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THE AERODYNAMIC ANALYSIS OF THE GYROPLANE ROTATING-WING SYSTEM

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N O T I C E

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SUMMARY

An aerodynamic analysis of the gyroplane rotating-wing system is presented herein. This system consists of a freely rotating rotor in which opposite blades are rigidly connected and allowed to rotate or feather freely about their span axis. Equations have been derived for the lift, the lift-drag ratio, the angle of attack, the feathering angles, and the rolling and pitching moments of a gyroplane rotor in terms of its basic parameters. Curves of lift-drag ratio against lift coefficient have been calculated for a typical case, showing the effect of varying the pitch angle, the solidity, and the average blade-section drag coefficient. The analysis expresses satisfactorily the qualitative relations between the rotor characteristics and the rotor parameters. As disclosed by this investigation, the aerodynamic principles of the gyroplane are sound, and further research on this wing system is justified.

INTRODUCTION

From considerations of safe flight, it is desirable that an airplane should be able to fly slowly under good control without tending to stall or spin, and should be capable of descending steeply and landing in a restricted area in the event of an engine failure. The N.A.C.A., in pursuance of its research on safety in flight, has intensively studied rotating-wing systems and found that they possess characteristics which conform closely to these safety requirements.

A preliminary analysis of the gyroplane rotating-wing system disclosed sufficient promise to justify further work. It was decided, therefore, to develop the detailed aerodynamic analysis of the gyroplane rotor as a

guide for further investigations. The analysis is based on the autogiro-rotor theory given in references 1 and 2 and experimentally verified by the data given in reference 3. The aerodynamic similarity between the gyroplane and autogiro is very close, the only difference being the type of blade motion used to eliminate rotor rolling moments in the two systems.

DESCRIPTION

The gyroplane rotor consists of four blades, the opposing blades being rigidly connected, which rotate freely under the influence of air forces about an approximately vertical axis. Each blade pair is held in bearings at the hub which permit the blades to oscillate or feather freely about the axis of the bearing; i.e., the feathering axis. The blade is usually offset, swept back. or both, to place the blade center of pressure behind the feathering axis and thus stabilize the feathering motion. Figure 1 shows the rotor analyzed in this paper; the blades are rectangular, and are offset and swept back from the feathering axis.

ANALYSIS

The aerodynamic analysis of the gyroplane is essentially similar to that of the autogiro. For this reason, the autogiro theory of Glauert and Lock (references 1 and 2) has been used as a guide in this development. The experimental verification of the autogiro analysis (reference 3) is considered indicative of the validity of this treatment of the gyroplane.

In the general case, the gyroplane rotor travels at a velocity V, and the plane perpendicular to the rotor axis is inclined at the angle α to the direction of flow of the undisturbed air. The aerodynamic analysis of the rotor will be made in two distinct parts: First, equations will be developed for the region between zero lift and the maximum lift coefficient, and second, a method for evaluating the rotor forces in the vertical-descent condition will be presented.

In the immediate neighborhood of the rotor, induced velocities are generated by the air forces acting on the

rotor. The resultant force acting on the rotor will be inclined but slightly to the rotor axis, so it will be assumed that the induced velocity is generated by the component of the resultant force along the axis. In the low-angle-of-attack condition it will be assumed that the induced velocity is constant in magnitude over the rotor disk. Then, from airfoil theory,

$$v = \frac{T}{2\pi R^2 \rho V'} \tag{1}$$

where v is the induced velocity

T is the rotor thrust

R is the rotor radius

ρ is the air density

V' is the resultant velocity at the rotor

The axial component u_z of the resultant velocity is

$$u_z = V \sin \alpha - v \tag{2}$$

and the component u_x of the resultant velocity in the plane of the disk is

$$u_x = V \cos \alpha \tag{3}$$

Let

$$u_z = \lambda \Omega R \tag{4}$$

where Ω is the rotor angular velocity and

$$u_x = \mu \Omega R \tag{5}$$

Then

$$V' = (u_z^2 + u_x^2)^{1/2} = \Omega R (\lambda^2 + \mu^2)^{1/2} \tag{6}$$

The thrust coefficient C_T will be defined by the equation

$$C_T = \frac{T}{\pi R^2 \rho \Omega^2 R^2} \tag{7}$$

Equation (2) may be written

$$\lambda \Omega R = V \sin \alpha - \frac{\frac{1}{2} C_T \Omega R}{(\lambda^2 + \mu^2)^{1/2}} \tag{8}$$

or, dividing by $\mu \Omega R$,

$$\tan \alpha = \frac{\lambda}{\mu} + \frac{\frac{1}{2} C_T}{\mu (\lambda^2 + \mu^2)^{1/2}} \tag{9}$$

The feathering of the blade pair is a periodic function of the angular position of the blade. The angle of feathering can then be expressed as a Fourier series in ψ, the angle of the blade from its downwind position. The blade position will be defined as the angular position of the blade tip projected onto the plane perpendicular to the rotor axis, and measured to the tip quarter-chord point. Since opposing blades have an equal and opposite feathering angle, the coefficients of the even multiples of ψ in the Fourier series will be zero. Then if θ is the instantaneous pitch angle of the blade,

$$\theta = a_0 - a_1 \cos \psi - b_1 \sin \psi - a_3 \cos 3\psi - b_3 \sin 3\psi \tag{10}$$

where a_0 is the pitch setting of the blade.

From figure 1 it is seen that the distance from the feathering axis of a blade element dr at r is $\epsilon R + \zeta r$, when the blade is offset a distance ϵR from the feathering axis and has a sweepback ζR at the blade tip. If U_T is the blade velocity component parallel to the rotor disk and perpendicular to the projection in the rotor disk of a radius drawn to the blade tip,

$$U_T = \Omega r + \mu \Omega R \sin \psi \tag{11}$$

The blade velocity component U_P is the component perpendicular to the rotor disk; then

$$U_P = \lambda \Omega R + (\zeta r + \epsilon R) \frac{d\theta}{dt} \tag{12}$$

If U is the resultant blade velocity in a plane perpendicular to the projection of the blade radius in the rotor disk, and φ the angle between U and the rotor disk,

$$U_T = U \cos \varphi \tag{13}$$

and

$$U_P = U \sin \varphi \tag{14}$$

The term $\lambda \Omega R$ is the principal part of U_P and is found to be less than 3 percent of the tip speed for any reasonable set of values of the rotor characteristics. It follows that in any part of the rotor disk in which the resultant velocity is large U_P is small in comparison with U_T. It will consequently be assumed that $\sin \varphi = \varphi$ and $\cos \varphi = 1$. Then

$$\left.\begin{aligned} U_T &= U \\ U_P &= \varphi U \end{aligned}\right\} \tag{15}$$

In the evaluation of the elementary air forces on the blade, it is assumed that the resultant force on a blade element lies in a plane perpendicular to the projection of the blade radius in the rotor disk, and depends only upon the resultant velocity in that plane.

The thrust on a blade element dr at a radius r is

$$dT_1 = \frac{1}{2} \rho U^2 c \, dr \, C_L \tag{16}$$

where dT is the element of thrust on one blade

c is the blade chord (assumed constant)

C_L is the lift coefficient of the blade element

The total thrust on the rotor is obtained by integrating thr thrust along the radius and taking the average value around the disk. It is considered advisable to allow for tip losses by assuming that the outer tip of the blade develops no thrust; this outer part is assumed to have a span of one half the tip chord, and the radius to this part is designated BR. A further correction is required to express in the thrust equation the fact that the velocity U_T is negative in the region bounded by $r = -\mu R \sin \psi$ and in that region the angle of attack of the blade elements requires an expression differing from that used over the rest of the disk. An additional term in the thrust integral is used to correct the expression for the angle of attack. It is assumed, probably with small error, that when

the velocity is directed from the trailing edge toward the leading edge of the airfoil, the lift curve has the same slope as for normal flow and the lift coefficient may be expressed in an equivalent manner. The total thrust T on the rotor may be expressed

$$T = \frac{b}{2\pi} \int_0^{\pi} d\psi \int_0^{BR} \frac{1}{2} \rho \, c \, U^2 \, C_L \, dr$$

$$+ \frac{b}{2\pi} \int_{\pi}^{2\pi} d\psi \int_{-\mu R \sin\psi}^{BR} \frac{1}{2} \rho \, c \, U^2 \, C_L \, dr$$

$$+ \frac{b}{2\pi} \int_{\pi}^{2\pi} d\psi \int_{0}^{-\mu R \sin\psi} \frac{1}{2} \rho \, c \, U^2 \, C_L' \, dr \qquad (17)$$

where b is the number of blades

C_L' is the lift coefficient in the reversed-velocity region

On the straight-line portion of the lift curve

$$C_L = a \, \alpha_r$$

and $$C_L' = a \, \alpha_r' \qquad (18)$$

where a is the lift-curve slope in radian measure

α_r is the angle of attack of the blade element for normal flow, measured from zero lift

α_r' is the angle of attack of the blade element for reversed flow, measured from zero lift

Also $$\alpha_r = \theta + \varphi$$

$$\alpha_r' = -\theta - \varphi \qquad (19)$$

The signs of α_r and α_r' are determined by the convention that a positive angle of attack gives a positive element of thrust, and α_r or α_r' is the acute angle between the blade chord and the resultant air flow.

Substituting (18) and (19) in (17),

$$T = \frac{b}{2\pi} \int_0^{\pi} d\psi \int_0^{BR} \tfrac{1}{2} \rho\, c\, a\, U^2 (\theta + \varphi)\, dr$$

$$+ \frac{b}{2\pi} \int_{\pi}^{2\pi} d\psi \int_{-\mu R \sin\psi}^{BR} \tfrac{1}{2} \rho\, c\, a\, U^2 (\theta + \varphi)\, dr$$

$$+ \frac{b}{2\pi} \int_{\pi}^{2\pi} d\psi \int_0^{-\mu R \sin\psi} \tfrac{1}{2} \rho\, c\, a\, U^2 (-\theta - \varphi)\, dr \qquad (20)$$

Collecting and rearranging, and substituting for U and φ from (15),

$$T = \frac{b}{2\pi} \int_0^{2\pi} d\psi \int_0^{BR} \tfrac{1}{2} \rho\, c\, a\, (\theta\, U_T^2 + U_T\, U_P)\, dr$$

$$- \frac{b}{\pi} \int_{\pi}^{2\pi} d\psi \int_0^{-\mu R \sin\psi} \tfrac{1}{2} \rho\, c\, a\, (\theta\, U_T^2 + U_T\, U_P)\, dr \qquad (21)$$

Substitute for U_T and U_P from (11) and (12), and for $d\theta/dt$ from (10), noting that $d\psi/dt = \Omega$. During integration it will be assumed that terms of higher order in μ than the fourth are negligible, and it will be shown later by inspection that a_n and b_n are of the order μ^n. The integration and simplification of (21) results in

$$T = \frac{1}{2} \rho c a b\, \Omega^2 R^3 \left\{ \frac{1}{2} \lambda (B^2 + \frac{1}{2} \mu^2) + a_0 \left(\frac{1}{3} B^3 + \frac{1}{2} \mu^2 B - \frac{4}{9\pi} \mu^3\right) \right.$$

$$\left. + \frac{1}{2} \mu a_1 \left(\epsilon B - \frac{4}{3\pi} \mu\epsilon + \frac{1}{2} \zeta B^2 - \frac{1}{8} \mu^2 \zeta\right) - \frac{1}{2} \mu b_1 \left(B^2 + \frac{1}{4} \mu^2\right) \right\} \qquad (22)$$

and

$$C_T = \frac{1}{2} \sigma a \left\{ \frac{1}{2} \lambda (B^2 + \frac{1}{2} \mu^2) + a_0 \left(\frac{1}{3} B^3 + \frac{1}{2} \mu^2 B - \frac{4}{9\pi} \mu^3\right) \right.$$

$$\left. + \frac{1}{2} \mu a_1 \left(\epsilon B - \frac{4}{3\pi} \mu\epsilon + \frac{1}{2} \zeta B^2 - \frac{1}{8} \mu^2 \zeta\right) - \frac{1}{2} \mu b_1 \left(B^2 + \frac{1}{4} \mu^2\right) \right\} \qquad (23)$$

where $\sigma = \frac{bc}{\pi R}$ and (represents, for rectangular blades, the ratio between total blade area and swept-disk area.

The aerodynamic torque on the element dr may be written

$$dQ_1 = \frac{1}{2} \rho U^2 c r dr C_L \varphi - \frac{1}{2} \rho U^2 c r dr \delta \qquad (24)$$

where δ is the average profile-drag coefficient of the blade section.

In numerical work, δ should be assigned a value that averages the high drag coefficients at large angles of attack and the smaller coefficients at low angles. A value greater by 50 percent than the minimum is suggested, inasmuch as the large angles of attack occur only at low velocities.

In the evaluation of equation (24) tip losses will be accounted for, as in the thrust equation, by integrating to the radius BR instead of to R. The drag term will, however, be integrated to the tip, since the drag is more likely to be augmented than diminished where the thrust disappears.

Summing the entire torque and taking the average value, equation (24) becomes

$$Q = \frac{b}{2\pi} \int_0^{\pi} d\psi \int_0^{BR} \frac{1}{2} \rho c a \left\{ \theta U_T U_P + U_P^2 \right\} r dr$$

$$+ \frac{b}{2\pi} \int_{\pi}^{2\pi} d\psi \int_{-\mu R \sin\psi}^{BR} \frac{1}{2} \rho c a \left\{ \theta U_T U_P + U_P^2 \right\} r dr$$

$$- \frac{b}{2\pi} \int_{\pi}^{2\pi} d\psi \int_0^{-\mu R \sin\psi} \frac{1}{2} \rho c a \left\{ -\theta U_T U_P + U_P^2 \right\} r dr$$

$$- \frac{b}{2\pi} \int_0^{\pi} d\psi \int_0^{R} \tfrac{1}{2} \rho c \delta U_T^2 r \, dr - \frac{b}{2\pi} \int_{\pi}^{2\pi} d\psi \int_{-\mu R \sin\psi}^{R} \tfrac{1}{2} \rho c \delta U_T^2 r \, dr$$

$$+ \frac{b}{2\pi} \int_{\pi}^{2\pi} d\psi \int_0^{-\mu R \sin\psi} \tfrac{1}{2} \rho c \delta U_T^2 r \, dr \tag{25}$$

In a steady state of rotation, the torque must be zero. Rearranging and equating to zero,

$$Q = \frac{b}{2\pi} \int_0^{2\pi} d\psi \int_0^{BR} \tfrac{1}{2} \rho c a \left\{ \theta U_T U_P + U_P^2 \right\} r \, dr$$

$$- \frac{b}{2\pi} \int_0^{2\pi} d\psi \int_0^{R} \tfrac{1}{2} \rho c \delta U_T^2 r \, dr$$

$$- \frac{b}{\pi} \int_{\pi}^{2\pi} d\psi \int_0^{-\mu R \sin\psi} \tfrac{1}{2} \rho c a U_P^2 r \, dr$$

$$+ \frac{b}{\pi} \int_{\pi}^{2\pi} d\psi \int_0^{-\mu R \sin\psi} \tfrac{1}{2} \rho c \delta U_T^2 r \, dr = 0 \tag{26}$$

Integrating and collecting, and neglecting terms of higher order than μ^4, as in the thrust expression,

$$0 = \frac{1}{2} \lambda^2 (B^2 - \frac{1}{2} \mu^2) + \lambda (\frac{1}{3} a_0 B^3 - \frac{1}{4} \mu b_1 B^2 + \frac{4}{3} \epsilon \mu^2 a_1 + \frac{1}{4} \mu^3 a_1 \zeta)$$

$$+ \frac{1}{2} \mu a_0 a_1 (\frac{1}{3} \zeta B^3 + \frac{1}{2} \epsilon B^2)$$

$$+ (\frac{1}{4} \zeta^2 B^4 + \frac{2}{3} \epsilon \zeta B^3 + \frac{1}{2} \epsilon^2 B^2) (\frac{1}{2} a_1^2 + \frac{1}{2} b_1^2)$$

$$- \frac{1}{16} \mu^2 \epsilon^2 (3 a_1^2 + b_1^2) - \frac{\delta}{4a} (1 + \mu^2 - \frac{1}{8} \mu^4) \tag{27}$$

The unknowns in (27) are μ, λ, and the blade-motion coefficients a_1, b_1, a_3, and b_3. The solution for λ as a function of μ may be obtained by expressing the blade-motion coefficients as functions of μ and λ. The following consideration of the blade motion will be utilized to express the blade-motion coefficients in the desired form.

The dynamic equation for the oscillation of a blade pair about the feathering axis may be written

$$I_p \frac{d^2\theta}{dt^2} = \int_0^{BR} (\epsilon R+\zeta r)\left(\frac{dT_1}{dr}\right)_\psi dr - \int_0^{BR} (\epsilon R+\zeta r)\left(\frac{dT_1}{dr}\right)_{\psi+\pi} dr \qquad (28)$$

where $\left(\frac{dT_1}{dr}\right)_\psi$ is the thrust on a blade element dr when the blade position is ψ

$\left(\frac{dT_1}{dr}\right)_{\psi+\pi}$ is the thrust on a blade element dr when the blade position is $\psi + \pi$

I_p is the moment of inertia of the blade pair about the feathering axis

From equations (10) and (21),

$$\left(\frac{dT_1}{dr}\right)_\psi = \frac{1}{2}\rho c a \Big\{(a_0 - a_1 \cos\psi - b_1 \sin\psi - a_3 \cos 3\psi - b_3 \sin 3\psi)(\Omega r + \mu\Omega R \sin\psi)^2 + (\Omega r + \mu\Omega R \sin\psi)(\lambda\Omega R + [\epsilon\Omega R + \zeta\Omega r][a_1 \sin\psi - b_1 \cos\psi + 3a_3 \sin 3\psi - 3b_3 \cos 3\psi])\Big\} \qquad (29)$$

and $\left(\frac{dT_1}{dr}\right)_{\psi+\pi}$ is identical with $\left(\frac{dT_1}{dr}\right)_\psi$ with the sign of each trigonometric function reversed.

The reversal of flow over a portion of the retreating blade is neglected in the equation for the feathering, since normally both the forces and moment arms in that portion will be very small.

From equation (10),

$$\frac{d^2\theta}{dt^2} = \Omega^2(a_1\cos\psi + b_1 \sin\psi + 9a_3\cos 3\psi + 9b_3 \sin 3\psi) \tag{30}$$

Substitute (29) and (30) in (28); integrate and collect. Then

$$\left\{ -2\, a_1 \left(\frac{1}{3}\,\epsilon B^3 + \frac{1}{4}\,\zeta B^4\right) - \frac{1}{2}\,\mu^2\, a_1\left(\epsilon B + \frac{1}{2}\,\zeta B^2\right)\right.$$

$$\left. -2b_1\left(\frac{1}{2}\,\epsilon^2 B^2 + \frac{2}{3}\,\epsilon\zeta B^3 + \frac{1}{4}\,\zeta^2 B^4\right) + \frac{1}{2}\,\mu^2 a_3\left(\epsilon B + \frac{1}{2}\,\zeta B^2\right)\right\}\cos\psi$$

$$+\left\{4\,\mu\, a_0\left(\frac{1}{2}\,\epsilon B^2 + \frac{1}{3}\,\zeta B^3\right) + 2\,\mu\lambda\left(\epsilon B + \frac{1}{2}\,\zeta B^2\right)\right.$$

$$+ 2a_1\left(\frac{1}{2}\,\epsilon^2 B^2 + \frac{2}{3}\,\epsilon\zeta B^3 + \frac{1}{4}\,\zeta^2 B^4\right) - 2b_1\left(\frac{1}{3}\,\epsilon B^3 + \frac{1}{4}\,\zeta B^4\right)$$

$$\left. - \frac{3}{2}\,\mu^2 b_1\left(\epsilon B + \frac{1}{2}\,\zeta B^2\right)\right\}\sin\psi$$

$$+\left\{\frac{1}{2}\,\mu^2\, a_1\left(\epsilon B + \frac{1}{2}\,\zeta B^2\right) - 2a_3\left(\frac{1}{3}\,\epsilon B^3 + \frac{1}{4}\,\zeta B^4\right)\right.$$

$$\left. - 6b_3\left(\frac{1}{2}\,\epsilon^2 B^2 + \frac{2}{3}\,\epsilon\zeta B^3 + \frac{1}{4}\,\zeta^2 B^4\right)\right\}\cos 3\psi$$

$$+\left\{\frac{1}{2}\,\mu^2 b_1\left(\epsilon B + \frac{1}{2}\,\zeta B^2\right) + 6a_3\left(\frac{1}{2}\,\epsilon^2 B^2 + \frac{2}{3}\,\epsilon\zeta B^3 + \frac{1}{4}\,\zeta^2 B^4\right)\right.$$

$$\left. - 2\, b_3\left(\frac{1}{3}\,\epsilon B^3 + \frac{1}{4}\,\zeta B^4\right)\right\}\sin 3\psi$$

$$= \frac{2I_p}{\rho c a R^4}\left(a_1\cos\psi + b_1 \sin\psi + 9a_3\cos 3\psi + 9b_3 \sin 3\psi\right) \tag{31}$$

Let $\dfrac{2I_p}{\rho c a R^4} = \gamma$

$$\frac{1}{3}\epsilon B^3 + \frac{1}{4}\zeta B^4 = K_1$$

$$\frac{1}{2}\epsilon B^2 + \frac{1}{3}\zeta B^3 = K_2$$

$$\epsilon B + \frac{1}{2}\zeta B^2 = K_3$$

$$\frac{1}{2}\epsilon^2 B^2 + \frac{2}{3}\epsilon\zeta B^3 + \frac{1}{4}\zeta^2 B^4 = K_4$$

Substitute the above in (31) and equate the coefficients of corresponding trigonometric terms; then

$$\left.\begin{aligned}
a_1(\gamma+2K_1+\tfrac{1}{2}\mu^2 K_3) &= -2b_1K_4+\tfrac{1}{2}\mu^2 a_3K_3 \\
b_1(\gamma+2K_1+\tfrac{3}{2}\mu^2 K_3) &= 4\mu a_0K_2+2\mu\lambda K_3+2a_1K_4+\tfrac{1}{2}\mu^2 b_3K_3 \\
a_3(9\gamma+2K_1+\mu^2K_3) &= \tfrac{1}{2}\mu^2 a_1K_3 - 6b_3K_4 \\
b_3(9\gamma+2K_1+\mu^2K_3) &= \tfrac{1}{2}\mu^2 b_1K_3 + 6a_3K_4
\end{aligned}\right\} \qquad (32)$$

The coefficients ϵ and ζ will normally be of the order of 0.1 or smaller. The term K_4 is then of the second order with respect to K_1, K_2, and K_3. A first approximation to the solution of (31) may be obtained by neglecting K_4. Then

$$b_1 = \frac{4K_2\,\mu\,a_0 + 2K_3\,\mu\,\lambda}{\gamma + 2K_1 + \frac{3}{2}\mu^2 K_3 - \dfrac{\mu^4 K_3^{\,2}}{4(9\gamma + 2K_1 + \mu^2 K_3)}} \qquad (33)$$

$$b_3 = \frac{\mu^2 K_3\, b_1}{2(9\gamma + 2K_1 + \mu^2 K_3)} \qquad (34)$$

$$a_1 = \frac{-2K_4\ b_1}{(\gamma + 2K_1 + \frac{1}{2}\mu^2\ K_3) - \frac{\mu^4\ K_3^{\ 2}}{4(9\gamma + 2K_1 + \mu^2\ K_3)}} \tag{35}$$

$$a_3 = \frac{\mu^2\ K_3\ a_1}{2(9\gamma + 2K_1 + \mu^2\ K_3)} - \frac{3\mu^2\ K_3\ K_4\ b_1}{(9\gamma + 2K_1 + \mu^2\ K_3)^2} \tag{36}$$

Equations (33) to (36), inclusive, show that a_1, b_1, a_3, and b_3 are linear functions of λ. Substitution of these equations in the torque equation results in a quadratic in λ, the only unknown, making the solution for λ simple. The larger, or positive, value of λ obtained in the solution corresponds to a positive angle of attack; the smaller, or negative, value corresponds to a negative angle of attack.

The aerodynamic pitching moment is easily determined from a consideration of the thrust variation with ψ. If M is the pitching moment about an axis passing through the rotor axis

$$M = \frac{-b}{2\pi} \int_0^{2\pi} d\psi \int_0^{BR} \left(\frac{dT_1}{dr}\right) r \cos\psi\ dr \tag{37}$$

Substituting from (16), (18), and (19), integrating and collecting

$$M = \frac{1}{2}\ bc\rho a\ \Omega^2 R^4 \left\{\frac{1}{8}\ a_1(B^4 + \frac{1}{2}\ \mu^2 B^2) + \frac{1}{8}\ b_1(\frac{4}{3}\epsilon\ B^3 + \zeta B^4)\right\} \tag{38}$$

The reversed flow is again ignored here as being of negligible importance.

Similarly, the rolling moment L' may be written

$$L' = \frac{-b}{2\pi} \int_0^{2\pi} d\psi \int_0^{BR} \left(\frac{dT_1}{dr}\right) r \sin\psi\ dr \tag{39}$$

and

$$L' = -\frac{1}{2}\, bc\rho a\, \Omega^2 R^4 \left\{\frac{1}{4}\,\mu\lambda B^2 + \frac{1}{3}\,\mu a_0 B^3 + \frac{1}{8}\, a_1\left(\frac{4}{3}\,\epsilon B^3 + \zeta B^4\right)\right.$$

$$\left. - \frac{1}{8}\, b_1\left(B^4 + \frac{3}{2}\mu^2 B^2\right)\right\} \tag{40}$$

The energy losses in the rotor arise only from the generation of thrust and the profile drag of the blades. Then

$$VD = vT + \frac{b}{2\pi}\int_0^{2\pi} d\psi \int_0^R \frac{1}{2}\,\rho\, c\,\delta\, U^3\, dr$$

$$- \frac{b}{\pi}\int_{\pi}^{2\pi} d\psi \int_0^{-\mu R \sin\psi} \frac{1}{2}\,\rho\, c\,\delta\, U^3\, dr \tag{41}$$

the second integral being added to account for the reversed velocities. But

$$L = T\cos\alpha \tag{42}$$

Then

$$\frac{D}{L} = \frac{vT}{VT\cos\alpha} + \frac{1}{VT\cos\alpha} \times \frac{b}{2\pi}\int_0^{2\pi} d\psi \int_0^R \frac{1}{2}\,\rho\, c\,\delta\, U^3\, dr$$

$$- \frac{1}{VT\cos\alpha} \times \frac{b}{\pi}\int_{\pi}^{2\pi} d\psi \int_0^{-\mu R \sin\psi} \frac{1}{2}\,\rho\, c\,\delta\, U^3 dr \tag{43}$$

Integrating and simplifying,

$$\frac{D}{L} = \frac{\frac{1}{2}\, C_T}{\mu(\mu^2 + \lambda^2)^{1/2}} + \frac{\sigma\,\delta\left(1 + 3\mu^2 + \frac{3}{8}\,\mu^4\right)}{8\mu\, C_T} \tag{44}$$

The lift coefficient of the rotor C_{L_r} may be expressed in terms of C_T from the following equation:

$$L = C_{L_r}\,\frac{\rho}{2}\, V^2\, \pi\, R^2 = T\cos\alpha$$

$$= C_T\, \pi\, R^2\, \rho\, \Omega^2\, R^2 \cos\alpha \tag{45}$$

$$C_{L_r} = C_T \frac{2\,\Omega^2 R^2}{V^2} \cos\alpha = \frac{2\,C_T \cos^3\alpha}{\mu^2} \tag{46}$$

Substituting for C_T in (44)

$$\frac{D}{L} = \frac{C_{D_r}}{C_{L_r}} = \frac{\sigma\delta(1+3\mu^2+\frac{3}{8}\mu^4)\cos^3\alpha}{4\mu^3\,C_{L_r}} + \frac{C_{L_r}\,\mu}{4\cos^3\alpha(\lambda^2+\mu^2)^{1/2}} \tag{47}$$

where C_{D_r} is the rotor drag coefficient. At small angles of incidence, λ is negligible with respect to μ, and $\cos\alpha$ is nearly unity. Then

$$C_{D_r} = \frac{\sigma\,\delta(1 + 3\,\mu^2 + \frac{3}{8}\mu^4)}{4\mu^3} + \frac{C_{L_r}^{\,2}}{4} \tag{48}$$

illustrating that at small angles of attack the drag coefficient can be expressed as the sum of a profile and an induced-drag coefficient.

The preceding equations (specifically (9), (23), (27), (33), (34), (35), (36), (38), (40), and (44)) determine completely the low-angle-of-attack operation of a gyroplane rotor when its physical dimensions and constants are known. The first step in the application of these equations is to determine a_1, b_1, a_3, b_3 as functions of λ for an assumed series of values of μ ranging from 0.07 to 0.6. The next step is to solve the torque equation for λ, after which the angle of attack, lift coefficient, and drag coefficient may be determined from the equations given. The rotor loading determines V for a given lift coefficient, and the tip speed can then be found from μ, α, and V.

In the high-angle-of-attack range, say from 50° to 90°, the equations previously developed give erroneous results. It is suggested that the drag coefficient at an angle of attack of 90° be calculated by the following method, based on an empirical relationship obtained in wind-tunnel experiments (reference 4).

Figure 2 shows a curve obtained from reference 4 that defines the relationship between the thrust coefficient of a propeller based on speed of translation and the thrust coefficient based on the velocity of the air in the neigh-

borhood of the propeller. The expressions are

$$f = \frac{T}{2 \pi R^2 \rho V^2} \tag{49}$$

and

$$F = \frac{T}{2 \pi R^2 \rho u_z^2} \tag{50}$$

where u_z is the axial flow at the propeller and f and F are thrust coefficients. Since

$$u_z = \lambda \Omega R \tag{51}$$

(from equation (12)), equation (50) becomes

$$F = \frac{C_T}{2 \lambda^2} \tag{52}$$

and if C_{D_r}' is the rotor drag coefficient at 90° angle of attack,

$$C_{D_r}' = \frac{T}{\frac{1}{2} \rho V^2 \pi R^2} = 4f \tag{53}$$

Equation (52) is evaluated from equations (23) and (27); the proper value of $1/f$ is then obtained from figure 2, from the branch of the curve labeled "windmill decelerating state," and C_{D_r}' follows at once.

Some indication has been obtained from an isolated test (reference 5) that in the high-angle-of-attack range the resultant force coefficient C_R is constant and equal to C_{D_r}'; C_{L_r} and C_{D_r} then become equal to $C_R \cos \alpha$ and $C_R \sin \alpha$, respectively, where α may be calculated from (9). The use of this relationship is not recommended without more general verification.

LIST OF SYMBOLS

Velocities:

V, velocity of translation of rotor

ΩR, tip speed of rotor

v, induced axial velocity at rotor

V', resultant velocity at rotor

u_x, component of V' in plane of disk

u_z, axial component of V'

U, resultant velocity at blade element perpendicular to blade span axis

U_T, component of U parallel to disk

U_P, component of U perpendicular to disk

Forces:

T, rotor thrust

L_r, rotor lift

D_r, rotor drag

D_r', rotor drag at 90° angle of attack

Moments:

Q, rotor torque about axis of rotation

M, rotor pitching moment

L', rotor rolling moment

Angles:

ψ, azimuth angle of blade

α, angle of attack of rotor

α_r, angle of attack of blade element

Angles (cont.):

Φ, acute angle between U and plane of disk

θ, instantaneous blade pitch angle

a_o, pitch setting of blade

Rotor constants:

a, lift curve slope of blade profile

I_p, moment of inertia of blade pair about feathering axis

c, blade chord

R, rotor radius

γ, mass constant of blade pair $= \frac{c\ \rho\ a\ R^4}{I_p}$

ϵR, offset of blade from feathering axis

ζR, sweepback of blade tip from feathering axis

σ, solidity or ratio between total blade area and disk area

δ, average profile drag coefficients of blade profile

B, factor expressing allowance to be made in integrating along radius to account for tip losses

Coefficients:

C_T, thrust coefficient $= \frac{T}{\pi\ R^2\ \rho\ \Omega^2\ R^2}$

C_{L_r}, rotor lift coefficient $= \frac{L_r}{\frac{1}{2}\ \rho\ V^2\ \pi\ R^2}$

f, propeller thrust coefficient based on undisturbed velocity $= \frac{T}{2\ \pi\ R^2\ \rho\ V^2}$

Coefficients (cont.):

F, propeller thrust coefficient based on local velocity at propeller $= \dfrac{T}{2 \pi R^2 \rho u_z^2}$

C_{D_r}', rotor drag coefficient at 90° angle of attack $= \dfrac{D_r'}{\frac{1}{2} \rho V^2 \pi R^2}$

Miscellaneous:

μ, ratio between component of speed of translation in plane of disk and tip speed

λ, ratio between axial component of resultant translational velocity and tip speed

EXAMPLES

In order to illustrate the influence of the rotor parameters on the over-all performance of the rotor, curves of L/D as functions of C_{L_r} have been calculated for a typical rotor having the following characteristics:

$$\epsilon = 0$$

$$\zeta = 0.10$$

$$\sigma = 0.10$$

$$a_0 = 0.0698 \text{ rad.} = 4^\circ$$

$$\delta = 0.0120$$

$$a = 5.00$$

$$\gamma = 0.004$$

$$B = 0.950$$

Figures 3 to 5, inclusive, show the effect of varying a_0,

σ, and δ. The calculated longitudinal and lateral positions of the rotor center of pressure in terms of percent of the radius are shown in figure 6 for the typical rotor.

DISCUSSION

The development of an aerodynamic theory of the gyroplane in a mathematical form necessarily involves simplifications and assumptions. The major sources of error in this theory are the assumptions made concerning the uniformity of the inflow and the equality of $\tan \varphi$ with φ. The inflow probably varies materially over the rotor area, considering the form and relative positions of the blade tip vortices. The influence of the uniform inflow is a rough average of the influence of the nonuniform inflow, however, and should introduce no serious errors in the expressions for the net forces. The angle φ is large only when the resultant velocity is small, so that again the errors in the net forces are small.

Errors of lesser importance are introduced by the assumptions that the aerodynamic force on the blade element is independent of velocities along the blade radius, and that the tip losses are calculable by the method given. Some energy will be dissipated in the skin friction between the blade and the radial air flow, but since any computation of this energy loss would be an approximation it was thought best to neglect it. It seems reasonable to expect this factor to be small. Tip losses have been taken into account approximately, although the accuracy of the assumption made concerning the effective radius (BR) is uncertain.

The treatment given in this paper considers the simplest form of a blade - one with constant chord and pitch angle; a similar treatment, however, can be applied to any blade in which the chord or pitch angle is a given function of the radius. It is only necessary to substitute the given function for c and a_0 before integrating from 0 to R along the radius, and the result obtained will express the desired relation. It should be remembered, however, that this aerodynamic analysis in its simplified form is of doubtful value quantitatively, although its qualitative accuracy should be satisfactory.

The illustrative examples presented in figures 3 to 5 show the type of variation in lift-drag ratio to be expected with changes in the rotor constants. It is interesting to note that a small solidity is advantageous only at very low lift coefficients. The increase in lift-drag ratio with pitch angle is somewhat misleading, since with normal airfoils the pitch angle can be increased but slightly beyond 4° without adverse effects upon the autorotation. Figure 6 shows the variation in pitching and rolling centers of pressure with lift coefficient; the rolling moment arises from the fact that the center of thrust is at a greater distance from the hub on the retreating blade, so that for swept-back blades the thrust twisting moment on this blade balances the twisting moment of a smaller thrust on the opposite blade.

The application of the aerodynamic principles presented here is essentially a structural problem. The blade pair is stressed in bending and tension, and yet must be held in the hub in bearings that permit free rotation. Torsion in the blades must be considered in relation to possible vibrations. No insuperable difficulties are anticipated, however, since the obstacles to be overcome are for the most part similar to those successfully dealt with in the autogiro.

CONCLUSIONS

The gyroplane is aerodynamically sound, and its promise justifies further research.

The aerodynamic theory of the gyroplane here developed expresses satisfactorily the qualitative relations between the rotor characteristics and the design parameters of the rotor.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 6, 1934.

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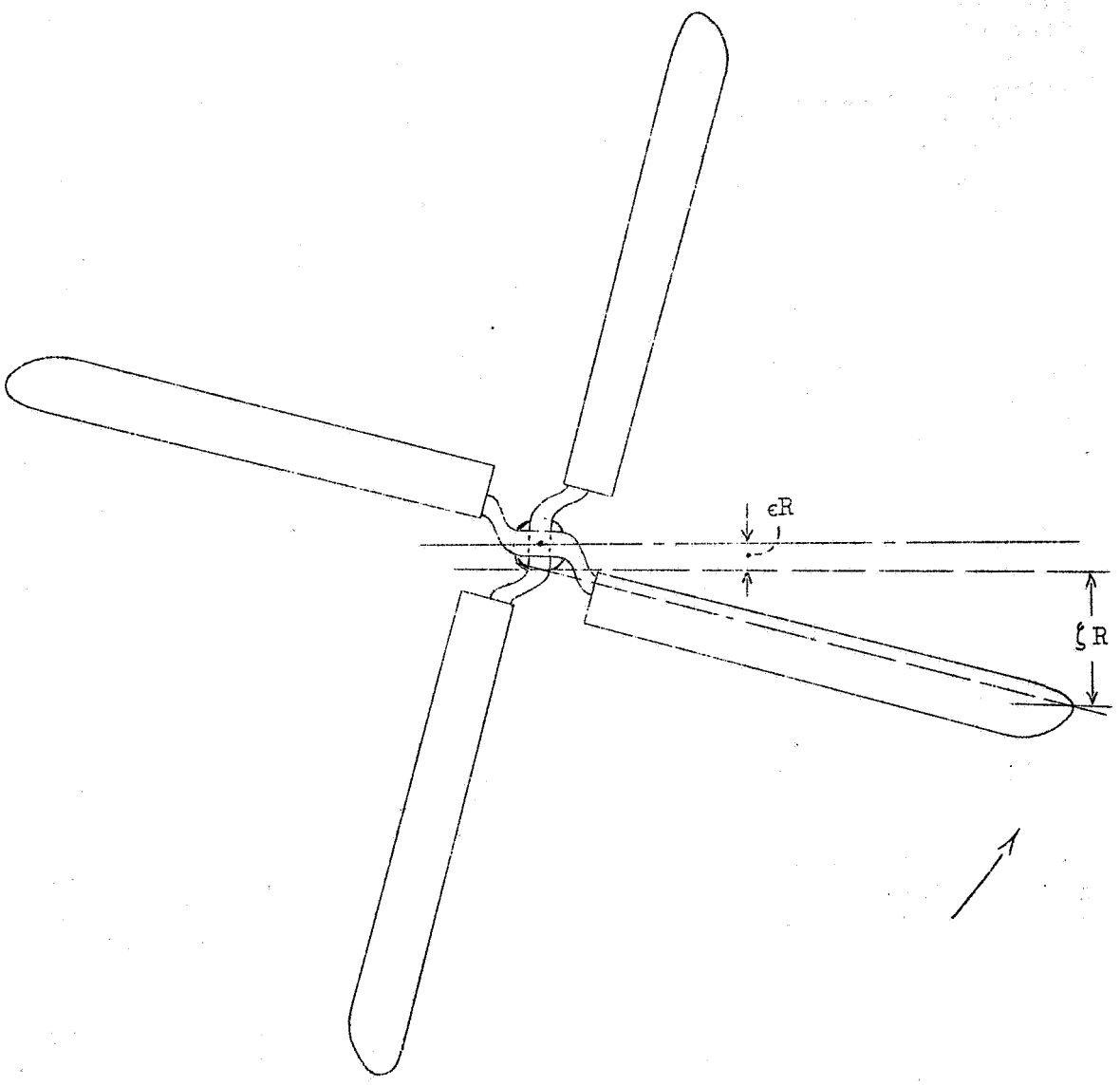

Figure 1.-Gyroplane rotor

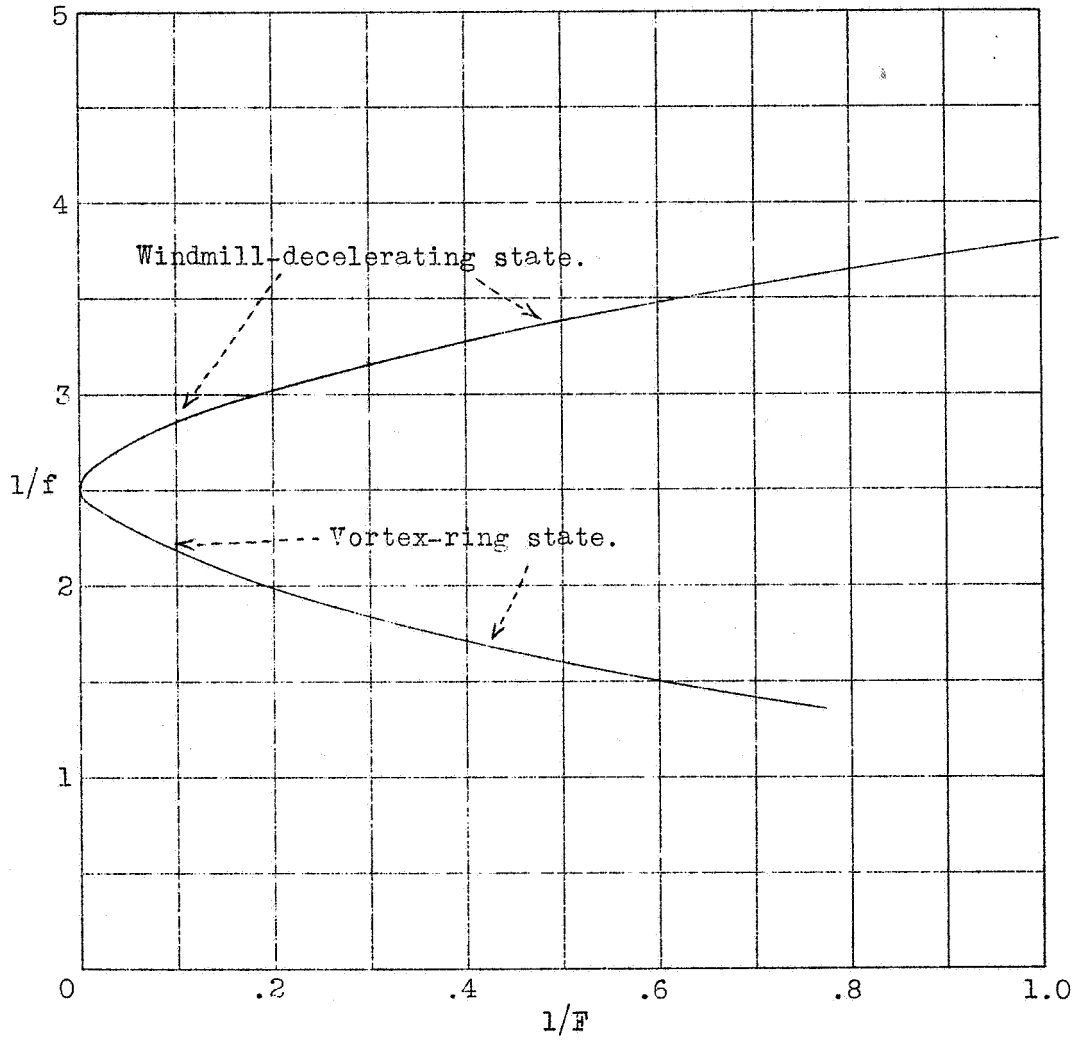

Figure 2.-Propeller thrust coefficients based on resultant velocity and velocity of translation.

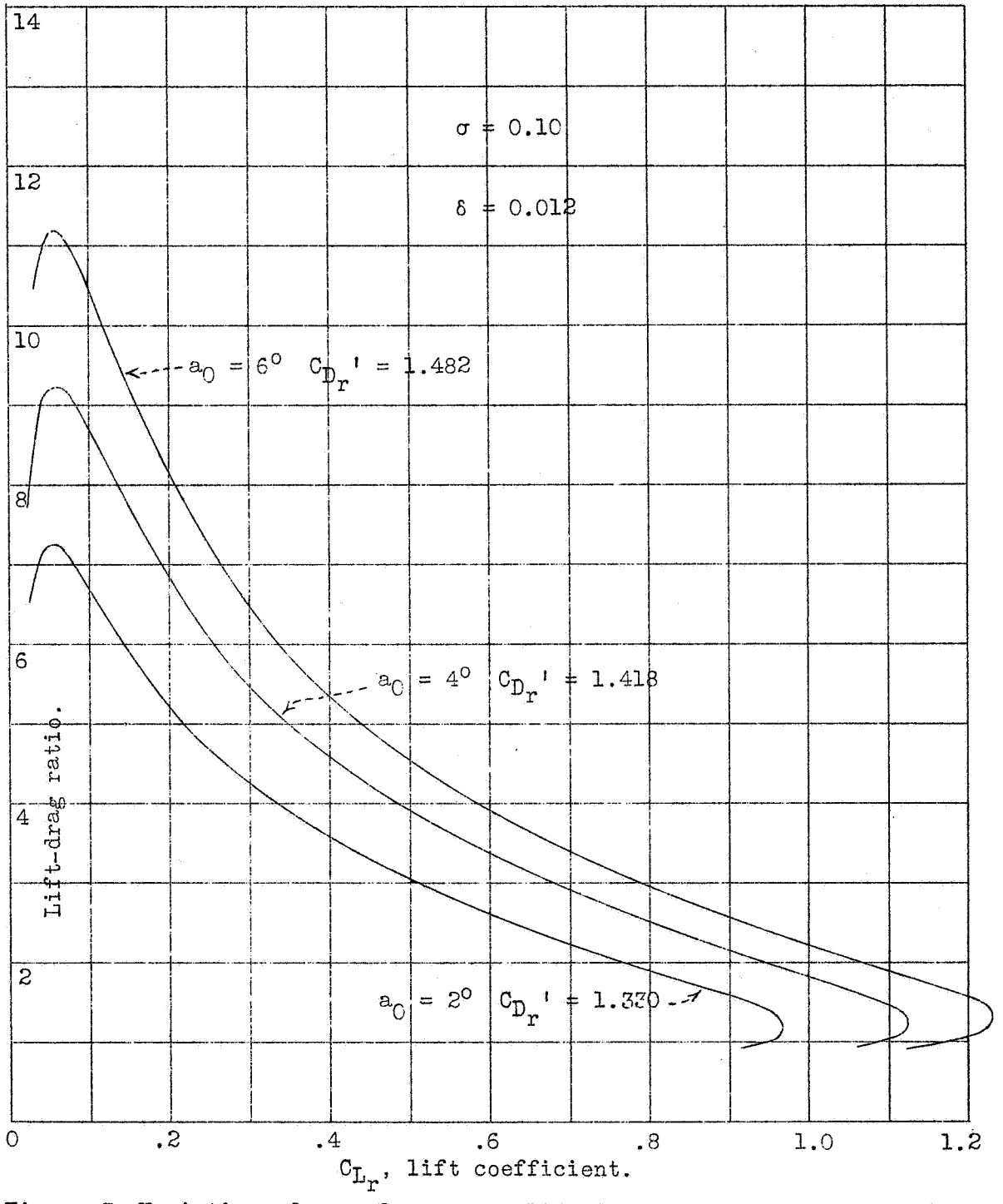

Figure 3.-Variation of gyroplane rotor lift-drag ratio with pitch angle.

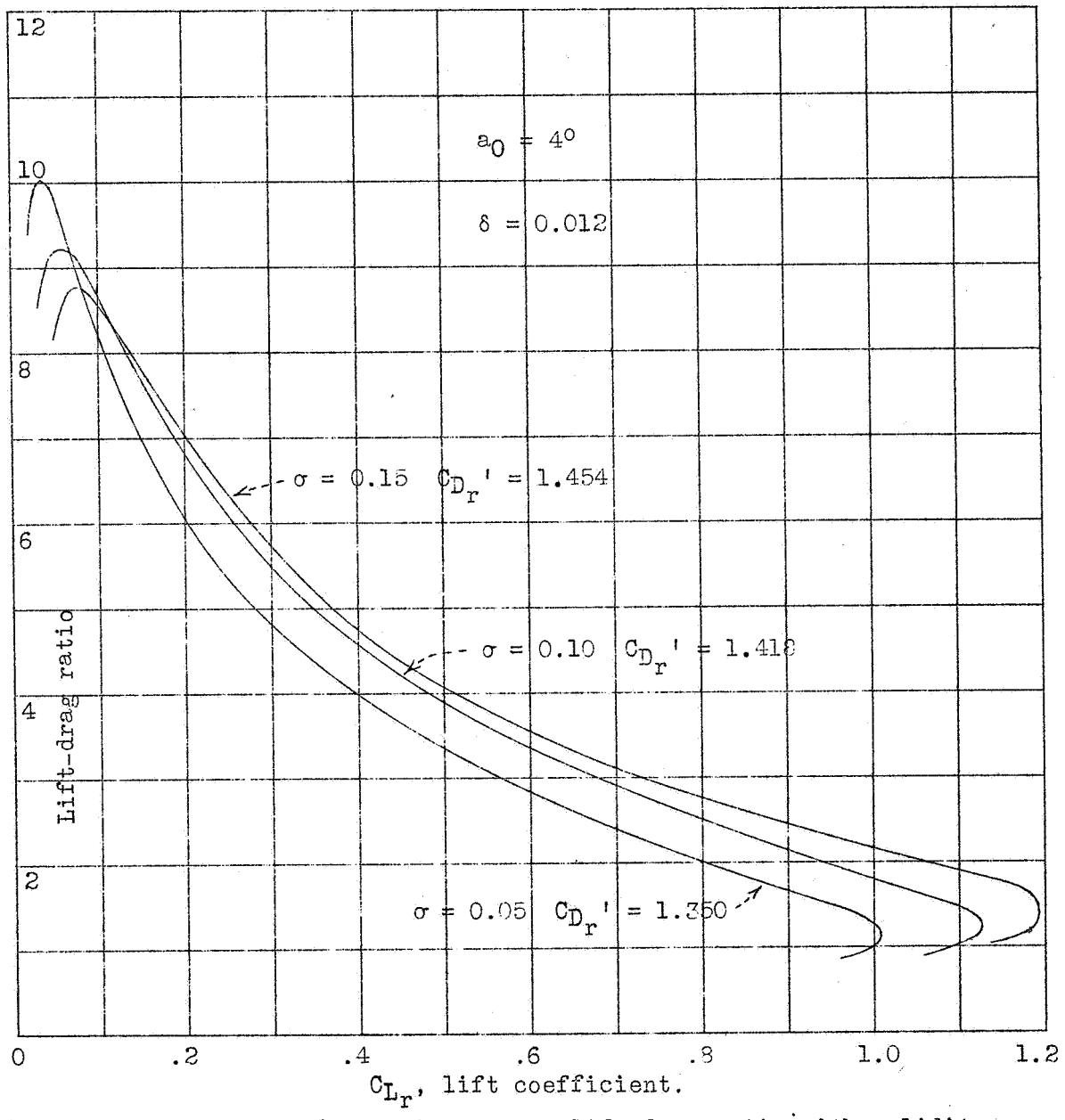

Figure 4.-Variation of gyroplane rotor lift-drag ratio with solidity.

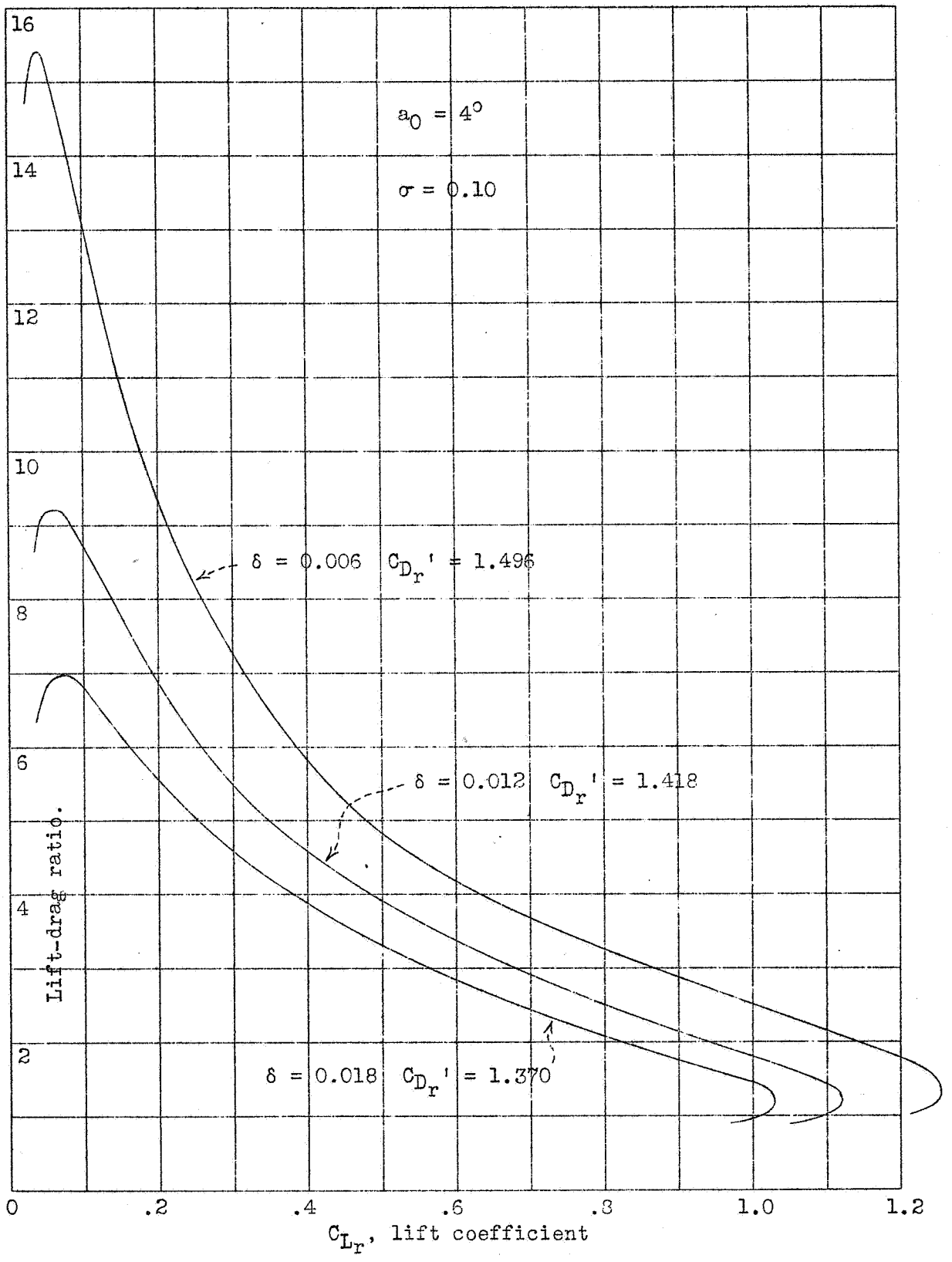

Figure 5.-Variation of gyroplane rotor lift-drag ratio with blade-section average profile drag coefficient.

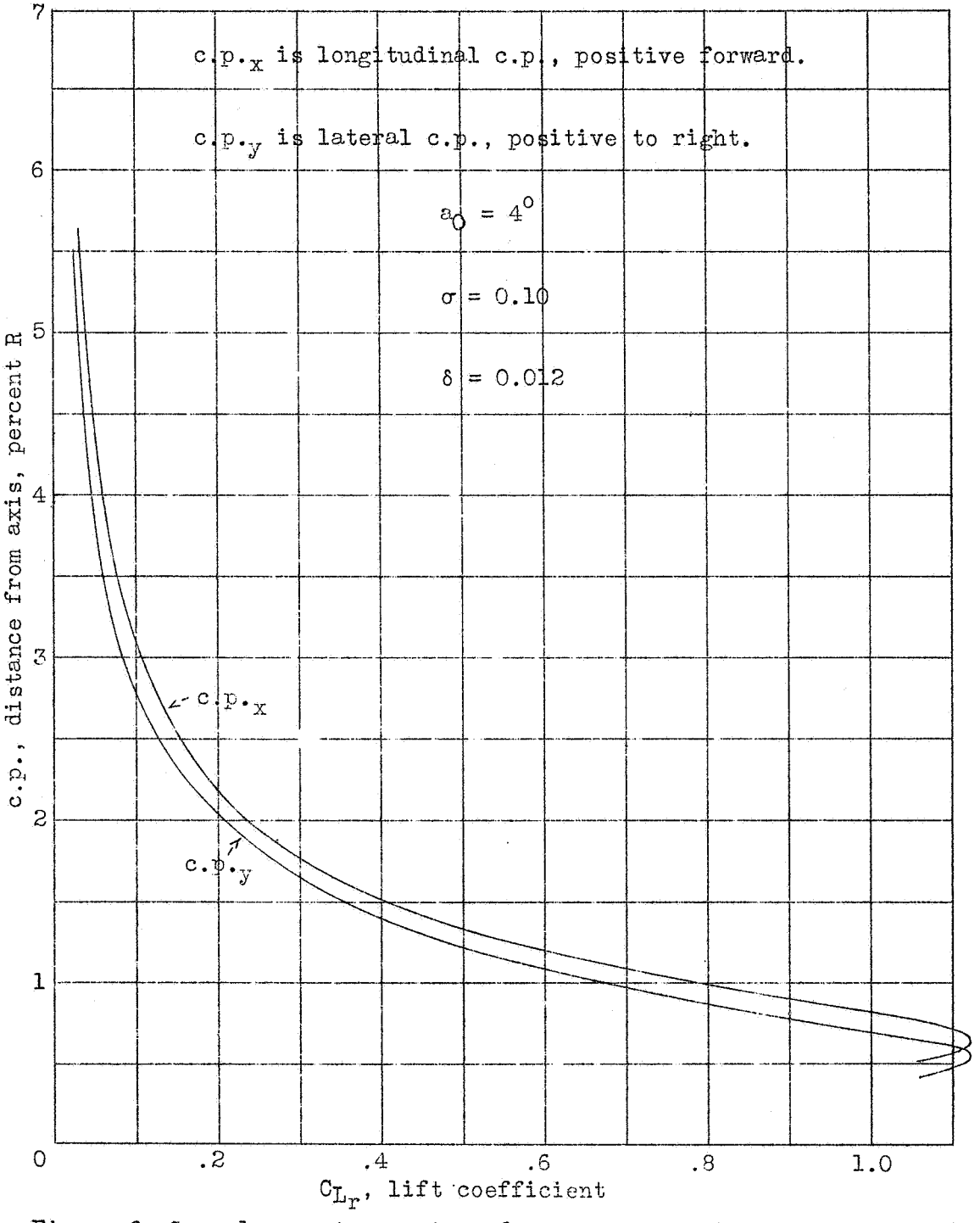

Figure 6.-Gyroplane rotor center-of-pressure travel as a function of rotor lift coefficient.