Bellcomm

955 L'Enfant Plaza North, S.W.
Washington, D. C. 20024

B71 09030

date: September 30, 1971

to: Distribution

from: T. J. Rudd

subject: A Hydroelastic Model for a General Elliptical Tank - Case 320

ABSTRACT

This memorandum deals with some extensions to Goldman's hydroelastic model for a liquid propellant contained in an elliptical tank. These extensions were specifically required to make the model suitable for use in a vibration study of the Saturn S-IVB stage.

The extended model can handle configurations where the propellant level is above (or below) the lower bulkhead. Also the location of the boundary points (mass points) can be chosen by the user. This greatly simplifies the calculation of the stiffness matrix of the propellant tank.

A computer program, ETANK2, for computing the liquid mass matrix corresponding to this extended model is described and a sample problem is solved using this program.

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MEMORANDUM FOR FILE

INTRODUCTION

In the longitudinal vibration analysis of liquid fueled rockets a hydroelastic model is used to give an accurate representation of the liquid propellant.

A hydroelastic model was developed in [1] for the case of a liquid propellant contained in an elliptical tank. For use in an S-IVB longitudinal vibration study the model was generalized to handle:

(a) liquid levels greater than half full and

(b) variable boundary point (mass point) locations.

The second extension is valuable as it greatly simplifies the stiffness calculation for the propellant tank and, therefore, for the total vehicle.

These extensions and a computer program (ETANK2), written to compute the liquid mass matrix corresponding to this extended model, are described in this memorandum. A sample problem is solved illustrating the use of the program.

THEORETICAL BACKGROUND

Following a development similar to that given in [1] or [2] it can be shown that the liquid mass matrix, M, has the form

$$M = \pi R^3 \rho \, [\overline{G}Q + (\overline{G}Q)^T] \quad . \tag{1}$$

Here R is the maximum tank radius (in a plane parallel to the free surface), ρ is the liquid mass density, $\bar{G}$ is a surface integration matrix and Q is a matrix relating the liquid velocity potential (ϕ) to the normal velocity (v) at the N boundary points, i.e.,

$$\phi_i = \sum_{j=1}^{N} Q_{ij} v_j \quad (i = 1,N) \ . \tag{2}$$

The matrix $\bar{G}$ consists of two parts; a matrix $\bar{G}_1$ which is the surface integration matrix for the lower bulkhead and a matrix $\bar{G}_2$ which is the corresponding matrix for the wetted area of the upper bulkhead. The calculation of $\bar{G}_1$ or $\bar{G}_2$ is parallel to that given in [1] for $\bar{G}$, and will not be discussed here.

The matrix Q is obtained through a finite difference solution of Laplace's equation. In [1] this was carried out using a uniform mesh for the finite difference grid. Consequently the layout of the boundary points (which are determined by the intersection of the grid lines with the tank well) is fixed. In practice this layout is not always convenient for vibration analysis. For instance it may happen that two of the points are relatively close together. This can lead to ill-conditioning of the tank stiffness matrix. Furthermore, every time the liquid level is changed the boundary point locations change and a new stiffness matrix must be computed. In the program ETANK2. these problems are overcome by using a non-uniform mesh for the finite difference grid. The details of how Q is calculated using this type of grid are given in [3].

CAPABILITY OF PROGRAM

The program, ETANK2., is designed to compute the mass matrix for a liquid contained in an elliptical tank of the type shown in Figure 1. The tank consists, in general, of two unequal partial ellipsoids with centers at O1 and O2. The liquid is assumed to be incompressible with a free surface anywhere between the top and bottom of the tank. The boundary point locations can be chosen arbitrarily except when the free surface is in the upper bulkhead. Then boundary

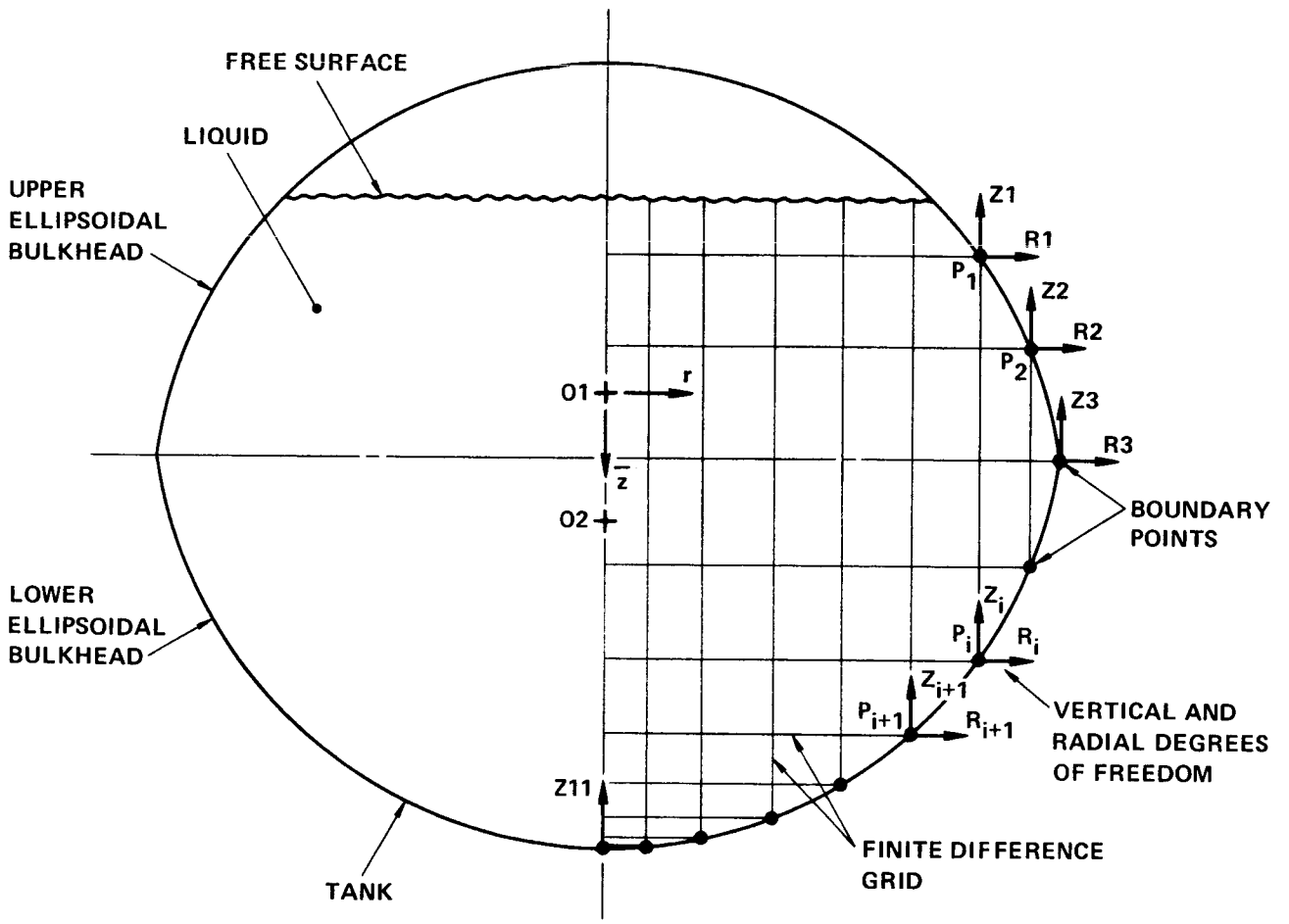


FIGURE 1 - ELLIPTICAL TANK SYSTEM SHOWING BOUNDARY POINTS AND DEGREES OF FREEDOM

points must be assigned on the wetted surface of the upper bulkhead, which are vertically above corresponding points on the lower bulkhead. No boundary point is required at the free surface and the free surface need not intersect the upper bulkhead on a grid line.

PROGRAM DESCRIPTION

A flow chart showing the main steps in the computation is shown in Appendix A. Here the variables λ and z are the normalized values of r and $\bar{z}$ for the chosen boundary points; α and β are the horizontal and vertical mesh distances for the finite difference grid.

In order to conserve computer storage and reduce running time a "Block Gaussian" elimination scheme [3] has been used in the computation of the Q matrix. In this scheme the Q matrix is obtained by inverting a series of matrices whose maximum dimension does not exceed the dimension of Q itself. (In general one has to invert a matrix dimension equal to the total number of grid points to obtain Q.) The programming details for this reduction differ slightly depending upon whether the liquid free surface is in the lower or upper bulkhead. The details for the former are given in Appendix B.

INPUT FORMAT

The input to the program is in the NAMELIST format as follows:

$INPUT

A1 = Semi-major axis of lower ellipsoid,

B1 = Semi-minor axis of lower ellipsoid,

A2 = Semi-major axis of upper ellipsoid,

B2 = Semi-minor axis of upper ellipsoid,

Zϕ2 = Distance of origin O2 below O1,

N = Total number of boundary points plus one for the free surface,

NA = Number of boundary points on the lower ellipsoid (plus one if the free surface is in the lower ellipsoid),

ZEE = Single order array, of dimension N, of boundary point locations below the origin O1. (These must be listed starting with the bottom most value and proceeding around the tank periphary to the free surface value),

WL = Density of liquid.

$END.

A sample problem illustrating the use of the program is shown in Appendix C.

ACKNOWLEDGMENT

I wish to thank J. S. Vandergraft for providing the programming details of Appendix B.

2031-TJR-jf

T. J. Rudd

Attachments
References
Appendices A, B and C

REFERENCES

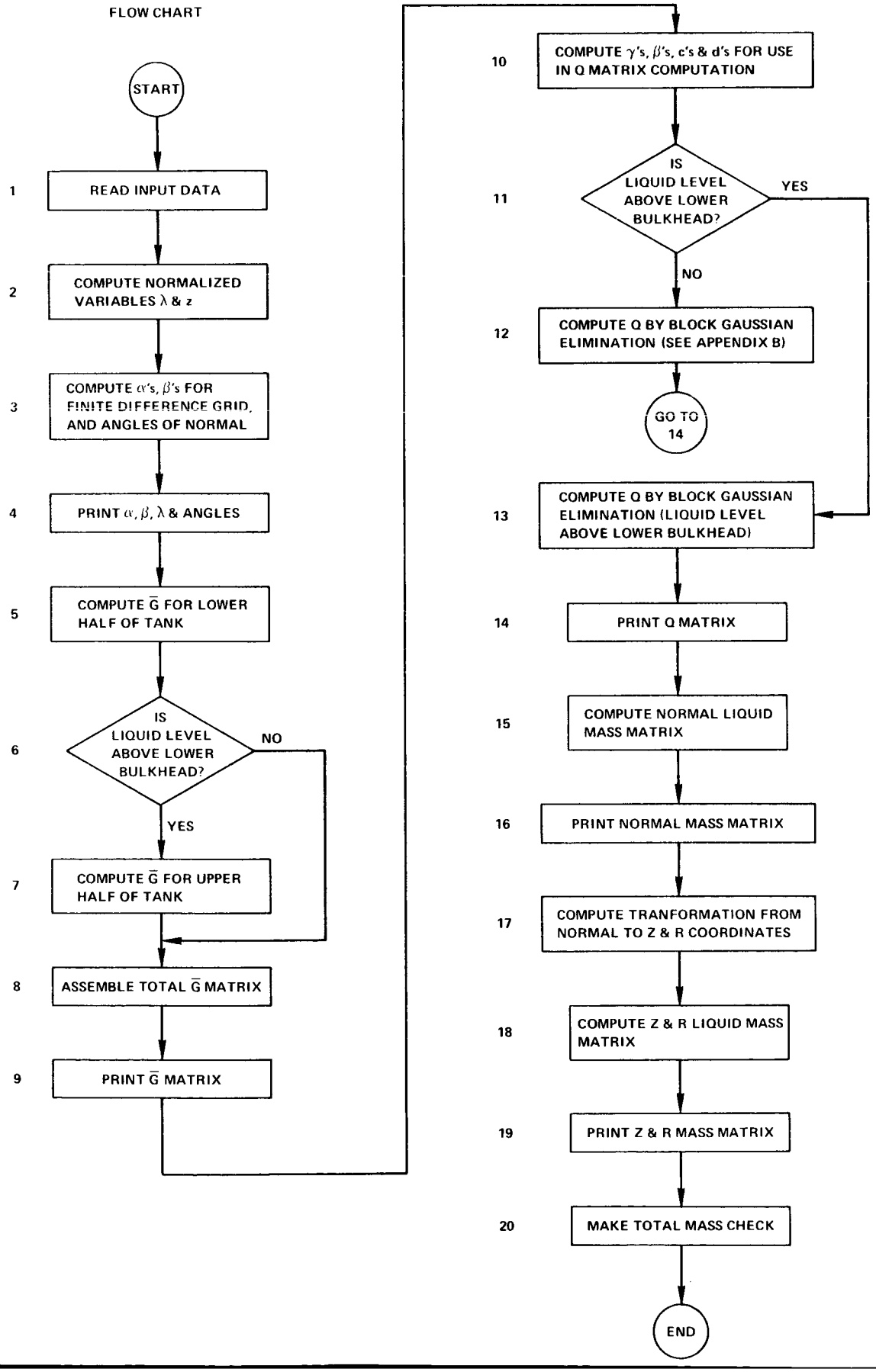
1. Goldman, R. L., "Longitudinal Vibration Analysis of Partially-Filled Ellipsoidal Tanks by Finite Differences", TR 70-6C, RIAS, Baltimore, Maryland, August 1970.

2. Rudd, T. J., "A Hydroelastic Model for a Cylindrical Tank with an Inverted Bulkhead", Bellcomm Memorandum for File B71 09029, Case 320, September 30, 1971.

3. Vandergraft, J. S., "A Finite Difference Method for Solving Laplace's Equation, Using a Non-Uniform Mesh", Bellcomm Technical Memorandum, TM 71-2031-1, Case 320, March 23, 1971.

APPENDIX-A

FLOW CHART



APPENDIX B

BLOCK GAUSSIAN ELIMINATION SCHEME

(Case of Liquid Free Surface in Lower Bulkhead)

The main step in this process is to compute

$$\tilde{A}_{k+1} = A_{k+1} - B_{k+1}\,\tilde{A}_k^{-1}\,C_k \;,\; k = 1,2,\ldots,N-2$$

where A_{k+1} is a (k+1) x (k+1) diagonal matrix, and B_k, C_k are bi-diagonal matrices defined by the vectors γ, δ, b, c (see Reference [3]). Using one temporary array (t_{ij}), of size at least (N-1) x (N-1), the reduction is programmed as follows:

$$\text{Set } t_{11} = \left(\frac{2}{\alpha_1^2} + \frac{1}{\beta_1\beta_2}\right)^{-1} = A_1^{-1}$$

For k = 1, 2, ..., N-2

$$\tilde{A}_k^{-1} \cdot C_k \begin{cases} t_{j,k+1} = t_{ik}\,c_k & i = 1, 2, \ldots, k \\ t_{ij} = t_{i,j-1}\,c_{j-1} + t_{ij}\;\gamma_{k-j+1} \;, & \begin{cases} j = k, k-1, \ldots, 2 \\ i = 1, 2, \ldots, k \end{cases} \\ t_{i1} = \gamma_k t_{ij} & i = 1, 2, \ldots, k \end{cases}$$

If k = N-2, go to final reduction

$$B_{k+1}(\tilde{A}_k^{-1}C_k)\begin{cases} t_{k+1,j} = b_{k+1}t_{kj}\ , \ j = 1, 2, \ldots, k+1 \\ t_{ij} = b_i t_{i-1} + \delta_{k-i+1}t_{ij} \ \begin{cases} i = k, k-1, \ldots, 2 \\ j = 1, 2, \ldots, k+1 \end{cases} \\ t_{1j} = \delta_k t_{1j}\ , \ j = 1, 2, \ldots, k+1 \end{cases}$$

$$A_{k+1} - B_{k+1}\tilde{A}_k^{-1}C_k \begin{cases} t_{11} = \dfrac{2}{\alpha_1^2} + \dfrac{1}{\beta_{k+1}\beta_{k+2}} - t_{11} \\ t_{ii} = \dfrac{1}{\alpha_{i-1}\alpha_i} + \dfrac{1}{\beta_{k-i+2}\beta_{k-i+3}} - t_{ii}\ , \ i = 2,3, \ldots, k+1 \\ t_{ij} = -t_{ij},\ i \neq j,\ i,j = 1, 2, \ldots, k+1 \end{cases}$$

Replace T by T^{-1}, increase k.

Final Reduction

($T = \tilde{A}_{N-2}^{-1}C_{N-2}$ computed above)

$$B_{N-1}(\tilde{A}_{N-2}^{-1}C_{N-2}) \begin{cases} t_{N-1,j} = -\dfrac{\cos\Omega_{N-1}}{\alpha_{N-2}}\, t_{N-2,j}\ , \ j = 1, 2, \ldots, N-1 \\ t_{i,j} = -\dfrac{\cos\Omega_i}{\alpha_{i-1}}\, t_{i-1,j} - \dfrac{\sin\Omega_i}{\beta_{N-1}}\, t_{ij},\ \begin{cases} i = N-2, N-3, \ldots, 2 \\ j = 1, 2, \ldots, N-1 \end{cases} \\ t_{1j} = \dfrac{-1}{\beta_{N-1}}\, t_{1j}\ , \ j = 1, 2, \ldots, N-1 \end{cases}$$

$$\tilde{A}_{N-1}^{-1} \begin{cases} t_{11} = \dfrac{1}{\beta_{N-1}} - t_{11} \\[2ex] t_{ii} = \dfrac{\cos\Omega_i}{\alpha_{i-1}} + \dfrac{\sin\Omega_i}{\beta_{N-1}} - t_{ii} \ , \ i = 2, 3, \ldots, N-1 \\[2ex] t_{ij} = -t_{ij}, \ i \neq j, \ i,j = 1, 2, \ldots, N-1 \end{cases}$$

Replace T by T^{-1}. Then $Q = (q_{ij})$ where

$$q_{ij} = t_{N-i,\ N-j} \quad , \ i,j = 1, 2, \ldots, N-1$$

APPENDIX C

SAMPLE PROBLEM

The configuration for this problem is shown in Figure 1. The dimensions of the tank correspond to that of the S-IVB liquid oxygen tank. The propellant is liquid oxygen and the free surface is 20 inches above 01 (i.e., ZEE(N) = -20.). The number of mass points is 11 which leads to 21 degrees of freedom for the Z&R mass matrix.

The computer print out is shown on the following pages, note that for the Z&R mass matrix the ordering of the degrees of freedom is: Z1, R1, Z2, R2, Z3, R3... Z10, R10, Z11.

The term CMASS = 493.861 lbf in/sec^2 is the total mass of the liquid found from the Z&R mass matrix. This can be used as a check on the accuracy of the computed mass matrix. For the configuration considered here the theoretical mass,

$$\text{TMASS} = \frac{0.000106\pi}{3}\left\{2(88.75)^2(390-88.75) - (27.5)^2(390-27.5)\right\}$$

$$= 496.348 \text{ lbf in/sec}^2$$

Hence the computed mass is 0.6% low.

SAMPLE PROBLEM LIQUID LEVEL 20 IN (ABOVE C1)

```
$XGT          TPF$.ABS15
 A1=130.,B1=130.,A2=130.,B2=130.,Z02=82.5,
 ZEF(1)=130.,128.8,125.2,122.08,113.,99.28,82.5,62.0,41.25,20.5,0.0,-20.,
 N=12,NA= 9,WL=.000106,
 $END
```

	ALPHA	BETA	LAMBDA	OMEGA
(1)	.1429465+00	.1683134+00	.0000000	.1570726+01
(2)	.1409509+00	.1662855+00	.1429465+00	.1434818+01
(3)	.7833942-01	.1361108+00	.2838974+00	.1298207+01
(4)	.1569117+00	.1112896+00	.3624369+00	.1219935+01
(5)	.1594118+00	.7365231-01	.5213485+00	.1053644+01
(6)	.1341819+00	.2530785-01	.6807603+00	.8690133+00
(7)	.1118996+00	.2920134-01	.8149422+00	.6975107+00
(8)	.7315625-01	.9733796-02	.9268418+00	.4971507+00
(9)	.0000000	.1622298+00	.1000000+01	.3228801+00
(10)	.0000000	.0000000	.0000000	.4971507+00
(11)	.0000000	.0000000	.0000000	.6975107+00
(12)	.0000000	.0000000	.0000000	.9083038+00

OMEGAS = .3228869+00 BETA(NA)= .1622330+00

MATRIX GRAR

ROW = 1							
1	.1167865+00	.2924216-01	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	11
ROW = 2							
1	.2924216-01	.1183303+00	.2959587-01	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	11
ROW = 3							
1	.0000000	.2959587-01	.1205353+00	.2959588-01	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	11
ROW = 4							
1	.0000000	.0000000	.2959588-01	.1183302+00	.2924216-01	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	11
ROW = 5							
1	.0000000	.0000000	.0000000	.2924216-01	.1065292+00	.2393446-01	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	11
ROW = 6							
1	.0000000	.0000000	.0000000	.0000000	.2393446-01	.8738295-01	6
7	.1957014-01	.0000000	.0000000	.0000000	.0000000	.0000000	11
ROW = 7							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.1957014-01	6
7	.6472501-01	.1295025-01	.0000000	.0000000	.0000000	.0000000	11
ROW = 8							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.1295025-01	.3295494-01	.4448285-02	.0000000	.0000000	.0000000	11
ROW = 9							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.4448285-02	.2031042-01	.5133284-02	.0000000	.0000000	11
ROW = 10							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.5133284-02	.1370004-01	.1666413-02	.0000000	11
ROW = 11							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.1666413-02	.1801943-02	.0000000	11

'Q'

MATRIX ERROR

ROW = 1							
1	.2583948+00	.1048300+00	.1579363+00	.1012659+00	.1392129+00	.1128480+00	6
7	.8395914-01	.4245107-01	.2860669-01	.1964421-01	.3026106-02	.0000000	11
ROW = 2							
1	.1747898+00	.3679256+00	.2588092+00	.2503929+00	.2079934+00	.1658601+00	6
7	.1224324+00	.6162984-01	.4148734-01	.2845571-01	.4382130-02	.0000000	11
ROW = 3							
1	.1011132+00	.2803707+00	.4230092+00	.3144739+00	.2480235+00	.1951274+00	6
7	.1429985+00	.7174863-01	.4825809-01	.3306840-01	.5091230-02	.0000000	11
ROW = 4							
1	.1334494+00	.2524050+00	.2924592+00	.4382048+00	.2954281+00	.2253304+00	6
7	.1033511+00	.8159012-01	.5481404-01	.3751318-01	.5773665-02	.0000000	11
ROW = 5							
1	.1457149+00	.2304063+00	.2542780+00	.3226939+00	.4412894+00	.2855148+00	6
7	.1999798+00	.9868759-01	.6610499-01	.4509906-01	.6935710-02	.0000000	11
ROW = 6							
1	.1403279+00	.2181542+00	.2369477+00	.2919103+00	.3354748+00	.4302804+00	6
7	.2563453+00	.1226648+00	.8161304-01	.5531425-01	.9493018-02	.0000000	11
ROW = 7							
1	.1366950+00	.2108741+00	.2271655+00	.2767307+00	.3066772+00	.3302028+00	6
7	.4021810+00	.1036867+00	.1065668+00	.7095178-01	.1085040-01	.0000000	11
ROW = 8							
1	.1346646+00	.2067021+00	.2221569+00	.2693711+00	.2947788+00	.3071903+00	6
7	.3140793+00	.2755832+00	.1513787+00	.9524679-01	.1441688-01	.0000000	11
ROW = 9							
1	.1340875+00	.2055906+00	.2207715+00	.2673668+00	.2916779+00	.3017968+00	6
7	.3011366+00	.2219316+00	.2202762+00	.1189184+00	.1769458-01	.0000000	11
ROW = 10							
1	.1333953+00	.2042713+00	.2191397+00	.2650352+00	.2881610+00	.2960476+00	6
7	.2896981+00	.2003659+00	.1674905+00	.2251951+00	.2929958-01	.0000000	11
ROW = 11							
1	.1331673+00	.2038770+00	.2186539+00	.2643444+00	.2871344+00	.2944183+00	6
7	.2866948+00	.1957726+00	.1604248+00	.1800131+00	.9943171-01	.0000000	11

MATRIX M MASS

ROW = 1							
1	.4405728+02	.4077687+02	.3440968+02	.3328369+02	.2852064+02	.2274595+02	6
7	.1067706+02	.8467019+01	.5341853+01	.3058939+01	.5889175+00	.0000000	11
ROW = 2							
1	.4077687+02	.7142967+02	.6234346+02	.5525408+02	.4566625+02	.3581611+02	6
7	.2605513+02	.1517821+02	.8309626+01	.5686683+01	.9143963+00	.0000000	11
ROW = 3							
1	.3440968+02	.6234346+02	.8598794+02	.7041127+02	.5377156+02	.4138961+02	6
7	.2989481+02	.1506286+02	.9524068+01	.6512968+01	.1044141+01	.0000000	11
ROW = 4							
1	.3328369+02	.5525408+02	.7041127+02	.8809789+02	.6825928+02	.4956366+02	6
7	.3526717+02	.1767344+02	.1112923+02	.7600033+01	.1219637+01	.0000000	11
ROW = 5							
1	.2852064+02	.4566625+02	.5377156+02	.6825928+02	.7946519+02	.5814135+02	6
7	.3896500+02	.1923663+02	.1205041+02	.8204278+01	.1315648+01	.0000000	11
ROW = 6							
1	.2274595+02	.3581611+02	.4138961+02	.4956366+02	.5814135+02	.6351237+02	6
7	.4294598+02	.1999313+02	.1236374+02	.8359243+01	.1337982+01	.0000000	11
ROW = 7							
1	.1067706+02	.2605513+02	.2989481+02	.3526717+02	.3896500+02	.4294598+02	6
7	.4382076+02	.2089498+02	.1213942+02	.8049520+01	.1281959+01	.0000000	11
ROW = 8							
1	.8467019+01	.1517821+02	.1506286+02	.1767344+02	.1923663+02	.1999313+02	6
7	.2089498+02	.1522756+02	.8808121+01	.5491425+01	.8620407+00	.0000000	11
ROW = 9							
1	.5341853+01	.8309626+01	.9524068+01	.1112923+02	.1205041+02	.1236374+02	6
7	.1213942+02	.8808121+01	.7496271+01	.4796759+01	.7126215+00	.0000000	11
ROW = 10							
1	.3058939+01	.5686683+01	.6512968+01	.7600033+01	.8204278+01	.8359243+01	6
7	.8049520+01	.5491425+01	.4796759+01	.4986154+01	.8470649+00	.0000000	11
ROW = 11							
1	.5889175+00	.9143963+00	.1044141+01	.1219637+01	.1315648+01	.1337982+01	6
7	.1281959+01	.8620407+00	.7126215+00	.8470649+00	.2845182+00	.0000000	11

MATRIX TR

ROW = 1							
1	.6346154+00	.7728281+00	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21
ROW = 2							
1	.0000000	.0000000	.4769231+00	.8789450+00	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21
ROW = 3							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.1000000+01	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21
ROW = 4							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	-.4769231+00	.8789450+00	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21
ROW = 5							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	-.6346154+00	.7728281+00	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21
ROW = 6							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	-.7636923+00	.6455804+00	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21
ROW = 7							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	-.8692307+00	.4944066+00	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21
ROW = 8							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	-.9396769+00	.3437071+00	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21

ROW = 9

1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	-.2630769+00	.2692264+00	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21

ROW = 10

1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	-.9907692+00	.1355594+00	.0000000	.0000000	.0000000	.0000000	21

ROW = 11

1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	-.1000000+01	.0000000	.0000000	.0000000	21

MATRIX ZRK MS

ROW = 1							
1	.1775943+02	.2159802+02	.1234164+02	.2274501+02	.0000000	.2183691+02	6
7	-.1007373+02	.4656588+02	-.1148631+02	.1398791+02	-.1102385+02	.9318969+01	12
13	-.9199520+01	.5232561+01	-.5045942+01	.1846841+01	-.3264852+01	.9126834+00	18
19	-.2300565+01	.3147715+00	-.3737361+00	.0000000	.0000000	.0000000	21
ROW = 2							
1	.2153862+02	.2630165+02	.1502952+02	.2769865+02	.0000000	.2659277+02	6
7	-.1220769+02	.2260873+02	-.1398791+02	.1703433+02	-.1342473+02	.1134847+02	12
13	-.1120368+02	.6372169+01	-.6146897+01	.2249065+01	-.3975904+01	.1111456+01	18
19	-.2801629+01	.3833255+00	-.4551320+00	.0000000	.0000000	.0000000	21
ROW = 3							
1	.1234164+02	.1502952+02	.1624708+02	.2994254+02	.0000000	.2973303+02	6
7	-.1256785+02	.2316191+02	-.1382147+02	.1683165+02	-.1304503+02	.1102750+02	12
13	-.1080151+02	.6143640+01	-.5902093+01	.2160197+01	-.3816724+01	.1066958+01	18
19	-.2687075+01	.3676520+00	-.4366967+00	.0000000	.0000000	.0000000	21
ROW = 4							
1	.2274501+02	.2769865+02	.2994254+02	.5518259+02	.0000000	.5479647+02	6
7	-.2316191+02	.4266623+02	-.2547927+02	.3101987+02	-.2404133+02	.2032312+02	12
13	-.1998627+02	.1132242+02	-.1087726+02	.3981133+01	-.7034029+01	.1966350+01	18
19	-.4952143+01	.6775640+00	-.8037041+00	.0000000	.0000000	.0000000	21
ROW = 5							
1	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	6
7	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	12
13	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	18
19	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	21
ROW = 6							
1	.2183691+02	.2659277+02	.2973303+02	.5479647+02	.0000000	.8398794+02	6
7	-.3358076+02	.6188764+02	-.3412426+02	.4155617+02	-.3160893+02	.2672032+02	12
13	-.2596549+02	.1476019+02	-.1414519+02	.5177213+01	-.9172410+01	.2564130+01	18
19	-.6452848+01	.8828940+00	-.1044141+01	.0000000	.0000000	.0000000	21
ROW = 7							
1	-.1007373+02	-.1220769+02	-.1256785+02	-.2316191+02	.0000000	-.3358076+02	6
7	.2005896+02	-.3092968+02	.2065954+02	-.2515898+02	.1805220+02	-.1526026+02	12
13	.1462023+02	-.8315785+01	.7915356+01	-.2897062+01	.5111805+01	-.1428996+01	18
19	.3591173+01	-.4913528+00	.5816730+00	.0000000	.0000000	.0000000	21
ROW = 8							
1	.1656588+02	.2260873+02	.2316191+02	.4266623+02	.0000000	.6188764+02	6
7	-.3092968+02	.6805953+02	-.3807448+02	.4636672+02	-.3326929+02	.2812389+02	12
13	-.2094403+02	.1532557+02	-.1458760+02	.5339138+01	-.9420797+01	.2633566+01	18
19	-.6618349+01	.9055383+00	-.1071994+01	.0000000	.0000000	.0000000	21

SAMPLE PROBLEM LIQUID LEVEL 20 IN (ABOVE C1)

ROW = 9							
1	-.1146631+02	-.1398791+02	-.1382147+02	-.2547927+02	.0000000	-.3412426+02	6
7	.2065954+02	-.3607448+02	.3200355+02	-.3897359+02	.2817825+02	-.2382023+02	12
13	.2149415+02	-.1222558+02	.1146412+02	-.4195927+01	.7365012+01	-.2058875+01	18
19	.5156560+01	-.7057983+00	.8349307+00	.0000000	.0000000	.0000000	21
ROW = 10							
1	-.1396791+02	.1703453+02	.1683165+02	.3101987+02	.0000000	.4155617+02	6
7	-.2015698+02	.4631672+02	-.3897359+02	.4746164+02	-.3431519+02	.2900804+02	12
13	-.2017556+02	.1488419+02	-.1396089+02	.5109758+01	-.9969036+01	.2507278+01	18
19	-.6281969+01	.8595139+00	-.1016770+01	.0000000	.0000000	.0000000	21
ROW = 11							
1	-.1102565+02	-.1342473+02	-.1304503+02	-.2404133+02	.0000000	-.3160893+02	6
7	.1605220+02	-.3328929+02	.2817825+02	-.3431519+02	.3704206+02	-.3131317+02	12
13	.2050661+02	-.1021531+02	.1433839+02	-.5247926+01	.9993461+01	-.2542060+01	18
19	.6324961+01	-.8653963+00	.1021936+01	.0000000	.0000000	.0000000	21
ROW = 12							
1	.9316909+01	.1134847+02	.1102750+02	.2032312+02	.0000000	.2672032+02	6
7	-.1520026+02	.2612389+02	-.2382023+02	.2900804+02	-.3131317+02	.2647031+02	12
13	-.2409949+02	.1370746+02	-.1212083+02	.4436287+01	-.7687075+01	.2148908+01	18
19	-.5346749+01	.7315550+00	-.8637748+00	.0000000	.0000000	.0000000	21
ROW = 13							
1	-.9199519+01	-.1120308+02	-.1080131+02	-.1990627+02	.0000000	-.2598549+02	6
7	.1462662+02	-.2694433+02	.2149415+02	-.2617536+02	.2850861+02	-.2409949+02	12
13	.3310931+02	-.1683212+02	.1705604+02	-.6242601+01	.1016234+02	-.2840865+01	18
19	.6932204+01	-.9484943+00	.1114319+01	.0000000	.0000000	.0000000	21
ROW = 14							
1	.5232561+01	.6372160+01	.6143640+01	.1132242+02	.0000000	.1478019+02	6
7	-.8015765+01	.1532557+02	-.1222558+02	.1488819+02	-.1021531+02	.1370746+02	12
13	-.1083212+02	.1071146+02	-.9701244+01	.3550706+01	-.5780203+01	.1615845+01	18
19	-.3943000+01	.5394906+00	-.6338092+00	.0000000	.0000000	.0000000	21
ROW = 15							
1	-.5045942+01	-.6144897+01	-.5902093+01	-.1087726+02	.0000000	-.1414519+02	6
7	.7913356+01	-.1456760+02	.1146412+02	-.1396089+02	.1433839+02	-.1212083+02	12
13	.1703604+02	-.9701244+01	.1342866+02	-.4914961+01	.7960094+01	-.2226907+01	18
19	.5109268+01	-.6990622+00	.8095225+00	.0000000	.0000000	.0000000	21
ROW = 16							
1	.1046841+01	.2249865+01	.2160197+01	.3981133+01	.0000000	.5177213+01	6
7	-.2097062+01	.5339138+01	-.4195927+01	.5109758+01	-.5247926+01	.4436287+01	12
13	-.6242601+01	.3550706+01	-.4914961+01	.1798901+01	-.2915632+01	.8150596+00	18
19	-.1870019+01	.2558604+00	-.2962895+00	.0000000	.0000000	.0000000	21

ROW = 17

1	-.3264852+01	-.3975904+01	-.3816724+01	-.7034029+01	.0000000	-.9172410+01	6
7	.5111805+01	-.9420797+01	.7365012+01	-.8969036+01	.9093461+01	-.7687075+01	12
13	.1016234+02	-.5780203+01	.7966094+01	-.2915632+01	.6952920+01	-.1943676+01	18
19	.4577004+01	-.6262366+00	.6863093+00	.0000000	.0000000	.0000000	21

ROW = 18

1	.9126834+00	.1111456+01	.1066958+01	.1966350+01	.0000000	.2564130+01	6
7	-.1428996+01	.2633566+01	-.2058875+01	.2507278+01	-.2542060+01	.2148908+01	12
13	-.2840865+01	.1615845+01	-.2226907+01	.8150596+00	-.1943676+01	.5433510+00	18
19	-.1279493+01	.1750633+00	-.1918565+00	.0000000	.0000000	.0000000	21

ROW = 19

1	-.2300565+01	-.2601629+01	-.2687075+01	-.4952143+01	.0000000	-.6452848+01	6
7	.3591173+01	-.6618349+01	.5158500+01	-.6281969+01	.6324961+01	-.5346749+01	12
13	.6932304+01	-.3943000+01	.5104268+01	-.1870019+01	.4577004+01	-.1279493+01	18
19	.4894526+01	-.6696808+00	.8392459+00	.0000000	.0000000	.0000000	21

ROW = 20

1	.3147715+00	.3633255+00	.3676520+00	.6775640+00	.0000000	.8828940+00	6
7	-.4913528+00	.9055383+00	-.7057983+00	.8595139+00	-.8653963+00	.7315549+00	12
13	-.9484943+00	.5394906+00	-.6990622+00	.2558604+00	-.6262366+00	.1750633+00	18
19	-.6696808+00	.9162732-01	-.1148276+00	.0000000	.0000000	.0000000	21

ROW = 21

1	-.3737361+00	-.4551320+00	-.4360967+00	-.8037041+00	.0000000	-.1044141+01	6
7	.5816730+00	-.1071994+01	.8349307+00	-.1016770+01	.1021806+01	-.8637748+00	12
13	.1114319+01	-.6338092+00	.8095225+00	-.2962895+00	.6863093+00	-.1918565+00	18
19	.8392459+00	-.1148276+00	.2845182+00	.0000000	.0000000	.0000000	21

ZV= 1.00 .00 1.00 .00 1.00 .00 1.00 .00 1.00 .00
1.00 .00 1.00 .00 1.00 .00 1.00 .00 1.00 .00
1.00

CMASS= .4938605+03

NO2=21,NARR(1)=28,29,26,27,24,25,22,23,20,21,18,19,16,17,14,15,12,13,
10,11,8,
NPOS=60,
$END

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From: T. J. Rudd

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