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NUMERICAL SOLUTIONS OF THREE-DIMENSIONAL NAVIER-STOKES EQUATIONS FOR CLOSED-BLUFF BODIES

By

Jamshid S. Abolhassani

and

Surendra N. Tiwari, Principal Investigator

Progress Report
For the period ended October 31, 1986

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665

Under
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Dr. Robert E. Smith, Jr.
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(NASA-CR-182359) NUMERICAL SOLUTIONS OF THREE-DIMENSIONAL NAVIER-STOKES EQUATIONS FOR CLOSED-BLUFF BODIES Progress Report, period ending 31 Oct. 1986 (Old Dominion Univ.) 31 p CSCL 20D G3/34

N88-16990

Unclas
0124721

November 1986

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Norfolk, Virginia 23508

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FOREWORD

This is a progress report on the research project "Numerical Solutions of Three-dimensional Navier-Stokes Equations for Closed-Bluff Bodies." The period of performance on this research was from January 1 through August 31, 1986. The project is supported by the NASA/Langley Research Center, Hampton, Virginia, and monitored by Dr. Robert E. Smith, Jr., of the Analysis and Computations Division (Computer Applications Branch), NASA/Langley Research Center.

NUMERICAL SOLUTIONS OF THREE-DIMENSIONAL NAVIER STOKES EQUATIONS FOR CLOSED-BLUFF BODIES

Jamshid S. Abolhassani[1]

Surendra N. Tiwari[2]

ABSTRACT

Methods of grid adaption are reviewed and a method is developed with the capability of adaption to several flow variables. This method is based on a variational approach and is an algebraic method which does not require the solution of partial differential equations. Also the method has been formulated in a such way that there is no need for any matrix inversion. The method is used in conjunction with the calculation of hypersonic flow over a blunt nose. The equations of motion are the compressible Navier-Stokes equations where all viscous terms are retained. They are solved by the MacCormack time-splitting method and a movie has been produced which shows simultaneously the transient behavior of the solution and the grid adaption. The results are compared with the experimental and other numerical results.

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INTRODUCTION

In hypersonic flows about bluff bodies, the temperature, pressure and density increase almost explosively across the shock wave and the curved shock wave is closer to the body. Numerical simulations of this phenomena is a great challenge to the computational fluid dynamics researchers. Presently, there is a great deal of interest in improving the quality of numerical simulation techniques and adaptive gridding is one way to achieve this goal.

Grid generation is the very first step in the numerical solutions of partial differential equations for complex geometries. Basically, grid generation is the numerical generation of boundary-fitted curvilinear coordinates. The second step is the construction of difference equations for partial differential equations. It is apparent that the accuracy of the finite difference solution depends on the fineness of the mesh. Therefore, the finer the grid, the more accurate the numerical solution will be. Also, the accuracy of the solution depends on the resolution of the solution gradient. Presence of large gradients causes the error to be large in the difference approximation of the derivatives. In the presence of shockwave, more artificial diffusion must be added to retain adequate smoothness of the solution. Therefore, there is a need for schemes that can resolve these large gradients without adding any additional grid points. An adaptive scheme moves the grid points to regions of high gradients when locations of these gradients are not known as a priori. Also, an adaptive method reduces the total number of grid points required to achieve a given accuracy, but it takes more computer time. In some instances, the computer time requirement makes this method undesirable. The ideas used in the construction of adaptive grid techniques are limited by one's imaginations; any scheme that works in the sense of providing a better solution is a good one. The ultimate answer to

numerical solutions of partial differential equations may well be dynamically adaptive grids rather than more elaborate difference representations and solution methods [1].

Adaptive methods have been used in the solutions of ordinary differential equations. Variable-step initial-value problems are solved by adjusting the step size as the integration advances in order to control the local truncation error [2]. Adaptive methods have been implemented also for solving the equations of motion in conjunction with the method of lines [3]. In this case, the time step was adjusted automatically to control the local error. Similarly, adaptive methods have been to solve boundary value problems [4-9]. An optimal grid for a two-point boundary-value problem can be determined either implicitly or explicitly. In the implicit approach, the weight function depends upon the solution. As a result, the original boundary-value problem is converted into an augmented system in which the dependent variables and the mesh are computed simultaneously. In the explicit approach, the weight function does not depend on the solution. Instead, it depends upon a previously calculated solution. Even for a linear boundary-value problems, the implicit approach requires that one solves a nonlinear two-point boundary-value problem. The implicit techniques do not preserve the linear-nonlinear character of the original problem. Moreover, even for the nonlinear problem, the augmented system is usually more difficult to solve than the original problem. On the other hand, the explicit technique preserves the linear-nonlinear character of the original two-point boundary-value problem.

Adaptive schemes are divided into two basic categories: differential and algebraic. The differential method is based on the variational approach. Brackbill and Saltzman [10-13] have developed a technique for constructing adaptive grids using a variational approach. In their scheme, a function

which contains a measure of grid smoothness, orthogonality and volume variation is minimized by using the variational principal. The smoothest grid can be generated by solutions of Laplace equations which are better known as elliptic systems [14, 15]. This approach ignores the effects of orthogonality and requires too much CPU time. The method has been modified for better efficiency by dropping the second derivative terms in one coordinate direction [16]. This makes the equations parabolic and, therefore, they can be solved by marching techniques. A method which considers the orthogonality and volume variation is developed by Steger and Sorenson [17]. This method is widely known as the hyperbolic method; it can be solved by any non-iterative marching techniques. The variational approach provides a solid mathematical basis for the adaptive method, but the Euler-Lagrange equations must be solved in addition to the original governing differential equations. On the other hand, the algebraic method requires much less computational effort, but the grid may not be smooth.

Rai and Anderson [18-23] have developed an algebraic technique where the grid movement is governed by estimates of the local error in the numerical solution. This is achieved by requiring the points in the large error regions to attract other points and points in the low error region to repel other points. Nakahashi and Deiwert [24] have formulated an algebraic method which is based on the variational principal. A spring analogy is used to redistribute the grid points in an optimal sense to reduce the over all solution error. In this case, operator splitting and one sided controls for the orthogonality and smoothness are used to make the method practical, robust and efficient. Dwyer [25-28] has used an adaptive method in which the points are moved along one set of the original coordinate lines in response to the evolving gradients in the physical solution. The analysis showed that the

percentage change in a dependent variable can be determined as a priori. An order of magnitude improvement in speed was obtained, but some problems with excessive skewness were encountered.

Generally, dynamic adaption can be performed in two ways, one is to keep the computational space fixed and include the grid speed in the flow field equations. This is an ideal method to use for unsteady flows. The second way is to set the grid speed equal to zero and interpolate the solution onto the new grid after each adaption. This is equivalent to solving a sequence of boundary-value problems, which is an economical way to treat steady flows whose solutions are approached asymptotically. It is generally sufficient to adapt just a few times during the course of the computation. In this approach, the grid distribution at time N+1 is determined from time N. Dwyer adapted the grids after each integration step or after a selected number of steps [25]. However, the grid speed can be obtained by postulating a law which is based on some solution properties. Then, these equations can be integrated with the governing partial differential equations to yield a new grid distribution [29]. The advantage of this technique is that the grid location and grid speed are time accurate.

The literature survey indicates most techniques adapt to just one variable. This means that the weight function is based on the solution of one variable only. However, the solution of equations of motion produce several dependent variables. Viscous-hypersonic flow over a blunt-body have large gradient in pressure, velocity, etc in different part of the flow field. There is a large gradient in pressure near the shock region; at the same time, there is a large gradient in velocity near the solid body. Therefore, there is a need for the development of efficient grid adaption method which utilizes several variables simuntanously.

METHODS OF GRID ADAPTION

One reason to use grid adaption is to minimize the error over some domain by rearranging the grid. Calculus of variation can be used to perform this minimization. In general, a weighted integral which is a measure of some grid or solution property over some domain can be expressed as

$$I = \int_{\forall} W \, d\forall, \tag{1}$$

where W is the weight function and it is minimized. The selection of W may vary from problem to problem. There is a collection of definitions for W in Ref. 30. The weight function can be based on grid properties such as cell volume, the average of the square of diagonal lengths, the cell area/volume ratio and/or cell skewness. There exists a differential equation which minimizes the integral I in Eq. (1). This differential equation is called Euler-Lagrange differential equation [31] and it constitutes the grid generation system. The Euler-Lagrange equations can be found in Refs. 10-13.

Multidimensional Adaption

Brackbill and Saltzman [10-13] have developed a technique based on a variational approach. In their scheme, a function which contains measures of grid smoothness, orthogonality, and volume variation is minimized. To maximize the smoothness of the grid, the following integral must be minimized

$$I_s = \int_{\forall} \sum_{i=1}^{3} \nabla \xi^i \cdot \nabla \xi^i \, d\forall. \tag{2}$$

This is simply the sum of the squares of cell-edge lengths. Similarly, the orthogonality can be acquired by minimizing the integral I_o given by

$$I_o = \int_{\forall} g^{3/2} (\nabla \xi^i \cdot \nabla \xi^i) \, d\forall. \qquad (i,j,k) \text{ cyclic} \tag{3}$$

This integral vanishes for an orthogonal grid. The inclusion of the Jacobin of the transformation in the weight function is somewhat arbitrary and causes orthogonality to be better in the regions with the larger cell. The concentration or cell volume variation can be obtained by minimizing the integral

$$I_W = \int_{\forall} W\ J\ d\forall, \tag{4}$$

where W is a specified weight function. This causes the cells to be small where the weight function is large. The grid generation system which provides smoothness, orthogonality and concentration is obtained by minimizing the total integral I which is a linear combinations of I_S ,I_O and I_W

$$I = I_S + \lambda_O\ I_O + \lambda_W\ I_W. \tag{5}$$

The competing features such as smoothness, orthogonality, and cell volume variation can be stressed by proper choice of the coefficients λ_O and λ_W. For example, a large λ_O will result in a nearly orthogonal grid at the cost of the smoothness and concentration. The Euler-Lagrange equations for the sums of those individual integral form the system of partial differential equations from which the coordinate system is generated. These equations are quasi-linear, second-order partial differential equations with the coefficients which are quadratic functions of the first derivatives [12]. This variational formulation is equivalent to Winslow's method [14] where λ_O and λ_W are set equal to zeros. The Euler equations are those given by Winslow, and their solution maximizes the smoothness. This is also used by Thompson et al. [15]. The additional terms alter other characteristics of the mapping in a similar way. The cell-size variation and skewness can be controlled by proper selection of I_S, I_O and I_W. The use of a variational approach provides a solid mathematical basis for the adaptive gridding. But, the Euler-Lagrange

equations must be solved in addition to the governing equations of fluid motion.

One Dimensional Adaption

The Euler-Lagrange equation for Eq. (5) is a general and is capable of adapting grids in multidimensions simultaneously. When the solution varies predominately in a single direction, one-dimensional adaption can be applied with the grid points constrained to move along one family of fixed curvilinear coordinate lines. The fixed family of lines is established by generating a full multidimensional grid using the grid generating techniques. The points generated for these initial grids, together with some interpolation procedure, e.g., cubic or linear interpolation, serve to define the fixed lines along which the points will move during the adaption. This is done explicitly; therefore there is no need to solve any differential equations.

A technique called equidistribution is developed to improve the solutions of boundary value problems [4-9]. This has proven to be effective and efficient. This technique is used to minimize the error by redistributing the grid such that a weight function is constant over each interval. The Euler-Lagrange equation is

$$X_\xi W = \text{constant.} \tag{6}$$

This minimizes the following integral

$$I_1 = \int_0^1 W(\xi)\, X_\xi^2 \, d\xi \, . \tag{7}$$

This equation represents the energy of a system of springs with the spring constant $W(\xi)$, spanning each grid interval. The weight function is associated with the grid points themselves, not with their locations. An

alternative viewpoint results by integrating over x instead of ξ, i.e. summing over the grid intervals rather than over the grid points. This can be expressed as

$$I_2 = \int_0^1 \frac{\xi_x^2}{W(x)} dx \ . \tag{8}$$

The Euler-Lagrange equation for this is given by Eq. (6). The quantity ξ_x is considered to represent the point density. This variational problem represents a minimization over the density of the grid points subjected to a weight function. This can produce smooth point distribution. Here the weight function W(x) is associated with the location of points. If the weight function is associated with the points themselves rather than their locations, then $W = W(\xi)$. The integral for which Eq. (6) is the Euler-Lagrange equation for the following integral

$$I_3 = \int_0^1 \left[\frac{\xi_x}{W(\xi)}\right]^2 dx. \tag{9}$$

ξ_x is a measure of the smoothness of the point distribution, with the emphasis placed on the smoothness in the certain regions. This is inversely proportional to the weight function $W(\xi)$. Equation (6) is the Euler-Lagrange equation for the integrals in Eqs. (7-9) which can be written as

$$\xi(x) = \int_0^X W(\xi)\, dx. \tag{10}$$

Equation (10) can be written in terms of the arc length as

$$\xi(s) = \int_0^s W(\xi)\, dx. \tag{11}$$

The weight function is used to reduce the point spacing where W is large. The weight function should be set to some measure of the error. White [5] has suggested the following form of the weight function

$$W = [\alpha + |U^{(n)}|^2]^{1/2n} , \tag{12}$$

where α is constant. With n=1 and α=0, this becomes

$$W = |U_x| . \tag{13}$$

A combination of Eqs. (6) and (13) yields

$$U_\xi = \text{Constant.} \tag{14}$$

This choice replaces the points so that the same change in the solution occurs at each grid interval. This is simply the solution gradient. Taking n=1 and α=1 yields

$$W = \sqrt{1+ |U_x|^2} . \tag{15a}$$

A combination of Eqs. (6) (15a) results in

$$\sqrt{X_\xi^2 + U_\xi^2} = S_\xi = \text{Constant.} \tag{15b}$$

This produces a uniform distribution of arc lengths on the solution curve. White's results [5] indicated that the arc length form was favored. The disadvantages of this method is that the weight function near the solution extreme, i.e. U_x=0 locally, are treated as a flat region. Concentration near the solution extreme can be achieved by incorporating some effect of the second derivative (U_{xx}) into the weight function as [25]

$$W = 1 + \alpha\ f(U_x)\ \beta\ g(U_{xx}), \tag{16a}$$

where α and β are positive parameters. Therefore, Eq. (10) can be written as;

$$\xi(x) = \frac{\int_0^x W(x)\,dx}{\int_0^1 W(x)\,dx} . \tag{16b}$$

With the second derivative terms included, the value of α, β must be continually updated to keep the same relational emphasis or concentration according to this form. Therefore, a system of 2x2 should be solved for each fixed grid line [27-28]. It will be shown later that through the reformulation of Eq. (16b), the matrix inversion can be entirely avoided.

One-Dimensional Adaption With Several Variables

The solution of the flow equations consists of several variables. Therefore, the weight function should be a function of more than one variable. It is desirable to devise a scheme in which grids can adapt to several variables with control on the magnitude of adaption for each variable. In the case of high speed flow, velocity has large gradients in some regions whereas pressure is constant or vice versa. In general the weight function can be expressed as

$$W = 1 + \sum_{i=1}^{N} b_i f_i \tag{17}$$

where N is the number of variables, b_i is constant, f_i is some variable or its derivative, and 1 is for uniformity. A substitution of Eq. (17) into Eq. (11) results in

$$\xi(S) = \frac{S + \sum_{i=1}^{N} b_i F_i (S)}{S_{max} + \sum_{i=1}^{N} b_i F_i (S_{max})} , \tag{18}$$

where

$$F_i\ (S) = \int_0^S f_i\ (t)\ dt\ .$$

It should be noted that Eq. (18) is for adapting along a fixed grid line. The quantities b_i and f_i should be positive to ensure the monotoncity of $\xi(s)$. In order to keep the same relative emphasis on the concentration along each grid line, b_i should be computed based on some percentage of the grid being allocated to each variable. The percentage of grid points assigned to a particular function f_i can be expressed as

$$R_j = \frac{b_j\ F_j(S_{max})}{S_{max} + \sum_{i=1}^{N} b_i\ F_i\ (S_{max})}\ . \qquad j=1,2,\ldots,N. \tag{19}$$

A rearrangement of Eq. (19) results in

$$[A_{ji}]\ \{b_j\} = \{c_j\}\ , \tag{20}$$

where

$$A_{ji} = R_j\ \frac{F_i(S_{max})}{F_j(S_{max})} \qquad i \neq j$$

$$= R_j - 1 \qquad i = j,$$

$$C_j = -\ R_j\ S_{max}/F_j\ (S_{max})\ .$$

Therefore, along each fixed grid line a matrix of NxN (N is number of variables) should be inverted. This can be avoided by the reformulation of Eq. (18). The crucial steps are outlined here. Equation (18) can be rewritten as

$$\xi(S) = \frac{S}{S_{max} + \sum_{i=1}^{N} b_i F_i(S_{max})} + \sum_{i=1}^{N} \frac{b_i F_i(S)}{S_{max} + \sum_{j=1}^{N} b_j F_j(S_{max})} . \tag{21}$$

Similarly, Eq. (19) can be rewritten as

$$b_j = \frac{R_j}{F_j(S_{max})} \left[S_{max} + \sum_{i=1}^{N} b_i F_i(S_{max})\right] . \tag{22}$$

A substitution of Eq. (22) into (21) results in

$$\xi(S) = \frac{S + \left[S_{max} + \sum_{j=1}^{N} b_i F_j(S_{max})\right] \sum_{i=1}^{N} \frac{R_i}{F_i(S_{max})} F_i(S)}{S_{max} + \left[S_{max} + \sum_{i=1}^{N} b_i F_i(S_{max})\right] \sum_{j=1}^{N} R_j} \tag{23}$$

A summation of Eq. (19) over all j yields

$$\sum_{j=1}^{N} R_j = \frac{\sum_{j=1}^{N} b_j F_j(S_{max})}{S_{max} + \sum_{i=1}^{N} b_i F_i(S_{max})} . \tag{24}$$

Rearrangement of Eq. (24) results in

$$\sum_{i=1}^{N} b_i F_i(S_{max}) = \frac{S_{max} \sum_{i=1}^{N} R_i}{1 - \sum_{i=1}^{N} R_i} . \tag{25}$$

A substitution of Eq. (25) into Eq. (23) yields

$$\xi(S) = \frac{S}{S_{max}} \left[1 - \sum_{i=1}^{N} R_i\right] + \sum_{i=1}^{N} R_i \frac{F_i(S)}{F_i(S_{max})} . \tag{26}$$

This reformulation avoids the need for continuous updating the b_i's to keep the same relative emphasis on concentration. Applying this equation, grids can be adapted with more than one variable without any need for matrix inversions.

GOVERNING EQUATIONS OF MOTION

The governing equations for a thermal fluid system are the conservation of mass, momentum, and energy. These equations are developed for an arbitrary region assuming the system is in continuum. Equations of motion for a viscous, compressible, unsteady, heat conducting fluid can be written as [32]

Continuity: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0 ,$$

Momentum: $$\frac{\partial \rho \bar{u}}{\partial t} + \nabla \cdot (\rho \bar{u} \bar{u} - \bar{\bar{\tau}}) = 0 , \tag{27}$$

Energy: $$\frac{\partial E}{\partial t} + \nabla \cdot (E\bar{u} + \bar{q} - \bar{u} \bar{\bar{\tau}}) = 0 .$$

where E is the total energy per unit volume given by $(E=e+\bar{u}\cdot\bar{u}/2)$ and e is the internal energy per unit volume. The fluid is assumed to be Newtonian and the bulk viscosity is neglected. The viscosity is computed from the Sutherland law of viscosity. For simplicity, equations of motion can be written into a compact vector form as

$$\frac{\partial U}{\partial t} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = 0 , \tag{28}$$

where

$$U = \begin{Bmatrix} \rho \\ \rho v \\ \rho w \\ E \end{Bmatrix} , \quad G = \begin{Bmatrix} \rho v \\ \rho v v - \tau_{yy} + P \\ \rho v w - \tau_{yz} \\ E v = \dot{q}_y - \phi_y + P v \end{Bmatrix} , \quad H = \begin{Bmatrix} \rho w \\ \rho v w - \tau_{zy} \\ \rho w w - \tau_{zz} + P \\ E w + \dot{q}_z - \phi_z + P w \end{Bmatrix} .$$

For the sake of generality, governing equations are transformed from a physical domain into a computational domain. Resulting equations are:

$$\frac{\partial U}{\partial t} + \begin{Bmatrix} \eta_y \\ \eta_z \end{Bmatrix} \left(\frac{\partial G}{\partial \eta}, \frac{\partial H}{\partial \eta}\right) + \begin{Bmatrix} \zeta_y \\ \zeta_z \end{Bmatrix} \left(\frac{\partial G}{\partial \zeta}, \frac{\partial H}{\partial \zeta}\right) = 0. \tag{29}$$

The transformation coefficient can be computed from a functional relation between the computational coordinates and the physical coordinates.

METHOD OF SOLUTION

A time marching method is used to compute the solution. This allows us to capture the possible transient features. The method employs an explicit second-order accurate time-split predictor-corrector algorithm [33]. In a compact form, it can be expressed as

$$U_{j,k}^{n+1} = \left[L_\eta\ (\Delta t_\eta)\right]\ \left[L_\zeta\ (\Delta t_\zeta)\right]\ \left[L_\eta\ (\Delta t_\eta)\right], \tag{30}$$

where

$$\Delta\ t_\eta = \frac{1}{2}\ \Delta\ t_\zeta.$$

The method has a time-step stability limit, but there is no rigorous nonlinear stability analysis available for it. However, there is a conservative time-step which is based on a linearized form of the equations. It can be expressed as

$$\Delta t < \text{Min}\ \left[\frac{|V|}{\Delta\eta} + \frac{|W|}{\Delta\zeta} + c\ \left[\frac{1}{\Delta\eta^2} + \frac{1}{\Delta\zeta^2}\right]\right]^{1/2} \tag{31}$$

where c is the local speed of sound.

In the hypersonic region, there exits a large gradient which requires a

very fine mesh to resolve it. Most center difference methods admit a solution which has sawtooth or plus-minus waves with the shortest wave length that the mesh can support. In the of case nonlinear problems, these short waves interact, vanish, and reappear again as distorted long wave or oscillation. These oscillation eventually blow up the solution, if they are not resolved. The oscillations of "low frequency" can be suppressed by adding a fourth order damping. A common damping used is the pressure dampening. This can be expressed in computational coordinates as

$$- \alpha_\ell \, \Delta t_\ell \, \delta_\ell^3 \frac{\partial}{\partial \delta} \left[\frac{|V_\ell| + c}{4P} \cdot \frac{\partial^2 P}{\partial^2 \delta_\ell} \frac{\partial U}{\partial \delta_\ell} \right] \tag{32}$$

where V_ℓ is contravariant velocity.

RESULTS AND DISCUSSIONS

The main objective of this study is to investigate finite difference methods in which the mesh network adapts to the solution dynamically to obtain an accurate solution for hypersonic flow. A computer program has been written to utilize Eq. (26) for grid adaption. Presently, this code is being run on the Network Operating System (NOS) and the Vector Processing System (VPS) at NASA Langley Research Center. Hypersonic flow over a blunt nose is a typical test case in computational fluid dynamics. This problem inherits a detached shock which should be accurately resolved. The location and the magnitude of the shock are not known as a priori; therefore the grid should be adapted as the solution progresses. This problem is used to analyze and verify the adaptive method. Equations of motion are solved by the MacCormack method [33] for a hypersonic flow over a small-radius blunt-body with an inclined-plate after the body (Fig. 1). The blunt leading edge is a part of the panel holder which was tested at Langley Research Center [34]. Results are obtained at

following conditions: M_∞=6.8, Re_∞=220,000, P_∞=9.26 lb/ft^2, U_∞=6510 ft/sec, γ=1.38, R=1771 ft^2/sec^2/°R and wall temperature of 540°.

Two tests have been performed: static adaption and dynamic adaption. For static adaption, the solution has been obtained by fixed grids shown in Fig. 2. The grids are adapted to two variables, the first and the second derivatives of pressure, and the results are shown in Fig. 3. In this case, twenty percent of the grids are allocated to first and second derivatives of pressure (R1=R2=20%). For the same case, Fig. 4 shows adaption with R1=R2=50%. It is noted that in Fig. 4 entire grids are allocated to the first and the second derivatives of the pressure. This is explains the large voids in the constant pressure regions. Figures 3 and 4 lack grid resolution in the vicinity of the solid boundaries. This is due to the constant pressure near the solid boundaries. But Eq. (26) can adapt to several variables. Figure 5 shows the grids which are adapted to two variables, pressure and velocity. The weight function consists of the first derivatives of pressure and velocity and the second derivative of pressure. Twenty percent of grids are allocated to each function and forty percent of grids are for the uniformity. This avoids the creation of any void. It is noted that grids are clustered near the shock and the solid body. Therefore, it is possible to resolve the pressure as well as the velocity gradients in the boundary layer region. In chemically reacting flows, some of the grids can be allocated for resolving the gradients of the chemical species.

The procedure has been applied dynamically to the same problem. Figure 6a shows the initial grid distribution for this problem. Figures 6 and 7 show sequences of grid distributions at different times. In this case, grids are adapted to six variables; these are the first and second derivatives of pressure, temperature and velocity. Ten percent of grids are allocated to

first derivatives equally and five percent to their second derivatives. Fifty-five percent of grids are also allocated to the uniformity. A movie has been produced of this work which shows the dynamic adaption; a few frames are shown in Figs. 7a-7b. It demonstrates how grids are attracted toward high gradient region and repel from low gradient regions. The full paper will cover formulations, boundary conditions, grid generation and the vectorizing of the proposed scheme in more details. Also, the timing of the scheme will be reported. Currently, this work is being extended to the blunt nose with the after body.

REFERENCES

1. Thompson, J. F.," A Survey of Dynamically-Adaptive Grids in the Numerical Solutions of Partial Differential Equations," AIAA Paper 84-1606, AIAA 17th Fluid Plasma Dynamics and Laser conference, Snowmass, Colorado, June 25-27, 1984.

2. Gear, C. W., Numerical Initial Value Problems in Ordinary Differential Equations, Prentice-Hall, Englewood Cliffs, New Jersey, 1971.

3. Abolhassani, J. S., "Applications of the Method of Lines for Solutions of Navier-Stokes Equations Using a Nonuniform Grid Distribution," Master Thesis, School of Engineering, Old Dominion University, Norfolk, Virginia 23508, May 1983.

4. White, A. B. Jr., "On the Numerical Solution of Initial/Boundary Value Problems in One Space Dimension," SIAM Journal of Numerical Analysis, Vol. 19, 1982, pp. 683-697.

5. White, A. B., Jr., "On Selection of Equidistribution Meshes for Two-Point Boundary-Value Problems," SIAM Journal of Numerical Analysis, Vol. 16, No. 3, 1979, pp. 472-502.

6. Pereyra, V. and Sewell, E. G., "Mesh Selection for Discrete Solution of Boundary-Value Problems in Ordinary Differential Equations," Numerische Mathematik, Vol. 2, No. 3, 1975, pp. 261-268.

7. Russell, R. D. and Christiansen, J., "An Adaptive Mesh Selection Strategies for Solving Boundary Layer Problems," SIAM Journal of Numerical Analysis, Vol. 15, No. 1, February 1978, pp. 59-80.

8. De Rivas, E. K., "On the Use of Nonuniform Grids in Finite Difference Equations," Journal of Computational Physics, Vol. 10, 1972, pp. 202-210.

9. Denny, V. E. and Landis, R. B., "A New Method for Solving Two-Point Boundary Problems Using Optimal Node Distribution," Journal of Computational Physics, Vol. 9, 1972, pp. 120-137.

10. Brackbill, J. U., "Coordinate System Control: Adaptive Mesh," Numerical Grid Generation, Joe F. Thompson, Editor, Elsevier Science Publishing Company, 1982, pp. 277-294.

11. Brackbill, J. U. and Saltzman, J., "An Adaptive Computation Mesh for the Solution of Singular Perturbation Problems," Numerical Grid Generation Techniques, Robert E. Smith, Editor, NASA Conference Publication 2166, 1980, p. 193-196.

12. Brackbill, J. U. and Saltzman, J. S., "Adaptive Zoning for Singular Problems in Two Dimensions," Journal of Computational Physics, Vol. 46, No. 3, 1982, pp. 342-368.

13. Saltzman, J. U. and Brackbill, J. S., "Application and Generalization of Variational Methods for Generating Adaptive Meshes," Numerical Grid Generation, Edited By J.F. Thompson, Elsevier Science Publishing Company, 1985.

14. Winslow, A. M., "Numerical Solution of the Quasilinear Poisson Equation in a Nonuniform Triangle Mesh," Journal of Computational Physics, Vol. 1, No. 2, November 1966, pp. 149-172.

15. Thompson, J. F., Thames, F. C. and Mastin, C. W., "Automatic Numerical Generating of Body-Fitted Curvilinear System for Field-Containing any Number of Arbitrary Two-Dimensional Bodies," Journal of Computational Physics, Vol. 15, 1974, pp. 299-319.

16. Nakamura, S., "Marching Grid Generation Using Parabolic Partial Differential Equations," Numerical Grid Generation, J. F. Thompson, Editor, North-Holland, 1982, pp.775-786.

17. Steger, J. L. and Sorenson, R. L., "Use of Hyperbolic Partial Differential Equations to Generate Body Fitted Coordinates," Numerical Grid Generations Techniques, Robert E. Smith, Editor, NASA Conference Publication 2166, 1980, pp.463-478.

18. Rai, M. M. and Anderson, D. A., "Grid Evolution in Time Asymptotic Problems," Numerical Grid Generation Techniques, Robert E. Smith, Editor, NASA Conference Publication 2166, 1980, pp.409-430.

19. Rai, M. M. and Anderson, D. A., "The Use of Adaptive Grid in Conjunction with Shock Capturing Methods," AIAA paper 81-1012, St. Louis, Missouri, 1981.

20. Rai, M. M. and Anderson, D. A., "Application of Adaptive Grids to Fluid Flow Problems with Asymptotic Solutions, AIAA Journal, Vol. 20, 1982, pp 496-502.

21. Anderson, D. A. and Rai, M. M., "The Use of Solution Adaptive Grids in Solving Partial Differential Equations," Numerical Grid Generation, By J. F. Thompson, Editor, Elsevier Science Publishing Company, 1982, pp. 317-338.

22. Anderson, D. A. and Steinbrenner, J., "Generating Adaptive Grids with a Conventional Grid Scheme," AIAA paper 86-0427, Presented at the AIAA 24th Aerospace Science meeting, Reno, Nevada, January 6-9, 1986.

23. Anderson, D. A., "Adaptive Grid Method for Partial Differential Equations," ASME Applied Mechanics, Bioengineering and Fluids Engineering Conference, Houston, Texas, June 20-22, 1983.

24. Nakahashi, K. and Deiwert, G. S., "A Self Adaptive-Grid Method with Application to Airfoil Flow," AIAA seventh Computational Fluid Dynamic Conference, Cincinnati, Ohio, July 15-17, 1985; (AIAA paper 85-1525) July 1985.

25. Dwyer, H. A. , Kee, R. J. and Sanders, B. R., "An Adaptive Grid for Problem in Fluid Mechanics and Heat Transfer, "AIAA paper 79-1464, Williamsburg, Virginia, July 1979.

26. Dwyer, H. A., "A Discussion of Some Criteria for the Use of Adaptive Gridding," Adaptive Computational Method for Partial Differential Equations, Edited by Ivo Babuska, Jagdish Chandra, Joseph E. Flaherty, SIAM Publishing Company, 1983, also AIAA paper 83-1706, AIAA 16th Fluid and Plasma Dynamics Conference, Danvers, Mass., 1983.

27. Dwyer, H. D., Smooke, M. D. and Kee, R. J., "Adaptive Grinding for Finite Difference Solutions to Heat and Mass Transfer problems," Numerical Grid Generation, J. F. Thompson, Editor, Elsevier Science Publishing Company, 1982, pp. 339-356.

28. Dwyer, H. A., "Grid Adaption for Problems in Fluid Dynamics," AIAA Journal, Vol. 22, No. 12, December 1984, pp. 1705-1712.

29. Hindman, R. G. and Spencer, J., "A New Approach to Truly Adaptive Grid Generation," AIAA paper 83-0450, Reno, Nevada, January 1983.

30. Thompson, J. F., Warsi, Z. U. A. and Mastin, C.W., Numerical Grid Generation: Foundations and Applications, North-Holland Publishing, New York, 1985.

31. Weinstock, R., Calculus of Variations, McGraw-Hill Book Company, New York, 1952.

32. Anderson, D. A., Tannehill, J. C. and Pletcher, R. H., Computational Fluid Mechanics and Heat Transfer, McGraw-Hill Book Company, New York, 1984.

33. Smith, R. E., "Two-Boundary Grid Generation for the Solutions of the Three-Dimensional Compressible Navier-Stokes Equations," NASA Technical Memorandum 83123, May 1981.

34. Deveikis, W. D. and Hunt, L. R., "Loading and Heating of a Large Flat Plate at Mach 7 in the Langley 8-Foot High Temperature Structure Tunnel," NASA TM-D-7275, 1975.

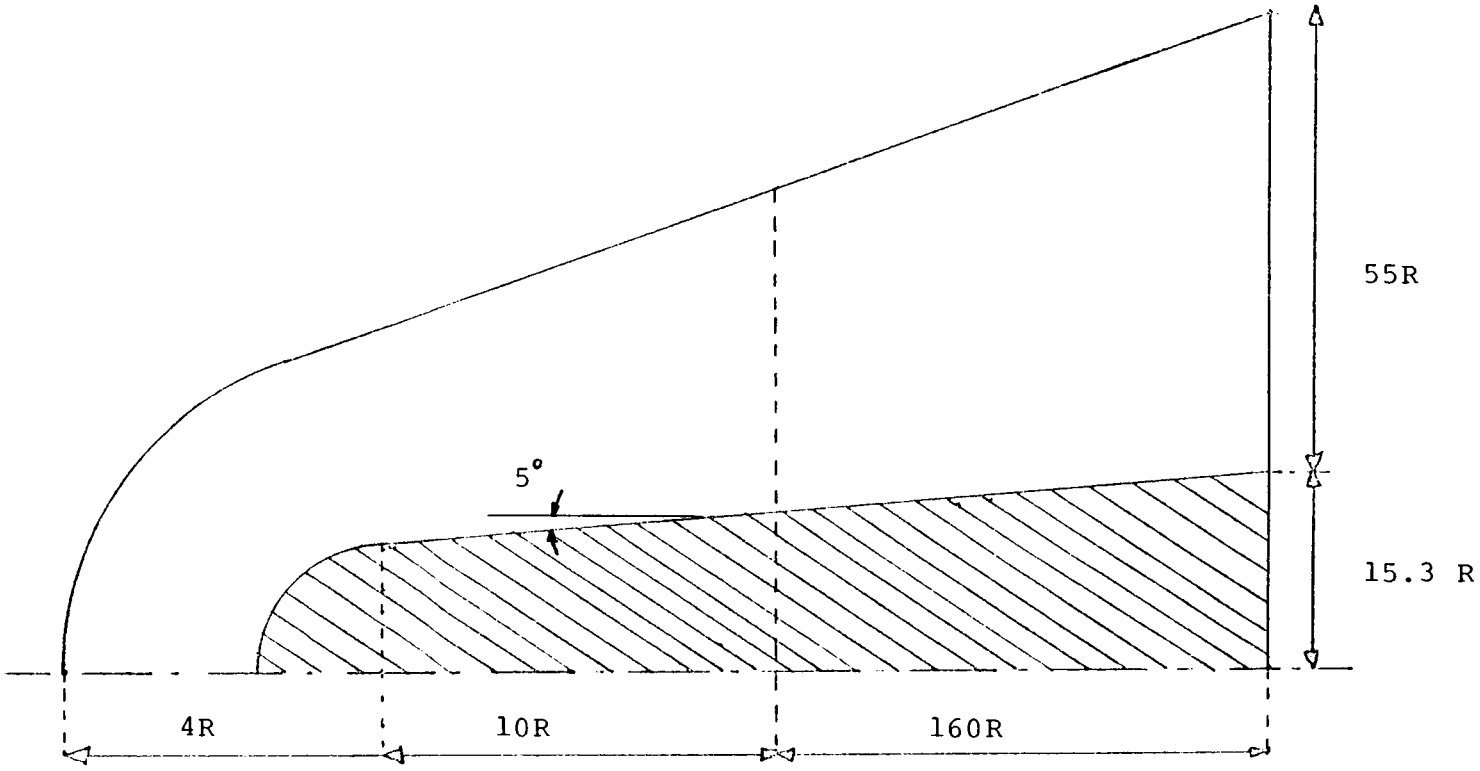

Fig. 1 Physical Geometry of the Blunt Leading-Edge of a Panel Holder (R=3/8")

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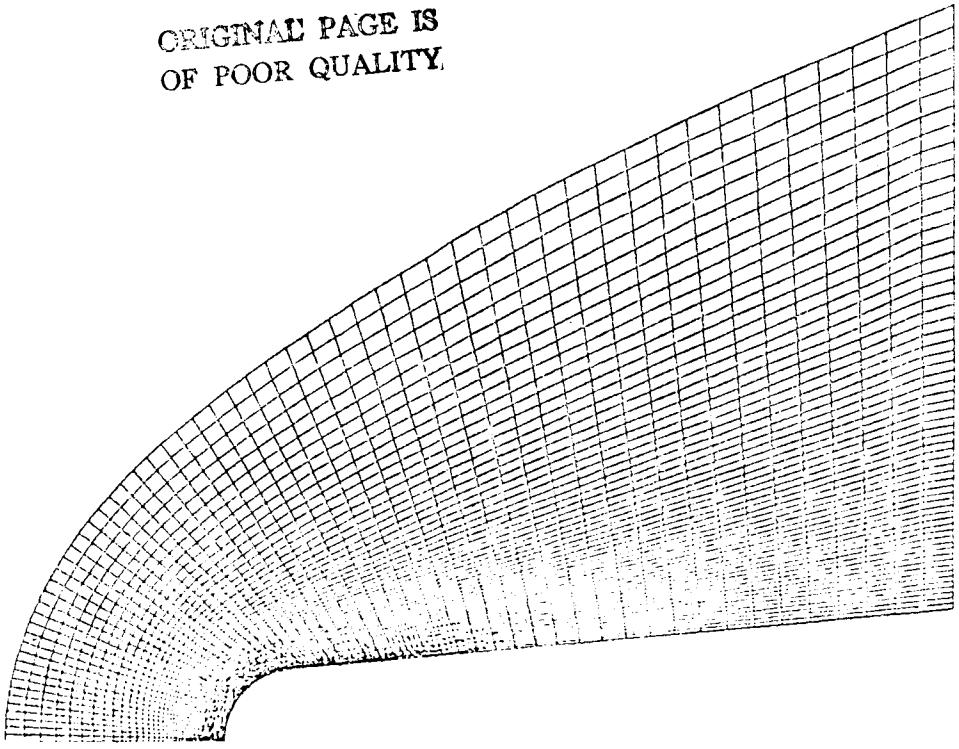

Fig. 2 Initial Grid Distribution

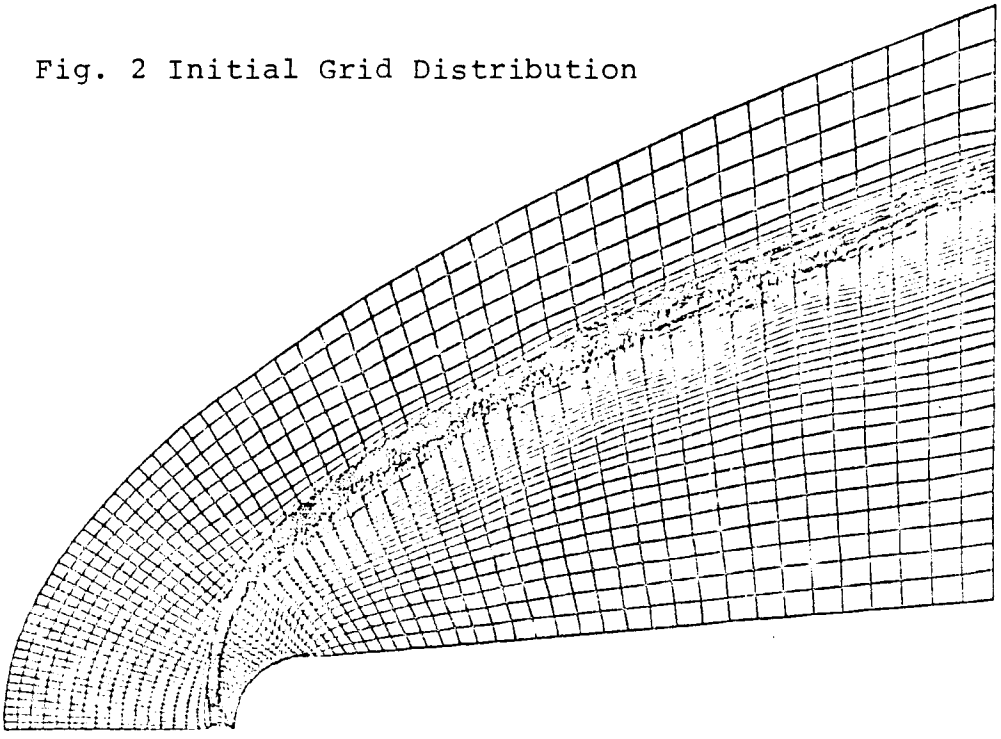

Fig. 3 Adapted Grid ($R_1 = R_2 = 20\%$)

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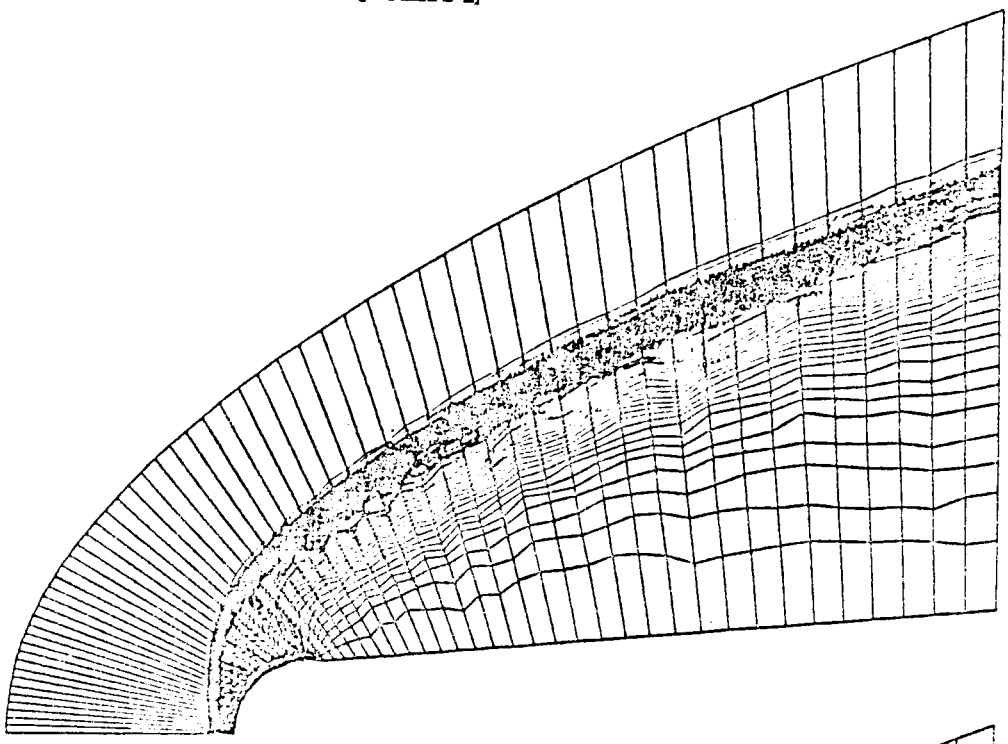

Fig. 4 Adapted Grid(R_1=R_2=50%)

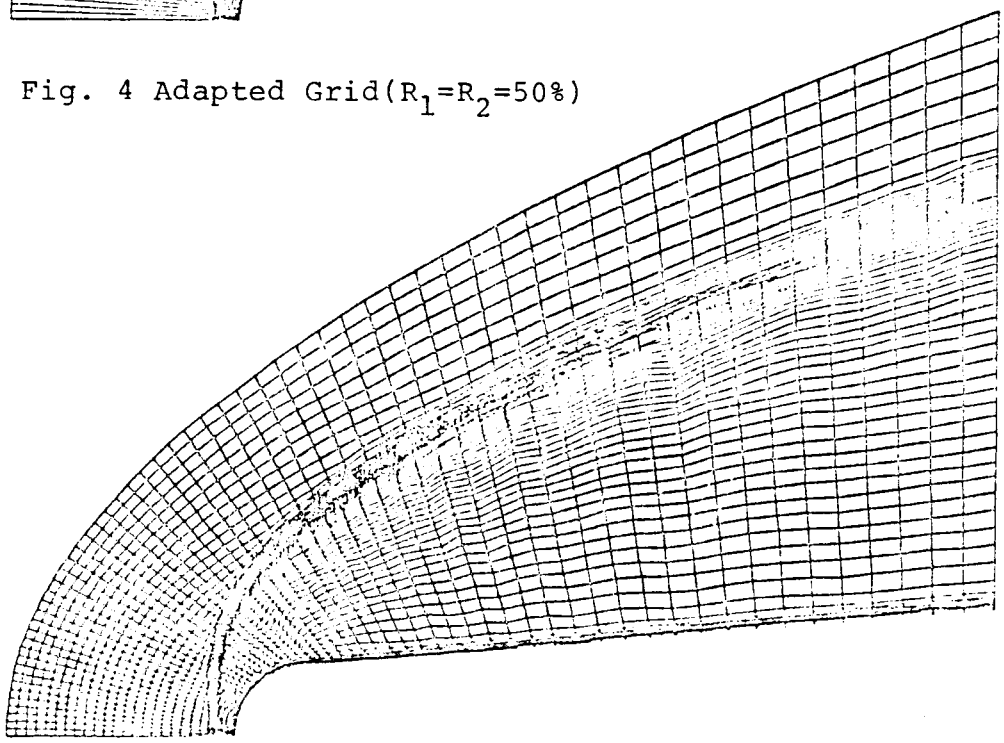

Fig. 5 Adapted Grid (R_1=R_2=R_3=20%)

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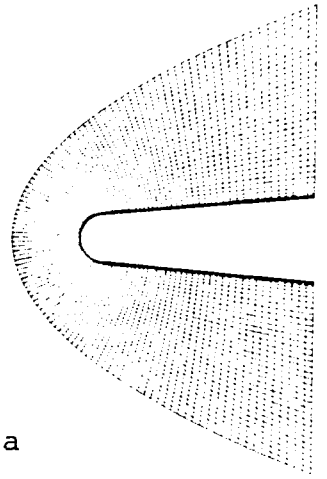

Fig. 6a

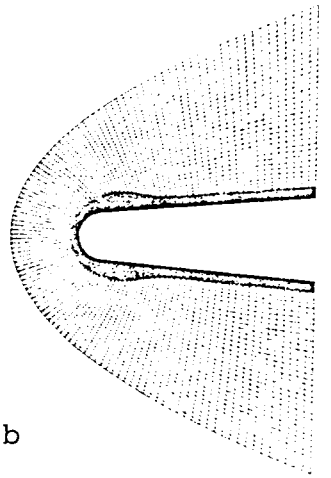

Fig. 6b

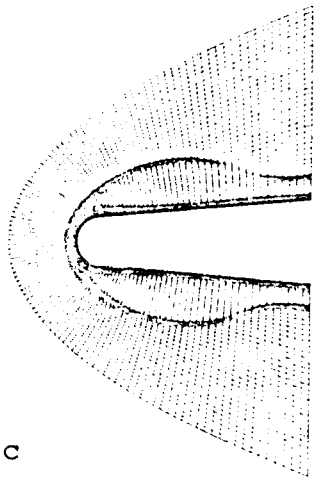

Fig. 6c

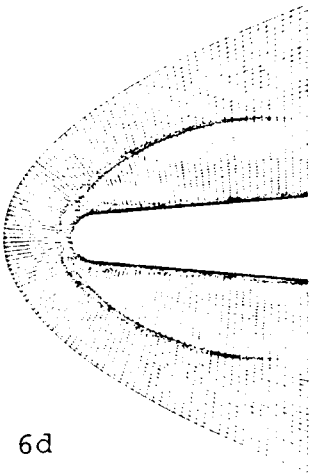

Fig. 6d

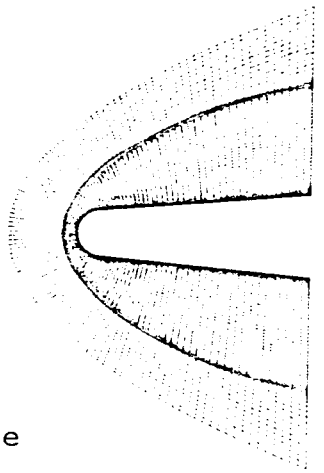

Fig. 6e

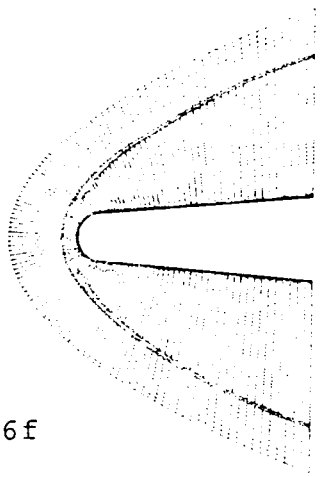

Fig. 6f

Fig. 6 Adapted Grid (Dynamic)

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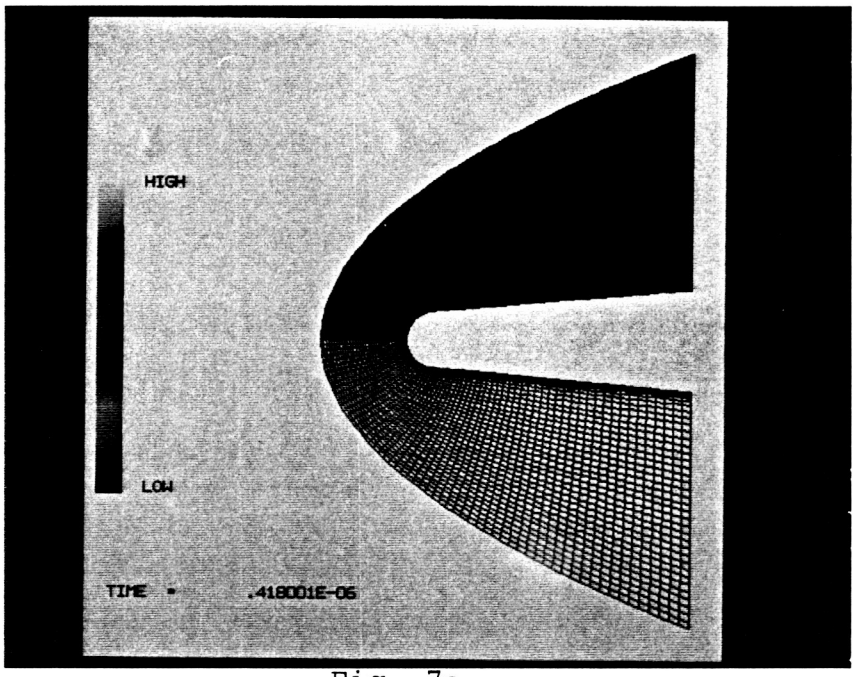

Fig. 7a

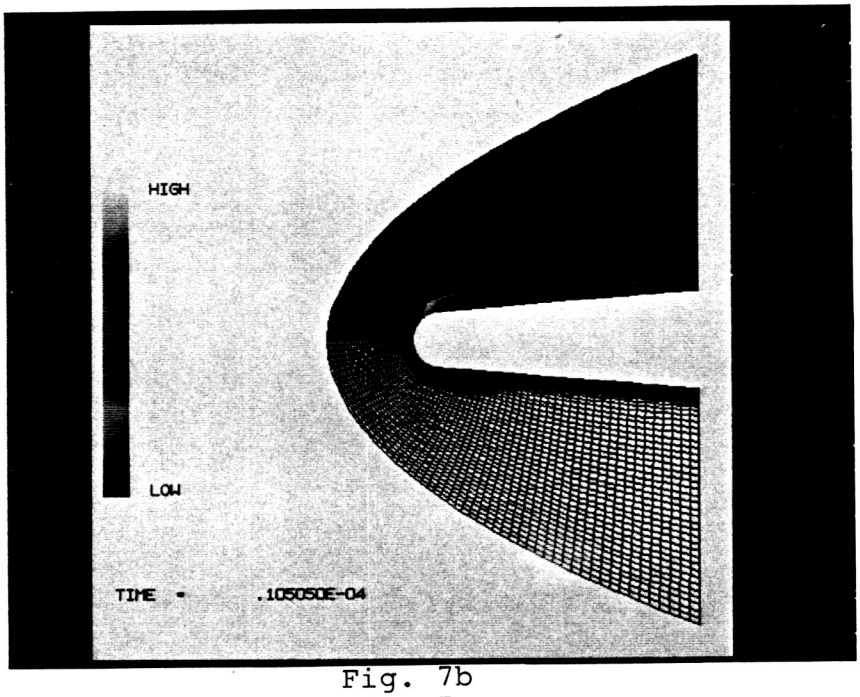

Fig. 7b

Fig. 7 Adapted Grid (dynamic)

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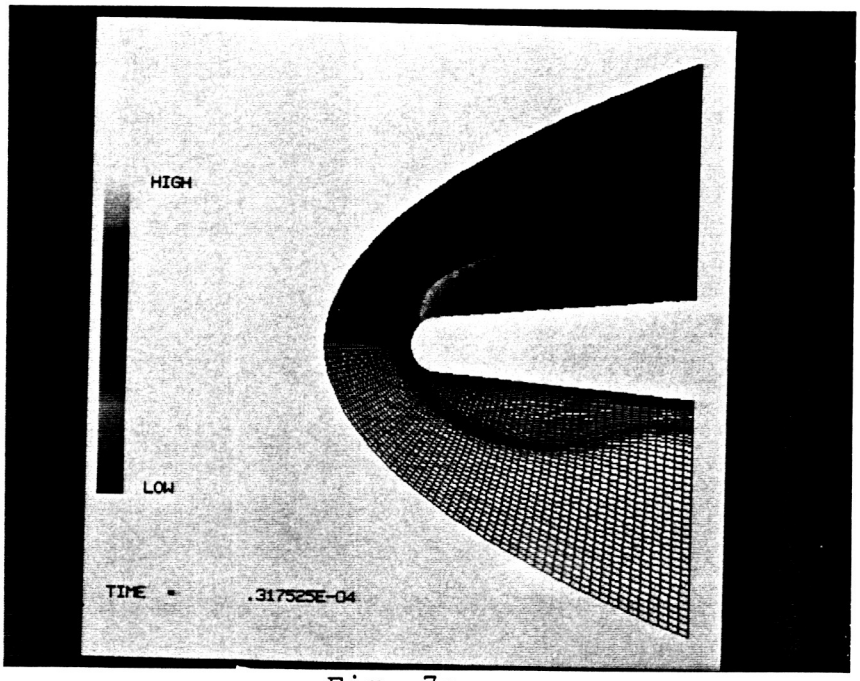

Fig. 7c

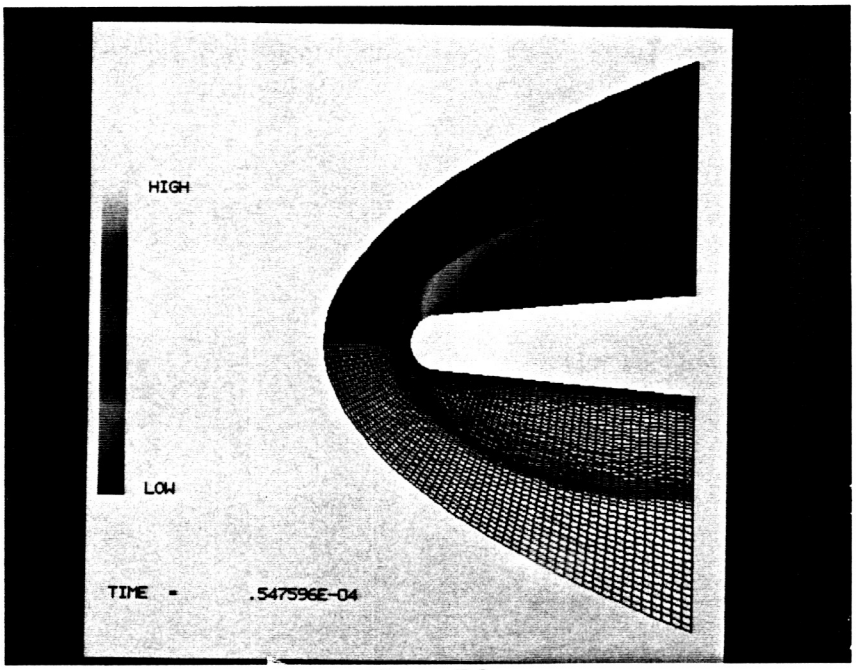

Fig. 7d

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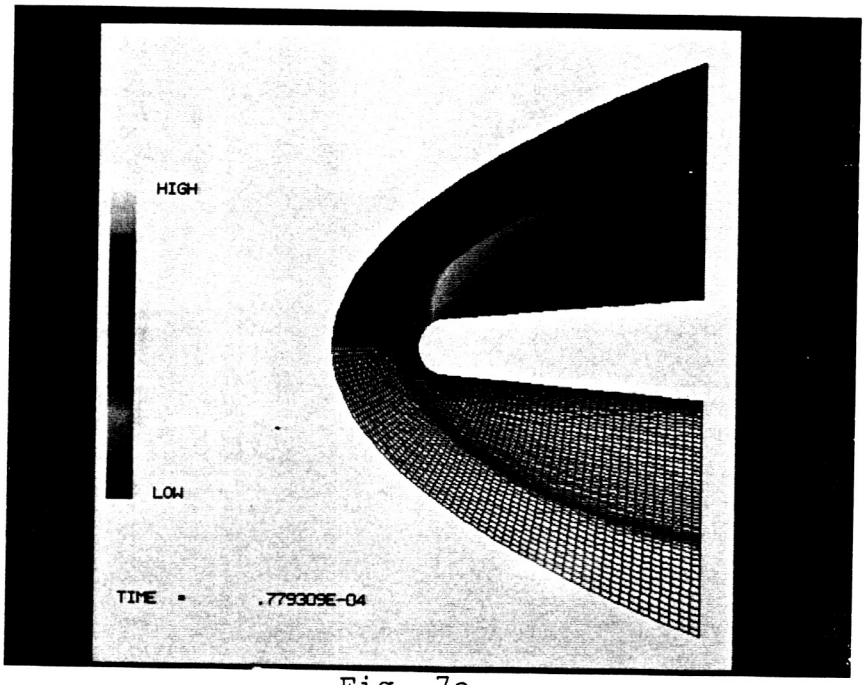

Fig. 7e

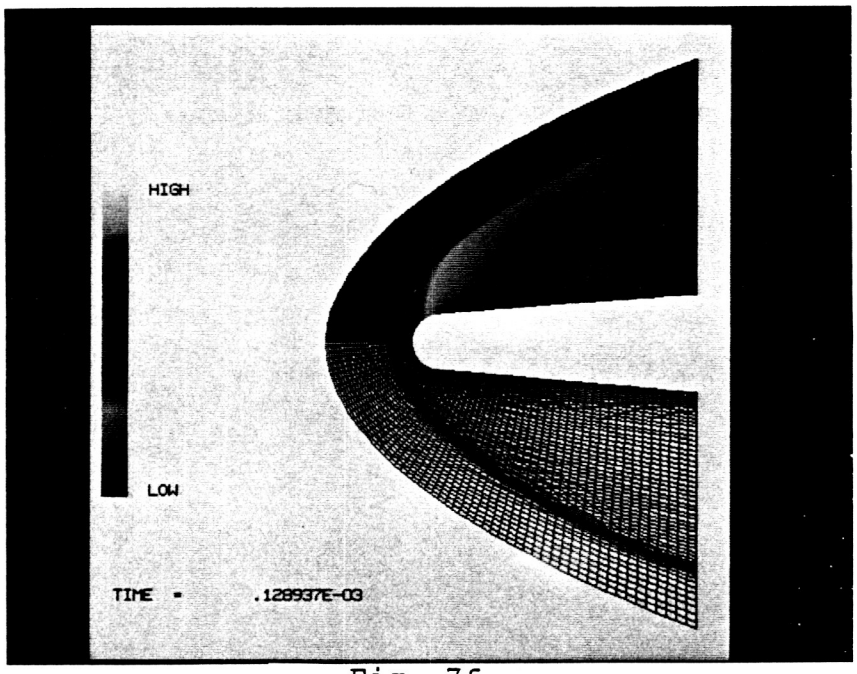

Fig. 7f