108466 N95-14880

EFFECT OF STRATIFICATION AND GEOMETRICAL SPREADING ON SONIC BOOM RISE TIME [1]

S2-02
29157
p. 20

Robin O. Cleveland, Mark F. Hamilton and David T. Blackstock
Applied Research Laboratories,
The University of Texas at Austin, Austin, TX 78713-8029,
and
Mechanical Engineering Department,
The University of Texas at Austin, Austin, TX 78712-1063

SUMMARY

Sonic boom propagation is not steady, even in a nonturbulent atmosphere. The shock overpressure Δp is affected by geometrical spreading, stratification of the atmosphere, and even the N shape of the waveform. Nevertheless, for purposes of predicting shock profile and rise time, it has commonly been assumed that the shock is in steady state. For example, molecular relaxation, which is a major factor controlling sonic boom rise time, is strongly dependent on relative humidity. Because humidity varies with altitude, rise time varies as the sonic boom propagates downward. The question is whether rise time depends only on local conditions or is also affected by the history of humidity variation along the propagation path. Kang [1, Chap. 7.2] argues that shocks respond to change in humidity quickly enough that they are in effect always in steady state. In other words only local conditions are important. Robinson [2, Chap 5.2] however disagrees with this hypothesis. Raspet *et. al.* [3] found that perturbed 100 Pa shocks (step waveform) require propagation distances of order 1 km for the rise time to return to within 10% of its steady shock value.

The purpose of our investigation is to determine the effect of unsteadiness (not associated with turbulence) on rise time. The unsteadiness considered here is due to (1) geometrical spreading, (2) stratification, which includes variation in density, temperature, and relative humidity, and (3) N

[1]Work supported by NASA

PRECEDING PAGE BLANK NOT FILMED

PAGE 18 INTENTIONALLY BLANK

shaped waveform. A very general Burgers equation, which includes all these effects, is the propagation model for our study. The equation is solved by a new computational algorithm in which all the calculations are done in the time domain.

The present paper is a progress report in which some of the factors contributing to unsteadiness are studied, namely geometrical spreading and variation in relative humidity. The work of Pierce and Kang [4], which motivated our study, is first reviewed. We proceed with a discussion of the Burgers equation model and the algorithm for solving the equation. Some comparison tests to establish the validity of the algorithm are presented. The algorithm is then used to determine the distance required for a steady-state shock, on encountering an abrupt change in relative humidity, to reach a new steady state based on the new humidity. It is found that the transition distance for plane shocks of amplitude 70 Pa is about 4 km when the change in relative humidity is 10%. Shocks of amplitude 140 Pa require less distance. The effect of spherical and cylindrical spreading is also considered. We demonstrate that a spreading shock wave never reaches steady state and that its rise time will be less than the equivalent steady state shock. Finally we show that an N wave has a slightly short rise time than a step shock of the same amplitude.

OUTLINE

- Introduction
- Review of Pierce-Kang result
- Burgers equation model
- Time-domain algorithm
- Validation tests of algorithm
- Rise time as a function of
    1. Change in relative humidity
    2. Geometrical spreading
    3. Waveform

REVIEW OF THE PIERCE-KANG PREDICTION OF SONIC BOOM RISE TIME

Pierce and Kang [4] calculated sonic boom rise time by solving a model propagation equation called Burgers' equation [5]. An important contribution was to augment the classical Burgers equation with terms that describe molecular relaxation due to both nitrogen and oxygen. By assuming that the sonic boom shock near the ground is in steady state, Pierce and Kang simplified their Burgers equation and solved it numerically. Rise time predictions obtained from their solution were then compared with rise time data from a large number of sonic boom measurements taken at Edwards Air Force Base in 1987 [6]. In their initial comparison almost all the measured rise times lay well above the predicted curve [7], as shown in the left-hand plot of Fig. 1. The discrepancy was attributed to atmospheric turbulence. Later, however, when they made a correction for pressure doubling at the ground, they found their prediction to fall in the middle of the data [8]; see the right-hand plot in Fig. 1. Although the data would seem to corroborate their prediction, the role of turbulence casts doubt on this conclusion. Model experiments done at our laboratory [9, 10] indicate that turbulence almost always increases rise time, rarely decreases it. If turbulence does have this effect, then molecular relaxation should be expected to provide a lower bound for the data, not approximate a mean for the data. Why then does the Pierce-Kang prediction not serve as a lower bound?

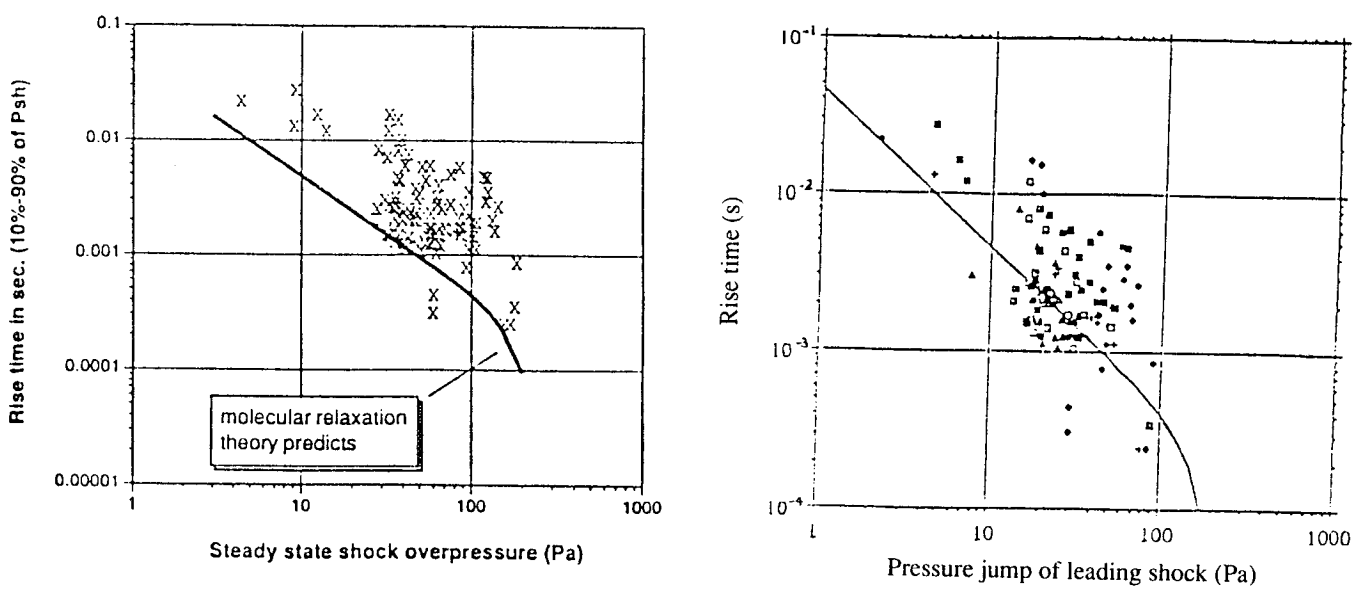

Figure 1: The left-hand plot shows Kang and Pierce's initial prediction [7]. The right-hand plot shows the corrected prediction [8].

Our hypothesis to answer this question is based on results found in Refs. [2, 3]. We suspect that the shock wave at the head of a sonic boom does not respond quickly enough to variation in atmospheric conditions (and to other changes that affect the profile, such as geometrical spreading and even wave shape), to justify the steady-state assumption. If our hypothesis is correct, then to improve on the Pierce-Kang prediction requires that more than local conditions be taken into account. Past history along the propagation path must be significant. Figure 2 shows profiles of temperature and relative humidity measured during a sonic boom experiment [1, pp 157–161]. It is seen that conditions can change rapidly, particularly during the lower part of the propagation path. To determine whether the sonic boom profile can respond quickly to changes of this order, we have calculated the effect on rise time of an abrupt change in atmospheric conditions. Thus far we have concentrated on changes in relative humidity. For purposes of this paper, temperature and pressure are fixed at their ground level values.

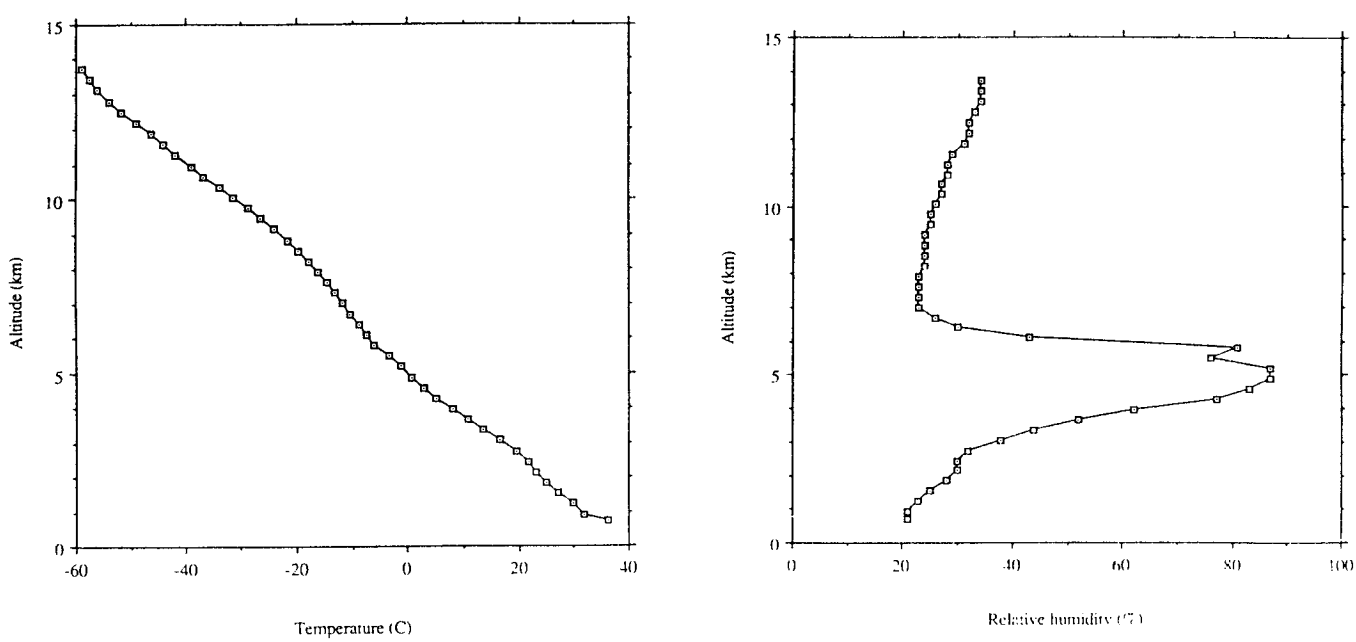

Figure 2: Atmospheric conditions measured during the sonic boom experiment.

THE BURGERS EQUATION

The "classical" Burgers equation is the standard model equation for plane finite-amplitude waves in a thermoviscous medium:

$$\frac{\partial p}{\partial x} - \frac{\beta}{2\rho_0 c_0^3}\frac{\partial p^2}{\partial t'} = \delta_{\mathrm{TV}}\frac{\partial^2 p}{\partial t'^2} \,. \tag{1}$$

Here p is acoustic pressure, t time, $t' = t - x/c_0$ retarded time, x distance, c_0 small-signal sound speed, ρ_0 ambient density, β coefficient of nonlinearity, and δ_{TV} the thermoviscous loss coefficient. Pierce [11] added terms to account for relaxation processes. Each relaxation process ν is characterized by a relaxation time τ_ν and a change in small-signal sound speed $(\Delta c)_\nu$ due to the relaxation. In operator notation Pierce's "augmented Burgers equation" may be written

$$\frac{\partial p}{\partial x} - \frac{\beta}{2\rho_0 c_0^3}\frac{\partial p^2}{\partial t'} = \delta_{\mathrm{TV}}\frac{\partial^2 p}{\partial t'^2} + \sum_\nu \frac{(\Delta c)_\nu \tau_\nu}{c_0^2}\frac{\frac{\partial^2}{\partial t'^2}}{1+\tau_\nu\frac{\partial}{\partial t'}}\, p \,. \tag{2}$$

Equation 2 is still for plane waves. If geometrical spreading is included, the equation becomes

$$\frac{\partial p}{\partial x} + \frac{a}{x}p - \frac{\beta}{2\rho_0 c_0^3}\frac{\partial p^2}{\partial t'} = \delta_{\mathrm{TV}}\frac{\partial^2 p}{\partial t'^2} + \sum_\nu \frac{(\Delta c)_\nu \tau_\nu}{c_0^2}\frac{\frac{\partial^2}{\partial t'^2}}{1+\tau_\nu\frac{\partial}{\partial t'}}\, p \,, \tag{3}$$

where the spreading factor a is 0 for plane waves, $\frac{1}{2}$ for cylindrical waves, and 1 for spherical waves. This is the equation we have solved numerically to obtain the results reported in this paper.

Burgers' equation may be further generalized to include effects of (1) stratification and (2) diffraction. Stratification may be included by scaling the dependent variable p and stretching the independent range variable x [12]. The Burgers equation for this case is not considered in this report but will be taken up in the future. To include diffraction effects, one must use the KZK equation, which is a multi-dimensional form of Burgers' equation; see, for example, Refs. [13, 14]. As a spinoff from the present work, relaxation effects have been included in a computer code that solves the KZK equation, but no formal report of the results has yet been given.

Solutions of the generalized Burgers equation that are not in steady state involve solving a partial differential equation. Except for a few rare cases the solution can only be obtained numerically and it is common to use some sort of marching scheme. A time waveform is digitized with M samples and then small steps are taken in the propagation direction. At each step absorption and nonlinearity are solved in series. It is popular to do the absorption effects in the frequency domain as this requires M complex multiplications. However in the frequency domain the nonlinear term involves a convolution — which requires of the order of M^2 operations. To speed up the code the nonlinear distortion can be applied in the time domain as it requires only order M operations. To go between the time and frequency domain one can use the fast fourier transform which requires order $M \log M$ operations. Algorithms like the Pestorius [15] code flip-flop between the time and frequency domains at each step to take advantage of computing absorption in the frequency domain and nonlinear distortion in the time domain, the price being the use of the FFT.

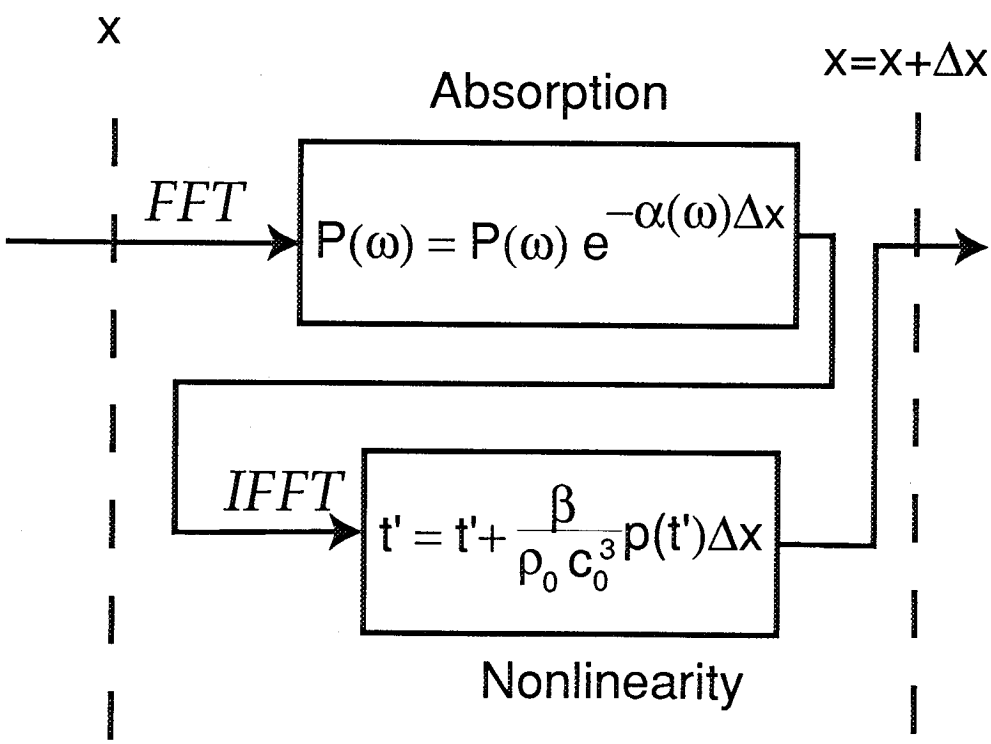

Figure 3: Pestorius type approach to solving the Burgers equation.

TIME DOMAIN ALGORITHM

It would be nice to stay in one domain but without having to pay the price of M^2 operations for a convolution. Lee and Hamilton [13, 14] have developed a method of computing the absorption in the time domain. They approximate the absorption with a finite difference equation. This yields a tridiagonal matrix system which can be solved in order M operations. The code they developed was used to solve the KZK equation, which is a generalization of the Burgers equation to include diffraction effects. We have extended the code to account for molecular relaxation and spreading effects. The work presented here does not, however, include diffraction effects.

Apart from its numerical advantage the fully time domain algorithm has the nice property that it can propagate pulses. Because the FFT isn't used, the endpoints of the waveform need not match to make a periodic waveform. Therefore step shocks and N waves are easily dealt with. The algorithm is particularly suited to finding a steady state solution. Raspet *et. al* [3] had to use a square pulse to find the steady state behavior of a shock. Square pulses have a limited propagation range before they turn into sawtooth waves. In the time domain code a pure shock can be propagated with out difficulty.

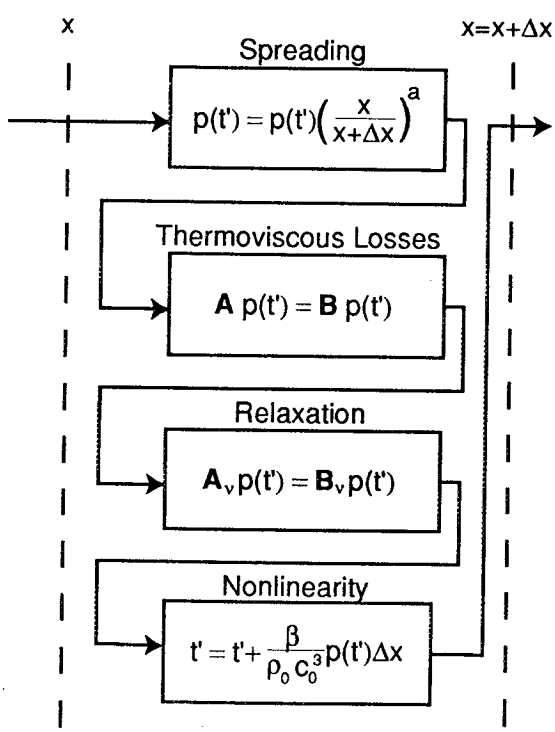

Figure 4: Time domain approach to solving the Burgers equation.

A number of cases were run to test the validity of the code. The first was to obtain the steady-state solution of the classical Burgers equation for a thermoviscous fluid. The known analytical solution for the steady shock is the hyperbolic tangent function. Figure 5 shows how a shock front is propagated with the time domain code; σ is the distance variable. The first figure shows the initial profile, chosen because it looked interesting. The other figures show how the profile develops. The final figure, $\sigma = 2$, shows that the numerical result agrees very well with the analytical solution.

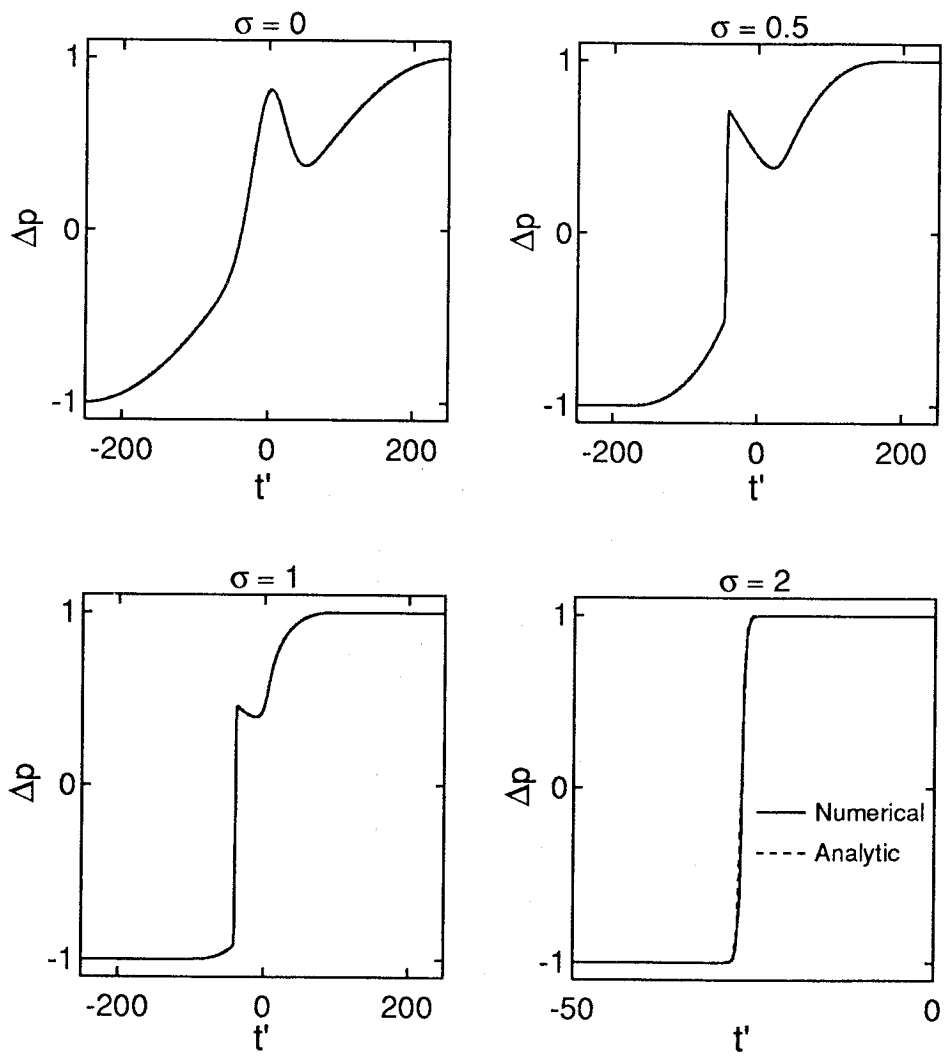

Figure 5: Propagation of a shock in a thermoviscous medium.

STEADY STATE: NONLINEARITY AND ONE RELAXATION PROCESS

We now determine whether the relaxation part of the code behaves correctly. Polykova *et. al.* [16] obtained the steady-state solution for a finite amplitude wave in a medium with one relaxation process but no thermoviscous losses. Their result (denoted PSK in Fig 6) is

$$\frac{t - t_0}{\tau_\nu} = \ln \frac{(1 + p/p_0)^{D-1}}{(1 + p/p_0)^{D+1}} ,$$

where

$$D = \frac{(\Delta c)_\nu \rho_0 c_0}{p_0 \beta} .$$

Figure 6 shows the result from the propagation program in a monorelaxing fluid. For the values chosen relaxation was not enough to stop the waveform from becoming multivalued. In the analytical result weak shock theory was used to ensure a single valued function. Multivaluedness was prevented in the numerical algorithm by including a small amount of thermoviscous losses.

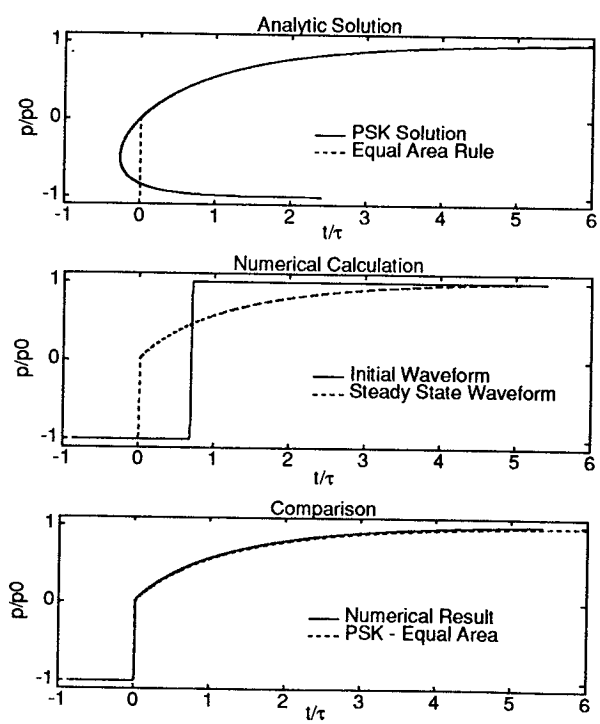

Figure 6: The top figure shows the analytical result for the steady-state solution in relaxing medium with no thermoviscous effects; $D = 0.5$. The middle figure shows the initial and steady-state profiles obtained by the time domain code. The bottom figure compares the analytical and numerical steady-state profiles.

The last verification test was against Kang's result. A plane shock front is sent into a standard atmosphere with a relative humidity of 10%. This allows us to compare results with a steady-state result in Kang's thesis. In our calculation the shock started out with a hyperbolic tangent profile and was then propagated until the profile no longer changed. Figure 7 compares the two results.

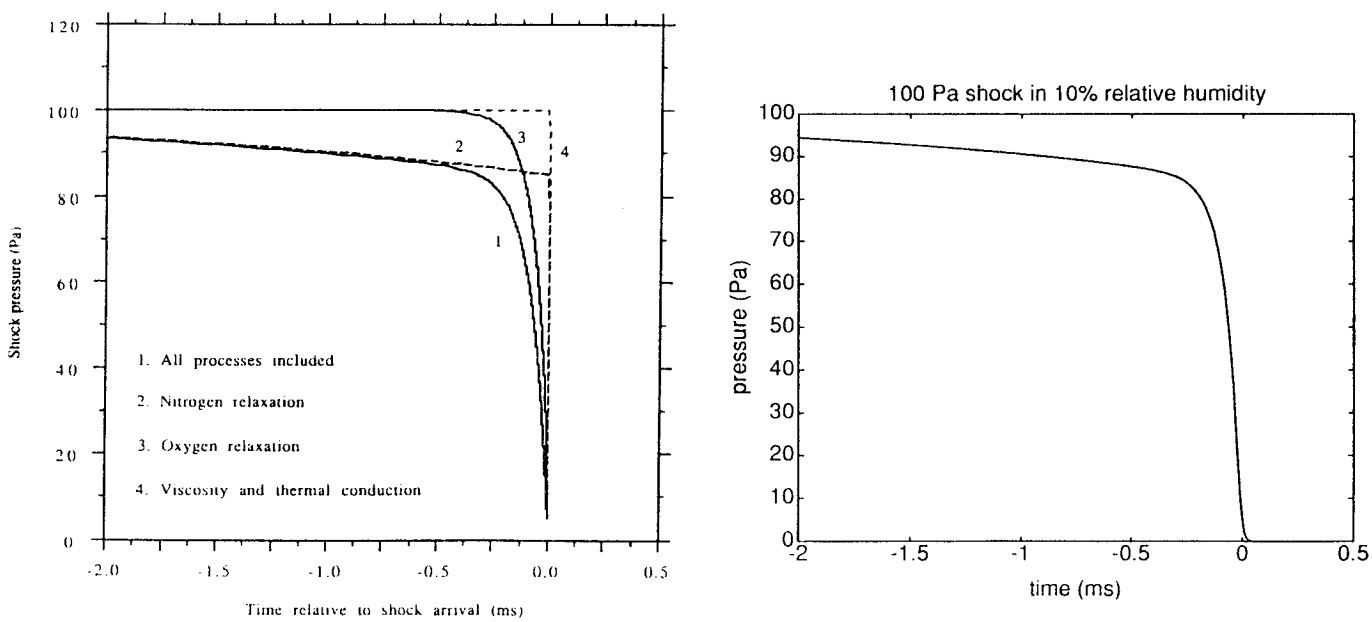

Figure 7: Steady-state solution in air; $T = 20\,°\mathrm{C}$, $P_0 = 1$ atm, and a relative humidity of 10%. On the left is Kang's profile [1, Fig. 5.8]. On the right is the profile from the time domain code.

The last verification test was against Kang's result. A plane shock front is sent into a standard atmosphere with a relative humidity of 10%. This allows us to compare results with a steady-state result in Kang's thesis. In our calculation the shock started out with a hyperbolic tangent profile and was then propagated until the profile no longer changed. Figure 7 compares the two results.

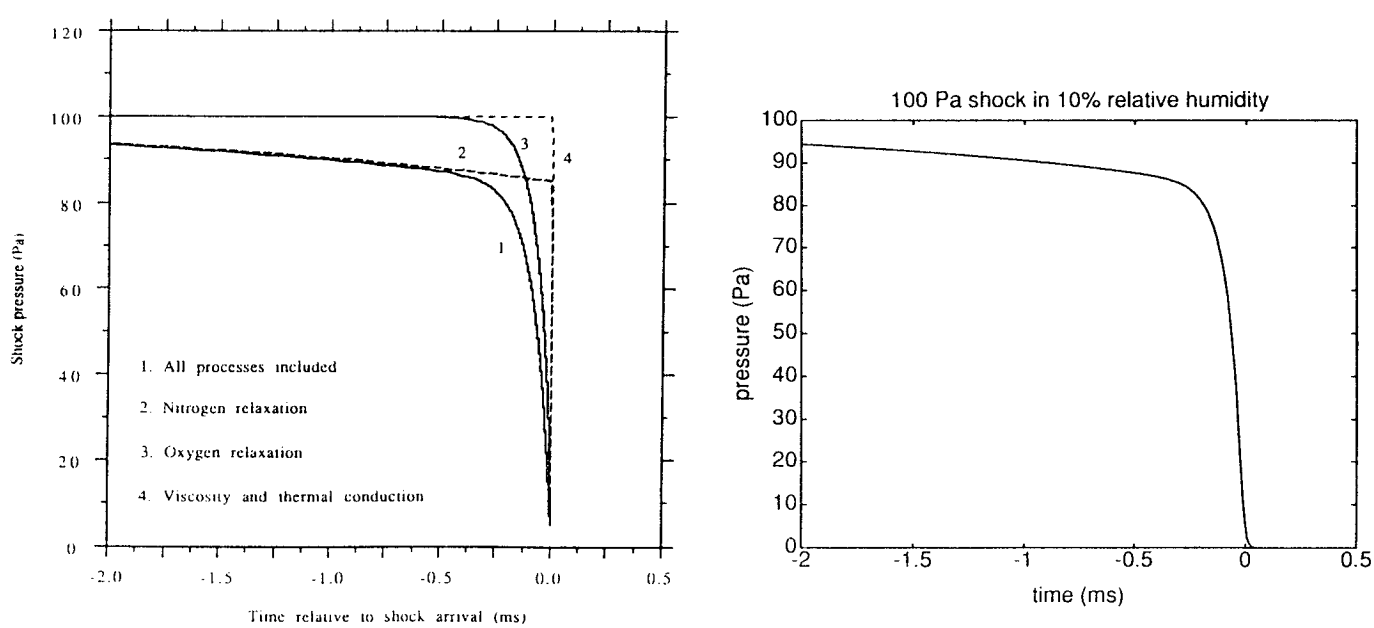

Figure 7: Steady-state solution in air; $T = 20\,°\mathrm{C}$, $P_0 = 1$ atm, and a relative humidity of 10%. On the left is Kang's profile [1, Fig. 5.8]. On the right is the profile from the time domain code.

The last verification test was against Kang's result. A plane shock front is sent into a standard atmosphere with a relative humidity of 10%. This allows us to compare results with a steady-state result in Kang's thesis. In our calculation the shock started out with a hyperbolic tangent profile and was then propagated until the profile no longer changed. Figure 7 compares the two results.

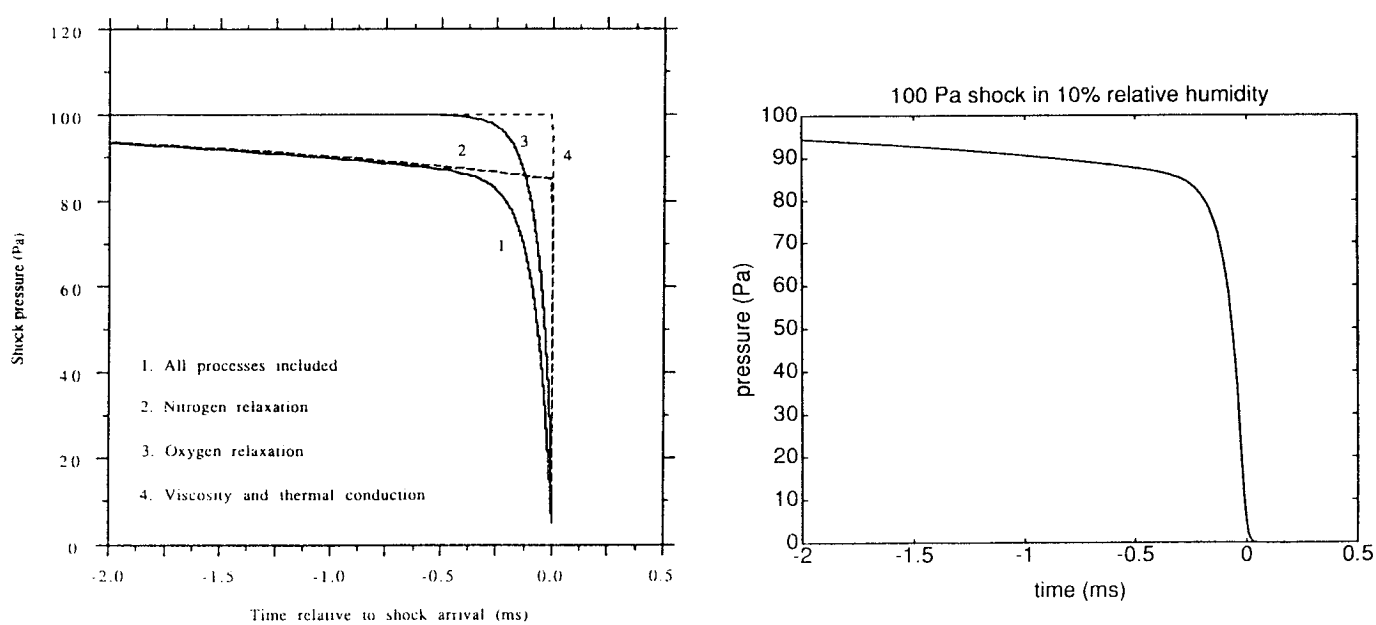

Figure 7: Steady-state solution in air; $T = 20\,°\mathrm{C}$, $P_0 = 1$ atm, and a relative humidity of 10%. On the left is Kang's profile [1, Fig. 5.8]. On the right is the profile from the time domain code.

EFFECTS ON RISE TIME

We will now use the time domain code to investigate the behavior of the rise time of shocks in the air. In all calculations the temperature is 20 °C, and the pressure is 1 atm.

First we examine how long it takes waveform to recover from a small but abrupt change in relative humidity. A plane wave shock is propagated in air of given relative humidity until it reaches steady state. This steady-state waveform is then used as the input waveform for an atmosphere with another relative humidity.

Second we investigate the effect of spreading on the rise time of a shock front. Shocks that are in steady state are propagated as spreading waves; because the amplitude of the shock decreases the rise time is expected to increase. We present results for the combined effects of spreading and a change in relative humidity. Finally we investigate the difference between the rise time of an N wave and a step shock.

TRANSITION DISTANCE FOR 70 Pa SHOCKS

We use the term transition distance to describe how far a shock needs to travel to go from one steady-state profile to another. A somewhat similar term "healing distance" is commonly used in literature related to turbulence for the distance a perturbed shock needs to return to its original state [3]. In this case we shall look at transition distances due to a change in relative humidity.

Figure 8 shows rise time as a function of propagation distance for a plane step shock of amplitude 70 Pa which starts in a medium of 20% relative humidity. The relative humidity of the second atmosphere is 10%, 20% or 30%. The results show the transition distance to be at least 5 km. The plot in Fig. 9 shows the rise time for a shock initially in an atmosphere of 50% relative humidity. Transition distances are greater than 2 km.

The initial fluctuations in the rise time are due to rather gross changes in the profile. The changes are such that the 10% to 90% definition is not a very suitable measure of rise time. Similar fluctuations were observed by Raspet *et. al.* [3].

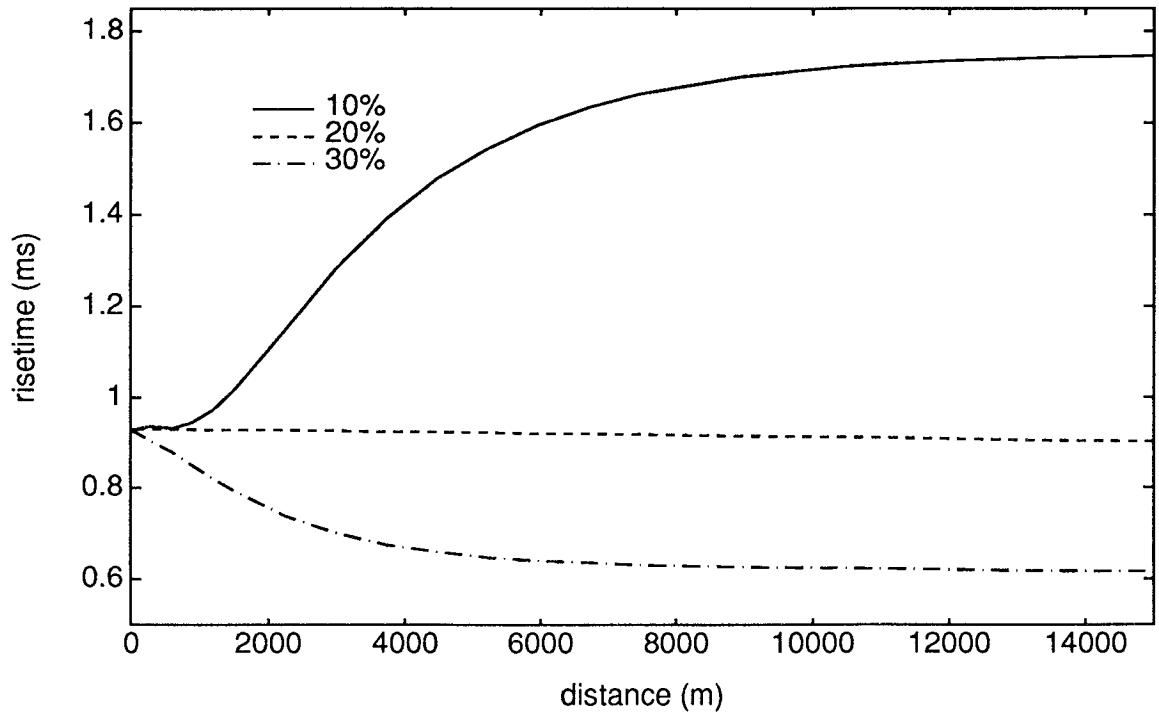

Figure 8: Change in rise time for a waveform leaving a medium of 20% relative humidity.

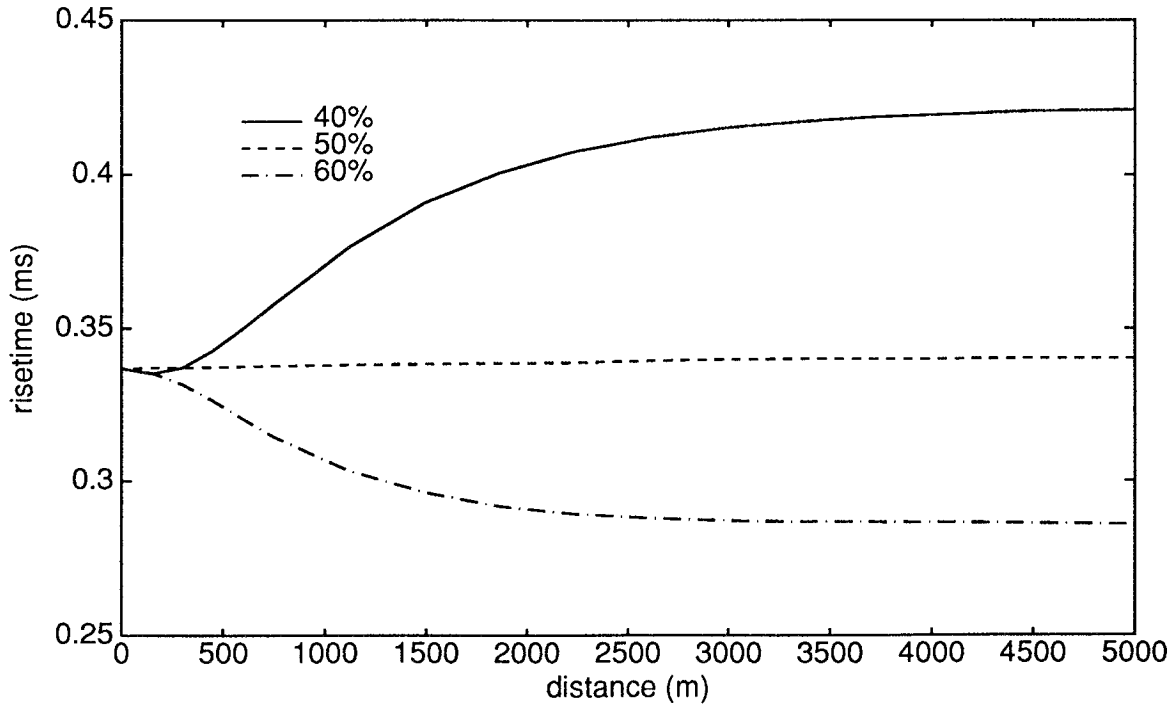

Figure 9: Change in rise time for a waveform leaving a medium of 50% relative humidity.

TRANSITION DISTANCE FOR 140 Pa SHOCKS

Figure 10 shows the behavior of a shock wave of amplitude 140 Pa which starts off in air of 20% relative humidity. The transition distance needed when the new relative humidity is 30% is about 2 km. However the transition distance is at least 6 km when the new relative humidity is 10%.

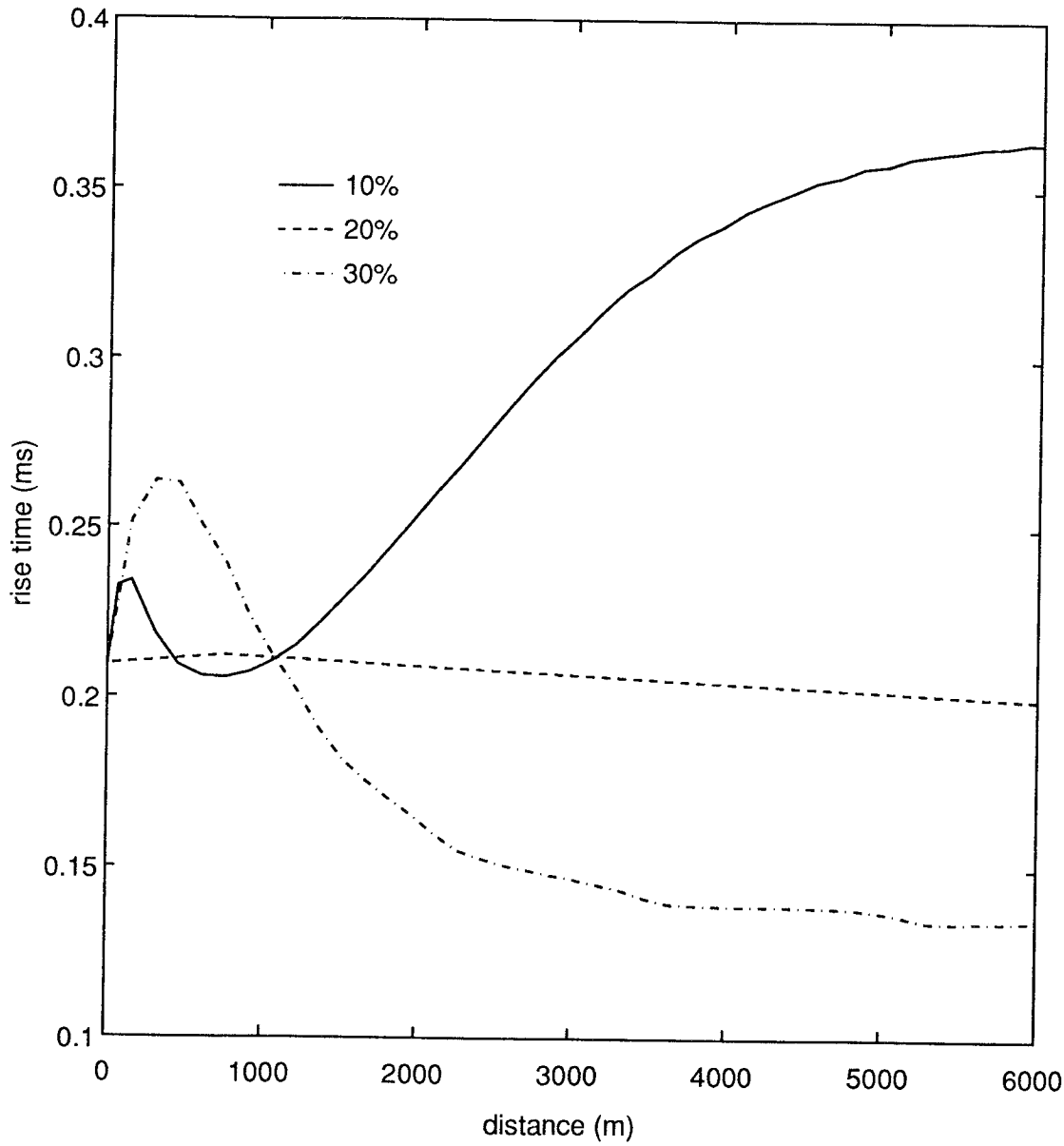

Figure 10: Change in rise time for a 140 Pa shock leaving a medium of 20% relative humidity.

SPREADING

The amplitude of a spherically spreading shock should decrease as

$$\Delta p = \frac{x_0}{x}\Delta p_0 \ , \tag{4}$$

and for cylindrical spreading,

$$\Delta p = \sqrt{\frac{x_0}{x}}\Delta p_0 \ .$$

The steady state rise time found from the analytical solution of the classical Burgers equation is

$$\Delta\tau = \ln(9)\frac{4\delta_{\mathrm{TV}}\rho_0 c_0^3}{\beta\Delta p} \tag{5}$$

As the amplitude of the waveform decreases the rise time increases because the nonlinear steepening effects are weaker. However it is not clear that a spreading waveform will be in steady state. This would require the absorption mechanism to respond immediately to the spreading. Naugol'nykh [18] argued that a spreading shock in a thermoviscous medium should have a rise time that is shorter than the steady-state value because the absorption mechanism can't work fast enough.

If a spreading shock remains in steady state then, from Eq. (5) the rise time should vary inversely as the pressure jump. Since for spherically spreading waves the pressure varies inversely with distance, Eq. (4), we have

$$\Delta\tau \propto x \ .$$

To investigate the validity of this relation we started with the hyperbolic tangent profile appropriate for a plane step shock. The shock was then propagated as a spreading wave. Figure 11 shows the initial waveform and how the shock diffuses as it loses amplitude. Figure 12 compares the steady state prediction of the rise time to the numerically calculated rise time. Note that the steady state prediction always overestimates the rise time. Absorption cannot act quickly enough to diffuse the profile before more amplitude decrease, due to spreading, occurs.

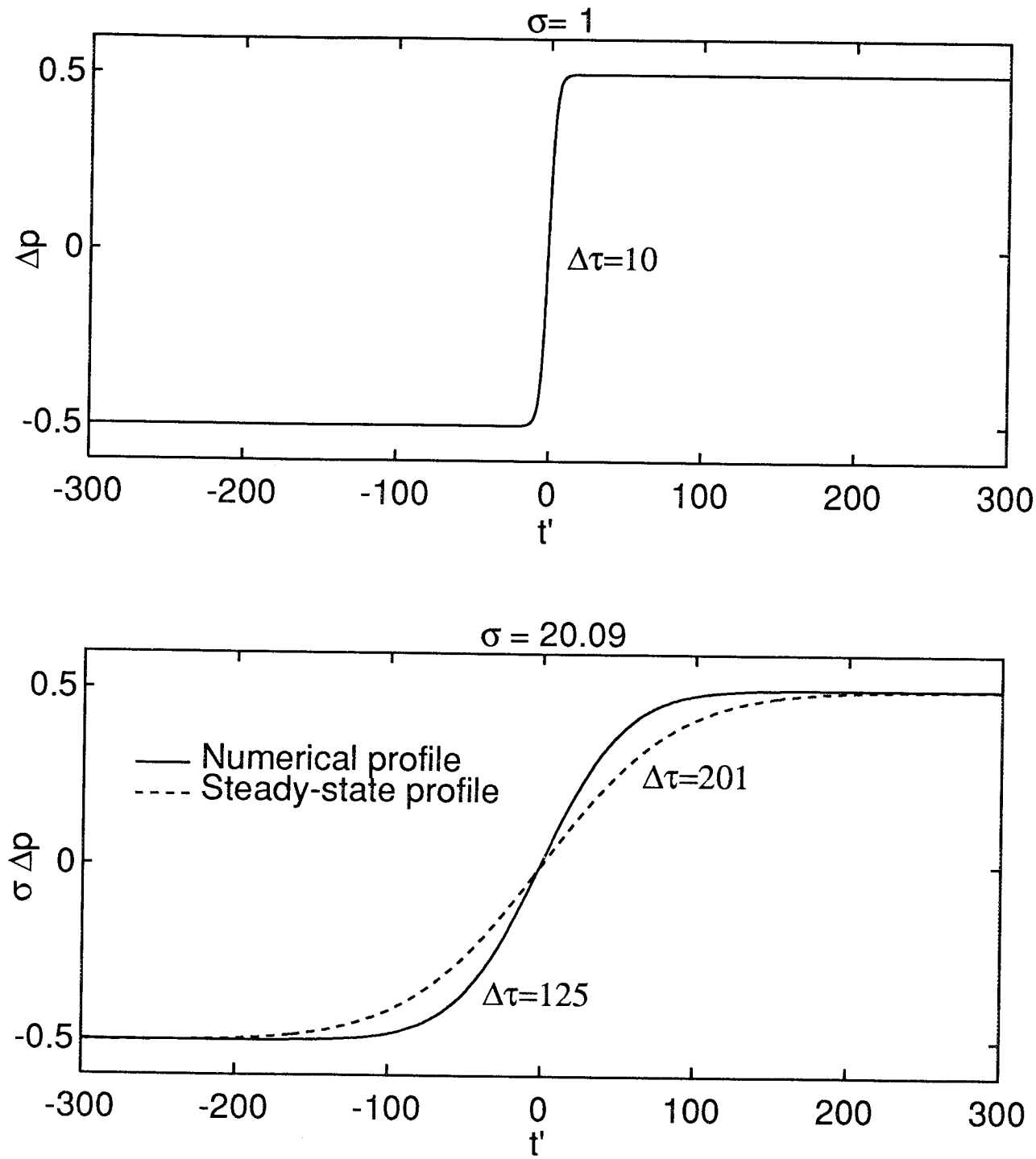

Figure 11: A shock front that starts off in steady-state in a thermoviscous medium is propagated as a spherical wave. The upper plot shows the initial waveform. The lower plot shows the waveform at 20 times the source radius; the steady-state waveform is also shown.

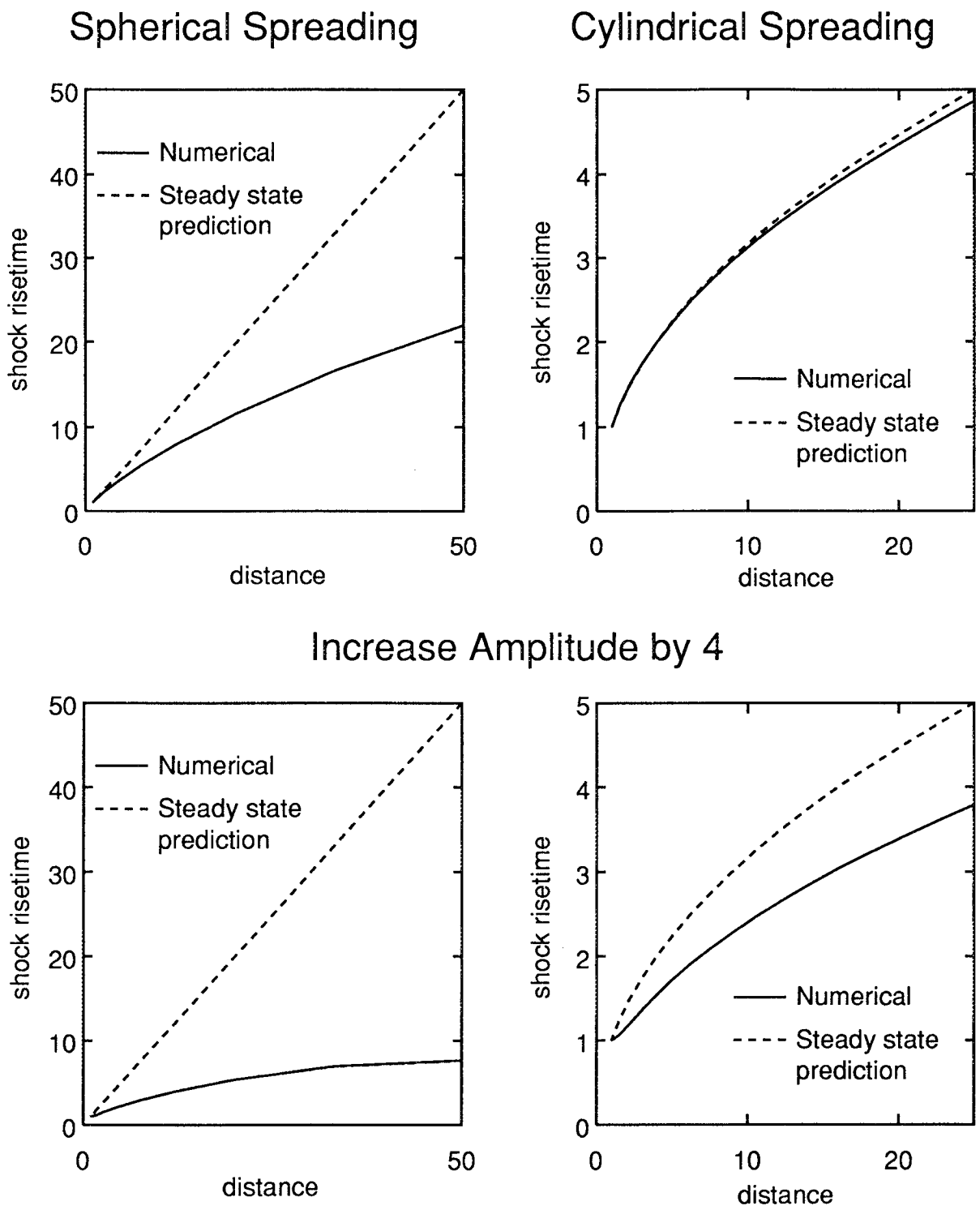

Figure 12: The rise time of a step shock in a thermoviscous medium. The waveform starts off in steady-state and is propagated as either a spherically or cylindrically spreading wave. Two different initial amplitudes are used. Rise time is normalized to the initial rise time and distance is normalized to the source radius.

EFFECT OF SPREADING ON TRANSITION DISTANCE

We now investigate the combined effects of spreading and a change in relative humidity. A source is assumed to be 15 km away and the shock front is propagated at an angle corresponding to a sonic cone from an aircraft flying at Mach 2. Cylindrical spreading is used. Figure 13 shows the rise time curves and compares them to the plane wave curves taken from Fig. 8. In Fig. 14 we compare the rise time curves to those of spreading waves in a homogeneous atmosphere, which are the effective asymptotes. We see that the waveform is never in steady state but that the asymptotes are reached in 6–8 km.

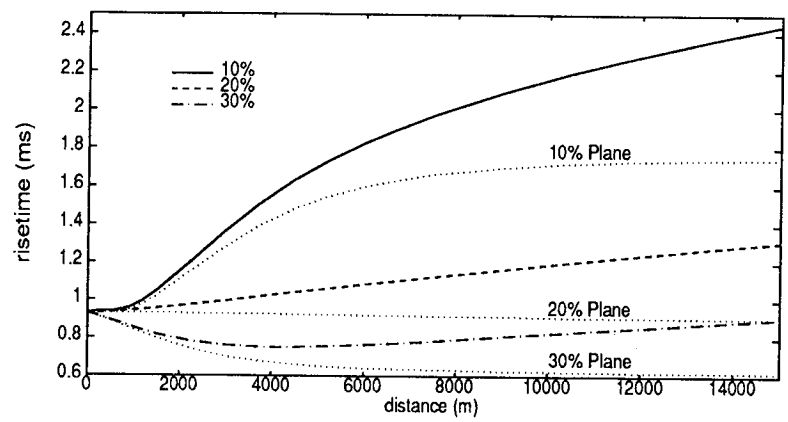

Figure 13: The rise time of a shock that is cylindrically spreading; the shock starts from a steady-state profile for an atmosphere with 20% relative humidity. The dotted lines show the results for plane waves as shown in Fig. 8. Note that the steady-state rise time is not achieved.

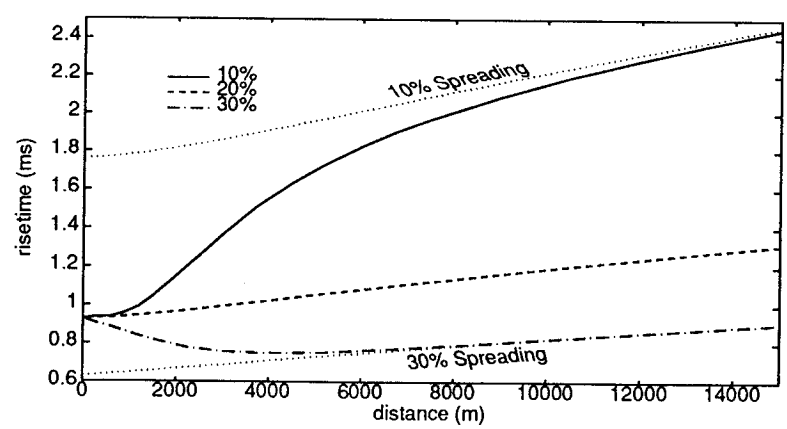

Figure 14: The same curves as in Fig. 13, except that the dotted lines show the results for cylindrically spreading waves in a homogeneous atmosphere; these are the asymptotic results.

EFFECT OF PROFILE ON THE RISE TIME

Finally we compare rise time for an N wave with that for a steady-state step shock. Figure 15 shows the curves for a plane N wave that has propagated a long distance in a medium with a relative humidity of 50%. The amplitude of the N wave is 48 Pa. We see that a steady-state shock of the same amplitude has a longer rise time. However a steady-state shock of amplitude 50.4 Pa has the same rise time. It appears that the profile corresponds to that of a higher amplitude shock because the amplitude of the N wave decreases with propagation distance. Just as was seen with spreading waves, the absorption cannot keep up with the loss in shock amplitude.

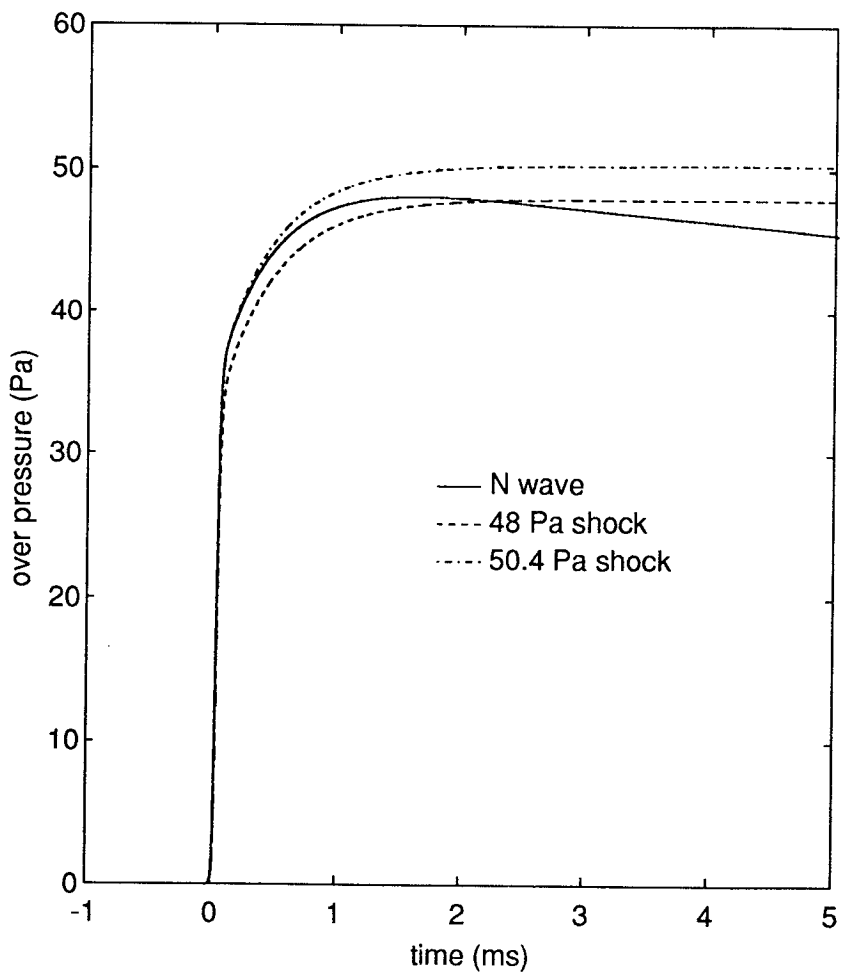

Figure 15: The solid line is an N wave with duration of 125 ms. It has propagated for 30 km. The dashed line is a step-shock of the same amplitude. The dot-dash line is a shock of the same rise time as the N wave. This shock has an amplitude of 50.4 Pa.

CONCLUSION

Our study indicates that assuming a steady-state profile for the head shock of a sonic boom tends to cause an overestimate of the rise time. First, we have found that substantial propagation distance is required for a shock to respond to abrupt but small changes in relative humidity. When the absorption increases, the actual rise time is always less than the steady-shock value. Second, neglect of geometrical spreading results in an overestimate of rise time. Third, modeling the N wave as a step shock also raises the rise time, although the effect is small. All of these results are consistent with the conclusion that for a decaying shock dissipation is always "struggling to catch up with nonlinearity."

References

[1] J. Kang, "Nonlinear acoustic propagation of shock waves through the atmosphere with molecular relaxation," Ph. D. Thesis, Department of Mechanical Engineering, Pennsylvania State University (1991).

[2] L. D. Robinson "Sonic boom propagation through an inhomogeneous windy atmosphere" Ph. D. dissertation, Department of Physics, University of Texas at Austin, (1991).

[3] R. Raspet, H. Bass, L. Yao and W. Wu, "Steady state risetimes of shock waves in the atmosphere," in NASA CP 3172: High-Speed Research Workshop on Sonic Boom, NASA Langley Research Center, Hampton, Virginia, C. M. Darden, ed., Vol. I, pp. 109-116 (1992).

[4] A. D. Pierce and J. Kang, "Molecular relaxation effects on sonic boom waveforms," in *Frontiers of nonlinear acoustics: Proceedings of the 12th ISNA*, M. F. Hamilton and D. T. Blackstock, eds. (Elsevier Applied Science, London, 1990), pp. 165-170.

[5] D. T. Blackstock, "Thermoviscous attenuation of plane, periodic, finite-amplitude sound waves," *J. Acoust. Soc. Am.* **36**, 534-542 (1964).

[6] R. A. Lee and J. M. Downing, "Sonic boom produced by United States Air Force and United States Navy aircraft: measured data," AL-TR-1991-0099, Biodynamic Environment Branch, Biodynamics and Bioengineering Division, Armstrong Laboratory, Wright-Patterson Air Force Base, Ohio (1991).

[7] A. D. Pierce and V. W. Sparrow "Relaxation and turbulence effects on sonic boom signatures," First Annual High-Speed Research Workshop, Williamsburg, Virginia (1991).

[8] Allan D. Pierce "Relaxation and turbulence effects on sonic boom signatures," in NASA CP 101321: Second Annual High-Speed Research Workshop on Sonic Boom, Moffett Field, California, pp. 1-18 (1993).

[9] B. Lipkens, "Experimental and theoretical study of the propagation of N waves through a turbulent medium," Ph. D. dissertation, Department of Mechanical Engineering, University of Texas at Austin (1993).

[10] B. Lipkens and D. T. Blackstock, "Model experiment to study the effect of turbulence on rise time and waveform of N waves," in NASA CP 3172: High-Speed Research Workshop on Sonic Boom, NASA Langley Research Center, Hampton, Virginia, C. M. Darden, ed., Vol. I, pp. 97-108 (1992).

[11] A. D. Pierce, *Acoustics: An introduction to its physical principles and applications* (McGraw-Hill, New York, 1981).

[12] R. O. Cleveland and D. T. Blackstock, "Waveform freezing of sonic booms in the atmosphere," *J. Acoust. Soc. Am.* **92** (4), 2331 (A) (1992).

[13] Y-S. Lee and M. F. Hamilton, "Nonlinear effects in pulsed sound beams," in *Ultrasonics International 91 Conference Proceedings*, D. E. Gray, ed. (Butterworth-Heinemann, Oxford, 1991), pp 177-180.

[14] Y-S. Lee and M. F. Hamilton, "Time-domain modeling of pulsed finite-amplitude sound beams," submitted to *J. Acoust. Soc. Am.* in March 1994.

[15] F. M. Pestorius, "Propagation of plane acoustics noise of finite amplitude," Technical Report ARL-TR-73-23, Applied Research Laboratories, The University of Texas at Austin (August 1973), AD 778 868.

[16] A. L. Polyakova, S. I. Soluyan, and R. V. Khokhlov, "Propagation of finite amplitude disturbances in a relaxing medium," *Sov. Phys. Acoust.* **8**, 78–82 (1962).

[17] M. F. Hamilton and E. A. Zabolotskaya, draft of "Dispersion," Chap. 5 in *Nonlinear Acoustics* to be published by Academic Press, New York.

[18] K. A. Naugol'nykh, "Transition of a shock wave into a sound wave," *Sov. Phys. Acoust.* **18**(4), 475–477 (1973).