Magnetic Bearing Stiffness Control Using Frequency Band Filtering

H. Ming Chen  
Mechanical Technology Incorporated  
968 Albany-Shaker Road  
Latham, New York 12110, U.S.A.

Active magnetic bearings can be implemented with frequency band-reject filtering that decreases the bearing stiffness and damping at a small bandwidth around a chosen frequency. The control scheme has been used for reducing a rotor dynamic force, such as an imbalance force, transmitted to the bearing stator. This study reveals that the scheme creates additional system vibration modes at the same frequency. It also shows that the amount of force reduction is limited by the stability requirement of these modes.

Introduction

The attractive-type active magnetic bearings (AMBs) usually have four quadrants of electromagnets (ref. 1). A pair of opposite quadrants independently control the journal motion in one direction by a Proportional-Integral-Derivative (PID) controller (ref. 2). Steady-state (bias) currents are induced in the quadrants so that the total control current in each quadrant never changes polarity. This provides a base for linear feedback control and greatly simplifies the control circuitry (ref. 3).

There appears to be a growing interest in applying the AMBs not only for their advantageous basic bearing function, but also for their potential to be the rotor force isolator (ref. 4). An AMB can be made extra "soft" at narrow frequency bands, and the rotor forces to ground at these frequencies can be dramatically reduced at the sacrifice of large rotor runouts. For example, the well-publicized AMB control feature, "auto-balancing," was designed to reduce imbalance force to ground with the rotor rotating about its inertia axis (ref. 2). However, the creation of these stiffness "valleys" also creates instability problems for the rotor-AMB system. Herein this type of instability problem is addressed analytically while an experimental project is in progress.

Nomenclature

AMB  active magnetic bearing
a  phase-lead network zero parameter
B  frequency bandwidth of band-pass filter
b  phase-lead network pole parameter
C  radial air gap
Cd  proportional feedback gain
\begin{align*}
C_v & \quad \text{phase-lead feedback gain} \\
C_e & \quad \text{integral feedback gain} \\
C_p & \quad \text{band-pass filter gain} \\
C_{po} & \quad \text{band-pass filter stability threshold gain} \\
F & \quad \text{regulating magnetic force} \\
F_{1,3} & \quad \text{magnetic force per pole due to bias current in quadrant 1 or 3} \\
FODE & \quad \text{first-order differential equation} \\
f & \quad \text{band-pass factor} \\
G & \quad \text{power amplifier gain} \\
I_{1,3} & \quad \text{bias current in quadrant 1 or 3} \\
i & \quad \text{regulating current} \\
j & \quad \sqrt{-1} \\
K & \quad \text{AMB stiffness} \\
K_i & \quad \text{current stiffness} \\
K_m & \quad \text{magnetic (negative) stiffness} \\
LHS & \quad \text{left hand side} \\
M & \quad \text{mass supported by AMB} \\
PID & \quad \text{proportional-integral-derivative} \\
Q & \quad \text{integrator output} \\
R & \quad \text{nondimensional frequency parameter defined at a stiffness valley} \\
RHS & \quad \text{right hand side} \\
S & \quad \text{Laplace variable} \\
t & \quad \text{time} \\
U & \quad \text{real stiffness at } \omega_c \text{ after filter implementation} \\
u & \quad \text{real stiffness at } \omega_c \text{ before filter implementation} \\
u_r & \quad \text{RHS stiffness} \\
u_L & \quad \text{LHS stiffness} \\
V & \quad \text{imaginary stiffness at } \omega_c \text{ after filter implementation} \\
v & \quad \text{imaginary stiffness at } \omega_c \text{ before filter implementation} \\
Y & \quad \text{AMB journal displacement in Y-direction} \\
Y_p & \quad \text{band-pass filter output} \\
Z & \quad \text{phase-lead network output} \\
\Delta C_d & \quad \text{net proportional feedback gain contributing to real stiffness} \\
\omega & \quad \text{exciting frequency} \\
\omega_c & \quad \text{band-pass filter center frequency} \\
\omega_L & \quad \text{LHS slope frequency} \\
\omega_o & \quad \text{integrator cut-off frequency}
\end{align*}
Assuming the bias currents are much larger than the regulating currents and the air gap is much larger than the journal's normal excursion, the two perpendicular axes of a radial AMB can be controlled independently (ref. 3). A control scheme for each axis is presented in figure 1. There are four parallel loops processing a journal displacement measurement. The top three loops, including a phase-lead network \((b > a)\), form a conventional PID controller. The fourth loop comprises a typical second-order band-pass filter with the center frequency at \(\omega_c\), bandwidth \(B\), and gain \(C_p\) (ref. 5). The filter can have a fixed center frequency or it may vary the center frequency with rotor speed. The latter is called a tracking filter and its implementation was explained by SKF (ref. 6). The filter output is subtracted from the displacement measurement that is fed into the phase-lead network. Also, the output multiplied by a gain \(\Delta C_d\) is added to the basic PID signals that control the power amplifier. The gain \(\Delta C_d\) is defined below.

\[
\Delta C_d = C_d - \frac{K_m}{(K_i G)}
\]

where

\[
C_d = \text{proportional loop gain, volt/m}
\]

\[
G = \text{amplifier gain, A/volt}
\]

\[
K_m = \text{magnetic stiffness}
\]

\[
= 2 \left( \frac{F_1}{C} + \frac{F_3}{C} \right) \cos \theta, \text{lb/m}
\]

\[
K_i = \text{current stiffness}
\]

\[
= 2 \left( \frac{F_1}{I_1} + \frac{F_3}{I_3} \right), \text{lb/amp}
\]

\[
C = \text{radial air gap, m}
\]

\[
I_1, I_3 = \text{bias current of quadrants 1 or 3, respectively, A}
\]

\[
F_1, F_3 = \text{magnetic forces per pole due to bias currents, N}
\]

The parameter \(K_m\) is the "negative spring" effect of the AMB magnetic field. The part of the proportional gain to overcome this effect is \(K_m/(K_i G)\). Therefore, \(\Delta C_d\) is the net proportional gain contributing to the AMB stiffness.

Assuming the power amplifiers are current sources in the bandwidth of interest \((G = \text{constant})\), the AMB regulating force across the quadrants 1 and 3 is

\[
F = K_i i + K_m Y
\]

From figure 1, the regulating current is

\[
i = G \left[ - C_d Y - C_v Z - C_e Q + \Delta C_d C_p Y_p \right]
\]
where
\[ Z = \frac{(S + a)}{(S + b)} (Y - C_p Y_p) \]  
(4)
\[ Q = \frac{1/(S + \omega_o)}{Y} \]  
(5)
\[ Y_p = \frac{BS/(S^2 + BS + \omega_c^2)}{Y} \]  
(6)

Incorporating equations (4), (5), and (6) into equation (3), which in turn is incorporated into equation (2), the AMB reaction transfer function can be expressed by equation (7).

\[-F/Y = \frac{KiG[(AC_d + CV(S + a)/(S + b))f + Ce/(S + \omega_o)]}{(S + \omega_o)} \]  
(7)

where
\[ f = 1 - C_p BS/(S^2 + BS + \omega_c^2) \]

The filter gain \( C_p \) ranges from 0 to 1. For AMB without frequency band rejection, i.e., \( C_p = 0 \), the complex stiffness as a function of exciting frequency is

\[-F/Y = \frac{KiG[(AC_d + CV(ab + \omega^2)/(b^2 + \omega^2)) + KiGCe\omega_o/(\omega^2 + \omega_o^2)]}{(S + \omega_o)} + j \omega[KiGCV(b - a)/(b^2 + \omega^2) - KiGCe/(\omega^2 + \omega_o^2)] \]  
(8)

The real part of equation (8) is the AMB stiffness and the imaginary part is the AMB damping. The second terms of both parts are the main contributors to the AMB "static stiffness" (for small \( \omega \)). The stiffness and damping of a typical AMB are presented in figure 2. It may appear unusual that the bearing damping can be negative, but as long as no rotor natural vibration mode exists in the frequency range with negative damping, there should be no dynamic problem.

Figure 3 shows the magnitude of the complex stiffnesses for the same AMB with a frequency band rejection. The filter creates a stiffness valley with a depth proportional to \( C_p \). With \( C_p \) approaching one, a dynamic force exerted on the rotor at the center frequency can be mostly balanced by the rotor inertia force. Only a small part will be resisted by the AMB and thus transmitted to the bearing stator or ground. In the following section, a potential stability problem of creating such a stiffness valley will be discussed.

**AMB STABILITY AT FILTER CENTER FREQUENCY**

Figure 4 is a zoomed-in view of the stiffness valley of figure 3. The local complex stiffness decreases and increases sharply but continuously around a center frequency \( \omega_c \) in a small bandwidth \( B \).

Let

\[ M = \text{rotor mass associated with the AMB} \]
\[ u, v = \text{real and imaginary parts of complex stiffness, respectively, at} \ \omega_c \ \text{before the valley was created} \]
and
\[ \sqrt{\frac{\mu}{M}} > \omega_c \]
which implies a natural frequency exists above the filter center frequency.

Imagine applying a dynamic force to the mass at a frequency \( \omega \) that is slowly increasing across the valley. There will be (if the valley is deep enough) an exciting frequency, \( \omega_L \), associated with a LHS slope stiffness, \( \mu_L \), such that,
\[ \sqrt{\frac{\mu_L}{M}} = \omega_L \]

Therefore, \( \omega_L \) is a resonance frequency. Similarly, there is a RHS slope resonance frequency:
\[ \sqrt{\frac{\mu_R}{M}} = \omega_R \]

These discussions are best illustrated by an example shown in figure 5. Note that these two resonance modes are not due to the filter circuitry alone. They are also related to the mass which the AMB sees. It is of less concern how well the modes are damped. Presumably, if the exciting force frequency, such as the rotor speed, is not drifting away from the filter center frequency, these modes can only be excited by impact type loads. Since the filters are usually implemented with narrow bandwidth, they will not be excited easily as long as they are reasonably damped and no persistant impact load exists. It is a major concern, however, that these modes may not be stable, i.e., associated with positive growth factor or negative damping. This can happen when the gain \( C_p \) is made large or approaching one. For example, the LHS slope mode at 59.4 Hz in figure 5 is unstable (growth factor = 120) with \( C_p = 1 \). It is therefore important in implementing this type of filter to know a threshold stable gain \( C_p \), which is determined in the following analysis.

Let
\[ \Delta \omega = \omega_c - \omega \]
and
\[ R = (B/2)/\Delta \omega \]

The band-reject factor in equation (7) is
\[ f = 1 - C_p(jB\omega)/[(\omega_c^2 - \omega^2) + j\omega] \]
\[ = [1 + (1 - C_p)R^2 - jC_pR]/(1 + R^2) \]

The complex stiffness around \( \omega_c \) is approximately
\[ -F/Y = (u + jv) f = U + jV \]

where
\[ U = u [1 - C_pR^2/(1 + R^2)] + v C_pR/(1 + R^2) \]
\[ V = v [1 - C_pR^2/(1 + R^2)] - u C_pR/(1 + R^2) \]
Both U and V are functions of R, which in turn is a function of the exciting
frequency near \( \omega_0 \). According to equation (12), the RHS mode \( R < 0 \) is always more
damped than the LHS mode. Note that at the center frequency \( R = \infty \), equations (11) and
(12) become

\[
U = u (1-C_p)
\]

and

\[
V = v (1-C_p)
\]

Also note that for effective force isolation, the frequency \( \sqrt{u(1 - C_p)/\mu} \) should
be one-third of \( \omega_c \) or less. To determine the threshold gain \( C_{po} \) for stability, it may
not be overly conservative to require

\[
V > 0
\]

which implies by equation (12)

\[
C_{po} \leq (1 + R^2)/[(u/v)R + R^2]
\]

(13)

In the range of 0 to 1, the minimum value of \( C_{po} \) occurs at the LHS slope where

\[
R = v/u + \sqrt{v^2/u^2 + 1}
\]

(14)

Incorporating equation (14) into equation (13), the relationship between \( C_{po} \) and \( v/u \) is
plotted in figure 6. For normal AMB applications with \( v/u < 1 \), the gain value of \( C_p \)
should be less than 0.83 according to this plot.

STABILITY OF ROTOR-AMB SYSTEM USING FREQUENCY BAND FILTERING

When two or more radial AMBs are supporting a rotor, the mass that each AMB sees is
different at different critical modes. The location of the filter center frequency
relative to these critical frequencies has a definite influence on the stability prob-
lem mentioned above. Since the influence is not straight forward, it would be appropri-
ate to investigate the stability problem in a rotor-AMB dynamic system as follows.

In a conventional rotordynamics approach, the rotor is modeled as sections of
circular beams using a finite element method. Concentrated masses and inertias are
assigned at the nodes of the beam elements for any attachments to the rotor. Gyroscopic
effect is included. For simplicity, circular orbits can be assumed and are adequate for
most AMB amplifications. For each radial AMB there are two independently controlled axes.
For each axis, there is a set of first-order differential equations (FODEs) represent-
ing the AMB dynamics. For example, the control scheme of figure 1 can be represented by
four FODEs according to equations (4), (5), and (6), which include the frequency band
filtering. The rotor-bearing coupling terms exist in equations (2) and (3).

Combining the dynamic equations of the rotor and AMBs, an electromechanical system
model can be formulated for eigenvalue evaluation. The formulation procedure is
straight forward and will not be presented here. An example eigenvalue analysis for a
simple rotor supported by two identical AMBs (fig. 7) is used to demonstrate this system
approach. The key AMB control parameters are identical for all four controlled axes
(table I). The stiffness and damping of this AMB as functions of frequency have been
plotted in figure 2.
Without the filtering, the system has natural modes at 3000 cpm, 10,000 cpm and 35,000 cpm. The rotor-AMB system has been analyzed for the filter centered at 30 Hz and 60 Hz separately. The additional LHS mode, which is less stable than the RHS one, is presented in table II for different values of $C_p$. The predicted values of $C_{po}$ using equations (13) and (14), as noted on table II, are consistently conservative.

CONCLUSIONS

A frequency band-reject scheme for reducing dynamic force to stator at a selected frequency may create AMB instability problems.

From the generic bearing point of view, this study has shown the following:

1. Depending on the mass supported by the AMB, there can be a natural vibration mode corresponding to the stiffness somewhere on each slope of the stiffness valley. Thus the frequencies of the induced modes are close to the filter center frequency.

2. There is a limit how deep the stiffness valley can be made without causing these modes to be unstable. The limit is related to the local damping-to-stiffness ratio before the valley is created. A conservative limit in terms of the ratio has been established.

From the rotor-AMB system point of view, the AMB mass varies with the rotor critical mode shapes. A rigorous approach to determine the stability is to find the eigenvalues of the electromechanical system.

REFERENCES


TABLE I - RADIAL AMB PARAMETERS

| \( K_i \) | 53.5 N/A |
| \( K_m \) | 2.27\times10^5 N/m |
| \( a \) | 163.3 rad/sec |
| \( b \) | 978.8 rad/sec |
| \( \omega_0 \) | 3.14 rad/sec |
| \( C_{d} \) | 5921 volt/m |
| \( C_{v} \) | 5626 volt/m |
| \( C_{e} \) | 31500 volt/m-sec |
| \( C_{p} \) | 0 to 1 |
| \( \omega_c \) | 188.5 rad/sec or 377.0 rad/sec |
| \( B \) | 7.54 rad/sec or 15.08 rad/sec |

TABLE II - ADDITIONAL LHS MODE

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<tr>
<th>( C_{p} )</th>
<th>Frequency (cpm)</th>
<th>log. decrement</th>
<th>( C_{p} )</th>
<th>Frequency (cpm)</th>
<th>log. decrement</th>
</tr>
</thead>
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<td>0.126</td>
<td>0.0</td>
<td>3600</td>
<td>0.126</td>
</tr>
<tr>
<td>0.4</td>
<td>1798</td>
<td>0.045</td>
<td>0.4</td>
<td>3597</td>
<td>0.064</td>
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<tr>
<td>0.5</td>
<td>1798</td>
<td>0.025</td>
<td>0.5</td>
<td>3597</td>
<td>0.049</td>
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<tr>
<td>0.6</td>
<td>1797</td>
<td>0.006 *</td>
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<td>3596</td>
<td>0.034</td>
</tr>
<tr>
<td>0.8</td>
<td>1796</td>
<td>-0.034 *</td>
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<td>3595</td>
<td>0.003 *</td>
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<tr>
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<td>-0.072 *</td>
<td>1.0</td>
<td>3593</td>
<td>-0.027 *</td>
</tr>
</tbody>
</table>

* marginal or unstable modes
AN ACTIVE MAGNETIC BEARING CONTROL SCHEME
WITH FREQUENCY BAND-REJECT FILTERING

Figure 1

STIFFNESS AND DAMPING OF A TYPICAL ACTIVE MAGNETIC BEARING

Figure 2
COMPLEX STIFFNESS OF A TYPICAL ACTIVE MAGNETIC BEARING WITH BAND-REJECT FILTERING

- Complex Stiffness, |F/M| (N/m x 10^4)
- Center Frequency = 30 Hz
- Q-Factor = 25
- C_p = 0.6

**Figure 3**

ZOOMED-IN VIEW OF A STIFFNESS VALLEY

- Complex Stiffness, |F/M| (N/m x 10^4)
- R Decreasing
- R = 1
- R = 8
- B/2
- LHS Slope
- RHS Slope

**Figure 4**
NATURAL FREQUENCY VS. EXCITING FREQUENCY

Local AMB Stiffness and Damping

Notes:
- \( M = 5.49 \text{ kg} \)
- \( u = 1.75 \times 10^6 \text{ N/m} \)
- \( v/u = 0.5 \)
- \( \omega_c = 60 \text{ Hz} \)
- Q-Factor = 25
- \( C_p = 1.0 \)

GF = Growth Factor

Exciting Frequency (Hz)

Figure 5

\( C_{po} \) VS \( v/u \)

Figure 6
Figure 7