UAA-015

U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
Software Analysis and Management System

Confidence Ellipse Research Software

EAAF

Technical Memorandum No. 6

August 8, 1985

by

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AD-A166 479

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JPL D-2786

86 4 8 069

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER JPL D-2786	2. GOVT ACCESSION NO. AD-A166 479	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CONFIDENCE ELLIPSE RESEARCH SOFTWARE		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER No. 41
7. AUTHOR(s) Dr. Janet Myhre, Will Duquette, Daniel Hockman		8. CONTRACT OR GRANT NUMBER(s) NAS7-918
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematical Analysis Research Corp. 4239 Via Padova Claremont, CA 91711		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS RE 182 AMEND # 187
11. CONTROLLING OFFICE NAME AND ADDRESS Commander, USAICS ATTN: ATSI-CD-SF Ft. Huachuca, AZ 85613-7000		12. REPORT DATE 8 August, 1985
		13. NUMBER OF PAGES 42
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Jet Propulsion Laboratory California Institute of Technology 4800 Oak Grove Drive, ATTN: 126-200 Pasadena, CA 91109		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Distribution

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Prepared under contract to JPL for the U. S. Army Intelligence Center and School's Combat Developer's Support Facility

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

CONFIDENCE ELLIPSE, ELLIPTICAL ERROR PROBABLE (EEP), COVARIANCE MATRIX, BASIC, STATISTICAL TESTS, MATHEMATICAL SIMULATION

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is one of a series of algorithm analysis reports performed for the US Army Intelligence Center and School covering selected algorithms in existing or planned Intelligence and Electronic Warfare (IEW) systems. This report documents the software used in the analysis of ellipse combination and testing which was reported in report 40 of this series.

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86 4 8 069

PREFACE

The work described in this publication was sponsored by the United States Army Intelligence Center and School. The writing and publication of this paper was supported by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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UNCLASSIFIED

MATHEMATICAL ANALYSIS RESEARCH CORPORATION

NO. 41

CONFIDENCE ELLIPSE RESEARCH SOFTWARE

8 AUGUST 1985

4239 VIA PADOVA, CLAREMONT, CA 91711

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PROGRAM LISTINGS

A. GENELLIPSE

B. ELLSIM

C. ELLIPSTUFF

I. INTRODUCTION

This document describes ~~several~~ three programs used by MARC in studying and testing confidence ellipses. All of these programs were developed on a Hewlett Packard Series 200 computer, the HP9836, and are written in Hewlett Packard's BASIC 3.0 programming language. This a very powerful version of BASIC, and consequently many of the programming constructs found in these programs are non-standard. Also, the graphics commands are peculiar to this BASIC.

The programs are called Genellipse, Ellsim, and Ellipstuff. The listings and comments are current as of August 9, 1985. The programs themselves are subject to change without notice.

Genellipse is an "ellipse graphing and combination" program. It allows the user to specify up to twenty confidence ellipses, combine ellipses, graph ellipses, and test ellipses for combination. It has been used primarily to explore the geometric properties of the combination method, but has also been used to create figures for a number of other reports.

Ellsim is a "confidence ellipse simulation" program. It has been used to explore the robustness and properties of the statistical test (which is used to decide whether or not to combine two ellipses). In it, two normal data distributions are specified, corresponding to two emitters. Confidence ellipses are generated, tested, and combined, and various descriptive statistics are compiled. In addition, there is a routine to calculate the power of the statistical test in certain cases.

Ellipstuff is an "ellipse routine library," which contains a large number of routines to make working with ellipses easier. As such, both Genellipse and Ellsim contain all of these routines, but they are also listed separately. If more programs need to be written, these are the routines to build them with.

Each of these programs is listed below, along with a short description of their use. This is a supplement to the report "Testing and Combination of Confidence Ellipses: A Geometric Analysis," submitted to JPL by MARC on August 5, 1985. Refer here any questions concerning confidence ellipses, the combination method, or the statistical test used.

II. GUIDE TO GENELLIPSE: Ellipse Graphing Program

Genellipse stands for General Ellipse graphing, and can also combine ellipses and test two ellipses for combination. The program makes extensive use of the Ellipstuff Library Routines.

When Genellipse is executed, the following menu is displayed on the screen:

```
=================================================================

    GENERAL ELLIPSE GRAPHER
    -----------------------------------------------------
      (A) -- Enter Ellipse Mean Point (Center)
      (B) -- Enter Ellipse Shape (Covariance Matrix)
      (C) -- Enter Ellipse Shape (Axes and Orientation)
      (D) -- Combine Two Ellipse (JPL Method)
      (E) -- Graph an Ellipse
      (F) -- Clear Graphics Screen
      (G) -- Choose Plotter
      (H) -- Set Graphics Screen Bounds
      (I) -- Calculate Acceptance Test Statistic (Chi-square)
      (J) -- Draw Axes
      (X) -- Exit Program

    Enter your choice:

=================================================================
```

These options will be dealt with one by one.

(A) -- Enter Ellipse Mean Point (Center)

This option prompts the user to enter the center point for one or more ellipses. The program first asks for the ellipse (1-20), and then for the mean point. This will repeat until all means have been entered. When the user is through entering means, pressing the return key in response to the question "Getting mean point for Ellipse #" will return the user to the menu shown above.

(B) -- Enter Ellipse Shape (Covariance Matrix)

There are two ways of specifying the shape of a confidence ellipse. The first is through a covariance matrix. This option will ask for the ellipse number, and then prompt for the elements of the matrix. It repeats until all matrices have been entered, just as option A does.

(C) -- Enter Ellipse Shape (Axes and Orientation)

The other way of specifying the shape of a confidence ellipse is geometrically. This option prompts for the ellipse number, just as those above, and then asks for the axes lengths and orientation in degrees of the ellipse. It repeats until all ellipses have been entered, just as option A does.

(D) -- Combine Two Ellipses (JPL Method)

This option combines confidence ellipses using the method described in "Testing and Combination of Confidence Ellipses: A Geometric Analysis." The option prompts for the numbers of the two ellipses to be combined, and then for the number of the resultant ellipse. For example, suppose the user had entered ellipses 1, 2, 3, and 7 out of 20, and wanted to combine 2 and 3. The resultant ellipse could be given number 4, 5, 6, or 8 through 20. Further, if either of ellipses 1 or 7 were no longer needed, the resultant ellipse could be given either of numbers 1 or 7 as well. This would, of course, erase the ellipses originally stored in these slots.

(E) -- Graph an Ellipse

This option prompts for the number of the ellipse the user wishes to graph, and continues asking for ellipses until all desired have been graphed. The process is similar to that in option A. The ellipse will be graphed on the current graphics device (See option G). Other options related to graphing are G, H, and J.

(F) -- Clear Graphics Screen

If the CRT screen is the current graphics device, choosing this option will erase all ellipses currently drawn on it.

(G) -- Choose Plotter

This options allows the user to select the current graphics device. This program is currently written to graph on the CRT screen and on an HP7470A two-pen plotter. Consequently, if selected, the program will ask "Do you want to (D)raw, (E)rase, Plot with Pen (0), (1), (2)?" Choosing "D" will cause it to do all graphing on the screen. Choosing "E" will cause it to graph on the screen in "black" -- so that an ellipse may be erased without clearing the whole screen. Choosing (1) or (2) will cause it to graph on the plotter, using the specified pen. Choosing (0) causes the plotter to put away the pen that its using, and then sets the CRT screen to be the graphics device.

(H) -- Set Graphics Screen Bounds

This option is used to scale the graphics screen. Default scaling is -50 to 50 on the X-axis and -50 to 50 on the Y-axis. When this option is chosen, it will first present the "limits" of any ellipse: that is, how far the ellipse extends in the X and Y directions, so that a reasonable screen size may be chosen. The program prompts for the number of each

ellipse, just as in option A. Press the return key after all desired ellipse limits have been seen. The program will then ask for screen bounds: Minimum X value, Maximum X value, Minimum Y value, Maximum Y value.

(I) -- Calculate Acceptance Test Statistic (Chi-square)

This option prompts the user for two ellipse numbers, and then performs the statistical test described in "Testing and Combining Confidence Ellipses: A Geometrical Analysis." The value of the test statistic is printed on the screen. If it is less than or equal to the 95% Chi-square value, 5.991, then the test accepts; otherwise the test rejects. (Actually, the test may be run at any confidence level desired. However, the ellipses used in this program are assumed to be 95% confidence ellipses, and if the test is performed at any other level the geometrical results shown in "Testing and Combining..." concerning the test will not necessarily hold true.

(J) -- Draw Axes

This option draws a set of axes on the current graphics device, and labels them according to the current screen bounds (see Options G and H).

III. GUIDE TO ELLSIM: Ellipse Simulation Program

Ellsim stands for Ellipse Simulator, but can also find analytical estimates of statistical power. It uses the Ellipstuff Library extensively.

See the report "Testing and Combining Confidence Ellipses: A Geometric Analysis" for a description of simulation and how it has been applied to the ellipse combination problem. This is the program used to generate the results in section V of that report. Note that in this program, sensor error is assumed to follow a bivariate normal distribution about the true location of the emitter. When location estimates are derived from lines of bearing, however, this assumption may be unrealistic.

When the program is run, it will present the user with the following menu of choices:

```
=================================================================

    Ellipse Combination Program Driver

    (A) -- Specify True Covariance Matrices
    (B) -- Specify True Mean Parameters
    (C) -- Specify Observations
    (D) -- Call Simulation Generator
    (E) -- Call Power Generator
    (X) -- Exit Program


    Enter your choice:

=================================================================
```

Options A, B, and C are used to specify the two data distributions.

Option A prompts for the means of two bivariate normal distributions. Setting the means to be equal is equivalent to having only one emitter. Setting them apart is equivalent to having two emitters.

Option B prompts for the covariance matrices for the two bivariate normal distributions.

Option C prompts for the sample sizes to be used for each distribution. Note that any confidence ellipses generated will have covariance matrices equal to those specified in option B, divided by these sample sizes.

Options D and E call the Simulation Generator and the Power Generator respectively. These will be discussed individually.

(D) -- The Simulation Generator

Choosing this option leads to another menu: the simulation menu. It appears as follows:

Enter the Letter of your choice:

(A) Reset the Random Number Seed
(B) Simulate using true Covariance Matrices
(C) Simulate using estimated Covariance Matrices
(D) Display Results on the Printer
(X) Exit Program

Enter Your Choice:

These options will be dealt with one by one

(A) -- Reset the Random Number Seed

This option allows the user to start the random number process with a given seed; this is generally not necessary, but is useful for purposes of debugging.

(B) -- Simulate using true Covariance Matrices

This option will prompt the user for the number of simulations (generally 100 or more), and then proceeds in this manner: for each simulation it generates the number of observations specified by the sample size entered earlier. It estimates the emitter locations from these observations, and calculates confidence ellipses. It tests to see if these ellipses may be combined. Finally, it checks to see if the combined ellipse contains the true location of the emitter(s). When all simulations are done, it compiles these results. See "Testing and Combining Confidence Ellipses" for more information.

(C) -- Simulate using estimated Covariance Matrices

The process here is the same as that outlined for option B, except for one addition: the covariance matrices used in the confidence ellipses and statistical test are estimated. However, the formulas for the ellipses and test assume that the covariance matrices are known. Thus, this option is used to explore what happens if estimated matrices are mistakenly used. Note that the estimates are made using the "S^2" statistic. This is the usual way of estimating variance-covariance from a data set, but is different from the methods used in most if not all of the position fixing algorithms we have seen.

(D) -- Display Results on the Printer

When either option B or option C has been completed, the results of the simulations are shown in the screen. If a hardcopy is desired, selecting this option will cause the results of the last simulation run to be output on the printer.

(X) -- Exit Program

This option will exit the Simulation Generator and return to the original menu.

(E) -- The Power Generator

The power of a statistical test is essentially the probability that the test will reject when it ought to. That is, it is the probability that the statistical test will say that there are two emitters when in fact there are two emitters. In the problem at hand, however, the power is not a single quantity; in fact, there is a different power value for each pair of emitters. If the emitters are close together, the power of the test will be low; if they are far apart, the power will be close to 1.

This option works in the following way, for convenience sake. It uses the covariance matrices and sample sizes specified from the main menu, but allows the user to enter the distance between the two emitters. It calculates the power, and then asks for another distance. To return to the main menu, enter 0 for the distance.

IV. GUIDE TO ELLIPSTUFF: Ellipse Routine Library

Ellipstuff is a library of subprograms and functions written to facilitate confidence ellipse research for JPL. It covers such things as defining ellipses, combining ellipses, testing ellipses for combination, and graphing ellipses.

To use Ellipstuff, simply include it in within a program. Genellipse and Ellsim are examples of this.

Confidence ellipses and how they are stored

A confidence ellipse is defined by two things: a point estimate, or mean, and a covariance matrix. Thus, both of these pieces of information must be stored for each ellipse. In addition, it is often necessary to have the inverse of the covariance matrix on hand as well. Ellipstuff stores ellipses in a matrix with 40 slots, allowing the storage of 20 covariance matrices with their inverses. In general most of the Ellipstuff routines deal with the inverses themselves, but in case it is necessary to use them explicitly, the ellipses are stored in slots 1 through 20 and the inverse covariances are stored in slots 21 through 40. By convention, the inverse of the covariance matrix for the ellipse in, say, slot 3, is stored in slot 23, and so on. Examine the routines for more programming information.

Ellipstuff User Routines

The slots specified in the following routines should be between 1 and 20 inclusive.

Get_ell_mean(Ellipse)
: This command prompts the user to input the mean (center point) for the ellipse in slot Ellipse.
Example: Get_ell_mean(1)

Get_covariance(Ellipse)
: This command prompts the user to input the covariance matrix for the ellipse in slot Ellipse.
Example: Get_covariance(2)

Get_axes(Ellipse)
: This command prompts the user to input the shape of the ellipse (lengths of the semi-minor and semi-major axes, and the orientation) which is then converted to a covariance matrix.
Example: Get_axes(3)

Test(Ellipse1, Ellipse2, Work, Test_stat)

This command runs a chi-square test on the ellipses in slots Ellipse1 and Ellipse2 in order to see if they may be combined. The value of the test-statistic is returned in Test_stat. Work is the number of any unused slot, to be used for scratch work.

Example: Test(3,4,20,Some_variable)

Combine_ellipse(Ellipse1, Ellipse2, Combo)

This command combines the ellipses and point estimates in slots Ellipse1 and Ellipse2, and stores the combined ellipse in slot Combo.

Example: Combine_ellipse(3,4,5)

Draw_ellipse(Ellipse, Xmin, Xmax, Ymin, Ymax,Prob_constant)

This draws the ellipse in slot Ellipse on the current plotting device (see Choose_plotter). Xmin, Xmax, Ymin, and Ymax specify the screen dimensions. The default values are (-50,50,-50,50). Prob_constant is zero minus the chi-square cutoff associated with the confidence level. (The cutoff is P=-2LN(1-Alpha), where Alpha is the confidence level. For some reason, all of the programming was done in terms of -P=2LN(1-Alpha). Thus, for 95% ellipses, Prob_constant = -5.991).

Example: Draw_ellipse(2,-10,10,-20,20,-5.991)

Disp_extremes(Ellipse)

This command displays the extreme x and y points calculated by Get_bound for the ellipse in slot Ellipse. This is designed to aid in choosing the Xmin, Xmax, Ymin, and Ymax values required by Draw_ellipse.

Example: Disp_extremes(1)

Display_cov(Ellipse)

This command displays the covariance matrix of the ellipse in slot Ellipse on the crt screen (not graphically). It may also be used to display the inverses in slots 21 through 40.

Example: Display_cov(11)

Choose_plotter

When first initialized, Ellipstuff assumes that all graphing will be done on the CRT. Choose_plotter is called to allow the program user to select which device to use. Note: this program was written on a system with two graphics devices--a CRT screen and an HP7470A two pen plotter. Choose_plotter presents the user with 5 options:

D)raw -- graph on the CRT in the normal fashion (DEFAULT).

E)rase -- graph on the CRT in "black". This may be used to erase things.

1) -- graph on the plotter using Pen 1.

2) -- graph on the plotter using Pen 2.

0) -- put away the current plotter pen and select the CRT.

Example: Choose_plotter

Ellipstuff Low Level Routines

These are routines which are used in building user routines. Since these must deal with inverse covariance matrices also, the slots specified may run from 1 to 40. The user will generally not use these, except when adding to the Ellipstuff module.

Invert(Source, Destination)
This command inverts the covariance matrix in slot Source and puts the inverse in slot Destination.
Example: Invert(3, 23)

Add_covariance(Ellipse1, Ellipse2, Summer)
This command adds the covariance matrices stored in slots Ellipse1 and Ellipse2, and stores them in slot Summer.
Example: Add_covariance(1,2,3)

Get_bounds(Ellipse, Prob_constant)
This command calculates and saves the extreme x and y points for the ellipse in slot Ellipse.
Example: Get_bounds(3, -5.991)

A. GENELLIPSE: Ellipse Graphing Program

Genellipse is an "ellipse graphing and combination" program. It allows the user to specify up to twenty confidence ellipses, combine ellipses, graph ellipses, and test ellipses for combination. It has been used primarily to explore the geometric properties of the combination method, but has also been used to create figures for a number of other reports.

```
100     !*************************************************************!
110     !*                    GENERAL ELLIPSE GRAPHER                *!
120     !*                                                           *!
130     !*  This program uses the general ellipse generation and graph-*!
140     !*  ing routines found in the file ELLIPSTUFF.  Its purpose   *!
150     !*  is to graph general ellipses and combinations of ellipses. *!
160     !*                                                           *!
170     !*              Will Duquette          May 13, 1985          *!
180     !*************************************************************!
190    COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_axis(
20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
191    REAL Xmin,Xmax,Ymin,Ymax,Prob_constant
200 Init_prog:!
210    GINIT
220    GCLEAR
230    ! These variables define the size of the screen.  For best results,
240    ! the lengths of the X and Y axes should be about the same.
250    Xmin=-50
260    Xmax=50
270    Ymin=-50
280    Ymax=50
281    Prob_constant=2*LOG(.05)
290     ! Display the menu
300 Menu_top:!
310    Clearscreen
320    PRINT "GENERAL ELLIPSE GRAPHER"
330    PRINT
340    PRINT "-------------------------------------------------------"
350    PRINT "  (A) -- Enter Ellipse Mean Point (Center)"
360    PRINT "  (B) -- Enter Ellipse Shape (Covariance Matrix)"
370    PRINT "  (C) -- Enter Ellipse Shape (Axes and Orientation)"
380    PRINT "  (D) -- Combine Two Ellipses (JPL Method)"
390    PRINT "  (E) -- Graph an Ellipse"
400    PRINT "  (F) -- Clear Graphics Screen"
401    PRINT "  (G) -- Choose Plotter"
410    PRINT "  (H) -- Set Graphics Screen Bounds"
420    PRINT "  (I) -- Calculate Acceptance Test Statistic (Chi-square)"
421    PRINT "  (J) -- Draw Axes"
430    PRINT "  (X) -- Exit Program"
440    PRINT
450    PRINT "Enter your choice:"
460    DISP "Choose an option:";
470    INPUT Option$
480    SELECT Option$
490    CASE "A","a"       ! Get ellipse mean points.
500       Clearscreen
510       PRINT "Getting mean point for Ellipse#";
520       REPEAT
530          Ellipse=0
540          DISP "Enter the Ellipse# (1-20)";
550          INPUT Ellipse
560          PRINT Ellipse
570          IF Ellipse<=20 AND Ellipse>0 THEN CALL Get_ell_mean(Ellipse)
580       UNTIL Ellipse=0
590    CASE "B","b"       ! Get ellipse covariance matrices
600       Clearscreen
610       PRINT "Getting Shape (Covariance Matrix) for Ellipse#";
620       REPEAT
630          Ellipse=0
640          DISP "Enter the Ellipse# (1-20)";
650          INPUT Ellipse
660          PRINT Ellipse
670          IF Ellipse<=20 AND Ellipse>0 THEN CALL Get_covariance(Ellipse)
680       UNTIL Ellipse=0
690    CASE "C","c"       ! Get ellipse axes and orientation
```

```
700       Clearscreen
710       PRINT "Getting Shape (Axes and Orientation) for Ellipse#";
720       REPEAT
730          Ellipse=0
740          DISP "Enter the Ellipse# (1-20)";
750          INPUT Ellipse
760          PRINT Ellipse
770          IF Ellipse<=20 AND Ellipse>0 THEN CALL Get_axes(Ellipse)
780       UNTIL Ellipse=0
790    CASE "D","d"      ! Combine ellipses
800       Clearscreen
810       PRINT "Combining two ellipses"
820       PRINT
830       Ellipse1=0
840       Ellipse2=0
850       Ellipse3=0
860       PRINT "Ellipse 1 #";
870       DISP "Enter the first Ellipse# (1-20)";
880       INPUT Ellipse1
890       PRINT Ellipse1
900       PRINT "ellipse 2 #";
910       DISP "Enter the second Ellipse# (1-20)";
920       INPUT Ellipse2
930       PRINT Ellipse2
940       PRINT "Combined Ellipse #";
950       DISP "Enter the combined Ellipse# (1-20)";
960       INPUT Ellipse3
970       PRINT Ellipse3
980       IF Ellipse1<=20 AND Ellipse1>0 AND Ellipse2<=20 AND Ellipse1>0 AND Ell
ipse3<=20 AND Ellipse3>0 AND Ellipse3<>Ellipse1 AND Ellipse3<>Ellipse2 THEN
990          Combine_ellipse(Ellipse1,Ellipse2,Ellipse3)
1000      END IF
1010      PRINT "New Mean Point: ",Xbar(Ellipse3),", ",Ybar(Ellipse3)
1020      PRINT
1030      PRINT "New Covariance Matrix:"
1040      Display_cov(Ellipse3)
1050      Pauseabit
1060   CASE "E","e"      ! Graph ellipses
1061      REPEAT
1070         Clearscreen
1080         PRINT "Graphing Ellipse#";
1100         Ellipse=0
1110         DISP "Enter Ellipse# (1-20)";
1120         INPUT Ellipse
1130         PRINT Ellipse
1140         IF Ellipse<=20 AND Ellipse>0 THEN
1150            PRINT
1160            PRINT "Center Point  (";Xbar(Ellipse);", ";Ybar(Ellipse);")"
1170            PRINT
1180            PRINT "Covariance Matrix:"
1190            Display_cov(Ellipse)
1200            PRINT "Major: ";Major_axis(Ellipse);"  Minor: ";Minor_axis(Ellip
se)
1210            PRINT "Orient: ";Angle(Ellipse)
1220            Invert(Ellipse,Ellipse+20)
1230            Get_bounds(Ellipse,Prob_constant)
1250            Draw_ellipse(Ellipse,Xmin,Xmax,Ymin,Ymax,Prob_constant)
1260            ALPHA ON
1280         END IF
1281      UNTIL Ellipse=0
1290   CASE "F","f"      ! Clear the graphics screen
1300      GCLEAR
1301   CASE "G","g"      ! Choose the plotter device
1302      Choose_plotter
1310   CASE "H","h"      ! Set the screen boundaries
1320      Clearscreen
```

```
1330      PRINT "Setting Screen Boundaries...."
1340      PRINT
1350      REPEAT
1360         Ellipse=0
1370         DISP "Display Extreme Points for Ellipse# (1-20)";
1380         INPUT Ellipse
1390         IF Ellipse>0 AND Ellipse<=20 THEN
1400            Invert(Ellipse,Ellipse+20)
1410            Get_bounds(Ellipse,Prob_constant)
1420            Disp_extremes(Ellipse)
1430         END IF
1440      UNTIL Ellipse=0
1450      PRINT
1460 Get_x:!
1470      PRINT "Minimum X value: ";
1480      DISP "Enter the minimum X value";
1490      INPUT Xmin
1500      PRINT Xmin
1510      PRINT "Maximum X value: ";
1520      DISP "Enter the maximum X value";
1530      INPUT Xmax
1540      PRINT Xmax
1550      IF Xmin>=Xmax THEN Get_x
1560 Get_y:!
1570      PRINT "Minimum Y value: ";
1580      DISP "Enter the minimum Y value";
1590      INPUT Ymin
1600      PRINT Ymin
1610      PRINT "Maximum Y value: ";
1620      DISP "Enter the maximum Y value";
1630      INPUT Ymax
1640      PRINT Ymax
1650      IF Ymin>=Ymax THEN Get_y
1660   CASE "I","i"          ! Calculate the chi-square acceptance test statistic
1670      Clearscreen
1680      PRINT "Calculating Acceptance Test"
1690      PRINT
1700      Ellipse1=0
1710      Ellipse2=0
1720      Swork=0
1730      PRINT "Ellipse 1 #";
1740      DISP "Enter the first Ellipse# (1-20)";
1750      INPUT Ellipse1
1760      PRINT Ellipse1
1770      PRINT "ellipse 2 #";
1780      DISP "Enter the second Ellipse# (1-20)";
1790      INPUT Ellipse2
1800      PRINT Ellipse2
1810      PRINT "Scratch Work #";
1820      DISP "Enter the scratch work # (1-20)";
1830      INPUT Swork
1840      PRINT Swork
1850      IF Ellipse1<=20 AND Ellipse1>0 AND Ellipse2<=20 AND Ellipse1>0 AND Swo
rk<=20 AND Swork>0 AND Swork<>Ellipse1 AND Swork<>Ellipse2 THEN
1860         Test(Ellipse1,Ellipse2,Swork,Test_stat)
1870      END IF
1880      PRINT
1890      PRINT "The Test statistic is ";Test_stat
1900      PRINT
1910      PRINT
1920      Pauseabit
1921   CASE "J","j"    ! Draw in the axes
1922     GRAPHICS ON
1923     CSIZE 2
1925     MOVE 0,0
1926     DRAW 0,100
```

```
1927      MOVE 100,0
1928      DRAW 0,0
1929      LABEL "(";Xmin;",";Ymin;")"
1930      LORG 7
1931      MOVE 100,0
1932      LABEL Xmax
1933      LORG 3
1934      MOVE 0,100
1935      LABEL Ymax
1936      LORG 4
1937      MOVE 50,0
1938      CSIZE 3
1940      LABEL "X-Axis (in kilometers)"
1941      MOVE 0,50
1942      DEG
1943      LDIR 270
1944      LABEL "Y-Axis (in kilometers)"
1945      LORG 1
1946      LDIR 0
1947      RAD
1949    CASE "X","x"          we can stop now
1950       Clearscreen
1951       PRINT "That's all, folks!"
1960       STOP
1970    CASE ELSE
1980       PRINT CHR$(7)
1990    END SELECT
2000    GOTO Menu_top
2010    END
5000    !
5010    !  SUBROUTINES:  Taken from ELLIPSTUFF
5020    !
5030    SUB Invert(Srce,Dest)
5040       ! This routine inverts any covariance matrix in Matrx and places
5050       ! the inverted matrix in Dest.
5060       COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
5070       Det=Matrx(1,1,Srce)*Matrx(2,2,Srce)-Matrx(1,2,Srce)*Matrx(1,2,Srce)
5080       Matrx(1,1,Dest)=Matrx(2,2,Srce)/Det
5090       Matrx(2,2,Dest)=Matrx(1,1,Srce)/Det
5100       Matrx(1,2,Dest)=-Matrx(1,2,Srce)/Det
5110       Matrx(2,1,Dest)=-Matrx(2,1,Srce)/Det
5120    SUBEND ! End of SUB Invert
5130    !
5140    !  GET_BOUNDS
5150    !
5160    SUB Get_bounds(Ellipse,Prob_constant)
5170       !  This subroutine calculates the X and Y limits for the given ellipse
.
5180       COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
5190       REAL Temp1,Temp2,Temp3,Temp4,Temp5
5210       Temp1=Matrx(2,2,Ellipse+20)*Prob_constant
5220       Temp2=Matrx(1,2,Ellipse+20)*Matrx(1,2,Ellipse+20)-Matrx(1,1,Ellipse+20
)*Matrx(2,2,Ellipse+20)
5230       Temp3=(Temp1/Temp2)^(.5)
5240       Temp4=Matrx(1,1,Ellipse+20)*Prob_constant
5250       Temp5=(Temp4/Temp2)^(.5)
5260       Emin_y(Ellipse)=-Temp5+Ybar(Ellipse)
5270       Emax_y(Ellipse)=Temp5+Ybar(Ellipse)
5280       Emin_x(Ellipse)=-Temp3+Xbar(Ellipse)
5290       Emax_x(Ellipse)=Temp3+Xbar(Ellipse)
5300    SUBEND  ! End of GET_BOUNDS
5310    !
5320    !  CHOOSE_PLOTTER
5330    !
```

```
5340   SUB Choose_plotter
5350      ! Subroutine to choose the desired plotter device
5360      REPEAT
5370         Go_on=-1
5380         DISP "Do you want to (D)raw, (E)rase, Plot with Pen (0), (1), (2)?"
;
5390         LINPUT Message$
5400         SELECT Message$
5410         CASE "D","d"
5420            PLOTTER IS 3,"INTERNAL"
5430            GRAPHICS ON
5440            PEN 1
5450            Go_on=1
5451         CASE "0"
5452            PLOTTER IS 705,"HPGL"
5453            GRAPHICS ON
5454            PEN 0
5455            Go_on=1
5460         CASE "1"
5470            PLOTTER IS 705,"HPGL"
5480            GRAPHICS ON
5490            PEN 1
5500            Go_on=1
5501         CASE "2"
5502            PLOTTER IS 705,"HPGL"
5503            GRAPHICS ON
5504            PEN 2
5505            Go_on=1
5510         CASE "E","e"
5520            PLOTTER IS 3,"INTERNAL"
5530            GRAPHICS ON
5540            PEN -1
5550            Go_on=1
5560         CASE ELSE
5570            PRINT CHR$(7)
5580         END SELECT
5590      UNTIL Go_on=1
5600   SUBEND
5610   !
5620   ! GET_ELL_MEAN
5630   !
5640   SUB Get_ell_mean(Ellipse)
5650      ! This subroutines prompts the user for the mean of an ellipse.
5660      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
5670      Clearscreen
5680      PRINT TABXY(1,10);"WHAT IS X-BAR  for ELLIPSE ";Ellipse;"?";
5690      INPUT Xbar(Ellipse)
5700      PRINT Xbar(Ellipse)
5710      PRINT TABXY(1,11);"WHAT IS Y-BAR  for ELLIPSE ";Ellipse;"?";
5720      INPUT Ybar(Ellipse)
5730      PRINT Ybar(Ellipse)
5740      Pauseabit
5750   SUBEND
5760   !
5770   !  GET_COVARIANCE
5780   !
5790   SUB Get_covariance(Ellipse)
5800      !  This routine gets the covariance matrix for an ellipse
5810      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
5820      Clearscreen
5830      FOR K=1 TO 2
5840         PRINT TABXY(1,3+K+6);"ENTER ELEMENT (";K;",";K;") IN THE COVARIANCE
 MATRIX FOR ELLIPSE#";Ellipse;": ";
5850         INPUT Matrx(K,K,Ellipse)
```

```
5860          PRINT Matrx(K,K,Ellipse)
5870       NEXT K
5880       PRINT TABXY(1,3+9);"ENTER ELEMENT (1,2) IN THE COVARIANCE MATRIX FOR E
LEMENT#";I;": ";
5890       INPUT Matrx(1,2,Ellipse)
5900       Matrx(2,1,Ellipse)=Matrx(1,2,Ellipse)
5910       PRINT Matrx(1,2,Ellipse)
5920       Pauseabit
5930    SUBEND
5940    !
5950    ! PAUSEABIT
5960    !
5970    SUB Pauseabit
5980       ! Pause and wait for a carriage return
5990       DISP "Type ENTER to continue...";
6000       INPUT Garbage$
6010    SUBEND
6020    SUB Clearscreen
6030       ! Clear the screen
6040       PRINT CHR$(12)
6050    SUBEND
6060    !
6070    ! GET_AXES
6080    !
6090    SUB Get_axes(Ellipse)
6100       ! This subroutine gets an ellipse in terms of the axes and the
6110       ! angle of orientation.  These are converted into a covariance
6120       ! matrix.
6130       COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
6140       Prob_constant=2*LOG(.05)
6150       Clearscreen
6160       PRINT "WHAT IS THE RADIUS OF THE MAJOR AXIS OF ELLIPSE ";Ellipse;" : "
;
6170       INPUT Major_axis(Ellipse)
6180       PRINT Major_axis(Ellipse)
6190       PRINT "WHAT IS THE RADIUS OF THE MINOR AXIS OF ELLIPSE ";Ellipse;" : "
;
6200       INPUT Minor_axis(Ellipse)
6210       PRINT Minor_axis(Ellipse)
6220       PRINT "WHAT IS THE ANGLE (IN DEGREES) OF THE MAJOR AXIS OF ELLIPSE ";E
llipse;" : ";
6230       INPUT Angle(Ellipse)
6240       PRINT Angle(Ellipse)
6250       ! Convert to Covariance Matrix
6260       Theta(Ellipse)=PI*Angle(Ellipse)/180
6270       Matrx(1,1,Ellipse)=(Major_axis(Ellipse)*COS(Theta(Ellipse)))^2+(Minor_
axis(Ellipse)*SIN(Theta(Ellipse)))^2
6280       Matrx(2,2,Ellipse)=(Major_axis(Ellipse)*SIN(Theta(Ellipse)))^2+(Minor_
axis(Ellipse)*COS(Theta(Ellipse)))^2
6290       Matrx(2,1,Ellipse)=((Major_axis(Ellipse))^2-(Minor_axis(Ellipse))^2)*C
OS(Theta(Ellipse))*SIN(Theta(Ellipse))
6300       Matrx(1,2,Ellipse)=Matrx(2,1,Ellipse)
6310       FOR J=1 TO 2
6320          FOR K=1 TO 2
6330             Matrx(J,K,Ellipse)=Matrx(J,K,Ellipse)/(-Prob_constant)
6340          NEXT K
6350       NEXT J
6360       PRINT
6370       Pauseabit
6380    SUBEND
6390    !
6400    ! DISPLAY_COV
6410    !
6420    SUB Display_cov(Ellipse)
6430       ! Displays the covariance matrix for an ellipse
```

```
6440      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
6450      FOR I=1 TO 2
6460         FOR J=1 TO 2
6470            PRINT Matrx(I,J,Ellipse);"   ";
6480         NEXT J
6490         PRINT
6500      NEXT I
6510   SUBEND
6520   !
6530   ! ADD_COVARIANCE
6540   !
6550   SUB Add_covariance(First,Second,Summer)
6560      ! This subroutine can be used to add Summer=First+Second
6570      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
6580      INTEGER I,J
6590      FOR I=1 TO 2
6600         FOR J=1 TO 2
6610            Matrx(I,J,Summer)=Matrx(I,J,First)+Matrx(I,J,Second)
6620         NEXT J
6630      NEXT I
6640   SUBEND
6650   !
6660   ! COMBINE_ELLIPSE
6670   !
6680   SUB Combine_ellipse(First,Second,Combo)
6690      ! This routine finds the "JPL" combination of the First and Second
6700      ! ellipses.
6710      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
6720      REAL Sx1,Sx2
6730      ! Calculate the new covariance matrix.
6740      Invert(First,First+20)
6750      Invert(Second,Second+20)
6760      Add_covariance(First+20,Second+20,Combo+20)
6770      Invert(Combo+20,Combo)
6780      ! Calculate the new mean point.
6790      Sx1=Matrx(1,1,First+20)*Xbar(First)+Matrx(1,2,First+20)*Ybar(First)
6800      Sx1=Sx1+Matrx(1,1,Second+20)*Xbar(Second)+Matrx(1,2,Second+20)*Ybar(Se
cond)
6810      Sx2=Matrx(2,1,First+20)*Xbar(First)+Matrx(2,2,First+20)*Ybar(First)
6820      Sx2=Sx2+Matrx(2,1,Second+20)*Xbar(Second)+Matrx(2,2,Second+20)*Ybar(Se
cond)
6830      Xbar(Combo)=Matrx(1,1,Combo)*Sx1+Matrx(1,2,Combo)*Sx2
6840      Ybar(Combo)=Matrx(2,1,Combo)*Sx1+Matrx(2,2,Combo)*Sx2
6850   SUBEND
6860   !
6870   ! DISP_EXTREMES
6880   !
6890   SUB Disp_extremes(Ellipse)
6900      ! This routine displays the extreme x and y values for the specified
6910      ! ellipse
6920      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
6930      PRINT "E# ";Ellipse;":  ";
6940      PRINT "Xmin ";Emin_x(Ellipse);
6950      PRINT ";  Xmax ";Emax_x(Ellipse);
6960      PRINT ";  Ymin ";Emin_y(Ellipse);
6970      PRINT ";  Ymax ";Emax_y(Ellipse)
6980   SUBEND
6990   SUB Draw_ellipse(Ellipse,Xmin,Xmax,Ymin,Ymax,Prob_constant)
7000      ! This routine draws one (1) ellipse on the current plotter device.
7010      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
7020      INTEGER Sign
```

```
7030      REAL Xpoint,Upoint,Ypoint,Temp1,Temp2,Temp3,Temp4,Temp5
7060      GRAPHICS ON
7070      ! Draw top half of the ellipse
7080      Sign=1
7090      GOSUB Draw_half
7100      ! Draw bottom half of the ellipse
7110      Sign=-1
7120      GOSUB Draw_half
7130      ! Finish up
7140      MOVE (Xbar(Ellipse)-Xmin)*100/(Xmax-Xmin),(Ybar(Ellipse)-Ymin)*100/(Ym
ax-Ymin)
7150      DRAW (Xbar(Ellipse)-Xmin)*100/(Xmax-Xmin),(Ybar(Ellipse)-Ymin)*100/(Ym
ax-Ymin)
7160      ! Okey, dokey, we're done here.
7170      SUBEXIT
7180 Draw_half:    ! Draw half of the ellipse
7190      !  If Sign=1, then draw top half;  if Sign=-1, then bottom half.
7200      FOR Xpoint=Emin_x(Ellipse) TO Emax_x(Ellipse) STEP .1
7210         GOSUB Draw_1_point
7220      NEXT Xpoint
7230      Xpoint=Emax_x(Ellipse)
7240      GOSUB Draw_1_point
7250      RETURN
7260      !
7270      !  Compute each point and draw the new line.  (It's here since we call
7280      !  it twice
7290      !
7300 Draw_1_point:  !
7310      Upoint=Xpoint-Xbar(Ellipse)
7320      Temp1=Matrx(1,2,Ellipse+20)*Upoint
7330      Temp2=Temp1*Temp1-Matrx(2,2,Ellipse+20)*(Matrx(1,1,Ellipse+20)*Upoint*
Upoint+Prob_constant)
7340      IF Temp2<10^(-10) THEN Temp2=0
7350      Ypoint=(-Temp1+Sign*SQR(Temp2))/Matrx(2,2,Ellipse+20)+Ybar(Ellipse)
7360      IF Xpoint<=Emin_x(Ellipse)+.001 THEN
7370         MOVE (Xpoint-Xmin)*100/(Xmax-Xmin),(Ypoint-Ymin)*100/(Ymax-Ymin)
7380      ELSE
7390         DRAW (Xpoint-Xmin)*100/(Xmax-Xmin),(Ypoint-Ymin)*100/(Ymax-Ymin)
7400      END IF
7410      RETURN
7420   SUBEND
7430   !
7440   ! TEST
7450   !
7460   SUB Test(First,Second,Work,Test_val)
7470      ! This routines calculates the acceptance test criteria for First and
7480      ! Second.  Work is used as a "scratchpad".
7490      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
7500      Add_covariance(First,Second,Work)
7510      Invert(Work,Work+20)
7520      Diffx=Xbar(First)-Xbar(Second)
7530      Diffy=Ybar(First)-Ybar(Second)
7540      Temp1=Matrx(1,1,Work+20)*Diffx+Matrx(1,2,Work+20)*Diffy
7550      Temp2=Matrx(2,1,Work+20)*Diffx+Matrx(2,2,Work+20)*Diffy
7560      Test_val=Diffx*Temp1+Diffy*Temp2
7570   SUBEND
```

B. ELLSIM: Ellipse Simulation Program

Ellsim is a "confidence ellipse simulation" program. It has been used to explore the robustness and properties of the statistical test (which is used to decide whether or not to combine two ellipses). In it, two normal data distributions are specified, corresponding to two emitters. Confidence ellipses are generated, tested, and combined, and various descriptive statistics are compiled. In addition, there is a routine to calculate the power of the statistical test in certain cases.

```
1000     !-----------------------------------------------------------!
1010     !                    Ellipse Simulation Driver              !
1020     !                                                           !
1030     !     This is a simple simulation program for the ellipse   !
1040     !     combination test.  Only the Normal distribution is    !
1050     !     supported.                                            !
1060     !                                                           !
1070     !     Original:  1/17/85                Updated:  7/29/85   !
1080     !-----------------------------------------------------------!
1090     !
1100     !
1110     ! 7/29/85
1120 Top_of_program:    !
1130     ! Use Fast Math card
1140     CONTROL 32,2;1
1150     ! Specify common variables between the different subprograms
1160     COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_axi
s(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
1170     COM /Driver/ Obs(2)
1180 Driver_menu:     !
1190     Clearscreen
1200     PRINT "Ellipse Combination Program Driver"
1210     PRINT
1220     PRINT "(A) -- Specify True Covariance Matrices"
1230     PRINT "(B) -- Specify True Mean Parameters"
1240     PRINT "(C) -- Specify Observations"
1250     PRINT "(D) -- Call Simulation Generator"
1260     PRINT "(E) -- Call Power Generator"
1270     PRINT "(X) -- Exit Program"
1280     PRINT
1290 Get_choice:    !
1300     INPUT "Enter your choice:",Option$
1310     SELECT Option$
1320       CASE "A","a"
1330         Clearscreen
1340         CALL Get_covariance(1)
1350         CALL Get_covariance(2)
1360       CASE "B","b"
1370         Clearscreen
1380         CALL Get_ell_mean(1)
1390         CALL Get_ell_mean(2)
1400       CASE "C","c"
1410         Clearscreen
1420         GOSUB Get_obs
1430       CASE "D","d"
1440         Clearscreen
1450         CALL Sim_ellipse
1460       CASE "E","e"
1470         Clearscreen
1480         CALL Power_ellipse
1490       CASE "X","x"
1500         GOTO End_program
1510       CASE ELSE
1520         PRINT CHR$(7)
1530         GOTO Get_choice
1540     END SELECT
1550     GOTO Driver_menu
1560     !-----------------------------------------------------------!
1570     !                      SUBROUTINES                          !
1580     !-----------------------------------------------------------!
1590     !
1600     ! Get the number of TRIALS (Main Menu Choice "B")
1610     !
1620 Get_obs:    !
1630     FOR Imat=1 TO 2
```

```
1640        PRINT "How many observations for ellipse #";Imat;": ";
1650        INPUT Obs(Imat)
1660        PRINT Obs(Imat)
1670      NEXT Imat
1680 RETURN
1690 End_program:!
1700      END
1710      !-----------------------------------------------------------!
1720      !   SUBPROGRAMS AND FUNCTIONS
1730      !
1740   SUB Invert(Srce,Dest)
1750      ! This routine inverts any covariance matrix in Matrx and places
1760      ! the inverted matrix in Dest.
1770      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
1780      Det=Matrx(1,1,Srce)*Matrx(2,2,Srce)-Matrx(1,2,Srce)*Matrx(1,2,Srce)
1790      Matrx(1,1,Dest)=Matrx(2,2,Srce)/Det
1800      Matrx(2,2,Dest)=Matrx(1,1,Srce)/Det
1810      Matrx(1,2,Dest)=-Matrx(1,2,Srce)/Det
1820      Matrx(2,1,Dest)=-Matrx(2,1,Srce)/Det
1830   SUBEND ! End of SUB Invert
1840   !
1850   !  GET_BOUNDS
1860   !
1870   SUB Get_bounds(Ellipse,Prob_constant)
1880      !  This subroutine calculates the X and Y limits for the given ellipse
.
1890      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
1900      REAL Temp1,Temp2,Temp3,Temp4,Temp5
1910      Temp1=Matrx(2,2,Ellipse+20)*Prob_constant
1920      Temp2=Matrx(1,2,Ellipse+20)*Matrx(1,2,Ellipse+20)-Matrx(1,1,Ellipse+20
)*Matrx(2,2,Ellipse+20)
1930      Temp3=(Temp1/Temp2)^(.5)
1940      Temp4=Matrx(1,1,Ellipse+20)*Prob_constant
1950      Temp5=(Temp4/Temp2)^(.5)
1960      Emin_y(Ellipse)=-Temp5+Ybar(Ellipse)
1970      Emax_y(Ellipse)=Temp5+Ybar(Ellipse)
1980      Emin_x(Ellipse)=-Temp3+Xbar(Ellipse)
1990      Emax_x(Ellipse)=Temp3+Xbar(Ellipse)
2000   SUBEND  ! End of GET_BOUNDS
2010   !
2020   !  CHOOSE_PLOTTER
2030   !
2040   SUB Choose_plotter
2050      ! Subroutine to choose the desired plotter device
2060      REPEAT
2070         Go_on=-1
2080         DISP "Do you want to (D)raw, (E)rase, Plot with Pen (0), (1), (2)?"
;
2090         LINPUT Message$
2100         SELECT Message$
2110         CASE "D","d"
2120            PLOTTER IS 3,"INTERNAL"
2130            GRAPHICS ON
2140            PEN 1
2150            Go_on=1
2160         CASE "0"
2170            PLOTTER IS 705,"HPGL"
2180            GRAPHICS ON
2190            PEN 0
2200            Go_on=1
2210         CASE "1"
2220            PLOTTER IS 705,"HPGL"
2230            GRAPHICS ON
2240            PEN 1
```

```
2250            Go_on=1
2260         CASE "2"
2270            PLOTTER IS 705,"HPGL"
2280            GRAPHICS ON
2290            PEN 2
2300            Go_on=1
2310         CASE "E","e"
2320            PLOTTER IS 3,"INTERNAL"
2330            GRAPHICS ON
2340            PEN -1
2350            Go_on=1
2360         CASE ELSE
2370            PRINT CHR$(7)
2380         END SELECT
2390      UNTIL Go_on=1
2400   SUBEND
2410   !
2420   ! GET_ELL_MEAN
2430   !
2440   SUB Get_ell_mean(Ellipse)
2450      ! This subroutines prompts the user for the mean of an ellipse.
2460      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
2470      Clearscreen
2480      PRINT TABXY(1,10);"WHAT IS X-BAR  for ELLIPSE ";Ellipse;"?";
2490      INPUT Xbar(Ellipse)
2500      PRINT Xbar(Ellipse)
2510      PRINT TABXY(1,11);"WHAT IS Y-BAR  for ELLIPSE ";Ellipse;"?";
2520      INPUT Ybar(Ellipse)
2530      PRINT Ybar(Ellipse)
2540      Pauseabit
2550   SUBEND
2560   !
2570   !  GET_COVARIANCE
2580   !
2590   SUB Get_covariance(Ellipse)
2600      !  This routine gets the covariance matrix for an ellipse
2610      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
2620      Clearscreen
2630      FOR K=1 TO 2
2640         PRINT TABXY(1,3+K+6);"ENTER ELEMENT (";K;",";K;") IN THE COVARIANCE
 MATRIX FOR ELLIPSE#";Ellipse;": ";
2650         INPUT Matrx(K,K,Ellipse)
2660         PRINT Matrx(K,K,Ellipse)
2670      NEXT K
2680      PRINT TABXY(1,3+9);"ENTER ELEMENT (1,2) IN THE COVARIANCE MATRIX FOR E
LEMENT#";I;": ";
2690      INPUT Matrx(1,2,Ellipse)
2700      Matrx(2,1,Ellipse)=Matrx(1,2,Ellipse)
2710      PRINT Matrx(1,2,Ellipse)
2720      Pauseabit
2730   SUBEND
2740   !
2750   ! PAUSEABIT
2760   !
2770   SUB Pauseabit
2780      ! Pause and wait for a carriage return
2790      DISP "Type ENTER to continue...";
2800      INPUT Garbage$
2810   SUBEND
2820   SUB Clearscreen
2830      ! Clear the screen
2840      PRINT CHR$(12)
2850   SUBEND
2860   !
```

```
2870   ! GET_AXES
2880   !
2890   SUB Get_axes(Ellipse)
2900      ! This subroutine gets an ellipse in terms of the axes and the
2910      ! angle of orientation.  These are converted into a covariance
2920      ! matrix.
2930      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
2940      Prob_constant=2*LOG(.05)
2950      Clearscreen
2960      PRINT "WHAT IS THE RADIUS OF THE MAJOR AXIS OF ELLIPSE ";Ellipse;" : "
;
2970      INPUT Major_axis(Ellipse)
2980      PRINT Major_axis(Ellipse)
2990      PRINT "WHAT IS THE RADIUS OF THE MINOR AXIS OF ELLIPSE ";Ellipse;" : "
;
3000      INPUT Minor_axis(Ellipse)
3010      PRINT Minor_axis(Ellipse)
3020      PRINT "WHAT IS THE ANGLE (IN DEGREES) OF THE MAJOR AXIS OF ELLIPSE ";E
llipse;" : ";
3030      INPUT Angle(Ellipse)
3040      PRINT Angle(Ellipse)
3050      ! Convert to Covariance Matrix
3060      Theta(Ellipse)=PI*Angle(Ellipse)/180
3070      Matrx(1,1,Ellipse)=(Major_axis(Ellipse)*COS(Theta(Ellipse)))^2+(Minor_
axis(Ellipse)*SIN(Theta(Ellipse)))^2
3080      Matrx(2,2,Ellipse)=(Major_axis(Ellipse)*SIN(Theta(Ellipse)))^2+(Minor_
axis(Ellipse)*COS(Theta(Ellipse)))^2
3090      Matrx(2,1,Ellipse)=((Major_axis(Ellipse))^2-(Minor_axis(Ellipse))^2)*C
OS(Theta(Ellipse))*SIN(Theta(Ellipse))
3100      Matrx(1,2,Ellipse)=Matrx(2,1,Ellipse)
3110      FOR J=1 TO 2
3120         FOR K=1 TO 2
3130            Matrx(J,K,Ellipse)=Matrx(J,K,Ellipse)/(-Prob_constant)
3140         NEXT K
3150      NEXT J
3160      PRINT
3170      Pauseabit
3180   SUBEND
3190   !
3200   ! DISPLAY_COV
3210   !
3220   SUB Display_cov(Ellipse)
3230      ! Displays the covariance matrix for an ellipse
3240      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3250      FOR I=1 TO 2
3260         FOR J=1 TO 2
3270            PRINT Matrx(I,J,Ellipse);"   ";
3280         NEXT J
3290         PRINT
3300      NEXT I
3310   SUBEND
3320   !
3330   ! ADD_COVARIANCE
3340   !
3350   SUB Add_covariance(First,Second,Summer)
3360      ! This subroutine can be used to add Summer=First+Second
3370      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3380      INTEGER I,J
3390      FOR I=1 TO 2
3400         FOR J=1 TO 2
3410            Matrx(I,J,Summer)=Matrx(I,J,First)+Matrx(I,J,Second)
3420         NEXT J
3430      NEXT I
```

```
3440   SUBEND
3450   !
3460   ! COMBINE_ELLIPSE
3470   !
3480   SUB Combine_ellipse(First,Second,Combo)
3490      ! This routine finds the "JPL" combination of the First and Second
3500      ! ellipses.
3510      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3520      REAL Sx1,Sx2
3530      ! Calculate the new covariance matrix.
3540      Invert(First,First+20)
3550      Invert(Second,Second+20)
3560      Add_covariance(First+20,Second+20,Combo+20)
3570      Invert(Combo+20,Combo)
3580      ! Calculate the new mean point.
3590      Sx1=Matrx(1,1,First+20)*Xbar(First)+Matrx(1,2,First+20)*Ybar(First)
3600      Sx1=Sx1+Matrx(1,1,Second+20)*Xbar(Second)+Matrx(1,2,Second+20)*Ybar(Se
cond)
3610      Sx2=Matrx(2,1,First+20)*Xbar(First)+Matrx(2,2,First+20)*Ybar(First)
3620      Sx2=Sx2+Matrx(2,1,Second+20)*Xbar(Second)+Matrx(2,2,Second+20)*Ybar(Se
cond)
3630      Xbar(Combo)=Matrx(1,1,Combo)*Sx1+Matrx(1,2,Combo)*Sx2
3640      Ybar(Combo)=Matrx(2,1,Combo)*Sx1+Matrx(2,2,Combo)*Sx2
3650   SUBEND
3660   !
3670   ! DISP_EXTREMES
3680   !
3690   SUB Disp_extremes(Ellipse)
3700      ! This routine displays the extreme x and y values for the specified
3710      ! ellipse
3720      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3730      PRINT "EV ";Ellipse;":  ";
3740      PRINT "Xmin ";Emin_x(Ellipse);
3750      PRINT ";  Xmax ";Emax_x(Ellipse);
3760      PRINT ";  Ymin ";Emin_y(Ellipse);
3770      PRINT ";  Ymax ";Emax_y(Ellipse)
3780   SUBEND
3790   SUB Draw_ellipse(Ellipse,Xmin,Xmax,Ymin,Ymax,Prob_constant)
3800      ! This routine draws one (1) ellipse on the current plotter device.
3810      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3820      INTEGER Sign
3830      REAL Xpoint,Upoint,Ypoint,Temp1,Temp2,Temp3,Temp4,Temp5
3840      GRAPHICS ON
3850      ! Draw top half of the ellipse
3860      Sign=1
3870      GOSUB Draw_half
3880      ! Draw bottom half of the ellipse
3890      Sign=-1
3900      GOSUB Draw_half
3910      ! Finish up
3920      MOVE (Xbar(Ellipse)-Xmin)*100/(Xmax-Xmin),(Ybar(Ellipse)-Ymin)*100/(Ym
ax-Ymin)
3930      DRAW (Xbar(Ellipse)-Xmin)*100/(Xmax-Xmin),(Ybar(Ellipse)-Ymin)*100/(Ym
ax-Ymin)
3940      ! Okey, dokey, we're done here.
3950      SUBEXIT
3960 Draw_half:    ! Draw half of the ellipse
3970      !  If Sign=1, then draw top half;  if Sign=-1, then bottom half.
3980      FOR Xpoint=Emin_x(Ellipse) TO Emax_x(Ellipse) STEP .1
3990         GOSUB Draw_1_point
4000      NEXT Xpoint
4010      Xpoint=Emax_x(Ellipse)
4020      GOSUB Draw_1_point
```

```
4030      RETURN
4040      !
4050      !  Compute each point and draw the new line.  (It's here since we call
4060      !  it twice
4070      !
4080 Draw_1_point:  !
4090      Upoint=Xpoint-Xbar(Ellipse)
4100      Temp1=Matrx(1,2,Ellipse+20)*Upoint
4110      Temp2=Temp1*Temp1-Matrx(2,2,Ellipse+20)*(Matrx(1,1,Ellipse+20)*Upoint*
Upoint+Prob_constant)
4120      IF Temp2<10^(-10) THEN Temp2=0
4130      Ypoint=(-Temp1+Sign*SQR(Temp2))/Matrx(2,2,Ellipse+20)+Ybar(Ellipse)
4140      IF Xpoint<=Emin_x(Ellipse)+.001 THEN
4150         MOVE (Ypoint-Ymin)*100/(Ymax-Ymin),(Ypoint-Ymin)*100/(Ymax-Ymin)
4160      ELSE
4170         DRAW (Xpoint-Xmin)*100/(Xmax-Xmin),(Ypoint-Ymin)*100/(Ymax-Ymin)
4180      END IF
4190      RETURN
4200   SUBEND
4210   !
4220   ! TEST
4230   !
4240   SUB Test(First,Second,Work,Test_val)
4250      ! This routines calculates the acceptance test criteria for First and
4260      ! Second.  Work is used as a "scratchpad".
4270      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
4280      Add_covariance(First,Second,Work)
4290      Invert(Work,Work+20)
4300      Diffx=Xbar(First)-Xbar(Second)
4310      Diffy=Ybar(First)-Ybar(Second)
4320      Temp1=Matrx(1,1,Work+20)*Diffx+Matrx(1,2,Work+20)*Diffy
4330      Temp2=Matrx(2,1,Work+20)*Diffx+Matrx(2,2,Work+20)*Diffy
4340      Test_val=Diffx*Temp1+Diffy*Temp2
4350   SUBEND
4360     SUB Sim_ellipse
4370     !  Ellipse Combination Simulation Program
4380     !  1/16/85   Update 7/18/85
4390     !
4450     !  Specify the common variables
4460     !
4470     COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_axi
s(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
4480     COM /Driver/ Obs(2)
4490     !  Variable Definitions
4500     !
4510     INTEGER Ellnum,Imat,Xmin,Xmax,Ymin,Ymax
4520     INTEGER Mobservations(2)
4530     DIM Sx(2,10),Xs(5000),Ys(5000)
4560     !
4570     !  Use of MATRX Array
4580     !
4590     ! MATRIX 1 is an initial matrix
4600     ! MATRIX 2 is an initial matrix
4610     ! MATRIX 3 is the theoretical combination of 8 and 9
4620     ! MATRIX 4 is the estimate of 8
4630     ! MATRIX 5 is the estimate of 9
4640     ! MATRIX 6 is the combination of 4 and 5
4650     ! MATRIX 7 is the sum of 3 and 4 (for test)
4652     !   Note that if the True Variance-Covariance is used instead of the
4653     !   the estimate, this is equal to the sum of 8 and 9.
4654     ! MATRIX 8 is MATRIX 1 divided by sample size
4655     ! MATRIX 9 is MATRIX 2 divided by sample size
4660     ! MATRIX 21-29 are the inverses of 1-9
4670     !
4680     !  Initialization
```

```
4690      !
4700      GOSUB Init_sub1
4710      !
4720      !  Main Program Loop
4730      !
4740 Main_loop: !
4750      Clearscreen
4760      PRINT "Ellipse Simulations"
4770      PRINT
4780      PRINT "Enter the Letter of your choice:"
4790      PRINT "    (A) Reset the Random Number Seed"
4800      PRINT "    (B) Simulate using true Covariance Matrices"
4810      PRINT "    (C) Simulate using estimated Covariance Matrices"
4820      PRINT "    (D) Display Results on the Printer"
4830      PRINT "    (X) Exit Program"
4831 Get_option:     !
4840      INPUT "Enter Your Choice:",Mainchoice$
4860      SELECT Mainchoice$
4870        CASE "A","a"
4880          GOSUB Get_seed
4890        CASE "B","b"
4900          Use_true=1
4910          GOSUB Simulate
4920        CASE "C","c"
4930          Use_true=0
4940          GOSUB Simulate
4950        CASE "D","d"
4960          GOSUB Display_results
4970        CASE "X","x"
4980          GOTO End_sub1
4990        CASE ELSE
5000          PRINT CHR$(7)
5010          GOTO Get_option
5020      END SELECT
5030      GOTO Main_loop
5040      !-----------------------------------------------------------!
5050      !    Utility Subroutines                                    !
5060      !-----------------------------------------------------------!
5070      !
5080      !  Initialize Program
5090      !
5100 Init_sub1:     !
5101      RANDOMIZE
5110      Prob_constant=2*LOG(.05)    ! Confidence level parameter
5111      FOR I=1 TO 2
5112        FOR J=1 TO 2
5113          Matrx(I,J,8)=Matrx(I,J,1)/Obs(1)
5114          Matrx(I,J,9)=Matrx(I,J,2)/Obs(2)
5115        NEXT J
5116      NEXT I
5117      Xbar(8)=Xbar(1)
5118      Ybar(8)=Ybar(1)
5119      Xbar(9)=Xbar(2)
5120      Ybar(9)=Ybar(2)
5122      ! Set parameters
5230 RETURN
5340      !
5350      !  Generate the new ellipses using random observations
5360      !
5370 Generate_new: !***FLAG***
5380      FOR Imat=1 TO 2
5390        N=Obs(Imat)
5400        ! Generate the X and Y values
5410        Xsum=0
5420        Ysum=0
5430        Cc=SQR(Matrx(1,1,Imat))
```

```
5440        A=Sigma(1,2,Imat)/Cc
5450        Bee=SQR(Matrx(2,2,Imat)-A*A)
5460        FOR I=1 TO M
5470          U1=RND
5480          U2=RND
5490          X1=SQR(-2*LOG(U1))*SIN(2*PI*U2)
5500          Y1=SQR(-2*LOG(U2))*COS(2*PI*U1)
5510          Xs(I)=Cc*X1+Xbar(Imat)
5520          Ys(I)=A*X1+Bee*Y1+Ybar(Imat)
5530          Xsum=Xsum+Xs(I)
5540          Ysum=Ysum+Ys(I)
5550        NEXT I
5560        Xbar(Imat+3)=Xsum/M
5570        Ybar(Imat+3)=Ysum/M
5580        ! Calculate the variances
5590        Matrx(1,1,Imat+3)=0
5600        Matrx(2,2,Imat+3)=0
5610        Matrx(1,2,Imat+3)=0
5620        FOR I=1 TO M
5630          Matrx(1,1,Imat+3)=Matrx(1,1,Imat+3)+(Xs(I)-Xbar(Imat+3))*(Xs(I)-Xba
r(Imat+3))
5640          Matrx(2,2,Imat+3)=Matrx(2,2,Imat+3)+(Ys(I)-Ybar(Imat+3))*(Ys(I)-Yba
r(Imat+3))
5650          Matrx(1,2,Imat+3)=Matrx(1,2,Imat+3)+(Xs(I)-Xbar(Imat+3))*(Ys(I)-Yba
r(Imat+3))
5660        NEXT I
5670        Matrx(1,1,Imat+3)=Matrx(1,1,Imat+3)/((M-1)*M)
5680        Matrx(2,2,Imat+3)=Matrx(2,2,Imat+3)/((M-1)*M)
5690        Matrx(1,2,Imat+3)=Matrx(1,2,Imat+3)/((M-1)*M)
5700        Matrx(2,1,Imat+3)=Matrx(1,2,Imat+3)
5710      NEXT Imat
5720 RETURN  ! Generate_new
5730      !
5740      !  Generate the inverses of our matrices
5750      !
5760 Gen_inverses:  !
5830 Patch1:    !
5840      IF Use_true=0 THEN Patch2
5850      ! To use the true matrices, copy them from 8 and 9 into 4 and 5
5860      FOR Imat=8 TO 9
5870        FOR I=1 TO 2
5880          FOR J=1 TO 2
5890            Matrx(I,J,Imat-4)=Matrx(I,J,Imat)
5900          NEXT J
5910        NEXT I
5920      NEXT Imat
5930 Patch2:    !
6151      Combine_ellipse(8,9,3)
6152      Combine_ellipse(4,5,6)
6170 RETURN   ! Gen_inverses
6430      !-----------------------------------------------------------!
6440      !                Program Subroutines                        !
6450      !-----------------------------------------------------------!
6460      !
6470      ! GET SEED  (Menu Menu Choice "C")
6480      !
6490 Get_seed:    !
6500      Clearscreen
6510      PRINT "ENTER A SEED (1 to 2^31-2): ";
6520      INPUT Seed
6530      Seed$=VAL$(Seed)
6540      PRINT Seed$
6550      RANDOMIZE Seed
6560      PRINT
6570      Pauseabit
6580 RETURN
```

```
6590      !
6600      ! SIMULATE
6610      !
6620 Simulate:    !
6630      Clearscreen
6640      PRINT TABXY(1,17);"*************** HOW MANY SIMULATIONS D  YOU WISH TO R
UN? ";
6650      INPUT Nsimulations
6660      PRINT Nsimulations
6670      N_accept=0
6680      N_reject=0
6690      Anotin1=0
6700      Anotin2=0
6710      Anotinb=0
6720      Rnotin1=0
6730      Rnotin2=0
6740      Rnotinb=0
6750      Pauseabit
6760      Clearscreen
6770      PRINT TABXY(5,3);"NUMBER OF SIMULATIONS= 0"
6780      PRINT TABXY(5,4);"****************************"
6790      PRINT TABXY(5,5);"ACCEPTED ERROR ELLIPSES= 0"
6800      PRINT TABXY(5,6);"REJECTED ERROR ELLIPSES= 0"
6810      PRINT TABXY(5,9);"ACCEPTED BREAKOUT"
6820      PRINT TABXY(5,10);"Target One is NOT in Combined Ellipse= 0"
6830      PRINT TABXY(5,11);"Target Two is NOT in Combined Ellipse= 0"
6840      PRINT TABXY(5,12);"Neither Target is in Combined Ellipse= 0"
6850      PRINT TABXY(5,14);"REJECTED BREAKOUT"
6860      PRINT TABXY(5,15);"Target One is NOT in Combined Ellipse= 0"
6870      PRINT TABXY(5,16);"Target Two is NOT in Combined Ellipse= 0"
6880      PRINT TABXY(5,17);"Neither Target is in Combined Ellipse= 0"
6890      FOR Isim=1 TO Nsimulations
6900        GOSUB Generate_new
6910        GOSUB Gen_inverses
6940        One_in=Matrx(1,1,26)*(Xbar(6)-Xbar(8))^2+Matrx(2,2,26)*(Ybar(6)-Ybar(
8))^2
6950        One_in=One_in+2*Matrx(1,2,26)*(Xbar(6)-Xbar(8))*(Ybar(6)-Ybar(8))
6960        Two_in=Matrx(1,1,26)*(Xbar(6)-Xbar(9))^2+Matrx(2,2,26)*(Ybar(6)-Ybar(
9))^2
6970        Two_in=Two_in+2*Matrx(1,2,26)*(Xbar(6)-Xbar(9))*(Ybar(6)-Ybar(9))
6971        Test(4,5,7,Test2)
6980        !
6990        !
7000        PRINT TABXY(28,3);Isim
7010        IF Test2<-Prob_constant THEN
7020          ! Accept as same
7030          N_accept=N_accept+1
7040          PRINT TABXY(30,5);N_accept
7050          IF One_in>-Prob_constant THEN
7060            Anotin1=Anotin1+1
7070            PRINT TABXY(44,10);Anotin1
7080          END IF
7090          IF Two_in>-Prob_constant THEN
7100            Anotin2=Anotin2+1
7110            PRINT TABXY(44,11);Anotin2
7120          END IF
7130          IF One_in>-Prob_constant AND Two_in>-Prob_constant THEN
7140            Anotinb=Anotinb+1
7150            PRINT TABXY(44,12);Anotinb
7160          END IF
7170        ELSE
7180          N_reject=N_reject+1
7190          PRINT TABXY(30,6);N_reject
7200          IF One_in>-Prob_constant THEN
7210            Rnotin1=Rnotin1+1
7220            PRINT TABXY(44,15);Rnotin1
```

```
7230        END IF
7240        IF Two_in>-Prob_constant THEN
7250          Rnotin2=Rnotin2+1
7260          PRINT TABXY(44,16);Rnotin2
7270        END IF
7280        IF One_in>-Prob_constant AND Two_in>-Prob_constant THEN
7290          Rnotinb=Rnotinb+1
7300          PRINT TABXY(44,17);Rnotinb
7310        END IF
7320      END IF
7330    NEXT Isim
7340    PRINT TABXY(5,19);"*** SIMULATION COMPLETE ***";
7341    BEEP
7342    BEEP
7343    BEEP
7344    BEEP
7350    Pauseabit
7360 RETURN
7370    !
7380    ! Display results
7390    !
7400 Display_results:!
7410    Clearscreen
7420    PRINTER IS 9;WIDTH 132
7421    PRINT
7422    PRINT "*****************************************************************"
7423    PRINT
7424    PRINT
7430    PRINT "Results:  ";
7440    IF Use_true=1 THEN
7450      PRINT "Using TRUE Covariance Matrices"
7460    ELSE
7470      PRINT "Using ESTIMATED Covariance Matrices"
7480    END IF
7490    PRINT
7491    FOR Imat=1 TO 2
7492      PRINT "Base Distribution #";Imat
7493      PRINT "Mean: (";Xbar(Imat);", ";Ybar(Imat);");"
7494      PRINT "Observations: ";Obs(Imat)
7495      PRINT "Covariance Matrix:"
7496      PRINT "* ";Matrx(1,1,Imat);"        ";Matrx(1,2,Imat);"  *"
7497      PRINT "* ";Matrx(2,1,Imat);"        ";Matrx(2,2,Imat);"  *"
7498      PRINT
7499    NEXT Imat
7500    FOR Imat=4 TO 5
7501      PRINT "Ellipse #";Imat-3
7502      PRINT "Mean: (";Xbar(Imat);", ";Ybar(Imat);");"
7504      PRINT "Covariance Matrix:"
7505      PRINT "* ";Matrx(1,1,Imat);"        ";Matrx(1,2,Imat);"  *"
7506      PRINT "* ";Matrx(2,1,Imat);"        ";Matrx(2,2,Imat);"  *"
7507      PRINT
7508    NEXT Imat
7590    PRINT "Last Combined Ellipse:"
7600    PRINT "Mean: (";Xbar(6);", ";Ybar(6);");"
7610    PRINT "Covariance Matrix:"
7620    PRINT "* ";Matrx(1,1,6);"        ";Matrx(1,2,6);"  *"
7630    PRINT "* ";Matrx(2,1,6);"        ";Matrx(2,2,6);"  *"
7640    PRINT
7650    PRINT "Simulation Results:"
7660    PRINT "# of simulations: ";Nsimulations
7661    PRINT
7662    PRINT "                    Totals:      % of Total:      % of Category"
7670    PRINT "Accepted:           ";N_accept;"   ";100*N_accept/Nsimulations
7680    PRINT " Target 1 NOT in: ";Anotin1;
7681    PRINT "   ";100*Anotin1/Nsimulations;"   ";100*Anotin1/N_accept
7690    PRINT " Target 2 NOT in: ";Anotin2;
```

```
7691     PRINT "   ";100*Anotin2/Nsimulations;"   ";100*Anotin2/N_accept
7700     PRINT " Neither one in:  ";Anotinb;
7701     PRINT "   ";100*Anotinb/Nsimulations;"   ";100*Anotinb/N_accept
7710     PRINT "Rejected:          ";N_reject;"   ";100*N_reject/Nsimulations
7720     PRINT " Target 1 NOT in: ";Rnotin1;
7721     PRINT "   ";100*Rnotin1/Nsimulations;"   ";100*Rnotin1/N_reject
7730     PRINT " Target 2 NOT in: ";Rnotin2;
7731     PRINT "   ";100*Rnotin2/Nsimulations;"   ";100*Rnotin2/N_reject
7740     PRINT " Neither one in:  ";Rnotinb
7741     PRINT "   ";100*Rnotinb/Nsimulations;"   ";100*Rnotinb/N_reject
7750     PRINT
7751     PRINT
7753     PRINT CHR$(12)
7760     PRINTER IS 1;WIDTH 80
7770 RETURN
7780 End_sub1:!
7790     SUBEND
7800     SUB Power_ellipse
7810    ! Target Ellipse Chi-square Test Power Calculation
7820    ! Specify common variables
7830     COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_axi
s(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
7840     COM /Driver/ Obs(2)
7850    ! Variable Definition
7870    DIM New_mat(2,2,2),Delta(2)
7880    DIM Table(62,2)
7900    ! Initialize Program
7910    GOSUB Init_sub2
7920    ! MENU LOOP
7930 Menu_loop:   !
7940    Clear_screen
7950    GOSUB Power_stuff
7960    SUBEXIT
7970     !
7980     !  INITIALIZE THE PROGRAM
7990     !
8000 Init_sub2:  !
8010     Prob_constant=2*LOG(.05)
8020     ! Load in the Power Table
8030     RESTORE Power_data
8040     FOR I=1 TO 62
8050       FOR J=1 TO 2
8060         READ Table(I,J)
8070       NEXT J
8080     NEXT I
8170 RETURN
8180     !
8190     !  Power calculations:  find the non-centrality parameter, Lambda,
8200     !  to use with the tables.
8210     !
8220 Power_stuff:!
8230     PRINT "Power Calculations:  Non-centrality Parameter"
8240     PRINT
8250     FOR I=1 TO 2
8260       FOR J=1 TO 2
8270         Power_mat(I,J,1)=Matrx(I,J,1)/Obs(1)+Matrx(I,J,2)/Obs(2)
8280       NEXT J
8290     NEXT I
8300     !compute inverse of power_mat
8310     Det=Power_mat(1,1,1)*Power_mat(2,2,1)-Power_mat(1,2,1)*Power_mat(1,2,1)
8320     Power_mat(1,1,2)=Power_mat(2,2,1)/Det
8330     Power_mat(2,2,2)=Power_mat(1,1,1)/Det
8340     Power_mat(1,2,2)=-Power_mat(1,2,1)/Det
8350     Power_mat(2,1,2)=-Power_mat(2,1,1)/Det
8360     PRINT "Enter the differences in the Mean components:"
8370     PRINT
```

```
8380      PRINT "Mean difference along the X axis: ";
8390      INPUT "X Difference:",Delta(1)
8400      PRINT Delta(1)
8410      PRINT "Mean difference along the Y axis: ";
8420      INPUT "Y Difference:",Delta(2)
8430      PRINT Delta(2)
8440      ! Calculate parameters
8450      Lambda=0
8460      FOR I=1 TO 2
8470        FOR J=1 TO 2
8480          Lambda=Lambda+Power_mat(I,J,2)*Delta(I)*Delta(J)
8490        NEXT J
8500      NEXT I
8510      PRINT "Lambda = ";Lambda
8520      PRINT
8530      ! Get Linear Interpolation Result
8540      GOSUB Linear_interp
8550      PRINT "Linear Power: ";Li_power
8560      ! Get Lagrange Interpolation Result
8570      GOSUB Lagrange_interp
8580      PRINT "Lagrange Power: ";La_power
8590      Pauseabit
8600      IF Delta(1)<>0 OR Delta(2)<>0 THEN 8360
8610 RETURN
8620      !
8630      ! Linear Interpolation
8640      !
8650 Linear_interp:    !
8660      ! Find  Bounding Values
8670      Search=1
8680      IF Lambda>=39 THEN
8690        Li_power=1
8700        RETURN
8710      END IF
8720      WHILE Lambda>Table(Search,1)
8730        Search=Search+1
8740      END WHILE
8750      IF Lambda=Table(Search,1) THEN
8760        Li_power=Table(Search,2)
8770        RETURN
8780      END IF
8790      Lambda_h=Table(Search,1)
8800      Lambda_l=Table(Search-1,1)
8810      Power_h=Table(Search,2)
8820      Power_l=Table(Search-1,2)
8830      IF Power_h=Power_l THEN
8840        Li_power=Power_h
8850        RETURN
8860      END IF
8870      Power1=Power_h*(Lambda-Lambda_l)/(Lambda_h-Lambda_l)
8880      Power2=Power_l*(Lambda_h-Lambda)/(Lambda_h-Lambda_l)
8890      Li_power=Power1+Power2
8900 RETURN
8910      ! Lagrange Interpolation
8920 Lagrange_interp:    !
8930      Search=1
8940      IF Lambda>=39 THEN
8950        La_power=1
8960        RETURN
8970      END IF
8980      WHILE Lambda>Table(Search,1)
8990        Search=Search+1
9000      END WHILE
9010      IF Lambda=Table(Search,1) THEN
9020        La_power=Table(Search,2)
9030        RETURN
```

```
9040      END IF
9050      Summer=0
9060      FOR I=Search-3 TO Search+2
9070        Prod=Table(I,2)
9080        FOR J=Search-3 TO Search+2
9090          IF J<>I THEN
9100            Prod=Prod*(Lambda-Table(J,1))/(Table(I,1)-Table(J,1))
9110          END IF
9120        NEXT J
9130        Summer=Summer+Prod
9140      NEXT I
9150      La_power=Summer
9160 RETURN
9170     !Chi-square(2) Power Table.  1st column is non-centrality parameter,
9180     !2nd is power.  0.05 significance label.  From Selected Tables in
9190     !Mathematical Statistics, Volume 1.
9200 Power_data:   !
9210     DATA .0,.05
9220     DATA .1,.0576
9230     DATA .2,.0653
9240     DATA .3,.0733
9250     DATA .4,.0814
9260     DATA .5,.0896
9270     DATA .6,.0980
9280     DATA .7,.1065
9290     DATA .8,.1151
9300     DATA .9,.1239
9310     DATA 1,.1327
9320     DATA 1.2,.1507
9330     DATA 1.4,.1691
9340     DATA 1.6,.1877
9350     DATA 1.8,.2065
9360     DATA 2.0,.2255
9370     DATA 2.2,.2447
9380     DATA 2.4,.2639
9390     DATA 2.6,.2831
9400     DATA 2.8,.3024
9410     DATA 3.0,.3215
9420     DATA 3.5,.3690
9430     DATA 4,.4154
9440     DATA 4.5,.4604
9450     DATA 5,.5037
9460     DATA 6,.5840
9470     DATA 7,.6554
9480     DATA 8,.7176
9490     DATA 9,.7707
9500     DATA 10,.8154
9510     DATA 11 .8526
9520     DATA 12,.8832
9530     DATA 13,.9080
9540     DATA 14,.9280
9550     DATA 15,.9440
9560     DATA 16,.9567
9570     DATA 17,.9667
9580     DATA 18,.9745
9590     DATA 19,.9805
9600     DATA 20,.9852
9610     DATA 21,.9888
9620     DATA 22,.9916
9630     DATA 23,.9937
9640     DATA 24,.9953
9650     DATA 25,.9965
9660     DATA 26,.9974
9670     DATA 27,.9981
9680     DATA 28,.9986
9690     DATA 29,.9989
```

```
9700    DATA 30,.9992
9710    DATA 31,.9994
9720    DATA 32,.9996
9730    DATA 33,.9997
9740    DATA 34,.9998
9750    DATA 35,.9998
9760    DATA 36,.9999
9770    DATA 37,.9999
9780    DATA 38,.9999
9790    DATA 39,1.0000
9800    DATA 40,1.0000
9810    DATA 41,1.0000
9820    DATA 42,1.0000
9830 End_sub2:   !
9840    SUBEND
```

C. ELLIPSTUFF: Ellipse Routine Library

Ellipstuff is an "ellipse routine library," which contains a large number of routines to make working with ellipses easier. As such, both Genellipse and Ellsim contain all of these routines, but they are also listed separately. If more programs need to be written, these are the routines to build them with.

```
10    ! TEST PROGRAM FOR ELLIPSE ROUTINE LIBRARY
11    ! LIBRARY SUBROUTINES BEGIN ON LINE 1000.
12    ! THIS TEST PROGRAM GETS AN ELLIPSE FROM THE USER AND DISPLAYS IT ON
13    ! THE SCREEN.
20      GCLEAR
30      GRAPHICS ON
31      REAL Prob_constant
32      Prob_constant=2*LOG(.05)
40      Get_ell_mean(1)
50    ! Get_covariance(1)
60      Get_axes(1)
70      Invert(1,21)
80      Get_bounds(1,Prob_constant)
81      Choose_plotter
90      Draw_ellipse(1,-50,50,-50,50,Prob_constant)
100     STOP
110     END
1000  ! ELLIPSE ROUTINE LIBRARY....
1010  ! This file contains standard ellipse routines and data variables,
1020  ! including the following:
1030  !     -- Entry of ellipses by covariance matrices.
1040  !     -- Entry of ellipses by axes and orientation.
1050  !     -- Display of ellipses on screen and plotter.
1060  !     -- 2x2 Matrix inversion routine for use with Matrx.
1070  !     -- Matrx, an array which stores covariance matrices and their
1080  !        inverses (up to 10 matrices).
1090  !     -- Axes/orientation to Covariance matrix conversion routine.
1100  !
1110  ! USING MATRX
1120  !  Matrx is designed to hold covariance matrices and their inverses
1130  !  for 10 ellipses.  In general, Matrx(1)...Matrx(20) are the covariance
1140  !  matrices and Matrx(21)...Matrx(40) are the corresponding inverses.
1150  !  This is the convention assumed by a number of these routines.
1160  !  In cases where the inverse is calculate first, put in Matrx(25), say,
1170  !  and then call Invert(25, 5).  This will put the covariance matrix in
1180  !  Matrx(5).
1190  !
1200  ! THE ROUTINES ARE CALLED AS FOLLOWS:
1210  !  Invert(Srce,Dest)
1220  !     This command will invert the covariance matrix in Matrx(Srce) and
1230  !     put the result in Matrx(Dest)
1240  !  Get_Bounds(Ellipse,Prob_constant)
1250  !     This command will get the extreme points of the ellipse.  Note that
1260  !     the ellipse's covariance matrix must have been inverted.
1270  !  Draw_Ellipse(Ellipse,Xmin,Xmax,Ymin,Ymax,Prob_constant)
1280  !     This will draw the given ellipse.  The remaining variables define
1290  !     the screen.  Note that Get_bounds must have been executed.
1300  !  Get_Ell_Mean(Ellipse)
1310  !     This is an input routine to read in Xbar and Ybar for the given
1320  !     ellipse.
1330  !  Get_Covariance(Ellipse)
1340  !     This is an input routine to read in the covariance matrix for the
1350  !     given ellipse.
1360  !  Get_Axes(Ellipse)
1370  !     This is an input routine which reads in the axes and orientation
1380  !     of the ellipse, and converts this to covariance matrix form.
1390  !  Choose_Plotter
1400  !     This prompts the user to draw the ellipse on the plotter or the
1410  !     screen.
1420  !  Display_Cov(Ellipse)
1430  !     This command displays the given covariance matrix.
1431  !  Test(Ellipse1,Ellipse2,Work,Test_Stat)
1432  !     This calculates the acceptance test criteria for the two ellipses.
1433  !     Work is a Matrx entry used as working space.
1434  !  Add_Covariance(Ellipse1,Ellipse2,Summer)
```

```
1435  !     This adds any two Matrx entries into a third Matrx entry.
1436  !  Combine_Ellipse(Ellipse1,Ellipse2,Combo)
1437  !     This combines any two Matrx entires into a third Matrx entry.
1438  !  Disp_Extremes(Ellipse)
1439  !     This displays the extreme X and Y bounds for the ellipse.
1441  !  Also included are Pauseabit and Clearscreen.
1450  !
1460  !=================================================================!
1470   SUB Invert(Srce,Dest)
1480      ! This routine inverts any covariance matrix in Matrx and places
1490      ! the inverted matrix in Dest.
1500      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
1510      Det=Matrx(1,1,Srce)*Matrx(2,2,Srce)-Matrx(1,2,Srce)*Matrx(1,2,Srce)
1520      Matrx(1,1,Dest)=Matrx(2,2,Srce)/Det
1530      Matrx(2,2,Dest)=Matrx(1,1,Srce)/Det
1540      Matrx(1,2,Dest)=-Matrx(1,2,Srce)/Det
1550      Matrx(2,1,Dest)=-Matrx(2,1,Srce)/Det
1560   SUBEND ! End of SUB Invert
1600   SUB Get_bounds(Ellipse,Prob_constant)
1610      !  This subroutine calculates the X and Y limits for the given ellipse
.
1620      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
1630      REAL Temp1,Temp2,Temp3,Temp4,Temp5
1650      Temp1=Matrx(2,2,Ellipse+20)*Prob_constant
1660      Temp2=Matrx(1,2,Ellipse+20)*Matrx(1,2,Ellipse+20)-Matrx(1,1,Ellipse+20
)*Matrx(2,2,Ellipse+20)
1670      Temp3=(Temp1/Temp2)^(.5)
1680      Temp4=Matrx(1,1,Ellipse+20)*Prob_constant
1690      Temp5=(Temp4/Temp2)^(.5)
1700      Emin_y(Ellipse)=-Temp5+Ybar(Ellipse)
1710      Emax_y(Ellipse)=Temp5+Ybar(Ellipse)
1720      Emin_x(Ellipse)=-Temp3+Xbar(Ellipse)
1730      Emax_x(Ellipse)=Temp3+Xbar(Ellipse)
1740   SUBEND ! End of GET_BOUNDS
1780   SUB Choose_plotter
1790      ! Subroutine to choose the desired plotter device (also contained
1791      ! in GRAPHER).
1800      REPEAT
1810         Go_on=-1
1820         DISP "Do you want to (D)raw, (E)rase, Plot with Pen (0), (1), (2)?"
;
1830         LINPUT Message$
1840         SELECT Message$
1850         CASE "D","d"
1860            PLOTTER IS 3,"INTERNAL"
1870            GRAPHICS ON
1880            PEN 1
1890            Go_on=1
1891         CASE "0"
1892            PLOTTER IS 705,"HPGL"
1893            GRAPHICS ON
1894            PEN 0
1895            PLOTTER IS 3,"INTERNAL"
1896            GRAPHICS ON
1898            PEN 1
1899            Go_on=1
1900         CASE "1"
1910            PLOTTER IS 705,"HPGL"
1920            GRAPHICS ON
1930            PEN 1
1940            Go_on=1
1941         CASE "2"
1942            PLOTTER IS 705,"HPGL"
1943            GRAPHICS ON
```

```
1940            PEN 2
1945            Go_on=1
1950         CASE "E","e"
1960            PLOTTER IS 3,"INTERNAL"
1970            GRAPHICS ON
1980            PEN -1
1990            Go_on=1
2000         CASE ELSE
2010            PRINT CHR$(7)
2020         END SELECT
2030      UNTIL Go_on=1
2040   SUBEND
2080   SUB Get_ell_mean(Ellipse)
2090      ! This subroutines prompts the user for the mean of an ellipse.
2100      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
2110      Clearscreen
2120      PRINT TABXY(1,10);"WHAT IS X-BAR  for ELLIPSE ";Ellipse;"?";
2130      INPUT Xbar(Ellipse)
2140      PRINT Xbar(Ellipse)
2150      PRINT TABXY(1,11);"WHAT IS Y-BAR  for ELLIPSE ";Ellipse;"?";
2160      INPUT Ybar(Ellipse)
2170      PRINT Ybar(Ellipse)
2180      Pausebit
2190   SUBEND
2230   SUB Get_covariance(Ellipse)
2240      !  This routine gets the covariance matrix for an ellipse
2250      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
2260      Clearscreen
2270      FOR K=1 TO 2
2280         PRINT TABXY(1,3+K+6);"ENTER ELEMENT (";K;",";K;") IN THE COVARIANCE
 MATRIX FOR ELLIPSE#";Ellipse;": ";
2290         INPUT Matrx(K,K,Ellipse)
2300         PRINT Matrx(K,K,Ellipse)
2310      NEXT K
2320      PRINT TABXY(1,3+9);"ENTER ELEMENT (1,2) IN THE COVARIANCE MATRIX FOR E
LEMENT#";I;": ";
2330      INPUT Matrx(1,2,Ellipse)
2340      Matrx(2,1,Ellipse)=Matrx(1,2,Ellipse)
2350      PRINT Matrx(1,2,Ellipse)
2360      Pausebit
2370   SUBEND
2410   SUB Pausebit
2420      ! Pause and wait for a carriage return
2430      DISP "Type ENTER to continue...";
2440      INPUT Garbage$
2450   SUBEND
2460   SUB Clearscreen
2470      ! Clear the screen
2480      PRINT CHR$(12)
2490   SUBEND
2530   SUB Get_axes(Ellipse)
2540      ! This subroutine gets an ellipse in terms of the axes and the
2550      ! angle of orientation.  These are converted into a covariance
2560      ! matrix.
2570      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
2580      Prob_constant=2*LOG(.05)
2590      Clearscreen
2600      PRINT "WHAT IS THE RADIUS OF THE MAJOR AXIS OF ELLIPSE ";Ellipse;" : "
;
2610      INPUT Major_axis(Ellipse)
2620      PRINT Major_axis(Ellipse)
2630      PRINT "WHAT IS THE RADIUS OF THE MINOR AXIS OF ELLIPSE ";Ellipse;" : "
;
```

```
2640      INPUT Minor_axis(Ellipse)
2650      PRINT Minor_axis(Ellipse)
2660      PRINT "WHAT IS THE ANGLE (IN DEGREES) OF THE MAJOR AXIS OF ELLIPSE ";E
llipse;" : ";
2670      INPUT Angle(Ellipse)
2680      PRINT Angle(Ellipse)
2690      ! Convert to Covariance Matrix
2700      Theta(Ellipse)=PI*Angle(Ellipse)/180
2710      Matrx(1,1,Ellipse)=(Major_axis(Ellipse)*COS(Theta(Ellipse)))^2+(Minor_
axis(Ellipse)*SIN(Theta(Ellipse)))^2
2720      Matrx(2,2,Ellipse)=(Major_axis(Ellipse)*SIN(Theta(Ellipse)))^2+(Minor_
axis(Ellipse)*COS(Theta(Ellipse)))^2
2730      Matrx(2,1,Ellipse)=((Major_axis(Ellipse))^2-(Minor_axis(Ellipse))^2)*C
OS(Theta(Ellipse))*SIN(Theta(Ellipse))
2740      Matrx(1,2,Ellipse)=Matrx(2,1,Ellipse)
2750      FOR J=1 TO 2
2760         FOR K=1 TO 2
2770            Matrx(J,K,Ellipse)=Matrx(J,K,Ellipse)/(-Prob_constant)
2780         NEXT K
2790      NEXT J
2800      PRINT
2810      Pausesbit
2820   SUBEND
2860   SUB Display_cov(Ellipse)
2870      ! Displays the covariance matrix for an ellipse
2880      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
2890      FOR I=1 TO 2
2900         FOR J=1 TO 2
2910            PRINT Matrx(I,J,Ellipse);"   ";
2920         NEXT J
2930         PRINT
2940      NEXT I
2950   SUBEND
2990   SUB Add_covariance(First,Second,Summer)
3000      ! This subroutine can be used to add Summer=First+Second
3010      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3020      INTEGER I,J
3030      FOR I=1 TO 2
3040         FOR J=1 TO 2
3050            Matrx(I,J,Summer)=Matrx(I,J,First)+Matrx(I,J,Second)
3060         NEXT J
3070      NEXT I
3080   SUBEND
3120   SUB Combine_ellipse(First,Second,Combo)
3130      ! This routine finds the "JPL" combination of the First and Second
3140      ! ellipses.
3150      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3160      REAL Sx1,Sx2
3170      ! Calculate the new covariance matrix.
3180      Invert(First,First+20)
3190      Invert(Second,Second+20)
3200      Add_covariance(First+20,Second+20,Combo+20)
3210      Invert(Combo+20,Combo)
3220      ! Calculate the new mean point.
3230      Sx1=Matrx(1,1,First+20)*Xbar(First)+Matrx(1,2,First+20)*Ybar(First)
3240      Sx1=Sx1+Matrx(1,1,Second+20)*Xbar(Second)+Matrx(1,2,Second+20)*Ybar(Se
cond)
3250      Sx2=Matrx(2,1,First+20)*Xbar(First)+Matrx(2,2,First+20)*Ybar(First)
3260      Sx2=Sx2+Matrx(2,1,Second+20)*Xbar(Second)+Matrx(2,2,Second+20)*Ybar(Se
cond)
3270      Xbar(Combo)=Matrx(1,1,Combo)*Sx1+Matrx(1,2,Combo)*Sx2
3280      Ybar(Combo)=Matrx(2,1,Combo)*Sx1+Matrx(2,2,Combo)*Sx2
3290   SUBEND
```

```
3330   SUB Disp_extremes(Ellipse)
3340      ! This routine displays the extreme x and y values for the specified
3350      ! ellipse
3360      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3370      PRINT "Ell ";Ellipse;":  ";
3380      PRINT "Xmin ";Emin_x(Ellipse);
3390      PRINT ";  Xmax ";Emax_x(Ellipse);
3400      PRINT ";  Ymin ";Emin_y(Ellipse);
3410      PRINT ";  Ymax ";Emax_y(Ellipse)
3420   SUBEND
3430   SUB Draw_ellipse(Ellipse,Xmin,Xmax,Ymin,Ymax,Prob_constant)
3440      ! This routine draws one (1) ellipse on the current plotter device.
3450      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3460      INTEGER Sign
3470      REAL Xpoint,Upoint,Ypoint,Temp1,Temp2,Temp3,Temp4,Temp5
3480      Get_bounds(Ellipse)
3490      Invert(Ellipse,Ellipse+20)
3500      GRAPHICS ON
3510      ! Draw top half of the ellipse
3520      Sign=1
3530      GOSUB Draw_half
3540      ! Draw bottom half of the ellipse
3550      Sign=-1
3560      GOSUB Draw_half
3570      ! Finish up
3580      MOVE (Xbar(Ellipse)-Xmin)*100/(Xmax-Xmin),(Ybar(Ellipse)-Ymin)*100/(Ym
ax-Ymin)
3590      DRAW (Xbar(Ellipse)-Xmin)*100/(Xmax-Xmin),(Ybar(Ellipse)-Ymin)*100/(Ym
ax-Ymin)
3600      ! Okey, dokey, we're done here.
3610      SUBEXIT
3620 Draw_half:    ! Draw half of the ellipse
3630      !  If Sign=1, then draw top half;  if Sign=-1, then bottom half.
3640      FOR Xpoint=Emin_x(Ellipse) TO Emax_x(Ellipse) STEP .1
3650         GOSUB Draw_1_point
3660      NEXT Xpoint
3670      Xpoint=Emax_x(Ellipse)
3680      GOSUB Draw_1_point
3690      RETURN
3700      !
3710      !  Compute each point and draw the new line.  (It's here since we call
3720      !  it twice
3730      !
3740 Draw_1_point:  !
3750      Upoint=Xpoint-Xbar(Ellipse)
3760      Temp1=Matrx(1,2,Ellipse+20)*Upoint
3770      Temp2=Temp1*Temp1-Matrx(2,2,Ellipse+20)*(Matrx(1,1,Ellipse+20)*Upoint*
Upoint+Prob_constant)
3780      IF Temp2<10^(-10) THEN Temp2=0
3790      Ypoint=(-Temp1+Sign*SQR(Temp2))/Matrx(2,2,Ellipse+20)+Ybar(Ellipse)
3800      IF Xpoint<=Emin_x(Ellipse)+.001 THEN
3810         MOVE (Xpoint-Xmin)*100/(Xmax-Xmin),(Ypoint-Ymin)*100/(Ymax-Ymin)
3820      ELSE
3830         DRAW (Xpoint-Xmin)*100/(Xmax-Xmin),(Ypoint-Ymin)*100/(Ymax-Ymin)
3840      END IF
3850      RETURN
3860   SUBEND
3900   SUB Test(First,Second,Work,Test_val)
3910      ! This routines calculates the acceptance test criteria for First and
3920      ! Second.  Work is used as a "scratchpad".
3930      COM /Ellipses/ Matrx(2,2,40),Xbar(20),Ybar(20),Major_axis(20),Minor_ax
is(20),Theta(20),Angle(20),Emin_x(20),Emax_x(20),Emin_y(20),Emax_y(20)
3940      Add_covariance(First,Second,Work)
3950      Invert(Work,Work+20)
```

```
3960      Diffx=Xbar(First)-Xbar(Second)
3970      Diffy=Ybar(First)-Ybar(Second)
3980      Temp1=Matrx(1,1,Work+20)*Diffx+Matrx(1,2,Work+20)*Diffy
3990      Temp2=Matrx(2,1,Work+20)*Diffx+Matrx(2,2,Work+20)*Diffy
4000      Test_val=Diffx*Temp1+Diffy*Temp2
4010    SUBEND
```