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INTRODUCTION

The tests described in this report constitute the second part of an investigation of the strength and stiffness characteristics of noncircular aluminum alloy sections loaded to failure in torsion. The first part (reference 1) covered tests of rectangular bars of several proportions and square, rectangular, and streamline tubing. Within the elastic range it was shown that torsional stiffness and maximum shearing stresses could be predicted quite closely by means of existing formulas. First yield and ultimate strengths in torsion were shown to be a function of either the shearing properties of the material or the shear-buckling resistance of the tube walls. Approximate methods of evaluating these strengths for rectangular bars and tubular sections were indicated.

This second series of torsion tests covers a cruciform and an I-section in which, as before, shearing stresses were of principal importance. In addition, several extruded flanged members of I, Z, and channel section were included in which significant longitudinal stresses were also developed. Longitudinal stresses result either from restraint of the warping which tends to occur in noncircular members on sections normal to the axis of twist or from the unequal straining under large angles of twist of fibers at different distances from the center of rotation. The combination of shearing stresses with longitudinal stresses from two sources, one of which is not linear with torque, obviously complicates any analysis of torsional behavior for large angles of twist or for loads up to failure. Although ultimate strength data for some of the specimens tested are probably of more academic than practical interest because of the large deformations required to produce failure, a study of the behavior observed should be helpful in formulating a better understanding of the general torsion problem for noncircular sections and in evaluating the relative importance of some of the factors involved.

Methods of computing stresses and rotations in composite rectangular or flanged sections under torque may be obtained from numerous sources. (See references 2 to 7.) For simple torsion with no restraint of cross-sectional warping and small angles of twist, the problem is one of determining suitable stiffness and stress factors for substitution in the ordinary torsion formulas. (See table II.) Mathematical analysis supplemented by experimental application of the membrane analogy has been used in determining these factors and consequently some differences exist in the work of various investigators.

If warping of cross sections normal to the axis of twist is restrained, the problem is complicated by the fact that the longitudinal stresses developed carry a portion of the torque and an analysis on the basis of simple torsion alone is not adequate. For I-sections the effect of the longitudinal stresses is analogous to that produced by lateral bending of the flanges in their own planes, and this similarity has been used as a basis for the formulas given in references 2, 3, and 4. In the case of channel or Z-sections, however, longitudinal stresses are produced in the webs as well as in the flanges, and the lateral bending approach does not lead to a complete solution of the stress problem. Reference 5 presents formulas for longitudinal stresses and twists in open sections, based upon analyses given in references 8 to 11, and gives the necessary stiffness and stress coefficients for the I-, Z-, and channel sections used in these tests. (See tables III and IV.)

In thin open sections, having little torsional stiffness, large angles of twist will also result in longitudinal stresses. These stresses, however, are a function of the distance from the center of rotation and the angle of twist (reference 2) and consequently have a different distribution over the cross section than those caused by restraint of warping. They result from the fact that the outermost fibers must be stretched to take a helical path around the axis of twist; whereas the fibers at the center have no such tendency to change length. The summation of these longitudinal stresses on any cross section must be zero, the tensions in the outside fibers being balanced by compressions at the center of twist.

OBJECT

The object of these tests was to determine the behavior of composite rectangular or flanged aluminum alloy sections under torsion loadings to failure and to compare, as far as possible, the stresses and rotations observed with those computed by available methods.

SPECIMENS

The sketches included in table II show the cross-sectional dimensions of the test specimens. The cruciform section was machined from 3/4- by 3/4-inch rolled 24S-T bar; the $1\frac{1}{2}$-inch deep I-section was machined from 3/4-by $1\frac{1}{2}$-inch rolled 24S-T bar. Figure 6 shows these specimens after failure. The machined test section was 16 inches long in both cases, the over-all length 28 inches.

The $2\frac{1}{2}$-inch deep I-sections were extrusions (Die No. K-9008) of an experimental high-strength alloy X74S-T. The channel sections were obtained by machining down the flanges of one of these I-sections. The Z-sections were obtained by machining down the flanges of an extruded I-section of 24S-T. Over-all lengths of these specimens ranged from 14 to 44 inches.

Table I gives a summary of mechanical properties for the materials used.

PROCEDURE

The torsion tests were all made in an Amsler torsion machine, having a maximum capacity of 1200 lb-ft. For all but the ultimate strength tests, a 240-lb-ft capacity range was used. The cruciform and I-sections, machined from solid bars, were tested by inserting the unmachined ends of the specimens between the $2\frac{1}{2}$-inch wide by 4-inch long flat grips of the machine. For the clamped-end tests of the extruded sections, steel or aluminum filler plugs of the type shown in figure 13 were bolted to the webs and the testing machine grips bore directly upon these. Care was taken to see that the flanges of the specimens did not project above the filler plugs.

All specimens but those of channel section were twisted about an axis through the shear centers, which was the axis of least torsional stiffness. In the cruciform, I-, and Z-sections the shear centers coincided with the centers of gravity for the cross sections. In the case of the channel specimens, however, the computed position of the shear center (see table IV) was about 0.3 inch outside the web and the opening in the testing machine heads was not of such proportions as to permit twisting about this shear center. Several other arbitrary axes of twist were used, however, as shown in figure 15.

The two types of filler plugs shown in figure 13 were tried in an effort to determine whether or not abruptness in change of section at the clamped ends would have any noticeable effect upon ultimate torsional

strengths. The short, square-end plugs which were used in most cases, were of steel. The tapered plugs were of aluminum.

The simply supported end tests of the extruded sections were made in the manner shown in figure 1 using a set of special adapters designed by Mr. R. L. Templin (Aluminum Research Laboratories Drawing No. 2225). The ends of the specimens were inserted through a close-fitting opening between four strips of 0.064-inch high-strength aluminum alloy sheet, mounted 90° apart as shown. Since the only restraint against longitudinal warping of the end cross sections was that resulting from the torsional stiffness of the individual sheet strips, a condition very closely approaching simply supported ends was obtained. No attempt was made to carry any tests of this type to ultimate failure.

Rotations were measured by means of troptometers graduated in 0.5° and readable by estimation to 0.1°. Gage lengths varied with the length of the specimens and in numerous cases more than one gage length was investigated on the same specimen to illustrate the nonuniformity of twist when restraint of cross-sectional warping was involved.

Strains were measured by means of Huggenberger tensometers (type A, multiplication ratio approx. 1200) using gage lengths of 1/2 and 1 inch. The locations of these gage lengths are shown in the figures. Longitudinal stresses were obtained by multiplying the unit strains by moduli of elasticity of 10,400,000 psi for the X74S-T specimens and 10,600,000 psi for the 24S-T specimens. Diagonal tensile and compressive stresses on the gage lines inclined 45° to the axes of twist and shearing stresses normal to the axis of twist were determined from the relation,

$$\sigma = \tau = \frac{E\,\epsilon}{1 + \mu} = 3/4\ E\,\epsilon$$

where

σ diagonal tensile or compressive stress, psi

τ shearing stress normal to axis of twist, psi

E Young's modulus of elasticity, psi

ϵ average of diagonal tensile and compressive strains, inch per inch

μ Poisson's ratio (1/3)

In general, no attempt was made to measure strains for torques exceeding the elastic range of the material. In order to obtain measurements at all the points desired within the elastic range it was necessary to reload most specimens a number of times.

RESULTS AND DISCUSSION

The results of these tests have been presented for the most part in graphical form as torque-twist and torque-stress curves and stress-distribution diagrams. Wherever possible, computed values of twist and stress have been included for comparison. Such data as were obtained on torsional yield and ultimate strengths are summarized in tables V and VI, along with values of computed stresses and ratios of maximum applied torques to computed values corresponding to the ultimate strengths of the material in direct stress or shear. The latter comparisons emphasize how greatly the stress patterns developed for small angles of twist must be altered when the same type of torsion loadings are carried to failure. Figures 6, 13, and 14 show a number of specimens after the completion of the tests.

Table II summarizes the basic formulas for simple torsion and gives the values of stiffness factor J and stress factor C computed for the test sections by several methods. That of reference 2, which is generally considered least accurate because it neglects "end effects" for individual rectangles, as well as the "junction effect" at intersections, was used here because of its simplicity and because it gave computed values which were generally in best agreement with observed behavior. The differences in J values obtained by references 2 and 4, the latter based upon a recent application of the membrane analogy, were negligible for the cruciform and machined I-sections, a fact of considerable practical importance because it shows to what extent the neglect of end and junction effects in the method of reference 2 may be compensating. For the extruded sections the differences obtained by the two methods were greater, ranging from about 9 percent for the channels and Z-sections to 14 percent for the I-sections. These larger differences appear to be due to significant junction effects neglected by the one method since generous fillets were used at the intersection of the thin rectangles. For larger sections these effects will generally be less pronounced.

The J values computed by the method of reference 3 are consistently higher than those obtained by the other two methods and on the basis of these tests are the least satisfactory.

The stress coefficients C, given in table II, are equal to the thickness of the rectangular sections and may be used in equation (2) to obtain a measure of the shearing stresses that are distributed uniformly over the greater portion of the long sides. The coefficients C_1 and C_2 make possible an estimate of the maximum shearing stresses which may be developed near the junction of the component rectangles. These coefficients are based upon applications of the membrane analogy and, of course, cannot readily be checked by direct stress measurements.

Stresses of this type are so localized, moreover, that they have almost no influence upon over-all yield characteristics.

Tables III and IV summarize the formulas and coefficients used in computing longitudinal stresses and rotations in the specimens in which cross-sectional warping was restrained, or in which angles of twist were large. Their application to cases involving restraint of warping necessitated some estimate as to where complete restraint of warping might have occurred. It was concluded after some consideration of measured and computed twists for different assumed effective lengths, L, that for the extruded sections complete restraint of warping might be assumed reasonably at the midpoint of the grips (2 in. behind face). For the machined I-section, this action was arbitrarily assumed at the end of the fillets or slightly over 1 inch from the end of the uniform machined section.

Detailed comparisons of observed and computed behavior are given in the following paragraphs.

A. Cruciform Section

Figure 2 gives the results of the only measurements made on this section. Longitudinal stresses in the ends of the rectangles would undoubtedly have been significant for large angles of twist, but such measurements were not taken. Measured angles of twist within the elastic range were about 6 percent greater than computed, using J values based upon either the methods of reference 2 or reference 4. (See table II.) The corresponding difference based on the method of reference 3 was about 17 percent.

Table V indicates that the shearing stresses computed for first yielding were in the vicinity of 18,000 psi, which is a reasonable value for the shearing proportional limit for 24S-T bar having the properties indicated in table I. It is quite apparent from this result, however, that the shearing stresses at the inside corners of the section were so localized as to have no appreciable effect upon first-yield characteristics.

Ultimate failure occurred under a torque of 132 lb-ft by a shearing and splitting fracture near the end of the machined length as shown in figure 6. This torque was approximately 2.2 times that computed by equation (2) (table II) to be necessary to develop a stress equal to 65 percent of the tensile strength or a value according to reference 12 that corresponds closely to the ultimate shearing strength of the material. This result indicates a considerably greater margin of torsional strength than would be expected without an appreciable change in the type of action from that obtained within the elastic range. It was shown in reference 1 that the ultimate torsional strength of a 3/4-by-

3/4-inch rolled 24S-T bar (same lot of material as used for cruciform section of present tests) was 87 percent of that computed on the assumption of a uniform distribution of shear stress over the cross section equal to the shearing ultimate strength of the material. It is of interest to point out that the observed ultimate torque for the cruciform section was 84 percent of that computed on the same basis, indicating the possibility of extending this approximate method of predicting ultimate torsional strengths to other than single rectangles.

B. Machined I-Section

The results of this test are presented in figures 3, 4 and 5. In addition to a determination of the torque-twist characteristics and ultimate torsional strength, data on shearing stresses and on longitudinal stresses resulting from restraint of cross-sectional warping and large angles of twist are shown.

Within the elastic range measured angles of twist over an 8-inch gage length at the center of the specimen were within 1 percent of those computed by equation (1) (table II) using J values based on either reference 2 or reference 4. The measured twists were about 8 percent greater than those computed according to reference 3. A comparison of the torque-twist curves for 8- and 16-inch gage lengths in figure 3 indicates a slight difference in twist as a result of restraint of warping. The average computed twist by equation (5) (table III), assuming complete restraint of warping at the end of the fillet (see sketch in fig. 3) was within 1 percent of that computed by equation (1) for the 8-inch gage length and only 6 percent less than that computed by equation (1) for the 16-inch gage length.

Although restraint of warping had little effect upon the average measured twists shown, the corresponding longitudinal stresses were of significant magnitude. Figure 4 shows values as high as 14,000 psi on the edges of the flanges at section a-a. On section d-d, 2½ inches closer to the center of the specimen, however, the stresses were less than 4000 psi. It will be noted that the stresses on the inside edges of the flanges were consistently greater than those on the outside edges, which is in accordance with the warping factors given in table IV for equation (3). The values of stress computed by this latter equation for section a-a on the basis of complete restraint of warping at the end of the fillet were considerably higher than measured, although such a result was not surprising in view of the variable section at the end of the test length. At section c-c, on the uniform portion of the specimen, the maximum computed longitudinal stress was 6800 psi, as compared to measured values of from 6000 to 7300 psi on the four inside edges.

Figure 5 shows a number of torque-stress curves for both longitudinal

and shearing stresses. The average measured shear at the center of the web was in quite satisfactory agreement with the computed value obtained by equation (2) (table II) using C and J values from any of the methods of table II. The average shears at the center of the flange were about 11 percent greater than that measured at the center of the web, which is a difference on the order of that indicated by the ratio of the C and C_1 values indicated for reference 4. The difference between web and maximum flange stresses was probably greater than these data indicate, however, because of the limitations imposed by the lengths of the gage lengths used on the flanges. The ratio of the C and C_1 values given for reference 3 may be more indicative of the stress condition actually obtained.

First apparent yielding occurred under a torque of approximately 80 lb-ft which, according to table V, corresponds to computed shearing stresses of the order of 18,000 psi, or the same as those indicated for yielding of the cruciform section. Here again, the effect of stress concentrations on first-yield characteristics was not discernible. The computed longitudinal stress shown in table V for section a-a is sufficiently below the tensile yield strength of the material to indicate that this type of action did not contribute to first yielding. This fact is even more evident when it is realized that the maximum longitudinal stresses measured on section a-a were only about 60 percent of those computed.

Failure occurred in the machined I under a torque of 638 lb-ft by a combination shear and tensile fracture as shown in figure 6. This torque was approximately 3.4 times that computed as necessary to develop an ultimate shear strength equal to 65 percent of the tensile strength of the material or 4.4 times that necessary to develop longitudinal stresses equal to the ultimate strength in tension. These results emphasize the inadequacy of the ordinary stress formulas for predicting ultimate torsional strengths. The method proposed in reference 1 of estimating strengths for rectangular sections on the assumption of a uniform distribution of shear stress equal to the ultimate shear strength of the material is not so applicable to this I-section as was the case with the cruciform section. The observed maximum torque was only about 60 percent of this computed value; whereas for all other cases where the method has been tried, the ratio was about 85 percent. The combination of longitudinal with shearing stresses apparently decreased the torque-carrying capacity over that which might be expected for shear alone.

Figure 3 shows the relation observed between torque and longitudinal stresses at the center of the specimen as a result of large angles of twist. Although the stresses indicated for a torque of 180 lb-ft, or 28 percent of the maximum, were only about 6000 psi, it is evident from the shape of the torque-stress curve that a rapid increase in stress must have been obtained as torques approached the ultimate. Computed

longitudinal stresses according to equation (4) (table III) were considerably less than measured. For an average twist of 3° per in., for example, the computed flange stress was found to be about 3700 psi as compared to about 6000 psi measured. Additional strain measurements, giving a more complete picture of the stress distribution, would probably have been helpful in explaining this discrepancy.

C. Extruded I-Section

Specimens of this section were subjected to a variety of tests in different lengths and with different end conditions. Although the cross-sectional area was about 25 percent greater than that of the machined I-section, the torsional stiffness, based on J values, was only about one-third as great. Consequently, the influence of longitudinal stresses was much more pronounced than in the case previously considered. Figures 7 to 12 present the results of these tests.

The torque-twist curves in figure 7, covering only a limited portion of the elastic range, indicate a nonlinear relation between torque and twist, a behavior not observed in the foregoing tests. This action must be attributed mainly to the influence of longitudinal stresses accompanying large angles of twist. The departure from linearity was greatest, it will be noted, in the tests involving the greatest twists. A second fundamental difference in behavior will be noted in that end condition had a significant effect upon angles of twist and that the latter varied not only with length of specimen but also with gage length on any one specimen. None of these factors were significant in the tests of the machined cruciform and I-sections. Angles of twist were not a function of gage length, and the agreement between observed twists and computed values based upon equation (1) and the J values for simple torsion (table II) eliminated the need for considering longitudinal stresses.

The comparison between measured and computed angles of twist for the simply supported end tests shown in figure 7 is of interest because of the material difference between the J values shown in table II for the various methods of computation. For a torque of 20 lb-ft the measured twist on a 10-inch gage length at the center was practically the same as that computed using the J value of reference 2. The measured twists for this torque were 25 percent and 15 percent greater than computed by references 3 and 4. On the basis of this observation the J value of reference 2 was used in all subsequent computations of stresses and twists for the extruded I-sections.

The differences between the angles of twist measured for gage lengths A and B shown in figure 7 indicate that a condition of simply supported ends was not obtained completely. The difference between the average twist per inch on gage length B (10-in. gage) and gage length A

(30-in.gage) was about 5 percent, the twist being greater on the shorter gage length where there were no appreciable longitudinal stresses.

A comparison of the average measured twists shown in figure 7 for the 10-inch gage lengths B, C, and D on approximately 36-inch long specimens having simply supported and clamped ends emphasizes the extent to which longitudinal stresses resulting from restraint of warping may influence torsional stiffness. The additional effect of variations in length of specimen where restraint of warping is involved is indicated by the measurements on gage length E.

The computed torque-twist relations shown in figure 7 for the clamped-end tests were obtained by means of equation (5) (table III) and the stiffness factors, C_{BT}, given in table IV. The use of this equation involves some assumption as to the effective length, L, between sections of completely restrained warping. After some comparisons between observed angles of twist and computed values for different values of L, it was decided that complete restraint of warping might reasonably be assumed at the midpoint of the testing machine grips (2 in. behind face). The necessity for such an assumption reflects in no way upon the general reliability of the method of computation used. In tests in which the section of complete restraint of warping is definitely known, as in the tests of reference 5, equation (5) has been shown to give computed values of twist in good agreement with those measured.

In view of the good agreement shown in figure 7 between measured and computed twists for the clamped-end tests, it should be indicated to what extent this agreement was due to the choice of sections of completely restrained warping. The following computed relations between average unit twist and effective length of specimen were obtained:

Complete restraint of warping assumed at	Average twist for torque of 50 lb-ft (deg per in.)		
Gage length ⟶	C	D	E
Face of grips	1.19	0.75	0.32
Midpoint of grips (2 in. behind face)	1.32	.94	.50
End of grips (4 in. behind face)	1.45	1.12	.68

Figure 8 shows a comparison between measured and computed shearing stresses at the center of specimens of different lengths and with different end conditions. The greatest shearing stresses were measured in the simply supported end tests in which longitudinal stresses were

least pronounced. The distribution of shear across the width of flanges and web investigated was essentially the same in all cases, however, a peak stress being observed in the flanges directly over the web, with the stresses on all other gage lines being approximately the same. As shown in the figure, these uniformly distributed stresses were in reasonably good agreement with those computed. It should be emphasized that these computed shearing stresses do not correspond to the full torque of 50 lb-ft applied but were based on only that portion of the torque that was resisted by torsion shear. These reduced torques were computed in the following manner. Equation (6) (table III) gives the unit twist at the center of a specimen in which cross-sectional warping is restrained at the ends. The ratio of this twist to that determined by equation (1) (table II) gives the percentage of the applied torque carried by torsion shear. The remainder is carried by transverse shear in the flanges. For the clamped-end test of the 36-inch long specimen, the torque carried by torsion shear at the center was computed to be 62 percent of the total. For the 16-inch long clamped-end specimen the corresponding value was 26 percent. These reduced torques were used in equation (2) (table II) to compute the shearing stresses in the webs and flanges shown in figure 8. The inclusion of stresses resulting from transverse shear in the flanges would have increased the computed values by less than 5 percent.

An additional adjustment should have been made for the fact that longitudinal stresses resulting from large angles of twist also reduced the torque carried by torsion shear. This reduction would amount to about 15 percent for the simply supported end test, based upon the ratio of measured to computed twists for a torque of 50 lb-ft, and 7 percent for the clamped-end test of the 36-inch long specimen.

The shearing stresses shown in figure 8 for gage line 3, directly over the web, were about 30 percent greater than the average for the outstanding portions of the flanges. The ratio of maximum to average stress was probably considerably higher, corresponding more nearly to the C and C_1 factors attributed to reference 3 (table II) than to those according to reference 4. Unfortunately, the strain measurements do not provide a check upon the reliability of these stress factors.

Figures 9 and 10 present data on the longitudinal stresses in the extruded I-sections for a torque of 50 lb-ft. For the $36\frac{1}{4}$-inch long specimen tested with simply supported ends, the stresses resulted mainly from large angles of twist. The outstanding edges of the flanges were all subjected to tension whereas the greater portion of the web was subjected to compression. The dotted lines connecting the measured points on the flanges at various sections along the length were patterned after the distribution measured at the center of the specimen. The summation of the longitudinal stresses on any section should equal zero, although particular pains were not taken to balance the tensile and compressive stress areas of the diagrams.

Although tensile stresses were observed in all flanges near the end of the specimen in the simply supported end test, the differences in individual values indicated some restraint of warping; whereas at the center the stresses were more nearly equal. Their magnitudes, moreover, were in good agreement with that computed on the basis of equation (4) (table III). Longitudinal stresses resulting from large angles of twist were less pronounced in the clamped-end tests, although their effect in the 36-inch long specimen was readily measurable at the center of the span.

Two sets of computed longitudinal stress curves resulting from restraint of cross-sectional warping have been shown in figure 10, one for 100 percent restraint at the midpoint of the grips, which was the basis for the computed twists shown in figure 7; the other for 100 percent restraint at the face of the grips. Equation (3) of table III and the stiffness and warping factors, C_{mr} and u, given in table IV, were used in these computations. For the 36-inch long specimen with clamped ends, the assumption of 100 percent restraint of warping at the face of the grips provided a closer agreement with measured stresses; for the 16-inch long specimen the measured stresses were slightly higher than when computed on the basis of either assumption. In general, however, it appears that a reasonably satisfactory estimate of longitudinal stresses of this type may be made.

Figure 11 shows some torque-longitudinal stress curves which indicate the nonlinearity with torque of stresses resulting from large angles of twist as compared to the linearity of stresses resulting from restraint of warping. It should be noted that the longitudinal stresses for section a-a in the clamped-end test have been plotted to a scale $2\frac{1}{2}$ times that used in plotting the other stresses. Otherwise, a proper indication of the relative importance of the different types of stress shown at the different sections and for different end conditions may not be obtained.

The ultimate torsional strength data obtained for the extruded I-sections tested with clamped ends are shown in figure 12. Linear relations between torque and twist were observed within the elastic range for all but the longest specimen, where the longitudinal stresses resulting from large angles of twist caused the specimen to stiffen up noticeably. Unfortunately, twist measurements were not continued in this test as in the others until general yielding was obtained.

The computed stresses given in table V for the estimated torques for first yielding indicate that longitudinal stresses probably caused yielding of the 6-inch long specimen; whereas both longitudinal and shearing stresses probably contributed to yielding in the case of the 16-inch long specimen. As shown in table VI, the ultimate torques were far in excess of the torques computed to develop the strengths of

the material in either shear or tension, using the basic stress formulas shown to be applicable within the elastic range.

Figures 13 and 14 show the specimens after tests to failure. Longitudinal stresses were undoubtedly carrying a much greater percentage of the applied torque at failure than was the case in the early stages of the tests. Otherwise, length of specimen would not have had an appreciable influence upon ultimate torsional strength. The web buckling evident in the longer specimens apparently resulted mainly from the compressive forces set up in the center to balance the tensions in the extreme fibers, although some other action would also appear necessary to account for the triangular buckle patterns observed. Figure 14 shows a longitudinal fracture of a flange directly over the web, indicative of the tendency of the flanges to pull in toward the axis of twist.

It does not appear that any particular significance can be attached to the values of ultimate torque obtained. Neither is it possible to draw any pertinent conclusion from the difference in results obtained with the two types of filler plug in view of the question as to the effective length of specimen when the tapered plugs were used. Perhaps the most interesting practical result is the demonstration of the toughness and resistance to plastic deformations without fracture of the high-strength wrought aluminum alloy used.

D. Extruded Channel Section

Tests of this section were not so extensive as described for the extruded I-section and, because of the fact that the section was not twisted about the theoretical shear center, the results were not so susceptible to analysis.

The measured twists shown in figure 15 for the simply supported end test averaged about 7 percent less than when computed according to reference 2, which is a greater difference than was found in the corresponding test of the extruded I-section. Any restraint of warping at the ends would be more significant in the channel test, however, because of the shorter length of specimen. Longitudinal stresses resulting from large angles of twist would also be expected to result in somewhat greater stiffness than when computed for simply supported ends. Despite the presence of these factors known to decrease torsional stiffness, the measured angles of twist ranged from 3 to 9 percent greater than when computed according to references 3 and 4.

A comparison of the torque-twist curves in figure 15 for the clamped-end tests shows the influence of position of axis of twist upon torsional stiffness. Had it been possible to obtain twist about the shear center it seems reasonable to believe that a fair agreement with

the computed twist shown would have been obtained. Under the conditions of test imposed, however, torque was resisted by transverse shears and bending moments as well as by longitudinal stresses resulting from restraint of warping and ordinary torsion shear.

Figure 16 shows shear-stress distribution diagrams in the webs and flanges for simply supported and clamped ends and for twists about different axes. The distribution for the simply supported end test was in reasonably good agreement with that computed. The stress patterns for the clamped-end tests were essentially the same although the magnitudes decreased with increasing distance between the shear center and the axis of twist. This decrease in observed torsion-shear stress was consistent with the decreased angles of twist indicated in figure 15.

Longitudinal stresses are shown in figures 17 and 17a. The values for the simply supported end test, which should be the result mainly of large angles of twist, show a perplexing distribution in that stresses in the outstanding toe of the flanges were less than in the web. This result is contrary to that observed for the extruded I-sections and that indicated by equation (4). The distribution at the center of the specimen in the clamped-end test of figure 17, however, appears more reasonable.

A comparison of the longitudinal stresses for the clamped-end tests shown in figure 17a emphasizes to what extent the action was dependent upon the location of the center of twist. The farther the center of rotation was moved from the shear center, the greater the influence of transverse shears and bending moments in resisting the applied torques and the smaller the maximum longitudinal stresses.

None of the channel sections was twisted to ultimate failure.

E. Extruded Z-Section

Figures 18 to 21 present the results of these tests. Since the specimens were twisted about an axis through the shear center, comparisons of measured and computed behavior similar to those for the extruded I-section tests have been made.

As shown in figure 18, the relation between observed angles of twist and those computed by reference 2 was essentially the same for the simply supported end test as that shown for the channel section in figure 15. Some effects of longitudinal stresses resulting from large angles of twist and restraint of warping were indicated. Quite satisfactory agreement between measured and computed twists was obtained in the clamped-end tests.

The shear stresses shown in figure 19 likewise indicate a behavior close to that computed. The torque used in computing the stresses for the clamped-end test was 81 percent of the total, a percentage obtained by a comparison of values of unit twist determined by equation (1) (table II) and equation (6) (table III).

Figures 20 and 21 show the distribution of longitudinal stresses resulting both from large angles of twist and from restraint of warping. It is evident that some end restraint was obtained in the simply supported end test. For clamped ends, however, the measured stress distribution at the ends was intermediate between that computed, assuming complete restraint of warping at the face of the grips and at the midpoint of the latter. The discrepancy between measured and computed stresses at the center is due to effects of large angles of twist. The maximum longitudinal stresses resulting from the latter cause were larger in the clamped-end test than in the simply supported end test. The computed maximum tensile stress according to equation (4) (table III) was about 7500 psi for the simply supported end test; whereas the measured values averaged only about 3000 psi. This considerable difference between measured and computed longitudinal stresses resulting from large angles of twist, also noted in the channel tests, raises a question as to the general applicability of equation (4) to unsymmetrical sections.

Figure 18 shows the torque-twist curve obtained in the ultimate torque test of a 36-inch long specimen with clamped ends. Unfortunately, twist measurements were not continued until yield occurred. The influence of longitudinal stresses resulting from large angles of twist is plainly evident from the nonlinearity of the torque-twist curve. As shown in table VI, the ultimate torque carried was far greater than that necessary to develop the ultimate strengths of the material in tension or shear. Figure 14 shows the Z-section after failure (middle specimen of group).

CONCLUSIONS

The following conclusions are based upon the results of the torsion tests of noncircular 24S-T and X74S-T aluminum alloy sections described herein:

1. The behavior of flanged or composite rectangular sections under torques carried to failure involves the following considerations:

(a) Shearing stresses and twists normally associated with torsion

(b) Longitudinal stresses resulting from restraint of cross-sectional warping

(c) Transverse shear and bending stresses resulting from twist about other than the shear center

(d) Longitudinal stresses resulting from the unequal straining under large angles of twist of fibers at different distances from the center of rotation

(e) Buckling resistance under longitudinal compressive stresses

(f) Yield and ultimate strengths of the material in tension and shear

2. The first two types of action have been covered by torsion theory, and reasonable agreement between measured and computed behavior may be obtained in most cases for relatively small angles of twist within the elastic range.

3. Twist about other than the shear center results in transverse shear and bending stresses in addition to stresses of types (a) and (b). Although methods of computing type (c) stresses have not been given here, it should be noted that their effect in the case of the channel tests described was to decrease angles of twist and values of longitudinal stress.

4. Longitudinal stresses resulting from large angles of twist may apparently be predicted with reasonable accuracy for thin symmetrical sections such as the extruded I-section tested. The applicability of the same method to unsymmetrical channel or Z-sections, or to sections in which relatively high torsion-shear stresses are produced, however, has not been established by these tests.

5. Although ultimate torsional strengths of thin composite rectangular sections may involve buckling resistance under longitudinal compressive stresses as well as the ultimate strengths of the material in tension and shear, these factors do not appear susceptible to complete rational analysis.

6. General formulas for computing shearing and longitudinal stresses and corresponding angles of twist within the elastic range are summarized in tables II and III. Application of these formulas requires evaluation of the following section elements:

(a) Torsion factor, J

(b) Shear-stress coefficients, C

(c) Torsion-bending factor, C_{BT}

(d) Unit warping factor, u

7. Several methods of computing values of J for cross sections composed of rectangular areas have been proposed. (See table II.) The method of reference 2 ($J = \Sigma 1/3\ bt^3$, where b is length and t, thickness of each rectangle) is the simplest and, as far as the angles of twist observed in these tests were concerned, provided the best agreement between measured and computed behavior for simple torsion.

8. The values of stress coefficient C given in table II gave computed shearing stresses on the long sides of the component rectangles in reasonably good agreement with measured values. It was not possible, however, to obtain an experimental check on the coefficients C_1 and C_2 given for estimating shearing stresses at fillets and at junctions of intersecting rectangles. Stress concentrations of this nature may be significant from the standpoint of fatigue strength, but they had no measurable effect in these tests upon torsional stiffness or first-yield characteristics.

9. The measured longitudinal stresses resulting from restraint of cross-sectional warping in the extruded I- and Z-sections, twisted about their shear centers, were in satisfactory agreement with computed values obtained by the general equations of table III and the values of C_{BT} and u given in table IV. A similar agreement would probably have been obtained for the channel sections if twists had been produced about the shear center.

10. Stresses and rotations resulting from restraint of warping may be difficult to estimate in design because of the uncertainty regarding the degree of restraint likely to be obtained. Even in tests such as those described, where particular pains were taken to obtain definite end conditions, variations from idealized, simply supported and completely restrained ends were observed. Figure 10 shows that in the simply supported end test of the extruded I-section the longitudinal stresses resulting from restraint of cross-sectional warping were only 4 percent of the corresponding values for the clamped-end test, indicating the effectiveness of the special adapter shown in figure 1.

11. First over-all yielding in torsion depends upon the relative importance of shear and longitudinal stresses and may be expected when either or both of these types of stress, computed by the methods outlined and neglecting the effect of stress concentrations, approach the yield strengths of the material.

12. The ultimate torsional strength of the cruciform section, in which shearing stresses were of primary importance throughout the entire load range, was about 85 percent of that computed assuming a uniform

distribution of shear stress at failure equal to the ultimate shear strength of the material (65 percent of tensile strength). This result was in accord with reference 1, describing ultimate torsion tests of single solid rectangles. The same approximate method of analysis was not applicable to the flanged sections tested, however, because of the presence of significant longitudinal stresses from the beginning of the tests.

13. The ultimate torques for the flanged sections ranged from 4.2 to 7.3 times those computed to develop the estimated shear strengths of the material using the formulas applicable in the elastic range. The corresponding ratios of observed to computed torques based on longitudinal stresses equal to the tensile strengths of the material ranged from 2.1 to 5.6. These results and the appearance of the specimens after failure (shown in figs. 6, 13, and 14) emphasize the difficulty of formulating definite rules for the prediction of ultimate torsional strengths of flanged sections.

14. The large angles of twist necessary to produce failure indicate the toughness of flanged sections of high strength aluminum alloys under torsion loadings.

15. The most obvious conclusion to be drawn from these tests would seem to be that ultimate strengths of thin flanged sections in torsion seldom if ever will be a limiting factor in structural design.

Aluminum Research Laboratories,
Aluminum Company of America,
New Kensington, Penna., September 14, 1945.

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TABLE I.- MECHANICAL PROPERTIES OF MATERIAL

Form	Alloy	Yield strength (set = 0.2 percent) (psi)		Tensile strength (psi)	Elongation in 2 in. (percent)
		Tension	Compression		
3/4- by 3/4-in. rolled bar for cruciform section	24S-T	57,700	48,300	72,000	16.7
3/4- by $1\frac{1}{2}$-in. rolled bar for machined I-section	24S-T	46,000	39,800	66,800	21.1
Extrusion (Die No. K-9003) for I- and channel sections	X74S-T	69,500	67,900	75,900	17.9
Extrusion (Die No. K-9003) for Z-section	24S-T	49,900	40,000	62,200	19.2

Tension tests made in accordance with Standard Methods of Tension Testing of Metallic Materials (E8-44) ASTM Book of Standards, pt. I, 1944.

TABLE II.- COMPARISON OF VALUES OF J AND SHEAR-STRESS COEFFICIENTS C FOR TORSION SPECIMENS

Formulas for simple torsion:

$$\theta = \frac{T}{JG} \tag{1}$$

where

θ twist, radians per in.

T torque, lb-in.

G modulus of elasticity in shear, psi (3,900,000 for X74S-T) (4,000,000 for 24S-T)

J torsion factor, in.^{4}

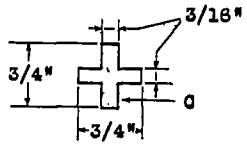
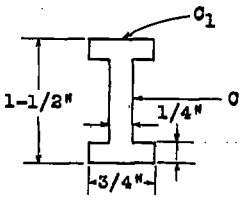
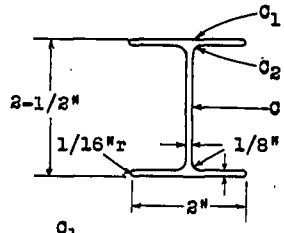
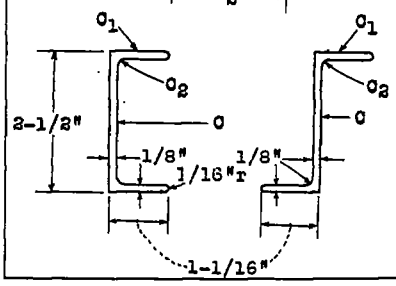
$$\tau = \frac{TC}{J} \tag{2}$$

where

τ shear stress, psi

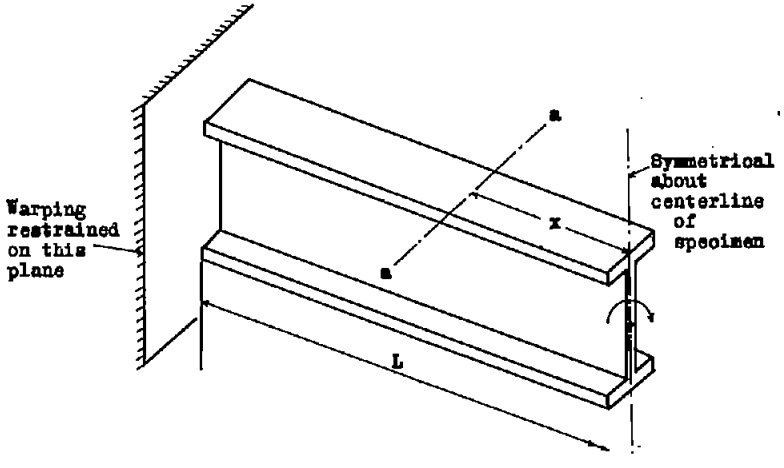
C stress coefficient

VALUES OF J AND C FOR VARIOUS METHODS OF COMPUTATION

Test section	$J = \Sigma\, 1/3\, bt^3$ [1] C = t (reference 2)	Roark (reference 3)	Lyse and Johnston (reference 4)
	J = 0.00288 C = 0.188	J = 0.00320 C = 0.188	J = 0.00291 C = 0.188
	J = 0.0130 C = 0.250	J = 0.0138 C = 0.250 C_1 = 0.296	J = 0.0129 C = 0.250 C_1 = 0.275
	J = 0.00406 C = 0.125	J = 0.00511 C = 0.125 C_1 = 0.218 C_2 = 0.265	J = 0.00472 C = 0.125 C_1 = 0.172
	J = 0.00285 C = 0.125	J = 0.00332 C = 0.125 C_1 = 0.185 C_2 = 0.219	J = 0.00314 C = 0.125 C_1 = 0.155

[1] b, width, and t, thickness of individual rectangles

TABLE III.- FORMULAS FOR COMPUTING LONGITUDINAL STRESSES AND TWIST IN NONCIRCULAR SECTIONS UNDER TORQUE



1. Stress resulting from restraint of warping at ends (reference 5):

$$\sigma = \frac{ET}{Ga}\,\frac{\sinh\frac{x}{a}}{\cosh\frac{L}{2a}}\,u \tag{3}$$

where

σ longitudinal stress at section a-a, psi

E Young's modulus, psi (10,400,000 for X74S-T) (10,600,000 for 24S-T)

x distance of section a-a from center, in.

L length of specimen between restrained ends, in.

$a = \sqrt{\frac{EC_{BT}}{GJ}}$ where C_{BT} is given in table IV

u unit warping factor, also given in table IV

2. Secondary stress resulting from large angles of twist (reference 2):

$$\left[\sigma = E\,\frac{(\theta r)^2}{2} - \epsilon_0\right] \tag{4}$$

where

σ longitudinal stress, psi

θ twist, radians per in.

r distance from center of twist, in.

ϵ_0 unit compressive strain calculated from condition that sum of longitudinal stresses on any section equals zero ($\Sigma\sigma = 0$)
For machined I, $\epsilon_0 = 0.159^2$
For extruded I, $\epsilon_0 = 0.639^2$
For extruded channel and Z, $\epsilon_0 = 0.526^2$

3. Twist of member in which warping at ends is restrained (reference 5):

$$\psi = \frac{T}{JG}\left[x - a\,\frac{\sinh\frac{x}{a}}{\cosh\frac{L}{2a}}\right] \tag{5}$$

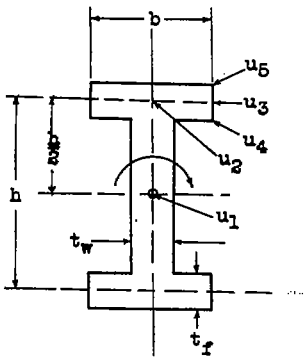
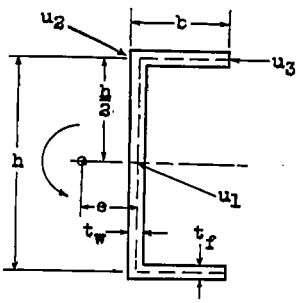
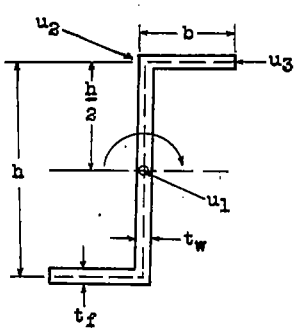
where

ψ twist of section a-a relative to center, radians

4. Twist at center of member in which warping at ends is restrained (reference 2):

$$\theta = \frac{T}{JG}\left[1 - \frac{1}{\cosh\frac{L}{2a}}\right] \tag{6}$$

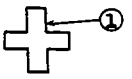
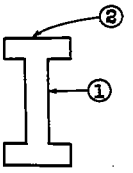
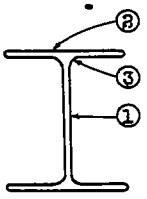
TABLE IV.- VALUES OF C_{BT} AND u FOR EQUATIONS 3, 5, AND 6 OF TABLE III

Test section	General[1]	Computed for test sections	
		Machined I	Extruded I
	$u_1 = 0$	0	0
	$u_2 = 0$	0	0
	$u_3 = \pm\frac{hb}{4}$	-	± 1.188
	$u_4 = \pm\frac{hb}{4} \pm \frac{bt}{4}$	± 0.281	-
	$u_5 = \pm\frac{hb}{4} \mp \frac{bt}{4}$	± 0.188	-
	[2]$C_{BT} = \frac{I_f h^2}{2} + \frac{1}{72}\left[A_f^3 + \frac{A_w^3}{2}\right]$	0.00716	0.235
	$u_1 = 0$	0	
	$u_2 = \pm\frac{h}{2}\left(e + \frac{tw}{2}\right)$	± 0.499	
	$u_3 = \mp\frac{h}{2}(b - e)$	∓ 0.762	
	$C_{BT} = \frac{I_f h^2}{2}\left(4 - 6\frac{e}{b}\right)$	0.0544	
	$e = \frac{3bA_f}{A_w + 6A_f}$	0.36	
	$u_1 = \pm\frac{hb}{2}\frac{A_f}{A}$	± 0.272	
	$u_2 = \pm\frac{h}{2}\left(\frac{A_f b}{A} + \frac{tw}{2}\right)$	± 0.344	
	$u_3 = \mp\frac{hb}{2}\left(1 - \frac{A_f}{A}\right)$	∓ 0.917	
	$C_{BT} = \frac{I_f h^2}{2}\left[4 - 6\frac{A_f}{A}\right]$	0.0774	

[1]Taken from reference 5, $I_f = b^3 t_f/12$: $A_f = bt_f$: $A_w = ht_w$: $A = A_w + 2A_f$.

[2]Second term is thickness factor included for machined I-section; neglected for extruded sections.

TABLE V.- DATA ON FIRST YIELDING IN TORSION

Test section and location of computed shearing stresses	Length[1] (in.)	Torque at first yielding[2] (lb-ft)	Average twist (deg per in.)	Corresponding shearing stresses[3] (psi)			Corresponding longitudinal stress[4] (psi)
				Reference 2	Reference 3	Reference 4	
	16	24	1.5	(1) 18,800	(1) 16,900	(1) 18,600	
	16	80	1.05	(1) 18,400	(1) 17,400 (2) 20,600	(1) 18,600 (2) 20,500	36,600 (sec. a-a, fig. 4)
	6	500	1.5	(1) 14,800	(1) 11,800 (2) 20,500 (3) 24,900	(1) 12,700 (2) 17,500	85,000 (Face of grips)
	16	240	2.35	(1) 23,000	(1) 18,300 (2) 32,000 (3) 38,800	(1) 19,900 (2) 27,400	93,500 (Face of grips)

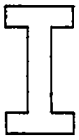
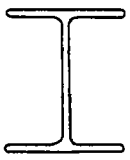
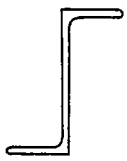
[1]Length of uniform section between fillets or grips.

[2]Estimated proportional limit on torque-twist curves (figs. 2, 3, and 12).

[3]Computed by equation (2), table II, using J and C values shown. For 6-in. length of extruded I-section, torsion-shear moment = 8 percent of total torque (equation (6), table III). For 16-in. length of extruded I-section, torsion-shear moment = 26 percent of total.

[4]Computed by equation (3), table III, assuming complete restraint of warping at end of fillet of machined I or at midpoint of grips for extruded I.

TABLE VI.- DATA ON ULTIMATE TORSIONAL STRENGTHS

Test section	Length[1] (in.)	Ultimate torque (lb-ft)	Average twist (deg per in.)	Average shortening (in. per in.)	Ratio of ultimate torque to computed torque corresponding to ultimate strength of material	
					Shear[2]	Tension[3]
	16	132	107	0.062	2.2	--------
	16	638	47	.094	3.4	[a]4.4
	3 1/2	970	7	.053	7.3	--------
	6	940	16	.031	7.1	[b]2.1
	14 1/2	[c]650	13	.056	4.9	--------
	16	644	15	.047	4.8	3.3
	36	564	10	.023	4.2	4.3
	36	371	10	.021	4.8	5.6

[1]Length of uniform section between fillets or grips.

[2]Computed torques for shear stress equal to 65 percent of tensile strength, obtained by equation (2), table II, using values of J and C under method of reference 2.

[3]Computed torques for longitudinal stress equal to tensile strength, obtained by equation (3), table III.

[a]Stress at section a-a, fig. 4. Complete restraint of warping assumed at end of machined length.

[b]Stress at face of grips for all extruded I- and Z-sections. Complete restraint of warping assumed at midpoint of grips.

[c]Average of two tests.

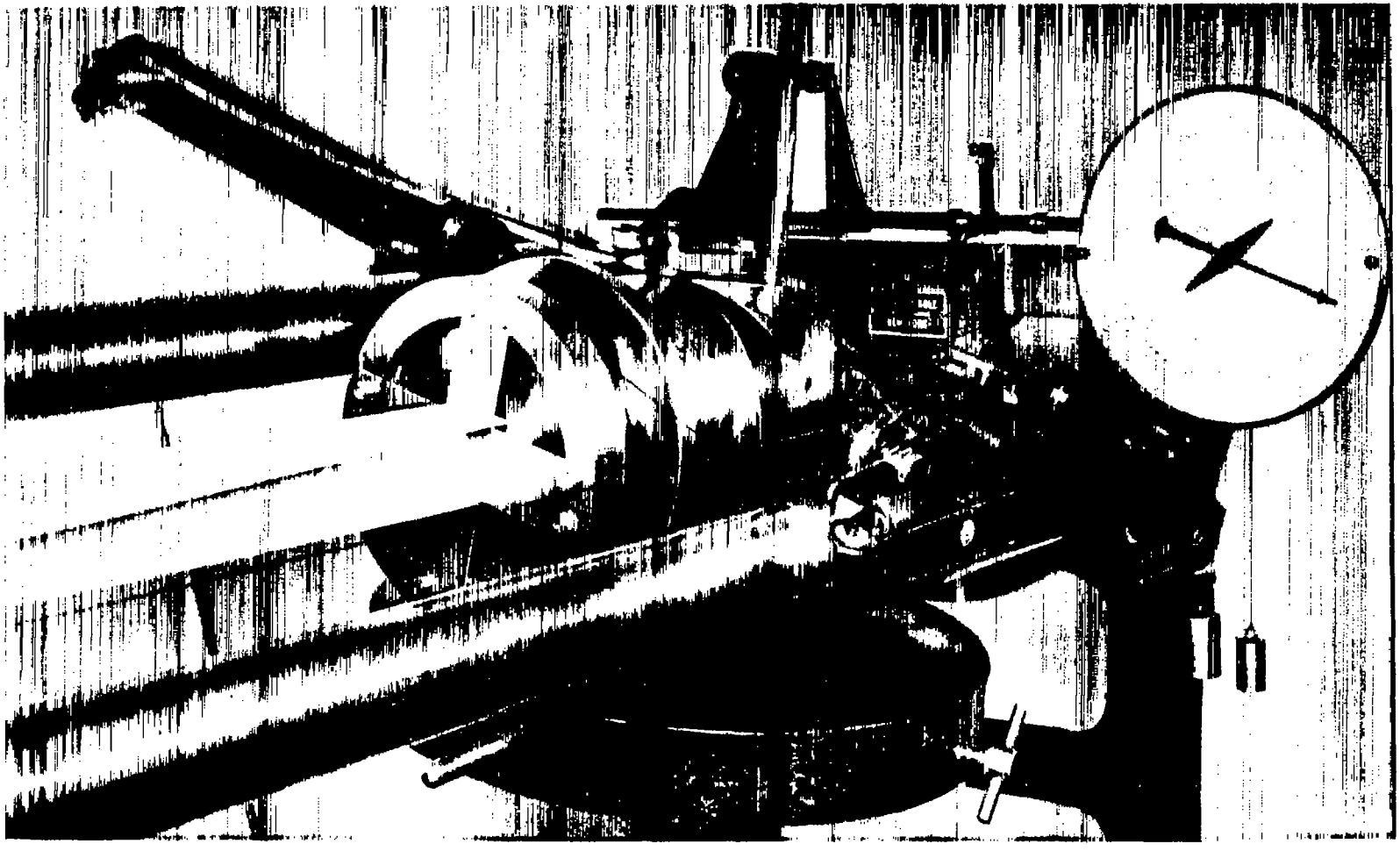


Figure 1.- Torsion test of I section with simply-supported ends.

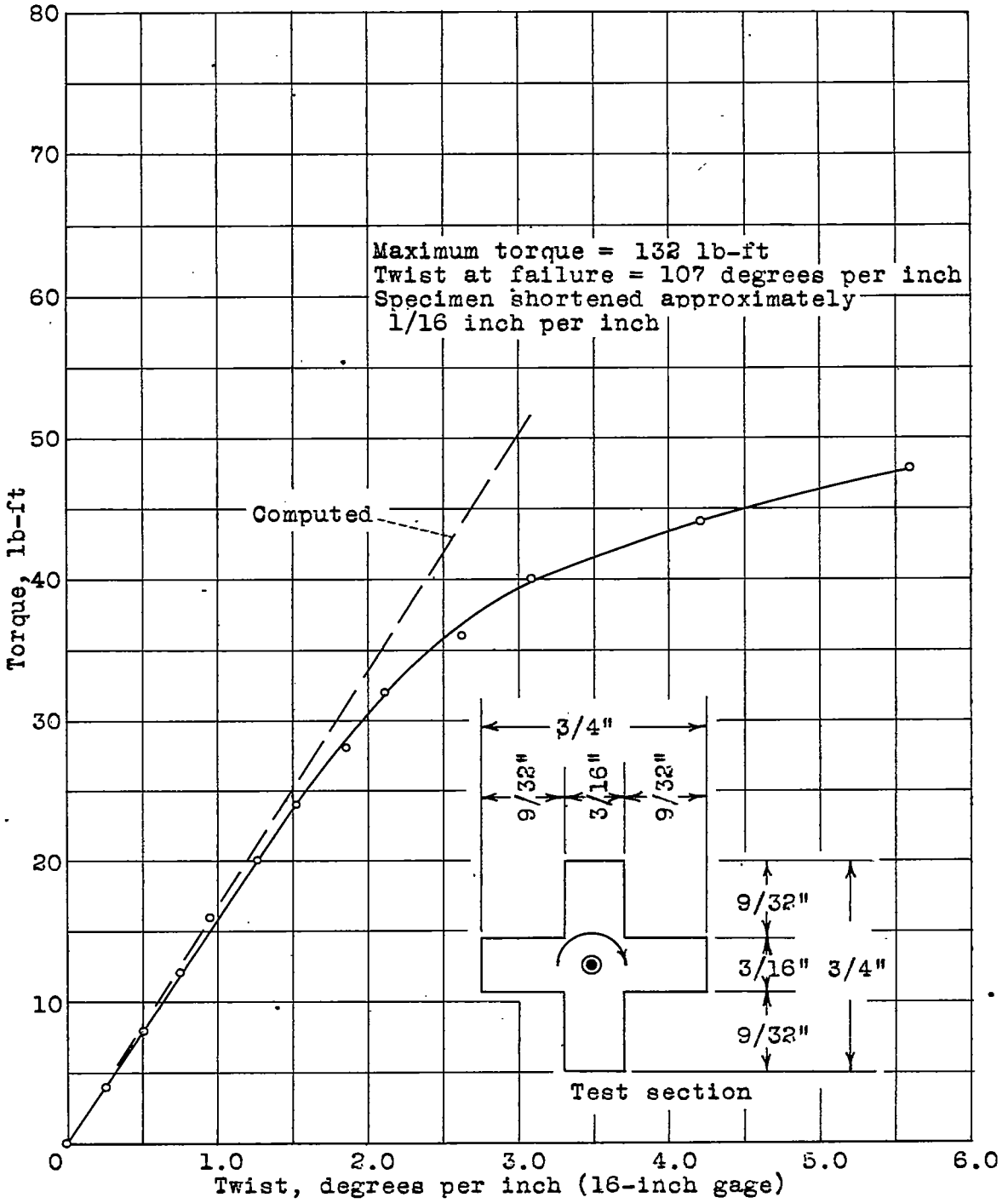


Figure 2.- Torque-twist curve for cruciform section, 24S-T.

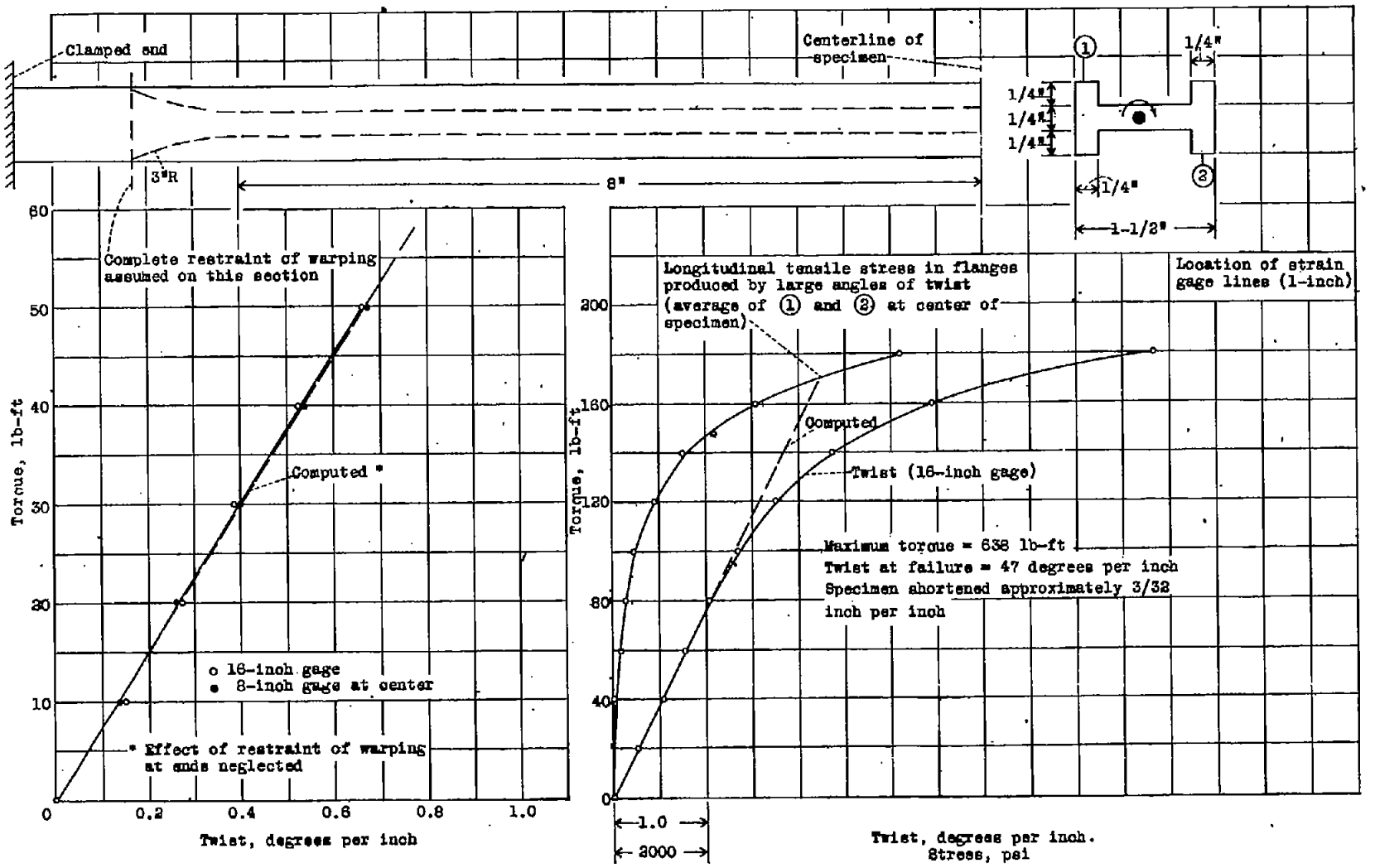
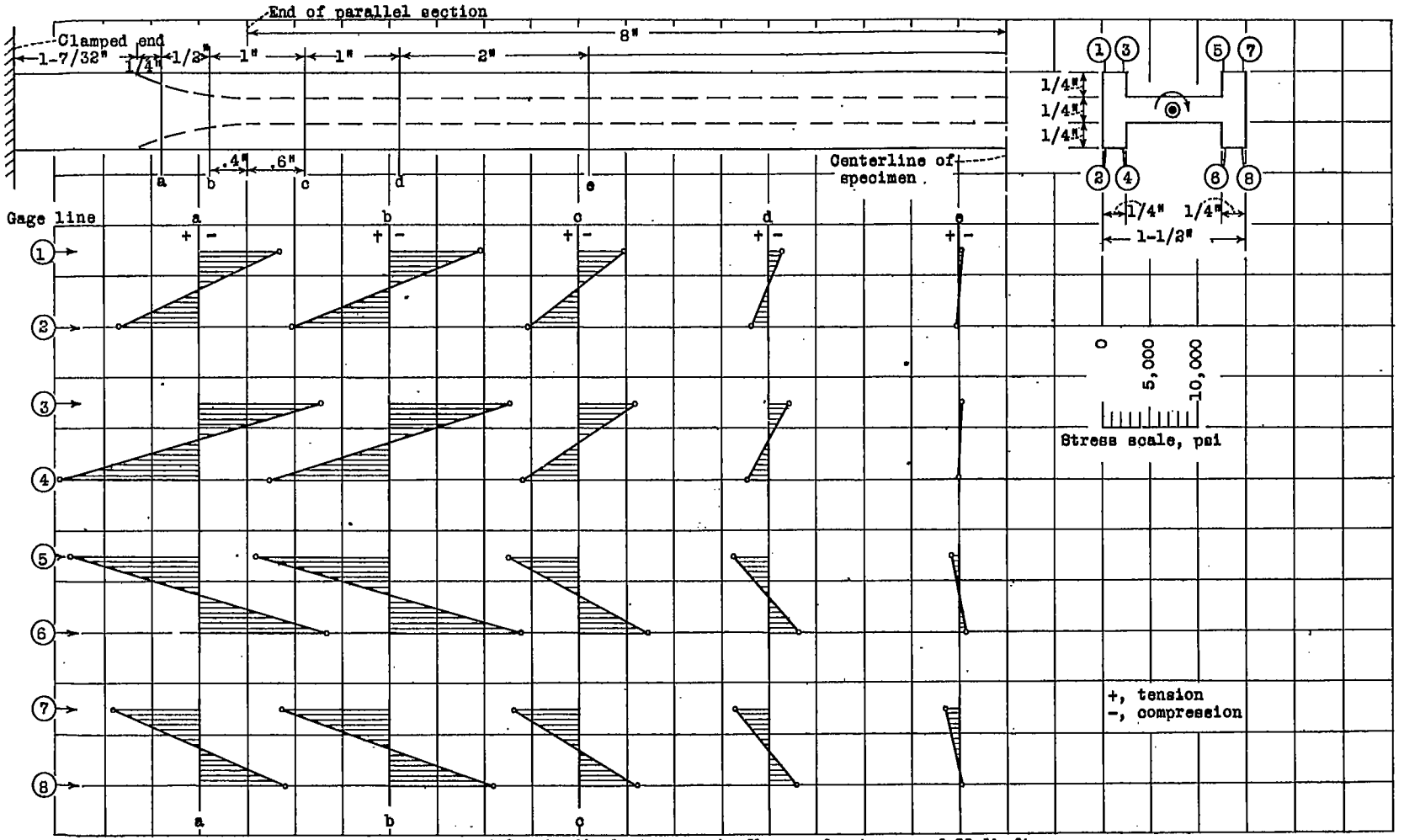


Figure 3.- Torque-twist and stress curves for I-section, 24S-T.



Measured longitudinal stresses in flanges for torque of 50 lb-ft

Figure 4.- Longitudinal stress distribution in I-section under torque, 24S-T.

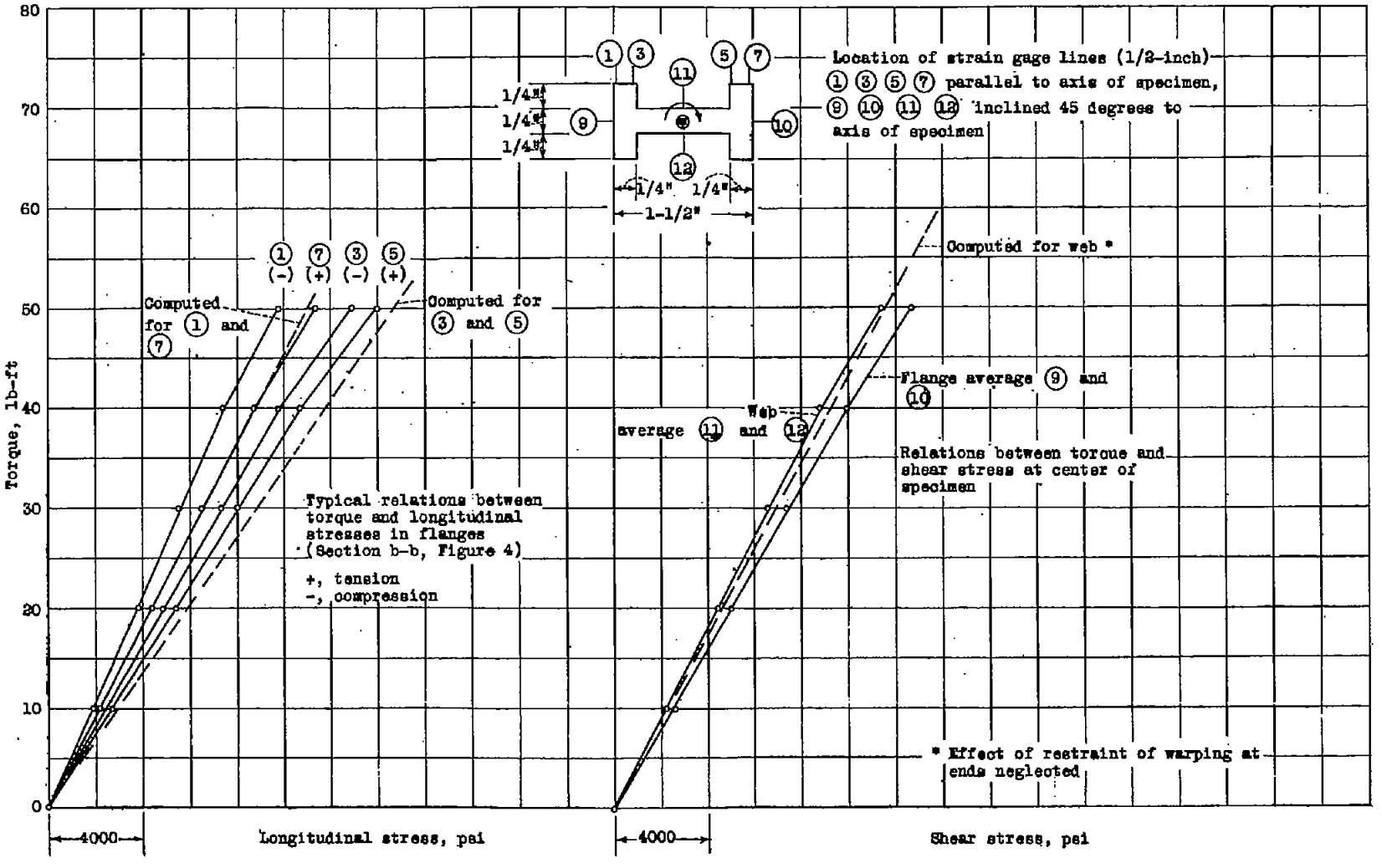


Figure 5.- Torque-stress curves for I-section, 24S-T.

Figure 6.- I and cruciform sections after torsion test (specimens machined from rolled 24S-T bar).

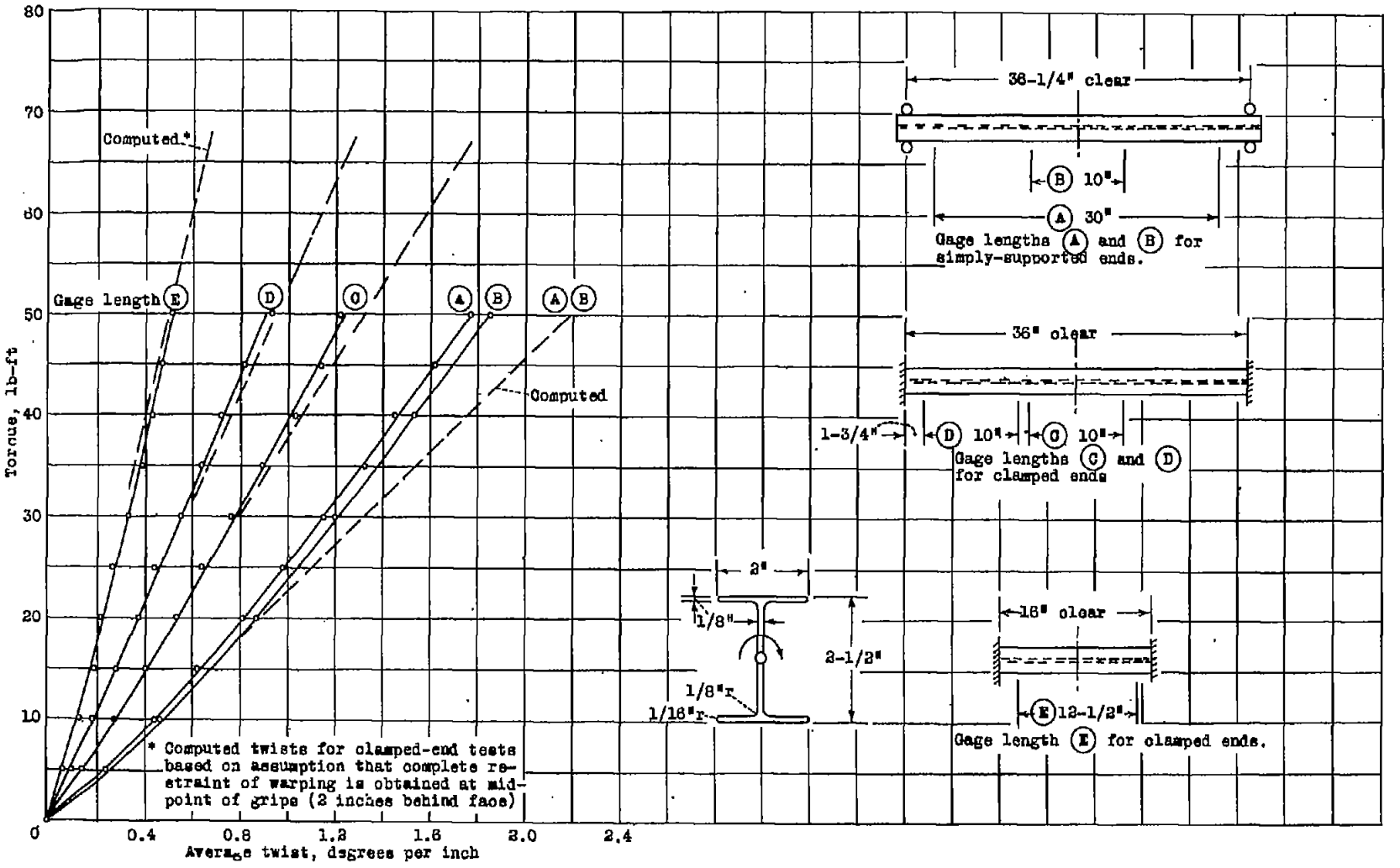


Figure 7.- Torque-twist curves for I-section, X74S-T.

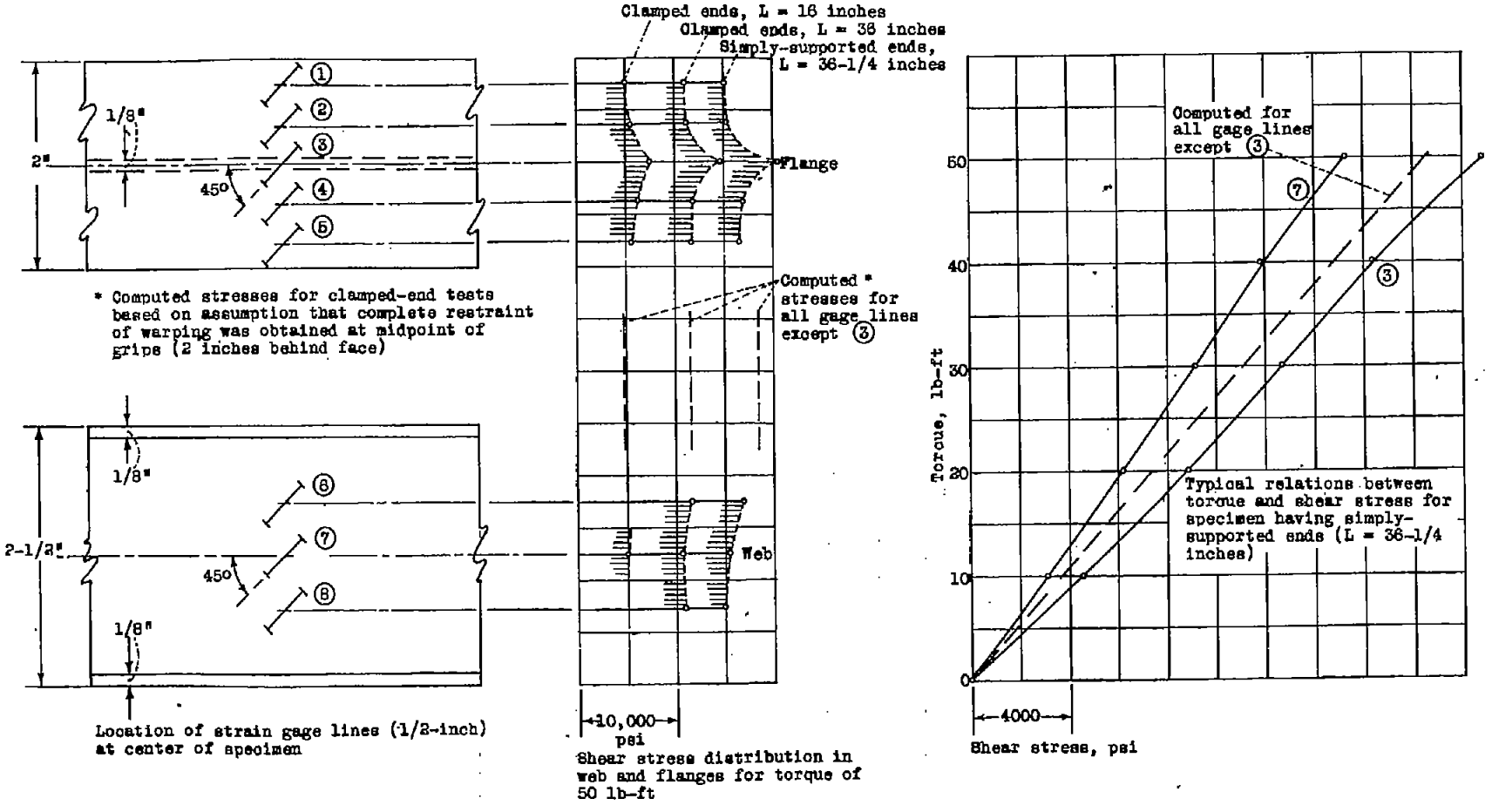
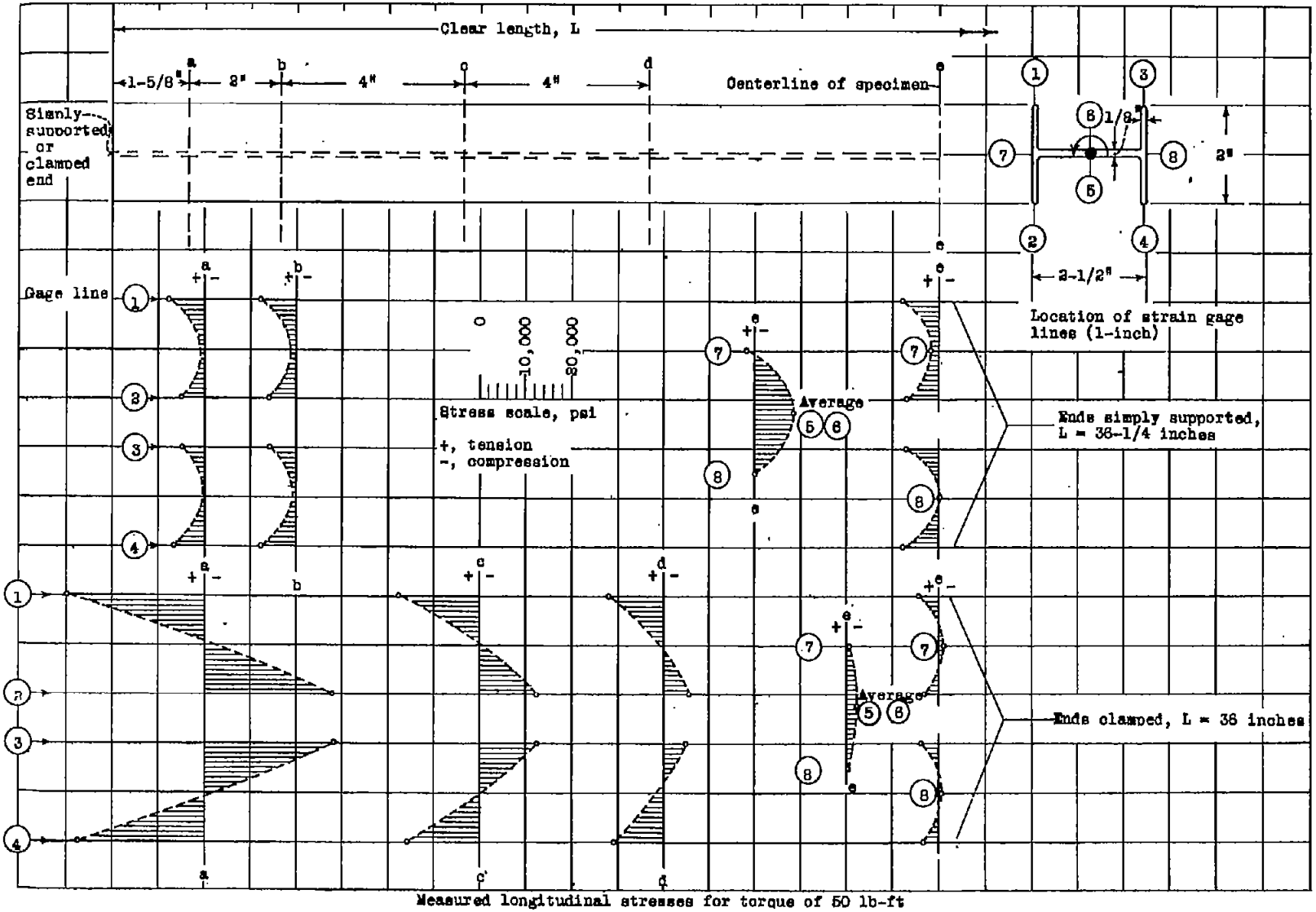


Figure 8.- Shear stresses in I-section under torque, X748-T.



Measured longitudinal stresses for torque of 50 lb-ft

Figure 9.- Longitudinal stresses in I-section under torque, X74S-T.

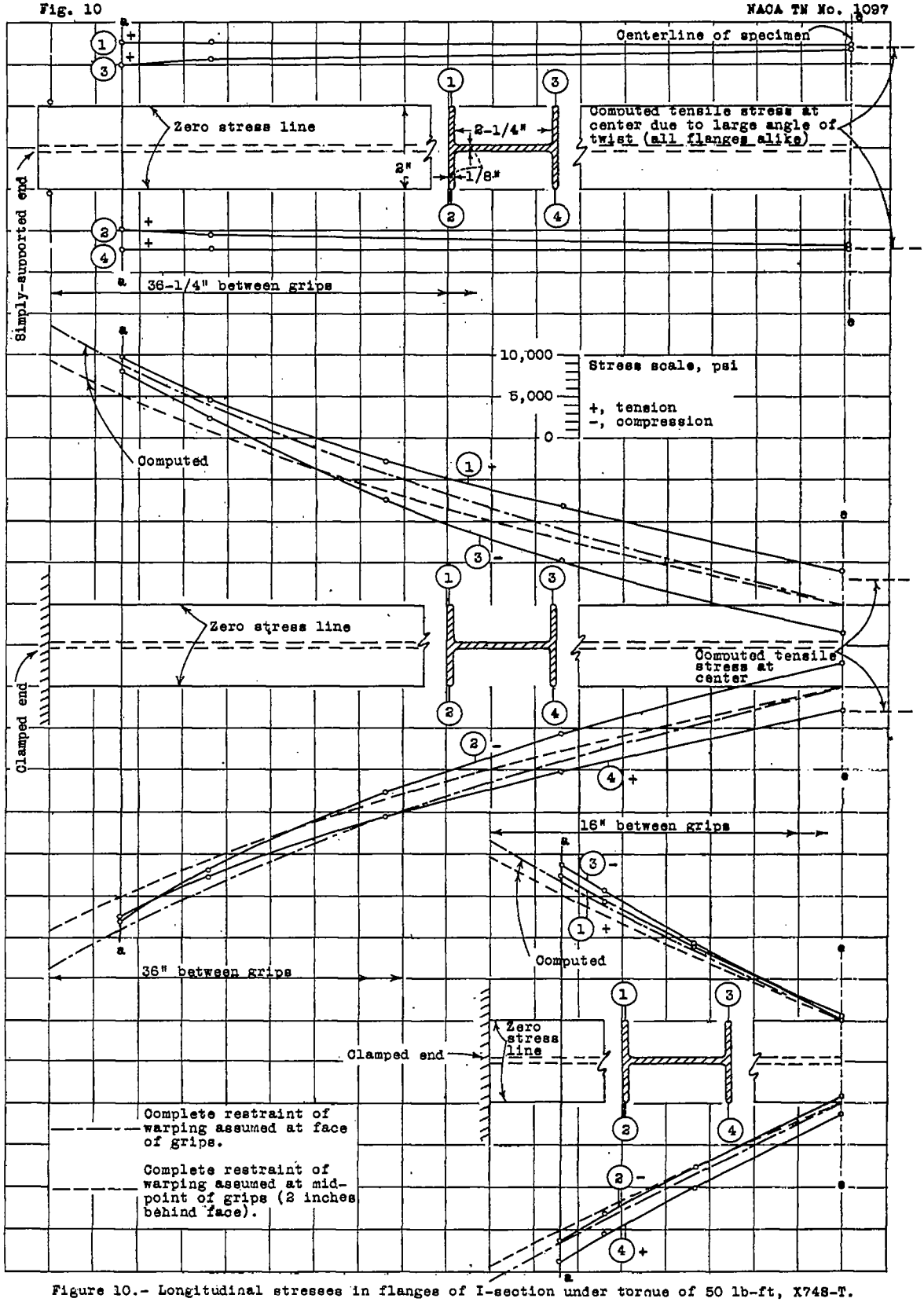


Figure 10.- Longitudinal stresses in flanges of I-section under torque of 50 lb-ft, X748-T.

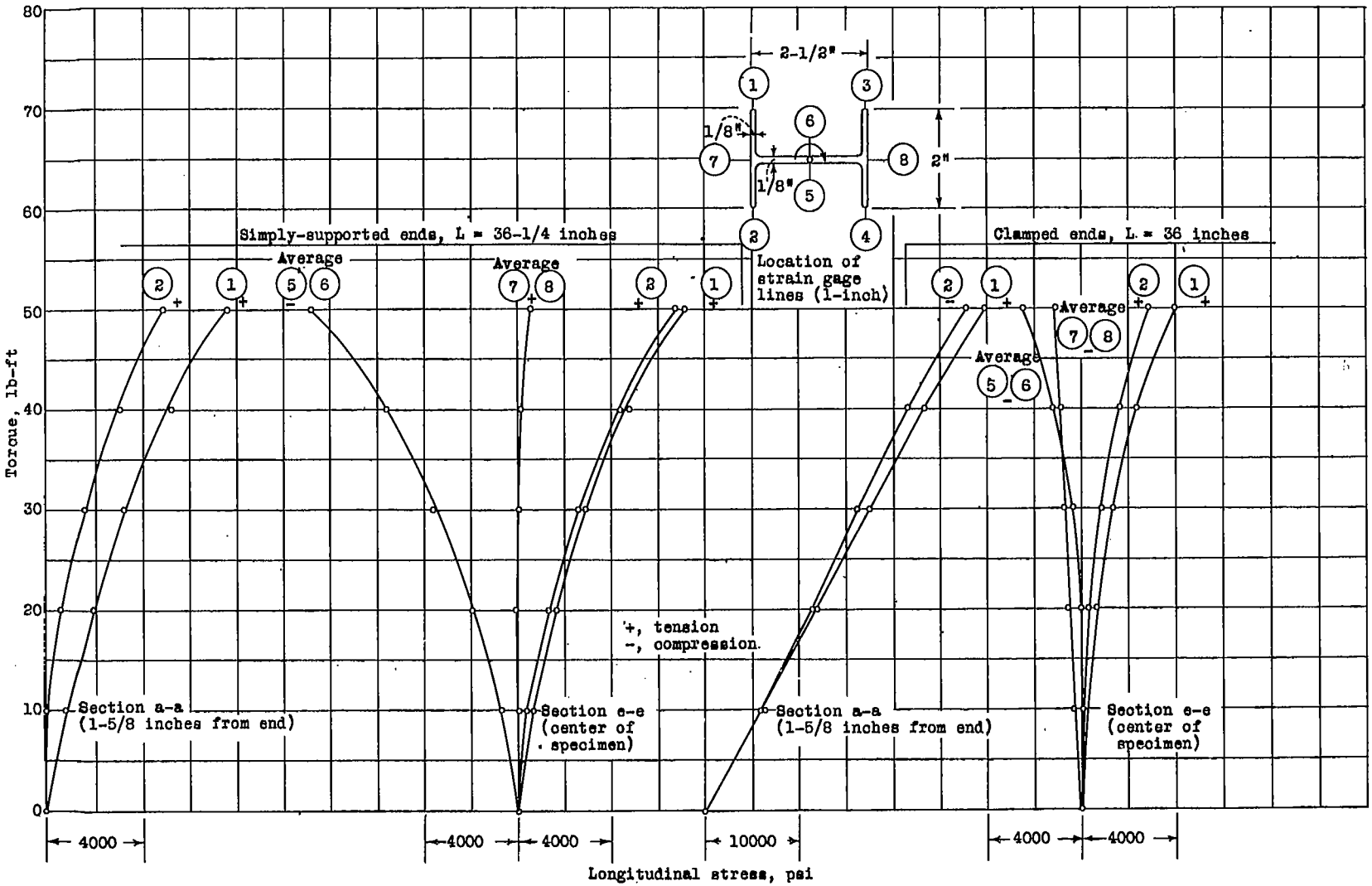


Figure 11.- Torque-stress curves for I-section, X74S-T.

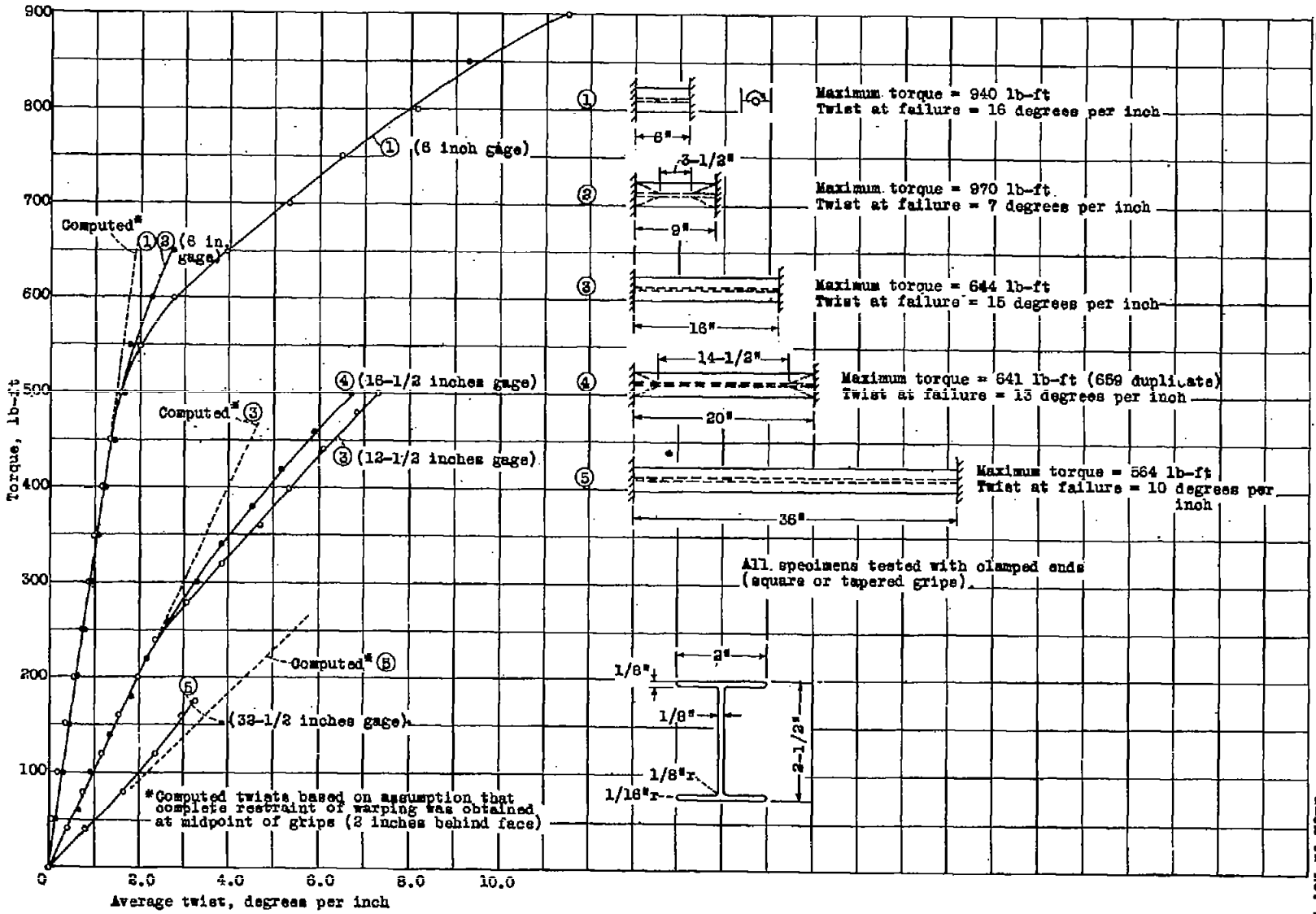


Figure 12.- Torque-twist curves and ultimate torques for I-sections, X74S-T.

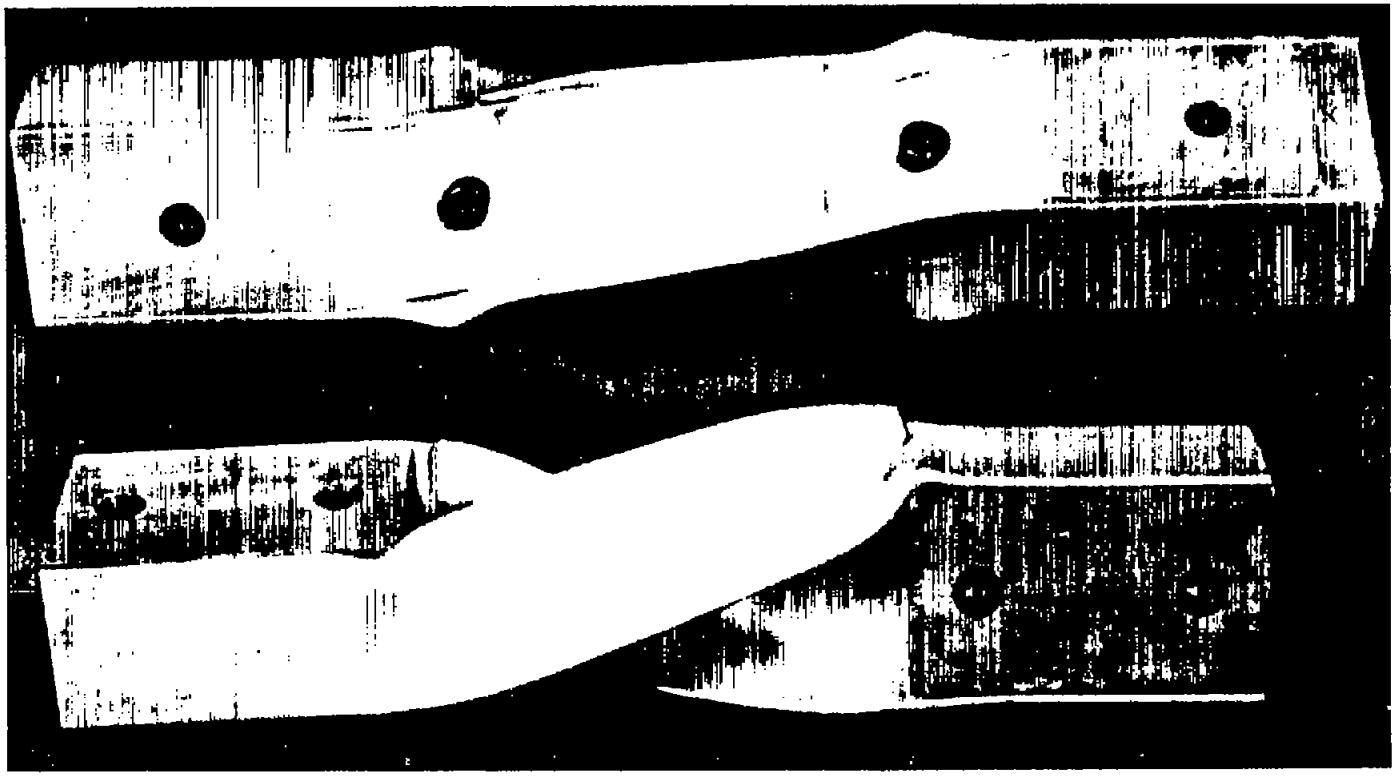


Figure 13.- I sections after torsion test, showing two types of filler plugs used in ends for gripping specimens.

Figure 14.- I and Zee sections after torsion test.

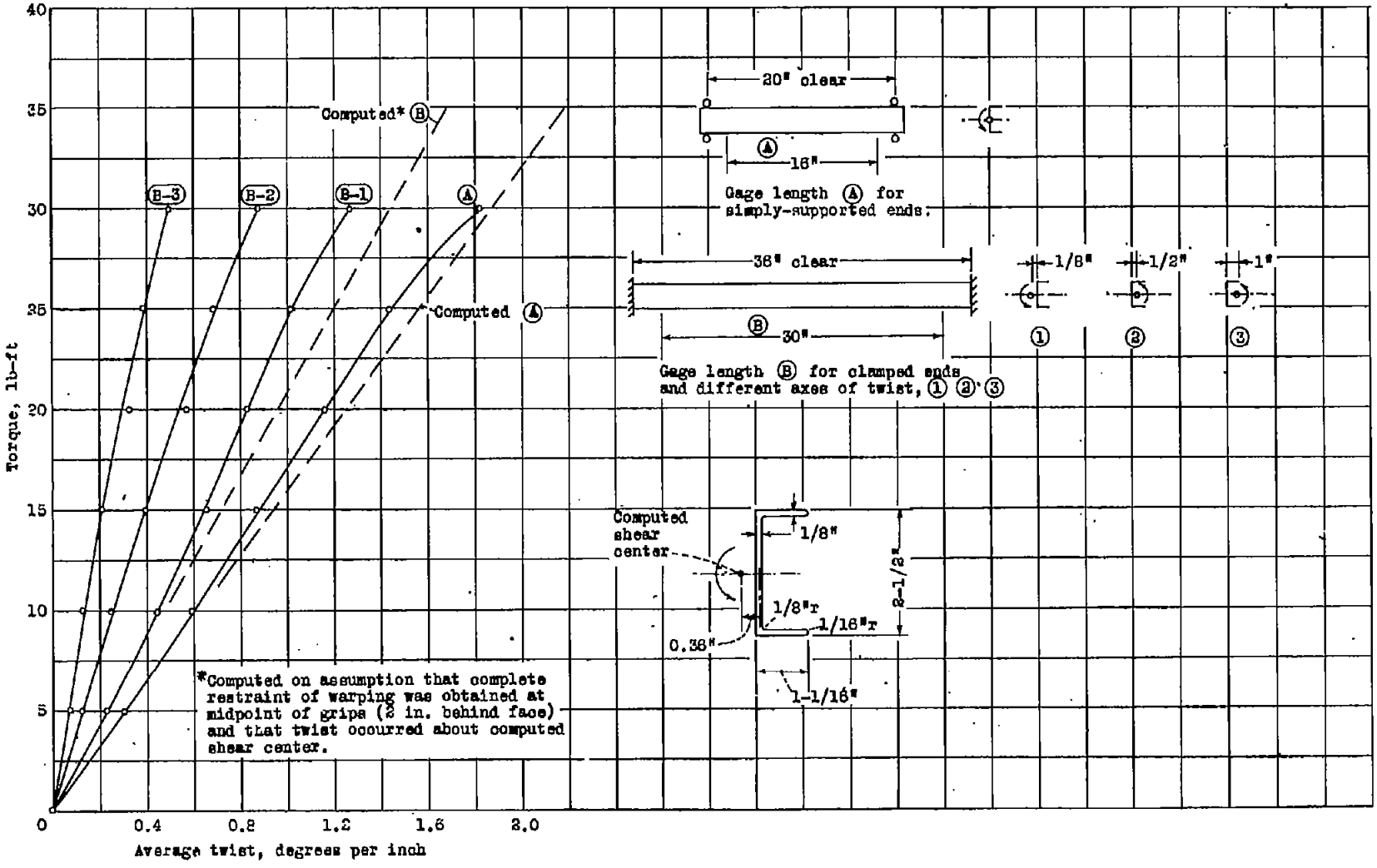


Figure 15.- Torque-twist curves for Channel section, X74S-T.

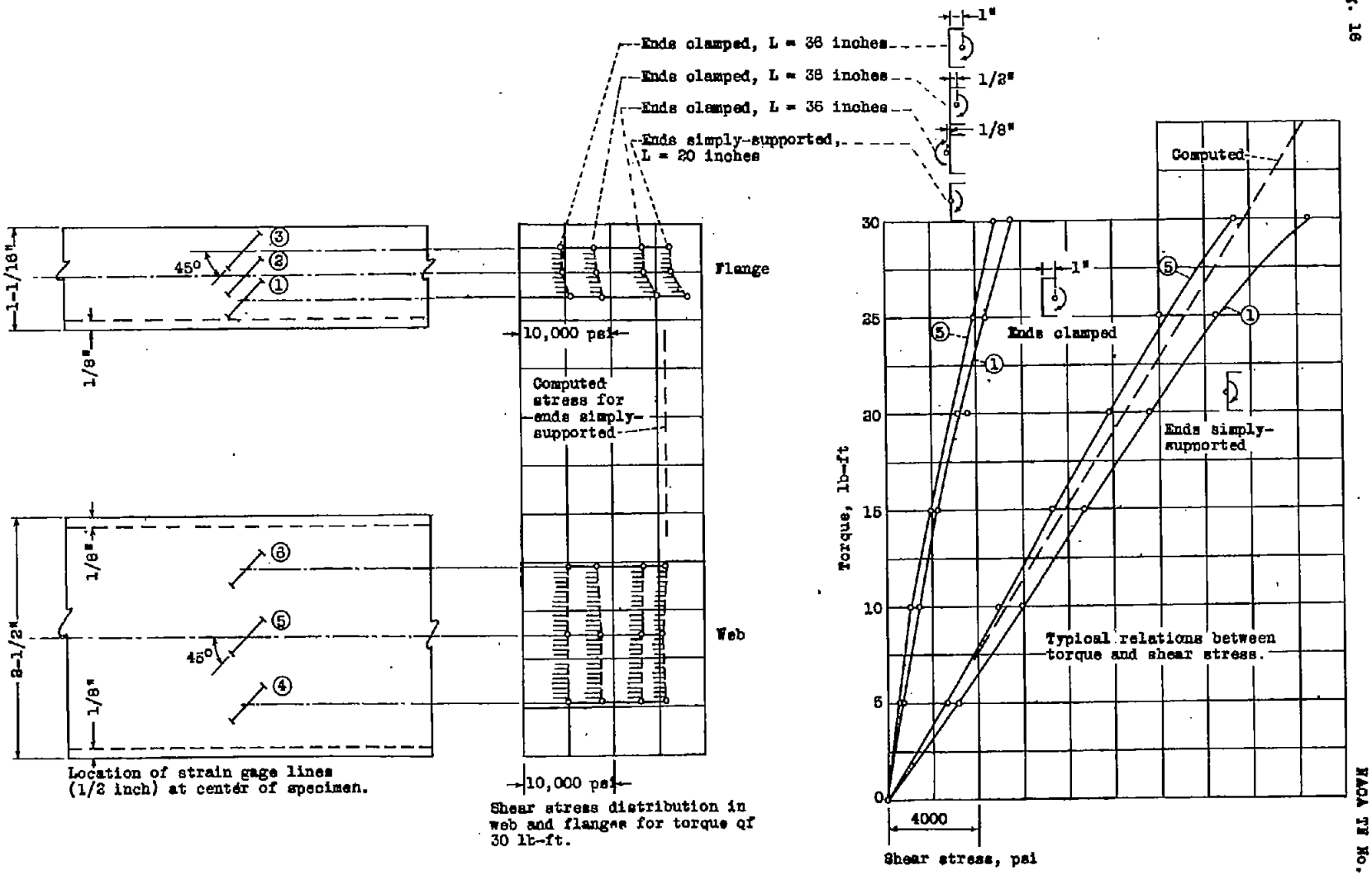
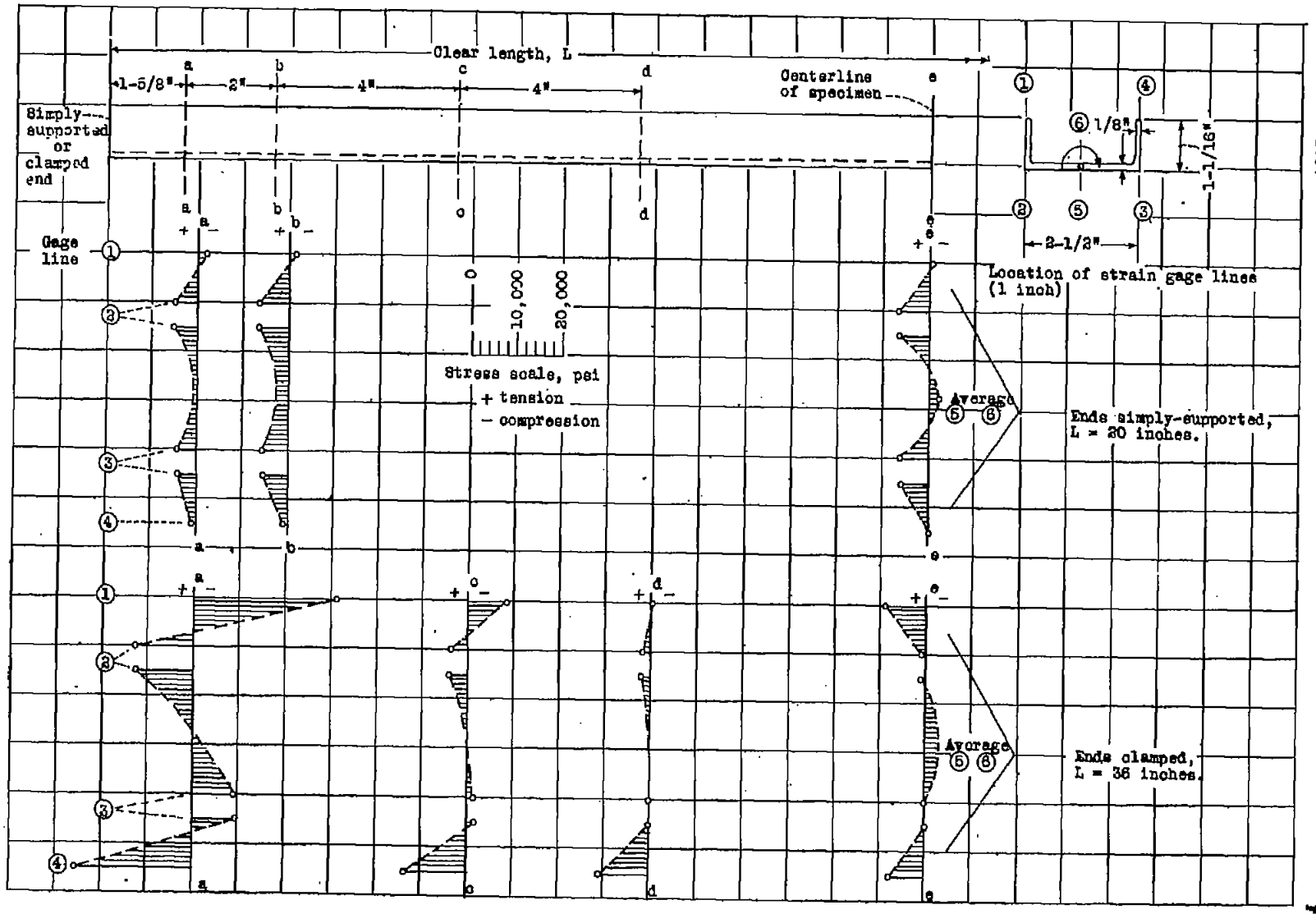
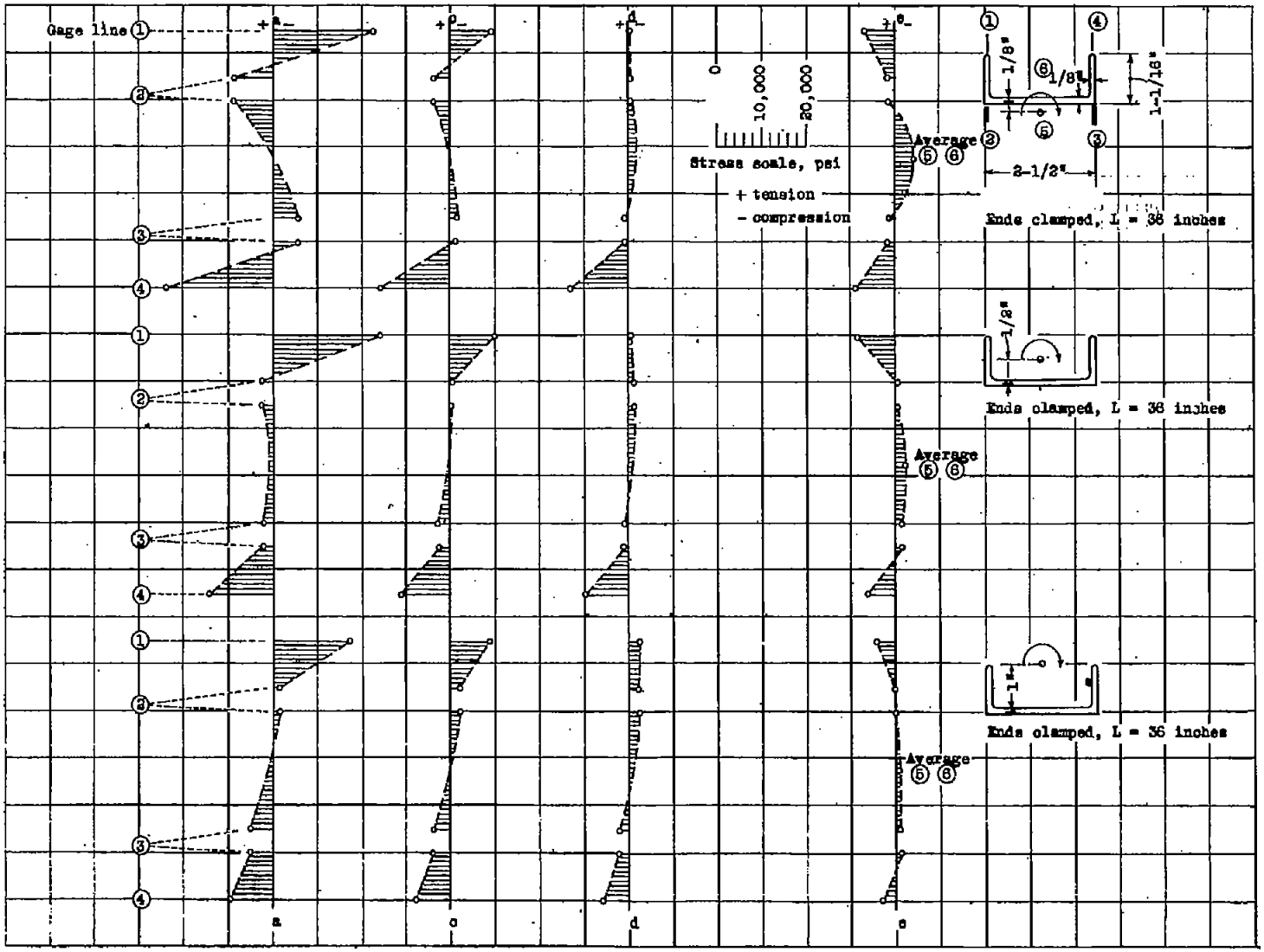


Figure 18.- Shear stresses in Channel section under torque, X74S-T.



Measured longitudinal stresses for torque of 30 lb-ft.

Figure 17.- Longitudinal stresses in Channel section under torque, X74S-T.



Measured longitudinal stresses for torque of 30 lb-ft.

Figure 17a.- Longitudinal stresses in Channel section under torque, X748-T.

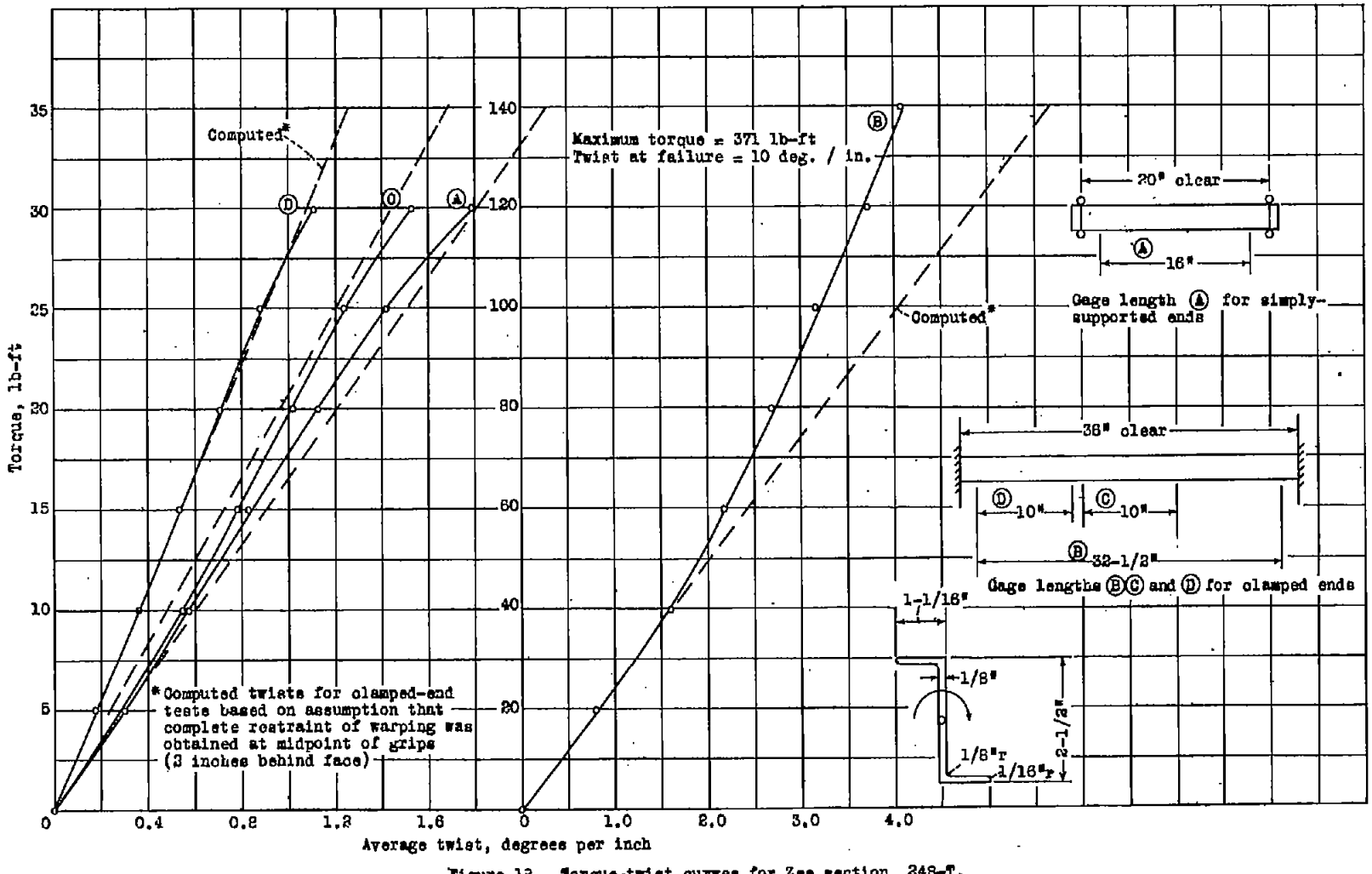
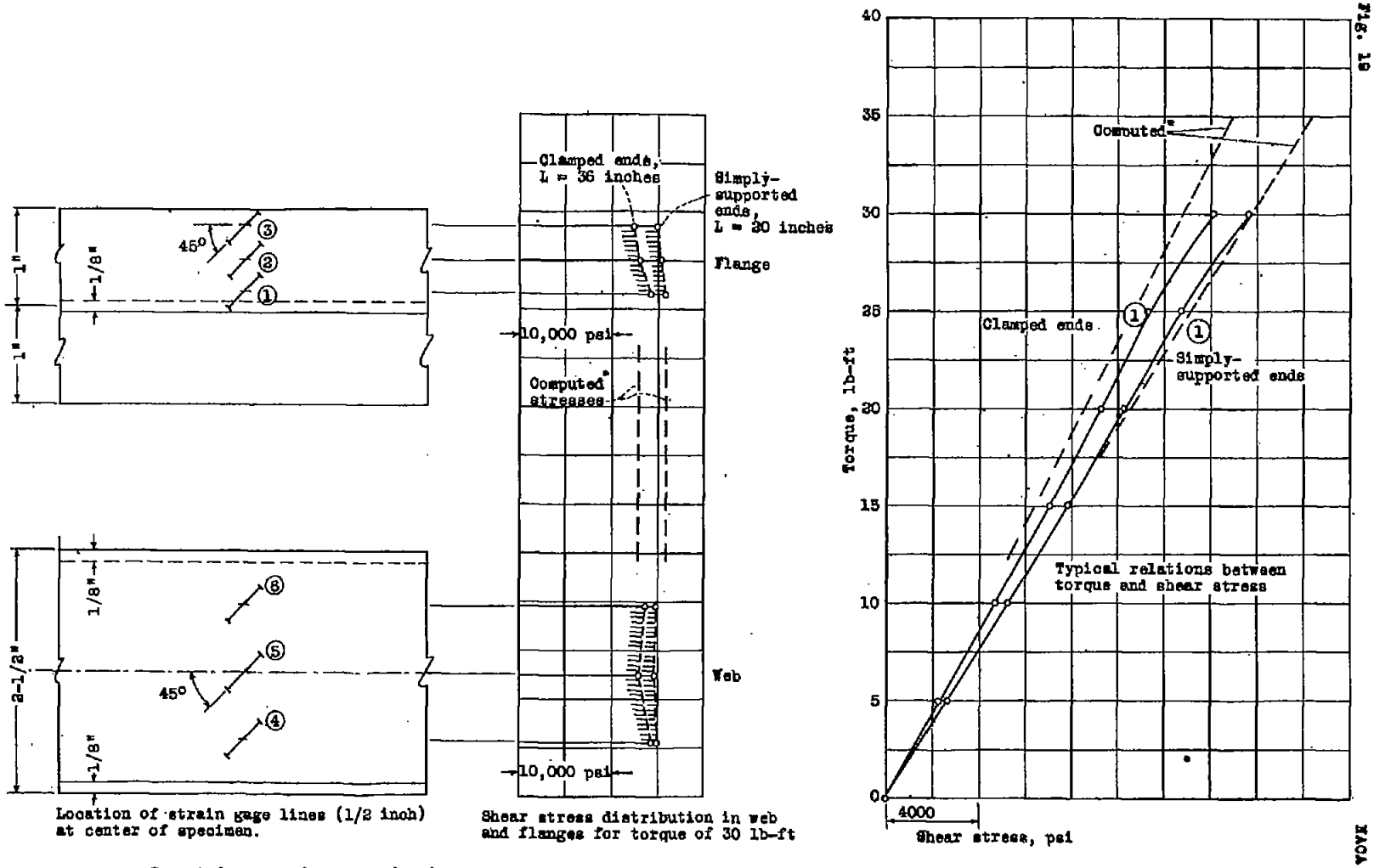


Figure 18.- Torque-twist curves for Zee section, 24S-T.

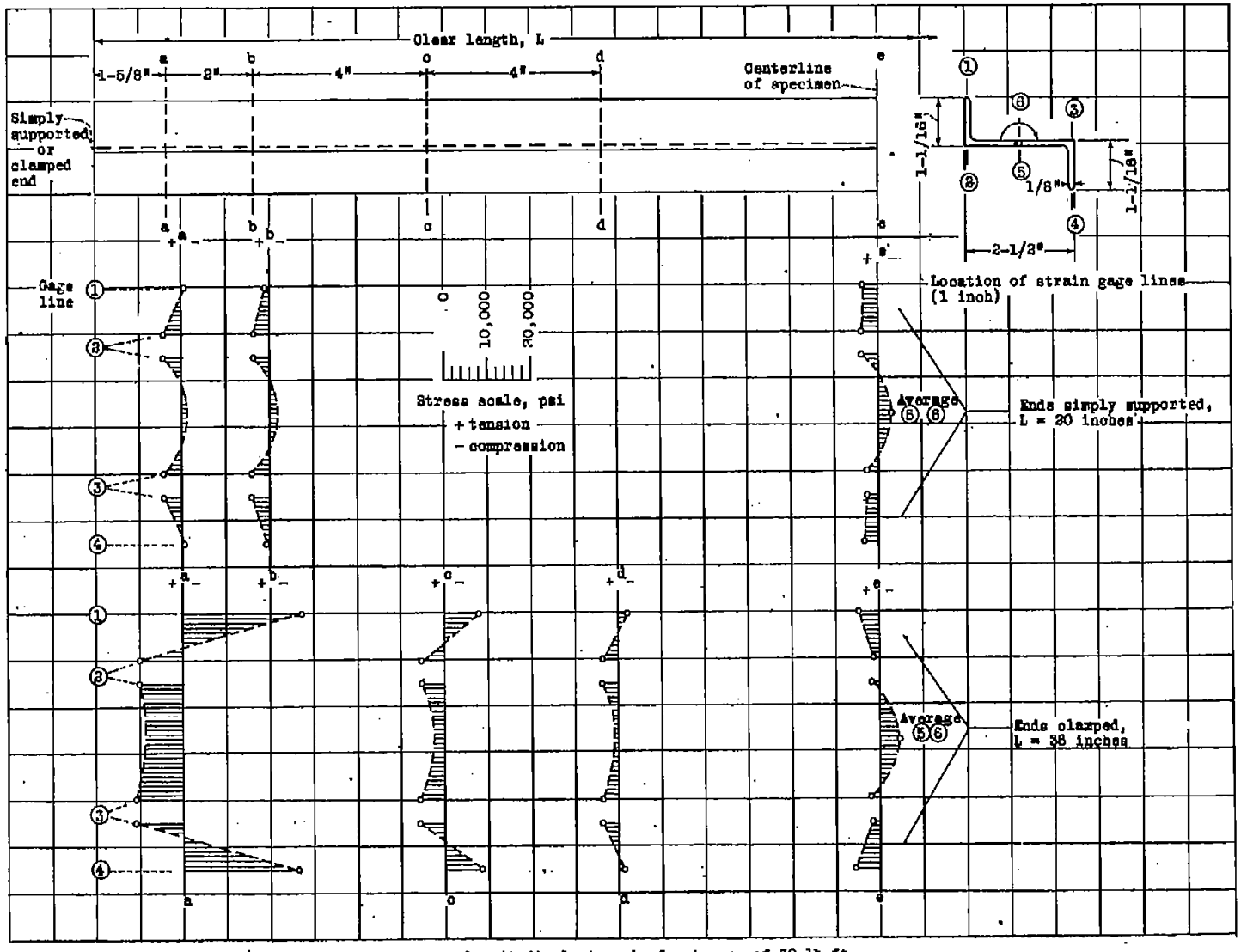


Location of strain gage lines (1/2 inch) at center of specimen.

Shear stress distribution in web and flanges for torque of 30 lb-ft

* Computed stress for clamped-end test based on assumption that complete restraint of warping was obtained at midpoint of grips (2 inches behind face).

Figure 19.- Shear stresses in Zee section under torque, 24S-T.



Measured longitudinal stresses for torque of 30 lb-ft

Figure 20.- Longitudinal stresses in Zee section under torque, 24S-T.

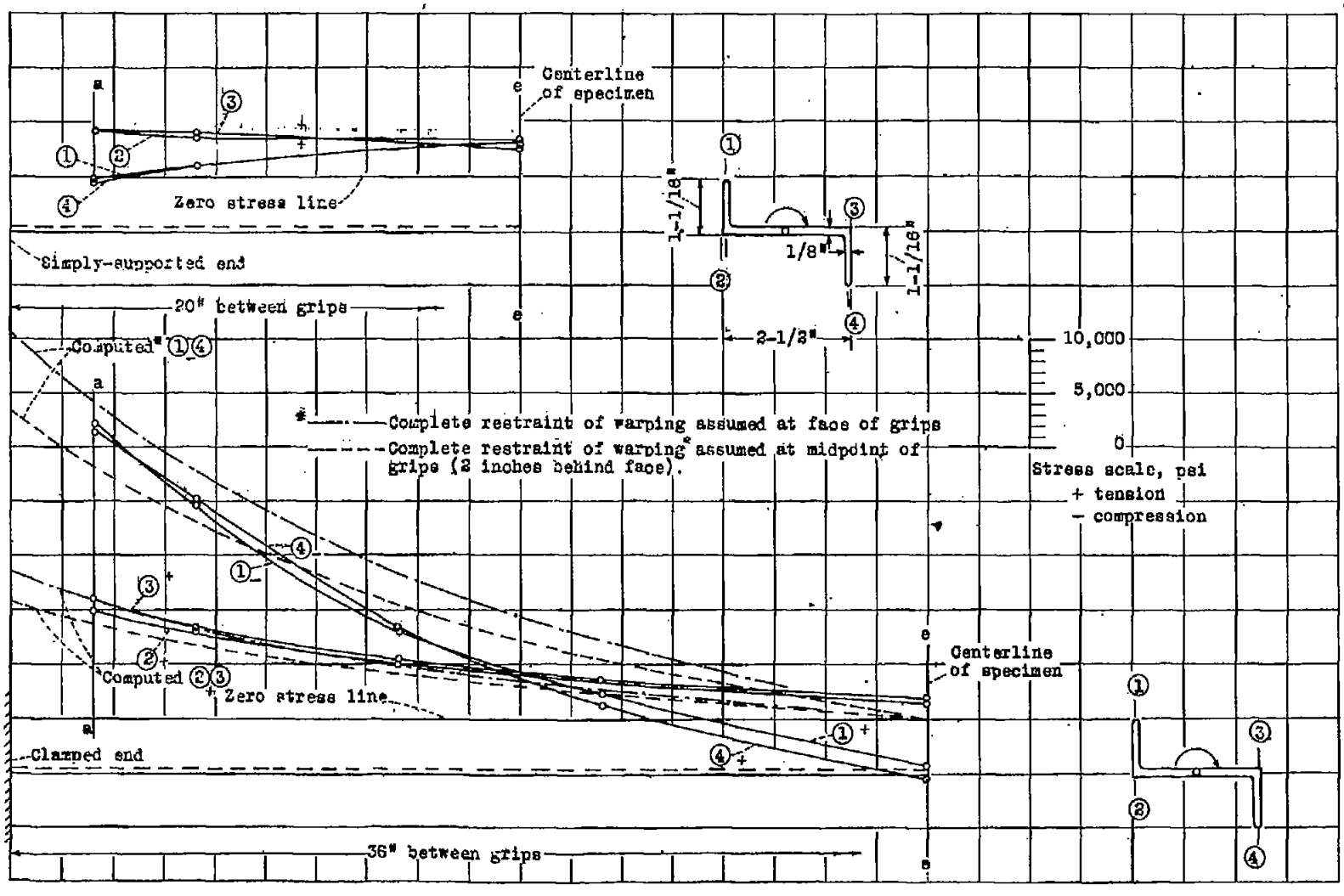


Figure 21.- Longitudinal stresses in Zee section under torque of 30 lb-ft, 24S-T.