STABILITY ANALYSIS OF POSITION AND FORCE CONTROL FOR ROBOT ARMS

NAGW-1333

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CIRSSE Document #53

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May 10, 1990

Keywords. Force control, robot control, hybrid force/position control, impedance control, compliant motion

Abstract. A stability analysis for robot manipulators under the influence of external forces is presented. Several control objectives are considered: rejecting the external force as a source of disturbance, complying to the external force as a generalized mass–spring–damper system, and actively controlling the external force when a dynamic model for the environment is available.

1. Introduction

In most robotic applications, the robot is subject to a variety of external forces (in addition to the inter–link and base constraint forces). These forces may be due to disturbances, interaction with a work piece, collision with obstacles etc. The performance of the robot controller in this type of situations is critical to the success of the overall operation. This problem has generated a lot of interest recently [1–8] but the stability analysis and control design is based on either open loop linearization or exact cancellation of the nonlinear arm dynamics. Insight can be gained by the linearized analysis, but the results are necessarily local in nature, and a global analysis is always preferable for nonlinear control systems. The method of the exact compensation for the arm dynamics requires the complete knowledge of the arm dynamical model. In practice, this information is typically inexact and the real time computation load presents implementation difficulties. This paper analyzes the force interaction problem when the control law has the simple structure of the proportional–derivative (PD) feedback plus gravity compensation. The Lyapunov method is used for the stability analysis. with the choice of the Lyapunov function candidates motivated by the energy consideration. This approach has gained popularity in the position control problem of a single unconstrained robot arm [9–12]. There has been recent extension to multiple–arm control [13], and attitude control [14] Other modern control methods can be considered in this framework [15], though the generality will not be pursued here.

We will consider three scenarios:

(1) The external force is unmodeled and regarded as a disturbance. The control objective is to maintain a set point or to track a desired trajectory, while rejecting the external force.
(2) The external force is unmodeled but the arm is to accommodate it in the specified directions.
(3) The external force is generated by the interaction between the arm end effector and the external environment. The control objective is either a pure position control or compliant force control in some specified directions and position control in the others.

In the first case, we prove two results that agree with intuition. If the external force converges to a steady state, then the arm also converges to a steady state, with the set point error proportional to the disturbance force. If the external force is bounded but possibly persistently time varying, then the state of the arm is also bounded with the bound proportional to the bound of the disturbance force. In both cases, the error bound can be made arbitrarily small with a high enough proportional gain.

In the second case, we require the arm to accommodate the external force by acting as a generalized mass–spring–damper at its end effector. This approach is very similar to the impedance control technique [7], but instead of requiring the exact compensation of the arm dynamics and the external force (which requires the full model information and the force feedback to the joint torques), *the desired impedance is used as a trajectory planning tool, and therefore any stable tracking algorithm can be used.* We also show global stability when an approximate force compensation is added as an outer loop.

In the third case, the arm is assumed to be in contact with a mass–spring–damper environment. When the arm is only under the position control (with no consideration of the force of interaction), a steady state will be reached as long as the contact is maintained for *all time*. When the force is controlled in some subspace, there are two plausible approaches. First the impedance control idea is applied and the desired trajectory is modified according to a mass–damper equation until the actual force is equal to the desired force. Any stable position control law can then be used to track the desired trajectory. This method is useful for industrial arms where direct joint torque control is not possible (the low level servo loop is fixed). When the position control law is of the PD type, the effective closed loop system has the proportional–integral–derivative (PID) structure, and the desired damping has to be chosen based on the environmental stiffness to ensure stability. For the case that the joint torque can be commanded (e.g., via direct current control), force feedback control can be directly designed. When the environment is infinitely rigid, direct unfiltered force feedback tends to be non–robust with respect to the time delay in the force measurement. The integral force control law is proposed as a remedy, and coincidentally, it becomes the same control law obtained in the first approach. If the environment is truly infinitely rigid, the integral feedback gain can be chosen to be an arbitrary positive definite matrix. For the good transient response and disturbance rejection, one would like to choose this gain as large as possible. However, we show that if the environment contains any flexibility at all, no matter how small, the integral gain must be chosen very small to avoid instability. In other words, the closed loop system is not robust with respect to the unmodeled flexible dynamics unless the integral gain is chosen very small. Since flexibility is always present in the environment and the manipulator, this result appears to place a fundamental limitation on the performance of the force control when the environment is rigid. In contrast to the past explanation of the instability phenomenon for the direct force feedback applied to rigid environment, which suggests the culprit to be the unmodeled dynamics (perhaps from sensors and actuators) [2], our results offer two alternative explanations: time delay in the force measurement in the direct force feedback case, and excessive gain in the integral force feedback case.

This paper is organized as follows. In section 2, a general model of a rigid robot arm subject to an external force is presented. The external force rejection property of the PD position control law is considered in section 3. The compliant control scheme based on a desired impedance at the arm end effector is discussed in section 4. Section 5 deals with the case that the external force is due to the interaction with a mass–spring–damper type of environment.

To simplify the presentation, throughout this paper, a force in the Cartesian space means torque and force. A Cartesian force (resp. velocity, acceleration) is denoted by a column of two vectors, torque (resp. angular velocity, angular acceleration) and force (resp. translational velocity, translational acceleration), which we call a spatial force (resp. velocity, acceleration).

2. Model for a Rigid Arm Subject to External Force

Consider an N-link rigid arm with the force exerted by the arm end effector f:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + k(\theta) = \tau - J^T f \tag{2.1}$$

where θ, $\dot{\theta}$, $\ddot{\theta}$ are the joint displacement, velocity, and acceleration vectors, $M(\theta)$ is the inertia matrix, $C(\theta,\dot{\theta})\dot{\theta}$ is the Coriolis and centrifugal torque, $k(\theta)$ is the gravity load, τ is the control torque, and J is the Jacobian from the joint velocity to the point of disturbance. The external force f may be the result of an unmodeled disturbance force (e.g., a human being pushing on the end effector) or the force of constraint when the arm is in contact with an environment. In the latter case, if the environment is a generalized mass–spring–damper system, the constrained force may be modeled by

$$M_c\alpha_c + b_c + k_c = A^T f \tag{2.2}$$

where α_c is the acceleration of the environment at a point C, b_c is the sum of Coriolis, centrifugal, gyroscopic and damping forces, k_c is the sum of the gravity force and spring force, M_c is the effective mass of the environment, A is the Jacobian from the robot tip to the point C. When the arm is in contact with the environment, the kinematic constraint is given by

$$\dot{J}\dot{\theta} + J\ddot{\theta} = A\alpha_c + a \tag{2.3}$$

$$J\dot{\theta} = Av_c \quad . \tag{2.4}$$

Several special cases are worth noting.

(1) If the arm is unconstrained but holds a payload in its end effector, then (2.2) is the payload dynamical equation about some point C and A is a square nonsingular matrix representing the Jacobian from the arm tip to C. If C is the center of the mass, then (2.2) is just the Newton's equation and Euler's equation.

(2) If the environment is a rigid surface and the arm is pushing an object along the surface, then A is a "tall" matrix, the range of which is the direction that the arm tip is free to move. Eq. (2.2) is the dynamic equation of the object that the arm is pushing. A^T now has a null space, $\mathcal{N}(A^T)$, the dimension of which does not change. Due to the infinite rigidity assumption, the force in $\mathcal{N}(A^T)$ does no work and only affects the force of constraint.

(3) If the environment behaves like a generalized mass–spring–damper, then $b_c = D_e v_c$ and $k_c = K_e(x_c - x_{c_{ref}})$ where D_e and K_e are the generalized damping and stiffness matrices, and v_c and x_c are the velocity and position (suitably parameterized) of the environment. A is the square nonsingular Jacobian matrix from the arm tip to the environment.

(4) When multiple robot arms (or fingers) grasp (either rigidly or with degrees of freedom such as in a point contact with friction) an object, the dynamic model is of the same form as (2.2) — (2.4) . A is again a tall matrix. The null space of A^T now represents the subspace of contact force that only contributes to the internal force but does no work. If the grasp is not rigid, then f is constrained to be zero in some subspace.

(5) There can be combination of the cases above. For example, an arm pushing a block along a soft surface, moving a flexible object along a hard surface, multiple arms handling an object that is in contact with a surface etc. The dynamical model can be generalized to include all these cases.

The dynamic model given in this section is the basis for the stability analysis in the subsequent sections. They are an approximation to the reality, due to the unmodeled effect of backlash, friction, actuator/sensor dynamics and saturation, sampling etc. However, within a reasonable operating range, the approximation is useful in predicting the arm's behavior.

3. Rejection of Disturbance Force

The performance of the PD robot control under an unmodeled external force disturbance is analyzed in this section. The problem is divided into three cases:

(1) The arm is under the set point control and $f(t)$ tends to a steady state as $t \to \infty$.

(2) The arm is under the set point control and $f(t)$ is a general time varying but uniformly bounded signal.
(3) The arm is under the general tracking control and $f(t)$ is a general time varying, uniformly bounded disturbance.

Only the PD plus gravity control law [9] is discussed in detail. The PD feedback terms can be in either the joint coordinate or the task (end effector) coordinate. Other types of control laws such as computed torque [16], operational space exact linearization [17,18,19], PID control [20,21], energy Lyapunov function based [11,12,22,23] and many others, can be analyzed in the same fashion and are not covered here.

First consider the case that $f \to f_{ss}$. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} + U \tag{3.1}$$

where the first term is the total arm kinetic energy and U has the interpretation of an artificial potential energy. In the unconstrained arm case ($f = 0$), two common choices of U are [9]

$$U = \frac{1}{2}\Delta\theta^T K_p \Delta\theta \tag{3.2a}$$

$$U = \frac{1}{2}\Delta x^T K_p \Delta x \tag{3.2b}$$

where $\Delta\theta = \theta - \theta_d$ and $\Delta x = x - x_d$, x is some parameterization of the task space. By taking the derivative of V along the solution trajectory (with $f = 0$), it can be shown that the following control laws

$$\tau = -K_p\Delta\theta - K_v\dot{\theta} + k(\theta) \tag{3.3a}$$

$$\tau = -J^T K_p \Delta x - J^T K_v \dot{x} + k(\theta) \tag{3.3b}$$

would render $\dot{V}$ negative semidefinite. Then by using the Invariance Principle [24], global asymptotic stability about ($\theta = \theta_d, \dot{\theta} = 0$) can be ascertained.

When $f \neq 0$, we do not expect global asymptotic stability, but it is intuitive that $f \to f_{ss}$ should result in the arm reaching a steady state, also. This behavior is proved below. For simplicity, consider the case $f = f_{ss}$. Modify U to

$$U = \frac{1}{2}(\Delta\theta - K_p^{-1}J^T(\theta_{ss})f_{ss})^T K_p(\Delta\theta - K_p^{-1}J^T(\theta_d)f_{ss}) \tag{3.4a}$$

$$U = \frac{1}{2}(\Delta x - K_p^{-1}f_{ss})^T K_p(\Delta x - K_p^{-1}f_{ss}) \tag{3.4b}$$

where θ_{ss} is given by

$$K_p(\theta_{ss} - \theta_d) + J^T(\theta_{ss})f_{ss} = 0 \quad . \tag{3.5}$$

Assume that (3.5) has a solution, θ_{ss}. This is true when K_p is sufficiently large. In the task space feedback case, there is no such requirement. In $\dot{V}$, the extra terms generated by f (from (2.1) and (3.4)) cannot be overbounded by the $-\dot{\theta}^T K_v \dot{\theta}$ term. Hence, the previous stability proof fails. To remedy the situation, we use a modified Lyapunov function candidate proposed in [12]:

$$V = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} + U + c\Delta q^T M(\theta)\dot{\theta} + c\Delta q^T K_v \Delta q \tag{3.6}$$

where Δq is either one of the following, depending on whether the joint level or the task level error is used:

$$\Delta q = \Delta\theta - K_p^{-1}J^T(\theta_d)f_{ss} \quad \text{or} \tag{3.7a}$$

$$\Delta q = \Delta x - K_p^{-1}f_{ss} \tag{3.7b}$$

and c is an arbitrary positive constant. By using the PD control law given by (3.3) , and after some algebra, $\dot{V}$ can be overbounded by

$$\dot{V} \leq -c\lambda_p \|\Delta q\|^2 - \lambda_v \|\dot{\theta}\|^2 + c\eta \|\Delta q\| \|\dot{\theta}\|^2 \quad . \tag{3.8}$$

Since $\dot{V}$ consists of a negative definite term and an indefinite higher (third) order term, $(\Delta q = 0, \dot{\theta} = 0)$ is locally exponentially stable, with the size of the domain of convergence inversely proportional to c. Since c is not used in the control law and can be chosen arbitrarily small, the domain of convergence is in fact the entire state space (the same argument used in [12]). Hence, we have the global asymptotic stability about the point $(\Delta q = 0, \dot{\theta} = 0)$. This means that the arm will always reach a steady state configuration, but will incur a position error of

$$\theta_{ss} - \theta_d = -K_p^{-1} J(\theta_{ss})^T f_{ss} \quad \text{in the joint level feedback case} \tag{3.9a}$$

$$x_{ss} - x_d = -K_p^{-1} f_{ss} \quad \text{in the task level feedback case.} \tag{3.9b}$$

If it is known that f_{ss} occurs only in certain subspace, then K_p can be selected large in the corresponding direction to reduce the steady state error. This analysis of course remains valid when the external force is applied at a point other than the end effector. Indeed, if the location of the disturbance is known, a hybrid joint level position and disturbance rejection controller can be used:

$$\tau = -K_p \Delta\theta - K_v \dot{\theta} - J^T (K_{p_1} \Delta x + K_{v_1} \dot{x}) + k(\theta) \tag{3.10}$$

where Δx and $\dot{x}$ now correspond to the position error and velocity at the point of disturbance, and can be obtained via the forward kinematics calculation, and J is the corresponding Jacobian.

Next consider the case that f is a general bounded time varying signal. Define

$$\overline{f} = \limsup_t f(t) \quad . \tag{3.11}$$

Now repeat the same Lyapunov stability analysis as above with f_{ss} replaced by $\overline{f}$. Then in the $\dot{V}$ equation (3.8) , there is an additional term proportional to $(c\|\Delta q\| + \|\dot{\theta}\|)\|f(t) - \overline{f}\|$. This is a first order term. We can no longer state that the arm will reach a steady state. Indeed, if f is persistently time varying, such a behavior is not expected. What we *can* conclude is that V is uniformly bounded, and the ultimate bound of V is proportional to $\limsup_t \|f(t) - \overline{f}\|$. The detailed stability analysis follows that in [15].

4. Hybrid Position/Force Control with Unmodeled External Force

In the last section, we have considered position control under unmodeled external force. In this section, we extend our analysis to the case that the arm is equipped with a force/torque sensor and the control objective is position control in some directions and compliant force control in others.

The approach here is very similar to the impedance control [7] in that the control objective is to make the arm tip appear like a mass-spring-damper system. The difference is that [7] includes an exact linearization control law so that the desired behavior is obtained *exactly* if the full model information is available and the external force is directly canceled, while here, we consider any stable position control law, and the desired end effector impedance just generates the desired trajectory for tracking. The stability analysis presented in the last section is also useful in this section, under the assumption that f is uniformly bounded in t. If the external environment possesses its own dynamics, then the uniform boundedness of f cannot be asserted a priori. This case will be considered in the next section.

The desired trajectory is generated from the following desired impedance behavior:

$$M_{des}\alpha_{b_{des}} + D_{des} v_{b_{des}} + K_{des}(x_{b_{des}} - x_{b_{ref}}) = -(B^T f - f_{b_{des}}) \tag{4.1}$$

where M_{des}, D_{des}, K_{des} are the desired mass, damping and stiffness, $\alpha_{b_{des}}$, $v_{b_{des}}$, $x_{b_{des}}$ are the desired acceleration, velocity and position (again, suitably parameterized) of a point b, $x_{b_{ref}}$ is the reference point for the desired spring, $f_{b_{des}}$ is the desired force of interaction at the point b, and B is the Jacobian between the tip of the arm and the point b, i.e.,

$$B = \begin{bmatrix} I & 0 \\ \tilde{r}_{tb} & I \end{bmatrix}$$

where r_{tb} is the vector from the arm tip to the point b, and $\tilde{r}$ transforms a vector r to the cross product operation $r\times$.

If τ is given by

$$\tau = J^T f + k(\theta) + C(\theta, \dot{\theta})\dot{\theta} + M(\theta)u$$

then $\ddot{\theta} = u$. u can be chosen in several ways to ensure the desired trajectory generated by (4.1) is followed asymptotically. This is commonly known as the impedance control [7]. However, any other stable tracking control laws can be used to track the desired trajectory generated by (4.1) .

When the tracking control law is of the PD type (2.3) as in Section 2, then the same analysis can be used to assure stable tracking. In particular, when the applied force converges to a steady state, then the desired arm tip trajectory converges to a set point, with a position error proportional to the steady state force.

Experiments were conducted on a PUMA 560 arm. where the force/torque output is used to drive the trajectory planner (4.1) , and the Unimation controller is used to track the desired trajectory (by running in the Internal Alter Mode in VAL II). For a range of M_{des}, D_{des} and K_{des}, a reasonable behavior is obtained when force and torque are applied to the end effector. If D_{des} is set to be too small, the end effector visibly jerks, this is due to the inability of the arm control to track a fast moving desired trajectory.

To obtain a closer approximation of the desired mass–spring–damper behavior prescribed by (4.1) , one can augment a nominal position control torque τ_o by a force compensation:

$$\tau = \tau_o + J^T f \quad . \tag{4.2}$$

In the case of constant f, there will be no steady state tracking error, since this just reduces to the standard set point control problem. There may be a concern that the direct force feedback may be susceptible to the time delay in the force measurement since an algebraic loop is formed. If f is uniformly bounded in time, at least the stability will not be affected. Indeed, if instead of the true f, an approximate f_{appr} is used in the control law (4.2) , then the situation is like in the pure position control case with no force feedback, but in the Lyapunov stability analysis, f will be replaced by $f - f_{appr}$. Hence, the tracking performance will deteriorate by an amount corresponding to the size of the force compensation error, but the stability is not affected. In fact, if f_{appr} is not too far off, including it in the control law is better than no force compensation at all. When the environment has its own dynamics (say a mass–spring–damper system), the above analysis involving the force feedback is no longer valid. Indeed, we will show in the next section that unfiltered force feedback in the case of very stiff environment is very non–robust with respect to the time delay.

By setting M_{des}, D_{des}, and K_{des} to different values, different behavior of the arm end effector can be obtained.

(1) By choosing $D_{des} = 0$, the arm acts as a spring.
(2) By choosing $K_{des} = 0$, the arm acts a damper. If a pulse of external force is applied, a displacement proportional to the integral of the applied force will result. This case is also useful in guarded approach to an obstacle. If $f_{b_{des}}$ is set to the desired force in the direction of the obstacle, then during the free motion (before contact) the desired velocity will ramp up to a maximum. The motion continues until the force of contact increases to the specified $f_{b_{des}}$. The stability issue related to this scenario is discussed in the next section.

(3) With $K_{des} = 0$ and $f_{b_{des}} = 0$, the arm is in the compliance mode. The arm will move in the direction of the applied force and remains in the new configuration once the force is removed. This mode is useful in positioning the arm by dragging the arm tip by hand.
(4) By choosing B other than the identity, the arm complies at a point other than the tip of the arm. In this way, one can obtain an active remote center compliance (RCC) device, which is useful for tasks involving insertions [25].

5. Position/Force Control with External Force Model

When the arm's end effector interacts with an environment that has its own dynamics then the boundedness assumption on the external force, which was used in the last section, can no longer be asserted a priori. In this section, the dynamics of the environment is assumed to be given by (2.2) . The following control objects are considered:

(1) The arm is under the position set point control.
(2) The arm is under force control in some subspace and position control in its complement.

Only joint level PD plus gravity compensation set point control law is considered. Generalization to other control laws can be done in a similar manner.

Assume that the environment is passive in the sense that

$$v_c^T b_c \geq 0 \tag{5.1}$$

$$k_c(x_c) = \nabla_{x_c} U_c(x_c) \quad . \tag{5.2}$$

This is satisfied if b_c models the Coriolis, centrifugal, gyroscopic, and damping effects, and the force field on the environment is conservative (e.g., spring force). For simplicity, assume there is no gravity load on the environment (otherwise, it needs to be compensated for in the control law).

Consider the Lyapunov function candidate

$$V = \frac{1}{2} v_c^T M_c v_c + \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta} + U_c + U \tag{5.3}$$

where the first two terms are the kinetic energies of the environment and the arm, respectively, and U is the artificial potential energy as given in (3.2a). By using (2.1) and (2.2) , and the control law (3.3a), the derivative of V along the solution can be computed as:

$$\dot{V} = v_c^T(-b_c - k_c + A^T f) + \dot{\theta}^T(-K_v \dot{\theta} - J^T f) + v_c^T k_c \quad .$$

By using the kinematic relationship in (2.4) and the assumption that the environment is passive, it follows that $\dot{V} \leq -\dot{\theta}^T K_v \dot{\theta}$, and $\dot{\theta}(t) \to 0$ as $t \to \infty$. From the kinematic constraint, $v_c \to 0$, also. Hence, by using the Invariance Principle [24], the arm and the environment always reach a steady state, but will incur a steady state error given by

$$\begin{aligned} \Delta\theta_{ss} &= -K_p^{-1} J^T(\theta_{ss}) f_{ss} \\ k_c(x_{ss}) &= A^T f_{ss} \quad . \end{aligned} \tag{5.4}$$

If the environment is nonrigid in all directions, then A is square invertible, and the steady state joint error is given by

$$\Delta\theta_{ss} = -K_p^{-1} J^T(\theta_{ss}) A^{-T} k_c(x_{c_{ss}}) \quad . \tag{5.5}$$

This can be used to solve for $\Delta\theta_{ss}$, since x_{ss} is related to θ_{ss} via the forward dynamics. If the environment cannot move, then A is a tall. full rank matrix, where $\mathcal{N}(A^T)$ is the subspace in which the environment is infinitely rigid. In this case, the steady state joint error converges to a manifold (called the jam manifold in [13]):

$$\mathcal{S} = \{\theta : \theta - \theta_d + K_p^{-1} J^T(\theta_{ss}) A^{-T} k_c(x_{c_{ss}}) = 0\} \quad .$$

All configurations in S are equilibria of the closed loop system. Some preliminary investigation of the jam manifold was given in [26], but a general characterization is lacking at the present time. Even though the jam manifold is not present when the environment is flexible, in reality, the flexible model is only valid for a finite displacement, beyond which, the rigid model is more appropriate; therefore, if the steady state error in (5.5) is too large, the rigid model should be used.

So far, only the position control law has been discussed. When the environment is uncertain, an important mode of operation is position control in some directions and force control in some other directions. This is what is generally known as the hybrid force/position control problem. A number of algorithms have been proposed (see [2, Chapters 8 and 9] for a survey). All of them are based on the partition of the contact force into components in a position control subspace and an orthogonal force control subspace. However, a rigorous treatment of the closed loop stability of the hybrid force/position control when the environment is flexible is currently unavailable. Here we will only address the stability of the force control loop, and show that the control gain should be selected based on the environmental stiffness, even if there is no unmodeled dynamics which was suggested in [2] as being responsible for the instability phenomenon observed in apply force control to a stiff environment. Based on this result, two alternative explanations for the instability are suggested.

We first partition $A^T f$ in (2.2) into two orthogonal components:

$$A^T f = \begin{bmatrix} \tilde{E} \\ E^T \end{bmatrix} f = \begin{bmatrix} f_E \\ f_P \end{bmatrix}$$

where the columns of E form a basis of the position control space and columns of $\widetilde{E^T}$ form a basis of the force control subspace which is $\mathcal{N}(E^T)$. Define $\tilde{E} = \widetilde{E^T}^{\dagger}$ which is the annihilator of E. Note that E plays the same role as the nonzero columns of the selection matrix in [2] and others. A is non-singular, therefore,

$$f = \widetilde{E^T}(\tilde{E}\widetilde{E^T})^{-1} f_E + E(E^T E)^{-1} f_P \quad .$$

As the dual to force decomposition, the arm tip velocity can also be partitioned into two components:

$$v = A \begin{bmatrix} v_E \\ v_P \end{bmatrix} = \begin{bmatrix} \widetilde{E^T} & E \end{bmatrix} \begin{bmatrix} v_E \\ v_P \end{bmatrix} \quad . \tag{5.6}$$

The arm tip acceleration α can be similarly partitioned:

$$\alpha = \begin{bmatrix} \widetilde{E^T} & E \end{bmatrix} \begin{bmatrix} \alpha_E \\ \alpha_P \end{bmatrix} + a \quad . \tag{5.7}$$

where a is the centrifugal and Coriolis acceleration due the configuration dependence of $\widetilde{E^T}$ and E. Note that a is quadratic in velocity.

When the arm is not in contact with the environment, the following desired impedance may be used to drive the arm into contact with the environment and to exert a specified force:

$$M_{des}\alpha_{E_{des}} + D_{des} v_{E_{des}} = -(f_E - f_{E_{des}}) \tag{5.8}$$

where $\alpha_{E_{des}}$ and $v_{E_{des}}$ are the desired acceleration and velocity for the tip of the robot in the force control subspace.

The rationale behind (5.8) is based on the following:

(1) When the arm is not in contact with the environment, $f_E = 0$. Then $v_{E_{des}}$ will ramp up to the maximum approach velocity $D_{des}^{-1} f_{E_{des}}$. It is assumed that if the arm moves in the subspace $\mathcal{N}(E^T)$, it will eventually come in contact with the environment.

(2) Once the arm is in contact with the environment, *if* f_E ramps up to $f_{E_{des}}$, then $v_{E_{des}}$ and $\alpha_{E_{des}}$ will decrease to zero.

The question is: How is the step (2) above assured? We will answer this question below.

Assume that the environment is a generalized mass–spring–damper system:

$$M_E\alpha_E + D_E v_E + K_E(x_E - x_{E_{ref}}) = f_E \quad . \tag{5.9}$$

For simplicity, let $M_{des} = 0$ (the desired velocity is proportional to the force error). Now substitute for f_E in the desired impedance equation by using (5.9) . Then the desired velocity is

$$v_{E_{des}} = -D_{des}^{-1}(M_E\alpha_E + D_E v_E + K_E(x_E - x_{E_{ref}}) - f_{E_{des}}) \quad . \tag{5.10}$$

Integrate both sides to get the desired position set point:

$$x_{E_{des}} = -D_{des}^{-1}(M_E v_E + D_E(x_E - x_{E_{ref}}) + K_E\int_0^t (x_E(s) - x_{E_{ref}} - K_E^{-1}f_{E_{des}})\,ds) \quad . \tag{5.11}$$

Consider a control law of the form:

$$\tau = J^T F + \tau_g \tag{5.12}$$

where $\begin{bmatrix}\tilde{E}\\ E^T\end{bmatrix} F = \begin{bmatrix}F_E\\ F_P\end{bmatrix}$, f_E is for the force control, F_P is for the position control, and τ_g provides the gravity compensation. If F_E is a task level PD position control law given by (3.3 b) with the position set point given by (5.11) , then

$$\begin{aligned} F_E &= -K_p(x_E - x_{E_{des}}) - K_v v_E \\ &= -(K_v + K_p D_{des}^{-1}M_E)v_E - K_p(I + D_{des}^{-1}D_E)(x_E - x_{E_{ref}} - K_E^{-1}f_{E_{des}}) \\ &\quad - K_p D_{des}^{-1}K_E\int_0^t (x_E(s) - x_{E_{ref}} - K_E^{-1}f_{E_{des}})\,ds - K_p h \end{aligned} \tag{5.13}$$

where h is a constant vector. Note that when the environment is rigid ($K_E \to \infty$), F_E is simply the integral feedback of the force error:

$$F_E(t) = -K_p D_{des}^{-1}\int_0^t (f_E(s) - f_{E_{des}}(s))\,ds - K_p x_{E_{ref}} \quad . \tag{5.14}$$

In general, the control law (5.13) is a constant gain PID position controller with a bias.

To analyze the closed loop stability, we first write the dynamic equation for the tip acceleration α:

$$\alpha = JM^{-1}(J^T(F - f) - C(\theta,\dot{\theta})\dot{\theta}) + \dot{J}\dot{\theta} \quad . \tag{5.15}$$

In terms of components in the force and position control subspaces, we have

$$\begin{aligned} \begin{bmatrix}\alpha_E\\ \alpha_P\end{bmatrix} &= \begin{bmatrix}(\tilde{E}\widetilde{E^T})^{-1}\tilde{E}\\ (E^TE)^{-1}E^T\end{bmatrix} JM^{-1}J^T \begin{bmatrix}\widetilde{E^T}(\tilde{E}\widetilde{E^T})^{-1} & E(E^TE)^{-1}\end{bmatrix}\begin{bmatrix}F_E - f_E\\ F_P - f_P\end{bmatrix} \\ &\quad + \begin{bmatrix}(\tilde{E}\widetilde{E^T})^{-1}\tilde{E}\\ (E^TE)^{-1}E^T\end{bmatrix}(-JM^{-1}C(\theta,\dot{\theta})\dot{\theta} + \dot{J}\dot{\theta} - a) \quad . \end{aligned} \tag{5.16}$$

Clearly, the position and force control loops are coupled. This is to be expected since the dynamics in the two subspace are coupled due to the effective arm tip mass matrix $(JM^{-1}J^T)^{-1}$. (When the environment is infinitely rigid, the α_P equation is decoupled from the force control; this will be shown later.) The stability of the general hybrid force/position control in this setting will be considered in the future. Here, we will focus only on the stability of the force control loop by assuming that the position loop is tightly controlled so that $F_P - f_P$ does not affect α_E. In this case, we have (assume a non–singular configuration):

$$\tilde{E}\widetilde{E^T}(\widetilde{E^T}JM^{-1}J^T\widetilde{E^T})^{-1}\tilde{E}\widetilde{E^T}\alpha_E = (F_E - f_E) + \eta \quad .$$

where η contains only terms that are quadratic in velocity. Now substitute for F_E using (5.13) and for f_E using (5.9) , then we have

$$(M_E + \tilde{E}\widetilde{E^T}(\widetilde{E^T}JM^{-1}J^T\widetilde{E^T})^{-1}\tilde{E}\widetilde{E^T})\alpha_E + (D_E + K_v + K_pD_{des}^{-1}M_E)v_E +$$
$$(K_E + K_p(I + D_{des}^{-1}D_E))(x_E - x_{E_{ref}} - K_E^{-1}f_{E_{des}}) + K_pD_{des}^{-1}K_E\int_0^t (x_E(s) - x_{E_{ref}} - K_E^{-1}f_{E_{des}})\,ds$$
$$= h_1 + \eta$$

where h_1 is a constant.

The linearized closed loop system is of the form

$$A\dddot{x} + B\ddot{x} + C\dot{x} + Dx = \text{constant}$$

with A, B, C, D, all positive definite matrices (assume the gains are chosen so that this is true). It is shown in Appendix I that if $\|D\|$ is sufficiently small, then the unforced system is exponentially stable. With a constant forcing term, $\dot{x}$, $\ddot{x}$, and $\dddot{x}$ all tend to zero as $t \to \infty$. Therefore, α_E, v_E and $x_E - x_{E_{ref}} - K_E^{-1}f_{E_{des}}$ all tend to zero, which implies f_E tends to $f_{E_{des}}$ as required.

If D is too large, then the system becomes unstable (e.g., consider the scalar case). For our case,

$$D = K_pD_{des}^{-1}K_E \quad .$$

The significance of D_{des} lies the in the speed with which the arm approaches the environment. Large D_{des} means a slow approach and small D_{des} means a fast approach.

If the environment is stiff (large K_E), large D_{des} is needed for a stable behavior since small D_{des} will cause large integral gain, and hence instability. This instability occurs during the contact phase and will typically cause a break of the contact, but then the position control law becomes stable and the arm would start to approach the environment again. A persistent oscillation would then result with the arm tip "bouncing" off the environment. Hence, the speed of approach to the environment has to be limited in order to guarantee a stable operation.

If the environment is soft (small K_E), much smaller D_{des} can be tolerated without affecting the stability, hence, the performance as measured by the speed of approach will be better than the rigid environment case.

The above prediction has been observed in the experiments with a PUMA 560 under VAL II force control as described earlier. Even though the analysis is done for the control law (5.12) , the same reasoning holds true for joint level feedback control laws such as the one used in PUMA. A blackboard was used as the rigid surface and a cardboard box as the soft surface. As expected, the smallest acceptable value of D_{des} is drastically different for the two types of surfaces; the difference in D_{des} is by an order of magnitude. For D_{des} at the boundary of the stability region, persistent oscillation *without* breaking off contact is observed: as D_{des} is lowered beyond that point, bouncing oscillation begins. In contrast to the instability noted in [2, Chapter 8], where the cause is attributed to high frequency unmodeled dynamics, here the culprit is in the effective integral gains.

When the environment is stiff, α_E in (5.17) is approximately zero. Therefore,

$$f_E = F_E + \eta \quad . \tag{5.17}$$

Furthermore, the arm motion is decoupled from the force control as seen by rewriting (5.15) as

$$\alpha = E\alpha_P = JM^{-1}J^T(F - f) - JM^{-1}C(\theta,\dot{\theta})\dot{\theta} + \dot{J}\dot{\theta} - a \quad . \tag{5.18}$$

Again assume arm non-singularity, and multiply both sides by $E^T(JM^{-1}J^T)^{-1}$, we have

$$E^T(JM^{-1}J^T)^{-1}E\alpha_P = (F_P - f_P) + E^T(JM^{-1}J^T)^{-1}(-JM^{-1}C(\theta,\dot{\theta})\dot{\theta} + \dot{J}\dot{\theta} - a) \quad . \tag{5.19}$$

Since the Coriolis acceleration is independent of F_E and f_E (by assuming α_E and v_E are zero), F_E and f_E are decoupled from the motion. However, the motion does affect the force through η (d'Alembert force) in (5.17) .

Continue with the rigid environment assumption (i.e., $\alpha_E = 0$), if a stable position control law is used so that $\eta(t) \to 0$ as $t \to \infty$, then a plausible force control law is

$$F_E = K_F(f_E - f_{E_{des}}) + f_{E_{des}} \quad . \tag{5.20}$$

If the eigenvalues of K_F are all different from 1, then f_E converges to $f_{E_{des}}$ asymptotically (with rate determined by η). Furthermore, large K_F means improved transient response and noise rejection (see [13] for a more detailed discussion). However, any arbitrarily small time delay in the force feedback loop will cause instability for any $K_F > I$, since the discrete time equation is of the form

$$\Delta f_E((N+1)T) = K_F \Delta f_E(NT) + \eta((N+1)T)$$

where $\Delta f_E \triangleq f_E - f_{E_{des}}$. This problem is typical to feedback systems containing algebraic loops (f_E is fed directly back to f_E), and in part explains the instability phenomena observed in the past in direct force control in a rigid environment (such as noted in [2]).

A remedy for this problem was proposed in [13] by modifying the force control law to

$$F_E = f_{E_{des}} + \mathcal{C}(f_E - f_{E_{des}}) \tag{5.21}$$

where $\mathcal{C}$ is a strictly proper linear filter such that $\mathcal{I} - \mathcal{C}$ has zeros only in the open left half plane and $\mathcal{C}$ has a pole at the origin. The simplest choice of $\mathcal{C}$ is just an integrator (integral force feedback is also used in [8]):

$$F_E = f_{E_{des}} - K_F \int_0^t (f_E(s) - f_{E_{des}}(s))\, ds \quad . \tag{5.22}$$

As noted earlier, when $K_E = \infty$, this is the same as the control law (5.14) (with $K_F = K_p D_{des}^{-1}$) based on the first approach when $M_{des} = 0$. Therefore, for the infinitely rigid environment, any $D_{des} > 0$ can be used. However, for any K_E large but finite, our earlier analysis indicates that D_{des} should be selected very large to ensure stability. A similar requirement exists for the integral force feedback control (5.22) , also. For this case, the closed loop system is described by

$$\begin{aligned}(\tilde{E}\widetilde{E^T}(\widetilde{E^T} J M^{-1} J^T \widetilde{E^T})^{-1} \tilde{E}\widetilde{E^T} + M_E)\alpha_E + (D_E + K_F M_E)v_E + (K_E + K_F D_E)(x_E - x_{E_{ref}} - K_E^{-1} f_{E_{des}}) \\ + K_F K_E \int_0^t (x_E(s) - x_{E_{ref}} - K_E^{-1} f_{E_{des}})\, ds = h + \eta\end{aligned} \tag{5.23}$$

where h is a constant. Again, based on the linearized system, the integral gain $K_F K_E$ must be sufficiently small to ensure stability. This indicates a discontinuous behavior at $K_E = \infty$: any K_F (and D_{des}, in the first approach) can be used when $K_E = \infty$, but as soon as K_E becomes finite, no matter how large, then K_F must be sufficiently small (or D_{des} sufficiently large). This result is not entirely surprising since the very rigid case can be considered as a singularly perturbed system with the infinitely rigid system as the unperturbed case (the fast poles are pushed off to infinity). As soon as the perturbation becomes nonzero, the poles at infinity emerge into the complex plane, either from the right or the left, depending on the integral gain. This implies that the infinitely rigid model is *not* robust with respect to the unmodeled flexible dynamics, unless the integral gain is chosen sufficiently small. Since flexibility is always present (in force/torque sensor, in joint motors, etc.), the force integral feedback gain should always be selected sufficiently small to ensure the closed loop stability.

6. Conclusion

Stability issues involving the control of a robot arm under the influence of external forces are discussed in this paper. Several different scenarios are considered: position control with the external force as an unmodeled disturbance, compliant control for a bounded external force in some subspace, and compliant control for a force due to the interaction with an environment whose dynamical behavior can be modeled. In each of these cases, a stability analysis using the Lyapunov's method is presented. A new explanation of instability is suggested in the case that the environment has flexibility and the gains are inappropriately chosen. When the environment is stiff in the force control subspace, robust (in time delay) stability can be achieved via the integral force feedback. However, the integral feedback gain should be chosen sufficiently small to account for possible flexibility in the system. A natural direction for generalization is to adaptively estimate the environmental stiffness and select the gain accordingly. This topic is currently under investigation as the next stage of this research.

Acknowledgment

This research is supported in part by the New York Center of Advanced Technology at Rensselaer Polytechnic Institute, and in part by the National Aeronautics and Space Administration through the Center of Intelligent Robotic Systems for Space Exploration at Rensselaer Polytechnic Institute under grant NAGW 1333.

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Appendix

Appendix I Stability Condition for a Third Order Vector Differential Equation.

LEMMA 1. *Consider the system*

$$A\dddot{x} + B\ddot{x} + C\dot{x} + Dx = 0$$

where $x \in \mathbf{R}^n$, A, B, C, D are positive definite matrices. Without loss of generality, assume A, B, C, D, are all symmetric matrices.

If $\|D\|$ is sufficiently small, then the zero equilibrium is globally exponentially stable.

PROOF: Define $z = [x, \dot{x}, \ddot{x}]^T$. Consider the quadratic form

$$V(z) = \frac{1}{2} z^T P z$$

where

$$P = \begin{bmatrix} dC + cD & dB & dA \\ dB & C + cB & cA \\ dA & cA & A \end{bmatrix}.$$

Clearly, for c sufficiently small, the lower block diagonal $2n \times 2n$ submatrix is positive definite. For each such chosen c, there exists a range for d, say, $d \in [0, d_1(c)]$, such that P is positive definite. For c and d chosen as above, $V(z)$ is a Lyapunov function candidate.

The derivative of $V(z)$ along the solution, denoted by $\dot{V}(z)$, is also a quadratic form:

$$\dot{V}(z) = -z^T Q z$$

where

$$Q = \begin{bmatrix} 2dD & 0 & -D \\ 0 & 2(cC - dB) & dA \\ -D & dA & 2(B - cA) \end{bmatrix}.$$

If we choose $d = \delta c$ such that $C > \delta B$, then for c sufficiently small, the lower diagonal $2n \times 2n$ submatrix, denoted by Q_1, is positive definite. Hence, if D is sufficiently small in the sense that

$$\|D\|^2 < 2d\lambda_{\min}(D)\lambda_{\min}(Q_1)$$

then Q is negative definite. It follows from the standard Lyapunov argument that the zero equilibrium is globally exponentially stable. ∎