Proc. Space Helium Dewar Conf. 1984

NASA-TM-111339

NASA
7N-31-TM
8332
P-6

OPTIMIZATION OF COOLED SHIELDS IN CRYOGENIC INSULATIONS

John C. Chato
Jeyhoon M. Khodadadi

Department of Mechanical and Industrial Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801 USA

Peter Kittel

NASA/Ames Research Center
Moffett Field, CA 94035 USA

INTRODUCTION

It has been shown that the optimization of an insulation system with or without cooled shields corresponds to the minimization of the entropy production rate [1]. In contrast to a previously developed optimization constrained by boiloff rate [2,3], we developed a simple, unconstraint technique assuming that equipment cooling loads provide adequate boiloff rates to cool the shields. Details of the method will be presented elsewhere [4] and we present here a summary of the results.

DISCUSSION

The optimization technique yielded two independent, algebraic equations for the temperature, T_s, and the distance from the cold surface, x, of a cooled shield between a parallel, hot surface at T_H and a cold one at T_c separated by an overall distance t. The thermal conductivity of the insulation is assumed to follow the relation,

$$k = k_1 T^m + k_2 T^n. \tag{1}$$

The optimization equations are as follows.

For the temperature:

$$\frac{(T_c/T_H)^m F(m,T_s/T_c) + \gamma R^n F(n,T_s/T_c)}{[(T_c/T_H)^{m-1} D(m,T_s/T_c) + \gamma (T_c/T_H)^{n-1} D(n,T_s/T_c)]^2}$$

$$= \frac{F(m,T_s/T_H) + \gamma F(n,T_s/T_H)}{[D(m,T_s/T_H) + \gamma D(n,T_s/T_H)]^2} \qquad (2)$$

For the location:

$$\frac{x}{t - x} = - \frac{(T_c/T_H)^{m-1} D(m,T_s/T_c) + \gamma (T_c/T_H)^{n-1} D(n,T_s/T_c)}{D(m,T_s/T_H) + \gamma D(n,T_s/T_H)} \qquad (3)$$

The dimensionless heat extraction rate at the shield is:

$$\frac{q_s t}{A k_H T_H} (m + 1) [1 + \gamma (n + 1)/(m + 1)]$$

$$= (q_H t/A k_H T_H - q_c t/A k_H T_H)(m + 1) [1 + \gamma (n + 1)/(m + 1)]$$

$$= \frac{1 - (T_s/T_H)^{m+1} + \gamma[1 - (T_s/T_H)^{n+1}]}{1 - x/t}$$

$$- \frac{(T_s/T_H)^{m+1} - (T_c/T_H)^{m+1} + \gamma[(T_s/T_H)^{n+1} - (T_c/T_H)^{n+1}]}{x/t} \qquad (4)$$

Here k_H is k at T_H and $\gamma \equiv [(m + 1)k_2 T_H^n]/[(n + 1)k_1 T_H^n]$ is a weighting factor indicating the relative importance of the second term in Eq. (1). The functionals D and F are defined in the nomenclature. The results obtained using Eq. (1) can be approximated by an equivalent single term function $k = k_3 T^b$ with b having a value between m and n. Typical results are shown in the figures taken from Ref. [4].

Figures 1 and 2 show the entropy production rates, S, relative to the thermodynamically minimum possible rates, S_{min}, as a function of the overall temperature ratio, T_c/T_H, and the number of optimal shields, for the two temperature functions $k_1 T$ and $k_1 T^3$. It can be deduced that shields become more effective with small temperature ratios, T_c/T_H and

with high values of the exponent of the temperature (m,n, or b). The maximum number of shields which significantly reduce the entropy production rate, hence the heat leak, is about three in the practical ranges of the parameters. At high values of T_C/T_H even one shield may be superfluous.

Figures 3 and 4 show the temperatures and locations of three optimal shields, with a cubic temperature function of the thermal conductivity, as functions of the overall temperature ratio. Both the temperature ratios and relative locations with respect to the cold wall decrease with decreasing T_C/T_H.

Another result of this work was that the entropy production rate is relatively insensitive to x/t and T_S/T_H and deviations of ±20 percent can easily be tolerated.

Finally, we have shown that if two different insulations have to be used on the two sides of a cooled shield, then the better insulation should be on the warm side.

REFERENCES

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3. Bejan, A., Entropy Generation through Heat and Fluid Flow, Wiley-Interscience, N.Y., 1982, Ch. 9.
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NOMENCLATURE

A	Area of heat flow, m^2
b,m,n	Exponents of the thermal conductivity function
D(a,B)	$= B^{a+1} - B^a - 1 + B^{-1}$
F(a,B)	$= (a+1)B^a - aB^{a-1} - B^{-2}$
k	Thermal conductivity, W/m-K; with subscripts, coefficients in Eq. (1)
q	Heat flow, W

S	Entropy production rate, W/K
t	Distance between hot and cold walls, m
T	Absolute temperature, K
x'/L	Distance of optimal shield from cold wall, x', with respect to overall thickness, L, of insulation
x	Distance from cold wall, m
γ	Weighting factor

SUBSCRIPTS

c	Cold Wall
H	Hot Wall
s	Shield

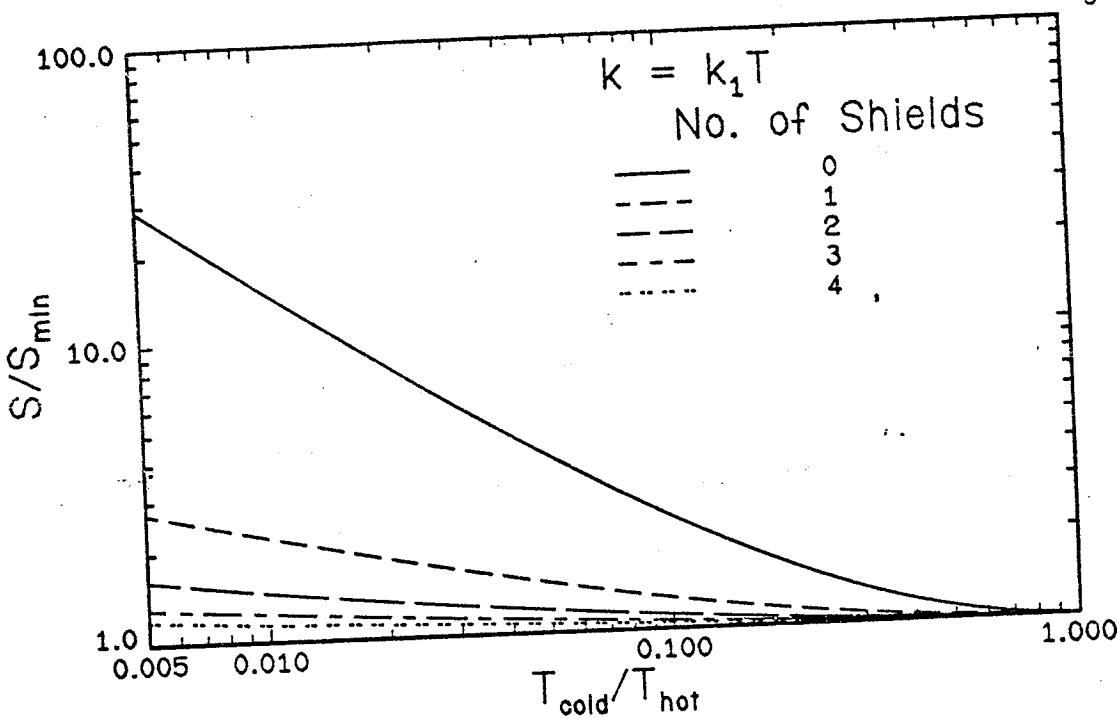

Fig. 1 The effect of optimally cooled shields on the entropy production rates for $k = k_1T$.

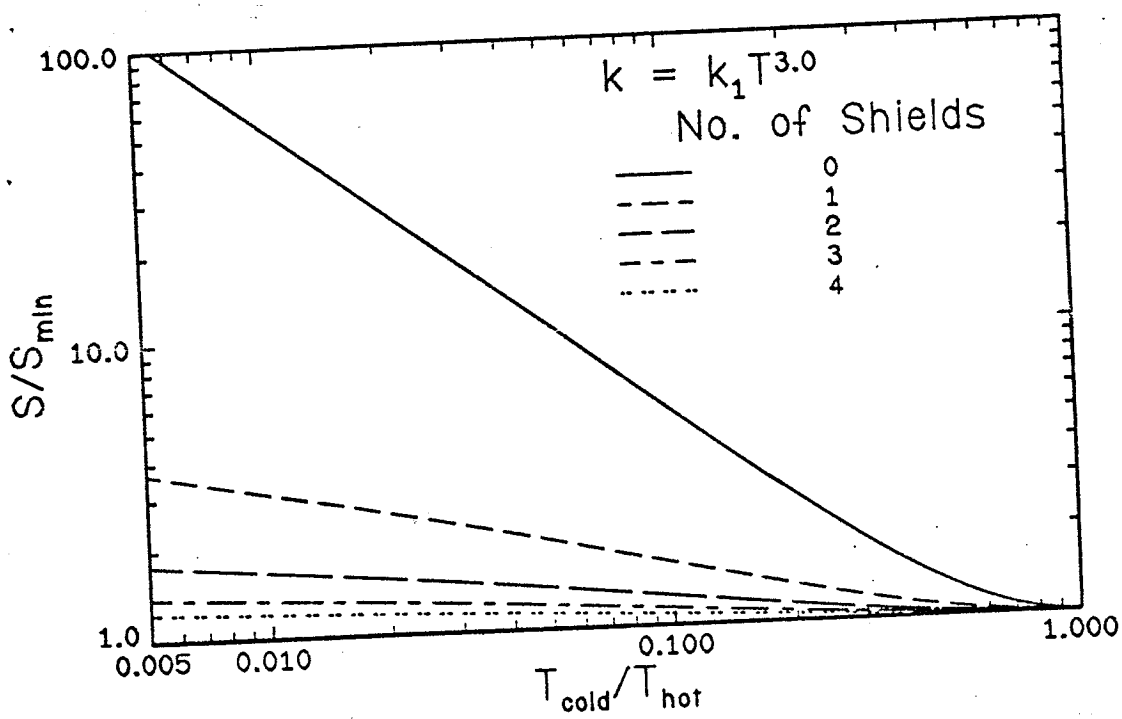

Fig. 2 The effect of optimally cooled shields on the entropy production rates for $k = k_1T^3$.

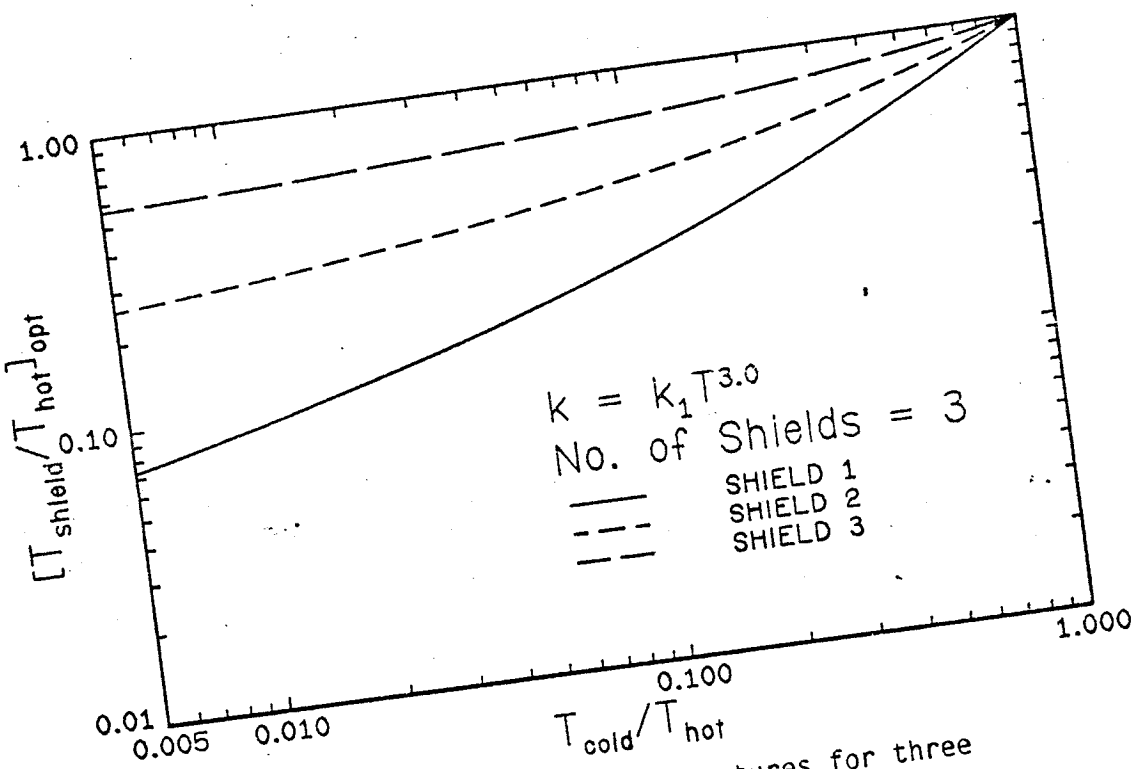

Fig. 3 Optimal shield temperatures for three shields with $k = k_1 T^3$.

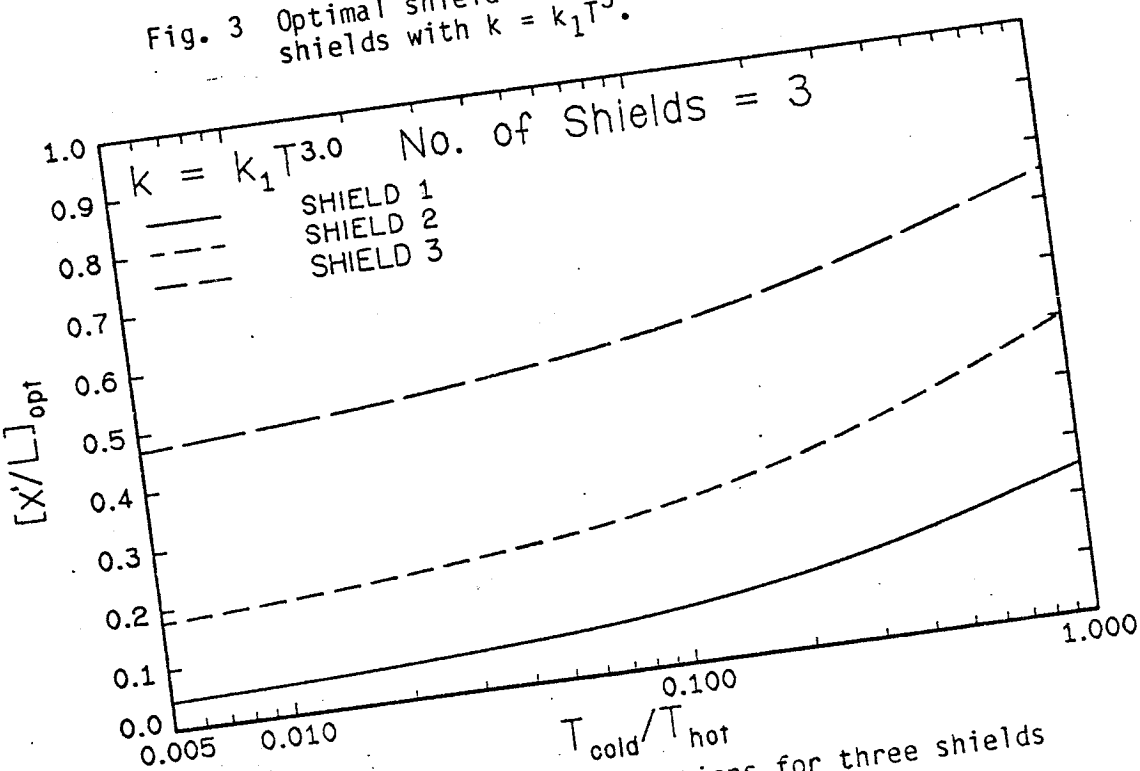

Fig. 4 Optimal shield locations for three shields with $k = k_1 T^3$.