Bellcomm
955 L'Enfant Plaza North, S.W.
Washington, D. C. 20024

B71 09028

date: September 29, 1971

to: Distribution

from: S. Kaufman

subject: Investigation of a High Gain Antenna Pendulum - Case 320

ABSTRACT

The purpose of this investigation is to ascertain the feasibility of a proposed high gain antenna pendulum for the Lunar Roving Vehicle (LRV). The investigation was limited to non-steady accelerations encountered when the LRV traverses a level lurain whose roughness is characterized by a power spectrum equivalent to a mid-range smooth mare. It is concluded that the proposed pendulum design is not practical; in fact, the pendulum excursions exceed those of the LRV.

(NASA-CR-123224) INVESTIGATION OF A HIGH GAIN ANTENNA PENDULUM (Bellcomm, Inc.) 15 p

N79-72719

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FF No. 602
CR-123224
(NASA CR OR TMX OR AD NUMBER) (CATEGORY)
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MEMORANDUM FOR FILE

INTRODUCTION

The purpose of this investigation is to ascertain the feasibility of a proposed high gain antenna pendulum for the Lunar Roving Vehicle (LRV). The function of the pendulum is to assist the antenna in acquiring earth by aligning itself along the lunar vertical axis. This investigation is limited to non-steady accelerations encountered while traversing a level lurain whose roughness is characterized by a power spectrum equivalent to a mid-range smooth mare (Reference 1). No attempt has been made to evaluate quasi-static accelerations that result from such maneuvers as braking, accelerating, and turning.

The equations of motion were programmed on the UNIVAC 1108 digital computer. Five computer runs were made by varying the damping ratio (0, 0.1, 0.2, 0.5, and 1.0 critical damping). Although the .5 damping ratio gave the best results for the proposed design (35 inch pendulum) the pendulum elevation angle (θ in Figure 1) exceeded LRV excursions as the LRV negotiated a 32 second traverse.

EQUATIONS OF MOTION OF A SPHERICAL PENDULUM

As can be seen from Figure 1, there are only two degrees-of-freedom for a spherical pendulum (θ, ϕ). However, if one examines the ϕ equation of motion (p. 81, Reference 2), assuming non-constant ϕ angular momentum

$$(mR^2\sin^2\theta\ddot{\phi} + 2mR^2\sin\theta\cos\theta\dot{\theta}\dot{\phi} = F(\phi)) \quad , \qquad (1)$$

one finds a potential singularity existing for the digital computer about $\theta = 0$ and π. It is for this reason that this natural coordinate system was abandoned in favor of the basic X, Y, Z cartesian system and the Lagrange multiplier λ. Before we formulate these equations, let us first write a relationship between the moving xyz and the basic XYZ systems and then construct a damping model for the pendulum as follows:

$$\begin{Bmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{Bmatrix} = \begin{bmatrix} \cos\phi\cos\theta & \sin\phi\cos\theta & \sin\theta \\ -\sin\phi & \cos\phi & 0 \\ -\cos\phi\sin\theta & -\sin\phi\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \bar{I} \\ \bar{J} \\ \bar{K} \end{Bmatrix} \qquad (2)$$

$$T_\theta = \frac{\dot{\theta}}{|\dot{\theta}|}\left(c + b|\dot{\theta}|^n\right) \quad , \qquad (3)$$

where $\bar{i}\ \bar{j}\ \bar{k}$ and $\bar{I}\ \bar{J}\ \bar{K}$ are unit vectors along the moving (xyz) and basic (X Y Z) coordinates axes, respectively. T_θ is the damping torque for θ motions and b,c,n are constants, which characterize the damping model.

The pendulum motion is constrainted along z. This constraint can be expressed as follows:

$$(X\dot{X} + Y\dot{Y} + Z\dot{Z}) = 0 \quad . \qquad (4)$$

Next, the Lagragian is formulated as follows:

$$L = 1/2m(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) - mgZ \qquad (5)$$

where m is the mass of the pendulum and g the moon's gravity.

The three Lagrange equations of motion together with the derivative of the constraint equation are given below (with the help of equation (2)) in matrix notation.

$$\begin{bmatrix} m & 0 & 0 & X \\ 0 & m & 0 & Y \\ 0 & 0 & m & Z \\ X & Y & Z & 0 \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} -ma_X - \cos\phi\cos\theta T_\theta/R \\ -ma_Y - \sin\phi\cos\theta T_\theta/R \\ -ma_Z - \sin\theta T_\theta/R - mg \\ -(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) \end{Bmatrix} \tag{6}$$

where

R = pendulum length,

a_x, a_y, a_z = input accelerations at pendulum gimbal,

$\cos\theta = -Z/R$, $\sin\theta = (X^2+Y^2)^{1/2}/R$,

$\cos\phi = X/(X^2+Y^2)^{1/2}$, and $\sin\phi = Y/(X^2+Y^2)^{1/2}$.

If $X^2 + Y^2 = 0$, then $\cos\phi$ and $\sin\phi$ can be defined by replacing X and Y with their derivitives. If $\dot{X}^2 + \dot{Y}^2 = 0$ also, then $\cos\phi$ and $\sin\phi$ are assumed to be unchanged from their values at the previous time step.

After solution of equation (6), the derivatives and displacements are computed as follows:

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix}_{i+h} = \begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix}_i + h \begin{Bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{Bmatrix}_i \tag{7}$$

and

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_{i+h} = \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_{i} + h/2\left(\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix}_{i+h} + \begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix}_{i}\right)$$

where h is the time interval. The pendulum invariant length is next computed

$$R_{i+h} = (X^2 + Y^2 + Z^2)^{1/2}_{i+h} \tag{8}$$

and the displacements corrected as follows:

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_{i+h} = R/R_{i+h} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_{i+h} \tag{9}$$

INPUT DATA AND RESULTS

Computer runs were based on the following input data:

$m = .03$ lb. sec.^{2}/inch,

$R = 35$ inches,

$g = 61.8$ inches/sec.^{2}

$c = 0$

$n = 1$ (linear damping)

The damping coefficient b was obtained by the following expression:

$$b = mR^2 (g/R)^{1/2}\ 2\beta \tag{10}$$

where $(g/R)^{1/2}$ is the natural frequency (1.33 radians/sec) of the pendulum for small motions and β is the so-called damping ratio. Computer runs were made for β = 0, .1, .2, .5, and 1.0. The input accelerations every .025 seconds were obtained from another computer run of 32 second duration using the digital program ROVER (References 3 and 4). For this run the LRV was made to traverse essentially a straight course at full throttle (approximately 13 km/hr speed) over a lurain whose power spectrum is equivalent to a mid-range smooth mare (Reference 1).

The output of ROVER in binary format contains among others things a 3x3 direction cosine matrix, six velocities and six accelerations about the body axes of the cg. of the vehicle. These accelerations were transferred to the gimbal of the pendulum as follows:

$$\begin{Bmatrix} a_X \\ a_Y \\ a_Z \end{Bmatrix} = [B]\left(\begin{Bmatrix} a_{x0} \\ a_{y0} \\ a_{z0} \end{Bmatrix} + \left(\left(\begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_z & 0 \end{bmatrix}\right)^2 + \begin{bmatrix} 0 & -\alpha_z & \alpha_y \\ \alpha_z & 0 & -\alpha_x \\ -\alpha_y & \alpha_x & 0 \end{bmatrix}\right)\begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix}\right) \tag{11}$$

where

[B] is the direction cosine matrix,

a_{x0}, a_{y0}, a_{z0} are total linear accelerations along body axes at cg.,

b_y, b_y, b_y are body coordinates to gimbal from cg.,

α_x, α_y, α_z are rotational accelerations about cg.,

ω_x, ω_y, ω_z are rotational velocities.

The results of elevation pendulum angles for all five runs are summarized in Table I along with the elevation angle of the LRV. These angles for the first four runs together with the elevation angle of the LRV are plotted against time in the Appendix. It can be shown that if the length of the pendulum (and hence its period) is increased by an order of magnitude the transmission of the gimbal accelerations to the pendulum would be vastly reduced. However, for the proposed design, this is not practical. It can be concluded that the proposed pendulum design is not feasible while the rover is traversing a mid-range smooth mare lurain.

S. Kaufman

2031-SK-jf

S. Kaufman

Attachments
Figure 1
Table 1
References
Appendix

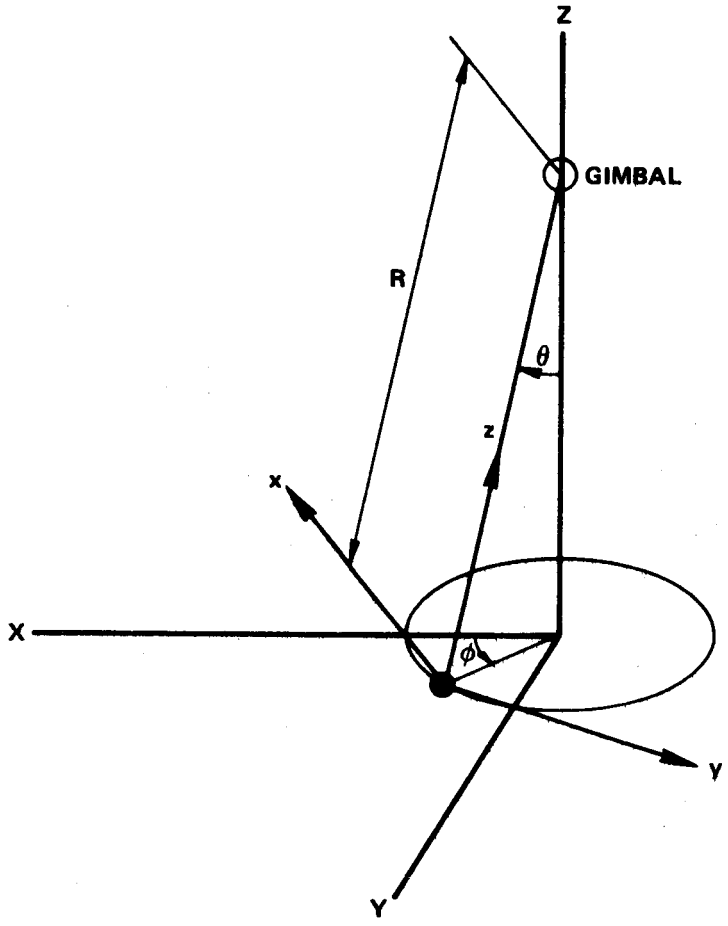


FIGURE 1 - SPHERICAL PENDULUM

TABLE 1

COMPARISON OF PENDULUM AND LRV
ELEVATION ANGLES

Damping Ratio - β	Pendulum Elevation Angle - Degrees		
	Maximum	Mean	Standard Deviation
0	30.2	9.76	6.46
.1	22.2	8.15	4.68
.2	19.3	7.87	4.08
.5	14.9	7.69	3.51
1.0	20.6	6.73	4.08
	6.96	2.57	1.36
	LRV Elevation Angle - Degrees		

The above statistics approximates a

Raleigh distribution $f(X) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}$

Mean $= \sigma\sqrt{\pi/2}$

Std. dev. $= \sigma\sqrt{2 - \frac{\pi}{2}}$

Probability function $= \int_0^x f(y)dy = (1 - e^{-x^2/2\sigma^2})$

REFERENCES

1. LRV Operations Handbook, Appendix A (Performance Data), NASA LS006-002-2H, Manned Spacecraft Center, June 10, 1971.

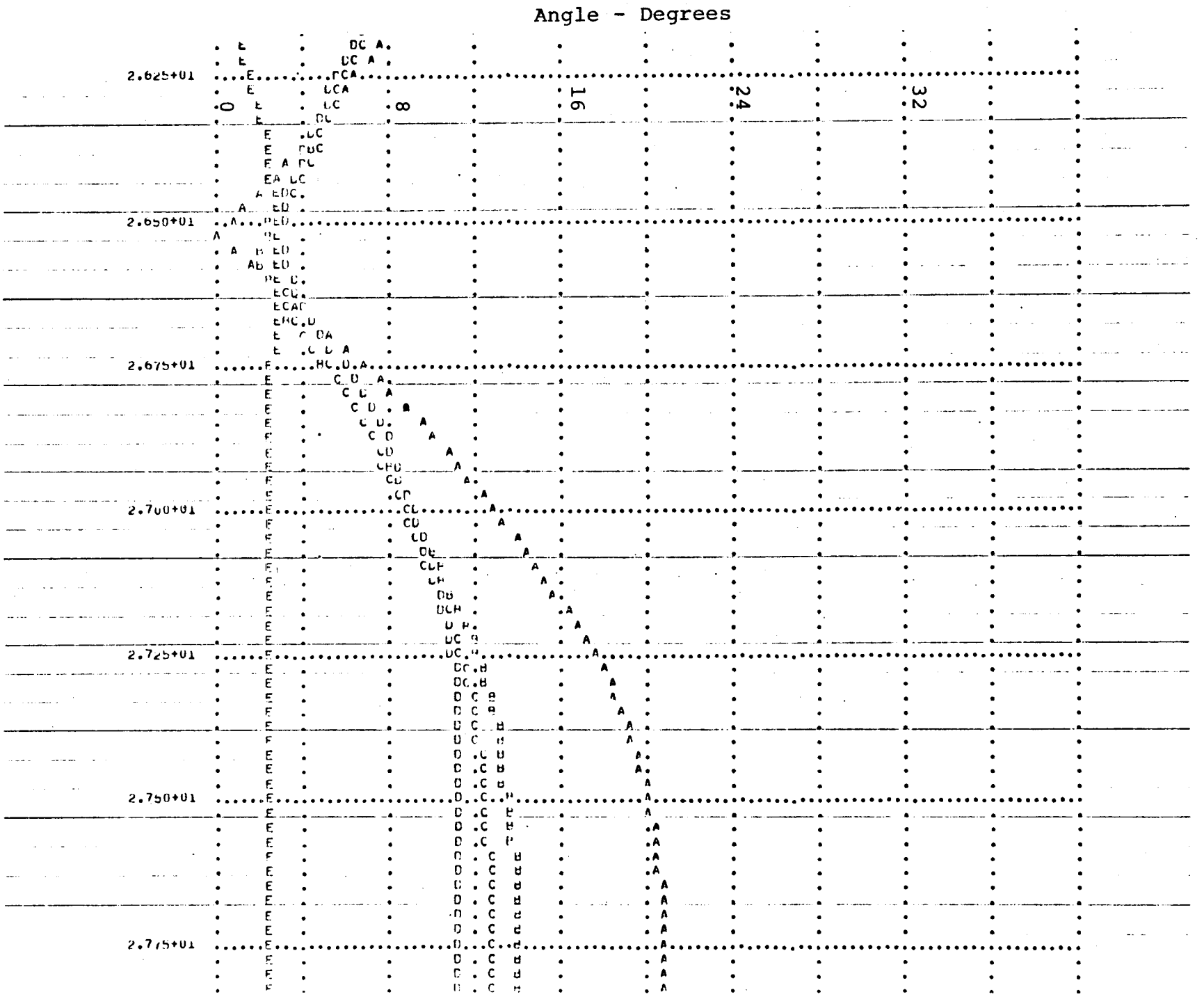
2. J. C. Slater and N. H. Frank, Mechanics, McGraw-Hill, 1947.

3. S. Kaufman, A Digital Program for Stability and Performance Characteristics of a Four-Wheel Vehicle, Bellcomm Memorandum for File B71 09027, Case 320,

4. S. Kaufman, The Dynamics and Performance of a Four-Wheel Vehicle, Bellcomm Memorandum for File B71 02032, Case 320, February 16, 1971.

APPENDIX

The Appendix contains a plot of angle in degress vs. time for a portion of the 32 second traverse. The portion chosen (26.25 - 31.0 seconds) contains the maximum pendulum elevation angles. Five curves are shown: four graphs of the pendulum elevation angle for $\beta = 0$ (curve A), $\beta = .1$ (curve B), $\beta = .2$ (curve C) and $\beta = .5$ (curve D), and a graph of the elevation angle of the LRV (curve E). The elevation angle of the LRV is the angle between the vertical body axis and vertical inertial axis or the inverse cosine of the (3, 3) term of the direction cosine matrix. The author is indebted to Mr. R. D. Weiksner of Bellcomm for these curves.


Angle - Degrees
0
8
16
24
32
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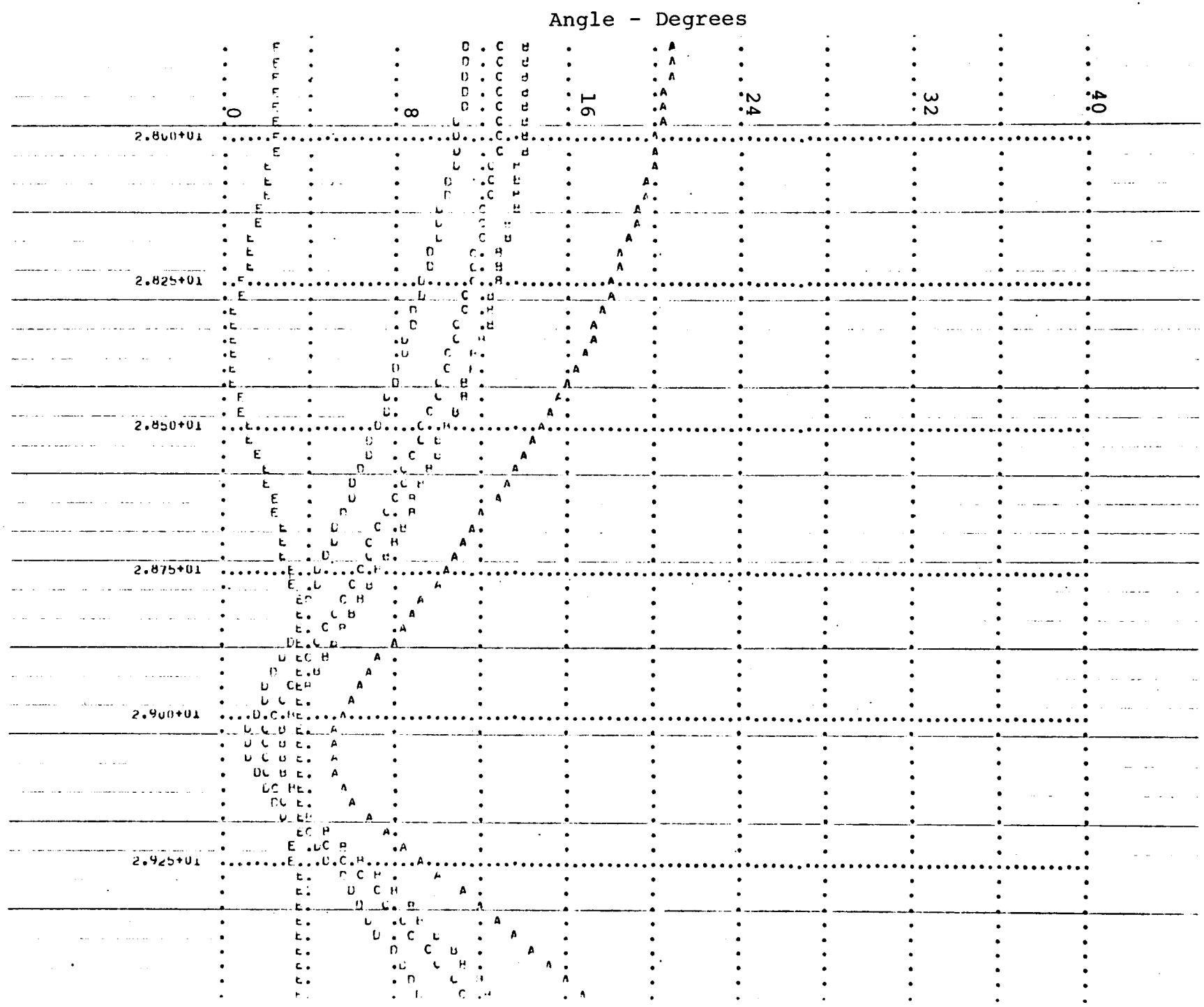
2.650+01
2.675+01
2.700+01
2.725+01
2.750+01
2.775+01
Time - Seconds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8
16
24
32
40
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2.825+01
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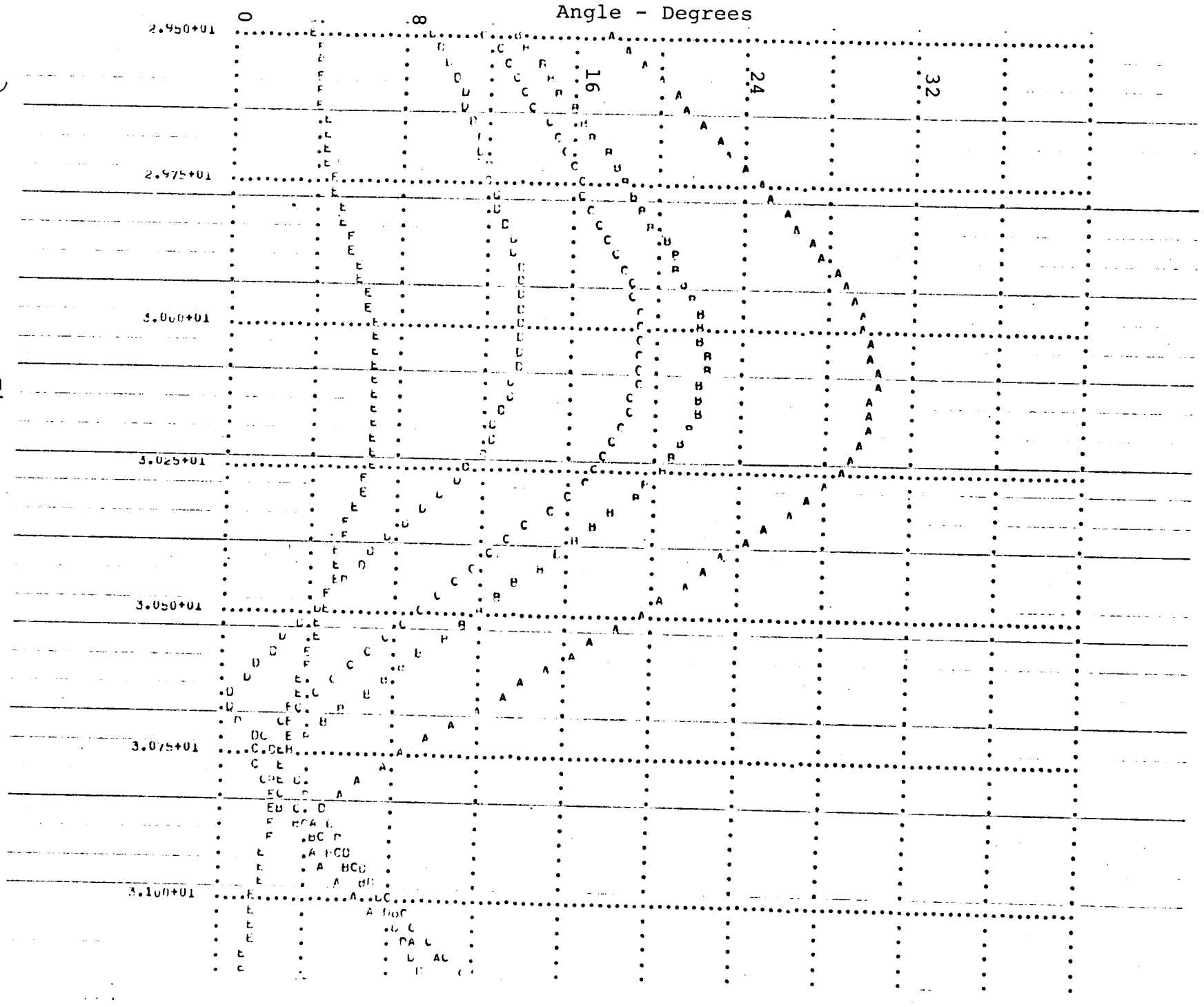
2.875+01
2.900+01
2.925+01
Time - Seconds


Angle - Degrees
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8
16
24
32
Time - Seconds
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2.975+01
3.000+01
3.025+01
3.050+01
3.075+01
3.100+01

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