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U.S. Army Intelligence Center and School Software Analysis and Management System

The Need to Track Sample Size in Conjunction With Memo No. 72, Dated 04/01/86

Technical Memorandum No. 12

MARC

Mathematical Analysis Research Corporation

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02 April 1986

National Aeronautics and Space Administration

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This memo extends the results for Tech Memo # 11. By use of simulation, two EEP ellipses, one or both inappropriately (by use of the wrong statistic) normalized, are combined. The probability (obtained by simulating the ellipse combination 1000 times) of the resultant ellipse containing the target is then compared to the theoretical result.

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U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
Software Analysis and Management System

The Need To Track Sample Size In Conjuction With Memo No. 72, Dated 04/01/86

Technical Memorandum No. 12

02 April 1986

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PREFACE

The work described in this publication was performed by the Mathematical Analysis Research Corporation (MARC) under contract to the Jet Propulsion Laboratory, an operating division of the California Institute of Technology. This activity is sponsored by the Jet Propulsion Laboratory under contract NAS7-918, RE182, A187 with the National Aeronautics and Space Administration, for the United States Army Intelligence Center and School.

The Need To Track Sample Size (In Conjunction With Memo No. 72, Dated 04/01/86)

ATTACHMENTS: APPENDICES I, II, III.

The role of sample size (number of Lines Of Bearing) depends on the set of algorithms one uses. Some mixes of algorithms can produce undesirable results because of the way sample size is handled (or not handled.) The most obvious place to look for this source of problems is mismatching of the Chi-square and F distributions. In this memo we examine two issues:

(1) How do ellipses based on F-distributions combine with other ellipses? The theory for ellipse combination is based on the ellipses having known covariance and hence Chi-square cut-off values.

(2) What is the impact of converting F confidence levels as if they were Chi-square?

The conclusion of this memo is that both of the sources of error work in the same direction, namely making the resultant error ellipse size too small to have 95% confidence of capturing the emitter. The smaller the sample sizes of the reported ellipses the greater the problem. Furthermore, corrections for these errors require knowledge of sample size.

The two issues given above are discussed in more detail in the sections that follow. Appendices with simulation results and showing size variation follow.

I. COMBINATION OF ELLIPSES WITH ESTIMATED COVARIANCE

MARC used simulation to study this problem. The simulation used and results found will be discussed at the end of this section. With hindsight it is possible to explain why the results turned out the way they do. We present this explanation in question-answer format before showing the simulation results:

(i) What is the most significant difference between the ellipse that combination theory calls for and the ellipse the F-test generates?

Answer: The theory assumes that location error is being accounted for. With small sample size, however, ellipse size estimates vary so much that ellipse size error dominates.

(ii) Isn't this difference also an issue before ellipse combination?

Answer: Yes. The method of handling it is to make the ellipse bigger. By making it enough bigger one can regain 95% confidence.

(iii) If bigger ellipses are being combined won't the result be bigger, enough bigger to still have 95% confidence?

Answer: The combined ellipse is certainly bigger than it would be if the source ellipses were not based on the F. Our simulations suggest that this is not quite big enough. The problem is that we are exposed to a 5% risk of having any individual incoming ellipse be too small. If two ellipses are being combined then this a total of two 5% risks or almost a 10% risk.

(iv) Does having one of the source ellipses too small always cause a problem?

Answer: No, not always. Simulation yielded about 92% correct or 8% in error in our example. The reason it is a problem most of the time is probably that when an incoming ellipse is smaller than it should be it is weighted heavily in determination of resultant location and error ellipse size. The 'bad apple' becomes the driver in the problem.

(v) How do you know that the difference between 8% error observed and 5% error expected doesn't reflect some other problem such as bias?

Answer: Two methods were used to guarantee that the correct problem was being isolated. First a 'controlled experiment' or simulation was run with the Chi-square. Second and more importantly, two simulations were run for the same sensor-emitter setup with only sensor accuracy changed. It is easily shown that bias and other effects are a second order effects. To study behavior uninfluenced by them it is sufficient to make sensor accuracy good enough. To guarantee this MARC first performed the simulation with a one degree standard deviation. Then to prove this was small enough a second simulation was performed with a .1 degree standard deviation.

(vi) How do you know that you weren't using cases that the acceptance test would have rejected?

Answer: This would still be a problem although admittedly a different one. MARC intends to investigate this further. Our belief is, however, that study will show that this type of rejection will only occur when either

a) two ellipses are too small

or b) one is too small and the another is naturally small.

The simulations primarily reflect the case of one ellipse too small and the other ellipse being reasonable sized as this is more likely in our problem.

A description of the individual simulations follows.

1000 runs each.
Emitter Location: (20,20)
Ellipse 1 is based on 5 LOBs from (0,0),(5,0),(10,0),(15,0),(20,0).
Ellipse 2 is based on 5 LOBs from (10,0),(15,0),(20,0),(25,0),(30,0).
(Note although the locations used overlap different LOBs were generated.)
Fix Method: (Weighted) Perpendicular with an estimated variance on angular measurements using the F-distribution.
95% Cut-off Level for the individual ellipses (based on the F)=19.1
Percent of combined ellipses capturing the emitter using a 19.1 cutoff

i) 91.9% when the standard deviation = 1 degree
ii) 91.9% when the standard deviation = .1 degree

(note that this agreement is a coincidence)
Cutoff that yields a 95% chance of capturing the emitter

i) 27.9 when the standard deviation = 1 degree
ii) 29.8 when the standard deviation = .1 degree

Note that the square root of 27.9/19.1 and 29.8/19.1 yield the percentage of further growth in the ellipse size (radii length) to have a 95% chance of capture. Computing these equal 1.209 and 1.249 respectively.

A number of graphed examples of the ellipses corresponding to a sequence of the 1 degree runs are included in the appendix. A listing of the cutoff scores is also included. (These are the numbers that should be compared with 19.1 for the F cutoff or 5.99 for the Chi-square.)

II. IMPROPER CONVERSION

Suppose that a fix source is reporting to a fusion center. Further suppose that the source is reporting an ellipse based on

i) 5 LOBs

ii) a 50% F cut-off value appropriate for sample size 5

The fusion center would report an ellipse that is 1.59 times smaller on each axis than it should be.

The method of arriving at this is to note that the Chi-square conversion to a 95% ellipse assumes that the cutoff level must go from 1.39 to 5.99. In fact it must go from 1.76 to 19.1. Thus the Chi-square conversion process scales up ellipse size by the square root of 5.99/1.39 or 2.075 when it should scale up by the square root of 19.1/1.76 or 3.29. (1.59=3.292/2.075)

The incorrectly converted cut off value would be 1.76*5.99/1.39=7.58. Note that in the simulations discussed in the previous section this would only have captured the emitter

i) 77.9% of the time when the standard deviation = 1 degree

ii) 79.0% of the time when the standard deviation = .1 degree

APPENDICES I

Sorted cut-off level from 1000 simulations with angular standard deviation of one degree.

(1 - 4)	.000518399354139	.00107077331116	.00220549175486	.00488801884855
(5 - 8)	.0068566877792	.00845171449431	.0105978497784	.0115295460189
(9 - 12)	.0218209202903	.0239995638037	.0240503579056	.0247919949375
(13 - 16)	.0253506361558	.0271702873482	.0281838775087	.0330599938544
(17 - 20)	.03711181512	.0379512156511	.0411569621387	.0432184661264
(21 - 24)	.0432684328364	.0446931087287	.0469502379127	.046986259162
(25 - 28)	.0541034633624	.05917541138	.0626594696163	.0658545154898
(29 - 32)	.066882278719	.0694862189601	.0726785442113	.0749056605071
(33 - 36)	.0845665196705	.0853785150267	.0916353569206	.0933002882982
(37 - 40)	.0978194958035	.0981524403629	.0987033731869	.0990057166235
(41 - 44)	.105124346294	.106185884661	.108053203833	.108549668962
(45 - 48)	.110089355935	.113911327139	.118673023854	.126417257335
(49 - 52)	.127546358101	.128422202598	.128816912055	.131250461957
(53 - 56)	.150922567534	.153633406389	.154018019094	.169075806207
(57 - 60)	.17103203562	.177732628579	.179655371556	.187158905989
(61 - 64)	.193033960122	.200711463457	.2094332523	.215489564711
(65 - 68)	.220679788629	.223164801733	.223317226311	.224641088078
(69 - 72)	.226670239796	.23387787024	.235062892862	.235197381246
(73 - 76)	.237753608921	.240900057036	.244107081584	.254059049693
(77 - 80)	.256693259001	.272731328 01	.274025910464	.283001192498
(81 - 84)	.289357063024	.296690808252	.300998720062	.301447586134
(85 - 88)	.301883052955	.303122007742	.308385028579	.326984981872
(89 - 92)	.327412926247	.333207790386	.333247997757	.338023572438
(93 - 96)	.346557798951	.346578694899	.352752945145	.353648241801
(97 - 100)	.354422291704	.355874232709	.375974562843	.378908940597
(101 - 104)	.379781371756	.380432956052	.380817827115	.38160650768
(105 - 108)	.396973469577	.400765362796	.402155145235	.403008676379
(109 - 112)	.403321293963	.403816435 45	.403873218364	.412251423075
(113 - 116)	.412465777723	.414076276079	.418185797977	.422354519392
(117 - 120)	.42448223135	.427874529324	.445330725871	.447523803445
(121 - 124)	.449846696943	.456453366858	.467256854416	.468350454638
(125 - 128)	.470728150759	.476799813009	.47704523074	.479906888084
(129 - 132)	.483571130244	.488870325746	.49002624293	.490986977503
(133 - 136)	.496706154292	.497958249226	.50022671817	.502120137282
(137 - 140)	.504443940024	.506365947284	.509413876267	.511602188354
(141 - 144)	.513964670428	.519461810546	.521489238967	.523266266558
(145 - 148)	.524140793654	.526328003351	.527146556511	.527703357523
(149 - 152)	.527925536546	.534229431893	.535611099198	.546434280704
(153 - 156)	.550948961676	.553750198946	.56116731805	.565332652673
(157 - 160)	.572514613813	.573302746375	.575006717258	.583624550728
(161 - 164)	.58492611868	.584989889298	.588629125379	.591347565778
(165 - 168)	.591732240384	.593902789857	.597068424686	.602696996644
(169 - 172)	.606092349953	.608848357332	.610874305707	.61257856697
(173 - 176)	.621238102456	.622270164319	.626529427569	.628179047347
(177 - 180)	.637932117742	.645723060514	.647497033415	.663918118914
(181 - 184)	.66638091354	.673408586234	.676353737448	.68309288965
(185 - 188)	.685442579078	.688193078562	.689928068904	.690884414949
(189 - 192)	.693033872093	.697176522696	.697245193908	.698196097682
(193 - 196)	.699098829331	.70188703401	.705081204034	.710001797767
(197 - 200)	.714921994125	.718420472341	.721543375832	.722275777125
(201 - 204)	.722628900089	.735205024613	.741085577653	.742166038928
(205 - 208)	.744591708081	.746389741505	.746486773318	.758530893608
(209 - 212)	.759593339745	.761109224358	.765525657458	.775739673029
(213 - 216)	.778363340355	.787313223959	.788498772395	.789604535661
(217 - 220)	.7926057638	.805078215315	.806301481278	.811737867329

(221 - 224)	.81208689042	.812200079965	.818727716477	.826320349387
(225 - 228)	.829173730416	.837818561279	.845648849794	.847779232787
(229 - 232)	.857273596009	.859133405928	.863279203099	.888077743697
(233 - 236)	.890151158964	.893169090953	.896081288128	.897665338736
(237 - 240)	.900486144188	.901282365875	.903086552127	.904080230615
(241 - 244)	.906052922768	.914868534809	.930548382419	.930688345228
(245 - 248)	.931887868512	.935472248892	.939024681857	.938106263378
(249 - 252)	.941291975209	.945634158952	.946131937827	.946555137484
(253 - 256)	.948347229251	.954293225455	.957545666794	.957938849937
(257 - 260)	.965612888651	.965730178866	.967235019902	.97307145 05
(261 - 264)	.98123437121	.987655122367	.989409351309	.991783040937
(265 - 268)	.992991119056	1.000228663463	1.00152986014	1.00181786727
(269 - 272)	1.00458801295	1.01130758736	1.0166789133	1.01701671582
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(317 - 320)	1.24619187218	1.24971247505	1.25460340873	1.26086355775
(321 - 324)	1.26462285196	1.27037120473	1.29598117155	1.29708652134
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(565 - 568)	3.08880979872	3.10915955356	3.1125621834	3.12018879019
(569 - 572)	3.12825908463	3.13445062504	3.13540028611	3.1509250672
(573 - 576)	3.16239369932	3.1648279484	3.16800410982	3.16961827979
(577 - 580)	3.18841232647	3.2110812906	3.23016553807	3.24632702104
(581 - 584)	3.25364052351	3.27254375247	3.27537200769	3.27741118383
(585 - 588)	3.32383345038	3.3245739698	3.32701435766	3.35294404714
(589 - 592)	3.36014381481	3.37081819309	3.38296187252	3.38503149778
(593 - 596)	3.40475115314	3.40763274427	3.42810051143	3.47858741356
(597 - 600)	3.47880177346	3.48456011971	3.51442204529	3.52566299297
(601 - 604)	3.52791937531	3.54490736171	3.54558522889	3.54751744725
(605 - 608)	3.55771214486	3.57141968099	3.57242333353	3.62945057064
(609 - 612)	3.66155792445	3.66579860702	3.66776346392	3.67252441071
(613 - 616)	3.69274482051	3.70110563807	3.70303154923	3.70470945551
(617 - 620)	3.7067031532	3.70939633997	3.74751748168	3.76273139225
(621 - 624)	3.76820045922	3.77119435865	3.79815477477	3.81099400532
(625 - 628)	3.82904696456	3.83532045553	3.83749229824	3.87354448467
(629 - 632)	3.89861243693	3.91113212051	3.9117771132	3.92371968269
(633 - 636)	3.94163493006	3.96184913688	3.97024410998	3.97819105406
(637 - 640)	4.0049717249	4.01570961691	4.0258764715	4.02657318183
(641 - 644)	4.05384466113	4.08864656911	4.09153767365	4.10660308305
(645 - 648)	4.10792517125	4.11878879547	4.14037329025	4.145289633
(649 - 652)	4.15428756453	4.16454151977	4.17127033765	4.19780718937
(653 - 656)	4.21012804906	4.23154654377	4.2622495465	4.27068247775
(657 - 660)	4.29632143412	4.3413040897	4.40018120337	4.43637027546
(661 - 664)	4.48868174323	4.51308783927	4.52725878724	4.52813875045
(665 - 668)	4.55515130027	4.55797162449	4.56033625446	4.56756031042
(669 - 672)	4.58070368777	4.51849139202	4.63389471337	4.63933858047

(673 - 676)	4.68449275246	4.71676616193	4.72090493226	4.73878710137
(677 - 680)	4.7423766293	4.78052795386	4.78899053062	4.84191801054
(681 - 684)	4.8433178709	4.84774087175	4.87068850771	4.89412429574
(685 - 688)	4.91992010987	4.96881635392	4.98465172839	5.00522414069
(689 - 692)	5.04318024312	5.06865704159	5.10961828961	5.11247929604
(693 - 696)	5.13068033525	5.19657327703	5.2487645398	5.255681783
(697 - 700)	5.29094493905	5.30649406724	5.36297652342	5.36485782864
(701 - 704)	5.37039688931	5.38087727135	5.39746646788	5.45809428214
(705 - 708)	5.47038008062	5.47551329902	5.48415410422	5.49333763567
(709 - 712)	5.52084478503	5.55428641086	5.55520962866	5.56005582935
(713 - 716)	5.56690064369	5.57016217054	5.58878441284	5.6[illegible]357390579
(717 - 720)	5.663602221	5.72735901248	5.7698706236	5.82713443924
(721 - 724)	5.82937469581	5.83985578629	5.86142238818	5.86538061629
(725 - 728)	5.88866799373	5.89105896355	5.89237166211	5.92048253751
(729 - 732)	6.02037463407	6.02211919587	6.07524001041	6.11480078975
(733 - 736)	6.15644030473	6.17220162465	6.17573557662	6.23092892364
(737 - 740)	6.23689068552	6.25393054372	6.29625063523	6.31312026212
(741 - 744)	6.31967578746	6.32787[illegible]	6.34116521878	6.35184638226
(745 - 748)	6.37257616159	6.42699744426	6.43094001248	6.45155255801
(749 - 752)	6.46145552255	6.48921627954	6.52003861884	6.54709699557
(753 - 756)	6.57323710487	6.59193520297	6.62153650491	6.65121816545
(757 - 760)	6.66237968159	6.66364709153	6.68387314531	6.78279563714
(761 - 764)	6.85183864[illegible]39	6.85864974295	6.89402091743	6.95866858861
(765 - 768)	6.97355603243	6.99177709043	7.00315052805	7.09116979282
(769 - 772)	7.10784960582	7.17502103548	7.18138030902	7.18771340[illegible]
(773 - 776)	7.27037261[illegible]28	7.3003924017	7.32381783814	7.[illegible]
(777 - 780)	7.40354343527	7.46878135443	7.58385028454	7.52[illegible]2620841
(781 - 784)	7.66075806246	7.73710589742	7.81385417778	7.95358303215
(785 - 788)	8.02647437603	8.13703599787	8.15705385718	8.17237201682
(789 - 792)	8.18407843284	8.20186572946	8.20886469286	8.2111031901
(793 - 796)	8.2789765682	8.28605017858	8.31047667064	8.39853603216
(797 - 800)	8.5009871971	8.56252644083	8.56680525107	8.59877220351
(801 - 804)	8.61[illegible]0506893	8.70202766981	8.74903988986	8.81068828113
(805 - 808)	8.82882258536	8.88181623807	8.89131404569	8.9011139834
(809 - 812)	8.93774693908	8.94716184167	9.00251037826	9.04508323604
(813 - 816)	9.07351515627	9.12631783056	9.2667628183	9.26858116593
(817 - 820)	9.28124284654	9.29707684088	9.33603466498	9.36757432858
(821 - 824)	9.36893743915	9.38297453196	9.39399521509	9.40274515273
(825 - 828)	9.40361499443	9.41633644927	9.44459152064	9.48811348205
(829 - 832)	9.53502524576	9.61120147206	9.68598299186	9.76328734247
(833 - 836)	9.79753625178	9.82177439958	9.87782551739	9.92865785678
(837 - 840)	9.94884878901	9.97087102887	9.97820153901	10.0219306682
(841 - 844)	10.0950223286	10.2203488138	10.3623886715	10.3769374999
(845 - 848)	10.4558231555	10.5872442775	10.6728566735	10.675985653
(849 - 852)	10.8586646664	10.9375083386	11.0003099154	11.0552990557
(853 - 856)	11.1398268183	11.1864063426	11.5251059162	11.6014174338
(857 - 860)	11.7122877425	11.7305374478	11.7716316578	11.9073978763
(861 - 864)	11.9150666193	11.9343732304	11.9831301077	12.0588383412
(865 - 868)	12.4108557614	12.4151[illegible]717	12.6560684038	12.7178804879
(869 - 872)	12.7326638067	12.7888623559	12.83094979	12.8420066963
(873 - 876)	12.8443111604	12.8864936841	13.0735086518	13.0858310513
(877 - 880)	13.4145553401	13.4186375683	13.5084494949	13.9097283401
(881 - 884)	14.0122328207	14.0367573243	14.196696313	14.2572001611
(885 - 888)	14.3160229165	14.5920346886	14.694198429	14.9377705197
(889 - 892)	15.2047163755	15.3249022014	15.4792847408	15.5610965355
(893 - 896)	15.5703772878	15.5990730838	15.6031304571	15.6157905372
(897 - 900)	15.7450492122	15.7469987475	15.9675137653	16.1073543883

(901 - 904)	16.1839040373	16.2483953084	16.2613636087	16.342547026
(905 - 908)	16.5735244873	16.8247337015	16.8750565306	16.9086028477
(909 - 912)	16.9193826989	16.9385899486	17.0311912642	17.149743042
(913 - 916)	17.4572798	17.6453220412	17.7921227752	17.8644481704
(917 - 920)	17.8796877893	18.4438863707	18.8605577048	19.1572757709
(921 - 924)	19.5322728306	19.6203628418	19.6596340136	19.8103208181
(925 - 928)	20.1267452379	20.1523999336	20.2951493637	20.6756620122
(929 - 932)	21.0644468385	21.0804152579	21.0844478592	21.1926566532
(933 - 936)	21.8279132719	22.4606858209	22.6102190318	22.7981025706
(937 - 940)	22.8780984306	23.104429208	23.6134700793	23.9069838009
(941 - 944)	23.9992102884	24.5454569383	25.2523804273	26.7800066613
(945 - 948)	26.8537281466	26.9701874562	27.3582685623	27.7761072787
(949 - 952)	27.79400857	27.9232255676	28.6474493792	29.5800709164
(953 - 956)	29.6222993602	30.0549740781	30.1743403932	31.1479998935
(957 - 960)	31.8767385405	31.9584663148	32.978835979	33.3021160338
(961 - 964)	33.6420174765	33.9976506076	35.9002560969	35.9615261356
(965 - 968)	37.0820012155	37.3590236986	37.9783364093	38.5513199622
(969 - 972)	40.0608480557	40.6084643207	40.7663090982	41.3893196972
(973 - 976)	41.7954807691	42.6046954682	44.4483334486	44.4842189466
(977 - 980)	48.3399342526	48.5702337064	49.150580118	49.7634595914
(981 - 984)	52.4523631273	52.6676891977	53.1465720201	62.3097156196
(985 - 988)	66.854734646	68.1757181456	72.7320335471	80.6498109258
(989 - 992)	85.3828692104	88.1873542057	91.9467272387	92.0738197672
(993 - 996)	94.5228364312	95.015211162	134.072221745	144.019822671
(997 - 1000)	155.820137048	251.783097879	345.008990801	710.560286471

APPENDICES II

Deviation of one tenth of a degree.

(1 - 4)	.00184381035052	.00590549319437	.0090106647873	.0112778971469
(5 - 8)	.0142838173823	.0146930487769	.0228811718247	.0266243970657
(9 - 12)	.0321631808865	.0371120937299	.0439215277191	.0442053316393
(13 - 16)	.0461843507197	.0570783690458	.0625362645314	.0690341719708
(17 - 20)	.0744827319563	.075029412531	.0805618712597	.0809213533378
(21 - 24)	.0826271389604	.0833831161267	.0846783009439	.0850654324377
(25 - 28)	.0904863775215	.0960800563147	.0971370510656	.0980444434536
(29 - 32)	.10029804762	.10484857108	.107578972402	.11322976568
(33 - 36)	.116488834091	.117764789016	.127670731175	.128534958826
(37 - 40)	.129799651389	.136044597862	.138381401879	.145341921357
(41 - 44)	.146982004144	.15070306733	.153723814229	.155598743332
(45 - 48)	.15697316019	.157482617838	.165729885652	.171700603511
(49 - 52)	.173391720239	.182205691973	.186751181395	.188005902987
(53 - 56)	.188066884581	.189313379052	.194568443441	.218819633501
(57 - 60)	.218902621343	.223588961538	.227515086664	.246240880265
(61 - 64)	.251228205244	.252770626493	.254259775493	.254281779584
(65 - 68)	.264608629749	.265761461683	.266915363157	.267179707804
(69 - 72)	.269315213809	.275017555696	.275855080418	.280990463768
(73 - 76)	.281762392772	.294664431551	.296223896575	.296447891085
(77 - 80)	.298125884869	.299956724943	.302041604096	.305603493425
(81 - 84)	.312374224734	.325594595754	.329752220891	.343803598653
(85 - 88)	.348121569213	.349169128054	.355031983951	.355461905601
(89 - 92)	.356455400554	.372973130391	.373012386458	.374135249772
(93 - 96)	.374696787148	.379600906897	.385951263638	.388063506103
(97 - 100)	.391481268777	.395376288757	.395569636269	.396741934453
(101 - 104)	.402461286988	.412117257312	.412777348599	.417350385776
(105 - 108)	.42003263594	.429435357537	.430250004257	.432715187018
(109 - 112)	.4373463606	.437533128263	.440960484761	.445044464271
(113 - 116)	.457453764727	.457907702492	.464431531288	.465443997259
(117 - 120)	.467984833762	.473218170398	.473324936772	.475985958763
(121 - 124)	.477627379178	.480117349789	.481575964321	.483619933458
(125 - 128)	.501601760713	.503235919114	.504173558852	.510032273155
(129 - 132)	.513924193278	.517037552538	.519122751988	.520627941455
(133 - 136)	.52386774351	.524388444678	.524924793099	.527498495764
(137 - 140)	.528105181515	.528327365694	.529408333496	.529828631751
(141 - 144)	.533180297752	.53745670599	.540035363592	.545856161243
(145 - 148)	.547341744582	.552791241660	.553240657512	.556212514687
(149 - 152)	.562264800209	.562944192549	.570662252331	.574651575904
(153 - 156)	.57764934723	.581261396848	.592231618894	.595435706281
(157 - 160)	.596935793196	.597729957473	.598308949753	.607800932881
(161 - 164)	.611746989545	.612008946761	.614248214381	.622009886817
(165 - 168)	.625899126695	.627236322902	.629482486414	.629821908685
(169 - 172)	.63925786865	.644653833721	.649949613556	.653217756727
(173 - 176)	.65900619799	.663123258354	.667015026664	.672355178144
(177 - 180)	.673920484253	.676715211204	.68139725509	.68309988638
(181 - 184)	.686651150046	.690146259442	.691100830578	.691431056319
(185 - 188)	.70197110481	.712228258167	.714380343513	.72962059013
(189 - 192)	.72991507666	.737715964477	.740631365751	.749842551669
(193 - 196)	.759324714015	.762798798207	.770933487861	.776455143781
(197 - 200)	.778501731322	.782691041645	.786849361743	.789108301973
(201 - 204)	.801401905624	.802469902919	.803389913641	.808416602048
(205 - 208)	.808647615729	.811755472356	.816949284329	.819194372242
(209 - 212)	.825894105176	.846363989924	.846576837404	.850569959489
(213 - 216)	.854214720743	.856154964539	.85688883471	.859378046446
(217 - 220)	.860700498088	.861393689653	.87628794268	.881007823891

(221 - 224)	.884649380269	.897635238211	.905296909819	.90561865104
(225 - 228)	.90577153192	.911241847053	.914069679264	.917673683494
(229 - 232)	.924112678733	.935[illegible]93683834	.936886875542	.94500641347
(233 - 236)	.945941800399	.946499529904	.959962579665	.960954558148
(237 - 240)	.966454501905	.982825137336	.985522313672	.993340537743
(241 - 244)	1.00361236076	1.00614697937	1.00888801726	1.0088949953
(245 - 248)	1.01231449528	1.01805899351	1.02223295765	1.02334697635
(249 - 252)	1.02533521316	1.03024358765	1.03079855788	1.03502312534
(253 - 256)	1.04025072841	1.0409100329	1.04094275837	1.04204603445
(257 - 260)	1.04214983167	1.05504601625	1.05520493033	1.05557086073
(261 - 264)	1.05645799682	1.06375017649	1.06599027021	1.07015624151
(265 - 268)	1.07421260331	1.07998653747	1.08239101246	1.09172847776
(269 - 272)	1.09336468186	1.09542947127	1.09791947876	1.10518195192
(273 - 276)	1.10777457703	1.12901605676	1.13005110261	1.1[illegible]54275679
(277 - 280)	1.14648402121	1.15006431205	1.15232230438	1.152933188
(281 - 284)	1.16170222996	1.1652334017	1.17371863357	1.17562808471
(285 - 288)	1.17799025306	1.17960457851	1.18214026428	1.18423614259
(289 - 292)	1.19746526673	1.20036165301	1.20877862929	1.2178545393
(293 - 296)	1.22721672147	1.2329606236	1.23759620998	1.252524503 22
(297 - 300)	1.25651972937	1.26153232459	1.26431162175	1.26605373725
(301 - 304)	1.26641875418	1.26849177765	1.27721378209	1.28692032762
(305 - 308)	1.29791570888	1.30992771526	1.31429460246	1.32402122539
(309 - 312)	1.3291456851	1.33429715989	1.33560071457	1.33573159191
(313 - 316)	1.3503650943	1.35210927378	1.359365445 96	1.35959831307
(317 - 320)	1.36592366689	1.38210014752	1.38544152701	1.40852493978
(321 - 324)	1.41178045759	1.41510215707	1.4201693704	1.42686061029
(325 - 328)	1.43190758035	1.43848182653	1.44047393646	1.46438044041
(329 - 332)	1.46721628327	1.46910561973	1.46966208113	1.47018773997
(333 - 336)	1.47613733412	1.48850561038	1.48907110151	1.49283657468
(337 - 340)	1.50142100737	1.50809477492	1.51791161923	1.5236344785
(341 - 344)	1.52379591726	1.52636283079	1.52725048878	1.53029919924
(345 - 348)	1.54160732505	1.541945397 31	1.542601753 76	1.5486556311
(349 - 352)	1.55168733981	1.56101977817	1.56219644814	1.56349067282
(353 - 356)	1.56373172201	1.5665954488	1.56853795897	1.57661714124
(357 - 360)	1.57882071282	1.58044786032	1.58053582536	1.58256181493
(361 - 364)	1.58408739523	1.58890822319	1.59508670754	1.60284562071
(365 - 368)	1.60290086872	1.60518594856	1.61206380036	1.61854223663
(369 - 372)	1.62128177494	1.64839691953	1.65238420324	1.65854023471
(373 - 376)	1.66174507248	1.67907102812	1.67933631382	1.67953452504
(377 - 380)	1.68311412001	1.68354127056	1.69563292487	1.69725475377
(381 - 384)	1.70068176298	1.70276304299	1.70308253637	1.70543274576
(385 - 388)	1.70761894859	1.71300927209	1.71321384989	1.72291148319
(389 - 392)	1.72872327159	1.73083512634	1.74421396558	1.75041969573
(393 - 396)	1.75423364433	1.75841333192	1.75851346894	1.75967461653
(397 - 400)	1.77248416472	1.77441306313	1.77990141515	1.79670843796
(401 - 404)	1.80019172234	1.80754763893	1.82165273989	1.82759362847
(405 - 408)	1.82764368519	1.8297004294	1.83509321818	1.83865847914
(409 - 412)	1.85521554803	1.86280241709	1.87806437638	1.88175376508
(413 - 416)	1.88317477568	1.88727776398	1.89377892354	1.89712310686
(417 - 420)	1.90459163415	1.91817101278	1.92216546791	1.92237333327
(421 - 424)	1.92982414395	1.94128245015	1.94813387871	1.95269384395
(425 - 428)	1.95396615743	1.96706986446	1.96839162405	1.96845170798
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(857 - 860)	10.9947337857	11.1052954354	11.4769889791	11.5392132102
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(873 - 876)	12.9064068219	12.9620098983	13.0312536287	13.1421616752
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(889 - 892)	14.4468795485	14.537664875	14.6025844118	14.6744429838
(893 - 896)	14.8324578503	14.9839994349	15.215389534	15.6961819936
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(953 - 956)	31.984595118	32.7409534048	33.3761967653	33.4431982613
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(973 - 976)	49.17280525	49.3719106266	50.6278090844	53.1079640397
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(985 - 988)	77.8770812496	83.307381396	112.110161026	118.950389965
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APPENDICES III

A sequence of ellipse from the 1 degree simulation to show the variability of ellipse size.

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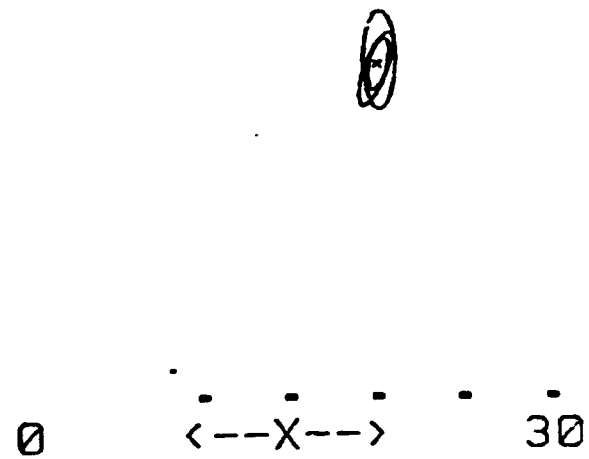
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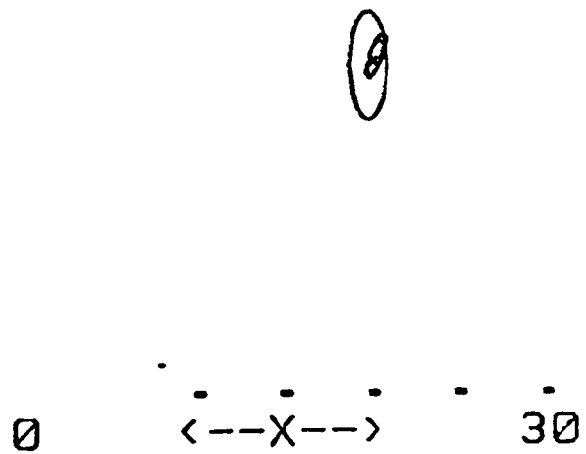
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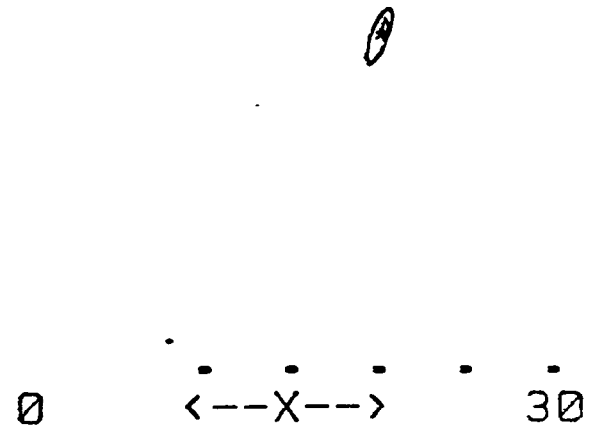
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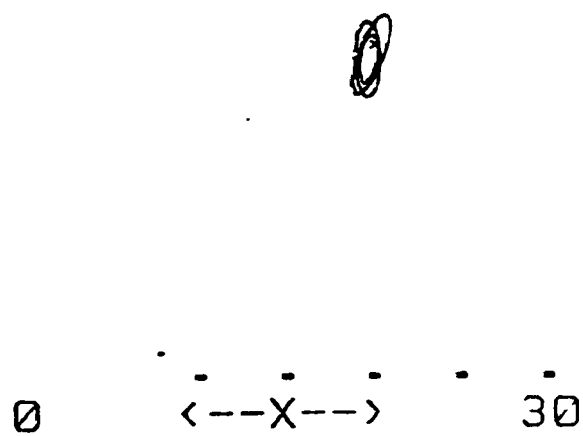
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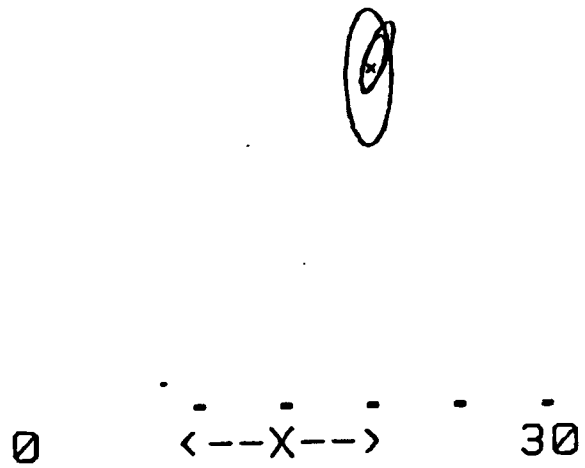
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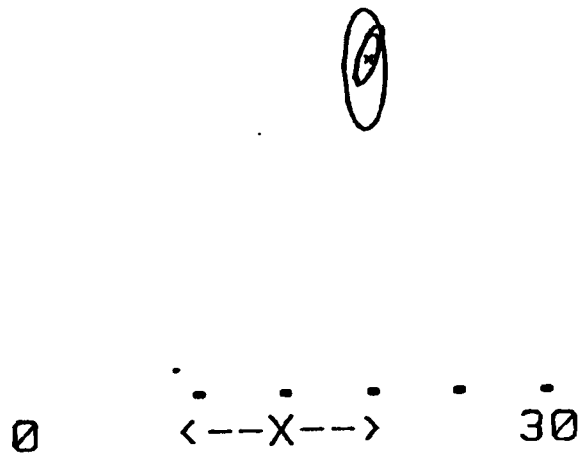
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