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Structural Control Experiments Using $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Power Flow Approaches

(NASA-CR-192686) STRUCTURAL CONTROL EXPERIMENTS USING H2 AND H(INFINITY) POWER FLOW APPROACHES (MIT) 23 p

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Outline

Motivation

Dynamics and Power Flow Formulation

- Wave Model Based
- Measurement Based

Control Formulations

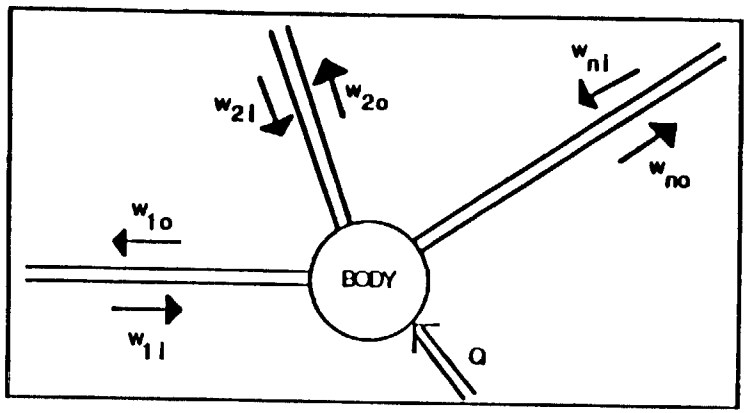
- H_2
- H_∞

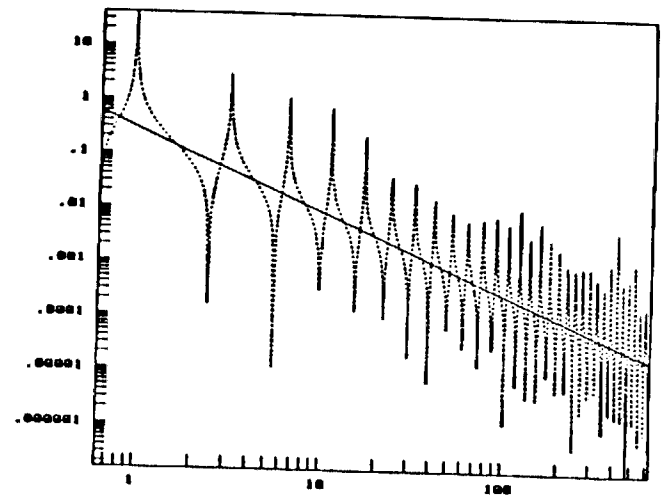
Experiment

- Setup
- H_2
- H_∞

Conclusions

Motivation





- Local models can be more easily and accurately obtained.
- Models of the direct, rather than reverberant field are more benign in frequency allowing easier formulation of control.
- Point sensors and actuators are better suited to alteration of point properties, such as transmission and reflection, than distributed properties, such as modes.
- Local models require use of local control objectives such as energy absorption and isolation.

Definitions

- The vector Q(ω) contains the control inputs

$$w_o(\omega) = S(\omega)\, w_i(\omega) + \psi(\omega) Q(\omega)$$

- The cross-sectional variables are partitioned into those which can

$$y_j(\omega) = \begin{Bmatrix} u_j \\ f_j \end{Bmatrix} = \begin{bmatrix} Y_{ui} & Y_{uo} \\ Y_{fi} & Y_{fo} \end{bmatrix} \begin{Bmatrix} w_i \\ w_o \end{Bmatrix} = Y_j(\omega) w_j(\omega)$$

  be actuated, f_j, and those which can be sensed, u_j.

- The feedback has the form

$$Q = F(\omega) w_i = G(\omega)\, u$$

- Power incoming to the active junction is defined as negatively flowing. Therefore, minimization of power flow results in more power incoming to than outgoing from the junction (*i.e.*, power absorption).

- An incoming wave mode spectrum, Φ, is assumed to give the formulation an idea of the frequency distribution of incoming power.

Power Flow: Junction

- Assembly of the member power matrices and reordering the wave mode vector yields:

$$P_{AVG} = \frac{1}{2}[w_i^H \; w_o^H]P_j\begin{bmatrix} w_i \\ w_o \end{bmatrix}$$

- Substituting of junction dynamics yields :

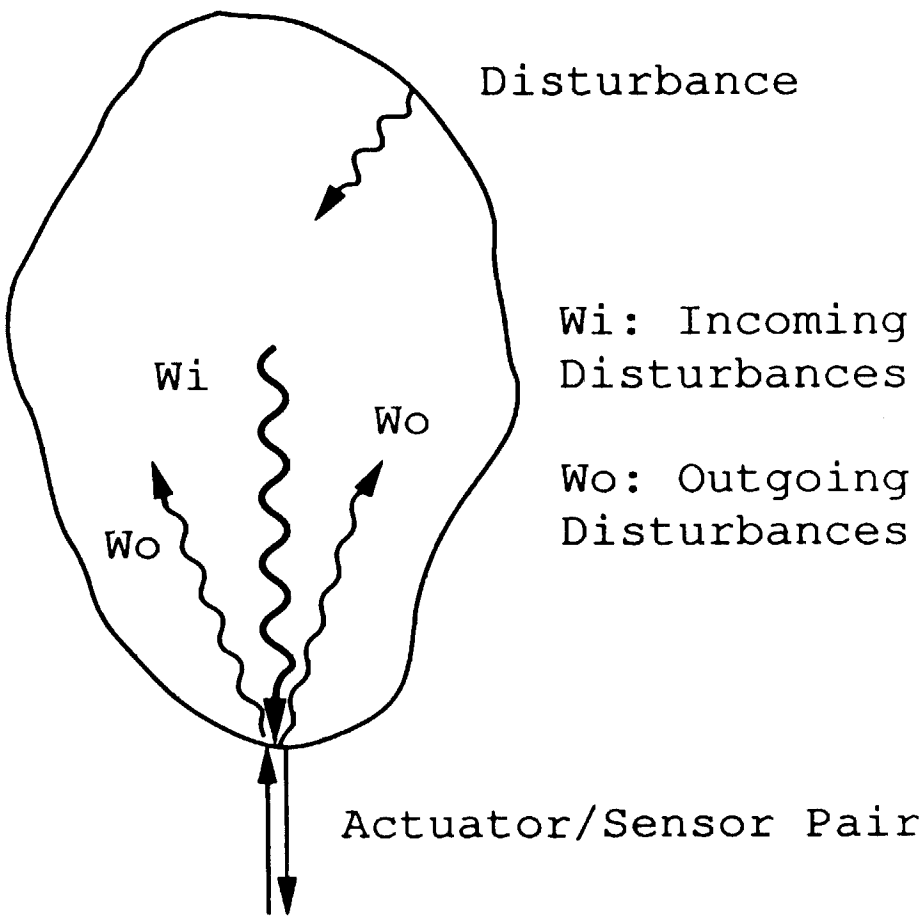
$$\begin{aligned} P_{AVG} = \frac{1}{2}\{ & w_i^H\left[P_{ii} + S^H P_{io}^H + P_{io}S + S^H P_{oo}S\right]w_i \\ & + \left[w_i^H\left[P_{io} + S^H P_{oo}\right]\psi Q + Q^H \psi^H\left[P_{io}^H + P_{oo}S\right]w_i\right] \\ & + Q^H \psi^H P_{oo}\psi Q\} \end{aligned}$$

describing homogeneous dissipation, interactive and nonhomogeneous characteristics.

- If no excitation exists at the junction, P_{OL} can be used to analyze the power dissipation properties of the junction.

$$P_{AVG} = \frac{1}{2}w_i^H[I \;\; S^H]P_j\begin{bmatrix} I \\ S \end{bmatrix}w_i = \frac{1}{2}w_i^H P_{OL}w_i$$

Arbitrary Structure: Statistical Approach



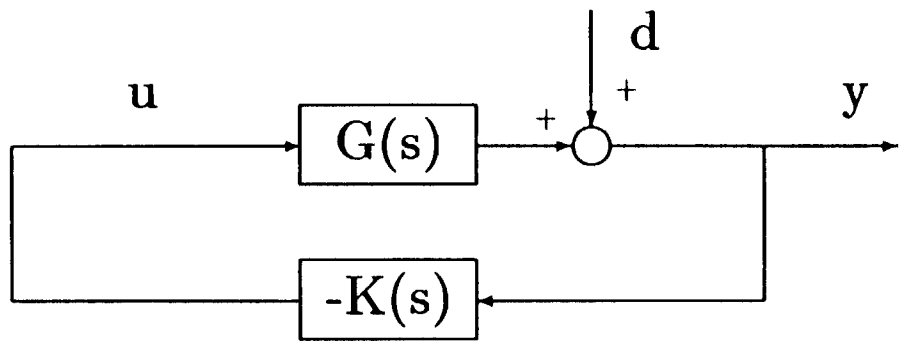
- Local model at actuator/sensor pair is desired.

Dereverberated Structural Model

- Total Response = Direct Field + Reverberant Field
- Benefits of dereverberated mobility model:
  - Can apply to any structure.
  - Accurate model of local dynamics.
- Easily computed:
  - From wave model (for a waveguide.)
  - From transfer function (experimental or analytic) by "Cepstral" analysis.
  - By averaging the logarithm of the transfer function.

Power Flow

- Find dereverberated mobility model $G(s)$, and include an additive disturbance at the output.



$$Power = \frac{1}{2}\int_{-\infty}^{\infty}\left(u(j\omega)^H y(j\omega) + y(j\omega)^H u(j\omega)\right)\frac{d\omega}{2\pi}$$

$$P(\omega) = u(j\omega)^H y(j\omega) + y(j\omega)^H u(j\omega)$$

Optimization Options

- Unconstrained: Maximize power dissipation at each frequency to give:

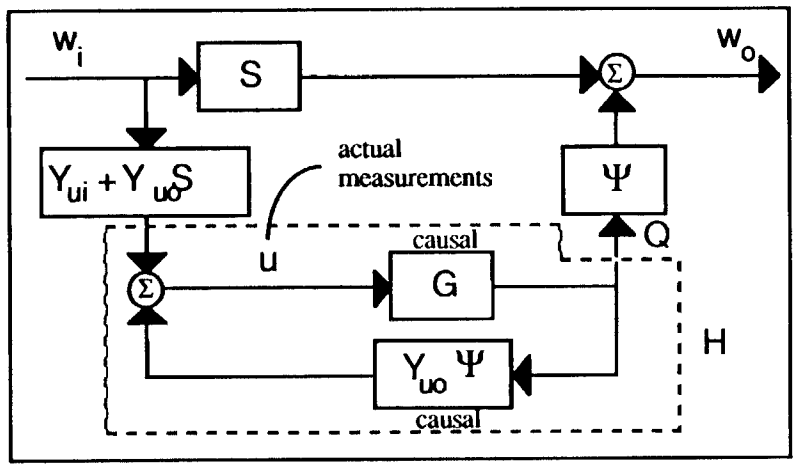
$$K = \frac{1}{G(-s)^T}$$

  - This is in general noncausal and cannot be implemented.

- $\mathcal{H}_2$ Optimization:
  - Wiener-Hopf (Miller *et al.*) or LQG.
  - Causal, but no guarantee of stability.
  - May add power at certain frequencies to achieve greater dissipation at other frequencies.

- $\mathcal{H}_\infty$ Optimization: Minimize worst case power flow.
  - Guaranteed dissipation at all frequencies $\Rightarrow$ Guaranteed stability (positive real compensator).

Control: General Formulation



Cross-sectional state feedback to mimic wave mode feedforward

- Define a cost composed of junction power flow plus control effort

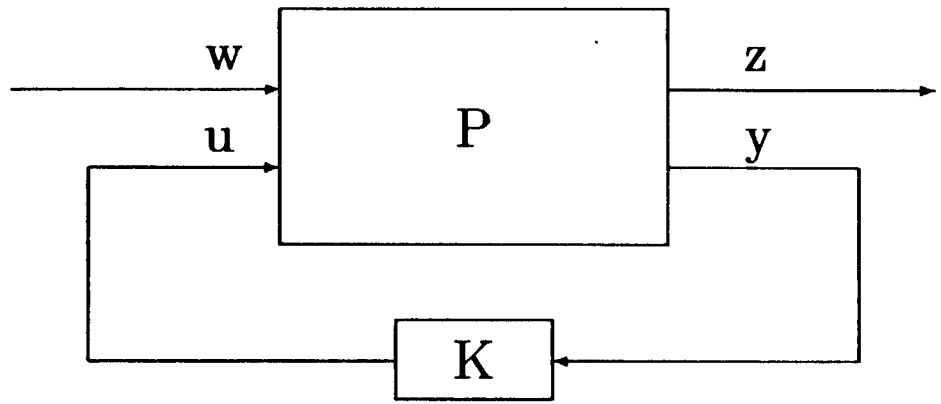
$$J = \frac{1}{2} E \left\{ \int_{-\infty}^{\infty} \left(w^H P_j w + Q^H R Q \right) d\omega \right\}$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} trace\left(P_j(\omega) \Phi_{ww}(\omega) + R(\omega) \Phi_{QQ}(\omega) \right) d\omega$$

where

$$\Phi_{ww}(\omega) = E(w\, w^H) \qquad and \qquad \Phi_{QQ}(\omega) = E(Q Q^H)$$

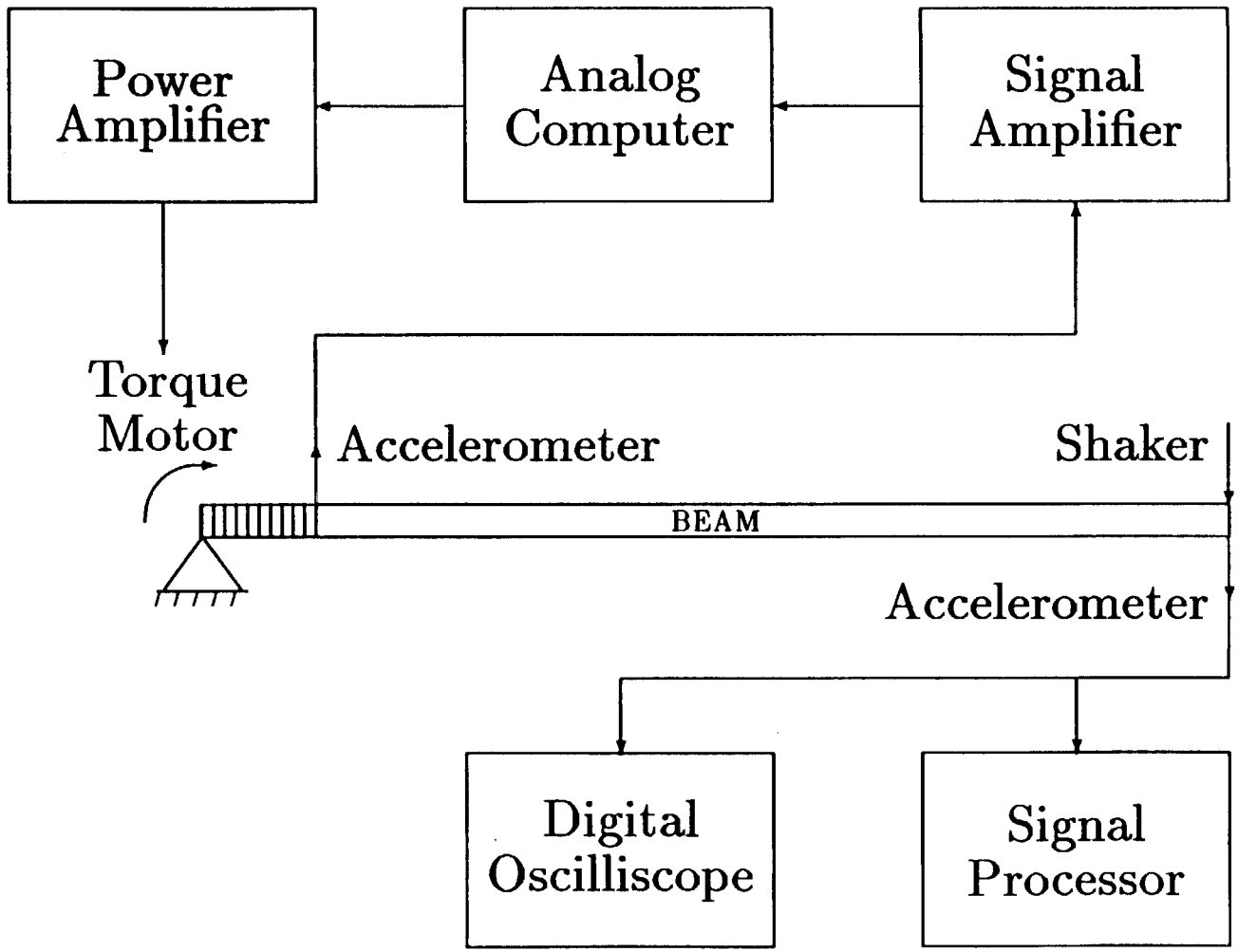
$\mathcal{H}_\infty$ Problem



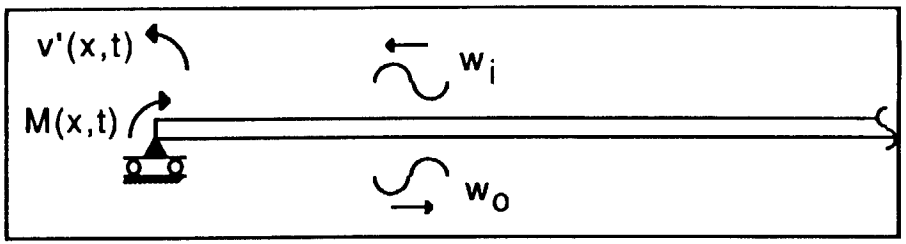
- Find $K(s)$ to minimize $\mathcal{H}_\infty$ norm of transfer function from w to z.
- Define $z = G_1 G_0^{\sim} u + G_1 w$, and $H(s) = \dfrac{z(s)}{w(s)}$, then $H^* H$ is the reflected power at each frequency for unit input.
- Then:

$$P = \begin{bmatrix} G_1 & G_1 G_0^{\sim} \\ G_0 & G \end{bmatrix}$$

Schematic of Experimental Setup



Example: Local Dynamics and Power Flow



- A Bernoulli-Euler beam

$$EI\frac{\partial^4 v(x,t)}{\partial x^4} + \rho A\frac{\partial^2 v(x,t)}{\partial t^2} = 0$$

supports four wave modes (two propagating, two evanescent)

$$v(x,t) = w_{lp}e^{ikx+i\omega t} + w_{le}e^{kx+i\omega t} + w_{rp}e^{-ikx+i\omega t} + w_{re}e^{-kx+i\omega t}$$

WAVENUMBER:

$$k = \sqrt[4]{\frac{\rho A}{EI}}\sqrt{\omega} = c_0\sqrt{\omega}$$

SCATTERING MATRIX:

$$S = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

WAVE GENERATION MATRIX:

$$\psi_M = \frac{1}{2EIk^2}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

Example: Control at a Pinned Junction

- The noncausal solution is

$$M = \sqrt{2}\, c_0 EI \sqrt{-s}\,[1\ 0]\begin{Bmatrix} v' \\ -EIv''' \end{Bmatrix}$$

- Causal fixed-form solution, choosing a $s^{1/2}$ form, is

$$M = \sqrt{2}\, c_0 EI \sqrt{s}\,[1\ 0]\begin{Bmatrix} v' \\ -EIv''' \end{Bmatrix}$$

- The Wiener-Hopf solution is

$$M = -c_0 EI\left(\sqrt{2}\sqrt{s} - \frac{8\, s^{7/4}}{(\omega_n + 5s)\sqrt[4]{\omega_n}}\right)[1\ 0]\begin{Bmatrix} v' \\ -EIv''' \end{Bmatrix}$$

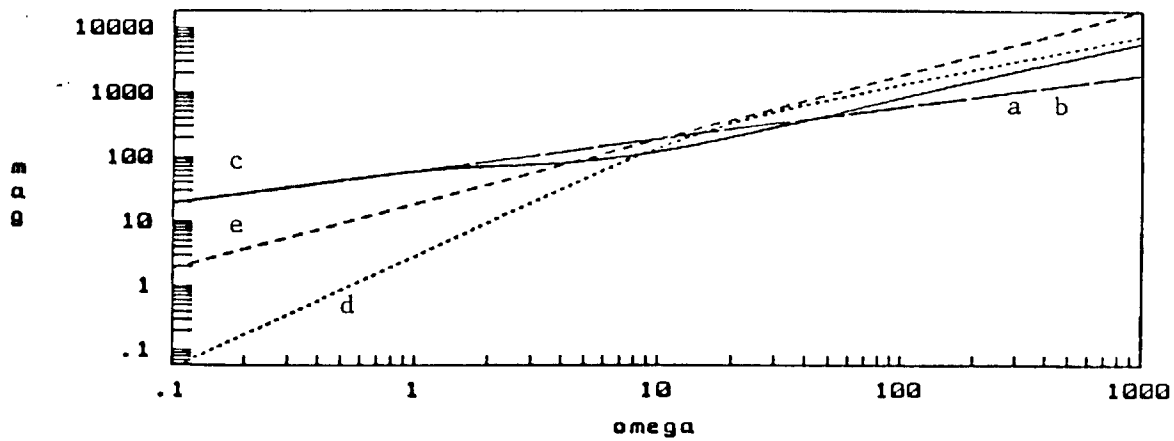
- A positive-real approximation can be used to insure stability.

$$M = \frac{8c_0 EI}{5\sqrt[4]{\omega_n}}\,\frac{s^2}{s + 92}\,\frac{1}{\sqrt[4]{s}}[1\ 0]\begin{Bmatrix} v' \\ -EIv''' \end{Bmatrix}$$

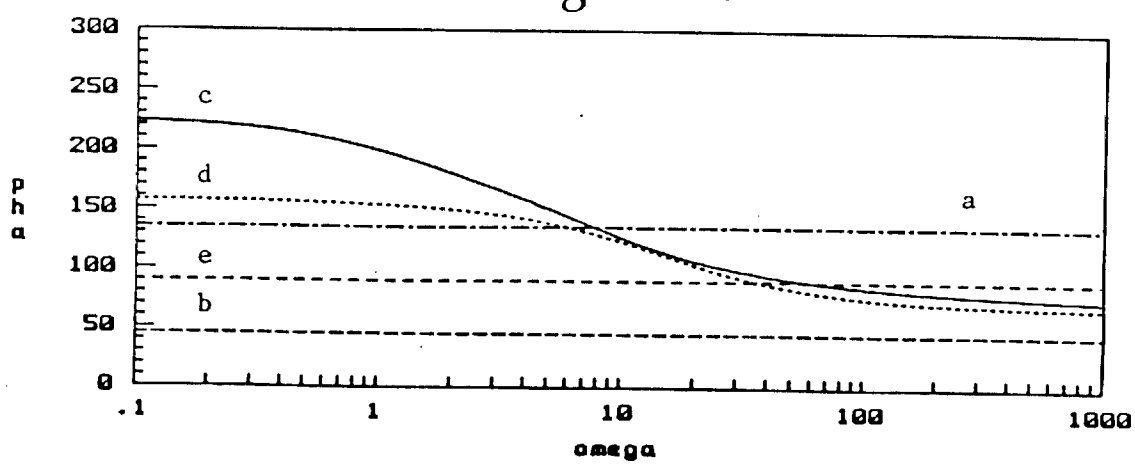
- Rate feedback optimized for 62 rad/sec.

$$M = 3s[1\ 0]\begin{Bmatrix} v' \\ -EIv''' \end{Bmatrix}$$

Example: Compensator Transfer Functions



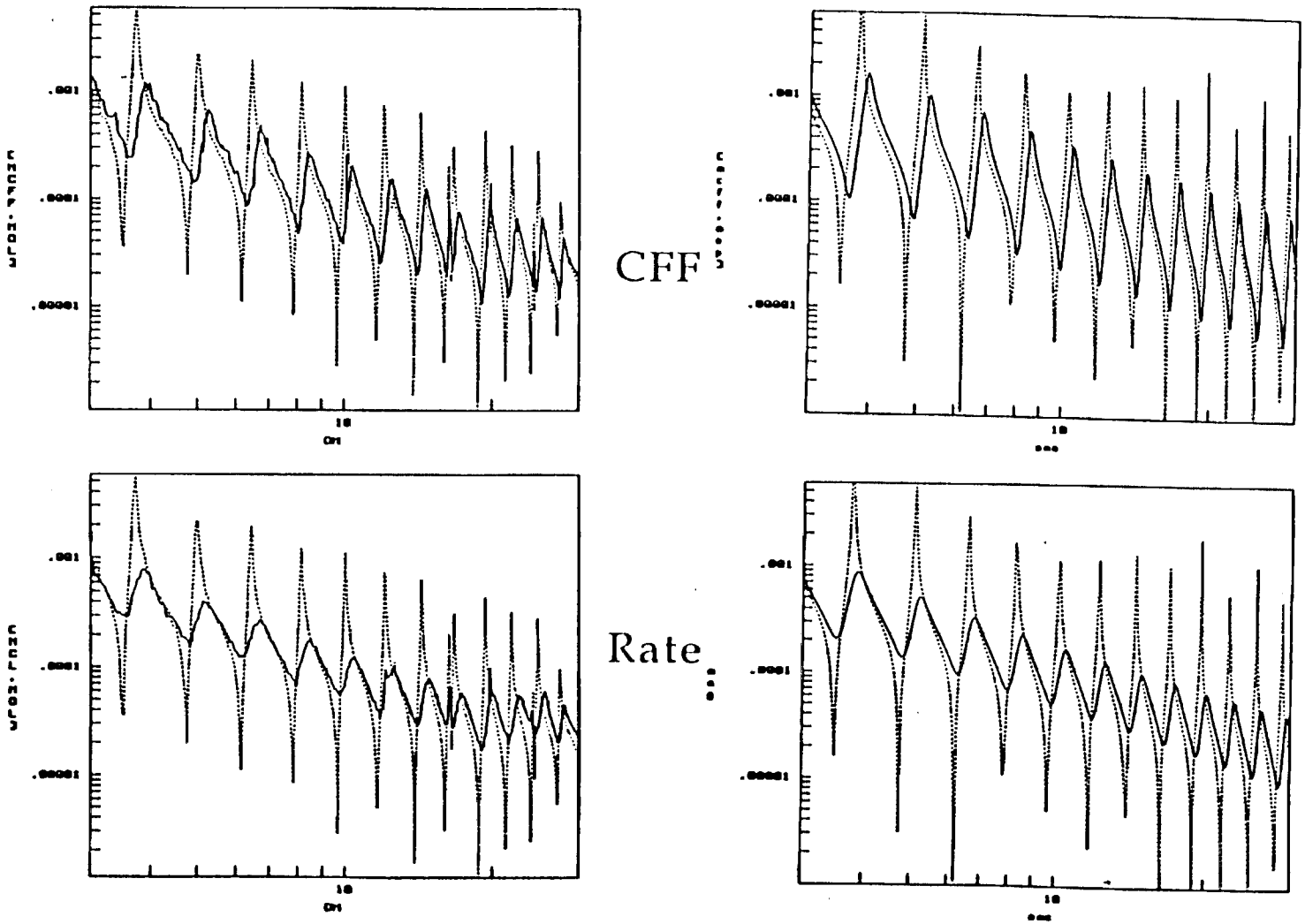
Magnitude



Phase

a) noncausal b) causal fixed-form c) Wiener-Hopf
d) Wiener-Hopf approximation e) rate feedback

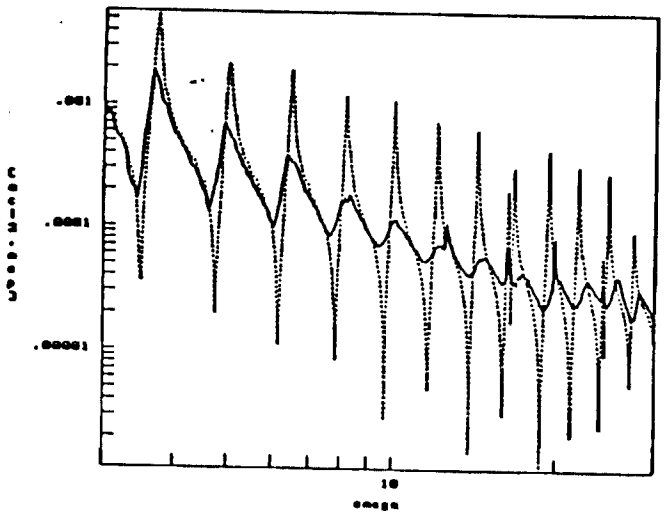
Example: Experimental Results



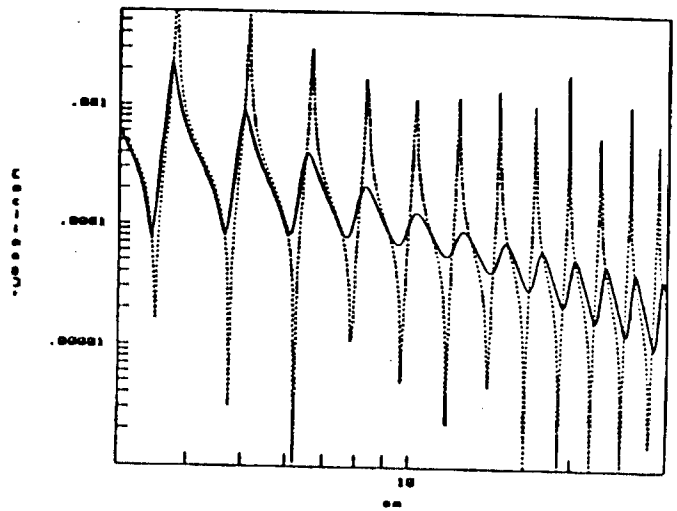
Experimental Transfer Function

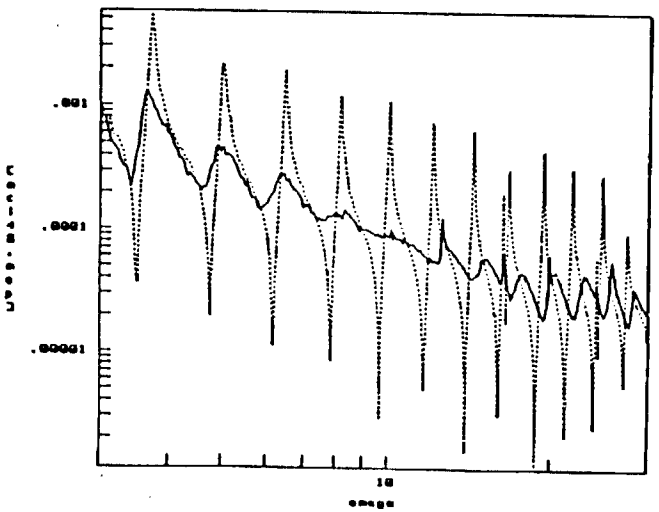
Theoretical Transfer Functionn

Example: Experimental Results (cont'd)

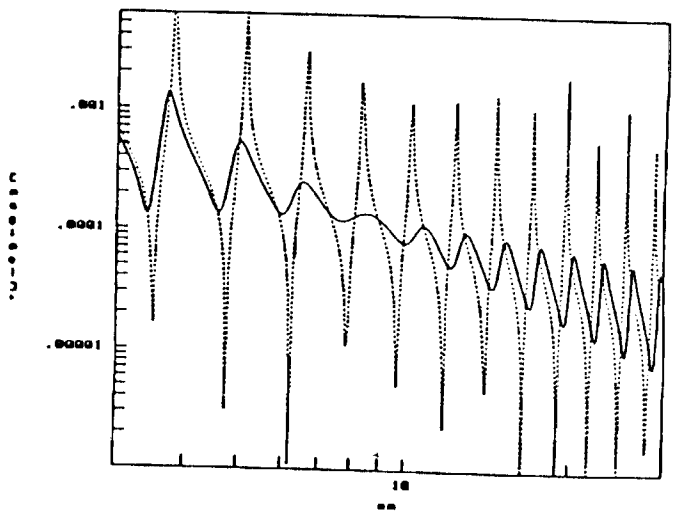


W-H app





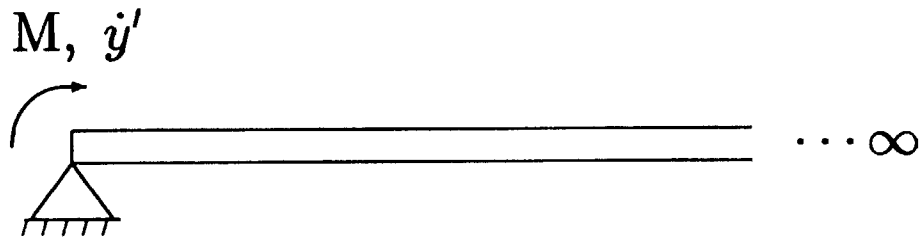
W-H max



Experimental Transfer Function

Theoretical Transfer Function

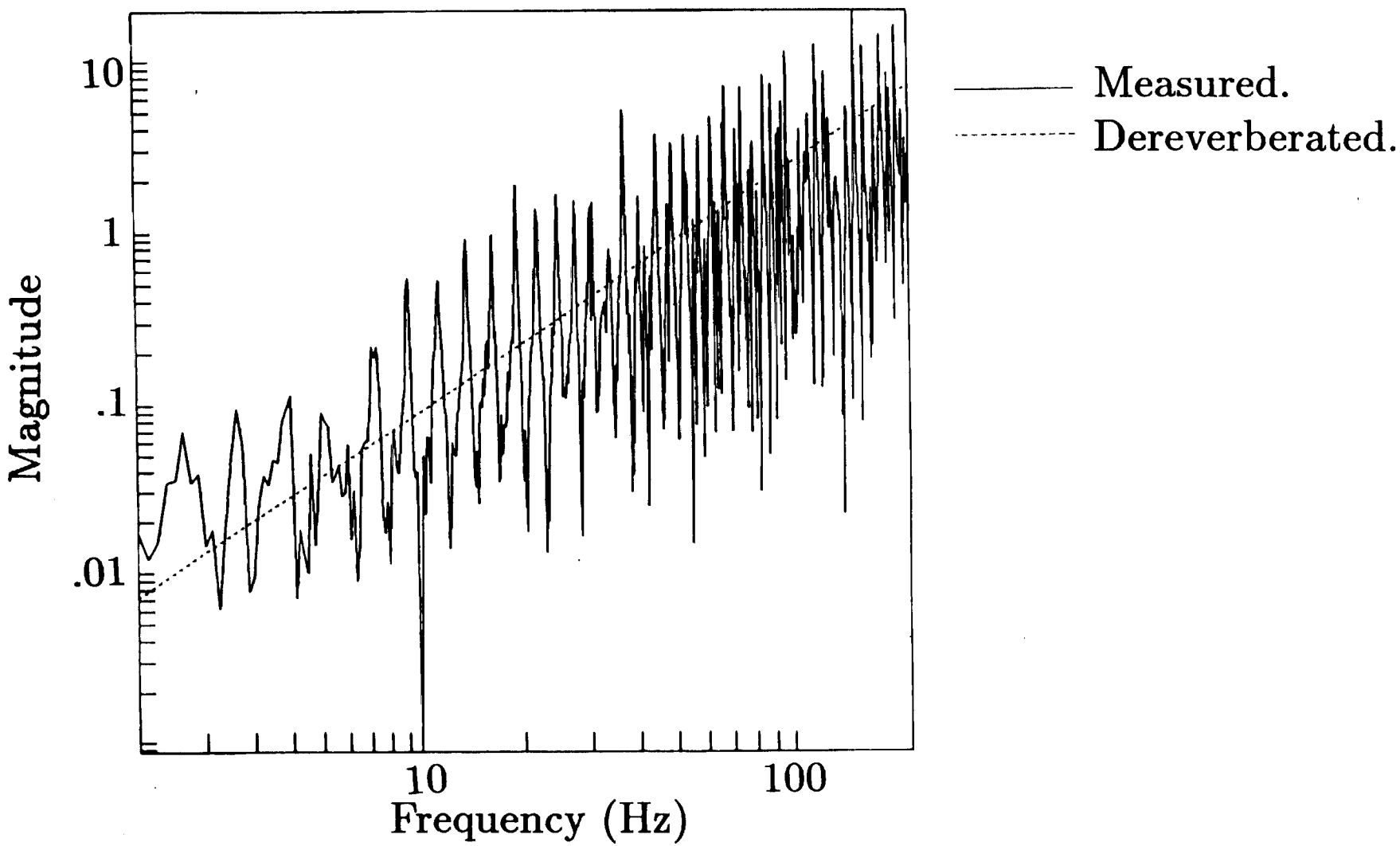
Example: Bernoulli-Euler Beam



- The dereverberated mobility for a semi-infinite pinned Bernoulli-Euler beam is

$$G(s) = \frac{\sqrt{s}}{\sqrt{2}(\rho A)^{1/4}(EI)^{3/4}}$$

Open Loop Transfer Function



Example: Compensator Designs

- The unconstrained (noncausal) compensator that dissipates the maximum power possible is

$$K(s) = \frac{\sqrt{2}(\rho A)^{1/4}(EI)^{3/4}}{\sqrt{-s}}$$

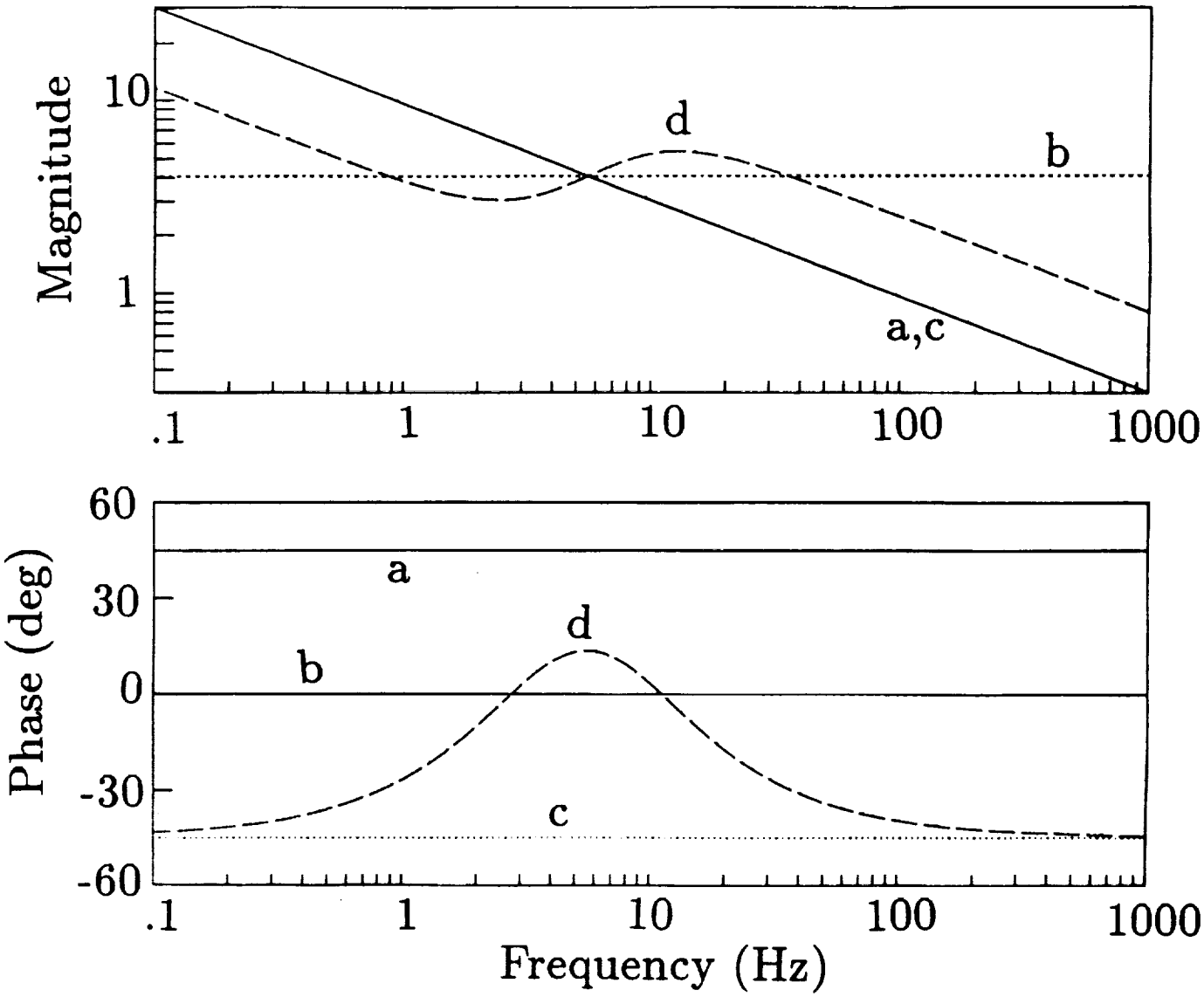
- The $\mathcal{H}_\infty$ compensator with equal weighting at all frequencies is

$$K(s) = \frac{\sqrt{2}(\rho A)^{1/4}(EI)^{3/4}}{\sqrt{s}}$$

- With a weighting function to emphasize a narrow frequency region near 35 rad/sec, the approximate optimal $\mathcal{H}_\infty$ compensator is

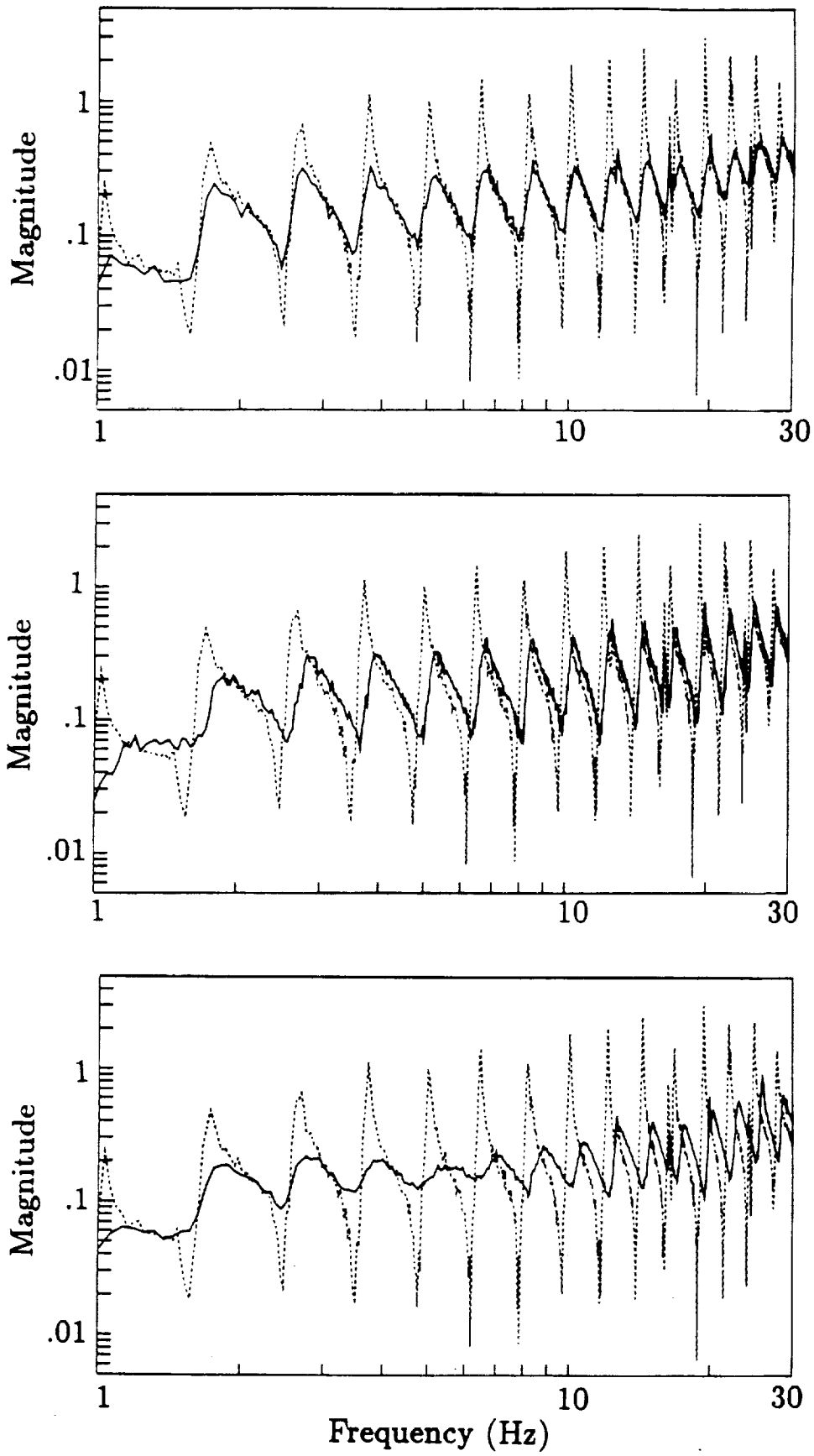
$$K(s) = \frac{\sqrt{2}(\rho A)^{1/4}(EI)^{3/4}}{\sqrt{s}} \cdot \left(2.62\frac{s^2 + 38.5s + 466}{s^2 + 100s + 3210}\right)$$

Optimal Compensators



(a) Unconstrained optimum
(b) Velocity feedback
(c) $\mathcal{H}_\infty$ design with no weighting
(d) $\mathcal{H}_\infty$ design with weighting at 35 rad/sec

Experimental Response



Velocity Feedback

Unweighted $\mathcal{H}_\infty$ Design

Weighted $\mathcal{H}_\infty$ Design

Conclusions

- The wave control solution indicates desirable compensators in the set of all positive real compensators.
- Better performance can be achieved when more than one unique type of actuator is used at a given cross-section.
- To lower the cost, the Wiener-Hopf solution may amplify waves in one frequency range in order to more effectively attenuate waves in another. This can lead to instability.
- Fractional compensators are relatively easy to implement.
- H_∞ minimization of reflected power flow guarantees stabilizing (positive-real) compensator.