# ANALYTICAL AND EXPERIMENTAL VIBRATION ANALYSIS OF A FAULTY GEAR SYSTEM

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## SUMMARY

A comprehensive analytical procedure was developed for predicting faults in gear transmission systems under normal operating conditions. A gear tooth fault model is developed to simulate the effects of pitting and wear on the vibration signal under normal operating conditions. The model uses changes in the gear mesh stiffness to simulate the effects of gear tooth faults. The overall dynamics of the gear transmission system is evaluated by coupling the dynamics of each individual gear-rotor system through gear mesh forces generated between each gear-rotor system and the bearing forces generated between the rotor and the gearbox structure. The predicted results were compared with experimental results obtained from a spiral bevel gear fatigue test rig at NASA Lewis Research Center. The Wigner-Ville Distribution (WVD) was used to give a comprehensive comparison of the predicted and experimental results. The WVD method applied to the experimental results were also compared to other fault detection techniques to verify the WVD's ability to detect the pitting damage, and to determine its relative performance. Overall results show good correlation between the experimental vibration data of the damaged test gear and the predicted vibration from the model with simulated gear tooth pitting damage. Results also verified that the WVD method can successfully detect and locate gear tooth wear and pitting damage.

## I. INTRODUCTION

In the last two decades, the use of gear transmissions in both defense and commercial applications has substantially increased. With the demand for higher power and performance, premature failures in transmissions often result in financial losses, and sometimes even lead to catastrophic consequences. In the aerospace industry, one of the major concerns is with gear fatigue failures in rotorcraft transmission systems. Large vibrations in gear transmission systems usually result in excessive gear tooth wear and possible tooth crack formation which, in turn, leads to premature gear failure. Thus, it is important to understand the dynamics of a transmission system over a variety of fault conditions, as well as under nominal conditions. With this, methods can be explored to detect and assess the magnitude of the gear damage

present. Due to limitations in the number and types of experiments that can be performed, the only practical means of obtaining this type of data is through analytical simulations.

The major objective of the research reported herein is to develop and verify a model to predict the vibration of a transmission system with the effects of gear surface pitting and wear. To simulate the vibration of the transmission system, the equations of motion were established individually for each rotor-gear-bearing system. The effects of tooth wear or surface pitting are simulated by changes in the magnitude and phase of the mesh stiffness. These localized changes in the gear mesh are incorporated into each gear-rotor model for dynamic simulation [Choy 1991,1993 and Kahraman 1990]. The dynamics of each gear-rotor system are coupled with each other through the gear mesh interacting forces and the bearing support forces. The global vibrations of the system are evaluated by solving thetransient dynamics of each rotor system simultaneously with the vibration of the casing. In order to minimize the computational effort, the number of degrees of freedom of the system are reduced by using a modal synthesis procedure [Choy 1991,1993].

To verify the analytical model with experimental data, the dynamics of a single spiral bevel pinion with various degrees of gear tooth damage was simulated. Results from the model were compared to experimental results using a joint time-frequency analysis method. This approach was chosen because of the large amount of information represented in the joint time-frequency results which cannot be represented separately in either the time domain or the frequency domain. The joint time-frequency analysis will provide an instantaneous frequency spectrum of the system at every instant of the revolution of the pinion while a Fourier Transform can only provide the average vibration spectrum of the signal obtained during one complete revolution. In other words, the time-changing spectral density from the joint time-frequency spectra will provide information concerning the frequency distribution concentrated at that instant around the excited instantaneous frequency which cannot be obtained in a regular vibration frequency spectrum. The joint time-frequency analysis approach applies the Wigner-Ville Distribution (WVD) [Boashash 1987, Claasen 1980, and Shin 1993] on the time vibration signal of the system. Some success has been achieved in applying the WVD to gear transmission systems [Forrester 1990 and McFadden 1991] to recognize faults at various locations of the gear. Other fault detection techniques, including frequency domain analysis [Randall 1982 and Taylor 1980] and several time discriminant methods such as: *FM*4 [Stewart 1977], $NA4^*$, and $NB4^*$ [Zakrajsek 1993,1994 and Decker 1994] are also used to compare with and verify the WVD approach.

Based on results of this study, some conclusions are made on the ability of the developed model to simulate gear tooth surface damage, and the ability of the WVD method to detect damage sufficient to verify the model.

## II. ANALYTICAL PROCEDURE

The dynamics of the ith individual gear-shaft system can be evaluated through the equations of motion for the vibrations of a individual rotor-bearing-gear system as shown in figure 1 [Choy 1991,1993], given in matrix form, as

$$[M]\{\ddot{W}_i\} + [K_s]\{W_i\} = \{F_{bi}(t)\} + \{F_{gi}(t)\} + \{F_{ui}(t)\} \tag{1}$$

where $[M]$ and $[K_s]$ are respectively the mass and shaft stiffness matrices of the rotor, $\{W_i\}$ is the general displacement vector of the ith rotor in its local coordinate system, and, $\{F_{bi}(t)\}$, $\{F_{gi}(t)\}$, and $\{F_{ui}(t)\}$ are respectively the force vectors acting on the ith rotor system due to bearing forces, gear mesh interactions, and mass-imbalances. In this model, the dynamics between the gearbox and the rotor are coupled through the bearing forces, which are evaluated by the relative motion between the rotor and the gearbox. The interactions between each individual rotor are coupled through the gear forces generated by the relative motion of the two mating gears at the mesh point.

The equations of motion of the gearbox with $p$ rotor systems can be expressed as

$$[M_c]\{\ddot{W}_c\} + [K_c]\{W_c\} = \sum_{i=1}^{p}[T_{ci}]\{F_{bi}(t)\} \tag{2}$$

where $[T_{ci}]$ represents the coordinate transformation between the ith rotor and the gearbox.
The bearing forces $\{F_{bi}(t)\}$ for the ith rotor can be evaluated as

$$\{F_{bi}(t)\} = [C_{bi}]\left(\{\dot{W}_i\} - [T_{ic}]\{\dot{W}_{ci}\}\right) + [K_{bi}]\left(\{W_i\} - [T_{ic}]\{W_{ci}\}\right) \tag{3}$$

where $[C_{bi}]$ and $[K_{bi}]$ are respectively the damping and stiffness of the bearing, $[T_{ic}]$ is the coordinate transformation matrices for the gearbox with respect to the ith rotor, and $W_{ci}$ is the casing displacements at the rotor locations.
The gear forces generated from the gear mesh interaction [Choy 1991,1993] can be written as

$$\{F_{gi}(t)\} = \{F_{ri}(t)\} + \{F_{ti}(t)\} \tag{4}$$

where $\{F_{ri}(t)\}$ is the vector containing the gear forces and moments resulting from the relative rotation between the two mating gears and $\{F_{ti}(t)\}$ is the vector containing gear forces and moments due to the translational motion between the two gears [Choy 1988 and Boyd 1989].

In order to calculate the transient/steady state dynamics of the system, all the coupled rotor and casing equations of motion have to be solved simultaneously. To minimize the computational effort, the modal transformation [Choy 1991,1993] procedure was applied to reduce the degrees of freedom of the global equations of motion. Using the undamped mode shapes of the rotor system and the undamped mode shapes of the gearbox, the rotor and the gearbox displacements were transformed into the modal coordinates to reduce the number of degrees of freedom in the system to minimize the computational effort. The modal equations of motion for the rotor systems and the gearbox vibration were solved simultaneously for the modal accelerations. A numerical integration scheme was used to integrate the accelerations to obtain velocities and displacements at each time step for transient calculations.

## III. EXPERIMENTAL STUDY

Using the spiral bevel gear fatigue test rig, illustrated in figure 2, the resulting fatigue damage on the pinion is illustrated in figures 3 and 4. The primary purpose of this rig is to study the effects of gear tooth design, gear materials, and lubrication types on the fatigue strength of aircraft quality gears [Handschuh 1992]. Because spiral bevel gears are used extensively in helicopter transmissions to transfer power between nonparallel intersecting shafts, the use of this fatigue rig for diagnostic studies is practical. Vibration data from an accelerometer mounted on the pinion shaft bearing housing was captured using a personal computer with an analog to digital conversion board and antialiasing filter. The 12-tooth test pinion, and the 36-tooth gear have: 0.5141-in. diametral pitch, 35° spiral angle, 1-in. face width, 90° shaft angle, and 22.5° pressure angle. The pinion transmits 720 hp at nominal speed of 14 400 rpm. The test rig was stopped several times for gear damage inspection. The test was ended at 17.79 operational hours when a broken portion of a tooth was found during one of the shutdowns.

## IV. VIBRATION SIGNATURE ANALYSIS PROCEDURE

Three major methodologies: (A) the joint time-frequency approach, (B) the frequency domain approach, and (C) the time domain techniques, were used in this study. The following is a description of the three methodologies:

(A) Joint Time-Frequency Technique

To examine the vibration signal in a joint time-frequency domain, the Wigner-Ville method [Boashash 1987 and Claasen 1980] was used in this study. While the Fast Fourier Transform (FFT) technique can provide the spectral contents of the time signal, it cannot distinguish time phase change during a complete cycle of operation. In other words, it assumes that the time signals are repeatable for each time data acquisition window without considering the effects of any magnitude and phase changes during the sampling period. The Wigner-Ville distribution will provide an interactive relationship between time and frequency during the period of the time data window. The comprehensive representation of the vibration signal using the WVD method is the primary reason that it was used to compare the predicted and experimental vibration results. The WVD (Wigner-Ville Distribution), in a discrete form, can be written as:

$$W_x(nT, f) = 2T \sum_{i=-1}^{L} x(nT + iT)\, x^*(nT - iT) \cdot e^{-j4\pi f i \pi} \tag{5}$$

where

| | |
|---|---|
| $W(t,f)$ | the Wigner-Ville distribution in both the time domain $t$ and frequency domain $f$ |
| $x(t)$ | the time signal |
| $T$ | the sampling interval |
| $L$ | the length of time data used in the transform |

To allow sampling at the Nyquist rate and eliminate the concentration of energy around the frequency origin due to the cross product between negative and positive frequency [Boashash 1987 and Claasen 1980], the analytic signal was used in evaluating the WVD. The analytic signal $s(t)$ is defined as

$$s(t) = x(t) + jH[x(t)] \tag{6}$$

where $H[x(t)]$ is the Hilbert transform of $x(t)$. However, an alternative approach can be used to calculate the analytic signal using the frequency domain definition. The analytic signal $s(t)$ can be evaluated by calculating the FFT of the time signal $x(t)$, then setting the negative frequency spectrum to zero. The analytic signal can be obtained by evaluating the inverse FFT of the spectrum.
To simplify the computational effort, the WVD can be evaluated using a standard FFT algorithm. Adopting the convention that the sampling period is normalized to unity, it is necessary only to evaluate the WVD at time zero. Hence

$$W_x(0, f) = 2 \sum_{i=-L}^{L} k(i) e^{-j4\pi f i} \tag{7}$$

where $k(i) = s(i)s^*(i)$. Equation (13) can be evaluated using the discrete FFT algorithm.

In order to avoid a repetition in the time domain WVD, a weighting function [McFadden 1991] was added to the time data before the evaluation process. Such a process may decrease the resolution of the distribution, but it will eliminate the repetition of peaks in the time domain and the interpretation of the result is substantially easier.

(B) Frequency Domain Technique

The frequency spectrum is found by applying a Discrete Fourier Transform (DFT) on the time averaged signal $x(t)$, such that the spectral components are

$$X(k) = T\sum_{i=0}^{N-1} x(t)\exp\left(\frac{-j2\pi ik}{N}\right) \tag{8}$$

where

| | |
|---|---|
| $x(t)$ | time domain signal |
| $X(k)$ | frequency domain signal |
| $T$ | sampling time interval |
| $N$ | number of data points |

The frequency components were examined in the frequency domain and compared with those obtained at various stages of the fault development in the spiral bevel pinion.

(C) Time Domain Techniques

Four different time domain techniques for early detection of gear tooth damage were used in this study. All of the time domain techniques were applied to the vibration signal after it was time synchronously averaged. These techniques are: *FM*4, *NA*4*, and *NB*4*. They are defined as follows:

*FM*4 [Stewart 1977] is an isolated fault detection parameter for the gear tooth, and is given by the normalized kurtosis, of the resulting difference signal as

$$FM4 = \frac{N\sum_{i=1}^{N}\left(d_i - \bar{d}\right)^4}{\left[\sum_{i=1}^{N}\left(d_i - \bar{d}\right)^2\right]^2} \tag{9}$$

where

| | |
|---|---|
| $d(t)$ | $A(t)$ - $R(t)$ |
| | mean value of $d(t)$ |
| $A(t)$ | original time synchronous signal |
| $R(t)$ | regular meshing components plus their first order side bands |
| $N$ | total number of data points in the time signal |

*NA*4* [Zakrajsek 1993,1994 and Decker 1994] is a general gear fault detection parameter, with trending capabilities. A residual signal is constructed by removing regular meshing components from the time averaged signal. The first order sidebands stay in the residual signal and the fourth statistical moment of the residual signal is then divided by the averaged variance of the residual signal, raised to the second power. The average variance is the mean value of the variance of all previous records in the run

ensemble. This allows *NA*4 to compare the current gear vibration with the baseline of the system under nominal conditions. *NA*4 is given by the quasi-normalized kurtosis equation shown below:

$$NA4*(M) = \frac{N\sum_{i=1}^{N}\left(r_i - r\right)^4}{\left\{\frac{1}{M}\sum_{j=1}^{N}\left[\sum_{i=1}^{N}\left(r_{ij} - \bar{r}_j\right)^2\right]\right\}^2} \tag{10}$$

where

| | |
|---|---|
| $r$ | residual signal |
| $r$- | mean value of residual signal |
| $N$ | total number of data points in one time record |
| $i$ | data point number in time record |
| $j$ | time record number |
| $M$ | current time record number in run ensemble |

An enhancement to this parameter is given by $NA4^*$, in which the value of the averaged variance is "locked" when the instantaneous variance exceeds a predetermined value [Decker 1994]. This provides *NA*4 with enhanced trending capabilities, in which the kurtosis of the current signal is compared to the variance of the baseline signal under nominal conditions.

*NB*4 is another parameter similar to *NA*4 that also uses the quasi-normalized kurtosis given in equation (4). The major difference is that instead of using a residual signal, *NB*4 uses the envelope of the signal bandpassed about the mesh frequency. Again, as with $NA4^*$, $NB4^*$ is an enhancement to the *NB*4 parameter, in which the value of the average variance is "locked" when the instantaneous variance exceeds a predetermined value. The equation for $NB4^*$ is given below:

$$NB4*(M) = \frac{N\sum_{i=1}^{N}\left(s_i - \bar{s}\right)^4}{\left\{\frac{1}{M}\sum_{j=1}^{N}\left[\sum_{i=1}^{N}\left(s_{ij} - \bar{s}_j\right)^2\right]\right\}^2} \tag{11}$$

and

$$s(t) = \text{magnitude of } \left\{b(t) + i\left\{H\left[b(t)\right]\right\}\right\} \tag{12}$$

where

| | |
|---|---|
| $b(t)$ | time averaged signal bandpassed filtered about the meshing frequency |
| $H[b(t)]$ | the Hilbert Transform of $b(t)$ |
| $N$ | total number of data points in one time record |
| $i$ | data point number in time record |
| $j$ | time record number |
| $M$ | current time record number in run ensemble |

## V. DISCUSSIONS OF RESULTS

A series of photos showing the deterioration of the pinion teeth at various stages of the test are shown in figures 3 and 4. Figure 3(a) shows the initiation of a small pit on one of the pinion teeth during the first shutdown, at about 5.5 hr into the test. As the test progressed, the rig was shut down seven more times to examine the severity of the pitting and its relationship with the corresponding vibrations. Figures 3(b), (c), and (d) show the increase of the damaged area at the pinion tooth as the elapsed time increased to 6.55, 8.55, and 10.03 hr, respectively. Note that in figure 4(a), at 12 hr, the damage of the pinion tooth increased to 75 percent of the tooth surface. At this stage, pitting also initiated on the adjacent tooth and continued to grow as the time increased to 14.53 hr (fig. 4(b)). At 16.16 hr, the damage has grown to three adjacent teeth as shown in figure 4(c). The test was terminated when a tooth fracture was found on one of the three heavily pitted teeth at 17.79 hr, as shown in figure 4(d).

Figures 5 and 6 show the WVD, the time signal (to the left of the WVD), and frequency spectra (below the WVD), of the spiral bevel pinion vibration at various stages of damage, corresponding to the photographs given in figures 3 and 4. Figure 5(a) shows the vibration signature due to initial pitting, as shown in figure 3(a), at around 200° from the triggering point of the gear at 5.5 hr of running time. As time progressed to 6.55 hr, more advanced pitting is occurring as shown in figure 3(b), the WVD amplitude increases following by a short term amplitude and phase change at 260° as illustrated in figure 5(b). This phenomena is also evident in the frequency spectrum with the existence of sizable sideband components. Due to the speed increase (after the shutdown) at 8.55 hr, the overall WVD amplitude increases substantially as shown in figure 5(c). At the running time of 10.03 hr, as seen in figure 3(d), pitting become more advanced as can be detected by the initiation of a cross pattern in the WVD (fig. 5(d)). The pitting of the tooth is more pronounced at 12.03 hr, as shown in figure 4(a), and can be verified by the distinct cross pattern in the WVD shown in figure 6(a). The high concentration in the WVD energy at 14.53 hr combined with the further increase in sideband amplitudes in figure 6(b) shows the initiation of the pitting process on the neighboring teeth, as illustrated in figure 4(b). At 16.16 hr, as seen in figure 6(c), the WVD pattern changes, as two cross patterns start to appear, showing more advanced damage to the adjacent teeth as illustrated in figure 4(c). The discontinuity of the mesh frequency component with time, as shown by the WVD in figure 6(d), is mainly due to the short term phase change caused by the fractured tooth and the pitting of the two adjacent teeth, as shown in figure 4(d). This phenomenon is also confirmed by the substantial increase in the sideband amplitude in the frequency spectrum in figure 6(d).

In order to simulate the dynamics of the gear system with gear tooth damage, two mesh stiffness models, with combinations of stiffness reduction and phase delay, were introduced. A simple stiffness reduction and phase change model, shown in figure 7(a), was used to simulate the single tooth surface pitting damage at 12 hr, as shown in figure 4(a). The resulting experimental vibration signature due to this damage is given in figure 6(a). The mesh stiffness model given in figure 7(b) consisted of a combination of stiffness and phase changes over three consecutive teeth, to approximate the multiple tooth pitting damage at 17.79 hr, as seen in figure 4(d). The resulting experimental vibration signature due to this extensive damage is given in figure 6(d). Additional frictional effects were also added to the model to simulate the roughness of the tooth surface due to pitting. The simulated vibration signatures of the pinion gear are given in figure 8. Comparing figures 8(a) and 6(a), for the single tooth damage case at 12 hr, one may notice the similarities between both the frequency spectra and the WVD display. The unevenness in the experimental time signal is mainly due to the modulation of frequencies due to other excitations in the test rig, which are not included in the model. For the tooth break-off case at 17.79 hr, both the numerical WVD (fig. 8(b)) and the experimental WVD (fig. 6(d)), show a large cross pattern at the sixth tooth pass location due to the tooth fracture. Some discrepancies have been detected between the experimental and the numerical time signal at the fourth and fifth tooth pass locations. The experimental time signal consists of some higher frequency, smaller amplitude vibration modulation, which were not simulated. This additional modulated signal excited the 14 times rotational speed component, as shown in the experimental frequency spectrum in figure 6(d), and, in turn, is responsible for the small differences created in the WVD.

The *FM*4 parameter, as seen in figure 9(a), shows a possible reaction as the pitting started to occur, however, it does not provide any coherent indication of the severity of the pitting as the damage increased. In addition, it does not provide any information to distinguish the pitting on single or multiple teeth.

Results from $NA4^*$ are illustrated in figure 9(b). It is obvious from the figure that $NA4^*$ provides a very good indication of the pitting development on the pinion tooth. The magnitude of the parameter increases to a nondimensional value of 7 after shutdown no. 2 at 6.55 hr, and further to a value of 17 when the pitting covers 75 percent of the tooth surface at 8.5 hr. As expected, the "locked" denominator in $NA4^*$ provides a robust indication as the pitting progresses [Decker 1994 and Zakrajsek 1994].

The $NB4^*$ method, as shown in figure 9(c), displays a very similar trend to that of $NA4^*$, with a more robust indication to the severity of the damage. However, both $NA4^*$ and $NB4^*$ did not provide any type of indication as the damage spread to other teeth, and finally tooth fracture.

From the results, it is clear that the analytical model has the ability to accurately model gear tooth pitting damage. The WVD method not only verified the analytical results, but also provided detailed diagnostic information on the damage. The WVD method provides information as to the location of the damage on the gear, and also the magnitude of the damage present. This information, however, is currently only discernable by the human eye. A post-processing scheme, such as a neural network algorithm, is needed to process the data provide by the WVD in an automated fashion.

## VI. CONCLUSIONS

1. A modal synthesis methodology was developed to simulate the dynamics of gear transmission systems. While the computational efforts have been greatly reduced by the modal transformation, the numerical results generated maintain good accuracy.

2. Gear tooth damage due to wear and pitting can be approximated by amplitude and phase changes in the gear mesh stiffness model. The gear mesh model can easily be incorporated into the global transmission system for dynamic simulations.

3. The WVD method provides a comprehensive representation of the vibration signal. It was successfully used to verify the analytical model.

4. The WVD method provides detailed information that can be used to detect and locate gear tooth faults.

5. The $NA4^*$ and $NB4^*$ parameters show good reactions to the initial pitting damage and very nice indications for the growth and severity of the pitting damage. However, their indications for the tooth fracture are unclear.

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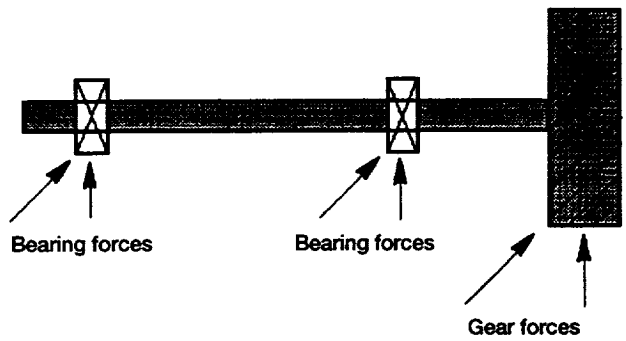


Figure 1.—Schematic of a rotor-gear bearing system.

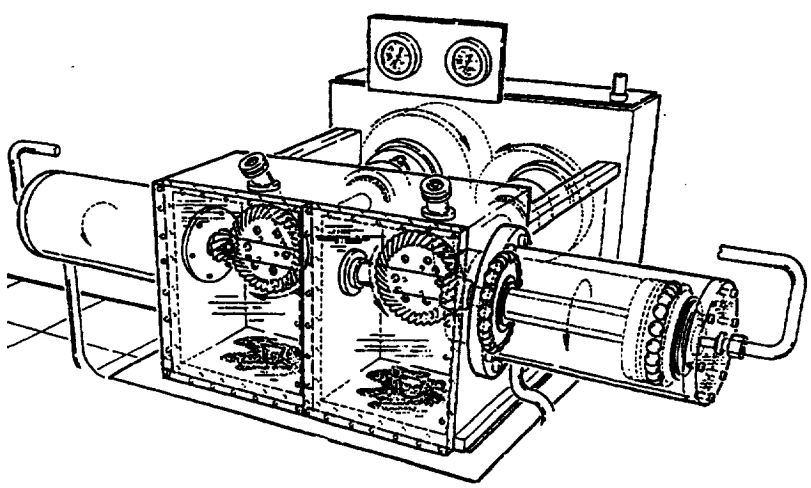

Figure 2.—Spiral bevel gear test rig at NASA Lewis Research Center.

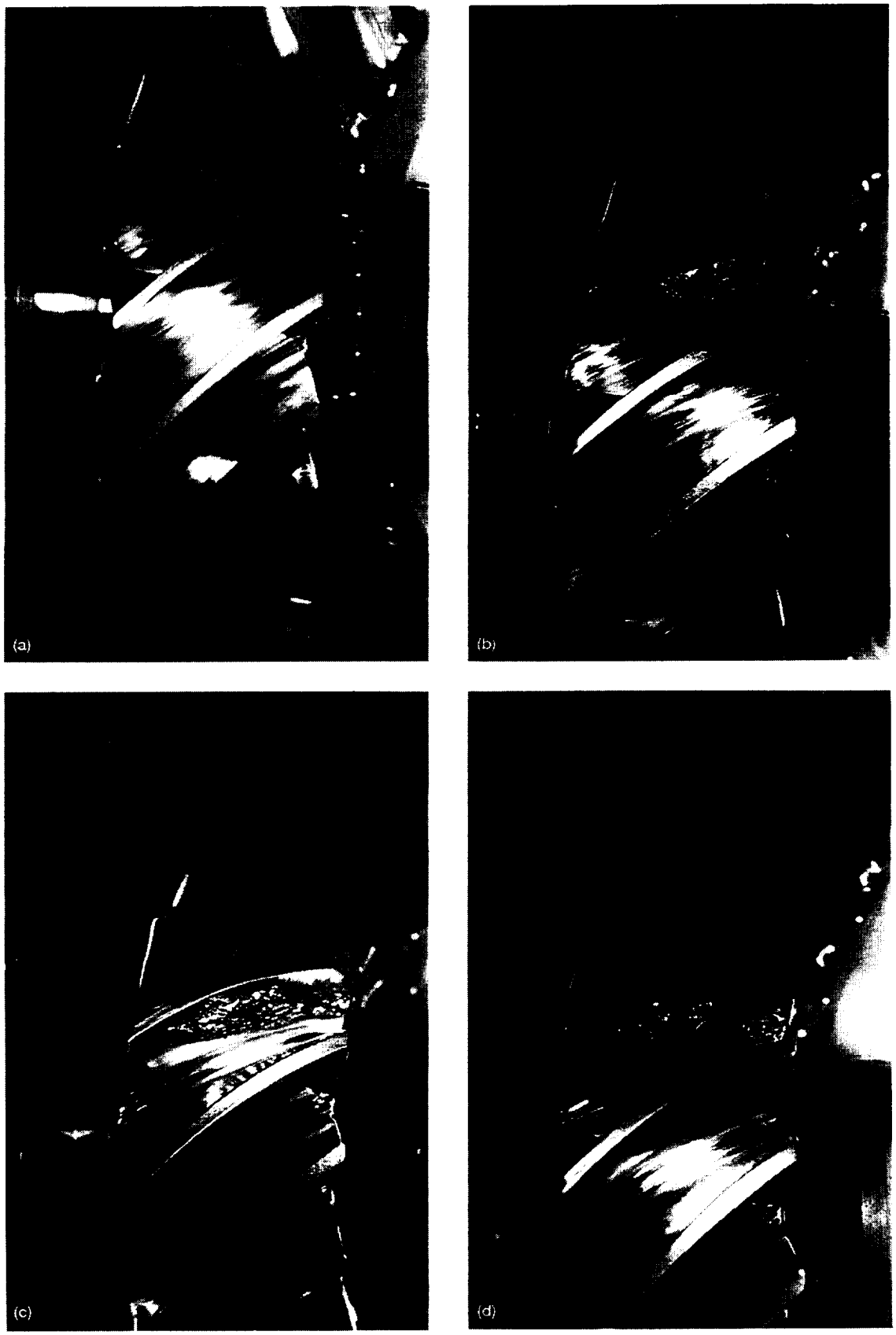


Figure 3.—Pictures of the damaged pinion teeth. (a) 5.5 hours. (b) 6.55 hours. (c) 8.55 hours. (d) 10.03 hours.

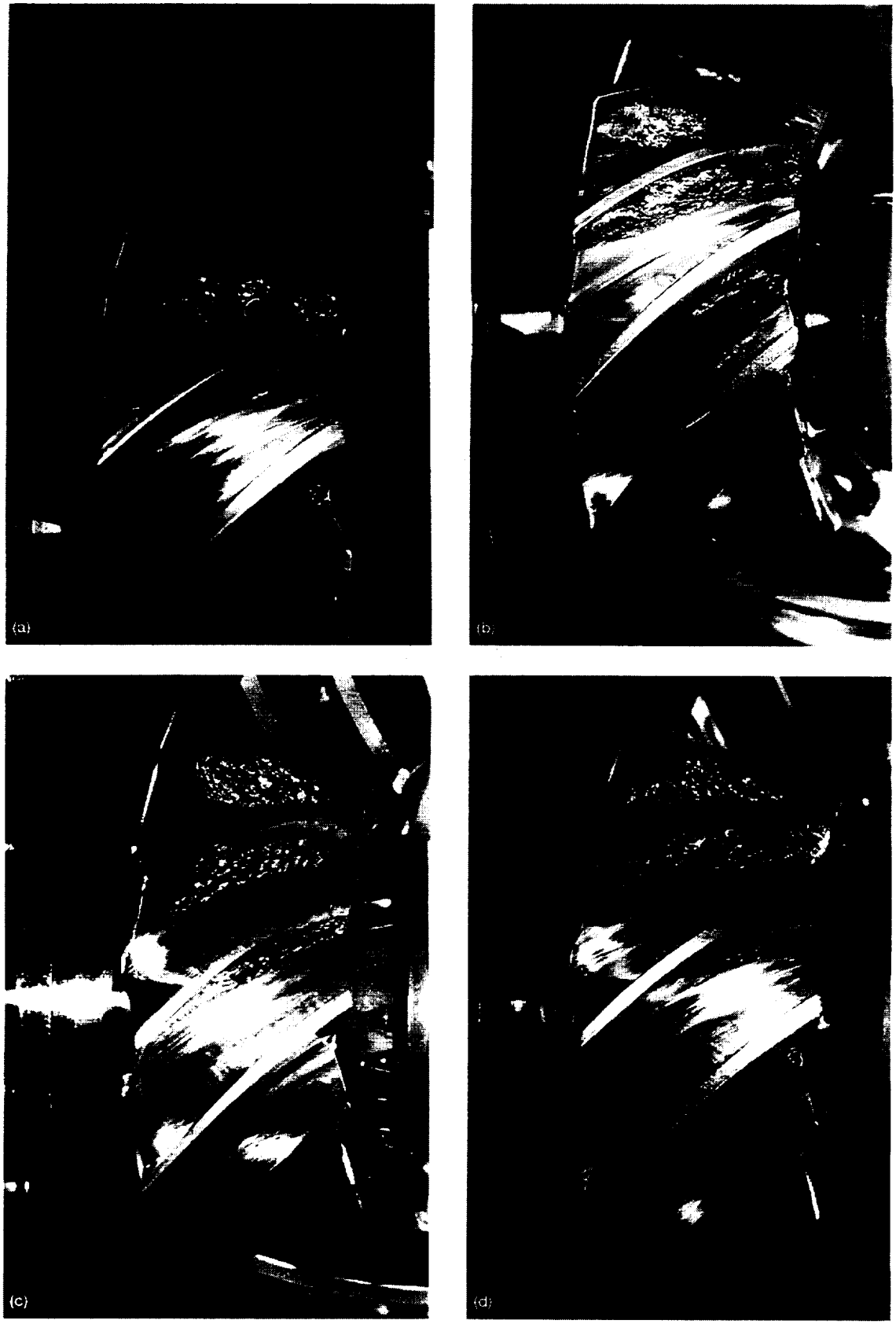


Figure 4.—Pictures of the damaged pinion teeth. (a) 12.03 hours. (b) 14.53 hours. (c) 16.16 hours. (d) 17.79 hours.

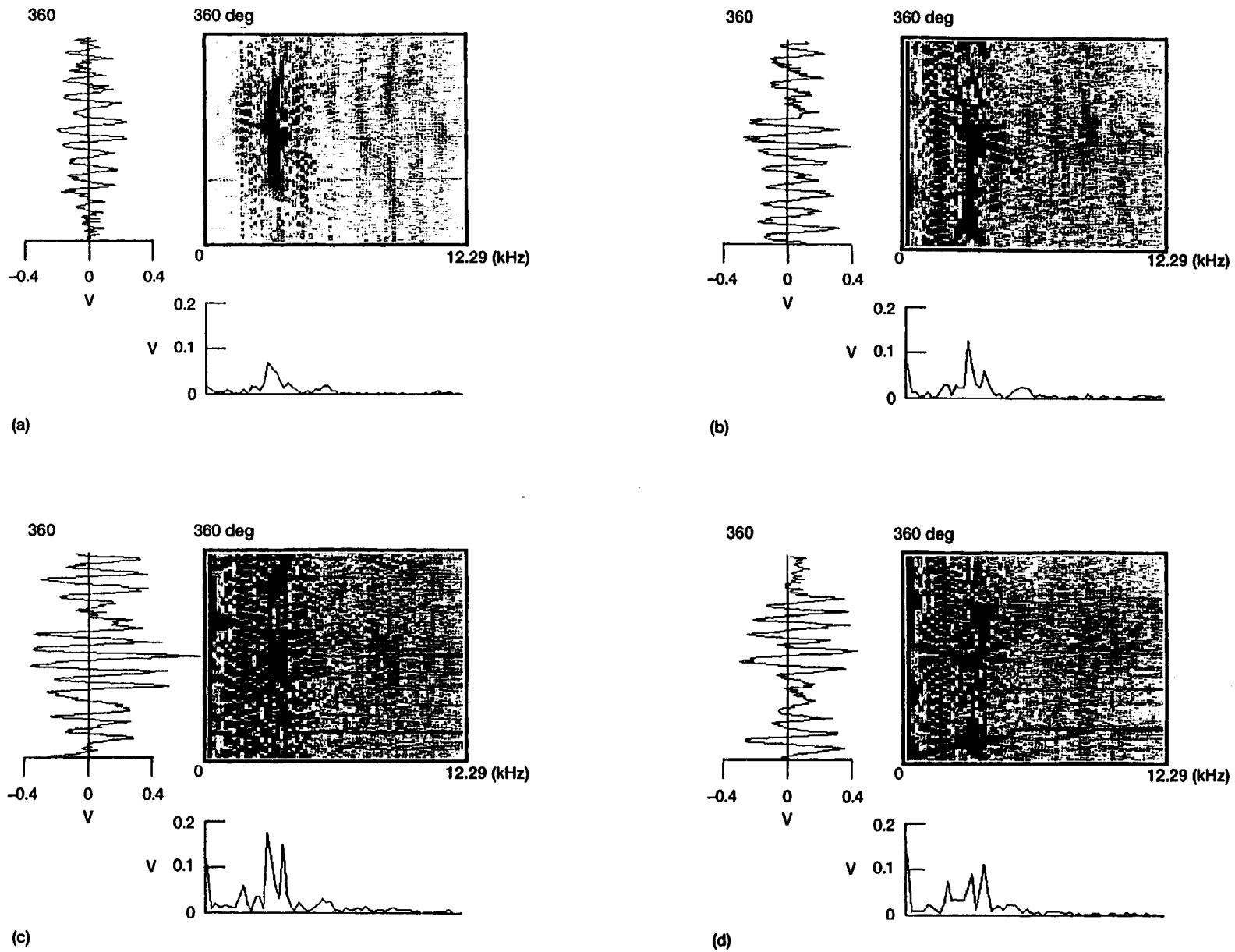


Figure 5.—Time signal, WVD plot, and frequency spectra of the damaged pinion teeth from experimental study. (a) 5.5 hours. (b) 6.55 hours. (c) 8.55 hours. (d) 10.03 hours.

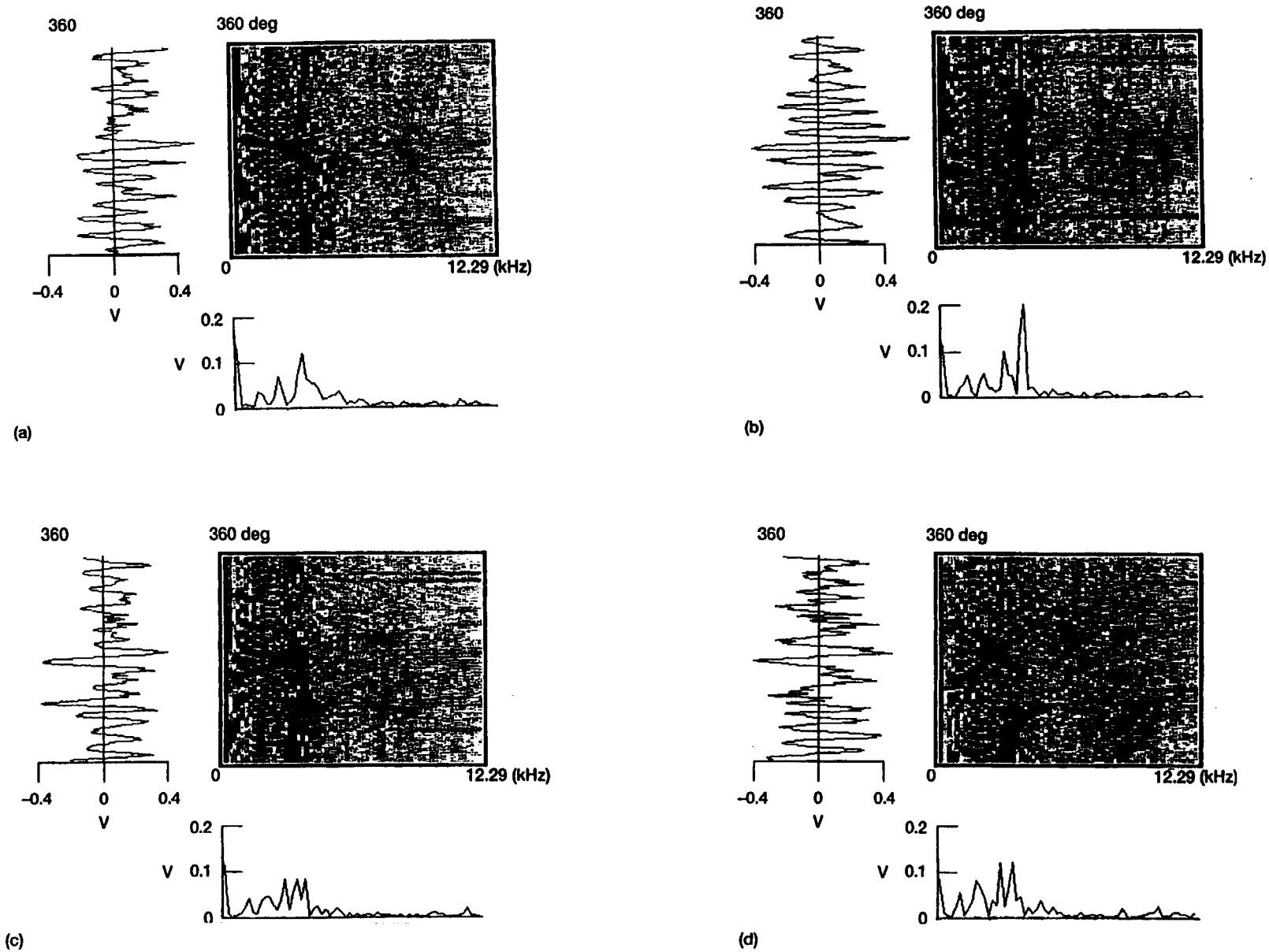


Figure 6.—Time signal, WVD plot, and frequency spectra of the damaged pinion teeth from experimental study. (a) 12.03 hours. (b) 14.53 hours. (c) 16.16 hours. (d) 17.79 hours.

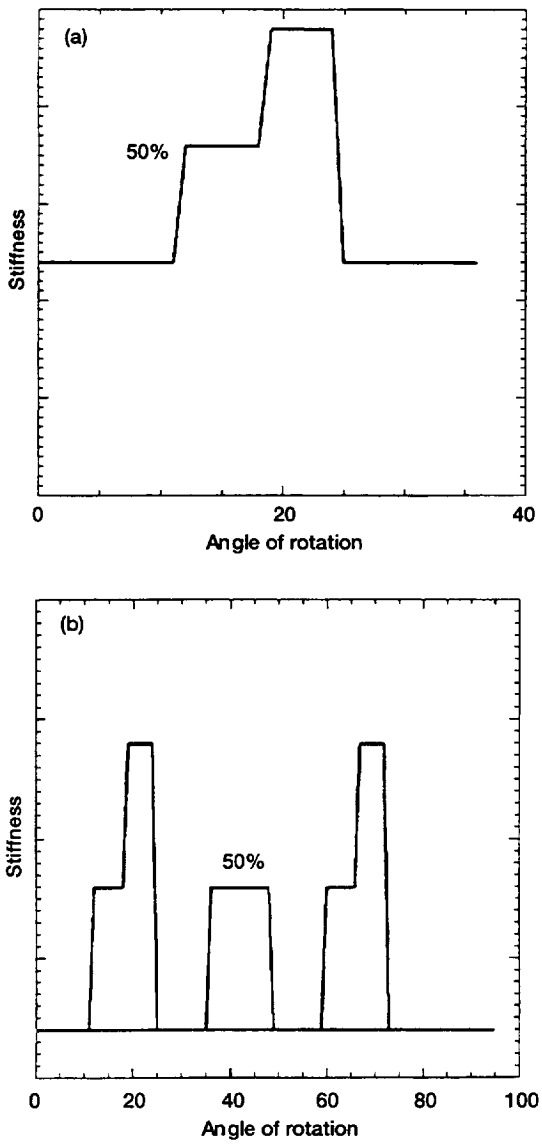


Figure 7.—Gear mesh stiffness models to simulate damage on pinion gear teeth. (a) Single tooth damage model. (b) Multiple tooth damage model.

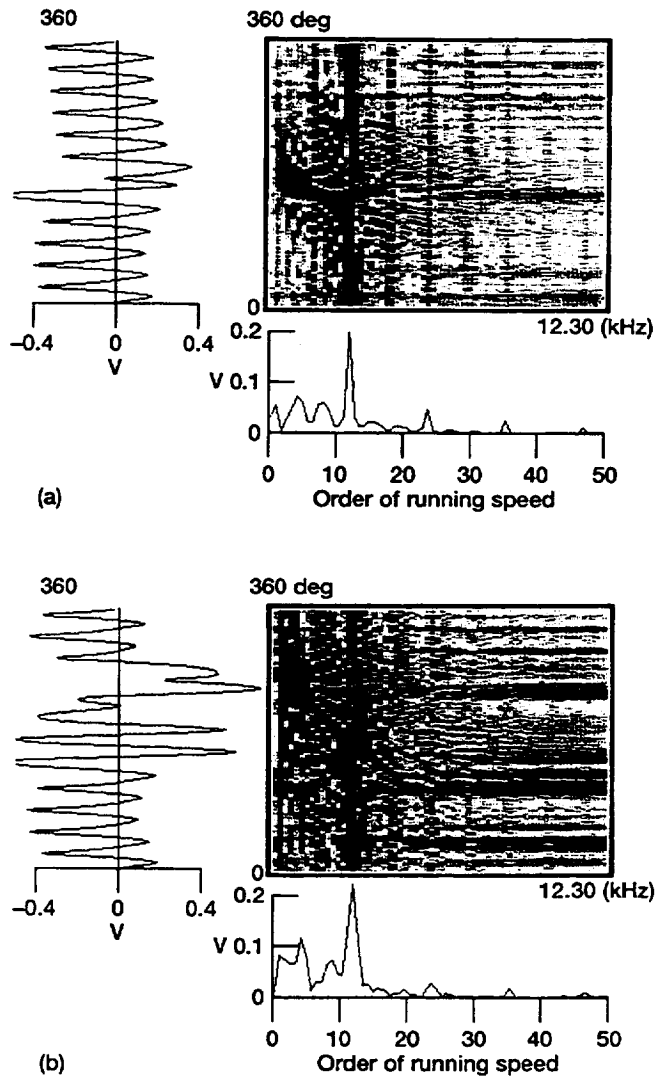


Figure 8.—Time signal, WVD plot, and frequency spectra of the damaged pinion teeth from analytical simulations. (a) 12.03 hours (single tooth damage). (b) 17.79 hours (damage on three consecutive teeth).

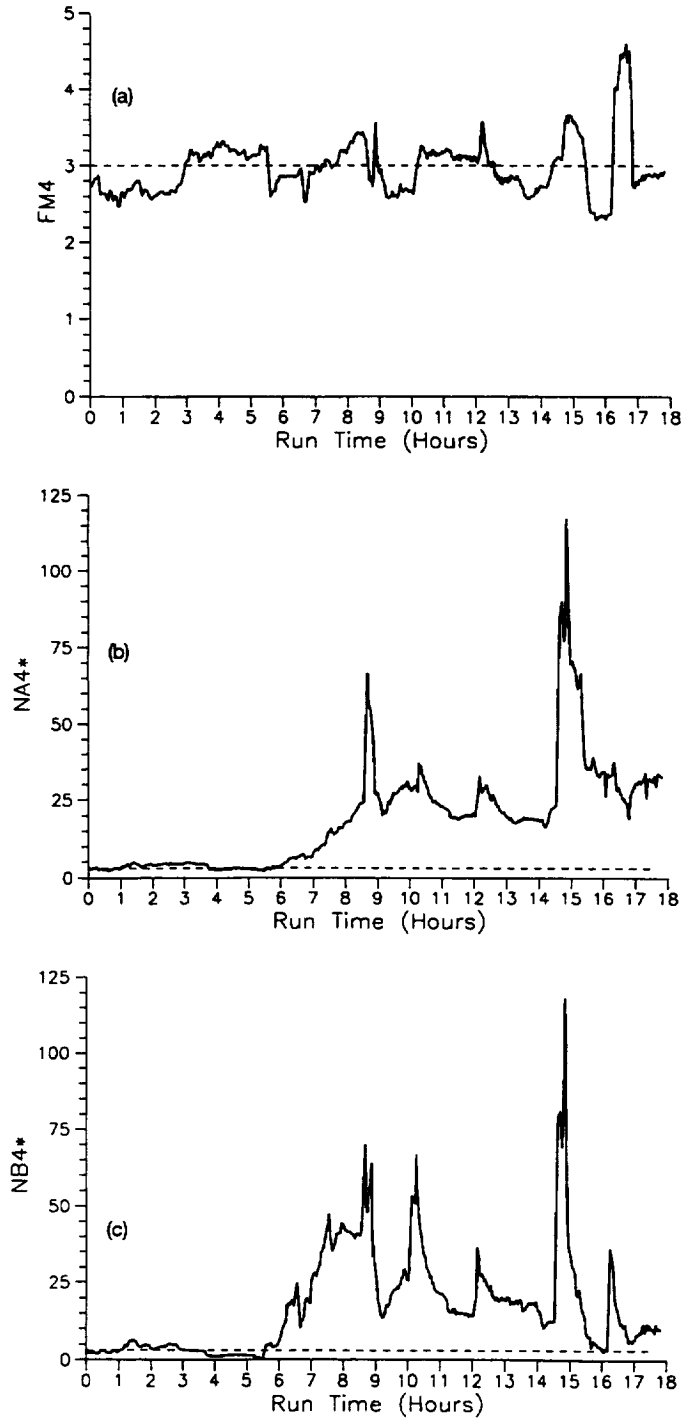


Figure 9.—Results from method. (a) FM4. (b) NA4*. (c) NB4*.

# REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

**1. AGENCY USE ONLY** (*Leave blank*)

**2. REPORT DATE**
October 1994

**3. REPORT TYPE AND DATES COVERED**
Technical Memorandum

**4. TITLE AND SUBTITLE**
Analytical and Experimental Vibration Analysis of a Faulty Gear System

**5. FUNDING NUMBERS**
WU–505–62–36
1L162211A47A

**6. AUTHOR(S)**
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**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
NASA Lewis Research Center
Cleveland, Ohio 44135–3191
and
Vehicle Propulsion Directorate
U.S. Army Research Laboratory
Cleveland, Ohio 44135–3191

**8. PERFORMING ORGANIZATION REPORT NUMBER**
E–9045

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
National Aeronautics and Space Administration
Washington, D.C. 20546–0001
and
U.S. Army Research Laboratory
Adelphi, Maryland 20783–1145

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**
NASA TM–106689
ARL–TR–574

**11. SUPPLEMENTARY NOTES**
Prepared for the 1994 Fall Technical Workshop sponsored by the American Gear Manufacturers Association, St. Louis, Missouri, October 24–26, 1994.
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**12a. DISTRIBUTION/AVAILABILITY STATEMENT**
Unclassified - Unlimited
Subject Category 37

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** ***(Maximum 200 words)***

A comprehensive analytical procedure was developed for predicting faults in gear transmission systems under normal operating conditions. A gear tooth fault model is developed to simulate the effects of pitting and wear on the vibration signal under normal operating conditions. The model uses changes in the gear mesh stiffness to simulate the effects of gear tooth faults. The overall dynamics of the gear transmission system is evaluated by coupling the dynamics of each individual gear-rotor system through gear mesh forces generated between each gear-rotor system and the bearing forces generated between the rotor and the gearbox structure. The predicted results were compared with experimental results obtained from a spiral bevel gear fatigue test rig at NASA Lewis Research Center. The Wigner-Ville Distribution (WVD) was used to give a comprehensive comparison of the predicted and experimental results. The WVD method applied to the experimental results were also compared to other fault detection techniques to verify the WVD's ability to detect the pitting damage, and to determine its relative performance. Overall results show good correlation between the experimental vibration data of the damaged test gear and the predicted vibration from the model with simulated gear tooth pitting damage. Results also verified that the WVD method can successfully detect and locate gear tooth wear and pitting damage.

**14. SUBJECT TERMS**
Dynamics; Modeling; Gear; Fatigue; Diagnostics; Failure prediction

**15. NUMBER OF PAGES**
18

**16. PRICE CODE**
A03

**17. SECURITY CLASSIFICATION OF REPORT**
Unclassified

**18. SECURITY CLASSIFICATION OF THIS PAGE**
Unclassified

**19. SECURITY CLASSIFICATION OF ABSTRACT**
Unclassified

**20. LIMITATION OF ABSTRACT**