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TORQUE-SPEED CHARACTERISTICS FOR HIGH-SPECIFIC-WORK TURBINES

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SUMMARY

This report presents an investigation of turbine torque-speed characteristics in a general form using the ideal specific work output corresponding to the turbine static- to total-pressure ratio as the normalizing parameter. These characteristics are first obtained using reference single-, two-, and three-stage analytical efficiency curves in the high-specific-work turbine operating range. Comparison of the results with available experimental data is then made to verify the trends of the curves presented.

These analytical curves are used to provide a basis for estimating starting torque margin as a function of design-point requirements. An example turbodrive application is also described to illustrate the manner in which the torque-speed curves can be used to establish the effects of staging on these characteristics. These studies showed that adding stages and reducing the criticalness of design with respect to the relation of specific work output to blade speed result in improved margin of starting torque over that at design speed.

INTRODUCTION

In many turbopump applications there exists a need for knowledge of turbine torque-speed characteristics. This need arises in order to estimate (1) the starting characteristics of the turbopump and (2) the stress of such parts as shafting, gearing, and blading. Unfortunately, very little reference torque-speed data are available for such purposes. Furthermore, little information is available concerning the fundamental studies of these characteristics.

As a consequence, an investigation of turbine torque-speed characteristics in general form was made at the Lewis laboratory, with the results presented herein. The report first discusses the basis for the selection of the form used in generalizing the correlation. This is followed by an analytical treatment of the problem for single-, two-, and

three-stage turbines using the material in reference 1 in the high-specific-work turbine operating range as a basis. Comparison of the analytical results is then made with available experimental results.

The analytical results are subsequently used in the development of a figure to be used in estimating margin of starting torque over that of design in terms of design velocity diagram characteristics. Finally, an example is presented to illustrate one manner in which the material in the report can be applied to the turbopump starting problem.

SYMBOLS

c_p specific heat at constant pressure, Btu/(lb)($^{\circ}$R)

g acceleration due to gravity, 32.17 ft/sec^2

H total enthalpy rate, Btu/sec

h specific enthalpy, Btu/lb

J mechanical equivalent of heat, 778.2 ft-lb/Btu

n number of turbine stages

p pressure, lb/sq ft

r mean-section radius, ft

T absolute temperature, $^{\circ}$R

U mean-section blade speed, ft/sec

V absolute gas velocity, ft/sec

V_j ideal gas velocity corresponding to static- to total-pressure ratio across turbine, ft/sec

W relative gas velocity, ft/sec

w weight-flow rate, lb/sec

γ ratio of specific heats

η_s efficiency based on static- to total-pressure ratio across turbine

θ_{cr} squared ratio of critical velocity at turbine inlet to that at NACA standard sea-level temperature

λ speed-work parameter, $U^2/gJ\ \Delta h'$

ν blade-jet speed ratio, U/V_j

τ torque, ft-lb

ω rotative speed, radians/sec

Subscripts:

a first stage

d design

id ideal

S stage

s static

t total

u tangential component

0 turbine inlet

1 first-stage stator exit

2 first-stage rotor exit

3 turbine exit

Superscripts:

' absolute total state

— over-all

DEVELOPMENT

Basis for Correlating Torque with Speed

In the investigation presented herein, it is desired to express the turbine torque in terms of velocity diagram characteristics. Figure 1 presents a schematic diagram of a turbine with the station nomenclature used in the report. A typical set of velocity diagrams is also included, in this case, for the first stage. Mean-section velocity diagrams will

be used throughout this report as representing the average of those occurring from hub to tip. Also, as indicated in figure 1, the mean-section radius is considered constant through the turbine. If the first stage is used as an example, the specific work output $\Delta h_a'$ can be related to the velocity diagram quantities by the equation

$$\Delta h_a' = \frac{U\ \Delta V_{u,a}}{gJ} \tag{1}$$

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The total work output $\Delta H_a'$ can then be expressed by the equation

$$\Delta H_a' = \Delta h_a' w$$

or, by using equation (1),

$$\Delta H_a' = \frac{wU\ \Delta V_{u,a}}{gJ} \tag{2}$$

The total stage work output can also be related to the stage torque τ_a by

$$\Delta H_a' = \frac{\tau_a \omega}{J}$$

or, since $U = r\omega$,

$$\Delta H_a' = \frac{\tau_a U}{Jr} \tag{3}$$

Combining equations (2) and (3) finally yields

$$\frac{\tau_a}{r\left(\frac{w}{g}\right)} = \Delta V_{u,a} \tag{4}$$

From equation (4) it is evident that $\Delta V_{u,a}$ is the diagram quantity, variations of which are a measure of changes in the stage torque. In the multistage turbine case then, the total change in tangential velocity $\Delta V_{u,t}$, which is the sum of those occurring across each stage, is the diagram quantity, variations of which are a measure of changes in the turbine torque. This quantity, $\Delta V_{u,t}$, is therefore used herein for this purpose.

In correlating $\Delta V_{u,t}$ with the turbine speed let the rotor mean-section blade speed U be used. Thus, without consideration of a

general form, the torque-speed characteristics can be determined for a particular turbine by a graphical presentation of $\Delta V_{u,t}$ as a function of U. Since a number of points would be obtained for a given speed because of the variation of torque with pressure ratio, a series of lines would be required, each for a different value of pressure ratio. An example of such a plot is presented in figure 2. The quantity $\Delta V_{u,t}$ is presented as a function of U in equivalent terms (NACA standard conditions assumed at turbine inlet) by dividing both these quantities by $\sqrt{\theta_{cr}}$ or the ratio of critical velocity of the gas at the turbine inlet to that at standard conditions. The ratio of static pressure at the turbine exit to total pressure at the turbine inlet is shown as the independent parameter. The data used for this figure were obtained from the results of the investigation of the transonic turbine reported in reference 2.

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The generalization of the torque-speed relation such as that presented in figure 2 can be made by normalizing the ordinate and abscissa in such a manner as to minimize or eliminate the spread of the curves due to variation in the static- to total-pressure ratio. Such a normalizing parameter is the ideal specific work output $\Delta\bar{h}'_{id,s}$ corresponding to the static- to total-pressure ratio across the turbine p_3/p'_0. The relation between these two parameters is

$$\Delta\bar{h}'_{id,s} = c_p T'_0 \left[1 - \left(\frac{p_3}{p'_0}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{5}$$

The parameter $\Delta\bar{h}'_{id,s}$ (Btu/lb) can be converted to the proper normalizing dimensions (ft/sec) by putting it in the form $\left(gJ\ \Delta\bar{h}'_{id,s}\right)^{1/2}$. The results of normalizing the curves in figure 2 in this manner are presented in figure 3 where the torque parameter $\Delta V_{u,t}/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ is presented as a function of the speed parameter $U/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ for various pressure ratios and speeds. In this figure it can be seen that, except for the lowest values of p_3/p'_0, the original curves resolve into a generalized basic curve. This generalization into one curve occurs because, for the given turbine geometry, the specification of a value of $U/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ in turn determines similar velocity diagrams as speed is varied (to be discussed in more detail in the next section). Thus, for similar diagrams the torque parameter $\Delta V_{u,t}/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ would also be constant.

It might be noted that for this example the deviation of the low-pressure-ratio curves (0.25 and 0.30) from the general curve in figure 3 occurs as a result of choking in the rotor. This choking then changes the velocity diagram from that at higher pressure ratios because of the supersonic expansion downstream of the rotor throat with more and more of the pressure being merely used to increase the axial component of velocity.

The speed parameter $U/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ used as the abscissa in the torque-speed relation is, except for a factor of $\sqrt{2}$, the same as the well-known blade-jet speed ratio ν. Also, if $\bar{\eta}_s$ is originally known as a function of ν for a particular turbine, a conversion to the form used in figure 3 can be easily made by the relations

$$\frac{U}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}} = \nu\sqrt{2} \tag{6a}$$

and

$$\frac{\Delta V_{u,t}}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}} = \frac{U\ \Delta V_{u,t}}{gJ\ \Delta\bar{h}'_{id,s}}\ \frac{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}}{U}$$

or

$$\frac{\Delta V_{u,t}}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}} = \frac{\bar{\eta}_s}{\nu\sqrt{2}} \tag{6b}$$

since

$$U\ \Delta V_{u,t} = gJ\ \Delta\bar{h}' \tag{7}$$

and

$$\bar{\eta}_s = \frac{\Delta\bar{h}'}{\Delta\bar{h}'_{id,s}} \tag{8}$$

An example of such a conversion is presented in figure 4. Figure 4(a) shows a reproduction of the variation in $\bar{\eta}_s$ with ν from figure 218 of reference 3. The curve presented in figure 4(a) covers a range of ν from 0 to 0.5 and is for the single-stage case with a stator-exit angle of 17°. Figure 4(b) shows the results of using equations (6a) and (6b) to convert the curve of figure 4(a) into the form used in presenting the generalized torque-speed characteristics.

Use of $\bar{\eta}_s$ - $\bar{\lambda}$ Relation in Analytically Obtaining Generalized Torque-Speed Characteristics

In reference 2, an analytical investigation of the static efficiency characteristics of single-stage and multistage turbines is presented as a function of a speed-work parameter $\bar{\lambda}$. The over-all value of this parameter is defined as the ratio of the square of the mean-section blade speed to the over-all required specific work output, or

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$$\bar{\lambda} = \frac{U^2}{gJ\ \Delta\bar{h}'} \tag{9}$$

Similarly, an equation for the stage speed-work parameter λ_S can be written as

$$\lambda_S = \frac{U^2}{gJ\ \Delta h_S'} \tag{10}$$

In using $\bar{\lambda}$ in establishing an analytical correlation of torque with speed, single-, two-, and three-stage turbines will be considered. Figure 5 is a reproduction of figure 5 of reference 1 where $\bar{\eta}_s$ is presented as a function of $\bar{\lambda}$ for single-stage and multistage turbines. These curves are obtained under the following assumed conditions:

(1) The specific work output and mean-section blade speed of each stage are the same. This results in the relation

$$\bar{\lambda} = \frac{\lambda_S}{n} \tag{11}$$

(2) Impulse conditions, defined as equal rotor blade inlet and exit relative tangential velocities, exist in the range of λ_S from 0 to 0.5. This range will be used in this report because it represents the design range encountered in high-specific-work turbine applications.

In order to use the curves in figure 5 to obtain the relation between $\Delta V_{u,t}/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ and $U/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$, the relation between these two parameters and the parameters $\bar{\lambda}$ and $\bar{\eta}_s$ must first be determined. Now,

$$\frac{\Delta V_{u,t}}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}} = \sqrt{\frac{\Delta V_{u,t}^2}{gJ\ \Delta\bar{h}'}\ \frac{\Delta\bar{h}'}{\Delta\bar{h}'_{id,s}}} \tag{12a}$$

and

$$\frac{U}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}} = \sqrt{\frac{U^2}{gJ\ \Delta\bar{h}'}\ \frac{\Delta\bar{h}'}{\Delta\bar{h}'_{id,s}}} \tag{12b}$$

Combining equations (7) and (9) yields another equation for $\bar{\lambda}$:

$$\bar{\lambda} = \frac{gJ\ \Delta\bar{h}'}{\Delta V_{u,t}^2} \tag{13}$$

So substituting equations (8), (9), and (13) into equations (12) yields

$$\frac{\Delta V_{u,t}}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}} = \sqrt{\frac{\bar{\eta}_s}{\bar{\lambda}}} \tag{14a}$$

and

$$\frac{U}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}} = \sqrt{\lambda\eta_s} \tag{14b}$$

By using the curves in figure 5 with equations (14a) and (14b) the desired torque-speed curves can be computed. The results of such calculations are presented in figure 6 for the single-, two-, and three-stage turbine cases in the λ_s range from 0 to 0.5. For the single-stage turbine, the torque-speed curve approaches a straight line and, incidentally, is in very good agreement with the similar curve in figure 4(b). For the two- and three-stage turbines the indicated torque parameters are considerably greater than those for the single-stage case and curve up slightly as the speed parameter is reduced. These curves shown in figure 6 will be used in a subsequent section to study turbine starting torque margin characteristics.

It might be noted that equation (14b) offers an insight as to why the specification of $U/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ specified similar diagrams as the blade speed was varied in the example presented in the section concerned with the basis for correlating torque with speed. Since a specified value of $\bar{\lambda}$ defines similar diagrams as blade speed is varied and, in addition, specifies a value of $\bar{\eta}_s$ from figure 5, it then follows from equation (14b) that a specification of $U/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ also determines similar diagrams.

Comparison of Analytical Curves with Experimental Data

Before analytical curves such as those presented in figure 6 can be considered valid for use in studying such off-design problems as starting torque margin, comparison with experimental results must first be made. This is required particularly for these curves because the original analysis (ref. 1) represents a design-point study, but the results are being used herein for a study of off-design characteristics.

Figure 7 presents a comparison of the analytical results with available experimental data obtained from single-, two-, and three-stage turbines. As indicated by the figure, these turbines were all designed for λ_S values of approximately 0.5 or less. The data shown were taken over a range of speed at approximately design static- to total-pressure ratio.

Figure 7 shows that reasonable correlation of analytical and experimental results was obtained. Therefore, it can be concluded that the analytical curves can be used with confidence in representing the generalized torque-speed characteristics for turbines designed for λ_S values of 0.5 or less. Such a representation for turbines designed for λ_S values greater than 0.5 may not be too reliable because of increased incidence losses incurred as the turbine speed is reduced.

APPLICATION

Starting Torque Margin as Function of Design Speed-Work Parameter

The previous section presented an analytically obtained relation between turbine torque and speed in a general form using the ideal specific work output corresponding to the static- to total-pressure ratio across the turbine as the normalizing parameter. Such a relation can be used to provide a basis for estimating starting torque margin as a function of design-point requirements, expressed herein in terms of the over-all design speed work parameter $\bar{\lambda}_d$. The curves in figure 5 together with equation (15) can be used for this purpose. The desired relation is the ratio of starting torque to design torque as a function of $\bar{\lambda}_d$ for a given ideal specific work output. The equation for the ordinate is

$$\frac{(\Delta V_{u,t})_{U=0}}{(\Delta V_{u,t})_{U=U_d}} = \frac{\left(\dfrac{\Delta V_{u,t}}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}}\right)_{\dfrac{U}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}}=0}}{\left(\dfrac{\Delta V_{u,t}}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}}\right)_{\dfrac{U}{\sqrt{gJ\ \Delta\bar{h}'_{id,s}}}=\text{Design value}}} \tag{15}$$

The numerator on the right side of equation (15) can be obtained from figure 6. The denominator can be obtained by first selecting the $\bar{\eta}_s$ corresponding to the particular value of $\bar{\lambda}_d$ from figure 5, computing $U/\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$ by equation (14b), and finally selecting the proper value from figure 6.

The results of these calculations are presented in figure 8 where $(\Delta V_{u,t})_{U=0}/(\Delta V_{u,t})_{U=U_d}$ is presented as a function of $\bar{\lambda}_d$. This figure shows that, for all three types of turbines considered, increasing the value of $\bar{\lambda}_d$ increases the margin of starting torque to that of design. For example, if $\bar{\lambda}_d$ is 0.2, the torque ratio is 1.32 for the single-stage case. However, if $\bar{\lambda}_d$ is increased to 0.4, the torque ratio increases to 1.66. Thus, it is evident that if high starting torque margin is desired more conservative turbine designs (increased $\bar{\lambda}_d$) are required. Incidentally, the upper limits imposed on the three curves represent a λ_s value of 0.5, an upper limit used throughout this report.

Further inspection of figure 8 shows that as the number of stages is increased the slope of the curves is also increased, which results in a considerable increase in torque margin. For example, if $\bar{\lambda}_d$ is 0.15 the torque ratio is seen to increase from 1.23 for the single-stage case to 1.75 and 2.20 for the two- and three-stage turbines, respectively. Thus, staging is another way to increase the torque margin.

It might be noted that in many applications, a turbopump drive, for example, as the number of stages is changed, $\bar{\lambda}_d$ may also change, thus requiring a different approach to the torque margin problem. An example of such a situation will now be described.

Example Turbodrive Application

In illustrating the effects of staging on the torque-speed characteristics of an example turbodrive application, let the ideal specific work $\Delta\bar{h}'_{id,s}$ and mean-section blade speed U be specified. Values selected for calculation purposes are $\Delta\bar{h}'_{id,s} = 2160$ Btu per pound and $U = 1260$ feet per second.

In determining a design line to impose on figure 5 using these requirements, let it be noted that the design-speed parameter $U/\sqrt{gJ\,\Delta\bar{h}'_{id,s}}$ is specified as 0.171. Therefore, from equation (14b) the product $\bar{\eta}_s\bar{\lambda}_d$ is also constant at 0.0292. By using this value a design line can be imposed on figure 5. Since the figure is presented as a log-log plot, the design line as shown is straight with a slope of -45°.

The effect of staging for the example case can be seen from figure 5. As the number of stages is increased, the required $\bar{\lambda}_d$ is reduced. This reduction occurs as a result of the increased efficiency due to staging. The increase in efficiency permits an increase in specific work output, which thus reduces the required turbine weight flow to deliver the same total power; a very desirable feature.

By using the design value of $U/\sqrt{gJ\,\Delta\bar{h}'_{id,s}}$ and the curves in figure 5, torque-speed curves can be constructed for single-, two-, and three-stage turbines. The results of such a construction are presented in figure 9 where the ratio of torque to design torque is presented as a function of the ratio of speed to design speed. Here again, turbine staging yields a significant increase in the starting torque margin with the margin increasing from 14 percent for the single-stage turbine to 33 and 50 percent for the two- and three-stage turbines, respectively. These increases are less than those obtained in the previous section where the comparison was made for a given $\bar{\lambda}_d$.

CONCLUDING REMARKS

This report has presented the results of an investigation of turbine torque-speed characteristics using the ideal specific work output corresponding to the turbine static- to total-pressure ratio as the normalizing parameter. Single-, two-, and three-stage turbines were considered to indicate the starting torque margin for these turbines as well as the comparable torque characteristics for an example turbopump application.

These studies showed that adding stages and reducing the criticalness of design with respect to the relation of specific work output to blade speed result in improved margin of starting torque over that at design speed.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, July 29, 1958

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3. Stodola, A.: Steam and Gas Turbines. Vol. I. McGraw-Hill Book Co., Inc., 1927. (Reprint, Peter Smith (New York), 1945.)

4. Johnston, I. H., and Sansome, G. E.: Tests on an Experimental Three-Stage Turbine Fitted with Low Reaction Blading of Unconventional Form. Rep. R.218, British NGTE, Jan. 1958.

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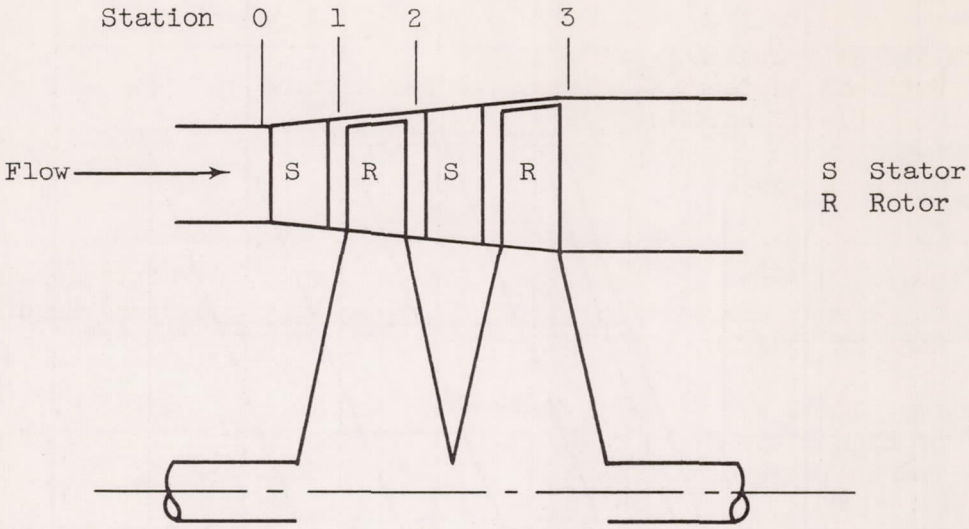
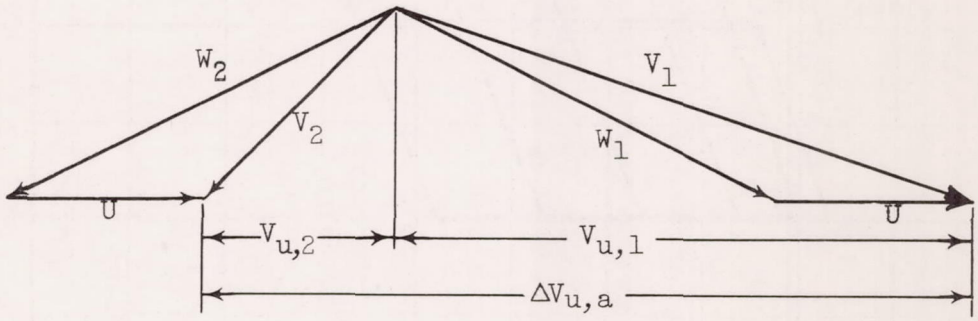


Schematic diagram of turbine with station nomenclature



Typical first-stage velocity diagrams

Figure 1. - Station nomenclature and example velocity diagrams.

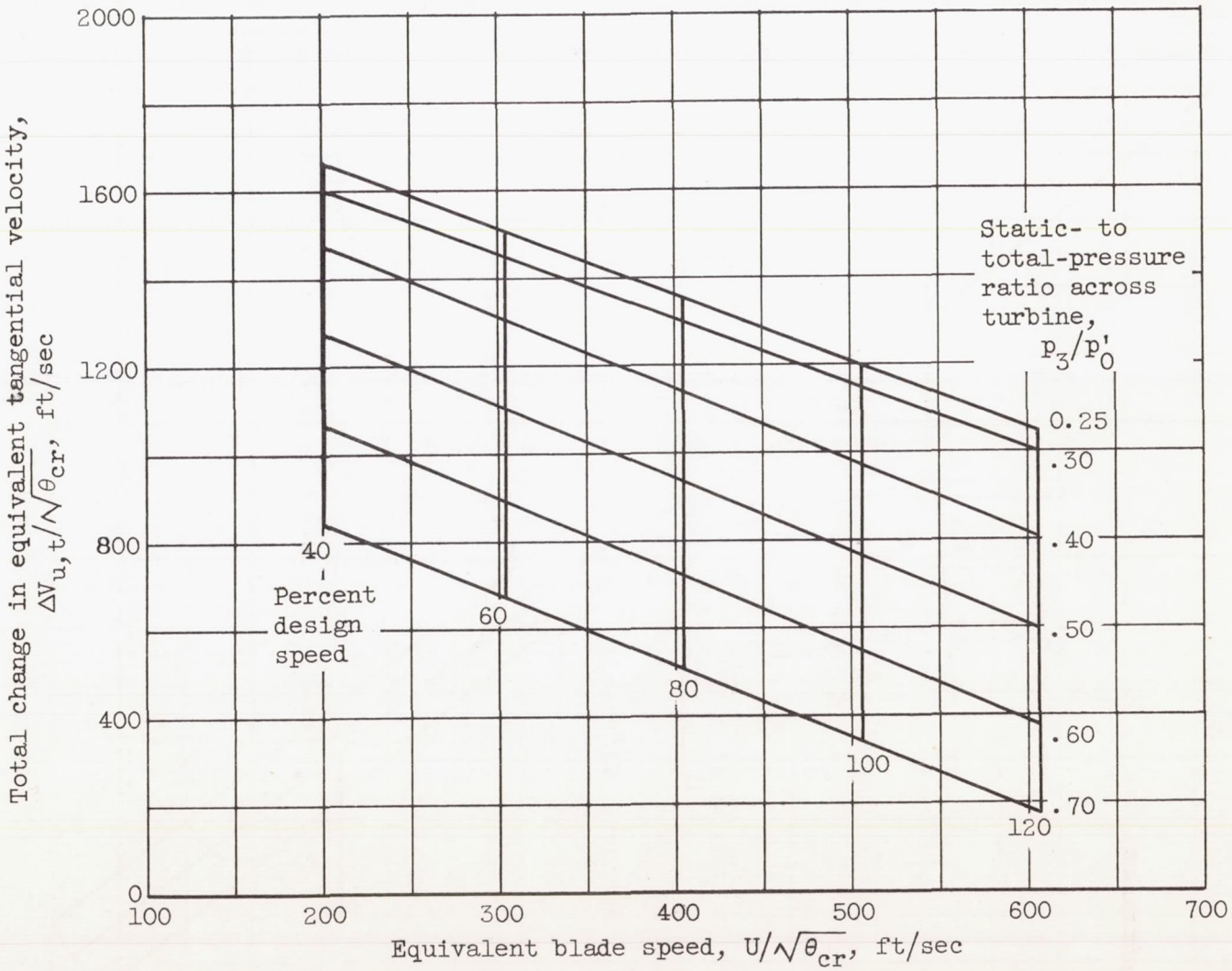


Figure 2. - Absolute torque-speed characteristics obtained from testing single-stage turbine of reference 2.

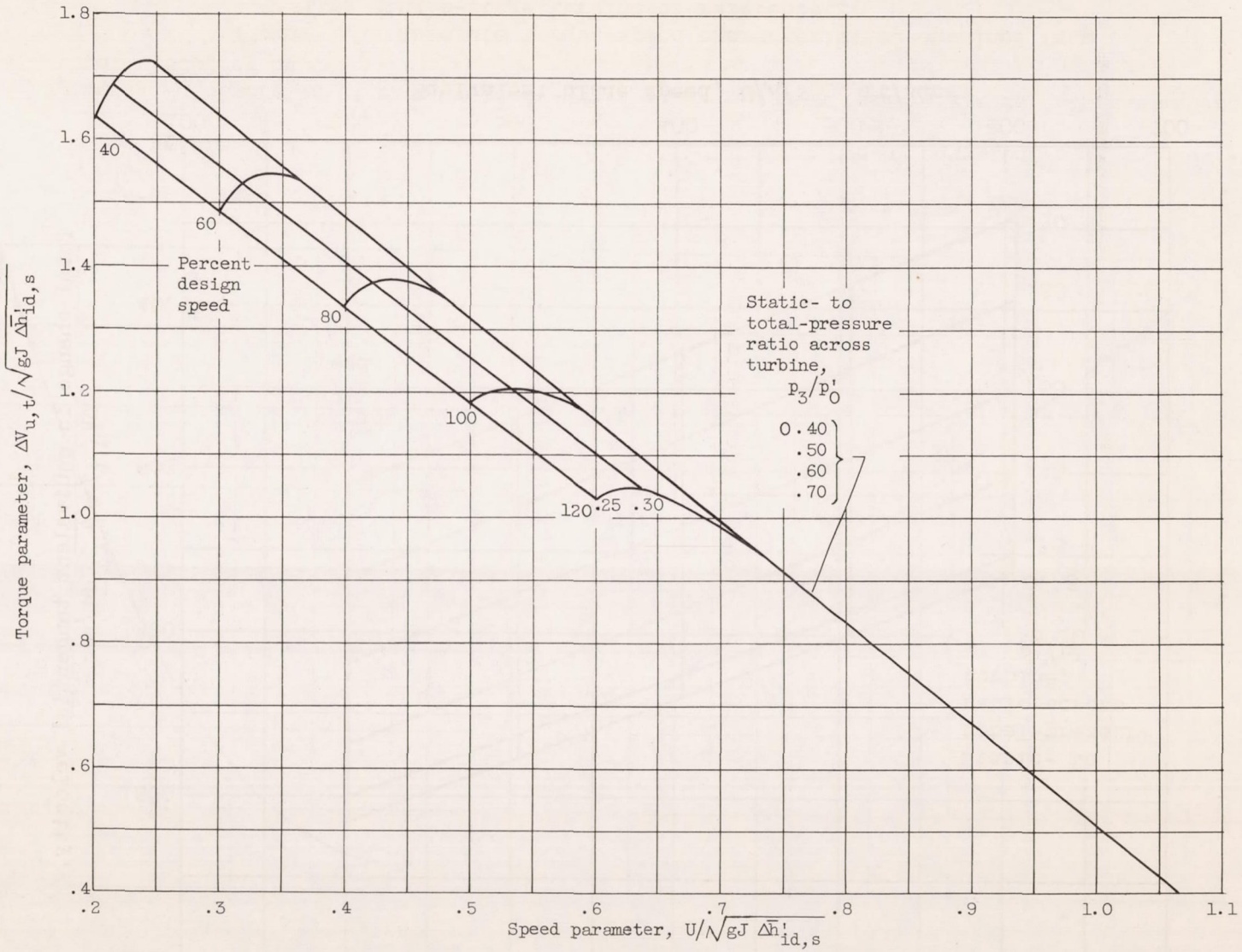
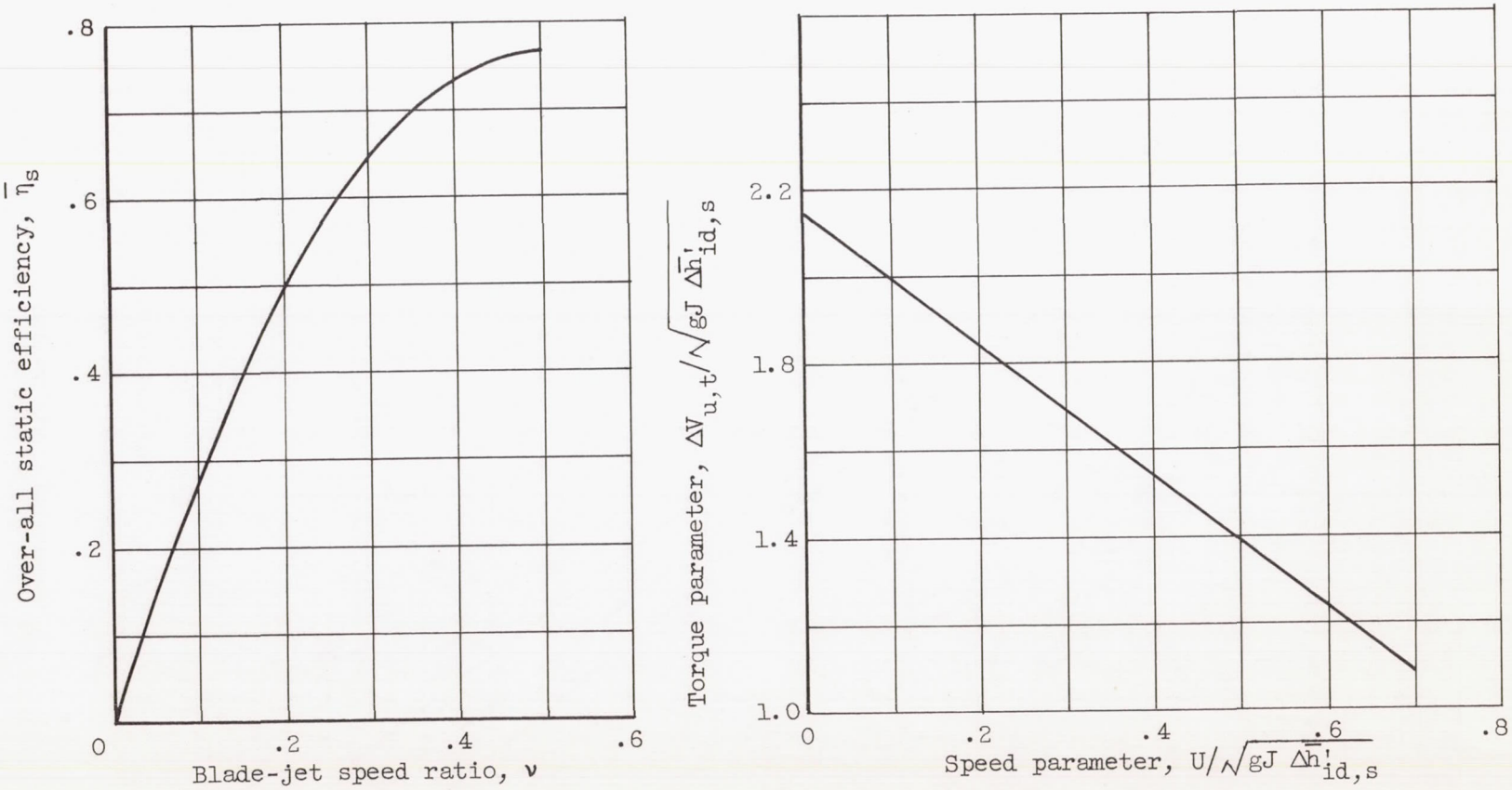


Figure 3. - Torque-speed characteristics of single-stage turbine of reference 2 when normalized by $\sqrt{gJ\ \Delta\bar{h}'_{id,s}}$.



(a) Reference curve.

(b) Conversion by equations (6a) and (6b).

Figure 4. - Conversion of example blade-jet speed ratio data from reference 3 to parameters used in subject report.

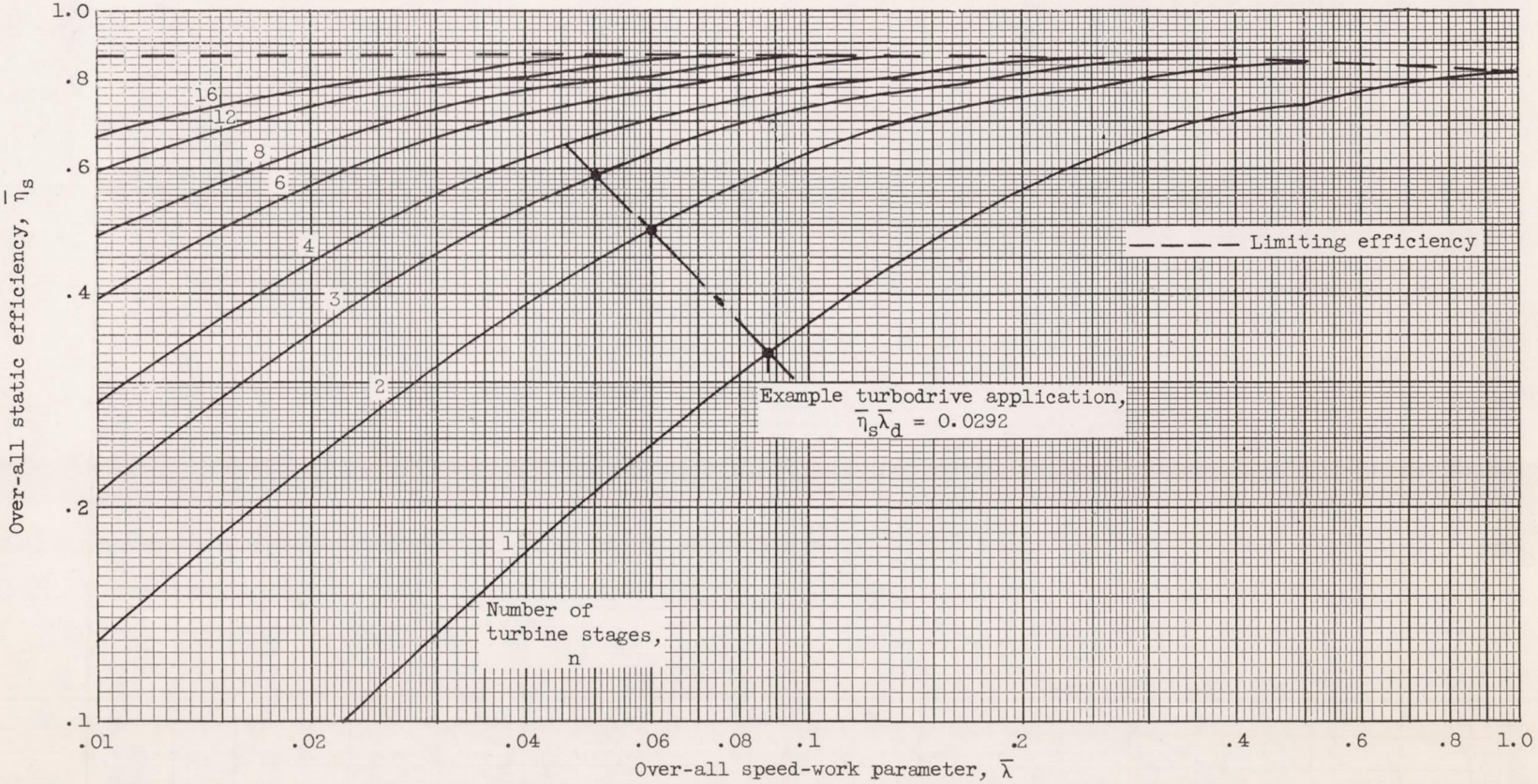


Figure 5. - Efficiency - speed-work relation of reference 1.

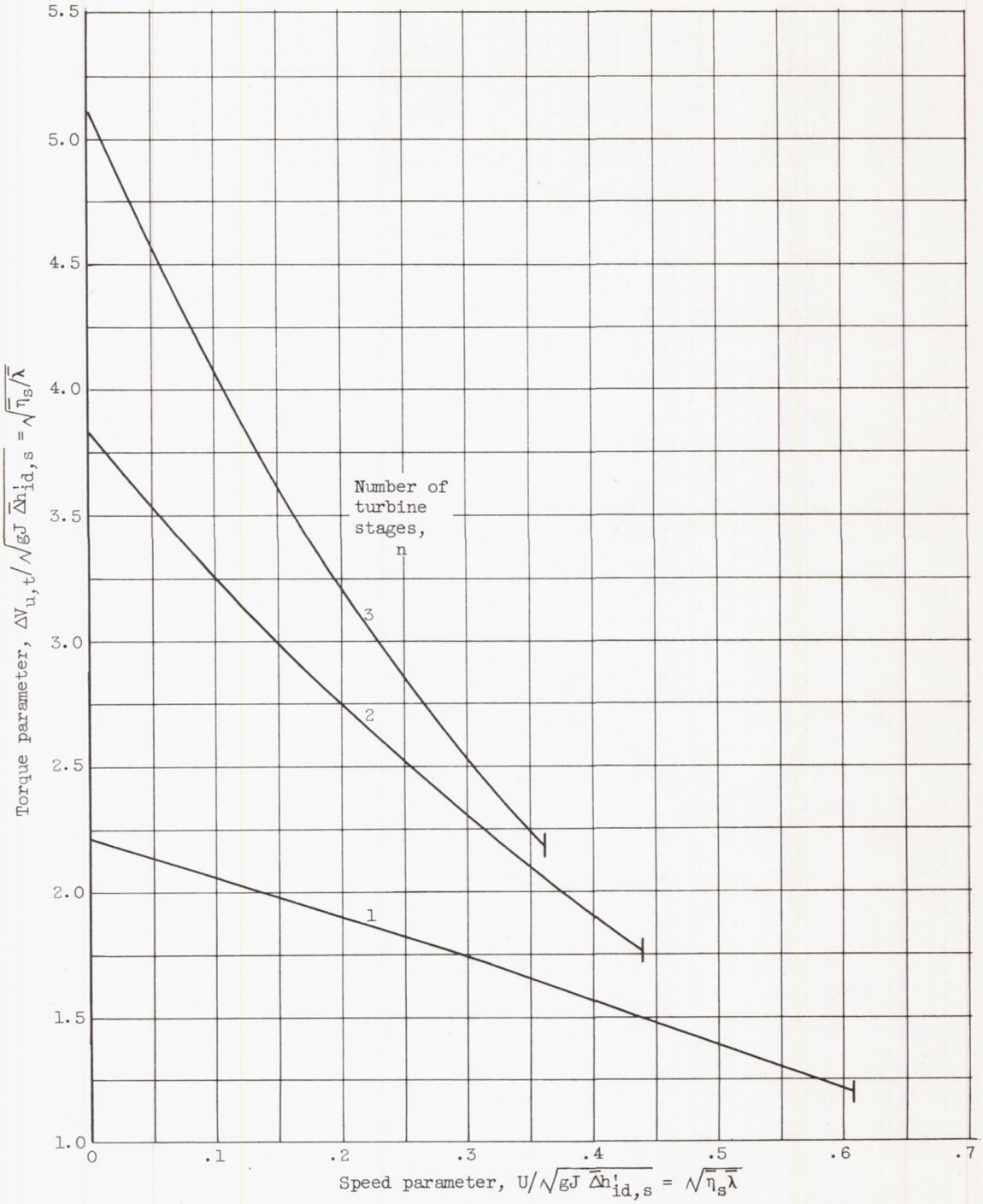


Figure 6. - Analytically obtained turbine torque-speed characteristics for λ_S range from 0 to 0.5.

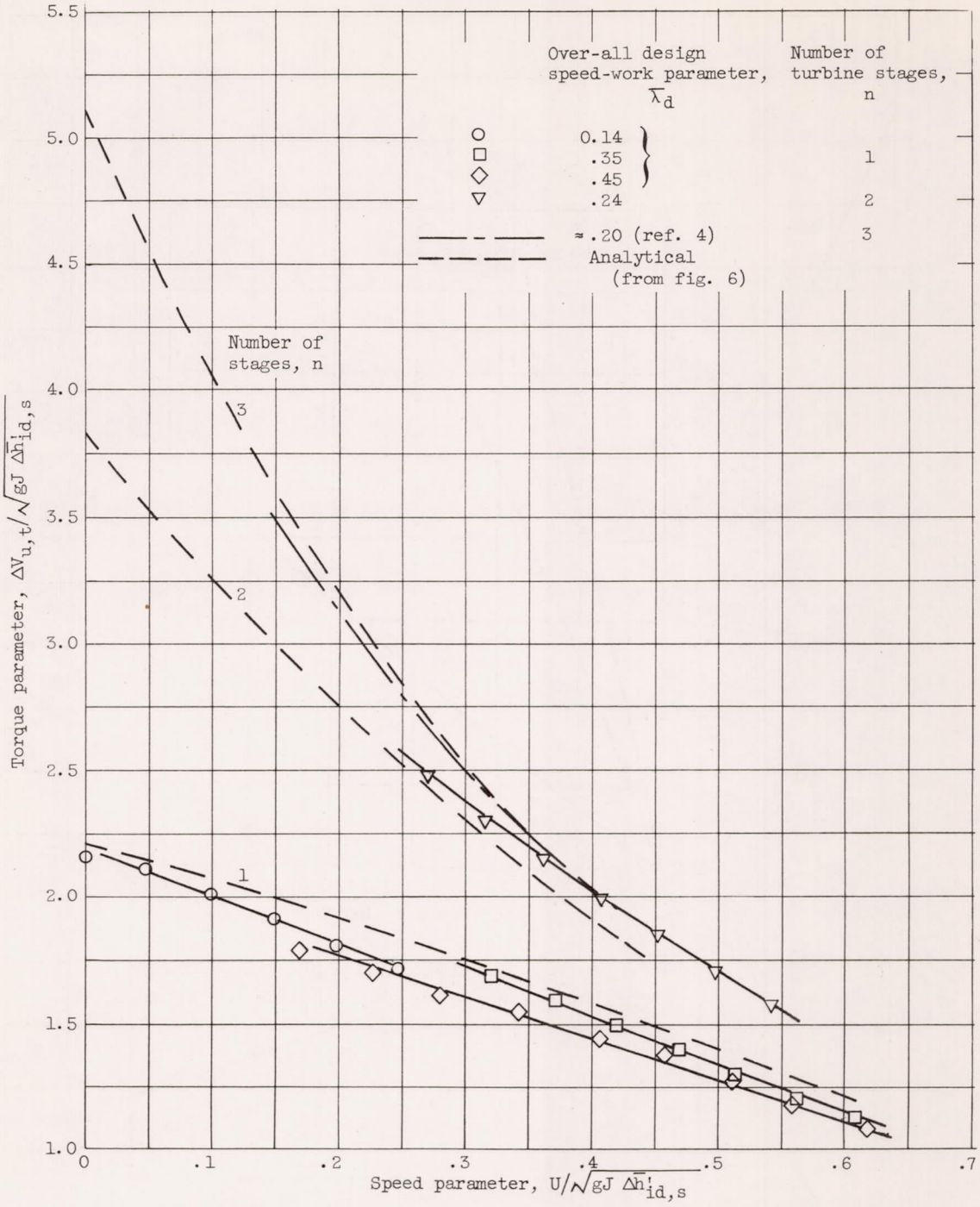


Figure 7. - Comparison of analytically obtained torque-speed characteristics with experimental results.

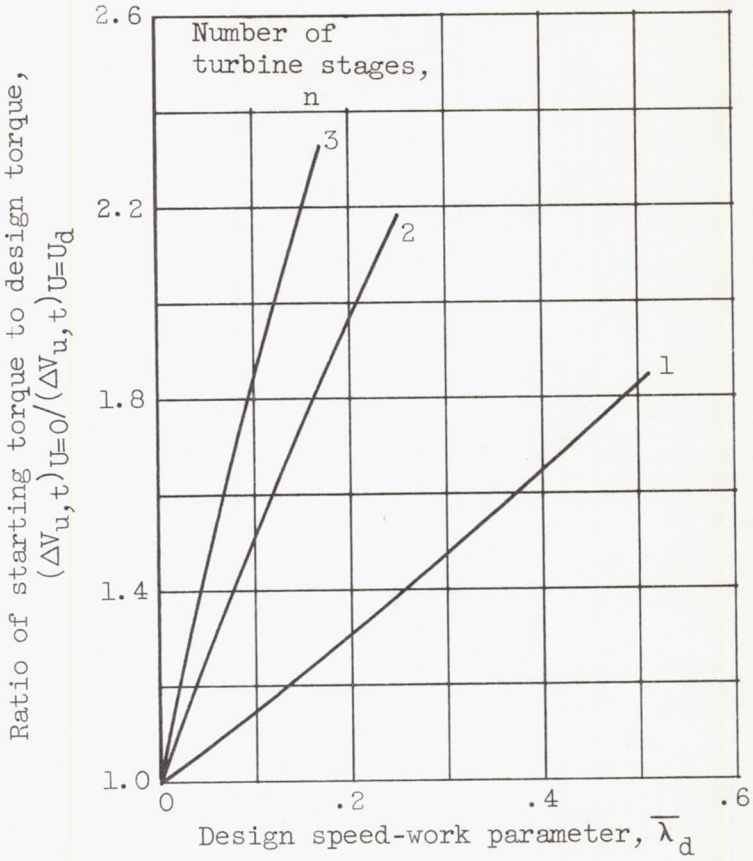


Figure 8. - Analytically obtained ratio of starting-to-design torque as function of design speed-work parameter.

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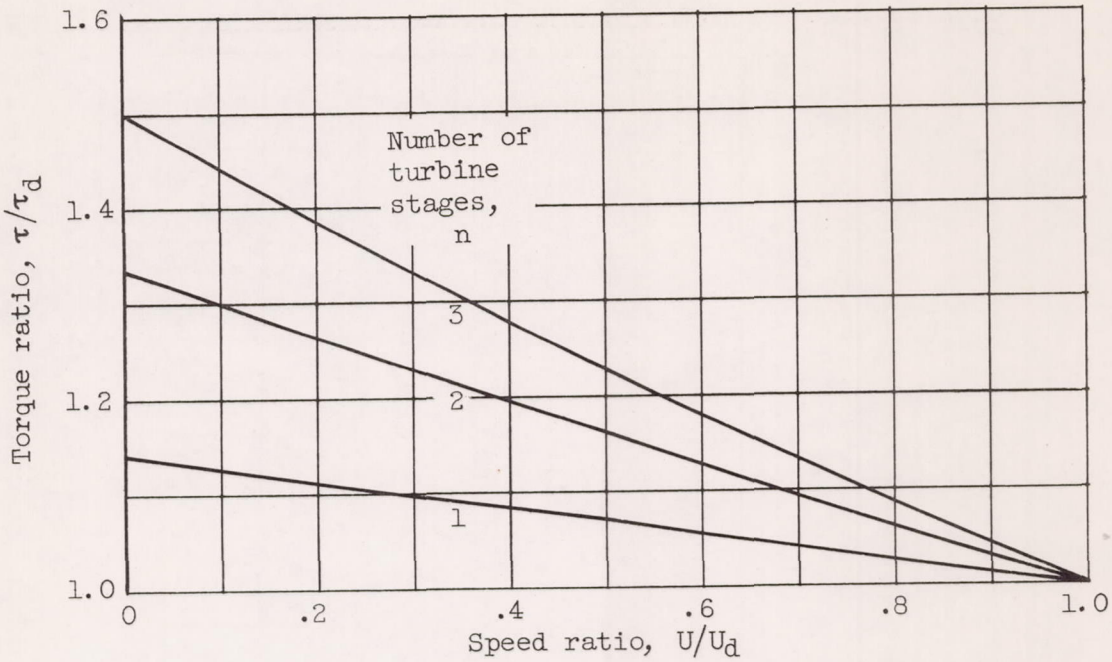

Figure 9. - Torque-speed characteristics for example turbodrive application.