03	5.7817382E 01	-0.	-2.4229736E 01	1.8143717E 00
6	-3.0416667E 01	1.1060743E 04	-1.7916667E 01	-1.1465454E 00	5.8116800E 01	-1.5828450E-01	-1.8920898E-02
7	-6.3958333E 01	2.1101517E 02	2.0312500E 00	-2.6254018E 00	1.0420203E 00	5.8797200E-02	-1.7720540E-01
8	6.8124999E 01	-8.3026933E 01	6.2499999E-01	2.5716909E 00	-2.3886016E-01	-5.5541992E-02	1.3977051E-01
9	5.3812499E 02	0.	-2.2130208E 02	3.1138356E 00	-0.	-1.8589274E 00	3.9306640E-01
10	-2.2895833E 02	0.	9.7916666E 01	-2.5101471E 00	-0.	4.6793620E-01	-1.2695312E-01
11	-3.3958333E 01	6.0332566E 02	-3.1250000E-01	-1.4633942E 00	4.5233533E 00	3.0924479E-02	-1.1230469E-01
12	-2.3541667E 01	-2.1095363E 02	1.0937500E 00	-1.1050415E 00	-1.0378377E 00	2.3803711E-02	-8.2397460E-02
13	-1.4602273E-01	0.	-1.1093750E-01	1.9697709E-05	-0.	-5.3622159E-03	3.3927224E-03
14	-4.6875000E-01	0.	3.0681818E-01	-2.0278931E-03	-0.	2.6134144E-04	3.7730824E-04
15	-5.2499999E 05	0.	-2.7152343E 05	-3.2638549E 03	-0.	-1.2792816E 04	8.0944824E 03
16	-1.8125000E 04	-3.0564950E 06	-1.6406250E 03	-7.4005126E 02	-1.2987115E 04	1.6174316E 01	-5.5541992E 01
17	3.1620312E 06	0.	-1.7576953E 06	1.5787315E 04	-0.	-2.5466919E 03	-2.2940063E 03
18	-9.3749999E-03	0.	-4.9218750E-02	-1.8230438E-03	-0.	-2.3529053E-03	8.3618164E-04
19	-1.6562500E-01	0.	-7.0312500E-02	-1.1309433E-02	-0.	2.1972656E-04	-1.0986328E-03
20	-2.5312500E 01	1.0828125E 03	-2.3437500E-01	-1.1859131E 00	5.4011450E 00	2.7160645E-02	-8.0871582E-02
21	-3.0312500E 01	4.5019730E 02	7.0312500E-01	-1.3987350E 00	1.2119270E 00	3.2043457E-02	-1.1047363E-01
22	-1.4687500E 02	0.	-2.2890625E 01	-3.1159592E 00	-0.	-1.1270142E 00	6.1462402E-01
23	1.1709375E 03	0.	-5.5906250E 02	8.2069397E 00	-0.	-1.0848999E 00	-5.9600830E-01
24	2.6125000E 01	7.5774300E-01	7.2406250E 01	7.8660583E-01	2.9612280E-02	-1.7456055E-02	2.8564453E-01
25	1.1640000E 00	0.	-2.1407812E-01	8.4794235E-04	-0.	-8.5693359E-05	-1.8310547E-07
26	2.5625000E-03	9.1193025E-01	2.3437500E-05	8.9694976E-05	-3.9259050E-04	-2.0751953E-06	4.6997070E-06
27	1.7737500E-01	0.	8.9512109E-01	-8.4800720E-06	-0.	-3.9056396E-04	-9.1552734E-08
28	-3.9390625E 01	0.	1.3715105E 04	-1.2442760E 01	-0.	-8.1481934E-02	2.7443237E 00
29	-2.8640625E 01	0.	1.1295867E 04	-1.0363207E 01	-0.	-5.4367065E-02	3.6290695E 01
30	4.2968750E 00	0.	2.6171719E 03	-2.2744789E 00	-0.	-8.3160400E-03	-3.4043442E 01
31	1.5625000E-03	0.	-1.3707109E 02	1.2952556E-01	-0.	-6.2561035E-05	-4.8762665E-01
32	-1.1645000E 00	0.	1.0158281E 00	-1.7274017E-03	-0.	8.8745117E-05	1.5425110E-03
33	1.0937500E-04	-9.1158725E-01	-6.1289062E-03	-5.2986145E-06	4.0029950E-04	2.4414062E-07	-1.8646240E-05
34	-1.7650000E-01	0.	8.6050780E-02	-9.1458129E-04	-0.	3.9051819E-04	-1.5365601E-05

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THE MATRIX OF PARTIALS Table 3 (+ 2 σ perturbations) [redacted]AL

	N_E	T_E	R_E	$\dot{N}_E$	$\dot{T}_E$	$\dot{R}_E$	
1	5.5725524E 01	5.1475105E 02	7.3754370E 00	1.5402841E 00	2.2499196E 01	-4.4816024E-02	0.
2	1.8400350E 03	0.	2.1935096E 02	8.0383941E 01	-0.	-4.0014307E 01	0.
3	-8.5227272E 01	2.2058374E 03	1.6389860E 01	-3.7748163E 00	9.6419562E 01	-1.2804578E-02	0.
4	-5.0833333E 01	3.1603009E 02	-4.1666666E-01	-2.0413462E 00	1.3816357E 01	4.2521159E-02	0.
5	1.2947917E 03	0.	2.4203125E 02	5.5176366E 01	-0.	-2.4877116E 01	0.
6	-2.8125000E 01	1.3555037E 03	1.2187500E 01	-1.1477152E 00	5.9260233E 01	-1.6499837E-01	0.
7	-6.7291666E 01	2.4326143E 01	2.7083333E 00	-2.6259104E 00	1.0649407E 00	5.5541992E-02	0.
8	6.4791666E 01	-5.7302033E 00	1.3020833E 00	2.5711059E 00	-2.5038296E-01	-5.8797200E-02	0.
9	6.8958333E 01	0.	5.8229167E 01	3.0140940E 00	-0.	-1.8841553E 00	0.
10	-6.0000000E 01	0.	-1.2291667E 01	-2.4676513E 00	-0.	4.7668457E-01	0.
11	-3.6458333E 01	1.0452887E 02	3.6458333E 00	-1.4633433E 00	4.5739466E 00	3.0924479E-02	0.
12	-2.6666667E 01	-2.4263346E 01	1.7708333E 00	-1.1050161E 00	-1.0607580E 00	2.3600260E-02	0.
13	5.1988636E-02	0.	9.5262784E-01	1.0757099E-03	-0.	-6.1590021E-05	0.
14	-8.8068182E-03	0.	2.1924716E-01	-6.2314813E-04	-0.	7.2132457E-06	0.
15	4.9531250E 04	0.	2.7315234E 06	-7.0268630E 02	-0.	-4.1961669E 01	0.
16	-2.0468750E 04	2.6423250E 02	-1.1328125E 03	-7.4001311E 02	1.0106857E 01	1.6174316E 01	0.
17	1.9218750E 04	0.	-1.1691797E 06	1.7731666E 03	-0.	-1.7089844E 01	0.
18	2.8125000E-02	0.	4.0234375E-01	-1.3721466E-03	-0.	3.0517578E-06	0.
19	-2.5312500E-01	0.	-1.5625000E-03	-1.1611938E-02	-0.	2.2583008E-04	0.
20	-3.0312500E 01	-5.4745200E-02	7.8125000E-01	-1.1866760E 00	-2.1122120E-03	2.2277832E-02	0.
21	-3.5312500E 01	1.6901095E-02	1.7187500E 00	-1.3985443E 00	7.5899250E-04	3.2348633E-02	0.
22	-6.2500000E 01	0.	4.1921875E 02	-2.8657532E 00	-0.	5.2185059E-02	0.
23	5.5312500E 01	0.	-3.5148437E 02	2.5049973E 00	-0.	-5.0048828E-02	0.
24	2.2125000E 01	7.5688580E-01	7.3156250E 01	7.8627014E-01	2.9612260E-02	-1.9531250E-02	0.
25	0.	0.	0.	0.	-0.	0.	0.
26	2.3125000E-03	0.	7.4218749E-05	8.9208603E-05	-0.	-1.9989014E-06	0.
27	0.	0.	0.	0.	-0.	0.	0.
28	-3.8656250E 01	0.	1.3714969E 04	-1.2442404E 01	-0.	-8.2305908E-02	0.
29	-2.8390625E 01	0.	1.1295820E 04	-1.0363049E 01	-0.	-5.4916382E-02	0.
30	4.0625000E 00	0.	2.6172187E 03	-2.2742271E 00	-0.	-8.3618164E-03	0.
31	-7.1874999E-02	0.	-1.3705703E 02	1.2949695E-01	-0.	-8.3923340E-05	0.
32	-2.5312500E-03	0.	8.0164062E-01	-8.8012695E-04	-0.	2.1972656E-06	0.
33	-3.7500000E-04	3.3538750E-04	-6.0312500E-03	-5.2947998E-06	7.7274050E-06	2.2888183E-07	0.
34	1.2500000E-04	0.	9.8131250E-01	-9.2339516E-04	-0.	-4.8828124E-07	0.

[redacted]

THE MATRIX OF PARTIALS

Table 4 (-2 σ perturbations) [redacted]

	N_A	T_A	R_A	$\dot{N}_A$	$\dot{T}_A$	$\dot{A}$	$\dot{W}$
1	-7.2115384E 01	7.8708426E 03	-4.0974650E 00	-1.5428183E 00	2.1197185E 01	2.7743253E-02	1.1844235E-01
2	1.8038680E 04	-0.	-1.0216619E 04	1.0472331E 02	0.	-3.9113718E 01	2.4211323E 00
3	7.2115384E 01	1.8930245E 04	4.0974650E 00	3.7752165E 00	9.4423321E 01	1.0670482E-02	3.3932132E-01
4	4.7708333E 01	4.5135633E 03	1.0937500E 00	2.0408122E 00	1.3096883E 01	-4.5979817E-02	1.6235351E-01
5	1.0333125E 04	-0.	-5.7144271E 03	5.5720977E 01	0.	-2.3816732E 01	1.6750081E 00
6	3.0416667E 01	1.1060743E 04	1.7916667E 01	1.1465454E 00	5.8116800E 01	1.5828450E-01	1.8920898E-02
7	6.3958333E 01	2.1101517E 02	-2.0312500E 00	2.6254018E 00	1.0420203E 00	-5.8797200E-02	1.7720540E-01
8	-6.8124999E 01	-8.3026933E 01	-6.2499999E-01	-2.5716909E 00	-2.3886016E-01	5.5541992E-02	-1.3977051E-01
9	5.0395833E 02	-0.	-2.2156250E 02	2.1304576E 00	0.	-1.8377685E 00	3.6763509E-01
10	-1.7812500E 02	-0.	9.8385416E 01	-7.7641805E-02	0.	4.1178385E-01	6.2052409E-02
11	3.3958333E 01	6.0332566E 02	3.1250000E-01	1.4633942E 00	4.5233533E 00	-3.0924479E-02	1.1230469E-01
12	2.3541667E 01	-2.1095363E 02	-1.0937500E 00	1.1050415E 00	-1.0378377E 00	-2.3803711E-02	8.2397460E-02
13	-2.4715909E-01	-0.	-1.0568182E-01	-3.0145818E-03	0.	-5.2839799E-03	3.2811945E-03
14	-4.2272727E-01	-0.	3.0724432E-01	5.2778070E-04	0.	1.9586736E-04	6.0757723E-04
15	-5.5703124E 05	-0.	-2.6546875E 05	-3.8452530E 03	0.	-1.2798920E 04	8.1446838E 03
16	1.8125000E 04	-3.0564950E 06	1.6406250E 03	7.4005126E 02	-1.2987115E 04	-1.6174316E 01	5.5541992E 01
17	3.0917187E 06	-0.	-1.7519531E 06	1.3449058E 04	0.	-2.4903869E 03	-2.3797607E 03
18	-3.6250000E-01	-0.	-3.2812500E-02	-9.8838806E-03	0.	-2.1881103E-03	1.1932373E-03
19	-2.1562500E-01	-0.	-6.0937500E-02	-8.7280273E-03	0.	1.3732910E-04	-3.0517578E-04
20	2.5312500E 01	1.0828125E 03	2.3437500E-01	1.1859131E 00	5.4011450E 00	-2.7160645E-02	8.0871582E-02
21	3.0312500E 01	4.5019730E 02	-7.0312500E-01	1.3987350E 00	1.2119270E 00	-3.2043457E-02	1.1047363E-01
22	-5.0937500E 01	-0.	-2.5937500E 01	1.3224411E 00	0.	-1.2341309E 00	9.7503662E-01
23	9.9812500E 02	-0.	-5.5664062E 02	2.1427155E 00	0.	-9.6282959E-01	-8.8714600E-01
24	-2.6125000E 01	7.5774300E-01	-7.2406250E 01	-7.8660583E-01	2.9612280E-02	1.7456055E-02	-2.8564453E-01
25	1.1619375E 00	-0.	-2.1420312E-01	7.9558181E-04	0.	-8.5357666E-05	-1.0070801E-06
26	-2.5625000E-03	9.1193025E-01	-2.3437500E-05	-8.9694976E-05	-3.9259050E-04	2.0751953E-06	-4.6997070E-06
27	1.7632812E-01	-0.	8.9506250E-01	-3.2602310E-05	0.	-3.9028930E-04	-3.8146973E-07
28	8.0062500E 01	-0.	1.5117264E 04	-1.4073784E 01	0.	6.9381714E-02	3.3118439E 00
29	5.0500000E 01	-0.	1.2231074E 04	-1.1614565E 01	0.	4.8873901E-02	3.8728500E 01
30	5.3593750E 00	-0.	2.6548848E 03	-2.3189030E 00	0.	-2.3956299E-03	-3.4889297E 01
31	2.3437500E-02	-0.	-1.3682109E 02	1.3130512E-01	0.	5.7983398E-05	-4.8632812E 01
32	-1.1671562E 00	-0.	1.0260078E 00	-1.7198334E-03	0.	8.8378906E-05	1.5743408E 03
33	-1.0937500E-04	-9.1158725E-01	6.1289062E-03	5.2986145E-06	4.0029950E-04	-2.4414062E-07	1.8646240E-05
34	-1.7557812E-01	-0.	8.5613281E-02	-8.4940147E-04	0.	3.8931274E-04	-6.5307617E-06

[redacted]

THE MATRIX OF PARTIALS

Table 5 (-2σ perturbations) [redacted]

	N_E	T_E	R_E	N_E	T_E	R_E	
1	-5.5725524E 01	5.1475105E 02	-7.3754370E 00	-1.5402841E 00	2.2499196E 01	4.4816024E-02	-0.
2	2.3328234E 03	-0.	1.9613199E 02	1.0000149E 02	0.	-4.0261862E 01	-0.
3	8.5227272E 01	2.2058374E 03	-1.6389860E 01	3.7748163E 00	9.6419562E 01	1.2804578E-02	-0.
4	5.0833333E 01	3.1603009E 02	4.1666666E-01	2.0413462E 00	1.3816357E 01	-4.2521159E-02	-0.
5	1.2220833E 03	-0.	2.3541667E 02	5.3082351E 01	0.	-2.4450887E 01	-0.
6	2.8125000E 01	1.3555037E 03	-1.2187500E 01	1.1477152E 00	5.9260233E 01	1.6499837E-01	-0.
7	6.7291666E 01	2.4326143E 01	-2.7083333E 00	2.6259104E 00	1.0649407E 00	-5.5541992E-02	-0.
8	-6.4791666E 01	-5.7302033E 00	-1.3020833E 00	-2.5711059E 00	-2.5038296E-01	5.8797200E-02	-0.
9	4.1250000E 01	-0.	5.6562500E 01	2.0344798E 00	0.	-1.8601481E 00	-0.
10	6.2499999E-01	-0.	-1.3750000E 01	-3.4688313E-02	0.	4.2419433E-01	-0.
11	3.6458333E 01	1.0452887E 02	-3.6458333E 00	1.4633433E 00	4.5739466E 00	-3.0924479E-02	-0.
12	2.6666667E 01	-2.4263346E 01	-1.7708333E 00	1.1050161E 00	-1.0607580E 00	-2.3600260E-02	-0.
13	-3.0965909E-02	-0.	9.5447443E-01	-1.9571131E-03	0.	2.5801225E-05	-0.
14	5.5113636E-02	-0.	2.1619318E-01	1.9393227E-03	0.	-4.5221502E-05	-0.
15	2.9687500E 04	-0.	2.7351953E 06	-1.2804413E 03	0.	-2.8381347E 01	-0.
16	2.0468750E 04	2.6423250E 02	1.1328125E 03	7.4001311E 02	1.0106857E 01	-1.6174316E 01	-0.
17	-4.3750000E 04	-0.	-1.1649219E 06	-5.6138992E 02	0.	4.5928955E 01	-0.
18	-2.4062500E-01	-0.	3.5000000E-01	-9.4264983E-03	0.	2.1972656E-04	-0.
19	-2.1562500E-01	-0.	-6.0937500E-02	-9.0244293E-03	0.	1.9226074E-04	-0.
20	3.0312500E 01	-5.4745200E-02	-7.8125000E-01	1.1866760E 00	-2.1122120E-03	-2.2277832E-02	-0.
21	3.5312500E 01	1.6901095E-02	-1.7187500E 00	1.3985443E 00	7.5899250E-04	-3.2348633E-02	-0.
22	4.7812500E 01	-0.	4.1320312E 02	1.5702820E 00	0.	-3.9367676E-02	-0.
23	-1.0281250E 02	-0.	-3.5210937E 02	-3.5575485E 00	0.	8.2397461E-02	-0.
24	-2.2125000E 01	7.5688580E-01	-7.3156250E 01	-7.8627014E-01	2.9612260E-02	1.9531250E-02	-0.
25	-1.5937500E-03	-0.	-2.1093750E-04	-5.3173065E-05	0.	1.4648437E-06	-0.
26	-2.3125000E-03	-0.	-7.4218749E-05	-8.9208603E-05	0.	1.9989014E-06	-0.
27	-7.9687500E-04	-0.	-1.0546875E-04	-2.3300171E-05	0.	6.4086914E-07	-0.
28	8.2031250E 01	-0.	1.5116887E 04	-1.4073248E 01	0.	6.9030762E-02	-0.
29	5.2656250E 01	-0.	1.2230404E 04	-1.1613853E 01	0.	4.8492432E-02	-0.
30	6.3437500E 00	-0.	2.6546953E 03	-2.3186150E 00	0.	-2.1820068E-03	-0.
31	7.1874999E-02	-0.	-1.3683086E 02	1.3130913E-01	0.	8.2397461E-05	-0.
32	-2.6562500E-03	-0.	8.1182812E-01	-8.7189865E-04	0.	2.7160644E-06	-0.
33	3.7500000E-04	3.3538750E-04	6.0312500E-03	5.2947998E-06	7.7274050E-06	-2.2888183E-07	-0.
34	2.2656250E-03	-0.	9.8064062E-01	-8.5867309E-04	0.	-1.3427734E-06	-0.

[redacted]

FIGURE 1. KEY TO TABLES 2 and 4

	DEPENDENT VARIABLES						
	N_A	T_A	R_A	$\dot{N}_A$	$\dot{T}_A$	$\dot{R}_A$	W
1 . . . 12	gyro errors		Φ^T_{AP} (sensors)				W_P^T (sensors)
13 . . . 23	accelerometer errors						
24	launch azimuth error						
25 26 27	initial actual errors			Φ^T_{AA}			W_A^T
28 . . . 31	powered flight errors			Φ^T_{AP} (powered flight)			W_P^T (powered flight)
32 33 34	initial estimate errors			Φ^T_{AE}			W_E^T

The superscript T indicates the transpose of the matrix.

FIGURE 2. KEY TO TABLES 3 and 5

DEPENDENT VARIABLES

	N_E	T_E	R_E	$\dot{N}_E$	$\dot{T}_E$	$\dot{R}_E$
1 . . . 12	gyro errors					
					Φ_{EP} T (sensors)	
13 . . . 23	accelerometer errors					
24	launch azimuth error					
25 26 27	initial actual errors				Φ_{EA} T	
28 . . . 31	powered flight errors					
					Φ_{EP} T (powered flight)	
32 33 34	initial estimated errors				Φ_{EE} T	