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PRESSURE DISTRIBUTION ON LIFTING SURFACES

E. A. KRASILSHCHIKOVA

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Let us consider the problem of the flow around a thin, slightly cambered wing of finite span moving forward in a straight line with velocity $u > a$ (where a is the speed of sound in the undisturbed stream) in its linearized form [1].

Using the solution to the problem found by us earlier [2,3] let us find the pressure distribution on the wing surface.

Wings of arbitrary planform are sketched in figures 1 and 2. Let the equation of the leading-edge arc E_1E be given in characteristic coordinates [2] in the form $y = \psi_0(x)$; the equations of the tips ED and E_1D_1 in the form, respectively, $y = \psi(x)$ and $y = \psi_1(x)$; the equations of the trailing edges DT and D_1T_1 in the form $y = \chi(x)$ and $y = \chi_1(x)$.

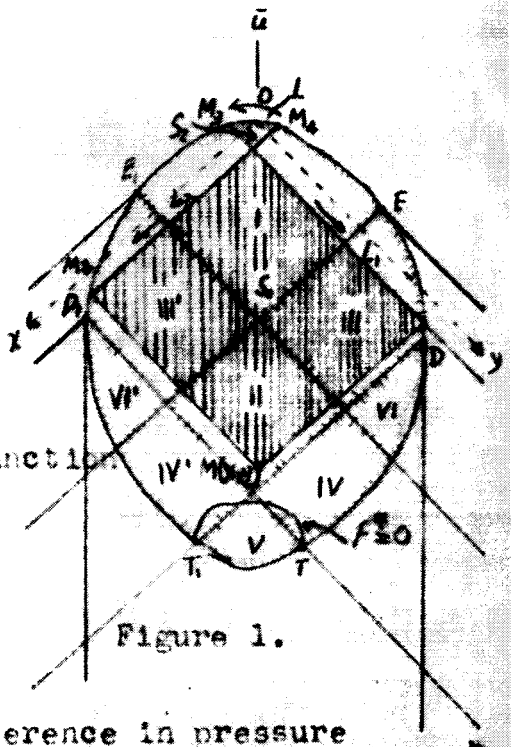

Figure 1.

Let us express the pressure on the wing surface with the aid of the function which is given on the wing

$$A(x,y) = \frac{-u\beta_0(x,y)}{\sqrt{u^2/a^2 - 1}}$$

where β_0 is the angle of attack of a wing element.

From Bernoulli's integral the difference in pressure below and above the wing $p_l(x,y) - p_u(x,y) = p(x,y)$ in characteristic coordinates is

$$p(x,y) = 2u\rho\left[\varphi_{0x}(x,y) + \varphi_{0y}(x,y)\right]$$

where φ_0 is the velocity potential, ρ is the density of the undisturbed stream.

Let us divide the wing surface into nine characteristic regions as shown in the figures.

We will denote by the letters K and M with an index the ends of the line segments parallel to the coordinate axes which lie in the x,y plane. It is clear that the named segments refer to the lines of intersection of the characteristic cones of the wave equation with vertices in the x,y plane with the x,y plane itself.

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Region I is the region where there is no tip effect. This region of the wing lies beyond the characteristic cones with vertices at E_1 and E.

Region II is the region where there is a tip effect, but no effect from the vortex system of the wing. This region lies inside the cones with vertices at E_1 and E and beyond the cones with vertices at D_1 and D. The pressure difference at the point $M(x,y)$ of Region II where the lines M_1M_3 and M_2M_4 intersect on the wing (as shown in figure 1) is expressed by

$$
\begin{aligned}
p(x,y) = & -\frac{u\rho}{\pi}\iint_{S_1} D(\xi,\eta;\ x,y)\,d\xi\,d\eta + \frac{u\rho}{\pi}\iint_{S_2} D(\xi,\eta;\ x,y)\,d\xi\,d\eta \\
& + \frac{u\rho}{\pi}\int_{L=\overset{\frown}{M_4M_3}} B\left[\xi,\ \psi_1(\xi);\ x,y\right]\left\{1-\frac{d\psi_1(\xi)}{d\xi}\right\}d\xi \\
& - \frac{u\rho}{\pi}\left\{1-\frac{d\bar{\psi}(y)}{dy}\right\}\int_{L_1=M_3M_4} B\left[\bar{\psi}(y),\eta;\ x,y\right]d\eta \\
& - \frac{u\rho}{\pi}\left\{1-\frac{d\psi_2(x)}{dx}\right\}\int_{L_2=M_4M_3} B\left[\xi,\ \psi_2(x);x,y\right]d\xi \qquad (1)
\end{aligned}
$$

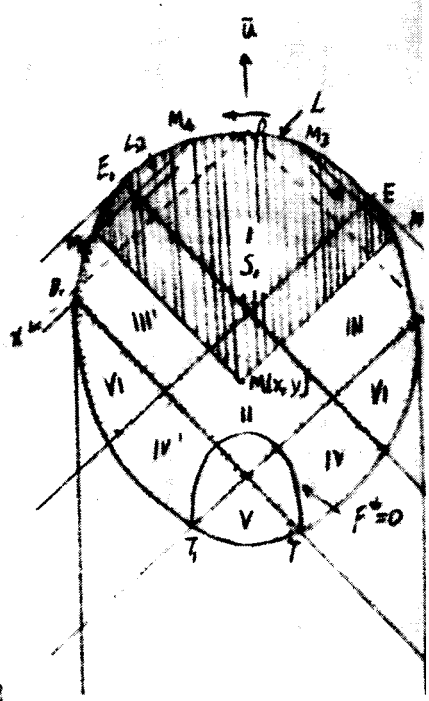
where S_1 is the region of the wing bounded by the lines MM_1, MM_3, M_1M_3, and M_2M_4; S_2 is bounded by M_1M_3, M_2M_4 and the arc $L = \overset{\frown}{M_4M_3}$; the function $x = \bar{\psi}(y)$ is the equation of the tip edge ED and

$$
D(\xi,\eta;\ x,y) = \frac{A_\xi(\xi,\eta)+A_\eta(\xi,\eta)}{\sqrt{(x-\xi)(y-\eta)}};\quad B(\xi,\eta;x,y) = \frac{A(\xi,\eta)}{\sqrt{(x-\xi)(y-\eta)}}
$$

If the lines M_1M_3 and M_2M_4 do not intersect on the wing, as shown in figure 2, then the pressure difference is expressed by

$$
\begin{aligned}
p(x,y) = & -\frac{u\rho}{\pi}\iint_{S_1} D(\xi,\eta;x,y)\,d\xi\,d\eta \\
& -\frac{u\rho}{\pi}\int_{L=\overset{\frown}{M_3M_4}} B\left[\xi,\ \psi_1(\xi);x,y\right]\left\{1-\frac{d\psi_1(\xi)}{d\xi}\right\}d\xi \\
& -\frac{u\rho}{\pi}\left\{1-\frac{d\bar{\psi}(y)}{dy}\right\}\int_{L_1=M_3M_1} B\left[\bar{\psi}(y),\eta;x,y\right]d\eta \\
& -\frac{u\rho}{\pi}\left\{1-\frac{d\psi_2(x)}{dx}\right\}\int_{L_2=M_4M_2} B\left[\xi,\ \psi_2(x);x,y\right]d\xi \qquad (2)
\end{aligned}
$$

where S_1 is the region bounded by the lines MM_1, M_1M_3, MM_2, M_2M_4 and the arc $L = \overset{\frown}{M_3M_4}$. The arrows in the figures show the direction of integration over the contour integrals.



The pressure difference in Region III which lies inside the characteristic cone with vertex at E and beyond the cones with vertices at E_1, D and D_1 is calculated from formula (2) in which the last term is set equal to zero. The pressure difference in Region III is also expressed by formula (2) if the penultimete term is set equal to zero.

Region IV is that region lying inside the cones with vertices at E_1, E, and D and beyond the cone with vertex at D_1. Correspondingly, Region IV' is defined. At $M(x,y)$ of Region IV for which the lines M_1M_3 and M_2M_4 intersect on the wing, the pressure difference is expressed by formula (1) if the penultimate term is set equal to zero. At M for which the lines M_1M_3 and M_2M_4 do not intersect on the wing, the pressure difference is expressed by formula (2) if the penultimate term is also put equal to zero. The pressure difference is expressed analogously for points of Region IV' by means of formulas (1) and (2) if the last terms are set equal to zero.

The pressure difference in Region V which lies within the characteristic cones with vertices at E, E_1, D, and D_1 is

$$p(x,y) = \frac{-u\rho}{\pi} \iint_{S_1} D(\xi,\eta;x,y)\,d\xi\,d\eta + \frac{u\rho}{\pi} \iint_{S_2} D(\xi,\eta;x,y)\,d\xi\,d\eta$$

$$+ \frac{u\rho}{\pi} \int_{L=\overset{\frown}{M_4M_3}} B\left[\xi, \psi_1(\xi);x,y\right]\left\{1 - \frac{d\psi_1(\xi)}{d\xi}\right\} d\xi \qquad (3)$$

if the lines M_1M_3 and M_2M_4 intersect on the wing and if the lines do not intersect, then the formula is

$$p(x,y) = \frac{-u\rho}{\pi} \iint_{S_1} D(\xi,\eta;x,y)\,d\xi\,d\eta$$

$$\frac{-u\rho}{\pi} \int_{L=\overset{\frown}{M_4M_3}} B\left[\xi, \psi_1(\xi);x,y\right]\left\{1 - \frac{d\psi_1(\xi)}{d\xi}\right\} d\xi \qquad (4)$$

In Region VI which lies inside the cones with vertices at E and D and beyond the cones with vertices at E_1 and D_1, also in Region VI', the pressure difference is expressed by formula (4). The formula for the pressure in Region I has the same form. Thus, if the point M in which the pressure is desired, is found in one of the Regions II (as shown in the figure), IV (correspondingly IV') or V then in order to establish the region and contour of integration in the formulas for the pressure, it is necessary to do the following: From M draw two lines MM_1 and MM_2 upstream to intersect with the tips or trailing edges of the wing. From the points of intersection M_1 and M_2 once more draw lines M_1M_3 and M_2M_4 upstream of the wing to intersect the leading edge EE_1 in the points M_3 and M_4.

If the point $M(x,y)$ is found in Regions III or VI (correspondingly III' or VI') then draw from M the lines MM_4 and MM_1 upstream; the line MM_4 immediately intersects the leading edge E_1E in M_4 and the line MM_1 intersects the tip ED in the case of Region III or the trailing edge DT in the case of Region VI. From the point of intersection M_1 again draw a line M_1M_3 to intersect with the leading edge E_1E.

If the tips ED and E_1D_1 are straight lines parallel to the basic stream or the wing planform is such that the points E and D and also E_1 and D_1 coincide, then the pressure formulas (1) and (2) are naturally simplified because the last two terms reduce to zero.

If the wing surface is such that the function $D(\xi,\eta;x,y) \equiv 0$ (for example, in a plane wing) then only the contour integrals remain in the pressure formulas.

The pressure formulas show that there may exist a place on the wing where $F^*(x,y) = 0$ where the pressure is zero. Downstream from this geometrical place the pressure difference becomes negative.

For example, if $D \equiv 0$, then $F^* = 0$ will be found in the region of the wing lying inside the characteristic cones with vertices at E and E_1. If, with this, the tips of the wings are straight lines parallel to the basic stream, or the points E and D coincide, then the equation $F^*(x,y) = 0$ assumes the simple form

$$F^* = \bar{\psi}_1[\psi_2(x)] - \bar{\psi}(y) = 0$$

in Region VI;

$$F^* = \bar{\psi}_1[\psi_1(x) - \bar{\chi}(y)] = 0$$

in Region IV'; and

$$F^* = \bar{\psi}_1[\chi_1(x)] - \bar{\chi}(y) = 0$$

in Region V;

where $x = \bar{\psi}_1(y)$ and $x = \bar{\chi}(y)$ are the respective equations of the arcs E_1E and DT in a form solved relative to the x coordinate.

In all these cases when the pressure difference on the wing is expressed only by means of curvilinear integrals according to formulas (1) - (4), distributed over the arc $\widetilde{M_3M_4}$ of the wing contour, the curve $F^* = 0$ may be easily constructed

by geometric means, keeping in mind that the curve of zero pressure is the locus of such points $M(x,y)$ on the wing surface for which the points M_3 and M_4 on the wing contour coincide, i.e., the arc L shrinks to a point.

The results are generalized to the case when the leading edge E_1E is not given by one equation $y = \psi_1(x)$ but consists of segments of smooth curves given by the equations $y = \psi_{1k}(x)$ where $k = 1,2,\ldots,n$. In this case for actual calculation the formulas for pressure of the surfaces and the contour integrals should be separated in corresponding parts. The same remark applies to the tips and trailing edge.

1. L.I.Sedov: Theory of twodimensional motion of an ideal fluid, 1939
2. E.A.Krasilshchikova, DAN, 58, No. 4, 1947
3. E.A.Krasilshchikova, DAN, 58, No. 6, 1947

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