(NASA-CR-199283) ADVANCED COMPILATION TECHNIQUES IN THE PARADIGM COMPILER FOR DISTRIBUTED-MEMORY MULTICOMPUTERS (Illinois Univ. at Urbana-Champaign) 10 p
Advanced Compilation Techniques in the PARADIGM Compiler for Distributed-Memory Multicomputers


Abstract

The PARADIGM compiler project provides an automated means to parallelize programs, written in a serial programming model, for efficient execution on distributed-memory multicomputers. A previous implementation of the compiler based on the PTD representation allowed symbolic array sizes, affine loop bounds and array subscripts, and variable number of processors, provided that arrays were single- or multi-dimensionally block distributed. The techniques presented here extend the compiler to also accept multidimensional cyclic and block-cyclic distributions within a uniform symbolic framework. These extensions demand more sophisticated symbolic manipulation capabilities. A novel aspect of our approach is to meet this demand by interfacing PARADIGM with a powerful off-the-shelf symbolic package, Mathematica™. This paper describes some of the Mathematica™ routines that performs various transformations, shows how they are invoked and used by the compiler to overcome the new challenges, and presents experimental results for code involving cyclic and block-cyclic arrays as evidence of the feasibility of the approach.

1 Introduction

Distributed-memory multicomputers offer significant advantages over shared-memory multiprocessors in terms of cost and scalability. Unfortunately, extracting all the computational power from these machines requires users to write efficient software for them, which is a laborious process. One major reason for this difficulty is the absence of a global address space. As a result, the programmer has to distribute "code and data across processors and manage communication among tasks explicitly."

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are implemented using stable and wide-spread tools like yacc and lex instead of being built from scratch, we argue that many parallelizing compilers in the future, in particular research prototypes developed in academic environments, will rely on off-the-shelf symbolic packages for most of their symbolic manipulations. Several reasons account for this:

- New compilers are required to perform increasingly complex symbolic tasks [9, 15, 22].
- As compiler algorithms become more complex, there is a need to focus more on their designs rather than on their implementation details.
- Many powerful symbolic packages are readily available (e.g., Mathematica, Maple), and some provide external interfaces (e.g., MathLink) with reasonable performance and robustness.

Clearly, the performance is not comparable with a C implementation optimized for special cases, but compilation time has not been a major problem in our experience. Furthermore, it is not a good practice to fully tune compiler features until their impact on optimizing real codes has been tested [8]. The approach presented in this paper allows quick prototyping and testing of new algorithms with real programs. Once the effectiveness of an algorithm is verified, the critical features affecting efficiency and compilation time can be identified and rewritten in C.

The rest of this paper is organized as follows. Section 2 describes some of the Mathematica packages that we have written to meet the symbolic demands of the compiler. Section 3 shows how PARADIGM performs computation partitioning and communication generation by building inequality expressions from the source program and passing them to the Fourier-Motzkin package in Mathematica to obtain necessary symbolic expressions. Some results for two programs involving cyclic and block-cyclic array distributions, respectively, appear in Section 4. Finally, concluding remarks are made in Section 5.

2 Mathematica Tools in PARADIGM

This section describes a set of tools that we have built using Mathematica and shows how PARADIGM, which uses an internal representation based on Parafrase-2, is interfaced to Mathematica via MathLink. These tools are the result of only three man-months and represent roughly 1000 lines of Mathematica code and 700 lines of C code. The key to this quick turnaround is the extensive use of built-in Mathematica functionality as well as public domain Mathematica packages.

2.1 Linking PARADIGM with Mathematica

An overview of the interactions between PARADIGM and Mathematica is shown in Figure 2. This follows a typical client/server model in which the server (Mathematica) waits for a “symbolic request,” processes it, and returns the result to the client. Both run as separate UNIX processes and communicate with each other using MathLink. The underlying communication mechanism can be either UNIX pipes, when they are running on the same machine, or TCP/IP sockets for remote execution.

Efficient forward and reverse conversion routines transfer data between PARADIGM’s internal representation (based on Parafrase-2) and Mathematica expressions (PSToMath and MathToP2 in the figure). Conversion time is kept to a minimum using several techniques:

- The structure of the expression is never lost. We use MathLink API function calls to send and receive a tokenized stream.
- Variable names are transformed so that they encode a pointer to the symbol table. This eliminates symbol table look-ups when expressions return from Mathematica.
- Mathematica also pre-processes the result (PreMathToP2 in the figure) so that functions have the correct number of arguments and easy-to-decode names following a preset convention.

An Example: Simplifying Loop Bounds. To illustrate the interactions between PARADIGM and Mathematica we use a simple example, namely the simplification of loop bounds. After certain loop transformations, such as loop normalization, bound expressions can become very complex, and in particular, when arrays with multiple levels of indirection are involved, it is difficult for conventional compilers to “clean up the mess.”

Figure 3 shows our solution to this problem using Mathematica. The PARADIGM process first opens a connection with the Mathematica process and then traverses the hierarchical representation of the loop nest and finds the lower and upper bound expressions. Each of these expressions is transformed into a Mathematica expression using PSToMath, and a Mathematica built-in function, Simplify, is applied to them.
PARADIGM Process

Math = OpenMath()
p = FirstLoop
While (NotNull(p))
    IB = lowerBound(p)
    SendMath("PreMathToP2")
    SendMath("Simplify")
    SendMath(PreMathToMath(IB))
    lowerBound(p) = MathToP2[Receive(Math)]
    p = NextLoop(p)

CloseMath(Math)

Mathematica Process

MathLink

While (NotEnd)
    Receive Request
    Process Request
    Send Reply

Figure 3: Simplifying Loop Bounds

by the server. The result is sent back to the client where it
is transformed into a PARADIGM expression by MathToP2
and inserted into the appropriate data structures.

A simple optimization that reduces the number of in-
teractions between the processes is to generate a list with
all the lower and upper bound expressions and send it for
simplification as one stream. This is particularly important
when the granularity of the Mathematica operations is small
as it is in this case.

2.2 Symbolic Fourier-Motzkin Elimination (FME)

The Fourier-Motzkin method solves systems of linear in-
equalities with real variables using an elimination method.
In the context of compilers it has important applications, in-
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of the integer matrix that describes the access pattern into the product of other matrices with special characteristics. In our implementation, we used a Mathematica package\(^3\) to obtain the Smith Normal Form and Hermite Column or Row Form of a matrix. Then, we can use the unimodular package described above to compute the new loop bounds. Currently PARADIGM is not taking advantage of this transformation, but it will in the near future.

2.4 Other Tools of Interest

In this section we briefly describe some other Mathematica tools that we have written that are useful in other areas of the compilation process.

Integer Area Estimation For applications like automatic data partitioning [14] or static buffer allocation it is crucial to have compile-time estimates of computation and/or communication requirements. Good estimates require the computation of the integer volume of a polyhedron, but this is in general too complex. Fortunately, in most practical cases we only need an estimate of this volume, and heuristics like the one proposed in [1] are sufficient. We have a preliminary 2-D implementation to estimate the integer area, which extends the work in [1] by handling affine loop bounds. The integer area is approximated by the "real" area plus half the points in the boundary. Using a Mathematica package\(^4\), we obtain all the vertices of the polyhedron from the loop inequalities. Then, using built-in functionality, the vertices are ordered clockwise and the "real" area is obtained through triangulation. Finally, the perimeter is computed by applying the GCD rule to consecutive points.

We can also apply this method in cases when symbolic terms only scale the polyhedron without changing its shape. In that case, we instantiate the symbolic terms for several values, compute the integer area with the previous method for each instantiation, and obtain an interpolating polynomial with the symbolic variable for these values using built-in functionality.

In cases where symbolic terms change the shape of the polyhedron, we can apply FME to obtain a bounding box as described in [4]. Assuming an n-dimensional space and using FME to project n - 1 dimensions, we can obtain real bounds for the remaining dimension. Rotating the dimensions and repeating this process we can compute bounds for every dimension.

We are currently working on extensions to handle more than two dimensions and symbolic terms more effectively.

Graphic Visualization The built-in plotting functions in Mathematica are used to visualize iteration space and array access patterns in two or three dimensions. By merging plots for different processors (using different colors for each), the sets for a particular communication or the load distribution after partitioning a computation can be shown. Masks in plotting functions can be used to represent non-convex regions or filter certain types of points. By combining these plotting functions with FME, any convex set defined by linear inequalities can be visualized. This tool proved to be invaluable for debugging the compiler.

\(^3\) Thanks to Brian L. Evans at Georgia Tech for providing the Lattice Theory package.

\(^4\) Thanks to Komei Fukuda and Ichiro Mitsuokhi of University of Tsukuba, Tokyo for providing the n-dimensional vertex enumeration package.

C++ Code Generation We have extended Mathematica's capabilities to output structured C/C++ loops, conditions, local declarations, and function headers and calls from Mathematica expressions. Together with the built-in function Splice and a template file, it is relatively easy to generate simple C++ code. We are using this capability to generate inspector code for irregular problems and link it with PARADIGM's irregular run-time support library, PI-LAR [18].

3 Compilation Techniques

This section will explain how PARADIGM compiles a loop nest containing computations on regularly distributed arrays. The arrays are allowed to have an arbitrary number of dimensions, each of which can have a block, cyclic, or block-cyclic distribution, or be replicated along a processor dimension, or be sequentialized on a single processor. The array subscripts and loop bounds are allowed to be affine functions of index variables of enclosing loops. Because of the symbolic nature of the approach, the number of processors need not be specified at compile time, allowing the resulting program to execute on an arbitrarily sized system. For simplicity, global references are not translated into local address spaces and, therefore, arrays are not scaled down as more processors are used. We are currently looking for an efficient way that can handle this local translation effectively and uniformly for all supported array distributions.

The data distribution and alignment of each array dimension is either automatically generated by PARADIGM [14] or provided by the user through standard HPF directives. Given the data distributions, the compiler still must carry out two major tasks, namely the partitioning of the computation across processors and the generation of communication code to transfer data among processors. How the compiler performs these tasks is the main focus of this section.

3.1 Computation Partitioning

The key to the symbolic analyses and transformations required for both of the aforementioned tasks is the use of the Mathematica tools described previously. The symbolic FME package is the main engine used to generate various iteration sets for different data access patterns.

To exploit parallelism in the source program, a compiler must somehow distribute computations across processors. PARADIGM partitions loops using the owner computes rule: a processor p only executes those iterations for which the left-hand side (lhs) array reference of an assignment statement is local to (owned by) p. Since determining ownership for each access at run time is prohibitively costly, the compiler, through a process called loop bounds reduction [16], derives new loop bounds confined to only p's local iterations. This is to say that the new loop must only traverse the ACCESS set \([22]\) of the lhs reference. Consider the loop:

\[
\begin{align*}
d & = L,U \\
& \cdots A(2i+3) \\
& \cdots \\
& \end{align*}
\]

end do

where A(\(l : u\)) is block distributed on P processors indexed by \(p, 0 \leq p \leq P - 1\). By definition, ACCESS(A(s(i)), p) is the set of all iterations \(i\) such that \(A(a(s(i)))\) is stored in \(p\). This set is obtained from the polytope \(R\) returned by FME when
called with the inequalities $I$ constraining the loop variable $i$, which is also the polytope's only axis variable in this example ($V = i$). Two types of inequalities can be distinguished: loop inequalities and data inequalities. The former arises from loop bounds and specifies constraints on the loop variables and the relationships among them, while the latter is due to data distribution and describes the relationships between processor coordinates and subscript functions (hence loop variables, unless the subscript is a constant). In this example,

$$I = \begin{cases} L \leq i \leq U & \text{(loop inequality)} \\ \frac{b}{b} + 1 \leq 2i + 3 \leq \frac{b}{b} + 1 + b - 1 & \text{(data inequality)} \end{cases}$$

where $b = \lceil \frac{i+1}{p} \rceil$ is the block size of distribution. FME returns:

$$R = \left\{ \left[ \max\left( L, \frac{3i + b - b}{2} \right) \right] \leq i \leq \left[ \min\left( U, \frac{3i + b + b}{2} \right) \right] \right\}$$

where $b = \lceil \frac{i+1}{p} \rceil$ is the block size of distribution. FME returns:

$$R = \left\{ \left[ \max\left( L, \frac{-3i + b - b}{2} \right) \right] \leq i \leq \left[ \min\left( U, \frac{-3i + b + b}{2} \right) \right] \right\}$$

and the output is:

$$V = i, i$$

and the output is:

$$U = \left\{ \min(L_n, L_n) \leq n \leq \max(U_n, U_n) \right\}$$

Once $R$ is found, $\text{ACCESS}(A(s(i)), p)$ is simply the set of $i$'s in $R$. A purely cyclic distribution is just cyclic(1), so this case is covered by setting $b = 1$ in the above expressions. These results easily extend to multi-dimensional arrays distributed on multi-dimensional processor meshes.

Some special cases are worth mentioning. If an array dimension $A(..., 1 : u, ...)$ is replicated, then its data inequality is $l \leq s(i) \leq u$ and involves no processor indices. If an entire array dimension is sequenalized (or collapsed) on a particular processor $r$, then instead of a data inequality, the mask "IF $p = r$ THEN" is applied to the $\text{ACCESS}$ set.

To perform loop bounds reduction, PARADIGM just needs to extract the loop inequalities from the the original loop bounds and the data inequalities corresponding to the $\text{lhs}$ reference and its distribution, and send them to FME. Then, it uses the polytope returned to construct the new loop.

Using the previous results for the $\text{ACCESS}$ set of a block distributed array $A$, the sequential loop:

$$\text{do } i = L, U \text{ end do}$$

becomes:

$$\text{do } i = \left[ \max\left( L, \frac{-3i + b - b}{2} \right) \right], \left[ \min\left( U, \frac{-3i + b + b}{2} \right) \right] \text{ end do}$$

after loop bounds reduction, where $\text{myp}()$ returns the processor's index in the mesh. Similarly, for a cyclic(1) distribution the new loop is:

$$\text{do } i = \left[ \max\left( L, \frac{-3i + b - b}{2} \right) \right], \left[ \min\left( U, \frac{-3i + b + b}{2} \right) \right] \text{ end do}$$

and so the loop becomes:

$$\text{do } i = L, U \text{ end do}$$

When there are multiple $\text{lhs}$ array references, a conservative union $U$ of their $\text{ACCESS}$ sets is used to form the reduced loop bounds, while each assignment statement is masked with the corresponding $\text{ACCESS}$ set of its $\text{lhs}$. For example, in the following loop:

$$\text{do } i = L, U \text{ end do}$$

let both $A$ and $B$ have a block-cyclic distribution, and let their $\text{ACCESS}$ sets be $\{L_n \leq n \leq U_n, L_n \leq i \leq U_n\}$ and $\{L_n \leq n \leq U_n, L_n \leq i \leq U_n\}$, respectively. Their union is:

$$U = \left\{ \min(L_n, L_n) \leq n \leq \max(U_n, U_n) \right\}$$

and so the loop becomes:

$$\text{do } i = L, U \text{ end do}$$

Although one of the processors may have an incomplete last block, this causes no problems as the loop inequalities will provide a tighter upper bound.
do n = min(L_{nA}, L_{nB}), max(U_{nA}, U_{nB})
do i = min(L_{iA}, L_{iB}), max(U_{iA}, U_{iB})

if ((L_{nA} \leq n \leq U_{nA}) and (L_{iA} \leq i \leq U_{iA})) then
  A(2i + 3) = ...
end if

if ((L_{nB} \leq n \leq U_{nB}) and (L_{iB} \leq i \leq U_{iB})) then
  B(i - 1) = ...
end if
end do

Since block and cyclic distributions are special cases of block–cyclic, this method works for all types of regular distributions. In particular, if some of the lhs terms are block distributed, then before finding their ACCESS sets, they are first cast into a block-cyclic form by adding a block term to the bounds in their data inequality, and adding an extra data inequality, 0 \leq n \leq 0. However, if all of the lhs terms are block distributed, then this transformation is not necessary.

### 3.2 Communication Generation

If a processor \( p \) does not own all of the elements of a right-hand side (rhs) array reference \( R \) required by a statement that it executes, then this data must be sent from its owner, say \( q \), to \( p \) via inter-processor communication. To reduce communication overheads, array elements can be combined into a single larger message instead of being communicated individually. This optimization is generally known as message vectorization [16].

The approach is the basically similar to the work of An- court [4] which describes regions requiring communication using a set of linear inequalities and generates loops to scan these regions through a Fourier-Motzkin projection. Dependence analysis done by Parareal-2 is used to determine the communication point, which is the outermost loop at which the combining can be applied.

Therefore, the main role of the compiler is to obtain relevant linear inequalities from the loop bounds, data decompositions, and array references in the source program, send these inequalities to FME, and from its results generate the scanning loops to pack/send and receive/unpack, and insert this code at the communication point.

The set of iterations for which a processor \( p \) must receive elements of a rhs reference \( R \) from its owner processor \( q \) is called the COMM set of \( R \), denoted COMM(\( R, p, q \) (In previous work [22], this was referred to separately as the SEND set of \( q \) and the RECEIVE set of \( p \).) To obtain the COMM set, the inequalities \( I \) needed the loop inequalities\(^4\), the data inequalities (parameterized by \( p \)) of the lhs reference, and the data inequalities (parameterized by \( q \)) of the rhs reference \( R \). The resulting polytope \( \mathcal{R} \) is used to both pack/send data (by setting \( q = \text{myr}() \)) and receive/unpack data (by setting \( p = \text{myr}() \)).

This process is demonstrated using the HPF program in Figure 4. The processor grid is a \( P_1 \times P_2 \) mesh\(^5\) (hence \( 0 \leq p_1 \leq P_1 - 1 \) and \( 0 \leq p_2 \leq P_2 - 1 \)). The lhs array \( A \) is distributed by cyclic(5) on the first mesh dimension and sequentialized (\( p_2 = 0 \)) on the second mesh dimension; i.e., only the processors whose second coordinate \( p_2 \) is 0 will own parts of \( A \). The first dimension of the rhs array \( B \) is distributed by cyclic(3) on the first mesh dimension, while the second array dimension is cyclic(7) on the second mesh dimension.

First, the compiler uses data dependence analysis to determine that communication can take place outside the entire loop nest. Then, it calls FME with inequalities derived from the loop (as explained below), and extracts the COMM set from the solution \( \mathcal{R} \) returned to construct the communication code to pack/send and receive/unpack data. The input sent to FME is:

\[
\begin{align*}
3 \leq i & \leq 40 & \text{(i-loop)} \\
2 \leq j & \leq i - 1 & \text{(j-loop)} \\
0 \leq n & \leq \left\lceil \frac{120 - 1 - 3q_1}{3} \right\rceil & \text{(lhs)} \\
5p_1 + 1 + 5 \times 4n & \leq 4i + 5 \leq 5p_1 + 1 + 5 \times 4n + 5 - 1 & \text{(lhs)} \\
I & = \left\{ \begin{array}{l}
0 \leq m_1 \leq \left\lceil \frac{120 - 1 - 3q_1}{3} \right\rceil & \text{(rhs)} \\
3q_1 + 1 + 3 \times 4m_1 & \leq 2i + j - 1 & \text{(rhs)} \\
3q_1 - 1 + 3 \times 4m_1 & \leq 3 & \text{(rhs)} \\
0 \leq m_2 & \leq \left\lceil \frac{120 - 1 - 7q_2}{7} \right\rceil & \text{(rhs)} \\
7q_2 + 1 + 7 \times 2m_2 & \leq 3i - 2j + 1 & \text{(rhs)} \\
7q_2 + 1 + 7 \times 2m_2 + 7 - 1 & \leq 7 & \text{(rhs)} \\
\end{array} \right.
\end{align*}
\]

\( V = n, m_1, m_2, i, j \)

The loop inequalities come directly from the loop bounds. A pair of data inequalities comes from each of the three array dimensions involved (one from \( \text{lhs} \) and two from \( \text{rhs} \)); each pair consists of an inequality bounding the block number and one bounding the subscript function. The block numbers \( m_1 \) and \( m_2 \) are involved in the inequalities. Since the \( \text{lhs} \) determines the sender \((q_1, q_2)\), the processor coordinates involved in these inequalities are \( q_1 \) and \( q_2 \). Similarly, the block number for the \( \text{rhs} \) is \( n \), and the processor coordinate involved is only \( p_1 \) because the \( \text{lhs} \) determines the receiver \((p_1, p_2)\) and \( A \) is distributed only along the first processor dimension. Although \( p_2 \) is not involved in the inequalities, the compiler takes into account the fact that only processors with coordinate \( p_2 = 0 \) own parts of the \( \text{lhs} \) and hence are potential receivers, and generates code accordingly (shown in Figure 5). The loops
(PACK/SEND phase: processor \(q_1, q_2\)) sends to processor \((p_1, p_2 = 0)\)

\[ q_1 = \text{my} p_1() \]
\[ q_2 = \text{my} p_2() \]
\[ p_2 = 0 \]
do 20 \( p_1 = 0, P_1 - 1 \)
if ((\( p_1 \neq q_1 \))) or (\( p_2 \neq q_2 \)) then
len = 0
\do 10 \( n = \text{ceiling}(max(0, \text{real}(4 - 5p_1 + 4q_1)/(5P_1)), \)
real(16 - 35p_1 + 24q_1 + 8q_2)/(35P_1), \)
real(16 - 35p_1 + 28q_2)/(35P_1), \)
\( (12 - 5p_1)/(5P_1)) \)
floor(min(real(3356 - 15p_1 - 35p_2)/(5P_1), real(236 - 5p_1)/(5P_1), real(1124 - 5p_1)/(5P_1), real(284 - 5p_1 - 28q_2)/(5P_1), \)
real(284 - 5p_1 - 15q_1)/(5P_1), \)
real(284 - 5p_1 - 21q_2)/(5P_1) \)
do 10 \( m = 0, Pi - 1 \)
do 10 \( i = \text{ceiling}(max(0, \text{real}(4 - 3q_1)/(3P_1), \)
real(-4 - 5p_1 + 4q_1)/(3P_1), \)
real(-4 - 5p_1 + 8q_2)/(3P_1), \)
\( (16 - 5p_1)/(3P_1)) \)
floor(min(real(39 - q_1)/(P_1), real(1124 - 5p_1 - 35q_1)/(3P_1), real(-8 + 5p_1 + 5nP_1 - 6q_1)/(6P_1), \)
real(2 - 9q_1 + 14q_2)/(9P_1)) \)
do 10 \( m = 0, P_1 - 1 \)
done 

send(msgid(5i,g_2), buffer, len*4, (pi,p_2) )
end if
20 continue

(RECEIVE/UNPACK phase: processor \((p_1, p_2 = 0)\)) receives from processor \((q_1, q_2)\)

\[ p_1 = \text{my} p_1() \]
\[ p_2 = \text{my} p_2() \]
if (\( p_2 \).eq. 0) then
\do 40 \( q_1 = 0, P_1 - 1 \)
do 40 \( q_2 = 0, P_2 - 1 \)
if ((\( p_1 \neq q_1 \))) or (\( p_2 \neq q_2 \)) then
len = 0
recv(msgid(q_1,g_2), buffer, BUFFERSIZE)
\do 30 \( n = \text{ceiling}(max(0, \text{real}(4 - 3q_1)/(3P_1), \)
real(-4 - 5p_1 + 4q_1)/(3P_1), \)
real(-4 - 5p_1 + 8q_2)/(3P_1), \)
\( (16 - 5p_1)/(3P_1)) \)
floor(min(real(39 - q_1)/(P_1), real(1124 - 5p_1 - 35q_1)/(3P_1), real(-8 + 5p_1 + 5nP_1 - 6q_1)/(6P_1), \)
real(2 - 9q_1 + 14q_2)/(9P_1)) \)
do 10 \( m = 0, P_1 - 1 \)
do 10 \( m = 0, P_1 - 1 \)
done 

buffer(len) = 5(2i + j - 1, 3j - 2; + 1)
len = len+1
30 continue 
end if
40 continue 
end if

(EXECUTION phase)
if (\( p_2 \).eq. 0) then
\do 50 \( n = \text{ceiling}(max(0, \text{real}(4 - 3q_1)/(3P_1), \)
real(-4 - 5p_1 + 4q_1)/(3P_1), \)
real(-4 - 5p_1 + 8q_2)/(3P_1), \)
\( (16 - 5p_1)/(3P_1)) \)
floor(min(real(39 - q_1)/(P_1), real(1124 - 5p_1 - 35q_1)/(3P_1), real(-8 + 5p_1 + 5nP_1 - 6q_1)/(6P_1), \)
real(2 - 9q_1 + 14q_2)/(9P_1)) \)
do 10 \( m = 0, P_1 - 1 \)
do 10 \( m = 0, P_1 - 1 \)
done 

A(2i + j - 1, 3j - 2; + 1) = buffer(len)
len = len+1
50 continue 
end if

Figure 5: SPMD Code for the Example Loop
in the receive/unpack phase have the same bound expressions as those in the pack/send phase, since both come from the same polytope. Therefore, we have only included their abbreviated bound expressions along with the variables of which these expressions are functions. The figure also includes the execution phase, which is the result of the loop bounds reduction procedure described previously.

In this example, the communication code essentially traverses the entire processor space since the program has an all-to-many communication pattern. However, the communication pattern is often not as complicated. For example, many stencil computations exhibit one-to-one (shift) communication patterns when their arrays are properly aligned. In such cases, it is useful to find receiving and sending processor sets. Then, the send phase only has to scan through the receiving processor set instead of the entire processor space. Similarly, the receive phase only scans through the sending processor set for candidate senders. The sending processor set can be found simultaneously with the COMM set in a single invocation to FME. To do this, FME is called with the same input that we used to find the COMM set as before, but with two additions. A processor inequality \( 0 < q < P \) is added to \( I \) and the variable \( q \) is prepended to the list \( V \), as \( q \) is now the first polytope axis. Note that this set is parameterized by the receiver \( p \). The same steps can be followed to find the receiving processor set: simply replace \( q \) by \( p \) and proceed as before. Similarly, this set is parameterized by the sender \( q \).

4 Results

Two small scientific programs, LU and POTENG, were compiled using the techniques described in the previous sections. The communication generation is not fully integrated with the rest of the compiler yet, so we manually added the communication code generated by Mathematica. LU is a standard LU matrix factorization code, and POTENG computes the potential energy in the molecular dynamic simulation program (MDG) of the Perfect Benchmarks [20]. These programs were chosen because cyclic or block-cyclic distributions are required to obtain reasonable performance. Both programs were run on an IBM SP-2 using MPIF [12].

LU has three 1024 \( \times \) 1024 2-D arrays, two distributed in a column-cyclic manner (U and the input matrix) while the other one (L) in a row-cyclic manner. The computation consists mainly of two doubly-nested loops (one for L and the other for U), both enclosed in an outermost loop. In each iteration of the outermost loop the program computes a column of L and a row of U, where L and U are triangular matrices. Both operations are performed in parallel and with reasonable load balance due to the cyclic distributions. Communication occurs outside the doubly-nested loops but inside the outermost loop. The communication pattern involves a broadcast of a row and a column in each iteration of the outermost loop. The broadcast initiator, the size of the broadcast message, and the number of receiving processes depend on the outermost iteration. This makes it difficult to identify the broadcast. The speedup for up to 32 processors is shown in Figure 6. We can see in this figure that it is critical to identify the communication pattern as a broadcast, and therefore be able to use a MPI collective communication primitive instead of point-to-point communication. We are currently extending the high-level communication detection features of PARADIGM to handle such cases.

POTENG has six 1-D arrays, three of size \( N \), where \( N \) is the number of molecules to be simulated; and the other three of size \( 3N \), where 3 is the number of atoms in a water molecule. A cyclic distribution is selected for the first three arrays while a cyclic(3) distribution (i.e., block-cyclic with block size of 3) for the second.

This subroutine has two main loops. The first one has statements of the form:

\[
\text{double precision } x, y, z, c1, c2
\]

\[
do \ i = 1, N \ \\
\ \\
\text{xm}(i) = c1 \cdot x(3i - 1) + c2 \cdot (x(3i - 2) + x(3i))
\]

while the second one is a computationally intensive triangular loop. The cyclic(3) distribution becomes a natural choice to satisfy both communication and load balancing requirements (in agreement with the decision made by PARADIGM's automatic data partitioner [14]).

Prerequisite steps for the parallelization of the triangular loop are the following:

- Inlining three small subroutine calls.
- Fairly complex induction variable elimination.
- Array privatization of five small arrays.
- Detecting two simple reductions.

In our case we used the induction variable elimination of Parafrase-2 and manually performed the other steps. All of these steps are automatically performed by the Polaris compiler [8].

Communication operations generated by PARADIGM were combined outside the triangular loop, and reductions were performed using MPI collective communication primitives. Figure 7 plots the speedup for \( N = 3000 \) for up to 32 processors. The encouraging speedup shown in the figure is a good indication that the distributions chosen are correct and that our compilation techniques are effective.

Figure 6: Speedup of LU

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{speedup.png}
\caption{Speedup of LU}
\end{figure}
5 Conclusions and Future Work

The techniques presented in this paper allows PARADIGM to deal with all of the common regular data decompositions (block, cyclic, and block-cyclic) within a unified framework. By leveraging off the symbolic power of existing packages like Mathematica, PARADIGM has immediate access to symbolic capabilities that ultimately allows a much wider range of compilable programs than was possible before.

The work presented does not translate global references to the local address space of a processor; we are working on better ways to handle local translations uniformly for any of the supported data distributions. We are also extending previous work [14] on automatic detection of high-level communication to handle more complicated cases such as that encountered in LU.

References


