NASA Technical Memorandum 81695

(NASA-TM-81695) ELASTOHYDRODYNAMIC LUBRICATION THEORY (NASA) 56 p HC A04/MF A01 CSCL 11B
N83-12432
Unclas
G3/37 01111

# Elastohydrodynamic Lubrication Theory

Bernard J. Hamrock
*Lewis Research Center*
*Cleveland, Ohio*

and

Duncan Dowson
*The University of Leeds*
*Leeds, England*

NOV 1982
RECEIVED
NASA STI FACILITY
ACCESS DEPT.

October 1982

NASA

# ELASTOHYDRODYNAMIC LUBRICATION THEORY*

This chapter is devoted to the method used by the authors to solve the elastohydrodynamic lubrication problem for elliptical contacts (Hamrock and Dowson, 1976a). The problem has been discussed earlier in some detail and the relevant effects examined separately, so it remains only to seek a solution that reconciles the hydrodynamic and elastic equations. The problem is to calculate the pressure distribution in the contact and at the same time to allow for the effects that this pressure will have on the properties of the fluid and on the geometry of the elastic solids. The solution will also provide the shape of the lubricant film, particularly the minimum clearance between the solids. Thermal effects are, however, neglected. The variation of lubricant viscosity with temperature and the thermal expansion of the lubricant and the solids are not considered.

## 7.1 Reynolds Equation

The coordinate system to be used in defining the elastohydrodynamic problem is shown in Figure 7.1. The appropriate

*Published as Chapter 7 in Ball Bearing Lubrication by Bernard J. Hamrock and Duncan Dowson, John Wiley & Sons, Inc., Sept. 1981.

Reynolds equation for the coordinate system $(\tilde{x},\tilde{y})$ can be written from equation (5.43) as

$$\frac{\partial}{\partial\tilde{x}}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial\tilde{x}}\right)+\frac{\partial}{\partial\tilde{y}}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial\tilde{y}}\right)=12u\,\frac{\partial(\rho h)}{\partial\tilde{x}} \tag{5.43}$$

where $u = (u_a + u_b)/2$ is the mean surface velocity or the entraining velocity in the $\tilde{x}$ direction. Letting

$$\left.\begin{aligned} X = \frac{\tilde{x}}{b},\ Y = \frac{\tilde{y}}{a},\ \bar{\rho} = \frac{\rho}{\rho_0} \\ \bar{\eta} = \frac{\eta}{\eta_0},\ H = \frac{h}{R_x},\ P = \frac{p}{E'} \end{aligned}\right\} \tag{7.1}$$

we can write equation (5.43) as

$$\frac{\partial}{\partial X}\left(\frac{\bar{\rho}H^3}{\bar{\eta}}\frac{\partial P}{\partial X}\right)+\frac{1}{k^2}\frac{\partial}{\partial Y}\left(\frac{\bar{\rho}H^3}{\bar{\eta}}\frac{\partial P}{\partial Y}\right)=12U\left(\frac{b}{R_x}\right)\frac{\partial(\bar{\rho}H)}{\partial X} \tag{7.2}$$

where $U = u\eta_0/E'R_x$ is a dimensionless speed parameter and $k = 1.03\,(R_y/R_x)^{0.64}$ is an ellipticity parameter.

Equation (7.2) is a nonhomogeneous partial differential equation in two variables. This is generally a difficult equation to solve, and the degree of complexity depends on the form of the parametric functions $\bar{\rho}$, $\bar{\eta}$, and $H$ and on the boundary conditions. In elastohydrodynamic lubrication the parameters are of the most difficult form. Therefore equation (7.2) is solved by approximate numerical methods. Before proceeding, however, it is necessary to write expressions for the dimensionless film

thickness H, the dimensionless density $\bar{\rho}$, and the dimensionless viscosity $\bar{\eta}$.

The influence of pressure on the density of fluids has been discussed in Section 5.5. From this section the dimensionless density can be written as

$$\bar{\rho} = 1 + \frac{A_2 PE'}{1 + B_2 PE'} \tag{5.57}$$

where $A_2$ and $B_2$ are constants that depend only on the fluid considered.

The viscosity of fluids has been discussed in Section 5.4, where attention was drawn to the work of Roelands (1966). For isothermal conditions the dimensionless viscosity of the fluid can be written from Section 5.4 as

$$\bar{\eta} = \left(\frac{\eta_\infty}{\eta_0}\right)^{\left[1-(1+p/c)^{Z_1}\right]} \tag{5.52}$$

where

$\eta_\infty = 6.31\times10^{-5}$ N s/m$^2$ ($9.15\times10^{-9}$ lbf s/in$^2$)

$c = 1.96\times10^8$ N/m$^2$ (28 440 lbf/in$^2$)

$\eta_0$ = viscosity of lubricant at atmospheric pressure

$Z_1$ = viscosity-pressure index, a dimensionless constant

These dimensions should also be used to define the constants in equation (5.52).

## 7.2 Film Shape

The film thickness can be written simply as

$$h(\tilde{x},\tilde{y}) = h_0 + S(\tilde{x},\tilde{y}) + \tilde{\delta}(\tilde{x},\tilde{y}) \tag{7.3}$$

where

$h_0$ = constant

$S(\tilde{x},\tilde{y})$ = separation due to geometry of undeformed ellipsoidal-solids

$\tilde{\delta}(\tilde{x},\tilde{y})$ = elastic deformation

The separation due to the geometry of the two undeformed ellipsoidal solids shown in Figure 2.18 can be described by an equivalent ellipsoidal solid near a plane. The geometrical requirement is that the separation of the two ellipsoidal solids in the initial and equivalent situations should be the same at equal values of $\tilde{x}$ and $\tilde{y}$. Therefore from Figure 7.1 the separation due to the undeformed geometry of the two ellipsoids can be written as

$$S(\tilde{x},\tilde{y}) = \frac{(\tilde{x} - \tilde{m}b)^2}{2R_x} + \frac{(\tilde{y} - \bar{\ell}a)^2}{2R_y} \tag{7.4}$$

where

$\tilde{m}$ = constant used to determine length of inlet region

$\bar{\ell}$ = constant used to determine width of side-leakage region

For illustration the mesh described in Figure 7.1 is used, where $\bar{m} = 4$ and $\bar{\ell} = 2$. However, the equations developed are written in general form.

Figure 7.2 illustrates the film thickness and its components for an ellipsoidal solid near a plane. Substituting equation (7.4) into equation (7.3) while using equation (7.1) to make this equation dimensionless gives

$$H = H_0 + \frac{b^2(X - \tilde{m})^2}{2R_x^2} + \frac{a^2(Y - \bar{\ell})^2}{2R_xR_y} + \frac{\tilde{\delta}(X,Y)}{R_x} \tag{7.5}$$

where $H_0$ is a constant that is initially estimated.

The way in which the elastic deformation of an equivalent ellipsoidal solid near a plane can be calculated has been outlined in Section 5.7. Therefore from Figure 7.1 and the results of Section 5.7 the elastic deformation can be written as

$$\tilde{\delta}_{\alpha,\beta}(X,Y) = \frac{2}{\pi} \sum_{j=1,2,\ldots}^{2\bar{\ell}X\bar{c}} \sum_{i=1,2,\ldots}^{(\tilde{m}+\bar{n})X\bar{d}} P_{i,j}\bar{D}_{i*,j*} \tag{7.6}$$

where

$\bar{c}$ = number of divisions in semimajor axis

$\bar{n}$ = constant used to determine length of outlet region

$\bar{d}$ = number of divisions in semiminor axis

ORIGINAL PAGE IS
OF POOR QUALITY

$$i^* = |\alpha - i| + 1$$

$$j^* = |\beta - j| + 1$$

$$\bar{D} = b\left(X + \frac{1}{2\bar{d}}\right) \ln \left\{ \frac{k\left(Y + \frac{1}{2\bar{c}}\right) + \left[k^2\left(Y + \frac{1}{2\bar{c}}\right)^2 + \left(X + \frac{1}{2\bar{d}}\right)^2\right]^{1/2}}{k\left(Y - \frac{1}{2\bar{c}}\right) + \left[k^2\left(Y - \frac{1}{2\bar{c}}\right)^2 + \left(X + \frac{1}{2\bar{d}}\right)^2\right]^{1/2}} \right\}$$

$$+ a\left(Y + \frac{1}{2\bar{c}}\right) \ln \left\{ \frac{\left(X + \frac{1}{2\bar{d}}\right) + \left[k^2\left(Y + \frac{1}{2\bar{c}}\right)^2 + \left(X + \frac{1}{2\bar{d}}\right)^2\right]^{1/2}}{\left(X - \frac{1}{2\bar{d}}\right) + \left[k^2\left(Y + \frac{1}{2\bar{c}}\right)^2 + \left(X - \frac{1}{2\bar{d}}\right)^2\right]^{1/2}} \right\}$$

$$+ b\left(X - \frac{1}{2\bar{d}}\right) \ln \left\{ \frac{k\left(Y - \frac{1}{2\bar{c}}\right) + \left[k^2\left(Y - \frac{1}{2\bar{c}}\right)^2 + \left(X - \frac{1}{2\bar{d}}\right)^2\right]^{1/2}}{k\left(Y + \frac{1}{2\bar{c}}\right) + \left[k^2\left(Y + \frac{1}{2\bar{c}}\right)^2 + \left(X - \frac{1}{2\bar{d}}\right)^2\right]^{1/2}} \right\}$$

$$+ a\left(Y - \frac{1}{2\bar{c}}\right) \ln \left\{ \frac{\left(X - \frac{1}{2\bar{d}}\right) + \left[k^2\left(Y - \frac{1}{2\bar{c}}\right)^2 + \left(X - \frac{1}{2\bar{d}}\right)^2\right]^{1/2}}{\left(X + \frac{1}{2\bar{d}}\right) + \left[k^2\left(Y - \frac{1}{2\bar{c}}\right)^2 + \left(X + \frac{1}{2\bar{d}}\right)^2\right]^{1/2}} \right\} \tag{7.7}$$

Equation (7.8) and Figure 7.1 clarify the meaning of equation (7.6). The elastic deformation at the center of the rectangular area $\tilde{\delta}_{9,5}$ (Figure 7.1) caused by the pressure on the

ORIGINAL PAGE IS
OF POOR QUALITY

various rectangular areas in and around the contact ellipse can be written as

$$\tilde{\delta}_{9,5} = \frac{2}{\pi}\begin{pmatrix} P_{1,1}\bar{D}_{9,5} + P_{2,1}\bar{D}_{8,5} + \cdots + P_{35,1}\bar{D}_{27,5} \\ P_{1,2}\bar{D}_{9,4} + P_{2,2}\bar{D}_{8,4} + \cdots + P_{35,2}\bar{D}_{27,4} \\ \vdots \\ P_{1,20}\bar{D}_{9,16} + P_{2,20}\bar{D}_{8,16} + \cdots + P_{35,20}\bar{D}_{27,16} \end{pmatrix} \tag{7.8}$$

## 7.3 Phi ($\Phi^*$) Solution

Having defined the density, viscosity, and film thickness, we can return to the problem of solving for the pressure in the Reynolds equation. It is well known (e.g., Whomes, 1966) that the dimensionless pressure $P$ of the Reynolds equation plotted as a function of $x$ exhibits a very localized pressure field with high values of $\partial P/\partial X$ and $\partial^2 P/\partial X^2$. Such a condition with high gradients is not welcomed when performing numerical analysis by relaxation methods. Therefore, to produce a much gentler curve, a parameter $\Phi^*$ is introduced where

$$\Phi^* = PH^{3/2} \tag{7.9}$$

ORIGINAL PAGE IS
OF POOR QUALITY

The pressure P is small at large values of film thickness H and large when the film thickness is small. This substitution also has the advantage that it eliminates all terms containing derivatives of the products of H and P or H and $\Phi^*$. Therefore from equation (7.9) and expansion of terms within equation (7.2) the Reynolds equation becomes

$$H^{3/2}\left[\frac{\partial}{\partial X}\left(\frac{\bar{\rho}}{\bar{\eta}}\frac{\partial\Phi^*}{\partial X}\right)+\frac{1}{k^2}\frac{\partial}{\partial Y}\left(\frac{\bar{\rho}}{\bar{\eta}}\frac{\partial\Phi^*}{\partial Y}\right)\right]-\frac{3\Phi^*}{2}\left\{\frac{\partial}{\partial X}\left(\frac{\bar{\rho}}{\bar{\eta}}H^{1/2}\frac{\partial H}{\partial X}\right)\right.$$

$$\left.+\frac{1}{k^2}\frac{\partial}{\partial Y}\left(\frac{\bar{\rho}}{\bar{\eta}}H^{1/2}\frac{\partial H}{\partial Y}\right)\right\}=\frac{12Ub}{R_x}\frac{\partial(\rho H)}{\partial X} \qquad (7.10)$$

Relaxation methods are usually used to solve equation (7.10) numerically. In the relaxation process the first step is to replace the differentials in equation (7.10) by finite difference approximations. The area to be considered is shown in Figure 7.1. The relaxation method relies on the fact that a function can be represented with sufficient accuracy over a small range by a quadratic expression. With this standard finite central-difference representation equation (7.10) can be rewritten as

$$A^*_{i,j}\Phi^*_{i+1,j}+B^*_{i,j}\Phi^*_{i,j-1}+C^*_{i,j}\Phi^*_{i-1,j}+D^*_{i,j}\Phi^*_{i,j+1}$$

$$-L^*_{i,j}\Phi^*_{i,j}-M^*_{i,j}=0 \qquad (7.11)$$

ORIGINAL PAGE IS
OF POOR QUALITY

where

$$\mu^* = \frac{\bar{\rho}}{\bar{\eta}}$$

$$A^*_{i,j} = 3\mu^*_{i+1,j} + \mu^*_{i-1,j}$$

$$B^*_{i,j} = \left(\frac{\bar{c}}{\bar{d}k}\right)^2 \left(\mu^*_{i,j+1} + 3\mu^*_{i,j-1}\right)$$

$$C^*_{i,j} = \mu^*_{i+1,j} + 3\mu^*_{i-1,j}$$

$$D^*_{i,j} = \left(\frac{\bar{c}}{\bar{d}k}\right)^2 \left(3\mu^*_{i,j+1} + \mu^*_{i,j-1}\right)$$

$$L^*_{i,j} = 4\left(\mu^*_{i+1,j} + \mu^*_{i-1,j}\right) + 4\left(\frac{\bar{c}}{\bar{d}k}\right)^2 \left(\mu^*_{i,j+1} + \mu^*_{i,j-1}\right)$$

$$+ \frac{3}{2H^{3/2}_{i,j}} \left\{ \mu^*_{i+1,j} \left(H_{i+1,j}\right)^{1/2} \left(3H_{i+1,j} - 4H_{i,j} + H_{i-1,j}\right) \right.$$

$$+ \mu^*_{i-1,j}\left(H_{i-1,j}\right)^{1/2}\left(H_{i+1,j} - 4H_{i,j} + 3H_{i-1,j}\right)$$

$$+ \left(\frac{\bar{c}}{\bar{d}k}\right)^2 \left[ \mu^*_{i,j+1}\left(H_{i,j+1}\right)^{1/2}\left(3H_{i,j+1} - 4H_{i,j} + H_{i,j-1}\right) \right.$$

$$\left.\left. + \mu^*_{i,j-1} \left(H_{i,j-1}\right)^{1/2}\left(H_{i,j+1} - 4H_{i,j} + 3H_{i,j-1}\right)\right]\right\}$$

$$M^*_{i,j} = \frac{24Ub}{\bar{d}R_x H^{3/2}_{i,j}} \left(\bar{\rho}_{i+1,j}H_{i+1,j} - \bar{\rho}_{i-1,j}H_{i-1,j}\right)$$

Central differences are used in obtaining equation (7.11) because they produce better estimates than forward or backward differences. Central differences tend to average; forward and backward differences tend to "overshoot" and "undershoot" the value of the function. More details of the derivation of equation (7.11) are given in Hamrock (1976).

An example of a nodal structure that can be used in evaluating the pressure distribution in an elastohydrodynamic elliptical contact is shown in Figure 7.3. A nodal structure is considered to be suitable when the minimum film thickness within the conjunction does not change materially either when additional nodes are placed in the semimajor and semiminor axes or when the distances from the center of the contact to the edges of the computing zone are extended. Note that because of the dimensionalization of $\tilde{x}$ and $\tilde{y}$ the Hertzian contact area is represented by a circle of radius unity regardless of the ellipticity parameter.

From Figure 7.3 the features of this nodal structure are

$$\left.\begin{aligned} \tilde{m} = 4,\ \bar{n} = 1.15,\ \bar{\ell} = 1.6 \\ \bar{c} = 5,\ \bar{d} = 13 \end{aligned}\right\} \qquad (7.12)$$

These values are used to define the nodal structure for most of the results presented in Chapters 8 and 9. The only exception is for a number of high-speed cases when the constant used to

determine the inlet distance $\tilde{m}$ has to be extended from 4 to 6, with the other values being held constant.

## 7.4 Boundary and Initial Conditions

The following boundary conditions have been adopted:

(1) At the edges of the rectangular zone of computation (Figure 7.3) the pressure is zero. This implies that $\Phi^*$ is also zero. Specifically this means that along the bottom of Figure 7.3 $\Phi^*_{i,1} = 0$, along the left side $\Phi^*_{1,j} = 0$, along the top $\Phi^*_{i,16} = 0$, and along the right side $\Phi^*_{67,j} = 0$.

(2) At the cavitation boundary

$$P = \frac{\partial P}{\partial X} = \frac{\partial P}{\partial Y} = 0$$

This condition, commonly known as the Reynolds condition, is satisfied by simply resetting $\Phi^*_{i,j}$ to zero whenever it occurs as a negative value.

The pressure outside the Hertzian contact area is initially assumed to be zero, and therefore $\Phi^*$ is also equal to zero. That is, $\Phi^* = 0$ when

$$(X - \tilde{m})^2 + (Y - \bar{\ell})^2 \geq 1$$

The pressure inside the Hertzian contact area is initially assumed to be Hertzian. Therefore from equation (3.5) the initial value of $\Phi^*$ inside the contact area is

$$\Phi^* = \frac{3FH^{3/2}}{2\pi abE'}\left[1 - (X - \tilde{m})^2 - (Y - \bar{\ell})^2\right]^{1/2} \qquad (7.13)$$

where

$$(X - \tilde{m})^2 + (Y - \bar{\ell})^2 < 1$$

## 7.5 Relaxation Method

Equation (7.11) represents a system of simultaneous equations. There will be as many unknown values of $\Phi^*$ as there are nodal points and equations. This system of equations can thus be solved by the Gauss-Seidel iterative method. In calculating each nodal value of $\Phi^*$ this method uses the most recently calculated values at the surrounding nodes involved in the application of the finite difference equation (7.11). That is, nodal values calculated on that cycle are used if available. A cycle is one complete set of calculations for all the nodes in the structure. The concept of the Gauss-Seidel approach is that the best information available should be used instead of waiting until the next cycle. If subscript $n$ is the iterant and $\Phi^*_{i,j}$ is the particular solution to be found, the relaxation formula can be expressed as

ORIGINAL PAGE IS
OF POOR QUALITY

$$\Phi^*_{i,j,n+1} = \Phi^*_{i,j,n} - \lambda_c \Big\{ M^*_{i,j} - A^*_{i,j}\Phi^*_{i+1,j,n} - B^*_{i,j}\Phi^*_{i,j-1,n+1} - C^*_{i,j}\Phi^*_{i-1,j,n+1} - D^*_{i,j}\Phi^*_{i,j+1,n} + L^*_{i,j}\Phi^*_{i,j,n} \Big\} \Big/ L^*_{i,j} \tag{7.14}$$

where $\lambda_c$ is a relaxation factor.

Therefore equation (7.14) is used, starting with node (2,2) and then continuing with (3,2), . . ., $(\overline{M},2)$ followed by (2,3), (3,3), . . ., $(\overline{M},3)$ and ending with $(2,\overline{N})$, $(3,\overline{N})$, . . ., $(\overline{M},\overline{N})$, where

$$\left.\begin{aligned} \overline{M} &= (\tilde{m} + \overline{n})\overline{d} - 1 \\ \overline{N} &= 2\widetilde{\ell c} - 1 \end{aligned}\right\} \tag{7.15}$$

The relaxation procedure described by equation (7.14) is continued until

$$\sum_{j=2,3,\ldots}^{\overline{N}} \sum_{i=2,3,\ldots}^{\overline{M}} \frac{\left|\Phi^*_{i,j,n+1} - \Phi^*_{i,j,n}\right|}{\Phi^*_{i,j,n+1}} < 0.1$$

The relaxation method provides values of $\Phi^*_{i,j}$ for every node within the nodal structure. Once a solution has been obtained, the dimensionless pressure can be computed from the relationship

$$P_{i,j} = \Phi^*_{i,j}(H_{i,j})^{-3/2} \tag{7.16}$$

With these new values of the dimensionless pressure, revised values of the dimensionless viscosity, density, and film thickness can be evaluated. Thus the coefficients of equation (7.14) ($A^*_{i,j}$, $B^*_{i,j}$, $C^*_{i,j}$, $D^*_{i,j}$, $L^*_{i,j}$, and $M^*_{i,j}$) may also change. Accordingly it is necessary to reenter the relaxation loop. This process is continued until the following inequality is satisfied:

$$\sum_{j=2,3,\ldots}^{\overline{N}} \sum_{i=2,3,\ldots}^{\overline{M}} \frac{\left|P_{i,j,n+1} - P_{i,j,n}\right|}{P_{i,j,n+1}} < 0.1$$

## 7.6 Normal Applied Load and Flow Rate

The initial value of the constant $H_0$ in equation (7.5) has to be estimated, but thereafter the task is to find the correct value. To do this, the integrated normal load must be evaluated, where

$$\overline{F} = E'ab \int_0^{\overline{m}+\overline{n}} \int_0^{2\overline{\ell}} P \, dY \, dX \tag{7.17}$$

According to Simpson's rule this double integral can be written as

$$\bar{F} = \frac{4E'ab}{9\bar{c}\bar{d}} \sum_{i=2,3,\ldots}^{\tilde{m}+\bar{n}-1} 2^{q_1} \left( \sum_{j=2,3,\ldots}^{2\bar{\ell}-1} 2^{q_2} P_{i,j} \right) \qquad (7.18)$$

where

$q_1 = 0$ if $i$ is odd

$q_1 = 1$ if $i$ is even

$q_2 = 0$ if $j$ is odd

$q_2 = 1$ if $j$ is even

A new value of $H_0$ can be estimated from the following expression and the current values of $H_0$ and $\bar{F}$ and the applied load:

$$\bar{H}_0 = H_0 \left( \frac{F}{\bar{F}} \right) \qquad (7.19)$$

The film thickness can now be recalculated at each node for the new value of $\bar{H}_0$ from equation (7.5), and reentry into the relaxation process is required. This process is continued until the following convergence is satisfied:

$$\frac{|\bar{F} - F|}{F} < 0.0005$$

Once this convergence criterion has been satisfied, the values of pressure and film thickness in and around the contact are established.

The mass flow rate per unit width in the $\tilde{x}$ and $\tilde{y}$ directions shown in Figure 7.1 can be written as

$$q_{\tilde{x}} = u\rho h - \frac{\rho h^3}{12\eta}\frac{\partial p}{\partial \tilde{x}}$$

$$q_{\tilde{y}} = -\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial \tilde{y}}$$

These equations can be written in dimensionless form as

$$\bar{Q}_x = U\bar{\rho}H - \frac{\bar{\rho}H^3}{12\bar{\eta}}\left(\frac{R}{b}\right)\frac{\partial P}{\partial X} \tag{7.20}$$

$$\bar{Q}_y = -\frac{\bar{\rho}H^3}{12\bar{\eta}}\left(\frac{R}{a}\right)\frac{\partial P}{\partial Y} \tag{7.21}$$

Furthermore the total flow rates and flow angles at any node can be written as

$$\bar{Q} = \left(\bar{Q}_x^2 + \bar{Q}_y^2\right)^{1/2} \tag{7.22}$$

$$\gamma_a = \tan^{-1}\left(\frac{\bar{Q}_y}{\bar{Q}_x}\right) \tag{7.23}$$

## 7.7 Flow Charts

Figures 7.4 and 7.5 are flow charts for the numerical solution on the digital computer of the equations developed in the analysis. Figure 7.4 is the flow chart for the main program. There are essentially three loops within the main program: the relaxation loop, the pressure loop, and the normal-load loop. In the relaxation loop $\Phi^*_{i,j,n+1}$ is generated. In the pressure loop the new values of $\Phi^*_{i,j,n+1}$ of the relaxation loop result in new values of pressure $P_{i,j,n+1}$, which in turn result in new values of film thickness $H_{i,j}$, viscosity $\bar{\eta}_{i,j}$, and density $\bar{\rho}_{i,j}$. The final loop, the normal-load loop, ensures that the integrated normal applied load agrees with the initially specified value.

Figure 7.5 is the flow chart for the film thickness subroutine SUB6. A number of calculations occur only once and need not be repeated on reentering this subroutine. With each new pressure distribution the elastic deformation is recalculated, and this subroutine is left with a new film thickness and therefore a new value of $\Phi^*_{i,j,n+1}$.

## 7.8 Closure

In this chapter a procedure for the numerical solution of the complete, isothermal elastohydrodynamic lubrication problem

for elliptical contacts has been outlined. This procedure calls for the simultaneous solution of the elasticity and Reynolds equations. In the elasticity analysis the contact zone was divided into equal rectangular areas, and it was assumed that a uniform pressure was applied over each area. In the numerical solution of the Reynolds equation the parameter $\Phi^*$ equals $PH^{3/2}$, where $P$ is the dimensionless pressure and $H$ the dimensionless film thickness, was introduced in order to ease the relaxation process. The nodal structure and the boundary and initial conditions were given and a Gauss-Seidel relaxation method was used. The computer program has three major loops: the relaxation loop, the pressure loop, and the normal-load loop. The last loop requires the integrated hydrodynamic load-carrying capacity to be in agreement with the applied load within some specified tolerance. When all three loops converge, the pressure and film thickness in and around the elliptical contact area are established.

## SYMBOLS

| | |
|---|---|
| $A$ | constant used in equation (3.113) |
| $A^*$, $B^*$, $C^*$, $D^*$, $L^*$, $M^*$ | relaxation coefficients |
| $A_v$ | drag area of ball, $m^2$ |
| $a$ | semimajor axis of contact ellipse, m |
| $\bar{a}$ | $a/2\bar{m}$ |
| $B$ | total conformity of bearing |
| $b$ | semiminor axis of contact ellipse, m |
| $\bar{b}$ | $b/2\bar{m}$ |
| $C$ | dynamic load capacity, N |
| $C_v$ | drag coefficient |
| $C_1,\ldots,C_8$ | constants |
| $c$ | 19,609 $N/cm^2$ (28,440 $lbf/in^2$) |
| $\bar{c}$ | number of equal divisions of semimajor axis |
| $D$ | distance between race curvature centers, m |
| $\tilde{D}$ | material factor |
| $\bar{D}$ | defined by equation (5.63) |
| De | Deborah number |
| $d$ | ball diameter, m |
| $\bar{d}$ | number of divisions in semiminor axis |
| $d_a$ | overall diameter of bearing (Figure 2.13), m |
| $d_b$ | bore diameter, m |
| $d_e$ | pitch diameter, m |
| $d_e'$ | pitch diameter after dynamic effects have acted on ball, m |
| $d_i$ | inner-race diameter, m |
| $d_o$ | outer-race diameter, m |

| | |
|---|---|
| $E$ | modulus of elasticity, $N/m^2$ |
| $E'$ | effective elastic modulus, $2\Big/\left(\frac{1-\nu_a^2}{E_a}+\frac{1-\nu_b^2}{E_b}\right)$, $N/m^2$ |
| $E_a$ | internal energy, $m^2/s^2$ |
| $\tilde{E}$ | processing factor |
| $\tilde{E}_1$ | $[(\tilde{H}_{min} - H_{min})/H_{min}] \times 100$ |
| $\mathscr{E}$ | elliptic integral of second kind with modulus $(1 - 1/k^2)^{1/2}$ |
| $\bar{\mathscr{E}}$ | approximate elliptic integral of second kind |
| $e$ | dispersion exponent |
| $F$ | normal applied load, N |
| $F^*$ | normal applied load per unit length, N/m |
| $\tilde{F}$ | lubrication factor |
| $\bar{F}$ | integrated normal applied load, N |
| $F_c$ | centrifugal force, N |
| $F_{max}$ | maximum normal applied load (at $\psi = 0$), N |
| $F_r$ | applied radial load, N |
| $F_t$ | applied thrust load, N |
| $F_\psi$ | normal applied load at angle $\psi$, N |
| $\mathscr{F}$ | elliptic integral of first kind with modulus $(1 - 1/k^2)^{1/2}$ |
| $\bar{\mathscr{F}}$ | approximate elliptic integral of first kind |
| $f$ | race conformity ratio |
| $f_b$ | rms surface finish of ball, m |
| $f_r$ | rms surface finish of race, m |
| $G$ | dimensionless materials parameter, $\alpha E$ |
| $G^*$ | fluid shear modulus, $N/m^2$ |
| $\tilde{G}$ | hardness factor |
| $g$ | gravitational constant, $m/s^2$ |

| | |
|---|---|
| $g_E$ | dimensionless elasticity parameter, $W^{8/3}/U^2$ |
| $g_V$ | dimensionless viscosity parameter, $GW^3/U^2$ |
| $H$ | dimensionless film thickness, $h/R_x$ |
| $\hat{H}$ | dimensionless film thickness, $H(W/U)^2 = F^2h/u^2\eta_0^2R_x^3$ |
| $H_c$ | dimensionless central film thickness, $h_c/R_x$ |
| $H_{c,s}$ | dimensionless central film thickness for starved lubrication condition |
| $H_f$ | frictional heat, N m/s |
| $H_{min}$ | dimensionless minimum film thickness obtained from EHL elliptical-contact theory |
| $H_{min,r}$ | dimensionless minimum film thickness for a rectangular contact |
| $H_{min,s}$ | dimensionless minimum film thickness for starved lubrication condition |
| $\tilde{H}_c$ | dimensionless central film thickness obtained from least-squares fit of data |
| $\tilde{H}_{min}$ | dimensionless minimum film thickness obtained from least-squares fit of data |
| $\bar{H}_c$ | dimensionless central-film-thickness - speed parameter, $H_cU^{-0.5}$ |
| $\bar{H}_{min}$ | dimensionless minimum-film-thickness - speed parameter, $H_{min}U^{-0.5}$ |
| $\bar{H}_0$ | new estimate of constant in film thickness equation |
| $h$ | film thickness, m |
| $h_c$ | central film thickness, m |
| $h_i$ | inlet film thickness, m |

| | |
|---|---|
| $h_m$ | film thickness at point of maximum pressure, where $dp/dx = 0$, m |
| $h_{min}$ | minimum film thickness, m |
| $h_0$ | constant, m |
| $I_d$ | diametral interference, m |
| $I_p$ | ball mass moment of inertia, m N $s^2$ |
| $I_r$ | integral defined by equation (3.76) |
| $I_t$ | integral defined by equation (3.75) |
| J | function of k defined by equation (3.8) |
| J* | mechanical equivalent of heat |
| $\bar{J}$ | polar moment of inertia, m N $s^2$ |
| K | load-deflection constant |
| k | ellipticity parameter, a/b |
| $\bar{k}$ | approximate ellipticity parameter |
| $\tilde{k}$ | thermal conductivity, N/s °C |
| $k_f$ | lubricant thermal conductivity, N/s °C |
| L | fatigue life |
| $L_a$ | adjusted fatigue life |
| $L_t$ | reduced hydrodynamic lift, from equation (6.21) |
| $L_1,\ldots,L_4$ | lengths defined in Figure 3.11, m |
| $L_{10}$ | fatigue life where 90 percent of bearing population will endure |
| $L_{50}$ | fatigue life where 50 percent of bearing population will endure |
| $\ell$ | bearing length, m |
| $\bar{\ell}$ | constant used to determine width of side-leakage region |
| M | moment, Nm |

| | |
|---|---|
| $M_g$ | gyroscopic moment, Nm |
| $M_p$ | dimensionless load-speed parameter, $WU^{-0.75}$ |
| $M_s$ | torque required to produce spin, N m |
| m | mass of ball, N $s^2$/m |
| m* | dimensionless inlet distance at boundary between fully flooded and starved conditions |
| $\tilde{m}$ | dimensionless inlet distance (Figures 7.1 and 9.1) |
| $\bar{m}$ | number of divisions of semimajor or semiminor axis |
| $m_W$ | dimensionless inlet distance boundary as obtained from Wedeven, et al. (1971) |
| N | rotational speed, rpm |
| n | number of balls |
| n* | refractive index |
| $\bar{n}$ | constant used to determine length of outlet region |
| P | dimensionless pressure |
| $P_D$ | dimensionless pressure difference |
| $P_d$ | diametral clearance, m |
| $P_e$ | free endplay, m |
| $P_{Hz}$ | dimensionless Hertzian pressure, N/$m^2$ |
| p | pressure, N/$m^2$ |
| $p_{max}$ | maximum pressure within contact, $3F/2\pi ab$, N/$m^2$ |
| $p_{iv,as}$ | isoviscous asymptotic pressure, N/$m^2$ |
| Q | solution to homogeneous Reynolds equation |
| $Q_m$ | thermal loading parameter |
| $\bar{Q}$ | dimensionless mass flow rate per unit width, $q\eta_0/\rho_0 E' R^2$ |
| $q_f$ | reduced pressure parameter |
| $q_x$ | volume flow rate per unit width in x direction, $m^2$/s |

| | |
|---|---|
| $q_y$ | volume flow rate per unit width in $y$ direction, $m^2/s$ |
| $R$ | curvature sum, m |
| $R_a$ | arithmetical mean deviation defined in equation (4.1), m |
| $R_c$ | operational hardness of bearing material |
| $R_x$ | effective radius in $x$ direction, m |
| $R_y$ | effective radius in $y$ direction, m |
| $r$ | race curvature radius, m |
| $\left.\begin{matrix} r_{ax}, r_{bx}, \\ r_{ay}, r_{by} \end{matrix}\right\}$ | radii of curvature, m |
| $r_c, \phi_c, z$ | cylindrical polar coordinates |
| $r_s, \theta_s, \phi_s$ | spherical polar coordinates |
| $\bar{r}$ | defined in Figure 5.4 |
| $S$ | geometric separation, m |
| $S^*$ | geometric separation for line contact, m |
| $S_0$ | empirical constant |
| $s$ | shoulder height, m |
| $T$ | $\tau_0/p_{max}$ |
| $\tilde{T}$ | tangential (traction) force, N |
| $T_m$ | temperature, °C |
| $T_b^*$ | ball surface temperature, °C |
| $T_f^*$ | average lubricant temperature, °C |
| $\Delta T^*$ | ball surface temperature rise, °C |
| $T_1$ | $(\tau_0/p_{max})_{k=1}$ |
| $T_v$ | viscous drag force, N |
| $t$ | time, s |
| $t_a$ | auxiliary parameter |
| $u_B$ | velocity of ball-race contact, m/s |

| | |
|---|---|
| $u_c$ | velocity of ball center, m/s |
| $U$ | dimensionless speed parameter, $\eta_0 u/E'R_x$ |
| $u$ | surface velocity in direction of motion, $(u_a + u_b)/2$, m/s |
| $\bar{u}$ | number of stress cycles per revolution |
| $\Delta u$ | sliding velocity, $u_a - u_b$, m/s |
| $v$ | surface velocity in transverse direction, m/s |
| $W$ | dimensionless load parameter, $F/E'R^2$ |
| $w$ | surface velocity in direction of film, m/s |
| $X$ | dimensionless coordinate, $x/R_x$ |
| $Y$ | dimensionless coordinate, $y/R_x$ |
| $X_t$, $Y_t$ | dimensionless grouping from equation (6.14) |
| $X_a$, $Y_a$, $Z_a$ | external forces, N |
| $Z$ | constant defined by equation (3.48) |
| $Z_1$ | viscosity pressure index, a dimensionless constant |
| $x, \tilde{x}, \bar{x}, \bar{x}_1$ | coordinate system |
| $y, \tilde{y}, \bar{y}, \bar{y}_1$ | |
| $z, \tilde{z}, \bar{z}, \bar{z}_1$ | |
| $\alpha$ | pressure-viscosity coefficient of lubrication, $m^2/N$ |
| $\alpha_a$ | radius ratio, $R_y/R_x$ |
| $\beta$ | contact angle, rad |
| $\beta_f$ | free or initial contact angle, rad |
| $\beta'$ | iterated value of contact angle, rad |
| $\Gamma$ | curvature difference |
| $\gamma$ | viscous dissipation, $N/m^2$ s |
| $\dot{\gamma}$ | total strain rate, $s^{-1}$ |
| $\dot{\gamma}_e$ | elastic strain rate, $s^{-1}$ |
| $\dot{\gamma}_v$ | viscous strain rate, $s^{-1}$ |

| | |
|---|---|
| $\gamma_a$ | flow angle, deg |
| $\delta$ | total elastic deformation, m |
| $\delta^*$ | lubricant viscosity temperature coefficient, $^oC^{-1}$ |
| $\delta_D$ | elastic deformation due to pressure difference, m |
| $\delta_r$ | radial displacement, m |
| $\delta_t$ | axial displacement, m |
| $\delta_x$ | displacement at some location x, m |
| $\bar{\delta}$ | approximate elastic deformation, m |
| $\tilde{\delta}$ | elastic deformation of rectangular area, m |
| $\epsilon$ | coefficient of determination |
| $\epsilon_1$ | strain in axial direction |
| $\epsilon_2$ | strain in transverse direction |
| $\zeta$ | angle between ball rotational axis and bearing centerline (Figure 3.10) |
| $\zeta_a$ | probability of survival |
| $\eta$ | absolute viscosity at gauge pressure, N s/m$^2$ |
| $\bar{\eta}$ | dimensionless viscosity, $\eta/\eta_0$ |
| $\eta_0$ | viscosity at atmospheric pressure, N s/m$^2$ |
| $\eta_\infty$ | $6.31 \times 10^{-5}$ N s/m$^2$(0.0631 cP) |
| $\theta$ | angle used to define shoulder height |
| $\Lambda$ | film parameter (ratio of film thickness to composite surface roughness) |
| $\lambda$ | equals 1 for outer-race control and 0 for inner-race control |
| $\lambda_a$ | second coefficient of viscosity |
| $\lambda_b$ | Archard-Cowking side-leakage factor, $(1 + 2/3\ \alpha_a)^{-1}$ |
| $\lambda_c$ | relaxation factor |

| | |
|---|---|
| $\mu$ | coefficient of sliding friction |
| $\mu^*$ | $\bar{\rho}/\bar{\eta}$ |
| $\nu$ | Poisson's ratio |
| $\xi$ | divergence of velocity vector, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$, $s^{-1}$ |
| $\rho$ | lubricant density, $N\ s^2/m^4$ |
| $\bar{\rho}$ | dimensionless density, $\rho/\rho_0$ |
| $\rho_0$ | density at atmospheric pressure, $N\ s^2/m^4$ |
| $\sigma$ | normal stress, $N/m^2$ |
| $\sigma_1$ | stress in axial direction, $N/m^2$ |
| $\tau$ | shear stress, $N/m^2$ |
| $\tau_0$ | maximum subsurface shear stress, $N/m^2$ |
| $\tilde{\tau}$ | shear stress, $N/m^2$ |
| $\tilde{\tau}_e$ | equivalent stress, $N/m^2$ |
| $\tilde{\tau}_L$ | limiting shear stress, $N/m^2$ |
| $\Phi$ | ratio of depth of maximum shear stress to semiminor axis of contact ellipse |
| $\Phi^*$ | $PH^{3/2}$ |
| $\Phi_1$ | $(\Phi)_{k=1}$ |
| $\phi$ | auxiliary angle |
| $\phi_T$ | thermal reduction factor |
| $\psi$ | angular location |
| $\psi_\ell$ | limiting value of $\psi$ |
| $\Omega_i$ | absolute angular velocity of inner race, rad/s |
| $\Omega_o$ | absolute angular velocity of outer race, rad/s |
| $\omega$ | angular velocity, rad/s |
| $\omega_B$ | angular velocity of ball-race contact, rad/s |
| $\omega_b$ | angular velocity of ball about its own center, rad/s |

| | |
|---|---|
| $\omega_c$ | angular velocity of ball around shaft center, rad/s |
| $\omega_s$ | ball spin rotational velocity, rad/s |

Subscripts:

| | |
|---|---|
| a | solid a |
| b | solid b |
| c | central |
| bc | ball center |
| IE | isoviscous-elastic regime |
| IR | isoviscous-rigid regime |
| i | inner race |
| K | Kapitza |
| min | minimum |
| n | iteration |
| o | outer race |
| PVE | piezoviscous-elastic regime |
| PVR | piezoviscous-rigid regime |
| r | for rectangular area |
| s | for starved conditions |
| x,y,z | coordinate system |

Superscript:

| | |
|---|---|
| $(\overline{\phantom{x}})$ | approximate |

## REFERENCES


Abbott, E. J. and Firestone, F. A. (1933) Specifying Surface Quality, Mech. Eng., 55, 569-572.

Agricola, G. (1556) De Re Metallica, Basel.

Aihara, S. and Dowson, D. (1979) "A Study of Film Thickness in Grease Lubricated Elastohydrodynamic Contacts," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics', D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 104-115.

Allan, R. K. (1945) Rolling Bearings, Sir Isaac Pitman & Sons, London.

Alsaad, M., Bair, S., Sanborn, D. M., and Winer, W. O. (1978) "Glass Transitions in Lubricants: Its Relation to Elastohydrodynamic Lubrication (EHD)," J. Lubr. Technol., 100(3), 404-417.

Amontons, G. (1699) "De la resistance caus'ee dans les machines," Memoires de l'Academie Royal, A, Chez Gerard Kuyper, Amsterdam, 1706, 257-282.

Anderson, W. J. (1978) "The Practical Impact of Elastohydrodynamic Lubrication," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 217-226.

Anderson, W. J. and Zaretsky, E. V. (1968) "Rolling-Element Bearings." Mach. Des. (Bearing Reference Issue), 40(14), 22-39.

Anderson, W. J. and Zaretsky, E. V. (1973) "Rolling-Element Bearings - A Review of the State of the Art," Tribology Workshop sponsored by National Science Foundation, Atlanta, Ga., Oct. 19-20, 1972.

Archard, J. F. (1968) "Non-Dimensional Parameters in Isothermal Theories of Elastohydrodynamic Lubrication." J. Mech. Eng. Sci., 10(2), 165-167.

Archard, J. F. and Cowking, E. W. (1965-66) "Elastohydrodynamic Lubrication at Point Contacts," Proc. Inst. Mech. Eng., London, 180(3B), 47-56.

Archard, J. F. and Kirk, M. T. (1961) "Lubrication at Point Contacts" Proc. R. Soc. London, Ser. A, 261, 532-550.

Archard, J. F. and Kirk, M. T. (1964) "Film Thickness for a Range of Lubricants Under Severe Stress," J. Mech. Eng. Sci., 6, 101-102.

Ausherman, V. K., Nagaraj, H. S., Sanborn, D. M., and Winer, W. O. (1976) "Infrared Temperature Mapping in Elastohydrodynamic Lubrication," J. Lubr. Technol., 98(2), 236-243.

Baglin, K. P. and Archard, J. F. (1972) "An Analytic Solution of the Elastohydrodynamic Lubrication of Materials of Low Elastic Modulus," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 13.

Bair, S. and Winer, W. (1979) "Shear Strength Measurements of Lubricants at High Pressures," J. Lubr. Technol. 101(3), 251-257.

Bamberger, E. N. (1967) "The Effect of Ausforming on the Rolling Contact Fatigue Life of a Typical Bearing Steel," J. Lubr. Technol., 89(1), 63-75.

Bamberger, E. N. (1972) "The Thermomechanical Working of Electro-Slag Melted M-50 Bearing Steel," R72AEG290, General Electric Co., Cincinnati, Ohio.

Bamberger, E. N., Harris, T. A., Kacmarsky, W. M., Moyer, C. A., Parker, R. J., Sherlock, J. J., and Zaretsky, E. V. (1971) Life Adjustment Factors for Ball and Roller Bearings. American Society of Mechanical Engineers, New York.

Bamberger, E. N., Zaretsky, E. V., and Singer, H. (1976) "Endurance and Failure Characteristics of Main-Shaft Jet Engine Bearing at $3x10^6$DN," J. Lubr. Technol., 98(4), 580-585.

Barus, C. (1893) "Isotherms, Isopiestics, and Isometrics Relative to Viscosity," Am. J. Sci., 45, 87-96.

Barwell, F. T. (1974) "The Tribology of Wheel on Rail," Tribol. Int., 7, (4), 146-150.

Barwell, F. T. (1979) "Bearing Systems - Principles and Practice," Oxford University Press, Oxford.

Bell, J. C. and Kannel, J. W. (1970) "Simulation of Ball-Bearing Lubrication with a Rolling-Disk Apparatus," J. Lubr. Technol., 92, 1-15.

Bell, J. C., Kannel, J. W., and Allen, C. M. (1964) "The Rheological Behaviour of the Lubricant in the Contact Zone of a Rolling Contact System," J. Basic Eng., 86(3), 423-432.

Bisson, E. E. and Anderson, W. J. (1964) "Advanced Bearing Technology," NASA SP-38.

Biswas, S. and Snidle, R. W. (1976) "Elastohydrodynamic Lubrication of Spherical Surfaces of Low Elastic Modulus," J. Lubr. Technol., 98(4), 524-529.

Blok, H. (1952) Discussion of paper by E. McEwen. Gear Lubrication Symposium. Part I. The Lubrication of Gears, J. Inst. Petrol., 38, 673.

Blok, H. (1965) "Inverse Problems in Hydrodynamic Lubrication and Design Directives for Lubricated Flexible Surfaces," Proceedings of International Symposium on Lubrication and Wear, D. Muster and B. Sternlicht, eds., McCutchan, Berkeley, 1-151.

Brewe, D. E., Coe, H. H., and Scibbe, H. W. (1969) "Cooling Studies with High-Speed Ball bearings Operating in Cool Hydrogen Gas," Trans. ASLE, vol. 12, 66-76.

Brewe, D. E. and Hamrock, B. J. (1977) "Simplified Solution for Elliptical-Contact Deformation Between Two Elastic Solids," J. Lubr. Technol. 99(4), 485-487.

Brewe, D. E., Hamrock, B. J., and Taylor, C. M. (1979) "Effect of Geometry on Hydrodynamic Film Thickness," J. Lubr. Technol., 101(2), 231-239.

Brown, P. F. and Potts, J. R. (1977) "Evaluation of Powder Processed Turbine Engine Ball Bearings," PWA-FR-8481, Pratt & Whitney Aircraft Group, West Palm Beach, Fla. (AFAPL-TR-77-26.)

Cameron, A. (1954) "Surface Failure in Gears," J. Inst. Petrol., vol. 40, 191.

Cameron, A. (1966) The Principles of Lubrication, Wiley, New York.

Cameron, A. and Gohar, R. (1966) "Theoretical and Experimental Studies of the Oil Film in Lubricated Point Contact," Proc. R. Soc. London, Ser. A., 291, 520-536.

Carburi, M. (1777) "Monument Elevé a la Gloire de Pierre-le-Grand, ou Relation Des Travaux et des Moyens Mechaniques Qui ont été employés pour transporter à Petersbourg un Rocher de trois millions pesant, destiné à servir de base à la Statue équestre de cet Empereur; avec un Examen Physique et Chymique de meme Rocher," Paris, (Bookseller: Nyon aîné, Libraine, rue Saint-Lean-de-Beauvois; Printer: Imprimeur-Libraire, rue de la Harpe, vis-à-vis la rue S. Severin).

Castle, P. and Dowson, D. (1972) "A Theoretical Analysis of the Starved Contact," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 131.

Cheng, H. S. (1967) "Calculation of Elastohydrodynamic Film Thickness in High-Speed Rolling and Sliding Contacts," Mechanical Technology Technical Report MTI-67TR24, May 1967.

Cheng, H. S. (1970) "A Numerical Solution to the Elastohydrodynamic Film Thickness in an Elliptical Contact," J. Lubr. Technol., 92(1), 155-162.

Cheng, H. S. and Orcutt, F. K. (1965-66) "A Correlation Between the Theoretical and Experimental Results on the Elastohydrodynamic Lubrication of Rolling and Sliding Contacts," Elastrohydrodynamic Lubrication, Symposium, Leeds, England, Sept. 21-23, 1965, General Papers. Institution of Mechanical Engineers, London, 111-121.

Cheng, H. S. and Sternlicht, B. (1964) "A Numerical Solution for the Pressure, Temperature, and Film Thickness Between Two Infinitely Long, Lubricated Rolling and Sliding Cylinders, Under Heavy Loads," J. Basic Eng. 87(3), 695-707.

Chiu, Y. P. (1974) "An Analysis and Prediction of Lubricant Film Starvation in Rolling Contact Systems," ASME Trans., 17(1), 22-35.

Clark, R. H. (1938) "Earliest Known Ball Thrust Bearing Used in Windmill," English Mechanic, 30 (Dec.) 223.

Coulomb, C. A. (1785) "Théorie des Machines Simples, en ayant égard au frottement de leur parties, et a la roideur des cordages," Academic Royale des Sciences, Mem. Math. Phys., X, Paris, 161-342.

Crook, A. W. (1957) "Simulated Gear-Tooth Contact: Some Experiments Upon Their Lubrication and Sub-Surface Deformation," Proc. Inst. Mech. Eng., London, 171, 187.

Crook, A. W. (1958) "The Lubrication of Rollers, I," Phil. Trans. R. Soc. London, Ser. A, 250, 387-409.

Crook, A. W. (1961) "Elasto-Hydrodynamic Lubrication of Rollers, Nature," 190, 1182.

Crook, A. W. (1963) "The Lubrication of Rollers, IV - Measurements of Friction and Effective Viscosity," Phil. Trans. R. Soc. London, Ser. A, 255, 281-312.

Dalmaz, G. and Godet, M. (1973) "Traction, Load, and Film Thickness in Lightly Loaded Lubricated Point Contacts," J. Mech. Eng. Sci., 15(6), 400-409.

Dalmaz, G. and Godet, M. (1978) "Film Thickness and Effective Viscosity of Some Fire Resistant Fluids in Sliding Point Contacts," J. Lubr. Technol., 100(2), 304-308.

Denhard, W. G. (1966) "Cost Versus Value of Ball Bearings," Gyro-Spin Axis Hydrodynamic Bearing Symposium, Vol. II, Ball Bearings. Massachusetts Institute of Technology, Cambridge, Mass., Tab. 1.

Desaguliers, J. T. (1734) A Course of Experimental Philosophy, 2 Volumes, London, Volume I, with 32 copper plates.

Dowson, D. (1962) "A Generalized Reynolds Equation for Fluid-Film Lubrication," Int. J. Mech. Sci., 4, 159-170.

Dowson, D. (1965) "Elastohydrodynamic Lubrication - An Introduction and a Review of Theoretical Studies," Institute of Mechanical Engineers, London, Paper R1, 7-15.

Dowson, D. (1968) "Elastohydrodynamics," Proc. Inst. Mech. Eng., London, 182(3A), 151-167.

Dowson, D. (1975) "The Inlet Boundary Condition," Cavitation and Related Phenomena in Lubrication. D. Dowson, M. Godet, and C. M. Taylor, eds., Mechanical Engineering Publications, Ltd., New York, 143-152.

Dowson, D. (1976) "The Origins of Rolling Contact Bearings," T. Sakuri, ed., Proceedings of JSLE-ASLE International Lubrication Conference, Elsevier, Amsterdam, 20-38.

Dowson, D. (1979) History of Tribology, Longman, London and New York.

Dowson, D. (1981) "Lubrication of Joints," Chapter 13 in "The Biomechanics of Joints and Joint Replacements," Edited by D. Dowson and V. Wright, Mechanical Engineering Publications, Bury St. Edmunds, Suffolk. (To be published.)

Dowson, D. and Hamrock, B. J. (1976) "Numerical Evaluation of the Surface Deformation of Elastic Solids Subjected to a Hertzian Contact Stress," ASLE Trans., 19(4), 279-286.

Dowson, D. and Higginson, G. R. (195[illegible]) "A Numerical Solution to the Elastohydrodynamic Problem," J. Mech. Eng. Sci., 1(1), 7-15.

Dowson, D. and Higginson, G. R. (1961) "New Roller-Bearing Lubrication Formula," Engineering London, vol. 192, 158.

Dowson, D. and Higginson, G. R. (1964), "A Theory of Involute Gear Lubrication," Institute of Petroleum Gear Lubrication; Proceedings of a Symposium organized by the Mechanical Tests of Lubricants Panel of the Institute, (1964), Elsevier, 8-15.

Dowson, D. and Higginson, G. R. (1966) Elastohydrodynamic Lubrication, The Fundamentals of Roller and Gear Lubrication. Pergamon, Oxford.

Dowson, D., Saman, W. Y., and Toyoda, S. (1979) "A Study of Starved Elastohydrodynamic Line Contacts," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 92-103.

Dowson, D. and Swales, P. D. (1969) "The Development of Elastohydrodynamic Conditions in a Reciprocating Seal," Proceedings of Fourth International Conference on Fluid Sealing, Vol. 2, Paper 1, British Hydromechanics Research Association, 1-9.

Dowson, D. and Toyoda, S. (1979) "A Central Film Thickness Formula for Elastodydrodynamic Line Contacts." Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 104-115.

Dowson, D. and Whitaker, A. V. (1965-66) "A Numerical Procedure for the Solution of the Elastohydrodynamic Problems of Rolling and Sliding Contacts Lubricated by a Newtonian Fluid," Proc. Inst. Mech. Eng., London, 180(3B), 57.

Dyson, A. (1970) "Flow Properties of Mineral Oils in Elastohydrodynamic Lubrication," Phil. Trans. R. Soc. London, Ser. A, 258(1093), 529-564.

Dyson, A., Naylor, H., and Wilson, A. R. (1965-66) "The Measurement of Oil-Film Thickness in Elastohydrodynamic Contacts," Proceedings of Symposium on Elastohydrodynamic Lubrication, Leeds, England, Institution of Mechanical Engineers, London, 76-91.

Dupuit, A. J. E. J. (1839), "Résumé de Mémoire sur le tirage des voitures et sur le frottement de seconde espece," Competes rendus de l'Académie des Sciences, Paris, IX, 689-700, 775.

Eaton, J. T. H., ed. (1969) "A Trip Down Memory Lane," The Dragon, XLIV (5), 5-7.

ESDU (1965) "General Guide to the Choice of Journal Bearing Type," Engineering Sciences Data Unit, Item 65007, Institute of Mechanical Engineers, London.

ESDU (1967) "General Guide to the Choice of Thrust Bearing Type," Engineering Sciences Data Unit, Item 67033, Institution of Mechanical Engineers, London.

ESDU (1978) "Grease Life Estimation in Rolling Bearings," Engineering Sciences Data Unit, Item 78032, Institution of Mechanical Engineers, London.

ESDU (1978) "Contact Phenomena. I: Stresses, Deflections and Contact Dimensions for Normally-Loaded Unlubricated Elastic Components," Engineering Sciences Data Unit, Item 78035, Institution of Mechanical Engineers, London.

Evans, H. P., Biswas, S., and Snidle, R. W. (1978) "Numerical Solution of Isothermal Point Contact Elastohydrodynamic Lubrication Problems," Proceeding of First International Conference on Numerical Methods in Laminar and Turbulent Flow, Pentech Press, London, 639-656.

Evans, H. P. and Snidle, R. W. (1978) "Toward a Refined Solution of the Isothermal Point Contact EHD Problem." International Conference Fundmentals of Tribology, Massachusetts Institute of Technology, Cambridge, Mass., June 19-22, 1978.

Fein, R. S. (1968) Discussion on the Papers of J. K. Appeldorn and A. B. Metzner, J. Lubr. Technol., 90, 540-542.

Fellows, T. G., Dowson, D., Perry, F. G., and Plint, M. A. (1963) "Perbury Continuously Variable Ratio Transmission," in N. A. Carter, Ed. Advances in Automobile Engineering, Part 2; Pergamon Press, 123-139.

Foord, C. A., Hammann, W. C., and Cameron, A. (1968) "Evaluation of Lubricants Using Optical Elastohydrodynamics," ASLE Trans., 11, 31-43.

Foord, C. A., Wedeven, L. D., Westlake, F. J. and Cameron, A. (1969-70) "Optical Elastohydrodynamics," Proc. Inst. Mech. Eng., London, Part I, 184, 487-503.

Fromm, H. (1948), "Laminre Strömung Newtonscher und Maxwellscher Slüssigkeiten," Angew Math. Mech., 28(2), 43-54.

Furey, M. J. (1961) "Metallic Contact and Friction Between Sliding Surfaces," ASLE Trans., vol. 4, 1-11.

Gentle, C. R. and Cameron, A. (1973) "Optical Elastohydrodynamics at Extreme Pressure," Nature, 246(5434), 478-479.

Gohar, R. and Cameron A. (1966) "The Mapping of Elastohydrodynamic Contacts," ASLE Trans., 10, 215-225.

Goodman, J. (1912) "(1) Roller and Ball Bearings;" "(2) The Testing of Antifriction Bearing Materials," Proceedings of the Institute of Civil Engineers, CLXXXIX, Session 1911-12, Pt. III, pp. 4-88.

Greenwood, J. A. (1969) "Presentation of Elastohydrodynamic Film-Thickness Results." J. Mech. Eng. Sci., 11(2), 128-132.

Greenwood, J. A. and Kauzlarich, J. J. (1973) "Inlet Shear Heating in Elastohydrodynamic Lubrication," J. Lubr. Technol., 95(4), 401-416.

Grubin, A. N. (1949) "Fundamentals of the Hydrodynamic Theory of Lubrication of Heavily Loaded Cylindrical Surfaces," Investigation of the Contact Machine Components. Kh. F. Ketova, ed., Translation of Russian Book No. 30, Central Scientific Institute for Technology and Mechanical Engineering, Moscow, Chapter 2. (Available from Dept. of Scientific and Industrial Research, Great Britain, Transl. CTS-235, and from Special Libraries Association, Chicago, Trans. R-3554.)

Gunther, R. T. (1930), Early Science in Oxford, Volumes VI and VII, "The Life and Work of Robert Hooke," Vol. VII, Pt. II, 666-679, printed for the author at the Oxford University Press by John Johnson (Oxford).

Hall, L. F. (1957) "A Review of the Papers on the Lubrication of Rotating Bearings and Gears," Proceedings of Conference on Lubrication and Wear, Institution of Mechanical Engineers, pp. 425-429.

Hamilton, G. M. and Moore, S. L. (1971) "Deformation and Pressure in an Elastohydrodynamic Contact," Proc. R. Soc., London, Ser. A, 322, 313-330.

Halling, J. (1976) Introduction of Tribology, Wykeham Publ., London.

Hamrock, B. J. (1976) Elastohydrodynamic Lubrication of Point Contacts, Ph.D. Dissertation, University of Leeds, Leeds, England.

Hamrock, B. J. and Anderson, W. J. (1973) "Analysis of an Arched Outer-Race Ball Bearing Considering Centrifugal Forces," J. Lubr. Technol., 95(3), 265-276.

Hamrock, B. J. and Dowson, D. (1974) "Numerical Evaluation of Surface Deformation of Elastic Solids Subjected to Hertzian Contact Stress," NASA TN D-7774.

Hamrock, B. J. and Dowson, D. (1976a) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part I - Theoretical Formulation," J. Lubr. Technol., 98(2), 223-229.

Hamrock, B. J. and Dowson, D. (1976b) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part II - Ellipticity Parameter Results," J. Lubr. Technol., 98(3), 375-378.

Hamrock, B. J. and Dowson, D. (1977a) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part III - Fully Flooded Results," J. Lubr. Technol., 99(2), 264-276.

Hamrock, B. J. and Dowson, D. (1977b) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part IV - Starvation Results," J. Lubr. Technol., 99(1), 15-23.

Hamrock, B. J. and Dowson, D. (1978) "Elastohydrodynamic Lubrication of Elliptical Contacts for Materials of Low Elastic Modulus, Part I - Fully Flooded Conjunction," J. Lubr. Technol., 100(2), 236-245.

Hamrock, B. J. and Dowson, D. (1979a) "Elastohydrodynamic Lubrication of Elliptical Contacts for Materials of Low Elastic Modulus, Part II - Starved Conjunction," J. Lubr. Technol., 101(1), 92-98.

Hamrock, B. J. and Dowson, D. (1979b) "Minimum Film Thickness in Elliptical Contacts for Different Regimes of Fluid-Film Lubrication," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechnical Engineering Publications, Ltd., 22-27.

Hardy, W. B. and Doubleday, I. (1922a) "Boundary Lubrication - the Temperature Coefficient," Proc. R. Soc. London, Ser. A, 101, 487-492.

Hardy, W. B. and Doubleday, I. (1922b) "Boundary Lubrication - the Paraffin Series," Proc. R. Soc. London, Ser. A., 100, 550-574.

Harris, T. A. (1966) Rolling Bearing Analysis. Wiley, New York.

Harris, T. A. (1971) "An Analytical Method to Predict Skidding in Thrust-Loaded, Angular-Contact Ball Bearings," J. Lubr. Technol., vol. 93, 17-24.

Harrison, H. C. (1949) The Story of Sprowston Mill, Phoenix House, London.

Harrison, W. J. (1913) "The Hydrodynamical Theory of Lubrication with Special Reference to Air as a Lubricant," Trans. Cambridge Philos. Soc., xxii (1912-25), 6-54.

Harrison, G. and Trachman, E. G. (1972) "The Role of Compressional Viscoelasticity in the Lubrication of Rolling Contacts," J. Lubr. Technol., 94, 306-312.

Heathcote, H. L. (1921) "The Ball Bearing: In the Making, Under Test, and on Service," Proc. Instn. Automotive Engrs., London, 15, pp. 569-702.

Herrebrugh, K. (1968) "Solving the Incompressible and Isothermal Problem in Elastohydrodynamic Lubrication Through an Integral Equation," J. Lubr. Technol., 90(1), 262-270.

Hersey, M. D. (1966) Theory and Research in Lubrication - Foundations for Future Developments, Wiley, New York.

Hersey, M. S. and Hopkins, R. F. (1954) "Viscosity of Lubricants Under Pressure. Coordinated Data from Twelve Investigations." ASME, New York.

Hertz, H. (1881) "The Contact of Elastic Solids," J. Reine Angew. Math., 92, 156-171.

Hooke, C. J. (1977) "The Elastohydrodynamic Lubrication of Heavily Loaded Contacts," J. Mech. Eng. Sci., 19(4), 149-156.

Hirst, W. and Moore, A. J. (1974) "Non-Newtonian Behavior in Elastohydrodynamic Lubrication," Proc. R. Soc. London, Ser. A, 337, 101-121.

Houghton, P. S. (1976) Ball and Roller Bearings, Applied Science Publishers, Ltd., London.

Jacobson, B. (1970) "On the Lubrication of Heavily Loaded Spherical Surfaces Considering Surface Deformations and Solidification of the Lubricant." Acta Polytech. Scand., Mech. Eng. Ser. No. 54.

Jacobson, B. (1972) "Elasto-Solidifying Lubrication of Spherical Surfaces." American Society of Mechanical Engineers Paper No. 72-LUB-7.

Jacobson, B. (1973) "On the Lubrication of Heavily Loaded Cylindrical Surfaces Considering Surface Deformations and Solidification of the Lubricant," J. Lubr. Technol., 95(3), 321-27.

Jamison, W. E., Lee, C. C., and Kauzlarich, J. J. (1978) "Elasticity Effects on the Lubrication of Point Contacts," ASLE Trans., 21(4), 299-306.

Johnson, B. L. (1964) "A 'Stainless High Speed' Steel for Aerospace Applications," Metal Prog., 86(3), 116-118.

Johnson, B. L. (1965) "High Temperature Wear Resisting Steel," U.S. Patent No. 3,167,423, Jan. 1965.

Johnson, K. L. (1970) "Regimes of Elastohydrodynamic Lubrication." J. Mech. Eng. Sci., 12(1), 9-16.

Johnson, K. L. and Cameron, R. (1967) "Shear Behavior of Elastohydrodynamic Oil Films at High Rolling Contact Pressures," Proc. Ins. Mech. Eng., Part 1, 182, 307-319.

Johnson, K. L. and Roberts, A. D. (1974) "Observation of Viscoelastic Behaviour of an Elastohydrodynamic Lubricant Film," Proc. R. Soc. London, Ser. A, 337, 217-242.

Johnson, K. L. and Tevaarwerk, J. L. (1977) "Shear Behaviour of Elastohydrodynamic Oil Films," Proc. R. Soc. London, Ser. A, 356, 215-236.

Jones, A. B. (1946) "Analysis of Stresses and Deflections," New Departure Engineering Data, General Motors Corp., Bristol, Conn.

Jones, A. B. (1956) "The Mathematical Theory of Rolling-Element Bearings," Mechanical Design and Systems Handbook.

Kannel, J. W., Bell, J. C., and Allen, C. M. (1964) "Methods for Determining Pressure Distribution in Lubricated Rolling Contact," ASLE Paper 64-LC-23, Presented at ASME-ASLE Lubrication Conference, Washington, D.C., Oct. 13-16, 1964.

Kakuta, K. (1979) "The State of the Art of Rolling Bearings in Japan," Bull. Japan Soc. Prec. Eng., 13(4), 169-176.

Kapitza, P. L. (1955) "Hydrodynamic Theory of Lubrication During Rolling," Zh. Tekh. Fiz., 25(4), 747-762.

Koye, K. A. and Winer, W. O. (1980) " An Experimental Evaluation of the Hamrock and Dowson Minimum Film Thickness Equation for Fully Flooded EHD Point Contacts," International ASME/ASLE Lubrication Conference, San Francisco, August 1980.

Kunz, R. K. and Winer, W. O. (1977) Discussion 275-276, to Hamrock, B. J. and Dowson, D. "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part III - Fully Flooded Results," J. Lubr. Technol., 99(2), 264-275.

Lane, T. B. (1951) "Scuffing Temperatures of Boundary Lubricant Films," Br. J. Appl. Phys., 2, (Suppl. 1), 35-38.

Lane, T. B. and Hughes, J. R. (1952) "A Study of the Oil Film Formation in Gears by Electrical Resistance Measurements," Br. J. Appl. Phys., 3(10), 315-318.

Lamb, H. (1932) Hydrodynamics. Cambridge University Press.

Layard, A. H. (1849) Nineveh and Its Remains, Vols. I and II, John Murray, London.

Layard, A. H. (1853) Discoveries in the Ruins of Nineveh and Babylon, Vols. I and II, John Murray, London.

Lee, D., Sanborn, D. M., and Winer, W. O. (1973) "Some Observations of the Relationship Between Film Thickness and Load in High Hertz Pressure Sliding Elastohydrodynamic Contacts," J. Lubr. Technol., 95(3), 386.

Leibnitz, G. W. (1706) "Tentamen de natura et remedlie resistenziarum in machines," Miscellanea Berclinensia. Class. mathem, 1710, (Jean Boudot, Paris), 1, 307.

Lewicki, W. (1955) "Some Physical Aspects of Lubrication in Rolling Bearings and Gears," Engineer, 200 (5193), 176-178, and (5194), 212-215.

Lundberg, G. and Palmgren, A. (1947) "Dynamic Capacity of Rolling Bearings," Acta Polytech., Mech. Eng. Sci., 1(3).

Martin, H. M. (1916) "Lubrication of Gear Teeth," Engineering, London, 102, 199.

McEwen, E. (1952) "The Effect of Variation of Viscosity with Pressure on the Load Carrying Capacity of Oil Films Between Gear Teeth," J. Inst. Petrol., 38, 646.

Meldahl, A. (1941) "Contribution to the Theory of the Lubrication of Gears and of the Stressing of the Lubricated Flanks of Gear Teeth," Brown Boveri Review, 28(11), 374.

Merritt, H. E. (1935) "Worm-Gear Performance," Proc. Inst. Mech. Eng., London, 129, 127-158.

Meyer, D. R. and Wilson, C. C. (1971) "Measurement of Elastohydrodynamic Oil Film Thickness and Wear in Ball Bearings by the Strain Gage Method," J. Lubr. Technol., 93(2), 224-230.

Moes, H. (1965-66) "Communication, Elastohydrodynamic Lubrication," Proc. Inst. Mech. Eng., London, 180(3B), 244-245.

Moes, H. and Bosma, R. (1972) "Film Thickness and Traction in EHL at Point Contact," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Leeds, England, Institution of Mechanical Engineers, London, 149.

Moore, A. J. (1973) "Non-Newtonian Behaviour in Elastohydrodynamic Lubrication," Ph.D. Thesis, University of Reading.

Morgan, M. H. and Warren, H. L. (1960) Translation of Vitruvius: The Ten Books of Architecture, Dover, New York.

Morin, A. J. (1835) "Nouvelles expériences faites à Metz en 1833 sur le frottement, sur la transmission due movement par le choc, sur le résistance des milieun imparfaits a le pénétration des projectiles, et sur le frottement pendant le choc," Mem. Savans Etrang. (Paris), VI, 641-785; Ann. Min. X, (1836), 27-56.

Nagaraj, H. S., Sanborn, D. M., and Winer, W. O. (1977) "Effects of Load, Speed, and Surface Roughness on Sliding EHD Contact Temperature," J. Lubr. Technol., 99(4), 254-263.

Navier, C. L. M. H. (1823) "Memoire sur les lois du mouvement des fluides," Mem. Acad. R. Sci., 6(2), 389-440.

Needham, J. (1965) Science and Civilization in China, Vol. 4, Physics and Physical Technology, Part II, Mechanical Engieering, Cambridge University Press.

Newton, I. (1687) Philosophiae Naturales Principia Mathematica, Imprimature S. Pepys, Reg. Soc. Praeses, 5 Julii 1686. Revised and supplied with a historical and explanatory appendix by F. Cajori, edited by R. T. Crawford (1934), and published by the University of California Press, Berkeley and Los Angeles (1966).

Orcutt, F. K. and Cheng, H. S. (1966) "Lubrication of Rolling-Contact Instrument Bearings," Gyro-Spin Axis Hydrodynamic Bearing Symposium, Vol. 2, Ball Bearings, Massachusetts Institute of Technology, Cambridge, Mass., Tab. 5.

Pai, S. I (1956) Viscous Flow Theory, Vol. I - Laminar Flow. Van Nostrand Reinhold, New Jersey.

Palmgren, A. (1945) "Ball and Roiller Bearing Engineering," S. K. F. Industries, Philadelphia.

Parker, R. J. and Hodder, R. S. (1978) "Roller-Element Fatigue Life of AMS 5749 Corrosion Resistant, High Temperature Bearing Steel," J. Lubr. Technol., 100(2), 226-235.

Parker, R. J. and Kannel, J. W. (1971) "Elastohydrodynamic Film Thickness Between Rolling Disks with a Synthetic Paraffinic Oil to 589 K (600° F); NASA TN D-6411.

Parker, R. J. and Zaretsky, E. V. (1978) "Rolling-Element Fatigue Life of AISI M-50 and 18-4-1 Balls." NASA TP-1202.

Peppler, W. (1936) "Untersuchunge uber die Drukubertragung bei Balasteten und Geschmierten um Laufenden Achsparallelen Zylinder," Maschinenelemente-Tagung Archen 1935, 42; V. D. I. Verlag, Berlin, 1936.

Peppler, W. (1938) "Druchubertragung an Geschmeirten Zylindriachen Gleit und Wälzflächen," V. D. I. Forschungshaft, 391.

Petrov, N. P. (1883) "Friction in Machines and the Effect of the Lubricant," Inzh. Zh., St. Peterb., 1, 71-140; 2, 227-279; 3, 377-436; 4, 535-564.

Petrusevich, A. S. (1951) "Fundamental Conclusion from the Contact-Hydrodynamic Theory of Lubrication," dzo. Akad. Nauk. SSSR (OTN), 2, 209.

Piggott, S. (1968) "The Earliest Wheeled Vehicles and the Caucasian Evidence," Proc. Prehist. Soc., XXXIV, (8), 266-318.

Pirvics, J. (1980) "Numerical Analysis Techniques and Design Methology for Rolling Element Bearing Load Support Systems," in International Conference on Bearing Design: Historical Aspects, Present Technology and Future Problems; Century 2 - Emerging Technology, W. J. Anderson, ed., American Society of Mechanical Engineers, New York, 1980, 47-85.

Plint, M. A. (1967) "Traction in Elastohydrodynamic Contact," Proc. Inst. Mech. Eng., London, Part 1, 182(14), 300-306.

Poritsky, H., Hewlett, C. W., Jr., and Coleman, R. E., Jr. (1947) "Sliding Friction of Ball Bearings of the Pivot Type," J. Appl. Mech., 14(4), 261-268.

Pritchard, C. (1981) "Traction Between Rolling Steel Surfaces - A Survey of Railway and Laboratory Services," Proceedings of the 7th Leeds-Lyon Symposium on 'Friction and Traction, Leeds, September 1980, Mechanical Engineering Publications. (To be published.)

Ramelli, A. (1588) "Le Diverse et Artificiose Machine," Paris, France.

Ranger, A. P., Ettles, C. M. M., and Cameron, A. (1975) "The Solution of Point Contact Elastohydrodynamic Problem," Proc. R. Soc. London, Ser. A, 346, 277-244.

Reti, L. (1971) "Leonardo on Bearings and Gears," Scientific American, 224, (2), 101-110.

Reynolds, O. (1875) "On Rolling Friction," Phil. Trans. R. Soc., 166, Pt. 1, 155.

Reynolds, O. (1886) "On the Theory of Lubrication and Its Application to Mr. Beauchamp Tower's Experiments, Including an Experimental Determination of the Viscosity of Olive Oil," Philos. Trans. R. Soc. London, 177, 157-234.

Roelands, C. J. A. (1966) Correlational Aspects of the Viscosity-Temperature-Pressure Relationship of Lubricating Oils. Druk. V. R. B., Groningen, Netherlands.

Rowe, J. (1734) "All Sorts of Wheel-Carriage Improved," printed for Alexander Lyon under Tom's Coffee House in Russell Street, Covent Garden, London.

Sanborn, D. M. (1969) "An Experimental Investigation of the Elastohydrodynamic Lubrication of Point Contacts in Pure Sliding," Ph.D. Thesis, University of Michigan.

Schlatter, R. (1974) "Double Vacuum Melting of High Performance Bearing Steels," Ind. Heat. 41(9), 40-55.

Shaw, M. C. and Macks, E. F. (1949) Analysis and Lubrication of Bearings, McGraw-Hill, New York.

Sibley, L. B., Bell, J. C., Orcutt, F. K., and Allen, C. M. (1960) "A Study of the Influence of Lubricant Properties on the Performance of Aircraft Gas Turbine Engine Rolling Contact Bearings," WADD Technical Report, 60-189.

Sibley, L. B. and Orcutt, F. K. (1961) "Elasto-Hydrodynamic Lubrication of Rolling Contact Surfaces," Trans. Amer. Soc. Lub. Engrs., 4(2), 234.

Smith, F. W. (1959) "Lubricant Behavior in Concentrated Contact Systems - The Caster Oil - Steel System," Wear, 2(4), 250-264.

Smith, F. W. (1962) The Effect of Temperature in Concentrated Contact Lubrication. ASLE Trans. 5(1), 142-148.

Stokes, G. G. (1845) "On the Theory of Internal Friction of Fluids in Motion," Trans. Cambridge Philos. Soc. 8, 287-319.

Stribeck, R. (1901) " Kugellager fur beliebige Belastungen," Z. Ver. dt. Ing., 45(3), 73-125.

Stribeck, R (1907) "Ball Bearings for Various Loads" - translation by H. Hess, Trans. Am. Soc. Mech. Engrs., 29, 420.

Swingler, C. L. (1980) "Surface Roughness in Elastohydrodynamic Line Contacts," Ph.D. Thesis, University of London (Imperial College).

Tabor, D. (1962) "Introductory Remarks," in Rolling Contact Phenomena, J. B. Bidwell, ed., Elsevier, Amsterdam, 1-5.

Tallian, T. E. (1969) "Progress in Rolling Contact Technology," Report AL 690007, SKF Industries, King of Prussia, Pa.

Tallian, T., Sibley, L., and Valori, R. (1965) "Elastohydrodynamic Film Effects on the Load-Life Behavior of Rolling Contacts," ASMS Paper 65-LubS-11.

Theyse, F. H. (1966) "Some Aspects of the Influence of Hydrodynamic Film Formation on the Contact Between Rolling/Sliding Surfaces," Wear, 9, 41-59.

Thorp, N. and Gohar, R. (1972) "Oil Film Thickness and Shape for a Ball Sliding in a Grooved Raceway," J. Lubr. Technol., 94(3), 199-210.

Timoshenko, S. and Goodier, J. N. (1951) Theory of Elasticity, 2nd ed., McGraw-Hill, New York.

Trachman, E. G. and Cheng, H. S. (1972) "Thermal and Non-Newtonian Effects on Traction in Elastohydrodynamic Contacts," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 142-148.

Tower, B. (1883) "First Report on Friction Experiments (Friction of Lubricated Bearings)," Proc. Inst. Mech. Eng., London, 632-659.

Turchina, V., Sanborn, D. M., and Winer, W. O. (1974) "Temperature Measurements in Sliding Elastohydrodynamic Point Contacts," J. Lubr. Technol., 96(3), 464-471.

Ucelli, G. (1940) "Le Navi Di Nemi," La Liberia Dello Stato, Roma.

Valori, R. (1978) Discussion to Parker, R. J. and Hodder, R. S. (1978) Rolling-Element Fatigue Life of AMS 5749 Corrosion Resistant, High Temperature Bearing Steel," J. Lubr. Technol., 100(2), 226-235.

Van Natrus, L., Polly, J., and Van Vuuren, C. (1734 and 1736), Groot Volkomen Moolenbock, 2 Volumes, Amsterdam.

Varlo, C. (1772) "Reflections Upon Friction with a Plan of the New Machine for Taking It Off in Wheel-Carriages, Windlasses of Ships, etc., Together with Metal Proper for the Machine, the Full Directions for Making It."

Vaughan, P. (1794) "Axle Trees, Arms, and Boxes," British Patent No. 2006 of A.D. 1794, 1-2, accompanied by 11 diagrams on one sheet.

Wailes, R. (1954) The English Windmill, Routledge & Kegan Paul, London.

Wailes, R. (1957) "Windmills" in History of Technology, C. Singer, E. J. Holmyard, A. R. Hall, and T. I. Williams, eds., Volume III, Oxford University Press, pp. 89-109.

Weber, C. and Saalfeld, K. (1954) Schmierfilm bei Walzen mit Verformung, Zeits ang. Math. Mech. 34 (Nos. 1-2).

Wedeven, L. E., Evans, D., and Cameron, A. (1971) "Optical Analysis of Ball Bearing Starvation," J. Lubr. Technol., 93(3), 349-363.

Weibull, W. (1949) "A Statistical Representation of Fatigue Failures in Solids," Trans. Roy. Inst. Technol., (27), Stockholm.

Whomes, T. L. (1966) The Effect of Surface Quality of Lubricating Film Performance, Ph.D. Dissertation, University of Leeds, Leeds, England.

Wilcock, D. F. and Booser, E. R. (1957) Bearing Design and Application. McGraw-Hill, New York.

Willis, T., Seth, B., and Dave, M. (1975) "Evaluation of a Diffraction Method for Thickness Measurement of Oil-Filled Gaps," J. Lubr. Technol. 97(4), 649-650.

Wilson, A. R. (1979) "The Relative Thickness of Grease and Oil Films in Rolling Bearings," Proc. Inst. Mech. Eng., London, 193(17), 185-192.

Winn, L. W., Eusepi, M. W., and Smalley, A. J. (1974) "Small, High-Speed Bearing Technology for Cryogenic Turbo-Pumps," MTI-74TR29, Mechanical Technology, Inc., Latham, N.Y. (NASA CR-134615.)

Wolveridge, P. E., Baglin, K. P., and Archard, J. G. (1971) "The Starved Lubrication of Cylinders in Line Contact," Proc. Inst. Mech. Eng., London, 185(1), 1159-1169.

Zaretsky, E. V., Anderson, W. J., and Bamberger, E. N. (1969) "Rolling Element Bearing Life for 400° to 600° F." NASA TN D-5002.

Zaretsky, E. V., Parker, R. J., and Anderson, W. J. (1967) "Component Hardness Differences and Their Effect on Bearing Fatigue," J. Lubr. Technol., 87(1), 47-62.

ORIGINAL PAGE IS
OF POOR QUALITY

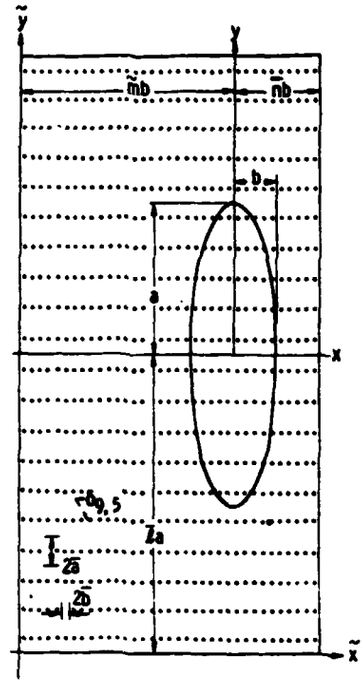


Figure 7.1. - Division of area in and around contact zone into equal rectangular areas.

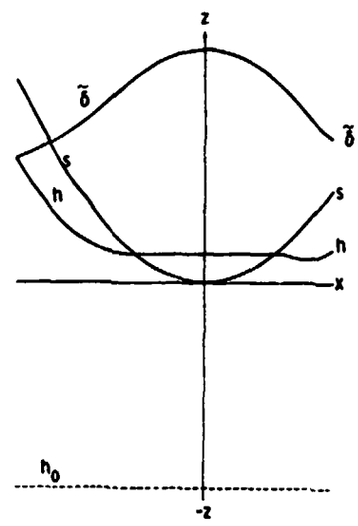


Figure 7.2. - Components of film thickness for ellipsoidal solid near plane.

ORIGINAL PAGE IS
OF POOR QUALITY

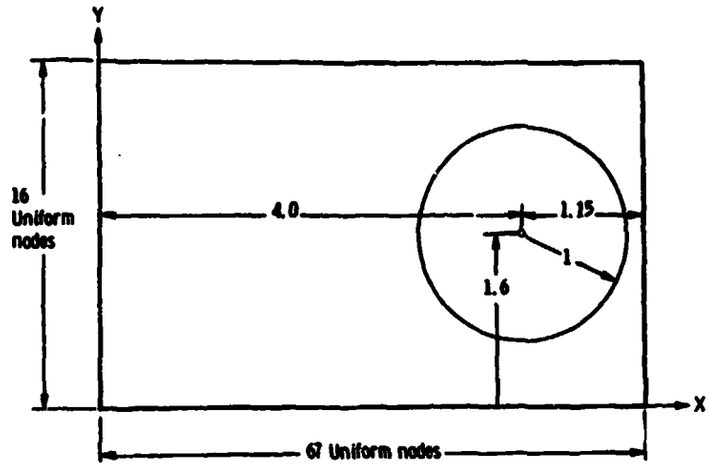


Figure 7.3. - Nodal structure used for numerical calculations.

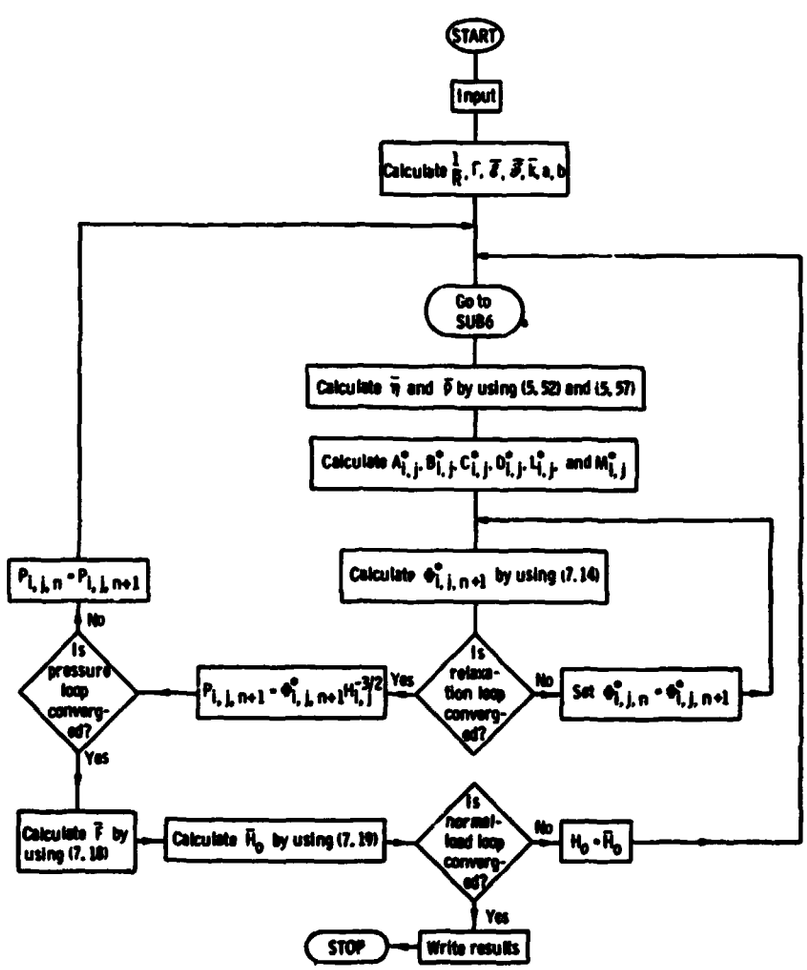


Figure 7.4. - Flow chart of main program.

ORIGINAL PAGE IS
OF POOR QUALITY

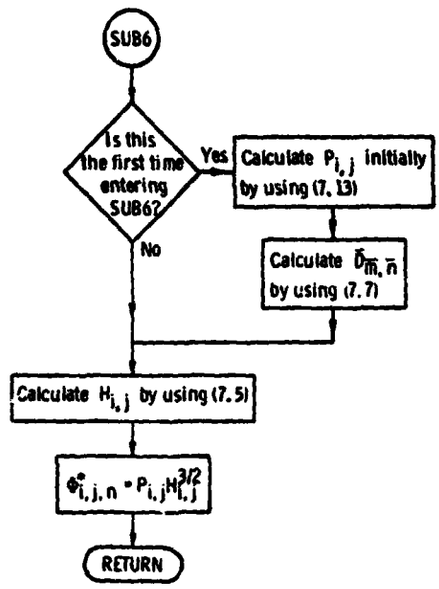


Figure 7.5. - Flow chart of subroutine SUB6.